Corrigendum
Involution Products in Coxeter Groups
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S.B. Hart and P.J. Rowley

In [1], Theorem 2.4 states a well-known result on Coxeter groups which gives conditions under which the stabilizer of a nonzero vector is a proper parabolic subgroup. However the hypothesis of this result is incorrectly stated in our paper: it holds for finite Coxeter groups but is not true in general for infinite Coxeter groups. We are grateful to an anonymous referee of a subsequent paper for pointing this out. As a consequence, the proof of Theorem 1.1 in [1], which uses Theorem 2.4, is incomplete. Here we complete the proof of Theorem 1.1 without recourse to Theorem 2.4.

Theorem 1.1 states that if X is a strongly real conjugacy class of a Coxeter group W (not necessarily finite), then there exists $w_* \in X$ such that $e(w_*) = 0$. That is to say, there are involutions σ, τ of W such that $w_* = \sigma\tau$ and $\ell(w) = \ell(\sigma) + \ell(\tau)$. At the point in the proof where Theorem 2.4 is used, we have established that zy is an element of X where z and y are involutions with the following properties. First, y is the central involution of some standard parabolic subgroup W_J of W. The involution z has the property that $\ell(gzg^{-1}) \geq \ell(z)$ for all $g \in W_J$. It follows that if $\ell(zr) < \ell(z)$ for any $r \in J$, then $rzr = z$ and $z \cdot \alpha_r = -\alpha_r$.

Now let $K = \{r \in J : \ell(zr) < \ell(z)\}$. From the above we know that $z \cdot \alpha_r = -\alpha_r$ for all $r \in K$. If K is nonempty then, as $\Phi_K^+ \subseteq N(z)$, Φ_K^+ is finite. Therefore W_K has a unique longest element w_K, which is an involution, and $N(w_K) = \Phi_K^+$. If $K = \emptyset$ we set $w_K = 1$. In all cases, since y is central in W_J and $w_K \in W_J$, we see that $w_K y = y w_K$ is an involution. Moreover $zr = rz$ for all $r \in K$, and thus zw_K is also an involution. Let $\sigma = zw_K$ and $\tau = w_K y$. Then $\sigma\tau = zy \in X$. Moreover z and y both act as -1 on Φ_K^+. Thus, by Lemma 2.2,

$$N(\sigma) = N(z) \setminus [-z \cdot N(w_K)] = N(z) \setminus N(w_K)$$

and

$$N(\tau) = N(y) \setminus [-y \cdot N(w_K)] = N(y) \setminus N(w_K) = \Phi_J^+ \setminus N(w_K).$$

Consider $r \in J$. If $r \in K$, then $\alpha_r \in N(w_K)$ and so $\alpha_r \notin N(z) \setminus N(w_K) = N(\sigma)$. On the other hand if $r \in J \setminus K$ then by definition of K, $\alpha_r \notin N(z)$ and hence $\alpha_r \notin N(\sigma)$, which is after all a subset of $N(z)$. Hence for all $r \in J$ we have $\alpha_r \notin N(\sigma)$. This implies that $N(\sigma) \cap \Phi_J^+ = \emptyset$, because every positive root in Φ_J^+ is a positive linear combination of some $\alpha_r, r \in J$. But $N(\tau) \subseteq \Phi_J^+$ and therefore $N(\sigma) \cap N(\tau) = \emptyset$. Hence, by Lemma 2.2, $\ell(\sigma\tau) = \ell(\sigma) + \ell(\tau)$. Setting $w_* = \sigma\tau$ we have $w^* \in X$ and $e(w_*) = 0$, so completing the proof of Theorem 1.1. □

References

[1] S. B. Hart and P. J. Rowley *Involution Products in Coxeter Groups*, J. Group Theory 14 (2011), no.2, 251 - 259.