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Electrical Power Grid Network Optimisation by Evolutionary Computing

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Abstract

A major factor in the consideration of an electrical power network of the scale of a national grid is the calculation of power flow and in particular, optimal power flow. This paper considers such a network, in which distributed generation is used, and examines how the network can be optimized, in terms of transmission line capacity, in order to obtain optimal or at least high-performing configurations, using multi-objective optimisation by evolutionary computing methods.

Keywords: Multi-objective, Optimisation, Evolutionary Computing, Electrical Power, Grid

1 Introduction

This work explores a possible method of addressing the configuration of large-scale electrical power networks, such as a national grid, using an approach based on evolutionary computing, which has been used previously in complex systems research such as emergent computation (Mitchell, 1999) and dynamics of complex networks (Aguilar-Hidalgo et al., 2012), and also directly in OPF research (Pandya and Joshi, 2008). As described by Allen *et al.*, (Allen et al., 2010), consideration of systems exhibiting complexity entails the construction of synergies between the studies of systems and their structures, and the ideas of neo-Darwinian evolutionary processes.

The essential problem in the architecture of national grid networks is that of power flow and optimal power flow (OPF) calculations of alternating current (AC) power, and these calculations are at the centre of Independent System Operator (ISO) power markets (Cain et al., 2013) in which AC OPF is solved over a number of different orders of magnitude of timescales, from minutes via hours, to annually and multi-year horizons, where the latter is for planning and investment while the former are for ensuring demand is met and for spot market pricing. The ISO produces and acquires load forecasts, receives offers of power from generating companies acting within a competitive auction market, and produces generation schedules consisting of required power units and a price, to meet demand within the constraints of the grid and generators.

Electrical power networks can be improved both technically and economically through the inclusion of distributed generation (DG) which may include renewable energy sources. DG units are lower output generators that provide incremental capacity at specific geographical locations, thus enhancing voltage support and improving network reliability while also acting economically as a hedge against a high price of centrally produced power, through locational marginal pricing (LMP). The operation of grids by ISOs as unbundled auction wholesale spot power markets that support real-time pricing provides a further incentive to roll-out DG, thus arises the need to define the type, number and location of extra DG units (Gautam and Mithulananthan, 2007).

The work presented here addresses the composition of a DG AC electrical power network based upon the IEEE 30 Bus Test Case which represents a portion of the American Electric Power System (in the Midwestern US) in December 1961, and which was downloaded from (Christie, 1993). This network, as shown in Figure 1, is amended to have six central fixed large-scale open cycle gas turbine (OCGT) electrical power stations, and twenty four variable distributed generators, powered either by renewable energy sources, being solar photovoltaic (PV) or micro-wind turbine, or by micro gas turbine. In particular, this work uses historical data of weather (in the form of actual solar PV and wind power generation), central power generation, and electrical energy demands, from Australia of 2010, thus providing a realistic simulation environment for both demand and renewable generation.

This work continues the investigation of optimising power networks by Oliver *et al.* (Oliver *et al.*, Expected July 2014), by looking at the power capacity of transmission lines, as well as considering the number and types of DG unit used in the network. In this way, the network connections undergo an optimising process, as well as the nodes (the buses) comprising the network. The aims are then to determine the composition of the power network in terms of the type, number and location of the non-central DG units, allowing transmission line power capacity to become further variables within the optimisation, with the goal of finding the smallest capital cost in meeting the demand for power, while keeping over- and under-production of power as low as possible, and of minimizing the average spot price and CO₂ emissions.

2 Background

The Plexos tool (Energy Exemplar Pty Ltd, 2013) is incorporated to provide both OPF and financial market simulations, in particular providing unit commitment (which generators should be used, bearing in mind their operating characteristics such as ramp-up time as well as power output and running costs), economic dispatch (which generators to use to meet demand from a cost viewpoint), transmission analyses (losses, congestion), and spot market operation. It also provides estimations of CO₂ emissions. The volume of lost load (VoLL) is the threshold price above which loads prefer to switch off, while the dump energy price is that below which generators prefer to switch off, and these along with market auctions also contribute to the ratio of power generated to power consumed. Transmission losses are also taken into account within Plexos through sequential linear programming.

Plexos is integrated with a multi-objective optimizing evolutionary algorithm (MOOEA) (Oliver *et al.*, 2013), thus establishing an optimization feedback loop, since Plexos gives optimal unit commitment for a given set of DG units, while the MOOEA is used to determine the optimal set of generators for the given demand profile and weather pattern. A MOOEA is used as they have a history of tackling non-linear (Nicolis, 1995) multi-objective and multi-dimensional optimization problems successfully, and since OPF for AC power is a non-linear problem while power markets require multi-part non-linear pricing. In the model used here, there are seventy two parameters that constitute the design vector applicable to each candidate solution, represented as one individual in the MOOEA, thus the problem is both non-linear and multi-dimensional. The simulation has a horizon of

one calendar year, represented as 365 steps of 1 day increments with a resolution to 30 minutes, from 01-Jan-2010.

A MOOEA (Deb, 2001) is generally a heuristic, stochastic means of searching very large non-linear decision or objective spaces in order to attempt to obtain (near) optimal or high-performing solutions (Jones et al., 2002) for problems upon which classical optimization methods do not perform well. EAs are characterized by populations of potential solutions that converge towards local or global optima through evolution by algorithmic selection as inspired by neo-Darwinian (Coello Coello, 2006) evolutionary processes. An initial population of random solutions is created and through the evaluation of their fitnesses for selection for reproduction, and by the introduction of variation through mutation and recombination (crossover), the solutions are able to evolve towards the optima. MOO produces a set of trade-off solution points (Fonseca and Fleming, 1995) since all objectives are optimised simultaneously, giving rise to individuals that cannot be improved upon in one OF dimension without being degraded in another. When each remaining solution in the population cannot be said to be better than any other in *all* OF dimensions, they are called non-dominated and are members of the local Pareto-optimal set, and are all of equal value and potential interest to the researcher. The non-dominated set of the entire feasible search space is the global Pareto-optimal set (Deb, 2001).

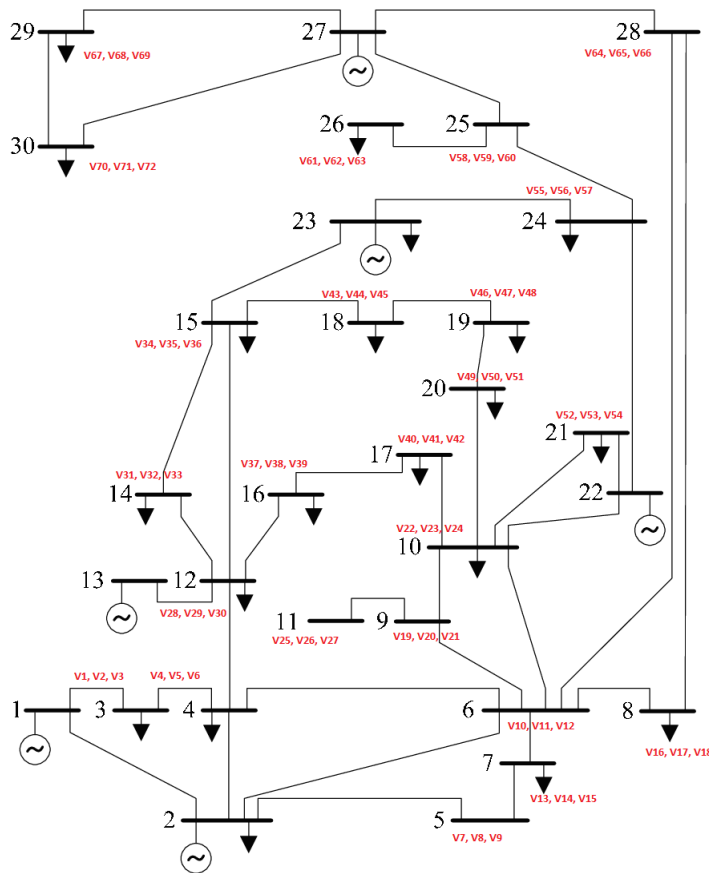


Figure 1. The IEEE 30-bus test system in single line diagram style, showing the location of DG units by bus, where the V-number is the variable for the number of units of the given DG type at that bus. See also Table 1.

3 Method

The MOOEA used here is a multi-objective optimizing genetic algorithm that self-adapts its control parameters, implemented in Java (Oliver et al., Expected July 2014), where the term self-adaptive is used in the sense of Eiben *et al.* (Eiben et al., 2006) following on from the work of Bäck (Bäck, 1992), to indicate control parameters that are encoded in the internal representation of each candidate solution along with the problem definition parameters applying to the objective functions (the *main* parameters), and that these control parameters are subject to change along with the *main* parameters due to mutation and crossover. This is different from a purely *adaptive* control parameter strategy as in that case the change is instigated algorithmically by some feedback at the higher level of the GA rather than the lower level of each chromosome/solution in the population. The *deterministic* approach is rule-based and is not considered adaptive.

The Plexos tool is used as the source of the values of the objective functions that are evaluated and selected for, that is to say, the fitness indicators, by the MOOEA, as depicted in Figure 2.

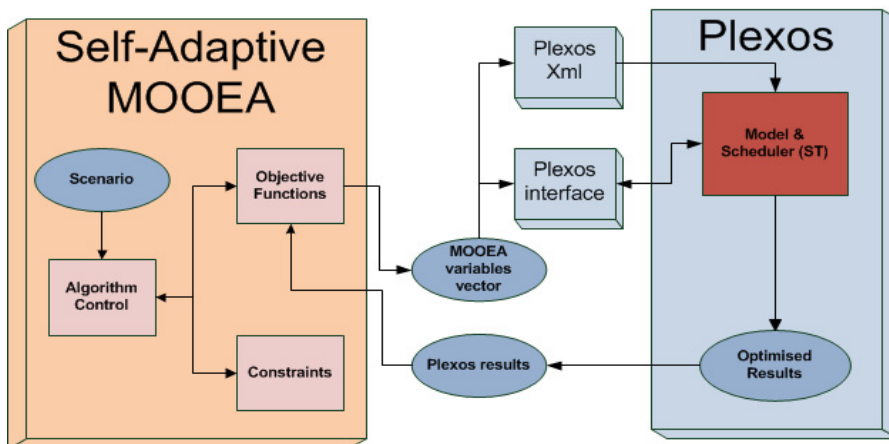


Figure 2. The integration of Plexos with the self-adaptive multi-objective optimisation algorithm.

The problem is defined as a set of potential DG units each of which may or may not be located at a given node (bus). The DG units are defined as (i) micro-gas turbine (ii) Wind turbine and (iii) Solar photovoltaic, where a unit of value 0 means the generator is not present at the location. The scenario allows for up to 5 units of each type to be located at any of the nodes defined as variable in the network diagram (Figure 1), which means that it is any except for the nodes 1, 2, 13, 22, 23 and 27, as these are the large fixed central OCGT power stations. Each transmission line between any two buses has a maximum flow capacity stated in megawatts (MW). The transmission line capacities are amended in the Plexos Xml model file which are sent to Plexos for each solution run.

The labels shown as V_n at the given nodes indicate the design variable number that defines the number of units of the given generator types at that bus, and as can be seen, each of the 3 variable types can be present potentially. As there are 24 nodes at which variable DG units can be located and 3 types of generator, the design vector of each candidate solution therefore consists of 72 variables. A candidate solution is therefore a vector of n decision variables: $= (x_1, x_2, \dots, x_n)$, where $n = 72$. This configuration thus allows a solution to have from 0 DG units up to a theoretical 360 (being 5 units of each of 3 DG types at the 24 nodes). Table 1 below shows the allocation of DG units by type to nodes, cross-referenced to its variable number (as shown in Figure 1), with the assumption that a given generator feeds in to one associated node only.

There are 4 objective functions defined, all of which are to be minimised simultaneously and the values for all of which come from Plexos, these being:

$min F(\sum u) = \text{sumU}$	Equation 1
$min F(\text{useDump}) = \text{useDump} $	Equation 2
$min F(\text{spotPrice}) = \mu_{\text{spotPrice}}$	Equation 3
$min F(\text{CO2em}) = \text{CO2em}$	Equation 4

in which the values represent respectively:

- i. The total number of DG units
- ii. The USE/DUMP energy (MWh)
- iii. Spot Price (\$/MWh)
- iv. CO₂ emissions (Kg)

Considering the values above, useDump, depending whether it is negative or positive, is either the un-served amount of energy due to under-production or the dump energy due to over-production, relative to demand. The spot price is the mean price achieved in the simulated market auctions over the course of the simulation in Plexos.

A hard constraint on the total number of DG units deployed, u , is applied in Equation 5, in order to investigate how the system transforms itself. Without such a constraint, which can be viewed as a limit to financial resources available as investment into DG, we would perhaps expect the system to maximize DG deployment as this provides a known benefit where cost is the only downside, and this would hide the effects that placement may have when otherwise. It is the number of DG units (and their placement) that is particularly of interest in these studies, and having the objective function for the total DG units is important as it ensures diversity in sumU, enabling plots such as Figure 5 to be possible. The intention of this rather low constraint for this case is to encourage the optimisation to find the best locations for the extra DG units, rather than simply adding more units overall, to better illustrate the potential of the method.

$$\sum_{i=1}^{72} u_i \leq 35 \tag{Equation 5}$$

The candidate solutions chosen by the MOOEA, using the results from Plexos, are thus selected due to the effect their chosen DG units have on the electrical network due to their operating characteristics and where they feed into the network, defined in the topology as shown in Figure 1.

The MOOEA, as described at the start of section 3 above, allows each new experiment to override its default initializer which creates an initial population of candidate solutions by generating variables under a uniform random distribution regime within the ranges of the defined variables, in this case $0 \leq u \leq 5$. The initializer used instead generates solutions that meet the hard constraint, by selecting for each solution a random value between 0 and the constraint, 35, and using this as the limit for that candidate solution. Each variable of that solution is then selected randomly, and is allocated a random value within its range, until the solution’s own limit is reached. In this way, solutions in the initial population will vary between 0 DG units and 35 with a uniform distribution.

In subsequent generations, solutions will evolve that may break the hard constraint, due to mutation and recombination operators acting on ‘fit’ parent solutions selected for breeding, and in this case the solutions will be retained in the population but repaired. Repairing in this context means that a failing solution’s vector of DG variables is changed until it falls within the constraint, by randomly choosing one of the variables, decrementing its DG unit count (when it has $u \geq 1$), and then repeating the process until the total falls within the constraint.

The MOOEA is configured to have a mixed *chromosome* consisting of a vector of 72 integers, for the DG *genes*, one per bus, with the self-adaptive control parameters encoded as real numbers. In addition, another 41 *genes* each contain the line maximum flow capacity (LC), in MW, of a given transmission line. In the first optimisation defined, the LC genes are fixed but one is changed to a new value, while in the following optimisation, all 41 are enabled to evolve. There is a fixed population of size 30, allowing 0 duplicate solutions in any single generation, with initial crossover and mutation probabilities of 0.9 and 0.009 ($\approx 1/(72+41)$) respectively. The MOOEA is allowed to run for 2,000 function evaluations (67 generations), with each generation taking around 3 hours elapsed time.

Table 1. Buses, their Variables and DG unit types

Micro-Gas		Micro-Wind		Solar PV	
Node	Var	Node	Var	Node	Var
n03	V01	n03	V02	n03	V03
n04	V04	n04	V05	n04	V06
n05	V07	n05	V08	n05	V09
n06	V10	n06	V11	n06	V12
n07	V13	n07	V14	n07	V15
n08	V16	n08	V17	n08	V18
n09	V19	n09	V20	n09	V21
n10	V22	n10	V23	n10	V24
n11	V25	n11	V26	n11	V27
n12	V28	n12	V29	n12	V30
n14	V31	n14	V32	n14	V33
n15	V34	n15	V35	n15	V36
n16	V37	n16	V38	n16	V39
n17	V40	n17	V41	n17	V42
n18	V43	n18	V44	n18	V45
n19	V46	n19	V47	n19	V48
n20	V49	n20	V50	n20	V51
n21	V52	n21	V53	n21	V54
n24	V55	n24	V56	n24	V57
n25	V58	n25	V59	n25	V60
n26	V61	n26	V62	n26	V63
n28	V64	n28	V65	n28	V66
n29	V67	n29	V68	n29	V69
n30	V70	n30	V71	n30	V72

4 Results

Some figures below show results in the form of parallel coordinates (\parallel -coords), the technique introduced and promulgated originally by Inselberg (Inselberg, 2009), and later used in the field of optimisation by (Fleming et al., 2005), (Siirtola, 2000), (Siirtola and R  ih  , 2006), and engineering design (Kipouros et al., 2008) and (Kipouros et al., 2013), in which each dimension is oriented parallel to the others, thus transforming an n -dimensional point into a 2-dimensional polygonal line that relates the values in each dimension. This technique enables highly multi-dimensional data to be plotted uniquely and without loss of information, and in these cases the whole design space of each solution, 72 variables, are plotted alongside their 4 objective function results. These plots were produced using the Parallax tool (Avidan and Avidan, 1999).

As a first experiment, the maximum flow capacity of just one line is altered and the results compared with a previous run in which all aspects are the same, including the seed for the pseudo-random number generator, except for the line capacity. In this case, line 11 is chosen, being that between the most highly connected bus, node 6, and node 9 which has less than half the connections,

and for which the line capacity is a low-ish 65 MW. The line’s capacity is doubled to 130 MW, a figure used by other transmission lines in the network, in the new network definition. The new results, for the higher line capacity, are termed R008 and the previous with original line capacity, R003.

The plot in Figure 3 shows the entire 72-variable set and the objective functions for the new result set (termed R008) with the higher line 11 capacity. This has some variables as always 0, hence these can be said to be of no relevance to further optimisation runs, allowing them to be removed in future, in order to improve optimisation performance.

The results of the objective function minimisations appear in Table 2, although sumU (the total number of DG units used) is not listed as this is always between 0 and 35, given the hard constraint. It can be seen that just changing the one line capacity from 65 MW to 130 MW improves each OF result.

Table 2. New and previous best objective function results for the first experiment

OF result	R008	R003
useDump	260.00	300.73
Spotprice	21.22	21.67
CO ₂	1,346,914.25	1,348,057.25

The plot in Figure 3 shows that the variable v11, which contains the number of units of Wind DG for node 6, when having the value 5, is on the many highly performing solutions, including the best solution of all. The R003 results shown in Figure 4 in a similar fashion to Figure 3, seem to indicate that the reasons for the improved performance in R008, is that the number of DG units for node 9 are no longer so important as variables v19 (node 9, Gas) and v20 (node 9, Wind) are no longer on the optimum path in R008, while for R003 both are at maximum (5). R008 also has fewer variables at 0, which seems to suggest the network load may be better balanced too. The scatter plot of Figure 5 shows the variation of the mean spot price against the total number of DG units (sumU), with the most converged points manually selected, and in Figure 6, the subset of those selected points in which v11 has 5 DG units, are highlighted.

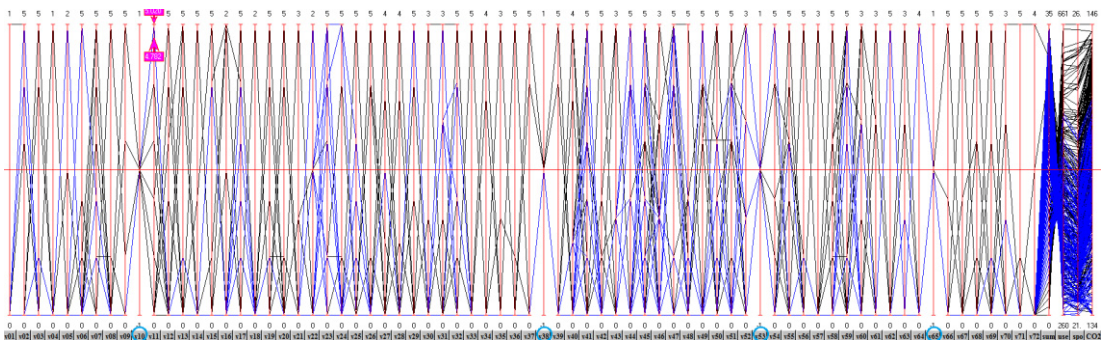


Figure 3. ||-coords plot for R008 showing all 72 variables and 4 OFs, with selection of results in which v11 has 5 units, and circled variables always 0 (v10, v38, v53, and v65).

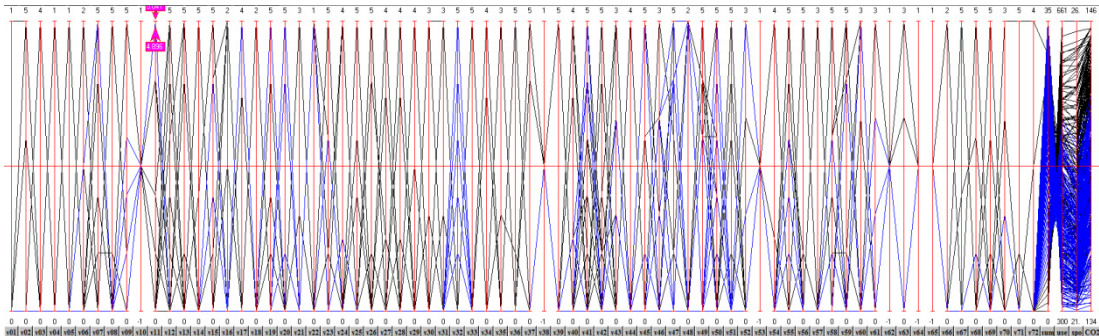


Figure 4. ||-coords plot for R003 showing all 72 variables and 4 OFs, with selection of results in which v11 has 5 units.

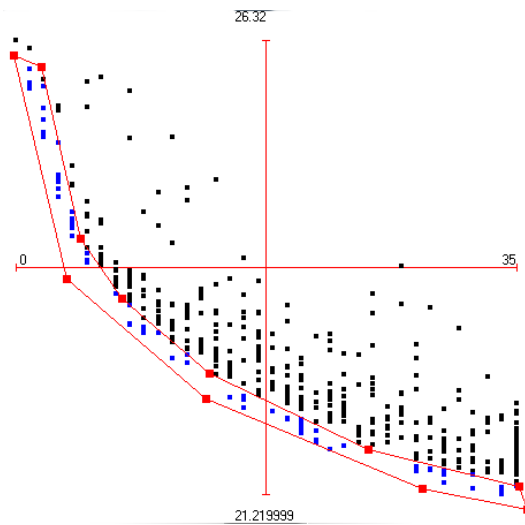


Figure 5. A scatter plot for R008, showing sumU on x-axis against spotPrice on y-axis, with the most converged points selected by hand using the polygon tool of Parallax.

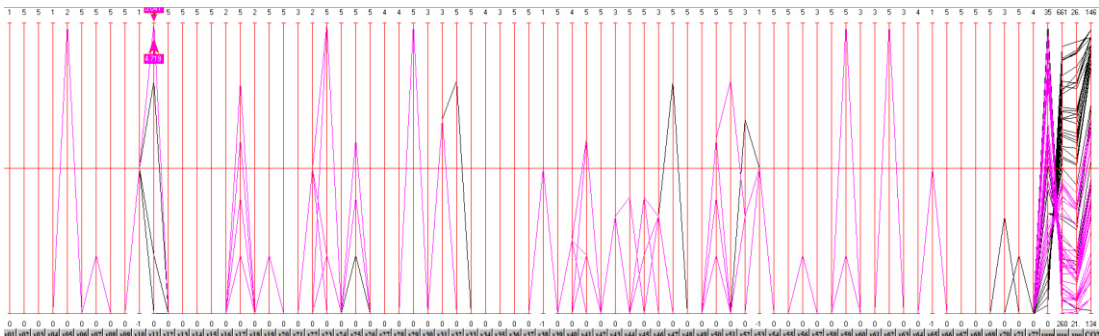


Figure 6. The set of points selected in Figure 5 are shown here with only those that have v11=5 selected, resulting in two apparent clusters, the lower set being the best performing.

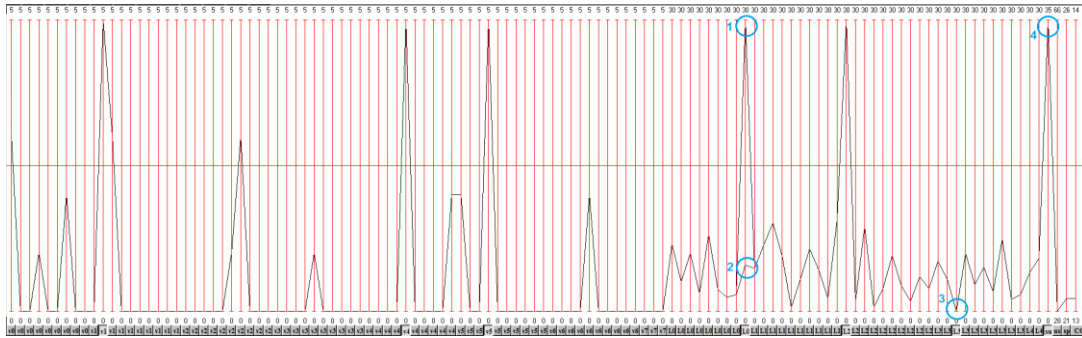


Figure 7. ||-coords plot in which all lines' capacities are allowed to evolve, showing the isolated solution having the best result for the useDump objective. This plot shows the 72 DG unit values and the 41 line capacity values, along with the 4 OF results.

A subsequent optimization was tried in which the maximum flow capacities of all lines were allowed to evolve along with the DG units. In this case, the 41 line capacity (LC) genes were initialised following a Gaussian distribution using the mean and standard deviation from the first optimisation, with limits applied for a minimum of 4 MW, a maximum of 300 MW. An additional hard constraint was applied on the total flow capacity, being equal to the original plus 20%. Figure 7 shows the isolated solution having the best result for the useDump objective with points of interest circled and numbered. Points 1 and 2 show that line 9 (between buses 6 and 7) has both a low (47MW) and high (300 MW) value for the same high-performing solution, indicating that it is not a critical path. Its original LC was set to 130 MW, which suggests that this could be optimised down to the lower value found, although this would need to be further explored. Point 3 shows line 32 (between buses 23 and 24) has evolved down to zero (from 16 in the datum design), indicating that this line might be able to be eliminated entirely. Point 4 shows that the total of DG units was 35, as would be expected in a high-performing solution, given the constraint on the total number of units allowed. Figure 8 below shows the solutions selected by having line 20 at 300 MW, which are all high-performing and some of which are the best performing for useDump and CO₂ as indicated by the brace at the bottom right of the image. Line 20 runs between buses 14 and 15.

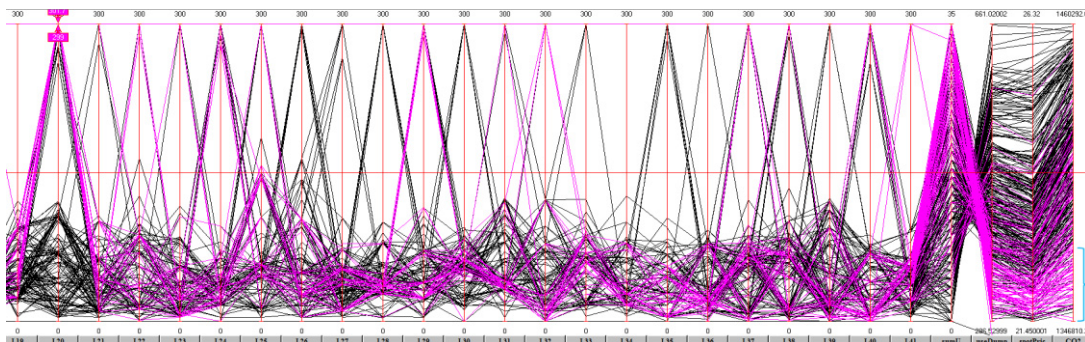


Figure 8. ||-coords plot in which Line 20 is selected for values of 300 MW, showing that these solutions are highly performing (indicated by the brace at the bottom right).

5 Conclusion

It has been shown that this methodology, using the MOOEA with Plexos and examining the results with a multi-dimensional visualisation, can be used to assist in the design of network topologies from the perspective of transmission line maximum power flow capacities, by allowing the optimisation process to determine the maximum flow capacities along with the types and locations of DG units. It also shows that this method could be used to assist in the determination of network topology from a bus-to-bus connection perspective, through elucidation of at least best and worst lines for transmission and therefore connectivity. It should be remembered that these results relate to particular weather patterns for a region in which this model power grid is imposed, and that the DG unit placement is realistic in that regard, considering micro-wind turbines and solar pv units.

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