ISSN 1745-8587

Birkbeck UNIVERSITY OF LONDON

Department of Economics, Mathematics and Statistics

Birkbeck Working Papers in Economics & Finance

BWPEF 1514

Portmanteau Tests for Linearity of Stationary Time Series

Zacharias Psaradakis
Birkbeck, University of London

Marián Vávra
National Bank of Slovakia

September 2015

▪ Birkbeck, University of London ▪ Malet Street ▪ London ▪ WC1E 7HX ▪

Portmanteau Tests for Linearity of Stationary Time Series

Zacharias Psaradakis

Department of Economics, Mathematics and Statistics

Birkbeck, University of London

Marián Vávra

Advisor to the Governor and Research Department

National Bank of Slovakia

September 2015

Abstract

This paper considers the problem of testing for linearity of stationary time series. Portmanteau tests are discussed which are based on generalized correlations of residuals from a linear model (that is, autocorrelations and cross-correlations of different powers of the residuals). The finite-sample properties of the tests are assessed by means of Monte Carlo experiments. The tests are applied to 100 time series of stock returns.

JEL classification: C12; C22; C52.

Key words: Autocorrelation; Cross-correlation; Nonlinearity; Portmanteau test; Stock returns.

1 Introduction

The problem of testing for neglected nonlinearity in time series models has attracted a great deal of interest in recent years. A multitude of statistical procedures designed to test the null hypothesis of linearity against nonlinear alternatives are available in the literature, including general portmanteau tests without a specific alternative as well as tests with fully specified parametric alternatives; Tong (1990) and Teräsvirta, Tjøstheim, and Granger (2010) provide useful overviews. Linearity tests have become an essential first step in model-building exercises since, due to the difficulties associated with the statistical analysis of nonlinear models, it is often desirable to establish the adequacy or otherwise of a linear data representation before exploring more complicated nonlinear structures.

The present paper contributes to this literature by considering portmanteau tests for linearity of stationary time series based on 'generalized correlations' of residuals from a finite-parameter linear model, that is to say autocorrelations and cross-correlations of different powers of the residuals. Such tests are similar in spirit to the popular test proposed by McLeod and Li (1983), which is based on the empirical autocorrelations of squared residuals. The McLeod–Li test is known to respond well to autoregressive conditional heteroskedasticity (ARCH) but tends to lack power against many other interesting types of nonlinearity.

In addition to tests based on the empirical autocorrelations of the second or higher power of residuals, we also investigate tests that involve empirical cross-correlations between residuals and their squares (or, more generally, cross-correlations between different powers of the residuals). Lawrance and Lewis (1985, 1987) put forward the idea of using such cross-correlations to identify nonlinear dependence and examined analytically the cross-correlation functions for certain types of nonlinear models. Their analysis, however, focused only on visual inspection of individual cross-correlations and they did not consider the effects of parameter estimation.

In what follows we tackle these problems by developing portmanteau tests based on the generalized correlations of residuals from linear models. The proposed tests are easy to implement and have chi-square asymptotic null distributions under general regularity conditions. Furthermore, tests based on cross-correlations are shown to be more powerful against many types of nonlinearity compared to the familiar test based on squared-residual autocorrelations.

The paper is organized as follows. In Section 2 we discuss residual-based generalized correlations and the associated portmanteau tests for linearity, and present some relevant asymptotic results. Section 3 examines the finite-sample properties of the proposed tests by means of Monte Carlo experiments. Section 4 presents an application to time series of stock returns. Section 5 summarizes and concludes.

2 Generalized Correlations and Portmanteau Statistics

Consider a second-order stationary, short-range dependent, real-valued time series $\{X_t\}$ with mean μ satisfying

$$X_t - \mu = \Psi(L)\varepsilon_t, \qquad t \in \mathbb{Z}, \tag{1}$$

where

$$\Psi(z) = 1 + \sum_{j=1}^{\infty} \psi_j(\boldsymbol{\delta}) z^j, \qquad z \in \mathbb{C},$$

$\{\psi_j(\boldsymbol{\delta})\}$ is an absolutely summable sequence of weights, assumed to be known functions of a finite-dimensional (row) vector $\boldsymbol{\delta}$ of unknown parameters, $\{\varepsilon_t\}$ is strictly stationary white noise, and L is the lag operator. A leading example of a parametric model of the form of (1) is the autoregressive moving average (ARMA) model. In this case, the transfer function $\Psi(z)$ is of the form

$$\Psi(z) = B(z)/A(z), \qquad z \in \mathbb{C}, \tag{2}$$

where, for some fixed $p, q \in \mathbb{N} \cup \{0\}$ such that $p + q > 0$, $A(z) = 1 - \sum_{i=1}^{p} \alpha_i z^i$, with $A(z) \neq 0$ for all $|z| \leqslant 1$, $B(z) = 1 + \sum_{i=1}^{q} \beta_i z^i$, and $\boldsymbol{\delta} = (\alpha_1, \ldots, \alpha_p, \beta_1, \ldots, \beta_q)$.

A time series satisfying (1) is typically characterized as *linear* if $\{\varepsilon_t\}$ consists of independent and identically distributed (i.i.d.) random variables. This is the notion of linearity found in McLeod and Li (1983) and Lawrance and Lewis (1985, 1987), among many others, and is the one considered in this paper. It is worth noting, however, that the i.i.d. requirement on $\{\varepsilon_t\}$ is not the only characterization of linearity encountered in the literature. Hannan (1973), for example, considers a time series to be linear if its best one-step-ahead linear predictor is the best predictor, both in the mean-square sense, which is equivalent to $\{\varepsilon_t\}$ in (1) being a square-integrable martingale-difference sequence relative to its natural filtration. This alternative characterization of linearity does not lend itself to the type of statistical tests considered in the sequel. A test for linearity of the best predictor is discussed in Terdik and Máth (1998).

The focus of attention here are the generalized correlations of the noise $\{\varepsilon_t\}$ in (1). For $r, s \in \mathbb{N}$ such that $\mathsf{E}(|\varepsilon_0|^{r+s}) < \infty$, we define the generalized correlations of $\{\varepsilon_t\}$ at lag k as

$$\rho_{rs}(k) = \{\gamma_{rr}(0)\gamma_{ss}(0)\}^{-1/2}\gamma_{rs}(k), \qquad k \in \mathbb{Z}, \tag{3}$$

where $\gamma_{rs}(k) = \operatorname{cov}(\varepsilon_0^r, \varepsilon_k^s)$. Thus, (3) gives the autocorrelations of $\{\varepsilon_t\}$ for $r = s = 1$, the autocorrelations of $\{\varepsilon_t^2\}$ for $r = s = 2$, and cross-correlations of the type considered by Lawrance and Lewis (1985, 1987) for $(r, s) \in \{(1, 2), (2, 1)\}$. If $\{X_t\}$ is linear, then $\rho_{rs}(k) = 0$ for all $k \neq 0$.

When an estimator $\hat{\boldsymbol{\theta}} = (\hat{\mu}, \hat{\boldsymbol{\delta}})$ of $\boldsymbol{\theta} = (\mu, \boldsymbol{\delta})$ is available, one may use residuals $\{\hat{\varepsilon}_t; t = 1, 2, \ldots, T\}$ (to be defined in a precise manner later) in place of the unobservable noise $\{\varepsilon_t\}$. For $r, s \in \mathbb{N}$, we define the empirical generalized correlations of the residuals at lag k as

$$\hat{\rho}_{rs}(k) = \{\hat{\gamma}_{rr}(0)\hat{\gamma}_{ss}(0)\}^{-1/2}\,\hat{\gamma}_{rs}(k), \qquad k = 0, \pm 1, \ldots, \pm(T-1), \tag{4}$$

where $\hat{\gamma}_{rs}(k) = T^{-1}\sum_{t=1}^{T-k} f_r(\hat{\varepsilon}_t) f_s(\hat{\varepsilon}_{t+k})$ for $k \geqslant 0$, $\hat{\gamma}_{rs}(k) = \hat{\gamma}_{sr}(-k)$ for $k < 0$, and $f_b(\xi_t) = \xi_t^b - T^{-1}(\xi_1^b + \cdots + \xi_T^b)$ for any collection of random variables $\{\xi_t\}$ and $b \in \mathbb{N}$.

Tests for linearity of $\{X_t\}$ may then be based on portmanteau test statistics of the form

$$Q_{rs}(m) = T \sum_{k=1}^{m} \hat{\rho}_{rs}^2(k), \tag{5}$$

for some $r, s, m \in \mathbb{N}$ such that $r + s > 2$ and $m < T$.

In order to develop some asymptotic distribution theory for residual-based generalized correlations and associated portmanteau tests, the following assumptions are made (in the sequel, limits in stochastic-order symbols are taken by letting $T \to \infty$):

A1: $\{\varepsilon_t\}$ are i.i.d. with $\mathsf{E}(\varepsilon_0) = 0$ and $0 < \mathsf{E}(\varepsilon_0^2) < \infty$.

A2: $\Psi(z)$ is holomorphic in an open neighbourhood of the closed disc $|z| \leqslant 1$, does not vanish at any $|z| \leqslant 1$, and is differentiable in $\boldsymbol{\delta}$.

A3: $\hat{\boldsymbol{\theta}} - \boldsymbol{\theta} = O_p(T^{-1/2})$.

A4: $\partial \tilde{\gamma}_{rs}(k)/\partial \boldsymbol{\theta} = O_p(T^{-1/2})$ for $k \in \{0, 1, \ldots, T-1\}$ and $r, s \in \mathbb{N}$ such that $r + s > 2$ and $\mathsf{E}[|\varepsilon_0|^{2(r+s)}] < \infty$, where $\tilde{\gamma}_{rs}(k) = T^{-1} \sum_{t=1}^{T-k} f_r(\varepsilon_t) f_s(\varepsilon_{t+k})$.

Assumption A1 amounts to linearity of $\{X_t\}$ in our setting. Under A2, $1/\Psi(z)$ has the convergent power series expansion $1/\Psi(z) = \phi_0(\boldsymbol{\delta}) - \sum_{j=1}^{\infty} \phi_j(\boldsymbol{\delta}) z^j$ for $|z| \leqslant 1$, with $\phi_0(\boldsymbol{\delta}) = 1$ and

$$\phi_j(\boldsymbol{\delta}) = \psi_j(\boldsymbol{\delta}) - \sum_{i=1}^{j-1} \phi_{j-i}(\boldsymbol{\delta}) \psi_i(\boldsymbol{\delta}), \qquad j \in \mathbb{N},$$

and, consequently, $\{X_t\}$ admits the autoregressive (AR) representation

$$X_t - \mu = \sum_{j=1}^{\infty} \phi_j(\boldsymbol{\delta})(X_{t-j} - \mu) + \varepsilon_t, \qquad t \in \mathbb{Z}.$$

Hence, given an estimator $\hat{\boldsymbol{\theta}}$ based on a finite stretch $(X_0, X_1, \ldots, X_T)$ of $\{X_t\}$, residuals may be defined as (cf. Kreiss (1991))

$$\hat{\varepsilon}_t = X_t - \hat{\mu} - \sum_{j=1}^{t} \phi_j(\hat{\boldsymbol{\delta}})(X_{t-j} - \hat{\mu}), \qquad t = 1, 2, \ldots, T.$$

Estimators of $\boldsymbol{\theta}$ satisfying assumption A3 may be obtained by quasi-maximum likelihood or instrumental-variables methods under suitable regularity conditions (see, e.g., Hannan (1973); Dunsmuir (1979); Hosoya and Taniguchi (1982); Kuersteiner (2001)). In the ARMA case specified by (2), assumptions A2–A4 hold true, under an i.i.d. assumption about $\{\varepsilon_t\}$, as long as the polynomials $A(z)$ and $B(z)$ have no zeros in common and $A(z)B(z) \neq 0$ for all $|z| \leqslant 1$.

We have the following result for the asymptotic distribution of a finite set of empirical generalized correlations of the residuals defined by (4) under the assumption that $\{X_t\}$ is linear.

Theorem 1 *Suppose that* $\{X_t\}$ *satisfies* (1) *and assumptions* A1–A4 *hold. Then, for any fixed* $m \in \mathbb{N}$ *and* $r, s \in \mathbb{N}$ *such that* $r + s > 2$ *and* $\mathsf{E}[|\varepsilon_0|^{2(r+s)}] < \infty$, *the asymptotic distribution of* $\sqrt{T}\,(\hat{\rho}_{rs}(1), \ldots, \hat{\rho}_{rs}(m))$, *as* $T \to \infty$, *is Gaussian with zero mean vector and identity covariance matrix.*

Proof: For a fixed $m < T$, a Taylor expansion of $\hat{\gamma}_{rs}(k)$ about $\boldsymbol{\theta}$ leads to

$$\hat{\gamma}_{rs}(k) = \tilde{\gamma}_{rs}(k) + \frac{\partial \tilde{\gamma}_{rs}(k)}{\partial \boldsymbol{\theta}}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})' + O_p(T^{-1}) = \tilde{\gamma}_{rs}(k) + O_p(T^{-1}), \quad k = 0, 1, \ldots, m.$$

Hence, the distribution of $\sqrt{T}(\hat{\gamma}_{rs}(1) - \gamma_{rs}(1), \ldots, \hat{\gamma}_{rs}(m) - \gamma_{rs}(m))$ is asymptotically the same as the distribution of $\sqrt{T}(\tilde{\gamma}_{rs}(1) - \gamma_{rs}(1), \ldots, \tilde{\gamma}_{rs}(m) - \gamma_{rs}(m))$. Furthermore, putting $\dot{f}_b(\varepsilon_t) = \varepsilon_t^b - \mathsf{E}(\varepsilon_0^b)$, $b \in \mathbb{N}$, and noting that $T^{-1}\sum_{t=1}^{T} \dot{f}_b(\varepsilon_t) = O_p(T^{-1/2})$ for $b \in \{r, s\}$, it is not difficult to show that $\tilde{\gamma}_{rs}(k) - T^{-1}\sum_{t=1}^{T} \dot{f}_r(\varepsilon_t)\dot{f}_s(\varepsilon_{t+k}) = o_p(T^{-1/2})$ for $0 \leqslant k \leqslant m$. Therefore, recalling that $\gamma_{rs}(k) = 0$ for all $k \neq 0$ under assumption A1, by an application of the central limit theorem for strictly stationary, finitely dependent sequences (e.g., Anderson (1971, Theorem 7.7.6)) to the normalized partial sum $T^{-1/2}\sum_{t=1}^{T}(\dot{f}_r(\varepsilon_t)\dot{f}_s(\varepsilon_{t+1}), \ldots, \dot{f}_r(\varepsilon_t)\dot{f}_s(\varepsilon_{t+m}))$ we may conclude that, as $T \to \infty$, the distribution of $\sqrt{T}\{\gamma_{rr}(0)\gamma_{ss}(0)\}^{-1/2}(\hat{\gamma}_{rs}(1), \ldots, \hat{\gamma}_{rs}(m))$ converges weakly to the standard normal distribution on $\mathbb{R}^m$. The assertion of the theorem follows from this result and the fact that $\hat{\gamma}_{bb}(0) = \tilde{\gamma}_{bb}(0) + O_p(T^{-1}) = \gamma_{bb}(0) + o_p(1)$ for $b \in \{r, s\}$. ■

We note that, for $r = s = 2$ and $\Psi(z)$ specified as in (2), the central limit theorem of McLeod and Li (1983) is retrieved from Theorem 1. It also readily seen that, under the conditions of Theorem 1, the asymptotic distribution of the portmanteau statistic $Q_{rs}(m)$ defined by (5) is chi-square with m degrees of freedom.

In Sections 3 and 4 we shall focus on tests with $r, s \in \{1, 2\}$. The use of higher values for (r, s) is, of course, possible but the asymptotic justification of the associated portmanteau tests requires finiteness of a fairly large number of moments (cf. Theorem 1). This requirement may be at odds with the characteristics of many economic and financial time series (e.g., equity returns, exchange rate returns, interest rates), for which it is often argued that they only possess unconditional moments of relatively low order (see, e.g., Koedijk, Schafgans, and de Vries (1990); Jansen and de Vries (1991); de Lima (1997)).

3 Monte Carlo Experiments

Monte Carlo experiments are carried out to investigate the finite-sample performance of portmanteau tests based on Q_{rs}. The following data-generating processes (DGPs) are considered in the simulations ($\mathbf{1}_{\{\cdot\}}$ denotes the indicator function):

M1: $X_t = 0.6X_{t-1} + \varepsilon_t$

M2: $X_t = 0.8X_{t-1} + 0.15X_{t-2} + 0.3\varepsilon_{t-1} + \varepsilon_t$

M3: $X_t = -0.5X_{t-1}\mathbf{1}_{\{X_{t-1} \leqslant 1\}} + 0.4X_{t-1}\mathbf{1}_{\{X_{t-1} > 1\}} + \varepsilon_t$

M4: $X_t = -0.5X_{t-1}\{1 - G(X_{t-1})\} + 0.4X_{t-1}G(X_{t-1}) + \varepsilon_t, \quad G(x) = (1 + e^{-x})^{-1}$

M5: $X_t = Y_t^2 + \varepsilon_t, \quad Y_t = 0.6Y_{t-1} + \eta_t$

M6: $X_t = 0.8\sqrt{|X_{t-1}|} + \varepsilon_t$

M7: $X_t = 0.6X_{t-1} + \sigma_t\varepsilon_t, \quad \sigma_t^2 = 0.1 + 0.8\sigma_{t-1}^2 + 0.1\sigma_{t-1}^2\varepsilon_{t-1}^2$

M8: $X_t = 0.6X_{t-1} + \sigma_t\varepsilon_t, \quad \sigma_t^2 = 0.25 + 0.6\sigma_{t-1}^2 + 0.5\sigma_{t-1}^2\varepsilon_{t-1}^2\mathbf{1}_{\{\varepsilon_t < 0\}} + 0.2\sigma_{t-1}^2\varepsilon_{t-1}^2\mathbf{1}_{\{\varepsilon_t \geqslant 0\}}$

M9: $X_t = 0.4X_{t-1} - 0.3X_{t-2} + 0.5X_{t-1}\varepsilon_{t-1} + 0.8\varepsilon_{t-1} + \varepsilon_t$

M10: $X_t = -0.3\varepsilon_{t-1} + 0.2\varepsilon_{t-2} + 0.4\varepsilon_{t-1}\varepsilon_{t-2} - 0.25\varepsilon_{t-2}^2 + \varepsilon_t$

These DGPs, borrowed from Lee, White, and Granger (1993), Barnett, Gallant, Hinich, Jungeilges, Kaplan, and Jensen (1997), and Hong and White (2005), cover a variety of linear and nonlinear processes used in the literature, namely AR (M1), ARMA (M2), threshold AR (M3), smooth-transition AR (M4), square AR (M5), fractional AR (M6), generalized ARCH (M7), threshold generalized ARCH (M8), bilinear (M9), and nonlinear moving average (M10). In all cases, $\{\varepsilon_t\}$ and $\{\eta_t\}$ are i.i.d. standard normal random variables independent of each other.

In the experiments, 2,000 independent artificial time series $\{X_t\}$ of length $100+T$, with $T \in \{200, 500\}$, are generated according to M1–M10. The first 100 data points of each series are then discarded in order to eliminate start-up effects and the remaining T data points are used to carry out linearity tests based on the statistics $Q_{12}(m)$, $Q_{21}(m)$ and $Q_{22}(m)$, with $m \in \{1, 2, \ldots, \langle\sqrt{T}\rangle\}$, where $\langle\cdot\rangle$ denotes the greatest-integer function. The tests are applied to least-squares residuals from an AR model for $\{X_t\}$ the order of which is determined by the Bayesian information criterion (BIC), defined according to Method 1 of Ng and Perron (2005), with the maximum allowable order set equal to $\langle 8(T/100)^{1/4}\rangle$.

The Monte Carlo rejection frequencies of tests of nominal level 0.05 are shown in Figures 1and 2. Under linear DGPs (M1, M2), all three portmanteau tests have empirical levels which do not differ significantly from the nominal level regardless of the sample size T and the number of generalized correlations m used to construct the test statistic. For six out of the eight nonlinear DGPs (M3, M4, M5, M6, M9, M10), at least one of the two cross-correlation tests Q_{12} and Q_{21} has higher rejection frequencies than the Q_{22} test (especially when $T = 200$). The Q_{22} test has a clear advantage in the case of time series generated according to M7 and M8; this is not perhaps surprising since Q_{22} is asymptotically equivalent to a Lagrange multiplier statistic for testing linearity against ARCH (see Luukkonen, Saikkonen, and Teräsvirta (1988)). Finally, we note that the power of the tests generally improves as T increases.

4 Empirical Application

Portmanteau tests for linearity are applied to a set of weekly stock returns, spanning the period 1993–2007 (781 observations), for 100 companies from the Standard & Poor's 500 Composite index. The selected series are part of the data set analyzed by Kapetanios (2009) and are such that the hypothesis of strict stationarity cannot be rejected for any of them (at 5% significance level). We wish to examine whether stock returns exhibit significant signs of nonlinear dependence beyond those associated with conditional heteroskedasticity and significant squared-residual autocorrelations.

The asymptotic P-values for tests based on $Q_{12}(m)$, $Q_{21}(m)$ and $Q_{22}(m)$, with $m \in \{10, 20\}$, are reported in Table 1. As in Section 3, the tests are applied to least-squares residuals from AR models the order of which is selected by the BIC. At 5% significance level, evidence against linearity is found in 97% of stock returns on the basis of the Q_{22} test, regardless of the value of m used. This arguably is not a very surprising finding since conditional heteroskedasticity is a characteristic feature of many asset returns. However, linearity is also rejected by at least one of the cross-correlation Q_{12}/Q_{21} tests in almost 80% of the cases. In light of the results of our simulation experiments, this suggests that the vast majority of the stock returns considered in our analysis may have nonlinear features beyond those associated with dynamic conditional heteroskedasticity.

The presence of nonlinearity in asset returns has important implications for pricing and risk management. For example, according to the Basel banking regulations, commercial banks are required to measure the market risk of their asset portfolios and hold capital in proportion to their risk position. Banks calculating their risk positions using a value-at-risk methodology based on ARCH-type volatility models may systematically overestimate or underestimate the downside risk, something which may impact negatively on their financial stability.

5 Summary

This paper considered portmanteau tests for linearity of stationary time series based on generalized correlations of residuals. The proposed tests are easy to implement, have a chi-square large-sample null distribution, and good size and power properties in finite samples. The simulation results indicated that the cross-correlation tests Q_{12} and Q_{21} are useful in identifying various types of nonlinearity and are generally more powerful than the popular Q_{22} test based on squared-residual autocorrelations. An application to time series of stock returns illustrated the practical use of the tests.

References

ANDERSON, T. W. (1971): *The Statistical Analysis of Time Series*. Wiley, New York.

BARNETT, W. A., A. R. GALLANT, M. J. HINICH, J. A. JUNGEILGES, D. T. KAPLAN, AND M. J. JENSEN (1997): "A single-blind controlled competition among tests for nonlinearity and chaos," *Journal of Econometrics*, 82, 157–192.

DE LIMA, P. J. F. (1997): "On the robustness of nonlinearity tests to moment condition failure," *Journal of Econometrics*, 76, 251–280.

DUNSMUIR, W. (1979): "A central limit theorem for parameter estimation in stationary vector time series and its application to models for a signal observed with noise," *Annals of Statistics*, 7, 490–506.

HANNAN, E. J. (1973): "The asymptotic theory of linear time-series models," *Journal of Applied Probability*, 10, 130–145.

HONG, Y., AND H. WHITE (2005): "Asymptotic distribution theory for nonparametric entropy measures of serial dependence," *Econometrica*, 73, 837–901.

HOSOYA, Y., AND M. TANIGUCHI (1982): "A central limit theorem for stationary processes and the parameter estimation of linear processes," *Annals of Statistics*, 10, 132–153.

Jansen, D. W., and C. G. de Vries (1991): "On the frequency of large stock returns: putting booms and busts into perspective," *Review of Economics and Statistics*, 73, 18–24.

Kapetanios, G. (2009): "Testing for strict stationarity in financial variables," *Journal of Banking and Finance*, 33, 2346–2362.

Koedijk, K., M. Schafgans, and C. de Vries (1990): "The tail index of exchange rate returns," *Journal of International Economics*, 29, 93–108.

Kreiss, J.-P. (1991): "Estimation of the distribution function of noise in stationary processes," *Metrika*, 38, 285–297.

Kuersteiner, G. M. (2001): "Optimal instrumental variables estimation for ARMA models," *Journal of Econometrics*, 104, 359–405.

Lawrance, A. J., and P. A. W. Lewis (1985): "Modelling and residual analysis of nonlinear autoregressive time series in exponential variables," *Journal of the Royal Statistical Society B*, 47, 165–202.

——— (1987): "Higher-order residual analysis for nonlinear time series with autoregressive correlation structures," *International Statistical Review*, 55, 21–35.

Lee, T.-H., H. White, and C. W. J. Granger (1993): "Testing for neglected nonlinearity in time series models: a comparison of neural network methods and alternative tests," *Journal of Econometrics*, 56, 269–290.

Luukkonen, R., P. Saikkonen, and T. Teräsvirta (1988): "Testing linearity in univariate time series models," *Scandinavian Journal of Statistics*, 15, 161–175.

McLeod, A. I., and W. K. Li (1983): "Diagnostic checking ARMA time series models using squared-residual autocorrelations," *Journal of Time Series Analysis*, 4, 269–273.

Ng, S., and P. Perron (2005): "A note on the selection of time series models," *Oxford Bulletin of Economics and Statistics*, 67, 115–134.

Teräsvirta, T., D. Tjøstheim, and C. W. J. Granger (2010): *Modelling Nonlinear Economic Time Series*. Oxford University Press, Oxford.

Terdik, G., and J. Máth (1998): "A new test of linearity of time series based on the bispectrum," *Journal of Time Series Analysis*, 19, 737–753.

Tong, H. (1990): *Non-linear Time Series: A Dynamical System Approach*. Oxford University Press, Oxford.

Figure 1: Rejection frequencies of the Q_{rs} tests: $T = 200$

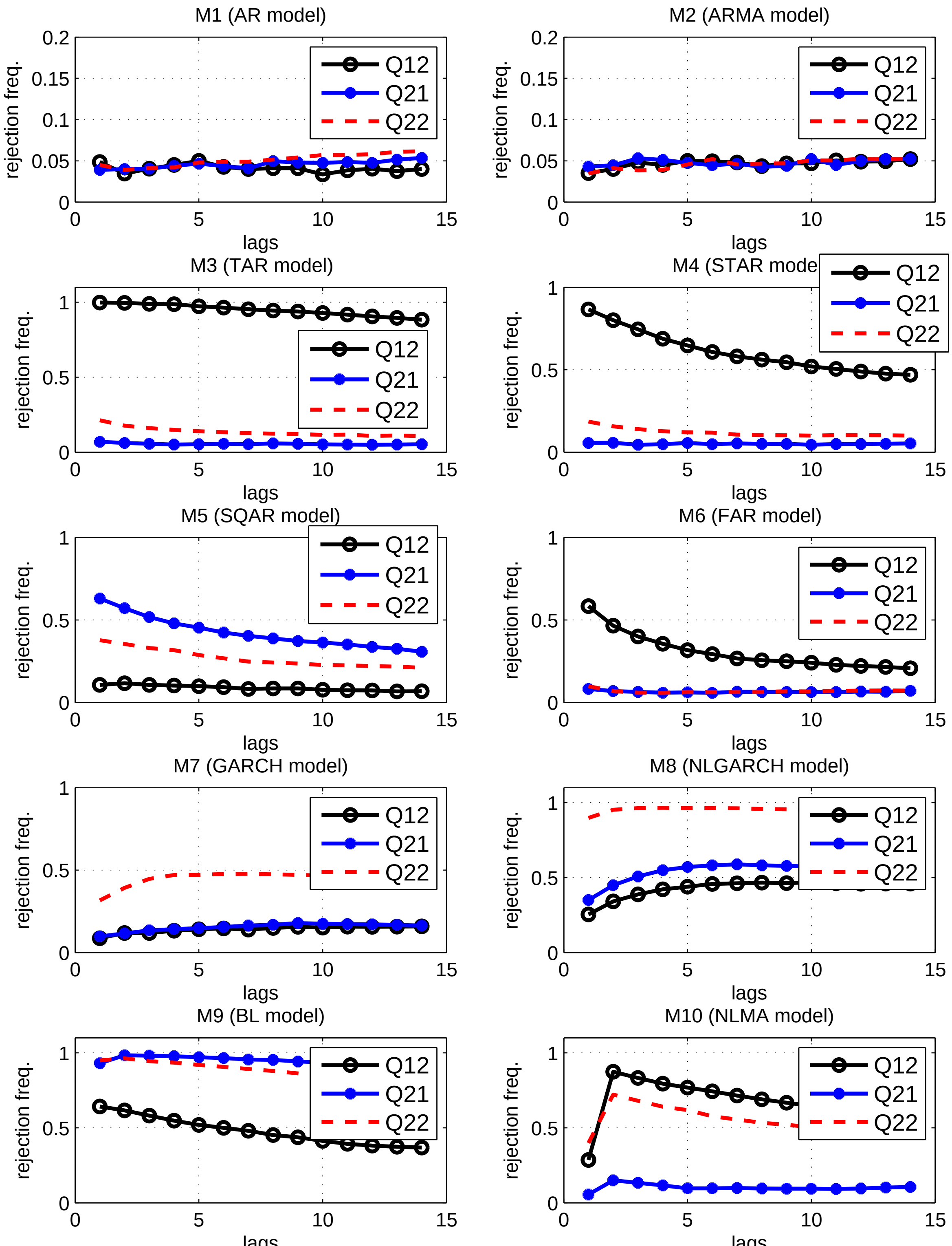

Figure 2: Rejection frequencies of the Q_{rs} tests: $T = 500$

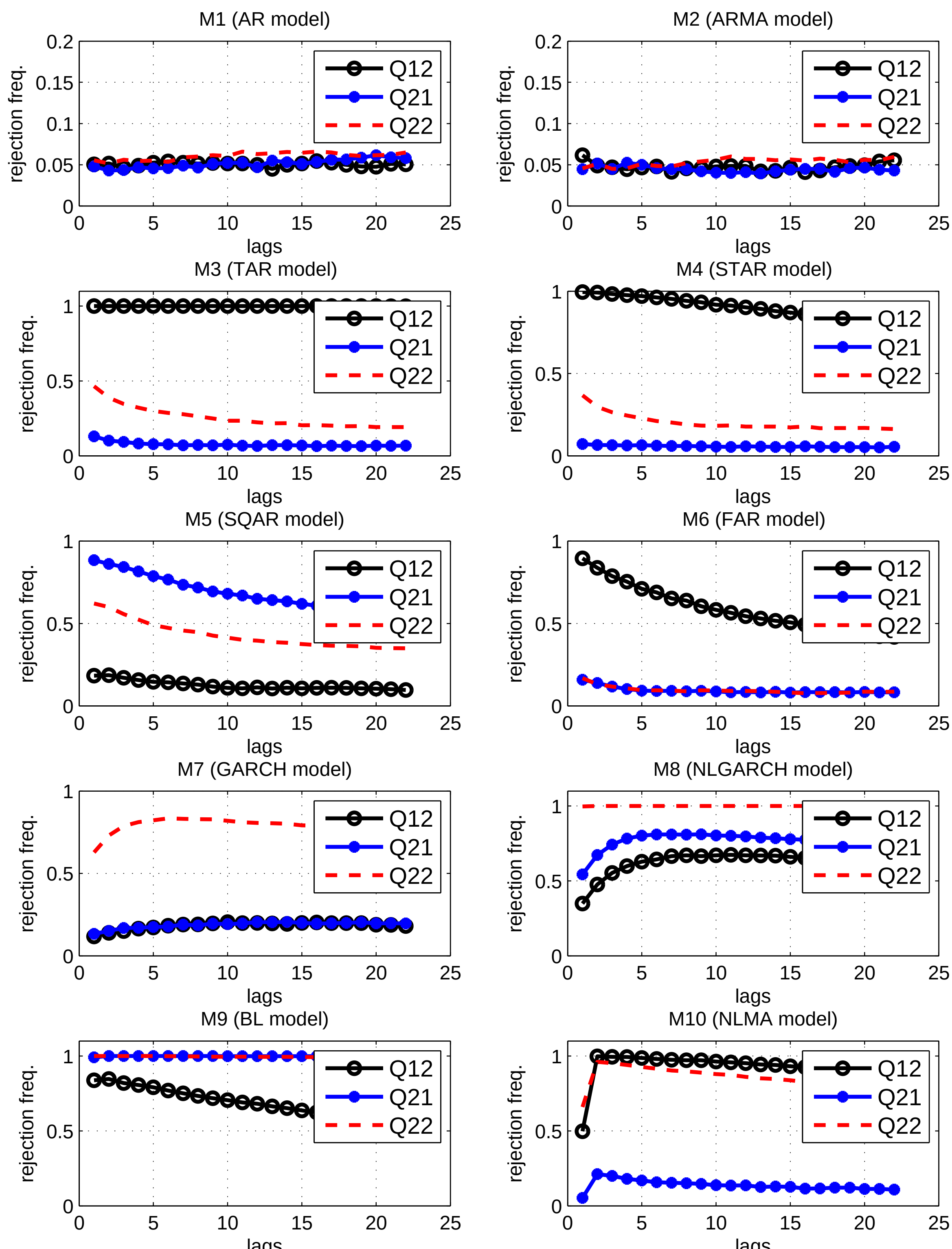

Table 1: P-values of the Q Tests

	$m=10$			$m=20$				$m=10$			$m=20$		
company	Q_{12}	Q_{21}	Q_{22}	Q_{12}	Q_{21}	Q_{22}	company	Q_{12}	Q_{21}	Q_{22}	Q_{12}	Q_{21}	Q_{22}
Alcoa Inc	0.02	0.00	0.00	0.00	0.00	0.00	Danaher Corp.	0.53	0.04	0.00	0.02	0.00	0.00
Apple Inc.	0.00	0.00	0.00	0.00	0.00	0.00	Walt Disney Co.	0.00	0.05	0.00	0.00	0.00	0.00
Adobe Systems Inc	0.05	0.02	0.00	0.01	0.00	0.00	Dow Chemical	0.43	0.02	0.00	0.03	0.14	0.00
Analog Devices Inc	0.00	0.00	0.00	0.00	0.00	0.00	Duke Energy	0.00	0.00	0.00	0.00	0.00	0.00
Archer-Daniels-Midland Co	0.09	0.00	0.00	0.00	0.00	0.00	Ecolab Inc.	0.00	0.00	0.00	0.00	0.00	0.00
Autodesk Inc	0.58	0.01	0.00	0.00	0.00	0.00	Equifax Inc.	0.39	0.12	0.14	0.18	0.00	0.01
American Electric Power	0.09	0.00	0.00	0.01	0.00	0.00	Edison Int'l	0.22	0.17	0.00	0.17	0.01	0.00
AES Corp	0.00	0.00	0.00	0.00	0.00	0.00	EMC Corp.	0.00	0.00	0.00	0.00	0.00	0.00
AFLAC Inc	0.00	0.64	0.00	0.00	0.17	0.00	Emerson Electric	0.03	0.08	0.00	0.01	0.02	0.00
Allergan Inc	0.03	0.21	0.00	0.01	0.03	0.00	Equity Residential	0.00	0.00	0.00	0.00	0.00	0.00
American Intl Group Inc	0.02	0.01	0.00	0.16	0.00	0.00	EQT Corporation	0.00	0.00	0.00	0.00	0.00	0.00
Aon plc	0.01	0.22	0.00	0.00	0.01	0.00	Eaton Corp.	0.92	0.02	0.00	0.69	0.00	0.00
Apache Corporation	0.06	0.08	0.00	0.01	0.50	0.00	Entergy Corp.	0.00	0.00	0.00	0.00	0.00	0.00
Anadarko Petroleum Corp	0.19	0.34	0.00	0.03	0.15	0.00	Exelon Corp.	0.00	0.00	0.00	0.00	0.00	0.00
Avon Products	0.36	0.41	0.00	0.16	0.58	0.00	Ford Motor	0.00	0.67	0.00	0.00	0.05	0.00
Avery Dennison Corp	0.04	0.25	0.00	0.02	0.07	0.00	Fastenal Co	0.40	0.85	0.00	0.45	0.09	0.00
American Express Co	0.84	0.09	0.00	0.01	0.17	0.00	Family Dollar Stores	0.25	0.66	0.00	0.64	0.54	0.00
Bank of America Corp	0.00	0.01	0.00	0.01	0.01	0.00	FedEx Corporation	0.03	0.92	0.00	0.08	0.16	0.00
Baxter International Inc.	0.05	0.00	0.00	0.01	0.00	0.00	Fiserv Inc	0.33	0.03	0.00	0.07	0.11	0.00
BBT Corporation	0.02	0.09	0.00	0.20	0.17	0.00	Fifth Third Bancorp	0.04	0.49	0.00	0.00	0.01	0.00
Best Buy Co. Inc.	0.60	0.32	0.00	0.33	0.37	0.00	Fluor Corp.	0.00	0.01	0.00	0.00	0.00	0.00

Bard (C.R.) Inc.	0.97	0.18	0.05	0.76	0.01	0.00	Forest Laboratories	0.00	0.05	0.00	0.00	0.00	0.00
Becton Dickinson	0.21	0.00	0.00	0.10	0.00	0.00	Frontier Communications	0.47	0.06	0.00	0.37	0.03	0.00
Franklin Resources	0.00	0.01	0.00	0.00	0.00	0.00	Gannett Co.	0.73	0.10	0.00	0.84	0.09	0.00
Brown-Forman Corp	0.01	0.25	0.00	0.02	0.00	0.00	General Dynamics	0.01	0.07	0.00	0.02	0.39	0.00
Baker Hughes Inc	0.00	0.00	0.00	0.08	0.00	0.00	General Electric	0.02	0.06	0.00	0.00	0.02	0.00
The Bank of NY Mellon	0.12	0.04	0.00	0.29	0.04	0.00	General Mills	0.78	0.90	0.66	0.47	0.69	0.00
Ball Corp	0.00	0.00	0.00	0.00	0.00	0.00	Genuine Parts	0.15	0.20	0.00	0.20	0.11	0.00
Boston Scientific	0.00	0.00	0.00	0.00	0.00	0.00	Gap (The)	0.02	0.00	0.00	0.00	0.00	0.00
Cardinal Health Inc.	0.12	0.54	0.01	0.00	0.89	0.00	Grainger Inc.	0.02	0.41	0.00	0.00	0.48	0.00
Caterpillar Inc.	0.00	0.00	0.00	0.00	0.00	0.00	Halliburton Co.	0.15	0.86	0.00	0.04	0.64	0.00
Chubb Corp.	0.04	0.27	0.00	0.12	0.12	0.00	Harman Int'l Industries	0.63	0.01	0.18	0.66	0.02	0.10
Coca-Cola Enterprises	0.71	0.01	0.00	0.00	0.00	0.00	Hasbro Inc.	0.92	0.78	0.01	0.31	0.90	0.01
Carnival Corp.	0.02	0.03	0.00	0.01	0.08	0.00	Huntington Bancshares	0.13	0.74	0.00	0.02	0.67	0.00
CIGNA Corp.	0.38	0.00	0.00	0.55	0.00	0.00	Health Care REIT	0.01	0.01	0.00	0.00	0.00	0.00
Cincinnati Financial	0.05	0.87	0.00	0.00	0.92	0.00	Home Depot	0.00	0.17	0.00	0.00	0.00	0.00
Clorox Co.	0.37	0.00	0.00	0.67	0.00	0.00	Hess Corporation	0.18	0.00	0.00	0.18	0.00	0.00
Comerica Inc.	0.63	0.02	0.00	0.96	0.04	0.00	Harley-Davidson	0.07	0.03	0.00	0.07	0.05	0.00
CMS Energy	0.19	0.03	0.01	0.28	0.00	0.01	Honeywell Int'l Inc.	0.10	0.01	0.00	0.00	0.00	0.00
CenterPoint Energy	0.00	0.00	0.00	0.00	0.00	0.00	Hewlett-Packard	0.00	0.24	0.00	0.00	0.00	0.00
Cabot Oil and Gas	0.00	0.01	0.00	0.00	0.00	0.00	Block H and R	0.34	0.28	0.00	0.59	0.00	0.00
ConocoPhillips	0.00	0.00	0.00	0.00	0.00	0.00	Hormel Foods Corp.	0.20	0.09	0.00	0.00	0.49	0.00
Campbell Soup	0.60	0.01	0.00	0.08	0.00	0.00	The Hershey Company	0.03	0.43	0.00	0.20	0.59	0.02
CSX Corp.	0.05	0.00	0.00	0.02	0.00	0.00	Intel Corp.	0.03	0.00	0.00	0.00	0.00	0.00
CenturyLink Inc	0.48	0.12	0.00	0.03	0.02	0.00	International Paper	0.02	0.01	0.00	0.05	0.00	0.00

Cablevision Systems Corp.	0.00	0.01	0.00	0.02	0.00	0.00	Interpublic Group	0.00	0.00	0.00	0.00	0.00	0.00
Chevron Corp.	0.01	0.00	0.00	0.02	0.00	0.00	Ingersoll-Rand PLC	0.43	0.00	0.00	0.00	0.00	0.00
Dominion Resources	0.04	0.04	0.00	0.01	0.00	0.00	Johnson Controls	0.01	0.00	0.00	0.00	0.00	0.00
Deere and Co.	0.00	0.72	0.00	0.00	0.00	0.00	Jacobs Engineering Group	0.00	0.00	0.00	0.00	0.00	0.00
D. R. Horton	0.07	0.09	0.00	0.00	0.00	0.00	Johnson and Johnson	0.01	0.00	0.00	0.00	0.00	0.00