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Enforcing Repayment: Social Sanctions versus Individual Incentives

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Enforcing Repayment:
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Abstract: We study repayment incentives generated through social sanctions and under pure individual liability. In our model agents are heterogeneous, with differing degrees of risk aversion. We consider a simple setting in which agents might strategically default from a loan program. We remove the usual assumption of exogenous social penalties, and consider the endogenous penalty of exclusion from an underlying social cooperation game, modeled here as social risk-sharing. For some types of agents social risk-sharing can be sustained by the threat of exclusion from this arrangement. These types have social capital and can be given a loan that bootstraps on the risk-sharing game by using the threat of exclusion from social risk-sharing to deter strategic default. We show that the use of such sanctions can only cover a fraction of types participating in social risk sharing. Further, coverage is decreasing in loan duration. We then show that an individual loan program augmented by a compulsory illiquid savings plan (such schemes are used by the Grameen Bank) can deliver greater coverage, and can even cover types excluded from social risk-sharing (i.e. types for whom social penalties are not available at all). Further, the coverage of an individual loan program has the desirable property of increasing with loan size as well as loan duration. Finally, we show that social cooperation enhances the performance of individual loans. Thus fostering social cooperation is beneficial under individual liability loans even though it has limited usefulness as a penalty under social enforcement of repayment. The results offer an explanation for the Grameen Bank's adoption of individual liability replacing group liability in its loan programs since 2002.

KEYWORDS: *Strategic default, social cooperation, social penalties, individual liability, loan coverage, loan duration, loan size.*

JEL CLASSIFICATION: O12

1 Introduction

A large body of literature analyzing collective lending programs in developing countries often assumes that compliance is born of the threat of social penalties.[(1)] The analysis typically assumes that an exogenous social penalty is available, and members of schemes such as group loans can harness this effectively to deter strategic default. It follows that group loans can foster repayment incentives even in the absence of useful collateral, which causes loan programs based on individual liability to fail.

However, in the last few years many loan programs in developing countries have moved away from group loans towards offering individual loans. The most prominent change has been in the contracts offered by the Grameen Bank, which originated the idea of group loans. Since 2002, the Grameen Bank has moved to "Grameen II" contracts, which are purely individual liability loans with certain compulsory illiquid savings requirements.[(2)] Other well known micro finance institutions such as Bolivia's BancoSol have similarly moved from group loans to individual loans across several loan categories.

In this paper, our first objective is to consider the effectiveness of social penalties in deterring strategic default once we remove the assumption that they are exogenously available. Social penalties themselves arise from the fact that agents are engaged in some long-term cooperation amongst themselves supported by appropriate threats of exclusion from the cooperative arrangement. Suppose now some agents are given a loan with strategic default penalized by exclusion from social cooperation.[(3)] The underlying social cooperation is sustained by the threat of exclusion - and now the same threat is also used to support the loan program. In other words, when a loan program is introduced, the social penalty

[(1)]See Ghatak and Guinnane (1999) for a survey. See also the discussion of this and related topics in Armendáriz de Aghion and Morduch (2005).

[(2)]See, for example, Dowla and Barua (2006) for a detailed exposition of the Grameen II programs.

[(3)]There is a question about who punishes an agent who defaults strategically on a loan. In a group lending program, it could be just the other group members who punish. Alternatively, some others closely connected to the group members might also punish. The strongest punishment arises if all members of society punish. Bloch, Genicot, and Ray (2008) study such punishments in the context of the social networks. In this paper we assume the strongest punishments (i.e. all members of society punish a deviating agent) to maximize the power of social penalties. If the punishments are any weaker, that would only strengthen our results on the superiority of individual liability loans. As we discuss later in the introduction (in the part headed "aspects of the model") as well as in section 2, our modeling accommodates the strongest penalties as part of an equilibrium.

remains the same as that in the original social cooperation game. However, the deviation payoff is higher because an agent who considers defaulting on the loan would simultaneously deviate from social cooperation (since he will be excluded from the latter in any case after the loan default).

In other words, ceratin penalties (such as exclusion from social cooperation) sustain social cooperation in a repeated game. When a loan program is introduced that relies on social penalties, it piggybacks on the same penalties to ensure that agents have the incentive to repay. Therefore a loan program that depends on social penalties increases the deviation payoff, but the penalties remain the same. Agents for whom the incentive constraint in the underlying game binds or is close to binding would, if given a loan, now have an incentive to deviate from paying the underlying social transfers as well as repaying the loan. Thus relying on social sanctions for repayment can dilute the effectiveness of the sanctions themselves.

The paper presents a simple model to illustrate this phenomenon. We assume the simple case of complete information, and consider the incentive for strategic default from a loan program. To endogenize social sanctions, we assume that social capital is the result of an underlying risk-sharing arrangement among agents with different degrees of risk aversion. The approach is similar to that in Bloch, Genicot, and Ray (2007), who, in a different context, study social capital as arising from a model mutual help. We then analyze the performance of a loan programme that exploits social penalties to enforce repayment by deterring strategic default.

We first consider a loan program that depends purely on social enforcement. We show that the threat of exclusion from social risk-sharing fails to deter deviation by types (the type of an agent is his degree of risk aversion) below a cutoff, which could be quite high. Essentially, a loan program using social penalties does not add to penalties, but adds to deviation benefits, creating greater incentive to deviate simultaneously from social risk-sharing as well as the loan program. This reduces the fraction of types covered by the loan program. Coverage is decreasing in loan duration, and the coverage of long duration loans are also decreasing in loan size.

Under complete information, the types that would deviate are known, and therefore excluded from the loan program. Thus the only issue is the coverage of a loan - i.e. the fraction of types who can be offered loans. Of course, under incomplete information on types the outcome would be worse: then deviation would no longer be an out-of-equilibrium phenomenon, and there would be types who would, after receiving a loan, deviate from both the loan program as well as social cooperation. In other words, a loan program using social sanctions would damage social cooperation itself.

Our second objective is to compare the repayment incentives arising from a social penalty discussed above to those arising from a purely individual loan. We show that a simple individual loan scheme augmented by a mandatory savings plan can deliver both incentive compatibility and budget balance while allowing greater coverage compared to schemes using social sanctions. Indeed, we show that such an individual loan scheme can even cover types *excluded* from social risk-sharing (i.e. types for whom social penalties are not available at all). Moreover, unlike schemes using social sanctions, the coverage of an individual loan program *increases* with loan size as well as loan duration. Thus not only is it possible to offer individual loans to a much larger fraction of types, it is also possible to offer loans of greater size and duration without violating repayment incentives.

In general, offering individual loans to an agent can alter the agent's incentive to participate in social cooperation. However, we show that an incentive compatible individual loan program does not harm social cooperation. Types with social insurance do not have an incentive to deviate from the insurance arrangement under an incentive compatible individual loan program.

Finally, we show that even though individual loans do not directly depend on social cooperation, such cooperation is useful because it enhances the performance of individual loans. Essentially, the incentive to deviate from an individual loan repayment program weakens when the income stream is smoothed through social insurance. Thus fostering social cooperation raises coverage of individual loans even though it has limited usefulness as a penalty under social enforcement of repayment. In his discussion of the "Grameen II" model (which is based purely on individual lending) Yunus (2008) notes that if a borrower cannot repay basic installments, "there is now no need for the bank to trigger actions to mobilize the group and the center pressure on her...Group solidarity is used for forward-looking joint-actions...rather than for the unpleasant task of putting unfriendly pressure on a friend." Our analysis shows how such benefits of social cooperation might arise when the resulting social capital is not used as a threat to extract repayments.

An interesting paper by Giné and Karlan (2009) study the results from a randomized control trial by a Philippine bank. The bank removed group liability from randomly selected group-screened lending groups. After three years, the authors find no increase in default as well as larger group sizes in the converted centers. Thus coverage increased, but performance did not worsen. Our results offer an explanation for this observation. More generally, our results provide an explanation for the movement away from group liability to individual liability in micro-credit programs.

Aspects of the Model

Before discussing related literature, it is worth noting two aspects of the model. First, consider the nature of social punishments. Since we want to test the limits of the power of social sanctions, we ensure that social sanctions have maximum power. Our model makes two assumptions to ensure maximum power of social sanctions. First, note that the ability of other agents to punish an agent arises from the fact that agents engage in a risk-sharing game. Therefore, the greater the benefit of an agent from risk sharing, the greater the power of social sanctions (which takes away this benefit by excluding the agent from future risk sharing) on that agent. To maximize the power of social enforcement, we then assume *complete* risk-sharing. Second, we assume that any deviation is met by full exclusion from risk sharing. In other words, *all other* members of society punish an agent who defaults strategically. Formally, this is achieved by the simple device of modeling a continuum of agents. Since each agent is of measure zero, others have no cost of punishing an agent who strategically defaults, and therefore the strongest punishment is possible in equilibrium. Thus in terms of modeling of loans under social penalties, we simply study the repayment incentive of an agent who faces full exclusion from social risk sharing under strategic default.[4] Further, note that if we relax either feature, that would weaken social penalties, and would enhance the relative performance of individual loans. In other words, our results show the superiority of individual-incentive based loans over social-penalty based loans even when the latter features the strongest possible penalties. Weakening this penalty would obviously reinforce our results.

[4]Note that we do not need to specify any further features of a group loan arrangement that borrowing agents are part of. Given a continuum of agents, whether a single agent participates in risk sharing or is excluded is payoff irrelevant for any other agent, and therefore punishing is always incentive compatible. Thus irrespective of which fraction of other agents punish a defaulter, we need not specify any further reason for them to carry out the punishment. And to maximize the punishment, we assume that all other members of society punish a defaulter.

The second aspect worth noting is the assumption of complete information. Given complete information, all types that cannot be made to repay (i.e. types that would potentially default) under social penalties are excluded from a loan program to start with. In other words, strategic default is an entirely out-of-equilibrium phenomenon. The only concern therefore is coverage - i.e. what fraction of types can qualify for a loan. However, if peer information were not complete and therefore screening were imperfect, there would be types who secure a loan and subsequently default in equilibrium. Note that this would happen even if social ties are strong (so that the risk-sharing is full and coverage of risk-sharing is high).

Related Literature

Several papers have analyzed enforcement by peers where the peers can impose exogenously given social sanctions on a defaulter. Besley and Coate (1995) show that group lending creates incentive for strategic default. They show that such problems can be resolved if borrowers can impose high enough social sanctions on others. In the analysis of cooperative design by Banerjee, Besley, and Guinnane (1994), monitoring a borrower by a non-borrowing co-signer depends on a sanction available to the latter. Similarly, the design of peer monitoring schemes in Armendáriz de Aghion (1999) depends on the ability of peers to impose a social sanction on a defaulter. In the model of Roy Chowdhury (2007), some borrowers are assumed to possess an amount of social capital that they stand to lose in the event of default from a group loan. In these papers, the sanction that peers can impose is exogenously given. In contrast, this paper considers social penalties arising from the threat of exclusion from a risk-sharing game, which is itself enforced by the threat of exclusion. Such endogenous incentives give rise to very different conclusions about the power of social sanctions.

In our model, social capital arises from a repeated game of risk sharing. Coate and Ravallion (1993) study risk sharing in a repeated game with two agents. Genicot and Ray (2003) and Bloch et al. (2008) study risk sharing in groups and networks. Bloch et al. (2007) study a model in which social capital arises from mutual help. As mentioned previously, while the context is different, our paper follows a similar approach in modeling social capital as arising endogenously through a simple risk-sharing game.

Our focus is on the nature and power of social penalties. Of course, group lending programs have other features such as sequential financing and monitoring by the lender. In a series of papers, Roy Chowdhury(2005; 2007) has investigated the role of such features. Further, Jain and Mansuri (2003) has considered the role played by collection of repayment through several small installments. It is worth noting that any incentive effects generated by these features have little to do with group lending - they could easily feature under individual liability as well. Indeed, Armendáriz de Aghion and Morduch (2000) offer evidence of the success of individual loans that use progressive/dynamic incentives, frequent repayments, and nontraditional collateral to guarantee a loan. Using data from Eastern Europe and Russia, they demonstrate that individual loans can generate repayment rates greater than 90 percent (and above 95 percent in Russia).

Finally, other papers have compared group and individual loans based on entirely different considerations. Conning (2005) shows that delegating the task of monitoring to other borrowers–who must also expend effort on their own projects–has an incentive diversification effect, which can make it attractive for the lender to delegate monitoring to borrowers. In this case, peers are given formal incentives to monitor by the lender. Here we consider endogenous sanctions arising from social interactions rather than peer enforcement when peers are formally contracted agents of the lender. Madajewicz (2005) shows that a group loan may yield a lower welfare compared to an individual loan for the wealthier among credit-constrained borrowers. In an environment with risk averse borrowers, joint liability has the usual advantage from peer monitoring, but there is also a second effect: choice of risky project by one borrower imposes a negative externality on others, so that the optimal choice of each is riskier. The paper shows that the second effect can dominate for higher levels of wealth, so that wealthier borrowers can get larger loans under individual liability. In this paper, on the other hand, the difference between social and individual enforcement arise from the fact that the former must exploit incentives that arise endogenously through social cooperation.

The rest of the paper is organized as follows. The next section sets up the model. Section 3 clarifies the underlying risk-sharing arrangement, which generates the scope for social penalties. Section 4 then analyzes the performance of a loan program that harnesses such penalties. Section 5 sets up and analyzes a simple individual loan program with forced saving. Finally, section 7 concludes by comparing the results from the two previous sections. Proofs not in the body of the paper are collected in the appendix.

2 The Model

There is a continuum of agents. Each agent draws a type ρ from some distribution F on $[0, \overline{\rho}]$. The utility function of an agent with wealth x and type ρ is

$$u(x\,;\rho) = \frac{1 - \mathrm{e}^{-x\rho}}{\rho}. \tag{1}$$

The type ρ of an agent is the degree of absolute risk aversion of the agent. Note that as $\rho \to 0$, $u(x\,;\rho) \to x$, the risk neutral utility function. We assume full information on types.

There are an infinite number of periods. In each period the wealth of an individual is 1 with probability p and 0 with probability $(1-p)$. Agents have no savings technology.(5) Agents have a common discount factor $\delta \in (0, 1)$.

An investment opportunity lasting T periods arises for some agents. For simplicity, we assume that a project needs an indivisible investment of size M each period and the gross return each period is $M(1+R)$ where $R > 0$. Note that we simply study the incentive for strategic default which arises after any return is realized - and therefore assume the simplest possible investment opportunity.

An external lender funds the project with a loan for T periods. In each period, the external lender offers a loan of M. For simplicity, we assume that the lender can obtain funds at a zero rate of interest and lends on a zero-profit basis. This implies that the lender requires a repayment of M for each loan. We also assume that the lender has access to a savings technology.(6)

We do the accounting as follows. The first loan made at time 0 and the first repayment date is $t = 1$, which is also when the second loan is given. Time $t = 2$ is then the date for the second repayment and third loan. Thus in general, the k-th loan is made in period $k-1$ and the repayment is due in period k. Therefore a T period loan consists of T **loan dates** $t = 0, \ldots, T-1$, and T **repayment dates** $t = 1, \ldots, T$.

(5)If agents could save and thereby smooth own consumption to an extent, that would only reduce the value of social insurance, thereby reducing the power of social sanctions.

(6)This is made use of under individual loans - see section 5 for details.

3 Risk Sharing and Social Capital

In this section we specify formally the social risk-sharing arrangement that forms the basis for social sanctions in our model. Types participating in social risk sharing are said to possess social capital. The extent of social capital for an agent of course depends on the agent's type - social capital is higher for higher values of ρ. As we show later, some types participating in social risk sharing can be given a loan with repayment incentives sustained by a social sanction which consists of excluding an agent from risk sharing (thus taking away the social capital of the agent).

As shown below, reducing risk sharing simply reduces the ability of social incentives to provide repayment incentives in a loan program. To maximize the power of social sanctions we assume that the risk sharing is full.

Recall that in each period an agent has an income of 1 or 0. Sharing risk consists of agents with income 1 contributing some share of this to a social pot which is then distributed equally across agents. Suppose agents share a fraction α of their income. In other words, all agents with an income of 1 pays α into a common pot, which is then equally distributed across agents. In any period, the payoff from conforming to this arrangement is $U(\alpha) = \mathrm{p}u((1-\alpha)+\mathrm{p}\alpha)+(1-\mathrm{p})u(\mathrm{p}\alpha)$. Therefore the total repeated game payoff from conforming is given by $U(\alpha)+\delta U(\alpha)+\delta^2 U(\alpha)+\ldots = U(\alpha)/(1-\delta)$. Deviation has an immediate gain only if the agent has an income of 1, and in future the agent loses the social insurance. Therefore the deviation payoff is $U^D = u(1\,;\rho)+\delta/(1-\delta)(\mathrm{p}u(1\,;\rho)+(1-\mathrm{p})u(0\,;\rho))$.

Note that the deviation payoff does not depend on α, while the payoff from conforming given by $U(\alpha)$ is maximized when risk sharing is full, i.e. $\alpha = 1$. Clearly, full risk sharing maximizes the net benefit of social risk sharing. In what follows, we assume that the underlying social risk sharing game is characterized by full risk sharing. This maximizes the benefit of cooperating, which therefore maximizes the power of social penalties.

Since risk sharing is full, by conforming, any agent receives a constant payoff p per period. Therefore the payoff from conforming is given by $u(\mathrm{p}\,;\rho)/(1-\delta)$. Deviation has an immediate gain only if the agent has an income of 1, in which case the immediate gain from deviation is given by $G = u(1\,;\rho) - u(\mathrm{p}\,;\rho)$. Following a deviation, the agent loses social insurance in future. Therefore, starting next period, the loss per period is

$L = u(\mathrm{p}\,;\rho) - (\mathrm{p}u(1\,;\rho) + (1-\mathrm{p})u(0\,;\rho))$. Let $\rho_{\min}$ be the solution to

$$G = \frac{\delta}{(1-\delta)} L. \tag{2}$$

The following result shows that the solution is unique, i.e. there is a unique type $\rho_{\min}$ that is indifferent between deviating and conforming. Types below this cutoff prefer to deviate while types above conform. Clearly, this cutoff is decreasing in δ.

Proposition 1. *There is a unique type $\rho_{\min} > 0$ such that the subset $[\rho_{\min}, \overline{\rho}]$ of types cooperate and share risk. Types below $\rho_{\min}$ are excluded from the risk sharing scheme. The type $\rho_{\min}$ is decreasing in δ.*

Figure 1 in the next section shows the cutoff $\rho_{\min}$ in an example. Since only types in the interval $[\rho_{\min}, \overline{\rho}]$ participate in risk sharing, any loan program based on social sanctions can cover at most these types. However, as we show next, coverage might be considerably lower.

4 Loan Program Under Social Sanctions

A loan program dependent on social sanctions works as follows. As explained in section 2, an external lender lends M at each loan date to each agent with an investment opportunity, and requires a repayment of M at each repayment date. The social sanction is then as follows. If at any $t \in \{1, \ldots, T\}$ a borrowing agent fails to make a repayment, the agent is excluded from social risk-sharing.

A loan program (M, T) is said to be sustainable under social sanctions for any type $\rho \in [0, \overline{\rho}]$ if the threat of exclusion form social risk-sharing is sufficient to deter this type from not making the required repayment at all dates $t \in \{1, \ldots, T\}$.

Three aspects of the scheme are worth emphasizing.

- As mentioned in the introduction, under complete information, loans are only offered to agents with types for which the loan is sustainable. Types that would potentially deviate (from the loan program as well as social risk sharing) after receiving a loan are precluded from the loan program. Therefore deviations are an entirely out-of-equilibrium phenomenon. The only issue is the coverage, i.e. for what fraction of types can a loan program be made incentive compatible.

- Next, note that each agent is measure zero, and therefore other agents suffer no loss of utility from excluding an agent from social cooperation. In other words, we need no further incentive for other agents to punish a deviator: such punishments are already compatible with an equilibrium. Since other agents are indifferent between punishing and not punishing a deviator, we are simply selecting the equilibrium in which such punishments take place. It is worth reiterating that this modeling device strengthens the power of social sanctions. If agents were non-atomistic so that exclusion of an agent from risk-sharing would reduce the utility of other agents, social sanctions would be less effective, as others would require incentives to penalize a deviator.

- We want to isolate the incentive effects arising purely from social sanctions. To this end, it is assumed that even after a default at $t < T$, the external lender still makes loans as scheduled at future dates up to T. We will later study the impact of exclusion from future loans as a punishment under individual liability loans.

  In our model, a default is detected with certainty, implying that this distinction arises only as a modeling device. However, such an arrangement can occur naturally in equilibrium in a richer model. Suppose instead of a certain return R, a project returns R with probability $q \in (0, 1)$ and 0 with the residual probability. Suppose that peers know the realized state while an external lender must incur some positive cost $c > 0$ to learn the realized state. A loan under social sanction then uses no monitoring by the lender and relies purely on social enforcement. Thus an agent who defaults strategically at period $t < T$ expects to be excluded from social risk-sharing but expects to receive loans in future as normal. An individual loan, in contrast, verifies the state at every date and penalizes strategic defaulters directly (through denial of future loans and transfers). For small values of c, this setting does not change any result qualitatively.

The next section derives the set of types who would deviate under a loan program that makes use of social sanctions to enforce repayment. Such types would then be excluded from the loan program. Section 4.2 then derives the main results for such loans.

4.1 Deviation Incentives

Under a T period loan, a borrower gets T opportunities to deviate. In this section we determine the set of types who would potentially deviate, and therefore cannot be covered by a loan program using social penalties.

The plan of the section is as follows. First, we derive the total loss from deviation and a crucial property of this loss. Next, we show that there is a unique cutoff type in each period such that types below the cutoff deviate and those above conform. Finally, proposition 2 presents the main result showing that the highest incentive to deviate occurs in the earliest repayment period, and therefore the first period's cutoff type is the global separator between deviating and conforming types.

Consider a $T > 1$ period loan program. For any $1 \leqslant t \leqslant T$, the payoff in period t from conforming is given by

$$V_t = \sum_{k=0}^{T-t} \delta^k u(\mathrm{p} + MR\,;\rho) + \frac{\delta^{T-t+1}}{1-\delta} u(\mathrm{p}\,;\rho).$$

Note that a type deviating from loan payment in any period $t \leqslant T$ also deviates from loan payment in every subsequent period until T. To see this, note that in any subsequent period, the type is not included in social cooperation - and therefore there is noting to be gained from conforming, but the extra payoff from deviation is lost.[7]

Using this, the payoff from deviating in period t is given by:

$$V_t^D = u(1 + M + MR\,;\rho) + \sum_{k=1}^{T-t} \delta^k \Big(\mathrm{p}u(1 + M + MR\,;\rho) + (1-\mathrm{p})u(M + MR\,;\rho) \Big) + \frac{\delta^{T-t+1}}{1-\delta} \Big(\mathrm{p}u(1\,;\rho) + (1-\mathrm{p})u(0\,;\rho) \Big).$$

Let

$$\mathbb{G}(X\,;\rho) \equiv u(1 + X + XR\,;\rho) - u(\mathrm{p} + XR\,;\rho), \tag{3}$$

$$\mathbb{L}(X\,;\rho) \equiv u(\mathrm{p} + XR\,;\rho) - \Big(\mathrm{p}u(1 + X + XR\,;\rho) + (1-\mathrm{p})u(X + XR\,;\rho) \Big). \tag{4}$$

(7) Formally, after deviation at $t > 0$, in any subsequent period the payoff from conforming is either $u(1 + MR\,;\rho)$ (with probability p) or $u(MR\,;\rho)$ (with probability $(1-\mathrm{p})$). The payoff from deviating in any such period, on the other hand, is $u(1 + M + MR\,;\rho)$ in the first case and $u(M + MR\,;\rho)$ in the second case. Clearly, deviating dominates conforming.

If an agent of type ρ deviates in any period $t \leqslant T$, the immediate gain is $\mathbb{G}(M\,;\rho)$. In every subsequent period up to period T, the agent incurs a loss of $\mathbb{L}(M\,;\rho)$. After period T, the loss in each period is $u(\mathrm{p}\,;\rho) - (\mathrm{p}u(1\,;\rho) + (1-\mathrm{p})u(0\,;\rho))$, which is simply $\mathbb{L}(0\,;\rho)$. Thus the total loss from deviation in any period $t \leqslant T$ is

$$
\begin{aligned}
\mathbb{TL}_t(M\,;\rho) &= \sum_{k=1}^{T-t} \delta^k \mathbb{L}(M\,;\rho) + \frac{\delta^{T-t+1}}{1-\delta}\mathbb{L}(0\,;\rho) \\
&= \frac{\delta}{1-\delta}\Big[(1-\delta^{T-t})\mathbb{L}(M\,;\rho) + \delta^{T-t}\mathbb{L}(0\,;\rho)\Big]. \qquad (5)
\end{aligned}
$$

The next result establishes a crucial property of the total loss from deviation: this loss is increasing in time. In other words, the loss from deviation is lower for earlier deviations.

Lemma 1. *For any $t \leqslant T$, $\mathbb{TL}_{t-1}(M\,;\rho) < \mathbb{TL}_t(M\,;\rho)$.*

This then suggests–as explained below–that it is the incentives at date 1 that determines overall incentive compatibility. However, before we can establish this claim, we need to show that there is a well-defined participation threshold type that separates deviators and conformers in each period. We show this through Lemmas 2 and 3 below.

Note that $\mathbb{TL}_t(M\,;\rho) - \mathbb{G}(M\,;\rho) = V_t - V_t^D$. Let $\widehat{\rho}_t$ be the type that is indifferent between conforming and deviating. It is the type for which gain and loss are exactly equal. Thus $\widehat{\rho}_t$ is given by the solution to $V_t = V_t^D$, which is the same as $\mathbb{G}(M\,;\rho) = \mathbb{TL}_t(M\,;\rho)$. Using equations (3) and (4), this can be written as follows:

$\widehat{\rho}_t$ is given by the solution to

$$
\frac{\delta}{1-\delta}\left[(1-\delta^{T-t})\frac{\mathbb{L}(M\,;\rho)}{\mathbb{G}(M\,;\rho)} + \delta^{T-t}\frac{\mathbb{L}(0\,;\rho)}{\mathbb{G}(M\,;\rho)}\right] = 1. \qquad (6)
$$

We now establish that this cutoff type exists and is unique. First, we need the following result:

Lemma 2. $\frac{\mathbb{L}(0\,;\rho)}{\mathbb{G}(M\,;\rho)}$ *is increasing in ρ.*

The following result now shows that there exists a unique cutoff $\widehat{\rho}_t$ which is indifferent between deviating and conforming in any period t. Only types above this threshold conform, and types below are excluded from the loan program.

Lemma 3. *For any $t \in \{0, \dots, T\}$, and for any $M, R \geqslant 0$ there is a unique solution $\widehat{\rho}_t$ to equation* (6)*. Further, $V_t(T\,;\rho) \gtreqless V_t^D(T\,;\rho)$ according as $\rho \gtreqless \widehat{\rho}_t$.*

Let $\rho^*(T)$ denote the repayment threshold under a T period loan program. This is given by

$$\rho^*(T) \equiv \max\{\widehat{\rho}_1, \dots, \widehat{\rho}_T\}.$$

This cutoff separates the types below who deviate in some period $t \leqslant T$, and therefore cannot be included in the loan program, from the types above who conform and can be covered by the loan. The result below now shows that the relevant threshold is the initial one. In other words, to determine the types to be excluded from the loan programme, we only need to consider types who would deviate in the very first period.

Proposition 2. ***(Participation Threshold)*** *For a loan program lasting T periods, the incentive to deviate is strongest in the initial period of the loan and therefore the participation threshold* $\rho^*(T)$ *is given by*

$$\rho^*(T) = \widehat{\rho}_1, \tag{7}$$

where $\widehat{\rho}_1$ *is the solution to equation* (6) *for* $t = 1$*.*

Proof: Consider the incentive of type $\widehat{\rho}_t$ in period $t-1$. The gain from deviation in period $t-1$ for this type is the same as that in period t, and given by $\mathbb{G}(M\,;\rho)$ (which is given by equation (3)). But from Lemma 1 the total loss at $t-1$ is lower. Therefore it is clear that type $\widehat{\rho}_t$ is among the types that strictly prefer to deviate in period $t-1$. In other words, $\widehat{\rho}_{t-1} > \widehat{\rho}_t$ for any $t \leqslant T$. This proves that $\widehat{\rho}_1 = \max\{\widehat{\rho}_1, \dots, \widehat{\rho}_T\}$.$\|$

4.2 Project Size, Loan Duration and Loan Coverage

This section presents the main results on the coverage of a loan program using social sanctions to enforce repayment. We first show that the coverage of such a loan program (i.e. the fraction of types for whom the loan program is incentive compatible) decreases in loan duration.

Proposition 3. (Loan Duration and Coverage) $\rho^*(T+1) > \rho^*(T)$, *which implies that loan coverage decreases in the duration of the loan program.*

Figure 1 below shows an example of coverage of a loan program. Recall that the loan cutoff ρ^* is given by the solution to equation (6) for $t = 1$. Further, from equation (2), the social insurance cutoff $\rho_{\min}$ is given by $\frac{\delta}{1-\delta}\frac{L}{G} = 1$ which is the same as equation (6) for $M = 0$. The figure plots the left hand side of equation (6) for $M = 0$ and $M > 0$. The points where these cross 1 gives ρ_{min} and ρ^* respectively. Note that the coverage of the loan program shrinks significantly compared to the coverage of the underlying social risk-sharing arrangement.

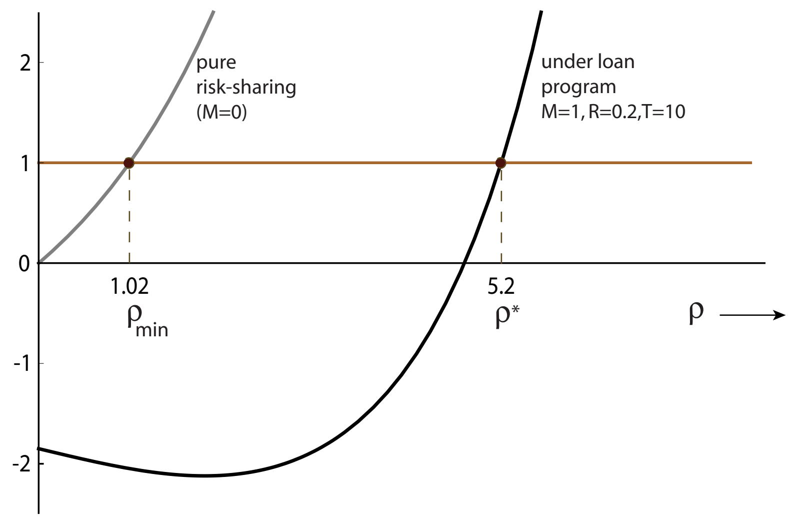

Figure 1: The coverage of social risk-sharing and that of a loan program under social enforcement. Recall that the cutoff under any loan program is given by equation (6) for $t = 1$. The figure plots the left hand side of this equation for both $M = 0$ (pure risk sharing) and under the loan program $M = 1$, $R = 0.2$, and $T = 10$. Further, p $= 0.5$ and $\delta = 0.75$. Type above the cutoff $\rho_{\text{min}} = 1.02$ participate in social risk-sharing, but the loan program is incentive compatible only for types above $\rho^* = 5.2$.

The next result considers the impact on coverage of varying loan size M. In general, it is not possible to sign the derivative of $\rho^*(T)$ with respect to M. This is because the first ratio in the left hand side of equation (6) is decreasing in M, while the second ratio is either increasing in M or decreasing first and then increasing in M. The overall effect is therefore ambiguous. However, as the result below shows, we can sign the derivative in the special case of a long-duration loan. If loan duration is long enough, we know from the previous result that coverage is already low. In this case, coverage decreases even further as the loan size M increases.

Proposition 4. (Loan Size and Coverage) *For any loan program (M, T). For values of T large enough (loan duration long enough), the coverage of the loan is decreasing in loan size M.*

5 Individual Loan Program

In an individual loan program, a defaulter is simply denied future loans, and any amount accumulated in a compulsory savings account can be used towards repayment of outstanding amounts. We now discuss such an individual loan program and show that in contrast with loans supported by social sanctions, it can cover *all* types - i.e. even cover types who do not participate in social risk sharing. Further, even when an individual loan program has less than full coverage, the coverage *increases* unambiguously with loan size as well as loan duration.

Finally, we show that even though an individual loan does not make use of social sanctions, it can indirectly benefit from stronger underlying risk-sharing: the incentive compatibility condition for types who do not participate in social risk sharing (types without social capital) is stricter than that for types who do participate in social risk sharing. Therefore participation in social risk sharing makes it easier to cover a type by an individual loan program.

5.1 The Loan Mechanism

As noted before, since 2002 the Grameen Bank offers purely individual loans. These "Grameen II" loans have a an associated special savings account in which a borrower must make compulsory deposits (of a certain percentage of the loan value). These savings have restricted liquidity, and accumulated amounts can be used towards repayment of outstanding amounts at the end of the loan.[(8)]

We construct a similar loan program, and define an individual loan as a finite sequence of loans with individual liability augmented by a compulsory illiquid savings account held with the lender. As assumed at the outset, agents do not have any savings technology, but the lender has a savings technology. The lender can therefore place any repayment amounts into a savings fund and hold it on behalf of an agent.

As before, loans of size M are made across T periods $t = 0, \ldots, T-1$. A loan given in period t requires repayment in period $t+1$. This implies that there are T corresponding repayment dates $t = 1, \ldots, T$. We assume that the loan program is designed so that the

[(8)]See, for example, Dowla and Barua (2006), chapter 3.

lender makes a zero profit. Therefore for a loan of size M made at date t, the required repayment at $t+1$ is also M.

Finally, the loan program also includes a compulsory savings clause. This requires a borrowing agent to deposit an amount rM each period, $r \in (0,1)$, into a savings account held with the lender. The borrower has no withdrawal privileges while the loan program is ongoing. If the borrower does not repay in any period $t \leqslant T$, the lender can seize the amount available in the account. At the end of the loan program, when all repayments are made, the borrower is given full access to the accumulated account. Let S denote the amount returned to the borrower at the end of the loan program. This is derived from budget balance below.

The purpose of the compulsory savings account is obvious. This serves as collateral. In the absence of some such provision, borrowers would definitely default in the last period. With the savings clause, they stand to lose the accumulated savings if they default. Therefore, incentive to repay can be sustained by accumulating the right amount of savings. However, the fact that such deposits must be made itself affects repayment incentives in any period before the last period. We derive below the condition under which the mechanism succeeds in sustaining repayments for all types of agents.

Analyzing repayment incentives We now analyze the repayment incentives of agents under an individual loan program. This is organized as follows. First, in section 5.2, we consider the repayment incentives for types with social capital (i.e. types $\rho \geqslant \rho_{\min}$ - these are the types that participate in social risk sharing). Second, in section 5.3, we consider the repayment incentives for types who do not participate in social risk sharing (types below $\rho_{\min}$). One immediate question is whether such a division is justified - as an individual loan could alter the incentive of a type to participate in social risk sharing. If this were the case, $\rho_{\min}$ would not be a meaningful separation point any longer. However, once we derive the incentive compatibility condition below for types who participate in risk sharing, we show (proposition 8 below) that an incentive compatible individual loan program does not alter the incentive to participate in social risk sharing, justifying the separation.

5.2 Repayment Incentives for Types with Social Capital

5.2.1 Deriving the incentive compatibility condition

Recall that a loan of size M is given each period, and gross return for the borrower each period is $M(1+R)$ where $R > 0$. The lender requires a repayment of $M + Mr$ at each repayment date, where Mr is placed in a savings account that the borrower cannot withdraw from. At the end of the loan program, the lender returns S to the borrower.

The expected utility from repaying the loan in each period starting from any period $t \leqslant T$ is given by

$$\begin{aligned} W_t &= u(M(R-r)+\mathrm{p}\,;\rho) + \ldots + \delta^{T-t-1}u(M(R-r)+\mathrm{p}\,;\rho) \\ &\quad + \delta^{T-t}u(M(R-r)+\mathrm{p}+S\,;\rho) + \frac{\delta^{T-t+1}}{1-\delta}u(\mathrm{p}\,;\rho) \\ &= \frac{1-\delta^{T-t}}{1-\delta}u(M(R-r)+\mathrm{p}\,;\rho) + \delta^{T-t}u(M(R-r)+\mathrm{p}+S\,;\rho) + \frac{\delta^{T-t+1}}{1-\delta}u(\mathrm{p}\,;\rho). \end{aligned}$$

The expected payoff from defaulting in period $t \leqslant T$ is

$$W_t^D = u(M+MR+\mathrm{p}\,;\rho) + \frac{\delta}{1-\delta}u(\mathrm{p}\,;\rho).$$

Now, for the last period,

$$W_T - W_T^D = u(M(R-r)+\mathrm{p}+S\,;\rho) - u(M+MR+\mathrm{p}\,;\rho).$$

Since $u(\cdot;\cdot)$ is increasing in the first argument, incentive compatibility requires $M(R-r)+\mathrm{p}+S \geqslant M+MR+\mathrm{p}$, implying $S \geqslant M+Mr$.

Given that repayment is incentive compatible, the budget balance condition is

$$S = TMr. \tag{8}$$

Therefore incentive compatibility requires that $(T-1)Mr \geqslant M$, which says that the net transfer in the last period must be at least M. Since $\delta < 1$, it is better for the agents to receive income earlier rather than later. Therefore the optimal arrangement is to set r so that $(T-1)Mr = M$, i.e.

$$S^* = M + Mr. \tag{9}$$

Now, using the value of $u(\cdot;\cdot)$ from equation (1), and S^* from above,

$$W_t - W_t^D = \frac{e^{-(p+M(1+R))\rho}}{\rho}\frac{1-\delta^{T-t}}{1-\delta}\left(1-\delta+\delta e^{M(1+R)\rho} - e^{M(1+r)\rho}\right)$$

Therefore the incentive compatibility condition for any ρ is independent of t and is given by

$$1-\delta+\delta e^{M(1+R)\rho} - e^{M(1+r)\rho} \geqslant 0. \tag{10}$$

This proves the following result.

Proposition 5. *Consider a type $\rho \in [\rho_{\min}, \overline{\rho}]$, and a budget balanced individual loan program (M, R, T). The following condition is necessary and sufficient for the loan program to be incentive compatible for type ρ: condition (10) holds for some $r \geqslant \frac{1}{T-1}$.*

Note that the incentive compatibility constraint (10) implies a maximum value of r for any given values of other parameters. So long as this maximum value exceeds $1/(T-1)$, we can find r such that the conditions stated in the result above are satisfied. Therefore we can write the incentive compatibility condition more compactly as the following:

Corollary 1. *A project (M, R, T) can be supported by an incentive compatible and budget balanced individual lending program for a type $\rho \geqslant \rho_{\min}$ if and only if $\overline{r}(\rho) > \frac{1}{T-1}$, where*

$$\overline{r}(\rho) = \frac{\ln\left[1-\delta+\delta e^{M(1+R)\rho}\right]}{\rho M} - 1. \tag{11}$$

5.2.2 Covering all types with social capital

The next result shows that $\overline{r}(\rho)$ is increasing in ρ and M. This then gives us a sufficient condition for all types under social risk-sharing to be covered by an individual loan program.

Lemma 4. *$\overline{r}(\rho)$ is strictly increasing in ρ and M.*

The result above proves that if $\overline{r}(\rho) > 1/(T-1)$ for $\rho = \widehat{\rho}$, it is also met for all $\rho > \widehat{\rho}$. Therefore an individual loan program can cover *all* types $\rho \geqslant \rho_{\min}$ so long as the condition holds for $\rho = \rho_{\min}$. A sufficient condition for this is that the condition holds at $\rho = 0$. Now, $\lim_{\rho \to 0} \overline{r}(\rho) = \delta(1+R) - 1$. This proves the following result.

Proposition 6. *Consider any project (M, R) with $M > 0$ and $R > 0$. The condition*

$$\delta(1+R) - 1 \geqslant \frac{1}{T-1}$$

is sufficient for this project to be supported for all types $\rho \in [\rho_{\min}, \overline{\rho}]$ by an incentive compatible and budget balanced individual loan program.

Figures 2 and 3 plot the function $\overline{r}(\rho)$ for different parameter values under which all types with social capital (i.e. all types above $\rho_{\min}$) are covered by an individual loan program.

5.2.3 Comparative statics of coverage

Finally, let us consider the case of $\overline{r}(\rho_{\min}) < 1/(T-1)$. In this case, there is scope for the coverage to change with loan size and duration, and we show that the coverage increases in both size and duration.

Now, in this case we have $\overline{r}(\rho) > 1/(T-1)$ only for types exceeding some cutoff $\widehat{\rho} > \rho_{\min}$, where the cutoff $\widehat{\rho}$ is the solution for ρ to[9]

$$\overline{r}(\rho) = \frac{1}{T-1}. \tag{12}$$

Whenever $\widehat{\rho} < \overline{\rho}$, we can support a loan program for types $(\widehat{\rho}, \overline{\rho}]$. The following result establishes that the loan coverage increases in loan size as well as loan duration.

Proposition 7. *$\widehat{\rho}$ is decreasing in both M and T.*

Proof: We have $\dfrac{\partial \widehat{\rho}}{\partial M} = - \left. \dfrac{\frac{\partial \overline{r}(\rho,\delta,M)}{\partial M}}{\frac{\partial \overline{r}(\rho,\delta,M)}{\partial \rho}} \right|_{\rho=\widehat{\rho}}$. From Lemma 4, the derivatives in the numerator and the denominator are both positive, and therefore $\frac{\partial \widehat{\rho}}{\partial M} < 0$. Finally, since $\overline{r}(\rho)$ is increasing in ρ, and right hand side of equation (12) is decreasing in T, $\widehat{\rho}$ is also decreasing in T.$\|$

[9] Formally, for any $\delta < 1/(1+R)$, $\exists\, \widehat{\rho} > 0$ such that $\forall \rho > \widehat{\rho}$, $\overline{r}(\rho) > 1/(T-1)$. This is because there is a unique solution $\widehat{\rho}$ to $\overline{r}(\rho) = \frac{1}{T-1}$. Since $\overline{r}(\rho)$ is strictly increasing in ρ (Lemma 4), the result follows.

The previous section showed that under social-incentive-based lending, the loan coverage *decreases* in duration T, and the coverage of long-duration loans *decreases* in loan size M as well. In cases other than long-duration loans, the effect of loan size on coverage is ambiguous. The result above shows that, in contrast, under individual lending, a higher project size M *increases* coverage unambiguously, and coverage also *increases* in the length T of the program.

5.2.4 Individual loans and social cooperation incentives

We have considered above the repayment incentives of types with social capital. To complete the analysis, we also need to show that types with social capital remain so even when they receive individual liability loans. In other words, we need to show that types with social insurance do not have an incentive to deviate from the insurance arrangement under an incentive compatible individual loan program. The following result establishes this.

Proposition 8. *An incentive compatible individual loan program does not alter social cooperation in risk sharing.*

Next, we consider the repayment incentive for types who do not participate in social risk-sharing. These types therefore do not have any social capital and cannot be covered at all by a loan program that depends on social sanctions for enforcement of repayment. However, as we show below, an individual lending program can cover these types as well.

5.3 Repayment Incentives for Types without Social Capital

Social incentives are of course unavailable for types who do not participate in social risk-sharing. Here we show that individual loans can cover these types as well, but repayment requires satisfying a stricter condition than that for types with social capital. This also proves that social capital is helpful in supporting individual loans.

First, we establish the stricter incentive compatibility condition in section 5.3.1 below. We then show, in section 5.3.2, that whenever some (but not all) types without social capital are covered by an individual loan program, coverage is increasing in both loan size and loan duration.

5.3.1 The incentive compatibility condition and comparison with the condition under social capital

The result below establishes the incentive compatibility condition for types not covered by social insurance.

Proposition 9. *Consider a type $\rho \in (0, \rho_{\min})$, and an individual loan program (M, T). The following condition is necessary and sufficient for the loan program to be incentive compatible for type ρ:*

$$\delta\left(\mathrm{e}^{M(1+R)\rho} - 1\right)(p + (1-p)\mathrm{e}^{\rho}) - \mathrm{e}^{\rho}\left(\mathrm{e}^{M(1+r)\rho} - 1\right) \geqslant 0 \tag{13}$$

for some $r \geqslant \dfrac{1}{T-1}$.

The proof proceeds through the following result, which is also of interest on its own. The result shows that the incentive compatibility condition is stricter for types without social capital compared to types with social capital.

Lemma 5. ***(Stricter incentive compatibility condition for types without social capital)*** *If condition* (13) *holds, this implies condition* (10) *holds as well, but the reverse is not true. In other words, the incentive compatibility condition for an individual loan program is stricter for types without social capital.*

Proof: Let $\phi(\mathrm{p}) \equiv \delta(\mathrm{e}^{M(1+R)\rho} - 1)(\mathrm{p} + (1-\mathrm{p})\mathrm{e}^{\rho}) - \mathrm{e}^{\rho}(\mathrm{e}^{M(1+r)\rho} - 1)$. This is simply the expression on the left hand side of condition (13). Note that

$$\phi(0) = \mathrm{e}^{\rho}\big(1 - \delta + \delta \mathrm{e}^{\,M(1+R)\rho} - \mathrm{e}^{\,M(1+r)\rho}\big).$$

Thus $\phi(0) \geqslant 0$ is the same as condition (10). Now, $\phi'(\mathrm{p}) = -\delta(\mathrm{e}^{\rho} - 1)(\mathrm{e}^{M(1+R)\rho} - 1) < 0$. Therefore $\phi(\mathrm{p}) \geqslant 0$ implies $\phi(0) > 0$, but $\phi(0) \geqslant 0$ does not imply $\phi(\mathrm{p}) \geqslant 0$. This proves that the incentive compatibility condition for an individual loan program is stricter for types without social capital.‖

Let us now outline the proof of Proposition 9. The formal proof is in the appendix. First, note that for an agent who is not socially insured, income is either 0 or 1. It is then easy to see that the deviation gain is highest if the income is 0 - the gain in utility from retaining the entire return of $M(1 + R)$ (rather than repaying $M(1 + r)$ and retaining $M(R - r)$) is the highest in this case. Incentive compatibility then requires this gain to be lower than the future loss at every $t < T$. As before, the transfers at T ensure that repayment is incentive compatible at T.

Now, if we consider the incentive compatibility condition at $t = T - 1$, we get condition (13). This establishes the necessity of the condition. This condition would also be sufficient if the loss from deviation is higher for earlier deviations (i.e. for deviations at $t < T - 1$). In this case, clearly, if incentive compatibility holds at $T - 1$, it also holds for all earlier periods. It turns out that loss from earlier deviations is higher if and only if condition (10) holds. But Lemma 5 above shows that condition (13) implies condition (10). Therefore condition (13) is also sufficient for incentive compatibility.

As in the case of types with social capital, the **incentive compatibility condition can be written more compactly as follows.** A project (M, R, T) can be supported by an incentive compatible individual lending program for a type $\rho \in (0, \rho_{\min}]$ if and only if

(14)
$$\widetilde{r}(\rho) \geqslant \frac{1}{T-1},$$

where

(15)
$$\widetilde{r}(\rho) = \frac{1}{M\rho} \ln\Big[(1-\mathrm{p})\delta \mathrm{e}^{\,(1+M(1+R))\rho} + \big(1 - (1-\mathrm{p})\delta\big)\mathrm{e}^{\rho} + \mathrm{p}\delta\big(\mathrm{e}^{\,M(1+R)\rho} - 1\big)\Big] - \frac{M+1}{M}.$$

5.3.2 Comparative statics of coverage

Note that the constraint obviously becomes more lax as T increases (so that the right hand side decreases). The next Lemma shows that $\widetilde{r}(\rho)$ is increasing in M.

Lemma 6. *$\widetilde{r}(\rho)$ is increasing in M.*

If the inequality $\widetilde{r}(\rho) \geqslant \frac{1}{T-1}$ is satisfied for all $\rho \in (0, \rho_{\min}]$, clearly further increase in M or T has no impact. Also, if the inequality is satisfied for no $\rho \in (0, \rho_{\min}]$, an increase in M or T would reduce the extent by which the right hand side exceeds the left hand side,but the constraint still might not hold for any $\rho \in (0, \rho_{\min}]$. However, suppose the inequality is satisfied for at least some values of $\rho \in (0, \rho_{\min}]$. As T increases, or M increases, the inequality is now satisfied for further values of $\rho \in (0, \rho_{\min}]$. This shows that, in the appropriate case, coverage is increasing in both loan size and loan duration.

5.4 Some Examples

Let us consider some examples of coverage under an individual lending program. From sections 5.2 and 5.3, we know that the incentive compatibility condition for a type ρ is given by

$$\begin{cases} \overline{r}(\rho) \geqslant \frac{1}{T-1} & \text{for } \rho \geqslant \rho_{\min}\text{, i.e. types with social capital,} \\ \widetilde{r}(\rho) \geqslant \frac{1}{T-1} & \text{for } \rho < \rho_{\min}\text{, i.e. types without social capital,} \end{cases}$$

where $\overline{r}(\rho)$ is given by equation 11, and $\widetilde{r}(\rho)$ is given by equation (15).

Figures 2 and **??** below show the functions $\overline{r}(\rho)$ and $\widetilde{r}(\rho)$ with ρ on the horizontal axis, and also show the line $1/(T-1)$. For any type $\rho \geqslant \rho_{\min}$ for which the curve $\overline{r}(\rho)$ is above the line the line $1/(T-1)$ can be covered by an individual lending program. Similarly, for any type $\rho < \rho_{\min}$ for which the curve $\widetilde{r}(\rho)$ is above the line the line $1/(T-1)$ can be covered. Note that therefore only the solid parts of the functions depicted are the relevant parts. In Figure 2, all types $[0, \overline{\rho}]$ can be covered by an individual loan - the relevant parts of both $\overline{r}(\rho)$ and $\widetilde{r}(\rho)$ are above $1/(T-1)$. In Figure 3, some types without social capital cannot be covered.

Note also that if we choose a much smaller T, $1/(T-1)$ would rise above even $\overline{r}(\rho)$, so that individual loans cannot be offered. As noted above (in section 6) for very short-duration loans, individual liability does not work since it is impossible for the savings program to accumulate enough to deter deviation.

Finally, note that in the case of types covered by social insurance, $\bar{r}(\rho)$ is increasing in ρ (Lemma 4) and therefore, if a loan program is incentive compatible for a type ρ, it is also incentive compatible for all higher types. However, in the case of types excluded from social insurance, $\tilde{r}(\rho)$ is non-monotonic in ρ and therefore it is possible that a loan program is incentive compatible for some types but not higher types. Figure 3 shows such an example.

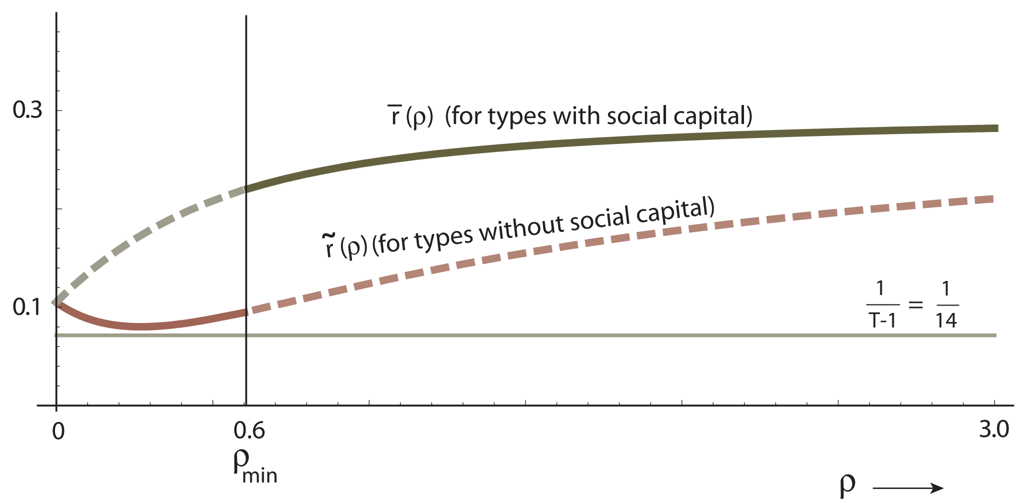

Figure 2: The figure is drawn for parameter values $M = 3, R = 0.3, T = 15$, and p $= 0.5, \delta = 0.85$. Note that both $\bar{r}(\rho)$ and $\tilde{r}(\rho)$ (the solid parts are the relevant parts) exceed $1/(T-1)$. Therefore all types–those with social capital and those without–are covered by the individual loan program.

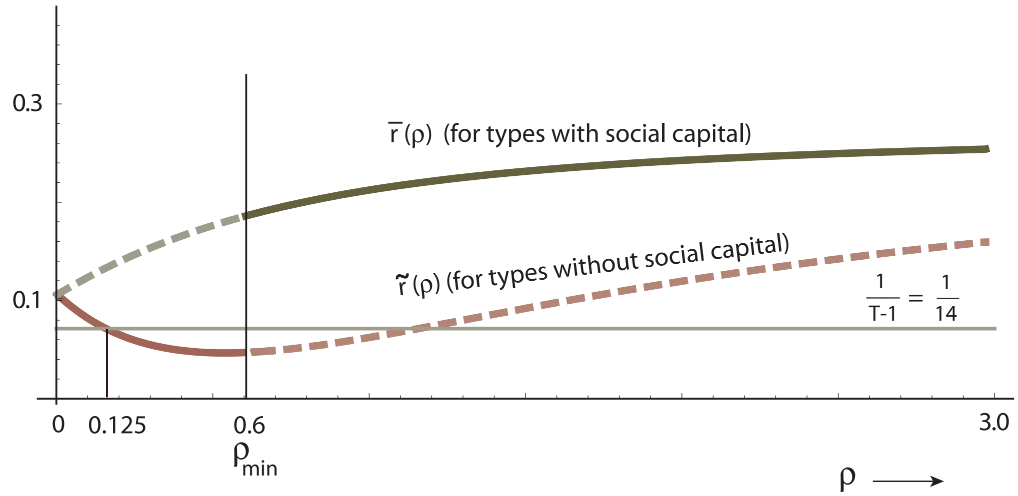

Figure 3: The only change from above is that now $M = 2$. All types with social capital are still covered by the individual loan program, while out of the types without social capital (types in the interval $(0, 0.6)$) only types $(0, 0.125)$ are covered.

6 Comparing Social and Individual Incentives

The analysis above shows that individual liability loans can cover all types including those who do not participate in social risk sharing, and these loans have the desirable property that coverage is increasing in both duration T and loan size M. Therefore offering longer-duration loans or larger loans present no loss of coverage.

These properties contrast with the coverage under loans with repayment enforced through social penalties. Such loans cannot cover types who do not participate in social risk sharing, and do not even cover all types who do participate in social risk sharing. The coverage for these loans is decreasing in duration, making long-duration loans problematic. Further, the impact of loan size on coverage is ambiguous in general, while for long-duration loans, loan coverage decreases in loan size.

The only clear advantage of loans based on social penalties arises for loans of very short duration. For very small values of T, individual loans do not work: neither the incentive compatibility condition in corollary 1 (for types with social capital) nor the condition

(14) (for types without social capital) can be satisfied. Basically, for very short-duration loans, individual liability does not work since it is impossible for the savings program to accumulate enough to deter deviation. Loans based on social penalty have exactly the opposite property: since the punishment can last even after the loan program is over, shorter duration reduces deviation payoff without limiting the punishment - so social penalties are very effective when the duration is very short.

7 Conclusion

Pioneered by the Grameen Bank, joint liability loans have generated much academic interest. However, since 2002, the Grameen Bank offers purely individual liability loans. Other well known micro finance institutions such as Bolivia's BancoSol have similarly moved from group loans to individual loans across several loan categories.

In this paper we compare the properties of loans based on social sanctions and individual liability loans for enforcing repayment. In discussing enforcement properties of such loans, the literature typically assumes that a defaulter can be punished by others using exogenously available social sanctions. However, social penalties typically arise from long term social interactions. We show that once we include such interactions in the model, the conclusions about the power of such sanctions can be quite different. Basically, social cooperation itself is sustained in equilibrium by ensuring that the benefit from cooperation exceeds the one-shot deviation payoff plus the post-deviation punishment payoff in future. Adding a loan program that relies on social penalties to engender repayment adds to the one-shot deviation payoff of an agent (who can now deviate from both the loan program and the underlying social cooperation game) but the post-deviation punishment payoff is still that in the underlying game. Thus adding a loan program dilutes the power of social penalties. In our complete information set-up, agents who would potentially deviate after getting a loan are simply excluded from the loan program. Thus deviation does not occur in equilibrium, and the only issue is the coverage of the loan program.

We show that a simple scheme of individual loans augmented by a compulsory illiquid savings provision (such schemes are used by the Grameen Bank) has several advantages over loans based on social penalties. First, such individual loans can cover a much greater proportion of types - and could even cover types who do not participate in social risk sharing (i.e. types for which social penalties are not available at all). Second, while the coverage of a loan program based on social sanctions *decreases* with the duration of the loan program, the coverage under an incentive compatible individual loan program *increases* with duration. Thus individual loan programs are much better suited for longer lasting projects. Third, under individual loans, the coverage is also unambiguously increasing in loan size, while under social sanctions the coverage in general changes in an ambiguous manner as loan size changes. An exception is the special case of long-duration loans, in which case the coverage is *decreasing* in loan size.

Further, interestingly, incentive compatible individual loans fit quite well with social cooperation. Such loans do not alter the incentive of agents to cooperate socially. More importantly, social cooperation helps raise the coverage of individual liability loans because the deviation gain is lower for an agent who participates in social risk-sharing. Thus social capital has very different roles to play under social sanction based loans and individual liability loans. Making use of social capital as a punishment mechanism can exclude a significant fraction of types from the coverage of a loan program. However, when not used as a penalty to enforce repayment, the same social capital can work as a positive factor in the background that helps raise the coverage of an individual loan program.

In essence, our results show that other than loans of a very short duration, individual liability loans can better deter strategic default compared to loans based on social sanction. In particular, micro-credit programs often advance loans of small size and long duration. Individual liability loans with appropriate savings requirements would have larger coverage for such loans compared to a loan program that relies on social penalties for enforcing repayment. This provides an explanation for the above mentioned move towards individual liability in leading micro-credit programs.

Appendix: Proofs

A.1 Proof of Proposition 1

Let us first note a case of Jensen's inequality that is used in this and a few other proofs. Let $f(x) = \mathrm{e}^{\rho x}$. Since $f(\cdot)$ is convex in x, Jensen's inequality implies $\mathrm{p}f(1) + (1-\mathrm{p})f(0) > f(\mathrm{p})$. This implies the following:

$$\mathrm{p}\mathrm{e}^{\rho} + (1-\mathrm{p}) - \mathrm{e}^{\mathrm{p}\rho} > 0. \qquad \textbf{(Jensen's Inequality)} \tag{A.1}$$

From equation (2), the value of $\rho_{\min}$ is given by $\dfrac{L}{G} = \dfrac{1-\delta}{\delta}$. Now,

$$\frac{L}{G} = \frac{(1-\mathrm{p})\mathrm{e}^{(1+\mathrm{p})\rho} + \mathrm{p}\mathrm{e}^{\mathrm{p}\rho} - \mathrm{e}^{\rho}}{\mathrm{e}^{\rho} - \mathrm{e}^{\mathrm{p}\rho}}.$$

Therefore

$$\frac{\partial L/G}{\partial \rho} = \frac{(1-\mathrm{p})\mathrm{e}^{(1+\mathrm{p})\rho}((1-\mathrm{p}) + \mathrm{p}\mathrm{e}^{\rho} - \mathrm{e}^{\mathrm{p}\rho})}{(\mathrm{e}^{\rho} - \mathrm{e}^{\mathrm{p}\rho})^2} > 0,$$

where the last inequality follows from (A.1). Finally, it is easy to check that L/G goes to 0 as $\rho \to 0$,[10] and increases without bound as ρ increases. Therefore $\rho_{\min}$ is positive and unique, and $L \gtreqless G$ according as $\rho \gtreqless \rho_{\min}$.

Finally, note that L/G is independent of δ, but the right hand side of the equation for $\rho_{\min}$ (given by $\dfrac{L}{G} = \dfrac{1-\delta}{\delta}$) decreases in δ. Since L/G is increasing in ρ, it is immediate that $\rho_{\min}$ is decreasing in δ. This completes the proof.$\|$

A.2 Proof of Lemma 1

Using the value of $\mathbb{TL}_t(M\,;\rho)$ from equation (5),

$$\begin{aligned}
\mathbb{TL}_t(M\,;\rho) - \mathbb{TL}_{t-1}(M\,;\rho) &= \frac{\delta}{1-\delta}\left((\delta^{T-t+1} - \delta^{T-t})\mathbb{L}(M\,;\rho) + \left(\delta^{T-t} - \delta^{T-t+1}\right)\mathbb{L}(0\,;\rho)\right) \\
&= \delta^{T-t+1}\left(\mathbb{L}(0\,;\rho) - \mathbb{L}(M\,;\rho)\right). \qquad \text{(A.2)}
\end{aligned}$$

First, $\mathbb{L}(0\,;\rho) = u(\mathrm{p}\,;\rho) - (\mathrm{p}u(1\,;\rho) + (1-\mathrm{p})u(0\,;\rho)) > 0$ (since $u(\cdot\,;\rho)$ is concave, the inequality follows from Jensen's inequality).

[10] As $\rho \to 0$, the derivative of the numerator with respect to ρ goes to 0, while that of the denominator goes to $(1-p)$. Using L'Hospital's rule, the result is immediate.

Now if $\mathbb{L}(M\,;\rho) \leqslant 0$, the right hand side of equation (A.2) is strictly positive, and the proof is complete. Next, suppose $\mathbb{L}(M\,;\rho) > 0$. Using the expression for $u(\cdot\,;\rho)$ from equation (1), and differentiating with respect to M,

$$\begin{aligned}
\mathbb{L}_M(M\,;\rho) &= R\,\mathrm{e}^{-(\mathrm{p}+MR)\rho} - (1+R)\left(\mathrm{p}\,\mathrm{e}^{-(1+M+MR)\rho} + (1-\mathrm{p})\,\mathrm{e}^{-(M+MR)\rho}\right) \\
&= -\mathrm{e}^{-(1+M+MR)\rho}\left((1-\mathrm{p})\,\mathrm{e}^{\rho}+\mathrm{p}\right) \\
&\qquad + R\left(\mathrm{e}^{-(\mathrm{p}+MR)\rho} - \mathrm{p}\,\mathrm{e}^{-(1+M+MR)\rho} - (1-\mathrm{p})\,\mathrm{e}^{-(M+MR)\rho}\right).
\end{aligned}$$

Note that the coefficient of R in the second term on the right hand side is exactly $\rho\,\mathbb{L}(M\,;\rho)$. Therefore,

$$\mathbb{L}_M(M\,;\rho) = -\mathrm{e}^{-(1+M+MR)\rho}\left((1-\mathrm{p})\,\mathrm{e}^{\rho}+\mathrm{p}\right) - \rho\,R\,\mathbb{L}(M\,;\rho).$$

Since $\mathbb{L}(M\,;\rho) > 0, \mathbb{L}_M(M\,;\rho) < 0$. Therefore $\mathbb{L}(0\,;\rho) - \mathbb{L}(M\,;\rho) > 0$. Therefore, the right hand side of equation (A.2) is again strictly positive. This completes the proof.$\|$

A.3 Proof of Lemma 2

Let $r_G(M\,;\rho) \equiv \dfrac{\mathbb{G}(M\,;\rho)}{u(\mathrm{p}\,;\rho)}$ and $r_L(0\,;\rho) \equiv \dfrac{\mathbb{L}(0\,;\rho)}{u(\mathrm{p}\,;\rho)}$. Clearly, $\dfrac{\mathbb{L}(0\,;\rho)}{\mathbb{G}(M\,;\rho)} = \dfrac{r_L(0\,;\rho)}{r_G(M\,;\rho)}$. We now show that $r_G(M\,;\rho)$ is strictly decreasing in ρ and $r_L(0\,;\rho)$ is strictly increasing in ρ.

Consider the derivatives of $r_L(0\,;\rho)$ and $r_G(M\,;\rho)$ with respect to ρ.

$$\frac{\partial r_L(0\,;\rho)}{\partial\rho} = \left(\frac{\delta}{1-\delta}\right)\left(\frac{\mathrm{p}\mathrm{e}^{(\mathrm{p}-1)\rho}}{(\mathrm{e}^{\mathrm{p}\rho}-1)^2}\right)\left(\mathrm{p}\mathrm{e}^{\rho} + (1-\mathrm{p}) - \mathrm{e}^{\mathrm{p}\rho}\right).$$

The first two terms are obviously strictly positive. The last term is positive from inequality (A.1). Therefore $\frac{\partial r_L(0\,;\rho)}{\partial\rho} > 0$. The proof now proceeds through the following Lemma.

Lemma 7. $\frac{\partial r_G(M\,;\rho)}{\partial\rho} < 0$ *for any* $M > 0$ *and* $R \geqslant 0$.

Proof: $\dfrac{\partial r_G(M\,;\rho)}{\partial\rho} = -\dfrac{\mathrm{e}^{-(1+M+LR)\rho}}{(\mathrm{e}^{\mathrm{p}\rho}-1)^2}Z(M,\rho)$, where

$$Z(M,\rho) = -MR\mathrm{e}^{(M+1)\rho} - (1+M+LR)\mathrm{e}^{2\mathrm{p}\rho} + (\mathrm{p}+MR)\mathrm{e}^{(1+\mathrm{p}+M)\rho} + ((1-\mathrm{p})+M+LR)\mathrm{e}^{\mathrm{p}\rho}.$$

It follows that if we can show $Z(M,\rho) > 0$ for any $M > 0$ and any $R \geqslant 0$, that would establish the result.

First, fix any $R \geqslant 0$.

Next, note that $Z(0, \rho) = \mathrm{e}^{\mathrm{p}\rho}(-\mathrm{e}^{\mathrm{p}\rho} + \mathrm{p}\mathrm{e}^{\rho} + (1-\mathrm{p})) > 0$, where the inequality follows from inequality (A.1).

Next, consider the derivative of Z with respect to M, denoted $Z_M(M, \rho)$.

$$Z_M(M, \rho) = R(\rho M + 1)(\mathrm{e}^{\mathrm{p}\rho} - 1)\mathrm{e}^{(1+M)\rho} + \mathrm{p}\rho\mathrm{e}^{(1+M+\mathrm{p})\rho} - (R+1)\mathrm{e}^{\mathrm{p}\rho}(\mathrm{e}^{\mathrm{p}\rho} - 1).$$

Claim: Given any $R \geqslant 0$, $Z_M(M, \rho) > 0$ for any $M > 0$ and any $\rho > 0$.

Proof of Claim: M enters the first two terms of Z_M which are both positive and strictly increasing in M. Therefore the derivative of Z_M with respect to M is strictly positive, i.e. $Z_{MM}(M, \rho) > 0$. Now, if we can show that Z_M is positive at $M = 0$, the proof would be complete.

$$Z_M(0, \rho) = R(\mathrm{e}^{\mathrm{p}\rho} - 1)\mathrm{e}^{\rho} + \mathrm{p}\rho\mathrm{e}^{(1+\mathrm{p})\rho} - (R+1)\mathrm{e}^{\mathrm{p}\rho}(\mathrm{e}^{\mathrm{p}\rho} - 1).$$

Consider the value of $Z_M(0, \rho)$ at $\rho = 0$. Clearly, $Z_M(0, 0) = 0$. Also, the derivative of $Z_M(0, \rho)$ with respect to ρ is given by

$$Z_{M\rho}(0, \rho) = \left(\mathrm{e}^{\rho} - \mathrm{e}^{\mathrm{p}\rho}\right)\left(\mathrm{e}^{\mathrm{p}\rho} - 1\right) R + \mathrm{e}^{\mathrm{p}\rho}\left(\mathrm{p}\rho\mathrm{e}^{\rho} - \mathrm{e}^{\mathrm{p}\rho} + 1\right).$$

The first term is obviously positive. In the second term, the coefficient of $\mathrm{e}^{\mathrm{p}\rho}$ is $Y(\rho) \equiv \mathrm{p}\rho\mathrm{e}^{\rho} - \mathrm{e}^{\mathrm{p}\rho} + 1$. This is 0 at $\rho = 0$, and $Y'(\rho) = \mathrm{p}(\mathrm{e}^{\rho} + \rho\mathrm{e}^{\rho} - \mathrm{e}^{\mathrm{p}\rho}) > 0$ for any $\mathrm{p} \in [0, 1]$. Therefore $Y(\rho) > 0$ for $\rho > 0$. It follows that $Z_{M\rho}(0, \rho) > 0$ for $\rho > 0$. Coupled with the fact that $Z_M(0, 0) = 0$, this implies that $Z_M(0, \rho) > 0$ for any $\rho > 0$. In turn, this, coupled with the fact that $Z_{MM}(M, \rho) > 0$, implies that $Z_M(M, \rho) > 0$ for any $M > 0$ and any $\rho > 0$. This completes the proof of the claim.

To continue with the proof of Lemma 7, we now know that given any $R \geqslant 0$, the function Z is positive at $M = 0$ and strictly increasing in M. It follows that $Z(M, \rho) > 0$ for all strictly positive values of M and ρ. Note also that beyond assuming $R \geqslant 0$, we have not restricted the value of R any further. This completes the proof of Lemma 7.‖

To continue with the proof of Lemma 2, we have now established that $r_G(M\,;\rho)$ is strictly decreasing in ρ and $r_L(0\,;\rho)$ is strictly increasing in ρ. Therefore the stated ratio is strictly increasing in ρ.‖

A.4 Proof of Lemma 3

Step 1 Using the form of the utility function (from equation (1)),

$$\frac{\mathbb{L}(M\,;\rho)}{\mathbb{G}(M\,;\rho)} = (1-\mathrm{p})\frac{(\mathrm{e}^{\,\rho}-1)}{\mathrm{e}^{\,(1-\mathrm{p}+M)\rho}-1} - 1. \tag{A.3}$$

ρ appears only in the coefficient of $(1-\mathrm{p})$, which is of the form $\dfrac{\mathrm{e}^{a\rho}-1}{\mathrm{e}^{b\rho}-1}$ where a, b are positive real numbers with $b \gtreqless a$ according as $M \gtreqless \mathrm{p}$. The following Lemma establishes some useful properties of such a ratio.

Lemma 8. *Let $Z_1(x) \equiv \mathrm{e}^{x\rho}-1$. Let a, b be positive real numbers. Then $\dfrac{\partial}{\partial\rho}\left(\dfrac{Z_1(a)}{Z_1(b)}\right) \lesseqgtr 0$ as $b \gtreqless a$. Further, if $b > a$, $\dfrac{\partial^2}{\partial\rho^2}\left(\dfrac{Z_1(a)}{Z_1(b)}\right) > 0$.*

Proof: Let $Z_2(x) \equiv \dfrac{x\mathrm{e}^{\,x\rho}}{\mathrm{e}^{\,x\rho}-1}$ and $Z_3(x) \equiv \dfrac{x^2\mathrm{e}^{\,x\rho}}{(\mathrm{e}^{\,x\rho}-1)^2}$. Then

$$\frac{\partial}{\partial\rho}\left(\frac{Z_1(a)}{Z_1(b)}\right) = \frac{Z_1(a)}{Z_1(b)}\Big(Z_2(a)-Z_2(b)\Big), \tag{A.4}$$

$$\frac{\partial^2}{\partial\rho^2}\left(\frac{Z_1(a)}{Z_1(b)}\right) = \frac{Z_1(a)}{Z_1(b)}\Big(Z_2(a)-Z_2(b)\Big)^2 + \frac{Z_1(a)}{Z_1(b)}\Big(Z_3(b)-Z_3(a)\Big). \tag{A.5}$$

Now,

$$\frac{\partial}{\partial x}Z_2(x) = \frac{\mathrm{e}^{\,x\rho}}{(\mathrm{e}^{\,x\rho}-1)^2}\Big(\mathrm{e}^{\,x\rho}-(1+x\rho)\Big) \;>\; 0,$$

and

$$\frac{\partial}{\partial x}Z_3(x) = \frac{x\mathrm{e}^{\,x\rho}}{(\mathrm{e}^{\,x\rho}-1)^2}\left(x\rho + 2\left(1-\frac{x\rho}{\mathrm{e}^{\,x\rho}-1}\right)\right) \;>\; 0,$$

where both inequalities follow from the fact that $\mathrm{e}^{\,x\rho} > 1+x\rho$. Therefore $Z_2(a)-Z_2(b) \lesseqgtr 0$ as $b \gtreqless a$ and for $b > a$, and $Z_3(b)-Z_3(a) > 0$. This completes the proof of Lemma 8.‖

We now continue with the proof of Lemma 3. The result above shows that for $M < \mathrm{p}$, the ratio $\mathbb{L}(M\,;\rho)/\mathbb{G}(M\,;\rho)$ increases in ρ. Further, for $b < a$, $\lim_{\rho\to\infty}\frac{Z_1(a)}{Z_1(b)} = \infty$, which implies that for $M < \mathrm{p}$, the ratio increases without bound as ρ increases.

For $M > \mathrm{p}$, the result shows that $\mathbb{L}(M\,;\rho)/\mathbb{G}(M\,;\rho)$ decreases in ρ , and the derivative with respect to ρ is increasing - i.e. the derivative becomes less negative as ρ increases.

Further, for $b > a$, $\lim_{\rho\to\infty} \frac{Z_1(a)}{Z_1(b)} = 0$. It follows that $\frac{\partial}{\partial\rho}\left(\frac{Z_1(a)}{Z_1(b)}\right) \to 0$ as ρ increases. Therefore for $M > \mathrm{p}$, the derivative of $\mathbb{L}(M\,;\rho)/\mathbb{G}(M\,;\rho)$ with respect to ρ falls to zero as ρ increases.

Step 2 First, consider the case $M \geqslant \mathrm{p}$. The left hand side of equation (6) is $\frac{\delta}{1-\delta}$ times a convex combination of the two ratios $\mathbb{L}(0\,;\rho)/\mathbb{G}(M\,;\rho)$ and $\mathbb{L}(M\,;\rho)/\mathbb{G}(M\,;\rho)$. From Lemma 2, $\mathbb{L}(0\,;\rho)/\mathbb{G}(M\,;\rho)$ is increasing in ρ. Further, from step 1, for $M < \mathrm{p}$, $\mathbb{L}(M\,;\rho)/\mathbb{G}(M\,;\rho)$ is increasing in ρ and for $M = \mathrm{p}$, $\mathbb{L}(M\,;\rho)/\mathbb{G}(M\,;\rho)$ is a constant function of ρ. Therefore for $M \leqslant \mathrm{p}$, the left hand side increases in ρ.

Now, as $\rho \to 0$, $\frac{\mathbb{L}(M\,;\rho)}{\mathbb{G}(M\,;\rho)} \to \frac{-M}{1-\mathrm{p}+M} < 0$, and $\frac{\mathbb{L}(0\,;\rho)}{\mathbb{G}(M\,;\rho)} \to 0$. Therefore, for ρ close to 0, the left hand side of equation (6) is below 1. Further, the left hand side is continuous in ρ and, as shown in step 1, rises without bound as ρ increases. Therefore there is a unique $\widehat{\rho}_t$ that solves equation (6).

Step 3 Next, consider the case $M > \mathrm{p}$. As ρ increases, $\mathbb{L}(0\,;\rho)/\mathbb{G}(M\,;\rho)$ increases without bound as before, and $\mathbb{L}(M\,;\rho)/\mathbb{G}(M\,;\rho)$ is negative, decreases at a decreasing rate, and is bounded below by -1. Further, the derivative of this ratio is the most negative at $\rho = 0$ at which point is equals $-\frac{1}{2}\frac{(M-\mathrm{p})(1-\mathrm{p})}{M-\mathrm{p}+1} > -\frac{1}{2}$. Therefore the left hand side of equation (6), if it decreases at all, is decreasing for values of ρ close to zero, but there exists $\rho_* \geqslant 0$ such that the left hand side of equation (6) is increasing in ρ for $\rho > \rho_*$. Since the left hand side is negative at $\rho = 0$, it is also negative at $\rho = \rho_*$. By the same argument as in step 2, it follows that there is a unique $\widehat{\rho}_t > \rho_*$ that solves equation (6).

Step 4 Finally, let us prove the second part of the Lemma. From the above we know that the left hand side of equation (6) rises from below 1. It then follows that $\mathbb{G}(M\,;\rho) \gtreqless \mathbb{TL}_t(M\,;\rho)$ according as $\rho \gtreqless \widehat{\rho}_t$ which in turn implies $V_t(T\,;\rho) \gtreqless V_t^D(T\,;\rho)$ according as $\rho \gtreqless \widehat{\rho}_t$.$\|$

A.5 Proof of Proposition 3

For ease of exposition, let us change the notation slightly and write the total program duration as an explicit argument of total loss - i.e. the total loss starting period t associated with a loan program of duration T is now written as $\mathbb{TL}_t(T, M; \rho)$. Further, let $\widehat{\rho}_t(T)$ denote the solution to equation 6 given duration T.

From Lemma 1, we know that given any program of duration T, and any $t \leqslant T$,

$$\mathbb{TL}_t(T, M; \rho) > \mathbb{TL}_{t-1}(T, M; \rho).$$

Further, it is clear that $\mathbb{TL}_t(T, M; \rho) = \mathbb{TL}_{t+1}(T+1, M; \rho)$. Therefore

$$\mathbb{TL}_1(T, M; \rho) = \mathbb{TL}_2(T+1, M; \rho) > \mathbb{TL}_1(T+1, M; \rho).$$

From Lemma 3, $\mathbb{TL}_1(T, M; \rho) = \mathbb{G}(M)$ at $\rho = \widehat{\rho}_1(T)$. It follows from above that at $\rho = \widehat{\rho}_1(T)$, $\mathbb{TL}_1(T+1, M; \rho) < \mathbb{G}(M)$. Now, the proof of Lemma 3 shows that for any given T, $\mathbb{TL}_1(T, M; \rho) \lesseqgtr \mathbb{G}(M)$ as $\rho \lesseqgtr \widehat{\rho}_1(T)$. It follows that $\widehat{\rho}_1(T+1) > \widehat{\rho}_1(T)$. Finally, using Proposition 2, this implies that $\rho^*(T+1) > \rho^*(T)$.$\|$

A.6 Proof of Proposition 4

In what follows, for economy of notation, we drop the argument and write $\rho^*(T)$ simply as ρ^*.

Let β denote the left hand side of equation (6).

1. As shown in the proof of Lemma 3, β is increasing in ρ at the solution $\widehat{\rho}_t$, and therefore at ρ^* (which is $\widehat{\rho}_t$ at $t = 1$). Let β_x denote the partial derivative of β with respect to x. The derivative of ρ^* with respect to M is simply $(-\beta_M/\beta_\rho)$ evaluated at ρ^*. Therefore if we can show that β is decreasing in M, this will establish that ρ^* is increasing in M.

2. The derivative of β (which is the left hand side of equation (6)) with respect to M is given by

$$\frac{\partial \beta}{\partial M} = \frac{\delta}{1-\delta}\left[(1-\delta^{T-t})\frac{\partial}{\partial M}\left(\frac{\mathbb{L}(M; \rho)}{\mathbb{G}(M; \rho)}\right) + \delta^{T-t}\frac{\partial}{\partial M}\left(\frac{\mathbb{L}(0; \rho)}{\mathbb{G}(M; \rho)}\right)\right].$$

3. $\frac{\mathbb{L}(M;\rho)}{\mathbb{G}(M;\rho)}$ is given by equation (A.3), from which it is clear that the ratio is decreasing in M.

4. Now consider the ratio $\frac{\mathbb{L}(0;\rho)}{\mathbb{G}(M;\rho)}$. Here only the denominator depends on M. Now,

$$\text{(A.6)} \qquad \frac{\partial}{\partial M}\mathbb{G}(M; \rho) = \mathrm{e}^{-MR\rho}\big((1+R)\mathrm{e}^{-(1+M)\rho} - R\mathrm{e}^{-\mathrm{p}\rho}\big).$$

Given finite R, the expression above is bounded below.[11] The expressions $\mathbb{L}(0\,;\rho)$ and $\mathbb{G}(M\,;\rho)$ are positive and bounded above. Therefore $\frac{\partial}{\partial M}\left(\frac{\mathbb{L}(0\,;\rho)}{\mathbb{G}(M\,;\rho)}\right)$ is bounded above.

5. Note that the expression for $\frac{\partial\beta}{\partial M}$ is a constant times the weighted sum of two partial derivative terms. Since $\delta < 1$, as T rises, the weight attached to the first of the two partial derivative terms rises to 1, and the weight attached to the second of the two partial derivative terms falls to 0. Then 3 and 4 above imply that for T large enough, $\frac{\partial\beta}{\partial M} < 0$. From 1, this implies that ρ^* increases in M, implying that coverage is decreasing in M.$\|$

A.7 Proof of Lemma 4

Let $M(1+R)\rho \equiv A$. Then $\bar{r}(\rho)$ can be written as $\phi(A)$, where

$$\phi(A) = (1+R)\frac{\ln[1-\delta+\delta \mathrm{e}^{A}]}{A} - 1.$$

Note that

$$\text{sign}\frac{\partial\phi(A)}{\partial A} = \text{sign}\frac{\partial\bar{r}(\rho)}{\partial\rho}. \tag{A.7}$$

Now, $\frac{\partial\phi(A)}{\partial A} = \frac{1+R}{A^2 B}H(A)$, where $H(A) \equiv A(B-(1-\delta)) - B\ln[B]$ and $B \equiv 1-\delta+\delta \mathrm{e}^{A}$. Thus $\text{sign}\frac{\partial\phi(A)}{\partial A} = \text{sign}\frac{\partial H(A)}{\partial A}$. At $A=0$, we have $B=1$. Therefore $H(0)=0$. Next, $\frac{\partial H(A)}{\partial A} = B-(1-\delta)+(A-\ln[B]-1)\frac{dB}{dA}$. Using $\frac{dB}{dA} = \delta \mathrm{e}^{A}$ and simplifying,

$$\frac{\partial H(A)}{\partial A} = \delta \mathrm{e}^{A}(A-\ln[B]) > 0.$$

Since $H(0)=0$ and $\frac{\partial H(A)}{\partial A} > 0$, it follows that $H(A) > 0$ for $A > 0$. Thus $\frac{\partial\phi(A)}{\partial A} > 0$ for $A > 0$. But $A > 0$ for $\rho > 0$. Using equation (A.7), this implies that $\frac{\partial\bar{r}(\rho)}{\partial\rho} > 0$ for $\rho > 0$.$\|$

A.8 Proof of Proposition 8

Define the following functions:

$$\begin{aligned}
\widehat{\mathbb{G}}(X\,;\rho) &\equiv u(1+X\,;\rho) - u(\mathrm{p}+X\,;\rho),\\
\widehat{\mathbb{L}}(X\,;\rho) &\equiv u(\mathrm{p}+X\,;\rho) - \Big(\mathrm{p}u(1+X\,;\rho) + (1-\mathrm{p})u(X\,;\rho)\Big).
\end{aligned}$$

[11] This can be verified easily by minimizing with respect to M. The minimized value is negative and finite.

Suppose a type participating in social insurance who receives and repays an individual loan contemplates deviation from social insurance in any period $t \leqslant T$. The immediate gain is $\widehat{\mathbb{G}}(M(R-r)\,;\rho)$ and the total future loss is given by

$$\widehat{\mathbb{TL}}_t(M\,;\rho) = \sum_{k=1}^{T-t-1} \delta^k \widehat{\mathbb{L}}(M(R-r)\,;\rho) + \delta^{T-t}\widehat{\mathbb{L}}(M(1+R)\,;\rho) + \frac{\delta^{T-t+1}}{1-\delta}\widehat{\mathbb{L}}(0\,;\rho).$$

Now, types $\rho \geqslant \rho_{\min}$ participate in social insurance where $\rho_{\min}$ is given by equation (2). Using current notation, this can be rewritten as

$$\widehat{\mathbb{G}}(0\,;\rho_{\min}) = \frac{\delta}{1-\delta}\widehat{\mathbb{L}}(0\,;\rho_{\min}). \tag{A.8}$$

If we can show that $\widehat{\mathbb{G}}(M(R-r)\,;\rho_{\min}) \leqslant \widehat{\mathbb{TL}}_t(M\,;\rho_{\min})$, that would imply that none of the types who participate in social insurance have an incentive to deviate from social insurance when the loan is introduced. Now,

$$\begin{aligned}\frac{\widehat{\mathbb{TL}}_t(M\,;\rho_{\min})}{\widehat{\mathbb{G}}(M(R-r)\,;\rho_{\min})} = \delta\left(\frac{1-\delta^{T-t-1}}{1-\delta}\right)\frac{\widehat{\mathbb{L}}(M(R-r)\,;\rho_{\min})}{\widehat{\mathbb{G}}(M(R-r)\,;\rho_{\min})}\\ + \delta^{T-t}\frac{\widehat{\mathbb{L}}(M(1+R)\,;\rho_{\min})}{\widehat{\mathbb{G}}(M(R-r)\,;\rho_{\min})} + \frac{\delta^{T-t+1}}{1-\delta}\frac{\widehat{\mathbb{L}}(0\,;\rho_{\min})}{\widehat{\mathbb{G}}(M(R-r)\,;\rho_{\min})}.\end{aligned}$$

Using the form of the utility function from equation (1), it can be easily verified that the following equalities hold:

$$\begin{aligned}\frac{\widehat{\mathbb{L}}(M(R-r)\,;\rho)}{\widehat{\mathbb{G}}(M(R-r)\,;\rho)} &= \frac{\widehat{\mathbb{L}}(0\,;\rho)}{\widehat{\mathbb{G}}(0\,;\rho)},\\ \frac{\widehat{\mathbb{L}}(M(1+R)\,;\rho)}{\widehat{\mathbb{G}}(M(R-r)\,;\rho)} &= \mathrm{e}^{\,-M(1+r)\rho}\frac{\widehat{\mathbb{L}}(0\,;\rho)}{\widehat{\mathbb{G}}(0\,;\rho)},\\ \frac{\widehat{\mathbb{L}}(0\,;\rho)}{\widehat{\mathbb{G}}(M(R-r)\,;\rho)} &= \mathrm{e}^{\,M(R-r)\rho}\frac{\widehat{\mathbb{L}}(0\,;\rho)}{\widehat{\mathbb{G}}(0\,;\rho)}.\end{aligned}$$

Using these, and using equation (A.8),

$$\begin{aligned}\frac{\widehat{\mathbb{TL}}_t(M\,;\rho_{\min})}{\widehat{\mathbb{G}}(M(R-r)\,;\rho_{\min})} &= \frac{1-\delta}{\delta}\left(\delta\left(\frac{1-\delta^{T-t-1}}{1-\delta}\right) + \delta^{T-t}\mathrm{e}^{\,-M(1+r)\rho} + \frac{\delta^{T-t+1}}{1-\delta}\mathrm{e}^{\,M(R-r)\rho}\right)\\ &= 1 + \delta^{T-t-1}\mathrm{e}^{\,-M(1+r)\rho}\left(1-\delta+\delta\mathrm{e}^{\,M(1+R)\rho} - \mathrm{e}^{\,M(1+r)\rho}\right).\end{aligned}$$

Therefore $\dfrac{\widehat{\mathbb{TL}}_t(M\,;\rho_{\min})}{\widehat{\mathbb{G}}(M(R-r)\,;\rho_{\min})} \geqslant 1$ if and only if $1-\delta+\delta\mathrm{e}^{\,M(1+R)\rho} - \mathrm{e}^{\,M(1+r)\rho} \geqslant 0$, which is the same condition as (10), which implies the rest. This completes the proof.‖

A.9 Proof of Proposition 9

Step 1: Loss and gain from deviation In each period an agent is supposed to repay the loan amount M and make a deposit of rM. Therefore the income gain from deviating in any period $t \leqslant T$ is $M(1+r)$. The gain in utility is therefore $\Delta u \equiv u(I + M(1+r)) - u(I)$ where $I = L(R-r)$ in the low income state and $I = 1 + L(R-r)$ in the high income state. Since $u(\cdot)$ is concave, Δu decreases in I: the same income gain leads to a higher utility gain in the low income state. It follows that the deviation gain is highest in the low income state. If the incentive to repay holds in this state, it also holds in the other (high income state).

Therefore the immediate gain from deviation we must consider is

$$\widetilde{\mathbb{G}} = u(L(1+R)\,;\rho) - u(L(R-r)\,;\rho).$$

Next, consider the loss from such deviation in future periods. If the deviation takes place in period $t < T$, then the agent does not get any further loans or lump sum payoffs. Therefore, at each of the next $T-t-1$ periods (i.e. for periods $t+1$ until period $T-1$) he loses $M(R-r)$ which is the amount he would have earned by conforming. Finally, the loss in period T is the final period payoff from conforming, given by $M(1+R)$.

Let

$$\widetilde{\mathbb{L}}(X) \equiv \mathrm{p}(u(1+X\,;\rho) - u(1\,;\rho)) + (1-\mathrm{p})(u(X\,;\rho) - u(0\,;\rho)).$$

As noted above, after deviation at t, the loss lasts for $T-t$ periods and the total loss is given by

$$\widetilde{\mathbb{TL}}_t = \delta\frac{1-\delta^{T-t-1}}{1-\delta}\widetilde{\mathbb{L}}(M(R-r)) + \delta^{T-t}\widetilde{\mathbb{L}}(M(1+R)).$$

As before, no deviation in the last period requires $(T-1)Mr \geqslant M$, implying that for any given r, we must have $r \geqslant 1/(T-1)$.

Step 2: Necessity Consider the incentive to deviate at period $T-1$ when there is only 1 period left. Clearly, a necessary condition for a loan to be incentive compatible is that the agent does not deviate at $T-1$. This condition is:

$$\widetilde{\mathbb{TL}}_{T-1} \geqslant \widetilde{\mathbb{G}}.$$

Using the form of the utility function, this is exactly the condition (13). This proves the necessity of condition (13).

Step 3: Sufficiency Next, we prove sufficiency.

$$\widetilde{\mathbb{TL}}_{t-1} - \widetilde{\mathbb{TL}}_t = \frac{\delta^{T-t}\,\mathrm{e}^{-(1+M+MR)\rho}\Big((1-\mathrm{p})\mathrm{e}^{\rho}+\mathrm{p}\Big)}{\rho}\Big(1-\delta+\delta\mathrm{e}^{\,M(1+R)\rho}-\mathrm{e}^{\,M(1+r)\rho}\Big).$$

Note that the above is positive if the term in the parenthesis at the end is positive. But this is the same as condition (10). From lemma 5, this condition holds automatically when condition (13) holds. Therefore if condition (13) holds, the total loss is increasing for earlier deviations. Therefore if incentive compatibility holds for deviation at $T-1$, it also holds for all earlier deviations. This proves that condition (13) is sufficient for the individual loan program to be incentive compatible. This completes the proof.$\|$

A.10 Proof of Lemma 6

Let $\psi(M) \equiv (1-\mathrm{p})\delta\mathrm{e}^{\,(1+M(1+R))\rho} + \big(1-(1-\mathrm{p})\delta\big)\mathrm{e}^{\rho} + \mathrm{p}\delta\big(\mathrm{e}^{\,M(1+R)\rho}-1\big)$. Note that

$$\psi'(M) = \delta\,\rho\,(1+R)\,\mathrm{e}^{\,M(1+R)\rho}(\mathrm{p}+(1-\mathrm{p})\mathrm{e}^{\rho}) > 0 \quad \text{and} \quad \psi''(M) = \rho\,(1+R)\,\psi'(M).$$

Now, $\widetilde{r}(\rho) = \dfrac{\ln[\psi(M)]}{M\,\rho} - \dfrac{M+1}{M}$. Therefore $\dfrac{\partial\widetilde{r}(\rho)}{\partial M} = \dfrac{1}{M^2}\xi(M)$, where

$$\xi(M) = 1 + \frac{M}{\rho}\frac{\psi'(M)}{\psi(M)} - \frac{\ln[\psi(M)]}{\rho}.$$

Clearly, $\operatorname{sign}\dfrac{\partial\widetilde{r}(\rho)}{\partial M} = \operatorname{sign}\xi(M)$.

Now, it can be checked easily that $\xi(0) = 0$. Further,

$$\begin{aligned}
\xi'(M) &= \frac{M}{\rho\,\psi(M)^2}\Big(\psi''(M)\psi(M) - (\psi'(M))^2\Big) \\
&= \frac{M\,\psi'(M)}{\rho\,\psi(M)^2}\Big(\rho\,(1+R)\psi(M) - \psi'(M)\Big) \\
&= \frac{M\,\psi'(M)}{\rho\,\psi(M)^2}\rho\,(1+R)\Big((1-\delta)\mathrm{e}^{\rho} + \mathrm{p}\,\delta(\mathrm{e}^{\rho}-1)\Big) \;>\; 0.
\end{aligned}$$

Since $\xi(0) = 0$ and $\xi'(\cdot) > 0$, it follows that $\xi(M) > 0$ for any $M > 0$. Therefore $\dfrac{\partial\widetilde{r}(\rho)}{\partial M} > 0.\|$

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