On $\mathcal{H}_\infty$ Estimation of Randomly Occurring Faults for A Class of Nonlinear Time-Varying Systems with Fading Channels

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***Abstract*—This paper is concerned with the finite-horizon $\mathcal{H}_\infty$ fault estimation problem for a class of nonlinear stochastic time-varying systems with both randomly occurring faults and fading channels. The system model (dynamical plant) is subject to Lipschitz-like nonlinearities and the faults occur in a random way governed by a set of Bernoulli distributed white sequences. The system measurements are transmitted through fading channels described by a modified stochastic Rice fading model. The purpose of the addressed problem is to design a time-varying fault estimator such that, in the presence of channel fading and randomly occurring faults, the influence from the exogenous disturbances onto the estimation errors is attenuated at the given level quantified by a $\mathcal{H}_\infty$-norm in the mean square sense. By utilizing the stochastic analysis techniques, sufficient conditions are established to ensure that the dynamic system under consideration satisfies the prespecified performance constraint on the fault estimation, and then a recursive linear matrix inequality approach is employed to design the desired fault estimator gains. Simulation results demonstrate the effectiveness of the developed fault estimation design scheme.**

***Index Terms*—Randomly occurring faults; $\mathcal{H}_\infty$ fault estimation; Fading channels; Nonlinear systems; Time-varying systems.**

I. Introduction

The past decade has seen a surge of research interest on the fault diagnosis and fault-tolerant control problems due primarily to the increasing security and reliability demand of modern control systems. Fault estimation, as a crucial stage for the implementation of the desired fault detection, can provide the accurate size and shape of the fault and has thus attracted a great deal of research attention. So far, a variety of fault estimation schemes have been proposed in the existing literature, see e.g. [4], [6], [9], [14], [17] and the references therein. Nowadays, in response to the rapidly growing complexity of industrial systems, the time-varying nature has gradually become an indispensable means of reflecting the fast changes in system dynamics. Accordingly, the fault estimation issues for time-varying systems over a finite horizon have started to receive some research attention with initial results scattered in the literature [12], [14], [19]. It should be noted that the results obtained so far have been mostly based on an assumption that the system is linear and the sensors are always well-conditioned so as to produce perfect measurements containing true signals only.

On the other hand, due to the prevalence of network technologies, the research on network-induced phenomena has been gaining a noticeable momentum especially for the filtering and control problems of networked systems. However, in comparison with those frequently investigated network-induced phenomena including packet dropouts [15], communication delays [1], [18], signal quantization [8] and randomly occurring nonlinearities (RONs) [2], [3], [5], [11], the channel fading problem in the control/estimation communities has not yet received adequate research attention despite its practical significance in wireless mobile communications. Note that the main reasons leading to signal fading are some special physical phenomena such as reflection, diffraction and scattering, which have a great impact on the signal power. If not dealt with properly, the network-induced channel fading would unavoidably deteriorate the performance of controlled systems or even cause the instability. Roughly speaking, fading may vary with time, geographical position or radio frequency, and is often modeled as a random process which reflects the random change of the amplitude and phase of the transmitted signal. Up to date, some initial results have been reported in the literature concerning the networked control systems with fading channels, see [10], [13], [16] and the references therein. Nevertheless, when it comes to the time-varying stochastic systems with fading measurements, the corresponding research problem for finite-horizon fault estimation has not been appropriately investigated and still remains open.

It is worth mentioning that, in the existing literature concerning finite-horizon fault estimation problems, it has been implicitly assumed that the occurred fault signals are instantaneous, that is, the actuator/sensor faults occur in a deterministic way. Such an assumption, unfortunately, is not always true. For example, in a networked control system, due to the bandwidth limitation of the shared links as well as the unpredictable variation of the network conditions, a number of network-induced intermittent phenomena (including electromagnetic interference, severe packet loss, data collision or temporary failure of the sensors/actuators) could be regarded as different kinds of faults when the reliability becomes a concern. Obviously, in terms of the random nature of the network load, these kinds of intermittent faults could be better modeled as randomly occurring faults (ROFs) whose occurrence probability can be estimated via statistical tests. In other words, the network-induced ROFs are typically time-varying and would act in a probabilistic fashion.

In this paper, we endeavor to investigate the finite-horizon estimation problem of ROFs for a class of nonlinear time-varying systems with fading channels. Sufficient conditions are established, via intensive stochastic analysis, to guarantee the existence of the desired time-varying fault estimator gains. Such fault estimator gains are obtained by solving a set of recursive linear matrix inequalities (RLMIs). A simulation example is finally presented to illustrate the effectiveness of the proposed design scheme. *The main contributions of this paper are highlighted as follows. 1) The system model addressed is quite comprehensive to cover time-varying parameters, Lipschitz-like nonlinearities as well as ROFs, hence reflecting the reality more closely. 2) This paper represents the first of few attempts to deal with the finite-horizon fault estimation problem with ROFs and fading channels. 3) The developed finite-horizon fault estimator design algorithm is dependent not only on the current available state estimate but also on the previous measurement, which is suitable for*

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online applications.

Notation. The notation used here is standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are real symmetric matrices, means that $X-Y$ is positive semi-definite (respectively, positive definite). M^T represents the transpose of the matrix M. 0 represents zero matrix of compatible dimensions. The n-dimensional identity matrix is denoted as I_n or simply I, if no confusion is caused. diag$\{\cdots\}$ stands for a block-diagonal matrix. $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|\,y\}$ will, respectively, denote expectation of the stochastic variable x and expectation of x conditional on y. $\mathrm{Prob}\{\cdot\}$ means the occurrence probability of the event "$\cdot$". $l_2[0,N]$ is the space of square summable sequences over $[0,N] := \{0,1,2,\cdots,N\}$. In symmetric block matrices, "$*$" is used as an ellipsis for terms induced by symmetry. The symbol $\otimes$ denotes the Kronecker product. Matrices, if they are not explicitly specified, are assumed to have compatible dimensions.

II. Problem Formulation

Consider the following class of discrete time-varying nonlinear stochastic systems defined on $k\in[0,N]$:

$$\begin{cases} x(k+1) = g(k,x(k)) + \alpha(k)D_f(k)f(k) + E_1(k)w(k) \\ \tilde{y}(k) = C(k)x(k) + E_2(k)v(k) \\ x(0) = \varphi_0 \end{cases} \tag{1}$$

where $x(k)\in\mathbb{R}^{n_x}$ represents the state vector; $\tilde{y}(k)\in\mathbb{R}^{n_y}$ is the process output; $w(k)\in\mathbb{R}^{n_w}$, $v(k)\in\mathbb{R}^{n_v}$ and $f(k)\in\mathbb{R}^{n_l}$ are, respectively, the disturbance input, the measurement noises and the fault signal, all of which belong to $l_2[0,N]$; and φ_0 is a given initial value. $D_f(k)$, $E_1(k)$, $C(k)$ and $E_2(k)$ are known, real, time-varying matrices with appropriate dimensions.

The nonlinear function $g(\cdot,\cdot)$ is assumed to satisfy $g(k,0)=0$ and the following condition:

$$\|g(k,x(k)+\sigma(k)) - g(k,x(k)) - A(k)\sigma(k)\| \leq b(k)\,\|\sigma(k)\|\,, \tag{2}$$

where $A(k)$ is a known matrix, $\sigma(k)\in\mathbb{R}^{n_x}$ is any vector and $b(k)$ is a known positive scalar.

Remark 1: The nonlinear description (2) with the system parameter $A(k)$ reflects the distance between the originally nonlinear model (1) and the nominal linear model. In fact, such a nonlinear description resembles the Lipschitz conditions on the nonlinear functions. In applications, the linearization technique is utilized to quantify the maximum possible deviation from the nominal model.

The dynamic characteristics of the fault vector $f(k)$ can be described as follows:

$$f(k+1) = A_f(k)f(k) \tag{3}$$

where $A_f(k)$ is a known matrix with appropriate dimensions.

The variable $\alpha(k)$ in (1), which accounts for the randomly occurring fault phenomena, is a Bernoulli distributed white sequences taking values on 0 or 1 with

$$\mathrm{Prob}\{\alpha(k)=1\} = \bar{\alpha}, \quad \mathrm{Prob}\{\alpha(k)=0\} = 1-\bar{\alpha}, \tag{4}$$

where $\bar{\alpha}\in[0,1]$ is a known constant.

Remark 2: The time-varying system (1) provides a way of accounting for the ROF phenomenon by resorting to the random variable $\alpha(k)$. At the kth time point, if $\alpha(k)=1$, the fault occurs; and if $\alpha(k)=0$, the system works normally. The fault obeying (3) may occur in a probabilistic way based on an individual probability distribution that can be specified *a prior* through statistical tests. Such a ROF concept could better reflect the probabilistically intermittent faults for the finite-horizon fault estimation problems, which render more practical significance for the time-varying systems (1).

In this paper, consider the case when an unreliable wireless network medium is utilized for the signal transmission. In this case, the phenomenon of fading channels becomes an issue that constitutes another focus of our present research. The measurement signal $y(k)$ with probabilistic fading channels is described by

$$y(k) = \sum_{s=0}^{l_k} \beta_s(k)\tilde{y}(k-s) + E_3(k)\xi(k) \tag{5}$$

with $l_k = \min\{l,\ k\}$. Here, l is the given number of paths. $\beta_s(k)$ $(s=0,1,\cdots,l_k)$ are channel coefficients which are mutually independent random variables taking values $[0,1]$ with mathematical expectations $\bar{\beta}_s$ and variances ν_s. $\xi(k)\in l_2[0,N]$ is also an external disturbance. For simplicity, we set $\{\tilde{y}(k)\}_{k\in[-l,-1]}=0$, i.e., $\{x(k)\}_{k\in[-l,-1]}=0$ and $\{v(k)\}_{k\in[-l,-1]}=0$.

Throughout the paper, we assume that $\alpha(k)$ and $\beta_s(k)$ $(s=0,\ldots,l_k)$ are uncorrelated random variables. The probabilistic fading measurement (5) is actually a weighted sum of the signals from channels of different delays where the weights are random variables taking values on the interval $[0,1]$. Such fading measurement includes the traditional packet dropouts and random communication delays as special cases. For example, $l=0$ corresponds to the case of probabilistically degraded measurements (without time-delays) and $l=1$ corresponds to the case that degraded measurement and one-step communication delay could occur simultaneously.

Letting $x_f(k) = \begin{bmatrix} x^T(k) & f^T(k)\end{bmatrix}^T$ and $z(k)=f(k)$, we have from (1), (3) and (5) that

$$\begin{cases} x_f(k+1) = (\bar{A}_f(k) + \tilde{\alpha}(k)\bar{D}_f(k))x_f(k) + \bar{E}_1(k)w(k) \\ \qquad + Hg(k,H^Tx_f(k)) \\ y(k) = \sum_{s=0}^{l_k}\beta_s(k)[\bar{C}(k-s)x_f(k-s) + E_2(k-s) \\ \qquad \times v(k-s)] + E_3(k)\xi(k) \\ z(k) = Lx_f(k) \end{cases} \tag{6}$$

where

$$\begin{aligned} \bar{A}_f(k) &= \begin{bmatrix} 0 & \bar{\alpha}D_f(k) \\ 0 & A_f(k)\end{bmatrix}, \quad \bar{D}_f(k) = \begin{bmatrix} 0 & D_f(k) \\ 0 & 0\end{bmatrix}, \quad H = \begin{bmatrix} I \\ 0\end{bmatrix}, \\ \tilde{\alpha}(k) &= \alpha(k) - \bar{\alpha}, \quad \bar{C}(k) = \begin{bmatrix} C(k) & 0\end{bmatrix}, \quad L = \begin{bmatrix} 0 & I\end{bmatrix}, \\ \bar{E}_1(k) &= \begin{bmatrix} E_1^T(k) & 0\end{bmatrix}^T. \end{aligned}$$

For the purpose of simplicity, for $-l\leq i\leq -1$, we assume that $C(i)=0$, $\tilde{y}(i)=0$ and $\begin{bmatrix} v^T(i) & \xi^T(i)\end{bmatrix}=0$. Based on the *actually* received signal $y(k)$, the following time-varying fault estimator is constructed for system (6):

$$\begin{cases} \hat{x}_f(k+1) = \bar{A}_f(k)\hat{x}_f(k) + Hg(k,H^T\hat{x}_f(k)) - K(k)\Big(y(k) \\ \qquad - \sum_{s=0}^{l}\bar{\beta}_s\bar{C}(k-s)\hat{x}_f(k-s)\Big) \\ \hat{z}(k) = L\hat{x}_f(k) \end{cases} \tag{7}$$

where $\hat{x}_f(k)\in\mathbb{R}^{n_x+n_l}$ is the estimate of the state $x_f(k)$, $\hat{z}(k)\in\mathbb{R}^{n_l}$ represents the estimate of the fault $f(k)$ and $K(k)$ is the gain matrix of the fault estimator to be designed.

Remark 3: It is worth pointing out that the constructed fault estimator (7) can be regarded as a Luenberger-type observer. In comparison with other kinds of estimators, the computational complexity with respect to (7) is relatively light as one parameter $K(k)$ needs to be designed, where $A(k)$ and $b(k)$ are involved. In addition, for the fault estimation purpose, the designed estimator should be physically implementable in practical engineering, and therefore the unknown (but bounded) disturbance inputs $w(k)$, $v(k)$ and $\xi(k)$ are excluded in (7).

For notational simplicity, we denote

$$\begin{aligned} m(k) \;&:=\; g(k,H^Tx_f(k))-g(k,H^T\hat{x}_f(k)) \\ &\quad -A(k)H^T(x_f(k)-\hat{x}_f(k)) \end{aligned} \tag{8}$$

Letting $e(k) := x_f(k)-\hat{x}_f(k)$, $\tilde{\beta}_s(k) := \beta_s(k)-\bar{\beta}_s$, $\eta(k)=\begin{bmatrix}x_f^T(k) & e^T(k)\end{bmatrix}^T$, $\tilde{z}(k)=z(k)-\hat{z}(k)$ and $\varpi(k)=\begin{bmatrix}w^T(k) & \xi^T(k)\end{bmatrix}^T$, we have the following dynamic system to be investigated:

$$\begin{cases} \eta(k+1)=\mathcal{Y}_l(k)+\tilde{\alpha}(k)\mathcal{D}_f(k)\eta(k)+\sum_{s=0}^{l_k}\tilde{\beta}_s(k) \\ \qquad\times\mathcal{C}(k-s)\eta(k-s)+\sum_{s=0}^{l_k}\tilde{\beta}_s(k) \\ \qquad\times\mathcal{E}_2(k-s)v(k-s) \\ \tilde{z}(k)=\mathcal{L}(k)\eta(k) \end{cases} \tag{9}$$

where

$$\begin{aligned} \mathcal{Y}_l(k) &= \mathcal{A}_f(k)\eta(k)+\mathcal{H}\mathcal{F}(\eta(k))+\sum_{s=1}^{l_k}\bar{\beta}_s\bar{\mathcal{C}}(k-s)\eta(k-s) \\ &\quad +\sum_{s=0}^{l_k}\bar{\beta}_s\mathcal{E}_2(k-s)v(k-s)+\mathcal{E}_1(k)\varpi(k), \\ \mathcal{F}(\eta(k)) &= \left[\left(g(k,H^Tx_f(k))-A(k)H^Tx_f(k)\right)^T \quad m^T(k)\right]^T, \\ \mathcal{A}_f(k) &= \mathrm{diag}\{A_H, A_H+\bar{\beta}_0K(k)\bar{C}(k)\},\ \mathcal{H}=\mathrm{diag}\{H,H\}, \end{aligned}$$

$$\begin{aligned} A_H &= \bar{A}_f(k)+HA(k)H^T,\ \mathcal{D}_f(k)=\mathbf{1}_2\otimes\begin{bmatrix}\bar{D}_f(k) & 0\end{bmatrix}, \\ \mathcal{C}(k-s) &= \begin{bmatrix}0 & 0\\ A_K & 0\end{bmatrix}, \mathcal{E}_1(k)=\begin{bmatrix}\bar{E}_1(k) & 0\\ \bar{E}_1(k) & K(k)E_3(k)\end{bmatrix}, \\ \bar{\mathcal{C}}(k-s) &= \mathrm{diag}\{0, A_K\},\ A_K=K(k)\bar{C}(k-s), \\ \mathcal{E}_2(k-s) &= \begin{bmatrix}0^T & (K(k)E_2(k-s))^T\end{bmatrix}^T,\ \mathcal{L}(k)=\begin{bmatrix}0 & L\end{bmatrix}. \end{aligned}$$

Our objective of this paper is to find a fault estimate $\hat{z}(k)$ $(0\le k\le N-1)$ such that, for the given positive scalar γ, the dynamic system (9) satisfies the following fault estimation performance requirement:

$$\begin{aligned} J \;:=\; &\mathbb{E}\left\{\sum_{k=0}^{N-1}\left(\|\tilde{z}(k)\|^2-\gamma^2\|\varpi(k)\|^2_{P_a}-\gamma^2\|v(k)\|^2_{P_b}\right)\right\} \\ &-\gamma^2\sum_{i=-l}^{0}\mathbb{E}\left\{\eta^T(i)P_{ci}\eta(i)\right\}<0, \\ &\forall(\{\varpi(k)\},\{v(k)\},\eta(0))\neq 0 \end{aligned} \tag{10}$$

where P_a, P_b and P_{ci} are known positive definite weighted matrices, $\|\varpi(k)\|^2_{P_a}=\varpi^T(k)P_a\varpi(k)$ and $\|v(k)\|^2_{P_b}=v^T(k)P_bv(k)$.

Remark 4: The fault estimation performance requirement (10) is adopted from the classical gain-based $\mathcal{H}_\infty$ control theory, which means that the influence from disturbances $\varpi(k)$, $v(k)$ and initial states $\eta(i)$ $(i=-l,-l+1,\cdots,0)$ onto the fault estimation error $\tilde{z}(k)$ over the given finite-horizon should be constrained by means of the given disturbance attenuation level γ.

III. Main results

In this section, let us investigate both the fault estimator performance analysis and design problems for system (9). Firstly, we propose the following finite-horizon fault estimation performance analysis results for a class of nonlinear time-varying systems with ROFs and fading channels.

For convenience of later analysis, we denote

$$\begin{aligned} \bar{\Gamma}(k) &= \begin{bmatrix}\Gamma_{ij}(k)\end{bmatrix}_{\{i=1,2,\ldots,5;j=1,2,\ldots,5\}},\ \bar{P}(k+1)=I_l\otimes P(k+1), \\ \bar{Q}(k,l) &= \mathrm{diag}\{Q(k-1,1),Q(k-2,2),\cdots,Q(k-l,l)\}, \\ \Gamma_{11}(k) &= \mathcal{A}_f^T(k)P(k+1)\mathcal{A}_f(k)+\bar{\alpha}(1-\bar{\alpha})\mathcal{D}_f^T(k)P(k+1)\mathcal{D}_f(k) \\ &\quad -P(k)+\nu_0\mathcal{C}^T(k)P(k+1)\mathcal{C}(k)+\sum_{j=1}^{l}Q(k,j), \\ \Gamma_{21}(k) &= \mathcal{H}^TP(k+1)\mathcal{A}_f(k),\ \Gamma_{31}(k)=(\Lambda_\beta\bar{\mathcal{C}}_l(k))^TP(k+1)\mathcal{A}_f(k), \\ \Gamma_{22}(k) &= \mathcal{H}^TP(k+1)\mathcal{H},\ \Gamma_{32}(k)=(\Lambda_\beta\bar{\mathcal{C}}_l(k))^TP(k+1)\mathcal{H}, \\ \Gamma_{33}(k) &= (\Lambda_\beta\bar{\mathcal{C}}_l(k))^TP(k+1)\Lambda_\beta\bar{\mathcal{C}}_l(k)-\bar{Q}(k,l)+(\bar{\Lambda}_\gamma\mathcal{C}_l(k))^T \\ &\quad \times\bar{P}(k+1)\bar{\Lambda}_\gamma\mathcal{C}_l(k),\ \Gamma_{42}(k)=(\bar{\Lambda}_\beta\bar{\mathcal{E}}_{2l}(k))^TP(k+1)\mathcal{H}, \\ \Gamma_{41}(k) &= (\bar{\Lambda}_\beta\bar{\mathcal{E}}_{2l}(k))^TP(k+1)\mathcal{A}_f(k)+\nu_0\mathcal{E}_2^T(k)P(k+1)\mathcal{C}(k)\bar{H}_3, \\ \Gamma_{43}(k) &= (\bar{\Lambda}_\beta\bar{\mathcal{E}}_{2l}(k))^TP(k+1)\Lambda_\beta\bar{\mathcal{C}}_l(k)+\bar{H}_0(\bar{\Lambda}_\gamma\hat{\mathcal{E}}_{2l}(k))^T\bar{P}(k+1) \\ &\quad \times\bar{\Lambda}_\gamma\mathcal{C}_l(k),\hat{P}(k+1)=I_{l+1}\otimes P(k+1), \end{aligned}$$

$$\begin{aligned} \Gamma_{44}(k) &= (\bar{\Lambda}_\beta\bar{\mathcal{E}}_{2l}(k))^TP(k+1)\bar{\Lambda}_\beta\bar{\mathcal{E}}_{2l}(k)+(\hat{\Lambda}_\gamma\bar{\mathcal{E}}_{2l}(k))^T\hat{P}(k+1) \\ &\quad \times\hat{\Lambda}_\gamma\bar{\mathcal{E}}_{2l}(k),\ \Gamma_{51}(k)=\mathcal{E}_1^T(k)P(k+1)\mathcal{A}_f(k), \\ \Gamma_{52}(k) &= \mathcal{E}_1^T(k)P(k+1)\mathcal{H},\ \Gamma_{53}(k)=\mathcal{E}_1^T(k)P(k+1)\Lambda_\beta\bar{\mathcal{C}}_l(k), \\ \Gamma_{54}(k) &= \mathcal{E}_1^T(k)P(k+1)\bar{\Lambda}_\beta\bar{\mathcal{E}}_{2l}(k),\ \Gamma_{55}(k)=\mathcal{E}_1^T(k)P(k+1)\mathcal{E}_1(k), \\ \phi(k) &= \rho(k)b^2(k)(\bar{H}_1^THH^T\bar{H}_1+\bar{H}_2^THH^T\bar{H}_2), \\ \bar{\mathcal{C}}_l(k) &= \mathrm{diag}\{\bar{\mathcal{C}}(k-1),\bar{\mathcal{C}}(k-2),\ldots,\bar{\mathcal{C}}(k-l)\}, \\ \bar{\mathcal{E}}_{2l}(k) &= \mathrm{diag}\{\mathcal{E}_2(k),\mathcal{E}_2(k-1),\ldots,\mathcal{E}_2(k-l)\}, \\ \mathcal{C}_l(k) &= \mathrm{diag}\{\mathcal{C}(k-1),\mathcal{C}(k-2),\ldots,\mathcal{C}(k-l)\}, \\ \hat{\mathcal{E}}_{2l}(k) &= \mathrm{diag}\{\mathcal{E}_2(k-1),\mathcal{E}_2(k-2),\ldots,\mathcal{E}_2(k-l)\}, \\ \Lambda_\beta &= \begin{bmatrix}\bar{\beta}_1I & \bar{\beta}_2I & \cdots & \bar{\beta}_lI\end{bmatrix},\bar{H}_0=\begin{bmatrix}0_{n_v,l\cdot n_v} & I_{l\cdot n_v,l\cdot n_v}\end{bmatrix}^T, \\ \bar{\Lambda}_\beta &= \begin{bmatrix}\bar{\beta}_0I & \bar{\beta}_1I & \cdots & \bar{\beta}_lI\end{bmatrix},\bar{H}_2=\begin{bmatrix}0_{n_x+n_l} & I_{n_x+n_l}\end{bmatrix}, \\ \bar{\Lambda}_\gamma &= \mathrm{diag}\{\sqrt{\nu_1}I,\sqrt{\nu_2}I,\ldots,\sqrt{\nu_l}I\},\bar{H}_1=\begin{bmatrix}I_{n_x+n_l} & 0_{n_x+n_l}\end{bmatrix}, \\ \hat{\Lambda}_\gamma &= \mathrm{diag}\{\sqrt{\nu_0}I,\sqrt{\nu_1}I,\ldots,\sqrt{\nu_l}I\},\bar{P}_b=\frac{\gamma^2}{l+1}I_{l+1}\otimes P_b, \\ \bar{H}_3 &= \begin{bmatrix}I_{n_v,2(n_x+n_l)} & 0_{l\cdot n_v,2(n_x+n_l)}\end{bmatrix}^T. \end{aligned}$$

Theorem 1: Consider the discrete time-varying nonlinear stochastic system described by (1)–(5). Let the disturbance attenuation level $\gamma>0$, the positive definite matrices $P_a>0$, $P_b>0$, $P_{ci}>0$ $(i=-l,-l+1,\ldots,0)$ and the gain matrices of the fault estimator $\{K(k)\}_{k\in[0,N-1]}$ in (7) be given. The fault estimator $\hat{z}(k)$ $(k=0,1,\ldots,N-1)$ satisfies the performance criterion (10) if there exist families of positive scalars $\{\rho(k)\}_{k\in[0,N-1]}$, positive definite matrices $\{P(k)\}_{k\in[0,N]}>0$ and $\{Q(i,j)\}_{i\in[-l,N],j\in[1,l]}>0$ satisfying

$$\begin{aligned} \Gamma(k) &= \bar{\Gamma}(k)+\mathrm{diag}\{\mathcal{L}^T(k)\mathcal{L}(k)+\phi(k),-\rho(k)I,0,-\bar{P}_b,-\gamma^2P_a\} \\ &< 0 \end{aligned} \tag{11}$$

and the initial condition

$$\gamma^2P_{c0}-P(0)>0,\gamma^2P_{-ci}-\sum_{j=i}^{l}Q(-i,j)>0\ (i=1,2,\ldots,l) \tag{12}$$

Proof: Consider the following Lyapunov-like functional candidate for system (9):

$$\begin{aligned} V(k) &= V_1(k)+V_2(k) \\ &= \eta^T(k)P(k)\eta(k)+\sum_{j=1}^{l}\sum_{i=k-j}^{k-1}\eta^T(i)Q(i,j)\eta(i) \end{aligned} \tag{13}$$

where $P(k)>0$ and $Q(i,j)>0$ are symmetric positive definite matrices with appropriate dimensions. Calculating the difference of

$V(k)$ along the solution of system (9) and taking the mathematical expectation, we have

$$\begin{aligned}&\mathbb{E}\{\Delta V_1(k)\} = \mathbb{E}\{V_1(k+1) - V_1(k)\}\\ =&\mathbb{E}\bigg\{\Big(\mathcal{Y}_l^T(k)P(k+1)\mathcal{Y}_l(k) - \eta^T(k)P(k)\eta(k) + \bar{\alpha}(1-\bar{\alpha})\\ &\times \eta^T(k)\mathcal{D}_f^T(k)P(k+1)\mathcal{D}_f(k)\eta(k) + \sum_{s=0}^{l}\nu_s(\mathcal{C}(k-s)\eta(k-s)\\ &+\mathcal{E}_2(k-s)v(k-s))^T P(k+1)(\mathcal{C}(k-s)\eta(k-s)\\ &+\mathcal{E}_2(k-s)v(k-s))\Big)\bigg\} \end{aligned} \tag{14}$$

Similarly, by noting the equation (13), one has

$$\mathbb{E}\{\Delta V_2(k)\} = \mathbb{E}\bigg\{\sum_{j=1}^{l}\eta^T(k)Q(k,j)\eta(k) - \eta_l^T(k)\bar{Q}(k,l)\eta_l(k)\bigg\} \tag{15}$$

where $\eta_l(k) = \begin{bmatrix}\eta^T(k-1) & \eta^T(k-2) & \cdots & \eta^T(k-l)\end{bmatrix}^T$. Therefore, by denoting

$$\begin{aligned} v_l(k) &= \begin{bmatrix} v^T(k) & v^T(k-1) & \cdots & v^T(k-l)\end{bmatrix}^T,\\ \tilde{\eta}(k) &= \begin{bmatrix}\eta^T(k) & \mathcal{F}^T(\eta(k)) & \eta_l^T(k) & v_l^T(k) & \varpi^T(k)\end{bmatrix}^T\end{aligned}$$

and combining (13)–(15), one immediately obtains

$$\mathbb{E}\{\Delta V(k)\} = \mathbb{E}\{\Delta V_1(k) + \Delta V_2(k)\} = \mathbb{E}\left\{\tilde{\eta}^T(k)\bar{\Gamma}(k)\tilde{\eta}(k)\right\} \tag{16}$$

Moreover, it follows from the constraint (2) that

$$\begin{aligned}&\|\mathcal{F}(\eta(k))\|^2\\ &\leq b^2(k)\eta^T(k)(\bar{H}_1^T H H^T \bar{H}_1 + \bar{H}_2^T H H^T \bar{H}_2)\eta(k)\end{aligned} \tag{17}$$

Hence we have

$$\begin{aligned}\mathbb{E}\{\Delta V(k)\} \leq \mathbb{E}\bigg\{&\tilde{\eta}^T(k)\bar{\Gamma}(k)\tilde{\eta}(k) - \rho(k)(\|\mathcal{F}(\eta(k))\|^2 - b^2(k)\\ &\times\eta^T(k)(\bar{H}_1^T H H^T \bar{H}_1 + \bar{H}_2^T H H^T \bar{H}_2)\eta(k))\bigg\}\end{aligned} \tag{18}$$

Summing up (18) on both sides from 0 to $N-1$ with respect to k, we obtain

$$\begin{aligned}&\sum_{k=0}^{N-1}\mathbb{E}\{\Delta V(k)\} = \mathbb{E}\{V(N)\} - \mathbb{E}\{V(0)\}\\ \leq\ & \mathbb{E}\bigg\{\sum_{k=0}^{N-1}\tilde{\eta}^T(k)\Gamma(k)\tilde{\eta}(k)\bigg\} + \mathbb{E}\bigg\{\frac{\gamma^2}{l+1}\sum_{s=0}^{l}\sum_{k=0}^{N-1}(\|v(k-s)\|_{P_b}^2\\ &-\|v(k)\|_{P_b}^2)\bigg\} - \mathbb{E}\bigg\{\sum_{k=0}^{N-1}(\|\tilde{z}(k)\|^2 - \gamma^2\|\varpi(k)\|_{P_a}^2\\ &-\gamma^2\|v(k)\|_{P_b}^2)\bigg\}\end{aligned} \tag{19}$$

It can be obtained from (11) and (12) that

$$\begin{aligned}&\mathbb{E}\bigg\{\sum_{k=0}^{N-1}\Big(\gamma^2\|\varpi(k)\|_{P_a}^2 + \gamma^2\|v(k)\|_{P_b}^2 - \|\tilde{z}(k)\|^2\Big)\\ &+\gamma^2\sum_{i=-l}^{0}\eta^T(i)P_{ci}\eta(i)\bigg\}\\ >\ & \mathbb{E}\{V(N)\} + \mathbb{E}\left\{\gamma^2\sum_{k=-l}^{0}\eta^T(i)P_{Ci}\eta(i) - V(0)\right\} \geq 0\end{aligned} \tag{20}$$

which is equivalent to (10), and the proof is now complete. ■

Remark 5: White noise disturbances are frequently encountered in practice where Kalman filter (KF) or extended Kalman filter (EKF) approaches can be used to deal with the state estimation problem. In H_∞ estimation, the noise sources are arbitrary deterministic signals with bounded energy or average power, and a H_∞ estimator is sought which ensures a prescribed upper-bound on the L_2-induced gain from the noise signals to the estimation error. Such a H_∞ estimation approach is particularly appropriate to applications where the statistics of the noise signals are not exactly known. In fact, the H_∞ estimator has been widely adopted in practical engineering due to its capability of providing a bound for the worst-case estimation error. It should be pointed out that the problem addressed in this paper is equipped with the following features: 1) the considered external disturbances are unknown but bounded and therefore do not possess known statistics; 2) the nonlinearities satisfy the given bounded conditions only; and 3) the plant under consideration is quite comprehensive that covers fading measurements, ROFs, nonlinearity and time-varying parameters. Unfortunately, the above features prevent the existing methods (such as KF, EKF) from being applied to the H_∞ state estimation problem for the underlying system in this paper, and the proposed scheme in this paper is particularly suitable for handling the addressed networked complex systems.

Based on the analysis results, we are now ready to solve the fault estimator design problem for system (9) in the following theorem.

For convenience of later analysis, we denote

$$\begin{aligned}\hat{\Gamma}_{11}(k) =&\mathrm{diag}\bigg\{-P(k) + \sum_{j=1}^{l}Q(k,j) + \mathcal{L}^T(k)\mathcal{L}(k) + \phi(k),\\ &-\rho(k)I\bigg\},\ H_0 = \begin{bmatrix}0 & I\end{bmatrix}^T,\ \Lambda_{\bar{\beta}_0} = \begin{bmatrix}0 & \bar{\beta}_0 I\end{bmatrix}^T,\\ \hat{\Gamma}_{22}(k) =&\mathrm{diag}\bigg\{-\bar{Q}(k,l), -\frac{\gamma^2}{l+1}P_b, -\frac{\gamma^2}{l+1}I_l\otimes P_b, -\gamma^2 P_a\bigg\},\\ \mathcal{A}_{f0}(k) =& I_2\otimes(\bar{A}_f(k) + HA(k)H^T),\ \mathcal{E}_{2K} = \bar{H}_K\tilde{\mathcal{E}}_{2l}(k),\\ \hat{\Gamma}_{31}(k) =& \begin{bmatrix}\sqrt{\nu_0}H_0K(k)\hat{C}(k) & 0\\ \mathcal{A}_{f0}(k) + \Lambda_{\bar{\beta}_0}K(k)\tilde{C}(k) & \mathcal{H}\\ \sqrt{\bar{\alpha}(1-\bar{\alpha})}\mathcal{D}_f(k) & 0\\ 0 & 0\end{bmatrix},\\ \tilde{\mathcal{E}}_{2l}(k) =&\mathrm{diag}\bigg\{\hat{\mathcal{E}}_2(k-1), \hat{\mathcal{E}}_2(k-2), \ldots, \hat{\mathcal{E}}_2(k-l)\bigg\},\\ \hat{\Gamma}_{32}(k) =& \begin{bmatrix}0 & \sqrt{\nu_0}\mathcal{E}_K & 0 & 0\\ \Lambda_\beta H_K\check{C}_l(k) & \beta_0\mathcal{E}_K & \Lambda_\beta\mathcal{E}_{2K} & \hat{\mathcal{E}}_{1K}\\ 0 & 0 & 0 & 0\\ \bar{\Lambda}_\gamma H_K\tilde{C}_l(k) & 0 & \bar{\Lambda}_\gamma\mathcal{E}_{2K} & 0\end{bmatrix},\\ \hat{\Gamma}_{33}(k) =&\mathrm{diag}\{I_3\otimes -R(k+1), -\bar{R}(k+1)\},\\ \hat{C}(k) =& \begin{bmatrix}\bar{C}(k) & 0\end{bmatrix},\ \tilde{C}(k) = \begin{bmatrix}0 & \bar{C}(k)\end{bmatrix},\\ \tilde{C}_l(k) =&\mathrm{diag}\bigg\{\hat{C}(k-1), \hat{C}(k-2), \ldots, \hat{C}(k-l)\bigg\},\\ \check{C}_l(k) =&\mathrm{diag}\bigg\{\tilde{C}(k-1), \tilde{C}(k-2), \ldots, \tilde{C}(k-l)\bigg\},\\ \hat{\mathcal{E}}_1(k) =&\mathbf{1}_2\otimes\begin{bmatrix}\bar{E}_1(k) & 0\end{bmatrix},\ \hat{\mathcal{E}}_2(k) = \begin{bmatrix}0 & E_2^T(k)\end{bmatrix}^T,\\ \hat{\mathcal{E}}_3(k) =& \begin{bmatrix}0 & E_3(k)\end{bmatrix},\ \bar{R}(k+1) = \bar{P}^{-1}(k+1),\\ H_K =& I_l\otimes H_0K(k),\ \bar{H}_K = I_l\otimes\bar{K}(k),\\ \mathcal{E}_K =& \bar{K}(k)\hat{\mathcal{E}}_2(k),\ \hat{\mathcal{E}}_{1K} = \hat{\mathcal{E}}_1(k) + H_0K(k)\hat{\mathcal{E}}_3(k).\end{aligned} \tag{21}$$

Theorem 2: Consider the discrete time-varying nonlinear stochastic system (1) with the time-varying fault estimator (7). For the given disturbance attenuation level $\gamma > 0$, the positive definite matrices $P_a > 0$, $P_b > 0$ and $P_{ci} > 0$ $(i = -l, -l+1, \ldots, 0)$, the fault estimator $\hat{z}(k)$ $(k = 0, 1, \ldots, N-1)$ satisfies the performance criterion (10) if there exist families of positive scalars $\{\rho(k)\}_{k\in[0,N-1]}$, positive definite matrices $\{P(k)\}_{k\in[0,N]} > 0$, $\{Q(i,j)\}_{i\in[-l,N],j\in[1,l]} > 0$, $\{R(k)\}_{k\in[0,N]} > 0$ and real-valued matrices $K(k)_{k\in[0,N-1]}$ satisfying

$$\hat{\Gamma}(k) = \begin{bmatrix} \hat{\Gamma}_{11}(k) & * & * \\ 0 & \hat{\Gamma}_{22}(k) & * \\ \hat{\Gamma}_{31}(k) & \hat{\Gamma}_{32}(k) & \hat{\Gamma}_{33}(k) \end{bmatrix} < 0 \quad (22)$$

and the initial condition

$$\gamma^2 P_{c0} - P(0) > 0, \gamma^2 P_{-ci} - \sum_{j=i}^{l} Q(-i,j) > 0 \ (i = 1, 2, \dots, l) \quad (23)$$

with the parameters updated by $P(k+1) = R^{-1}(k+1)$.

Proof: In order to avoid partitioning the positive define matrices $\{P(k)\}_{k\in[0,N]}$ and $\{Q(i,j)\}_{i\in[-l,N],j\in[1,l]}$, we rewrite the parameters in Theorem 1 in the following form:

$$\begin{aligned} \mathcal{C}(k-s) &= H_0 K(k)\hat{C}(k-s), \mathcal{A}_f(k) = \mathcal{A}_{f_0}(k) + \Lambda_{\bar{\beta}_0} K(k)\tilde{C}(k), \\ \bar{\mathcal{C}}(k-s) &= H_0 K(k)\tilde{C}(k-s), \mathcal{E}_1(k) = \hat{\mathcal{E}}_1(k) + H_0 K(k)\hat{\mathcal{E}}_3(k), \\ \mathcal{E}_2(k-s) &= \bar{K}(k)\hat{\mathcal{E}}_2(k-s), \bar{K}(k) = I_2 \otimes K(k). \end{aligned} \quad (24)$$

Noticing (24) and using the Schur Complement Lemma, (22) can be obtained by (11) after some straightforward algebraic manipulations. The proof of this theorem is now complete. ■

By means of Theorem 2, we can summarize the Finite-Horizon Fault Estimator Design (FHFED) algorithm as follows:

Algorithm FHFED

Step 1: Given the disturbance attenuation level γ, the positive definite matrices $P_a > 0$, $P_b > 0$ and $P_{ci} > 0$ $(i = -l, -l+1, \dots, 0)$.

Step 2: Set $k = 0$ and solve the matrix inequalities (23) and the recursive matrix inequalities (22) to obtain the values of matrices $P(0)$, $\sum_{j=i}^{l} Q(-i,j)$ $(i = 1, 2, \dots, l)$, $R(1)$ and the estimator gain matrix $K(0)$.

Step 3: Set $k = k+1$, update the matrices $P(k+1) = R^{-1}(k+1)$ and then obtain the estimator gain matrix $K(k)$ by solving the recursive matrix inequalities (22).

Step 4: If $k < N$, then go to Step 3, else go to Step 5.

Step 5: Stop.

Remark 6: In Theorem 2, the finite-horizon fault estimator is designed by solving a series of recursive matrix inequalities where both the current system measurement and previous system states are employed to estimate the current system state. Such a recursive process is particularly useful for online real-time implementation. It can be observed from Algorithm FHFED that the main results established contain all the information of the addressed general systems including the time-varying systems parameters, the occurrence probabilities of the random faults as well as the statistics characteristics of the channel coefficients. In the next section, a simulation example is provided to show the effectiveness of the proposed finite-horizon fault estimation technique.

IV. An Illustrative Example

In this section, we use a nonlinear pendulum in a network environment to demonstrate the effectiveness and applicability of the proposed method. Consider a pendulum system borrowed from [7]. It is assumed that two components of the system (that is, angle and angular velocity) are randomly perturbed by uncontrolled external forces. The equations of motion of the pendulum are described as follows:

$$\begin{aligned} \dot{\theta}(t) &= \lambda\bar{\theta}(t) + \alpha(t)((1-\lambda)\bar{\theta}(t) + \lambda\theta(t)) \\ \dot{\bar{\theta}}(t) &= -\frac{g\sin(\theta(t)) + (b/lm)\bar{\theta}(t) + (aml/4)\bar{\theta}^2(t)\sin(2\theta(t))}{\frac{2}{3}l - \frac{a}{2}ml\cos^2(\theta(t))} \\ &\quad -(aml\lambda/4)w(t) \\ y(t) &= \sin(\theta(t)) + \lambda\bar{\theta}(t) + \lambda v(t) \end{aligned} \quad (25)$$

where θ denotes the angle of the pendulum from the vertical, $\bar{\theta}$ is the angular velocity, $g = 9.8 \text{ m/s}^2$ is the gravity constant, m is the mass of the pendulum, $a = 1/(m+M)$, M is the mass of the cart, l is the length of the pendulum, b is the damping coefficient of the pendulum around the pivot, and w and v are the disturbance applied to the cart and measurement noise, respectively. In this simulation, the pendulum parameters are chosen as $m = 2$ kg, $M = 8$ kg, $l = 0.5$ m and $b = 0.7$ Nm/s, and the retarded coefficient $\lambda = 0.6$.

Since the nonlinear pendulum system is in a network environment, wireless channels are known to be sensitive to fading effects which serve as one of the most dominant features in wireless communication links. Letting $x_1(t) = \theta(t)$, $x_2(t) = \bar{\theta}(t)$, considering the fading channel phenomenon and discretizing the plant with a sampling period 0.04 s, we obtain the following discrete-time system model to be investigated:

$$\begin{cases} x(k+1) = g(k, x(k)) + \alpha(k)D_f(k)f(k) + E_1(k)w(k) \\ \tilde{y}(k) = C(k)x(k) + E_2(k)v(k) \\ y(k) = \sum_{s=0}^{l_k} \beta_s(k)\tilde{y}(k-s) + E_3(k)\xi(k) \end{cases}$$

The system data are given as follows:

$$\begin{aligned} g(k, x(k)) &= \begin{bmatrix} 0.48x_1(k) + 0.2x_2(k) + 0.12\sin(x_2(k)) \\ 0.03x_1(k) + 0.50x_2(k) \end{bmatrix}, \\ D_f(k) &= \begin{bmatrix} 0.4 + \sin(k) \\ 0.2 \end{bmatrix}, \ E_1(k) = \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix}, \ E_3(k) = 0.1, \\ C(k) &= \begin{bmatrix} -0.2 + 0.1\sin(5k) & 0.5 \end{bmatrix}, \ E_2(k) = 0.3 \end{aligned} \quad (26)$$

where $x_i(k)$ $(i = 1, 2)$ is the ith element of $x(k)$. The probability of randomly occurring fault is taken as $\bar{\alpha} = 0.9$. In view of (26), the other system parameters can be obtained as follows:

$$A(k) = \begin{bmatrix} 0.48 & 0.2 \\ 0.03 & 0.50 \end{bmatrix}, \quad b(k) = 0.2.$$

The order of the fading model is $l = 1$ and the probability density functions of channel coefficients are as follows

$$\begin{cases} \varrho(\beta_0(k)) = 0.0005(e^{9.89\beta_0(k)} - 1), & 0 \le \beta_0(k) \le 1, \\ \varrho(\beta_1(k)) = 8.5017e^{-8.5\beta_1(k)}, & 0 \le \beta_1(k) \le 1. \end{cases}$$

It can be obtained that the mathematical expectation $\bar{\beta}_s$ and variance ν_s $(s = 0, 1)$ are 0.8991, 0.1174, 0.0133 and 0.01364, respectively. The H_∞ performance level γ, the positive definite matrices P_a, P_b and P_{ci} $(i = -1, 0)$ are chosen as $\gamma = 1$, $P_a = I$, $P_b = I$, $P_{c0} = P_{-c1} = 5I$, respectively. By applying Algorithm FHFED, the desired fault estimate parameters are obtained and listed in Table I.

From (10), we can obtain that

$$J(N) := \frac{\mathbb{E}\left\{ \sum_{k=0}^{N-1} \left(\|\tilde{z}(k)\|^2 \right) \right\}}{\mathbb{E}\left\{ \sum_{k=0}^{N-1} \left(\|\varpi(k)\|_{P_a}^2 + \|v(k)\|_{P_b}^2 \right) + \bar{\eta}(0) \right\}} < \gamma^2, \quad (27)$$

where $\bar{\eta}(0) = \sum_{i=-l}^{0} \eta^T(i) P_{ci} \eta(i)$. To illustrate the effectiveness of the designed fault estimator, we introduce the index $J(N)$ to reflect the actual fault estimation performance.

TABLE I
FAULT ESTIMATE PARAMETERS

k	0	1	2	3	$\cdots$	29	30
$K(k)$	$\begin{bmatrix} 2.7544 \\ 2.7533 \\ 2.7552 \end{bmatrix}$	$\begin{bmatrix} 0.0021 \\ 0.0002 \\ -0.0008 \end{bmatrix}$	$\begin{bmatrix} -0.0067 \\ -0.0055 \\ -0.0301 \end{bmatrix}$	$\begin{bmatrix} -0.0020 \\ 0.0004 \\ -0.0011 \end{bmatrix}$	$\cdots$	$\begin{bmatrix} -0.0405 \\ -0.0268 \\ 0.0097 \end{bmatrix}$	$\begin{bmatrix} -0.0161 \\ 0.0052 \\ -0.0165 \end{bmatrix}$

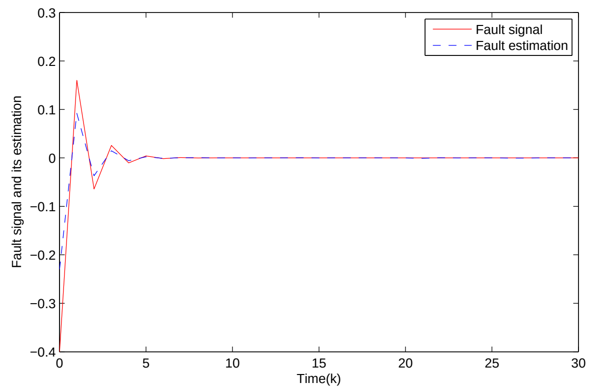

Fig. 1. Fault signal and its estimate with $A_f(k) = -0.4I$

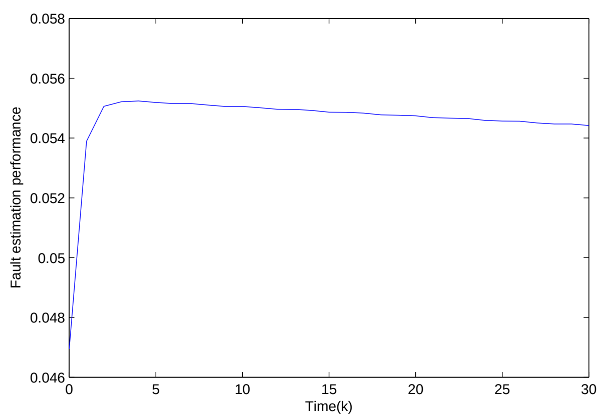

Fig. 2. Fault estimation performance $J(N)$ with $A_f(k) = -0.4I$

In the simulation, the initial value of the state is $x(0) = \begin{bmatrix} -0.55 & -0.16 \end{bmatrix}^T$ and the exogenous disturbance inputs are selected as $w(k) = 0.5e^{-2k}\sin(4k)$, $v(k) = 0.2e^{-4k}\cos(k)$ and $\xi(k) = \frac{4}{k+1}\cos(k)$. First, let the matrix $A_f(k) = -0.4I$. The fault to be estimated is $f(k) = 1$. Fig. 1 plots the simulation result on the fault signal and its estimate. Fig. 2 shows the evolution of the actual fault estimation performance in terms of the index $J(N)$ in (27), from which it can be seen that the index $J(N)$ $(N = 1, 2, ..., 30)$ is always less than the prescribed upper bound 1. The simulation results confirm that the approach addressed in this paper provides a good performance of fault estimation.

V. CONCLUSION

In this paper, we have dealt with the finite-horizon estimation problem of ROFs for a class of nonlinear time-varying systems with fading channels. Some uncorrelated random variables have been introduced, respectively, to govern the fault occurrence probability and fading measurements. By employing the stochastic analysis techniques, some sufficient conditions have been provided to ensure that the dynamic system under consideration satisfies the fault estimation performance constraint. Finally, an illustrative example has highlighted the effectiveness of the fault estimation technology presented in this paper.

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