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DUCTILITY, ITS MEASUREMENT AND RELEVANCE TO SHEET METAL FORMING OPERATIONS

by

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In sheet metal forming operations failure is identified by the onset of instability, which manifests itself as either splitting or buckling. An approximate analysis of the sheet metal forming process, and a study of formed industrial components suggests that the deformation which produces these complex parts is compounded from two 'pure' stress components, namely, plane stress and biaxial stress, predominantly, and thus the material parameters which are relevant to these modes must be determined and measured. The plane strain produced by the former is often of the form where circumferential strain is zero (Figure 1) and so the material properties in the through-thickness direction are important. For best performance here the flow strength in this direction must be high relative to the load required to draw in more metal and it is here that the strain ratio r, the ratio of width strain to thickness strain, measured at a prescribed elongation in the tensile test, is of great importance.

In the latter (deformation under a biaxial stress state) the position is not so clear. It is reasonable to assume that, in an operation exemplified by stretch-forming or bulging, (Figure 2) an elongation parameter would relate to the performance of the material, but unfortunately correlation between elongation parameters and these modes is poor(1), compared with that between r and ldr(2). The reason for this is twofold. Firstly, that failure by plane strain has a fairly precise meaning, and secondly, that r, by definition is measured in a stable part of the stress-strain curve. In the case of failure by biaxial stress, firstly, the stress ratio can vary from $x = \frac{1}{2}$ to $x = 1$, and secondly measurement of a parameter at instability is being attempted. Both these factors will tend to make correlations more diffuse.

Ideally, what seems to be required is a measurement of the ductility at instability under the stress system obtaining in the critical region of the component, but as the range of conditions is so wide, and the tensile test so popular, it has been the custom to measure ductility in uniaxial tension and from this attempt to predict complex stretching behaviour(3,4). Is this an incorrect procedure for the true strain at instability changes with x?

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One of the empirical equations for the stress-strain curve of a ductile metal is :

$$\sigma = K(B + \bar{\varepsilon})^n \tag{1}$$

suggested by Swift[5]. K, $\bar{\sigma}$ and $\bar{\varepsilon}$ and n have the usual meanings while B indicates the initial state of the material. This is shown in Figure 3. Differentiating this equation gives

$$\frac{d\bar{\sigma}}{d\bar{\varepsilon}} = \frac{\sigma}{z}$$

where $$z = \frac{B+\bar{\varepsilon}}{n}$$

The true instability stress and strain are defined as the stress and strain where

$$\frac{d\bar{\sigma}}{d\bar{\varepsilon}} = \frac{\bar{\sigma}}{z}$$

and so can be found by drawing a tangent to the stress-strain curve such that the length of the sub-tangent is z. This is also shown in Figure 3.

What is the relationship between these various parameters? The variation of critical subtangent with stress ratio is shown in Figure 14 and the relationship between critical subtangent and ε at instability is shown in Figure 15 for n = 0.2 and critical subtangent values of 1, 1.155, 1.8 and 2. The relationship is linear and must be so for all values of n. Bearing in mind the assumptions which have to be made, it is not unreasonable to use ε at instability in uniaxial tension as a measure of the behaviour of material in certain other more complex strain situations.

Now take the example of a circular diaphragm under internal pressure. Here, ε at instability is:

$0.727n + 0.364$ Obviously this will also give a linear relationship with ε at instability in uniaxial tension.

The problem then, with materials possessing σ, ε curves which can be closely approximated to equation (1), is the practical one of measuring ε at instability.

The construction shown in Figure 3 is theoretically sound, but in practice difficult, for the σ, ε curve tends to linearity after instability and so the tangential point becomes diffuse. The most popular approach at present is as follows: using a different empirical relationship:

If $\sigma = K\varepsilon^n$

then, a plot of $\log\sigma$ vs. $\log\varepsilon$ is linear, of slope n.

As $n = \varepsilon$ at instability,

Then the slope of the curve = ε

There are various ways of producing σ ε curves already reported, and it is unnecessary to enumerate them again.

However, ε is related to the engineering strain e as follows:

$$\varepsilon = \ln(1 + e)$$

and so it is possible to measure e in a tensile test and calculate n. However, there seems little point in calculating n, when e at instability will do as well, as long as measurement is possible. Many people measuring elongation for control purposes use uniform elongation as the ductility parameter.

Direct measurement of e_u need not be used for materials exhibiting $\sigma = K\varepsilon^n$ - type stress strain curves. Nelson and Winlock showed that if $\log\sigma$ vs. $\log\varepsilon$ curve was linear then:

$$n = \varepsilon_u = \frac{\log \sigma_u - \log \sigma_2}{\log \varepsilon_u - \log \varepsilon_2}$$

when σ_2 and ε_2 are the stress and strain at an arbitrary point below maximum load. This reduces to:

$$\frac{\sigma_u}{\sigma_2} = \left(\frac{\varepsilon_u}{\varepsilon_2}\right)^{\varepsilon u}$$

and substituting

$$\frac{P_u(1+e_u)}{A_o}$$, where A_o is the original cross sectional area of the testpiece, for σ_u, and similarly for σ_2, and $\ln(1+e_u)$ for ε_u, etc., then

$$\frac{P_u}{P_2} = \left[\frac{1+e_2}{1+e_u} \; \frac{\ln(1+e_u)}{\ln(1+e_2)}\right]^{\ln(1+e_u)}$$

From this equation curves of the type shown in Figure 4 can be plotted, and e_u read off for a given P_u/P_2 and e_2. It can readily be seen that this, when standardised will give rapid results; subsequent work on deep drawing steels has shown that this method agrees well with the log-log plot, the former giving higher results by about 0.002. However, this accuracy is dependant on the fit of the σ,ε curve to $\sigma = K\varepsilon^n$.

The slope of the plastic curve is indicated by other parameters, for instance the ratio of yield load to ultimate load taken directly from the tensile testing machine. This can be a useful parameter for a known material; the relationship betwen Ys/Us and e_u for a number of edd steels is shown in Figure 5. Again assuming $\sigma = K\varepsilon^n$ and defining yield stress as a proof stress, i.e., stress at $e = 0.002$, then:

$$Ys/Us = \left(\frac{k}{n}\right)^n$$

and

$$\frac{1}{n}\ln\left(Ys/Us\right) = \ln k - \ln n$$

The relationship for the same steels is shown in Figure 6. However, Ys/Us could be a dangerous parameter, for obviously it is possible to have materials with the same Ys/Us but widely differing elongations, as shown diagrammatically in Figure 7. To define the curve uniquely a knowledge of the elongation is required, and so this exercise is not entirely useful.

A further advantage in measuring uniform elongation is its generality - no assumptions are necessary regarding the shape of the σ, ε curve, the elongation to maximum load is sufficient definition. Unfortunately, instability shows in different ways in different materials, and this is demonstrated in Figure 8 which shows load-elongation curves for a number of different materials, with uniform and total elongations indicated by the horizontal lines. In the case of steel, copper, aluminium and magnesium, the uniform elongation would seem to be a not too difficult parameter to measure and use. Zinc should not really be included here, for it is effectively hot-working at the strain rate used, but is put in for interest. In the case of zirconium, the 'unstable' elongation is so 'stable' that it seems certain that in a sheet metal forming operation it would contribute substantially to the production of the part. With stainless steel in uniaxial tension fracture occurs under increasing load and so uniform elongation is total elongation.

The practical problems involved in the measurement of uniform elongation directly from scribed or gridded test pieces must not be forgotten. Figure 9 shows the variation in uniform elongation in test pieces of varying length produced from annealed commercial-purity aluminium by conventional milling techniques. giving an accuracy in cross-sectional area of ± 0.0005". The variation in 'uniform' elongation within one test piece and the variation from test piece to test piece for specimens cut adjacently from a sheet is clear. Figure 10 shows a similar experiment, where after milling the test pieces have been electropolished before gridding and testing. A slight improvement can be observed. Figure 11 shows the effect of test piece width (specimens again electropolished) on uniform elongation.

All measurements were made over 0.2" gauge lenths at 0.1" intervals as indicated in Figure 12.

It seems clear that for really accurate elongation measurements a test piece accuracy far in excess of that normally demanded is essential.

To sum up at this stage. Real materials do not always conform to empirical stress-strain relations and so the use of the strain hardening exponent $\underline{r}$ is not entirely satisfactory. The measurement of uniform elongation presents difficulties if a range of metals is being studied, and the effect of small variations in test piece dimensions is startling.

Is it then possible to devise a satisfactory method of determining the ductility of a material at instability? A suggestion for further investigation is as follows:

If P and A are the current load and cross sectional area of a test piece, l the gauge length, and σ and ε the instantaneous stress and strain :

$$\sigma = \frac{P}{A}$$

a $\therefore$
$$\frac{d\sigma}{\sigma} = \frac{dP}{P} - \frac{dA}{A}$$

$$d\varepsilon = \frac{dl}{l}$$

$$= -\frac{dA}{A}, \text{ for, } Al = \text{constant}$$

$\therefore$
$$\frac{d\sigma}{\sigma} = \frac{dP}{P} + d\varepsilon$$

at instability $\frac{dP}{P} = 0$

$\therefore$
$$\frac{d\sigma}{d\varepsilon} = \sigma$$

Figure 9 can therefore be plotted, showing the intersection of the stress strain curve with its derivative which is the condition obtaining at instability. This method does not depend upon any empirical assumptions, and the relationship between the true strain at instability derived in this way and the behaviour of the material in other deformation modes will be as valid as before, but more general in its application.

If necessary the σ, ε; curve can be plotted by one of the current methods, while the derivative can be obtained most easily from a simple computer program. Ideally, the value of ε at instability can be obtained from values of load and elongation, again from the appropriate program. This is exemplified in Appendix 1.

Finally, the reason for the lack of correlation between stretch forming and bulging, and uniaxial elongation parameter must be clarified. Firstly, the fracture behaviour is quite different in the two cases and so the cup height will vary in an, at present, unknown way, based on the fracture strain developed. It is also reasonable to think that fracture strain will be more variable with punch- rather than with fluid - stretching. The fracture strain developed will naturally be related to the way in which necking occurs and this varies considerably with stress ratio. Keeler and Backofen [5] in their punch-stretching experiments postulate two types of necking, firstly diffuse necking, in which flow occurs broadly and symmetrically about the loading direction, and is encountered when:

$$z = \frac{4(1-x+x^2)^{3/2}}{(1+x)(4-7x+4x^2)}$$

The other, described as local necking, is a thin band of material inclined at a given angle α determined by x, with the critical subtangent:

$$z = \frac{2(1-x+x^2)^{\frac{1}{2}}}{1+x}$$

Additionally, it is generally a height measurement which is made for correlation purposes, and this will be related in a complex fashion to the strain distribution, especially as most stretch forming tests are, finally, non-hemispherical.

It is most important that the effect of anisotropy on the work hardening behaviour in biaxial situations be resolved. It is not difficult to demonstrate[6] the disagreement between anisotropic plasticity theory and crystallography in the extreme case of sheet single crystals, but the most urgent requirement at the present time is more information than we possess[7] showing where these two approaches are complementary or in agreement.

REFERENCES

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3. See ref. 1.

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5. Keeler, S., and Backofen, W.A., Trans. ASM, 1963, 56, 25.

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7. Wilson, D.V., JIM, 1966, 94, 84.

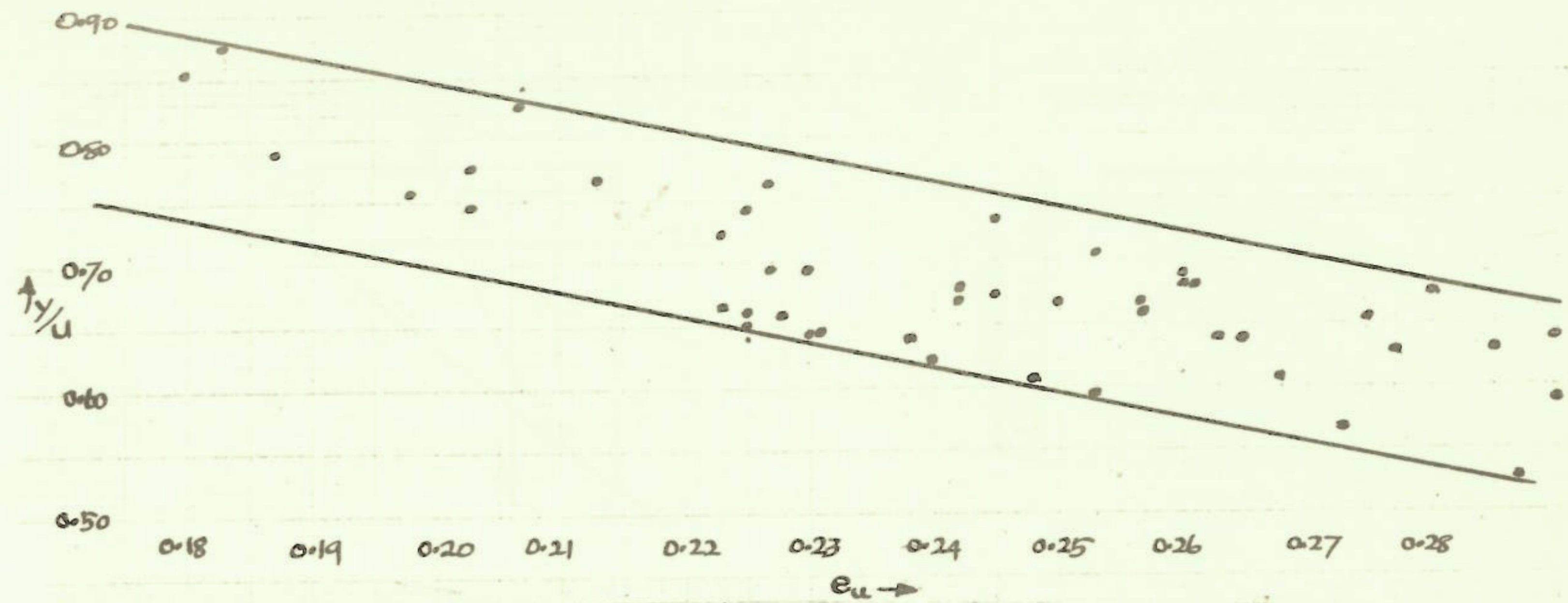

FIG. 5 RELATIONSHIP BETWEEN YS/US AND e_u FOR A NUMBER OF EDD STEELS

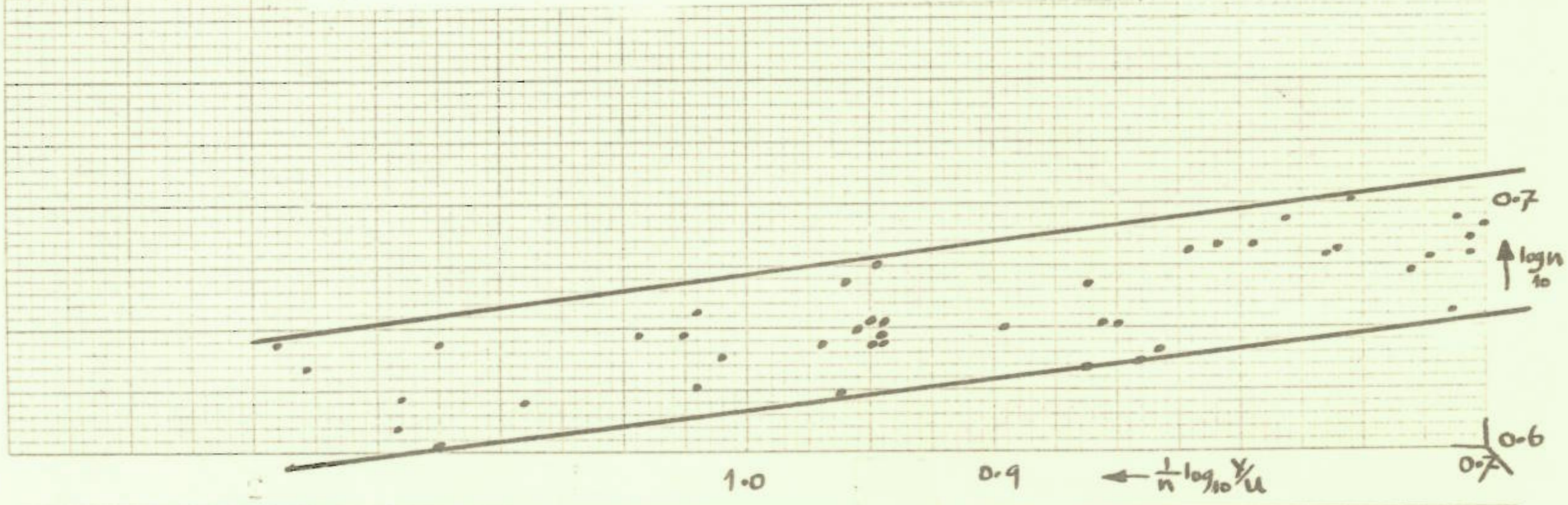

FIG. 6 RELATIONSHIP BETWEEN N AND YS/US FOR A NUMBER OF EDD STEELS

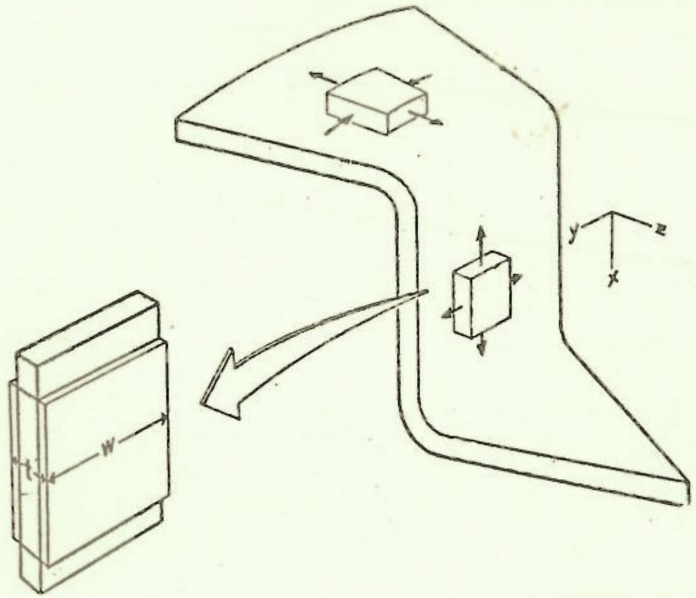

FIG. 1 ILLUSTRATION SHOWING $d\epsilon_y = 0$ FOR CUPDRAWING

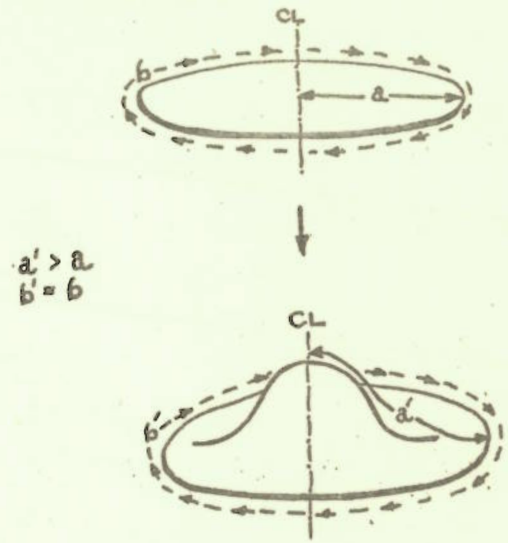

FIG. 2 A SIMPLE STRETCH-FORMING OPERATION

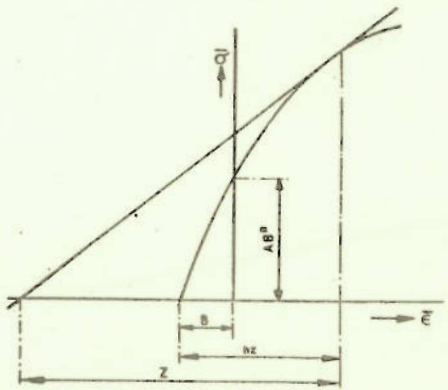

FIG. 3 GENERALISED STRESS-STRAIN CURVE

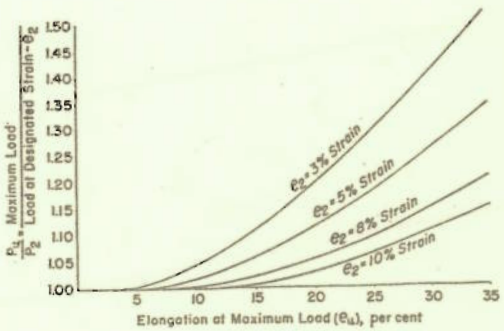

FIG. 4 CURVES FOR READING e_u FROM THE RATIO OF TWO LOADS

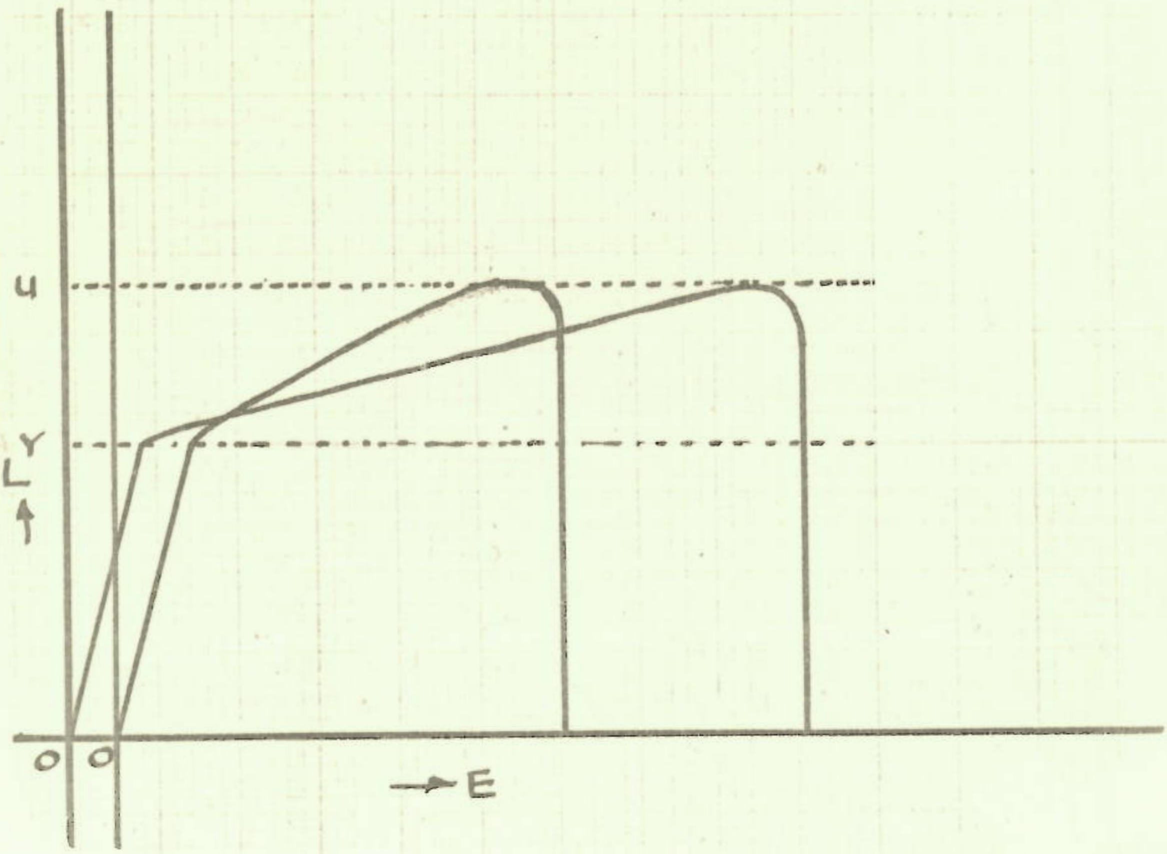

FIG. 7 HYPOTHETICAL LOAD-EXTENSION CURVES SHOWING SIMILAR YS/US BUT DIFFERENT ELONGATIONS

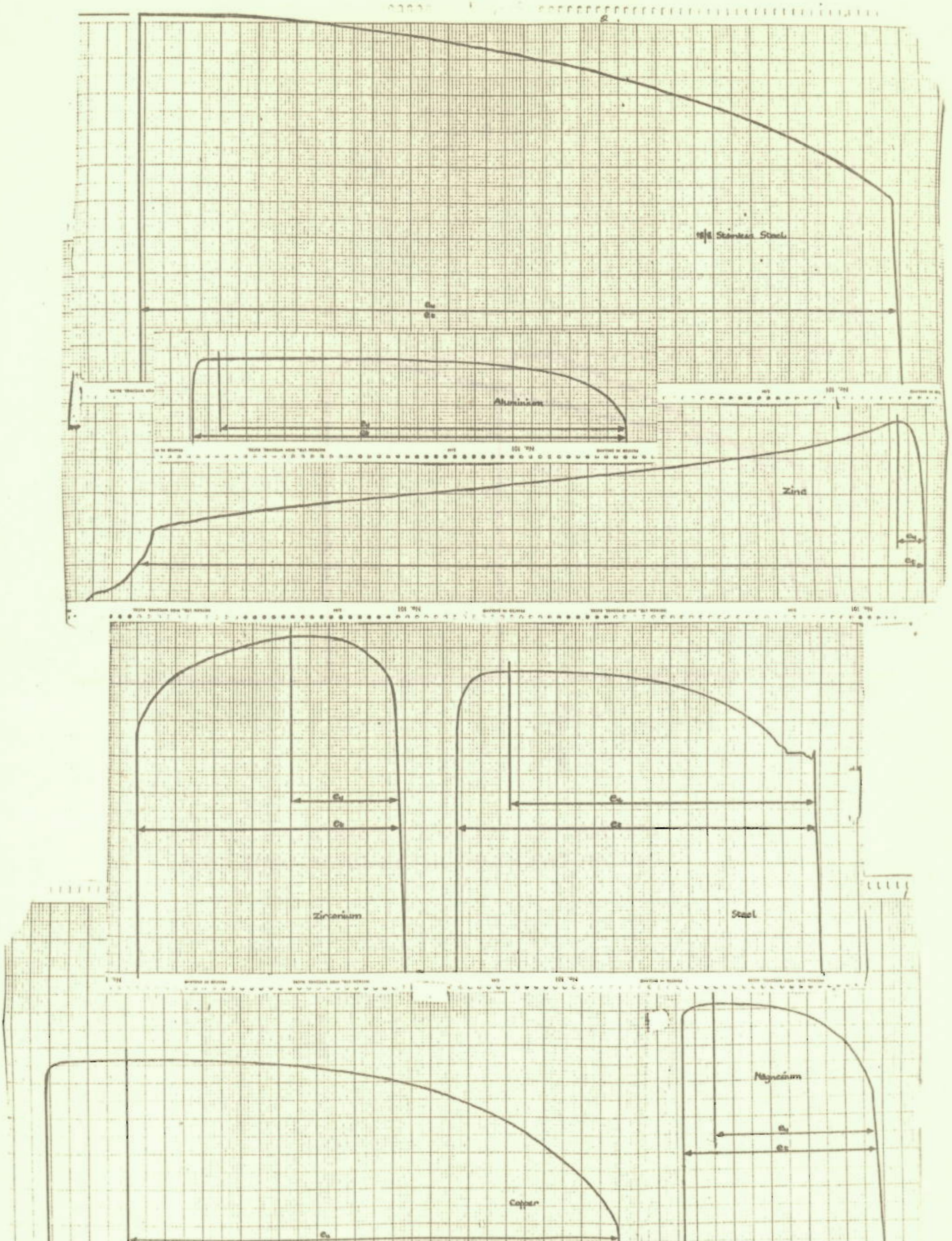

FIG. 8 LOAD EXTENSION CURVES FOR DIFFERENT METALS

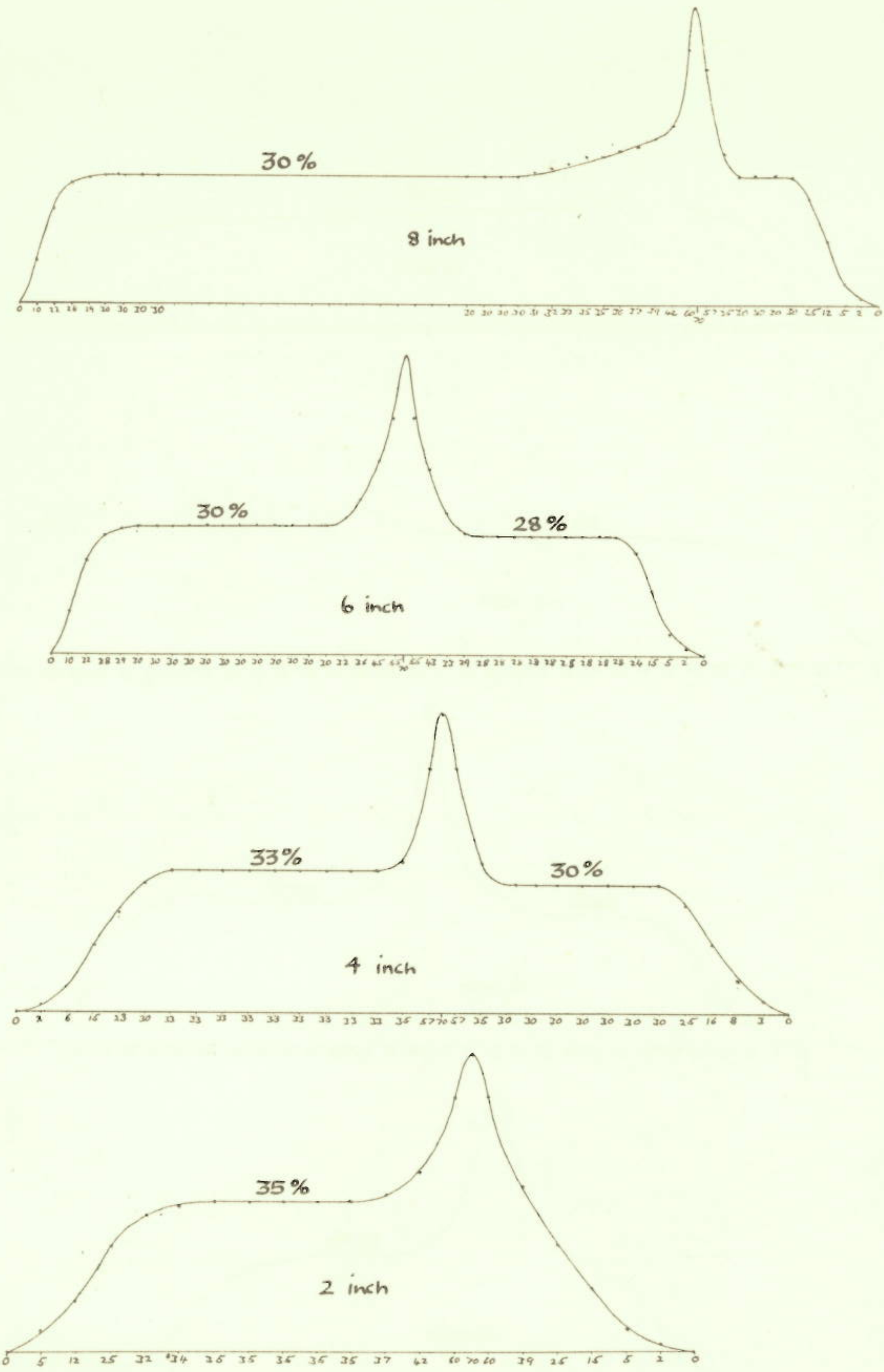

FIG. 9 VARIATION IN STRAIN IN MILLED ALUMINIUM TEST PIECES

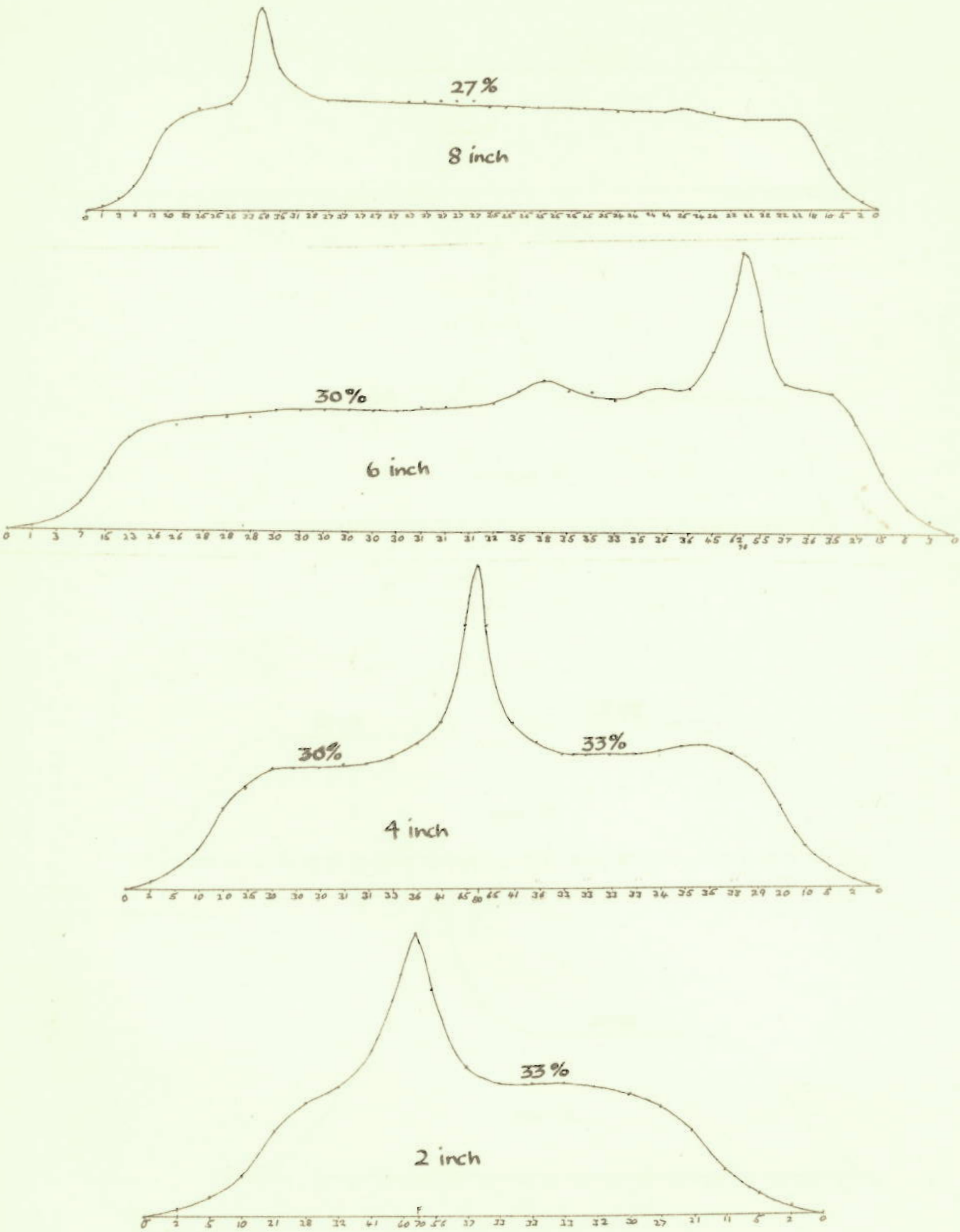

FIG. 10 VARIATION IN STRAIN IN MILLED AND ELECTROPOLISHED ALUMINIUM TEST PIECES

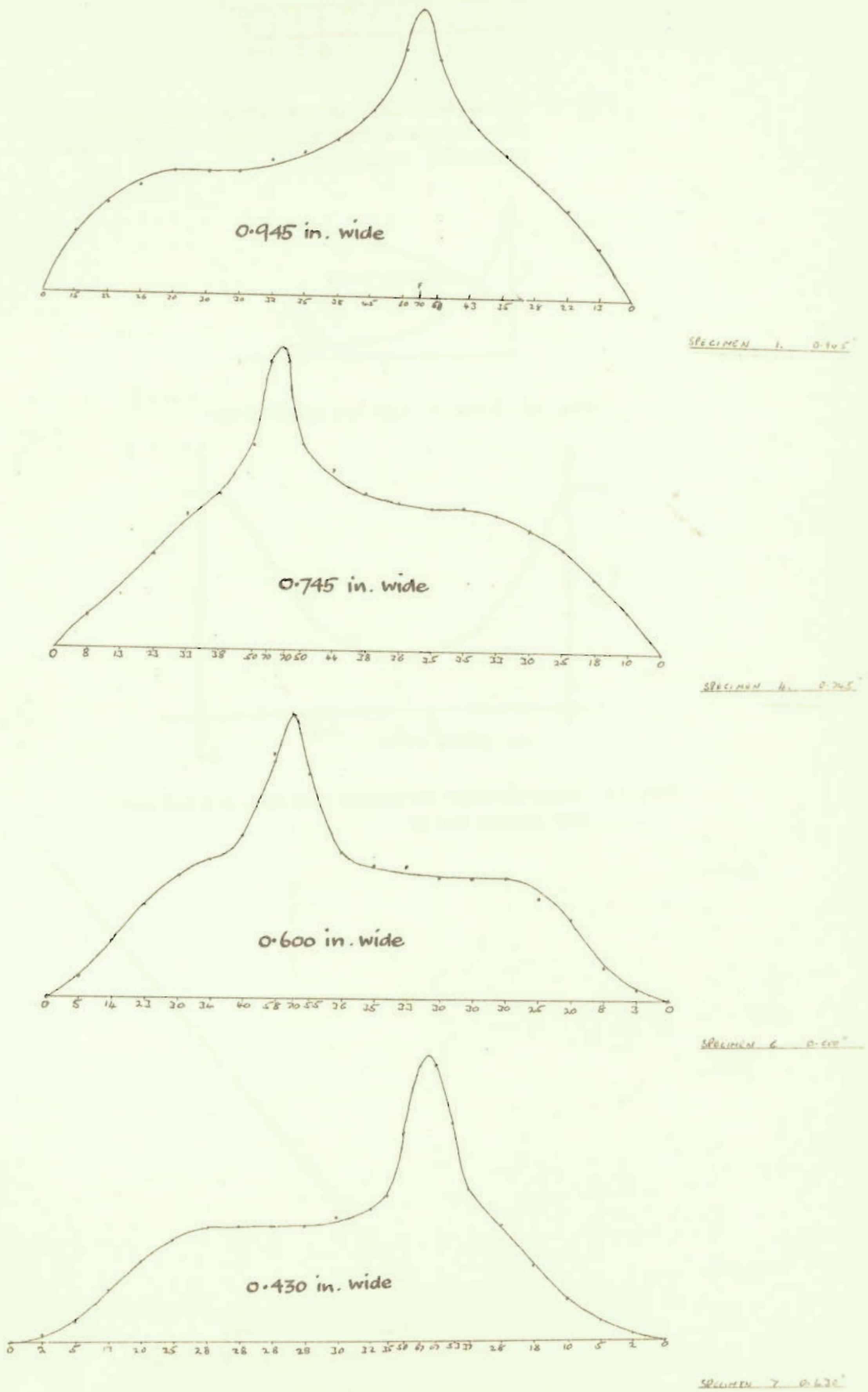

FIG. 11 EFFECT OF TEST PIECE WIDTH ON e_u

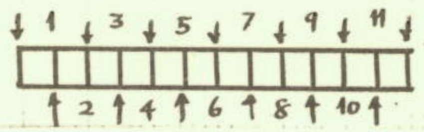

FIG. 12 REPRESENTATION OF MEASURING PROCEDURE USED FOR OBTAINING THE DATA USED FOR PLOTTING FIGURES 9, 10 AND 11.

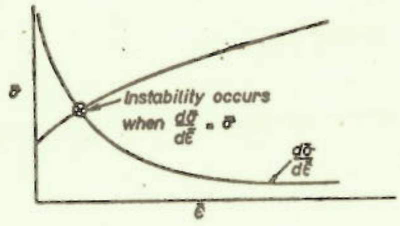

FIG. 13 σ vs. ϵ AND ITS DERIVATIVE

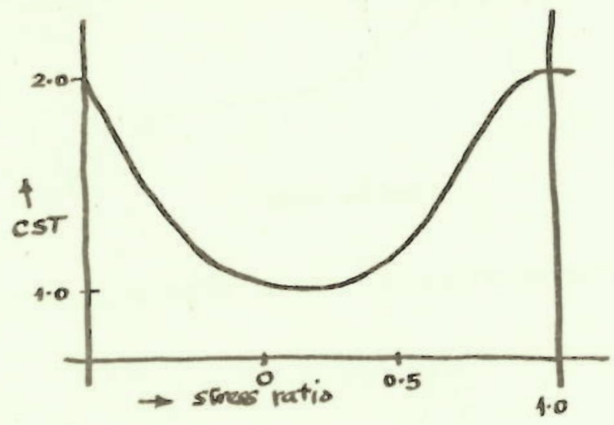

FIG. 14 RELATIONSHIP BETWEEN CRITICAL SUBTANGENT AND STRESS RATIO

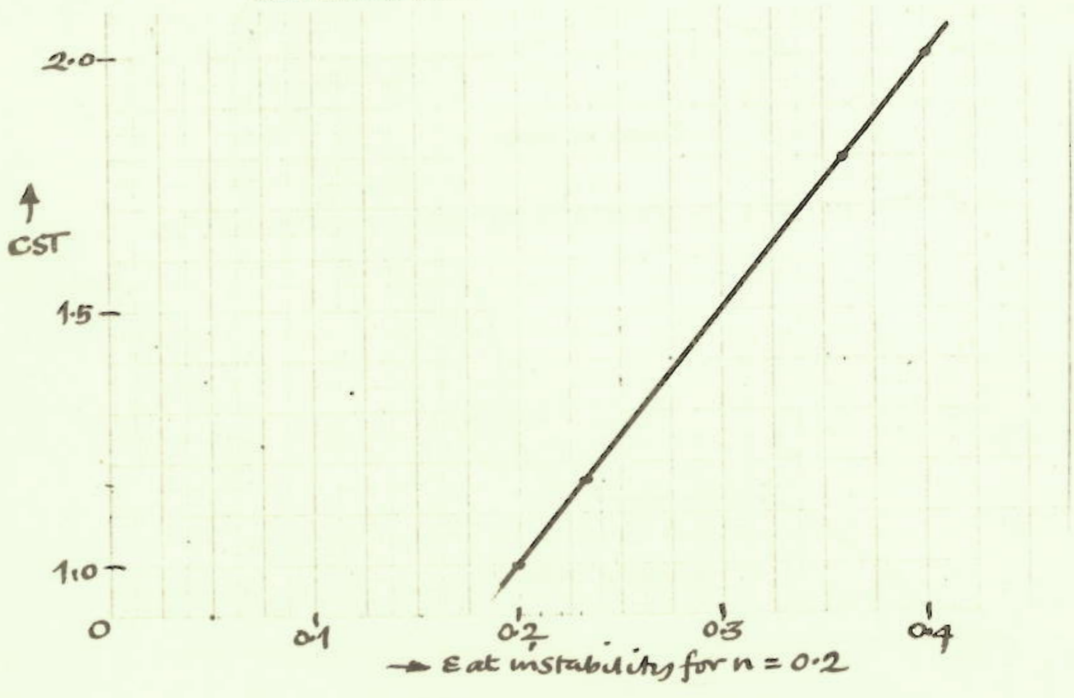

FIG. 15 LINEAR RELATIONSHIP BETWEEN CRITICAL SUBTANGENT AND ϵ AT INSTABILITY