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Numerical simulation of the edge stress singularity and the adhesion strength for compliant mushroom fibrils adhered to rigid substrates

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ABSTRACT

Bio-inspired adhesion of micropatterned surfaces due to intermolecular interactions has attracted much research interest over the last decade. Experiments show that the best adhesion is achieved with compliant “mushroom”-shaped fibrils. This paper analyses numerically the effects of different mushroom shapes on adhesion to a rigid substrate. When a remote stress is applied on the free end of a fibril perfectly bonded to a rigid substrate, the resultant stress distribution along the fibril is found to change dramatically between the straight punch and mushroom fibrils. A singular stress field is present at the edge of the fibril where it contacts the substrate and, in this work, the amplitude of the singularity is evaluated for fibrils perfectly bonded to a flat substrate so that sliding cannot occur there. This exercise is carried out for fibril geometries involving combinations of different diameters and thicknesses of the mushroom cap. By assuming a pre-existing detachment length at the corner where the stress singularity lies, we predict the adhesive strength for various mushroom cap shapes. Our study shows that a smaller stalk diameter and a thinner mushroom cap lead to higher adhesive strengths. A limited number of results are also given for other shapes, including those having a fillet radius connecting the stalk to the cap. The results support the rational optimisation of synthetic micropatterned adhesives.

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1. Introduction

Animals in nature possess different hairy contact structures such as straight punches, spherical and conical caps, toroidal suction cups, etc. The climbing abilities of geckos have inspired many researchers to develop reusable, reversible adhesives. Gecko feet are covered with millions of hierarchically structured hairs or setae with sizes ranging from millimetres to nanometres (Autumn et al., 2000; Gorb, 2007; Hiller, 1968). The smallest level of hierarchical structure is patterned with finer fibrils; these observations suggest that finer fibrils are associated with better adhesion (Arzt et al., 2003). This insight has led to formulation of the concept of “contact splitting” (Arzt et al., 2003). The present group (Huber et al., 2007; Huber et al., 2005a; Huber et al., 2005b; Huber et al., 2008; Orso et al., 2006), as well as Autumn et al. (Autumn et al., 2006; Autumn et al., 2002; Jin et al., 2012) and Jagota et al. (Jagota and

Bennison, 2002; Jagota et al., 2000) have done extensive research on this topic to understand the mechanism behind gecko adhesion. Experiments reveal that either intermolecular van der Waals forces (Autumn et al., 2002) or capillary forces (Huber et al., 2005b) play a major role in the adhesion mechanism.

Polydimethylsiloxane (PDMS) is one of the most widely used materials for the fabrication of gecko inspired adhesives. PDMS has a Poisson's ratio close to 0.5 and a Young's modulus ranging from approximately 100 kPa to approximately 10 MPa, depending on the amount of crosslinking. It is chemically inert, non-toxic and during preparation hardens quickly at elevated temperatures. It has been experimentally proven that PDMS surfaces patterned with fibrils offer better adhesion against a stiff smooth substrate than an unpatterned PDMS surface (Greiner et al., 2007; Hui et al., 2004; Sitti and Fearing, 2003; Yurdumakan et al., 2005).

Experiments with artificial patterned structures have shown that contact cap shape plays an important role in improving adhesion; compared to several different contact geometries, the mushroom fibril has generally been found to adhere best (Del Campo et al., 2007; Gorb et al., 2007; Greiner et al., 2007;

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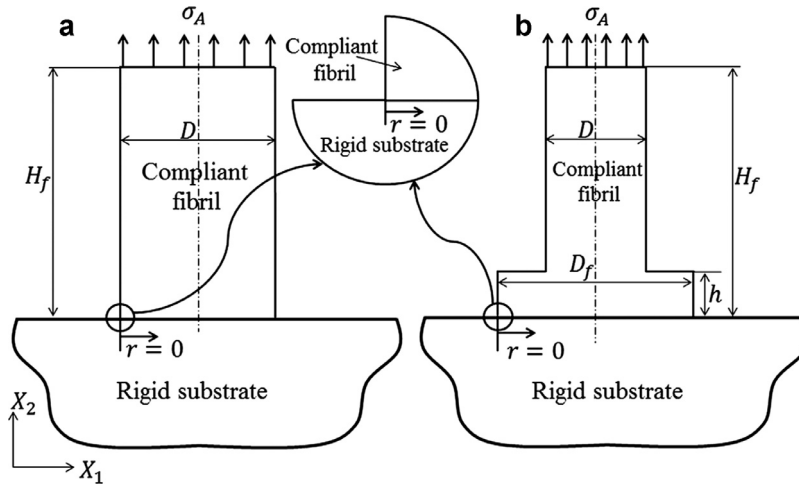


Fig. 1. Schematics of (a) a straight punch shaped fibril without a mushroom cap and (b) a fibril with a mushroom cap, both adhered to a rigid substrate.

Kim and Sitti, 2006). Adhesion also depends on other phenomena such as structural instability due to fibril buckling when they are compressed axially (Paretkar et al., 2013), misalignment of the adhering surfaces (Micchiché et al., 2014), surface roughness (Canas et al., 2012; Huber et al., 2007; Persson and Gorb, 2003; Persson and Tosatti, 2001) and backing layer thickness (Kim et al., 2007; Long et al., 2008). In most of the experiments exploring the adhesion of such patterned surfaces, compliant fibrils are pressed against a stiff spherical substrate and adhesive strength is measured during subsequent tensile loading. The fabrication and experimental exploration of such synthetic adhesives at the micrometre and nanometre scale are well established in the laboratory setting, and different parameters such as structure aspect ratio, fibril size and cap shape are well investigated. However, there is still a lack of theoretical models required for a better understanding of the adhesive interactions. The purpose of the present paper is to fill some of the gaps.

There have been on-going efforts by several researchers in the past years to understand the details of gecko adhesion through the development of various analytical models (Gao et al., 2005; Glassmaker et al., 2004; Glassmaker et al., 2005; Hui et al., 2004; Yao and Gao, 2006) and numerical simulations (Aksak et al., 2011; Aksak et al., 2014; Carbone and Pierro, 2012; Khaderi et al., 2014; Spuskanyuk et al., 2008). Spuskanyuk et al. (Spuskanyuk et al., 2008) addressed the influence of shape on adhesion and detailed the reason why mushroom fibrils show better performance than simple punch shapes. Aksak et al., 2011, demonstrated the influence of mushroom aspect ratio on adhesion, and Aksak et al., 2014 used a Dugdale cohesive zone model for mushroom like fibrils to predict the optimal shape for adhesion. They found that adhesion depends on the edge angle and the ratio of stalk radius to the outer fibril radius. Carbone and Pierro, 2012 have calculated the dependence of adhesive performance on the mushroom cap geometry and suggested an optimal shape for adhesion. Khaderi et al., 2014 provided a detailed analysis of the corner stress singularity at the edge of the fibril, its influence on the stress intensity factor for a small interface detachment near that edge, and the resulting influence on the detachment strength for a single compliant flat bottomed cylindrical fibril attached to a compliant or a rigid substrate.

In this work, we consider the corner stress singularity at the edge of a perfectly bonded compliant mushroom fibril on a rigid substrate where sliding of the fibril relative to the substrate is forbidden. In particular, we investigate how the mushroom cap geometry, including its thickness and diameter, influences the adhesive strength. We follow the approach introduced by

Akisanya and Fleck, 1997 to describe the corner stress singularity to explore the mechanics of detachment of 2D and 3D fibrils, thus extending the work of Khaderi et al., 2014 to mushroom caps. To evaluate the parameters of the corner stress singularity, we use finite element analysis, utilizing the commercial finite element software Abaqus (ABAQUS 6.11 Documentation, 2011), to solve for the stresses, strains and deformations in compliant fibrils adhered to a rigid substrate as shown in Fig. 1.

2. Analytical solution for the corner singularity

We consider a compliant fibril adhered to a rigid substrate without any interfacial crack. The fibril is treated as an incompressible, isotropic elastic solid, and the edge of the fibril always meets the substrate at right angles. The fibril material is forbidden to slide on the substrate at the interface between them. The boundary condition on the compliant material at the interface with the substrate is therefore one where the displacement is zero. When a tensile load is applied to the fibril as shown in Fig. 1, there will be a stress singularity at the fibril edge where it touches the substrate (Akisanya and Fleck, 1997). We treat both a straight punch fibril without (Fig. 1(a)) and with a mushroom cap (Fig. 1(b)). In the current paper we focus on this corner stress singularity to determine its strength and amplitude for the fibrils (straight punch and mushroom shape) shown in Fig. 1. Studies have been performed both for plane strain (2D) and axisymmetric cylindrical (3D) geometries. In addition, we provide a few results for a variation on the shape shown in Fig. 1.

We adopt the method of Akisanya and Fleck, (1997) and Khaderi et al., (2014). The most singular terms in the asymptotic normal (σ_{22}) and shear (σ_{12}) stress components along the interface between a compliant fibril and a rigid substrate (Khaderi et al., 2014) are

$$\sigma_{22} = H_1 r^{-0.406} \quad (1)$$

$$\sigma_{12} = 0.505 H_1 r^{-0.406} \quad (2)$$

where r is the distance from the fibril edge, and the directions X_1 and X_2 are shown in Fig. 1. The amplitude H_1 can be written in terms of the applied stress and one relevant dimension of the fibril. We choose the average stress σ_I on the interface between the fibril and the substrate as the measure of the applied stress and the width or diameter D_f , of the mushroom flange as the relevant dimension and obtain

$$H_1 = \sigma_I D_f^{0.406} \tilde{a} \quad (3)$$

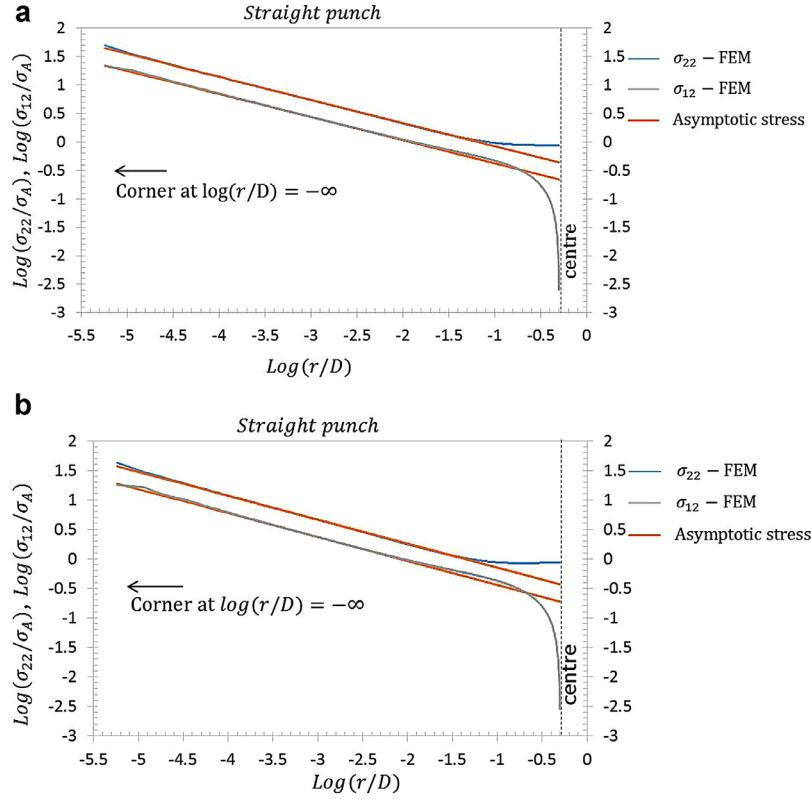


Fig. 2. Normalised normal (σ_{22}) and shear (σ_{12}) tractions for the straight punch (corner to centre) for (a) plane strain and (b) axial symmetry. The remote applied stress is denoted by σ_A .

where \tilde{a} is a dimensionless calibration parameter that depends on the geometry of the fibril, including whether it is plane strain or axisymmetric. The amplitude of the singularity scales directly with \tilde{a} as shown in Eq. 3. We will show that \tilde{a} controls detachment processes at the edge of fibril and that a low value is beneficial for good adhesion. For a mushroom cap fibril, the average interface stress, σ_I , is written as $\sigma_I = \sigma_A D/D_f$ for a plane strain slab and $\sigma_I = \sigma_A (D/D_f)^2$ for axial symmetry, where σ_A is the stress applied to fibril stalk. Note that when the fibril is a plane strain slab, D_f is the width of the mushroom cap and D is the width of the stalk, whereas in the axisymmetric cylinder case D_f is the diameter of the mushroom cap and D is the diameter of the stalk. For convenience we will refer to these parameters throughout as the diameter of the mushroom flange and the stalk even when discussing the plane strain results. We note that in the case of a straight punch without a mushroom cap, $D_f = D$, $\sigma_I = \sigma_A$ and $\tilde{a} = a_1$, where the straight punch is illustrated in Fig. 1(a), and a_1 is the calibration parameter utilised by Khaderi et al., 2014.

In order to verify our computational methods, we repeat the calculations of Khaderi et al., (2014) for the straight punch attached to a rigid substrate. For the finite element mesh we use a total of 123374 and 100501 linear quadrilateral hybrid elements for plane strain (Abaqus terminology element CPE4RH) and axial symmetry (Abaqus terminology element CAX4RH), respectively. A very fine mesh was used close to the corner of the compliant fibril to increase the accuracy of the results there. The finite element results for the normal and shear stress at the interface are plotted in logarithmic form (base 10) in Fig. 2 for both the plane strain and axisymmetric cases and marked “FEM”. According to Khaderi et al., (2014) $a_1 = 0.331$ for the plane strain punch and $a_1 = 0.278$ for the axisymmetric one. In logarithmic form (base 10), the asymptotic solutions for the plane strain case is thus

$$\log(\sigma_{22}/\sigma_A) = -0.480 - 0.406 \log(r/D)$$

and

$$\log(\sigma_{12}/\sigma_A) = -0.777 - 0.406 \log(r/D)$$

For axial symmetry it is

$$\log(\sigma_{22}/\sigma_A) = -0.556 - 0.406 \log(r/D)$$

and

$$\log(\sigma_{12}/\sigma_A) = -0.853 - 0.406 \log(r/D)$$

These 4 results are plotted in Fig. 2(a) for plane strain and Fig. 2(b) for axial symmetry and marked “asymptotic stress.” It can be seen that our finite element results agree very well with the asymptotic solution, thereby verifying the accuracy of our computational method.

With our computational method verified, we concentrate on the corner stress singularity for mushroom fibrils. With the mushroom cap diameter, D_f , held fixed we vary the diameter, D , of the fibril stalk to ascertain the influence of the ratio D_f/D . By proceeding in this way we are able to maintain element structure and size in the finite element mesh adjacent to the interface with the rigid substrate, where the accuracy of the results is most important. The same element types are used for mushroom capped fibrils as are utilised for the straight punches, and between 100,000 and 500,000 elements are used in the finite element models. For each value of D the thickness, h , of the mushroom cap is varied and more than 100 geometric configurations are studied through finite element solutions to quantify the amplitude, \tilde{a} , of the corner singularity in each case. The mesh close to the edge of the fibril is always very fine, ensuring the accuracy of the information there in the finite element solutions.

2.1. Predicting the adhesive strength

We now postulate that there is a small detachment or defect of length l at the edge of the fibril as seen in Fig. 3 such that the

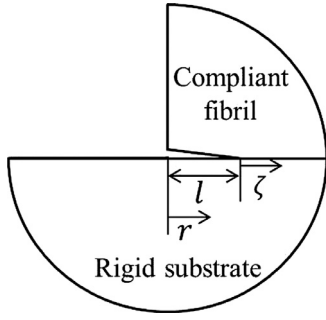


Fig. 3. Schematic of a small crack along the interface at the corner of the contact.

corner stress singularity controls its behaviour. This detachment can represent in an approximate manner the presence of an edge radius on the fibril due to the shape of the mould in which the fibril is formed. Such a detachment is equivalent to an interface crack having a stress singularity at its front, characterised by Mode I and Mode II stress intensity factors, K_I and K_{II} respectively, such that on the interface ahead of the detachment tip the leading order asymptotic stresses are given by

$$\sigma_{22} = \frac{K_I}{\sqrt{2\pi\zeta}} \quad \text{and} \quad \sigma_{12} = \frac{K_{II}}{\sqrt{2\pi\zeta}} \quad (4)$$

where ζ is the distance from the crack tip as shown in Fig. 3. The constants necessary to compute the stress intensity factors are adopted from Khaderi et al., (2014) and result in

$$K_I = 2.6H_1l^{0.094} = 2.6\sigma_1D_f^{0.406}\tilde{a}l^{0.094} \quad (5)$$

$$K_{II} = 0.8H_1l^{0.094} = 0.8\sigma_1D_f^{0.406}\tilde{a}l^{0.094} \quad (6)$$

We thus see that at a given location ahead of the detachment, the normal stress is 3.25 times the shear stress, showing that the detachment process is dominated by tension and is therefore almost a Mode I phenomenon. Upon detachment, the energy release rate is

$$G = \frac{1-\nu^2}{2E} (K_I^2 + K_{II}^2) = \frac{3}{8E} (K_I^2 + K_{II}^2) = \frac{2.8\sigma_1^2D_f^{0.81}l^{0.19}\tilde{a}^2}{E} \quad (7)$$

where E is Young's modulus and ν is Poisson's ratio, equal to 0.5 consistent with incompressibility. For detachment initiated at the edge to occur, the energy release rate must be equal to the adhesion energy, W . The value of the latter should be chosen to be consistent with the mode mixity, i.e., the ratio K_{II}/K_I ; however, the mixity is the same for all shapes and sizes of fibrils, so the dependence of the adhesion energy on mode mixity will not affect the relative adhesion strength of fibrils having different configurations. Furthermore, since detachment is nearly Mode I, it will be reasonably accurate to use the adhesion energy for purely tensile detachment as the value for W . When we set $G = W$ in Eq. 7, the stress, σ_I , is then equal to the interface strength, S^I , for edge initiated fibril detachment. This strength is thus given by

$$S^I = \frac{0.6\sqrt{EW}}{D_f^{0.406}l^{0.094}\tilde{a}} \quad (8)$$

The ratio of strengths of fibrils having the same diameter of mushroom cap and the same edge detachment length is therefore inversely proportional to the ratio of amplitudes of their stress singularities at the fibril edge. Specifically, when a straight punch fibril of diameter $D = D_f$ is compared ceteris paribus with one having a mushroom cap of diameter D_f , their strength ratio for

edge initiated detachment is given by

$$\frac{S^I}{S^{punch}} = \frac{a_1}{\tilde{a}}, \quad (9)$$

where, as noted above, a_1 is the calibration parameter quantified by Khaderi et al., (2014) for the straight punch and S^{punch} is the adhesive strength of the straight punch.

3. Results

3.1. Mushroom fibril

Fig. 4 depicts the stress distributions at the interface between a rigid substrate and a fibril with a mushroom cap where the cap diameter is twice that of the stalk ($D_f/D = 2$) and the ratio of the mushroom cap thickness to its diameter (h/D_f) is 0.008. Results are shown both for plane strain (Fig. 4(a)) and axial symmetry (Fig. 4(b)). It is seen that the stresses at the corner are reduced compared to the results in Fig. 2 at the same distance from the corner, while higher stress magnitudes now appear close to the centre of the fibril. This latter feature is somewhat misleading as the region near the corner has a stress singularity and thus there are extremely high stresses there, but not apparent on the scale encompassed by Fig. 4. Nevertheless, the fact that the stresses at the fibril centre appear to be the largest present when Fig. 4 is considered emphasises that the mushroom cap has reduced the amplitude of the corner singularity considerably. Again good agreement with the asymptotic analytical solution (Khaderi et al., 2014) is found, except near the fibril centre where the numerical stresses deviate strongly. This indicates that the corner singularity is not dominant at the fibril centre. The calibration coefficient, \tilde{a} , is extracted from the finite element stress solutions for the normal stress, σ_{22} , by fitting it to the asymptotic formula in logarithmic (base 10) form $\log(\sigma_{22}/\sigma_I) = \log\tilde{a} - 0.406\log(r/D_f)$. The particular mushroom fibril shape shown in Fig. 4 has calibration coefficient of $\tilde{a} = 1.3 \times 10^{-10}$ for a 2D plane strain fibril and $\tilde{a} = 1.6 \times 10^{-10}$ for the 3D axisymmetric fibril.

3.1.1. Varying the mushroom stalk diameter D

Fig. 5 shows the results for the influence of the mushroom cap diameter, D_f , relative to the stalk diameter, D , when $h/D_f = 0.008$ for axisymmetric fibrils. The equivalent plane strain results are given in the supplementary material in Fig. S1. In the calculations, the mushroom diameter D_f (which defines the contact area) is kept constant and D is varied to obtain a range from $D_f/D = 1.09$ to 2. Even for the smallest mushroom cap a significant reduction of the corner stress singularity over the straight punch is found. As the mushroom cap diameter is increased, the magnitude of the corner stress diminishes. Further increases in the mushroom diameter beyond twice the stalk diameter result in further reductions of the corner stress over many orders of magnitude.

3.1.2. Varying cap thickness h

The influence of the mushroom cap thickness h is reported for ten different values for axisymmetric fibrils in Fig. 6 when $D_f/D = 2$. The plane strain results are given in the supplementary material in Figure S2. Again, the mushroom diameter is kept constant and h is varied. It is observed that the amplitude of the corner stress singularity decreases when h/D_f decreases. The corner stress for thin caps, e.g., $h/D_f = 0.008$, is very low and the maximum stress visible in the plots lies close to the centre of the fibril. Any increase in h/D_f increases the corner stress, which approaches that of a straight punch as h increases.

Fig. 7 shows the calibration coefficient \tilde{a} plotted versus h/D_f for various values of D_f/D for plane strain (Fig. 7(a)) and axisymmetric (Fig. 7(b)) mushroom fibrils. In addition, the results

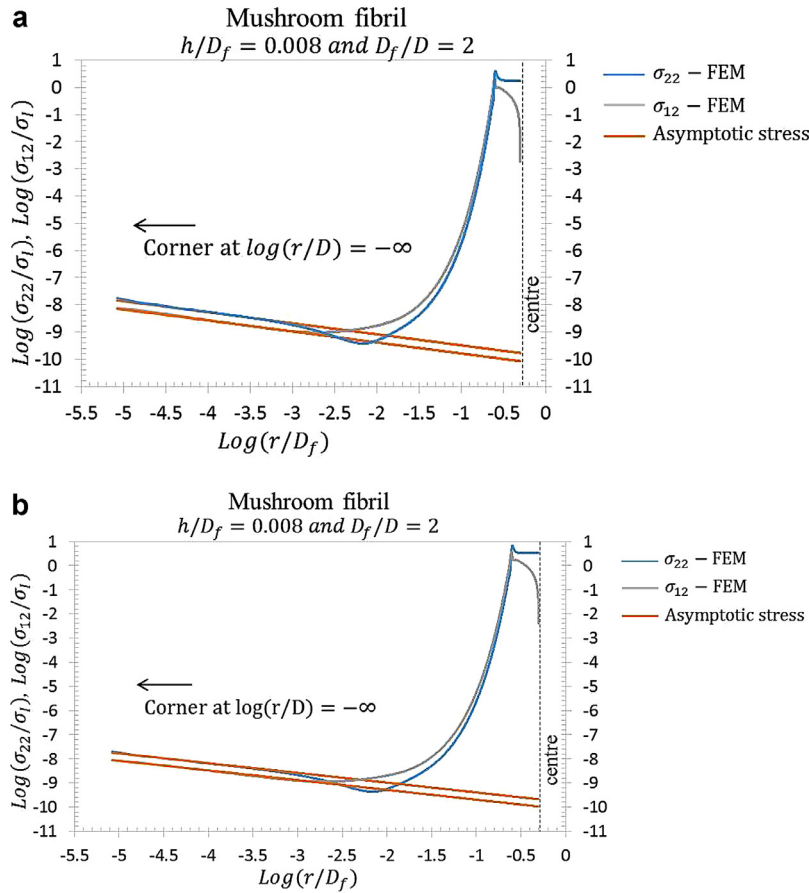


Fig. 4. Comparison of the finite element results and the theoretical asymptotic results for the normal and shear tractions on the interface between the substrate and a specific mushroom capped fibril. The results are plotted from near the edge (left) to the fibril centre (right) for (a) plane strain and (b) axial symmetry.

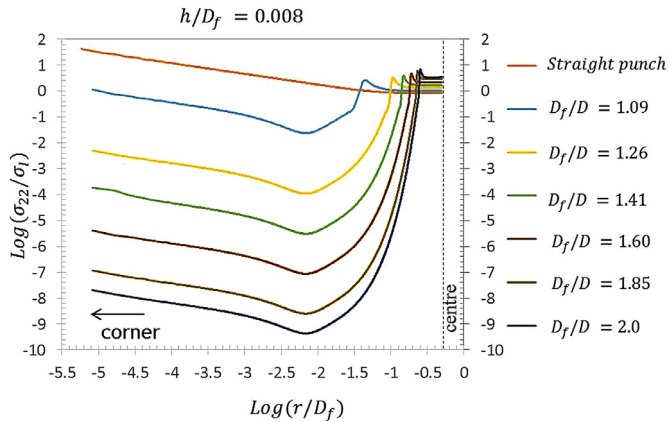


Fig. 5. Tensile stress along the interface between the substrate and a fibril having a mushroom cap for axial symmetry for various values of the mushroom cap diameter, D_f , divided by the diameter, D , of the fibril stalk. Results are shown for a mushroom cap that has a thickness, h , equal to 0.008 times its diameter.

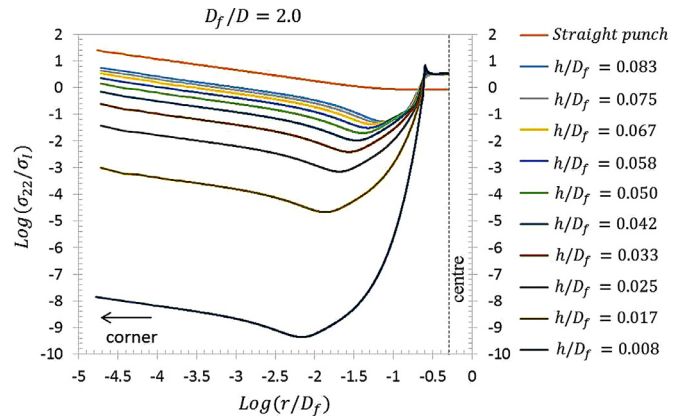


Fig. 6. Tensile stress along the interface between the substrate and a fibril having a mushroom cap for axial symmetry for various values of the ratio of the cap thickness to its diameter h/D_f . Results are shown for a fibril having a mushroom cap whose diameter is twice that of its stalk.

for straight punch fibrils are shown on the plots as horizontal dashed lines. On increase of the stalk diameter D and the mushroom cap thickness h the calibration coefficient \tilde{a} increases and approaches the value for that of a straight punch, i.e., 0.331 for plane strain and 0.278 for the axisymmetric case. Fig. 8 similarly depicts the calibration coefficient \tilde{a} as a function of the thickness, h , for fibrils having mushroom caps for axial symmetry, but now up to cap thicknesses that are comparable to the di-

ameter of the mushroom cap. The result for a straight punch is also shown as the horizontal dashed line. This plot clearly shows that as the mushroom cap becomes very thick, and therefore the lower segment of the fibril becomes indistinguishable from a straight punch, the behaviour reverts to that of the straight punch.

We have compiled all values of the calibration coefficients from our results and presented them in Tables 1 and 2.

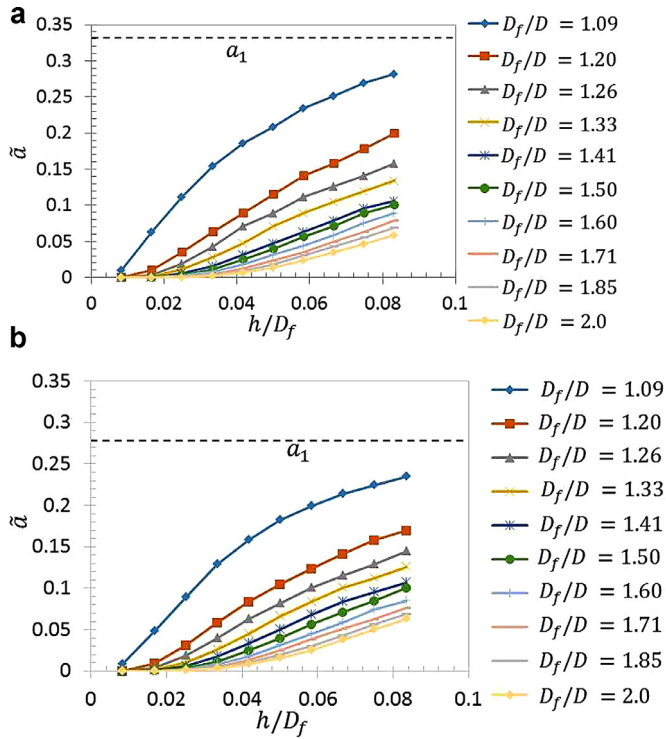


Fig. 7. Calibration coefficient, \bar{a} , plotted as a function of the ratio of mushroom cap thickness to diameter, h/D_f , for various ratios of fibril mushroom cap diameter to fibril stalk diameter D_f/D for (a) plane strain and (b) axial symmetry. The dashed horizontal lines indicate the value of the calibration coefficients a_1 for a straight punch.

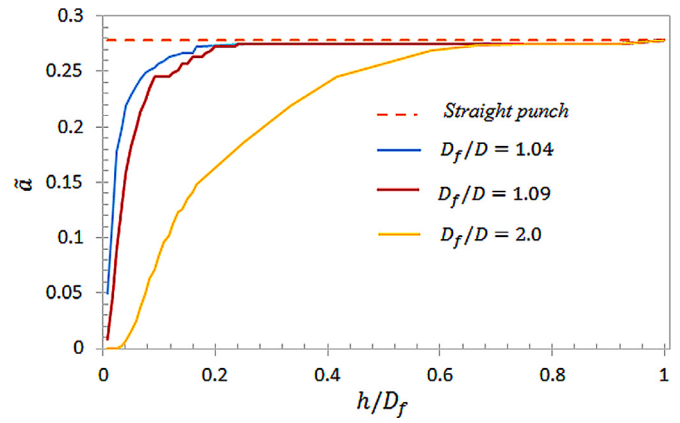


Fig. 8. Calibration coefficient \bar{a} plotted as a function of the ratio of mushroom cap thickness to diameter, h/D_f , for 3 values of the ratio of fibril mushroom cap diameter to fibril stalk diameter D_f/D for axial symmetry. The dashed horizontal lines indicate the value of the calibration coefficients a_1 for a straight punch. The plot is extended to $h/D_f = 1$ to show how the results for the mushroom capped fibril converge to that of the straight punch as the mushroom cap is thickened.

3.2. Adhesion strength

The adhesion strength controlled by edge initiated detachment for fibrils having mushroom caps was next calculated by using Eq. 9 so that the strength is normalised by that of a straight punch where both the fibril with the mushroom cap and the straight punch have the same diameter where they contact the substrate, the same detachment length, l , at the corner, the same Young's modulus and the same adhesion energy, W . That is, the diameter of the mushroom fibril cap is the same as the diameter of the straight punch. We note that the detachment length l is small

Table 1

Calibration coefficients \bar{a} for plane strain fibrils having a mushroom cap of width D_f and thickness h . The width of the fibril stalk is D .

| h/D_f | D_f/D | | | | | | | | | |
|---------|---------|---------|---------|----------|---------|----------|----------|---------|---------|---------|
| | 1.09 | 1.2 | 1.26 | 1.33 | 1.41 | 1.50 | 1.6 | 1.71 | 1.85 | 2 |
| 0.008 | 0.010 | 2.5E-04 | 4.0E-05 | 6.3E-06 | 1E-06 | 1.8E-07 | 3.2E-08 | 4.5E-09 | 7.9E-10 | 1.3E-10 |
| 0.017 | 0.063 | 0.010 | 4.0E-03 | 1.78E-03 | 6.3E-04 | 2.82E-04 | 1.26E-04 | 5.0E-05 | 2E-05 | 8.9E-06 |
| 0.025 | 0.112 | 0.035 | 0.020 | 0.011 | 0.0054 | 0.004 | 0.002 | 0.0011 | 0.0006 | 0.0004 |
| 0.033 | 0.155 | 0.063 | 0.043 | 0.028 | 0.0158 | 0.011 | 0.0079 | 0.0050 | 0.003 | 0.002 |
| 0.042 | 0.186 | 0.089 | 0.071 | 0.048 | 0.0316 | 0.025 | 0.0178 | 0.0126 | 0.009 | 0.007 |
| 0.050 | 0.209 | 0.115 | 0.089 | 0.071 | 0.0479 | 0.040 | 0.0316 | 0.0234 | 0.018 | 0.014 |
| 0.058 | 0.234 | 0.141 | 0.112 | 0.089 | 0.0631 | 0.056 | 0.0447 | 0.0355 | 0.030 | 0.023 |
| 0.067 | 0.251 | 0.158 | 0.126 | 0.105 | 0.0794 | 0.071 | 0.0589 | 0.0501 | 0.043 | 0.035 |
| 0.075 | 0.269 | 0.178 | 0.141 | 0.120 | 0.0955 | 0.089 | 0.0759 | 0.0631 | 0.055 | 0.047 |
| 0.083 | 0.282 | 0.200 | 0.158 | 0.135 | 0.1059 | 0.1 | 0.0891 | 0.079 | 0.069 | 0.059 |

Table 2

Calibration coefficients \bar{a} for axially symmetric fibrils having a mushroom cap of diameter D_f , and thickness h where D is the diameter of the fibril stalk.

| h/D_f | D_f/D | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| | 1.09 | 1.20 | 1.26 | 1.33 | 1.41 | 1.50 | 1.60 | 1.71 | 1.85 | 2.0 |
| 0.008 | 0.008 | 2.2E-04 | 4.0E-05 | 6.3E-06 | 1.0E-06 | 2.0E-07 | 3.2E-08 | 5.0E-09 | 1.0E-09 | 1.6E-10 |
| 0.017 | 0.048 | 0.009 | 4.0E-03 | 1.7E-03 | 7.1E-04 | 3.0E-04 | 1.3E-04 | 5.6E-05 | 2.2E-05 | 1.0E-05 |
| 0.025 | 0.089 | 0.032 | 0.019 | 0.010 | 0.006 | 0.004 | 0.002 | 0.0011 | 0.0007 | 0.0004 |
| 0.033 | 0.129 | 0.059 | 0.040 | 0.0263 | 0.018 | 0.012 | 0.008 | 0.005 | 0.0035 | 0.003 |
| 0.042 | 0.158 | 0.083 | 0.063 | 0.045 | 0.033 | 0.025 | 0.018 | 0.013 | 0.010 | 0.008 |
| 0.050 | 0.182 | 0.105 | 0.081 | 0.066 | 0.050 | 0.040 | 0.031 | 0.025 | 0.019 | 0.015 |
| 0.058 | 0.200 | 0.123 | 0.100 | 0.083 | 0.068 | 0.056 | 0.045 | 0.038 | 0.030 | 0.025 |
| 0.067 | 0.214 | 0.141 | 0.115 | 0.100 | 0.083 | 0.071 | 0.059 | 0.051 | 0.043 | 0.037 |
| 0.075 | 0.224 | 0.158 | 0.129 | 0.112 | 0.095 | 0.085 | 0.074 | 0.063 | 0.056 | 0.050 |
| 0.083 | 0.234 | 0.170 | 0.145 | 0.126 | 0.107 | 0.1 | 0.085 | 0.076 | 0.069 | 0.063 |

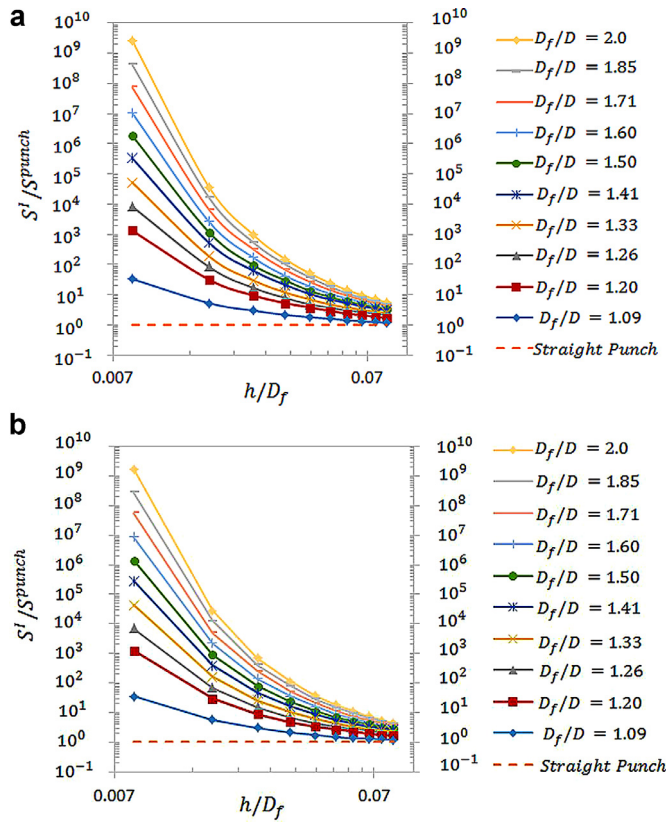


Fig. 9. Adhesion strength for edge initiated detachment for a fibril having a mushroom cap of thickness h and diameter D_f is shown for (a) plane strain and (b) axial symmetry. The adhesion strength, S^I , of the fibril with the mushroom cap is given in terms of the average stress on the interface and is normalised by the adhesion strength, S^{Spunch} of a straight punch fibril having the same diameter as the mushroom cap, with Young's modulus, adhesion energy and edge detachment length, l , the same for all fibrils.

($l \ll D_f$) so that its behaviour is controlled by the corner singularity. The results for adhesion strength are shown in Fig. 9 for both plane strain (Fig. 9(a)) and axisymmetric (Fig. 9(b)) fibrils, and are shown as functions of h/D_f for various values of D_f/D . The results for straight punches are shown as horizontal dashed lines.

It is observed that the fibril cap geometry plays an important role in promoting adhesive strength. As the mushroom cap diameter is increased and as the cap thickness is decreased, the adhesive strength rises. Therefore, both a thin mushroom cap and one with a large diameter enhance the adhesive strength of the fibril when strength is controlled by detachment from a defect at the corner. In fact we observe that the adhesive strength can be enhanced by 10 orders of magnitude over the straight punch strength S^{Spunch} when the mushroom cap diameter is twice that of its fibril stalk and its thickness is less than 1% of its diameter.

We note, however, that the benefit from the mushroom cap is limited by 3 sources. One source of the limitation is the inherent strength of the bond between the fibril and the substrate, which can lead to detachment commencing near the centre of the fibril rather than at the edge. The second source is that weak areas of bonding or of no bonding at all may be present near the centre of the fibril, representing adhesion defects that may initiate detachment. The third source of limitation is the strength of the fibril material itself, which may lead to the stalk of the fibril rupturing, since the mushroom shape leads to a stress in the stalk that is higher than the average stress at the interface with the substrate. In the current work we have not addressed the material strength. Therefore, we cannot qualify our results with quantified

limits from this source. However, the inherent strength of the bond between the fibril and the substrate can be addressed based on the following consideration. We observe from Figs. 4 and 5 and others that the stress at the interface for the mushroom fibril is almost uniform under the stalk. When the stress at that location reaches a critical value, S^C , we assume that the cohesive strength of the bond has been reached and detachment of the fibril occurs, if it has not already occurred through an alternative mechanism, such as propagation of the defect at the edge of the contact. Furthermore, detachment motivated by an adhesion defect at or near the centre of interface between the fibril may be considered explicitly and this is addressed below; in this case we consider an adhesion defect located at the peak stress visible in Figs. 4 and 5 just below the edge of the stalk of the fibril.

We now assume that detachment can occur by one of the three failure modes just described, namely that triggered when stress at the interface under the stalk reaches the critical value S^C . In Fig. 10 for comparison with this critical stress the adhesion strength, S^S , for fibrils having mushroom caps is shown in terms of the average stress on the mushroom fibril stalk instead of the average stress on the interface. The results are normalised by the adhesion strength of a straight punch fibril, S^{Spunch} , having the same diameter as the mushroom cap, and the same edge detachment length, Young's modulus and adhesion energy, and are shown for plane strain in Fig. 10(a) and for axial symmetry in Fig. 10(b). Results for straight punches are shown as full horizontal lines. The detachment strength associated with the stress under the stalk reaching the critical level S^C is shown as a horizontal dashed line marked "Cohesive Strength S^C " in the figure legend. The location of this dashed line on the ordinate is arbitrary as we do not select a specific value of the cohesive strength for any given interface. Thus the line can be adjusted up or down to represent the cohesive strength in any given case; however, in plotting the dashed lines in Fig. 10 we have assumed that the cohesive strength of the interface exceeds the strength of straight punches when detachment in that case initiates at the corner. The dashed line in Fig. 10 is to be used in the following manner. The detachment strength for mushroom fibrils cannot exceed the value represented by the dashed line; therefore, for any given combination of mushroom cap diameter and thickness, the detachment strength is given by the lower of the dashed line and the line representing the fibril detachment strength when detachment initiates at the corner of the mushroom cap. We conclude, therefore, that given our assumption regarding the cohesive strength behaviour, fibrils with very thin, large diameter mushroom caps will experience detachment that initiates near the centre of the fibril, and will have a detachment strength predicted approximately by the horizontal dashed lines in Fig. 10. In contrast, fibrils having thick, small diameter mushroom caps will experience detachment that initiates at the edge and their adhesive strength will be predicted by the full lines in Fig. 10.

When we inspect Figs. 5 and 6 we note that there is a peak in the normal traction at the interface that is located approximately under the edge of the fibril stalk. It is possible that the high stress under the edge of the fibril stalk will exceed the cohesive strength of the interface and therefore can initiate fibril detachment. Furthermore, if a detachment defect in the form of a region that is not adhered to is located there it is possible that fibril detachment will initiate due to that defect. To further characterise this possibility, in Fig. 11 we have plotted the ratio of the peak stress, σ_{peak} , to the average interface stress σ_I , as a function of h/D_f , for various values of D_f/D for axially symmetric fibrils having mushroom caps. It is convenient at this point to introduce the notation of a stress concentration factor for the stress at the interface under the edge of the fibril stalk as $k = \sigma_{peak}/\sigma_I$.

We note that as the mushroom cap is made thinner or its diameter is made larger the peak in stress under the edge of the

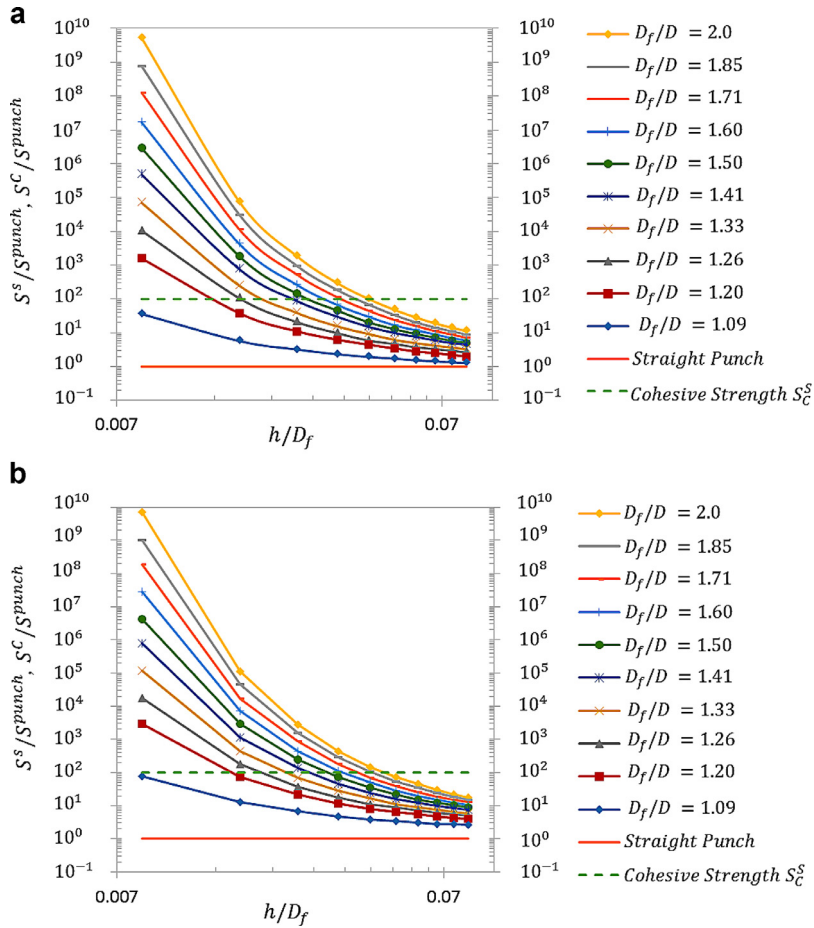


Fig. 10. Adhesion strength, S^S (full lines), for edge initiated detachment fibrils having mushroom caps of thickness h and diameter D_f is plotted for (a) plane strain and (b) axial symmetry. In this case the adhesion strength is given in terms of the average stress applied to the mushroom fibril stalk of diameter D and is normalised by the adhesion strength, S^{punch} , for a straight punch fibril having the same diameter as the mushroom cap, with Young's modulus, adhesion energy and edge detachment length, l , the same for all fibrils. The detachment strength associated with the stress under the stalk reaching a critical value S_c^ξ is represented by the horizontal dashed lines, marked "Cohesive Strength S_c^ξ ." The dashed line for Cohesive strength S_c^ξ is exemplary with an arbitrary position.

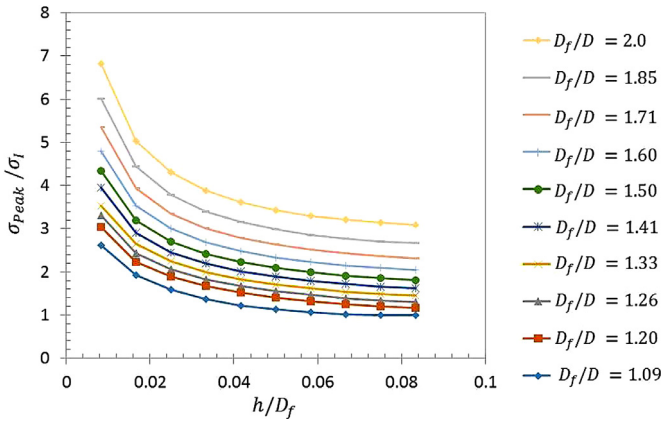


Fig. 11. The magnitude of the local peak stress σ_{peak} near the centre of the fibril as depicted in the plots of interface stress in Figs. 5 and 6. The peak stress is normalised by the average interface stress σ_l and is therefore given as the stress concentration factor k . The result is shown as a function of h/D_f for various values of D_f/D . Note that a thin mushroom cap and a thin stalk will promote detachment at the centre by raising the local peak stress there.

fibril stalk becomes more pronounced, and is more likely to promote fibril detachment. Therefore, the influence of this peak in stress on the likelihood of fibrillar detachment runs counter to that of the effect of the mushroom cap on edge detachment, whose likelihood is diminished by a thin, large diameter mushroom cap.

The influence of the peak in stress plotted in Fig. 11 may be investigated as follows. As a rudimentary treatment of the relevant fracture mechanics, we may consider a small detachment of length or diameter $2a$ at the location of the peak stress. There will be a stress singularity around the edge of this detachment that will control the tendency for it to spread. As an approximation we may estimate this stress intensity factor as if the detachment were present in an infinite body subject to a stress equal in magnitude to the peak in stress plotted in Fig. 11. The resulting value is

$$K_I = \sigma_{peak} \sqrt{\pi a}. \tag{10}$$

Note that we have not attempted to estimate a shape factor that would multiply the right hand side of Eq. 10 but instead have simply assumed that this factor is unity. As our treatment is highly approximate, we consider this to be satisfactory. With K_{II} assumed to be zero, we compute the energy release rate according to the first and second results on the right hand side of Eq. 7 and set it equal to the adhesion energy to predict fibril detachment initiated as such a defect. The predicted strength, S^P , in terms of the average interface stress, for fibril detachment initiated at a defect at the location of the peak stress is then

$$S^P = \frac{1}{k} \sqrt{\frac{8EW}{3\pi a}}, \tag{11}$$

where k is the stress concentration factor for the peak stress as introduced above in connection with Fig. 11.

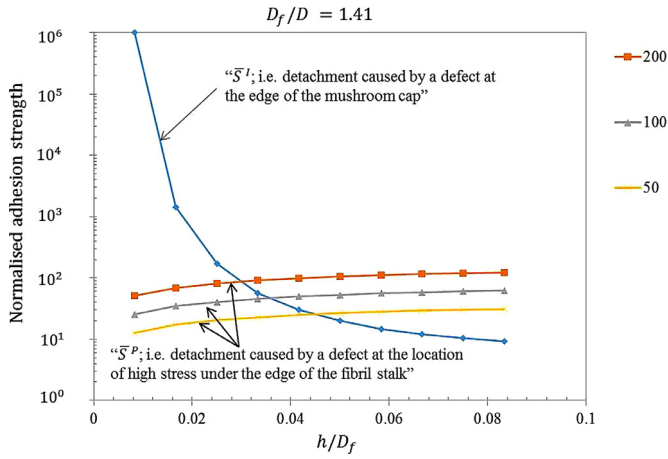


Fig. 12. Adhesion strength for a fibril having a mushroom cap predicted for detachment due to a defect at the edge of the mushroom cap (\bar{S}^I) and for detachment due to a defect at the location of high stress under the edge of the fibril stalk (\bar{S}^P). The ratio of fibril mushroom cap diameter to fibril stalk diameter $D_f/D = 1.41$ and the fibril is axisymmetric. The defect sizes are plotted for three different values of $(3.3D_f^{0.406}l^{0.094})/\sqrt{1.5\pi a} = 50, 100$, and 200 where l is the size of the defect at the mushroom cap edge and $2a$ is the size of the defect at the location of the high stress. These curves are marked accordingly as 50, 100 and 200.

The competition between detachment initiated at the edge of the mushroom cap and that initiated at a defect at the location of the peak stress under the edge of the fibril stalk is explored in Fig. 12 as follows. The non-dimensional strength for edge detachment follows directly from Eq. 8 as

$$\bar{S}^I = \frac{S^I D_f^{0.406} l^{0.094}}{0.6\sqrt{EW}} = \frac{1}{\bar{a}}. \quad (12)$$

From Eq. 11 the strength for fibril detachment initiated at a defect located at the peak stress under the edge of the fibril stalk is now expressed in this non-dimensional form as

$$\bar{S}^P = \frac{S^P D_f^{0.406} l^{0.094}}{0.6\sqrt{EW}} = \frac{D_f^{0.406} l^{0.094}}{0.3k\sqrt{1.5\pi a}}. \quad (13)$$

A comparison of \bar{S}^I and \bar{S}^P is plotted in Fig. 12. The curves for \bar{S}^P are marked “Detachment caused by a defect at the location of high stress under the edge of the fibril stalk.” Note that for illustration we have chosen three different values for $3.3D_f^{0.406}l^{0.094}/\sqrt{1.5\pi a}$ namely 50, 100 & 200, representing the relative sizes of the two defects of length l and $2a$ respectively and the diameter, D_f , of the mushroom cap. These curves are marked in Fig. 12 by their values 50, 100 & 200 respectively. These choices of 50, 100 & 200 are somewhat arbitrary and the reason for them will become clear below.

In Fig. 12 the curve for \bar{S}^I is marked “Detachment caused by a defect at the edge of the mushroom cap.” It can be seen that with the chosen relative sizes of the defects the plot for edge initiated detachment, within the domain of the figure, intersects with the plots for detachment commencing at the location of the peak stress. This is the reason for choosing the values $3.3D_f^{0.406}l^{0.094}/\sqrt{1.5\pi a} = 50, 100$ & 200 as the resulting intersections enable us to illustrate the relevant situation, which is as follows. For this explanation we focus on the case where either edge initiated detachment takes place or detachment occurs commencing at the location of peak stress with flaw sizes such that $3.3D_f^{0.406}l^{0.094}/\sqrt{1.5\pi a} = 200$. In this case, the adhesive strength of the fibril is predicted by the lower of the curve for the latter and the curve for edge initiated detachment in Fig. 12. This implies that for small values of h/D_f detachment will be initiated at the defect under the peak in stress at the edge of the fibril

stalk, whereas for large values of h/D_f detachment will be initiated at the edge of the mushroom cap. The reason for choosing the three different values $3.3D_f^{0.406}l^{0.094}/\sqrt{1.5\pi a} = 50, 100$ & 200 in Fig. 12 is to illustrate how the domains of edge initiated detachment and detachment initiated at the peak stress location vary as the relative sizes of a , l and D_f change. It can be seen that as $3.3D_f^{0.406}l^{0.094}/\sqrt{1.5\pi a}$ becomes larger, the domain in which edge initiated detachment occurs becomes more extensive; this outcome is a consequence of the fact that $3.3D_f^{0.406}l^{0.094}/\sqrt{1.5\pi a}$ becomes larger as a consequence of a diminishing, so that detachment caused by a defect at the location of the peak stress at the edge of the fibril stalk is less likely.

4. Discussion

Spuskanyuk et al., (2008) provided the initial study on how the corner stress singularity varies from a straight punch to a mushroom fibril for frictionless and sticking friction conditions. In the course of the work just described we have repeated their simulations and our results are in good agreement with theirs. They also reported how the stress varies along the substrate interface for the straight punch and the mushroom fibril, and a similar behaviour can be observed in our results in Figs. 2 and 4, respectively. The straight punch exhibits lower adhesive strength when compared to mushroom fibrils because the corner stress singularity acts as the driving force to initiate detachment. In the case of the mushroom fibril, the extra material present on either side of the stalk evidently reduces the corner singularity, which in turn improves the adhesive strength of the fibril. The results in Figs. 5 and 6 show that there is a zone of higher stress at the centre of the fibril under the edge of the stalk, from where detachment could now initiate. Experimentally, such a behaviour was found by Hossfeld et al., (2013) and by Micciché et al., (2014), who observed that detachment of mushroom fibrils started from close to the centre. We have only investigated this issue in a preliminary manner as we have not quantified the strength of the adhesive bond between the fibril and the substrate for any specific combination of materials. However, we have provided illustrative results that indicate the trade-off between detachment initiated near the centre of the fibril and that commencing at the edge of the mushroom cap.

The most systematic experimental study on how the contact shape of a compliant PDMS (polydimethylsiloxane) fibril adhered to a glass probe affects adhesion was published by Del Campo et al., (2007). They examined different geometries such as the straight punch, the spherical cap, the spatula cap, the straight cap with rounded edges, the mushroom cap and the concave cap. Mushroom fibrils showed superior adhesion when compared to other shapes. The adhesive strength of the straight punch and the mushroom fibril varied from approximately 30 to 60 kPa depending on the fibril radius and preload. For a mushroom fibril with $D_f/D = 1.29$, the adhesive strength varied from 30 to 185 kPa.

Carbone and Pierro, (2012) have calculated the dependence of adhesive performance on the mushroom cap geometry and suggested an optimal shape for adhesion. They erroneously assumed that the corner stress singularity involves inverse square root behaviour, a result that cannot be justified as it disagrees with the analysis of Khaderi et al., (2014) and our results. Direct comparison of their results with ours is difficult as they used a different mushroom design. As we have shown, mushroom fibrils having cap to stalk diameter ratios much less than 2 are very effective at raising adhesive strength; hence the larger mushroom designs assumed by Carbone and Pierro appear somewhat unrealistic and unnecessary.

Aksak et al., (2014) have presented a computational study on how the geometry of a wedge or conical shaped mushroom cap influences fibrillar adhesion by varying the angle (from 25° to 80°)

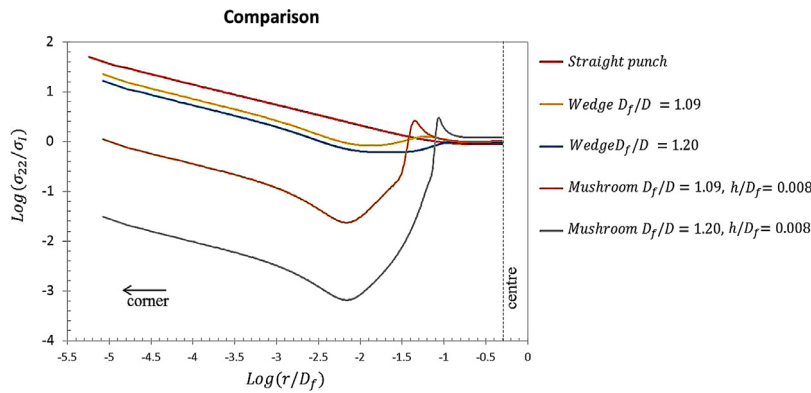


Fig. 13. Comparison of published fibril shapes and present study: Tensile stress along the interface for a straight punch, a mushroom fibril (as in the current paper) and a wedge shaped mushroom cap (by Aksak et al., 2014), all for the axisymmetric case. Adhesion controlled by edge detachment is superior for the mushroom shaped fibrils compared to that of the wedge shaped ones.

Table 3

Calibration coefficients \tilde{a} and normalised adhesion strength for axisymmetric fibrils with wedge shaped mushrooms as assumed by Aksak et al. (2014) (S'/S^{punch} Wedge). For comparison, we list the adhesion strength of mushroom cap fibrils (S'/S^{punch}) used in the current paper. The range of S'/S^{punch} values refers to the mushroom fibrils with different h/D_f .

| D_f/D | \tilde{a} . Wedge | S'/S^{punch} Wedge | S'/S^{punch} |
|---------|---------------------|----------------------|----------------|
| 1.09 | 0.151 | 1.84 | 1.2–35 |
| 1.2 | 0.125 | 2.21 | 1.63–1241 |

at which the wedge shaped mushroom cap meets the substrate. Using a Dugdale cohesive zone to model the detachment process, they found that the strongest adhesion was associated with a mushroom cap that meets the substrate at 45° and (in our notation) $D_f/D = 1.1$ or 1.2 . In addition, they assumed a thin disc of material appended to the tip with the same diameter as the fibril tip; this was found to compromise the adhesive strength of the fibril.

To make a direct comparison with the work of Aksak et al., we have chosen a mushroom fibril with wedge angle 45° and a thin disc of thickness $0.01D$ appended to it. Two cases were examined with different ratios of mushroom cap diameter to stalk diameter: $D_f/D = 1.09$ and 1.2 . The comparison of the stress distributions for wedge shape (by (Aksak et al., 2014)) and mushroom shape (as in the present study) can be seen in Fig. 13. The calibration coefficients and adhesion strength values are listed in Table 3. The range of values 1.2 to 35 and 1.63 to 1242 corresponds to the respective h/D_f . It can be observed from Fig. 13 and Table 3 that the mushroom fibrils (from the current study) have the capability to perform better, provided detachment does not occur earlier with a different mechanism. The stress peak close to the centre of mushroom fibrils seen in Fig. 13 can act as a crack initiation point, but the peak can be removed by improving the design which will be discussed in Section 4.1. Therefore, it is relatively straightforward to design mushroom shaped fibrils to outperform wedge shaped ones.

4.1. Improved shape for a fibril having a mushroom cap

As noted above, the results in Figs. 5 and 6 show that there is a peak in the normal traction at the interface between the fibril and the substrate that is located immediately under the edge of the fibril stalk. We have explored designs that will eliminate this elevation in the stress, and have found that it disappears when there is a fillet radius, as in Fig. 14, smoothing the transition from the fibril

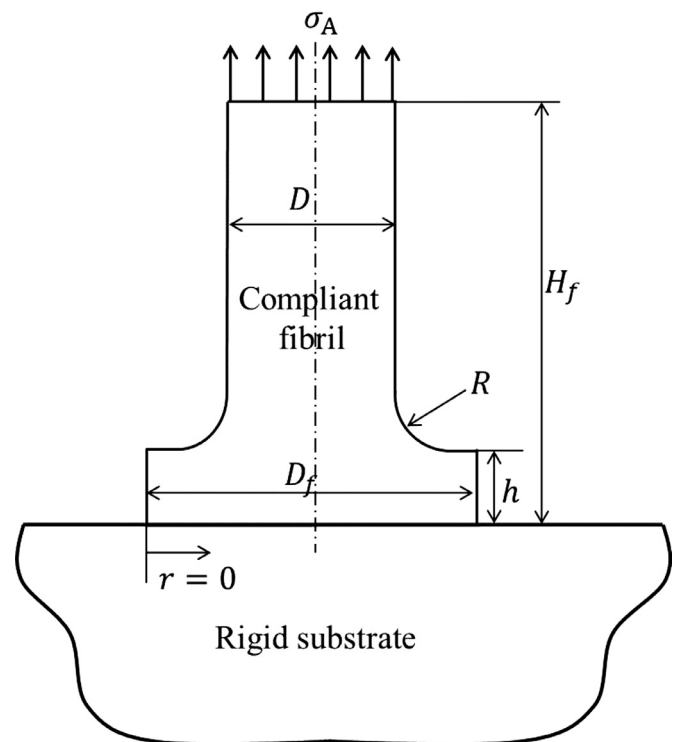


Fig. 14. Schematic of a mushroom fibril with a fillet of radius R where $R/D_f = 0.083$.

stalk to the mushroom cap. As an example of this improvement in the design, we carry out simulations for the stress at the interface between the substrate and fibrils having $D_f/D = 1.41$ and a fillet radius, R , such that $R/D_f = 0.083$. The results for the normal traction at the interface are shown in Fig. 15 for various values of h/D_f for axial symmetry. It can be seen that the stress elevation visible in Figs. 5 and 6 is absent in the results in Fig. 15. In this case the maximum normal traction under the fibril, other than at the edge of the cap, is simply twice the average stress on the interface.

The calibration coefficient for the edge singularity is extracted from the results in Fig. 15 by fitting the plots to the asymptotic solution as before. The results, designated \tilde{a}_{fillet} , are shown in Fig. 16 and are listed in Table 4, as are the calibration coefficients, \tilde{a} , for fibrils having mushroom caps without the fillet radius. It can be seen that the introduction of the fillet radius has increased the amplitude of the edge singularity compared to that prevailing in

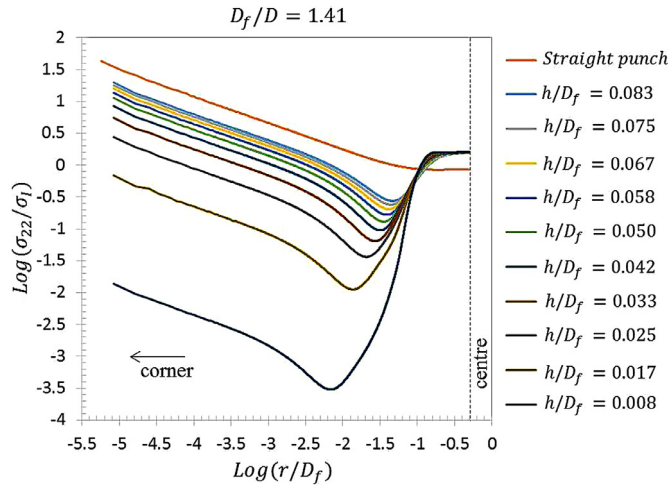


Fig. 15. Tensile stress along the interface between the substrate and an axisymmetric mushroom cap fibril having a value of the radius of the fillet, R , that is effective at eliminating the high stress under the edge of the fibril stalk. Results are shown for a fibril having a mushroom cap whose diameter is 1.41 times that of its stalk and for various values of the ratio of the cap thickness to its diameter h/D_f . The fillet radius is such that $R/D_f = 0.083$.

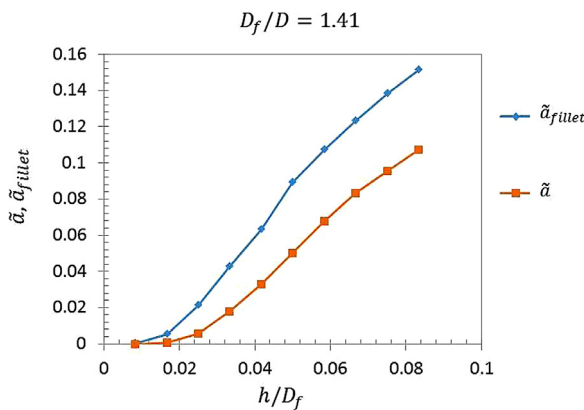


Fig. 16. Calibration coefficients, \tilde{a} , for mushroom fibrils without fillet radii and, \tilde{a}_{fillet} , for mushroom fibrils with fillet radii, both plotted as a function of the ratio of mushroom cap thickness to diameter, h/D_f , for a ratio of fibril mushroom cap diameter to fibril stalk diameter $D_f/D = 1.41$ for axisymmetric fibrils. The fillet radius is such that $R/D_f = 0.083$.

Table 4

Calibration coefficients \tilde{a} for axisymmetric fibrils having a fillet radius that is effective at eliminating the high stress under the fibril stalk for mushroom caps with $D_f/D = 1.41$ where D_f and D are the diameter of mushroom cap and the stalk respectively and h is the thickness of the mushroom cap. The fillet radius $R/D_f = 0.083$.

| h/D_f | \tilde{a}_{fillet} | S_{fillet}^l/S^{punch} |
|---------|----------------------|--------------------------|
| 0.008 | 1.05E-04 | 3.16E+03 |
| 0.017 | 5.25E-03 | 6.31E+01 |
| 0.025 | 2.14E-02 | 1.55E+01 |
| 0.033 | 4.27E-02 | 7.76E+00 |
| 0.042 | 6.31E-02 | 5.25E+00 |
| 0.050 | 8.91E-02 | 3.71E+00 |
| 0.058 | 1.07E-01 | 3.09E+00 |
| 0.067 | 1.23E-01 | 2.69E+00 |
| 0.075 | 1.38E-01 | 2.40E+00 |
| 0.083 | 1.51E-01 | 2.19E+00 |

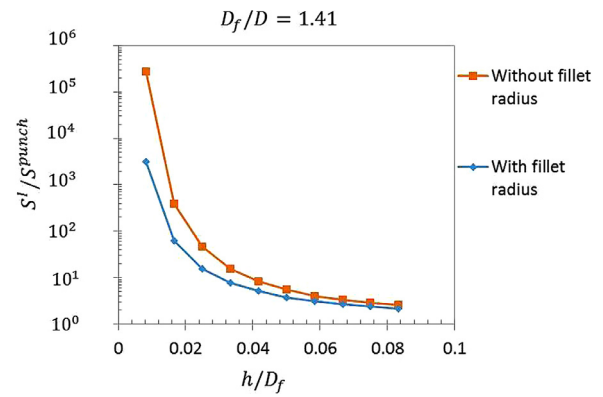


Fig. 17. Adhesion strength for fibrils with and without fillet radii plotted versus h/D_f for a ratio of fibril mushroom cap diameter to fibril stalk diameter $D_f/D = 1.41$ for axisymmetric mushroom fibrils. Detachment in these cases is initiated at the edge of the mushroom cap. The adhesion strength is normalised by that for a punch shaped fibril. The fillet radius is such that $R/D_f = 0.083$.

the absence of the fillet radius. It follows that the mushroom fibrils with the fillet radius will be more prone to edge detachment than the equivalent design lacking a fillet radius. However, the fillet radius, by eliminating the peak in stress that would otherwise occur under the edge of the stalk, diminishes the likelihood of detachment commencing near the centre of the fibril.

In Fig. 17 we have replotted the results in Fig. 16 to represent S^l/S^{punch} , the ratio of the adhesive strength of a fibril having a mushroom cap to that of a punch shaped fibril. One curve, marked “With fillet radius” gives the results for fibrils with the fillet radius connecting the mushroom cap and the stalk, while the results marked “Without fillet radius” are for the fibrils lacking the smooth transition from stalk to mushroom cap. It can be seen that the adhesive strength of the fibrils having a fillet radius is somewhat poorer than that for fibrils in which the fillet radius is absent. However, that conclusion is associated with detachment that commences at the edge of the mushroom. Due to the lack of a peak in the stress under the fibril stalk when there is a fillet radius present, the situation will be reversed when detachment commences near the centre of the fibril. In the case of the fibrils used for the calculations with results presented in Fig. 17, the adhesive strength for detachment commencing at the centre of the fibril, measured in terms of S^l , i.e. the average stress on the interface, will be $\sigma_o/2$, where σ_o is representative of the cohesive interaction strength between the fibril and the substrate. If we assume that this interaction is of the Dugdale type, σ_o will be equal to the cohesive stress attracting one surface to the other.

In fact, we can assume that a fillet radius can be introduced in all cases to smooth the transition from the fibril stalk to the mushroom cap no matter the values of the ratios D_f/D and h/D_f . Designs can then always be found with a sufficiently effective fillet radius such that detachment will commence near the centre of the fibril, and this will occur at a strength, measured in terms of the average stress at the interface, given by

$$S^l = \left(\frac{D}{D_f}\right)^2 \sigma_o. \quad (14)$$

Thus the best strength is achieved with the smallest possible mushroom cap diameter that eliminates edge detachment and therefore leads to a strength quite close to the cohesive stress.

5. Conclusion

In the current work we focus on the corner singularity for a mushroom fibril that is assumed to be perfectly bonded to a rigid

substrate so that no sliding can occur at the interface between the fibril and the substrate to which it is attached. The mushroom cap and stalk dimensions are varied and the calibration coefficient and the adhesive strength are calculated for the resulting combinations of mushroom cap geometry. In total, more than a hundred different combinations of fibril geometry are analysed in this study. The following conclusions can be drawn:

- In agreement with earlier work, straight punch fibrils exhibit a corner stress singularity. This explains the tendency of straight fibrils to detach from a corner location.
- In mushroom fibrils, the corner stress is reduced to a level that may preclude detachment initiating at that location. Instead, the stress near the centre of the fibril may initiate detachment there.
- The mushroom geometry strongly influences the stress distribution. The corner stress singularity amplitude is minimised for small cap thicknesses and small fibril stalk diameters.
- Optimisation of fibrillar adhesives will require mushrooms caps with minimum thickness and minimum stalk diameter, but limited by the tendency for detachment to occur near the centre of the fibril and by the material strength.
- Introduction of a fillet radius to smooth the transition from the fibril stalk to the mushroom cap was successful in removing the stress peak that otherwise occurs in mushroom fibrils on the interface immediately below the edge of the fibril stalk. However, the adhesive strength controlled by the edge singularity is reduced, which is a drawback. However, the adhesive strength associated with detachment commencing near the centre of the fibril is improved. The results suggest that an adhesive strength close to the cohesive stress at the interface can be achieved with careful design of the mushroom cap and the fillet radius. A detailed analysis is necessary to establish the best design.

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Supplementary materials

Supplementary material associated with this article can be found, in the online version, at [doi:10.1016/j.ijsolstr.2016.02.018](https://doi.org/10.1016/j.ijsolstr.2016.02.018).

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