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Common faith or parting ways? A time varying parameters factor analysis of euro-area inflation*

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Abstract

We analyze the interaction among the common and country specific components for the inflation rates in twelve euro area countries through a factor model with time varying parameters. The variation of the model parameters is driven by the score of the predictive likelihood, so that, conditionally on past data, the model is Gaussian and the likelihood function can be evaluated using the Kalman filter. The empirical analysis uncovers significant variation over time in the model parameters. We find that, over an extended time period, inflation persistence has fallen over time and the importance of common shocks has increased relatively to the idiosyncratic disturbances. According to the model, the fall in inflation observed since the sovereign debt crisis, is broadly a common phenomenon, since no significant cross country inflation differentials have emerged. Stressed countries, however, have been hit by unusually large shocks.

JEL classification: E31, C22, C51, C53.

Keywords: inflation, time-varying parameters, score driven models, state space models, dynamics factor models.

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1 Introduction

Inflation has fallen sharply and unexpectedly in most euro area countries following the sovereign debt crisis. At the outset, these developments were perceived as partly temporary, as they were mainly driven by a stabilization of energy prices, and partly idiosyncratic, as the fall in inflation was sharper in countries most hardly hit from the debt crisis (the so-called stressed countries: Greece, Ireland, Italy, Portugal and Spain). However, inflation weakness persisted long enough to raise fear of a prolonged period of low inflation (*lowflation*), prompting the ECB to deploy a range of unconventional measures, in the attempt to prevent the negative shocks that affected actual inflation to be passed through to the inflation expectations. Moreover, the deceleration of consumer prices has spread beyond the stressed countries, as inflation rates touched historically low levels in other countries, like Germany and the Netherlands. The issue goes beyond euro area borders since, depending on how long it takes for euro area inflation to respond to the monetary stimulus, it implies an asynchronous exit of the monetary policies from the unconventional measures across the world.

In this paper we develop a tool that, starting from national inflation rates, allows us to separate permanent from transitory shocks and common from idiosyncratic components. The model features time variation in the parameters so that it can capture the large number of regime shifts that inflation rates of the countries that form the European Monetary Union (EMU) have experienced since the 80s. Specifically, the model we propose is a (single) factor model with time-varying volatilities, in which each national inflation rate is separated in a stochastic trend that is common across countries, and a national component. The latter is less persistent than the common trend, but features a time-varying intercept that captures long lasting deviations of national inflation from the common trend. The model provides a real time decomposition of permanent and transitory shocks and as such could be used in real time to measure the extent of the imbalances within the euro area, as motivated by the argument developed in Corsetti and Pesaran (2012).

From a methodological point of view our work is related to two areas of the econometric literature. The first is the work by Creal et al. (2012) and by Harvey (2013) on score-driven models, i.e. models in which the parameters vary over time as function of the likelihood score. Delle Monache et al. (2015) show how the score-driven approach can be used to model parameter variation in a Gaussian state space model. In this paper we restrict the general approach therein developed, to a more parsimonious specification. The introduction of time variation

in the parameters makes these models highly nonlinear and possibly non Gaussian, so that computationally intensive simulation-based methods are typically required for estimation. In contrast, when the parameters are driven by the score, the model remains Gaussian, conditional on past data. In this case Delle Monache et al. (2015) develop a set of recursions that, running in parallel with the standard Kalman filter, allow the evaluation of the score vector at each point in time. Once the score is known, the model parameters can be updated. The likelihood function, which remains Gaussian, can then be evaluated by the means of the Kalman filter and maximized through standard procedures. A second stream of the econometric literature related to our work deals with dynamic factor models, see for example Giannone et al. (2008) and Camacho and Perez-Quiros (2010). Within this branch of the literature our paper is close to the studies that extend traditional dynamic factor models to nonlinear settings, like those by Del Negro and Otrok (2008), Mumtaz and Surico (2012) and Marcellino et al. (2013). There are a number of differences between our method and those just mentioned. The most important one is that all these papers adopt a Bayesian standpoint and rely on computationally intensive Bayesian methods to estimate the model parameters. In our setup, estimation can be carried out with straightforward maximization of the likelihood function, with some advantages in terms of computational simplicity.

Besides the methodological considerations, our work is relevant for applied economists with an interest on inflation modelling. The topic, which has always attracted much attention in the empirical literature, has received renewed interest in recent years due to the fact that the Great Recession had only a muted impact on consumer prices, an outcome that was at odds with the prediction of existing theoretical models (see Del Negro et al., 2013). In this respect, the inflation rates within the euro area provide an extremely interesting laboratory for nonlinear adaptive models. They have, in fact, undergone a number of breaks and regime shifts in the past thirty years, going from a decade of extreme heterogeneity (the 80s), through a period of rapid convergence (the 90s), to a decade of irreversibly fixed exchange rates and centralized monetary policy (the 2000s). In recent years, much like in the U.S., the euro area inflation proved to be relatively unresponsive to the Great Recession; it dropped very rapidly when the Sovereign Debt crisis spread through a large number of countries, bringing the euro area to the brink of deflation. In our model we capture this complicated narrative through three key ingredients: first, a common driving force that attracts national rates towards a common stochastic trend; second, the idiosyncratic cycles that account for potential heterogeneity across countries; third, time-varying coefficients and variances, so that the relative importance of these factors can

change over time. Our empirical setup is close in spirit to (but more general than) the models of trend inflation by Cogley (2002), Stock and Watson (2007) and Clark and Doh (2014), where trend/core inflation is modelled as a unit root process that underlies headline inflation. The main difference is that our trend inflation component is extracted from a panel of series, rather than through univariate models. Time variation in the model parameters separates our work from the measures of core inflation proposed by Cristadoro et al. (2005) and Giannone and Matheson (2007), where core inflation is defined in the frequency domain and extracted from a large dataset, but has time invariant characteristics as opposed to changing variance. The relevance of this latter feature will become clear in the empirical application as we find that the contribution of the persistent common component to individual inflation rates has risen over time. Similar differences arise with respect to the concept of Global inflation proposed by Ciccarelli and Mojon (2008) and Ferroni and Mojon (2014), where a common component is computed from a set of inflation rates as the first principal component or, alternatively, as the cross sectional mean. In their empirical applications the common component turns out (but does not need to) be persistent, attractive, and useful for predicting national inflation. We show that both the persistence and the relevance of this common component for explaining national inflation dynamics has changed frequently over time. Finally, heterogeneity across euro area inflation rates is the focus of the paper by Busetti et al. (2007). The main finding of this study is that the cross country inflation differentials can be very persistent in the European Monetary Union and it motivates the introduction in our model of national cycles that account for country specific idiosyncratic shocks.

Our empirical analysis uncovers significant time variation in the parameters, therefore validating our modelling framework. We find that since the 80s, inflation persistence has gradually fallen, as inflation rates have been disciplined first by the exchange rates agreement underpinning the EMS, then by the ECB monetary policy. At the same time the importance of common shocks has increased relative to that of the idiosyncratic ones, as a result of inflation convergence and common monetary policy. In more recent years the stressed countries have been hit by unusually large idiosyncratic shocks, which monetary policy has not been able to neutralize. This, however, has not resulted in significant inflation differentials since, when we take into account filter uncertainty, the idiosyncratic components are not significantly different from zero. We conclude that, despite some short term volatility in peripheral countries, the recent disinflation observed in the euro area is broadly a common phenomenon.

The rest of the paper is structured as follows. In Section 2 we present the model specifica-

tion. Section 3 describes the estimation strategy. In particular, we adapt to our specific setup the algorithm developed in Delle Monache et al. (2015) for general state space models with score driven parameters. In Section 4 we discuss the empirical analysis. Section 5 concludes.

2 A model of inflation trend and cycles

Our empirical setup is meant to capture the features of cross country inflation differentials in a currency union with national fiscal policies. In this institutional framework the nominal drift in the economy, which determines steady state inflation, nominal wage growth and nominal interest rates, is set by the single monetary policy and is common across countries. In our model this is driven by a random walk plus noise component that is shared by national inflation rates. At the same time, idiosyncratic productivity or fiscal shocks generate relative prices fluctuations, which are reflected in inflation differentials. However, a persistent deviation from the common trend is equivalent to a persistent real appreciation and leads to competitiveness losses. If too prolonged, inflation differentials may become not sustainable, as current account deficits and debt problems may ensue (see e.g. De Grauwe and Ji, 2012). In the absence of exchange rate adjustments, a relative price adjustment is bound to occur and national inflation rates can be expected to return towards the common trend. We therefore describe the behavior of the country specific component of national inflation rates through a time-varying intercept, which reflects longer lasting deviations from the common trend, plus an autoregressive component. We restrict the latter to have non-explosive roots, as explained in details below. The model can therefore be used to assess the *real time* state of convergence in the inflation rates, which is a key element of a well functioning monetary union (see e.g. Corsetti and Pesaran, 2012).

The model is described by the following equations:

$$\begin{aligned}
 \pi_{j,t} &= \mu_t + \psi_{j,t}, & j = 1, \dots, N, \quad t = 1, \dots, n, \\
 \mu_t &= \mu_{t-1} + \eta_t, & \eta_t \sim N(0, \sigma_{\eta,t}^2), \\
 \psi_{j,t} &= \gamma_{j,t} + \phi_{j,t}\psi_{j,t-1} + \kappa_{j,t}, & \kappa_{j,t} \sim N(0, \sigma_{\kappa,j,t}^2),
 \end{aligned} \tag{1}$$

where $\pi_{j,t}$ are the national inflation rates, μ_t is the common stochastic trend and $\psi_{j,t}$ the idiosyncratic components. We allow for time variation in all the elements of the model, namely the variance of the common stochastic trend $\sigma_{\eta,t}^2$, the intercept of the idiosyncratic process

$\gamma_{j,t}$, the autoregressive coefficients of the idiosyncratic component $\phi_{j,t}$, and the variance of the idiosyncratic process $\sigma_{j,t}^2$. A compact State Space representation of the above model is the following:

$$\begin{aligned} y_t &= Z\alpha_t, & t = 1, \dots, T, \\ \alpha_{t+1} &= T_t\alpha_t + \epsilon_t, & \epsilon_t \sim N(0, Q_t), \end{aligned} \quad (2)$$

where $y_t = [\pi_{1,t}, \dots, \pi_{N,t}]'$ is an $N \times 1$ vector of observed inflation rates, α_t is the $m \times 1$ vector of state variables with dimension $m = N + 2$, and Z , T_t and Q_t are the system matrices of appropriate dimension, namely

$$\begin{aligned} \alpha_t &= \begin{bmatrix} \mu_t & 1 & \psi_{1,t} & \dots & \psi_{N,t} \end{bmatrix}', & Z &= \begin{bmatrix} 1_{N \times 1} & 0_{N \times 1} & I_N \end{bmatrix}, \\ T_t &= \begin{bmatrix} I_2 & 0_{2 \times N} \\ 0_{N \times 1} & M_t \end{bmatrix}, & M_t &= \begin{bmatrix} \gamma_{1,t} & \phi_{1,t} & 0 & \dots & 0 \\ \gamma_{2,t} & 0 & \phi_{2,t} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ \gamma_{N,t} & 0 & \dots & 0 & \phi_{N,t} \end{bmatrix}, \\ Q_t &= \begin{bmatrix} \sigma_{\eta,t}^2 & 0 & \dots & \dots & 0 \\ 0 & 0 & \ddots & & \vdots \\ \vdots & \ddots & \sigma_{1,t}^2 & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \dots & \dots & 0 & \sigma_{N,t}^2 \end{bmatrix}. \end{aligned} \quad (3)$$

Notice that the second element of the diagonal Q_t is set to zero. This means that the breaking intercepts $\gamma_{j,t}$, rather than being driven by a set of random errors like the common level μ_t , will be driven by the score. This point will become clearer below, where we formalize the treatment of the dynamics of the time-varying parameters and discuss the model estimation.

3 Estimation

One way to model the time-varying elements in (3) is by specifying a law of motion where additional random shocks drive the changes in the parameters. In this case the Kalman filter loses its optimality and Bayesian simulation methods need to be used, see for example Del Negro and Otrok (2008) and Mumtaz and Surico (2012). The alternative approach is to consider an observation-driven model to account for parameters variation as in Koopman et al. (2010). In this framework the model remains Gaussian, conditionally on past data, and the likelihood

can be computed with the Kalman Filter and maximized with respect to the parameters of interest.

Recently, a new class of observation-driven models, the so called score-driven models, has been proposed by Creal et al. (2013) and Harvey (2013). The novelty of the approach is represented by the fact that the driver of the time-variation is the score of the conditional likelihood. This implies that, at each point in time, the parameters are updated in the direction that maximizes the local fit (i.e. the predictive likelihood). The intuition is that, when the score is zero, the likelihood is at its maximum, so that there is no need to change the parameters. Within this framework, Delle Monache et al. (2015) develop an algorithm that allows to compute the score and to update the parameters for a general Gaussian state space model.

We specialize the algorithm in Delle Monache et al. (2015) to our specific model (2)-(3). Collecting the time-varying parameters in the $k \times 1$ vector f_t , we posit the following law of motion

$$f_{t+1} = f_t + \Theta s_t, \quad t = 1, \dots, n. \quad (4)$$

The matrix Θ contains the static parameters that govern the speed at which the parameters are updated from one period to the next. The driving mechanism is represented by the scaled score vector of the conditional distribution, $s_t = \mathcal{I}_t^{-1} \nabla_t$, where $\nabla_t = \partial \ell_t / \partial f_t$ is the score, \mathcal{I}_t is a scaling matrix set to be equal to the information matrix $\mathcal{I}_t = -E_t(\partial^2 \ell_t / \partial f_t \partial f_t')$. Finally, ℓ_t is the likelihood function conditional on past observations $Y_{t-1} = \{y_{t-1}, \dots, y_1\}$, the current value of f_t and the vector of static parameters θ , namely $\ell_t = \log p(y_t | f_t, Y_{t-1}; \theta)$.

It is important to stress that the time-varying matrices in (3) are function of past observations only. This implies that the observation and the state vector are still conditionally Gaussian. Specifically, the conditional distribution of the observations and state are $y_t | f_t, Y_{t-1}; \theta \sim N(Za_t, F_t)$ and $\alpha_t | f_t, Y_{t-1}; \theta \sim N(a_t, P_t)$, respectively. Therefore, the log-likelihood function is equal to:

$$\ell_t = -\frac{1}{2} [\log(2\pi) + \log |F_t| + v_t' F_t^{-1} v_t], \quad (5)$$

and can be evaluated by the Kalman filter:

$$\begin{aligned} v_t &= y_t - Za_t, & F_t &= ZP_tZ', & K_t &= T_tP_tZ'F_t^{-1}, \\ a_{t+1} &= T_t a_t + K_t v_t, & P_{t+1} &= T_t P_t T_t' - K_t F_t K_t' + Q_t, & t &= 1, \dots, n. \end{aligned} \quad (6)$$

The dynamics of the model are completed by adding the recursions for the time-varying parameters. Therefore, at each point in time, the Kalman filter (6) needs to be augmented so that the score s_t can be computed and the vector f_t can be updated as in (4). Delle Monache et al. (2015) show that the score and the information matrix can be written as:

$$\begin{aligned}\nabla_t &= \frac{1}{2} \left[\dot{F}'_t(F_t^{-1} \otimes F_t^{-1})[v_t \otimes v_t - \text{vec}(F_t)] - 2\dot{V}'_t F_t^{-1} v_t \right], \\ \mathcal{I}_t &= \frac{1}{2} \left[\dot{F}'_t(F_t^{-1} \otimes F_t^{-1}) \dot{F}_t + 2\dot{V}'_t F_t^{-1} \dot{V}_t \right],\end{aligned}\tag{7}$$

where ‘ \otimes ’ denotes the Kronecker product, \dot{V}_t and \dot{F}_t denote the derivative of the prediction error, v_t , and of its variance, F_t , with respect to the vector f_t . Those are computed recursively as follows:

$$\begin{aligned}\dot{V}_t &= -Z\dot{A}_t, \\ \dot{F}_t &= (Z \otimes Z)\dot{P}_t, \\ \dot{K}_t &= (F_t^{-1} Z P_t \otimes I_m)\dot{T}_t + (F_t^{-1} Z \otimes T_t)\dot{P}_t - (F_t^{-1} \otimes K_t)\dot{F}_t, \\ \dot{A}_{t+1} &= (a'_t \otimes I_m)\dot{T}_t + T_t\dot{A}_t + (v'_t \otimes I_m)\dot{K}_t + K_t\dot{V}_t, \\ \dot{P}_{t+1} &= (T_t \otimes T_t)\dot{P}_t - (K_t \otimes K_t)\dot{F}_t + \dot{Q}_t \\ &\quad + 2N_m[(T_t P_t \otimes I_m)\dot{T}_t - (K_t F_t \otimes I_m)\dot{K}_t].\end{aligned}\tag{8}$$

We have that I_m denotes the identity matrix of order m , and $N_m = \frac{1}{2}(I_{m^2} + C_m)$, where C_m is the commutation matrix.¹ The filter (7)-(8) runs in parallel with usual the Kalman filter (5)-(6), together with the updating rule (4).

A distinctive feature of this setup is that at each point in time we update simultaneously the time-varying parameters and the state vector of the model. In this respect we differ from other methods proposed in the literature. In the two step approach of Koop and Korobilis (2014), for example, given an initial guess of the state vector (usually obtained by Principal Components), the time-varying parameters are computed using the forgetting factor algorithm developed in Koop and Korobilis (2013). Then, conditional on the time-varying parameters, the state vector is estimated through the Kalman filter. This procedure is iterated subject to a stopping rule. It can be shown that such approach is nested as special case of the general adaptive state space model by Delle Monache et al. (2015).

¹For any square matrix, A , of dimension m , the commutation matrix, C_m , is defined such that $C_m \text{vec}(A) = \text{vec}(A')$.

The matrix Θ is restricted to be block diagonal, with the diagonal elements depending on the static parameters collected in the vector θ . We opt for a very parsimonious specification and restrict the number of static parameters to three: one associated with the volatility of the common factor, one with the volatility of the idiosyncratic components and the last one with both the intercept and the autoregressive parameters of the idiosyncratic cycles.² The static parameters are estimated by maximum likelihood (ML), namely $\hat{\theta} = \arg \max \sum_{t=1}^n \ell_t(\theta)$, and the maximization is obtained numerically. Following Harvey (1989, p. 128) we have that $\sqrt{n}(\hat{\theta} - \theta) \rightarrow N(0, \Xi)$, where the asymptotic variance Ξ is evaluated by numerical derivative at the optimum as discussed in Creal et al (2013, sec. 2.3).³ The model is conditional Gaussian and the likelihood can be evaluated as usual prediction error decomposition (see Harvey, 1989, sec. 3.7.1). Note that our model requires a diffuse initialization, which is used when the state vector is non-stationary, and it is known to provide an approximation of the likelihood (Harvey, 1989, pp. 120-121). This implies that for (5), (7), and (8) $t = d + 1, \dots, n$, where $d = 1$ is the number of diffuse elements. In principle it would be possible to compute the diffuse likelihood via the augmented KF (see Durbin and Koopman, 2012, sec. 7.2.3) and therefore amend (5), (6), and (8) for $t = 1, \dots, d$. However, this is beyond the scope of this paper.

We impose some restrictions on the model parameters. In particular, we let long-run forecasts be bounded by constraining the dynamics of the idiosyncratic components $\psi_{j,t}$ not to be explosive, and this is achieved by restricting the AR coefficients $\phi_{j,t} \in (-1, 1)$. Furthermore, we require the volatilities to be positive. The restrictions are achieved by a transformation of the time-varying parameters through a so called *link function* that is invariant over time. In particular, we collect the parameters of interest in the vector:

$$\tilde{f}_t = \left[\sigma_{\eta,t}^2, \sigma_{1,t}^2, \dots, \sigma_{N,t}^2, \gamma_{1,t}, \dots, \gamma_{N,t}, \phi_{1,t}, \dots, \phi_{N,t} \right]', \quad (9)$$

and we define the link function:

$$\tilde{f}_t = g(f_t), \quad (10)$$

²Specifically, denote with θ_1 , θ_2 and θ_3 the parameters that govern the law of motion of, respectively, the volatility of the common factor, the volatilities of the idiosyncratic components and the idiosyncratic components (both the constants and the autoregressive coefficients). Then $\Theta = \text{diag}(\theta_1, \theta_2 \iota_{1 \times N}, \theta_3 \iota_{1 \times 2N})$, where ι denotes a row vector of ones. We also experimented with a specification that allowed for an independent updating coefficient for the constant and the autoregressive parameter in the idiosyncratic component, i.e. $\Theta = \text{diag}(\theta_1, \theta_2 \iota_{1 \times N}, \theta_3 \iota_{1 \times N}, \theta_4 \iota_{1 \times N})$, this gives results virtually identical to the ones presented here.

³Harvey (1989, pp.182-183) derives the asymptotic normality of non-stationary models with diffuse approximation for non-stationary model but fixed parameters.

such that the elements of \tilde{f}_t fall in the desired region. These restrictions are formalized as follows:

$$\sigma_{\eta,t}^2 = \exp(2\cdot), \quad \sigma_{j,t}^2 = \exp(2\cdot), \quad \sum_{j=1}^N \gamma_{j,t} = 0, \quad \phi_{j,t} = \tan(\cdot), \quad \forall j. \quad (11)$$

The first and the second constraints impose positive volatilities, the third one allows to identify the common trend from the idiosyncratic component and the last one leads to AR coefficients with stable roots. In practice, we model the following vector:

$$f_t = \left[\log \sigma_{\eta,t}, \log \sigma_{1,t}, \dots, \log \sigma_{N,t}, \tilde{\gamma}_{1,t}, \dots, \tilde{\gamma}_{N,t}, \arctan \phi_{1,t}, \dots, \arctan \phi_{N,t} \right]',$$

where $\gamma_{j,t} = \tilde{\gamma}_{j,t} - \frac{1}{N} \sum_{j=1}^N \tilde{\gamma}_{j,t}$. Note that, in general, the time-varying parameters collected in f_t enter linearly the system matrices T_t and Q_t , so that the first derivatives $\dot{T}_t = \partial \text{vec}(T_t) / \partial f_t'$ and $\dot{Q}_t = \partial \text{vec}(Q_t) / \partial f_t'$ turn out to be selection matrices. When restrictions are implemented, like those in (11), the derivative of \tilde{f}_t with respect to f_t must be taken into account. In this case the following general representation can be used:

$$\dot{T}_t = S_{1T} \Psi_{T,t} S_{2T}, \quad \dot{Q}_t = S_{1Q} \Psi_{Q,t} S_{2Q},$$

where the Jacobian matrices $\Psi_{Q,t}$ and $\Psi_{T,t}$ are equal to

$$\Psi_{T,t} = \begin{bmatrix} \Psi_{\gamma,t} & 0 \\ 0 & \Psi_{\phi,t} \end{bmatrix}, \quad \Psi_{Q,t} = \begin{bmatrix} 2\sigma_{\eta,t}^2 & 0 & \dots & 0 \\ 0 & 2\sigma_{1,t}^2 & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \dots & \dots & 2\sigma_{N,t}^2 \end{bmatrix}, \quad (12)$$

$$\Psi_{\gamma,t} = I_N - \frac{1}{N} \mathbf{1}_{N \times N}, \quad \Psi_{\phi,t} = \begin{bmatrix} 1 - \phi_{1,t}^2 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1 - \phi_{N,t}^2 \end{bmatrix},$$

and S_{1T} , S_{2T} , S_{1Q} , S_{2Q} are selection matrices. Specifically, S_{1T} is constructed starting from the identity matrix of dimension $(N+2)^2$ and selecting only the columns associated with nonzero entries in $\text{vec}(T_t)$, similarly S_{1Q} retains only the columns associated with nonzero entries of $\text{vec}(Q_t)$, whereas S_{2T} , and S_{2Q} identify the positions of the time-varying elements of T_t and

Q_t within the vector f_t , namely:

$$\begin{aligned} S_{2T} &= \begin{bmatrix} 0_{2N \times (k-2N)} & I_{2N} \end{bmatrix}; \\ S_{2Q} &= \begin{bmatrix} I_{N+1} & 0_{(N+1) \times (k-N-1)} \end{bmatrix}. \end{aligned}$$

Note that the filter for the time-varying parameters in (4) requires a starting value. We choose the initial values, f_1 by estimating the fixed parameters version of our model on a training sample (ten years of data) that we then discard. Specifically, we approximate the common factor by the cross-sectional average of the data (Pesaran, 2006)⁴ and then estimate an AR(1) model on the deviation of each country's inflation from the common component. This allows us to easily compute the initial values $\sigma_{\eta,1}^2$ and $\{\gamma_{j,1}, \phi_{j,1}, \sigma_{j,1}^2, \}_{j=1}^N$.

4 Empirical application

We model a panel of 12 inflation rates from a sample of EMU countries (Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, The Netherlands, Portugal and Spain) from 1980:Q1 to 2014:Q3. For each country $\pi_{j,t}$ is the annualized percentage change over the previous quarter of the headline index of consumer prices, $400 \times \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right)$. The data source is the OECD Main Economic Indicators database and seasonal adjustment is carried out with Tramo-Seats (Gomez and Maravall, 1996).

[Insert Figure 1 about here]

[Insert Figure 2 about here]

A plot of the data is presented in Figure 1 and Figure 2 reports the cohesion of the inflation series at various frequencies.⁵ Three features can be observed. First, there is an evident change in the level of inflation. In the majority of the countries considered here inflation was in fact at two digits in the early 80s; it then gradually converged through the Nineties to levels consistent with the Maastricht criteria⁶, and stabilized through the 2000s around the 2% rate

⁴Initializing the common factor estimates with the first principal component in the data (as in Ciccarelli and Mojon, 2010) gives very similar results.

⁵The cohesion measures the average pair-wise dynamic correlations at various frequencies (for more details see Croux et al., 2001).

⁶Formal break tests indeed detect a break in a large number of countries around 1992, see Corvoisier and Mojon (1995).

targeted by the ECB. This process of convergence towards low and stable rates of inflation is reflected into the high cohesion of the series at low frequencies. In fact, the pronounced disinflationary trend in the euro area is nested within a global tendency towards lower inflation rates, as documented, for example, by Mojon and Ciccarelli (2010) and Mumtaz and Surico (2012). Second, there is a strong decline in the volatility of inflation, which was generally much higher in the first part of the sample, with the temporary exception of the 2008/2009 biennium, when consumer prices were strongly affected by the oil price shock that followed the global financial crisis. A fall in the volatility of inflationary shocks is a common finding in the literature that uses time-varying structural VARs (see for example Benati and Mumtaz, 2007) and it bears important implications for the predictability of inflation, an issue to which we return below. Third, there is a marked increase in the co-movement of consumer prices among countries, as inflation rates become much more synchronized when the third stage of the EMU begins (1999), in particular at business cycle frequencies.⁷ We devote the rest of the section to explaining how our model sheds light on these stylized facts.

4.1 The level of inflation

In Figure 3 we present the data together with the estimate of the common stochastic trend μ_t and an alternative measure of co-movement, that is the first principal component of the inflation series. Ciccarelli and Mojon (2010) use principal components (PC) to obtain a measure of “global” inflation, essentially averaging across a relatively large panel of OECD inflation rates. The estimates presented in Figure 3 can therefore be seen as the euro-area counterpart of their global inflation concept. The first observation is that the common stochastic trend estimated by our model captures very well the downward trend in inflation from the high levels of the 80s to the moderate/low levels of the 90s. It is also very smooth as a consequence of the random walk structure that we have imposed on it. Second, the common stochastic trend turns out to be highly correlated with the PC trend. This result, a priori, is not obvious since the latter is obtained through a non-parametric estimator while our common trend obeys a defined law of motion described by the transition equations. A theoretical connection between the two estimators is established by Bai (2004), where it is shown that, if data are generated by a non-stationary factor model, then principal components deliver a consistent estimate of

⁷Interestingly, Figure 2 shows a lower cohesion at low frequencies in the post euro sample, after the common trend in the inflation series stabilizes. This highlights the presence of persistent deviations from the common trend, confirming the results in Busetti et al. (2007).

the non-stationary factors. This implies that, should a common stochastic trend be present in the data, the first principal component would end up capturing it, which is in fact what happens in our case.⁸ The common trend captures almost entirely the co-movement of the national inflation rates across all frequencies (see Figure 2).⁹ This highlights how a random walk specification for the common trend does not artificially constrain the common component to capture only the low frequency variations in the panel.

[Insert Figure 3 about here]

Figure 4 compares the estimated common factor with the observed year-on-year aggregate inflation, which is the official target of the European Central Bank. The aggregate factor captures well the persistent movements of aggregate inflation. Interestingly, being estimated in real time from quarterly data, it tends to lead the movements in the year-on-year variation.

[Insert Figure 4 about here]

In our model the country specific level of inflation is captured by the time-varying idiosyncratic cycles ($\psi_{j,t}$), which are shown in Figure 5. They track quite accurately the various steps that led from the European Monetary System in the late Seventies to the EMU. In particular, three clusters can be identified. The first one is composed of a set of countries whose idiosyncratic elements start out from negative levels in the Eighties, and then slowly converge to zero. These are continental countries that were founding members of the European Monetary System, i.e. Belgium, Germany and Luxembourg, together with Austria (whose currency did not take part in the Exchange Rate Mechanism, but was de facto pegged to West Germany's Deutsche mark, see Nyberg et al., 1983). Although the EMS arrangements did not grant a special role to any country, the low inflation policy that the Bundesbank had pursued since the Seventies (see Benati, 2011) spilled over to the main trading partners and acted as an attractor for the whole EMS, as testified by the fall of the common trend in the first part of

⁸Ciccarelli and Mojon (2010) find that their measure of global inflation acts as an attractor, i.e. they find that deviations of the specific inflation rates from this common force are temporary. Our common trend is an attractor by construction, as a result of the stationarity constraint that we impose on the autoregressive idiosyncratic processes.

⁹In addition, it is worth noting that once the common trend is removed from the national inflation rates the average pair-wise correlation in the panel drops from 0.68 to 0.03. Even though the latter is still significant according to Pesaran (2004)'s test for cross-sectional dependence (CSD), the low average pair-wise correlation suggests that the CSD of the deviations from the common trend is likely to be weak.

the sample. The second block is formed by countries whose idiosyncratic components start positive but converge to zero by the mid-Nineties. These are EMS members that managed to sustain temporarily higher inflation rates vis-a-vis Germany through some realignment of their exchange rates but whose price dynamics were eventually attracted to the common trend after the Maastricht criteria imposed a stronger nominal anchor (Italy, Ireland, the Netherlands, Portugal and, to some extent, Spain, which was not part of the EMS). The third cluster is formed by France and Finland, whose inflation rates roughly fluctuate around the common trend for the whole sample.¹⁰

[Insert Figure 5 about here]

Focusing on the period 2002-2014 (i.e. since the introduction of the euro notes in 2002) the national idiosyncratic components fluctuate roughly around zero, see Figure 6, hence testifying the ability of the centralized monetary policy to overall stabilize inflation rates in the various members of the EMU over this period. In the years since the global financial crisis, a slight downward trend in the national specific component is visible in the so-called vulnerable countries, especially Greece, Portugal, Spain and Ireland. Corsetti and Pesaran (2012) forcefully argue that real appreciation is the single most important indicator of macroeconomic imbalances (whatever the sources). In a currency union, real appreciation arises through inflation differentials. Our model can therefore be used to shed light on the persistent component of the relative inflation differentials, as captured by the breaking intercepts, $\gamma_{j,t}$. Turning to the post-euro sample Figure 7 unveils interesting patterns of divergence in the country specific inflation rates that would be otherwise obscured within broader long-term convergence in relative inflations. Greece, Ireland, Portugal, Spain and, to a smaller extent, Italy display a markedly persistent positive inflation differential in the run up to the sovereign debt crisis. The model also highlights how different responses to the European debt crisis have contributed to the realignment of inflation differentials. For instance, the sharp real depreciations in Ireland and Spain are in stark contrast with the slow correction in Greece.

[Insert Figure 6 about here]

[Insert Figure 7 about here]

¹⁰The behaviour of Greek inflation reflects the delay with which Greece met the necessary criteria for joining the EMU.

4.2 Volatility and persistence

Next, we look at the estimated time-varying volatilities, starting from the estimated volatility of the common component, shown in Figure 8. This variance displays a sharp downward trend since the Eighties up until 2009, when it temporarily increases, only to start falling soon after. Since the common component is a driftless random walk its variance can be interpreted as a measure of the (common) persistence present in the data. Its falling trajectory is therefore consistent with the decline in inflation persistence in the euro area highlighted by a number of studies within the Eurosystem Inflation Persistence Network (IPN), whose results are summarized in Altissimo et al. (2006). A number of papers within this literature (Robalo Marques, 2005, and O'Reilly and Whelan, 2005) argue that the evidence in favour of changes in inflation persistence is considerably weaker when the intercept of the inflation models is allowed to change over time. We stress that our model indeed allows for such a break in the country specific intercepts, yet even accounting for this feature the main result remains. The fall in inflation persistence is not at all specific to the euro area. It is a rather broader phenomenon usually identified either with stronger inflation anchoring by monetary policy, as argued by Benati and Surico (2007), or with more benign inflationary shocks. Stock and Watson (2007), for example, model U.S. inflation using a univariate model featuring a permanent and a transitory component, and allow for changing variances in both components. They find that the variance of the permanent component has fallen significantly over time. Benati (2008) also documents a reduction in inflation persistence in a large number of economies, including the euro area, and points out that these changes typically coincide with the adoption of an inflation target. The hypothesis that lower persistence stems from more benign shocks cannot, however, be completely ruled out. Comparing Figure 8 with Figure 3 it is in fact clear that the temporary rise in the variance of the common trend is due to the synchronized fall of inflation rates in 2009, which reflected the strong decline in oil prices at the inception of the financial crisis.

The fall in the volatility of the common permanent component has also strong implications for the predictability of inflation. Indeed it implies that inflation has become easier to forecast by naïve forecasting models, but also that more sophisticated models will have a harder time improving upon simple models, a point made by D'Agostino et al. (2006).

[Insert Figure 8 about here]

4.3 Co-movement

Given the fall in the volatility of common shocks discussed in the previous section, it would be tempting to conclude that there has been a fall in the degree of commonality of inflation across euro-area countries. This, however, would be erroneous because co-movement is determined by the relative importance of common and idiosyncratic shocks rather than by the absolute importance of the former. Synchronization of specific inflation rates has, instead, increased over time. We present evidence of this in Figures 9 and 10. Figure 9 shows at each point in time the cross sectional standard deviation of the country specific volatilities, which can be seen as a time-varying measure of dispersion across country specific inflation shocks (see also Del Negro and Otrok, 2008). Not surprisingly, not only the inflation levels have converged as observed above, but also the amplitude of the shocks has converged considerably over time. Figure 10 shows the share of the volatility of the common component in the overall forecast error for each of the 12 countries considered. Two main results stand out. First, in all the countries but Belgium, these ratios display an upward trend.¹¹ Given the decline in the volatility of the common trend documented above, this result implies that the idiosyncratic volatilities not only have diminished over the sample but also that their fall has been relatively faster than that of the common trend. In other words, in the context of a general fall in volatility (both common and idiosyncratic), co-movement across inflation rates has actually risen over time. The second result is that in all the stressed countries this trend has partly reversed after 2008. Such development can be better appreciated in Figure 11, which reports the same ratios of common to idiosyncratic variances over the 2002-2014 period. Over this shorter sample it is more evident that idiosyncratic variances have increased although with a different timing across countries. In Spain and Ireland, whose banking sectors were more directly hit by the 2008 financial crisis, there is a very sharp drop around that year. A similar, although more nuanced, trajectory, is visible for Greece and Portugal, where the crisis unfolded in 2010. Finally, in Italy the ratio starts bending in 2011, when the sovereign debt crisis struck the euro area.¹² Comparing these results with the ones on the idiosyncratic cycles discussed above, two conclusions can be drawn. First, peripheral countries in the euro area have been hit by relatively strong idiosyncratic shocks, which the ECB monetary policy had difficulties in offsetting. Second, this has not resulted in significant differences across countries in the

¹¹In the case of Germany the break visible at the beginning of the Nineties is due to the unification of West and East Germany.

¹²For a detailed account of the different stages of the euro-zone crisis see Shambaugh (2012).

levels of inflation, so that the disinflation observed since 2013, can largely be interpreted as a common phenomenon (see Figure 4).

[Insert Figures 9 about here]

[Insert Figures 10 about here]

[Insert Figure 11 about here]

5 Conclusions

In this paper we have analyzed inflation developments in the euro area through a factor model with time-varying parameters. The time variation in the model is driven by the score of the predictive likelihood, implying that the estimation can be carried out via maximum likelihood method. The model provides a real time decomposition of the permanent and transitory shocks to inflations' differentials across countries and could therefore be used in real time to measure the extent of the imbalances within the euro area, as discussed by Corsetti and Pesaran (2012). Our main findings are three. First, the inflation persistence, as measured by the variance of the common component, has decreased over time, in line with the findings obtained in the literature. Importantly, this result is not weakened by the presence of time-varying country-specific intercepts. Second, inflation commonality, estimated as a fraction of the common to idiosyncratic volatility, has risen as a result of the various steps that led to the EMU first, and of the common monetary policy in the last fifteen years. Third, the disinflation experienced since 2011 is largely a common phenomenon, since no significant cross country inflation differentials have emerged. Since 2008, however, more vulnerable countries have been hit by unusually large shocks, which were only imperfectly offset by the single monetary policy.

We conclude by briefly mentioning avenues for future research that the framework developed in this paper opens. First, the model could be used to analyze the time-varying effects of actual inflation on inflation expectations and possible feedbacks from the latter to the former. Second, it could be used to investigate the relevance of national inflation rates for forecasting the area wide inflation.

Figure 1: Data

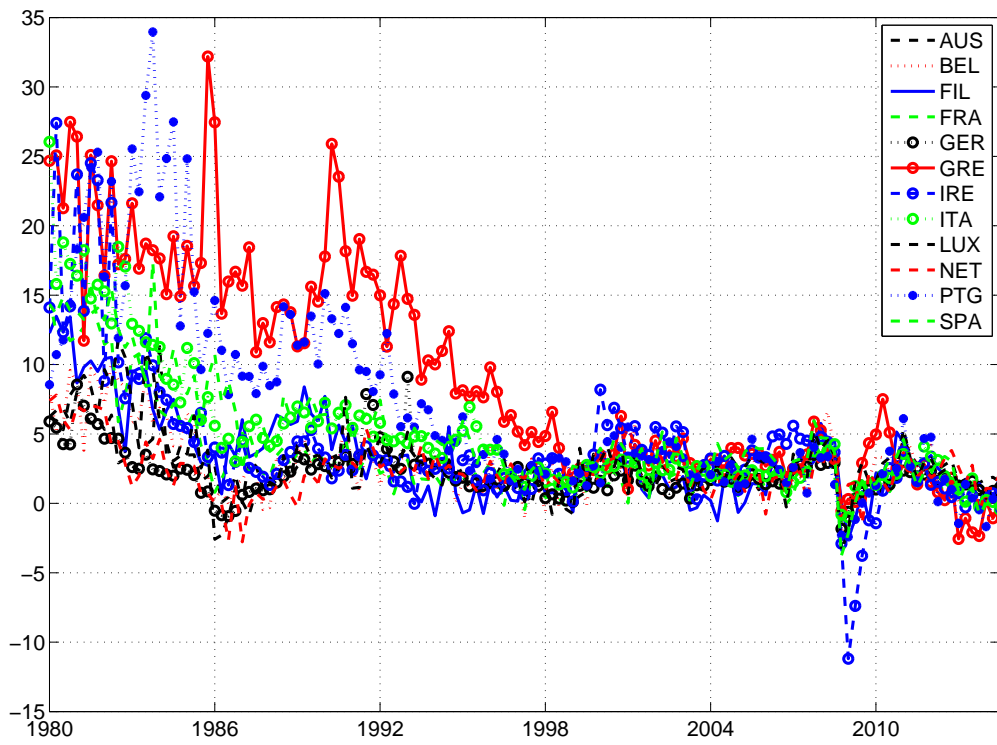


Figure 2: Cohesion

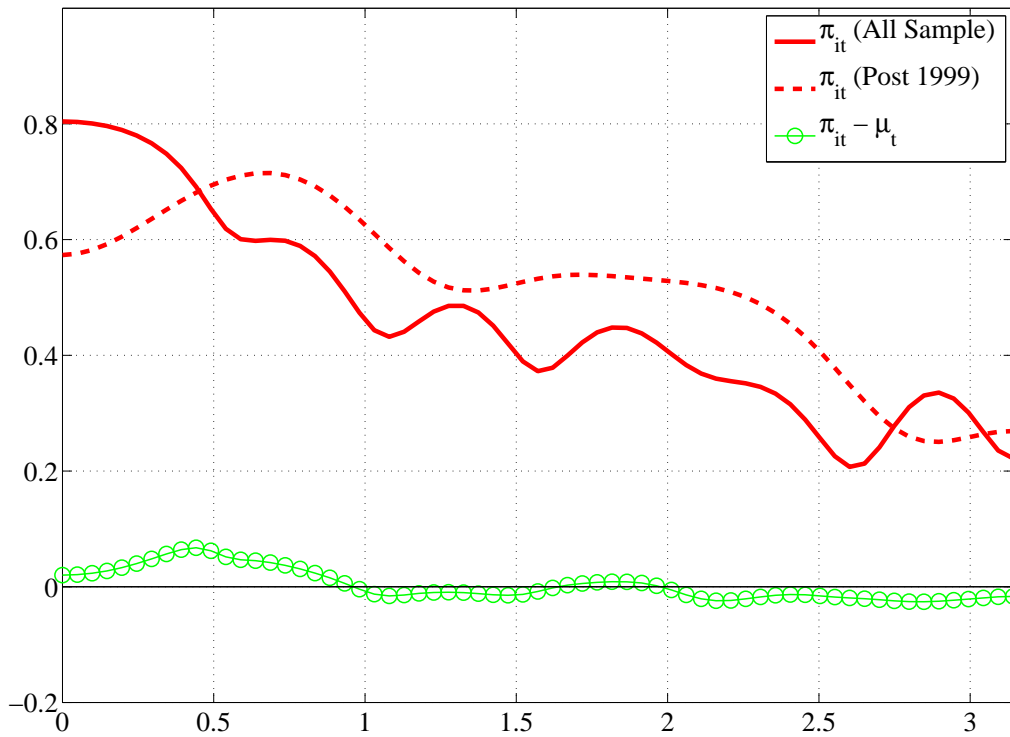


Figure 3: Common trend, first principal component and data

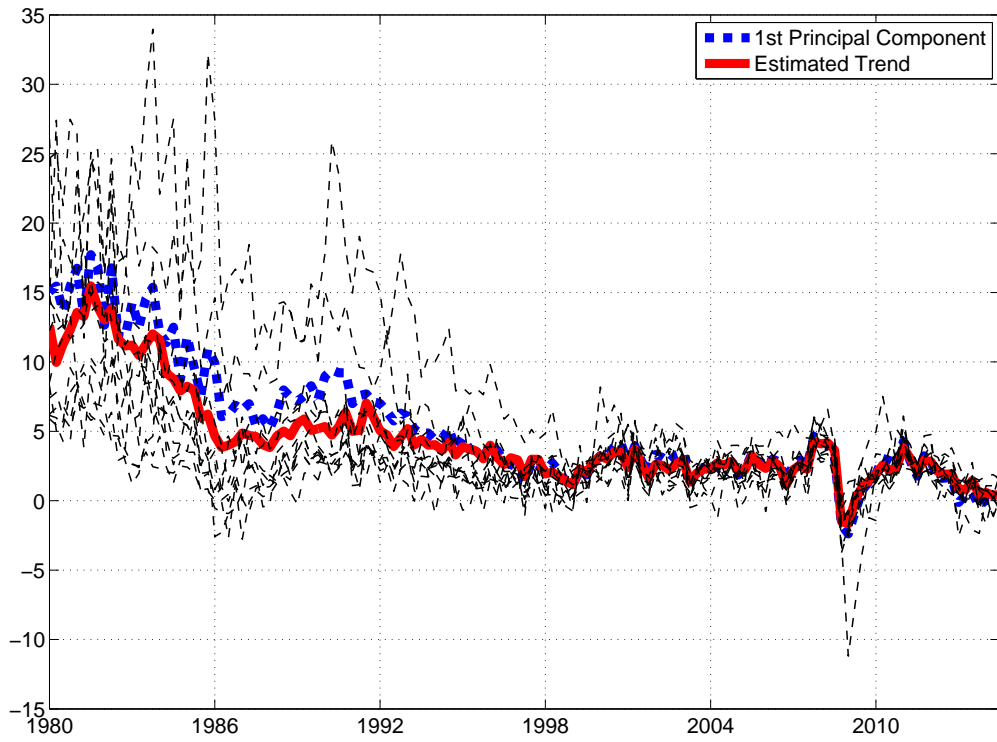


Figure 4: Actual (y-o-y) inflation and common trend (68% confidence interval)

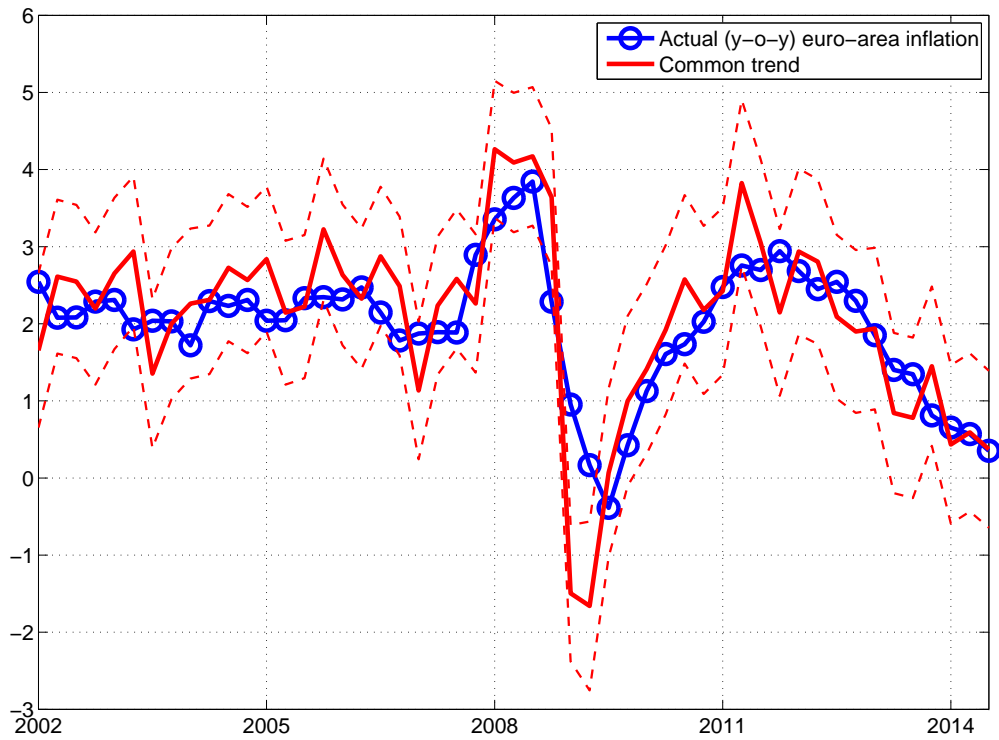
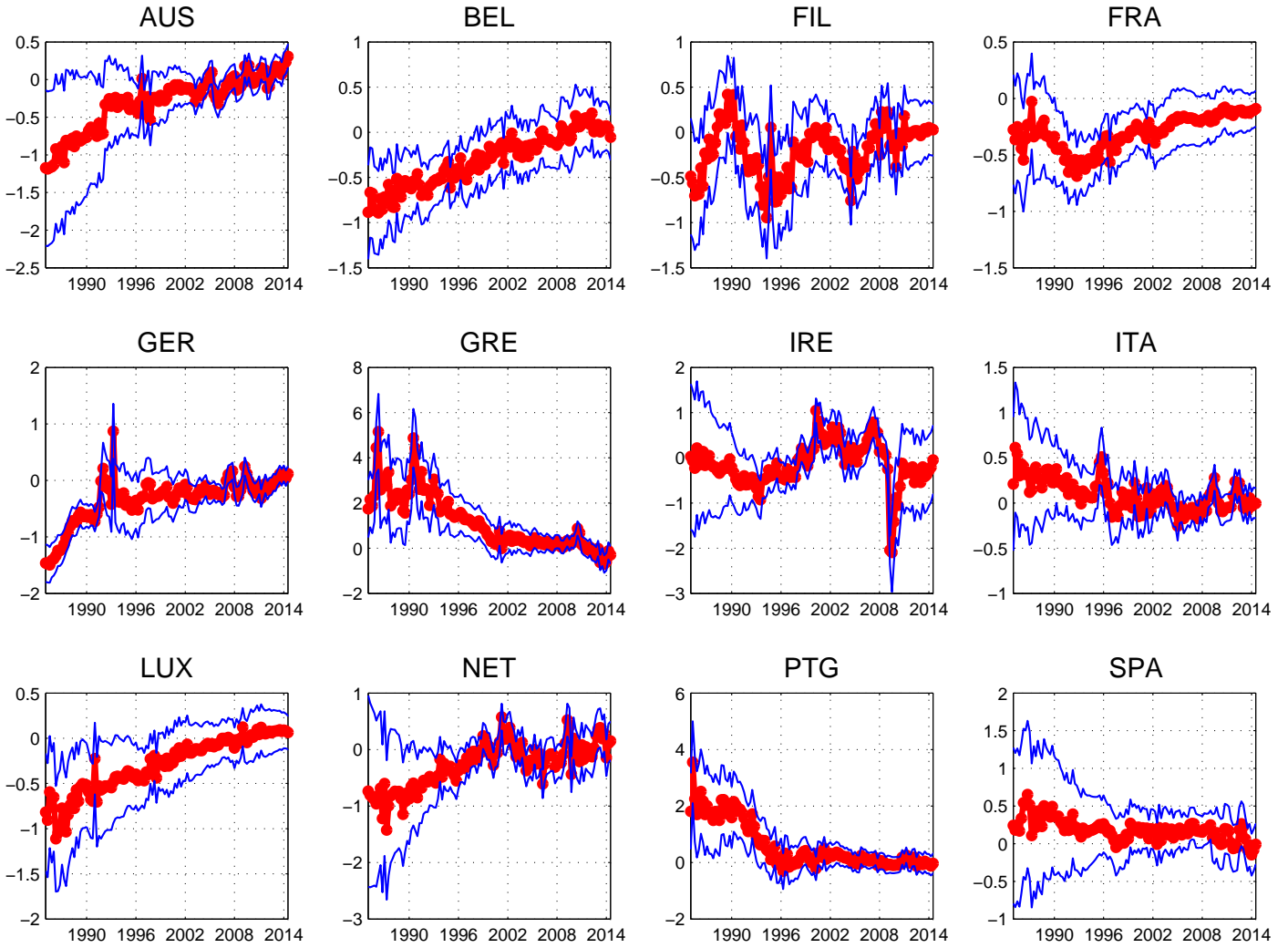
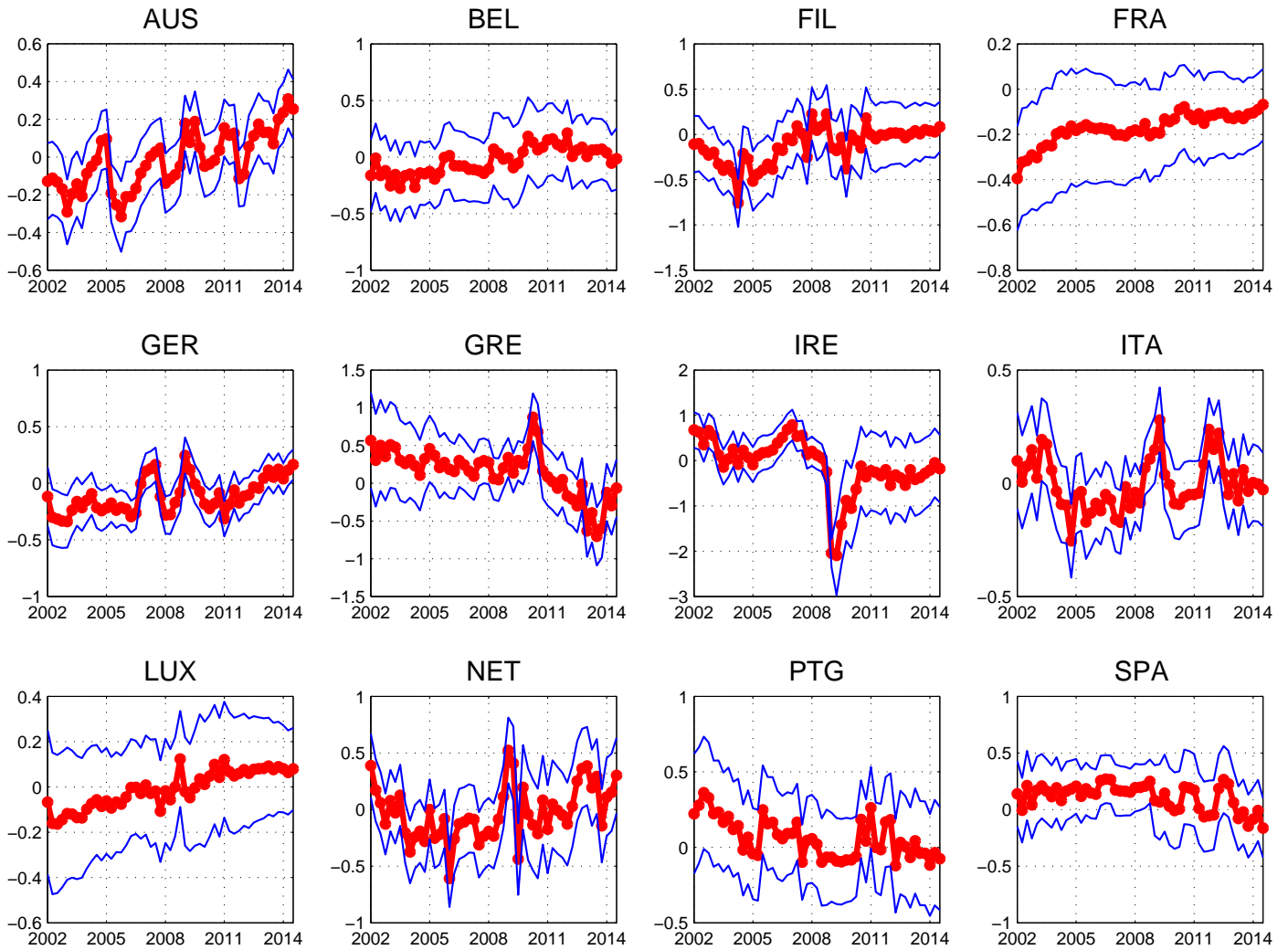


Figure 5: Idiosyncratic cycles $\psi_{i,t}$



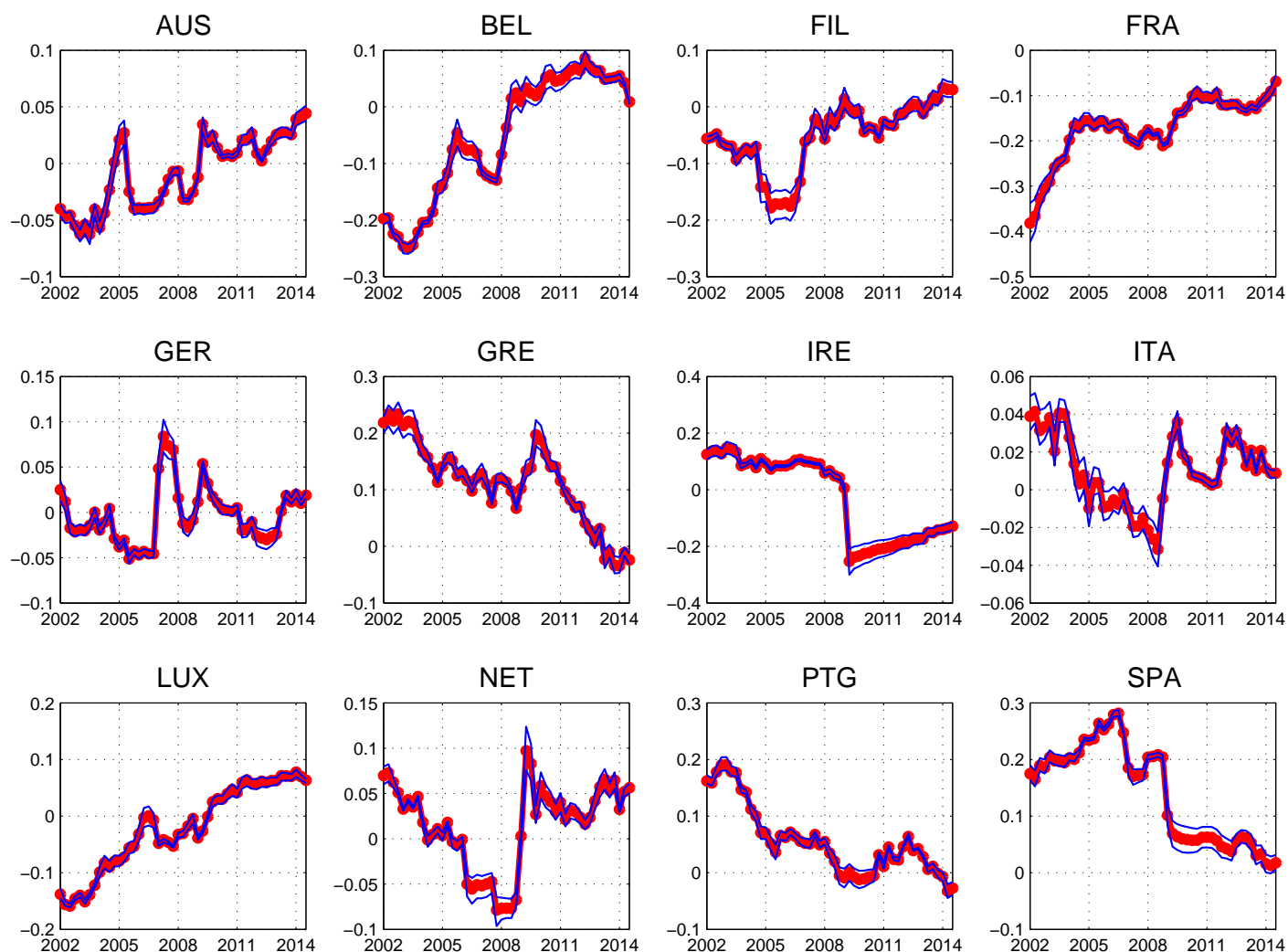
Note to Figure 5. The Figure shows the idiosyncratic cycles $\psi_{i,t}$ (red dotted line) together with the 68% confidence interval derived from the state covariance estimated with the Kalman filter.

Figure 6: Idiosyncratic cycles $\psi_{i,t}$ since the introduction of the euro



Note to Figure 6. The Figure shows the idiosyncratic cycles $\psi_{i,t}$ (red dotted line) together with the 68% confidence interval derived from the state covariance estimated with the Kalman filter.

Figure 7: Idiosyncratic intercepts $\gamma_{i,t}$



Note to Figure 7. The figure shows the maximum likelihood estimates of the idiosyncratic intercepts together with a 68% confidence interval. To compute the confidence interval we draw the static parameter from their distribution and run the filter 5000 times, we then report 16th and 84th percentile of the resulting empirical distribution at every period (see Hamilton, 1986).

Figure 8: Common volatility $\log \sigma_{\eta,t}$

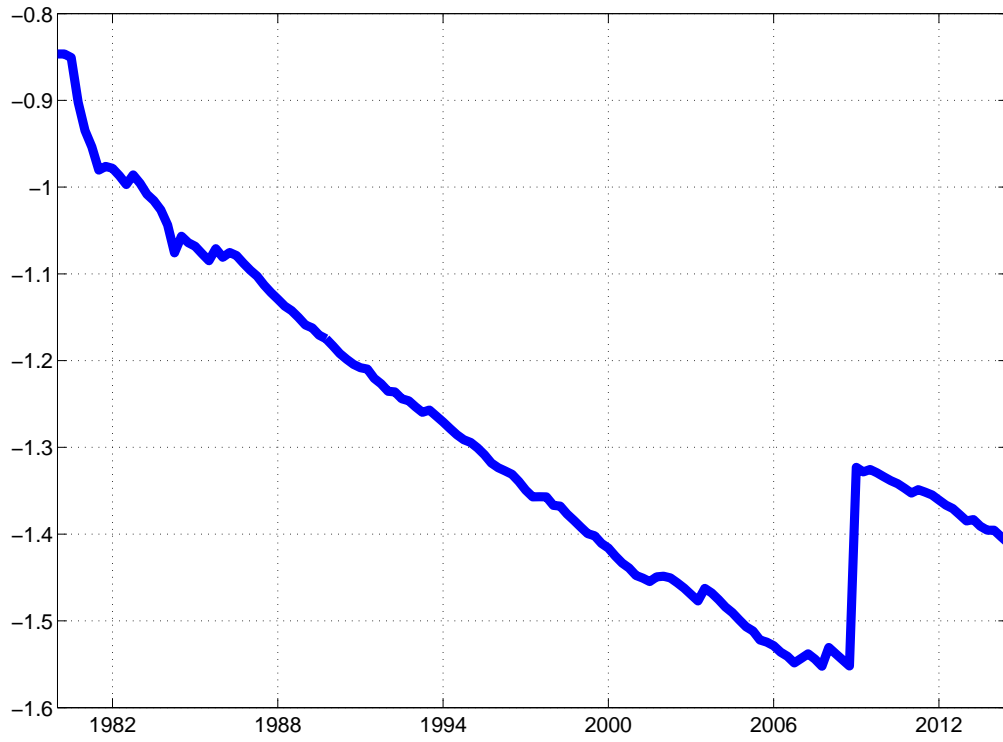


Figure 9: Cross sectional standard deviation of idiosyncratic volatilities

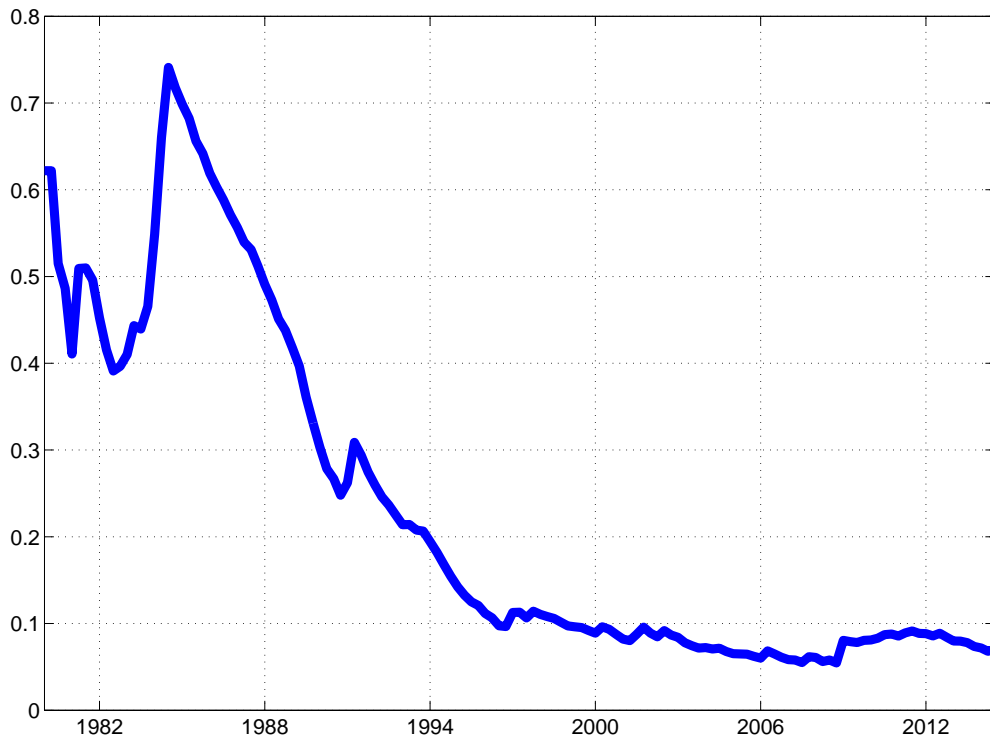


Figure 10: Share of common to overall volatility

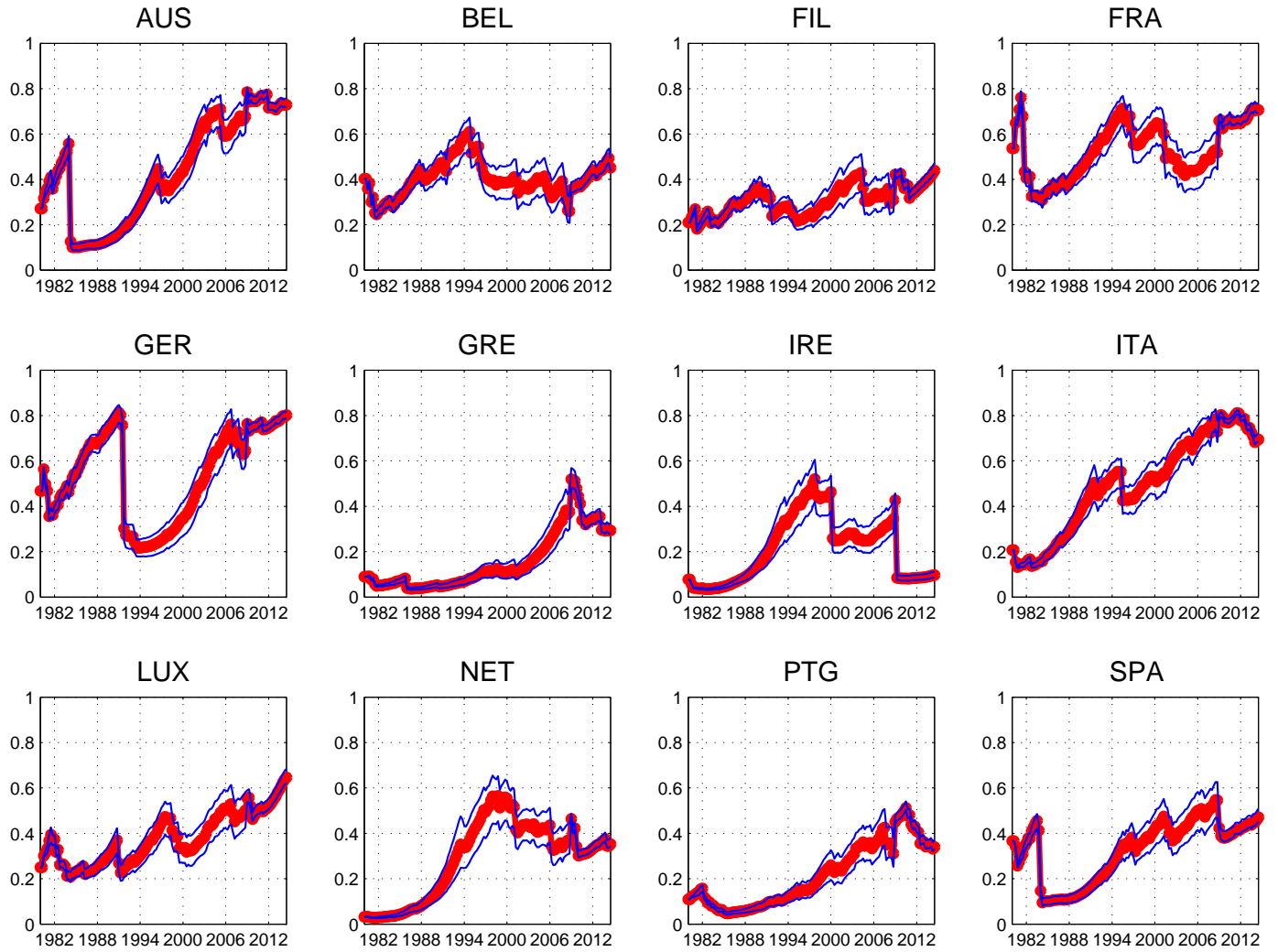
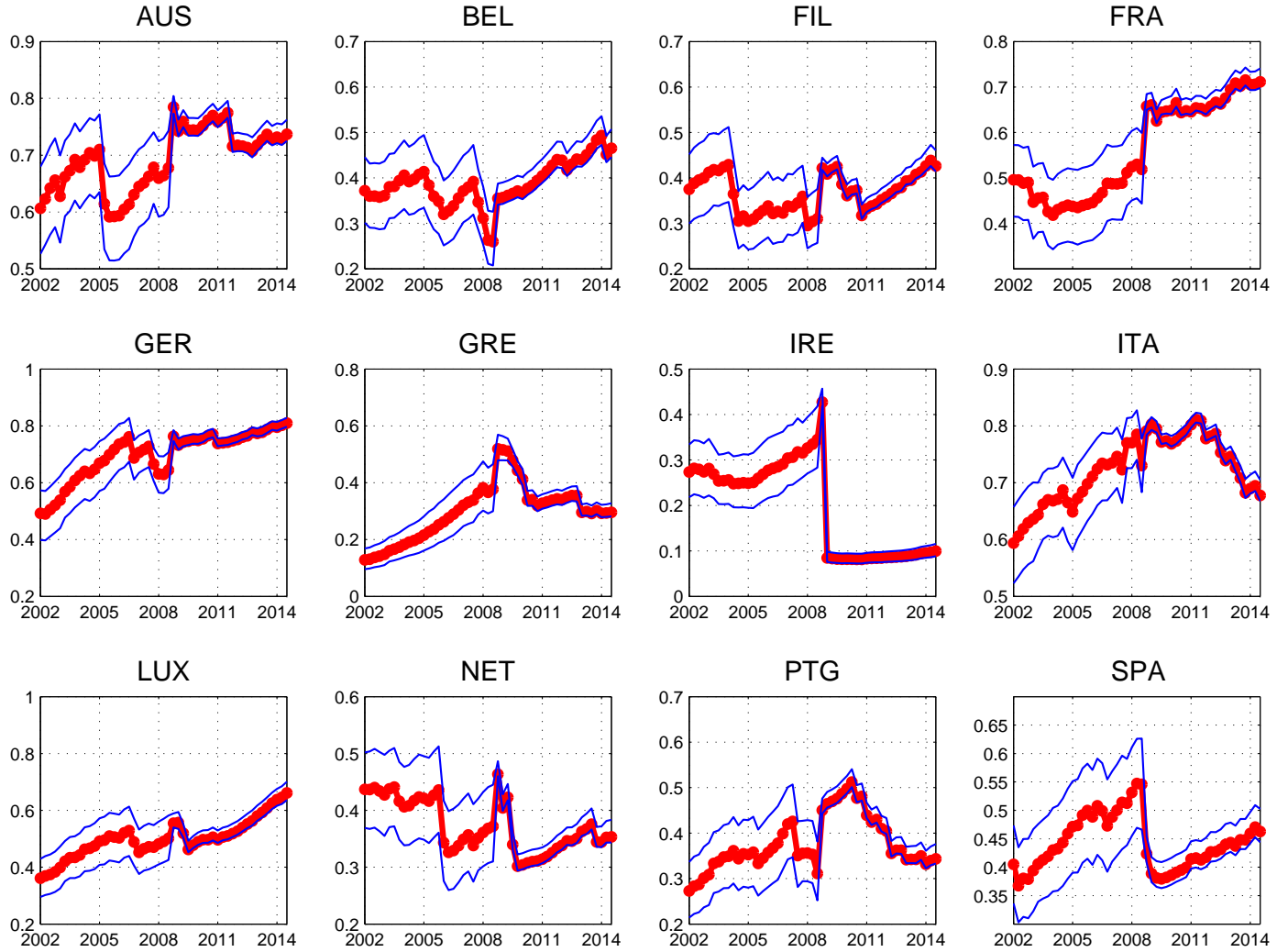


Figure 11: Share of common to overall volatility since the introduction of the euro



Note to Figure 11. The share is computed at each point in time as $\frac{\sigma_{\eta,t}^2}{\sigma_{\eta,t}^2 + \sigma_{\epsilon,t}^2}$, i.e. as the ratio of the variance of the common trend to the sum of the variance of the common trend and of the idiosyncratic cycle. To compute the confidence interval we draw the static parameter from their distribution and run the filter 5000 times, we then report 16th and 84th percentile of the resulting empirical distribution at every period (see Hamilton, 1986).

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