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Abstract

The effect of changes in demographic structure on medium-run trends of key macroeconomic variables is estimated using a Panel VAR of 21 OECD economies. The panel data variability assists the identification of direct effects of demographics, while the dynamic structure uncovers long-term effects. Young and old dependants are found to have a negative impact while workers contribute positively. We propose a theoretical model, highlighting the relationship between demographics, innovation and growth, whose simulations match our empirical findings. The current trend of population aging and reduced fertility is found to reduce output growth and real interest rates across OECD countries.

JEL Codes: E32, J11

Keywords: population age profile, medium-term, output growth, innovation, life-cycle, Kuznets cycles

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1. Introduction

The slow recovery after the great recession and the disappointingly slow growth of productivity in the last decade has fostered the debate on the medium to long-run prospects of developed economies and the danger of secular stagnation. This debate has centred on two main topics: the production of new ideas and the structural characteristics that can be important in shaping future economic conditions. There is disagreement about the rate of production of new ideas with Gordon (2012, 2014) being more pessimistic while, amongst others, Fernald and Jones (2014) and Brynjolfsson and McAfee (2011) being more optimistic. The importance and impact of structural characteristics are more widely accepted. Gordon (2012, 2014) and Fernald and Jones (2014), looking particularly at the U.S., stress the importance of educational attainment and demography. The effects of demographic changes on labour supply are often mentioned as one of the 'headwinds' of the observed slowdown in macroeconomic performance in advanced economies. Although important, this interpretation restricts the potential impact of demographic changes on the macroeconomy. In this paper we take a more general view, arguing that changes in demographic structure, defined as variations in proportions of the population in each age group from year to year, matter more generally for macroeconomic activity and may also be related to the production of ideas as recognized by Kuznets (1960). He argued that medium term variations in growth rates are linked with both population growth and associated evolution of demographic structures (Kuznets cycles).

As Figure 1 illustrates, advanced economies are undergoing major changes in the age profile of their populations due to reduced fertility and increased longevity. First, we observe that although the proportion of the population in the working age group $(20-59)$ is similar in 1970 and 2030, at 50% and 48% respectively, it initially increased to around 56% in 2003 before starting to decline again (age profiles for working age groups are hump-shaped). Second, the average proportion of the population aged 60+ across our sample is projected to increase from 16% in 1970 to 29% in 2030, with most of the corresponding decline experienced in the $0-19$ group. Moreover, United Nations (2015) reports that although "Europe has the greatest percentage of its population aged 60 or over (24 per cent) in 2015, rapid ageing will occur in other parts of the world as well, so that, by 2050, all major areas of the world except Africa will have nearly a quarter or more of their populations aged 60 or over".

Kuznets (1960) long ago stressed in detail the importance of demographic structure where the population is a producer, a saver and a consumer and changes in the demographic structure may affect the medium and long-term macroeconomic prospects. In this line of thought, age characteristics of population matters since different age groups (i) have different savings behaviour, according to the life-cycle hypothesis; (ii) have different productivity levels, according to the age profile of wages; (*iii*) work different amounts, the very young and very old tend not to work, with implications for labour input; (*iv*) contribute differently to the innovation process, with young and middle age workers contributing the most; and (*v*) provide different investment opportunities, as firms target their different needs. Given the scale of the age profile shifts observed and the relevance of increasing our understanding of the link between the economy's structural features and its future prospects, this paper investigates both empirically and theoretically the effects of changes in demographic structure on the macroeconomy by looking particularly at their impact on medium to long-term trends.

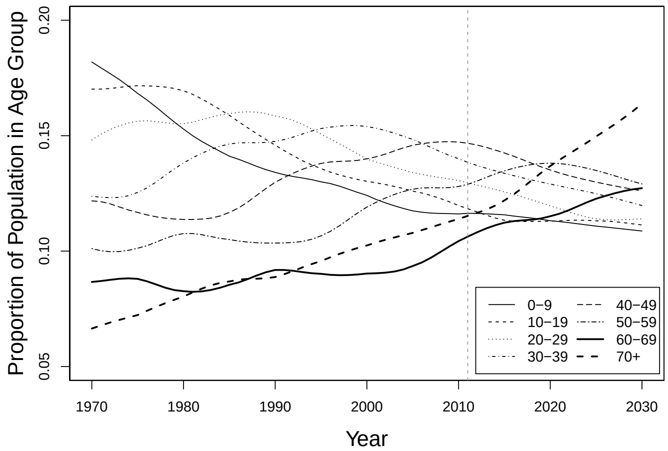

Figure 1: (Unweighted) Sample Mean Proportions in each Age Group by Year - Source: United Nations

In the first part of the paper we present empirical evidence on the short and long-term relevance of demographic structure for the macroeconomy. While the theoretical literature and most economic commentary on policy strongly emphasise the importance of demographic structure, the econometric evidence for its importance is less compelling. There are a number of reasons for this. First, changes in demographic structure are low frequency phenomena, difficult to distinguish from other low frequency trends that dominate economic time series. [1] Second, the vector of proportions in each age group is inevitably highly collinear, making precise estimation of the effect of each age group difficult. Hence it is common to impose strong restrictions on the effect of the age structure, for instance through the use a single variable, the dependency ratio. Finally, empirical models that impose a single (equation) relationship between a set of variables including demographics and a dependent variable of interest will be inevitably unsatisfactory to capture general equilibrium effects and will likely underestimate the marginal contribution of demographic transitions.

[1] Mueller and Watson (2015) discuss low-frequency econometrics. However, their concern, with the identification of low frequency trends in time-series is rather different from our concern with measuring the effect of slowly moving variables on macroeconomic activity using the cross-section variability.

Our empirical strategy attempts to address these issues by estimating a system of dynamic equations utilising a large panel of OECD countries over a sample period 1970-2007. The panel data variability assists the identification of the direct relationship between age groups and the core variables in our system, enabling us to use as much detail on demographics as data feasibility permits. The dynamic nature of our VAR allows us to estimate the long-run effects of the demographic movements transmitted through the whole system. As such our methodology uncovers any long-run association between the slowly changing demographic age profile and the macroeconomy. In our benchmark model, the demographic structure can plausibly be considered an exogenous process (the high birth rate that produced the baby-boomers after 1945 is unlikely to be influenced by growth rates 30 years later) and is represented by shares of age groups $(0-9,\ 10-19,\ldots,70+)$ in total population. The system of macroeconomic variables include: real output, investment, savings, hours worked, real interest rates, and inflation. We additionally introduce two exogenous controls: oil prices and population growth. We analyse two extensions to the benchmark case: in the first, the empirical model recognises the importance of innovation activities for capital and labour productivity and their impact on the macroeconomy. In the second, demographic structure is represented more parsimoniously and consists of three age groups (young, working-age and retirees).

We find that the variation in the age profiles across OECD countries has economically and statistically significant impact on all key macroeconomic variables. By allowing for the indirect feedbacks of the changes in age profile on the variables of interest we show that the long-term effects are stronger than their short-term counterparts. Crucially, we find that the impact of age profile changes follows a life-cycle pattern: that is, dependant cohorts tend to have a negative impact on all real macroeconomic variables including real returns and add positive inflationary pressures in the long run. The opposite is observed for working-age cohorts. Although we have not imposed any restrictions on the coefficients of each proportion of the population in different age groups we find that their joint significance is always stronger when combining working age groups or dependant groups. As such we estimate a more parsimonious model including just three age groups (young, adults and old dependants), which confirms our main results. Finally, the inclusion of patent applications as a proxy for innovative activities does not alter our benchmark results for the macroeconomic short and long-term dynamics. Moreover, we find evidence of demographic structure effects on innovative activities, with middle-aged workers (40-49) having a strong positive impact on patent applications, significantly higher than older workers, while dependants contribute negatively. Thus, the aging of the population may result in lower rates of innovation.

We use the United Nations (UN) population predictions and our long-run estimates to perform country specific prediction exercises. Firstly we contrast the impact of demographic changes during the previous and the current decade on trend output growth. We find that for all countries in our sample the changes in age profile will lead to a statistically and economically significant drop in trend growth. The average annual long-term output growth is expected to be reduced by 0.99 percentage points in Japan and 0.92 in the U.S. More generally, the expected average path of output growth and real interest rates from 2000 until 2030 highlights the downward pressure on these variables as a result of the decrease in working-age population and increase in proportion of retirees expected during this period. We finally evaluate how an economy would respond to a temporary increase in fertility using the dynamic properties of our VAR. We find a distinct life-cycle pattern with growth, investment and interest rates dropping as fertility increases, recovering as the proportion of working age population increases and falling again as the weight of old dependants increase.

In the second part of the paper we develop a theoretical model to match the observed life-cycle characteristics we find in the data and use it to study the main mechanisms through which demographic changes affect the macroeconomy. We construct an economic environment incorporating (*i*) life-cycle properties, allowing for three generations of the population (dependant young, workers and retirees) with investment in human capital and (*ii*) endogenous productivity and medium-term dynamics. As such, we can study the long-term interaction between demographic changes and savings, investment and innovation decisions, which in turn shape the evolution of output growth. To the best of our knowledge, this paper is the first to combine a life-cycle model with demographic transitions and medium run considerations.

Our model highlights three channels through which age profiles affect the macroeconomy. Firstly, changes in fertility and the implicit cost of taxing workers affect investment in human capital. Secondly, aging affects the saving decision of workers and through that real interest rates. Finally, reflecting our empirical findings the share of young workers impacts the innovation process positively and, as a result, a change in the demographic profile that skews the distribution of the population to the right, leads to a decline in innovation activity.

The calibrated theoretical model replicates most of the dynamics established in our panel VAR. A relative increase in the share of young dependants and retirees decreases output growth and investment while an increased share of workers does the opposite. A permanent increase in longevity (increase in life expectancy) leads to increased growth rates in the short-term as the decrease in the workers' marginal propensity to consume leads to lower real interest rate and an increase in innovative activity. However, as the share of young workers decreases, productivity in innovation decreases leading to a permanently lower output growth and share of investment over output. Finally, feeding the UN population predictions for different countries into the model, we match well the results of the prediction exercise conducted in the empirical part. Increases in average age and reduced fertility are found to be strong forces reducing long-term output growth and real rates across OECD countries.

Related Literature

Our work is related to a large empirical literature on the effects of demography on macroeconomic variables. Several studies that look at these effects, measure the changes in age structure by a single statistic; typically, the proportion of the population of working age (or dependency ratios) or a measure of life expectancy.[2] A number of other contributions, like ours, focus on a more granular representation of the age structure. Fair and Dominguez (1991) and Higgins (1998) use a low order polynomial function to describe the age distribution shares and examine the effect of demographics on various macro variables, the first concentrating on the U.S. and looking at consumption, money demand, housing investment and labour force participation while the second looking at savings, investment and current account, using 5-year averages, in a panel of countries. The impact of demographic profile is found to be highly significant. Lindh and Malmberg (1999) consider the impact of age structure in a transitional growth regression on a panel of 5-year periods in OECD countries. They find that growth of GDP per worker is strongly influenced by the age structure, with 50-64 year olds having a positive influence and the 65-plus age group a negative one. Finally, Feyrer (2007) considers the age structure of the workforce, rather than the population as a whole, and its impact on productivity and hence output. Our approach differs from these in at least two crucial ways: first, we use annual data rather than 5-year averages, and second, having a larger sample enable us to estimate a panel VAR rather than an individual equation thus allowing for interaction effects between key macro-variables and for the joint examination of these effects on key macroeconomic variables and growth. Incorporating those interactions generates a methodology that potentially captures general equilibrium effects. Miles (1999) has a careful discussion of the advantages and disadvantages of the use of different types of evidence to assess the impact of demographic change and argues for the use of calibrated general equilibrium models.[3]

On the theoretical side, the framework developed here incorporates demographic heterogeneity, building on Gertler (1999), Blanchard (1985) and Yaari (1965)[4] and endogenous productivity models, following Comin and Gertler (2006) and Romer (1990). As such, changes in demographic structure can be analysed in general equilibrium and a distinction

[2]A non-exhaustive list includes Higgins and Williamson (1997) studying the dependency hypothesis for Asia, Bloom, Canning, Fink, and Finlay (2007) looking at life expectancy and growth forecast and Gómez and Hérnandez de Cos (2008) looking at mature workers and global growth.

[3]Other interesting studies that focus on effects of demography are Jaimovich and Siu (2009) and Park (2010). The first examines the impact of demography, particularly differentiating volatility of hours worked across age groups, on business cycle volatility in the G7 countries. The second examines the impact of age distribution on stock market price-earnings ratios in the US.

[4]See Ferrero (2010), Carvalho and Ferrero (2013) and Sterk and Tenreyro (2013) for other contributions that incorporate demographic heterogeneity in a similar way.

between short-run (within a business cycle framework) and medium-run effects (that incorporates the interaction between macro variables and growth) can be explored. Our work is also related to the Alvin Hanson's recently popularised argument on whether mature economies are experiencing a long lasting stagnation due to permanently low demand. Most of this literature currently focuses on the effects of aggregate demand externalities in periods of financial deleveraging that may lead to prolonged periods of lower real rates of return (furthermore, these effects are amplified the lower the population and productivity growth rates, see Eggertsson and Mehrotra (2014) and Jimeno (2015)). By linking demographic changes and future low real interest rates and output growth, our results provide further indication that OECD economies are more likely to experience episodes where aggregate demand externalities may lead to stagnation in the following decades.

The remainder of the paper is organised as follows. The data and the econometric framework used are discussed in Section 2. Section 3 presents our empirical results, looking at the panel VAR estimates for the benchmark model and the extension including a measure of innovation activities. Section 4 introduces the theoretical framework while the simulation results are presented in Section 5. Finally, Section 6 concludes.

2. Data and econometric model

The annual dataset covers the period 1970-2007. Population data was obtained from the United Nations - World Population, 2010. Age compositions are calculated using the *de facto population* in the age group indicated and the percentage it represents with respect to the total population. It is important to note that UN populations data effectively captures those living in a specified country and not only its citizens, thus explicitly accounts for legal immigration. The annual data on savings and investment rates were calculated from nominal GDP, investment and savings series obtained from the OECD, which also supplied the data on hours worked. Annual data on policy rates and the Consumer Price Index (CPI) were obtained from the International Financial Statistics of the IMF. Per-capita GDP growth rates were calculated from per-capita real GDP obtained from Penn World Tables.

The 21 countries covered by our dataset are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Greece, Iceland, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and United States. For some countries data is not available over the whole period, so the panel is unbalanced. Data on hours are only available for Austria from 1995-2007, for Greece from 1983-2007 and for Portugal from 1986-2007. Savings and investment rates for Switzerland are only available from 1990-2007. All other countries have full datasets.[5]

[5]Though it would also be desirable to include Germany and Turkey as mature OECD economies, we

We have data for countries, $i = 1, 2, \ldots, N$, for years $t = 1, 2, ..., T$. For data on age structure Park (2010) uses age by year, and restricts the shape of their effect, but given the lack of data for many countries we use age by decade. With only 8 demographic proportions and a fairly large panel we chose not to restrict the age coefficients. Denote the share of age group $j = 1, 2, ..8$ $(0-9,\ 10-19, \ldots, 70+)$ in total population by $w_{j,i,t}$ and suppose the effect on the variable of interest, say $x_{i,t}$, takes the form

$$x_{i,t} = \alpha + \sum_{j=1}^{8} \delta_j w_{j,i,t} + u_{i,t}.$$

Since $\sum_{j=1}^{8} w_{j,i,t} = 1$, there is exact collinearity if all the demographic shares are included. To deal with this, we restrict the coefficients to sum to 0, use $(w_{j,i,t} - w_{8,i,t})$ as explanatory variables and recover the coefficient of the oldest age group from $\delta_8 = -\sum_{j=1}^{7} \delta_j$. We denote the 7 element vector of $(w_{j,i,t} - w_{8,i,t})$ as $W_{i,t}$.

Our benchmark estimation includes six endogenous variables. They are: the growth rate of the real GDP, $g_{i,t}$, the share of investment in GDP, $I_{i,t}$, the share of personal savings in GDP, $S_{i,t}$, the logarithms of hours worked per capita, $H_{i,t}$, the real short-term interest rate, $R_{i,t}$, and the rate of inflation, $\pi_{i,t}$. We denote the vector of these six variables as $Y_{i,t} = (g_{i,t},\ I_{i,t},\ S_{i,t},\ H_{i,t},\ rr_{i,t}, \pi_{i,t})'$.[6] In an extension (section 3.4) we analyse the link between demographic structure and innovation, incorporating a proxy for $R\&D$ activities. As such we include log of per capita residential patent applications ($R\&D^{PA}$), as reported by the World Bank, utilizing a vector of seven variables given by $Y_{i,t} = (g_{i,t},\ I_{i,t},\ S_{i,t},\ H_{i,t},\ rr_{i,t},\ R\&D^{PA}_{i,t},\ \pi_{i,t})$.

There are likely to be complicated dynamic interactions between our main six economic variables and literature suggesting an appropriate model for panel data is scarce. In Section 5 we present a theoretical general equilibrium model which allows for a range of these interactions, which we calibrate in terms of the deep parameters of the system, say θ. In principle, one might consider estimating a linearised version of the system which would be a structural system of the form

$$\Phi_0(\theta) Y_t = \Phi_1(\theta) E_t(Y_{t+1}) + \Phi_2(\theta) Y_{t-1} + \Gamma(\theta) W_t + \varepsilon_t. \tag{1}$$

If all the eigenvalues of $A(\theta)$ and $(I - \Phi_1(\theta) A(\theta))^{-1} \Phi_1(\theta)$ lie strictly inside the unit circle,

exclude Germany due to reunification and Turkey due to incomplete demographic data. However, we include predictions for Germany in the tables.

[6]The Schwarz Bayesian information criterion indicated that the specification excluding wealth gives a better fit. Hence, wealth, although potentially important to explain savings, has been excluded from the benchmark specification.

where $A(\theta)$ solves the quadratic matrix equation

$$\Phi_1(\theta) A(\theta)^2 - \Phi_0(\theta) A(\theta) + \Phi_2(\theta) = 0, \quad (2)$$

then there is a unique and stationary solution given by

$$\begin{aligned} Y_t &= A(\theta) Y_{t-1} + \Phi_0^{-1}(\theta) \Gamma(\theta) W_t + \Phi_0(\theta)^{-1} \varepsilon_t \text{ or} \\ Y_t &= a + AY_{t-1} + DW_t + u_t. \end{aligned} \quad (3)$$

Identifying and directly estimating the structural system is likely to be difficult. Therefore we estimate the solution or reduced form of the system and assume that conditional on the exogenous variables, it can be written as a VAR like (3). Notice that since the estimated matrices relative to A and D will be a complicated non-linear function of all the deep parameters, θ, it will be difficult to interpret the estimated coefficients. However, our objective in the empirical section is primarily to provide predictions of the long-run effect of the demographic variables and the same predictions would be obtained from any just identified structural model such as (3). Over-identifying restrictions, if available and correct, would increase the efficiency of the estimation, but given that we have a large panel that seems a secondary consideration. Instead we will compare the simulation results from our theoretical model of Section 5 with the main implications of the estimated empirical model.

The general model described above is for a single country. We consider a Panel VAR and thus we have vectors Y_{it} and W_{it} for countries $i = 1, 2, ..., N$. We assume slope homogeneity across countries but allow for intercept heterogeneity through a_i and estimate a one-way fixed effect panel VARX(1) of the form:

$$Y_{it} = a_i + AY_{i,t-1} + DW_{it} + u_{it}, \quad (4)$$

including additionally two controls: lagged oil price and population growth. The first accounts for common trends across countries and the second allows us to evaluate the macroeconomic dynamics induced by the composition of the demography (W_{it}) rather than the impact of an increase or decrease in the population. A is the 6×6 parameter matrix of lagged macroeconomic variables and D is the 6×7 matrix of coefficients of the demographic variables. Our estimate of the effect of the demographic variables is then the marginal effect after having controlled for lagged Y_{it}, the oil price and population growth.[7]

Slope heterogeneity is undoubtedly important and it can have unfortunate consequences

[7]Implicitly we are assuming either that all the variables are stationary or that a flexible unrestricted VAR will capture stationary combinations by differencing or cointegrating linear combinations. Phillips and Moon (1999) and Coakley, Fuertes, and Smith (2006) suggest that spurious regression may be less of a problem in panels.

in dynamic panels. Pesaran and Smith (1995) show that it biases the coefficient of the lagged dependent variable towards one and the coefficient of the exogenous variable towards zero. Moreover, estimating heterogeneous slopes based on relatively few degrees of freedom may result in poorly determined parameters and is likely to produce outliers. We found this to be the case when we experimented with VARs for each country. Finally, Baltagi and Griffin (1997) and Baltagi, Griffin, and Xiong (2000) show that the homogeneous estimators tend to have better forecasting properties. As a result, since our main aim is to predict the variables conditional on demographics, the homogeneous estimators may provide better predictors of this demographic contribution.

One of the key assumptions of our estimation is that the demographic structure is exogenous to the dynamics of the main macroeconomic variables. We find that the demographic variables show very low frequency variation relative to annual macroeconomic time-series and the elements are highly correlated, as such, their time series variability is not contributing much to the identification of D. In the robustness section we test for this exogeneity assumption by running a VAR with both vectors Y_{it} and W_{it} treated as endogenous. Thus, we estimate

$$\begin{bmatrix} Y_{it} \\ W_{it} \end{bmatrix} = a_i + \begin{bmatrix} A^{endo} & D^{endo} \\ B_1 & B_2 \end{bmatrix} \begin{bmatrix} Y_{i,t-1} \\ W_{i,t-1} \end{bmatrix} + u_{it} \tag{5}$$

where A^{endo} and D^{endo} are the counterpart of A and D when demographics is considered endogenous, B_2 is the parameter matrix of lagged demographic weights and B_1 is the parameter matrix that links past macroeconomic variables and demographic weights. The exogeneity test verifies whether matrix B_1 is equal to zero. Although some parameters in B_1 are precisely estimated and found to be different from zero, they are all rather small, thus changes in $Y_{i,t-1}$ do not significantly (economically) affect W_{it}, validating our exogeneity assumption. These results are discussed in the robustness section.

The panel variability is crucial for the identification of the short-run impact of demographics. The fixed effect (or within group) estimator, that we use can be written in the form of deviations from the country means of the variables. Thus it does not use the cross section (between) relationship linking the average values of the variables across countries. However in calculating the FE estimator even though we have removed the between group variation in the means, we benefit from the much greater range variance of the independent variables which is available in a panel. This panel effect allows us to obtain much more precise estimates of the demographic effects than would be available from any single time series. Although our sample only includes OECD countries, this cross-country variation results from the fact that some countries entered the demographic transition (i.e. Japan and Sweden) before others (i.e. Spain and Portugal).

Having estimated a_i, A and D, from the panel VAR, the long-run moving equilibrium for the system (ignoring population growth and oil prices) is then given by

$$Y_{it}^{*} = (I - A)^{-1} a_i + (I - A)^{-1} DW_{it},$$

where the effect of the demographic variables is given by $D_{LR} = (I - A)^{-1} D$, which reflects both the direct effect of demographics on each variable and the feedback between the endogenous variables. Therefore, the time series element allows us to consider, for instance, the effects of demography on savings to influence growth through the effect of savings on growth. We can isolate the long-run contribution of demography to each variable in each country by obtaining the demographic attractor for the economic variables at any moment in time

$$Y_{it}^{D} = (I - A)^{-1} DW_{it} = D_{LR} W_{it}. \tag{6}$$

We denote each element in matrix D_{LR}, which is a function of parameters in matrices A and D, $c_{ij}(A, D)$. In order to test whether each element or a sum of those elements in D_{LR} are significantly different from zero (e.g. $H_0 : D_{LR}(i, j) = 0$ or $H_0 : c_{ij}(A, D) = 0$) we utilize a non-linear Wald test.[8] Finally, it is important to distinguish between our long-run estimate and a long-run steady state. Our estimates provide a long-run forecast for the economic variables conditional on a particular vector of demographic shares after the completion of the endogenous adjustment of the economic variables and as such we are measuring the impact of demographics to the long-run trend of the key macroeconomic variables. However, as time passes the demographic structure might evolve towards a steady state demographic distribution. We do not model this process and thus are not providing an estimate of the effects of this convergence process of current demographic structure to its steady state onto the macroeconomy.

Before presenting our empirical results it is worth highlighting the main differences of our empirical specification relative to Higgins (1998) who also studies the quantitative effects of demographics on the macroeconomy. Firstly, Higgins (1998) directly estimates demographics on macroeconomic variables in a panel setting and thus for a variable of interest $y_{i,t}$ and demographic measure $d_{i,t}$, he postulates that $y_{i,t} = \beta d_{i,t} + \gamma x_{i,t-1}$.[9] Given that some of the dynamics of the variables of interest is short-term in nature, this estimation procedure typically includes some controls ($x_{i,t-1}$) in order to account for the business cycle variations in $y_{i,t}$ (variables used are, for instance, a productivity measure and output growth) and as such the parameter β should be interpreted as the effect of demographics on the trend of the macroeconomic variables of interest; therefore comparable to our matrix D_{LR}. A

[8] The Wald statistic is given by $c_{ij}(\hat{A}, \hat{D})^T [c'_{ij}(\hat{A}, \hat{D})(\hat{V}(A, D))c'_{ij}(\hat{A}, \hat{D})^T]^{-1} c_{ij}(\hat{A}, \hat{D}) \xrightarrow{\mathcal{D}} \chi^2_Q$, where $\hat{V}(A, D)$ is the estimated variance-covariance matrix and $c'_{ij}(\hat{A}, \hat{D})$ is the gradient of function $c_{ij}(\hat{A}, \hat{D})$.

[9] Interactive terms involving elements of $x_{i,t-1}$ and $d_{i,t}$ might also be used.

disadvantage of using this empirical strategy is that $x_{i,t-1}$ might not be sufficient to control for short-term dynamics and more importantly $y_{i,t}$ and $x_{i,t}$ may be endogenously determined and jointly affected by low frequency demographic changes. As advocated by Sims (1972), a VAR is a more appropriate empirical model in that it allows for a wide range of interactions to be considered. Our empirical model explores these interactions to measure the impact of demographics. Moreover, (as in our methodology variables are endogenous) we can study the effects of demographics on output growth instead of using it as a control variable. Secondly, given the correlation structure of the main variables, Higgins (1998) uses five year averages. Although demographic changes may reasonably be considered exogenous since these are mostly related to fertility choices occurred several decades before, using five year averages might introduce potential biases. Our estimation procedure uses yearly changes and given that it is appropriately accounting for endogeneity issues, we can explicitly test the validity of this exogeneity assumption. Finally, while we directly incorporate demographic weights, Higgins (1998) uses Fair and Dominguez's (1991) polynomial methodology, parameterising the effects of demographics.

3. Empirical Results

We start by presenting the results for our benchmark specification including the six main macroeconomic variables, namely output growth, investment, savings, per capita hours worked, real interest rates and inflation.

3.1. Panel VAR estimates - Benchmark model

We rely on the Schwarz Bayesian information criterion, SBC, to chose between possible specifications. On that basis, a one-way fixed effect model with country intercepts was preferred for every equation to a two-way fixed effect model with country and year intercepts, but without the oil price. This suggests that cross-section dependence or common trends is not a major problem with the model, but we investigate the robustness of our results to this below. A VARX(1) and a VARX(2) had almost identical SBCs. We used a VARX(1) keeping the model as parsimonious as possible (results for VARX(2) are qualitatively similar).

Table 1 presents the A matrix,[10] where each row represents an equation in the panel VAR representation. We note that hours worked, investment, savings and real rates are highly persistent and real output and inflation rate are moderately so. Basic Granger causality tests suggest that all our endogenous variables are Granger causal (with two lags) for most of the other variables in the system. As such, we seem to capture well the dynamic interac-

[10]Full estimates together with robust standard errors are shown in the Appendix.

tions between the main economic variables.[11] Finally, the matrix of correlations between the residuals of each equation of the VAR (presented in the Appendix) shows very strong contemporaneous correlations between the residuals of some of the equations possibly reflecting business cycle effects.

	g_{t-1}	I_{t-1}	S_{t-1}	H_{t-1}	rr_{t-1}	π_{t-1}
g_t	0.212*	-0.234*	0.056	0.009	-0.235*	-0.252*
I_t	0.18*	0.778*	0.025	0.011	-0.093*	-0.088*
S_t	0.043	-0.13*	0.806*	-0.008	-0.078*	-0.071*
H_t	0.288*	-0.026	0.047**	0.919*	-0.121*	-0.092*
rr_t	0.042	-0.266*	-0.14*	0.039	0.726*	0.152
π_t	0.039	0.214**	0.121**	-0.007	-0.109	0.561*

Note: $** = 10\%, * = 5\%$ levels of significance.

Table 1: Coefficients - A

Table 2 presents the matrix of short-term demographic impacts (D). As expected the individual coefficients are not well determined because of collinearity, but the hypothesis that the coefficients of the demographic variables are all zero is strongly rejected for all equations.[12] Generally the results look plausible, meaning dependant population as represented by the 0-9, 10-19 and 70+ have in general a negative impact on real output, investment, savings, hours worked and real rates while working population (20 - 60 groups) generally have a positive impact. Younger and older generations appear to have a positive impact on inflation whereas working age groups impact inflation negatively.

	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8
g	-0.063	0.197*	0.185*	0.004	-0.034	0.032	-0.084	-0.236
I	-0.069	0.035	0.061	-0.001	-0.058	0.043	0.181*	-0.191*
S	-0.063	0.146*	-0.023	0.099	0.050	0.18*	0.043	-0.431*
H	-0.172*	-0.029	0.068	0.206*	-0.048	0.100	0.002	-0.126
rr	-0.463*	-0.069	0.180	0.375*	0.259**	0.146	0.243	-0.671*
π	0.484*	0.141	-0.098	-0.403*	-0.341*	-0.097	-0.073	0.388

Note: $** = 10\%, * = 5\%$ levels of significance.

Table 2: Short-Run Demographic Impact - Matrix D

Table 3 shows the $(I - A)^{-1} D = D_{LR}$ matrix. We observe that allowing for the dynamics and interactions strengthens the general impact of demographics, the long-run effects are much larger. As expected statistical significance for each parameter is difficult to obtain

[11] Perhaps the most surprising feature is that lagged investment has a negative effect on growth. Nonetheless, there is a strong positive contemporaneous correlation between the growth and investment residuals (See Appendix). For OECD countries Bond, Leblebicioğlu, and Schiantarelli (2010) found a small positive effect in the bivariate relationship.

[12] Details are shown in Tables A.1 and A.2 in the Appendix.

(only 10 out of 48 elements of D_{LR} is significant) given that each element in D_{LR} is a function of 42 estimated parameters (matrix A and a column of matrix D). However, each demographic weight is related to the overall age structure, when one weight increases, others must be decreasing, thus it is important to evaluate the overall pattern of how demographics may affect the macroeconomy. As identified in the short-run impact matrix, dependants tend to have a negative impact on real output, investment, savings, hours worked per capita and real rates while working age population generally have a positive impact (the opposite for inflation). We thus test the joint significance of the parameter for workers, dependants and their difference. Note that most contributions that look at the impact of demographics also advocate that the relationship between demographics and the macroeconomy should consider the impact of the entire age structure. Fair and Dominguez (1991) in fact assumes a polynomial representation of such structure prior to estimation, while here we do not restrict each parameter during the estimation but find that their estimated covariances are such that the combination of parameters are more precisely estimated than each parameter separately, confirming the importance of looking generally at changes in demographic structure.

	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8
g	-0.040	0.118	0.093	0.046	0.108	-0.021	-0.304*	0.000
I	-0.506	0.195	0.251	0.228	0.023	0.241	0.399	-0.832
S	0.039	0.539**	-0.371	0.275	0.381	0.645**	-0.126	-1.382*
H	-2.024**	0.053	0.498	2.654*	0.295	1.217	-1.567	-1.126
rr	-0.895	-0.328	0.507	0.847	0.375	0.149	0.255	-0.910
π	1.117*	0.656**	-0.328	-0.979*	-0.749*	0.016	-0.073	0.339

Note: $** = 10\%, * = 5\%$ levels of significance.

Table 3: Long-Run Demographic Impact - D_{LR}

Table 4 shows the results of the joint significance tests (the same Wald test is used but the new non-linear function is $f_i(A, D) = c_{ij}(A, D) + c_{ik}(A, D)$ for elements j and k in row i). First, we find the impact of demographics on savings and interest rates gives support to the life-cycle hypothesis. Savings increase when the share of workers approaching retirement (30-59) increases and decrease substantially when the share of retirees increase. Moreover, when the share of dependants (old and young) increase while the weight of workers decrease, interest rates tend to decrease indicating the marginal propensity to consume out of income from workers might be falling. Life-cycle effects are also observed for hours worked per capita. The effect on hours and savings are particularly marked as these are highly persistent. Investment is negatively affected by young and old dependants and although we find that workers impact investment positively, the estimates are not precise. Long-term growth rates are positively affected by working population (20-50) and negatively affected by dependants (0-9 and 60-69 age groups). In summary our estimation provides evidence that societies with larger dependant age groups and smaller working age population faces in the long-term a statistically significant decline in hours worked, real rates, savings and investment and higher inflation.

	Workers	Dependants	Difference
g	$p(\sum_{j=2}^{5}\delta_j = 0)$=0.056	$p(\delta_1 + \delta_7 = 0)$=0.089	$p(\sum_{j=2}^{5}\delta_j = \delta_1 + \delta_7)$=0.050
I	$p(\delta_3 + \delta_4 = 0)$=0.276	$p(\delta_1 + \delta_8 = 0)$=0.057	$p(\delta_3 + \delta_4 = \delta_1 + \delta_8)$=0.078
S	$p(\sum_{j=4}^{6}\delta_j = 0)$=0.009	$p(\delta_7 + \delta_8 = 0)$=0.027	$p(\sum_{j=4}^{6}\delta_j = \delta_7 + \delta_8)$=0.011
H	$p(\delta_3 + \delta_4 = 0)$=0.026	$p(\delta_1 + \sum_{j=7}^{8}\delta_j = 0)$=0.026	$p(\delta_3 + \delta_4 = \delta_1 + \sum_{j=7}^{8}\delta_j)$=0.015
rr	$p(\delta_3 + \delta_4 = 0)$=0.073	$p(\delta_1 + \delta_8 = 0)$=0.128	$p(\delta_3 + \delta_4 = \delta_1 + \delta_8)$=0.072
π	$p(\delta_3 + \delta_4 = 0)$=0.001	$p(\delta_1 + \delta_8 = 0)$=0.020	$p(\delta_3 + \delta_4 = \delta_1 + \delta_8)$=0.003

Table 4: Joint Tests - p-values of Nonlinear Wald Test

3.2. Panel VAR estimates - Three generations

Given that our results show common patterns for dependant and working age groups we also analyze the link between demographics and the macroeconomic trends considering a less granular age structure. To this end, we reclassify demographic groups and estimate the model considering only three demographic groups at any given time. In particular, we aggregate age groups 0-9 and 10-19 as young dependants, age groups 20-29, 30-39, 40-49 and 50-59 as workers and 60-69, 70+ as older workers and retirees. Of course, this classification is somewhat coarse. Since the official retirement age in most OECD countries is around 65, there are some in the 60-69 age group who should actually be in the category of workers. Similarly, some in the 10-19 age group are also working. However, since our theoretical model has three groups (youngsters, workers and retirees), these estimates provide a closer link between theory and empirics.

We start by reporting (Table 5) the long-term demographic effects for these age groups (β's). We observe the same patterns as in the benchmark model: the working age group contributes positively to growth, investment, savings, hours worked and interest rates and the old dependants, particularly, contribute negatively. Reducing the number of demographic groups tends to produce more precise parameter estimates with half of the elements in the matrix being significant at the 10% level. We observe that the demographic effects (β's) on investment although with the expected sign are poorly estimated. Given that the pairs δ_1 and δ_2, and δ_7 and δ_8, had opposite effects on investment (see benchmark estimation) the effects of the 0-9 and 10-19 year old groups, and 60-70 and +70 age groups might be offsetting each other such that estimates become imprecise. Finally, although the effects on interest rates for the workers and old dependants are statistically insignificant, they have the expected sign and the hypothesis that $\beta_2 = \beta_3$ is rejected with 5% confidence.

Individual Country and Pooled Prediction Exercises

	β_1	β_2	β_3
g_t	0.040	0.103	-0.143*
I_t	0.068	0.091	-0.159
S_t	0.331*	0.226	-0.558*
H_t	-0.703*	1.704*	-1.001**
rr_t	-0.33**	0.627	-0.298
π_t	0.752*	-0.87*	0.119

Note: $** = 10\%, * = 5\%$ levels of significance.

Table 5: Long-Run Demographic Impact

Given that this empirical model is the closest to our theoretical setting we use our estimates to perform two distinct individual country predictions and a general (pooled across countries) response to an increase in fertility (baby-boomers). For the individual country exercises, we utilize the predicted future demographic structure as provided by the UN World Population Prospects (2010) and feed into our long-run reduced form model to project the effect of expected changes in demographic structure on the long-run values of our macroeconomic variables. Table 6 provides forecasts of the impact of changes in demographic structure on average annual GDP growth (in percentage points). The first (second) column shows the long-run growth rate in 2009 (2019) incorporating the effects of demographic changes from 2000-2009 (2010-2019). The results suggest that in all countries in our sample, as well as Germany, demographic changes over this decade depress long-term GDP growth. The magnitude of the drop is highly economically significant: for the US, for example, it is 0.92 percentage points and for Japan 0.99. The last column of the table shows the significance test; the predicted drop in trend GDP growth is statistically significant (highest p-values is 6.3%) for all countries.

We also present the predicted long-term trend of output growth and real rates for selected countries (the prediction for other countries is shown in the Appendix) for the period 2000 until 2030 in Figure 2. The initial point in the year 2000 is given by the respective average of the variable for the period 1970-2000. Then at each time $\tau \in [2001, 2030]$ we depict the resulting output growth and real rates incorporating the long-run effects of demographic changes measured from $t = 2000$ till τ. As it can be seen, demographic changes are expected to reduce growth and real interest rate in many OECD countries in the next decades; in some cases we observe negative real interest rates and output growth rates. The effect on trend output growth in 2030 is always significant at 10%. Given that β_3 is not as precisely estimated for real rates, and most of the predicted demographic changes imply a strong increase in the weight of old dependants (being compensated by decreases in weights for both the other groups), we find that the probability that real rates decrease in 2030 across countries is around 80% to 85%.

	2000-2009	2010-2019	Change	Prob(Change>0)
Australia	1.64%	0.95%	-0.69%	0.050
Austria	2.05%	1.37%	-0.68%	0.038
Belgium	2.03%	1.28%	-0.75%	0.056
Canada	1.57%	0.45%	-1.12%	0.047
Denmark	1.20%	0.64%	-0.57%	0.041
Finland	1.23%	0.18%	-1.05%	0.051
France	1.57%	0.73%	-0.83%	0.054
Germany	1.66%	0.76%	-0.91%	0.048
Greece	1.50%	0.88%	-0.63%	0.059
Iceland	2.56%	1.77%	-0.80%	0.043
Ireland	3.59%	2.83%	-0.76%	0.061
Italy	1.83%	1.23%	-0.60%	0.053
Japan	0.92%	-0.07%	-0.99%	0.050
Luxembourg	1.98%	1.62%	-0.37%	0.044
Netherlands	0.51%	-0.55%	-1.06%	0.046
New Zealand	2.64%	1.87%	-0.78%	0.043
Norway	2.77%	2.16%	-0.61%	0.042
Portugal	2.19%	1.38%	-0.80%	0.043
Spain	1.42%	0.75%	-0.67%	0.063
Sweden	0.44%	0.05%	-0.39%	0.048
Switzerland	1.54%	0.77%	-0.77%	0.042
United Kingdom	1.83%	1.43%	-0.40%	0.044
United States	1.93%	1.00%	-0.92%	0.051

Table 6: Average Predicted Impact on GDP Growth by Country

Our pooled exercise uses the VARX estimated equation, particularly matrices $\hat{A}$ and $\hat{D}$, and calculate the economy's response ($\{Y_t\}_{t=0}^{T=80}$) to an exogenous change to demographic weights (assuming a stationary age structure as starting point) produced by an increase in fertility that dies out slowly ($AR(1)$ with $\rho = 0.9$).[13] Figure 3 shows the responses to output growth, investment and real rates and the exogenous changes in our three age groups. We show the responses for $\tau = 10$ until $T = 79$ dropping the first 10 years such that some dynamics between the macroeconomic variables take effect after the initial change in weights (W). Furthermore, in order to capture the overall path of the main variables we also show the trend response, smoothing the discontinuities at decade 20 and 60 when the cohort impacted by the increase in fertility changes from one age group to the next (see the fourth plot in Figure 3). The results are in line with the long-run effects depicted in matrices D_{LR}. An increase in the weight of young dependants tend to depress output, investment and real rates. As the new cohorts enter the working population, output, investment and interest rates increase. After 60 years when the affected cohorts start joining the old dependant group the key variables fall. In section 5 we compare both prediction and pooled responses obtained empirically against the simulation results from our theoretical model.

[13]Note that this is not a classic impulse response of a random disturbance commonly used in VARs. The plots depict responses to changing the exogenous variables (which due to the link between demographic weights as population ages vary for all $t = 0$ to $t = 80$) in our VARX. Nonetheless, in this exercise we explore the short-term dynamics of our estimated system while in Figure 2 we focus directly on long-run effects.

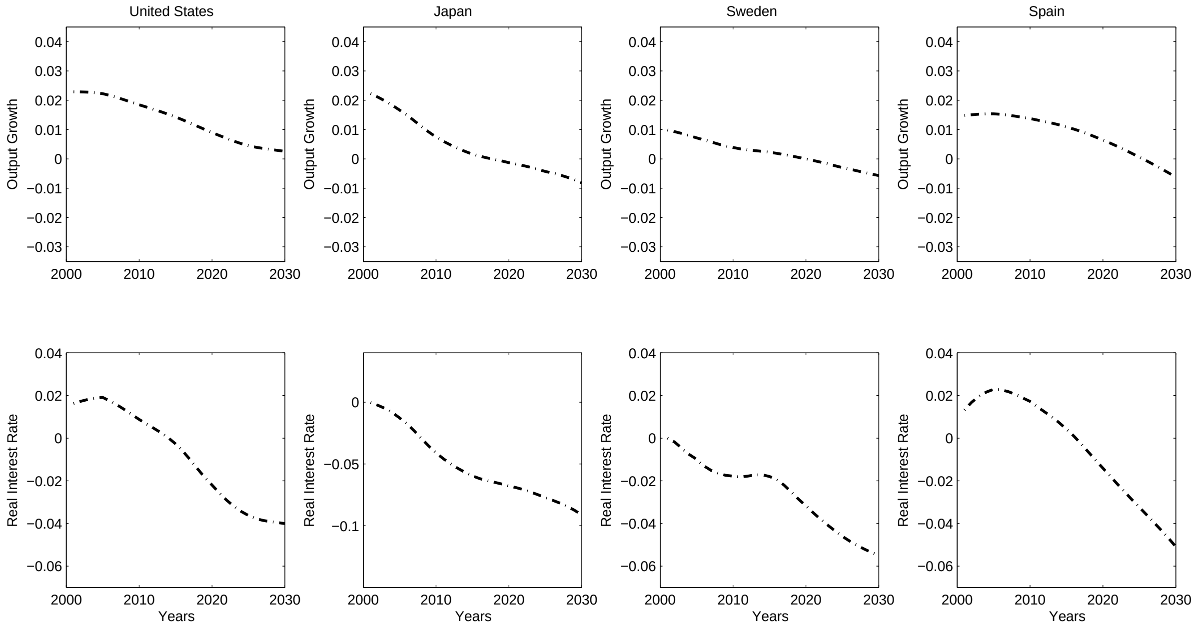

Figure 2: Impact of Predicted Future Demographic Structure

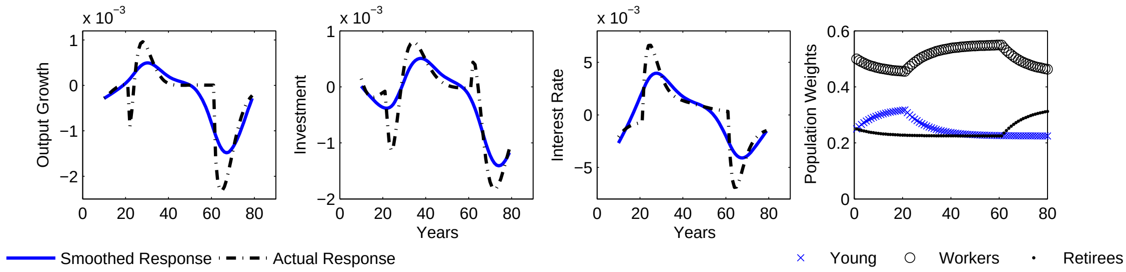

Figure 3: Response to an increase in fertility

3.3. Robustness

Controlling for time effects

As noted above the SBC chose the one-way (country) fixed effect model. There is the danger that if there are common trends correlated with the demographic variables, these trends may be wrongly attributed to the demographic variables. A two-way effects model (with country and year intercepts) avoids this issue by allowing for any global factor influencing all countries.

Tables A.4 and A.5 in the Appendix shows the long-term impact of demographic variables and the joint significance tests under a two-way fixed effects model, respectively. Comparing D_{LR} in this estimation with the same matrix obtained in the benchmark estimation reveals that long-term demographic effects are generally robust to the chosen effect. We do not observe any sign reversals and the significant cohorts impacting each macro variable remain the same. We now find that δ_5 is relevant for investment and as such the hypothesis that $\delta_3 + \delta_4 + \delta_5 = 0$ is rejected at 10% confidence level. The relationship between saving and demographics is not as strong as before given that δ_6 is no longer significant. We also observe some attenuation of the effect of demographics on inflation. We conclude that the impact of demographic variables on the macroeconomy identified by the model is not spuriously determined by common trends.

Per Capita Output Growth

In this alternative specification we replace output growth by the per capital output growth ($g^{pc}_{i,t}$). Tables A.6 and A.7 in the Appendix shows the long-term impact of demographic variables and the joint significance tests, respectively. In general, the alternative model with per capita output growth has similar demographic effects but estimates are less precise. Although we observe slightly higher p-values for the joint significant tests, the same conclusions are statistically supported by the data; economies with lower proportion of workers and higher proportion of dependants will experience lower per capita output growth, investment, savings and real interest rates.

Exclusion of individual countries

We test the robustness with respect to the selected countries by re-estimating the model on a dataset with each country excluded in turn. The results remain qualitatively similar, as are the tests as to whether the demographic variables are significant in each equation.

Structural Change

We also test for potential structural change by estimating the model on sub-periods of the entire dataset, and selecting the preferred model using the SBC. A single model over the whole period was preferred over models with structural breaks in any given year for the first four equations in the VAR - growth, investment, savings and hours worked. For the last two equations, interest rates and inflation there is evidence of structural breaks around the years 1982 and 1983.[14]

Exogeneity Test

[14]Estimating the model over the sample 1984-2007 yields inflation and real interest rate results that differ somewhat from the full-period estimation, indicating the possible presence of structural instability. The ranges of the demographic variables for these two periods are also somewhat different. The sub-sample has a vastly reduced variation in interest rates since the euro member countries in our sample shared a common rate for part of the period.

We test the exogeneity of demographic weights to the vector of macroeconomic variables by estimating a VAR with both vectors $Y_{i,t}$ and $W_{i,t}$ treated as endogenous, as depicted in equation (5), and verify whether matrix B_1 is equal to zero. The estimated matrix B_1 is shown in the Appendix. Although some parameters in B_1 are precisely estimated and found to be different than zero they are all considerably small. For instance, the highest significant parameter estimate is 0.011, measuring the effect of interest rates on the proportion of 50-60 year old individuals. That implies that a 100 basis point move in real rates would lead to a change in that proportion from 10% to 10.001%. We thus conclude that changes in $Y_{i,t-1}$ do not significantly (economically) affect $W_{i,t}$, validating our exogeneity assumption. We also find that the diagonal elements of B_2 are smaller but close to one, as demographic weights are very persistent and that matrices A^{endo} and B^{endo} are similar to matrices A and D obtained in the VARX estimation.

3.4. Panel VAR Estimation - Introducing Innovation variables

As mentioned in the Introduction, Kuznets (1960) discusses in detail the relevance of the native population growth, and hence presence of the young, for innovation activities, among other factors. Feyrer (2007) examines the link between productivity and demographic structure and finds strong and robust relationship between these two. In two other papers (Feyrer (2008), Feyrer (forthcoming)) he suggests two potential channels through which age structure can affect productivity: innovation and adoption of ideas through managerial and entrepreneurial activity.[15]

In this section, in order to account for possible dynamic interactions between demographic structure and innovation which in turn will affect technological progress, we re-estimate the model including an additional variable that proxies for $R\&D$ activity. To this end, we utilize World Development Indictors of the World Bank on residential patent applications, and include the log number of patent applications per capita, ($R\&D^{PA}$), as an additional variable in our system.[16]

Table 7 (left panel) gives the $(I - A)^{-1} D$ matrix of long-term demographic impacts with seven endogenous variables when eight demographics groups are included, while the right panel show the results when we estimate the model using three demographic groups. First, the introduction of innovation variable does not affect the main qualitative conclusions

[15] He shows that in the US innovators' median age is stable around 48 over the 1975-95 sample period whereas median age of managers who adopt ideas are lower around the age of 40 and the managerial median age is affected by the entry of the babyboom generation into the workforce over the years. He argues that changes in the supply of workers may have an impact on the innovation rate.

[16] Note that the data for residential patent applications for Australia and Italy are incomplete, therefore we exclude these countries in our estimations. We also interpolate residential patent applications data for Japan for the years 1981 and 1982 as there seemed to be anomaly in their data for these two years.

obtained in the benchmark estimation.[17] Second, the results for the 3 generations case indicate that young dependants and older generations contribute negatively to variations in patent applications whereas the workers (20-60) contribute positively. Third, in line with the evidence in Jones (2010) and Feyrer (2008) whereby the age distribution of patent applicants increases sharply from 30-35 years old, peaks around 45 years old and drops significantly after 50 years old, we find a strong and statistically significant positive effect of middle-aged workers (40 - 49) on $R\&D^{PA}$. We also strongly reject the $H_0 : \delta_5 + \delta_6 = \delta_7 + \delta_8$, as such, as society ages, with a decline in the proportion of middle-aged workers, the economy innovates less. As a robustness exercise we also estimate the model with the growth rate of patent application instead of patents per capita and find that mature workers (40 - 49) contribute positively while older workers (50-59) contribute negatively, confirming the importance of middle age workers relative to other age groups to the innovation process. Finally, using these long-term estimates and UN population predictions we find that the expected aging in the next two decades may lead to a drop in patent application per capita across OECD countries of around 20%.

	Benchmark								Three Generations		
	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8	β_1	β_2	β_3
g	-0.03	0.10	0.09	0.07	0.08	-0.05	-0.29**	0.03	0.03	0.10	-0.13**
I	-0.55	0.24	0.26	0.26	-0.05	0.24	0.43	-0.83	0.08	0.06	-0.14
S	-0.03	0.59	-0.36	0.32	0.40	0.78**	-0.19	-1.5*	0.31*	0.28	-0.6*
H	-1.71	-0.13	0.46	2.66*	0.43	0.93	-1.27	-1.38	-0.61**	1.73*	-1.12*
rr	-0.65	-0.75	0.66	0.83	0.51	-0.14	0.50	-0.95	-0.38**	0.68	-0.30
$R\&D^{PA}$	-1.88	-6.56	2.67	-0.08	8.78*	1.18	-1.09	-3.02	-3.84*	5.01*	-1.17
π	0.93*	0.91*	-0.37	-0.95*	-0.95*	0.36	-0.29	0.37	0.78*	-0.93*	0.14

Note: $** = 10\%, * = 5\%$ levels of significance.

Table 7: Long-Run Demographic Impact

4. Theoretical Model

In this section we propose a model that accounts for our main empirical findings and use it to perform different simulations studying the effects of demographic changes. Given that we are interested in those effects after the completion of the endogenous adjustments of the economic variables, our modelling framework focuses on demographic heterogeneity and medium-run dynamics, incorporating life-cycle properties and endogenous productivity to our economic environment. The economy consists of three main structures: a production sector, an innovation sector and households. The production sector comprises a final

[17] We perform the same joint significance test as in the benchmark case. Although p-values are slightly higher (particularly for the output growth equation) as each element in D_{LR} is now function of 56 estimated parameters, instead of 42 as in the benchmark model, the difference of the demographic impact of workers versus dependants remains significant for all variables (See the Appendix for details).

good producer, whose factors are differentiated goods (inputs), and input producers, whose production process employs capital, labour and a composite of intermediate goods. The number of input producers is endogenously determined, hence entry and exit is permitted. The composite of intermediate goods aggregates an endogenous set of product varieties, created by the innovation process. Product innovation consists of two joint processes. Product creation (prototypes) or R&D and product adoption, in which prototypes are made ready to be used in the production process as specialised intermediate goods. Within the household sector, individuals - who supply labour, accumulate assets and consume - exhibit life-cycle behaviour, albeit of a simple form. Individuals face three stages of life: young/dependant, worker and retiree. Finally, there is a zero expected profit financial intermediary to facilitate the allocation of assets between the household and the production and innovation sectors.

4.1. Production

The final good producer combines inputs from N_t^f firms, denoted by superscript j. Total output is thus given by

$$Y_{c,t} = \left[\int_0^{N_t^f} (Y_{c,t}^j)^{(1/\mu_t)} dj \right]^{\mu_t}, \tag{7}$$

where μ_t denotes the mark-up of input firms. We assume $\mu_t = \mu(N_t^f)$, $\mu'(\cdot) < 0$ and that profits of intermediate good firms $\Pi(\mu_t, Y_{c,t}^j)$ must equate operating costs given by $\Omega\tilde{\Psi}_t$, where $\tilde{\Psi}_t$ is a scaling factor defined to ensure we obtain a balanced growth path (see below).

Each firm j produces a specialised good using capital (K_t^j), labour (L_t^j) and an intermediate composite good (M_t^j). Production is given by

$$Y_{c,t}^j = \left[(U_t^j K_t^j)^{\alpha} (\xi_t L_t^j)^{(1-\alpha)} \right]^{(1-\gamma_I)} \left[M_t^j \right]^{\gamma_I}, \tag{8}$$

where U_t^j is the utilisation rate, γ_I the intermediate good share, $\xi_t L_t$ denotes the effective labour units employed in production and α the capital share of added value. The intermediate composite good used by firm j aggregates A_t specialised goods such that

$$M_t^j = \left[\int_0^{A_t} (M_t^{ji})^{(1/\vartheta)} di \right]^{\vartheta}. \tag{9}$$

Each producer of specialized good i acquires the right to market this good via the creation and adoption process. Total costs of production for firm j are then given by

$$TC = W_t \xi_t L_t^j + (r_t^k + \delta(U_t)) K_t^j + P_t^M M_t^j$$

Where W_t is the wage, r_t^k is the rent of capital, $\delta(U_t)$ is the capital depreciation rate, with

$\delta'(\cdot) > 0$, and P_t^M is the price of the intermediate composite good.

4.2. R&D and Adoption

The creation of intermediate good varieties is divided into two stages: R&D and conversion/adoption.

R&D

Let Z_t^p be the stock of invented goods for an innovator p. Then at every period an innovator spends S_t^p to add new goods to this stock. Each unit spent produces φ_t new goods. Thus, Z_{t+1}^p is given by

$$Z_{t+1}^p = \varphi_t S_t^p + \phi Z_t^p,$$

where ϕ is the implied product survival rate. In Comin and Gertler (2006) the productivity of new inventions φ_t is assumed to be given by $\varphi_t^{CG} = \chi Z_t[\tilde{\Psi}_t^{\rho}(S_t)^{1-\rho}]^{-1}$, where χ is a scale parameter. Thus, it depends on the aggregate stock of invented goods (Z_t), so there is a positive spillover as in Romer (1990), and on a congestion externality via the factor $[\tilde{\Psi}_t^{\rho}(S_t)^{1-\rho}]^{-1}$, such that, the R&D elasticity of new technology creation in equilibrium is ρ.[18] However, as Kremer (1993) discusses if each individual's chance of being lucky or smart enough to invent something is independent of population size, then the number of individuals working relative to total population will be important to determine the aggregate growth rate of invented goods in an economy.[19] Moreover, Jones (2010) and Feyrer (2008) analyse the age profile of inventors/innovators and show that young and middle-aged workers contribute the most to the pace of the innovation process. Our estimation results also suggest that groups of young/middle age workers (20-29 and 40-49) contribute positively to patent applications while dependants contribute negatively.[20] Consequently, innovation does not seem to be independent of the demographic structure and particularly the proportion of young/middle aged workers seems to correlate positively with innovation.

In order to incorporate the importance of the ratio of (young) workers in the innovation process we assume the productivity of innovation is given by $\varphi_t \equiv (\Gamma_t^{yw})^{\rho_{yw}} \chi Z_t[\tilde{\Psi}_t^{\rho}(S_t)^{1-\rho}]^{-1}$,

[18] As a way to ensure that the growth rate of new intermediate product is stationary, they also assume that the congestion effect depends positively on the scaling factor $\tilde{\Psi}_t$. Thus, everything else equal the marginal gain from R&D declines as the economy evolves.

[19] Similarly, in Kuznets (1960)' words "[...] since we have assumed the education, training, and other capital investment necessary to assure that the additions to the population will be at least as well equipped as the population already existing, the proportion of mute Miltons and unfulfilled Newtons will be no higher than previously. Population growth [...] would, therefore, produce an absolutely larger number of geniuses, talented men, and generally gifted contributors to new knowledge whose native ability would be permitted to mature to effective levels when they join the labor force."

[20] Liang, Wang, and Lazear (2014), although looking at entrepreneurship and not directly at R&D production shows that a high proportion of old workers prevents young workers gaining the necessary knowledge to start up a new business, thus reducing entrepreneurship.

where Γ_t^{yw} is a measure of the stock of workers relative to the rest of the population and ρ_{yw} controls the importance of workers to the aggregate productivity of innovation. If $\rho_{yw} = 0$, the innovation process is equivalent to the one assumed in Comin and Gertler (2006). We present the definition of Γ_t^{yw} when we discuss the population dynamics below.

Based on that the flow of the stock of invented products becomes

$$Z_{t+1}^p = (\Gamma_t^{yw})^{\rho_{yw}} \chi Z_t [(\tilde{\Psi}_t)^{\rho} (S_t)^{1-\rho}]^{-1} S_t^p + \phi Z_t^p. \tag{10}$$

We assume that innovators borrow S_t^p from the financial intermediary. Define J_t as the value of an invented intermediary good. Then, innovator p will invest S_t^p until the marginal cost equates the expected gain. Thus,

$$\phi E[J_{t+1}] = \frac{R_{t+1}}{\varphi_t}. \tag{11}$$

Where R_{t+1} is the interest rate. The realised profits of an innovator is

$$\Pi_t^{RD} = \phi J_t (Z_t^p - \phi Z_{t-1}^p) - S_{t-1} R_t. \tag{12}$$

Adoption

Let $A_t^q \subset Z_t^q$ denote the stock of converted goods ready to be marketed to firms. Adopters (q) obtain the rights of technology from innovators and make an investment expenditure (intensity) of Ξ_t to transform Z_t^q into A_t^q. This conversion process is successful with probability λ_t. We assume $\lambda_t = \lambda\left(\frac{A_t^q}{\tilde{\Psi}_t}\Xi_t\right)$ and $\lambda'(\cdot) > 0$, thus more intensity yields more adoptions. If unsuccessful the good remains in its invented form (prototype). A converted good can be marketed at every period to firms, thus its value, denoted V_t is given by

$$V_t = \Pi_{m,t} + (R_{t+1})^{-1} \phi E_t V_{t+1}, \tag{13}$$

where $\Pi_{m,t}$ is the profit from selling an intermediate good to input firms. We can now determine the value of a unadopted product (J_t). That is

$$J_t = \max_{\Xi_t} -\Xi_t + (R_{t+1})^{-1} \phi E_t [\lambda_t V_{t+1} + (1 - \lambda_t) J_{t+1}]. \tag{14}$$

The stock of unadopted goods at period t is given by $(Z_t^q - A_t^q)$. Thus, the flow of adopted goods for an adopter q is

$$A_{t+1}^q = \lambda_t \phi (Z_t^q - A_t^q) + \phi A_t^q. \tag{15}$$

The expenditure in consumption goods of adopters, financed by borrowing, is given by

$$\Xi_t(Z_t^q - A_t^q). \tag{16}$$

That way the profit of an adopter q is

$$\Pi_t^A = \int_0^{A_t^q} \Pi_{m,t} - \phi J_t(Z_t^p - \phi Z_{t-1}^p) - R_t \Xi_{t-1}(Z_{t-1}^q - A_{t-1}^q). \tag{17}$$

4.3. Household Sector

There are a continuum of agents of mass N_t. Individuals are born as dependants (young) and remain so from period t to period $t+1$ with probability ω^y and become a worker otherwise. Workers (w) at time t remain so in period $t+1$ with probability ω^r and retire otherwise. Once retired (r) the individual survives from period t to $t+1$ with probability $\gamma_{t,t+1}$. Let N_t^r be the mass of retirees, N_t^w the the mass of workers, and N_t^y the the mass of young. Furthermore, we assume $\tilde{n}_{t,t+1}N_t^y$ dependants are born at period t. As a result, population dynamics are as follows

$$N_{t+1}^y = \tilde{n}_{t,t+1}N_t^y + \omega^y N_t^y = (\tilde{n}_{t,t+1} + \omega^y)N_t^y = n_{t,t+1}N_t^y, \tag{18}$$

$$N_{t+1}^w = (1-\omega^y)N_t^y + \omega^r N_t^w, \tag{19}$$

$$N_{t+1}^r = (1-\omega^r)N_t^w + \gamma_{t,t+1}N_t^r. \tag{20}$$

We define the measure of the stock of workers (Γ_t^{yw}), which influence the innovation process, to be equal to

$$\Gamma_t^{yw} \equiv (1-\omega^y)\frac{N_t^y}{N_t} + (1-\lambda^y)\Gamma_{t-1}^{yw}, \tag{21}$$

where $0 < \lambda^y \leqslant 1$ denotes how much the previous stock of young that became workers before t are important for the measure of that stock at the current period. If $\lambda^y = 1$ the stock is made only of the ratio of young that just entered their working life and if $\lambda^y < 1$ then at time t the stock of young is augmented by the ratio of young that entered in their working life at time $t-h$ with the decaying weight of $(1-\lambda^y)^h$. As such, the stock of workers that contribute to innovation is particularly sensitive to the stock of young dependants that become workers (young workers) at each period, and less sensitive to more experienced workers, matching our empirical evidence.

We assume society collects transfers from workers that are then used to sustain the young and finance their educational investment. This expenditure will increase the effective labour units that will be supplied by the young when they become workers. In order to define the amount of investment in education at each period, society determines the social cost of obtaining resources from current period workers (a drop in their consumption at t) and the benefits of higher effective labour supply, which leads to higher workers' consumption in the following periods. The society (or benevolent government) then sets the educational investment to offset its marginal cost and benefit (see the Appendix for details). The young are thus passive in our model. Workers and retirees, on the other hand, decide their consumption to maximise welfare subject to a budget constraint.

As in Gertler (1999), we make two key assumptions to simplify the model. An individual faces two idiosyncratic risks during her lifetime: loss of wage income at retirement and time of death. The impact of uncertainty about time of death is eliminated by introducing a perfect annuity market allowing retirees to insure against this type of risk. That way, retirees turn their wealth over to perfectly competitive financial intermediaries which invest the proceeds and pay back a return of $R_t/\gamma_{t-1,t}$ for surviving retirees. The higher return than the market is financed by the asset holdings of retirees who did not survive.

The uncertainty about employment tenure is assumed not to affect workers since they are risk-neutral. In order to also incorporate a motive for consumption smoothing we assume individual preferences belong to the recursive non-expected utility family. Thus, for $z = \{w, r\}$ we assume agent j selects consumption and asset holdings to maximise

$$V_t^{jz} = \left\{(C^{jz})^{\rho_U} + \beta_{t,t+1}^z (E_t[V_{t+1}^j \mid z]^{\rho_U})\right\}^{1/\rho_U} \tag{22}$$

subject to

$$C_t^{jz} + FA_{t+1}^{jz} = R_t^z FA_t^{jz} + W_t \xi_t^j I^z + d_t^z - \tau_t^{jz} I^z \tag{23}$$

where $\beta_{t,t+1}^z$ is the discount factor, which is equal to β for workers and $\beta\gamma_{t,t+1}$ for retirees, R_t^z is the return on assets, which is equal to R_t for workers and $R_t/\gamma_{t-1,t}$ for retirees, W_t is the wage, ξ_t^j is the effective unit of labour supplied by worker j, and I^z is an indicator function that takes the value one when $z = w$ and zero otherwise, thus we assume retirees do not work and workers' labour supply is fixed,[21] FA_t^{jz} are the assets acquired from the financial intermediary and d_t^z is the dividend from the financial intermediary. Finally, τ_t^{jz} is the transfer a worker j makes to society for the expenditure on the young with the total transfer at time t given by $\tau_t = \int_0^{N_t^w} \tau_t^{jz}$.

Let ξ_t be the average effective units across workers at period t, or the current level of labour productivity/labour skill in the society. Each young who becomes a worker at the end of period t will provide ξ_{t+1}^y effective units. We assume

$$\xi_{t+1}^y = \rho_E \xi_t + \frac{\chi_E}{2} \left(\frac{I_t^y}{\xi_t}\right)^2 \xi_t, \tag{24}$$

[21] The framework can be extended to incorporate variable labour supply. See Gertler (1999) for details.

where $\rho_E < 1$ and denotes the obsolescence of labour skills and I_t^y is the total effective expenditure on the young and is defined as the ratio between total funds and their labour cost.

$$I_t^y = \frac{\tau_t}{W_t N_t^w}. \tag{25}$$

Based on the population dynamics we can now determine the evolution of workers effective labour units, that is

$$\xi_{t+1} \quad = \quad \omega_r \frac{N_t^w}{N_{t+1}^w}\xi_t + (1-\omega^y)\frac{N_t^y}{N_{t+1}^w}\xi_{t+1}^y. \tag{26}$$

4.4. Financial Intermediary

The financial intermediary sells assets to the households (FA_t^w, FA_t^r), holds the capital (K_t) and rents it to firms and lends funds (B_{t+1}) to innovators and adopters to finance their expenditure (given by S_t and $\Xi_t(Z_t - A_t)$, respectively). Finally, we assume it owns the innovators and adopters enterprises, receiving their dividends at the end of the period. Thus, financial intermediary profits are

$$\Pi_t^F = [r_t^k+1]K_t+R_tB_t-R_t(FA_t^w+FA_t^r)-K_{t+1}-B_{t+1}+FA_{t+1}^w+FA_{t+1}^r+\sum_x(\Pi_t^{RD}+\Pi_t^A), \tag{27}$$

where $B_{t+1} = S_t + \Xi_t(Z_t - A_t)$ and $FA_t = FA_t^w + FA_t^r$.

4.5. Equilibrium

The symmetric equilibrium is a tuple of endogenous predetermined variables $\{FA_{t+1}^z$, K_{t+1}, A_{t+1}, Z_{t+1}, $FA_{t+1}, B_{t+1}, \xi_{t+1}\}$ and a tuple of endogenous variables $\{C_t^z$, H_t^w, T_t^w, d_t^z, D_t^z, K_{t+1}, L_t, Y_t, Ξ_t, μ_t, N_t^f, S_t, V_t, J_t, λ_t, Π_t^{RD}, Π_t^A, Y_t, C_t, L_t, U_t, r_t^k, δ_t, R_t, Π_t^F, W_t, P_t^M, ε_t, τ_t, I_t^y, $\varsigma_t\}$ for $z = \{w, r\}$ obtained such that:

a. Workers and retirees, maximize utility subject to their budget constraint and investment in education is such that society's marginal cost and benefit is equated;

b. Input and final firms maximize profits, and firm entry occurs until profits are equal to operating costs;

c. Innovators and adopters maximise their gains;

d. The financial intermediary selects assets to maximize profits, and their profits are shared amongst retirees and workers according to their share of assets;

e. Consumption goods, capital, labour and asset markets clear;

taking as given the initial values of all the predetermined variables $\{FA_t^z,\ K_t,\ A_t,\ Z_t,\ \xi_t,\ FA_t, B_t\}$ and the exogenous predetermined variables $\{N_t^y,\ N_t^w,\ N_t^r,\ N_t\}$ specified by the population dynamics.

Population dynamics are controlled by four exogenous variables, fertility ($\tilde{n}_{t,t+1}$), the transition probabilities into and out of the workers population ($1-\omega^y$ and $1-\omega^r$) and the variable that controls longevity ($\gamma_{t,t+1}$). Based on that, we determine the evolution of young dependency ratio (ζ_t^y), old dependency ratio (ζ_t^r) and population growth as follows[22]

$$\begin{aligned} n_{t,t+1} &= \frac{\zeta_{t+1}^y}{\zeta_t^y}\left(\omega^r + \zeta_t^y(1-\omega^y)\right), \\ \zeta_{t+1}^r &= \left((1-\omega^r) + \gamma_{t,t+1}\zeta_t^r\right)\left(\omega^r + (1-\omega^y)\zeta_t^y\right)^{-1} \text{ and} \\ \frac{N_{t+1}}{N_t} &= n_{t,t+1}\left(1 + 1/\zeta_t^y + \zeta_t^r/\zeta_t^y\right)^{-1} + \left(\omega^r + (1-\omega^y)\zeta_t^y\right)\left(1+\zeta_t^r+\zeta_t^y\right)^{-1} \\ &\quad + \left(\frac{1-\omega^r}{\zeta_t^r} + \gamma_{t,t+1}\right)\left(1 + 1/\zeta_t^r + \zeta_t^y/\zeta_t^r\right)^{-1}. \end{aligned}$$

Another key population variable that is part of the link between demography and innovation is the stock of young workers, given by

$$\Gamma_t^{yw} = (1-\omega^y)\frac{\zeta_t^y}{1+\zeta_t^y+\zeta_t^r} + (1-\lambda^y)\Gamma_{t-1}^{yw}$$

As such, societies with a greater flow from young dependants into the working-age group (higher $(1-\omega^y)\zeta_t^y$) will have a bigger relative stock of young workers that contribute more heavily to innovation. Older societies, with higher old dependency ratios ζ_t^r, will have a smaller relative stock of young workers, innovating less.

The key equations that describe the individuals' behaviour are the consumption functions of workers and retirees depicted below

$$\begin{aligned} C_t^w &= \varsigma_t[R_t FA_t^w + H_t^w + D_t^w - T_t^w] \quad \text{and} \\ C_t^r &= \varepsilon_t\varsigma_t[R_t FA_t^r + D_t^r], \end{aligned}$$

where, H_t^w is the present value of gains from human capital, T_t^w is the present value of transfers, D_t^z is the present value of dividends for $z = \{w, r\}$, ς_t the marginal propensity of consumption of workers and $\varepsilon_t\varsigma_t$ the one for retirees (where $\varepsilon_t > 1$). The marginal propensities to consume at time t are function of both ω^r and $\gamma_{t,t+1}$ and are directly linked to the expected path of interest rates. Thus, these conditions incorporate the impact of life-cycle heterogeneity on aggregate demand and asset distribution.

[22] Also note that $N_{t+1}^w = N_t^w(\omega^r + (1-\omega^y)\zeta_t^y)$ and $N_{t+1}^r = N_t^r\left(\frac{1-\omega^r}{\zeta_t^r} + \gamma_{t,t+1}\right)$

Investment in human capital (I_t^y) is determined by

$$\varsigma_t^{-1/\rho_U} = \varsigma_{t+1}^{-1/\rho_U}\beta(1-\omega^y)\zeta_t^y \frac{W_{t+1}}{W_t}\chi_E \frac{I_t^y}{\xi_t}$$

and as such it reflects the trade-offs between the marginal propensity to consume of workers today and tomorrow and the expected growth rate of wages and crucially, it depends on the share of young relative to workers ζ_t^y. The greater the young dependency ratio, the greater the burden on the current generation of workers to finance human capital accumulation, depressing investment in education.

The remaining conditions that ensure **a.**, which define the value of the stock of human capital, the present value of transfers, the present value of the profits of financial intermediaries and the evolution of the marginal propensities to consume and of the human capital (effective labour unit ξ_t) are shown in the Appendix.

The equilibrium conditions that ensure firms behave optimally determine the equilibrium wage, the rent of capital, the utilisation rate, the intermediate good composite and their price, the number of firms (through the entry condition), the mark-up and the depreciation rate (See the Appendix for detail). The total production of final goods is given by

$$Y_{c,t} = (N_t^f)^{\mu_t - 1}\left[(U_t \frac{K_t}{\xi_t L_t})^{\alpha}(\xi_t L_t)\right]^{(1-\gamma_I)} [M_t]^{\gamma_I}.$$

The intermediate good composite M_t increases with the number of varieties A_t and the number of input producers N_t^f. Moreover, given that the mark-up μ_t is greater than one, as N_t^f increases total production increases. As such, demographic changes that increase aggregate demand, generate entry and higher $Y_{c,t}$.

The equilibrium conditions that refer to the innovation sector determine the stock of invented and adopted goods, the intensity of innovation efforts, the expenditure on adoption, its probability of success, the value of an invented and an adopted good, and finally, the profits of inventors and adopters. The two key equilibrium conditions are the ones that pin down the growth rate of invented goods $\left(\frac{Z_{t+1}}{Z_t}\right)$ and the value of adopted goods, V_t, shown below. The remaining conditions that ensure **c.** are presented in the Appendix.

$$\frac{Z_{t+1}}{Z_t} = (\Gamma_t^{yw})^{\rho_{yw}}\chi\left(\frac{S_t}{\tilde{\Psi}_t}\right)^{\rho} + \phi. \tag{28}$$

$$V_t = (1 - 1/\vartheta)\gamma_I \frac{Y_{c,t}}{\mu_t A_t} + (R_{t+1})^{-1}\phi E_t V_{t+1}. \tag{29}$$

The first equation depicts the direct link between innovation and demographics. As economies age, Γ_t^{yw} decreases and hence, holding the intensity of innovation efforts S_t constant, the economy will innovate less (innovation effort is less productive) depending on the size of

ρ_{yw}, which controls the importance of young workers for innovation. The second equation illustrate the link between innovation and aggregate conditions. Firstly, as final good production increases, the demand for the intermediate good composite M_t (which combines all specialised varieties) increases, and thus the gains from inventing and adopting new varieties increase (first term on the right hand side). Secondly, given that adopters and inventors borrow to fund investment in innovation, the equilibrium interest rate directly influences the gains from innovation (second term on the right hand side).

The equilibrium conditions that ensure **d.** are the arbitrage condition that links rent of capital to interest rates, the solution for financial intermediation realised profits and how they are distribution across individuals and finally the solution for total loans made to innovators and adopters. Equations are shown in the Appendix.

The market clearing equilibrium conditions determine equilibrium labour used in production, the dynamics of the capital stock, aggregate consumption, added value output from supply and demand sides and finally the asset market flows for retirees and workers. The asset flow equation links consumption and demographic flows (described by the transition probability $(1-\omega_r)$) to the evolution of the distribution of assets in the economy, as shown below

$$FA^r_{t+1} = R_t FA^r_t + d^r_t - C^r_t + (1-\omega^r)(R_t FA^w_t + W_t \xi_t L_t + d^w_t - C^w_t - \tau_t) = K_{t+1} + B_{t+1} - FA^w_{t+1}.$$

Added value output is given by

$$Y_t = Y_{c,t} - A_t^{1-\vartheta} M_t - \Omega \tilde{\Psi}_t = C_t + I_t + S_t + \Xi_t (Z_t - A_t) + \tau_t$$

Thus, final good production net of intermediate goods and entry costs must be equal to total expenditure on consumption, capital, innovation and educational investments.[23]

Finally, we must define $\tilde{\Psi}_t$ such that a balanced growth path obtains. Comin and Gertler (2006) select the current value of capital stock. Given that in their model the price of capital is determined at time t, $\tilde{\Psi}_t$ fluctuates accordingly ensuring stability. We simplify our model to consider only one sector and thus the price of capital and the value of the capital stock are constant at t, invalidating this choice of scaling factor. We therefore select the current value of adopted goods as our scaling factor. Thus,

$$\tilde{\Psi}_t \equiv V_t A_t. \tag{30}$$

[23] Details of the solution to all equilibrium conditions are provided in the Appendix.

4.6. Calibration and Steady State

All quantity variables of our model grow as a result of three main drivers, the exogenously given fertility rate ($\tilde{n}$), the endogenous growth of effective labour (ξ_t) and the endogenous process of invention and adoption of new intermediate goods (A_t), which increases the productivity of the other factors of production (capital and labour). It is convenient therefore to normalize certain variables relative to final goods output (which is used as the *numeraire*), obtaining then a system of equations that provide a stationary steady state given the set of parameters.[24]

We now discuss the parameter values selected to simulate our model economy. The standard parameters present in most macro models are presented first. Given our emphasis on medium-run dynamics, one period in the model is set to one year. We thus set the discount factor β equal to 0.96. Capital share (α) as usual is set to 0.33. We set depreciation (δ) to 0.08, capital utilisation (U) to 80% and the elasticity of the change in the depreciation rate with respect to utilisation to 0.33. The share of intermediate goods (γ_I) is set to 0.5 and we set mark-up in the consumption sector (μ) to 1.1. (all those parameters choices are in line with Comin and Gertler (2006)). Finally, following Gertler (1999) we set the intertemporal elasticity of substitution ($1/(1-\rho_U)$) equal to 0.25.

We next come to the parameters that govern the innovation process. We follow Comin and Gertler (2006) closely. We set obsolescence and productivity in innovation such that growth rate of output per working age person is 0.024 and share of research expenditures in total GDP is 0.012.[25] That way, $\phi = 0.97$, $\chi = 94.42$. The mark-up for specialised intermediate goods is set to 1.6. The elasticity of intermediate goods with respect to R&D (ρ) is set to 0.9. Average adoption time is set to 10 years thus $\lambda = 0.1$. The elasticity of this rate to increasing intensity (ϵ_λ) is set to 0.9. The price mark-up elasticity to entry (ϵ_μ) is set to 1.

Finally, we set the parameters that govern population dynamics. We initially assume individuals are young on average from age 0 till 20, thus setting probability of becoming a worker ($1-\omega^y$) equal to 0.05. Individuals work from age 21 to 65, thus setting the probability of retirement (1-ω^r) equal to 0.023, and then live in retirement on average from 66 until 75, thus setting γ equal to 0.9. That implies the ratio of young to workers is 48%, the ratio of retirees to workers is 20% and retirees hold around 16% of the assets. Finally, we assume workers remain part of the pool that influences invention with probability $(1-\lambda_y) = 2/3$ and that $\rho_{yw} = 0.9$. These two last parameters directly link demographic structure and

[24]The final de-trended system of equations is shown in the Appendix, with the definition of the new variables (all in lower case) all depicted (e.g. for aggregate consumption we have $c = \frac{C}{Y_{c,t}}$).

[25]Note that as opposed to here, in Comin and Gertler (2006) there are two sectors. Thus to obtain our measure we combine the total expenditure in both sectors in their calibration.

innovation, hence we verify how their variation affects our main results.

5. Results

We perform three sets of simulations to assess the impact of different demographic structures on the medium-run macroeconomy dynamics. The first simulation titled *baby-boomers* analyses the effect of increasing fertility holding longevity constant. The second set of simulations, titled *aging* looks at the effects of increasing longevity by increasing γ and firstly leaving population growth constant (hence fertility must reduce otherwise population naturally increases) and secondly holding fertility constant and thus allowing population to grow during the adjustment process. Finally, the third set of simulations, titled *prediction*, attempt to match the change in the demographic structure predicted for a selected number of countries in our sample during the next two decades and measure their impact on growth and real interest rates.

Simulation: Baby-boomers

In the first simulation results, presented in Figure 4, we analyse the effect of increasing fertility[26] for the first 10 periods, reducing back to the benchmark level after that. We can then analyse how the changes in age structure affect the economy through time, first with an increase in dependants, then an increase in workers and finally retirees, matching closely the responses observe in Section 3.2.[27] Initially the increase in fertility leads to a decrease in growth and investment. A high proportion of dependants is a cost to society, reducing the resources available for workers, and thus reducing savings and investment. Moreover, current workers also expect the growth rate to increase in the future when those youngsters join the labour force and accordingly increase their marginal propensity to consume, reducing savings further. As a result, during the fertility boom period, technological gains (g^A) and output growth are below their steady state level. The model therefore matches well the empirical results that show that 0-20 share of population has a negative impact on investment, savings and output growth.

As youngsters become workers[28] and fertility decreases the share of youngsters decrease

[26]Instead of shocking fertility directly, we alter the replacement rate, which we obtain by calculating the ratio between total birth ($\tilde{n}N_y$) and the proportion of childbearing women in the economy (40% of workers are women and women are assumed to be of childbearing are for 20 years on average.). This proportion is given by $20 * 0.4 * \frac{N_t^w \times 45}{20}$, assuming 40% of workers are women and bear child between the ages of 21 to 40.

[27]We also perform an additional *baby-boomers* simulation, setting $\rho_{yw} = 0.5$. Fluctuations are dampened but the main qualitative conclusions remain. See the Appendix for details.

[28]Note this happens at every period in the model since a proportion of $(1 - \omega_y)$ dependants become workers, in contrast to our empirical exercise depicted in Figure 3 whereby the proportion of workers only increases after 20 years, thus our responses here are smoother, closer to the trend case shown there.

(see periods 10 to 20) while the share of workers increase (thus, the share of retirees decreases). Society is then benefiting from the demographic dividends of the previous increase in fertility. As the proportion of young workers increase (Γ_{yw}), innovation increases and the growth rate of technology (or varieties) increases sharply, peaking 25 to 30 years after the fertility burst. This increase in growth is accompanied by both, an increase in investment and consumption. Finally, workers marginal propensity to consume continues to increase, leading to higher real interest rates. Hence, the increasing share of workers leads to higher growth, investment and real rates, matching the empirical estimates. Finally, as the proportion of young dependants does not change significantly, 30 to 40 years after the increase in fertility, and the proportion of workers decrease (thus the stock of young workers is reduced), innovation, technological gains and output growth decrease. At this point the share and consumption of retirees (who benefited from greater asset accumulation during the higher growth period) increase. Contrary to retirees, current workers are forced to increase their savings relative to the previous generation reducing real rates. Lower investment and innovation implies that as the share of retirees increase in the final stages of the adjustment output growth rates deviations (relative to the steady state level) become slightly negative. Overall the model matches well the main empirical findings although the increase in the proportion of old dependants generates a more sizable drop in output, investment and real rates in our empirical results.

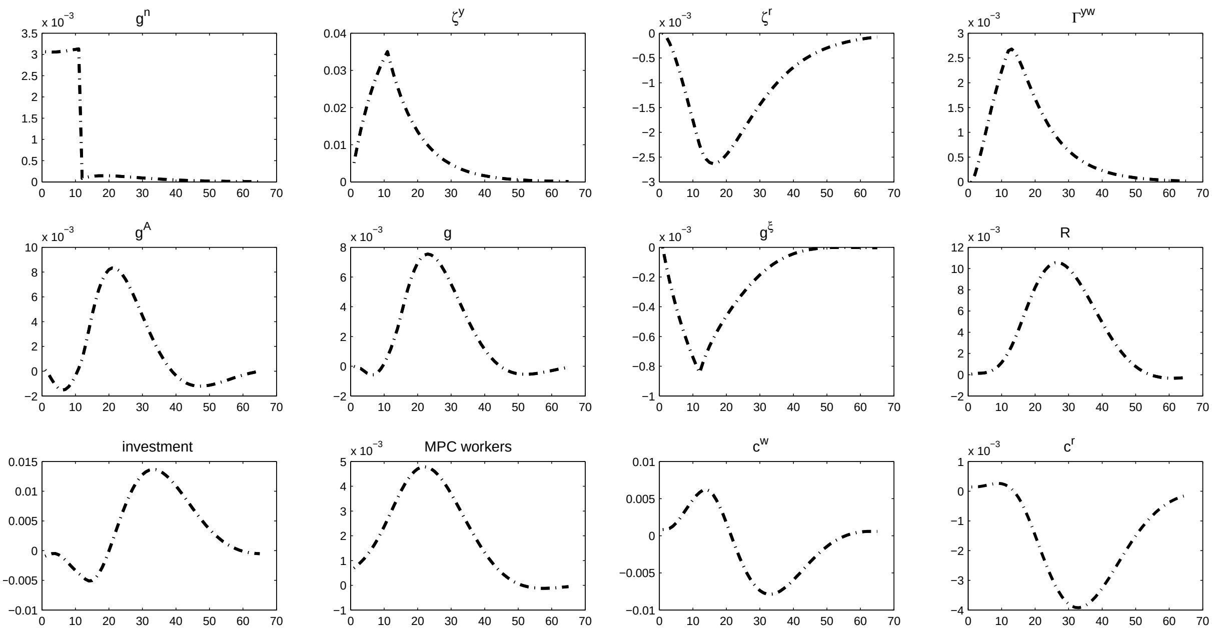

Figure 4: Simulation: *baby-boomers*

Simulation: Aging

Most economies during the period used in our estimation experienced a constant increase in life expectancy. That has resulted in a significant increase in the share of retirees in the population. In this set of simulations we smoothly increase the parameter γ such that the average retiree lives an additional 10 years, increasing societies' average age.[29] We consider two cases. The first holds population growth ($g^n = N_{t+1}/N_t$) constant. As longevity increases, ceteris paribus, population would also increase. Thus, in order to keep population constant, fertility must decrease during the adjustment process. Note that in our estimations demographic structure matters although we controlled for population growth, hence, by keeping population growth constant this simulation allows us to analyse the impact of shifting demographic structures due to aging as in the estimation. In the second case, we increase longevity but keep fertility constant thus, while population is growing, we also obtain a shift in demographic structure such that the share of retirees in the population increases. Results are displayed in Figure 5.

This set of simulations allow us to highlight the three main mechanisms through which demography impacts the economy. First, as longevity increases current workers are expected to live longer and thus have to accordingly adjust their savings, increasing asset accumulation during their working life. Workers' consumption, therefore, falls leading to a decrease in real rates. Those additional funds are allocated to investment in capital and innovation. Capital accumulation and technological gains increase, pushing the growth rate of output up. Therefore, life-cycle consumption adjustment, our first mechanism, leads to an increase in growth rates. Note that our model cannot generate a paradox of thrift such that greater desire to save decreases aggregate demand sufficiently to reduce resources such that no additional savings is done. As a result, additional resources always flow to the innovation sector increasing growth. Altering the aggregate demand features of the model may generate stronger negative effects on growth due to lower consumption. A second aspect of aggregate demand that is left out of our model which may also alter this mechanism is consumption demand composition. Both of these aggregate demand factors may decrease the positive response of growth we obtain in our model due to life-cycle consumption adjustments.

The second mechanism occurs through the adjustment of human capital accumulation due to the decrease in fertility, which only materialises when aging occurs under constant population growth - in this case the ratio of dependants to workers decrease. As workers must increase saving for retirement, the total investment in the education of young decreases. However, as the ratio of dependants decrease, the per capita investment in education increases, leading to a growth in human capital (g^ξ). As expected, that pushes the growth rate up. When fertility is kept constant, the decrease in workers resources lead to a small decrease in the growth of human capital.

[29]We set $\Delta\gamma_t = 0.9\Delta\gamma_{t-1} + 0.005$, thus γ increases at a decreasing rate from 0.9 to 0.95 in roughly 50 years.

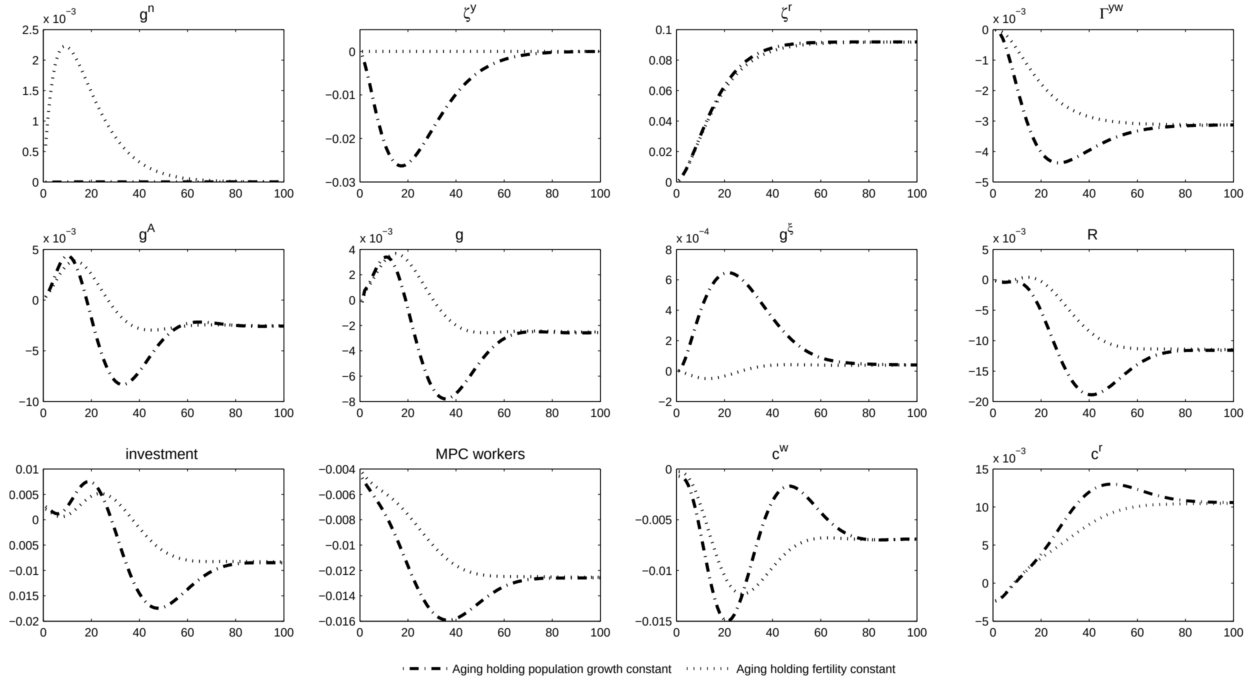

Figure 5: Simulation: *aging*

Finally, the third mechanism goes through the invention process. As our estimation results point out, as well as results from the literature of demographics and productivity and innovation (see Jones (2010) and Feyrer (2008)) and demographics and entrepreneurship (see Liang, Wang, and Lazear (2014)), young and middle age workers are relatively more important in the innovation process relative to other age groups. Our model accounts for that feature by assuming the stock of young workers relative to the total population impacts the productivity of the innovation process. Due to the aging of society this relative stock decreases, leading to a lower rate of invention and technological gains. This process is particularly strong when longevity is coupled with decreased fertility as we observed in most of the economies in the OECD.[30] If we shutdown this mechanism, by setting $\rho_{yw} = 0$, the productivity of innovation investment is independent of the demographic structure and the growth rate in the long run increases relative to its steady state level. In this case, the human capital accumulation and life-cycle consumption channels (when the paradox of thrift is not present) have a permanent and positive impact on the growth rate of technology (the results of the simulation setting $\rho_{yw} = 0$ and $\rho_{yw} = 0.5$, recall that $\rho_{yw} = 0.9$ in the benchmark model, are presented in the Appendix).

Note that the first mechanism, occurring through adjustments in consumption and sav-

[30]Note that in the long run, fertility is equal to its steady state level in both cases, only γ changes permanently. Thus, in both simulations in Figure 5, long-run growth decreases by the same magnitude.

ings as a result of life-cycle changes is strongly supported by our estimations. Population aging has been found to impact negatively interest rates. Our model indicates that this movement in interest rates is a result of workers lowering their marginal propensity to consume. Second, the third mechanism is also supported by our estimation results. Aging leads to lower patent application and thus to potentially lower contribution of innovation to growth. Moreover, as modelled here, this positive association between innovation and growth is stronger for populations with a relatively younger working population. Therefore, our theoretical model matches well the macroeconomic impacts of demographic changes but also incorporates the main channels that our empirical results give support to.

We also perform two additional simulations for robustness.[31] The first one alters λ_y, which determines the persistence of the effects of the stock of workers on innovation. Increasing λ_y decreases the amplitude of the fluctuations of the demographic changes but the main qualitative results are unchanged. The second alters the flow of the stock of workers. In the benchmark case, all youngsters who become workers influence innovation in the current period. In the alternative specification individuals (cohorts) who contribute to innovation must have at least 10 years of working experience. Thus, we set $\Gamma_t^{yw} \equiv (1-\omega^y)\frac{N_{t-10}^y}{N_t} + (1-\lambda^y)\Gamma_{t-1}^{yw}$. The simulation of this alternative specification shows a smaller response for the first 10 years, with a similar shaped response relative to the benchmark case occurring after that. Essentially, the macroeconomic effects of the demographic changes are delayed due to the assumed delayed effect of those changes on the innovation efforts.

Simulation: Prediction

In the final set of simulations we employ our model (we set $\rho_{yw} = 0.5$ for this simulations, reducing the sensitivity of innovation to the share of young workers) to analyse the effect of the predicted changes in the demographic structure on output growth and real interest rates for the next two decades in a subset of the countries in our sample, matching the prediction exercise done with the estimated model. We start by selecting three measures of expected population dynamics to feed into the model. The first is expected population growth ($g^n = N_{t+1}/N_t$). The second is the percentage point change in the share of workers, denoted Δs_w (following our empirical results we calculate that by obtaining the combined population with ages between 20 and 60 years old and dividing it by total population) and finally the third is the share of retirees (population with ages 60 and over divided by total population, denoted by Δs_r). In order to match these three measures $\{g^n, \Delta s_w, \Delta s_r\}$[32] we implicitly select three structural parameters, the fertility rate $\tilde{n}$, the longevity parameter γ and the probability a young dependant becomes a worker $(1-\omega_y)$.

[31] Results are shown in the Appendix.

[32] The share of workers in the population is given by $\frac{1}{1+\zeta_y+\zeta_r}$ and the share of retirees is given by $\frac{\zeta_r}{1+\zeta_y+\zeta_r}$, by setting those shares we are essentially selecting ζ_y and ζ_r, the young and retirees dependency ratios.

Period	Δs_w	Δs_r	g^n
2000-2005	0.5%	0.5%	1.053
2005-2011	-1.3%	2.0%	1.056
2011-2016	-1.4%	1.9%	1.043
2016-2021	-2.1%	2.2%	1.040
2021-2026	-1.3%	1.7%	1.037
2026-2031	-0.3%	0.8%	1.033

Table 8: *Prediction* Data Input: United States

As in the estimation exercise we use actual population data from 2000 till 2010 and United Nations predictions from 2011 till 2031. In the prediction exercise in the empirical section we use the long-run estimates to obtain the impact of demographic structure on the main macroeconomic variables. As such we select the average change of our three empirical measures of population dynamics for 5 year intervals such that some degree of endogenous feedback due to changes in demographic variables are captured in the theoretical simulation. As an example Table 8 shows the population measures we use for the six subperiods from 2000 till 2031 for the US.[33] That implies an agent in the U.S. at time t = 2000 gets to know that the yearly changes in population dynamics for the period 2005-20011 will be such that in those five years population will growth 5.6 percent, the ratio of workers will decrease by 1.3 percentage points and the ratio of retirees will increase by 2 percentage points. We do not calibrate the steady state of the model to match any of the countries in the sample - for all countries the initial point is the steady state of our model as discussed in the calibration section. Hence, we only focus on how the predicted changes in demographic structure and population growth impact the changes (or deviations from steady state) of the macroeconomic variables in the model.

Figure 6 shows the results for U.S., Japan, Sweden and Spain, matching Figure 2 in Section 3.2. Our model does a fairly good job in matching the predicted path of real rates and output growth for the countries in our sample (in the Appendix we show the estimation and theoretical simulation based on the UN predictions for four additional countries). The model does particularly well in matching the drop in real rates and growth expected for most of the countries during the 2010-2030 period, which occurs due to increase in aging and the drop in labour force as fertility is reduced. The model is less able to match the empirical predictions in the first few years (2000-2005). The main reason for the discrepancy is that while in the empirical predictions we directly show the long-run effect of yearly changes in demographics such that all macroeconomic interactions have taken place, in the model we input those changes and simulate those interactions, thus it takes some years until all the effects materialize. The differences in the first few years between theory and empirics can be substantial for countries in which increases in working population (dividends of demographic

[33]In order to use the first period of the prediction (2011) we stretch one subperiod to 6 years (2005-2011).

transition) were still occurring in 2000, as is the case in Spain. As we match those increases output initially increases and takes time to revert. Nonetheless, as aging and low fertility alter the demographic weights from 2005 onwards, increasing the share of old dependants sharply in all countries, the dynamics in our model generate the expected drop in output and real rates.

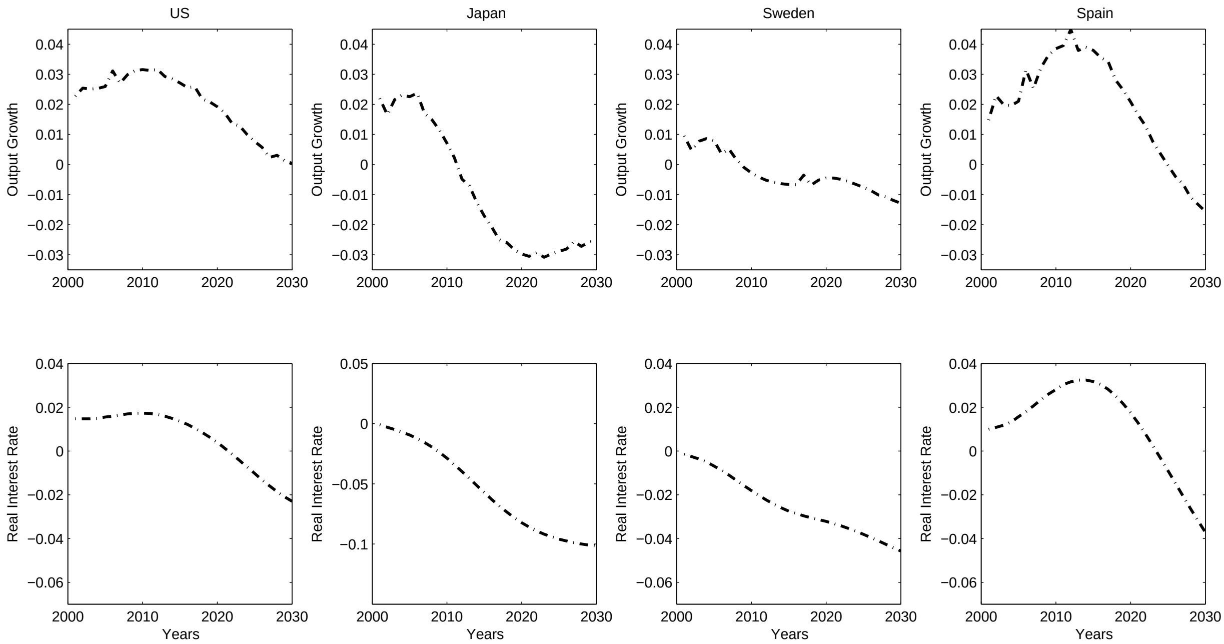

Figure 6: Simulation: *prediction*

6. Conclusions

Robert Gordon (2012) asks how much further could the frontier growth rate decline? We provide an analysis that measures this decline focusing on the impact of demographic structure on the macroeconomy. We start by presenting a parsimonious econometric model that aims to capture the impact of the demographic changes that currently affect nearly all developed economies on key macroeconomic variables of interest. The use of a panel VAR with six main macroeconomic variables, for 21 OECD countries over the period 1970-2007 allows us to obtain estimates of the long-run impact of demographic structure on the economy. Our results indicate that the age profile of the population has both economically and statistically significant impacts on output growth, investment, savings, hours worked per capita, real interest rates and inflation. The magnitude of the long-term impact is large. Demographic factors are predicted to depress average annual long-term GDP growth over the current decade, 2010-2019, by 0.75% in our sample of OECD countries. We also provide evidence of the link between demographic structure and innovation activity. We find that patent application is positively affected by middle-aged cohorts and negatively by retirees. We generally find our empirical results to be robust to time effects, to considering per capita output growth instead of overall output growth and the exclusion of individual countries.

Based on the empirical findings and the importance of considering the effects of demographic changes after all interactions between macroeconomic variables are allowed for, including their effect on innovation, we develop a theoretical model that incorporates life-cycle properties and endogenous productivity. Our model highlights three main channels by which demographics affects the macroeconomy: i) through life-cycle consumption decisions, ii) through incentives that alter human capital accumulation process and iii) through the influence of young workers on the innovation process. Our model is able to replicate most of our empirical findings, with the third channel being particularly important to generate reduced long-term output growth due to aging. Our empirical and theoretical results indicate that the current trend of population aging and reduced fertility, expected to continue in the next decades, may contribute to reduced output growth and real interest rates across OECD economies.

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Appendix - For Online Publication

"Demographic Structure and Macroeconomic Trends"

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Appendix A. Data

This provides a description of the data used in the empirical study.

- World Population Prospects: The 2010 Revision File 1A; Total population (both sexes combined) by five-year age group, major area, region and country, annually for 1950-2010 (thousands). (The data is the de facto population as of 1 July of the year indicated and in the age group indicated and the percentage it represents with respect to the total population.) United Nations, Population Division.
- Residential Patent Applications (annual): World Bank (2014), World Development Indicators.
- Trademark Applications (annual): World Bank (2014), World Development Indicators.
- Central Bank Discount Rates (annual): International Financial Statistics/IMF.
- Consumer Price Index (annual): International Financial Statistics/IMF.
- Households Savings Rate (annual): National Accounts, OECD.
- Hours worked (annual): Productivity Statistics, OECD.
- Gross Domestic Product (annual): National Accounts, OECD.
- Gross Fixed Capital Formation (annual): National Accounts, OECD.
- GDP per capita (annual): Penn World Tables.
- Spot Oil Price, West Texas Intermediate (Dollars per Barrel, annual, average): Dow Jones Company retrieved from FRED.

Appendix B. Additional Estimation Results

This appendix provides additional results on the estimations discussed in the main body of the paper.

Benchmark Panel VAR

	Growth (g)		Investment (I)		Savings (S)	
	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
g_{t-1}	0.21	0.07 *	0.18	0.03 *	0.04	0.05
I_{t-1}	-0.23	0.08 *	0.78	0.02 *	-0.13	0.05 *
S_{t-1}	0.06	0.05	0.03	0.01 *	0.81	0.02 *
H_{t-1}	0.01	0.02	0.01	0.01	-0.01	0.01
rr_{t-1}	-0.24	0.04 *	-0.09	0.02 *	-0.08	0.02 *
π_{t-1}	-0.25	0.06 *	-0.09	0.01 *	-0.07	0.02 *
$POIL_{t-1}$	-0.02	0.00 *	0.00	0.00	-0.01	0.00 *
$POIL_{t-2}$	0.02	0.00 *	0.00	0.00	0.00	0.00
$popGrowth$	1.94	0.86 *	0.42	0.54	1.18	0.80
$popGrowth_{t-1}$	-1.72	0.84 *	0.38	0.55	-0.74	0.92
δ_1	-0.06	0.08	-0.07	0.05	-0.06	0.05
δ_2	0.20	0.10	0.03	0.04	0.15	0.05 *
δ_3	0.19	0.06 *	0.06	0.04	-0.02	0.05
δ_4	0.00	0.07	-0.00	0.04	0.10	0.08
δ_5	-0.03	0.09	-0.06	0.05	0.05	0.07
δ_6	0.03	0.08	0.04	0.05	0.18	0.09 *
δ_7	-0.08	0.14	0.18	0.07 *	0.04	0.10
R^2	0.26		0.87		0.82	
$\Pr(\delta_j = 0)$	0.00		0.00		0.00	
obs	686		686		686	

Table A.1: Results for Growth, Investment and Savings

	Hours (H)		Real Rates (rr)		Inflation (π)	
	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
g_{t-1}	0.29	0.04 *	0.04	0.07	0.04	0.06
I_{t-1}	-0.03	0.04	-0.27	0.19	0.21	0.13
S_{t-1}	0.05	0.03	-0.14	0.11	0.12	0.11
H_{t-1}	0.92	0.01 *	0.04	0.07	-0.01	0.05
rr_{t-1}	-0.12	0.04 *	0.73	0.09 *	-0.11	0.05 *
π_{t-1}	-0.09	0.03 *	0.15	0.10	0.56	0.04 *
$POIL_{t-1}$	-0.01	0.00 *	0.00	0.00	-0.01	0.01 *
$POIL_{t-2}$	0.01	0.00 *	0.00	0.00	0.02	0.01 *
$popGrowth$	-0.51	1.43	-2.50	1.40	3.09	1.55 *
$popGrowth_{t-1}$	0.57	1.45	2.00	1.57	-2.87	1.49
δ_1	-0.17	0.06 *	-0.46	0.19 *	0.48	0.13 *
δ_2	-0.03	0.09	-0.07	0.13	0.14	0.09
δ_3	0.07	0.06	0.18	0.16	-0.10	0.08
δ_4	0.21	0.07 *	0.37	0.15 *	-0.40	0.15 *
δ_5	-0.05	0.07	0.26	0.10 *	-0.34	0.13 *
δ_6	0.10	0.09	0.15	0.22	-0.10	0.17
δ_7	0.00	0.07	0.24	0.36	-0.07	0.29
R^2	0.93		0.63		0.74	
$\Pr(\delta_j = 0)$	0.00		0.00		0.00	
obs	686		686		686	

Table A.2: Results for Hours, Real Interest Rate, and Inflation

	g	I	S	H	rr	π
g	1.000	0.541	0.441	0.451	-0.134	0.272
I	0.541	1.000	0.036	0.331	-0.067	0.157
S	0.441	0.036	1.000	0.258	-0.062	0.036
H	0.451	0.331	0.258	1.000	-0.009	0.158
rr	-0.134	-0.067	-0.062	-0.009	1.000	-0.762
π	0.272	0.157	0.036	0.158	-0.762	1.000

Table A.3: Residual Correlation Matrix

Robustness: Time Effects

	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8
g	-0.087	0.122	0.092	0.070	0.175	-0.064	-0.283*	0.000
I	-0.671*	-0.018	0.202	0.222	0.417	0.197	0.536	-0.832*
S	0.319	0.882*	-0.053	0.393	0.114	0.128	0.017	-1.382*
H	-2.309*	0.166	0.757	2.935*	0.887	0.486	-1.353	-1.126
rr	-0.564	-0.104	0.457	0.498	-0.083	0.430	0.097	-0.910
π	0.824*	0.333	-0.353	-0.892*	-0.194	-0.055	-0.101	0.339

Note: $*$ = 5% level of significance.

Table A.4: Long-Run Demographic Impact - D_{LR} (two-way effects)

	Workers	Dependants	Difference
g	$p(\sum_{j=2}^{5}\delta_j = 0)$=0.005	$p(\delta_1 + \delta_7 = 0)$=0.052	$p(\sum_{j=2}^{5}\delta_j = \delta_1 + \delta_7)$=0.008
I	$p(\delta_3 + \delta_4 = 0)$=0.229	$p(\delta_1 + \delta_8 = 0)$=0.005	$p(\delta_3 + \delta_4 = \delta_1 + \delta_8)$=0.012
S	$p(\sum_{j=4}^{6}\delta_j = 0)$=0.307	$p(\delta_7 + \delta_8 = 0)$=0.015	$p(\sum_{j=4}^{6}\delta_j = \delta_7 + \delta_8)$=0.061
H	$p(\delta_3 + \delta_4 = 0)$=0.000	$p(\delta_1 + \sum_{j=7}^{8}\delta_j = 0)$=0.001	$p(\delta_3 + \delta_4 = \delta_1 + \sum_{j=7}^{8}\delta_j)$=0.000
rr	$p(\delta_3 + \delta_4 = 0)$=0.094	$p(\delta_1 + \delta_8 = 0)$=0.150	$p(\delta_3 + \delta_4 = \delta_1 + \delta_8)$=0.079
π	$p(\delta_3 + \delta_4 = 0)$=0.002	$p(\delta_1 + \delta_8 = 0)$=0.0520	$p(\delta_3 + \delta_4 = \delta_1 + \delta_8)$=0.007

Table A.5: Joint Tests - p-values of Nonlinear Wald Test (two-way effects)

Robustness: Per Capita Output

	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8
g^{pc}	-0.003	0.113	0.068	0.030	0.099	-0.028	-0.244**	-0.035
I	-0.526	0.219	0.225	0.240	0.011	0.262	0.408	-0.838
S	0.064	0.501	-0.333	0.255	0.396	0.624**	-0.124	-1.383*
H	-2.04**	0.029	0.502	2.64*	0.292	1.256	-1.498	-1.182
rr	-0.915	-0.293	0.473	0.866	0.361	0.163	0.247	-0.903
π	1.14*	0.606**	-0.283	-1.006*	-0.732*	0.005	-0.049	0.320

Note: $** = 10\%, * = 5\%$ levels of significance.

Table A.6: Long-Run Demographic Impact - D_{LR} (Per Capita Output)

	Workers	Dependants	Difference
g^{pc}	$p(\sum_{j=2}^{5}\delta_j = 0)$=0.096	$p(\delta_7 = 0)$=0.090	$p(\sum_{j=2}^{5}\delta_j = \delta_7)$=0.039
I	$p(\delta_3 + \delta_4 = 0)$=0.290	$p(\delta_1 + \delta_8 = 0)$=0.056	$p(\delta_3 + \delta_4 = \delta_1 + \delta_8)$=0.078
S	$p(\sum_{j=4}^{6}\delta_j = 0)$=0.009	$p(\delta_7 + \delta_8 = 0)$=0.027	$p(\sum_{j=4}^{6}\delta_j = \delta_7 + \delta_8)$=0.011
H	$p(\delta_3 + \delta_4 = 0)$=0.023	$p(\delta_1 + \sum_{j=7}^{8}\delta_j = 0)$=0.024	$p(\delta_3 + \delta_4 = \delta_1 + \sum_{j=7}^{8}\delta_j)$=0.014
rr	$p(\delta_3 + \delta_4 = 0)$=0.075	$p(\delta_1 + \delta_8 = 0)$=0.129	$p(\delta_3 + \delta_4 = \delta_1 + \delta_8)$=0.073
π	$p(\delta_3 + \delta_4 = 0)$=0.001	$p(\delta_1 + \delta_8 = 0)$=0.018	$p(\delta_3 + \delta_4 = \delta_1 + \delta_8)$=0.002

Table A.7: Joint Tests - p-values of Nonlinear Wald Test (Per Capita Output)

Robustness: Exogeneity Test

Table A.8 shows the estimated coefficients of matrices A^{endo} (top left partition), B^{endo} (top right partition), B_1 (bottom left partition) and B_2 (bottom right partition) for the specification where W_{it} are considered endogenous.

	g	I	S	H	rr	π	W_1	W_2	W_3	W_4	W_5	W_6	W_7
g	0.21*	-0.23*	0.06	0.01	-0.23*	-0.25*	-0.06	0.24*	0.14	0.03	-0.05	0.03	-0.14
I	0.18*	0.78*	0.03	0.01	-0.09*	-0.09*	-0.07*	0.06	0.04	0.01	-0.08*	0.08	0.14*
S	0.04	-0.12*	0.81*	-0.01	-0.08*	-0.07*	-0.06	0.18*	-0.04	0.14*	0.03	0.2*	-0.05
H	0.29*	-0.02	0.05*	0.92*	-0.12*	-0.09*	-0.16*	-0.03	0.09	0.19*	-0.07	0.10	-0.06
rr	0.04	-0.27*	-0.14*	0.04	0.73*	0.15*	-0.47*	-0.03	0.18	0.41*	0.19	0.19	0.19
π	0.03	0.22*	0.12*	-0.01	-0.11*	0.56*	0.49*	0.13	-0.11	-0.42*	-0.32*	-0.10	-0.10
W_1	0.000	-0.002	-0.005*	-0.001	0.004*	0.006*	1.01*	-0.05*	0.04*	-0.01	0.02*	-0.01*	-0.04*
W_2	0.003	0.000	-0.002	0.003*	0.002	0.001	0.08*	0.99*	-0.09*	0.04*	-0.02*	0.05*	-0.05*
W_3	0.006*	-0.003	-0.002	0.002*	0.005*	0.004*	-0.06*	0.13*	0.97*	-0.05*	0.03*	0.01*	-0.04*
W_4	0.000	-0.004*	-0.005*	-0.002*	-0.001	0.000	0.01*	-0.03*	0.11*	0.99*	-0.07*	0.06*	-0.08*
W_5	0.003	0.004	-0.004*	0.002	0.011*	0.007*	-0.03*	0.05*	-0.06*	0.12*	0.97*	-0.05*	-0.01
W_6	0.005*	-0.003	0.002	0.000	-0.003*	0.000	0.02*	-0.01*	0.03*	-0.02*	0.09*	1*	-0.16*
W_7	0.007*	-0.005*	-0.006*	0.000	0.007*	0.005*	-0.03*	0.04*	-0.03*	0.04*	-0.03*	0.12*	0.94*

Note: $* = 5\%$ level of significance.

Table A.8: Exogeneity Test

Appendix C. Patent Application and Age Structure

Panel VAR Estimation - Introducing Innovation Variables

	Growth (g)		Investment (I)		Savings (S)	
	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
g_{t-1}	0.24	0.08 *	0.18	0.03 *	0.06	0.05
I_{t-1}	-0.26	0.08 *	0.77	0.02 *	-0.14	0.05 *
S_{t-1}	0.06	0.06	0.03	0.01 *	0.80	0.03 *
H_{t-1}	0.02	0.02	0.01	0.01	-0.00	0.01
rr_{t-1}	-0.24	0.05 *	-0.09	0.02 *	-0.08	0.02 *
$R\&D_{t-1}^{PA}$	0.01	0.01	-0.00	0.00	0.01	0.00
π_{t-1}	-0.26	0.06 *	-0.09	0.01 *	-0.07	0.02 *
$POIL_{t-1}$	-0.02	0.00 *	0.00	0.00	-0.01	0.00 *
$POIL_{t-2}$	0.01	0.00 *	0.00	0.00	0.00	0.00
$popGrowth_t$	1.75	0.85 *	0.05	0.51	0.88	0.81
$popGrowth_{t-1}$	-1.58	0.85	0.77	0.50	-0.54	0.99
δ_1	-0.05	0.08	-0.07	0.06	-0.07	0.06
δ_2	0.16	0.10	0.03	0.04	0.14	0.05 *
δ_3	0.21	0.06 *	0.08	0.04	-0.01	0.06
δ_4	0.00	0.08	-0.00	0.04	0.10	0.08
δ_5	-0.11	0.08	-0.07	0.05	0.05	0.07
δ_6	0.02	0.08	0.04	0.06	0.19	0.10
δ_7	-0.02	0.16	0.20	0.08 *	0.07	0.11
R^2	0.28		0.87		0.82	
$\Pr(\delta_j = 0)$	0.00		0.00		0.00	
obs	614		614		614	

Table A.9: Results for Growth, Investment and Savings

	Hours (H)		Real Rates (rr)		$R\&D$ ($R\&D^{PA}$)		Inflation (π)	
	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
g_{t-1}	0.30	0.04 *	0.06	0.08	0.12	0.33	0.02	0.07
I_{t-1}	-0.04	0.04	-0.30	0.21	0.38	0.24	0.22	0.15
S_{t-1}	0.05	0.03	-0.16	0.13	-0.30	0.15 *	0.13	0.12
H_{t-1}	0.92	0.01 *	0.05	0.09	-0.14	0.11	-0.01	0.06
rr_{t-1}	-0.12	0.04 *	0.71	0.10 *	-0.36	0.21	-0.10	0.06
$R\&D_{t-1}^{PA}$	0.01	0.01	0.02	0.01 *	-0.19	0.05 *	-0.03	0.00 *
π_{t-1}	-0.09	0.03 *	0.13	0.10	-0.06	0.17	0.57	0.05 *
$POIL_{t-1}$	-0.01	0.00 *	0.00	0.01	0.01	0.02	-0.02	0.01 *
$POIL_{t-2}$	0.01	0.00 *	0.00	0.00	0.01	0.03	0.02	0.01
$popGrowth_t$	-0.58	1.58	-2.28	1.55	1.13	8.80	2.73	1.54
$popGrowth_{t-1}$	0.60	1.58	1.58	1.57	0.21	9.99	-2.49	1.41
δ_1	-0.15	0.07 *	-0.41	0.20 *	0.15	0.71	0.47	0.14 *
δ_2	-0.05	0.09	-0.13	0.16	-0.73	0.54	0.14	0.10
δ_3	0.08	0.07	0.22	0.20	0.21	0.56	-0.10	0.09
δ_4	0.20	0.07 *	0.34	0.15 *	0.70	0.43	-0.40	0.15 *
δ_5	-0.05	0.08	0.25	0.10 *	0.99	0.50 *	-0.35	0.13 *
δ_6	0.11	0.10	0.17	0.24	-1.29	0.96	-0.11	0.17
δ_7	0.05	0.07	0.35	0.41	0.18	0.92	-0.13	0.33
R^2	0.93		0.62		0.08		0.73	
$\Pr(\delta_j = 0)$	0.00		0.00		0.00		0.00	
obs	614		614		614		614	

Table A.10: Results for Hours, Interest Rate, R&D and Inflation

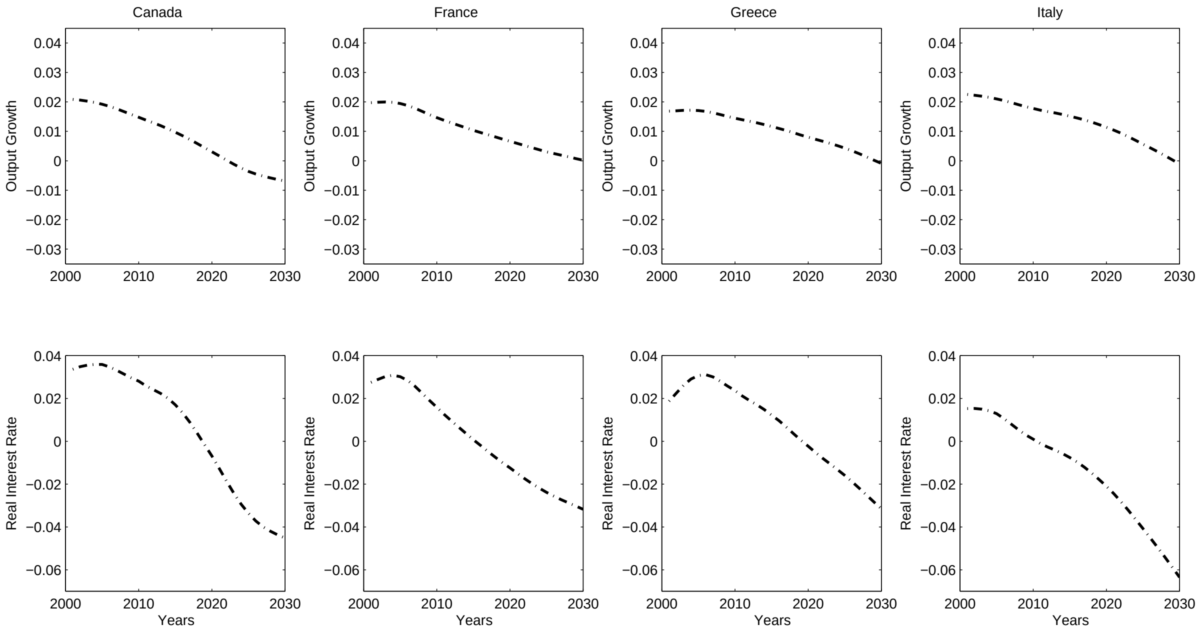

Figure A.1: Impact of Predicted Future Demographic Structure - Additional Countries

	Workers	Dependants	Difference
g	$p(\sum_{j=2}^{5}\delta_j=0)$=0.102	$p(\delta_1+\delta_7=0)$=0.136	$p(\sum_{j=2}^{5}\delta_j=\delta_1+\delta_7)$=0.085
I	$p(\delta_3+\delta_4=0)$=0.297	$p(\delta_1+\delta_8=0)$=0.051	$p(\delta_3+\delta_4=\delta_1+\delta_8)$=0.078
S	$p(\sum_{j=4}^{6}\delta_j=0)$=0.005	$p(\delta_7+\delta_8=0)$=0.000	$p(\sum_{j=4}^{6}\delta_j=\delta_7+\delta_8)$=0.001
H	$p(\delta_3+\delta_4=0)$=0.047	$p(\delta_1+\sum_{j=7}^{8}\delta_j=0)$=0.037	$p(\delta_3+\delta_4=\delta_1+\sum_{j=7}^{8}\delta_j)$=0.023
rr	$p(\delta_3+\delta_4=0)$=0.063	$p(\delta_1+\delta_8=0)$=0.153	$p(\delta_3+\delta_4=\delta_1+\delta_8)$=0.072
$R\&D^{PA}$	$p(\sum_{j=4}^{6}\delta_j=0)$=0.066	$p(\delta_2+\delta_8=0)$=0.211	$p(\sum_{j=4}^{6}\delta_j=\delta_2+\delta_8)$=0.000
π	$p(\delta_3+\delta_4=0)$=0.005	$p(\delta_1+\delta_8=0)$=0.060	$p(\delta_3+\delta_4=\delta_1+\delta_8)$=0.011

Table A.11: Joint Tests - p-values of Nonlinear Wald Test - Innovation Extension

Appendix D. Theoretical Model: Solution of Equilibrium Conditions

This appendix shows how the equilibrium conditions are determined.

We start by looking at the factor markets with the final and input firms decisions.

Production Sector

Firms in consumption and capital producing sectors maximise profits selecting capital, its utilisation, labour and intermediate goods demand.

Labour allocation is such that

$$(1-\alpha)(1-\gamma_I)Y_{c,t} \quad = \quad \mu_t W_t \xi_t L_t. \tag{A.1}$$

Capital stock and utilisation are such that

$$\alpha(1-\gamma_I)Y_{c,t} \quad = \quad \mu_t[r_t^k + \delta(U_t)]K_t, \tag{A.2}$$
$$\alpha(1-\gamma_I)Y_{c,t} \quad = \quad \mu_t \delta'(U_t)K_t U_t. \tag{A.3}$$

Where I_t is the investment in capital made by the financial intermediary, who holds all production and R& D assets. Intermediate goods are set such that

$$\mu_t M_t P_t^M = \gamma_I Y_{c,t} \tag{A.4}$$

where P_t^M is the price of intermediate goods.

In order to obtain this price one can minimise total cost of intermediary goods $\int_0^A \tilde{P}^M M^i di$ subject to (9) to obtain

$$P_t^M = \vartheta A_t^{1-\vartheta} \tag{A.5}$$

Combining (7) and (8) and defining total labour supply as $L_t \equiv \int_0^{N_t^f} L_t^j dj$ and total intermediate composite demand as $M_t \equiv \int_0^{N_t^f} M_t^j dj$, then[34]

$$Y_t = (N_t^f)^{\mu_t - 1}\left[(U_t \frac{K_t}{\xi_t L_t})^{\alpha}(\xi_t L_t)\right]^{(1-\gamma_I)}[M_t]^{\gamma_I} \text{ for } x = c, k. \tag{A.6}$$

Due to free entry the number of final good firms is such that their profits are equal to the operating costs. Using (7) total output per firm is given by $Y_t(N_t^f)^{-\mu_t}$, while their mark-up is given by $\frac{\mu_t - 1}{\mu_t}$, thus

$$\frac{\mu_t - 1}{\mu_t} Y_{c,t}(N_t^f)^{-\mu_t} = \Omega\tilde{\Psi}_t \tag{A.7}$$

Finally, let Y_t denote aggregate value added output. Y_t is equal to the total output net intermediate goods and operating costs. Thus, using (A.5)[35],

$$Y_t = Y_{c,t} - A_t^{1-\vartheta} M_t - \Omega\tilde{\Psi}_t. \tag{A.8}$$

On the expenditure side, output must be equal to consumption, investment and costs of R&D and adoption. Thus,

[34] Note that all firms select the same capital labour ratio.

[35] In order to net out intermediate goods one has to compute total expenditure on intermediate goods ($\int_0^A \tilde{P}^M M^i di$) minus the markup on intermediate goods ($\int_0^A (\tilde{P}^M - 1) M^i di$).

$$Y_t = C_t + I_t + S_t + \Xi_t(Z_t - A_t) + \tau_t. \tag{A.9}$$

Innovation Process

From conditions (10) and (15) one can easily determine the flow of the stock of invented (prototypes) and adopted goods, which are given by

$$\frac{Z_{t+1}}{Z_t} = \chi\left(\frac{S_t}{\tilde{\Psi}_t}\right)^{\rho} + \phi, \text{ and} \tag{A.10}$$

$$\frac{A_{t+1}}{A_t} = \lambda\left(\frac{A_t\Xi_t}{\tilde{\Psi}_t}\right)\phi[Z_t/A_t - 1] + \phi \tag{A.11}$$

Investment in R&D (S_t) is determined by (11), which using (10) becomes

$$S_t = R_{t+1}^{-1}\phi E_t J_{t+1}(Z_{t+1} - \phi Z_t). \tag{A.12}$$

Profits are given by the total gain in seeling the right to goods invented as a result of the previous period investment $S_{x,t-1}$ to adopters minus the cost of borrowing for that investment. Thus,

$$\Pi_t^{RD} = \phi J_t(Z_t - \phi Z_{t-1}) + S_{t-1}R_t$$

Thus, in perfect foresight equilibrium $\Pi_t^{RD} = 0$.

Investment in adoption (Ξ_t) is determined by solving (14). We thus obtain the following condition

$$\frac{A_t}{K_t}\lambda' R_t^{-1}\phi[V_{t+1} - J_{t+1}] = 1 \tag{A.13}$$

where $\frac{A_t}{K_t}\lambda' = \frac{\partial\lambda\left(\frac{A_t}{\tilde{\Psi}_t}\right)}{\partial\Xi_t\Xi_t}$. Assuming the elasticity of λ_t to changes in Ξ_t is constant, thus $\epsilon_\lambda = \frac{\lambda'}{\lambda_t}\frac{A_t\Xi_t}{K_t}$, then we obtain

$$\Xi_t = \epsilon_\lambda \lambda_t R_t^{-1}\phi[V_{t+1} - J_{t+1}] \tag{A.14}$$

Finally, the value of an invented good and an adopted good are given by

$$J_t = -\Xi_t + (R_{t+1})^{-1}\phi E_t[\lambda_t V_{t+1} + (1-\lambda_t)J_{t+1}], \text{ and} \tag{A.15}$$

$$V_t = (1 - 1/\vartheta)\gamma_I\frac{Y_{c,t}}{\mu_t A_t} + (R_{t+1})^{-1}\phi E_t V_{t+1} \tag{A.16}$$

where $\lambda_t = \lambda\left(\frac{A_t\Xi_t}{\tilde{\Psi}_t}\right)$ and $\Pi_{m,t} = (1-1/\vartheta)P_t^M M_t = (1-1/\vartheta)\gamma_I\frac{Y_{c,t}}{\mu_t A_t}$.

Profits for adopters are given by the gain from marketing specialised intermediated goods net the amount paid to inventors to gain access to new goods and the expenditures on loans to pay for adoption intensity.

$$\Pi_t^A = (1 - 1/\vartheta)\gamma_I \frac{Y_{c,t}}{\mu_t} - J_t(Z_t - \phi Z_{t-1}) - \Xi_{t-1}(Z_{t-1} - A_{t-1})R_t$$

Household Sector

Retiree j decision problem is

$$\max V_t^{jr} = \left\{(C_t^{jr})^{\rho_U} + \beta\gamma_{t,t+1}([V_{t+1}^{jr}]^{\rho_U})\right\}^{1/\rho_U} \tag{A.17}$$

subject to

$$C_t^{jr} + FA_{t+1}^{jr} = \frac{R_t}{\gamma_{t-1,t}} FA_t^{jr} + d_t^{jr}. \tag{A.18}$$

The first order condition and envelop theorem are

$$(C_t^{jr})^{\rho_U - 1} = \beta\gamma_{t,t+1}\frac{\partial V_{t+1}^{jr}}{\partial FA_{t+1}^{jr}}(V_{t+1}^{jr})^{\rho_U - 1}, \tag{A.19}$$

$$\frac{\partial V_t^{jr}}{\partial FA_t^{jr}} = (V_{t+1}^{jr})^{1-\rho_U}(C_t^{jr})^{\rho_U - 1}\frac{R_t}{\gamma_{t-1,t}}. \tag{A.20}$$

Combining these conditions above gives the Euler equation

$$C_{t+1}^{jr} = (\beta R_{t+1})^{1/(1-\rho_U)} C_t^{jr} \tag{A.21}$$

Conjecture that retirees consume a fraction of all assets (including financial assets, profits from financial intermediaries), such that

$$C_t^{jr} = \varepsilon_t \varsigma_t \left[\frac{R_t}{\gamma_{t-1,t}} FA_t^{rj} + D_t^{rj}\right]. \tag{A.22}$$

Combining these and the budget constraint gives

$$FA_{t+1}^{jr} = \frac{R_t}{\gamma_{t-1,t}} FA_t^{jr}(1 - \varepsilon_t\varsigma_t) + d_t^{jr} - \varepsilon_t\varsigma_t(D_t^{rj}).$$

Using the condition above the Euler equation and the solution for consumption gives

$$(\beta R_{t+1})^{1/(1-\rho_U)}\varepsilon_t\varsigma_t[\frac{R_t}{\gamma_{t-1,t}}FA_t^{rj}+D_t^{rj}]= \tag{A.23}$$
$$\varepsilon_{t+1}\varsigma_{t+1}\left[\frac{R_{t+1}}{\gamma_{t,t+1}}\left(\frac{R_t}{\gamma_{t-1,t}}FA_t^{jr}(1-\varepsilon_t\varsigma_t)+d_t^{jr}-\varepsilon_t\varsigma_t D_t^{rj}\right)+D_{t+1}^{jr}\right].$$

Collecting terms we have that

$$1-\varepsilon_t\varsigma_t = \frac{(\beta R_{t+1})^{1/(1-\rho_U)}\gamma_{t,t+1}}{R_{t+1}}\frac{\varepsilon_t\varsigma_t}{\varepsilon_{t+1}\varsigma_{t+1}}, \tag{A.24}$$
$$D_t^{jr} = d_t^{jr}+\frac{\gamma_{t,t+1}}{R_{t+1}}D_{t+1}^{jr}. \tag{A.25}$$

One can also show that $V_t^{jr}=(\varepsilon_t\varsigma_t)^{-1/\rho_U}C_t^{jr}$.

Worker j decision problem is

$$\max V_t^{jw}=\left\{(C_t^{jw})^{\rho_U}+\beta[\omega^r V_{t+1}^{jw}+(1-\omega^r)V_{t+1}^{jr}]^{\rho_U}\right\}^{1/\rho_U} \tag{A.26}$$

subject to

$$C_t^{jw}+FA_{t+1}^{jw}=R_tFA_t^{jw}+W_t\xi_t+d_t^{jw}-\tau_t^{jw}. \tag{A.27}$$

First order conditions and envelop theorem are

$$(C_t^{jw})^{\rho_U-1} = \beta[\omega^r V_{t+1}^{jw}+(1-\omega^r)V_{t+1}^{jr}]^{\rho_U-1}\left[\omega^r\frac{\partial V_{t+1}^{jw}}{\partial FA_{t+1}^{jw}}+(1-\omega^r)\frac{\partial V_{t+1}^{jr}}{\partial FA_{t+1}^{jw}}\right],$$
$$\frac{\partial V_t^{jw}}{\partial FA_t^{jw}} = (V_{t+1}^{jw})^{1-\rho_U}(C_t^{jw})^{\rho_U-1}R_t,\ \text{and} \tag{A.28}$$
$$\frac{\partial V_t^{jr}}{\partial FA_t^{jw}} = \frac{\partial V_t^{jr}}{\partial FA_t^{jr}}\frac{\partial FA_t^{jr}}{\partial FA_t^{jw}}=\frac{\partial V_t^{jr}}{\partial FA_t^{jr}}\frac{1}{\gamma_{t-1,t}}=(V_{t+1}^{jr})^{1-\rho_U}(C_t^{jr})^{\rho_U-1}R_t. \tag{A.29}$$

$\frac{\partial FA_t^{jr}}{\partial FA_t^{jw}}=\frac{1}{\gamma_{t-1,t}}$ since as individuals are risk neutral with respect to labour income they select the same asset profile independent of their worker/retiree status, adjusting only for expected return due to probability of death.

Combining these conditions above, and using the conjecture that $V_t^{jw}=(\varsigma_t)^{-1/\rho_U}C_t^{jw}$, gives the Euler equation

$$C_t^{jw} = \left((\beta R_{t+1}\mathfrak{z}_{t+1})^{1/(1-\rho_U)}\right)^{-1}[\omega^r C_{t+1}^{jw}+(1-\omega^r)\varepsilon_{t+1}^{\frac{-1}{\rho_U}}C_{t+1}^{jr}] \tag{A.30}$$
$$\text{where } \mathfrak{z}_{t+1}=(\omega^r+(1-\omega^r)\varepsilon_{t+1}^{(\rho_U-1)/\rho_U}).$$

Conjecture that retirees consume a fraction of all assets (including financial assets, human capital and profits from financial intermediaries), such that

$$C_t^{jw} = \varsigma_t[R_t FA_t^{jw} + H_t^{jw} + D_t^{jw} - T_t^{jw}]. \tag{A.31}$$

Following the same procedure as before we have that

$$\begin{aligned}
&\varsigma_t[R_t FA_t^{jw}+H_t^{jw}+D_t^{jw}](\beta R_{t+1}\mathfrak{z}_{t+1})^{1/(1-\rho_U)}= \\
&\quad \omega^r\varsigma_{t+1}\Big[R_{t+1}\Big(R_t FA_t^{jw}(1-\varsigma_t)+W_t\xi_t+d_t^{jw}-\tau_t^{jw}-\varsigma_t(H_t^{jw}+D_t^{jw}-T_t^{jw})\Big)+H_{t+1}^{jw}+D_{t+1}^{jw}-T_{t+1}^{jw}\Big]+ \\
&\quad\ \varepsilon_{t+1}^{\frac{-1}{\rho_U}}(1-\omega^r)\varepsilon_{t+1}\varsigma_{t+1}\Big[R_{t+1}\Big(R_t FA_t^{jw}(1-\varsigma_t)+W_t\xi_t+d_t^{jw}-\tau_t^{jw}-\varsigma_t(H_t^{jw}+D_t^{jw}-T_t^{jw})\Big)+D_{t+1}^{jr}\Big].
\end{aligned} \tag{A.32}$$

Collecting terms and simplifying we have that

$$\varsigma_t = 1 - \frac{\varsigma_t}{\varsigma_{t+1}} \frac{(\beta R_{t+1}\mathfrak{z}_{t+1})^{1/(1-\rho_U)}}{R_{t+1}\mathfrak{z}_{t,t+1}} \tag{A.33}$$

$$H_t^{jw} = W_t\xi_t + \frac{\omega^r}{R_{t+1}\mathfrak{z}_{t,t+1}} H_{t+1}^{jw} \tag{A.34}$$

$$T_t^{jw} = \tau_t^{jw} + \frac{\omega^r}{R_{t+1}\mathfrak{z}_{t,t+1}} T_{t+1}^{jw} \text{ and} \tag{A.35}$$

$$D_t^{jw} = d_t^{jw} + \frac{\omega^r}{R_{t+1}\mathfrak{z}_{t,t+1}} D_{t+1}^{jw} + \frac{(1-\omega)\varepsilon_{t+1}^{(\rho_U-1)/\rho_U}}{R_{t+1}\mathfrak{z}_{t,t+1}} D_{t+1}^{jr}. \tag{A.36}$$

Aggregation across households

Assume that for any variable X_t^{jz} we have that $X_t^z = \int_0^{N_t^z} X_t^{jz}$ for $z = \{w, r\}$, then

$$L_t = N_t^w, \tag{A.37}$$

$$H_t^w = W_t\xi_t L_t + \frac{\omega^r}{R_{t+1}\mathfrak{z}_{t,t+1}} H_{t+1}^w \frac{N_t^w}{N_{t+1}^w}, \tag{A.38}$$

$$T_t^w = \tau_t + \frac{\omega^r}{R_{t+1}\mathfrak{z}_{t,t+1}} T_{t+1}^w \frac{N_t^w}{N_{t+1}^w}, \tag{A.39}$$

$$D_t^w = d_t^w + \frac{\omega^r}{R_{t+1}\mathfrak{z}_{t,t+1}} D_{t+1}^w \frac{N_t^w}{N_{t+1}^w} + \frac{(1-\omega^r)\varepsilon_{t+1}^{(\rho_U-1)/\rho_U}}{R_{t+1}\mathfrak{z}_{t,t+1}} D_{t+1}^r \frac{N_t^w}{N_{t+1}^r}, \tag{A.40}$$

$$C_t^w = \varsigma_t[R_t FA_t^w + H_t^w + D_t^w - T_t^w], \tag{A.41}$$

$$D_t^r = d_t^r + \frac{\gamma_{t,t+1}}{R_{t+1}} D_{t+1}^r \frac{N_t^r}{N_{t+1}^r}, \tag{A.42}$$

$$C_t^r = \varepsilon_t\varsigma_t[R_t FA_t^r + D_t^r]. \tag{A.43}$$

Note that $\gamma_{t,t+1}$ is not shown in the last equation due to the perfect annuity market for retirees, allowing for the redistribution of assets of retirees who died at the end of the period.

Decision of Investment in Labour Skill

The marginal cost of increasing lump-sum taxes for worker j today to finance higher investment in young's education is given by

$$MC_t^{Ej} = -\frac{\partial V_t^{wj}}{\partial \tau_t^{wj}} = \frac{\partial V_t^{wj}}{\partial C_t^{wj}} = \varsigma_t^{-1/\rho_U} \tag{A.44}$$

The marginal benefit of increasing lump-sum taxes at time t for a young h who becomes a worker next period is

$$MB_t^{Eh} = \beta(1-\omega^y)\frac{\partial V_{t+1}^{wh}}{\partial \tau_t^{wj}} = \beta(1-\omega^y)\frac{\partial V_{t+1}^{wh}}{\partial \xi_{t+1}^y}\frac{\partial \xi_{t+1}^y}{\partial I_t^y}\frac{\partial I_t^y}{\partial \tau_t}\frac{\partial \tau_t}{\partial \tau_t^{wj}} \tag{A.45}$$

$$= \beta(1-\omega^y)\varsigma_{t+1}^{-1/\rho_U}\frac{W_{t+1}}{W_t}\chi_E\frac{I_t^y}{\xi_t} \tag{A.46}$$

Adding costs across all workers and benefits across all young at time t gives the condition that determines I_t^y. That is

$$\varsigma_t^{-1/\rho_U} = \beta(1-\omega^y)\varsigma_{t+1}^{-1/\rho_U}\zeta_t^y\frac{W_{t+1}}{W_t}\chi_E\frac{I_t^y}{\xi_t} \tag{A.47}$$

Financial Intermediary

Due to standard arbitrage arguments all assets must pay same expected return thus

$$E_t\left[r_{t+1}^k + 1\right] = R_t. \tag{A.48}$$

The flow of capital is then given by

$$K_{t+1} = K_t(1-\delta(U_t)) + I_t. \tag{A.49}$$

Also note that under a perfect foresight solution this equality holds without expectations, $\Pi_t^F = 0$ and thus $d_t^r = d_t^w = 0$. If $\Pi_t^F \neq 0$, then we assume profits are divided based on the ratio of assets thus $d_t^r = \Pi_t^F \frac{FA_t^r}{FA_t^r + FA_t^w}$ and $d_t^w = \Pi_t^F \frac{FA_t^w}{FA_t^r + FA_t^w}$.

Asset Markets

Asset Market clearing implies

$$FA_{t+1} = FA_{t+1}^w + FA_{t+1}^r = K_{t+1} + B_{t+1} \tag{A.50}$$

Finally, the flow of assets are given by

$$FA_{t+1}^{r} = R_t FA_t^r + d_t^r - C_t^r + (1-\omega^r)(R_t FA_t^w + W_t \xi_t L_t + d_t^w - C_t^w - \tau_t) \quad \text{(A.51)}$$
$$FA_{t+1}^{w} = \omega^r (R_t FA_t^w + W_t \xi_t L_t + d_t^w - C_t^w - \tau_t) \quad \text{(A.52)}$$

Appendix E. Theoretical Model: Equilibrium conditions

The equilibrium conditions that ensure **a.** are:

$$H_t^w = W_t \xi_t L_t + \frac{\omega^r}{R_{t+1}\mathfrak{z}_{t,t+1}} H_{t+1}^w \frac{N_t^w}{N_{t+1}^w} \quad \text{(A.53a)}$$
$$T_t^w = \tau_t + \frac{\omega^r}{R_{t+1}\mathfrak{z}_{t,t+1}} T_{t+1}^w \frac{N_t^w}{N_{t+1}^w} \quad \text{(A.53b)}$$
$$D_t^w = d_t^w + \frac{\omega^r}{R_{t+1}\mathfrak{z}_{t,t+1}} D_{t+1}^w \frac{N_t^w}{N_{t+1}^w} + \frac{(1-\omega^r)\varepsilon_{t+1}^{(\rho_U-1)/\rho_U}}{R_{t+1}\mathfrak{z}_{t,t+1}} D_{t+1}^r \frac{N_t^w}{N_{t+1}^r} \quad \text{(A.53c)}$$
$$D_t^r = d_t^r + \frac{\gamma_{t,t+1}}{R_{t+1}} D_{t+1}^r \frac{N_t^r}{N_{t+1}^r} \quad \text{(A.53d)}$$
$$C_t^w = \varsigma_t [R_t FA_t^w + H_t^w + D_t^w - T_t^w] \quad \text{(A.53e)}$$
$$C_t^r = \varepsilon_t \varsigma_t [R_t FA_t^r + D_t^r] \quad \text{(A.53f)}$$
$$\varsigma_t = 1 - \frac{\varsigma_t}{\varsigma_{t+1}} \frac{(\beta R_{t+1}\mathfrak{z}_{t+1})^{1/(1-\rho_U)}}{R_{t+1}\mathfrak{z}_{t,t+1}} \quad \text{(A.53g)}$$
$$1 - \varepsilon_t \varsigma_t = \frac{(\beta R_{t+1})^{1/(1-\rho_U)}\gamma_{t,t+1}}{R_{t+1}} \frac{\varepsilon_t \varsigma_t}{\varepsilon_{t+1}\varsigma_{t+1}} \quad \text{(A.53h)}$$
$$\tau_t = W_t N_t^w I_t^y \quad \text{(A.53i)}$$
$$\xi_{t+1} = (\omega^r + (1-\omega^y)\zeta_t^y)^{-1}(\omega^r \xi_t + (1-\omega^y)\zeta_t^y \xi_{t+1}^y) \quad \text{(A.53j)}$$
$$\xi_{t+1}^y = \rho_E \xi_t + \frac{\chi_E}{2}\left(\frac{I_t^y}{\xi_t}\right)^2 \xi_t. \quad \text{(A.53k)}$$
$$\varsigma_t^{-1/\rho_U} = \varsigma_{t+1}^{-1/\rho_U} \beta (1-\omega^y)\zeta_t^y \frac{W_{t+1}}{W_t} \chi_E \frac{I_t^y}{\xi_t} \quad \text{(A.53l)}$$

where $\mathfrak{z}_{t+1} = \omega^r + (1-\omega^r)\varepsilon_{t+1}^{(\rho_U-1)/\rho_U}$, H_t^w is the present value of gains from human capital, T_t^w is the present value of transfers, D_t^z is the present value of dividends for $z = \{w, r\}$, ς_t the marginal propensity of consumption of workers and $\varepsilon_t \varsigma_t$ the one for retirees.

The equilibrium conditions that ensure **b.** are:

$$(1-\alpha)(1-\gamma_I)Y_{c,t} = \mu_t W_t \xi_t L_t \tag{A.54a}$$

$$\alpha(1-\gamma_I)Y_{c,t} = \mu_t[r_t^k + \delta_t]K_t \tag{A.54b}$$

$$\alpha(1-\gamma_I)Y_{c,t} = \mu_t \delta_t'(U_t)K_t U_t \tag{A.54c}$$

$$\mu_t M_t P_t^M = \gamma_I Y_{c,t} \tag{A.54d}$$

$$P_t^M = \vartheta A_t^{1-\vartheta} \tag{A.54e}$$

$$Y_{c,t} = (N_t^f)^{\mu_t - 1}\left[(U_t \frac{K_t}{\xi_t L_t})^{\alpha}(\xi_t L_t)\right]^{(1-\gamma_I)}[M_t]^{\gamma_I} \tag{A.54f}$$

$$\frac{\mu_t - 1}{\mu_t} Y_{c,t}(N_t^f)^{-\mu_t} = \Omega \tilde{\Psi}_t \tag{A.54g}$$

$$\mu_t = \mu(N_t^f) \tag{A.54h}$$

$$\delta_t = \delta(U_t) \tag{A.54i}$$

The equilibrium conditions that ensure **c.** are:

$$\frac{Z_{t+1}}{Z_t} = (\Gamma_t^{yw})^{\rho_{yw}} \chi \left(\frac{S_t}{\tilde{\Psi}_t}\right)^{\rho} + \phi \tag{A.55a}$$

$$\frac{A_{t+1}}{A_t} = \lambda\left(\frac{A_t \Xi_t}{\tilde{\Psi}_t}\right)\phi[Z_t/A_t - 1] + \phi \tag{A.55b}$$

$$S_t = R_{t+1}^{-1}\phi E_t J_{t+1}(Z_{t+1} - \phi Z_t) \tag{A.55c}$$

$$\Xi_t = \epsilon_\lambda \lambda_t R_{t+1}^{-1}\phi[V_{t+1} - J_{t+1}] \tag{A.55d}$$

$$J_t = -\Xi_t + (R_{t+1})^{-1}\phi E_t[\lambda_t V_{t+1} + (1-\lambda_t)J_{t+1}] \tag{A.55e}$$

$$V_t = (1 - 1/\vartheta)\gamma_I \frac{Y_{c,t}}{\mu_t A_t} + (R_{t+1})^{-1}\phi E_t V_{t+1} \tag{A.55f}$$

$$\lambda_t = \lambda\left(\frac{A_t \Xi_t}{\tilde{\Psi}_t}\right) \tag{A.55g}$$

$$\Pi_t^{RD} = \phi J_t(Z_t - \phi Z_{t-1}) - S_{t-1}R_t \tag{A.55h}$$

$$\Pi_t^A = (1 - 1/\vartheta)\gamma_I \frac{Y_{c,t}}{\mu_t} - \phi J_t(Z_t - \phi Z_{t-1}) - \Xi_{t-1}(Z_{t-1} - A_{t-1})R_t \tag{A.55i}$$

The equilibrium conditions that ensure **d.** are:

$$E_t[r_{t+1}^k + 1] = R_{t+1} \tag{A.56a}$$

$$d_t^z = \Pi_t^F \frac{FA_t^z}{FA_t} \text{ for } z = r, w \tag{A.56b}$$

$$\Pi_t^F = [r_t^k + 1]K_t + R_t B_t - R_t(FA_t) - K_{t+1} - B_{t+1} + FA_{t+1} + \Pi_t^{RD} + \Pi_t^A \tag{A.56c}$$

$$B_{t+1} = S_t + \Xi_t(Z_t - A_t) \tag{A.56d}$$

The equilibrium conditions that ensure **e.** are:

$$L_t = N_t^w \tag{A.57a}$$

$$K_{t+1} = K_t(1 - \delta(U_t)) + I_t \tag{A.57b}$$

$$Y_t = Y_{c,t} - A_t^{1-\vartheta} M_t - \Omega \tilde{\Psi}_t \tag{A.57c}$$

$$Y_t = C_t + I_t + S_t + \Xi_t (Z_t - A_t) + \tau_t \tag{A.57d}$$

$$C_t = C_t^w + C_t^r \tag{A.57e}$$

$$FA_{t+1}^w + FA_{t+1}^r = K_{t+1} + B_{t+1} \tag{A.57f}$$

$$FA_{t+1}^r = R_t FA_t^r + d_t^r - C_t^r + (1 - \omega^r)(R_t FA_t^w + W_t \xi_t L_t + d_t^w - C_t^w - \tau_t) \tag{A.57g}$$

$$FA_{t+1} = FA_{t+1}^w + FA_{t+1}^r \tag{A.57h}$$

Also note that $FA_{t+1}^w = \omega^r (R_t FA_t^w + W_t \xi_t L_t + d_t^w - C_t^w - \tau_t)$.

Appendix F. Theoretical Model: Detrended equilibrium conditions

This section shows the detrended equilibrium conditions. Note that $\bar{x}$ denotes the steady state of variable x_t.

$$h_t^w = w_t + \frac{\omega^r}{R_{t+1}\mathfrak{z}_{t,t+1}} \frac{g_{t+1} h_{t+1}^w}{g_{t+1}^w} \text{ where } h_t^w = \frac{H_t^w}{Y_{c,t}}, w_t = \frac{W_t \xi_t L_t}{Y_{c,t}}, g_{t+1} = \frac{Y_{c,t+1}}{Y_{c,t}}, g_{t+1}^w = \frac{N_{t+1}^w}{N_t^w} \tag{A.58a}$$

$$\tilde{T}_t^w = \tilde{\tau}_t + \frac{\omega^r}{R_{t+1}\mathfrak{z}_{t,t+1}} \frac{g_{t+1}\tilde{T}_{t+1}^w}{g_{t+1}^w} \text{ where } \tilde{T}_t^w = \frac{T_t^w}{Y_{c,t}}, \tilde{\tau}_t = \frac{\tau_t}{Y_{c,t}} \tag{A.58b}$$

$$\tilde{D}_t^r = \tilde{d}_t^r + \frac{\gamma_{t,t+1}}{R_{t+1}} g_{t+1} \frac{\tilde{D}_{t+1}^r \zeta_t^r}{\zeta_{t+1}^r g_{t+1}^w} \text{ where } \tilde{D}_t^r = \frac{D_t^r}{Y_{c,t}}, \tilde{d}_t^r = \frac{d_t^r}{Y_{c,t}} \tag{A.58c}$$

$$\tilde{D}_t^w = \tilde{d}_t^w + \frac{\omega^r}{R_{t+1}\mathfrak{z}_{t,t+1}} \frac{g_{t+1}\tilde{D}_{t+1}^w}{g_{t+1}^w} + \frac{(1-\omega^r)\varepsilon_{t+1}^{(\rho_U-1)/\rho_U}}{R_{t+1}\mathfrak{z}_{t,t+1}} \frac{g_{t+1}\tilde{D}_{t+1}^r}{\zeta_{t+1}^r g_{t+1}^w} \text{ where } \tilde{D}_t^w = \frac{D_t^w}{Y_{c,t}}, \tilde{d}_t^w = \frac{d_t^w}{Y_{c,t}} \tag{A.58d}$$

$$c_t^w = \varsigma_t [R_t \frac{fa_t^w}{g_t} + h_t^w + \tilde{D}_t^w - \tilde{T}_t^w] \text{ where } fa_t^w = \frac{FA_t^w}{Y_{c,t-1}}, c_t^w = \frac{C_t^w}{Y_{c,t}} \tag{A.58e}$$

$$c_t^r = \varepsilon_t \varsigma_t [R_t \frac{fa_t^r}{g_t} + \tilde{D}_t^r] \text{ where } fa_t^r = \frac{FA_t^r}{Y_{c,t-1}}, c_t^r = \frac{C_t^w}{Y_{c,t}} \tag{A.58f}$$

$$1 - \varepsilon_t \varsigma_t = \frac{(\beta R_{t+1})^{1/(1-\rho_U)} \gamma_{t,t+1}}{R_{t+1}} \frac{\varepsilon_t \varsigma_t}{\varepsilon_{t+1}\varsigma_{t+1}} \tag{A.58g}$$

$$\varsigma_t = 1 - \frac{\varsigma_t}{\varsigma_{t+1}} \frac{(\beta R_{t+1}\mathfrak{z}_{t+1})^{\frac{1}{(1-\rho_U)}}}{R_{t+1}\mathfrak{z}_{t,t+1}} \tag{A.58h}$$

$$\mathfrak{z}_{t+1} = (\omega^r + (1-\omega^r)\varepsilon_{t+1}^{(\rho_U-1)/\rho_U}) \tag{A.58i}$$

$$g_{t+1}^w = \omega^r + (1-\omega^y)\zeta_t^y \tag{A.59a}$$

$$n_{t,t+1} = \frac{\zeta_{t+1}^y}{\zeta_t^y} \left(\omega^r + \zeta_t^y (1-\omega^y)\right) \tag{A.59b}$$

$$\zeta_{t+1}^r = \left((1-\omega^r) + \gamma_{t,t+1}\zeta_t^r\right)\left(\omega^r + (1-\omega^y)\zeta_t^y\right)^{-1} \text{ and} \tag{A.59c}$$

$$g_{t+1}^n = (n_{t,t+1}\zeta_t^y) + (\omega^r + (1-\omega^y)\zeta_t^y) + ((1-\omega^r) + \gamma_{t,t+1}\zeta_t^r)(1+\zeta_t^r+\zeta_t^y)^{-1} \text{ where } g_{t+1}^n = \frac{N_{t+1}}{N_t} \tag{A.59d}$$

$$g_{t+1}^\xi = (g_{t+1}^w)^{-1}(\omega^r + (1-\omega^y)\zeta_t^y \left(\rho_E + \frac{\chi_E}{2}(i_t^y)^2\right) \text{ where } g_{t+1}^\xi = \frac{\xi_{t+1}}{\xi_t}, i_t^y = \frac{I_t^y}{\xi_t} \tag{A.59e}$$

$$\tilde{\tau}_t = i_t^y w_t \tag{A.59f}$$

$$\varsigma_t^{-1/\rho_U} = \varsigma_{t+1}^{-1/\rho_U} \beta (1-\omega^y)\zeta_t^y \chi_E i_t^y \frac{w_{t+1}g_{t+1}}{w_t g_{t+1}^w} \tag{A.59g}$$

$$(1-\alpha)(1-\gamma_I)=\mu_t w_t \tag{A.60a}$$

$$\alpha(1-\gamma_I)=\mu_t[r_t^k+\delta(U_t)]k_t/g_t \text{ where } k_t=\frac{K_t}{Y_{c,t-1}} \tag{A.60b}$$

$$\alpha(1-\gamma_I)=\mu_t\delta_t' k_t U_t/g_t \tag{A.60c}$$

$$g_t=\frac{\mu_t}{\mu_{t-1}}g_t^M(g_t^A)^{1-\vartheta} \text{ where } g_t^M=\frac{M_t}{M_{t-1}}, g_t^A=\frac{A_t}{A_{t-1}} \tag{A.60d}$$

$$g_t=\frac{(N_t^f)^{\mu_t-1}}{(N_{t-1}^f)^{\mu_{t-1}-1}}\left(\frac{U_t k_t}{U_{t-1}k_{t-1}}g_{t-1}\right)^{\alpha(1-\gamma_I)}\left(g_t^\xi g_t^w\right)^{(1-\alpha)(1-\gamma_I)}(g_t^M)^{\gamma_I} \tag{A.60e}$$

$$\frac{\mu_t-1}{\mu_t}(N_t^f)^{-\mu_t}=b\Psi_t \text{ where } \Psi_t=\frac{\tilde{\Psi}_t}{Y_{c,t}} \tag{A.60f}$$

$$\mu_t=\mu(N_t^f)\approx\bar{\mu}\left(1+\frac{\epsilon_\mu}{\bar{N}^f}(N_t^f-\bar{N}^f)\right) \text{ where } \epsilon_\mu \text{ is the elasticity of } \mu(\cdot) \tag{A.60g}$$

$$\delta_t=\bar{\delta}+\delta_t'(U_t-\bar{U}) \tag{A.60h}$$

$$\delta_t'=\bar{\delta}'+\delta''(U_t-\bar{U}) \tag{A.60i}$$

$$\frac{z_{a,t+1}}{z_{a,t}}g_{t+1}^A=\chi\left(\frac{s_t}{\Psi_t}\right)^\rho+\phi \text{ where } z_{a,t}=\frac{Z_t}{A_t}, s_t=\frac{S_t}{Y_{c,t}} \tag{A.61a}$$

$$g_{t+1}^A=\lambda_t\phi[z_{a,t}-1]+\phi \tag{A.61b}$$

$$s_t=g_{t+1}R_{t+1}^{-1}\phi j_{t+1}\left(1-\phi\frac{z_{a,t}}{z_{a,t+1}g_{t+1}^A}\right) \text{ where } j_t=\frac{J_t Z_t}{Y_t} \tag{A.61c}$$

$$v_t=\frac{(1-1/\vartheta)\gamma_I}{\mu_t}+(R_{t+1})^{-1}\phi\frac{g_{t+1}}{g_{t+1}^A}v_{t+1} \text{ where } v_t=\frac{V_t A_t}{Y_t} \tag{A.61d}$$

$$\varpi_t=\epsilon_\lambda\lambda_t R_{t+1}^{-1}\phi\frac{z_{a,t}g_{t+1}}{g_{t+1}^A}\left[v_{t+1}-\frac{j_{t+1}}{z_{at+1}}\right] \text{ where } \varpi_t=\frac{\Xi_t Z_t}{Y_t} \tag{A.61e}$$

$$j_t=-\varpi_t+(R_{t+1})^{-1}\phi\frac{z_{a,t}g_{t+1}}{g_{t+1}^A}\left[\lambda_t v_{t+1}+(1-\lambda_t)\frac{j_{t+1}}{z_{a,t+1}}\right] \tag{A.61f}$$

$$\lambda_t=\lambda\left(\frac{\varpi_t}{z_{a,t}\Psi_t}\right)\approx\bar{\lambda}\left(1+\epsilon_\lambda\left(\frac{\varpi_t-\bar{\varpi}}{\bar{\varpi}}-\frac{z_{a,t}-\bar{z}_a}{\bar{z}_a}-\frac{\Psi_t-\bar{\Psi}}{\bar{\Psi}}\right)\right) \tag{A.61g}$$

$$\pi_t^A=\frac{(1-1/\vartheta)\gamma_I}{\mu_t}-\phi j_t\left(1-\phi\frac{z_{a,t-1}}{z_{a,t}g_t^A}\right)-R_t\varpi_{t-1}(1-1/z_{a,t-1})/g_t \tag{A.61h}$$

$$\pi_t^{RD}=\phi j_t\left(1-\phi\frac{z_{a,t-1}}{z_{a,t}g_t^A}\right)-R_t s_{t-1}/g_t \tag{A.61i}$$

where ϵ_λ is the elasticity of $\lambda(\cdot)$

$$r^k_{t+1} + 1 = R_{t+1} \tag{A.62a}$$

$$\tilde{d}^r_t = \pi^F_t \frac{fa^r_t}{fa_t} \text{ where } \pi^F_t = \frac{\Pi^F_t}{Y_{c,t}} \tag{A.62b}$$

$$\tilde{d}^w_t = \pi^F_t \frac{fa^w_t}{fa_t} \tag{A.62c}$$

$$b_{t+1} = s_t + \varpi_t(1 - 1/z_{a,t}) \text{ where } b_{t+1} = \frac{B_{t+1}}{Y_{c,t}} \tag{A.62d}$$

$$\pi^F_t = (Rk_t + 1)\frac{k_t}{g_t} + \frac{R_t}{g_t}b_t - \frac{R_t}{g_t}(fa_t) - k_{t+1} - b_{t+1} + (fa_{t+1}) + \pi^A_t + \pi^{RD}_t \tag{A.62e}$$

$$k_{t+1} = (1 - \delta(U_t))\frac{k_t}{g_t} + i_t \text{ where } i_t = \frac{I_t}{Y_{c,t}} \tag{A.63a}$$

$$y_t = (1 - \gamma_I/(\vartheta\mu_t)) - b\Psi_t \text{ where } y_t = \frac{Y_t}{Y_{c,t}} \tag{A.63b}$$

$$y_t = c_t + i_t + s_t + \varpi_t(1 - 1/z_{a,t}) + \tilde{\tau}_t \text{ where } c_t = \frac{C_t}{Y_{c,t}} \tag{A.63c}$$

$$c_t = c^w_t + c^r_t \tag{A.63d}$$

$$fa^w_{t+1} + fa^r_{t+1} = k_{t+1} + b_{t+1} \tag{A.63e}$$

$$fa^r_{t+1} = \frac{R_t}{g_t}fa^r_t + \tilde{d}^r_t - c^r_t + (1 - \omega^r)\left(\frac{R_t}{g_t}fa^w_t + w_t + \tilde{d}^w_t - c^w_t - \tilde{\tau}_t\right) \tag{A.63f}$$

$$fa_{t+1} = fa^w_{t+1} + fa^r_{t+1} \tag{A.63g}$$

$$\Psi_t = v_t \tag{A.63h}$$

$$fa^w_{t+1} = \omega^r\left(\frac{R_t}{g_t}fa^w_t + w_t + \tilde{d}^w_t - c^w_t - \tilde{\tau}_t\right)$$

Appendix G. Theoretical Model: Additional Simulations

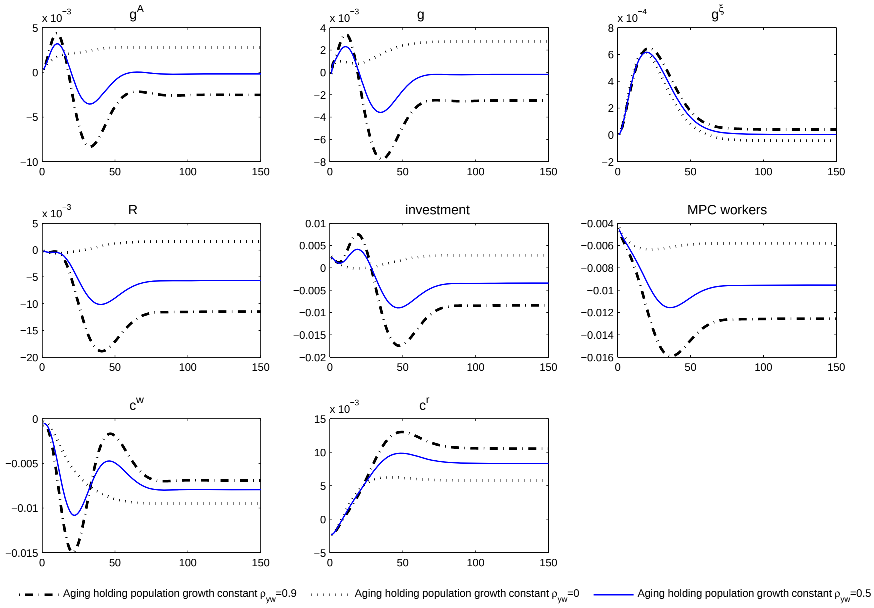

Figure A.2: Simulation: benchmark *aging* versus different ρ_{yw}

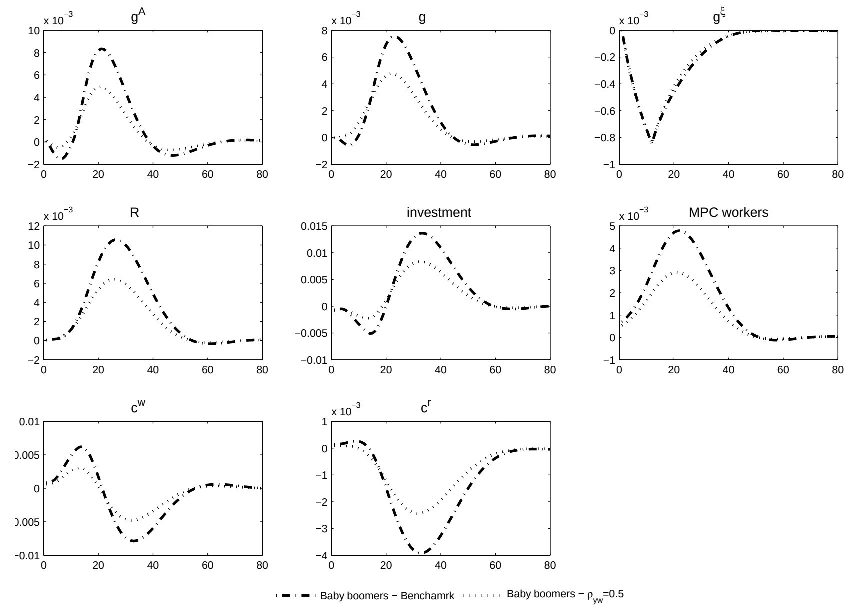

Figure A.3: Simulation: benchmark *Baby-boomers* versus $\rho_{yw} = 0.5$

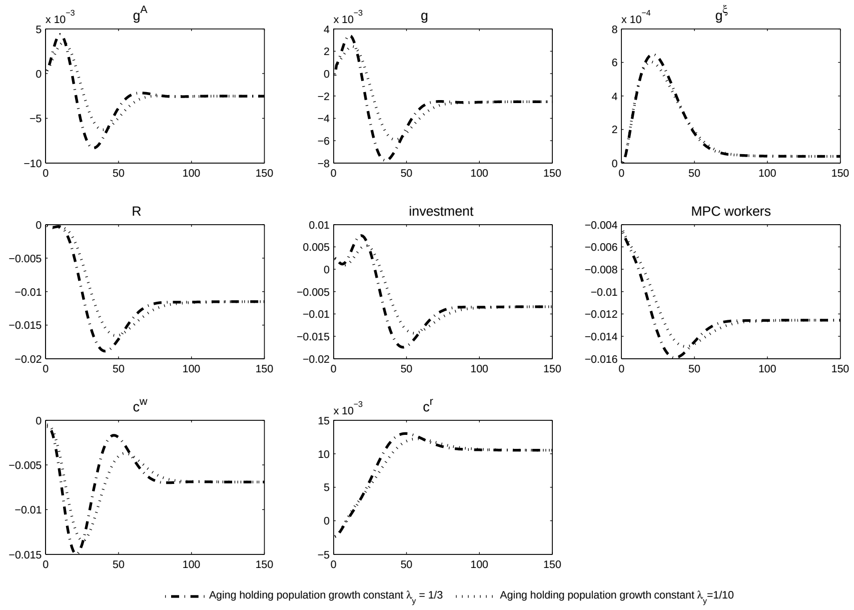

Figure A.4: Simulation: benchmark *aging* versus different $\lambda_y = 1/10$

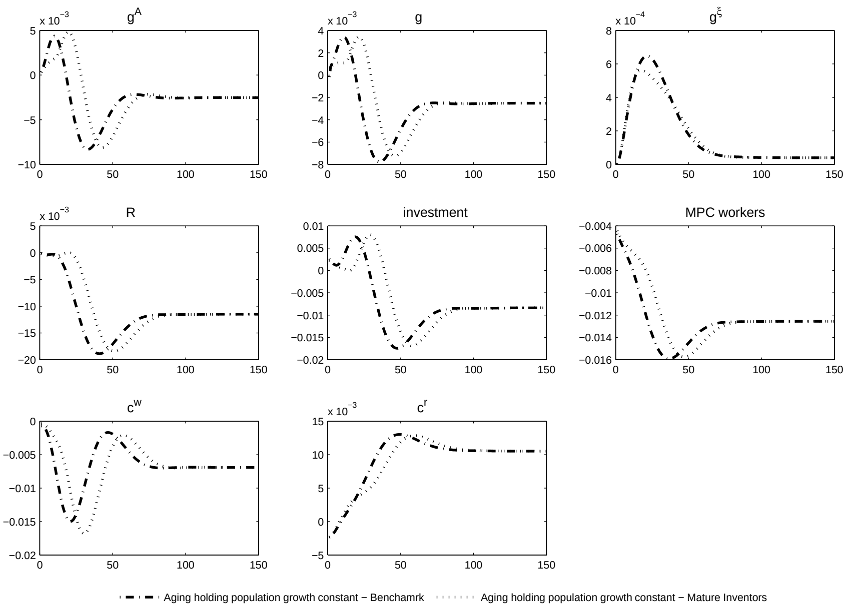

Figure A.5: Simulation: benchmark *aging* versus Delayed flow of inventors

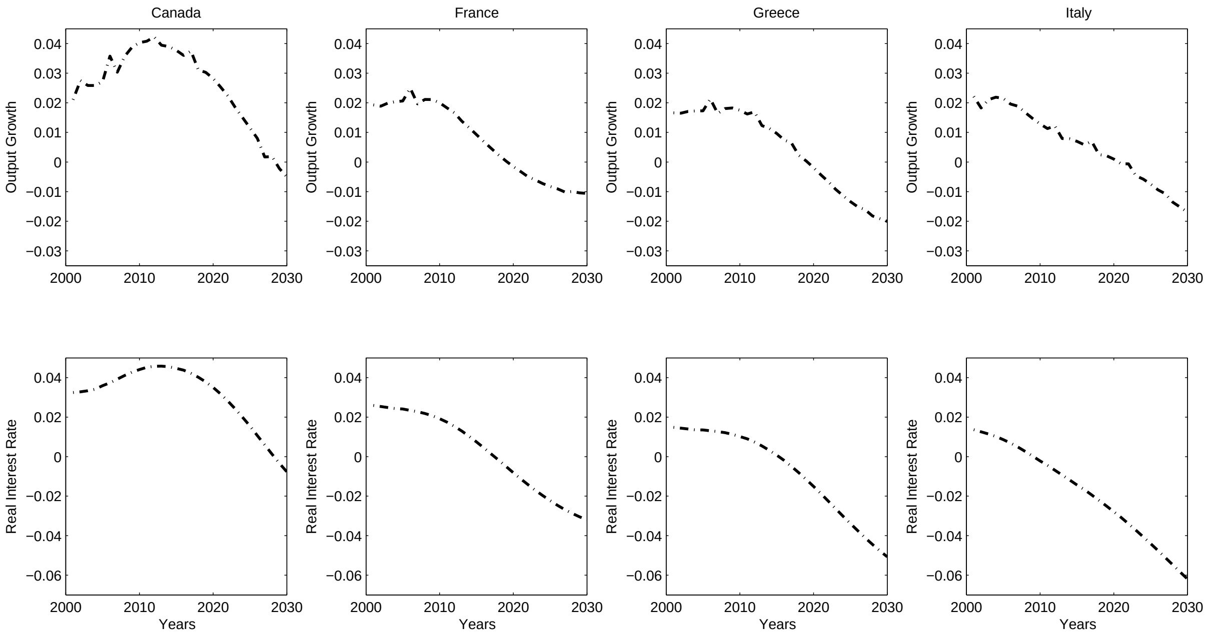

Figure A.6: Simulation: *prediction* - Additional Countries