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An analogue method for the analysis of current carrying semiconductor systems

- by -

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Contents

		Page
	List of symbols	
1.	Introduction	1
2.	Review of analogue treatment of equilibrium and quasi-equilibrium cases	1
3.	The current carrying system	3
4.	The recombination rate U	4
5.	Resistance network simulation of complete system	6
6.	Computation of individual terms	7
	6.1 $d\psi/dx$, $d\phi_p/dx$, $d\phi_n/dx$	7
	6.2 The recombination terms $U/\mu_n n$, $U/\mu_p p$	8
	6.3 $d^2\phi_p/dx$, $d^2\phi_n/dx$	9
7.	System parameters. Boundary conditions	9
	7.1 Impurity profile	9
	7.2 Lifetime profile	9
	7.3 Applied bias	9
8.	Equipment simplification : use of successive approximation technique	10
9.	Treatment of three dimensional systems	11
10.	Conclusions	12
11.	References	13

List of symbols

p, n	hole, electron concentration
q	(absolute value of) electronic charge
N_D, N_A	donor, acceptor atom concentration
ϵ	permittivity
I	total current/unit area
I_p, I_n	hole, electron current/unit area
μ_p, μ_n	hole, electron mobility
D_p, D_n	hole, electron diffusion constant
U	net hole/electron recombination rate under steady state conditions
ψ	electrostatic potential
E	electric field intensity
ϕ_p	quasi Fermi level for holes
ϕ_n	quasi Fermi level for electrons
n_i	hole/electron concentration in intrinsic material at thermal equilibrium
β =	q/KT
K	Boltzmann's constant
T	absolute temperature
x	distance coordinate in physical system
x^*	distance coordinate in analogue system
C_p, C_n	hole, electron capture rates
ϕ_t	energy level of recombination centre
p_1, n_1	hole, electron concentration for Fermi level coinciding with ϕ_t
i_s	saturation current of p-n junction diode or of diode group ('sinh term element')
R	resistance between adjacent nodes in resistance network

1. Introduction

In an earlier Internal Technical Memorandum[1], and in subsequent work[2], it has been demonstrated that a particular kind of resistance network, in which non-linear elements are associated with each mesh point, can be made to represent an exact analogue to a non-degenerate semiconductor system in the equilibrium or quasi-equilibrium state. The term 'exact' in this context implies that the difference equation which governs the potential distribution in the network becomes identical, for the limit of vanishing mesh interval, with the differential equation for the electrostatic potential within the semiconductor system, i.e. the Shockley-Poisson equation. From this type of analogue network information concerning the variation of maximum field intensity and of junction capacitance with applied bias voltages can be obtained for one, two and three dimensional configurations of p and n type regions of arbitrary geometry and impurity concentration profiles.

One limitation to the applicability of the analogue technique arises from the restriction to quasi-equilibrium conditions. This restriction precludes the investigation of situations in which current flow contributions to the carrier concentration pattern become significant - for example, in the case of strongly forward biassed p-n junctions, and of p-i-n junctions and transistors operating at high injection levels. In the present paper, the problems involved in an extension of the basic analogue method to the treatment of non-equilibrium situations are examined, and means for their solution are discussed. A review of the methods previously described and an illustration of the nature of their limitations is given in Section 2. This is followed, in Sections 3 to 7, by a detailed treatment of the case of a current carrying semiconductor system in one dimension which leads to a theoretically possible realization in terms of resistance-network/analogue computer techniques, which is, however, too complex to be considered practical. Section 8 discusses means for the simplification of the proposed schemes and leads to the description of a relatively simple system in which a significant reduction in equipment complexity has been made possible by the adoption of an operating mode based upon an iterative process of successive approximations. The extension of the technique to three dimensions is outlined in Section 9.

2. Review of analogue treatment of equilibrium and quasi equilibrium cases

As indicated above, and described in detail in[2], the solution of the Shockley-Poisson equation for the electrostatic potential, ψ, in a semiconductor system with given distributions of donor atom concentration, N_D, and acceptor atom concentration, N_A,

$$\nabla^2\psi = -\frac{q}{\epsilon}\left[N_D - N_A + p - n\right] \tag{A}$$

can be obtained by means of a particular type of resistance network.

In this network a p-n junction diode group, termed a 'sinh element' is associated with each mesh point in such a manner that its current-voltage characteristic correctly reproduces the function relating hole density, p, and electron density, n, to the difference between electrostatic potential, ψ, and Fermi level ϕ_o in the true equilibrium case, or to that between ψ and quasi Fermi levels ϕ_p, ϕ_n in the quasi-equilibrium case. The functions to be simulated are

$$p = n_i^2/n = n_i \exp\{\beta(\phi_o - \psi)\} \text{ for equilibrium}$$

$$\left.\begin{aligned} p &= n_i \exp\{\beta(\phi_p - \psi)\} \\ n &= n_i \exp\{\beta(\psi - \phi_n)\} \end{aligned}\right\} \text{ for quasi-equilibrium}$$

where n_i denotes the carrier density in intrinsic material, while $\beta = q/KT$.

The diode groups provide the required exponential dependence in the form of current flow between the mesh points of the resistance network and the ϕ_o, or the ϕ_p and ϕ_n reference potentials. The diode currents represent the physical analogues to the hole and electron concentrations. The addition of a system of constant current generators supplying each mesh point with a current of magnitude and sign proportional to the given impurity density at the corresponding point of the semiconductor system completes the simulation of equation (A) and results in a potential distribution over the resistance network which represents the required solution of that equation. In addition, the presence of currents which are proportional to the electron and hole densities integrated over the entire system makes possible the direct measurement of the dependence of these parameters upon applied bias potentials, and permits the direct evaluation of capacitance-bias relationships.

It is worth noting that an alternative method of simulating equation (1) with the aid of sinh term elements is feasible. The essential feature of this alternative consists of the use of diode groups in conjunction with an operational amplifier in accordance with standard analogue computing techniques. Compared with the resistance network, the analogue computer suffers from the following disadvantages in this particular application.

(i) Since the distance co-ordinate must be transformed to become the time axis, the analogue computer technique is restricted to the treatment of problems of one or at most two dimensions, the latter at the cost of greatly increased complexity.

(ii) The system geometry, i.e. the donor and acceptor atom concentration profiles, would have to be set up in terms of a time-programmed function.

(iii) The facility for static measurements provided by the simultaneous and continuous accessibility of the potential distribution at all mesh points of a resistance network would be lost.

3. The current carrying system

The current flowing within a semiconductor represents the sum of electron and hole currents, each of which is comprised of a drift and a diffusion component. For a one-dimensional system this leads to the equations[3]

$$
\begin{aligned}
I_p &= q\mu_p p\,E - q\,D_p \frac{dp}{dx} = -\,q\mu_p p \frac{d\phi_p}{dx} \\
I_n &= q\mu_n n\,E + q\,D_n \frac{dn}{dx} = -\,q\mu_n n \frac{d\phi_n}{dx}
\end{aligned}
\qquad (1)
$$

which indicate that the total hole current and the total electron current are each proportional to the product of the local carrier concentration and the gradient of the appropriate quasi-Fermi level.

The complication introduced into the analogue treatment of the system by the presence of current flow can thus be seen to consist of the necessity to accommodate quasi-Fermi levels which vary with position, in place of the constant quasi-Fermi levels which characterized the equilibrium and quasi-equilibrium cases. If ϕ_p and ϕ_n were known functions of x, this change would introduce little difficulty; in place of the two common reference potentials to which all sinh term element diode groups are connected in the equilibrium cases, distinct potentials $\phi_p(x^*)$, $\phi_n(x^*)$ would be applied to each diode group as indicated in Figure 1. As before, the resistance network potential $\psi(x^*)$ would represent the solution of the Poisson equation appropriate to this situation.

In practice, of course, ϕ_p and ϕ_n are not known as explicit functions of x. The spatial variation of the quasi-Fermi levels is governed by the requirement that the following conditions must be fulfilled:

(i) constancy of total current $I_p + I_n$

(ii) divergence of hole and electron current density vectors determined by prescribed local electron-hole recombination rates.

As shown in Reference 3, the complete analysis of the problem, i.e. the determination of $\phi_p(x)$, $\phi_n(x)$ and $\psi(x)$, requires the solution of the system of equations

$$\frac{d^2\psi}{dx^2} = -\frac{q}{\epsilon}(p - n + N_D - N_A) \qquad (2)$$

$$I_p = I - I_n = -q\mu_p p \frac{d\phi_p}{dx} \qquad (3)$$

$$\frac{dI_p}{dx} = -q\,U \qquad (4)$$

U, the net rate of electron-hole recombination, will itself depend upon p,n, and the density and energy levels of the centres involved in the recombination process.

Differentiation of (3) yields

$$\frac{dI_p}{dx} = -q\mu_p\left(\frac{dp}{dx}\frac{d\phi_p}{dx} + p\frac{d^2\phi_p}{dx^2}\right)$$

which, upon substitution in (4), and use of the Boltzmann relations for carrier densities, $p = n_i \exp[\beta(\phi_p - \psi)]$, $n = n_i \exp[\beta(\psi - \phi_n)]$ leads to

$$\frac{d^2\phi_p}{dx^2} = \frac{U}{\mu_p p} - \beta\frac{d\phi_p}{dx}\left(\frac{d\phi_p}{dx} - \frac{d\psi}{dx}\right) \qquad (5)$$

and

$$\frac{d^2\phi_n}{dx^2} = -\frac{U}{\mu_n n} - \beta\frac{d\phi_n}{dx}\left(\frac{d\psi}{dx} - \frac{d\phi_n}{dx}\right) \qquad (6)$$

As stated above, if functions $\phi_p(x)$, $\phi_n(x)$ and $\psi(x)$ obeying equations (2), (5), (6) can be found for given impurity concentrations $N_D(x)$, $N_A(x)$ and for a prescribed recombination function $U(x)$, then the problem of the current carrying semiconductor system is solved.

Before considering possible approaches to the solution of these equations by analogue methods, it will be useful to examine the nature of the recombination rate function, U, and of its dependence upon p and n.

4. The recombination rate U

The statistics of electron-hole recombination processes in semiconductors have been discussed by Hall(4) and by Shockley and Read(5). For recombination centres lying at a single energy level within the

forbidden gap, the steady state recombination rate U is given by the expression

$$U = \frac{pn - n_i^2}{\frac{1}{C_p}(n + n_1) + \frac{1}{C_n}(p + p_1)} \qquad (7)$$

Here C_p is the probability per unit time that a hole in the valence band will be captured by a recombination centre ('trap') for the case that all such centres are filled by electrons, while C_n is the corresponding capture probability for an electron in the conduction band. C_p, C_n represent the product of trap density and of effective cross section of a trap for hole or electron capture. n_1 is the electron concentration in the conduction band if the Fermi level coincides with the energy level of the trap.

From equation (7) various expressions for minority carrier lifetimes, appropriate to carrier concentrations which correspond to different degrees of deviation from equilibrium, can be derived. From the point of view of this study it will be more useful to preserve the general form of U given in equation (7).

The expressions U/p, U/n entering equations (5) and (6) can be transformed by means of the Boltzmann relations for p, p_1, n and n_1 in the following manner.

$$\frac{U}{p} = \frac{n - \frac{n_i^2}{p}}{\frac{1}{C_p}(n + n_1) + \frac{1}{C_n}(p + p_1)}$$

$$= \frac{e^{\beta(\psi-\phi_n)} - e^{\beta(\psi-\phi_p)}}{\frac{1}{C_p}\left[e^{\beta(\psi-\phi_n)} + e^{\beta(\psi-\phi_t)}\right] + \frac{1}{C_n}\left[e^{\beta(\phi_p-\psi)} + e^{\beta(\phi_t-\psi)}\right]} \qquad (8)$$

$$\frac{U}{n} = \frac{p - \frac{n_i^2}{n}}{\frac{1}{C_p}(n + n_1) + \frac{1}{C_n}(p + p_1)}$$

$$= \frac{e^{\beta(\phi_p-\psi)} - e^{\beta(\phi_n-\psi)}}{\frac{1}{C_p}\left[e^{\beta(\psi-\phi_n)} + e^{\beta(\psi-\phi_t)}\right] + \frac{1}{C_n}\left[e^{\beta(\phi_p-\psi)} + e^{\beta(\phi_t-\psi)}\right]} \qquad (9)$$

5. Resistance network simulation of complete system

As was shown above the complete solution of the potential and carrier concentration problem of a steady state, current carrying semiconductor system involves the determination of the three functions ψ, ϕ_p and ϕ_n. Each of these variables is governed by a differential equation of the form

$$\nabla^2\phi = f(\psi, \phi_p, \phi_n, U, N_D - N_A).$$

Since a resistance network provides analogue solutions to the difference approximation of this type of differential equation, an extension of the method used for the simulation of the quasi-equilibrium case suggests itself. In place of a single resistance network, three separate networks are set up, associated respectively with ψ, with ϕ_p, and with ϕ_n. Interconnections between these networks are provided in such a manner as to ensure that the current entering any node represents the corresponding term of the appropriate equation.

Figure 2 indicates schematically how this requirement can be met. Each ψ network node is connected to sinh term elements which are returned to the quasi-Fermi reference voltages ϕ_p, ϕ_n in the manner described previously. In consequence, the potential distribution set up in the ψ network will, as before, represent the solution to the Shockley-Poisson equation (equation (2)). However, since ϕ_p and ϕ_n are themselves now derived from resistance networks, it is necessary to interpose buffer stages, B, between the nodes of these networks and the sinh term elements to prevent the current flowing through these from affecting the ϕ_p and ϕ_n potential distributions.

The required derivatives $d\psi/dx$, $d\phi_p/dx$, and $d\phi_n/dx$ may be computed for a particular set of node points (x^*) from the values of ψ, ϕ_p, and ϕ_n at neighbouring node points $(x^* \pm 1, \pm 2$ etc.) in terms of finite difference approximations with an accuracy depending upon the highest order difference which is included in the computation. In the diagram, the three computing units 'D' perform these differentiations. An additional computing unit ('E') obtains the values of $U/\mu_p p$ and $U/\mu_n n$ from the values of ψ, ϕ_p, ϕ_n, C_p, C_n, and ϕ_t in accordance with equations (8) and (9). In this manner all the information required for the evaluation of the terms on the right hand side of equations (5) and (6) is assembled so that the two units shown schematically as 'C' can evaluate $d^2\phi_p/dx^2$, $d^2\phi_n/dx^2$ and provide currents proportional to these terms to the (x^*) nodes of the ϕ_p and ϕ_n networks.

The complete computation of all required values for a single set of nodes, equivalent to one point of the physical system, thus involves two buffer stages (B), three differentiating units (D), two 'C' units and one 'E' unit. As will be shown in the detailed description of these units below, in terms of conventional analogue computing equipment this would involve approximately 30 amplifiers and 4 multipliers per point (x^*).

While there can be little doubt that provision of equipment on such a scale is impracticable, so that alternative methods for achieving the desired result will be considered below, it is worth noting that the scheme outlined above would provide a means for the automatic and complete analysis of any semiconductor system of arbitrary impurity profile and arbitrary distribution and energy level of recombination traps. This is well illustrated by a comparison of the input and output data which are relevant to measurements on an actual semiconductor system with those corresponding to the analysis by means of the analogue. For example, an investigation of the voltage-current and the voltage-capacitance characteristics of a p-i-n diode will consist of the measurement of just the three variables involved, namely applied voltage, current, capacitance. The functional dependence found will be governed by the internal parameters describing the diode as a physical system:

Material parameters	n_i, μ_p, μ_n
Impurity profile	N_D, N_A as function of x, y, z
Distribution and characteristics of recombination traps	C_p, C_n, ϕ_t - possibly as function of x, y, z.

The actual measurements will yield the function $I = I(V)$, $C = C(V)$ which are completely determined by the parameters listed above.

It is clear that the analogue system described above will yield the same information on, i.e. $I(V)$ and $C(V)$, provided the structure of the physical system which it represents has been correctly simulated in terms of the parameters listed above. No further assumptions regarding the internal potentials, the quasi-Fermi level distribution, or that of any other internal parameter, are required.

6. Computation of individual terms

In this section, the realization, in terms of standard analogue computing equipment, of the functional units shown in Figure 2 will be discussed.

6.1 $d\psi/dx$, $d\phi_p/dx$, $d\phi_n/dx$

In terms of standard central difference notation, the first derivative of a function ψ at $x^* = 0$ in terms of the values of ψ at ±1, ±2, ... is given by

$$\frac{d\psi}{dx^*}(0) = \mu\delta\psi(0) - \frac{1}{6}\mu\delta^3\psi(0) + \frac{1}{30}\mu\delta^5\psi(0) \ \ldots$$

where $\mu\delta\psi = \frac{1}{2}(\delta\psi_{\frac{1}{2}} + \delta\psi_{-\frac{1}{2}})$

$$= \frac{1}{2}(\psi(1) - \psi(-1)), \text{ etc.}$$

This leads to the approximate expressions

$$\frac{d\psi}{dx}(0) = \frac{1}{2}(\psi(1) - \psi(-1)) \text{ if only first order differences are retained,}$$

or

$$\frac{d\psi}{dx}(0) = \frac{2}{3}(\psi(1) - \psi(-1)) - \frac{1}{12}(\psi(2) - \psi(-2))$$

if all terms beyond the third order difference are neglected.

The corresponding computing schemes are illustrated in Figures 3A and 3B. It will be noted that, in either case, one inverting and one summing amplifier are associated with each node if simultaneous differentiation is to be carried out at all nodes. For a single derivative the first order approximation requires two, the third order approximation three amplifiers.

6.2 The recombination terms $U/\mu_n n$, $U/\mu_p p$

The required expressions are those given in equations (8) and (9), which involve exponential factors of the form exp $\beta(\phi - \psi)$ in both numerator and denominator.

A convenient method* for this computation utilizes a combination of sinh term elements with operational amplifiers as illustrated in Figure 4. With the usual simplifying assumptions, the output voltage is given by

$$V_o = -R\left[i_s(e^{\beta V_i} - 1) + i_s(1 - e^{-\beta V_B})\right]$$

$$= -R\, i_s\, e^{\beta V_i}$$

if the constant bias potential V_B is chosen so as to make $\exp(-\beta V_B) \ll 1$.

With the aid of this method, the computing scheme for equations (8) and (9), i.e. unit 'E', may be set up as indicated in Figure 5.

The scheme involves the use of 4 sign-inverting amplifiers, 10 summing amplifiers and 2 multipliers.

* This method is based upon a suggestion by R.J.A. Paul.

6.3 $d^2\phi_p/dx$ and $d^2\phi_n/dx$

The remaining computing sub-units, C, which provide the current supplies to the individual nodes of the ϕ_p and ϕ_n networks in accordance with equations (5) and (6), comprise two multipliers, one sign inverter and four summing amplifiers as shown in Figure 6. The output from the final amplifiers must be provided by current generators of adequate impedance to ensure that the current remains unaffected by the potential existing at the node to which the supply is connected.

The equipment requirements represented by each 'C' unit amount to 4 summing amplifiers, 2 sign inverters and 2 multipliers.

7. System parameters. Boundary conditions.

7.1 Impurity profile

The impurity profile of the junction to be simulated is represented in the manner described in Reference 2. Into each node of the ψ network a current proportional to $(N_D - N_A)/n_i$ is passed. The proportionality factor, and the scale factor relating the spatial co-ordinates, x, to the network co-ordinates, x^*, are determined by the network mesh resistance, R, and the saturation current, i_s, of the sinh term element diode groups.

7.2 Lifetime profile

The minority carrier lifetime profile, i.e. the spatial variation of the parameters governing recombination processes, namely capture cross section and energy levels of traps, are set up in terms of C_p, C_n and ϕ_t. These may be constant or they may be prescribed functions of x. Equations (5) and (6) are based upon the assumption of a single trap level, but an extension to the case of recombination occurring via traps at different levels appears feasible. It is worth noting that the dependence of lifetime upon carrier densities, implicit in the Shockley-Reed expression, is correctly incorporated into the analogue by the computing scheme adopted.

By appropriate choice of ϕ_t, C_p, C_n at points of the analogue which correspond to an external boundary of the real system surface recombination processes can be included in the conditions governing the analogue solution.

7.3 Applied bias

The sign and magnitude of a bias voltage applied to the real system is represented in the analogue by the difference in quasi-Fermi levels at those points of the network which correspond to external contacts. Thus, for a p-n junction the applied bias will equal the value of ϕ_p at the p-region contact and of ϕ_n at the n-region contact. It will generally

be convenient to choose reference potential levels in such a way that the equilibrium Fermi level corresponds to ground potential, although this is not essential.

8. Equipment simplification: use of successive approximation technique

Although the analogue scheme described in the previous sections would provide the most convenient method for the analysis of the current carrying semiconductor system, its equipment requirements are on a scale which is obviously not practicable in terms of the numbers of amplifiers, multipliers, etc.* which would be involved. This is clearly shown in the following table which summarises these requirements for a one dimensional analogue comprising three resistance networks of N nodes each, i.e. representing N points of the physical system.

	No. of Units	No. of amplifiers, etc.
Buffer stages 'B'	2N	2N
Differentiating units 'D'	3N	6N
Computing units 'C'	2N	6N+2N multipliers
Computing units 'E'	N	14N+2N multipliers
		28N+2N multipliers.

Since the smallest useful analogue would have to possess at least, say, 16 nodes, this would imply a minimum complement of the order of 450 amplifiers and 64 multipliers even for such a system of very limited range of application. Evidently, if analogue systems of extended size and two and three dimensional representations are to be achieved, methods involving more realistic equipment requirements will have to be adopted.

One such method consists of the use of an iterative technique, utilising successive approximations. The adoption of such a procedure is feasible since the terms entering into the equations for $d^2\psi/dx^2$, $d^2\phi_p/dx^2$, and $d^2\phi_n/dx^2$ at a particular point x^* involve only the variables ψ, ϕ_p, ϕ_n and their derivatives at x^*. As the derivatives are computed in terms of finite difference, the only information required consists of the values of the three variables at the point x^* and at its neighbours $x^*\pm1$, $x^*\pm2$. Provided, therefore, that suitable storage devices are individually associated with each node of the ϕ_p and ϕ_n networks so that the result of each computing step is impressed upon the potential

* Although there can be no doubt regarding this impracticability as long as present-day operational amplifier techniques are considered, it appears feasible that such requirements could be met in the future by extensive use of solid circuit techniques.

distribution in these networks, only a single set of computing elements comprising 28 amplifiers and 4 multipliers will be required.

The simplest method of information storage for this case uses manually set current generators G_p, G_n to reproduce the output currents from the computer which represent $d^2\phi_p/dx^2$ and $d^2\phi_n/dx^2$. In addition, the use of the buffer stages (B), which ensure that the currents drawn by the sinh term elements do not affect the ϕ_p, ϕ_n potential distribution, can be avoided by the provision of simple, manually set, voltage sources which are adjusted to $\phi_p(x^*)$, $\phi_n(x^*)$, and to which the sinh term elements are connected.

The resulting arrangement is shown in Figure 6. The sequence of operations will be as follows.

(1) A suitable initial ϕ_p and ϕ_n distribution is set up by means of the G_p and G_n generators and the (ϕ_p) and (ϕ_n) sources. Such a distribution may be chosen to represent the quasi-equilibrium case for given bias conditions, or it may reproduce an approximate solution obtained by other means.

(2) The computing system is connected to a point x_i^* (i.e. to the nodes x_i^*, $x_i^* \pm 1$, $x_i^* \pm 2$ in the ϕ_p and ϕ_n networks, and to x_i^* in the ψ network. The new values obtained for $\phi_p(x_i^*)$ and $\phi_n(x_i^*)$ are noted.

(3) After disconnection of the computing system, the current and voltage generators at x_i^* are adjusted to the new values.

(4) Procedures (2) and (3) are repeated for $x_i^* + 1$ (or $x_i^* - 1$).

In this manner the entire network can be repeatedly traversed and the current and voltage generators at each node re-adjusted until self-consistency has been achieved, when the change-over from 'memory' to computer at any point will leave the potential distribution in the three resistance networks unchanged. These potential distributions then represent the required solutions for ψ, ϕ_p, and ϕ_n.

9. Treatment of three-dimensional systems

For a three dimensional system the basic equations to be solved, namely equations (1), (2), (5) and (6), assume the following form.

$$\underline{I}_p = q\mu_p p \underline{E} - q D_p \nabla p$$

$$= - q\mu_p p \nabla \phi_p \qquad (10a)$$

$$\underline{I_n} = q\mu_n n \underline{E} + q D_n \nabla n$$

$$= - q\mu_n n \nabla \phi_n \qquad (10b)$$

$$\nabla^2 \psi = - \frac{q}{\epsilon} (p - n + N_D - N_A) \qquad (11)$$

$$\nabla^2 \phi_p = \frac{U}{\mu_p p} - \beta \nabla \phi_p \cdot (\nabla \phi_p - \nabla \psi) \qquad (12a)$$

$$\nabla^2 \phi_n = - \frac{U}{\mu_n n} - \beta \nabla \phi_n \cdot (\nabla \psi - \nabla \phi_n) \qquad (12b)$$

The analogue treatment therefore becomes a straightforward extension of the method applied to the one-dimensional case. Three differentiating units 'D' will be associated with each of the networks and will provide the x, y and z components of the gradients. The second terms on the right hand side of equation (12) are then formed by the summation of the outputs from three multipliers. The generation/recombination term, which remains dependent only upon the values of the variables at the point under consideration, is derived as before.

10. Conclusions

In the preceding sections it has been established that the system of equations which determines potential profile, carrier distribution, and current flow in a semiconductor structure of arbitrary geometry under steady state conditions can be solved by means of a combination of resistance network and analogue computer techniques. While it is feasible, in principle, to design analogue systems which will perform such computations in a fully automatic manner, the equipment complexity and total requirements are such as to make this approach impracticable in terms of present day circuit techniques.

By adopting a method of operation which is based upon a procedure involving successive approximations, equipment requirements can be reduced to a level which is feasible and realistic even for three dimensional analogue systems without any loss of ultimate accuracy.

The development of analogue systems based upon the principles discussed in this paper will provide a novel and powerful method for the analysis and, ultimately, the design of semiconductor device geometries ranging from simple p-n junction diodes to complex three-dimensional solid circuit structures.

11. References

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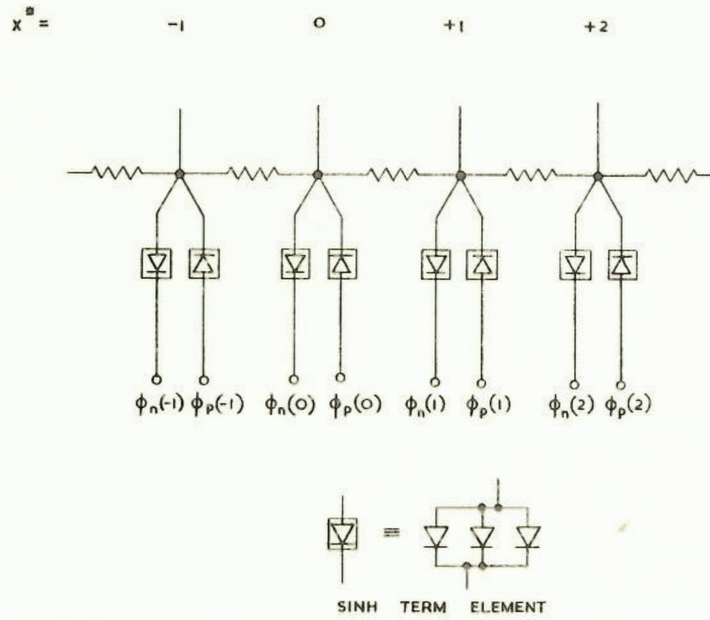


FIG. 1 ANALOGUE NETWORK FOR NON-EQUILIBRIUM SYSTEM (QUASI-FERMI LEVELS ϕ_p, ϕ_n POSITION DEPENDENT)

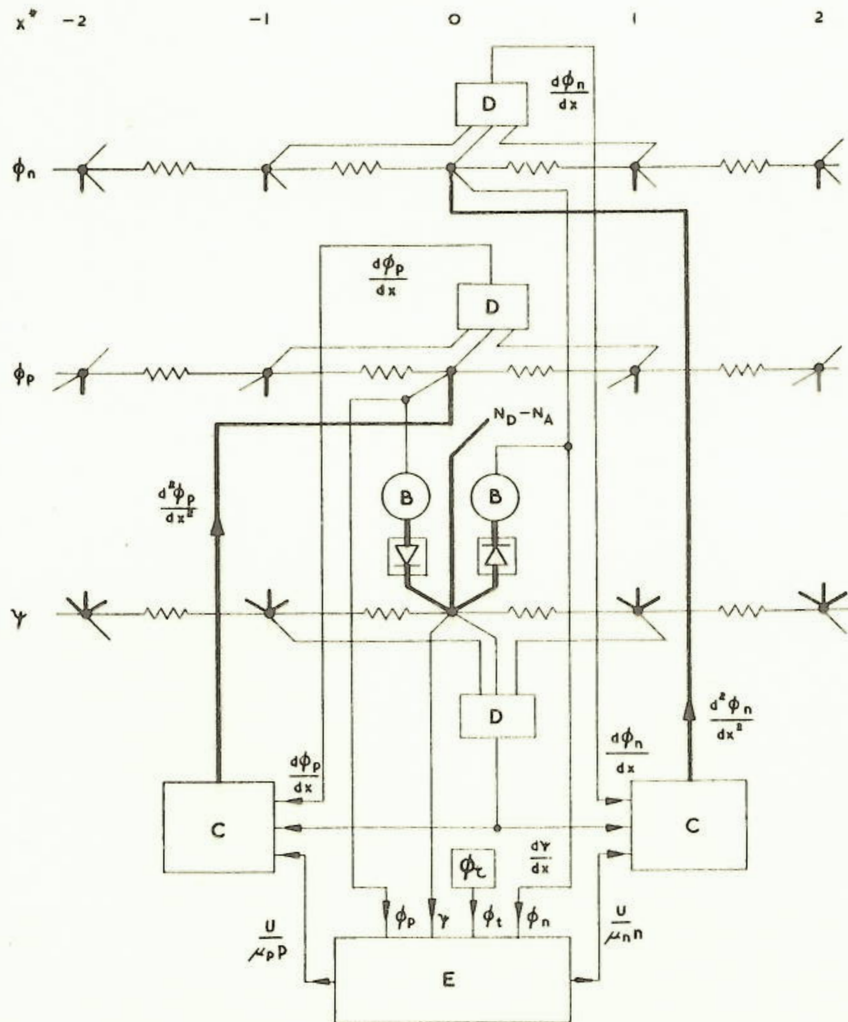


FIG. 2 INTERCONNECTION OF RESISTANCE NETWORKS TO PROVIDE CORRECT FUNCTIONAL RELATIONSHIP.

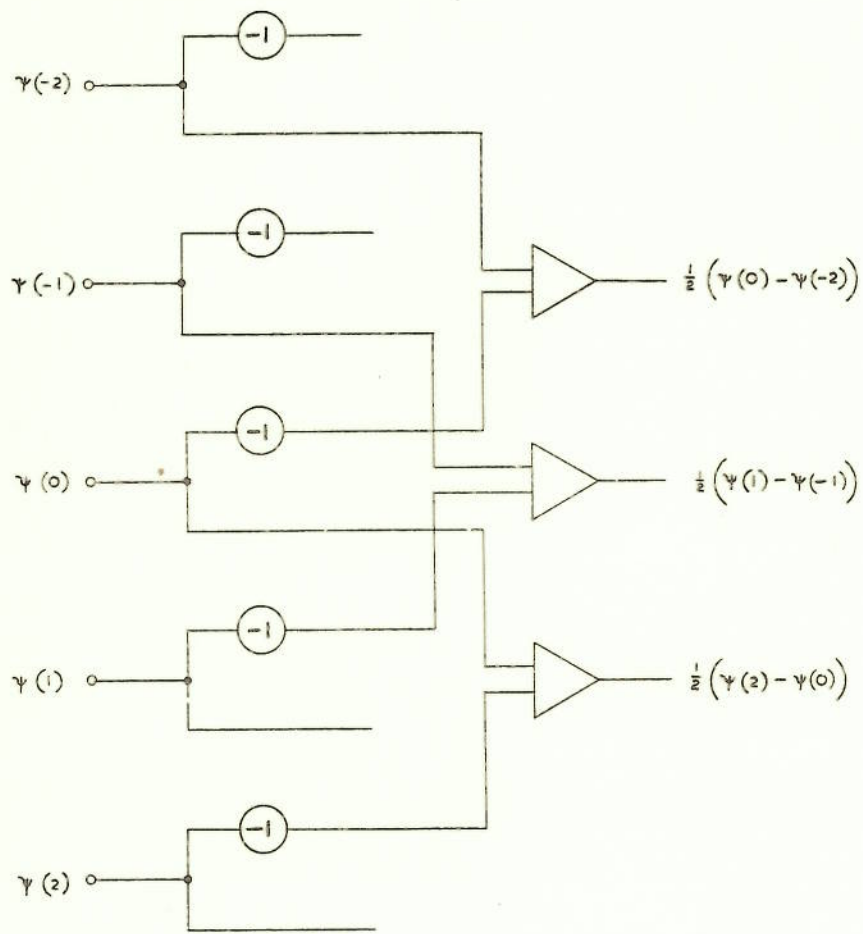


FIG. 3A

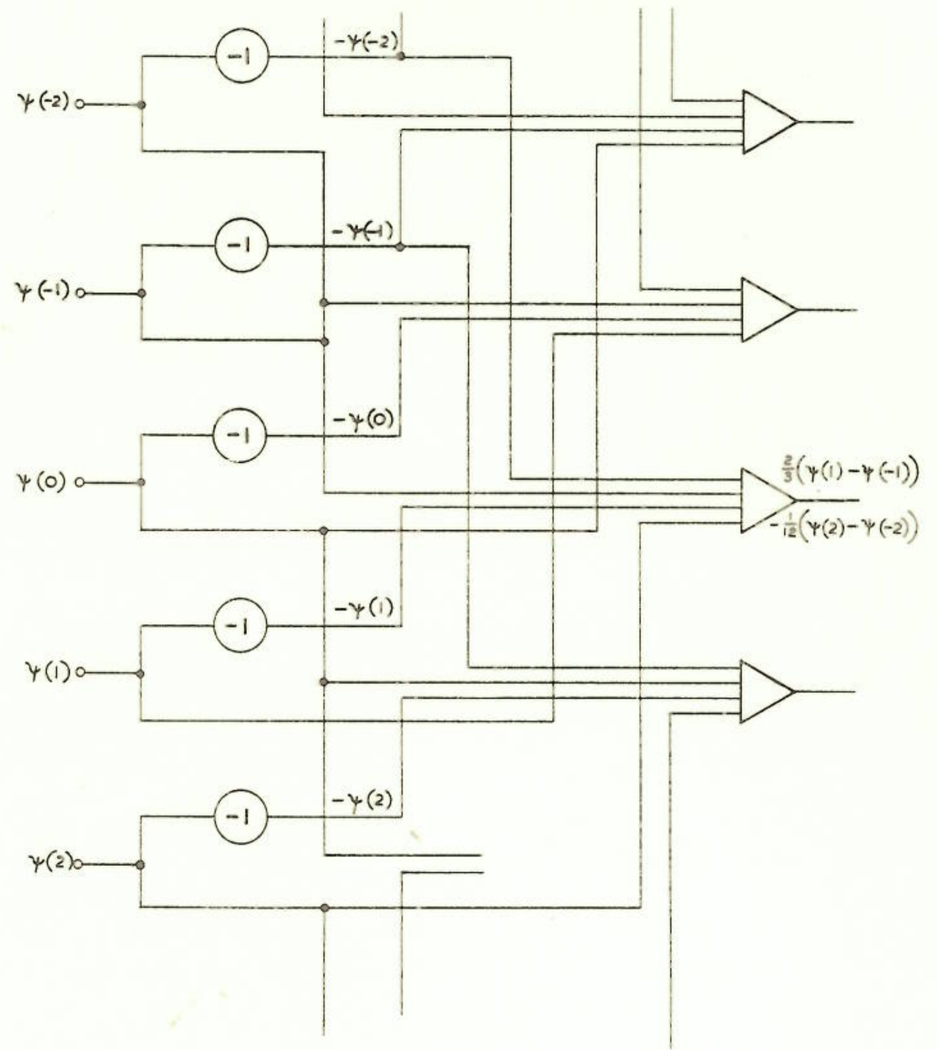
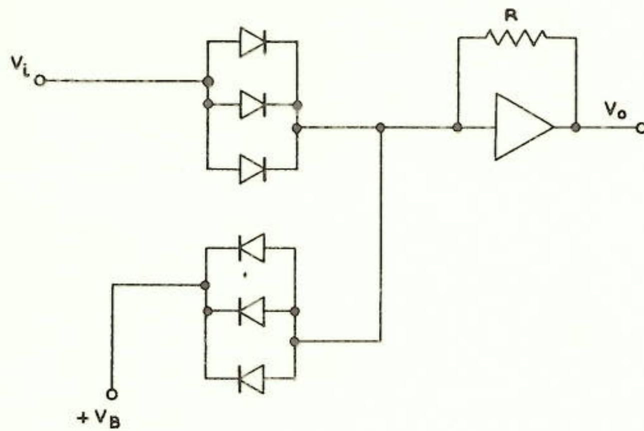


FIG. 3.B



$$\frac{V_o}{R} + i_s\left(e^{\beta V_i} - 1\right) - i_s\left(e^{-\beta V_B} - 1\right) = 0$$

IF $\beta V_B \gg 1$

$$V_o = -R\, i_s e^{\beta V_i}$$

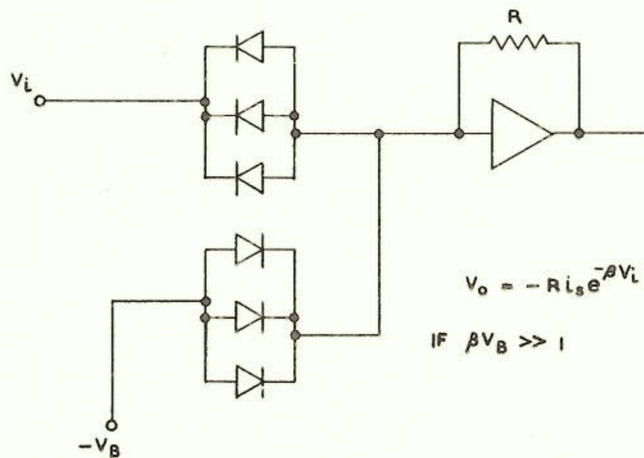


FIG. 4

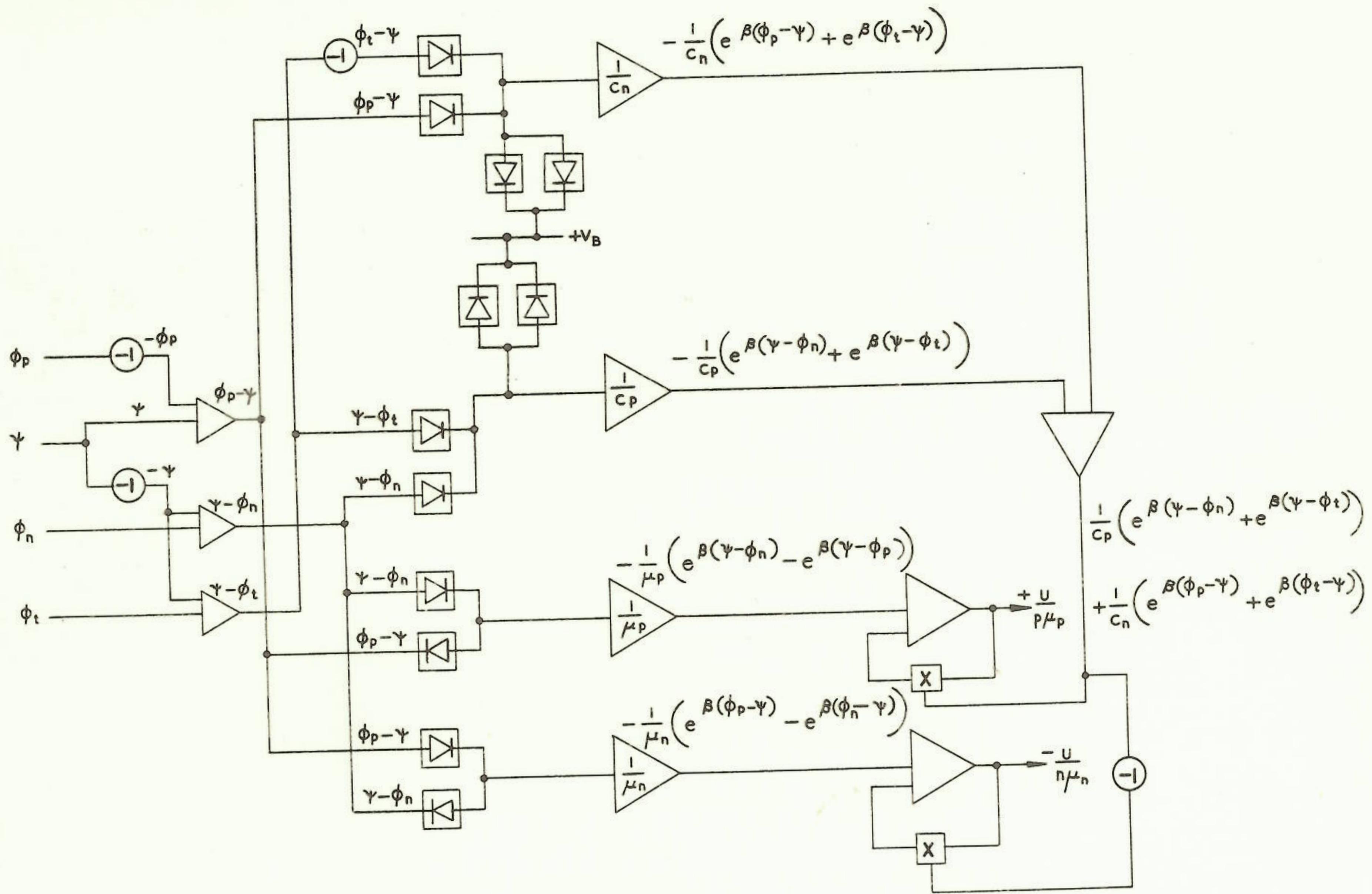


FIG. 5.

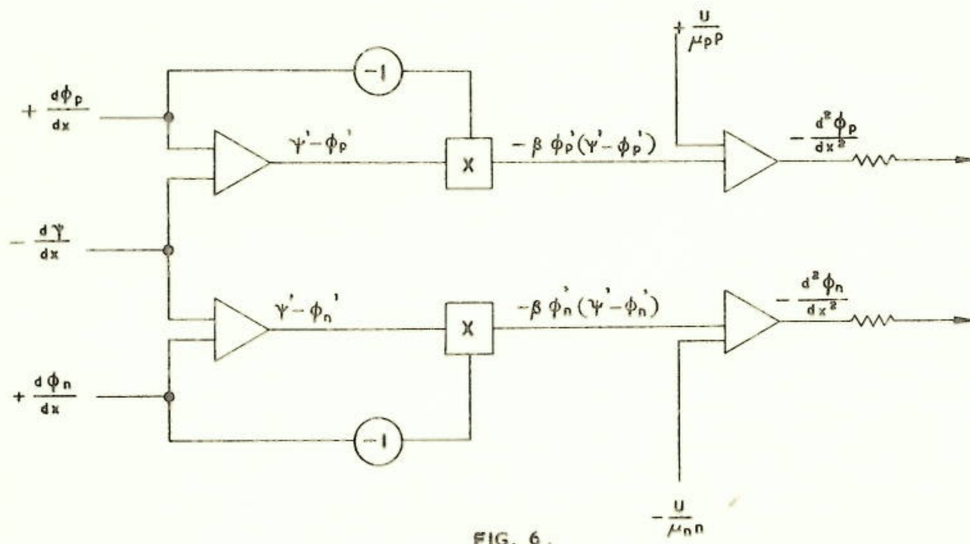


FIG. 6.

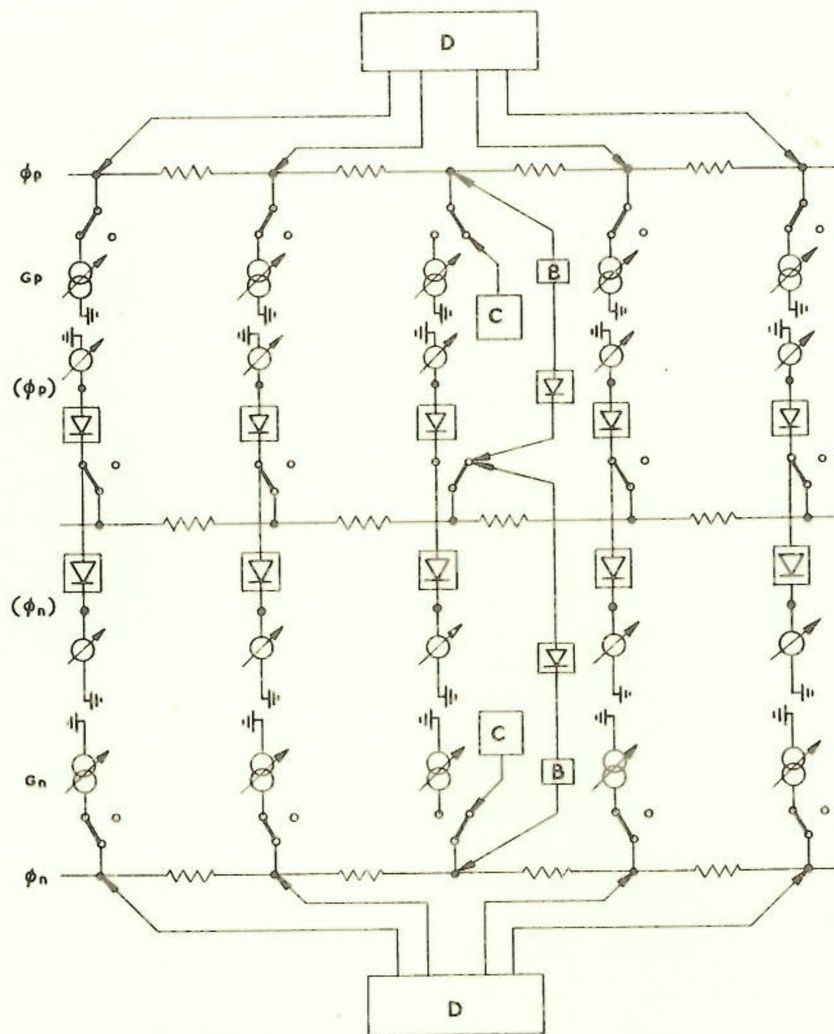


FIG. 7.