C.o.A.Memo Aero: 36

EFFECT OF ENGINE, TANK AND PROPELLANT SPECIFIC COST

ON SINGLE STAGE RECOVERABLE BOOSTER ECONOMICS

Dennis S. Carton[1] and B. Kalitventzeff[2]

Communication from the College of Aeronautics, Cranfield

(For Presentation at IVTH. European Symposium
on Space Technology. Rome June 1964.)

Abstract

Reusable first stages using hydrogen-oxygen, hydrogen-fluorine and kerosine-oxygen are compared with non-reusable stages using a solid in addition to the liquid combinations. The criterion used for comparison is the minimum specific cost of the "loaded and ready for launch" stage cost per unit of stage payload mass. A closed form relationship is used in which the empty stage mass without payload is taken to scale in part proportional to propellant mass, and in part to mass flow rate. The stage specific cost is proportional to specific cost of engine (or nozzle) tank and propellant. In the second part the hydrogen-oxygen combination is considered in more detail. The sensitivity of the results to changes in various specific costs including that of refurbishing are described.

Throughout, the stage velocity increments are compared in the 3000-6000 metres/second range with losses.

[1] Department of Aircraft Propulsion, Cranfield.

[2] Cranfield and Ecole Royale Militaire, Brussels.

Nomenclature

a	=	vehicle absolute acceleration, m/s^2.
c	=	effective exhaust velocity, m/s.
C	=	cost criterion $(\lambda_P + \lambda_T T)/\lambda_E$
$\overline{D}$	=	mean drag over flight N.
E	=	engine scaling constant $M_E/\dot{m}_P$ seconds.
$\overline{F}$	=	mean thrust over flight N.
G	=	mean effective component of gravitational field opposing vehicle acceleration, m/s^2.
g	=	local gravitational acceleration.
L	=	stage payload mass ratio M_L/M_α
M	=	mass, Kgm.
$\dot{m}$	=	dM/dt, Kgm/s.
P	=	stage propellant mass ratio M_P/M_α
R	=	reactor scaling constant $M_R/\frac{1}{2}\dot{m}_P c^2$, s^3/m^2
T	=	"tank" scaling constant M_T/M_P
v	=	velocity, m/s.
Δv	=	achieved velocity increment $v_\omega - v_\alpha$, m/s.
X	=	velocity loss factor due to drag $(1 - \overline{D}/\overline{F})$
λ	=	stage specific cost per unit mass of payload, £/Kgm.
λ_E	=	specific cost per unit mass of engine, £/Kgm.
λ_{ER}	=	specific cost per unit mass of reactor + engine, £/Kgm.
λ_P	=	specific cost, propellant, £/Kgm.
λ_T	=	specific cost, propellant container, £/Kgm.

Suffices

E	=	engine or solid propellant nozzle
L	=	stage payload
P	=	propellant
T	=	tank or solid propellant case
R	=	reactor
α	=	stage state at launch
ω	=	stage state at all burnt

Specific Cost Standard Exchange Rate (Rounded Values)

£1/Kgm. = \$ 1.25/lbm = 9 shilling/lbm.

\$ 1/lbm = £0.8/lbm.

INTRODUCTION

In this paper a technique is presented and used which permits the determination of first stage minimum specific cost and its variation with non optimum conditions for any arbitrary specification. For a given set of cost and scaling constants and specification the stage specific cost is defined by a value of initial acceleration, or burning time or thrust/mass ratio on engine or nozzle size within the stage.

As a continuous example the paper compares specific cost for 3 liquid and a solid propellant stage at various velocity increments. Placing in the analysis various costs involved with reuse permits an examination of the economies possible by this process. It is also shown that selection of the wrong engine or nozzle size or wrong initial acceleration can result in a significant economic penalty.

Earlier forms of optimisation have been particularly applied to the problem of determining the most advantageous distribution of effort between stages. In the papers of Malina and Summerfield, (ref.1) and of Vertregt (ref.2) a single structural factor is used for each stage which lumps together tanks, engine and other structure. Their structural factor is in fact defined as the ratio of the empty wet mass of a stage less its payload to the full mass of the stage less payload.

This simple form of stage description has proved most useful and has been widely operated and modified (ref.3). In fact its utility has inhibited more elaborate stage descriptions. More recently independent work at Cranfield (refs.4, 5, 6), at J.P.L. (ref.7) and Liege (ref.8) have enlarged the stage description to include the mass of thrust producing machinery.

As far as we can discover, it was Dr. Werner von Braun who first pointed out that, in spite of all the effort to the contrary, the minimisation of mass was an irrevalent exercise (ref.11). That the minimisation of cost was in fact the overriding factor. Immediately reported work on minimisation of cost followed the Malina and Summerfield trend and added propellant cost per unit mass and a monolithic specific structure cost. Builder (ref.9) wrote the first paper which specifically differentiated between the distribution of impulse between stages to give maximum payload ratio and that for minimum cost. It is interesting to note that using such costing processes but without optimisation can produce quite baffling results.

In August 1961 Schad (ref.10) categorically states that a liquid oxygen-hydrogen booster will always be cheaper than a solid propellant vehicle - even when the latter's effective exhaust velocity is assumed 3000 m/s. In November 1961 Thackwell proves that a solid propellant booster is only 0.7 the cost of a similar performance oxygen-hydrogen vehicle (ref.11). The discrepancy appears to have arisen because the former author assumes a fixed initial acceleration of 1.25 g_0 for all his computations and in consequence causes his solid propellant burning time to be 200 seconds - and the liquid 275 seconds. On the other hand, Thackwell's main "forced parameter" is burning time. He assumes for his main comparison that burning times for each are 150 seconds which in turn forces the hydrogen oxygen vehicle to have an initial acceleration of about 2 g_0.

In part of a recent paper (ref.12), the present authors tried to come to grips with the general problem of determining the optimum acceleration to produce a minimum cost stage. Such a solution was based particularly on the inclusion of numbers for specific mass and specific cost of propellant, engine and tank. In this paper the analysis is further expanded and now includes terms for reusable vehicle cost. It permits the comparison of optimum minimum cost reusable booster stages.

In this paper therefore we are assuming that a firm requirement will exist for a rocket propelled work horse first stage launcher for use by ELDO or its successor in the late 1970's. In size scale it is assumed that the payload will be 100 tonnes mass (comprising various upper stages and actual payload). It is also assumed that the overall requirement will involve something like 100 launches. The question of best velocity increment for the booster is left for discussion after the economics have been considered.

In this paper we are mainly concerned with the economics of first stage vehicles which may land, be refurbished, and used again. We have nevertheless set up a number of cost optimised "single shot" liquid and solid propellant models as standards, for both your benefit and our own. These are then used as yardsticks of comparison of cost per unit mass of launched payload by various single and multiple shot launchers.

Broadly speaking a closed form algebra has been set in order to describe minimum cost operations. The number of cycles of use to a full life is included. Most of the studies presented here are concerned with vehicles using liquid propellants. In consequence the engine is considered as a separate entity both in terms of specific mass and specific cost. A most important part of the procedure of this paper is the determination of optimum or "most economic" engine size. The problems involved in operating "optimum" solid propellant vehicles is also discussed.

Cost of Propellants

As we frequently show the cost per unit mass of payload is quite sensitive to propellant cost. Obviously a large amount of crystal ball gazing is involved. It is nevertheless important to demonstrate that guessing has been kept to a minimum, and that the quality of the basic information generates confidence.

Three liquid propellant combinations are considered in the first part of this paper. These are:

Oxygen and Kerosine
Oxygen and Hydrogen
Fluorine and Hydrogen.

It is thought that these are the only real contenders for a second or third era European Booster. One aspect of this paper is to consider their order of importance as economic sources of impulse.

The main source of data on the cost of these propellants is a paper by Dole and Margolis of the Rand Corporation (ref.13). Following the information from this and other sources we have taken as our Basic Price the cost of a liquid hydrogen-oxygen propellant mixture as £0.1 per kilogram.

This is identical to that estimated by Dole and Margolis. We assume that lower overheads on hydrogen production cancel out the advantageous supply of hydrogen gas in the U.S.A. (ref.14). Because hydrogen is some 25 times the cost of oxygen the optimum mixture ratio should be considerably biased toward oxygen. In fact, throughout this paper the mixture ratio giving maximum effective exhaust velocity is used.

For the fluorine-hydrogen mixture we have also been advised by Dole and Margolis. They suggest that this mixture will cost ten times that of oxygen-hydrogen. Their value is based on a fairly considerable annual demand and the assumption of a reduction in the U.S.A. market price by 2.5 (due to the propellant requirement). It appears to us therefore that in taking £1 per Kilogram for this combination we have shown it in a most adventitious light. In spite of this our study indicates that it is not an economic proposition for Booster applications where design starts from scratch. Incidentally, because fluorine costs only three times that of hydrogen, the cost optimisation of mixture ratio is not nearly so important as with oxygen-hydrogen.

The oxygen-kerosine combination has been taken with a certain amount of optimism compared to the figure of Dole and Margolis. It makes allowance for the fact that European/Australian overheads tend to be lower, and that this combination is at this time available at the launch sites. Increasing demand will lower the price at that point. Dole and Margolis suggest that the oxygen-hydrogen mixture should be six and a half times that of oxygen-kerosine. We have taken a value of £0.01 per kilogram and a ratio of 10. (Based on slightly more than a penny sterling per pound). With these two propellants the difference in cost is insignificant: it is in consequence only necessary to optimise their performance with respect to mixture ratio.

If the propellant prices discussed above are applied directly to each "flight vehicle" we shall have a minimum propellant cost for each example. In fact for each kilogram of propellant actually lifted from the launcher a lot more will have been liquified/manufactured/prepared. Some of this will be used in a flight vehicle in pre flight running and system checks. Yet again some will be lost in transfer, in storage or as vapour in pipes and tanks.

In order to have some idea of the possible magnitude of these propellant losses on the overall vehicle economics we have, for each propellant, and each example of a booster, taken propellant at minimum cost and propellant at ten times this cost. It is assumed that this gross assumption may make visible the effect of two separate methods of accounting and also to indicate how sensitive the end result is to propellant cost and propellant accounting method.

In the second part of the paper the sensitivity to propellant cost is considered in more delicate steps. The solid propellant cost is discussed later.

Engine Specific Mass and E.

Engine scaling for single shot liquid propellant stage

There is still a great deal of doubt about what are the most important factors that determine the change of engine specific mass (ESM) with change of size. A number of authors use ESM in optimisation without discussion (refs.4, 7, 8). They automatically assume that engine mass is proportional to thrust. In a number of papers estimates of how ESM alters with change of size is presented (refs.15, 16) or with detailed support (ref.17). It appears agreed that a "valley" exists in the curve, and its position in the thrust range and its depth have also been surmised. Baxter (ref.18) first pointed out this and suggested that it was in the 20-40 tonne force range. More recently values right up to 1000 tonnes have been shown (ref.15). It has been strongly suggested that these curves are very distorted by the fact that "very large engines have been very recently designed". But there is no evidence to support the contention that the sum of all the engine components scales in proportion to $(\text{thrust})^{1.25}$ (ref.19).

In this paper and elsewhere we have moved slightly away from the use of ESM to the concept that engine mass is more logically predominantly proportional to propellant mass flow rate or propellant volume flow rate. But a great deal of work still needs to be done in this area. This is particularly true when comparisons are made between different propellants, as in this paper, or in dealing with other changes within the propulsion system.

We have assumed that the E vs. engine size relationship is best. For a first guess it is assumed that "valley floor" values apply, and that the components which scale in the "square-cube" fashion are opposed in trend by some parts of the "machinery", e.g. gears, bearings, shafts, controls, etc. If at the conclusion of the first run it appears that the engine size is, as it were, running out of the valley, then small adjustments can be made.

The actual numbers selected as a basis for the work in this paper are derived from the Rolls Royce RZ 12 and the paper by Cleaver (ref.20). This gives a dry engine specific mass of 0.011 and E = 2.86 seconds based on mean flight effective exhaust velocity and a combustion pressure of 37 atmospheres.

In this paper allowance has to be made for a probable increase in "normal combustion pressure" for earth launched vehicles and also improvement in system and detail design. In consequence it is assumed that the dry engine value of E will improve by 20% in 20 years. E therefore equals 2.38 seconds. Here an extra mass due to power plant mass results in the final value of E = 2.5 seconds. This is taken for all propellant combinations. A value of combustion pressure = 68 atm. has been taken because it is the highest value for which detailed performance is available (ref.21).

In detail, engine dry mass should be used for estimating engine specific cost and wet mass for flight performance. In this paper only one value has been taken. This will be improved in subsequent work.

Tank Specific Mass and the Scaling Constant T.

Single Shot Vehicles

The absolute size of the vehicles considered in this paper, with first stage payloads of some 100 tonnes, allows the hydrogen tank to be used without extra flight carried external insulation. Following the trend of Sandorff (ref.22) a value of T = 0.045 has been used throughout for the oxygen-hydrogen and fluorine-hydrogen combinations. T = 0.035 is used for oxygen-kerosine. The sensitivity to a change in T is shown in figure 8.

Engine and Tank Specific Cost

The problem of settling a suitable area in order to cover the possible range of engine specific cost is quite difficult. Since for a European launcher the number of engines made will be spread over perhaps ten years. The greatest number of complete power plants required per year will not exceed perhaps 25 for single shot use, and say only 10 for reusable vehicles. In consequence production will be in the "one at a time" category and using craftsmens techniques rather than any form of repetition.

Most of the available information is U.S.A. based, and always includes as a single figure all the hardware (ref.9). The U.S.A. data is certainly based on a higher output of items per year. Equally their overheads are higher. In part 1 a value of £100 per kilogram is taken as not outrageous when compared with the "block values" of approximately £60 (Schad ref.10) and £120 (Thackwell ref.11). In part 2 the effect of changes of the engine specific cost are taken into account. Three values are used: the two extra are £50 and £20 per kilogramme. The direction of the extra values indicate that the authors feel that the original value may be slightly high.

The value of tank specific cost has been taken at £10/Kgm. It is assumed that material basic cost and manufacturing cost of liquid propellant tanks and solid propellant cases are the same per unit mass of material. The argument and studies below (Solid Propellant Engine Cost and Scaling) are taken to apply.

Specific Mass and Cost Changes. Reusable Stage.

Refurbishment and Landing

The mass of the part of the vehicle which actually lands is a very small part of the launching mass.

Since $L = 1 - P - PT - \frac{E\,a_\alpha}{c}$

$$\frac{\text{Landing Mass}}{\text{Initial Mass}} = PT + \frac{E\,a_\alpha}{c} = 1 - L - P.$$

Using Single Shot values (for an O.H. stage)

$$PT = 0.8 \times 0.06 = 0.048$$

$$\frac{E\,a_\alpha}{c} = \frac{2.5 \times 16.0}{3915} = 0.01$$

Which shows that the tank mass is about five times that of the engine. In consequence we assume that all the (undefined and undescribed) landing machinery is expressible as an increase in tank mass.

For the engine, the specific mass is increased by 5% to allow for the need for increased life and reliability.

For the tank, the specific mass is increased by 20%. In part this is an increase of ruggedness to withstand the multiple-landing load, and in part for the launching machinery. Obviously as the proportion allocated to the latter increases, then the proportion required by the former will decrease.

Cost and Scaling Constants for Solid Propellant Engines

A recent and most elaborate cost study of a series of solid propellant engines has been reported by Alexander and Fournier (ref.23). They conclude that the most economic case material is that of the highest yield strength. As best buy they select an 18% Nickel 7% Cobalt maraging steel with a convenient low temperature ageing cycle. It is the material having the maximum cost per unit mass. Their results also indicate an optimum pressure which differs between materials. Their study is planned to meet only one specification giving a payload of 500 tonnes, a velocity increment of 2000 m/s with a fixed burning time of 120 seconds.

The report of their work did not arrive on the scene in time to affect the main framework of the solid propellant study in this paper. Nevertheless it has acted as a useful confirmation of various aspects of this simpler and much wider ranging approach.

Values of the specific case mass T used in this analysis are 9%, 10% and 12% of the propellant mass and includes insulation but not the nozzle. These appeared to cover the range of values which might become current for the late 1970's, and assumed a combustion pressure of 68 atmospheres. The numbers were selected after consideration of a 1963 "State of the Art" report (ref.24) and ref.11.

The most careful estimate in ref.22 for a combustion pressure of 50 atmospheres suggests that $T = 0.09$ including nozzle, insulation etc. or 0.06 for the bare case. These are optimistic values based on a "fabrication on launch site" approach (not segments). They are very close to the lowest value used in this paper.

The value of E for the nozzle and thrust direction control was 0.3 seconds. It is based entirely on the note by Chuk Chin Ma (ref.25). Comparison with ref.22 indicates that in their view it is rather low and suggest something nearer to 0.6 seconds. Nozzle specific cost is taken to be £20/Kgm. This is just twice the number given to the case.

Propellant specific cost was fixed at £1/Kilogram after consideration of references 11 and 13. Only the single value is used in this analysis. This is mainly because evaporation, handling and line losses are zero and there is no pre flight qualification firing. Nor is any financial adjustment made for any items used on the ground in proof and reliability tests.

NUMERICAL RESULTS

Introduction

In both parts of the paper single stage boosters are compared. The all burnt velocity increments are 3000, 4000 and 6000 m/s. Separate allowance for velocity losses are made. That due to gravity through the burning time by assuming a mean trajectory inclination so that G - 8 m/s². An allowance for drag assumes that mean drag through the firing time is 5% of the mean thrust. The only comparison criteria used is the stage specific cost in pounds sterling per kilogram of stage payload. The stage payload is the sum of all the full upper stages and actual vehicle payload carried aloft by the booster.

The mass and cost of a non-reusable or single-shot stage is most conveniently described as follows:-

For a vehicle having a launching mass of unity,

$$M_\alpha = 1$$

The mass of propellant	=	P
the cost of propellant	=	$P \lambda_P$
The mass of tank (or case)	=	PT
the cost of tank (or case)	=	$PT \lambda_T$
The mass of engine (or nozzle)	=	$\frac{E a_\alpha}{c}$
the cost of engine (or nozzle)	=	$\frac{E a_\alpha \lambda_E}{c}$
The mass of payload	=	L
Stage specific cost	=	λ

$$\lambda = \frac{P(\lambda_P + T\lambda_T) + E\, a_\alpha\, \lambda_E/c}{L}$$

In both parts the main comparison is between the specific cost of vehicles designed for reuse, and those designed for single shot. Analysis of the former includes:

(i) A 5% increase in E to cover the extended life and reliability required.

(ii) A 20% increase in T which in part is the mass for the landing phase (not analysed) and the remainder for increased reggedness to stand the landing loads.

(iii) A 5% increase of both λ_E and λ_T to include a refurbishing cost and is included for each between flight inspection.

The number of flights expected from a reusable vehicle is 10 and 20. These numbers are quite conservative. It is considered that a firing frequency of 10 per year and a total of 100 launches falls reasonably within the state of the art. At the same time it will provide considerable stretch.

In part 1 comparison is made between three liquid and one solid propellant combination. The result of this comparison has therefore been taken for a range of initial accelerations including that which results in minimum cost. The complete difference in the analytical treatment between minimum cost liquid and acceleration or other criteria limited solid is included.

Comparison is also made between the low and high specific cost propellant to note its different effect on the various combinations and velocity increments.

In part II the main concern is to examine the sensitivity of the results to smaller changes in the engine tank and propellant specific masses and costs. Oxygen-hydrogen is the only propellant combination considered. This paper comes quite early in the history of optimised reusable vehicles based on cost analysis. It is therefore considered most important to determine the effect of certain changes upon the stage specific cost. The range of changes considered are:-

(i)	λp	=	0.1, 0.2 and 0.5
(ii)	T	=	0.045 and 0.06
(iii)	Refurbishing		5% and 25%.

<u>Numerical Results. Part 1.</u>

Figure 1 is a plot of stage specific cost vs. initial acceleration. Curves are shown for the three propellant combinations and for the solid propellant. The most optimistic accounting system has been used for the liquid propellants. Only the propellant price at the launcher is considered without any losses. The specified velocity increment is 3000 m/s for a "single shot" stage. It is interesting to see that the hydrogen-oxygen combination shows immediately as the cheapest buy with a specific launching cost of just over £5 per kilogram of payload.

The fluorine mixture is some 30% more expensive and Kerosine-oxygen 60% over. The magnitude of the initial accelerations which result in minimum cost is:

H.O.	1.4	g_0
H.F.	1.75	g_0
K.O.	1.25	g_0

This last is a most interesting coincidence since a number of U.S.A. vehicles (Saturn, Atlas, and Thor) and also Blue Streak use K.O. and have initial accelerations very near to this number. In fact we surmise that this is in order either to maximise the payload launched by available first stages with a degradation of velocity increment, or to maximise the payload launched by an available engine.

With an optimum initial acceleration of nearly 8 g_0 the specific cost of the "low T" (T = 0.09) solid propellant vehicle is almost identical to that of the "low propellant cost" Fluorine-Hydrogen combination. But of course there are all sorts of reasons why vehicles cannot be developed having initial accelerations anywhere near this value. These reasons include a serious problem of aerodynamic heating. Also very high values of final acceleration arise. In this particular example approximately 26 g_0. This tends to affect vehicle control and guidance and upsets instrumentation in the payload and may permanently distort a human. In order to show the differences that arise between solid and liquid propellant vehicles, the same set of stage specific costs have been plotted, in figure 1a it is plotted against all burnt acceleration. In figure 1b the specific cost is plotted against burning time. Each figure shows quite graphically the completely opposing trend of liquid and solid stages. In particular it shows that it is not possible to select a common "forced value" of either initial, or final acceleration, or burning time, in order to be fair to both solid and liquid systems. Therefore Schad (ref.10) is particularly unfair to the solid propellant vehicles whose cost he describes. All his computations are based on a fixed initial acceleration of 1.25 g_0 (ref.10, p.56). Reference back to figure 1 will indicate the extent of this bias. At this point the solid propellant cost curve is very steep, and rapidly approaching infinity! Because of this crucial sensitivity we have examined the relative effect of the various values on stage specific cost.

In Alexander and Fournier's most careful paper (ref.23), they have opted for a burning time of 120 seconds stating "this long burning time is required from aerodynamic considerations." Following this lead results in a "low T" specific cost of £10.5, an increase of almost £4 on the "real but unusable" minimum cost "low T" solid. This is now a little more than twice the "low cost propellant" Oxygen-Hydrogen combination. Such a stage would have initial and final accelerations of 1.75 g_0 and 8.00 g_0. Bringing the burning time down to 100 seconds reduces the cost by over a £1, but increases the accelerations to 2 g_0 and 9 g_0 respectively. But with the burning time at 80 seconds the low T solid cost is down to £8.3, the accelerations are now 2.7 g_0 and 11 g_0. It is only possible in this paper to indicate the manner of the interplay between specific cost, burning time and the end point accelerations. The manner in which these in turn affect the magnitude and altitude of $[\frac{1}{2}\rho v^2(\text{max})]$ the value of $[\frac{1}{2}\rho v^2(\text{at all burnt})]$ is treated in (ref.26).

Increasing the value of T from 0.09 to 0.1 at the 120 second burning time line increases the stage specific cost 10%.

In figure 2 all the values used are the same as in figure 1 with the single exception that the liquid propellant cost per kilogram has been multiplied by 10. It is assumed that this gross change will encompass a wide range of possible accounting systems with respect to propellant losses and flight vehicle preflight consumption. This considerable step affects the various combinations quite differently. It is most interesting to see that once again the H.O. combination works out most economically. Actually this factor of 10 on propellant cost only increases the H.O. cost by 50% to £7.5.

The K.O. combination is very little affected by propellant cost and is now the second best buy and only 15% more expensive than the H.O. The change due to propellant cost is from £8.25 up to £8.75. The most startling change is upon the economics of the fluorine combination. This high cost system is very sensitive to this form of gross propellant charge. In consequence it is now more than two and a half times the Kerosine combination and is more than three times the cost of the Hydrogen-Oxygen.

Comparing this with the unchanged solid propellant vehicle at its most optimistic - the 100 second burning time: low T vehicle is still more expensive than the K.O. And the Hydrogen-Oxygen system is still 25% cheaper. In fact, for the solid propellant vehicle to get onto terms cost-wise with the "high cost" H.O. would require the burning time to be reduced to 60 seconds and a maximum acceleration of 14 g_0.

As at this point our conclusions differ completely from those of Thackwell, we consider it important to indicate the main reasons for this. He uses a hardware cost for his oxygen-hydrogen vehicle of £120/Kgm. which includes engine and tank costs. This appears reasonable for U.S.A. specific cost for the engine, but tank specific cost we feel should be nearer the specific cost of his solid propellant case. A reduction of the average value to £80/Kgm. changes the cost ratio from 0.72 solid/liquid to 1.1!!

For the other two velocity increments considered in this paper, only low cost propellant is considered. In figure 3 the results are again given as specific cost vs. initial acceleration. This is a 4000 m/s all burnt and this is stretching the K.O. combination as the $\Delta v/c$ ratio is 1.36. In consequence it is almost twice the cost of the H.O. vehicle which once again is the minimum cost at approximately £8/Kgm. The fluorine combination is just over 25% more expensive. The solid propellant vehicle is also being stretched. For the optimum initial acceleration of 8.2 g_0 it is still 90% more expensive than the H.O.

For 6000 m/s. (figure 4) the solid propellant and K.O. combinations are unable physically to manage payloads. The H.O. system is still cheapest at a little less than £20/Kgm., whilst the fluorine combination is again 25% more expensive.

Reusable Stage Part 1 continued

A main concern of this paper, having set a good standard of cost determination for single shot or non-reusable first stage boosters, is to consider the economics of reuse. The closed form algebra developed for this includes terms for increased specific masses of engine and tank to include a "landing mechanism" and also to withstand the rigour of reuse. A cost associated with bringing the vehicle up to flight condition again after use, with separate numbers for engine and tank inspection. For reasons stated in the introduction, only lives of 10 flights and 20 flights are considered economically viable for Western European operations. In this part of the paper only one set of these constants is considered for the three liquid propellant combinations at each velocity increment. For Δv = 3000 m/s. and 10 flights to life in figure 5, a considerable reduction in cost per kilogram of launched stage payload. In fact the H.O. propellant combination is again cheapest and reuse reduces the cost by five. This is now to a figure of slightly more than £1/Kgm. The K.O. combination is at £1.4 and the fluorine combination £2.75. It is interesting to note that the optimum acceleration conditions are now more spread out.

With the value for fluorine being 2.90 g_0 it is interesting to note the increased cost if an acceleration of 1.3 g_0 is assumed for these calculations. It puts the cost up £3.6 whereas there is little change for H.O. at 1.3 g_0.

Considering now the 20 flights to life (figure 5a) there is little significant difference. Each of the costs is slightly reduced (compared to 10 flights), each of the optimum accelerations is increased. The H.O. propellant combination is least costly with £0.8/Kgm. K.O. is very close with £0.95. Fluorine is £2.4.

It is most interesting and revealing to compare the shape of the various initial acceleration vs. stage specific cost curves shown in part 1. They particularly indicate how much extra cost can accrue to a vehicle having the wrong engine size and consequently the wrong burning time and initial acceleration. This fact underlines the importance of including a separated engine - cost and mass within the analysis. It also largely points to the reason for the existence of this paper. A brief consideration of this factor shows that in figure 1 the optimum accelerations are all less than that of 1.75 g_0 for F.H. In figure 2 the high cost propellant example now pushes the optimum initial acceleration to 3.3 g_0. In figure 3 for Δv = 4000 m/s. the optimum acceleration of F.H. has moved up slightly (to 1.85 g_0) as has O.H. In figure 4 for Δv = 6000 m/s. the optimum values of O.H. has now reached 1.7 g_0, whilst the F.H. is above 2.

For the reusable stage the optimum acceleration increases both with velocity increment and number of flights to life.

For a 10 flight life and Δv = 3000 in figure 5 the optimum values of O.K. and O.H. show small increases, but for F.H. it has increased to 2.9 g_0. The maxumum values of the optimum acceleration are shown for a 20 flight life and Δv = 6000 for O.H. is 2.15 g_0 and F.H. 3.5 g_0.

Figure 8 shows the general trends discovered in part 1. Stage specific cost is now shown plotted against velocity increment. Only the optimum values of each liquid propellant combination are shown. For the low propellant cost the three liquid combinations are shown for N = 1 and the two high performance liquids for N = 20. The non-optimum solid propellant results are shown all for a case mass of 10% of the propellant mass. They show the general trend for the forced values of

$$t_b = 80 \text{ seconds}$$
$$t_b = 100 \text{ seconds}$$
$$a_\omega = 20\ g_0$$
$$a_\omega = 25\ g_0.$$

As a point of caution, and to indicate the extent of real propellant costs additional to the "low cost propellant" definition, a single point is shown for the F.H. "high cost propellant".

Numerical Results Part II.

In figures 9, 10, and 11 the stage specific cost is plotted against propellant specific cost. In each figure the effect of changes of engine specific cost, and tank specific cost are shown. Each figure is for a separate velocity increment. Only results for one shot vehicles are shown here. Comparison of reusable vehicles are shown and discussed later. Examination of each of the figures reveals that the relationships considered are almost linear. In addition the general trends are very similar for each.

First, at the lower portion of each graph where the characteristics for the low engine specific cost (λ_E = £20) are shown:-

(i) Doubling the propellant specific cost in each case increases the stage specific cost by 10%.

(ii) Increasing the specific tank mass (from T = 0.045 up to T = 0.06) increases the stage specific cost approximately by

+ 15% when Δv = 3000 m/s.
+ 20% when Δv = 4000 m/s.
+ 30% when Δv = 6000 m/s.

Second, for each figure at the top end, for the high engine specific cost (λ_E = £100) curves:-

(i) Doubling the propellant specific cost in each case now increases the stage specific cost by about 5%.

(ii) Increasing the specific tank mass has the following approximate effect on stage specific cost:

+ 10% when Δv = 3000 m/s.
+ 15% when Δv = 4000 m/s.
+ 25% when Δv = 6000 m/s.

In conclusion, the effect of a change of engine specific cost is extremely regular. For all of the figures, using the engine specific cost λ_E = £50 any constant T as a base, doubling the engine specific cost increases the stage specific cost by 55% for the low specific cost propellant and by 45% for the high specific cost propellant.

Referring to figure 12 which shows stage specific cost plotted against the number of flights to life. Plotted on here are two curves for each velocity increment. One set of curves is given for the "5% refurbishment cost" and the other for 25%. Broadly speaking it is shown that for both N = 10 and N = 20 multiplying the % refurbishment cost by 5 doubles the stage specific cost. This is quite an important trend and indicates the need for further attention to the allocation of this value.

Figures 13, 14, and 15 are sets of carpets plotted against stage specific cost. Each figure is for a particular velocity increment. On each is shown the trend of specific cost of the stage with the general changes considered in this part of the paper.

Everywhere the vertical scale is proportional to the stage specific cost. Within each parallelogram the horizontal scale is proportional to the propellant cost. The main horizontal scale is "number of flights to life" = N. The curved lines connect points of equal specific propellant and specific engine cost. Their interpretation at points between the parallelograms should be treated with reserve. Points within the parallelograms are accurate within the limits of the paper.

It is now possible to analyse the general trends which become apparent from the analysis. This now gives some "body" to the title of the paper. Quite generally the trends for reusable first stages are as follows:-

(i) as the number of flights to life is increased, the sensitivity to a change in engine specific cost decreases.

(ii) the sensitivity to a change in propellant specific cost increases.

The magnitude of the sensitivities is not affected by the velocity increment considered. Approximately, the other sensitivities are as follows:-

(iii) For N = 10
doubling engine specific cost increases stage specific cost by 40% (0.1) and 25% (0.5).

(iv) For N = 20
doubling engine specific cost increases stage specific cost by 35% (0.1) and 20% (0.5).
(Numbers in brackets above refer to propellant specific cost).

(v) For N = 10
doubling propellant specific cost increases stage specific cost by 35% (20) and 20% (100).

(vi) For N = 20
doubling propellant specific cost increases stage specific cost by 45% (20) and 30% (100).

(Numbers in brackets in (v) and (vi) above refer to engine specific cost).

All the numbers quoted above are within 3% of computed values.

DISCUSSION, DEFICIENCIES, CONCLUSIONS

Discussion

This paper has set out to present a quite straight forward method of using closed form algebra for the determination of minimum stage specific cost. The method is suitable for both solid and liquid propellant stages, and may be conveniently used to describe reusable stages.

As demonstrated in this paper, it may be used for broad sweep assessments. With more experience and design detail it may develop into a very powerful "selection" device.

The inclusion of an engine specific mass and also engine specific cost, are most important. It is at this point that the method described differs from those previously available. It is this inclusion which makes the method powerful, and the results so revealing and provoking.

The concept of "optimum engine size" is very clearly a most important one. The manner of its change with propellant type, with velocity increment and the number of flights to life underlines this fact.

The closed form algebra has been used in this paper to examine the merits on a cost basis of various propellant combinations.

Consideration was first given over the velocity range 3000 up to 6000 m/s. for a "single use" launching stage. The hydrogen-oxygen combination was at all points the lowest specific cost system. The hydrogen-fluorine combination was more expensive and most sensitive to propellant specific cost used.

At the lower velocity end of the spectrum the kerosine-oxygen combination as a known and developed propellant system was not overwhelmingly more expensive. It was least sensitive to propellant specific cost and in consequence most competitive under the "high propellant cost analysis". It appears that at lower velocity increments than are considered in this paper - 2500 - 1500 m/s. - the propellant may be worth continuing with. But such an arrangement would place added problems and cost upon the upper stages (which are not considered at all in this paper).

It appears that with the solid propellant booster, economic operation conflicts with aerodynamic heating and acceleration tolerance of payload and control. For the conditions considered in this paper the low velocity increment area is the only possible solution and this is not economically viable.

Within the economics described in this paper, reusable stages appear most attractive. Because only one stage is considered as recoverable in this paper economically the velocity increment should be as large as possible. For a velocity increment of 6000 m/s. it is three or four times cheaper to have a reusable stage with N = 20 using Hydrogen-Oxygen.

It is most important with reusable stages that the optimum engine size is used. Figure 16 is a plot of payload ratio L vs. stage specific cost. This shows for N = 1, 10, and 20 particularly how uneconomic are "payload ratio optimised" stages. For N = 1 the specific cost is doubled. For N = 20 the increase is from £1.3 up to £2 an increase of almost 50%. Of course the payload ratio is also affected.

Consider figure 16, N = 1 applied to launching a stage payload of 100 tonnes. At the maximum payload condition:

$L = 0.24 \qquad M_\alpha = 416$ tonnes

Stage Specific Cost = £16/Kgm. Stage Cost = $£1.6 \times 10^6$

At the minimum cost condition:

$L = 0.17$ hence $M_\alpha = 587$ tonnes.

Stage Specific Cost = £8/Kgm. ∴ Stage Cost = $£0.8 \times 10^6$

The move from maximum payload to minimum cost is (for liquid propellant stages) achieved by decreasing the proportionate engine size and stage initial acceleration. Although M_α increases the real engine size and cost decreases more rapidly thus producing the reducing stage cost.

In this paper consideration has not been given to any extraneous costs. In more elaborate analyses these may be included. Some of these will be sensitive to vehicle size, e.g. test and launch facilities. When these items are included in the cost analysis the optimum stage will not be exactly that of the minimum cost stage.

In the end this paper discusses and compares various payload "boosters". Of course, in use these will be only the first part of a multi-stage vehicle capable of placing the final payload into a variety of orbits. Two questions arise:-

(i) To what extent is it reasonable to consider the various first stages each by themselves?

(ii) To what extent is it reasonable to compare economics of different first stages for identical velocity increments?

Work is in hand at Cranfield to study the cost optimisation of multi-stage vehicles. These indicate that the optimum distribution between stages is sensitive to stage propellant performance and stage specific masses and costs. It follows therefore that for minimum specific cost operation, changing the propellant in the first stage will result in a change in the velocity increment contributed.

In this paper it is taken for granted that comparison of first stages for given velocity increment is reasonable. In European Space Technology it appears certain that boosters will be "workhorses". Through their lives they will be called upon to perform a variety of operations which it is not possible to guess at the outset. It is therefore the only possible approach to compare first stages at identical velocity increments. The problem of the optimum use of the stage for various needs can be left to those who have more precise end points, and a first stage already in existence.

In selecting the size or velocity increment to be contributed by a single shot first stage, there will be a tendency to be somewhat modest. Money in a "small velocity increment booster" may be considered politically convenient. But with a reusable first stage the trend is completely reversed. In fact as the planned number of flights to life increases so does the need to increase the size of the reusable part. In the limit maximum economy is achieved by not throwing anything away. This of course can mean one stage to orbit or separable reusable stages.

Deficiencies of the Present Analysis

This paper considers only those costs associated with vehicle manufacture and filling with propellant for flight use. It appears that the estimation of manufacturing cost is very contentious. Even for items actually being made. Estimation into the future contains further problems. This paper does not try to solve this problem, only to underline its importance in vehicle assessment. It certainly brings to notice those areas of greatest sensitivity. The following items of overall cost are excluded for a variety of reasons:-

(i) Research
(ii) Development
(iii) Major test facilities
(iv) Launch facilities
(v) Transport facilities
  To launch area from manufacturer
  To launch area from landing point
(vi) Reliability.

In this paper only a single lump mass is included as the "landing mechanism". Where significant differences exist between types of recovery system a more detailed approach is possible.

Throughout the analysis the liquid propellant mixture ratios used are, for each combination, the ones which give maximum effective exhaust velocity with maintained equilibrium. There is a very wide range of costs between propellants: for instance Fluorine is 75 times the cost of oxygen. In this paper the mixture ratios which reults in an overall effect of producing minimum stage specific cost ought to be used. This ratio is expected to be noticeably different from that actually used.

Conclusions

The analytical tool developed and used in this paper is one of considerable utility. The analysis actually reported herein quite briefly outlines this. Much remains to be done in its direct application and in the numerical validification of the various constants. This in particular refers to the treatment herein of drag, gravity, specific masses costs and staging. In detail more elaborate descriptions of propulsion are required in order to treat problems of combustion pressure and propellant system selection. These appear to be areas of fruitful research.

APPENDIX

1. Minimising Launch Vehicle Cost/Payload Mass for Single Stage Single Use Vehicles

For a specified velocity increment and a constant value of $\frac{dm}{dt}$ and hence a constant mean thrust, the vehicle dynamic equation has the form:-

$$\Delta v = -c\left(1 - \frac{\bar{D}}{\bar{F}}\right) \ln (1 - P) - \frac{c\,GP}{a_\alpha} \tag{1}$$

The initial mass of the vehicle is assumed to comprise the following discrete items:-

$$M_\alpha = M_L + M_P + M_T + M_E + M_R \tag{2}$$

The cost per unit mass of payload placed into the specified end velocity is

$$\lambda = \frac{\lambda_P M_P + \lambda_T M_T + \lambda_{ER} [M_E + M_R]}{M_L} \tag{3}$$

It is equation (3) which we are seeking to minimise. This may be re-arranged and written in the form of the mass scaling constants, the propellant ratio and initial acceleration as:

$$\lambda = \frac{P(\lambda_P + \lambda_T T) + (E + \frac{c^2}{2} R)\, \lambda_{ER}\, a_\alpha/c}{L} \tag{4}$$

The minimum value of (4) is obtained using (1) as a constraint either using the method ofLagrange or by the more cumbersome method of the calculus.

The initial acceleration and consequent engine size which result in a minimum cost vehicle is for a chemical rocket stage:-

$$\frac{a_\alpha}{G} = \frac{\left\{1 + \left(\frac{4X}{1-P}\right) [PC + 1 - (1+T)P] \frac{c\,PC}{EG}\right\}^{\frac{1}{2}} + 1}{2\left(\frac{X}{1-P}\right) [PC + 1 - (1+T)P]} \tag{5}$$

where $X = 1 - \bar{D}/\bar{F}$

and $C = \dfrac{\lambda_P + \lambda_T T}{\lambda_E}$

The form described in this appendix was first derived by the authors in reference 12. A single page describing the algebra between (4) and (5) may be obtained from Cranfield.

2. Minimising Stage Specific Cost for a Reusable Launch Stage

The specific tank mass is increased due to greater ruggedness and the inclusion of the landing mechanism. Then

$$T'' = T(1 + \Delta T)$$

Similar treatment is given to the engine. Values attributed to ΔT and ΔE are discussed in the text. The main extra cost for a reusable stage is that incurred in the refurbishing processes. It is assumed here that the cost of a single refurbishment is proportional to the complexity of the item inspected. Therefore it is taken that the refurbishing cost per uni mass is proportional to the specific cost of the item. Then for N flights of the stage, and N - 1 inspections, the new tank specific cost is given:

$$\lambda_T'' = \lambda_T \left[\frac{1 + \Delta\lambda_T (N - 1)}{N} \right]$$

Similar treatment is given to the engine. Values considered for $\Delta\lambda_T$ AND Δ are discussed in the text.

It is now possible to return to equation (4) above with the values applicable to a reusable stage. Equation (5) again gives the minimum cost condition for the reusable stage when the new values are used.

TABLE 1.				
Vehicle Designation	O.K.	O.H.	F.H.	S
Oxidant	Liq. Oxygen	Liq. Oxygen	Liq. Fluorine	Ammon. Perch.
Fuel	Kerosine	Hydrogen	Hydrogen	Al. $(H_2 C)_n$
Pc Atm.	68	68	68	68
$\bar{c}$ m/s. (I_s)	2960 (302)	3915 (400)	4120 (422)	2750 (281)
Engine Sp. Mass $\frac{M_E}{Thrust}$ $\left[\frac{Kgm}{Kgf}\right]$ $\left[\frac{lbm}{lbf}\right]$	0.0083	0.00625	0.006	0.00107
$E = (E.S.M.) \times I_s$	2.50	2.50	2.50	0.30 *
$T = {}^{M_T}/M_P$	0.035	0.045	0.045	0.09 etc.
λ_E £ per Kgm.	100	100	100	20 *
λ_T £ per Kgm.	10	10	10	10
λ_P (minimum)	0.01	0.1	1.0	1.0
λ_P (worst case)	0.1	1	10	-

* Includes Nozzle and actuation.

REFERENCES

1. F.J. Malina and M. Summerfield, "The Problem of Escape from the Ear
Journal I.A.S. 1947 14 (8).

2. M. Vertregt, "A Method for Calculating the Mass Ratios of Step Rock
Journal B.I.S. 1955 14 (1) 1956 15 (2).

3. D.J. Bell, "Review of Flight Optimisation along Synergic Paths in t
Period 1945-1960". Journal R.Ae.S. 1963 67 (2).

4. S.N. Roy Chaudhury, "Optimum Propulsion for Large Rocket Motors"
May 1958, Unpublished College of Aeronautics Report.

5. B. Kalitventzeff, "Propulsion Aspects of Earth Launched Chemical Rocket Vehicles".
June 1963, Unpublished College of Aeronautics Thesis.

6. D.S. Carton and B. Kalitventzeff, "Optimum Stage Initial Acceleratic and Velocity Distribution between Stages"
Journal B.I.S. 1964 17 .

7. J.R. Wrobel and R.R. Breshears, "Lunar Landing Vehicle Propulsion Requirements". Journal A.R.S. 1961 31 (11).

8. B. Fraeijs de Veubeke, "Overall Rocket Performance" in "Rocket Propulsion" by M. Barrere, A. Jaumotte, B.F. de Veubeke and J. Vandenkerckhove. Elsevier 1960. (Chapter II. page 232).

9. C.H. Builder, "General Solution for Optimisation of Staging of Multi-staged Boost Vehicles"
Journal A.R.S. 1959 29 (7).

10. J. Schad, "What Costs for Big Boosters"
Astronautics (Pubn. of A.R.S.) 1961 6 (8).

11. H.L. Thackwell, Jr., Paper at Third Meeting of the Joint Army-Navy-Air Force - NASA - ARPA Liquid Propellant Group, Nov. 1961.
(Report in Missiles and Rockets Nov. 13th. 1961).

12. D.S. Carton and B. Kalitventzeff, "Payload Achievement for Optimum and Non-Optimum Initial Acceleration of Earth Launched Thermal Rocket
Submitted to the British Aeronautical Research Council, Rockets Panel
9th. Dec. 1963.

13. S.H. Dole and M.A. Margolis, "Sources, Availability and Estimated Costs of Propellants". In "Chemistry of Propellants" Ed. S.S. Penner and Ducarme, Pergamon 1960.

14. T.J. Webster and K.C. Smith, "Liquid Hydrogen Production on an Industrial Scale"
Journal B.I.S. 1962 18 (5 and 6).

References (Continued)

15. D.M. Hammock, "Principles of Design"Handbook of Astronautics p.22-18 Ed. H.H. Koelle. McGraw-Hill 1961.

16. L.E. Maher, "Aerospace Applications of Rocket Engines" Journal B.I.S. 1964

17. J.E.P. Dunning, "The Optimum Size of a Rocket Engine" Journal R.Ae.S., 1960 65 (12).

18. A.D. Baxter, "Some Propulsion Problems of High Altitude Rockets" 1957. In "High Altitude and Satellite Rockets" A Symposium (No Editor) R.Ae.S., B.I.S. 1958.

19. K.J. Bossart, "Design Problems of Large Rockets" Ibid.

20. A.V. Cleaver, "The Blue Streak Rocket Propulsion System" Journal B.I.S. 1962 18 (7).

21. Propellant Performance Chart. Rocketdyne Publication 505-X Revised 1959 .

22. P.E. Sandorff, "Structural Considerations in Design for Space Boosters" Journal A.R.S. 1960 30 (11).

23. R.V. Alexander and E.A. Fournier,"Cost and Performance Considerations in the Selection of Structural Material for Ultra-Large-Size Booster Motors". Journal of Spacecraft and Rockets (A.I.A.A.) 1964 1 (1).

24. State of the Art - Launch Vehicles and Missiles Astronautics and Aerospace Engineering 1963 1 (10) page 52.

25. Chuk Chin Ma, "Advanced Uncooled Nozzles for Solid Propellant Rockets" Journal A.R.S., 1961, 31 (3).

26. J.F. White (Ed.), "Flight Performance Handbook for Powered Flight Operations". Wiley. New York 1963.

FIG. 1

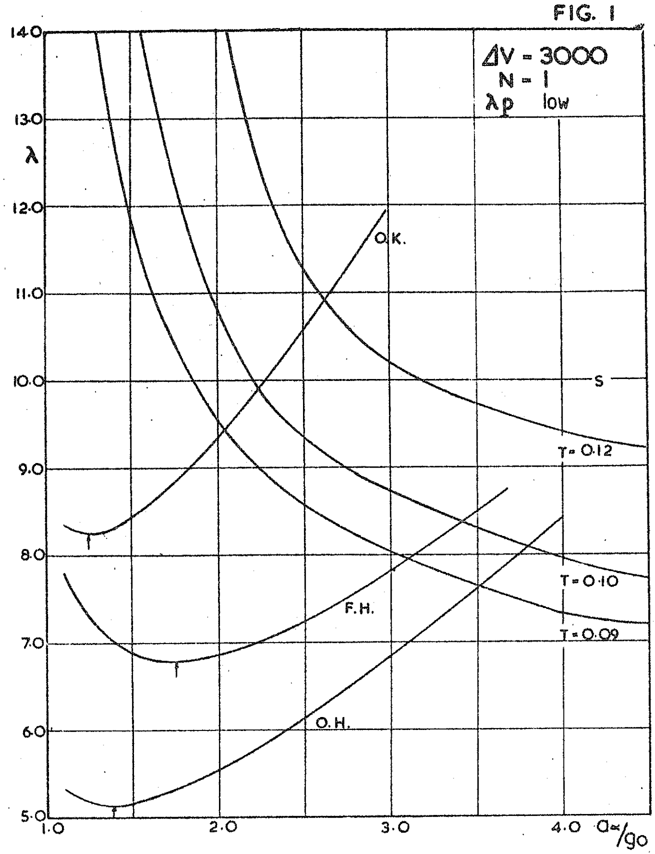

FIG. 1A

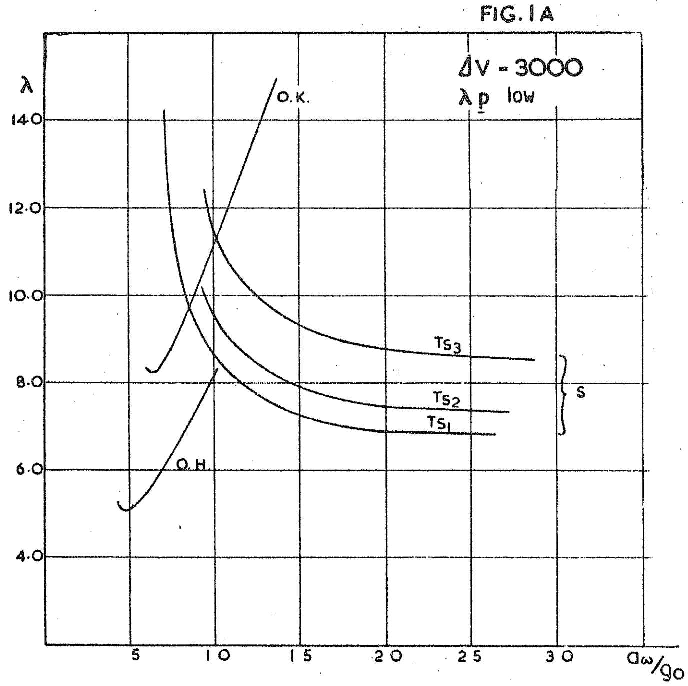

FIG. 1B

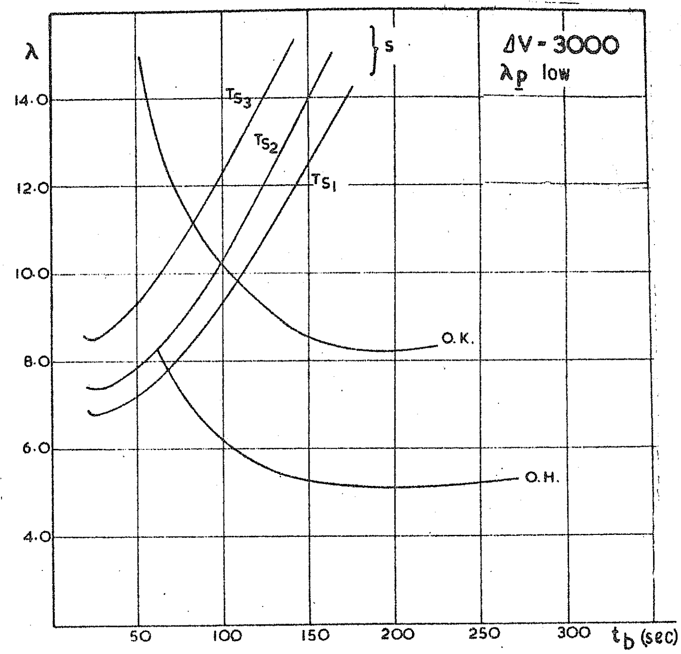

FIG. 2

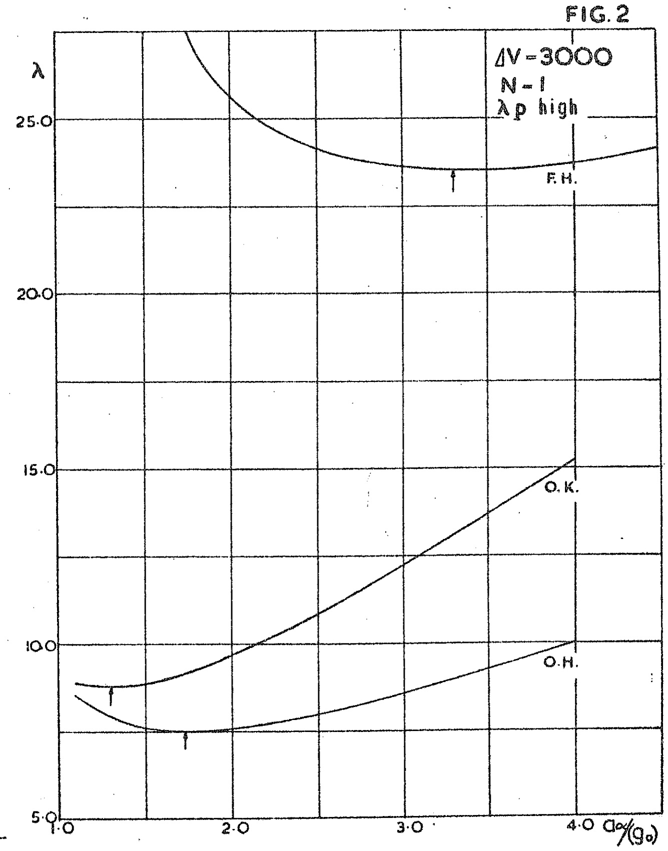

FIG 3

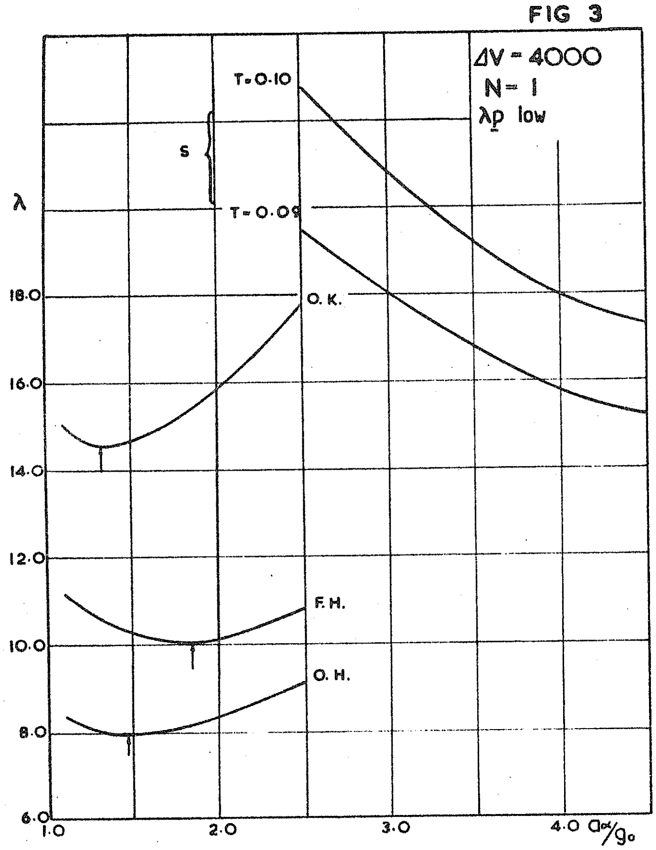

FIG 5

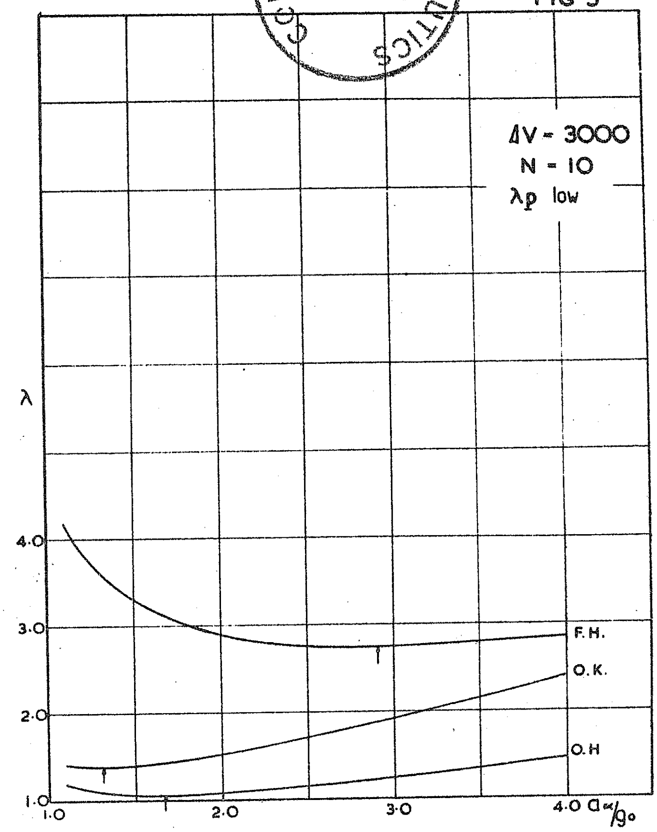

FIG. 4

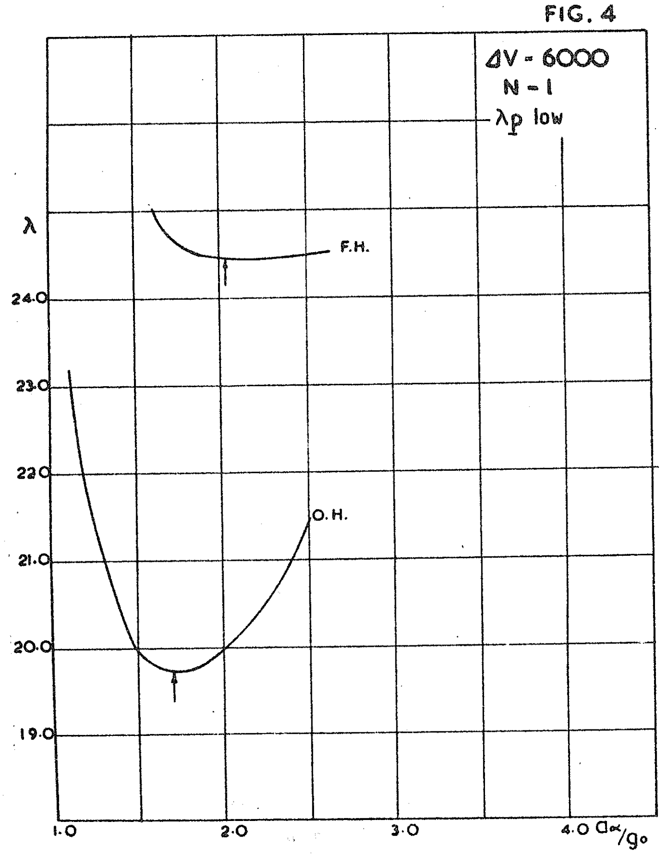

FIG. 5A

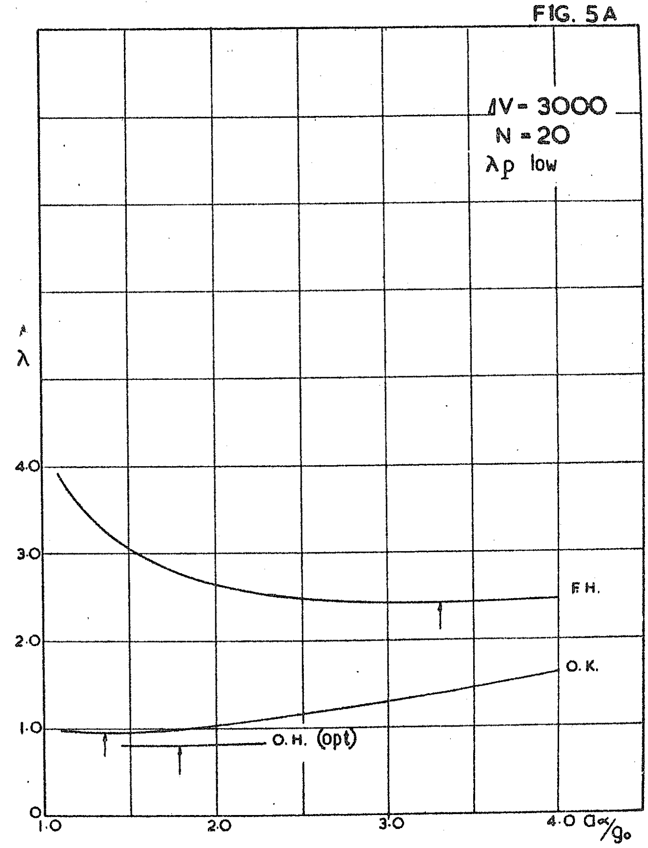

FIG. 6

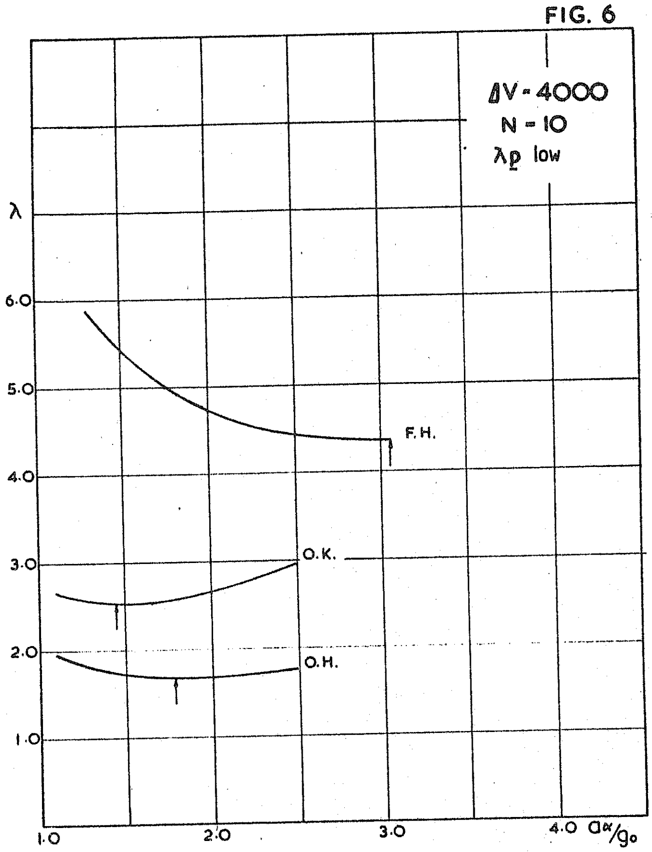

FIG. 7

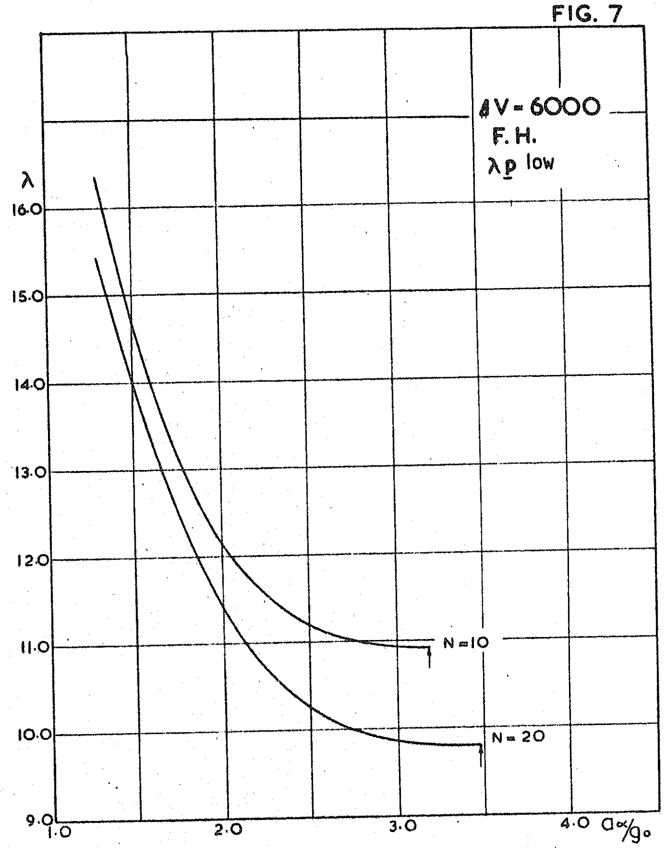

FIG. 6A

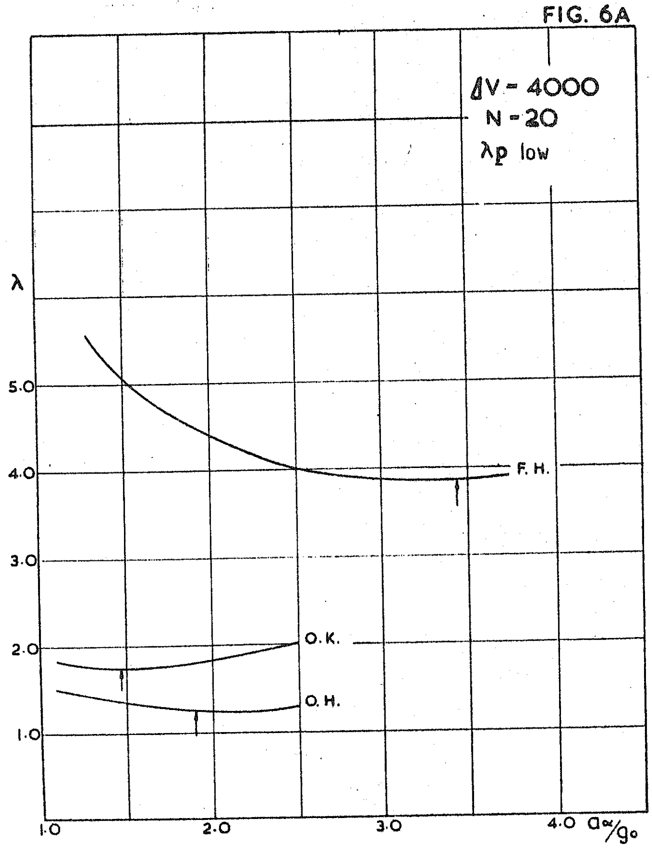

FIG. 7A

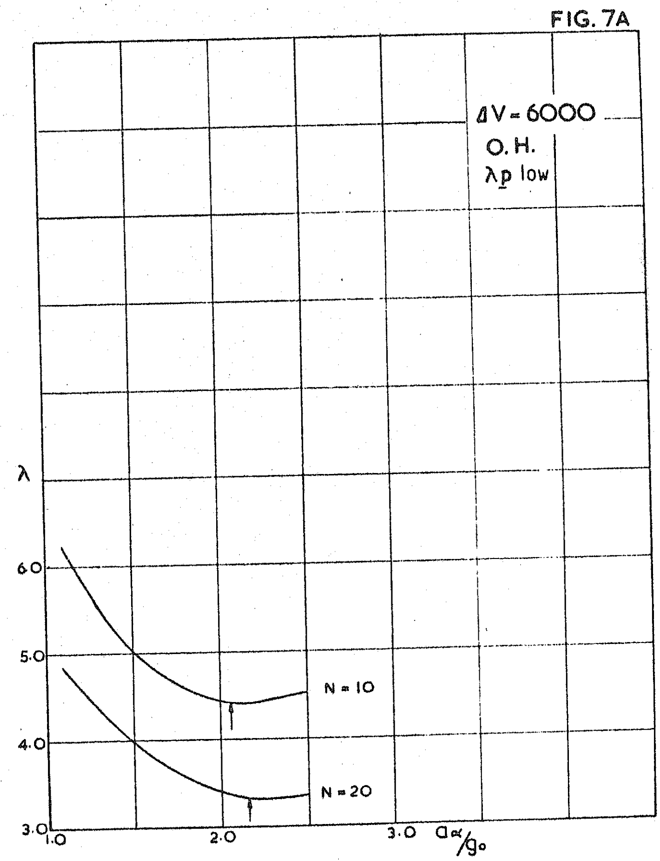

FIG. 8

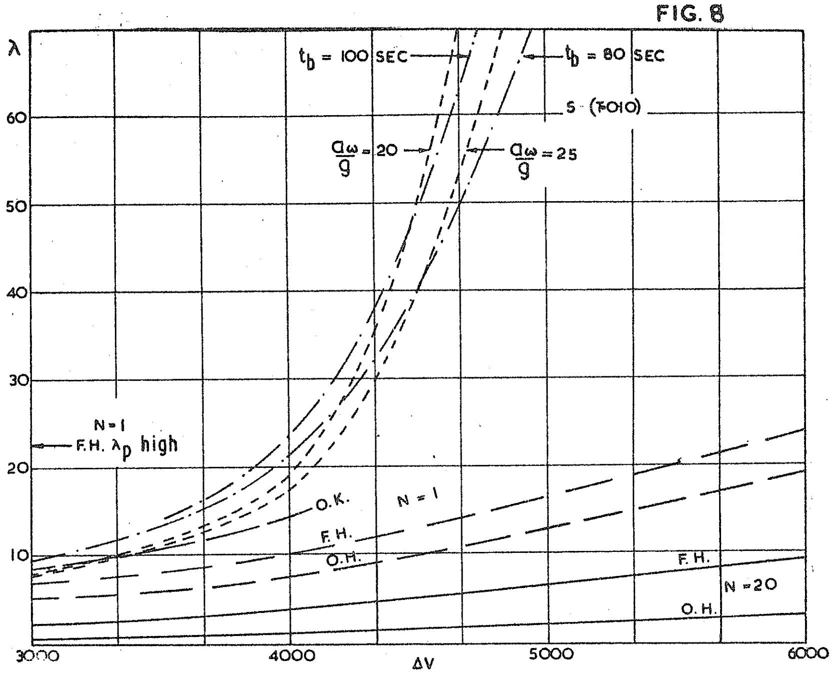

FIG. 9

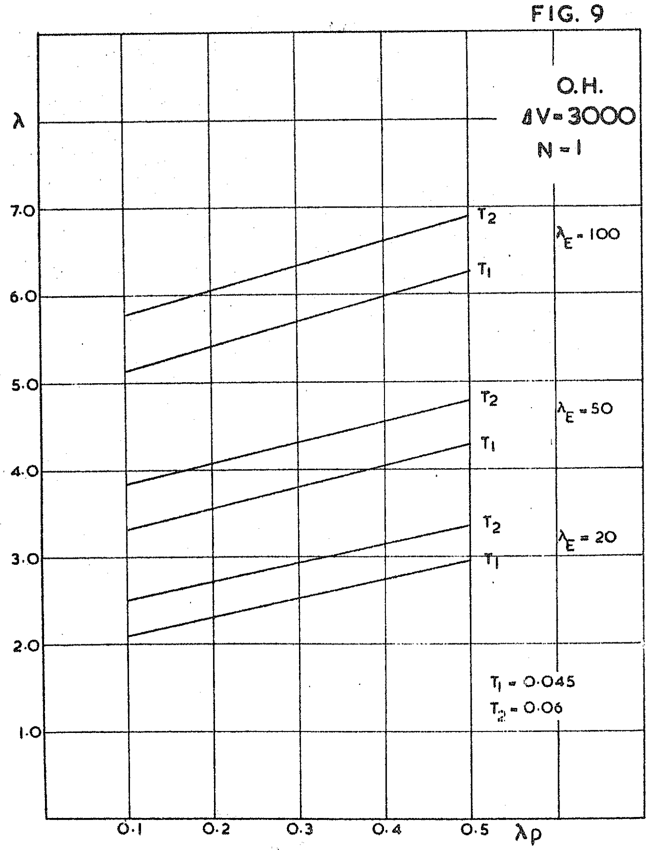

FIG. 10

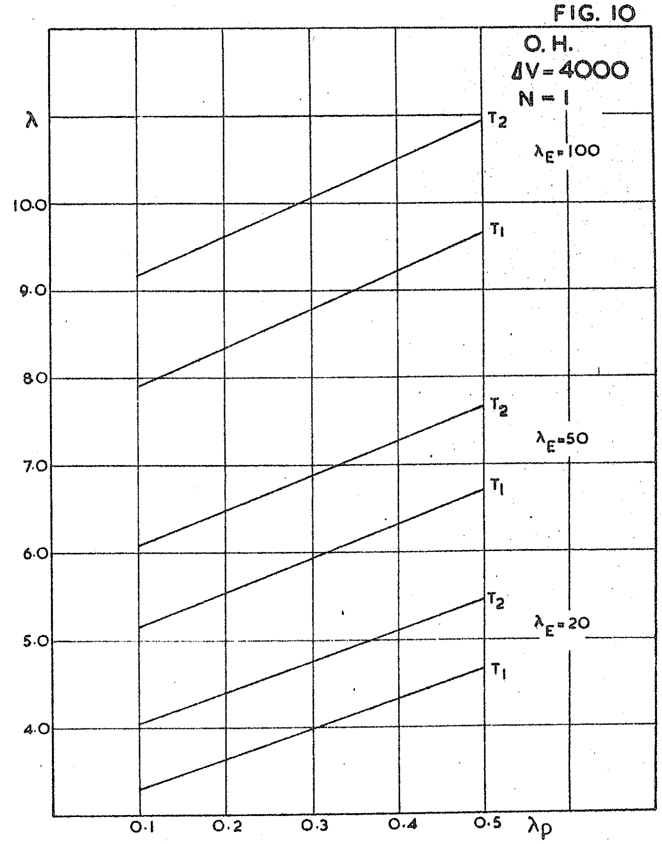

FIG. 11

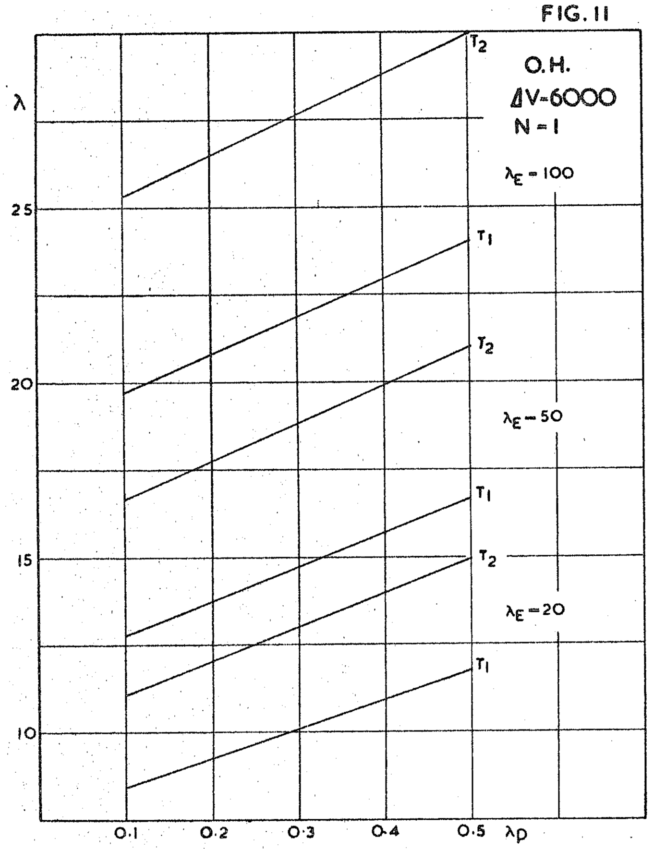

FIG. 12

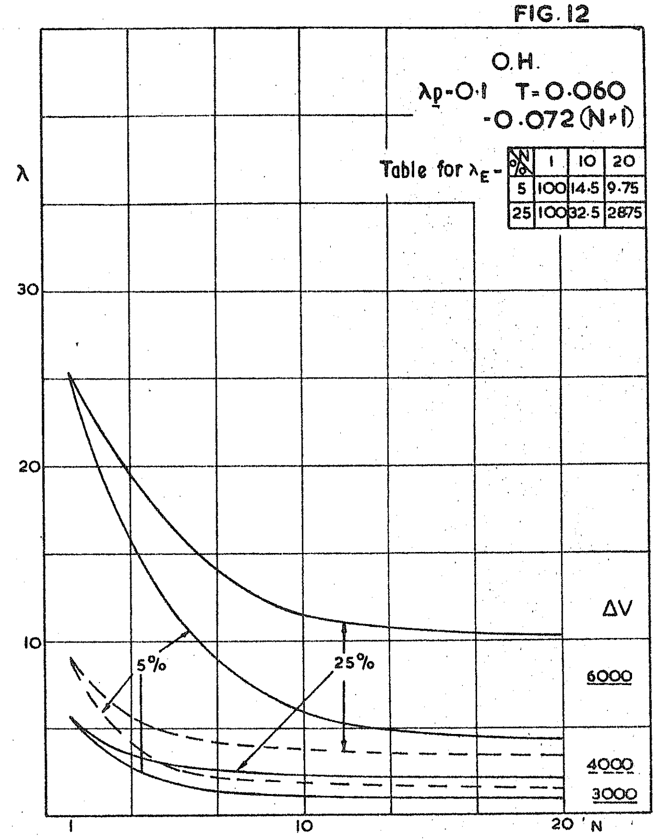

FIG. 13

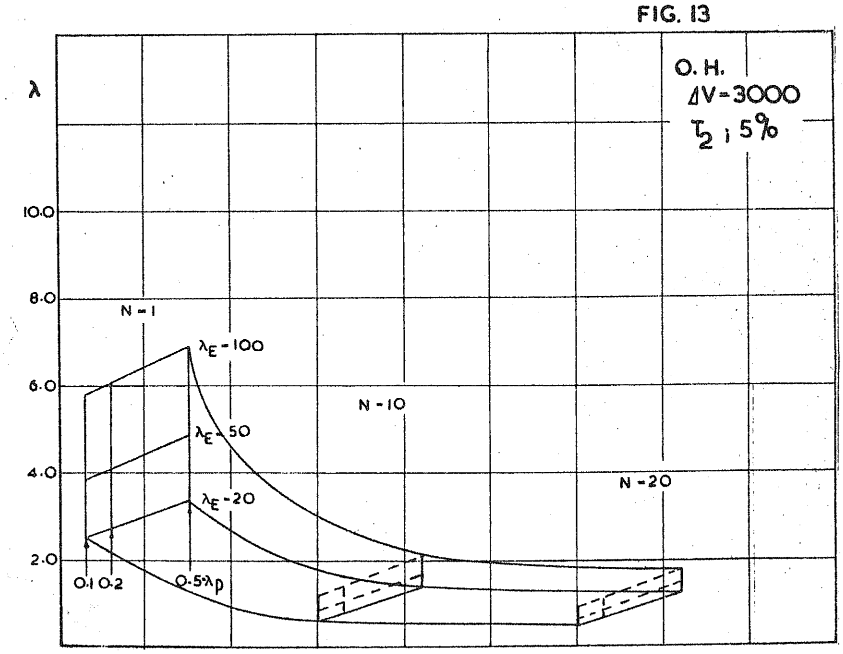

FIG. 14

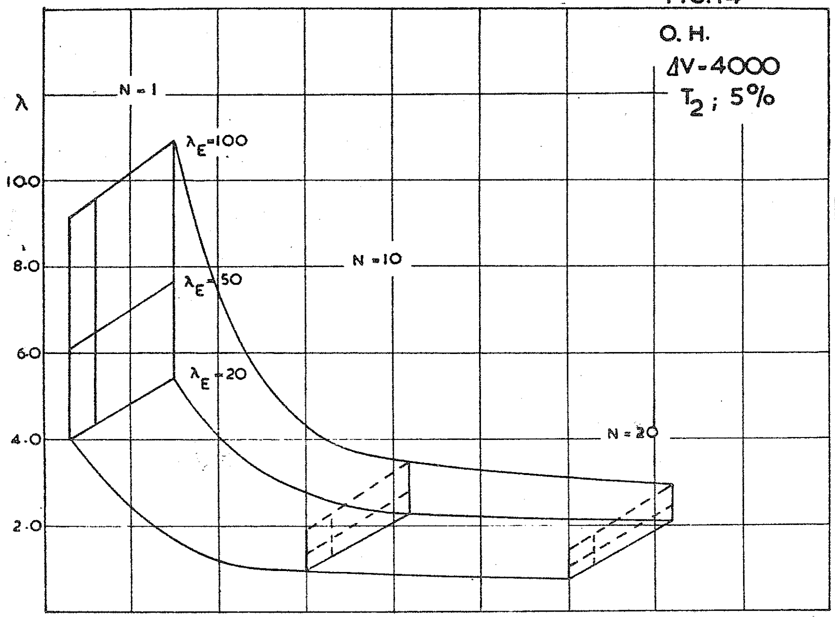

FIG. 15

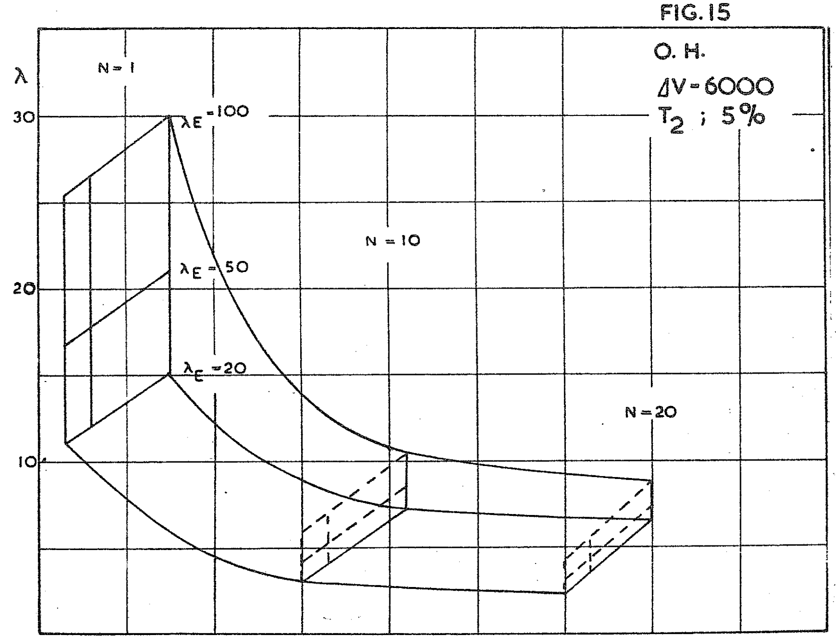

FIG. 16