

Forecasting U.S. Tourist Arrivals using Singular Spectrum Analysis

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Abstract

This paper introduces Singular Spectrum Analysis (SSA) for tourism demand forecasting via an application into total monthly U.S. Tourist arrivals from 1996-2012. The global tourism industry is today, a key driver of foreign exchange inflows to an economy. Here, we compare the forecasting results from SSA with those from ARIMA, Exponential Smoothing (ETS) and Neural Networks (NN). We find statistically significant evidence proving that the SSA model outperforms the optimal ARIMA, ETS and NN models at forecasting total U.S. Tourist arrivals. The study also finds SSA outperforming ARIMA at forecasting U.S. Tourist arrivals by country of origin with statistically significant results. In the process, we find strong evidence to justify the discontinuation of employing ARIMA, ETS and a feed-forward NN model with one hidden layer as a forecasting technique for U.S. Tourist arrivals in the future, and introduce SSA as its highly lucrative replacement.

Keywords: United States; Tourist arrivals; Tourism demand; Forecasting; Singular Spectrum Analysis; ARIMA; Exponential Smoothing; Neural Networks.

1 Introduction

Previous research has highlighted the importance of accurate demand forecasting to the tourism sector. The dependence of tourism on both investment and infrastructure development make a degree of advance planning essential, as many authors have recognised. Well informed investment decisions are vital for efficient resource allocation in both tourism and supporting sectors. The economic downturn and an increased awareness of world economic volatility have strengthened rather than weakened this need to forecast tourist demand accurately.

As discussed in the following section there is an extensive and high profile existing literature on forecasting tourism demand. This literature covers a wide range of different forecasting techniques, applied to a wide range of different countries or locations. The purpose of this paper is to add to this literature by introducing a new model for forecasting tourist arrivals and to apply it to inbound U.S. Tourist arrivals. Forecasting U.S. Tourist arrivals is both a demanding and important task, mainly because these data exhibit a high degree of fluctuation over time. Figure 1 depicts the time series for total monthly U.S. Tourist arrivals between January 1996 and November 2012. **A first look at the time series suggests signs of seasonality in U.S tourist arrivals.** The figure also shows that the tourism industry in the U.S. is experiencing rapid development in terms of demand. Since 2002 U.S. Tourist arrivals exhibit a strong upward trend. The need to

36 allocate resources for future growth is further evidence of the importance of developing accurate
37 demand forecasting for investors, managers and policy makers in the tourism sector.

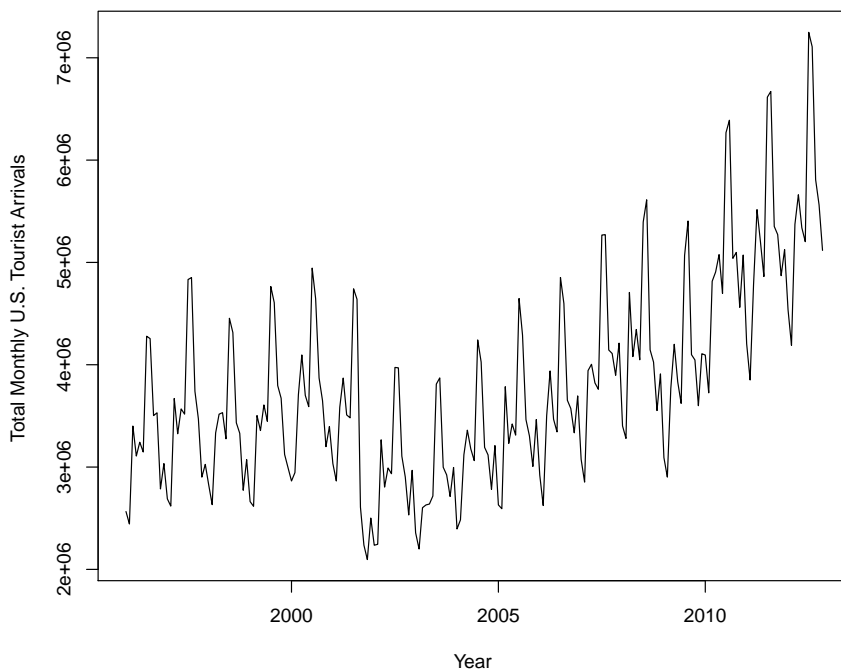


Figure 1: Total monthly U.S. Tourist arrivals time series (Jan. 1996 - Nov. 2012).

38 There are a number of components which define a good demand forecasting model for tourism
39 management. Firstly, the forecasting model has to be able to pick up strong variations in tourist
40 arrivals as most tourist demand time series show **increasing fluctuations with seasons**. Secondly,
41 given the **seasonal fluctuations**, the measure of forecasting accuracy based on the forecasting
42 error alone is not sufficient. It is important that the forecasting model is equally able to predict
43 the actual direction of change. If not, investment decisions and the resources allocated to tourism
44 could find themselves catering for a peak in demand but actually experiencing a trough. Thirdly,
45 a tourism demand forecasting model needs to be efficient both in the short and long run. **This**
46 **is because long term investments are needed to be able to supply to the short term demand**
47 **fluctuations**. In this paper we consider all these aspects as we introduce the Singular Spectrum
48 Analysis (SSA) technique for forecasting U.S. Tourist arrivals and compare its performance with
49 other forecasting models currently used to forecast tourism demand. In brief, the SSA technique
50 seeks to decompose the original time series, filter the noise and reconstruct a new time series
51 which is less noisy. We then use this newly reconstructed time series for forecasting future data
52 points.

53 The remainder of this paper is organized as follows. Section 2 provides a review of the
54 existing literature on the forecasting of tourism demand. Section 3 provides a review of the
55 main forecasting techniques employed. Section 4 introduces the data for U.S. inbound tourist
56 arrivals and discusses the measures for evaluating the forecasting performance. Section 5 reports
57 the empirical results from the SSA technique in comparison to other, previously employed,
58 forecasting techniques. Conclusions are provided in Section 6.

2 Literature Review

The existing literature on the forecasting of tourism demand is wide ranging both in terms of the different techniques employed and in terms of the different countries covered. A common theme in almost all of the papers also helps to explain the reasons behind this extensive interest in forecasting tourism demand. A large number of authors including Chan and Lim (2011), Chu (2008), Coshall and Charlesworth (2011) and, Goh and Law (2002) emphasise the importance of forecasting for investment and development planning in tourism. This message is re-enforced by authors, such as Gounopoulos, Petmezas, and Santamaria (2012) and Hui and Yuen (2002), who add that such forecasts are also important as a consequence of the vulnerability of tourism to large fluctuations in demand. Some authors also emphasise the importance of tourism to a particular economy to re-enforce the importance of accurately forecasting tourism demand. Examples include work by Jackman and Greenidge (2010) for Barbados and Chu (2011) for Macau. Those readers seeking a detailed review of the literature, the paper by Song and Li (2008) covers 121 studies produced from 2000 to the date of publication. This review article offers a further reason for the sustained and extensive interest in forecasting tourism demand. They found that no single forecasting model outperforms all other in all possible situations. This implies that the literature is not only of importance but also in need of further research. A more recent review of forecasting and the closely related issue of tourism demand modelling is included in the paper by Song et al. (2012).

Perhaps the most common form of study is one that assesses the performance of one or more forecast techniques relative to a set of alternatives. Alvarez-Diaz and Rossello-Nadal (2010) examine forecasts of UK tourist arrivals in the Balearics, using meteorological variables. They compare the performance of an ARIMA model and a non-causal autoregressive neural network, finding that the latter performs better. Assaf, Barros, and Gil-Alana (2011) examine persistence and seasonality in data for tourist arrivals into Australia. They compare the performance of three different forecasting models, two standard methods using stationarity of degrees 0 and 1 and a model with fractional degrees of integration. Athanasopoulos and De Silva (2012), in a study of tourist arrivals in Australia and New Zealand propose a model which captures time varying seasonality within a vector innovation time series model. They produce evidence that this model offers greater forecast accuracy than a number of alternatives. Cho (2003) investigates three different techniques (exponential smoothing, univariate ARIMA and artificial neural networks) to forecast tourist arrivals in Hong Kong, finding the artificial neural networks forecasts to be the most accurate.

Chu (2008) explores fractionally integrated ARMA models in forecasting tourism arrivals in Singapore, observing that they perform well in comparison to more traditional ARIMA models. Chu (2011) compares a piecewise linear model with autoregressive trend, seasonal ARIMA and fractionally integrated ARMA models in forecasting tourism demand for Macau, concluding the piecewise linear model to be the most accurate. Likewise, Gil-Alana (2005) considers forecasts using monthly data for tourist arrivals into the US using a procedure combining unit and fractional integration in seasonal variation. He finds evidence of long memory and mean reverting behaviour. Goh and Law (2002) use data for Hong Kong tourist arrivals to compare forecasts from a stochastic non-stationary seasonality model (SARIMA) and an intervention component model (MARIMA) with a selection of eight other time series models. Their results suggest the SARIMA and MARIMA models to have the highest forecast accuracy of the models analysed.

Greenidge (2001) uses a structural time series model to provide and evaluate forecasts for tourism arrivals in Barbados. Jackman and Greenidge (2010) further explore the structural time series model for tourist arrivals in Barbados, finding that it produces more accurate forecasts

106 than a number of alternatives. Hadavandi et al. (2011) present forecasts for tourism arrivals in
107 Taiwan using a hybrid artificial intelligence model, involving a fuzzy rule-based system, which
108 they found to be more accurate than a selection of three alternative approaches. Kim et al.
109 (2011) consider the performance of prediction intervals for tourism arrivals into Hong Kong
110 and Australia for a selection of time series forecasting models. They find an autoregressive bias
111 corrected bootstrap model to perform best of those tested. Lim and McAleer (2001) analysed the
112 performance of various different exponential smoothing models in forecasting tourist arrivals in
113 Australia, concluding that using models expressed in first differences increased forecast accuracy.
114 Shareef and McAleer (2007) evaluate the abilities of ARMA models to capture the effects of
115 volatility in the time series of tourism arrivals in the Maldives. Song et al. (2010) focus on a
116 different aspect of forecasting tourism demand - what is the appropriate measure of demand?
117 Using data for Hong Kong they find use of tourism arrivals to be more affected by income in
118 the country of origin and tourism expenditure to be more sensitive to prices. Wan, Wang, and
119 Woo (2013), also using tourist arrival data for Hong Kong, assess the properties of disaggregated
120 forecasts using a seasonal ARIMA model relative to aggregate forecasts. They find the sum of
121 disaggregated forecasts to provide greater accuracy than an aggregate forecast.

122 A very closely related strand in the literature seeks to combine two or more forecasting
123 models into a new hybrid model and to test whether this results in greater forecast accuracy.
124 Andrawis, Atiya, and El-Shishiny (2011) finds that, in forecasts of tourism arrivals into Egypt,
125 combining short and long term forecasts improves accuracy compared to the individual forecasts.
126 Cang (2011) examines tourism arrivals into the U.K. and examines three different forecasting
127 models support vector neural networks, seasonal ARIMA and an exponential smoothing model.
128 He finds that non-linear combinations of these models offer greater forecast accuracy than the
129 individual specifications. Coshall and Charlesworth (2011) consider a number of forecasting
130 models, both individually and in combination. Using data on UK outbound tourism they also
131 find that forecast accuracy is improved by using a combination of forecasts.

132 Shen, Li, and Song (2008) focus on outward leisure tourism from the U.S. and examine seven
133 different types of individual forecasting techniques. Their results also suggest that forecast accu-
134 racy is improved by combining forecasts. Shen, Li, and Song (2011) conduct a similar analysis of
135 UK outward tourism, using seven different individual forecasting methods and six combinations.
136 Again their findings suggest that forecast accuracy is improved by using combinations of fore-
137 casts. Song et al. (2011) develop a model to forecast Hong Kong tourist arrivals which combines
138 a structural time series model with a time varying parameter one. They find that, relative to a
139 number of time series models, their hybrid model exhibits greater forecast accuracy. Song, Gao,
140 and Lin (2013), again with respect to tourism arrivals in Hong Kong, consider a model which
141 combines quantitative forecasts which judgemental forecasting from an online survey. They find
142 that adding a judgemental component improves forecast accuracy.

143 A number of papers consider the implications of shocks to one or more forecasting models
144 of tourism demand. Gounopoulos, Petmezas, and Santamaria (2012) consider the forecasting of
145 the impact on tourism arrivals in Greece of macro-economic shocks. They compare a number
146 of different forecasting methods, finding an ARIMA model to be the most accurate and also
147 develop a VAR model. Smeral (2010) examines the effects on forecasts of outbound travel of
148 global recession for a sample of countries. Mao, Ding, and Lee (2010) use a cusp catastrophe
149 model to forecast the rates of recovery of tourist arrivals in Taiwan from the SARS epidemic.
150 Their results suggest that tourism from China and the U.S. recovered quickly but that from
151 Japan did not. In a similar vein Page, Song, and Wu (2012) estimate the negative effect of the
152 Swine flu epidemic on U.K. tourist arrivals using a time varying parameter model. Fourie and
153 Santana-Gallego (2011) use a gravity model to estimate and predict the impact of mega-sports

154 tourism events on tourist arrivals.

155 Studies which examine the determinants of demand for tourism are not analysis of fore-
156 casting models but are so closely related to the forecasting of tourism demand that they merit
157 consideration. Chan and Lim (2011) analyse seasonality in New Zealand tourism demand using
158 spectral analysis. They find different categories of inbound tourism share common cyclical be-
159 haviour. Naude and Saayman (2005) consider the determinants of tourist arrivals in 43 African
160 countries, finding tourism infrastructure and health risks to be of particular importance. Nelson
161 et al. (2011) estimate a demand model for visitors to Hawaii from mainland U.S. Their results
162 suggest home state income, airfares and (log) distance to be important. Seetanah, Durbarry,
163 and Ragodoo (2010) estimate tourism demand for South Africa using a gravity model. Their
164 analysis suggests prices, level of development and common borders to all be important deter-
165 minants. Seetaram (2010) uses dynamic panel cointegration to estimate demand elasticities for
166 tourism arrivals into Australia, finding demand to be inelastic in the short run but elastic in the
167 long run.

168 Volatility models are built upon an ARIMA model to which they add a second equation to
169 explain the conditional variance. Coshall (2009) provides a good overview of these techniques and
170 their application to forecasting tourism demand. The most commonly used specification is the
171 GARCH model, developed by Bollerslev (1986). This adds to the ARIMA model an equation to
172 explain the conditional variance. This equation models the current period conditional variance
173 in terms of lagged squared residuals (capturing the short run impact of past shocks) and longer
174 term effects from lagged values of the conditional variance. Extensions of the GARCH model
175 include the TGARCH (which use dummy variables to model asymmetric shocks) and EGARCH
176 models. For example, Kim and Wong (2006) use both the EGARCH and TGARCH models
177 to provide forecasts of tourism demand in Korea with asymmetric responses to news shocks.
178 Coshall (2009), in an application to UK outbound tourism, shows that forecasts using the
179 EGARCH model can be combined with those from an exponential smoothing model such that
180 the combined forecast is more accurate than either of the individual methods.

181 The use of Singular Spectrum Analysis (SSA) in the tourism sector was firstly evaluated by
182 Beneki et al. (2012) via an application into signal extraction and forecasting of U.K. Tourist
183 income. Here, we introduce SSA as a new model for forecasting tourism demand in the future.
184 The SSA technique is swiftly being recognized as a powerful, nonparametric time series analysis
185 and forecasting technique. The roots of SSA are closely associated with Broomhead and King
186 (1986a, 1986b). The applications of SSA are diverse and its growing success is evident in
187 many different fields (see for example, Lisi & Medio, 1997; Ghil et al. 2002; Hassani, Heravi,
188 & Zhigljavsky, 2009; Ghodsi et al. 2009; Hassani & Thomakos, 2010; Hassani, Heravi, &
189 Zhigljavsky, 2012; Hassani, Soofi, & Zhigljavsky, 2013; Beneki & Silva, 2013; Silva, 2013). As
190 noted above, there exists various different techniques which have been applied for forecasting
191 tourism demand in the past. In this paper we compare the forecasting results from SSA with
192 those obtained from ARIMA (Automatic-ARIMA), Exponential Smoothing (ETS) and Neural
193 Networks (nnetar). The ETS methodology gained its popularity through the sound performance
194 at the M3-competition, and the state space framework which now underlies the newly developed
195 ETS is widely applicable, like ARIMA, and provides a forecast with the backing of a good
196 stochastic model (Hyndman et al. 2002). Neural networks has frequently been adopted in
197 tourism demand forecasting as mentioned earlier on. It is important to note that here we use
198 the most basic version of Vector SSA along with optimal choices, and the optimal version of
199 ARIMA. Given the choice of forecasting methods we are comparing the forecasting accuracy
200 provided by both parametric and nonparametric time series analysis and forecasting techniques.
201 In comparison to parametric forecasting techniques, nonparametric techniques are not bound by

202 any of the parametric assumptions such as stationarity and normality. As such, nonparametric
203 models are able to provide a true approximation of the real situation. Also it is not the intention
204 of this paper to show SSA as the universally best model for forecasting tourist arrivals. Instead,
205 we are interested in introducing SSA as an alternative method, and further research is required
206 to compare SSA's performance against many other forecasting techniques.

207 **3 Forecasting Methods**

208 **3.1 Auto-Regressive Integrated Moving Average (ARIMA)**

209 We employ the most optimal version of Box and Jenkins (1970) ARIMA model. This optimal
210 version of ARIMA is provided via the forecast package in the R software and is more popular
211 as Automatic-ARIMA. This particular version was mainly developed to enable ease of use, and
212 provide swift, optimal forecasting results for those adopting the ARIMA method. A detailed
213 description of the algorithm and the optimality of Automatic-ARIMA can be found in Hyndman
214 and Khandakar (2008). The Automatic-ARIMA algorithm is summarized below. In doing so
215 we mainly follow Hyndman and Athanasopoulos (2013).

216 In terms of determining the number of differences d , required for the ARIMA modelling
217 process, the algorithm allows one to select this value using three different approaches; KPSS
218 unit root tests, Augmented Dickey-Fuller (ADF) test or the Phillips-Perron (PP) unit root
219 tests. It has been found that the KPSS tests lead to better forecasts in comparison to the ADF
220 and PP test when applied to the M3 competition data (Hyndman 2014). However, instead of
221 relying on these results alone, in this paper we consider modelling using all three approaches
222 and report the results based on the KPSS unit root tests from Kwiatkowski et al. (1992) for
223 the number of differences d required as it provided better forecasts for U.S. Tourist arrivals in
224 comparison to ADF and PP tests. It should also be noted that according to Hyndman and
225 Athanasopoulos (2013), when a time series is nonstationary, the Automatic-ARIMA forecasting
226 algorithm accounts for this by automatically taking first differences of the data until the data
227 are stationary.

228 Thereafter, p and q are calculated by minimizing the Akaike Information Criterion (AIC).
229 The optimal model will then be the model with the smallest AIC, and it is selected from
230 ARIMA(2, d , 2), ARIMA(0, d , 0), ARIMA(1, d , 0) and ARIMA(0, d , 1). Finally, the constant c
231 is included or set to zero in the model depending on whether $d = 0$ or whether $d \geq 1$. Log trans-
232 formations can be attained by setting lambda=0 where necessary to comply with the parametric
233 restrictions underlying ARIMA. Table 2 shows the ARIMA model parameters used for forecast-
234 ing total U.S. Tourist arrivals whilst the model parameters for forecasting U.S. Tourist arrivals
235 by country of origin are shown in Table 7.

236 **3.2 Exponential Smoothing (ETS)**

237 The ETS technique is an automatic forecasting model incorporating the foundations of expo-
238 nential smoothing and provided through the forecast package for the R software. Earlier models
239 of exponential smoothing did not provide a method for easily calculating prediction intervals
240 (Makridakis, Wheelwright, & Hyndman 1998). For a detailed description of ETS refer to Hynd-
241 man and Athanasopoulos (2013). In brief, the ETS model considers the error, trend and seasonal
242 components in choosing the best exponential smoothing model from over 30 possible options
243 by optimizing initial values and parameters using the MLE for example and selecting the best
244 model based on the AIC. As noted in Hyndman and Athanasopoulos (2013), both multiplicative

245 and additive models give the same point forecasts with varying prediction intervals. Here we
246 report the most favourable results for ETS by evaluating between point forecasts and prediction
247 intervals. The ETS model parameters for forecasting total U.S. Tourist arrivals are reported in
248 Table 3.

249 3.3 Neural Networks (NN)

250 Neural networks is a popular forecasting method for tourism demand according to the literature
251 review in Section 2. Here we use an automatic forecasting model known as nnetar and provided
252 through the forecast package in R. For a detailed explanation on how the nnetar model operates,
253 see Hyndman et al. (2013). In brief, nnetar is structured as a system of feed-forward neural
254 networks with lagged inputs and one hidden layer. The nnetar function trains 25 networks
255 by using random starting values and then obtains the average of the resulting predictions to
256 compute the forecasts. It should be noted that the simplest form of NN models contain no
257 hidden layers and are then equivalent to linear regression. One of the NN model parameters
258 are referred to as ‘weights’ and these are selected based on a loss function embedded into the
259 learning algorithm. This loss function could for example be the Mean Squared Error (MSE) or
260 even the Root Mean Squared Error (RMSE) which has been adopted for the loss function in
261 the SSA algorithm explained below. We leave the automatic NN forecasting model to select the
262 best parameters to suit the data. The resulting model parameters for NN model are shown in
263 Table 3.

264 3.4 Singular Spectrum Analysis (SSA)

265 The SSA technique is different and advantageous in comparison to classical time series methods
266 as the techniques which fall under the latter, forecasts both the signal and noise (assuming
267 that a time series consists of signal and noise) whilst SSA seeks to filter the noise and forecast
268 the signal. The univariate SSA technique has two variations known as Vector SSA (VSSA)
269 and Recurrent SSA (RSSA). In this paper we concentrate on the application of the VSSA
270 model for forecasting U.S. Tourist arrivals. The choice of VSSA over RSSA is motivated by two
271 factors. First and foremost, the total U.S. Tourist arrivals time series (see, Figure 1) shows
272 the presence of shocks around the periods of 2002 and 2008. Golyandina, Nekrutkin, and
273 Zhigljavsky (2001) found the VSSA model is more robust in comparison to the RSSA model,
274 and thus able to provide better forecasts in the presence of such shocks which create structural
275 breaks in a time series. This was later confirmed by Pepelyshev (2010) who also provides
276 a detailed comparison between recurrent and vector forecasting. Secondly, instead of solely
277 relying on Golyandina, Nekrutkin, and Zhigljavsky’s (2001) and Pepelyshev (2010) findings, we
278 compared the performance of both VSSA and RSSA models at forecasting total U.S. Tourist
279 arrivals (results are not reported here) and found no statistically significant difference in the
280 forecasting accuracy between the two methods, except at $h = 12$ steps ahead where the VSSA
281 model outperformed the RSSA model with statistically significant results. Thus, taking these
282 two factors into consideration, we selected the VSSA model as the more suitable counterpart
283 for forecasting U.S. Tourist arrivals. We use the R software to obtain the VSSA forecasts with
284 the aid of an automatic VSSA forecasting code. In brief, the SSA technique can be summarised
285 as follows. The SSA technique has two stages and two choices as mentioned in Hassani and
286 Mahmoudvand (2013). The two stages are referred to as Decomposition and Reconstruction,
287 whilst the two choices are known as the window length L and the number of eigenvalues r . Each
288 of the two stages include two separate steps known as Embedding, Singular Value Decomposition
289 (SVD) and, Grouping and Diagonal Averaging. A detailed description on the theoretical and

290 practical foundations of SSA can be found in Golyandina, Nekrutkin, and Zhigljavsky (2001)
 291 and Hassani (2007) which explains the process with examples. Here we provide a brief summary
 292 of the basic SSA process, and in doing so we mainly follow Hassani (2007).

293 Stage 1: Decomposition

294 1st step: Embedding

295 The first step is concerned with mapping a one dimensional time series $Y_N = (y_1, \dots, y_N)$
 296 into the multi-dimensional series X_1, \dots, X_K with vectors $X_i = (y_i, \dots, y_{i+L-1})' \in \mathbf{R}^L$, where
 297 $K = N - L + 1$. This process is referred to as embedding whilst the vectors X_i are called *L-lagged*
 298 *vectors*. The single choice of the embedding stage is the *window length* L , which is an integer
 299 such that $2 \leq L \leq N$. This step results in the trajectory matrix \mathbf{X} , which is also a Hankel
 300 matrix and takes the form: $\mathbf{X} = [X_1, \dots, X_K] = (x_{ij})_{i,j=1}^{L,K}$.

301 2nd step: Singular Value Decomposition (SVD)

302 Next we obtain the singular value decomposition of the trajectory matrix and represent it as
 303 a sum of rank-one bi-orthogonal elementary matrices. The eigenvalues of $\mathbf{X}\mathbf{X}'$ are denoted by
 304 $\lambda_1, \dots, \lambda_L$ in decreasing order of magnitude ($\lambda_1 \geq \dots \lambda_L \geq 0$) and by U_1, \dots, U_L the orthonor-
 305 mal system (that is, $(U_i, U_j) = 0$ for $i \neq j$ and $\|U_i\| = 1$ of the eigenvectors of the matrix $\mathbf{X}\mathbf{X}'$
 306 corresponding to these eigenvalues. Here, $\|U_i\|$ is the norm of the vector U_i , and (U_i, U_j) is the
 307 inner product of the vectors U_i and U_j . Set

$$d = \max(i, \text{ such that } \lambda_i > 0) = \text{rank } \mathbf{X}.$$

308 If we denote $V_i = \mathbf{X}'U_i/\sqrt{\lambda_i}$, then the SVD of the trajectory matrix can be written as:

$$\mathbf{X} = \mathbf{X}_1 + \dots + \mathbf{X}_d, \quad (1)$$

309 where $\mathbf{X}_i = \sqrt{\lambda_i}U_iV_i'$ ($i = 1, \dots, d$). The matrices \mathbf{X}_i are elementary matrices as they have
 310 rank 1, U_i and V_i denotes the left and right eigenvectors of the trajectory matrix. The collection
 311 $(\sqrt{\lambda_i}, U_i, V_i)$ is called the *i*-th eigentriple of the matrix \mathbf{X} , $\sqrt{\lambda_i}$ ($i = 1, \dots, d$) are the singular
 312 values of the matrix \mathbf{X} and the set $\{\sqrt{\lambda_i}\}$ is called the spectrum of the matrix \mathbf{X} . The expansion
 313 (1) is said to be uniquely defined if all the eigenvalues have a multiplicity of one.

314 Stage 2: Reconstruction

315 1st step: Grouping

316 At the first step in the second stage we split the elementary matrices \mathbf{X}_i into several groups
 317 and sum the matrices within each group. This is referred to as the grouping step. Denote
 318 $I = \{i_1, \dots, i_p\}$ as a group of indices i_1, \dots, i_p . Then the matrix \mathbf{X}_I corresponding to the group
 319 I can be defined as $\mathbf{X}_I = \mathbf{X}_{i_1} + \dots + \mathbf{X}_{i_p}$. The spilt of the set of indices $J = 1, \dots, d$ into the
 320 disjoint subsets I_1, \dots, I_m corresponds to the representation

$$\mathbf{X} = \mathbf{X}_{I_1} + \dots + \mathbf{X}_{I_m}. \quad (2)$$

321 The procedure of choosing the sets I_1, \dots, I_m is called the eigentriple grouping. For a given
 322 group I the contribution of the component \mathbf{X}_I into the expansion (1) is measured by the share
 323 of the corresponding eigenvalues: $\sum_{i \in I} \lambda_i / \sum_{i=1}^d \lambda_i$.

324 **2nd step: Diagonal Averaging**

325 Here we perform diagonal averaging in order to transform each matrix I into a time series, which
 326 is an additive component of the initial series Y_T . For example, suppose z_{ij} stands for an element
 327 of a matrix \mathbf{Z} , then the k -th term of the resulting series is obtained by averaging z_{ij} over all
 328 i, j such that $i + j = k + 2$. This procedure is also known as Hankelization of the matrix \mathbf{Z} .
 329 The output of the Hankelization of a matrix \mathbf{Z} is the Hankel matrix $\mathcal{H}\mathbf{Z}$, which is the trajectory
 330 matrix corresponding to the series obtained as a result of the diagonal averaging. In its turn,
 331 the Hankel matrix $\mathcal{H}\mathbf{Z}$ uniquely defines the series by relating the value in the diagonals to the
 332 values in the series. By applying the Hankelization procedure to all matrix components of (2),
 333 we obtain another expansion:

$$\mathbf{X} = \tilde{\mathbf{X}}_{I_1} + \dots + \tilde{\mathbf{X}}_{I_m} \quad (3)$$

334 where $\tilde{\mathbf{X}}_{I_1} = \mathcal{H}\mathbf{X}$. This is equivalent to the decomposition of the initial series $Y_N = (y_1, \dots, y_N)$
 335 into a sum of m series:

$$y_n = \sum_{k=1}^m \tilde{y}_n^{(k)} \quad (4)$$

336 where $\tilde{Y}_N^{(k)} = (\tilde{y}_1^{(k)}, \dots, \tilde{y}_N^{(k)})$ corresponds to the matrix \mathbf{X}_{I_k} .

337
 338 In the past, the selection of SSA choices of L and r , such that they are optimal, has been
 339 a major issue. However, in this paper, we use the Root Mean Squared Error (RMSE) criterion
 340 (see, Section 4.2) to determine the optimal L for decomposing the U.S. Tourist arrivals series,
 341 and the optimal r for reconstructing the less noisy series which can then be used for forecasting
 342 (it is also possible to use any other criteria for minimising the forecasting error as explained
 343 below in the forecasting algorithm). Accordingly, we look for the combination of L and r which
 344 provides the lowest RMSE, and this in turn represents the optimal decomposition and recon-
 345 struction choices for the SSA model.¹ Noteworthy is the fact that unlike with parameters of
 346 ARIMA, ETS and NN models, these optimal VSSA choices remain fixed for the respective hori-
 347 zon. The automated VSSA code is able to perform this task by evaluating all possible SSA
 348 choices for a given time series. The resulting optimal VSSA choices are presented in Table 3
 349 for total U.S. Tourist arrivals and Table 8 for U.S. Tourist arrivals by country of origin. As
 350 such, this paper adopts basic VSSA with optimal choices for forecasting U.S. Tourist arrivals.
 351 Below, we provide a concise explanation of the VSSA forecasting algorithm that has been used
 352 for obtaining forecasts for U.S. Tourist arrivals.

- 353 1. Consider a real-valued nonzero time series (for example, Total U.S. Tourist arrivals) $Y_N =$
 354 (y_1, \dots, y_N) of length N .
- 355 2. Divide the time series into two parts; $\frac{2}{3}^{rd}$ of observations for training and validating the
 356 VSSA model and $\frac{1}{3}^{rd}$ for testing the forecast accuracy. This is a common and widely
 357 accepted practice, visible in a majority of forecasting algorithms.
- 358 3. Use the training data to construct the trajectory matrix $\mathbf{X} = (x_{ij})_{i,j=1}^{L,K} = [X_1, \dots, X_K]$,
 359 where $X_j = (y_j, \dots, y_{L+j-1})^T$ and $K = N - L + 1$. Initially, we begin with $L = 2$ ($2 \leq L \leq$
 360 $\frac{N}{2}$) and in the process, evaluate all possible values of L for Y_N .
- 361 4. Obtain the SVD of \mathbf{X} by calculating $\mathbf{X}\mathbf{X}^T$ for which $\lambda_1, \dots, \lambda_L$ denotes the eigenvalues in
 362 decreasing order ($\lambda_1 \geq \dots \lambda_L \geq 0$) and by U_1, \dots, U_L the corresponding eigenvectors. The
 363 output of this stage is $\mathbf{X} = \mathbf{X}_1 + \dots + \mathbf{X}_L$ where $\mathbf{X}_i = \sqrt{\lambda_i} U_i V_i^T$ and $V_i = \mathbf{X}^T U_i / \sqrt{\lambda_i}$.

¹The optimal SSA code used in this study is available upon request.

- 364 5. Evaluate all possible combinations of r ($1 \leq r \leq L - 1$) singular values (step by step) for
365 the selected L and split the elementary matrices \mathbf{X}_i ($i = 1, \dots, L$) into several groups and
366 sum the matrices within each group.
- 367 6. Perform diagonal averaging to transform the matrix with the selected r singular values into
368 a Hankel matrix which can then be converted into a time series (the steps up to this stage
369 filters the noisy series). The output is a filtered series that can be used for forecasting.
- 370 7. Set $v^2 = \pi_1^2 + \dots + \pi_r^2$, where π_i represents the final component of the eigenvector U_i
371 ($i = 1, \dots, r$). Assume that, $e_L = (0, 0, \dots, 1)$ is not a component of the linear space \mathfrak{L}_r ,
372 which implies \mathfrak{L}_r is not a vertical space.
- 373 8. Consider the matrix $\Pi = \mathbf{V}^\nabla (\mathbf{V}^\nabla)^T + (1-v^2)AA^T$, where $A = (a_1, \dots, a_{L-1}) = \sum_{i=1}^r \pi_i U_i^\nabla / (1-$
374 $v^2)$ and $\mathbf{V}^\nabla = [U_1^\nabla, \dots, U_r^\nabla]$, where \mathbf{V}^∇ is the first $L - 1$ components.
- 375 9. Next, consider the linear operator $\theta^{(v)} : \mathfrak{L}_r \mapsto \mathbf{R}^L$, where $\theta^{(v)}U = \begin{pmatrix} \Pi U^\nabla \\ A^T U^\nabla \end{pmatrix}$.
10. Then, define vector Z_i after grouping and eliminating noise components, such that

$$Z_i = \begin{cases} \tilde{X}_i & \text{for } i = 1, \dots, K \\ \theta^{(v)} Z_{i-1} & \text{for } i = K + 1, \dots, K + h + L - 1, \end{cases}$$

376 where, \tilde{X}_i 's are the reconstructed columns of the trajectory matrix.

- 377 11. Construct the matrix $\mathbf{Z} = [Z_1, \dots, Z_{K+h+L-1}]$ and perform diagonal averaging to obtain
378 a new series $y_1, \dots, y_{N+h+L-1}$, where y_{N+1}, \dots, y_{N+h} forms the h terms of the SSA Vector
379 forecast.
- 380 12. Define a loss function \mathcal{L} .
- 381 13. When forecasting a series Y_N h -step ahead, the forecast error is minimised by setting
382 $\mathcal{L}(X_{K+h} - \hat{X}_{K+h})$ where the vector \hat{X}_{K+h} contains the h -step ahead forecasts obtained
383 using the VSSA forecasting algorithm.
- 384 14. Find the combination of L and r which minimises \mathcal{L} and thus represents the optimal VSSA
385 choices.
- 386 15. Finally use the optimal L to decompose the series comprising of the training and validation
387 set, and then select r singular values for reconstructing the less noisy time series, and use
388 this newly reconstructed series for forecasting the remaining $\frac{1}{3}^{rd}$ observations.

389 4 The Data and Measures for Evaluating Forecast Accuracy

390 4.1 The Data

391 This study uses monthly U.S. Tourist arrivals data from January 1996 to November 2012 ob-
392 tained via the U.S. Department of Commerce: Office of Travel & Tourism Industries². Table
393 1 provides some descriptives of the data. According to the data, average total tourist arrivals
394 into the U.S. between January 1996-November 2012 has been 3,798,000. The maximum number

²<http://travel.trade.gov/research/monthly/arrivals/>

395 of tourist arrivals during the period of concern is recorded at 7,249,000 in July 2012, whilst the
396 minimum number of arrivals was 2,096,000 in November 2001. On average, the lowest tourist ar-
397 rivals into U.S. has been recorded from Africa whilst Canada accounts for an average of 1,346,000
398 tourist arrivals which is the highest influx of tourists into U.S. from a single country. The skew-
399 ness statistic indicates that all time series analysed in this study are in fact skewed and not
400 normally distributed. An analysis of the kurtosis suggests that all the series have Platykurtic
401 distributions except for Italy which has a Leptokurtic distribution. Accordingly, this informa-
402 tion tells us that the Italian time series for tourist arrivals into U.S. has a high probability for
403 extreme values with thicker tails and values concentrated around the mean whilst all other time
404 series for U.S. Tourist arrivals have a lesser probability for extreme values in comparison to a
405 normal distribution and consist of values which have a wider spread around the mean. In order
406 to confirm the information provided through the skewness and kurtosis statistics, the data was
407 tested for normality using the Shapiro-Wilk test. Accordingly, it was found that Western Eu-
408 rope, Total Overseas, Asia and Central America were in fact normally distributed at a p -value
409 of 0.05, and that the skewness indicators are thus reliable. Note that in this paper we have
410 evaluated the ARIMA model with and without log transformations and the results reported in
411 the next section are the best outcomes.

Table 1: Descriptive statistics: U.S. Tourist arrivals (Jan. 1996 - Nov. 2012).

Series	Mean	Median	Min.	Max.	Std. Dev.	Skewness	Kurtosis
Total Arrivals	3798000	3590000	2096000	7249000	994944	0.86	0.63
<i>Arrivals by country</i>							
Canada	1346000	1261000	727300	2945000	417184	1.07	1.24
Mexico	491100	381000	67960	1668000	338299	1.25	0.74
Total Overseas	1961000	1922000	1119000	3089000	382831	0.34	-0.06
Western Europe	859700	848600	418800	1320000	187797	0.14	-0.50
Eastern Europe	39000	36170	17610	76360	11875	0.69	-0.15
Asia	550500	547200	246500	934300	106414	0.22	0.71
Middle East	51600	48300	22930	120200	17996	1.09	1.21
Africa	22870	22540	7869	48080	6863	0.63	0.51
Oceania	65190	65190	28090	165600	23470	1.26	1.81
South America	215200	212100	98580	420300	68877	0.62	0.06
Central America	59510	58540	29730	91860	12097	0.29	-0.14
Caribbean	97440	90670	48330	191100	31712	1.05	0.51
France	86290	78450	36920	201800	31954	1.20	1.39
Germany	136800	132800	54920	235600	39695	0.24	-0.80
Italy	51460	45640	17170	157400	23127	1.88	4.65
Netherlands	41180	38950	20340	90430	12554	1.26	2.23
Spain	36260	30700	13110	104600	16651	1.40	2.15
Sweden	25820	24780	11070	51560	7680	0.84	0.96
Switzerland	29090	27630	13270	74220	10514	1.24	2.22
United Kingdom	338400	349300	164300	475400	64735	-0.46	-0.34
Japan	331200	317700	141600	549100	80225	0.36	-0.21
South Korea	62490	56080	19510	130300	22956	0.78	0.24
PRC & Hongkong	46480	38570	11480	207000	28966	2.63	8.62
ROC (Taiwan)	27830	26480	9451	63400	10223	0.90	0.70
Australia	51380	44940	21000	142400	20462	1.41	2.17
Argentina	30780	31230	9279	64240	13845	0.25	-1.00
Brazil	65960	57190	18680	171000	34633	1.11	0.69
Colombia	31810	29370	11110	74670	12050	0.79	0.18
Venezuela	39330	36370	15780	86160	14841	0.93	0.51

412 Next we test the U.S. Tourist arrivals series for unit root problems as certain external shocks
413 such as recessions (for example) are infamous for making a time series nonstationary in mean
414 and variance, thereby creating a structural break in the series. In Table 2 we report the findings
415 from the Bai and Perron (2003) test for structural breaks in the U.S. Tourist arrivals series.
416 Whilst analysing the causes and reasons behind these structural breaks are beyond the mandate
417 of this paper, we will endeavour to outline certain observations. Firstly, we can see that based
418 on the Bai and Perron (2003) test, the time series relating to tourist arrivals from the Caribbean
419 is the only series that has not been affected by structural breaks. Secondly, except for Canada,
420 Mexico, Africa, Central America, Germany, Italy, Spain, United Kingdom, South Korea, PRC
421 & Hongkong, Australia and Colombia, all other time series considered in this study are affected
422 by a structural break in the year 2001. Interestingly, there are no structural breaks captured
423 beyond 2010, and thus we conclude that either the impact of 9/11 is masked in this level of
424 data, or its impact was not major enough to cause a structural break in tourist arrivals into U.S.
425 The National Bureau of Economic Research ³. reports the U.S. experienced its last recession

³<http://www.nber.org/cycles/cyclesmain.html>

426 beginning December 2007. The Bai and Perron (2003) test shows there has been a lagged impact
427 of this recession on U.S. Tourist arrivals with all series reporting structural breaks in 2010 with
428 the exception of Mexico, Western Europe, Africa, Germany, Italy, Netherlands, Spain, United
429 Kingdom, Japan, ROC (Taiwan), Colombia and Venezuela. Finally, in terms of U.S. Tourist
430 arrivals by country of origin, the most number of structural breaks visible in a time series is
431 seen in tourist arrivals from Brazil. The presence of structural breaks would also enable us to
432 ascertain the sensitiveness of the various forecasting techniques to such break points.

Table 2: Break points in U.S. Tourist arrivals time series.

Series	Structural Break
Total Arrivals	2001(8), 2004(3), 2007(2), 2010(2)
<i>Arrivals by country</i>	
Canada	2006(2), 2010(2)
Mexico	1998(6), 2006(3), 2009(12)
Total Overseas	2001(8), 2004(3), 2007(4), 2010(4)
Western Europe	2001(8), 2004(2), 2007(6)
Eastern Europe	2001(8), 2004(5), 2007(4), 2010(5)
Asia	2001(8), 2004(4), 2010(4)
Middle East	1999(2), 2001(8), 2006(5), 2010(5)
Africa	2008(4)
Oceania	2001(9), 2004(3), 2007(3), 2010(4)
South America	2001(8), 2007(5), 2010(5)
Central America	1998(6), 2001(8), 2007(4)
Caribbean	No structural break in series.
France	2001(8), 2007(3), 2010(3)
Germany	2000(10), 2007(2)
Italy	2007(6)
Netherlands	2001(8), 2007(3)
Spain	2007(5)
Sweden	2001(6), 2004(2), 2007(2), 2010(5)
Switzerland	2001(7), 2007(3), 2010(5)
United Kingdom	1998(6), 2008(12)
Japan	2001(8)
South Korea	2005(4), 2010(4)
PRC & Hongkong	2007(4), 2010(5)
ROC (Taiwan)	2001(8)
Australia	2005(4), 2010(4)
Argentina	2001(8), 2006(12), 2010(4)
Brazil	1999(1), 2001(7), 2005(4), 2007(11), 2010(5)
Colombia	2009(5)
Venezuela	2001(12), 2007(6)

433 In Table 3 we present the model parameters (SSA choices) for each of the forecasting tech-
434 niques considered in this study for forecasting total U.S. Tourist arrivals at horizons of $h = 1$,
435 3, 6 and 12 months ahead. It is important to note that each of the techniques have chosen the

436 model parameters (SSA choices) automatically using the respective algorithms (see, Section 3)
 437 to provide the best possible modelling and forecast for U.S. Tourist arrivals.

Table 3: Forecasting model parameters for total U.S. Tourist arrivals.

h	ARIMA	ETS(α, σ)	NN(p, P, k)	SSA(L, r)
1	order(2,0,1)seasonal(1,1,2)	(0.87,0.1245) ^M	NNAR(2,1,1)	(38,17)
3	order(2,0,1)seasonal(1,1,2)	(0.86,0.1241) ^M	NNAR(2,1,1)	(25,14)
6	order(2,0,1)seasonal(1,1,2)	(0.87,0.1239) ^M	NNAR(2,1,1)	(29,21)
12	order(2,0,1)seasonal(1,1,2)	(0.86,0.1231) ^M	NNAR(2,1,1)	(15,6)
24	order(2,0,1)seasonal(1,1,2)	(0.85,0.1586) ^M	NNAR(2,1,1)	(40,25)
36	order(2,0,1)seasonal(1,1,2)	(0.92,0.1531) ^M	NNAR(2,1,1)	(48,6)

Note:^M is an ETS model with multiplicative seasonality.

p is the number of lagged inputs, P is the automatically selected value for seasonal time series, and k is the number of nodes in the hidden layer. L is the window length and r is the number of eigenvalues.

438 Next, we consider the VSSA decompositions which is an integral part of the SSA process.
 439 The weighted correlation (w -correlation) statistic is used to show the appropriateness of the
 440 various decompositions achieved by SSA (see, Table 3 and Table 7). As mentioned in Golyand-
 441 ina, Nekrutkin, and Zhigljavsky (2001), the w -correlation statistic which shows the dependence
 442 between two time series can be calculated as:

$$\rho_{12}^{(w)} = \frac{\left(Y_N^{(1)}, Y_N^{(2)} \right)_w}{\| Y_N^{(1)} \|_w \| Y_N^{(2)} \|_w},$$

443 where $Y_N^{(1)}$ and $Y_N^{(2)}$ are two time series, $\| Y_N^{(i)} \|_w = \sqrt{\left(Y_N^{(i)}, Y_N^{(i)} \right)_w}$, $\left(Y_N^{(i)}, Y_N^{(j)} \right)_w = \sum_{k=1}^N w_k y_k^{(i)} y_k^{(j)}$
 444 ($i, j = 1, 2$), $w_k = \min\{k, L, N - k\}$ (here, assume $L \leq N/2$).
 445

446 Accordingly, if the w -correlation between two reconstructed components are close to 0, this
 447 implies that the corresponding series are w -orthogonal and in turn we know the two components
 448 are well separable (Hassani, Heravi, & Zhigljavsky, 2009). In Table 3 we calculate the w -
 449 correlations for all the decompositions by comparing the two components of signal and noise.
 450 Here, we use as signal the reconstructed series containing optimal r components and select the
 451 remaining r (which does not belong to the reconstruction) as noise. The results indicate that all
 452 w -correlations are close to 0 which in turn suggests that we have achieved a sound decomposition
 453 using the VSSA forecasting algorithm (see, Section 3.4). In other words, these w -correlations
 454 indicate that the VSSA forecasting algorithm works exceedingly well at separating the noise
 455 from the signal.

Table 4: W -correlations between signal and residuals for U.S. Tourist arrivals.

Series	1	3	6	12	24	36
Total U.S. Tourist Arrivals	0.007	0.009	0.009	0.012	0.008	0.009
<i>U.S. Tourist Arrivals by country</i>						
Canada	0.013	0.010	0.010	0.028	0.010	0.012
Mexico	0.020	0.020	0.021	0.047	0.032	0.035
Total Overseas	0.009	0.009	0.014	0.014	0.008	0.006
Western Europe	0.010	0.014	0.015	0.019	0.024	0.012
Eastern Europe	0.020	0.016	0.014	0.015	0.022	0.020
Asia	0.008	0.008	0.008	0.017	0.007	0.006
Middle East	0.027	0.047	0.044	0.029	0.022	0.024
Africa	0.019	0.020	0.015	0.031	0.013	0.010
Oceania	0.010	0.009	0.014	0.018	0.007	0.007
South America	0.012	0.019	0.023	0.016	0.020	0.023
Central America	0.013	0.016	0.014	0.021	0.012	0.016
Caribbean	0.021	0.021	0.031	0.051	0.034	0.019
France	0.014	0.027	0.040	0.015	0.014	0.015
Germany	0.015	0.015	0.014	0.017	0.017	0.017
Italy	0.026	0.026	0.026	0.016	0.035	0.024
Netherlands	0.016	0.018	0.018	0.018	0.027	0.014
Spain	0.030	0.014	0.031	0.027	0.016	0.027
Sweden	0.012	0.012	0.012	0.011	0.012	0.017
Switzerland	0.024	0.016	0.021	0.017	0.020	0.020
United Kingdom	0.013	0.016	0.015	0.013	0.012	0.016
Japan	0.009	0.015	0.008	0.009	0.07	0.012
South Korea	0.016	0.016	0.012	0.016	0.016	0.012
PRC & Hongkong	0.025	0.051	0.022	0.030	0.025	0.022
ROC (Taiwan)	0.019	0.031	0.025	0.025	0.015	0.015
Australia	0.011	0.011	0.011	0.011	0.011	0.011
Argentina	0.028	0.010	0.046	0.029	0.007	0.010
Brazil	0.025	0.023	0.026	0.027	0.030	0.027
Colombia	0.022	0.023	0.012	0.012	0.019	0.038
Venezuela	0.026	0.025	0.026	0.046	0.022	0.021

456 The U.S. Tourist arrivals series exhibits several seasonal patterns. In order to illustrate
457 SSA's capabilities at extracting various seasonal patterns in U.S. Tourist arrivals we present in
458 Figure 2, as an example, the in-sample decomposition of total U.S. Tourist arrivals at $h = 1$ step
459 ahead. Firstly we can observe the extracted trend in U.S. Tourist arrivals which corresponds
460 with the total arrivals pattern and clearly shows the general trend of increase and decreasing
461 tourist arrivals over time. Also interesting is the difference between the four month and twelve
462 month seasonal components. The 4 month seasonal component is increasing over time whilst
463 the 12 month seasonal component is seen to be decreasing over time. Furthermore, there is more
464 fluctuation in the 4 month seasonal component of total U.S. Tourist arrivals in comparison to
465 the 12 month component.

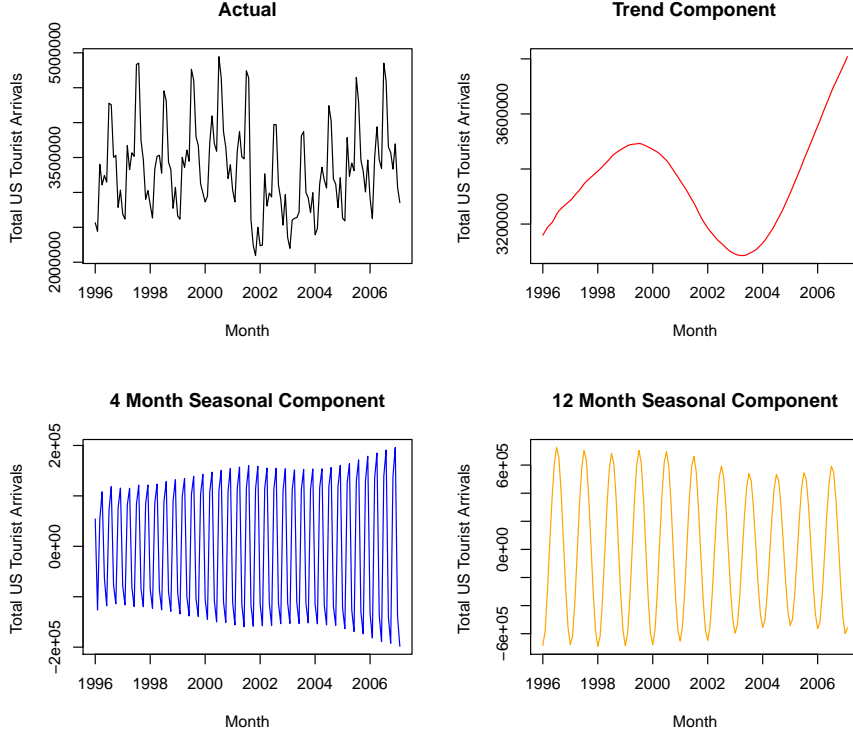


Figure 2: In-sample SSA decomposition of total monthly U.S. Tourist arrivals at $h = 1$ step ahead.

4.2 Measures for Evaluating the Forecast Accuracy

Root Mean Squared Error (RMSE)

The RMSE is now a popular measure for forecast accuracy, and is one of the most frequently cited measures in forecasting literature (see, for example, Zhang et al. 1998; Hassani, Heravi, & Zhigljavsky, 2009; Hassani, Heravi, & Zhigljavsky, 2012; Hassani & Mahmoudvand, 2013; Beneki & Silva, 2013). Here, in order to save space, we only provide the RMSE ratios of SSA to that of ETS:

$$\text{RRMSE} = \frac{SSA}{ETS} = \frac{\left(\sum_{i=1}^N (\hat{y}_{T+h,i} - y_{T+h,i})^2\right)^{1/2}}{\left(\sum_{i=1}^N (\tilde{y}_{T+h,i} - y_{T+h,i})^2\right)^{1/2}},$$

where, \hat{y}_{T+h} is the h -step ahead forecast obtained by SSA, \tilde{y}_{T+h} is the h -step ahead forecast from the ETS model, and N is the number of the forecasts. If $\frac{SSA}{ETS}$ is less than 1, then the SSA outperforms ETS by $1 - \frac{SSA}{ETS}$ percent.

Mean Absolute Percentage Error (MAPE)

The MAPE measure is also quoted in this paper as it is a widely understood criterion for evaluating forecast accuracy. In brief, the lower the MAPE result, the better the forecast.

$$\text{MAPE} = \frac{1}{N} \sum_{t=1}^N \left| 100 \times \frac{y_{T+h} - \hat{y}_{T+h,i}}{y_{T+h}} \right|,$$

where y_{T+h} represents the actual data corresponding to the h step ahead forecast, and $\hat{y}_{T+h,i}$ is the h step ahead forecasts obtained from a particular forecasting model.

Direction of Change (DC)

The DC criterion is a measure of the percentage of forecasts that accurately predict the direction of change (Hassani, Heravi, & Zhigljavsky, 2012). DC is an equally important measure, as the RMSE, for evaluating the forecasting performance of tourism demand models, because it is important that for example, when the actual series is illustrating an upwards trend, the forecast is able to predict that upward trend and vice versa. Here, the concept of DC is explained in brief, and in doing so we mainly follow Hassani, Heravi, and Zhigljavsky (2012). In the univariate case, for forecasts obtained using X_T , let D_{X_i} be equal to 1 if the forecast is able to correctly predict the actual direction of change and 0 otherwise. Then, $\tilde{D}_X = \sum_{i=1}^n D_{X_i}/n$ shows the proportion of forecasts that correctly identify the direction of change in the actual series.

5 Empirical Results

We select $\frac{2}{3}^{rd}$ of the data as in-sample for model training and validation, and set aside $\frac{1}{3}^{rd}$ of the data as out-of-sample for evaluating the forecasting accuracy. The data was forecasted at horizons of $h = 1, 3, 6, 12, 24$ and 36 steps ahead which corresponds to $1, 3, 6, 12, 24$ and 36 month ahead forecasts. These forecasting horizons have been considered because for the tourism industry, horizons beyond 12 months are considered to be long term. Moreover, both short and long run forecasts are vital for this sector as a country needs to be geared to accommodate the tourists and planning of large scale building works or the purchase of new aircrafts for example would require managerial decisions to be made well in advance. Therefore, in this paper we are effectively evaluating the performance of the forecasting models both in the short and long run in terms of obtaining forecasts for U.S. tourist arrivals. We first analyse total U.S. Tourist arrivals. Table 5 reports the RMSE and MAPE results for the out-of-sample forecasts of total U.S. Tourist arrivals using SSA, ARIMA, ETS and NN. In order to ensure the parametric models are correctly specified we carried out a Ljung-Box test on the residuals for autocorrelation and the results indicated that the residuals are independently distributed at a p -value of 0.05, and are there for not autocorrelated.

Table 5: Out-of-sample RMSE(MAPE) results for total U.S. Tourist arrivals.

h	ARIMA	ETS	NN	SSA	$\frac{SSA}{ARIMA}$	$\frac{SSA}{ETS}$	$\frac{SSA}{NN}$
1	601512 (9%)	760599 (13%)	1147080 (19%)	242601(4%)	0.40*	0.32*	0.21*
3	720751 (11%)	723556 (13%)	1124242 (19%)	316049(6%)	0.44*	0.44*	0.28*
6	738630 (12%)	1037666 (20%)	1180780 (19%)	445614(8%)	0.60*	0.43*	0.38*
12	937129 (14%)	1097366 (17%)	1385339 (23%)	517912(9%)	0.55*	0.47*	0.37*
24	1136616 (19%)	1300442 (20%)	1780513 (30%)	526323(9%)	0.46*	0.40*	0.30*
36	1002685 (17%)	1149585 (18%)	1684799 (24%)	605448(9%)	0.60*	0.53*	0.36*
Average	856221 (14%)	1011536 (17%)	1383792 (22%)	442325(8%)	0.52	0.44	0.32

Note: * indicates results are statistically significant based on Diebold-Mariano at $p = 0.05$.

507 Firstly, based on the MAPE criterion reported in Table 5, we can see that the Neural
508 Network model is the worst performer at all horizons with an overall average MAPE of 22% at
509 forecasting total U.S. Tourist arrivals. Interestingly the SSA technique is the only model which
510 reports MAPE values below 10% at all horizons and is in turn the model providing the most
511 accurate forecasts for total U.S. Tourist arrivals with a comparatively low average MAPE of
512 8%. Based on the MAPE we are also able to identify that the ARIMA model is the second best
513 model for forecasting total U.S. Tourist arrivals as its average MAPE of 14% is lower than the
514 ETS model's average MAPE of 17%. Moreover, it is interesting to note that the SSA model's
515 MAPE remains approximately constant over the forecasting horizons of $h = 12, 24$ and 36
516 months ahead, and thereby portrays SSA's capabilities of providing comparatively stable and
517 more accurate forecasts in the long run. The remainder of the analysis focusses on the RMSE
518 criterion for evaluating forecast accuracy.

519 It is evident from Table 5 that based on the RMSE criterion, SSA outperforms ARIMA, ETS
520 and Neural Networks comfortably by recording the lowest forecasting error for total U.S. Tourist
521 arrivals at all horizons. The RRMSE statistic shows that SSA is 60%, 56%, 40%, 45%, 54% and
522 40% better than ARIMA at forecasting total U.S. Tourist arrivals at $h = 1, 3, 6, 12, 24$ and 36
523 months ahead respectively. Likewise, in comparison to ETS, SSA is 68%, 56%, 57%, 53%, 60%
524 and 47% better at $h = 1, 3, 6, 12, 24$ and 36 steps ahead respectively. Finally we analyse the
525 forecasting results between SSA and the Neural Network model. Accordingly we can conclude
526 that the SSA model is 79%, 72%, 62%, 63%, 70% and 64% better than the feed-forward Neural
527 Network model at $h = 1, 3, 6, 12, 24$ and 36 months ahead respectively.

528 In order to ensure the results reported are not chance occurrences, we test the results further
529 for statistical significance using the modified Diebold-Mariano test found in Harvey, Leybourne,
530 and Newbold (1997). We find that all the RRMSE results are statistically significant at all
531 horizons and thus provides concrete evidence for the inferences we have made. Finally, from
532 Table 5 we can infer that when forecasting total U.S. Tourist arrivals, on average, the SSA
533 model is 48% better than ARIMA, 56% better than ETS and 68% better than Neural Networks
534 based on the forecasting accuracy. Note that initially, we normalized the data (where necessary)
535 and evaluated the forecasting accuracy of the ARIMA model. ARIMA's forecasting results for
536 U.S. Tourist arrivals were adversely affected following the data transformations. As such we
537 have reported the results sans transformation which gives the best possible outcome for the
538 ARIMA forecasts. The results from Table 5 also show that on average, ARIMA provides a
539 better forecasting accuracy in comparison to ETS and Neural Networks for U.S. Tourist arrivals
540 forecasting both in the short and long run, and is therefore chosen to be the second best model in
541 general for this purpose. The feed-forward Neural Network model with one hidden layer provides
542 the least favourable forecasts for total U.S. Tourist arrivals.

543 Thereafter, we use the Direction of Change (DC) criterion to evaluate the extent to which
544 the forecasts from all models are able to predict the actual direction of change in total U.S.
545 Tourist arrivals. Table 6 presents the DC results. Firstly, we see that only three outcomes are
546 in fact statistically significant for DC. However, based on the criterion itself we could infer that
547 SSA provides a more accurate prediction of direction of change in comparison to ARIMA at all
548 horizons when forecasting total U.S. Tourist arrivals, and on average, SSA is able to provide a
549 83% accurate direction of change prediction whilst ARIMA can only provide a 63% accurate
550 prediction of the direction of change in total U.S. Tourist arrivals. Likewise, in comparison to
551 both ETS and Neural Networks, SSA provides a better prediction of the direction of change
552 at all horizons. However ETS outperforms the ARIMA model in terms of DC at $h = 3$ and
553 24 months ahead and the DC predictions of the NN model is better than ETS at $h = 12$ and
554 24 steps ahead. Furthermore, at 36 steps ahead the SSA model obtains 100% accurate DC

555 predictions whilst ARIMA is able to report a significant 91% accuracy. Thus, it is clear that
 556 the SSA model stands out as the most superior model for forecasting total U.S. Tourist arrivals
 557 at all horizons based on the RMSE, RRMSE and DC criterions in comparison to the optimal
 558 ARIMA, ETS and Neural Network models. Furthermore, it is clear that the SSA model can
 559 pick up both short and long run fluctuations in total U.S. Tourist arrivals comparatively better
 560 than ARIMA, ETS and Neural Networks.

Table 6: Direction of change results for total U.S. tourist arrivals forecasts.

h	ARIMA	ETS	NN	SSA
1	0.74*	0.57*	0.48	0.87
3	0.70	0.73	0.57	0.85**
6	0.67	0.63	0.56	0.81
12	0.47	0.36	0.45	0.66
24	0.30	0.52	0.63	0.78*
36	0.91**	0.56	0.56	1.00*
Average	0.63	0.56	0.54	0.83

Note:* indicates results are statistically significant based on a t-test at $p = 0.05$.

** indicates results are statistically significant based on a t-test at $p = 0.10$.

561 As an example of the out-of-sample forecasting capabilities of the selected models, and also
 562 to show the accuracy of the DC results, in Figure 2 we provide a graphical representation of the
 563 forecasting results at $h = 24$ steps ahead for total U.S. Tourist arrivals. It is further evident from
 564 Figure 2 that both ETS and NN models experience great difficulty in picking up the seasonal
 565 fluctuations seen in the U.S. Tourist arrivals time series and that the NN model is indeed the
 566 worst performer in this case. The results from both Tables 5 and 6 proves that as the horizon
 567 increases from 1 month ahead to 24 months ahead, the forecasting performance of the parametric
 568 model (ARIMA), ETS and NN worsens immensely in comparison to that of the nonparametric
 569 model of SSA.

570 The initial results guided our interest towards evaluating the use of SSA for forecasting
 571 U.S. Tourist arrivals by country of origin. The total U.S. Tourist arrivals forecasting results
 572 show ARIMA to be the second best forecasting model in comparison to SSA, ETS and Neural
 573 Networks. As such, here we employ ARIMA as our benchmark as it is evident that ETS and
 574 feed-forward Neural Networks cannot provide accurate forecasts in this case. In Tables 7 and 8
 575 we present the ARIMA parameters and Vector SSA choices which were used for forecasting U.S.
 576 Tourist arrivals by country of origin. It should be noted that we have once again reviewed the
 577 models are correctly specified via a Ljung-Box test for the independent distribution of residuals.
 578 Where the residuals were not found to be independently distributed (for example, in some cases
 579 at $h = 24$ and 36 steps ahead this issue was experienced) we redefined the model parameters
 580 to ensure the model specification was valid. In most cases the test results indicated that the
 581 residuals were white noise at a p -value of 0.05, and that no further model review was required.

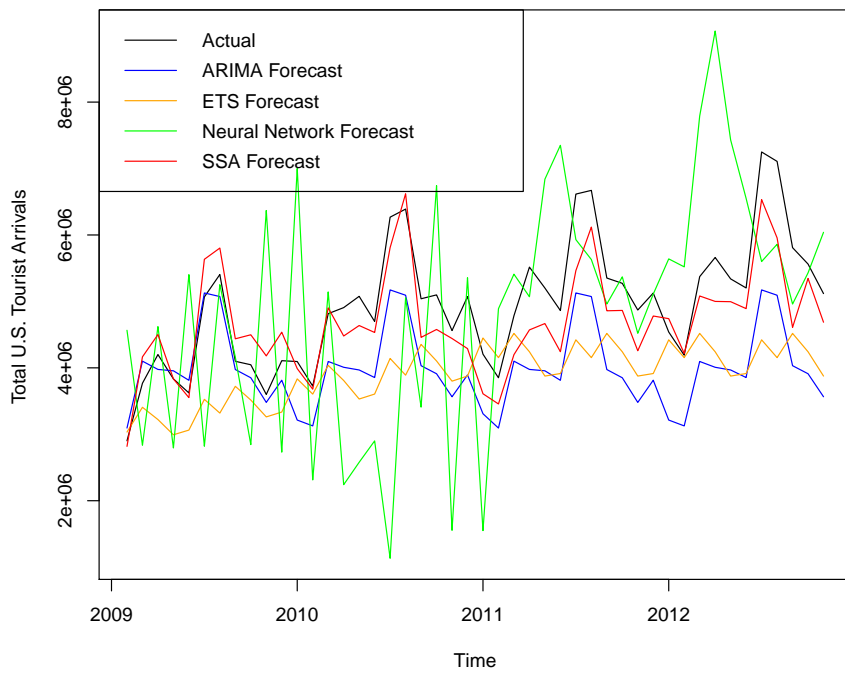


Figure 3: 24 months ahead forecast for U.S. Tourist arrivals (Feb. 2009 - Nov. 2012).

Table 7: U.S. Tourist arrivals by country of origin - ARIMA model parameters.

Series	1	3	6	12	24	36
Canada	(0,0,1) S	(3,0,3) S	(0,0,2) S	(0,0,2) S	(0,0,2) S	(0,0,2) S
Mexico	(1,1,3) S	(2,1,3) S	(2,1,3) S	(0,1,3) S	(2,1,3) S	(1,1,3) S
Total Overseas	(0,1,1) $S1$	(0,1,1) $S1$	(0,1,1) $S1$	(0,1,1) $S1$	(0,1,1) $S1$	(0,1,1) $S1$
Western Europe	(1,0,0) $S2$	(1,0,0) $S2$	(1,0,0) $S2$	(1,0,0) $S2$	(1,0,0) $S2$	(1,0,0) $S2$
Eastern Europe	(2,0,1) $S3$	(2,0,1) $S3$	(2,1,2) $S3$	(3,1,2) $S3$	(3,1,2) $S3$	(3,1,2) $S3$
Asia	(0,1,0) $S2$	(0,1,0) $S2$	(0,1,0) $S2$	(0,1,0) $S2$	(0,1,0) $S2$	(0,1,0) $S2$
Middle East	(2,0,1) $S1$	(2,0,1) $S1$	(2,0,1) $S1$	(2,0,1) $S1$	(2,0,1) $S1$	(2,0,1) $S1$
Africa	(2,0,3) $S4$	(2,0,3) $S4$	(2,0,3) $S4$	(2,0,3) $S4$	(2,0,3) $S4$	(2,0,3) $S4$
Oceania	(3,0,3) $S5$	(3,0,3) $S5$	(3,0,3) $S5$	(5,1,3) $S5$	(5,1,3) $S5$	(2,1,5) $S5$
South America	(0,1,2) $S2$	(0,1,2) $S2$	(0,1,2) $S2$	(1,1,2) $S2$	(1,1,2) $S2$	(1,1,2) $S2$
Central America	(2,1,1) $S6$	(2,1,1) $S6$	(2,1,1) $S6$	(2,1,1) $S6$	(2,1,1) $S6$	(2,1,1) $S6$
Caribbean	(0,0,1) $S3$	(0,0,1) $S3$	(0,0,1) $S3$	(0,0,1) $S3$	(0,0,1) $S3$	(0,0,1) $S3$
France	(1,1,1) $S1^\dagger$	(1,1,1) $S1^\dagger$	(1,1,1) $S1^\dagger$	(1,1,1) $S1^\dagger$	(2,0,0) $S1^\dagger$	(2,0,0) $S1^\dagger$
Germany	(2,1,3) $S1$	(2,1,4) $S1$	(2,1,4) $S1$	(2,1,4) $S1$	(4,1,5) $S1$	(2,1,4) $S1$
Italy	(2,0,2) $S3^\dagger$	(2,0,2) $S3^\dagger$	(2,0,2) $S3^\dagger$	(2,0,1) $S3^\dagger$	(2,0,5) $S3^\dagger$	(2,1,1) $S3^\dagger$
Netherlands	(4,0,4) $S4$	(4,0,4) $S4$	(4,0,5) $S4$	(4,0,4) $S4$	(4,0,4) $S4$	(4,0,4) $S4$
Spain	(3,0,3) $S5$	(3,0,3) $S5$	(2,1,3) $S5$	(2,1,1) $S5$	(1,1,1) $S5$	(1,1,1) $S5$
Sweden	(2,1,1) $S3$	(2,1,1) $S3$	(2,1,1) $S3$	(2,1,1) $S3$	(2,1,3) $S3$	(2,1,4) $S3$
Switzerland	(5,1,4) $S1$	(5,1,4) $S1$	(3,1,3) $S1$	(5,1,4) $S1$	(5,1,4) $S1$	(4,1,2) $S1$
UK	(2,1,4) $S2$	(0,1,3) $S2$	(1,1,1) $S2$	(2,1,5) $S2$	(2,1,5) $S2$	(2,1,4) $S2$
Japan	(2,1,2) $S2$	(3,1,4) $S2$	(2,1,2) $S2$	(2,1,2) $S2$	(2,1,2) $S2$	(2,1,5) $S2$
South Korea	(1,0,1) $S7^\dagger$	(1,0,1) $S7^\dagger$	(0,1,0) $S7^\dagger$	(0,1,0) $S7^\dagger$	(0,1,0) $S7^\dagger$	(0,1,0) $S7^\dagger$
PRC & Hongkong	(1,0,0) $S1^*$	(1,0,0) $S1^*$	(1,0,0) $S1^*$	(1,0,0) $S1$	(1,1,1) $S1$	(1,1,1) $S1$
ROC	(4,1,2) $S1$	(4,1,2) $S1$	(4,1,2) $S1$	(5,1,2) $S1$	(5,1,3) $S1$	(4,1,3) $S1$
Australia	(4,1,5) $S8$	(3,1,4) $S8$	(4,1,5) $S8$	(4,1,5) $S8$	(3,1,4) $S8$	(3,1,2) $S8$
Argentina	(1,1,1) $S2$	(1,1,1) $S2$	(1,1,1) $S2$	(1,1,1) $S2$	(1,1,1) $S2$	(1,1,1) $S2$
Brazil	(1,1,2) $S2$	(2,1,5) $S2$	(2,1,5) $S2$	(4,1,5) $S2$	(4,1,5) $S2$	(2,1,3) $S2$
Colombia	(2,0,4) $S7$	(2,0,5) $S7$	(4,0,4) $S7$	(4,0,3) $S7$	(3,0,3) $S7$	(2,1,3) $S7$
Venezuela	(3,1,1) $S5$	(3,1,1) $S5$	(3,1,5) $S5$	(3,1,1) $S5$	(3,1,1) $S5$	(3,1,1) $S5$

Note: S =seasonal(0,1,2), $S1$ =seasonal(2,0,2), $S2$ =seasonal(2,0,1), $S3$ =seasonal(1,1,2),

$S4$ =seasonal(2,1,2), $S5$ =seasonal(1,1,1), $S6$ =seasonal(2,1,1), $S7$ =seasonal(2,1,0), $S8$ =seasonal(0,1,1).

† ARIMA with drift. * ARIMA with non-zero mean.

Table 8: U.S. Tourist arrivals by country of origin - Vector SSA choices (L, r) .

Series	1	3	6	12	24	36
Canada	(22,16)	(28,19)	(28,19)	(16,9)	(33,20)	(36,15)
Mexico	(5,1)	(5,1)	(4,1)	(51,3)	(39,9)	(49,26)
Total Overseas	(28,16)	(29,16)	(20,10)	(18,11)	(38,25)	(50,29)
Western Europe	(29,17)	(23,14)	(23,12)	(19,14)	(21,13)	(28,21)
Eastern Europe	(14,13)	(22,11)	(23,11)	(20,14)	(18,13)	(49,5)
Asia	(29,23)	(25,22)	(29,23)	(23,11)	(31,28)	(49,40)
Middle East	(24,15)	(15,13)	(17,13)	(22,18)	(44,36)	(38,15)
Africa	(18,14)	(17,14)	(24,20)	(14,12)	(47,17)	(24,16)
Oceania	(39,25)	(42,27)	(31,19)	(34,12)	(33,27)	(33,27)
South America	(27,14)	(23,15)	(16,12)	(26,15)	(46,28)	(35,24)
Central America	(29,17)	(29,19)	(26,17)	(29,25)	(47,24)	(46,27)
Caribbean	(24,20)	(24,20)	(24,11)	(18,12)	(24,12)	(46,26)
France	(15,13)	(30,14)	(25,12)	(43,31)	(23,20)	(40,21)
Germany	(32,10)	(25,8)	(25,9)	(32,15)	(24,12)	(24,12)
Italy	(44,15)	(34,15)	(34,15)	(57,27)	(18,14)	(30,23)
Netherlands	(36,14)	(37,14)	(26,19)	(26,19)	(22,10)	(32,11)
Spain	(28,8)	(12,6)	(24,8)	(14,9)	(14,3)	(14,9)
Sweden	(39,11)	(39,11)	(39,11)	(38,15)	(23,20)	(24,15)
Switzerland	(15,12)	(44,38)	(16,13)	(31,21)	(26,17)	(26,17)
UK	(24,14)	(22,14)	(32,24)	(51,38)	(41,34)	(47,14)
Japan	(31,25)	(28,9)	(47,19)	(23,21)	(47,34)	(39,9)
South Korea	(32,18)	(27,17)	(28,25)	(31,21)	(39,15)	(50,36)
PRC and Hongkong	(40,18)	(16,13)	(41,21)	(25,15)	(50,21)	(42,34)
ROC	(40,21)	(40,31)	(37,33)	(37,33)	(37,16)	(37,16)
Australia	(55,19)	(37,21)	(37,12)	(36,12)	(49,33)	(48,33)
Argentina	(23,15)	(30,26)	(15,13)	(17,15)	(41,39)	(41,40)
Brazil	(26,15)	(14,11)	(46,12)	(39,24)	(39,22)	(50,12)
Colombia	(29,15)	(29,16)	(39,11)	(36,11)	(27,23)	(19,10)
Venezuela	(30,15)	(28,15)	(26,15)	(18,15)	(48,15)	(37,17)

582 Table 9 reports the results for out-of-sample forecasting of U.S. Tourist arrivals by country
583 of origin. We can infer from the RRMSE criterion that, SSA outperforms ARIMA at forecasting
584 U.S. Tourist arrivals at all horizons for all countries of origin with the exception of Mexico at h
585 $= 3$ steps ahead. Furthermore, it is clear from the results that on average, SSA is 53%, 49%,
586 44%, 47%, 46% and 41% better than ARIMA at horizons of $h = 1, 3, 6, 12, 24$ and 36 months
587 ahead respectively for forecasting U.S. Tourist arrivals by individual country of origin. These
588 results prove that by employing SSA to analyse and forecast the monthly U.S. Tourists arrivals
589 data by country of origin we can obtain significantly more accurate forecasts than those possible
590 with the optimal ARIMA for both short and long term fluctuations in tourist arrivals into the
591 U.S. from each country. We test the results further for statistical significance. Accordingly,
592 we find that except for tourist arrivals from Mexico, every other forecasting result obtained in
593 this study is statistically significant. This suggests that when forecasting tourist arrivals from
594 Mexico there is no difference between the forecasting accuracy of the ARIMA and SSA models.

Table 9: Forecasting results for U.S. Tourist arrivals by country of origin.

Origin	$\frac{SSA}{ARIMA}$					
	1	3	6	12	24	36
Canada	0.27*	0.32*	0.40*	0.37*	0.30*	0.36*
Mexico	0.98	0.96	1.07	0.99	0.93	0.77**
Total Overseas	0.44*	0.48*	0.42*	0.48*	0.33*	0.43*
Western Europe	0.46*	0.50*	0.53*	0.47*	0.43*	0.44*
Eastern Europe	0.34*	0.37*	0.42*	0.41*	0.42*	0.39*
Asia	0.54*	0.68*	0.72	0.91	0.80*	0.91
Middle East	0.55*	0.42*	0.47*	0.37*	0.38*	0.46*
Africa	0.28*	0.39*	0.45*	0.36*	0.26*	0.24*
Oceania	0.40*	0.43*	0.51*	0.53*	0.60*	0.75**
South America	0.43*	0.45*	0.56*	0.50*	0.49*	0.82
Central America	0.44*	0.45*	0.52*	0.46*	0.34*	0.46*
Caribbean	0.34*	0.38*	0.43*	0.34*	0.49*	0.61*
France	0.36*	0.45*	0.42*	0.36*	0.52*	0.42*
Germany	0.60*	0.51*	0.64*	0.61*	0.64*	0.60*
Italy	0.31*	0.37*	0.41*	0.38*	0.35*	0.44*
Netherlands	0.48*	0.53*	0.47*	0.44*	0.44*	0.43*
Spain	0.60*	0.78*	0.76**	0.62*	0.65*	0.93
Sweden	0.53*	0.62*	0.72*	0.69*	0.62*	0.47*
Switzerland	0.48*	0.50*	0.54*	0.50*	0.48*	0.42*
United Kingdom	0.52*	0.49*	0.61*	0.65*	0.72**	0.92
Japan	0.62*	0.83*	0.82	0.71**	0.66*	0.96
South Korea	0.48*	0.49*	0.73*	0.79*	0.88	0.91
PRC & Hongkong	0.51*	0.52*	0.56*	0.47*	0.73**	0.64*
ROC (Taiwan)	0.50*	0.44*	0.48*	0.58*	0.50*	0.40*
Australia	0.44*	0.45*	0.48*	0.49*	0.61*	0.59*
Argentina	0.54*	0.62*	0.75*	0.64*	0.61*	0.59*
Brazil	0.53*	0.53*	0.58*	0.53*	0.49*	0.49*
Colombia	0.34*	0.38*	0.41*	0.41*	0.35*	0.66*
Venezuela	0.42*	0.34*	0.34*	0.44*	0.53*	0.52*
Average	0.47	0.51	0.56	0.53	0.54	0.59

Note: * indicates results are statistically significant based on Diebold-Mariano at $p = 0.05$.

** indicates statistical significance at $p = 0.10$.

595 Interestingly, when forecasting U.S. Tourist arrivals from Mexico, the optimal SSA choice for
596 the number of eigenvalues, r is $r = 1$ at horizons of 1, 3 and 6 steps ahead. This in turn means
597 that the SSA model is relying on the trend alone to forecast future data points for Mexico.
598 As such we find it important to briefly comment on this fact. For this purpose, in Figure 4
599 we have selected the time series for Mexico and three other time series which were shown to
600 have structural breaks (see, Table 2). Upon closer analysis it is clear that whilst all four time
601 series shown here are affected by structural breaks, the time series for Mexico shows signs of a
602 major structural break shifting U.S. tourist arrivals from Mexico starting December 2009. The
603 magnitude of this break has implications on SSA's modelling capabilities especially as we do
604 not incorporate change point detection methods in this paper. This particular time series alone
605 provides some useful topics for future research as it suggests that SSA change point detection
606 should be incorporated into the tourist arrivals forecasting models so that the technique is
607 equipped to provide improved forecasts in the face of similar time series. In line with ensuring
608 equality between the other forecasting models adopted in this study, we have used the most

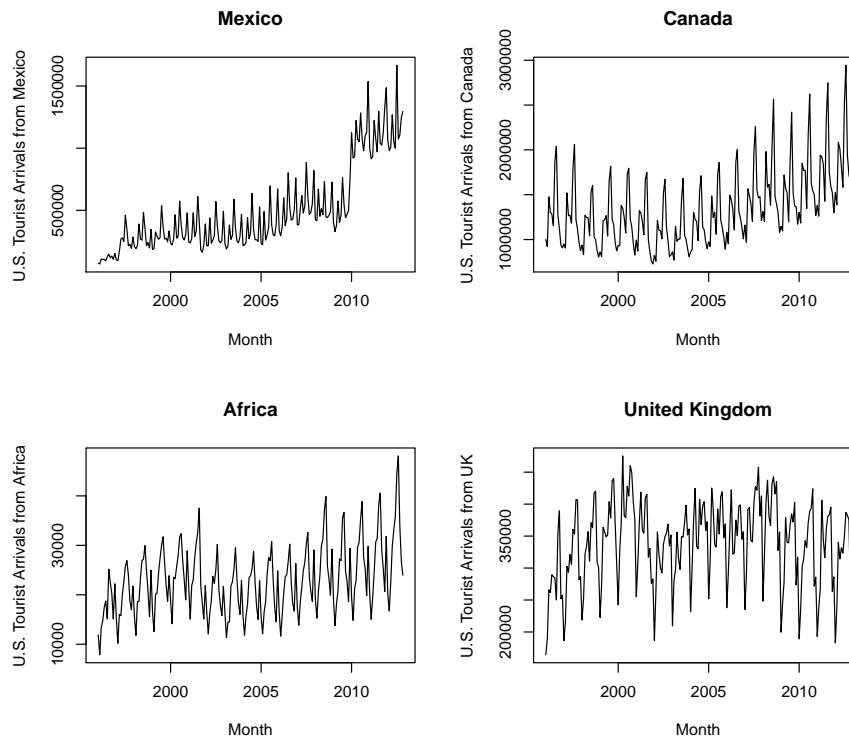


Figure 4: Selected U.S. Tourist arrivals time series (Jan. 1996 - Nov. 2012).

610 6 Conclusion

611 The starting point of this paper, as with many other authors, was the importance of accurate
 612 forecasts of tourism demand to investors, managers and policy makers. The existence of a
 613 high degree of volatility in tourism demand not only increases this need but creates a need
 614 for forecasting techniques that cope well with this volatility. We introduce Singular Spectrum
 615 Analysis (SSA) as a new model for forecasting inbound U.S. Tourist arrivals. The U.S. Tourist
 616 arrivals time series are analyzed in total and by country of origin.

617 Our analysis compared the forecasting accuracy of the newly proposed technique, the Vector
 618 SSA model, with the forecasting accuracy of the several different widely used forecasting models.
 619 These most were the accurate version of ARIMA, known as Automatic-ARIMA, an Exponential
 620 Smoothing model known as ETS and a feed-forward Neural Network model known as nnetar.
 621 Automatic-ARIMA, ETS and nnetar are all provided as automatic forecasting techniques within
 622 the forecast package within the R software. We found that the newly proposed U.S. Tourist
 623 arrivals forecasting model of SSA outperforms all three of these models (ARIMA, ETS and
 624 Neural Networks). The w -correlations (Table 4) also provide an explanation for one reason
 625 behind the outstanding performance recorded by the VSSA model as they indicate clearly that
 626 the VSSA forecasting algorithm is highly successful in separating the signal from the noise found
 627 in the U.S. Tourist arrivals series.

628 Moreover, this paper further uncovers substantial evidence to support the discontinuation of
 629 the use of ARIMA, ETS and feed-forward Neural Networks as a model for forecasting inbound

630 U.S. inbound tourist arrivals in the future, based on the RMSE, RRMSE and DC criteria along
631 with statistically significant results. The results also show that the basic Vector-SSA model with
632 the optimal decomposition is able to outperform the optimal ARIMA or, Automatic-ARIMA
633 model, ETS and nnetar of Hyndman and Khandakar (2008) in forecasting U.S. Tourist arrivals.
634 Our results indicate that the nonparametric SSA model is on average 49% more accurate than
635 the parametric model of ARIMA, 58% more accurate than ETS, and 69% more accurate than
636 the feed-forward Neural Network model (nnetar) at forecasting tourist arrivals. This provides
637 sound evidence for National Statistical Agencies in U.S. and elsewhere to consider introducing
638 SSA as a more reliable method of forecasting tourist arrivals. Furthermore, the SSA technique
639 has been shown here to provide not only the most accurate forecasts based on the lowest RMSE
640 values, but also with statistically significant results in comparison to the rest of the models.

641 This paper contributes to the literature on forecasting tourism demand in several ways.
642 Firstly, we show that the SSA technique can be used as a reliable demand forecasting technique
643 for tourism in the future, using its application to inbound tourist arrivals in the U.S. as an
644 example. In doing so, we also increase the number of options available for demand forecasting in
645 tourism. Secondly, we show that SSA outperforms the optimal ARIMA model of Hyndman and
646 Khandakar (2008). This is an important finding as ARIMA models are widely used in forecasting
647 tourism demand at present. The results are statistically significant and provide strong evidence
648 to support the discontinuation of ARIMA as a tourism demand forecasting technique for the
649 U.S. at least. Given the introduction of SSA and its strong performance with U.S. data it
650 would be interesting to see how well the model performs in forecasting tourism demand in other
651 nations. Thirdly, we also evaluate the SSA technique against an exponential smoothing and
652 neural network model which shows the basic Vector SSA to be superior. Whilst more research
653 work should be conducted on the comparison of SSA especially against neural networks in the
654 future, the initial evidence is supportive of the use of SSA.

655 Overall, given the importance of forecasting tourism demand and the important requirement
656 that such forecasts be able to cope well with volatility in demand, this paper offers a new
657 technique to forecasters in this area. The evidence from the U.S. data is that it offers the
658 prospect of better forecasting accuracy than the pick of those techniques previously employed.
659 Improvements in forecasting accuracy should provide a basis for more efficient resource allocation
660 by, in particular, investors and managers in tourism.

661 In terms of the implications of this paper for further research there are several. In this
662 paper we compared the performance of SSA to three of the most important existing alternative
663 techniques. It would be worthwhile extending this analysis in the future to a wider of alternative
664 techniques. The encouraging results from employing SSA to forecast U.S. inbound tourism
665 reported in this paper also suggests that it may be worthwhile in future research to build a
666 multivariate SSA model to forecast tourist arrivals. Here, it would be interesting to evaluate
667 the spatio-temporal correlations between tourist arrivals from various countries (as proposed
668 in [Sato, 2012](#)) so that this information could be used to enhance the multivariate SSA model
669 to enable more accurate forecasts. Finally, the use of hybrid models has been common in the
670 literature concerning the forecasting of tourism demand. It would be both interesting and of
671 potential value for future research to consider how the SSA technique performs in a hybrid
672 model. **Moreover, the presence of structural breaks in U.S. Tourist arrivals suggests that it
673 would also be interesting to evaluate the impact on the forecasts of replacing KPSS tests with
674 the Bai-Perron (2003) test for determining the number of differences in the ARIMA models.
675 The results from forecasting tourist arrivals from Mexico also makes it clear that future studies
676 should consider incorporating SSA change point detection for forecasting U.S. Tourist arrivals.
677 Finally, it is possible that different categories of tourism may be behaviourally different in a way**

678 that is relevant for other forecasting uses. In this paper, owing to data limitations we have not
679 been successful in analysing U.S. Tourist arrivals based on purpose of visit and future research
680 could benefit immensely if such data was made available by the relevant authorities.

681 References

- 682 Alvarez-Diaz, M., & Rossello-Nadal, J. (2010). Forecasting British tourist arrivals in the
683 Balearic Islands using meteorological variables. *Tourism Economics*, *16*, 153–168.
- 684 Andrawis, R. R., Atiya, A. F., & El-Shishiny, H. (2011). Combination of long term and short
685 term forecasts, with application to tourism demand forecasting. *International Journal of Fore-*
686 *casting*, *27*, 870–886.
- 687 Assaf, A. G., Barros, C. P., & Gil-Alana, L. A. (2011). Persistence in the short and long-term
688 tourist arrivals to Australia. *Journal of Travel Research*, *50*, 213–22.
- 689 Athanasopoulos, G., & de Silva, A. (2012). Multivariate exponential smoothing for forecasting
690 tourist arrivals. *Journal of Travel Research*, *51*, 640–652.
- 691 **Bai J., and Perron P. (2003). Computation and Analysis of Multiple Structural Change Models.**
692 ***Journal of Applied Econometrics*, *18*, 1–22.**
- 693 Beneki, C., Eeckels, B., and Leon, C. (2012). Signal extraction and forecasting of the UK
694 tourism income time series: A Singular Spectrum Analysis approach. *Journal of Forecasting*,
695 *31*, 391–400.
- 696 Beneki, C., & Silva, E. S. (2013). Analysing and forecasting European Union energy data.
697 *International Journal of Energy and Statistics*, *1*, 127–141.
- 698 **Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of***
699 ***econometrics*, *31*, 307–327.**
- 700 Box, G., & Jenkins, G. (1970). Time series analysis: Forecasting and control. San Francisco:
701 Holden-Day.
- 702 Broomhead, D. S., & King, G. P. (1986a). Extracting qualitative dynamics from experimental
703 Data. *Physica D: Nonlinear Phenomena*, *20*, 217–236.
- 704 Broomhead, D. S., & King, G. P. (1986b). On the qualitative analysis of experimental dynamical
705 systems. In *Nonlinear Phenomena and Chaos*. S., (ed.) Adam Hilger: Bristol, 113–144.
- 706 Cang, S. (2011). A non-linear tourism demand forecast combination model. *Tourism Eco-*
707 *nomics*, *17*, 5–20.
- 708 Chan, F., & Lim, C. (2011). Spectral analysis of seasonality in tourism demand. *Mathematics*
709 *and Computers in Simulation*, *81*, 1409–1418.
- 710 Cho, V. (2003). A comparison of three different approaches to tourist arrival forecasting.
711 *Tourism Management*, *24*, 323–330.
- 712 Chu, F. L. (2008). A fractionally integrated autoregressive moving average approach to fore-
713 casting tourism demand. *Tourism Management*, *29*, 79–88.
- 714 Chu, F. L. (2011). A piecewise linear approach to modeling and forecasting demand for Macau
715 tourism. *Tourism Management*, *32*, 1414–1420.
- 716 **Coshall, J. T. (2009). Combining volatility and smoothing forecasts of UK demand for inter-**
717 **national tourism. *Tourism Management*, *30*, 495-511.**

- 718 Coshall, J. T., & Charlesworth, R. (2011). A management orientated approach to combination
719 forecasting of tourism demand. *Tourism Management*, *32*, 759–769.
- 720 Croes, R. R., & Vanegas Sr, M. (2005). An econometric study of tourist arrivals in Aruba and
721 its implications. *Tourism Management*, *26*, 879–890.
- 722 Fourie, J., & Santana-Gallego, M. (2011). The impact of mega-sport events on tourist arrivals.
723 *Tourism Management*, *32*, 1364–1370.
- 724 Ghil, M., Allen, R. M., Dettinger, M. D., Ide, K., Kondrashov, D., Mann, M. E., Robertson, A.,
725 Saunders, A., Tian, Y., Varadi, F., & Yiou, P. (2002). Advanced spectral methods for climatic
726 time series. *Reviews of Geophysics*, *40*, 3.1-3.4.1.
- 727 Ghodsi, M., Hassani, H., Sanei, S., & Hicks, Y. (2009). The use of noise information for
728 detection of temporomandibular disorder. *Biomedical Signal Processing and Control*, *4*, 79–85.
- 729 Gil-Alana, L. A. (2005). Modelling international monthly arrivals using seasonal univariate
730 long-memory processes. *Tourism Management*, *26*, 867–878.
- 731 Goh, C., & Law, R. (2002). Modeling and forecasting tourism demand for arrivals with stochas-
732 tic nonstationary seasonality and intervention. *Tourism Management*, *23*, 499–510.
- 733 Golyandina, N., Nekrutkin, V., & Zhigljavsky, A. (2001). Analysis of time series structure:
734 SSA and related techniques. Chapman & Hall/CRC.
- 735 Gounopoulos, D., Petmezas, D., & Santamaria, D. (2012). Forecasting tourist arrivals in Greece
736 and the impact of macroeconomic shocks from the countries of tourists origin. *Annals of*
737 *Tourism Research*, *39*, 641–666.
- 738 Greenidge, K. (2001). Forecasting tourism demand: An STM approach. *Annals of Tourism*
739 *Research*, *28*, 98–112.
- 740 Hadavandi, E., Ghanbari, A., Shahanaghi, K., & Abbasian-Naghneh, S. (2011). Tourist arrival
741 forecasting by evolutionary fuzzy systems. *Tourism Management*, *32*, 1196–1203.
- 742 Harvey, D. I., Leybourne, S. J., & Newbold, P. (1997). Testing the equality of prediction mean
743 squared errors. *International Journal of Forecasting*, *13*, 281-291.
- 744 Hassani, H. (2007). Singular Spectrum Analysis (SSA): Methodology and comparison. *Journal*
745 *of Data Science*, *5*, 239–257.
- 746 Hassani, H., Heravi, H., & Zhigljavsky, A. (2009). Forecasting European industrial production
747 with Singular Spectrum Analysis. *International Journal of Forecasting*, *25*, 103–118.
- 748 Hassani, H., Heravi, H., & Zhigljavsky, A. (2012). Forecasting UK industrial production with
749 multivariate singular spectrum analysis. *Journal of Forecasting*, *32*, 395–408.
- 750 Hassani, H., & Mahmoudvand, R. (2013). Multivariate Singular Spectrum Analysis: A general
751 view and new vector forecasting approach. *International Journal of Energy Statistics*, *1*, 55–83.
- 752 Hassani, H., Soofi, A. S., & Zhigljavsky, A. (2013). Predicting inflation dynamics with singular
753 spectrum analysis. *Journal of the Royal Statistical Society A*, *176*, 743–760.
- 754 Hassani, H., & Thomakos, D. (2010). A review on Singular Spectrum Analysis for economic
755 and financial time series. *Statistics and its Interface*, *3*, 377–397.

- 756 Hui, T. K., & Yuen, C. C. (2002). A study in the seasonal variation of Japanese tourist arrivals
757 in Singapore. *Tourism Management*, *23*, 127–131.
- 758 Hyndman, R. J. (2014). Unit root tests and ARIMA models. Available via:
759 <http://robjhyndman.com/hyndsight/unit-root-tests/>
- 760 Hyndman, R. J. & Athanasopoulos G. (2013). Forecasting: principles and practice, OTexts:
761 Australia. OTexts.com/fpp.
- 762 Hyndman, R. J., Athanasopoulos G., Razbash, S., Schmidt, D., Zhou, Z., Khan, Y., &
763 Bergmeir, C. (2013). Package forecast: Forecasting functions for time series and linear models.
764 Available via: <http://cran.r-project.org/web/packages/forecast/forecast.pdf>
- 765 Hyndman, R. J., & Khandakar, Y. (2008). Automatic time series forecasting: The forecast
766 Package for R. *Journal of Statistical Software*, *27*, 1–22.
- 767 Hyndman, R. J., Koehler, A. B., Snyder, R. D., & Grose, S. (2002). A state space frame-
768 work for automatic forecasting using exponential smoothing methods. *International Journal of*
769 *Forecasting*, *18*, 439–454.
- 770 Jackman, M., & Greenidge, K. (2010). Modelling and forecasting tourist flows to Barbados
771 using structural time series models. *Tourism and Hospitality Research*, *10*, 1–13.
- 772 Kim, S. S., & Wong, K. K. (2006). Effects of news shock on inbound tourist demand volatility
773 in Korea. *Journal of Travel Research*, *44*, 457-466.
- 774 Kim, J. H., Wong, K., Athanasopoulos, G., & Liu, S. (2011). Beyond point forecasting: Evalua-
775 tion of alternative prediction intervals for tourist arrivals. *International Journal of Forecasting*,
776 *27*, 887–901.
- 777 Kwiatkowski, D., Phillips, P. C., Schmidt, P., & Shin, Y. (1992). Testing the null hypothesis
778 of stationarity against the alternative of a unit root. *Journal of Econometrics*, *54*, 159–178.
- 779 Lim, C., & McAleer, M. (2001). Forecasting tourist arrivals. *Annals of Tourism Research*, *28*,
780 965–977.
- 781 Lisi, F., & Medio, A. (1997). Is a random walk the best exchange rate predictor? *International*
782 *Journal of Forecasting*, *13*, 255-267.
- 783 Makridakis, S. G., Wheelwright, S. C., & Hyndman R. J. (1998). Forecasting: Methods and
784 applications. 3rd ed., Wiley: NY.
- 785 Mao, C. K., Ding, C. G., & Lee, H. Y. (2010). Post-SARS tourist arrival recovery patterns:
786 An analysis based on a catastrophe theory. *Tourism Management*, *31*, 855–861.
- 787 Naud, W. A., & Saayman, A. (2005). Determinants of tourist arrivals in Africa: a panel data
788 regression analysis. *Tourism Economics*, *11*, 365–391.
- 789 Nelson, L. A., Dickey, D. A., & Smith, J. M. (2011). Estimating time series and cross section
790 tourism demand models: Mainland United States to Hawaii data. *Tourism Management*, *32*,
791 28–38.
- 792 Page, S., Song, H., & Wu, D. C. (2012). Assessing the impacts of the global economic crisis
793 and swine flu on inbound tourism demand in the United Kingdom. *Journal of Travel Research*,
794 *51*, 142–153.

- 795 Pepelyshev, A. (2010). *Comparison of Recurrent and Vector forecasting*. UK-China workshop
796 on Singular Spectrum Analysis and its applications, Cardiff, September 20, 2010. Available via
797 http://ssa.cf.ac.uk/pepelyshev/pepelyshev_ssa_forecast.pdf.
- 798 Sato, A-H. (2012). Impact of the Great East Japan earthquake on hotel industry in Pacific
799 Tohoku prefectures. *Progress of Theoretical Physics Supplement*, 194, 165–172.
- 800 Seetanah, B., Durbarry, R., & Ragodoo, J. F. (2010). Using the panel cointegration approach to
801 analyse the determinants of tourism demand in South Africa. *Tourism Economics*, 16, 715–729.
- 802 Seetaram, N. (2010). Use of dynamic panel cointegration approach to model international
803 arrivals to Australia. *Journal of Travel Research*, 49, 414–422.
- 804 Shareef, R., & McAleer, M. (2007). Modelling the uncertainty in monthly international tourist
805 arrivals to the Maldives. *Tourism Management*, 28, 23–45.
- 806 Shen, S., Li, G., & Song, H. (2008). An assessment of combining tourism demand forecasts
807 over different time horizons. *Journal of Travel Research*, 47, 197–207.
- 808 Shen, S., Li, G., & Song, H. (2011). Combination forecasts of international tourism demand.
809 *Annals of Tourism Research*, 38, 72–89.
- 810 Silva, E. S. (2013). A combination forecast for energy related CO₂ emissions in the United
811 States. *International Journal of Energy and Statistics*, 1, 269–279.
- 812 Smeral, E. (2010). Impacts of the world recession and economic crisis on tourism: Forecasts
813 and potential risks. *Journal of Travel Research*, 49, 31–38.
- 814 Song, H., Dwyer, L., Li, G., & Cao, Z. (2012). Tourism economics research: A review and
815 assessment. *Annals of Tourism Research*, 39, 1653–1682.
- 816 Song, H., Gao, B. Z., & Lin, V. S. (2013). Combining statistical and judgmental forecasts
817 via a web-based tourism demand forecasting system. *International Journal of Forecasting*, 29,
818 295–310.
- 819 Song, H., & Li, G. (2008). Tourism demand modelling and forecasting - a review of recent
820 research. *Tourism Management*, 29, 203–220.
- 821 Song, H., Li, G., Witt, S. F., & Fei, B. (2010). Tourism demand modelling and forecasting:
822 How should demand be measured? *Tourism Economics*, 16, 63–81.
- 823 Song, H., Li, G., Witt, S. F., & Athanasopoulos, G. (2011). Forecasting tourist arrivals using
824 time-varying parameter structural time series models. *International Journal of Forecasting*, 27,
825 855–869.
- 826 Wan, S. K., Wang, S. H., & Woo, C. K. (2013). Aggregate vs. disaggregate forecast: Case of
827 Hong Kong. *Annals of Tourism Research*, 42, 434–438.
- 828 Zhang, G., Patuwo, B. E., & Hu, M. Y. (1998). Forecasting with artificial neural networks: the
829 state of the art. *International Journal of Forecasting*, 14, 35–62.