Noroozi et al.

The Dynamic elastic response to impulse synchronisation (DERTIS) of composite prosthetic energy storing and returning feet

Siamak Noroozi¹, Abdul Ghaffar Abdul Rahman², Khoo Shin Yee³, Saeed Zahedi⁴, Philip Sewell¹, Bryce Dyer¹ and Ong Zhi Chao³

Corresponding author: Siamak Noroozi, School of Design, Engineering & Computing, Bournemouth University, Poole, Dorset, BH12 5BB, UK. Email: snoroozi@bournemouth.ac.uk. Tel: +44(0)1202 965554.

¹School of Design, Engineering & Computing, Bournemouth University, UK

²Faculty of Mechanical Engineering, University Malaysia Pahang, Malaysia

³Department of Mechanical Engineering, Engineering Faculty, University of Malaya, Malaysia

⁴Chas A Blatchford & Sons Ltd., UK

Abstract

In this research it is proven that perfect synchronisation between the input impulse (human effort) and the ground reaction impulse/impact can result in a phenomenon called the Dynamic Elastic Response to Impulse Synchronisation (DERTIS) with favourable consequences to the behaviour of the energy storing and returning foot. An energy storing and returning composite prosthetic foot first saw use in elite competition at the 1988 Paralympic Games. Since its introduction, the prostheses have proven to be a source of controversy. In 2008 it was concluded that the use of such technology when running is advantageous when compared to able bodied athletes. However, these findings have since been disputed. There still remain unanswered questions regarding the level of contribution of energy storing and returning feet in the performance of amputee athletes. A series of investigations have previously been conducted to study the dynamic characteristics of a number of energy storing and returning composite prosthetic feet. It was found that if a sinusoidal input impulse, with a frequency close to one of the energy storing and returning foot's natural frequencies, could be applied it can make the energy storing and returning foot susceptible to resonance which, in the case of bending mode, if sustained can lead to a gain in height or increased velocity.

Keywords

Amputee, prosthesis, lower limb, dynamics, impulse synchronisation

Introduction

Lower limb prosthesis used in competitive athletics has proven to be a source of controversy¹. In 2008, bilateral amputee Oscar Pistorius was investigated for claims of having a technological advantage when competing against able bodied athletes². In 2012, bilateral amputee Alan Oliveira was ironically criticised by Pistorius himself directly after the 200m Paralympic Games final for using limbs that were allegedly advantageous due to their length. Both of these events centre on the use of energy storing and returning (ESR) composite prosthesis feet. The introduction of the Seattle foot in 1981 demonstrated the use of energy storing prosthetic feet in clinical prescription³. This comprised a flexible keel housed inside a polyurethane shell. When loaded, energy is retained within the structure as potential energy and a percentage of this is then returned to the user to assist their walking motion⁴. However, a significant progression in design was made when Van Philips conceived the Flexfoot in 1987³. This design is the basis of current sports lower limb prosthesis technology which then first saw use in elite competition at the 1988 Paralympic Games⁵.

An investigation concluded that the use of such technology when running is advantageous when compared to able bodied athletes⁶. However, Grabowski et al. (2010)⁷ concluded that this was not the case but that there is further work required in terms of quantifying the contribution which enables the amputee to achieve and sustain high speed.

Studies by Buckley (1999)⁸ and Bruggemann et al. (2008)⁶ have focused on physiological key performance indicators, which make the ESR technology contribution clouded. Dyer et al. (2011)⁹ evaluated a sample of sport stakeholders concluding that isolated, equipment specific assessments were recommended instead.

In able-bodied running, Weyard proposed that the dominant factor in sprinting speed was ground reaction force which then influenced limb cadence and stride length as secondary factors¹⁰. Hunter et al. (2005)¹¹ speculated that there is a link between ground reaction force, impulse and kinematics of running during acceleration.

It was speculated that if either stride length, limb cadence or ground reaction force exceed what is possible by human biology, enhanced running speed will result. Buckley et al. (1999)⁸ and Buckley and Juniper (2010)¹² described the mechanism of steady state running using ESR feet to a series of exchanges between potential, kinetic and strain energies of the leg and body mass system. However, no consideration has been given to the damping of such systems even though at the extremes of motion damping is the only force resisting biological forces generated by the athlete.

To better understand and be able to appreciate the contribution of ESR feet to running performance, Noroozi et al. (2013)^{13,14} conducted a series of investigations on the dynamics of ESR feet when attached to a mass. Noroozi et al. (2013)¹³ initially studied the experimental dynamic characteristics (natural frequencies, damping and mode shapes) of two composite ESR prosthetic feet when considered as a simple mass and ESR foot system. This research concluded that natural frequencies close to the typical running step frequency can be achieved. However, since running is impulse driven modal analysis did not give any information about the system response to impulse/impact.

Noroozi et al. (2013)¹⁴ also studied the dynamic elastic response (DER) of such systems to impulse due to ground reaction force when the ESR foot was dropped from a height. The theoretical underpinning for the DER to impulse of the ESR foot was proposed and validated using an analytical simulation and drop test experiments. It was concluded that every system has a unique and specific DER to impulse. It was observed that if a sinusoidal input impulse

with frequency close to the ESR foot's natural frequency is applied to the system it can make the ESR foot susceptible to resonance which if sustained can lead to a bouncing or a "trampolining" effect. It was also theoretically demonstrated that if this impulse can be synchronised with the frequency of human effort, it can result is storage or recovery of substantial amount of energy in the system that can be used at will by the runner (Figure 1). It was shown that if the loss of energy in one cycle could be compensated by additional input energy due to human effort and depending on the magnitude and the phase of this additional input there was three (i to iii) possible outcomes:

- Decaying amplitude (if effort is out of phase or less than the loss in 1 cycle) resulting in highly damped motion.
- ii) A steady state response (if effort is in phase and equal to loss in one cycle) resulting in a harmonic hopping, jumping or running motion/action.
- iii) Increasing potential energy in the system due to extra gain in height (if effort is in phase and energy is more than the loss in one cycle) which can be utilised to run faster, minimise energy consumption, have a faster take off velocity, gain more height and hence enhance performance.

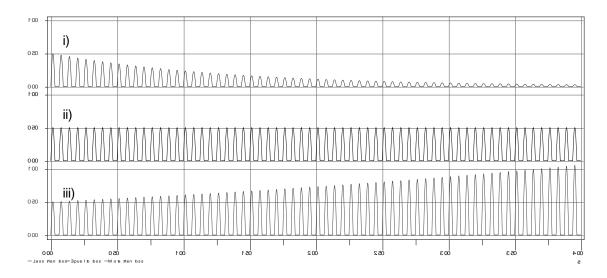


Figure 1. Three possible outcomes of DER of the system due to impulse/effort synchronisation¹³.

This research was further extended by examining the suitability of utilising Finite Element Analysis (FEA) as a design and analysis tool for studying the ESR¹⁵.

This paper validates the effect of impulse synchronisation proposed theoretically by Noroozi et al. (2013)^{13,14} on the mass and ESR foot system utilising simulation and experimental testing. Three commercially available composite ESR feet were studied in this investigation. Two Elite Blade (Chas A Blatchford & Sons Ltd, Basingstoke, UK) prostheses (Figure 2a and b) and one Flex Run (Ossur, Reykjavik, Iceland) prosthesis (Figure 2c) were investigated.

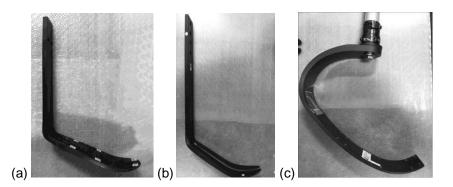


Figure 2. Blatchford Elite (a) solid toe and (b) split toe Blade and (c) Ossur Flex Run Blade used for the investigation.

Idealised model validation and hypothesis

An idealised mass and ESR foot system representing the body mass and composite foot system (Figure 3c) was developed, both experimentally (Figure 3a) and by using Working Model 2D (WM2D) simulation software (Design Simulation Technologies, Inc, Michigan, USA) (Figure 3b), to study the DER of the system to impulse. The mass and spring stiffness in this model are all generic to allow future parametric study. The experimental and simulated responses of the system to impulse due to a drop from a height are shown in Figures 4a and 4b respectively. A linear AC coupled piezoelectric force transducer, capable of measuring the change in the load due to inertia, was used between the floor and the bottom plate to capture experimentally in the vertical direction the Impulse (impact force and its duration). The output voltage from the transducer is positive when compressive forces are applied. A negative voltage means that the transducer is in tension or is settling.

In both cases the reaction to drop was in the form of a series of cyclic impulses with decaying amplitude, indicating that the simulation was an accurate representation of the experimental system. The decay in amplitude indicates the loss of energy after each impact. If no energy is added to the system in order to compensate for this loss of energy the motion soon reverts to a damped Simple Harmonic Motion (SHM) before eventually stopping. It must also be noted that the material and structural damping as well as air resistance play a significant role in dissipating useful energy.

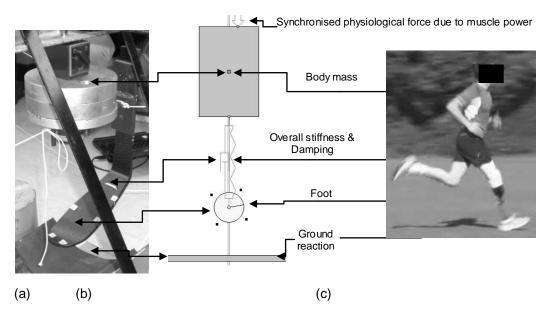


Figure 3. (a) experimental and (b) simulated mass spring damper representation of (c) body and prosthetic ESR system¹⁶.

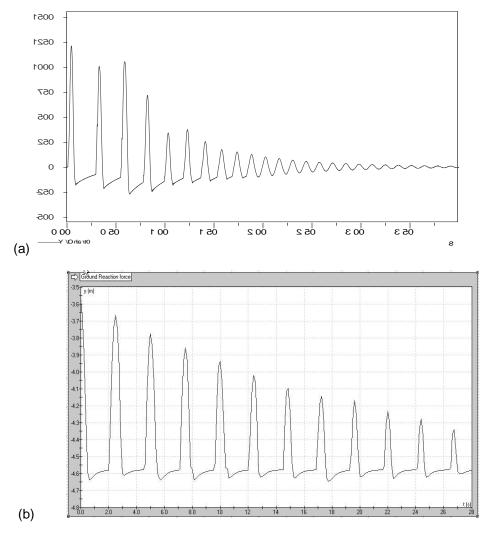


Figure 4. (a) experimental and (b) simulated drop test response.

Unlike a human, dead weights used in these drop tests have no intelligence or ability to generate a controlled cyclic excitation/input impulse in order to synchronise with ground reaction impulse needed to sustain this bouncing action. It is hypothesised that a perfect synchronisation between the excitation/input impulse (human effort) and the ground reaction

impulse can result in a phenomenon called the "Dynamic Elastic Response to Impulse Synchronisation" (DERTIS) with potentially favourable consequences.

Validation of DERTIS through simulation

Using the experimentally validated WM2D model of an idealised mass and ESR foot system (Figure 3b), it was possible to simulate and virtually test the DERTIS of the system in action. The two impulses that need to be synchronised in order to aid running are the ground reaction force (generated when the mass is falling due to gravity alone) and the human effort using muscle power (simulated using the force vector that is intuitively synchronised and triggered by the user based on visual feedback). This simulation highlights the human ability to control this motion through controlled energy transformation. The human effort or muscle force in this model was represented by a downward massless force applied to the body mass (Figure 3b) that only acts on the system when triggered by the click of a PC mouse. The duration of mouse click dictated the length of time in which the force acted on the system. This allowed the user to apply a constant force at a self-selected frequency and phase to control the motion.

Test procedure

The user was instructed to intuitively determine the required mouse click frequency and duration of click (to change the frequency and phase of the applied force) to:

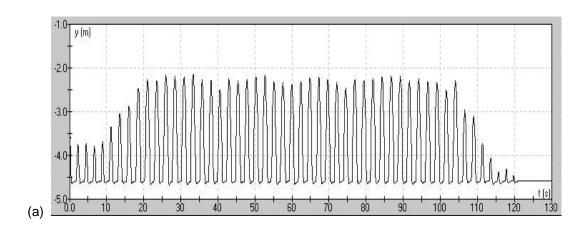
- (a) increase the amplitude of the bounce.
- (b) create a higher energy steady state impulse or bouncing action at a self selected impulse frequency.

(c) adjust the phase to consciously increase or decrease the amplitude of the motion of the mass.

Vertical displacement against time data was captured for analysis of the user's ability to control the level of displacement achieved.

Simulation results

Figure 5a and b shows the variation in the mass's vertical displacement, as a result of impulse synchronisation. The results also show an attempt by a human to create a steady state bouncing, or trampoline effect (Figure 5a) or fluctuate the energy state of the system at will (Figure 5b).



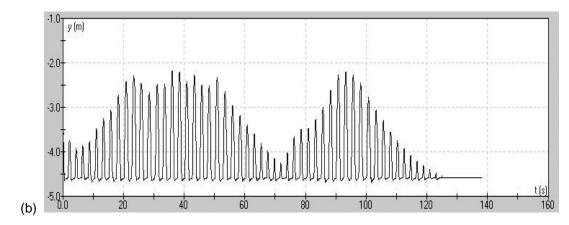


Figure 5. (a) Steady state and (b) variable dynamic elastic response to controlled impulse synchronisation.

Closer examination of the results shows that, if a cyclic energy input due to impulse from a massless force (body forces, muscle power, physiological inputs) with a controlled magnitude (more than, equal to or less than the loss of energy of the system in one cycle) is applied to the system it can result in a controlled output from the system. Depending on the relative magnitude and phase between the two impulses, either extra height or velocity can be gained and maintained (Figure 5a) or the motion can be suppressed and damped at will (Figure 5b).

This simulation demonstrated the human's unconscious ability to detect and synchronise their effort with any cyclic excitation force using eye and hand coordination. It also shows that higher energy state can be achieved and maintained both quickly and easily. The simulation, once setup with the specific parameters of a runner and prosthesis (e.g. mass of the amputee, stiffness of the prosthesis), can also potentially be used as a design tool that allows better tuning of mass to prosthesis spring rate, natural and impulse frequencies and general assessment of such systems.

Validation of DERTIS through experimentation

To validate the simulation of human's conscious and unconscious ability to detect and synchronise their effort with a cyclic excitation force an experiment was designed to test the DERTIS of a number of ESR feet. Each foot was mounted in a rig consisting of an aluminium frame housing two steel parallel guides where a flat guide/mounting plate with two linear bearings allowed free vertical movement of the load bearing plate up and down the guides (Figure 6). A mass of 15 kg was attached to the top of the mounting plate above each ESR foot. This resulted in the total mass of the moving element of the system to be just over 25 kg. It must be noted that a rigid mass is deemed an adequate model of a body due to the fact that when in the air the only external force acting on the body is gravity and all internal body forces due to muscle activity are all in the state of equilibrium. Two linear AC coupled piezoelectric force transducers were used to simultaneously measure both the ground reaction impulse and the input impulse due to human effort.

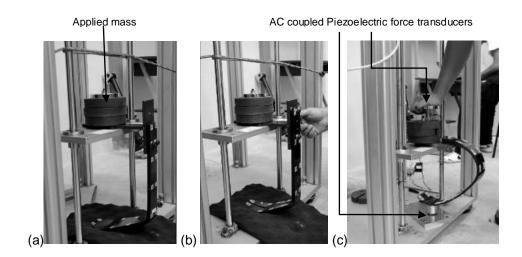


Figure 6. Blatchford Elite (a) solid toe and (b) split toe Blade and (c) Ossur Flex Run Blade mounted in the test rig.

Experimental procedure

Two types of experiment (qualitative and quantitative) were performed on each ESR, and each was repeated seven times to investigate, visualise and record the effect of DERTIS on different types of foot systems.

Qualitative synchronisation experiment. The objective of this experiment was to investigate, visualise and record if a human can detect the ground reaction impulse and its frequency and then synchronise their effort with this ground reaction impulse in order to store or release energy in the ESR foot at will. This was achieved by manually applying a cyclic vertical load by hand to the top of each mass and foot system. The motion response of each system to the applied cyclic load in each case was captured by a video camera. This was a qualitative experiment as it shows the DERTIS phenomenon in action. This also showed all ESR feet possess DERTIS to various degrees and its intensity is the function of the material, design and mass. This phenomenon was investigated under two initial conditions:

- In the first initial condition the foot/mass system was dropped from a height giving the
 user a few seconds to intuitively develop a feel for the frequency of the bounce. The
 user then began to apply a cyclic load to the system.
- 2. As discussed previously, it was observed by Noroozi et al. (2013)¹⁴ that if a sinusoidal input impulse with frequency close to the ESR foot's first bending natural frequency is applied to the system while stationary it can make the ESR foot susceptible to

resonance in the preferred mode which if sustained can lead to a bouncing or trampolining effect. By studying the captured footage it was possible to qualitatively assess the relative frequency of the input force to each system to further reinforce this point.

A modal analysis investigation was also conducted for each ESR foot and mass combination using the procedure described by Noroozi et al. (2013)¹³ to determine the first bending mode frequency of each system. An impact hammer, tri axial accelerometer, Me'Scope software (Vibrant Technology, California, USA) and an in house data acquisition and analysis software, developed within the DasyLab virtual instrument environment (Measurement Computing Corporation, Massachusetts, USA), was for this investigation. Figure 7 shows the first bending mode shapes and frequencies for each foot and mass system. The results show that the solid toe (5 Hz) and split toe (5.5 Hz) Blatchford feet have similar first bending mode frequencies while the Occur Flex Run (7 Hz) first bending mode frequency is higher. This indicates that a higher input impulse frequency will be required to initiate a sustained bounce.

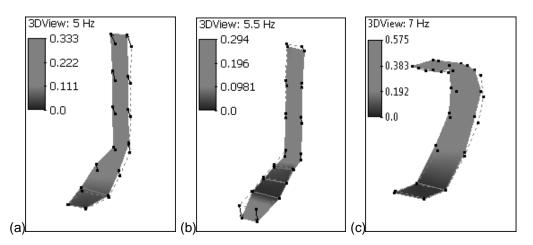


Figure 7. Blatchford Elite (a) solid toe and (b) split toe Blade and (c) Ossur Flex Run Blade first bending mode shapes and frequencies with a 25 kg mass attached to each foot.

Quantitative synchronisation experiment. In the second experiment, both the ground reaction force and the force generated by the muscle or arm activity of the person conducting the test were measured using two axial, AC coupled piezoelectric impulse/force transducers. The top transducer was used to measure the manually applied input force/impulse (human effort). The bottom transducer, placed between the ground and the toe region, measured the ground reaction force/impulse (Figure 6c). This data was captured after the foot/mass system had first been dropped from a height to initiate the dynamic elastic response due to ground reaction impulse, before any effort by the user to synchronise their effort with it in order sustain steady state or control the height or velocity at will.

Experimental results

Qualitative synchronisation experimental results. The qualitative analysis of the captured video footage clearly showed the effect of synchronisation between the two impulses involved on the DER of the mass and foot system (Figure 8). The analysis showed the human ability to control their input to maintain the motion. The captured footage shows that once the foot has been dropped from a height to initiate the DER to impulse the user has the instinct and the ability to sense the ground reaction impulse and its frequency and instinctively adjust their effort frequency in order to control the outcome, by increasing or decreasing the height or velocity at will (Figure 8a and b). It was also recognised that the impulse frequency is a function of the initial condition or the attained height and changes as the height changes, hence the need for active, real time and continuous synchronisation. If the input force is applied out of phase the vertical motion is quickly damped out. It was also seen that high energy low frequency high amplitude sinusoidal input force applied to a stationary system, by the user, can cause the initial

excitation of a stationary system (Figure 8c). It was clear that a low frequency high amplitude was required to initiate excitation in the Ossur Foot which validates the theory that if a sinusoidal input impulse with frequency close to the first or lowest bending mode of the ESR foot's natural frequency is applied to the system while stationary it can make the ESR foot susceptible to resonance which if sustained can lead to a bouncing or trampolining effect.

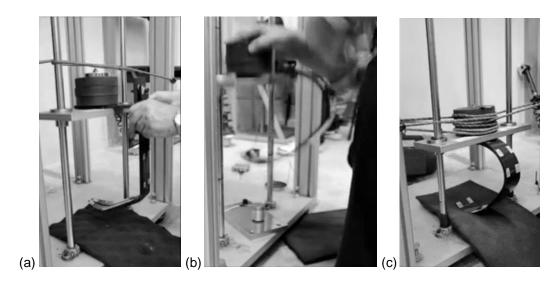


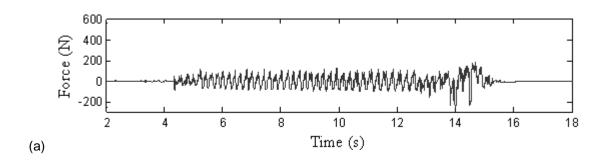
Figure 8. Video footage of the mass foot system typical response due to synchronised applied force for (a) Blatchford Elite split toe Blade, (b) Ossur Flex Run Blade and (c) initially stationary Ossur Flex Run Blade.

The force required to sustain steady state motion of such a heavy object (25 kg) was relatively small and could be applied using just two fingers. This could not have happened without the aid of the DERTIS. The energy supplied to the system using two fingers must have matched the loss of energy in one cycle in order to maintain the height or the energy state. The results from the transducer output also confirmed this finding (See Quantitative synchronisation experimental results section).

Quantitative synchronisation experimental results. Figure 9 shows typical outputs from the force transducers attached to the Ossur Flex Run foot. Figure9a shows the input force due to human effort in the form of a cyclic force or impulse applied by the user's arm and at a self selected frequency as an attempt to synchronise with the ground reaction impulse frequency (Figure9b). The traces have also been superimposed to show the relative magnitude and phase between the two inputs and the output impulses (Figure 9c).

Figure 9c show that the amplitude of the ground reaction force is larger than the input force as it contains all the forces involved (inertia, gravity, human effort). Figure 9c also show that once input impulse is synchronised (i.e. same frequency and phase) with the frequency of the ground reaction impulse, the amplitude of the ground reaction force increases significantly resulting in an extra gain in height or energy which is now stored in the mass due to its height or velocity.

When steady state is achieved, the input force required to maintain that energy state can drop to a nominal value equal to the loss of energy in one cycle, which is relatively small, in order to maintain the motion at that energy state. Theoretically, at this steady state stage the input impulse can remain constant while maintaining a high steady state performance.



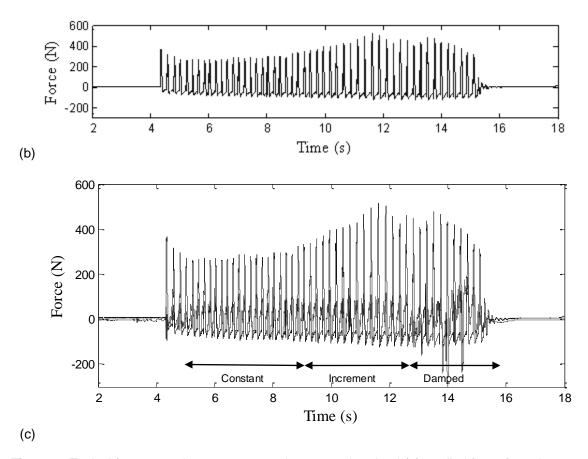


Figure 9. Typical force transducer responses due to synchronised (a) applied force from the user, (b) ground reaction force and (c) superimposition of two forces.

Figure 10 shows the effect of poor synchronisation between input impulse and the ground reaction impulse. In this case, both the mass and the force act as damper slowing or stopping the motion.

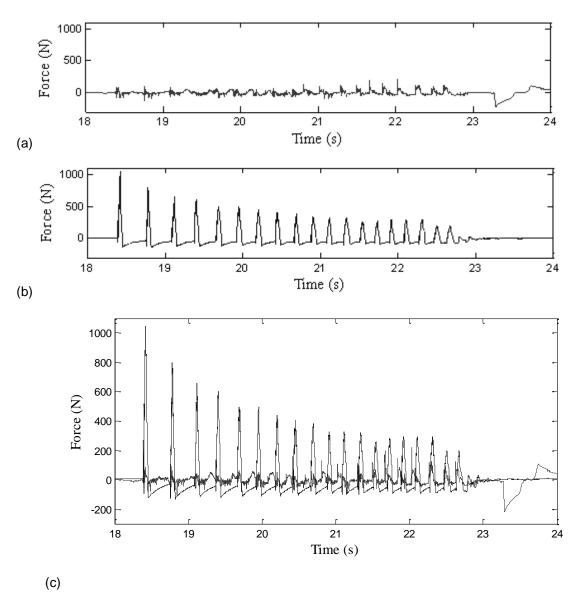


Figure 10. Typical force transducer responses due to unsynchronised (a) applied force from the user, (b) ground reaction force and (c) superimposition of two forces.

Discussion

Three different methods were used to demonstrate and validate the human ability to detect the dynamic elastic response to impulse of the ESR system and to synchronise with it (DERTIS) in this research. It is clear that every mass spring system has its own natural dynamic characteristic that is unique to that arrangement. The human body and brain can naturally feel and detect these natural characteristics. If these modes are identified and synchronised, the system goes into isolation where the inertia force and stiffness force cancel each other out. Therefore, any input or additional energy due to human effort, will go into the system unresisted resulting in extra deflection of the foot which means extra gain in strain energy which is then transferred to mass in the form of velocity or height. The differential values of this energy results in one of the three outcomes (damped, steady state or increasing amplitude and height). This also implies that when the person reaches their terminal velocity or a steady state that is natural to them and matches the dynamic elastic response of that system, potentially all they have to do is to maintain that state by applying a constant minimum effort at much lower levels of energy consumption and reduced fatigue.

The result of these simulation and experimental tests further supports and validates the theory and the simulation proposed previously ^{13,14}. It further substantiates the claim regarding the physical effect of impulse synchronisation on the DER of the ESR foot system.

Closer examination of the experimental data (input impulse and ground reaction impulse) highlights some key findings that may influence the way such ESR feet are prescribed to athletes. It may inform the way coaches need to explain the new mechanism of running, especially long distance running where it is expected that the steady state running action would maximise the potential effect of these feet.

Research is now required to investigate the full effects of DERTIS on bilateral and unilateral amputee runners. Future research will focus on measuring the runner's energy consumption, ground contact force and speed together with the dynamic characteristics of their prostheses to determine if an advantage can be gained from ESR feet. It may then be possible to develop a simple track side system that can estimate that maximum energy that can be available to the athlete based on his total mass and their running data. If this energy can be isolated for a given body mass and spring constant and then subtracted from the total energy consumption it can allow better and more objective assessment of athlete's relative level of fitness.

Conclusion

From this study it can be concluded that if synchronised cyclic energy, equal to the loss of energy in one cycle, can be applied to the ESR foot and mass system and depending on the relative phase, two distinct possible outcomes can occur: 1. extra gain in height, faster take off speed, higher kinetic energy more strain energy; 2. suppression of motion resulting in loss of energy due to damping and other losses.

Based on these findings, the dynamic elastic response to impulse synchronisation (DERTIS) of an ESR foot and mass system could potentially become a design parameter that allows better tuning of the amputee's mass to their ESR foot. With further acknowledgement of the existence of this phenomenon and subsequent prosthesis design optimisation, it is possible that the performance gap between able bodied and amputee athletes will narrow, notably over longer running distances.

Acknowledgement

The authors would like to thank Endolite® Chas A Blatchford & Sons Ltd for their kind donation of the blades used for this investigation and the continual support and collaboration throughout this investigation.

Funding

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Declaration of conflicting interests

The Authors declare that there is no conflict of interest.

References

- 1. Dyer B, Noroozi S, Sewell P, Redwood S. The Design of Lower-Limb Sports Prostheses: Fair Inclusion in Disability Sport. *Disability and Society* 2010; 25(5): 593-602.
- 2. Camporesi S. Oscar Pistorius, enhancement and post humans. *Journal of Medical Ethics* 2008; 34(9): 639.

- 3. Hafner B, Sanders J, Czerniecki J and Fergason J. Trans-tibial energy-storage-and-return prosthetic devices: A review of energy concepts and a proposed nomenclature. *Journal of Rehabilitation Research and Development* 2002; 39(1): 1-11.
- 4. Versluys R, Beyl P, Van Damme M, Desomer A, Van Ham R, and Lefeber D. Prosthetic feet: State-of-the-art review and the importance of mimicking human ankle-foot biomechanics. *Disability and Rehabilitation: Assistive Technology* 2009; 4(2): 65-75.
- 5. Nolan L. Carbon fibre prostheses and running in amputees: A Review. *Foot and Ankle Surgery* 2008; 14; 125-129.
- 6. Bruggemann P, Arampatzis A, Emrich F, Potthast W. Biomechanics of Double Transtibial Sprinting Using Dedicated Sprinting Prostheses. *Sports Technology* 2008; 1: 220-227.
- 7. Grabowski AM, McGowan CP, McDermott WJ, Beale MT, Kram R and Herr HM. Running-specific prostheses limit ground-force during sprinting. *Biology Letters* 2010; 6(2): 201-204.
- 8. Buckley J. Sprint Kinematics of Athletes with Lower Limb Amputations. *Archives of Physical Medicine and Rehabilitation* 1999; 80: 501-508.
- 9. Dyer B, Redwood S, Noroozi S, Sewell, P. The fair use of lower-limb running prostheses. Adapted Physical Activity Quarterly 2011; 28: 16-26.

- 10. Weyand P, Sternlight D, Bellizzi M, Wright S. Faster Top Running Speeds are achieved with Greater Ground Forces Not More Rapid Leg Movements. *Journal of Applied Physiology* 2000; 89:1991-1999.
- 11. Hunter J P, Marshall R N, McNair P J. Relationships Between Ground reaction force impulse and kinematics of Sprint-Running acceleration. *Journal of Applied Biomechanics* 2005; 21: 31-43.
- 12. Buckley J, Juniper M. Comments on point: Counterpoint: Artificial Limbs do/do not make artificially fast running speeds possible. *Journal of Applied Physiology* 2010; 108(4): 1016.
- 13. Noroozi S, Sewell P, Rahman AGA, Vinney J, Chao OZ, Dyer B. Modal analysis of composite prosthetic energy-storing-and-returning feet: an initial investigation. *Proceedings of the Institution of Mechanical Engineers, Part P: Journal of Sports Engineering and Technology* 2013: 227(1): 39-48.
- 14. Noroozi S, Sewell P, Rahman AGA, Vinney J, Chao OZ, Dyer B. Performance enhancement of bi-lateral lower-limb amputees in the latter phases of running events: an initial investigation. *Proceedings of the Institution of Mechanical Engineers, Part P: Journal of Sports Engineering and Technology* 2013: 227(2): 105-115.
- 15. Vinney J, Noroozi S, Rahman AGA, Sewell P, Chao OZ, Kuan KK, Dupac M. Analysis of Composite Prosthetic Energy-Storing-and-Returning (ESR) feet: A comparison between FEA and the experimental analysis. *International Journal of COMADEM* 2012; 15(3): 19-28.

16. Endolite® Chas A Blatchford and Sons. 'Elite Blade', http://www.endolite.co.uk/products/feet/eliteblade/eliteblade_video.html (2011, accessed August 2011).

Noroozi et al.

Appendix

Notation

DER Dynamic Elastic Response

ESR Energy-Storing-and-Returning

SHM Simple Harmonic Motion