Optimal PMU Placement using Topology Transformation Method in Power Systems

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Short running title: OPP using Topology Transformation Method

Abstract - Optimal phasor measurement units (PMUs) placement involves the process of minimizing the number of PMUs needed while ensuring the entire power system completely observable. A power system is identified observable when the voltages of all buses in the power system are known. This paper proposes selection rules for topology transformation method that involves a merging process of zero-injection bus with one of its neighbors. The result from the merging process influenced by the selection of bus selected to merge with the zero-injection bus. The proposed method will determine the best candidate bus to merge with zero-injection bus according to the three rules created in order to determine the minimum number of PMUs required for full observability of the power system. In addition, this paper also considered the case of power flow measurements. The problem formulated as integer linear programming (ILP). The simulation for the proposed method tested by using MATLAB for different IEEE bus systems. The explanation of the proposed method is demonstrated by using IEEE 14-bus system. The results obtained in this paper proved the effectiveness of the proposed method since the number of PMUs obtained are comparable with other available techniques.

Keywords: Integer linear programming; Phasor measurement unit; Power flow measurements; Power system measurements; Topology, Zero-Injection Bus

Introduction

As shown in the biggest blackout in North American history, one of the factors that caused the incident was the lack of real-time data gathering during the incident. This prevented the necessary steps from being taken before the incident happened, leading to the catastrophic blackout. Fifty million people in eight US states and two Canadian provinces were affected by the incident [1].

Following that incident, phasor measurement unit (PMU) became an interesting solution because of its ability to be used as a measurement tool that can provide synchronized phasor measurements [2]. Synchronized phasor measurements are achieved using the Global Positioning System (GPS), which makes it possible to obtain real-time data down to the microsecond [3, 4]. This knowledge encourages better monitoring of a power system because it allows one to detect, anticipate, and correct problems during irregular system conditions [2]. Hence, an efficient operation of power system increased by having a PMU installed in it. In spite of the fact that PMU can improve the monitoring of a power system, the cost of the PMU itself limits the number of PMUs that one can consider to install in the power system. Furthermore, it is not necessary to install PMU at all buses since the voltage phasor of the bus incident to the PMU installed bus can be computed with branch parameter and branch current phasor measurement [7, 8]. Thus, it proves by having optimal placement of PMUs in power system sufficient to make the whole network observable [5, 6]. However, this has not stopped the growth of interest for the development of PMU-based applications [9]. PMU applications for transmission system operation and control considered mature in recent years [10]. This has further encouraged engineers and researchers to find the best algorithm and method to identify the optimal PMU placement (OPP) in the power system for the intended PMU applications.

The PMU placement technique using spanning trees of a power system graph was proposed [11], from which the concept of “depth-of-unobservability” was then introduced. The simulated annealing method and graph theory were used to develop an algorithm that managed to minimize the size of the PMU set and ensured the observability of the system [12].

Bei Xu et. al [13] adopted integer linear programming (ILP) approach which allows easy analysis of network observability for mixed measurement sets based on conventional measurements. It was further enhanced through topology modification by merging the bus that has injection measurement with one of its neighbors [14]. Bei Gou [15] introduced a simpler algorithm that was then revised for the cases of redundant PMU placement, full observability and incomplete observability [16]. Dua et al [17] and Abbasy et al [18] overcome a single PMU loss by multiplied inequalities for every constraint with two which ensure every bus will be monitored by at least two PMUs. Meanwhile, measurement redundancy was considered and extended it to consider a practical limitation on the maximum number of PMU channels [19]. Branch and bound (B&B) method was proposed by Mohammadi-Ivatloo et al [20] to solve an OPP problem considering secondary voltage control. Nonlinear constraints were formed when considering an adjacent zero-injection bus based on the hybrid topology transformation. Differential evolution (DE) optimization was adopted by Al-Mohammed et al [21] to solve the OPP problem. Chakrabarti et al [22] used exhaustive search (ES) algorithm where the authors claimed it gave better results than the method used by Bei Xu et al [13] based on the uniform measurement redundancies obtained in the results. Mixed integer linear programming (MILP) was used to solve the OPP problem by considering PMU placement and maximum redundancy of the system simultaneously with the maintenance of system reliability [23]. Binary particle swarm optimization (BPSO) method was used in the research made by Ahmadi et al [24] and Rather et al [25], which is an extension of the conventional particle swarm optimization (PSO) method to solve OPP problems. PSO is a population-based search algorithm based on simulation of the social behavior of birds within a flock [26]. The two researches adopted different approaches: measurement redundancy [24]; measurement redundancy and cost [25].

The existence of zero-injection bus can also help reduce the number of PMUs needed. Most of the studies adapted merging method to deal with ZIB. However, there are two limitations when using merging method which are to identify the exact PMUs placement and the importance of selecting the right bus to merge. Hence, this paper proposes three rules to overcome these limitations. The three rules developed will evaluate the best candidate bus to merge with ZIB. The results obtained using the proposed method will give a definite PMU placement location. Additionally, the existence of power flow measurements is also adopted with the proposed method. Note that, the discussion made in this paper only involves PMU measurements. SCADA measurements are not considered in this paper.

This paper organized into seven sections including this section. Section 2 presents the objective function for PMU placement problem. Section 3 explores the PMU placement rules to determine the topological observability of power system. A detailed explanation of the proposed method explained in Section 4. Section 5 presents the case study for the proposed method by using IEEE 14-bus system. The simulation results obtained from MATLAB software for each IEEE bus systems presented in Section 6. Each result and the flow of the program highlighted in this section to ensure better understanding of the method presented. Section 7 concludes this paper by highlighting the key elements and the contribution of this paper.

PMU Placement Formulation

The objective in the OPP is to find the minimum number of PMUs required and its location in the power system to achieve full network observability. Thus, the objective function is formulated as below:

$$\min \sum_{k=1}^{N} x_k \qquad (1)$$

$$\text{subject to: } [A] \times [X] \geq [b]$$

where N is a number of system buses and [A] is a binary connectivity matrix. Entries for matrix [A] defined as follows:

$$A_{i,j} = \begin{cases} 1 & \text{if} \quad i = j \\ 1 & \text{if} \quad i \text{ and } j \text{ are connected} \\ 0 & \text{if} \quad \text{otherwise} \end{cases} \tag{2}$$

Meanwhile [X] is defined as a binary decision variable vector where $[X] = [x_1\ x_2\ x_3\ \cdots\ x_N]^T$ and $x_i \in \{0,1\}$:

$$x_i = \begin{cases} 1 & \text{if a PMU is installed at bus } i \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

$$[b] \text{ is a column vector where } [b] = [1\ 1\ 1\ \cdots\ 1]^T_{1\times N} \tag{4}$$

PMU Placement Rules

There are two types of observability analysis used to analyze the power system, which are numerical and topological observability. In this paper, a topological observability analysis is used. A power system achieves full observability if all buses in it are observable. A bus in the power system is identified as observable if its voltage can be directly or indirectly measured by using pseudo-measurements [27].

The ability of PMU to measure the voltage phasor at the installed bus and the current phasor of all the branches connected to the PMU installed bus can help determine the remaining parameters to use for indirect measurements. By using Ohm’s law and Kirchhoff’s Current Law (KCL), bus adjacent to PMU installed bus can have its voltage phasor and branch currents value known. Following are the PMU placement rules to identify bus as observable:

Rule 1 A bus that has a PMU installed on it will have its voltage phasor and all branches currents incident to it measured by the PMU.

Rule 2 By applying Ohm's law, the voltage phasor at one end of a branch current can be calculated if voltage phasor at the other end of branch current is known.

Rule 3 If the voltages at both ends of a branch are known, the branch current can be computed by using Ohm's law

In order to explain how these rules work, consider Fig. 1(a). If a PMU is placed on bus 1, the voltage phasor of bus 1 and the branch currents between 1-2 and 1-3 can be obtained (using Rule 1). Since branch 1–2 and 1–3 are now observed and are connected to the observed bus (bus 1), the voltage of bus 2 and 3 can be observed (Rule 2). By observing bus 2 and 3, branch current 2–3 can be observed (Rule 3).

A ZIB is another factor that can possibly reduce the number of PMUs required to achieve complete observability. There is no generator that injects power or a load that consumes power from this bus [9]. The sum of flows on all branch currents associated with ZIB is zero according to KCL. Network observability can be assessed with the presence of ZIB based on the rules below [29, 30]:

Rule 4 When buses incident to an observable ZIB are all observable except one, the unobservable bus can be identified as observable by applying the KCL at the ZIB

Rule 5 When buses incident to an unobservable ZIB are all observable, the ZIB will be identified as observable by applying the node equation

Rule 6 A group of unobservable ZIB which is adjacent to observable buses will be identified as observable by obtaining the voltage phasors of ZIB through the node equation

To explain these rules, consider Fig. 1(b). Bus i is a ZIB that incident to bus {1, 2, 3, 4}. For rule 4, consider that bus {i, 2, 3, 4} are observable and bus 1 is unobservable. By applying KCL at bus *i*, branch current *i*–1 can be calculated. For rule 5, consider bus {1, 2, 3, 4} are observable and bus *i* is unobservable. By applying the node equation in this situation, voltage phasor of bus *i* can be calculated. For rule 6, consider Fig. 1(c), where all buses are incident to the ZIB, bus {*i*, *j*} are observable. By using the node equation, both voltage phasors of bus {*i*, *j*} can be calculated. These rules allow buses incident to the ZIB to be observable without the need of placing a PMU on it. Therefore, it helps reduce the number of PMUs to be placed in the power system.

Power flow measurement can be used to determine other parameters in the power system. It allows one to determine other quantities provided certain quantities are known [31]. When power flow measurements are present, the voltage at the other end can be calculated by taking all the known real and reactive power flows at each bus including the voltage [2, 27, 28]. Previous studies have found that incorporated power flow measurement and ZIB together will further reduced number of PMUs needed. To reach this objective, the method proposed by Bei Xu et al [13] was used to deal with the existence of power flow measurement. According to research made by Bei Xu et al [13], the constraints involved with power flow measurement will be altered. The combination method introduced by Bei Xu et al [13] and the authors' proposed method will be incorporated when dealing with the OPP for the case of considering power flow measurement and ZIB.

Proposed Method

Topology transformation method involves the merging process of ZIB and one of its neighbors. This means the number of buses in a power system will be reduced by one for each available ZIB. Furthermore, the merging process causes the network topology of a power system to be modified and network equations need to be redefined to reflect the changes. As stated by Abbasy et al [18], the result from the merging process is different for each candidate bus available to merge with ZIB. The authors did not elaborate further how each merged bus was selected. In addition, if the results require a PMU to be placed at the merged bus, it is possible for the PMU to be placed at the original ZIB or at the bus it is merged with, or at both buses. These are the limitations that the proposed method will address by selecting the best candidate bus to merge with ZIB and to provide the exact location for PMU placing.

The proposed method considered the existence of ZIB and radial bus in a power system. Radial bus is referring to bus that has only one adjacent bus connected to it. Placing a PMU at a radial bus will ensure a maximum of two bus to be observed which is radial bus and its neighbor. Meanwhile placing a PMU at a bus that is adjacent to radial bus will ensure more than two bus observable. Thus, to ensure better network coverage, a PMU will be pre-assigned at a bus that is adjacent to radial bus. The proposed method consists of three rules for which every candidate bus will be evaluated in sequence. Following are the three rules:

1) ***Rule A*:** Merge ZIB with its adjacent bus that is radial bus

In the case where ZIB is incident to a radial bus, the merging process will take place between both buses. In the situation where after the merging process, the merged bus is connected to two or more buses, a PMU does not need to be pre-allocated. Meanwhile, if the merged bus is connected to two buses and one of them is a ZIB, a PMU must be pre-allocated to a bus that is not a ZIB.

Consider Fig. 2(a), where bus *i* is a ZIB and bus 2 is a radial bus. Bus 2 will be selected to merge with bus *i*. Bus {1, 3} will be connected to bus 2’ after the merging process and bus *i* is removed from the network. Since neither bus 1 nor bus 3 is a ZIB, it is not necessary to pre-allocate a PMU to either of these buses. In the case where bus 3 is a ZIB, a PMU must be pre-allocated at bus 1 to ensure bus 2’ is observable.

2) ***Rule B:*** If the adjacent bus of ZIB has the most number of bus connected to it, and one of its neighbor bus connected to the same ZIB, this adjacent bus will be selected to merge with the ZIB

This is to increase bus tendency to be picked as a PMU placement because of the better network coverage among other buses that are adjacent to the ZIB.

Consider Fig. 2(b), where bus *i* is a ZIB that is incident to bus {1, 2, 3}. The outward lines from bus {1, 2, 3} mean it is connected to more buses that are not illustrated in Fig. 2(b), to simplify the diagram. It can be seen that bus 1 is connected to more buses than any other bus that is incident to bus *i* follow by bus 3. However, since bus 2 and 3 are incident to each other and both are connected to the same ZIB, they will be considered to merge with bus *i*. To decide whether bus 2 or 3 that will be selected to merge with bus *i*, the bus that has the maximum number of neighbors among the buses involved will be chosen, and in this case bus 3 is the best candidate to be merged.

3) ***Rule C:*** Merge ZIB with its adjacent bus that has the most number of bus connected to it.

This scenario encourages better network coverage because it can reach more buses compared to the other adjacent buses when it is selected to merge with the ZIB.

Consider Fig. 2(c), where bus *i* is a ZIB that is incident to bus {1, 2}. As we can see, bus 1 has the maximum number of neighbors compared to bus 2. Hence, it is selected to merge with bus *i*. Like previous rules explained in this section, bus *i* is removed from the network after the topology transformation.

Note that, in all rules explained above, bus that has been merged is excluded for the next merging process. This means bus can only be merged once and will not be considered as a candidate bus for another merging process. Flowchart depicted in Fig. 3 shows how each bus is evaluated based on the rules above.

Case Studies

The effectiveness of the proposed method in solving the OPP problem is presented by using three experimental cases. All cases are elaborated in detail respectively by using IEEE 14-bus system illustrated in Fig. 4 and simulated by using MATLAB. Following are the three cases:

a) Case I: Ignoring conventional measurement for full network observability

For this case, ZIB and power flow measurements are not considered. In addition, no PMU is pre-allocated for the bus that is incident to the radial bus. By using (2), the binary connectivity matrix A is formed as follow:

$$[A] = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \tag{5}$$

The final inequality constraints of matrix A is formulated as follows:

$$f(x) = \begin{array}{lll} f_1 = x_1 + x_2 + x_5 & \geq 1 & (6) \\ f_2 = x_1 + x_2 + x_3 + x_4 + x_5 & \geq 1 & (7) \\ f_3 = x_2 + x_3 + x_4 & \geq 1 & (8) \\ f_4 = x_2 + x_3 + x_4 + x_5 + x_7 + x_9 & \geq 1 & (9) \\ f_5 = x_1 + x_2 + x_4 + x_5 + x_6 & \geq 1 & (10) \\ f_6 = x_5 + x_6 + x_{11} + x_{12} + x_{13} & \geq 1 & (11) \\ f_7 = x_4 + x_7 + x_8 + x_9 & \geq 1 & (12) \\ f_8 = x_7 + x_8 & \geq 1 & (13) \\ f_9 = x_4 + x_7 + x_9 + x_{10} + x_{14} & \geq 1 & (14) \\ f_{10} = x_9 + x_{10} + x_{11} & \geq 1 & (15) \\ f_{11} = x_6 + x_{10} + x_{11} & \geq 1 & (16) \\ f_{12} = x_6 + x_{12} + x_{13} & \geq 1 & (17) \\ f_{13} = x_6 + x_{12} + x_{13} + x_{14} & \geq 1 & (18) \\ f_{14} = x_9 + x_{13} + x_{14} & \geq 1 & (19) \end{array}$$

The above constraints imply that, for example, based on constraint (7), if a PMU is placed at bus 2, bus 1,2,3,4 and 5 are observable. The constraints (6)-(19) are then simulated using MATLAB and the result obtained from the simulation is:

$$[X] = [0\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 1\ 0\ 0\ 1\ 0]^T \tag{20}$$

Based on constraint (20), a PMU must be placed on bus 2, 8, 10, and 13 respectively in order to ensure the whole system is completely observable.

b) Case II: Existence of ZIB for full network observability

Based on Fig. 4, bus 7 is a ZIB and bus 8 is a radial bus. Since bus 8 is a radial bus, it is selected to be merged with the ZIB according to Rule A as mentioned in Section 4. This merging process means the constraint for bus 7 is removed from the equation and bus 8 is now connected directly to buses 4 and 9. Next, since this process involves a radial bus, a PMU must be pre-allocated to one of the buses that is incident to it.

However, since neither bus 4 nor bus 9 is a ZIB, a PMU is not pre-allocated to encourage more possible solutions. In the case where bus 4 is a ZIB, a PMU will be pre-allocated to bus 9 to ensure that bus 8’ is observable.

This topology transformation means the constraints for bus {4, 7, 8, 9} have changed. Note that the constraint for bus 7 is eliminated since it no longer exists after the topology transformation. Meanwhile, the constraints for bus {4, 8, 9} are updated to reflect the topology transformation made during the merging process.

$$f(x) = \begin{array}{lll} f_4 = x_2 + x_3 + x_4 + x_5 + x_{8'} + x_9 & \geq 1 & (21) \\ f_{8'} = x_4 + x_{8'} + x_9 & \geq 1 & (22) \\ f_9 = x_4 + x_{8'} + x_9 + x_{10} + x_{14} & \geq 1 & (23) \end{array}$$

From these newly formed constraints, a total of three PMUs need to be placed at bus {2, 6, 9} to ensure full observability of the network. Fig. 5 below shows the topology transformation concerning ZIB before and after the merging process.

c) ***Case III:*** Existence of ZIB and power flow measurements for full network observability

In this case, consider the power flow measurements exist on branch {1–5}, {6–11}, and {9–10}. When considering the existence of power flow measurements and ZIB in OPP, it is important that the power flow measurement is solved first followed by ZIB. If it is done opposite to the proposed method, the result of the merging could imbalance the topology thus leads an to infeasible solution. This is likely to happen in the situation where power flow is existed next to two ZIBs. Hence, for one to apply this proposed method, when dealing with power flow and ZIB, the power flow needs to be merged before ZIBs are merged.

As mentioned earlier in this paper, in the case of considering power flow measurements, if one of the voltage buses is known, the value of the voltage at the other end can be computed. Thus, the constraints that are related with the measured branch can be merged into a single constraint. The new merged constraint makes certain that as long as the bus voltage at one end of the branch is observable, the voltage at the opposite bus will also be observable. The following are the final constraints involved after the merging process. Note that the constraints for bus {5, 10, 11} are eliminated since they have merged with the opposite bus. Notice also that the new constraint for bus 9 (26) is the consequence of Equations (14) and (23).

$$f(x)=\begin{cases} f_{1'} = x_1 + x_2 + x_4 + x_5 + x_6 & \geq 1 \quad (24) \\ f_{6'} = x_5 + x_6 + x_{10} + x_{11} + x_{12} + x_{13} & \geq 1 \quad (25) \\ f_{9'} = x_4 + x_{8'} + x_9 + x_{10} + x_{11} + x_{14} & \geq 1 \quad (26) \end{cases}$$

From constraints (24)-(26), it can be seen that for full system observability two PMUs are required to be placed at bus 4 and 13.

Table 1 summarizes the number of PMUs required for each case using the IEEE 14-bus system described in this section. Notice that the number of PMUs required decreases when considering power flow measurement and ZIB.

Results and Discussion

The flow of the ILP method is depicted in Fig. 6. All simulations results obtained based on the assumption that each PMU has the maximum number of channels and the cost of each PMU is the same. Notice that for Case I, the radial bus is not excluded from the candidates for PMU placement as illustrated in the program flowchart in Fig. 6.

Table 2 shows the locations of ZIB and radial bus in each IEEE bus system simulated in this paper. Meanwhile, Table 3 presents the locations of power flow measurement introduced for the IEEE 14, 57, and 118-bus systems. Table 4 shows the comparison for the number of PMUs required for Case I, II, and III for each IEEE bus systems using the proposed method. From Table IV, without considering conventional measurements the number of PMUs required for all bus systems tested is obviously higher than the number of PMUs required when considering conventional measurements. One can consider the number of PMUs required for the IEEE 118-bus system. Notice that 32 PMUs are required for complete observability when ignoring conventional measurement. The number of PMUs required is reduced to 28 PMUs when considering ZIB. This is possible because of ZIB presence allows at least one bus to be calculated using pseudo-measurements by applying KCL at ZIB. Hence, the number of PMUs required is expected to be reduced by at least one for each ZIB available in the system depending on the location of the ZIB in each IEEE bus system. For example, in the IEEE 14-bus system with the introduction of one ZIB, the number of PMUs required is one less compared to the case when conventional measurement is ignored. However, it is interesting to note that this is not always the case. For example, in the IEEE 24-bus system, the number of PMUs required is only one less even with the presence of four ZIBs. However, one can conclude that the number of PMUs required are lower when ZIB is considered in the power system.

Consider the comparison between Case I and Case III for the IEEE 118-bus system in Table 4. It can be noted that the number of PMUs required is further reduced to 16, which is half the number required for Case I, and lower than the Case II in which ZIB is considered, which requires 28 PMUs. The existence of power flow measurement allows the voltage of the incident bus to be calculated if the voltage for one of the buses involved is known. This means it is enough to ensure one of the buses involved is observable by a PMU or pseudo-measurement as long the voltage for one of the buses is known. When combined with ZIB, the number of PMUs is expected to be further reduced since the method used is identical to that used for the case of considering ZIB.

Table 5 shows the full locations of the PMUs for all cases for every bus system simulated. As can be seen from the Table 5, PMUs are not placed at ZIB for the case of considering ZIB and power flow measurements. The decision to remove the constraints for ZIB and power flow measurements as the candidates for PMU placement has made this possible.

The simulation results for the case considering ZIB are compared with existing techniques in Table 6. Based on the comparison results above, the number of PMUs required for the proposed method is comparable and consistent across other methods used in existing techniques. It should be noted that the ILP method can provide the minimum number of PMUs required for the larger system.

The proposed method is specifically compared with the results obtained by Rather et al [25] for New England 39-bus system and IEEE 57-bus system as shown in Table 7 below. As can be noted from the table, measurement redundancy is larger when using the proposed method for both bus system network despite having the same number of PMUs installed in each bus system.

Conclusion

The simulation results confirm the method proposed in this paper can be used to solve the OPP problem. The rules created to deal with ZIB managed to produce comparable result with other existing methods. It also gives better measurement redundancy based on BOI and SORI value which evaluate the quality of PMUs placements set. In addition, the PMU locations given by this method is accurate unlike other merging technique. The proposed method also shows that it can be incorporated with power flow measurement to find optimal PMU placement. Furthermore, pre-assigned PMUs strategy help to reduce the total number of possible candidates for PMU placement and hence allow consideration to be given to other PMU placements in the power system. This paper will help the researchers as a platform to understand how to deal with ZIB in order to achieve OPP in power system since the rules developed are easy to implement and understand.

References

[1] Andersson G, Donalek P, Farmer R, Hatziargyriou N, Kamwa I, Kundur P, et al. Causes of the 2003 Major Grid Blackouts in North America and Europe, and

Recommended Means to Improve System Dynamic Performance. IEEE Trans Power Syst. 2005 Nov;20(4):1922–8.

[2] Xu Dongjie. Comparison of several PMU placement algorithms for state estimation. Eighth IEE International Conference on Developments in Power System Protection. IEE; 2004. p. 32–5.

[3] Morais H, Vancraeyveld P, Pedersen AHB, Lind M, Johannsson H, Ostergaard J. SOSPO-SP: Secure Operation of Sustainable Power Systems Simulation Platform for Real-Time System State Evaluation and Control. IEEE Trans Ind Informatics. 2014 Nov;10(4):2318–29.

[4] Wang Y, Wang C, Li W, Li J, Lin F. Reliability-Based Incremental PMU Placement. IEEE Trans Power Syst. 2014 Nov;29(6):2744–52.

[5] Ghosh D, Ghose T, Mohanta DK. Communication Feasibility Analysis for Smart Grid With Phasor Measurement Units. IEEE Trans Ind Informatics. 2013 Aug;9(3):1486–96.

[6] Gou B, Kavasseri RG. Unified PMU Placement for Observability and Bad Data Detection in State Estimation. IEEE Trans Power Syst. 2014 Nov;29(6):2573–80.

[7] Huang L, Sun Y, Xu J, Gao W, Zhang J, Wu Z. Optimal PMU Placement Considering Controlled Islanding of Power System. IEEE Trans Power Syst. 2014 Mar;29(2):742–55.

[8] Mazhari SM, Monsef H, Lesani H, Fereidunian A. A Multi-Objective PMU Placement Method Considering Measurement Redundancy and Observability Value Under Contingencies. IEEE Trans Power Syst. 2013 Aug;28(3):2136–46.

[9] Sanchez-Ayala G, Aguerc JR, Elizondo D, Lelic M. Current trends on applications of PMUs in distribution systems. 2013 IEEE PES Innovative Smart Grid Technologies Conference (ISGT). IEEE; 2013. p. 1–6.

[10] Bottura R, Borghetti A. Simulation of the Volt/Var Control in Distribution Feeders by Means of a Networked Multiagent System. IEEE Trans Ind Informatics. 2014 Nov;10(4):2340–53.

[11] Nuqui RF, Phadke AG. Phasor Measurement Unit Placement Techniques for Complete and Incomplete Observability. IEEE Trans Power Deliv. 2005 Oct;20(4):2381–8.

[12] Mili L, Baldwin T, Adapa R. Phasor measurement placement for voltage stability analysis of power systems. 29th IEEE Conference on Decision and Control. IEEE; 1990. p. 3033–8 vol.6.

[13] Bei Xu, Abur A. Observability analysis and measurement placement for systems with PMUs. IEEE PES Power Systems Conference and Exposition, 2004. IEEE; 2004. p. 1472–5.

[14] Bei X, Yoon YJ, Abur A. Optimal Placement and Utilization of Phasor Measurements for State Estimation. Final Proj Report, PSERC. 2005;1–6.

[15] Gou B. Optimal Placement of PMUs by Integer Linear Programming. IEEE Trans Power Syst. 2008 Aug;23(3):1525–6.

[16] Gou B. Generalized Integer Linear Programming Formulation for Optimal PMU Placement. IEEE Trans Power Syst. 2008 Aug;23(3):1099–104.

[17] Dua D, Dambhare S, Gajbhiye RK, Soman SA. Optimal Multistage Scheduling of PMU Placement: An ILP Approach. IEEE Trans Power Deliv. 2008 Oct;23(4):1812–20.

[18] Abbasy NH, Ismail HM. A Unified Approach for the Optimal PMU Location for Power System State Estimation. IEEE Trans Power Syst. 2009 May;24(2):806–13.

[19] Enshaee A, Hooshmand RA, Fesharaki FH. A new method for optimal placement of phasor measurement units to maintain full network observability under various contingencies. Electr Power Syst Res. 2012 Aug [cited 2015 Mar 15];89:1–10.

[20] Mohammadi-Ivatloo B, Hosseini SH. Optimal PMU placement for power system observability considering secondary voltage control. 2008 Canadian Conference on Electrical and Computer Engineering. IEEE; 2008. p. 000365–8.

[21] Al-Mohammed AH, Abido MA, Mansour MM. Optimal placement of synchronized phasor measurement units based on differential evolution algorithm. 2011 IEEE PES Conference on Innovative Smart Grid Technologies - Middle East. IEEE; 2011. p. 1–9.

[22] Chakrabarti S, Kyriakides E. Optimal Placement of Phasor Measurement Units for Power System Observability. IEEE Trans Power Syst. 2008 Aug;23(3):1433–40.

[23] Aghaei J, Baharvandi A, Rabiee A, Akbari MA. Probabilistic PMU Placement in Electric Power Networks: An MILP-Based Multi-objective Model. IEEE Trans Ind Informatics. 2015;11(2):1–1.

[24] Ahmadi A, Alinejad-beromi Y, Moradi M. Optimal PMU placement for power system observability using binary particle swarm optimization and considering measurement redundancy. Expert Syst Appl. 2011;38(6):7263–9.

[25] Rather ZH, Liu C, Chen Z, Thogersen P. Optimal PMU Placement by improved particle swarm optimization. 2013 IEEE Innovative Smart Grid Technologies-Asia (ISGT Asia). IEEE; 2013. p. 1–6.

[26] Havangi R, Taghirad HD, Nekoui MA, Teshnehlab M. A Square Root Unscented FastSLAM With Improved Proposal Distribution and Resampling. IEEE Trans Ind Electron. 2014 May;61(5):2334–45.

[27] Su C, Chen Z. Optimal Placement of Phasor Measurement Units with New Considerations. 2010 Asia-Pacific Power and Energy Engineering Conference. IEEE; 2010. p. 1–4.

[28] Saha Roy BK, Sinha AK, Pradhan AK. An optimal PMU placement technique for power system observability. Int J Electr Power Energy Syst. 2012 Nov [cited 2015 Mar 16];42(1):71–7.

[29] Aminifar F, Khodaei A, Fotuhi-Firuzabad M, Shahidehpour M. Contingency-Constrained PMU Placement in Power Networks. IEEE Trans Power Syst. 2010 Feb;25(1):516–23.

[30] Esmaili M. Inclusive multi-objective PMU placement in power systems considering conventional measurements and contingencies. Int Trans Electr Energy Syst. 2016 March;26(3):609-626.

[31] Meier A. Power Flow Analysis. Electric Power Systems. Hoboken, NJ, USA: John Wiley & Sons, Inc.; 2006. p. 195–228.

Table 1 Number of PMUs required for each case for IEEE 14-bus system.

Bus System Network	Number of PMUs		
	Case I (Ignoring conventional measurement)	Case II (ZIB)	Case III (ZIB and power flow measurements)
IEEE 14	4	3	2
PMU location	2, 8, 10, 13	2, 6, 9	4, 13

Table 2 Location of ZIB and radial bus

Bus system network	Location of ZIB	Location of radial bus
IEEE 14	7	8
IEEE 24	11, 12, 17, 24	7
IEEE 30	6, 9, 22, 25, 27, 28	11, 13, 26
NE-39	1, 2, 5, 6, 9, 10, 11, 13, 14, 17, 19, 22	30, 31, 32, 33, 34, 35, 36, 37, 38
IEEE 57	4, 7, 11, 21, 22, 24, 26, 34, 36, 37, 39, 40, 45, 46, 48	33
IEEE 118	5, 9, 30, 37, 38, 63, 64, 68, 71, 81	10, 73, 87, 111, 112, 116, 117

Table 3 Location of power flow measurements

Bus System	Number of power flow locations	Flow location
IEEE 14	3	1–5, 6–11, 9–10
IEEE 57	14	14–15, 15–45, 18–19, 21–22, 22–38, 24–26, 28–29, 30–31, 34–35, 36–40, 39–57, 47–48, 50–51, 53–54
IEEE 118	32	1–3, 5–6, 11–13, 16–17, 20–21, 22–23, 23–25, 27–28, 29–31, 34–43, 35–36, 41–42, 44–45, 46–48, 50–57, 51–52, 53–54, 56–58, 60–62, 65–66, 66–67, 68–81, 71–73, 75–118, 76–77, 77–82, 78–79, 86–87, 90–91, 95–96, 100–101, 114–115

Table 4 The number of PMUs required for case I, II and III

Bus System Network	Number of PMUs		
	Case I (Ignoring conventional measurement)	Case II (ZIB)	Case III (ZIB and power flow measurements)
IEEE 14	4	3	2
IEEE 24	7	6	N/A
IEEE 30	10	7	N/A
NE 39	13	8	N/A
IEEE 57	17	11	10
IEEE 118	32	28	16

Table 5 Location of PMUs for case I, II and III

Bus System Network	PMU location		
	Case I (Ignoring conventional measurement)	Case II (ZIB)	Case III (Power flow measurements)
IEEE 14	2, 8, 10, 13	2, 6, 9	4,13
IEEE 24	2, 3, 7, 10, 16, 21, 23	1, 2, 8, 16, 18, 23	N/A
IEEE 30	1, 7, 8, 10, 11, 12, 19, 23, 26, 30	1, 7, 10, 12, 19, 24, 30	N/A
NE-39	2, 6, 9, 12, 14, 17, 22, 23, 29, 32, 33, 34, 37	3, 8, 12, 16, 20, 23, 25, 29	N/A
IEEE 57	2, 6, 12, 15, 19, 22, 25, 27, 32, 36, 38, 41, 46, 50, 52, 55, 57	1, 6, 13, 19, 25, 29, 32, 38, 51, 54, 56	1, 3, 6, 9, 25, 32, 38, 41, 51, 53
IEEE 118	2, 5, 10, 12, 15, 17, 21, 25, 29, 34, 37, 41, 45, 49, 53, 56, 62, 64, 72, 73, 75, 77, 80, 85, 87, 91, 94, 101, 105, 110, 114, 116	3, 8, 11, 12, 17, 21, 25, 29, 33, 34, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 91, 94, 102, 105, 110, 114	8, 11, 12, 19, 32, 33, 40, 49, 59, 72, 74, 80, 85, 92, 105, 110

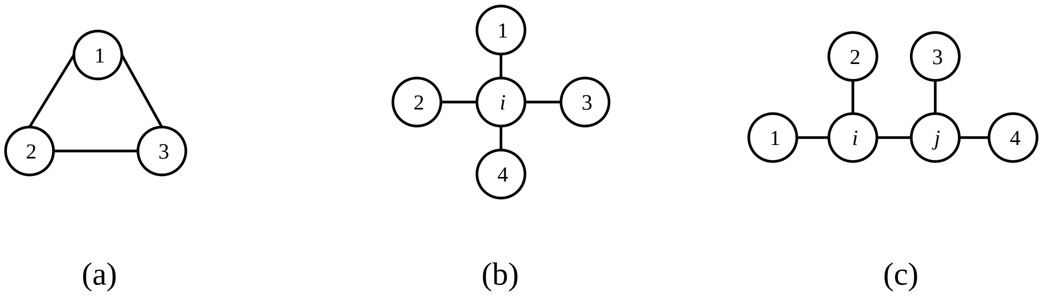
Table 6 Comparison between the proposed method and existing techniques for the case considering ZIB

Method:	Number of PMUs					
	IEEE 14	IEEE 24	IEEE 30	NE-39	IEEE 57	IEEE 118
Proposed	3	6	7	8	11	28
ILP [13]	3	N/A	N/A	N/A	12	29
ILP [18]	3	N/A	7	8	11	28
BPSO [24]	3	N/A	7	N/A	13	29
BPSO [25]	N/A	6	7	8	11	N/A
B&B [20]	N/A	N/A	7	9	12	29
DE [21]	3	N/A	7	8	11	N/A
ES [22]	3	6	7	8	N/A	N/A
ES [28]	3	6	7	8	11	28

Table 7 Comparison of BOI & SORI for case considering ZIB

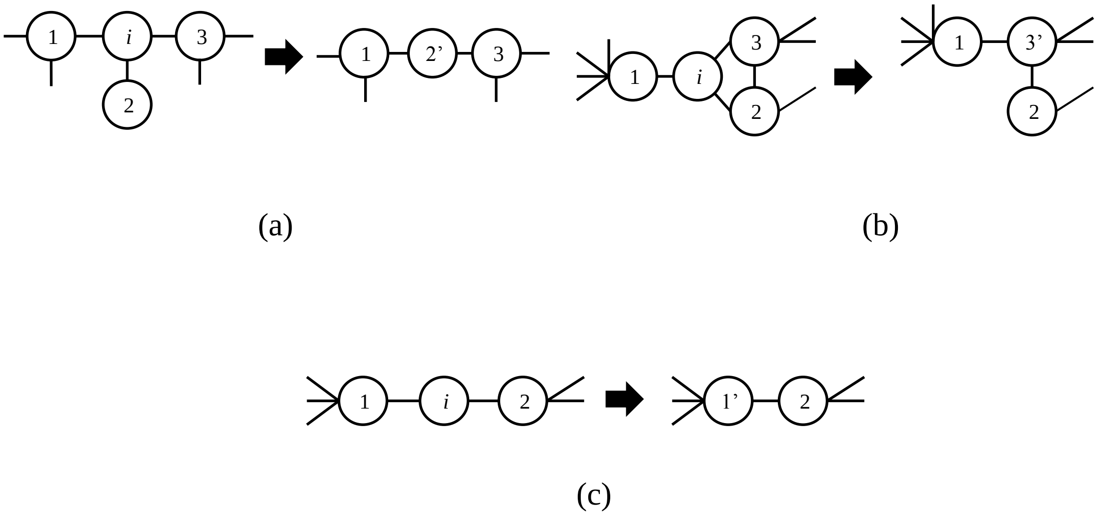
	Proposed Method		Ref [25]	
Bus System	NE 39-bus	IEEE 57-bus	NE 39-bus	IEEE 57-bus
No of PMU	8	11	8	11
PMU location	3,8,12,16,20,23,25,29	1,6,13,19,25,29,32,38,51,54,56	3,8,13,16,23,29,34,37	1,5,13,19,25,29,32,38,51,54,56
BOI*	1,2,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,2,1,1,1,1,2,1,2,1,1,1,1,1,1,1,1,1,1,1,1,1	1,1,1,1,1,1,2,1,1,1,1,1,1,2,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,2,1,1,1,1,1,1,1	1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,2,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1	1,1,1,1,1,1,1,1,1,1,1,1,1,2,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,2,1,1,1,1,1,1,1,1
SORI*	43	60	40	59

BOI = Bus Observability Index
SORI = Summation of Redundancy Index



(a) (b) (c)

Fig. 1 Modeling PMU placement rules



(a) (b)

(c)

Fig. 2 Modeling merging process

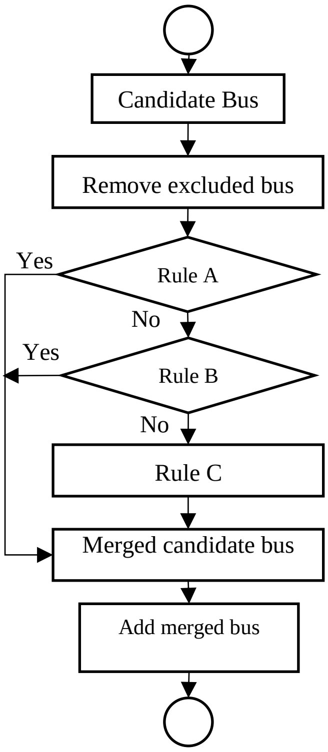


Fig. 3 Flowchart for rules evaluation for candidate bus

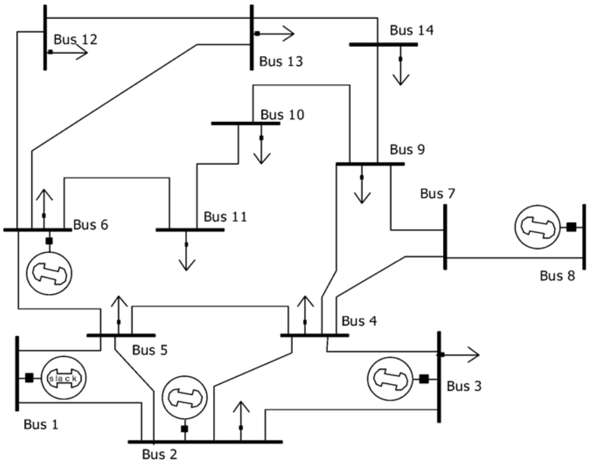


Fig. 4 IEEE 14-bus system. In this bus system, bus 7 is a ZIB and bus 8 is a radial bus [22]

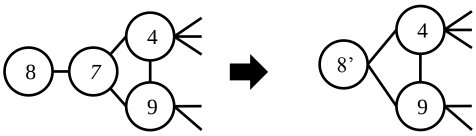


Fig. 5 Modeling ZIB for IEEE 14-bus system before (left) and after (right).

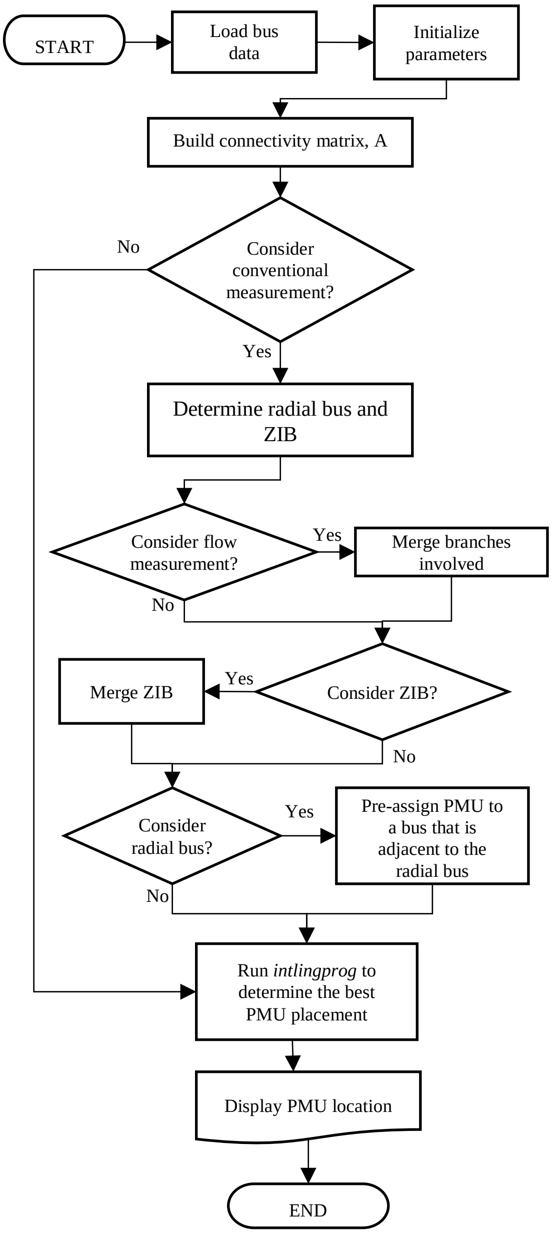


Fig. 6 Flowchart of the implemented MATLAB program