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# Thermobaric control of gravitational potential energy generation by diapycnal mixing in the deep ocean

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[1] Sources and sinks of gravitational potential energy (GPE) play a rate-limiting role in the large-scale ocean circulation. A key source is turbulent diapycnal mixing, whereby irreversible mixing across isoneutral surfaces is enhanced by turbulent straining of these surfaces. This has motivated international observational efforts to map diapycnal mixing in the global ocean. However, in order to accurately relate the GPE supplied to the large-scale circulation by diapycnal mixing to the mixing energy source, it is first necessary to determine the ratio,  $\xi$ , of the GPE generation rate to the available potential energy dissipation rate associated with turbulent mixing. Here the link between GPE and hydrostatic pressure is used to derive the GPE budget for a compressible ocean with a nonlinear equation of state. The role of diapycnal mixing is isolated and from this a global climatological distribution of  $\xi$  is calculated. It is shown that, for a given source of mixing energy, typically three times as much GPE is generated if the mixing takes place in bottom waters rather than in the pycnocline. This is due to GPE destruction by cabbelling in the pycnocline, as opposed to thermobaric enhancement of GPE generation by diapycnal mixing in the deep ocean. Citation: Oliver, K. I. C., and R. Tailleux (2013), Thermobaric control of gravitational potential energy generation by diapycnal mixing in the deep ocean, Geophys. Res. Lett., 40, 327-331, doi:10.1029/2012GL054235.

# 1. Introduction

[2] The ocean's global meridional overturning circulation (MOC) influences climate through the large-scale transport of heat, carbon, and nutrients. Intrinsic to this circulation is the formation and downwelling of dense water at high latitudes which, if balanced by the upwelling of more buoyant water elsewhere, constitutes a sink of gravitational potential energy (GPE). Two principal mechanisms supply the GPE required for a steady state MOC to exist: (a) downwelling of buoyancy by turbulent diapycnal mixing [e.g., *Munk and Wunsch*, 1998] and (b) wind-driven upwelling of dense water and downwelling of more buoyant water, chiefly in the Antarctic Circumpolar Current [e.g., *Toggweiler and Samuels*, 1998]. The former mechanism is required to sustain Antarctic overturning, since the downwelling Antarctic Bottom Water source is denser than the water that

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upwells in the Antarctic Circumpolar Current. This, together with the finding that mixing in the ocean has a highly heterogeneous distribution, has motivated much work to understand and to map turbulent diapycnal mixing in the global ocean [e.g., *Kunze et al.*, 2006; *St. Laurent and Simmons*, 2006].

[3] Turbulent diapycnal mixing describes the combined effect of fine-scale advection and irreversible mixing across isoneutral surfaces, which dissipates mechanical energy at the rate  $\varepsilon_K + \varepsilon_P$ . Here  $\varepsilon_K$  is the viscous dissipation rate and  $\varepsilon_P = \kappa_v N^2$ , where N is the buoyancy frequency, is the available potential energy (APE) dissipation rate that defines the effective turbulent diapycnal diffusivity  $\kappa_{\nu}$ . APE dissipation corresponds to the irreversible conversion of mechanical energy to internal energy through molecular diffusion, and has a signature in the irreversible entropy production rate [Tailleux, 2012a]. The concept of mixing efficiency,  $\Gamma$ , is commonly used to measure the relative importance of viscous versus nonviscous dissipation, defined either as  $\varepsilon_P/\varepsilon_K$  [e.g., *Oakey*, 1982], as adopted here, or as  $\varepsilon_P/(\varepsilon_P+\varepsilon_K)$  [e.g., Peltier and Caulfield, 2003]. As discussed by Tailleux [2009b, 2012b], the two possible definitions for  $\Gamma$  are each generalizable to a nonlinear equation of state because, although  $\varepsilon_P$  depends on the thermal expansion coefficient  $\alpha$ , it does not depend on the derivatives of  $\alpha$  with respect to temperature and pressure and is hence unaffected by cabbelling or thermobaricity. Physically, the turbulent fluxes for the materially conserved quantities (salinity and conservative temperature) are unaffected by a nonlinear equation of state, unlike the turbulent flux of buoyancy. Following Osborn [1980], oceanographers often assume that  $\Gamma$  has an upper bound of 0.2 in the case of mechanically-driven mixing (buoyancy-driven mixing can be much more efficient [Scotti and White, 2011]. Turbulent diapycnal mixing results in the reversible generation of GPE at the expense of internal energy at the rate  $\xi \varepsilon_P = \xi \Gamma \varepsilon_K$ , where  $\xi$  is the ratio of the rate of GPE generation by diapycnal mixing to the mixing energy transfer rate  $\varepsilon_{P}$ .

[4] Under a linear equation of state (EOS), buoyancy is a conservative quantity, so  $\xi=1$ . However, buoyancy may be created or destroyed under a nonlinear EOS, enhancing or reducing GPE generation for a given amount of mixing, and allowing for values of  $\xi$  greater and lower than unity. The seminal study of *Munk and Wunsch* [1998] provided an estimate for the mechanical energy source required to sustain the MOC under the assumption of a linear EOS. More recent studies have indicated that accounting for a nonlinear EOS would lead to a significant correction to this estimate, but have focused either on the role of isoneutral mixing [*Klocker and McDougall*, 2010] or have not isolated the role of diapycnal mixing [e.g., *Gnanadesikan et al.*, 2005]. *Fofonoff* [1998, 2001] did focus on diapycnal

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mixing, although not within the context of the energetics of the large-scale circulation, and showed that it is possible for diapycnal mixing to cause a net loss of GPE (i.e.,  $\xi < 0$ ) due to loss of buoyancy through cabbelling. Here, we advance upon this work by exploiting the link between GPE and the compressible work to express the GPE budget exactly for a compressible ocean, in a form that allows us to isolate the role of diapycnal mixing and derive an expression for  $\xi$  (section 2). We then use climatological data to show that, over most of the ocean,  $\xi$  differs significantly from the linear-EOS value of 1 (section 3). We show that GPE generation by diapycnal mixing is suppressed by cabbelling throughout the pycnocline, but that thermobaric buoyancy generation associated with the flux of heat to high pressures dominates over cabbelling the abyssal ocean, leading to values of  $\xi$  greater than 1 at depths greater than about 1500m. Finally, we explore the implications of the observed distribution of  $\xi$  for the energetics of the global MOC (section 4).

# 2. Gravitational Potential Energy in a Nonlinear Compressible Ocean

[5] The gravitational potential energy budget in the compressible ocean is given by

$$\frac{\mathrm{dGPE}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t} \int_{V} \rho g z \,\mathrm{d}V = \int_{V} \rho g w \,\mathrm{d}V, \tag{1}$$

where  $\rho$  is *in situ* density, *g* is acceleration due to gravity, and *w* is vertical velocity. *Gnanadesikan et al.* [2005] were perhaps the first to seek to generalize this idea to the fully nonlinear case, by proposing to regard the density flux  $\rho g w$  as being made up of different physical processes, which are represented by different physical parameterizations in general circulation models. We instead distinguish between adiabatic and diabatic processes by linking the GPE budget with the thermodynamic Pv work (compressible work), where *P* is the hydrostatic pressure and  $v=1/\rho$  is the specific volume. This link is made by manipulating the classical result

$$GPE = \int_{V} P' v \, dm - M_o g \overline{H}, \qquad (2)$$

where  $dm = \rho dV$  is the elementary mass of a fluid parcel, and  $\overline{H}$  is a mean ocean depth defined by the relation

$$M_o g \overline{H} = \int_{\mathcal{S}} (P_b - P_a) H \, \mathrm{d}x \, \mathrm{d}y,$$

where  $M_o = \int_S (P_b - P_a)g^{-1} dx dy$  is the mass of the ocean. Here  $P_b$  is bottom pressure,  $P_a$  is surface atmospheric pressure, and  $P' = P - P_a$ .

[6] The GPE budget is therefore

$$\frac{\mathrm{dGPE}}{\mathrm{d}t} = \int_{V} P' \frac{Dv}{Dt} \mathrm{d}m + \int_{V} v \frac{DP'}{Dt} \mathrm{d}m - g \frac{\mathrm{d}(M_{o}\overline{H})}{\mathrm{d}t}.$$
 (3)

[7] The first term in the right-hand side of (3) states that expansion increases GPE. The conceptual link with (1) is that expansion is associated with an isobaric upwelling of overlying water. The greater the hydrostatic pressure at which such expansion occurs, the greater the mass of overlying water that upwells and gains GPE. The second term represents the conversion between kinetic energy and APE. A negative correlation between v and DP'/Dt would imply the upwelling of buoyant water and downwelling of dense water, which would act to decrease GPE. The final term arises because

tendencies in the depth-integrated hydrostatic pressure and GPE of a water column are identical only at constant mass; any change of mass of the global ocean, or a net horizontal redistribution of mass across isobaths, leads to a correction. This term vanishes at steady state.

[8] Here we are interested in the effect of mixing. We have

$$\frac{Dv}{Dt} = \frac{1}{\rho} \left[ \alpha \frac{D\Theta}{Dt} - \beta \frac{DS}{Dt} \right] - \frac{1}{\rho^2 c_s^2} \frac{DP}{Dt}, \tag{4}$$

where  $\alpha$  and  $\beta$  are the thermal expansion and haline contraction coefficients defined relative to the variables ( $\Theta$ ,*S*,*P*),  $\Theta$ is *McDougall*'s [2003] conservative temperature and *S* is salinity, and  $c_s$  is the speed of sound. The integral effect of mixing is usually regarded as an isobaric process [e.g., *IOC et al.* 2010] i.e.  $\frac{DP}{dr} = -0$  so we can write

$$TOC \ et \ al., \ 2010$$
], i.e.,  $\frac{1}{Dt}\Big|_{\text{mixing}} = 0$ , so we can write

$$\frac{\mathrm{dGPE}}{\mathrm{d}t}\bigg|_{\mathrm{mixing}} = \int_{V} P' \bigg[ \alpha \frac{D\Theta}{Dt} - \beta \frac{DS}{Dt} \bigg] \mathrm{d}V \bigg|_{\mathrm{mixing}}, \qquad (5)$$

[9] We use the turbulent parameterization

$$\frac{D\Theta}{Dt}\bigg|_{\text{mixing}} = \nabla \cdot (\mathbf{K}\nabla\Theta), \quad \frac{DS}{Dt}\bigg|_{\text{mixing}} = \nabla \cdot (\mathbf{K}\nabla S).$$

where **K** is a turbulent diffusive tensor encapsulating the net effect of fine-scale advection and molecular diffusion. Using the condition of no diffusion through boundaries, so  $\int_{V} \nabla (\mathbf{KC}) dV = 0$  for arbitrary **C**, (5) can be rewritten as

$$\frac{\mathrm{dGPE}}{\mathrm{d}t}\bigg|_{\mathrm{mixing}} = -\int_{V} \mathbf{K}\nabla\Theta\cdot\nabla(P'\alpha)\mathrm{d}V + \int_{V} \mathbf{K}\nabla S\cdot\nabla(P'\beta)\mathrm{d}V,$$

which may be further rearranged to yield

$$\frac{\mathrm{dGPE}}{\mathrm{d}t}\bigg|_{\mathrm{mixing}} = \int_{V} \mathbf{K} (\beta \nabla S - \alpha \nabla \Theta) \cdot \nabla P' \mathrm{d}V + \int_{V} P' [\mathbf{K} \nabla S \cdot \nabla \beta - \mathbf{K} \nabla \Theta \cdot \nabla \alpha] \mathrm{d}V.$$
(6)

[10] The first term in the right-hand side of (6) represents GPE gain due to the downward flux of buoyancy associated with diapycnal mixing, and is positive wherever there is vertical mixing across stable stratification. With reference to (3), this entails expansion at high pressure and contraction at low pressure, indicating isobaric upwelling at intermediate pressures. The second term in (6) represents GPE gain due to the net generation of buoyancy associated with mixing. This term may either be positive or negative due to spatial variability in  $\beta$  and especially  $\alpha$ . For a linear EOS,  $\beta$  and  $\alpha$  are uniform so this term vanishes, and the rate of GPE gain due to mixing is equal to the mixing energy transfer rate,  $\Gamma \varepsilon_K$ 

$$\frac{\mathrm{dGPE}}{\mathrm{d}t} \bigg|_{\mathrm{linear}} = \int_{\mathrm{V}} \mathbf{K} (\beta \nabla \mathbf{S} - \alpha \nabla \Theta) \cdot \nabla \mathbf{P}' \mathrm{d}\mathbf{V}$$
$$= \int_{V} \kappa_{v} \rho N^{2} \mathrm{d}V = \int_{V} \Gamma \varepsilon_{K} \mathrm{d}V, \tag{7}$$

where  $\kappa_{\nu}$  is the vertical diapycnal diffusivity and  $N = \sqrt{g[\alpha \Theta_z - \beta S_z]}$ , where *z* subscripts indicate differentiation with respect to height. This is the formula considered by *Munk and Wunsch* [1998]. In the presence of nonlinearity in the EOS, both the diapycnal and isoneutral components of the second term in (6) make important contributions to the GPE budget. Here, we focus on the role of the diapycnal

component in order to explore the relationship between mixing energy and GPE generation. This is given by

$$\frac{\mathrm{dGPE}}{\mathrm{d}t}\Big|_{\kappa_{v}} = \frac{\mathrm{dGPE}}{\mathrm{d}t}\Big|_{\substack{\mathrm{linear}\\\mathrm{mixing}}} + \frac{\mathrm{dGPE}}{\mathrm{d}t}\Big|_{\kappa_{v}}$$
$$= \int_{V} \xi \Gamma \varepsilon_{K} \,\mathrm{d}V. \tag{8}$$

[11] The vertical diapycnal component of the second term in (6) is

$$\frac{\mathrm{dGPE}}{\mathrm{d}t}\bigg|_{\substack{\mathrm{nonlin}\\\kappa_{\nu}}} = \int_{V} P' \kappa_{\nu} (S_{z} \beta_{z} - \Theta_{z} \alpha_{z}) \mathrm{d}V.$$
(9)

[12] This yields

$$\xi = 1 + \xi^{\text{nonlin}}, \qquad \xi^{\text{nonlin}} = \frac{P'(S_z \beta_z - \Theta_z \alpha_z)}{\rho N^2}. \tag{10}$$

[13] A positive value of  $\zeta^{\text{nonlin}}$  would imply that nonlinearity in the EOS leads to an enhancement of GPE generation.  $\zeta^{\text{nonlin}}$  may be further decomposed  $\zeta^{\text{nonlin}} = \zeta^{\text{cab}} + \zeta^{\text{therm}}$ , where  $\zeta^{\text{cab}}$  and  $\zeta^{\text{therm}}$  are cabbelling and thermobaric components, respectively

$$\zeta^{\text{cab}} = \frac{P'\left(\beta_S(S_z)^2 - 2\alpha_S S_z \Theta_z - \alpha_\Theta(\Theta_z)^2\right)}{\rho N^2},\tag{11}$$

$$\xi^{\text{therm}} = \frac{gP'(-\beta_P S_z + \alpha_P \Theta_z)}{N^2}, \qquad (12)$$

where  $\Theta$ , *S*, and *P* subscripts indicate differentiation with respect to conservative temperature, salinity and pressure, respectively, and  $\beta_{\Theta} = -\alpha_S$  by definition. The cabbelling component is dominated by the final term in (11), and is negative in seawater because buoyancy, and therefore GPE, is destroyed by the mixing of waters of different temperatures ( $\alpha_{\Theta} > 0$ ). The thermobaric component is dominated by the final term in (12), and is positive where the vertical temperature gradient is positive. This is because warm water is less compressible than cold water ( $\alpha_P > 0$ ), so buoyancy is generated by fluxing heat to higher pressures.

[14] The formula for  $\xi$ , derived above for a non-Boussinesq ocean, is also valid under the Boussinesq approximation applied in ocean models. In Boussinesq fluids, the rate of GPE generation by mixing is represented by the term  $\int_V gz \frac{D\rho}{Dt} dV$ , which is also the Boussinesq approximation to the compressible work [*Nycander et al.*, 2007; *Tailleux*, 2012a], and an equivalent formula for  $\xi$  arises from the duality between the non-Boussinesq and Boussinesq equations at hydrostatic equilibrium [*de Szoeke and Samelson*, 2000].

# 3. Application to Global Hydrography

[15] Having derived expressions for  $\xi$  and its components, we now calculate climatological values for these terms throughout the global ocean using WOA 2005 practical salinities [*Antonov et al.*, 2006] and *in situ* temperatures [*Locarnini et al.*, 2006] and TEOS-10 calculations for absolute salinity, conservative temperature, and EOS variables  $\alpha$ ,  $\beta$ , and  $\rho$  [*IOC et al.*, 2010].

[16] The distributions of  $\xi$  and its components in the Atlantic and Pacific oceans are illustrated in Figure 1. Cabbelling associated with diapycnal mixing leads to loss of GPE throughout the global ocean, and is the dominant nonlinearity in the pycnocline. Where  $\xi^{cab} < -1$ , this GPE

loss is greater than GPE gain due to the downward buoyancy flux associated with diapycnal mixing. This is the case in parts of the lower pycnocline and near the Mediterranean outflow, both locations where strong vertical temperature and salinity gradients lead to relatively weak vertical density gradients, so a small downward buoyancy flux can be associated with strong cabbelling. At greater depths, the cabbelling term decreases in relative importance due to reduced temperature stratification.

[17] The thermobaric term leads to GPE gain associated with diapycnal mixing throughout the global ocean except where there is a negative vertical conservative temperature gradient (e.g., below Winter Water in the Southern Ocean). The mass of overlying water raised due to local expansion, associated with thermobaricity, increases with depth, explaining the increased importance of this term with depth. This also applies to the cabbelling term, but unlike  $\zeta^{cab}$ ,  $\zeta^{therm}$  is insensitive to the vertical conservative temperature gradient wherever temperature stratification dominates over salinity stratification. Where  $\zeta^{therm}>1$ , as is observed regionally at depths greater than ~4500m, this GPE gain is greater than GPE gain due to the downward buoyancy flux associated with diapycnal mixing.

[18] In summary,  $\xi$  deviates strongly from 1, the value that would be consistent with a linear EOS. The global mean at a depth of 500m is 0.5, with negative values in parts of the tropical pycnocline.  $\xi$  exceeds 2 in some bottom waters, for which 1.5 is a representative value. This indicates that diapycnal mixing is typically three times as effective at generating GPE if it occurs in bottom waters rather than the pycnocline.

### 4. Discussion

[19] Our results indicate that, in the low latitude pycnocline, diapycnal mixing results in a loss of GPE through cabbelling that is typically about 50% as great as the GPE gain due to the downward buoyancy flux associated with diapycnal mixing. Therefore, the nonlinearity in the EOS approximately doubles the mixing energy required to maintain a given GPE source here. However, the winddriven circulation provides an adiabatic GPE source to the pycnocline, and the relative importance of diapycnal mixing in the GPE budget likely increases with depth. The canonical estimate of Munk and Wunsch [1998], of a mixing energy transfer rate of approximately  $\int_{V} \varepsilon_{P} = 0.4$  TW ( $\varepsilon_{P} = \Gamma \varepsilon_{K}$ , with  $\Gamma=0.2$  and  $\int_{V} \varepsilon_{K} = 2.1$  TW) required to maintain the abyssal stratification, was made for depths of 1000-4000m. At these depths, the effect of cabbelling on the ratio,  $\xi$ , of the rate of GPE generation to the mixing energy transfer rate is small because of weak vertical temperature gradients (the region of the Mediterranean outflow is a notable exception), and thermobaricity dominates. Therefore, nonlinearity in the EOS does not reduce GPE generation by diapycnal mixing as is often assumed, but enhances GPE generation by ~20% (the mean value of  $\xi$  from 1000–4000m is 1.2), increasing to ~100% in bottom waters.

[20] Previous studies have emphasized the role of cabbelling but not thermobaricity in ocean energetics. For example, *Gnanadesikan et al.* [2005] found that cabbelling is a leading order sink of GPE in the global ocean, whereas *Tailleux* [2009a] explored the regime  $\xi < 1$ , which typically arises due to cabbelling, but did not consider the regime  $\xi > 1$ ,



**Figure 1.** Distribution of the ratio,  $\xi$ , of the GPE generation rate to the mixing energy transfer rate in the Atlantic Ocean at 18.5°W (left panels), the Pacific Ocean at 145.5°W (middle panels), and for global mean profiles (right panels). The components associated with the nonlinear EOS are GPE generation by cabbelling ( $\xi_{cab}$ , top row), which is negative, and GPE generation due to thermobaricity ( $\xi_{therm}$ , second row).  $\xi=1+\xi_{cab}+\xi_{therm}$  is presented in the bottom row. Contour intervals are 0.5, with white contours at  $\xi_{cab}<0$ ,  $\xi_{therm}<0$  or  $\xi<1$ . Intervals in shading are 0.125.

which can only arise due to thermobaric effects. Such an approach may be justifiable in studies that focus on a subset of vertically-integrated quantities dominated by the upper ocean, such as ocean heat transport [*Gnanadesikan et al.*, 2005]. However, the importance of cabbelling in the vertical integral masks its near-negligible role, in comparison with thermobaricity, in the abyssal ocean.

[21] The distinct roles of cabbelling and thermobaricity have been explicitly examined in the context of isoneutral mixing. *Klocker and McDougall* [2010] estimated the dianeutral transports that result from the nonlinear EOS due to a uniform isoneutral diffusivity, where a downward dianeutral transport corresponds to a loss of GPE. Thermobaric effects dominated over cabbelling in deep waters (isoneutral density greater than ~27.4kgm<sup>-3</sup>) in their study. Because isoneutral mixing entails mixing of waters of different temperatures, it invariably leads to a GPE sink associated with

cabbelling. However, the thermobaric effect of isoneutral mixing term opposes the thermobaric effect of diapycnal mixing, and leads to a globally integrated GPE sink. This may be understood if we consider that the thermobaric effect is dominated by the dependence of the compressibility of seawater on temperature, so that a flux of heat to greater pressures generates buoyancy and therefore GPE. Diapycnal mixing typically fluxes heat to higher pressures because vertical temperature gradients are positive over most of the ocean. However, the vertical temperature gradient along isoneutral surfaces is negative over much of the ocean, most notably the Antarctic Circumpolar Current, where pressure and conservative temperature on an isoneutral surface both increase as one moves equatorward. Therefore, eddies in the Antarctic Circumpolar Current flux heat to lower pressures, destroying buoyancy and therefore decreasing GPE. Klocker and McDougall [2010] estimated that isoneutral mixing leads to

approximately 6 Sv of dianeutral downwelling, in addition to the downwelling resulting from surface buoyancy exchange at high latitudes, which they suggested would be balanced by GPE input from diapycnal mixing. However, the heat that is fluxed upward by isoneutral mixing must, at steady state, be returned to the deep ocean by a combination of diapycnal mixing and advection, thus tending to negate the destruction of buoyancy by isoneutral mixing in the abyssal ocean. Therefore, much of the GPE lost due to isoneutral mixing is likely to be balanced by thermobaric GPE gain associated with diapycnal mixing and/or advection, without the requirement for an additional mixing energy source.

[22] Finally, we consider the impact of  $\xi$  on ocean dynamics. Fofonoff [1998, 2001] argued that diapycnal mixing would destabilize hydrographic structures with  $\xi < 0$  by converting GPE to kinetic energy, resulting in further mixing. This mechanism is unlikely to be seen in practice, since energy conversions associated with cabbelling/thermobaricity are typically between GPE and internal energy, as opposed to kinetic energy [McDougall et al., 2003], and indeed our climatology indicates that the  $\xi < 0$  regime is stable over significant regions of the ocean. A more straightforward link to dynamics is through the effect of diapycnal mixing on horizontal pressure gradients. From equation (2), we see that processes that do not alter the mass of a water column, such as diapycnal mixing, yield identical responses in vertically integrated hydrostatic pressure and in GPE. (More generally, GPE tendencies are proportional to depth-integrated steric height tendencies.) Modeling studies indicate that the steady state meridional overturning circulation within each ocean basin is approximately proportional to the vertically integrated meridional pressure gradient above an intermediate scale depth, with upper ocean flow from high to low pressure [de Boer et al., 2010, and references therein]. Provided that  $\bar{\xi}$ >0, this implies that regionally strengthened diapycnal mixing at lowor mid-latitudes should, at steady state, lead to enhanced upwelling in that region and a strengthened global MOC, a conclusion that is supported for the North Atlantic and for the Indo-Pacific basins by the linear-EOS experiments of Oliver and Edwards [2008]. However, where  $\xi < 0$ , diapycnal mixing tends to decrease local vertically integrated pressure, leading to the possibility that mixing at such locations tends to weaken the MOC.

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