University of Aberdeen

Business School

Department of Economics

Simultaneous Supplies of Dirty and Green Fuels with Capacity Constraints

Is there a Green Paradox?

Marc Gronwald, Ngo Van Long, Luise Röpke

Discussion Paper in Economics No 15-18

December 2015

ISSN 0143-4543

Simultaneous Supplies of Dirty and Green Fuels with Capacity Constraint: Is there a Green Paradox?

Marc Gronwald*
Ngo Van Long†
Luise Röpke‡

December 2015

*University of Aberdeen Business School, Aberdeen Centre for Research in Energy Economics and Finance, CESifo and ifo Institute. Address: University of Aberdeen Business School, Edward Wirght Building Block B, Dunbar Street, Aberdeen, AB24 3QY, United Kingdom. Email: mgronwald@abdn.ac.uk.

†Department of Economics, McGill University, Montreal H3A 2T7, Canada. Email: ngo.long@mcgill.ca.

‡Ifo Institute - Leibniz Institute for Economic Research, Poschinger Strasse 5, 81679 Munich, Germany, Email: roepke@ifo.de.

Simultaneous Supplies of Dirty and Green Fuels with Capacity Constraint : Is there a Green Paradox?

Abstract: This paper studies some possible unintended consequences of alternative climate policies, using a resource extraction framework with heterogeneous deposits and energy sources, thus extending the scope of the theory of the green paradox. A key feature of the model is that there is a capacity constraint on a green backstop resource. This feature implies the simultaneous use of the expensive backstop resource and the cheaper exhaustible resources, over some interval of time. The model considers two dirty exhaustible resources, reflecting the heterogeneity of energy sources with respect to cost structure and carbon content. The policies under consideration are taxation of the dirty resources and the promotion of the green resource via subsidies or capacity-increasing measures. We complement our analytical investigation by a numerical analysis of the welfare effects of the different policies, using specific functional forms of social damage functions. The evolution of the stock of atmospheric carbon is modeled under alternative assumptions about the accumulation of anthropogenic carbon in the atmosphere. The key findings that emerge from this paper, compared to a baseline scenario without policy intervention, are that (1) expanding the capacity of the renewable energy sector, without additional policy measures, can decrease social welfare, (2) both the capacity expansion and the subsidy on green energy lead to increases in short-term emissions, and (3) none of the analyzed policy measures leads to a decrease in the aggregate duration of the extraction of the exhaustible resources.

JEL-Classification: Q38, Q54, H23

Keywords: capacity constraints, green paradox, climate change

1 Introduction

Scientific evaluation of the severity of the threat of climate change has increased the priority accorded to policies aimed at mitigating carbon emissions. The most prominent developments toward decarbonization of the global economy are in the area of green energy production: The replacement of coal-fired power plants by wind turbines and solar power stations, as well as the use of biofuels as substitutes for fossil fuels in the transport sector. In many parts of the world and especially in the Western countries, various policy measures have been introduced in order to push these developments: Among these are the EU-wide energy policy goals, subsidies such as feed-in-tariffs at national levels, and various regional incentives for the production of green energy. The inevitable consequence of this decarbonization process is a complete reconstruction of the entire energy sector.

This reconstruction is not a simple matter. Investment projects in the development of green energies are not only large scaled and complex ventures, but also of a very long term nature; and there are various challenges that need to be met. For example, in the field of fuel development, increasing the use of biofuels is beset by many problems of food security and sustainability, as well as technological constraints. In short, the decarbonization process is limited in many respects and green energy cannot be used to the extent that many would wish.[1] In consequence, conventional fuels continue to be predominant. Strangely, however, even though, in addition to the above-mentioned problems, biofuels are not competitive, we observe that they are used simultaneously with conventional fuel types.

These two features - implementation difficulties and simultaneous use of resources with different costs - are also apparent in the context of renewable-source electricity. Transforming existing electricity transmission networks is extremely expensive, as is the provision of

[1] This is in contrast to the standard "backstop" literature. There, it is assumed that at a point in time, a backstop technology becomes (economically) available in unlimited amounts from then on being the only energy source used.

sufficient storage facilities for renewable-source electricity. But again, even though green electricity is considerably more expensive than conventional sources - the prime costs for producing wind and solar power being much greater than those for producing conventional thermal electricity - both types of electricity are being generated and used simultaneously.

Although these two facts - green energy is capacity-constrained and it is used simultaneously with conventional energy - are obvious, they are not adequately considered in the evaluation of climate policies regarding the resource extraction path and the respective climate consequences. The aim of this paper is to remedy this by using an extended Hotelling resource extraction model with a capacity-constrained backstop technology in order to analyze the effects of green policies. The effects of different policy measures on emission paths as well as their welfare consequences are studied both analytically and numerically. Our model is formulated to correspond to the concrete oil market example, and our numerical analysis involves oil market features allowing for an investigation of the use of conventional oil, unconventional oils, and biofuels. In consequence, this model is able to capture many empirically relevant problems of the transformation of the energy sector.

Our paper is related to two streams of literature. The first of these is the "green paradox" literature, which deals with the effects of green policies on the extraction decisions of carbon resource owners. Sinn's (2008) paper on this so-called green paradox is highly important as it sparked enormous research efforts (see, e.g., Gerlagh 2011, Hoel 2011, Grafton et al. 2012, van der Ploeg and Withagen 2012a, 2012b). Sinclair (1992, 1994) pointed out that a carbon tax should start at a high level and fall over time, contrary to the usual policy prescription (Nordhaus 2007). This is in the same spirit as Sinn (2008). In his paper, Sinn considers a scenario in which owners of carbon resources are confronted with green policies that are expected to become stricter over time. He shows that this can provide exhaustible resource owners with an incentive to accelerate rather than postpone the extraction of the carbon resource. Thus, a well-intended but poorly designed climate policy can have detrimental effects for the climate. In line with Sinn's argument, using a two-country model with country heterogeneity, Hoel (2011) found that lowering the costs of producing a substitute for carbon resources or imposing carbon taxes can have undesirable consequences since, under specified conditions, those policy measures can speed up the use of the carbon resource producing a green paradox result.[2] The green paradox literature is vast, but, to our knowledge, there has been no explicit consideration of the possibility of green paradox outcomes in a framework with capacity-constrained green resources, which is clearly more realistic. This is somewhat surprising as there is a literature on the order of resource extraction showing that this kind of constraints can have substantial effects on the optimal order of exploitations of deposits.[3]

Indeed, it is this very order of extraction literature that provides the second motivation for this paper. This stream of literature has its origin in Herfindahl (1967). However, the original finding that resources with different constant marginal extraction costs are extracted in strict order from low to high-cost, the so-called Herfindahl rule, has been repeatedly called into question. For example, Kemp and Long (1980) and Amigues et al. (1998) show in a general equilibrium setting, that when the inexhaustible substitute can be supplied only in constrained amounts, the extraction order deviates from the standard Herfindahl path. The contribution to the extraction order literature that is most relevant to this paper is Holland (2003), which finds similar results using a partial equilibrium model. Holland argues, in line with the resource literature, that resource owners base their extraction decision not only on marginal extraction costs, but also on the scarcity rent of the resources. Then, the crucial

[2] In an earlier paper with more than two countries, Hoel (2008) shows that if clean energy can be supplied at constant and positive marginal costs and without a capacity constraint, a policy of committing to subsidize the production of the clean energy will induce market participants to expect a lower price for fossil fuels in the future, leading to more extraction of fossil fuels sooner, resulting in the fossil fuel stock being exhausted sooner and hence producing a green paradox outcome. Thus, in the Hoel (2008) model, subsidizing clean energy increases carbon emissions (assuming that the subsidy is not accompanied by other policy measures).

[3] Chakravorty, Tidball and Moreaux (2008), for example, considered the optimal order of exploitations if non-renewable deposits have different carbon contents. They imposed overall capacity constraints on extractions, but did not address the issue of the green paradox.

feature of the model is that some extraction capacities are limited, which has important implications for the optimal order of resource extraction. In such a situation, energy from the inexhaustible resource may be used in parallel to and even strictly before some exhaustible resource stocks that have lower marginal costs. The resulting extraction patterns are similar to the ones we actually observe.

Based on a reinterpretation of Holland's (2003) model, we evaluate whether results obtained by Sinn (2008) and Hoel (2008, 2011) also hold in our model with two exhaustible resources and one capacity-constrained green backstop. We model the backstop technology in line with Dasgupta and Heal (1974), as a "perfectly durable commodity, which provides a flow of services at constant rate."We analyze different scenarios, for example, different taxation schemes on exhaustible resources or a marginal expansion of the green capacity, for their green paradox effects. We find conditions under which a green paradox outcome will arise.

The analysis employs the notions of a "weak green paradox" and a "strong green paradox" introduced by Gerlagh (2011) as well as an "overall green paradox" effect. The first refers to a short-term increase of anthropogenic emissions due to a policy measure, the overall green paradox effect refers to an overall increase, and the strong green paradox to overall increased social damages compared to a baseline scenario.

Our analytical results are complemented by a numerical welfare analysis, in which we formulate an explicit social damage function, analyze specific accumulation behavior of the anthropogenic carbon in the atmosphere, and investigate the situation where various deposits have different carbon content. Moreover, for the purpose of illustration, we introduce the example of an oil market with exhaustible resources being conventional and unconventional oil and a capacity-constrained green backstop technology. Based on different specifications of a green paradox, we numerically evaluate the overall welfare effect by looking at the social consequences of the various policy scenarios compared to a base case without policy intervention. While the strong green paradox is the most important effect for the analysis, the other green paradox effects are also worth analyzing since they provide additional insight into market behavior that has an impact on the short- and medium-term effects of a policy measure.

The remainder of the paper is organized as follows. In the next section, we derive a model of substitute production under a capacity constraint based on Holland (2003). Sections 3 and 4 present a policy analysis with an implicit determination of the endogenous variables and the comparative static analysis of the different policy scenarios and a welfare analysis, respectively. Section 5 discusses the policy relevance of this paper. Section 6 offers some concluding remarks.

2 A model of substitute production under capacity constraint

Assume that there are two deposits of fossil fuels, S_1 and S_2. The constant per unit extraction costs for these deposits are c_1 and c_2, respectively. There are no capacity constraints on the amount of extraction at any given point of time t, i.e., no upper bounds on $q_1(t)$ and $q_2(t)$. The cumulative extraction constraints are

$$\int_0^{\infty} q_i(t)dt \leq S_i \text{ for } i = 1, 2.$$

There is a clean energy that is a perfect substitute for the fossil fuels. Let $q_3(t)$ be the amount of clean energy produced at time t. Assume there is a capacity constraint on clean energy production: $q_3(t) \leq \overline{q}_3$. This means that at each point of time, the amount of green energy that can be produced is exogenously determined by the capacity constraint. Let c_3 be constant unit costs of production of the clean energy.

Let $Q(t) = q_1(t) + q_2(t) + q_3(t)$ denote the aggregate supply of energy from the three resources at time t, where some of these $q_i(t)$ may be zero. The utility of consuming $Q(t)$ is $U[Q(t)]$, where $U(\cdot)$ is a strictly concave and increasing function and $U'(0)$ can be finite or infinite. Moreover, assume $c_1 < c_2 < c_3 < U'(0)$.

The total welfare is

$$\int_0^\infty e^{-rt}\left[U[Q(t)] - \sum_{i=1}^{3} c_i q_i(t) - C[V(t)]\right] dt$$

where $V(t)$ is the volume of CO_2 in the atmosphere at time t, and $C(V)$ is the damage cost function with $C'(V) > 0$ and $C''(V) \geq 0$.

We assume that CO_2 emissions are proportional to the consumption of fossil fuels $q_1(t)$ and $q_2(t)$ and can be expressed as

$$\varepsilon_1(t) = \eta_1 q_1(t) \text{ and } \varepsilon_2(t) = \eta_2 q_2(t),$$

where η_1 and η_2 are positive coefficients.

Our first task is to characterize the equilibrium in the perfect competition situation. Consumer' demand is represented by the condition $p = U'(Q) \equiv \phi(Q)$, where $\phi(Q)$ is strictly decreasing. Inverting this function, we obtain the demand function

$$Q = D(p),\ D'(p) < 0.$$

The resource owners follow a Hotelling-like extraction path, maximizing the value of the resource stocks such that the resource rent increases at the rate of interest. The extraction order of the exhaustible resource stocks is based on the Herfindahl rule: The low-cost resource stock is strictly exhausted before the high-cost resource stock is extracted. Since the renewable resource owners do not have to optimize intertemporally, their supply behavior is different from that of the exhaustible resource owners. In the next subsection, we assume that the parameters of the models satisfy two conditions that ensure that the high-cost renewable energy will be produced simultaneously with extraction of the lowest cost deposit, and well before the lower cost stock S_2 enters into production. These conditions were first identified by Holland (2003). We impose these conditions so that the model reflects the current world energy market situation described in the introduction to this paper. Based on those conditions, the resulting extraction phases and prices can be outlined.

2.1 Extraction capacity and cost reversal

Based on Holland (2003), two conditions are imposed to ensure that both a binding capacity constraint of the renewable energy, as well as the cost reversal phenomenon, can be illustrated in the model. By "cost reversal", we mean that the higher cost renewable resource is produced well before the intermediate cost exhaustible resource begins to be extracted. In specifying the capacity constraint, we describe the real-world situation where even though in theory we have enough renewable energy resources, only a limited amount of that energy is practically available due to technological and economic constraints. To sharpen the consequences of this situation, we focus in the following analysis on the case where the capacity constraint is binding when green energy is produced. Then, at price $p = c_3$, the market demand $D(c_3)$ for energy exceeds the capacity output of the clean energy sector $\overline{q}_3$. This is stated in the following condition.

Condition 1: $D(c_3) > \overline{q}_3$

So, when $p(t)$ reaches c_3, the market demand must be met from both the clean energy sector and fossil fuel extraction.

Since the demand curve is downward sloping, Condition 1 implies that there exists a value $\overline{p} > c_3$ such that $D(\overline{p}) = \overline{q}_3$. Therefore, for all p in the range $[c_3, \overline{p}]$, the clean resource will always be produced at maximum capacity. The equilibrium price of energy can never exceed $\overline{p}$, even when $U'(0) = \infty$.[4]

The second condition is that the size of the high-cost exhaustible resource must be small enough such that the cost reversal of resource use described in the introduction can be illustrated with the present model. An analytical derivation of this condition can be found in Appendix A.

$$\textbf{Condition 2:}\ S_2 < S_2^{\max} \equiv \int_0^x D\left[c_2 + (c_3 - c_2)\, e^{r\tau}\right] d\tau - \frac{\overline{q}_3}{r} \ln \left[\frac{\overline{p} - c_2}{c_3 - c_2}\right]$$

where we define x by

$$x = \frac{1}{r} \ln \left[\frac{\overline{p} - c_2}{c_3 - c_2}\right].$$

From condition 2, we can show that if the size of deposit 2 is smaller than the threshold value $S_2^{\max}$, the equilibrium time path of extraction is continuous and production of green energy starts strictly before the extraction of the high-cost resource deposit S_2 begins (Holland 2003).[5]

2.2 Four phases of resource utilization and the price path

Based on Conditions 1 and 2, the equilibrium path of the energy price is continuous and the resource use pattern can be described as follows (see also Holland 2003, Figure 1).

[4]We will not consider the alternative case of $D(c_3) \leq \overline{q}_3$. In this case, at price $p = c_3$, market demand $D(c_3)$ is lower than (equals) capacity output $\overline{q}_3$. In case of $D(c_3) < \overline{q}_3$, the capacity constraint $q_3 \leq \overline{q}_3$ is never binding and green energy production could be anything up to $D(c_3)$ (completely replacing the exhaustible resources). For $D(c_3) = \overline{q}_3$, the capacity constraint is exactly binding. In both cases, despite the capacity constraint, we are in the standard backstop technology world: The energy price will rise along the Hotelling path until it reaches c_3, afterward it remains at $p = c_3$ forever. Before the price reaches c_3, the only supply is from the exhaustible resource deposits since the efficient level of supply of the renewable is $q_3(t) = 0$ when $p(t) < c_3$. In the razor edge case defined by $D(c_3) = \overline{q}_3$, Holland (2003) finds that, when the price just reaches c_3, the supply of renewable energy can be anything between zero and $\overline{q}_3$ and afterward the price will remain at c_3 forever.

[5]While Condition 1 can be understood as a necessary condition, Condition 2 can be understood as a sufficient condition for cost reversal. Moreover, the analyzed situations, based on the stated conditions, must be viewed as extreme cases. The model could also be designed to lead to a smooth increase in the production of green energy until the constraint is reached (which would be in accordance with actual observations in, for example, Germany). For simplicity and to sharpen our results, we believe it is useful to retain the strong assumptions. Determining a "dynamic capacity increase" would allow differentiating between constraints on existing production and natural capacity restrictions. Modeling such a differentiation would allow us to show a smooth and increasing use of green energy while maintaining the constrained situation.

Phase 1: Energy is supplied only by extraction from the low-cost deposit. This phase begins at time 0 and ends at an endogenously determined time $t_3 > 0$, such that the equilibrium price at time t_3 is equal to c_3. During this phase, the net price of the low-cost resource, $p(t) - c_1$, rises at a rate equal to the interest rate r.

Phase 2: Energy is simultaneously supplied by both extraction from the low-cost resource deposit S_1 and the (more costly) renewable energy running at its capacity level $\overline{q}_3$. This phase begins at time t_3 and ends at an endogenously determined time $T > t_3$. (In a limiting case, when Condition 2 holds with equality, we have $T = t_3$, meaning that Phase 2 degenerates to a single point.) The low-cost resource stock S_1 is entirely exhausted at time T. During this phase, the net price of the low-cost exhaustible resource, $p(t) - c_1$, also rises at a rate equal to the interest rate r.

Phase 3: Energy is simultaneously supplied by both extraction from the high-cost resource deposit S_2 and the (more costly) renewable energy running at its capacity level $\overline{q}_3$. This phase begins at time T and ends at an endogenously determined time $\overline{T}$. At time $\overline{T}$, the stock S_2 is completely exhausted. During this phase the net price of the higher cost exhaustible resource, $p(t) - c_2$, rises at a rate equal to the interest rate r. At time $\overline{T}$, the energy price reaches $\overline{p}$ (where $\overline{p}$ is defined by $D(\overline{p}) = \overline{q}_3$).

Phase 4: The only source of energy is green energy, available at capacity level $\overline{q}_3$. The price is constant at $\overline{p}$. This phase begins at time $\overline{T}$ and continues for ever (because the time horizon is infinite).

Note that from time t_3 on, where $p(t_3) = c_3$, the clean energy sector will supply $\overline{q}_3$ without any intertemporal considerations, and due to the assumption stated in Condition 1, there will not be enough energy to meet the demand $D(c_3)$. The shortfall, or residual demand, is met by extraction from the lowest-cost deposit available such that at t_3,

$$\overline{q}_3 + q_1(t_3) = D(c_3).$$

Or, in other words, only the residual demand must be met by the exhaustible resource, indicating that the existence of a constrained renewable resource alleviates the scarcity problem of the exhaustible resources.[6]

Holland (2003) does not provide explicit equations that specify how the length of various phases depends on parameters such as $c_1, c_2, c_3, \overline{q}_3, S_1$ and S_2. In what follows, we derive such equations, which help us obtain insightful comparative static results.

3 Policy scenario analysis

In the subsections below, we develop explicit expressions for determining the length of the various phases. Based on these, we investigate the conditions under which energy policy measures to alleviate climate change damages due to exhaustible resource use are effective when a capacity-constrained renewable energy source is available. Is it still true that a subsidy on renewable energy will harm the environment (Section 3.2.1)? Does a marginal expansion of the capacity help or hurt the mitigation efforts (Section 3.2.2)? Moreover, what are the effects of different ways of taxing exhaustible resource use (Sections 3.2.3 and 3.2.4)?

3.1 Implicit determination of the endogenous variables

Define y to be the length of Phase 3, i.e., the phase during which deposit 2 is extracted. Then $y \equiv \overline{T} - T$. Since total demand must equal total supply during $\left[T, \overline{T}\right)$ and deposit 2 must be exhausted during this interval, we can solve for y from the following equation

$$\int_T^{\overline{T}} D[p(t)]dt = S_2 + \left(\overline{T} - T\right)\overline{q}_3. \tag{1}$$

Since $q_2(t) > 0$ over the time interval $\left[T, \overline{T}\right)$, the Hotelling rule applied to deposit 2 must hold with equality such that

$$p(t) = c_2 + (\overline{p} - c_2)\, e^{r(t-T-y)} \tag{2}$$

[6]The reason deposit 2 is not extracted during the time interval $[t_3, T)$ is that any attempt to move extraction from S_2 to that interval to replace the high-cost clean energy would require curtailing consumption during the phase $\left[T, \overline{T}\right)$, which implies costs in terms of foregoing consumption smoothing.

with $t-\overline{T}=t-T-\left(\overline{T}-T\right)$.[7] Inserting this into Equation (1), together with $\tau=t-T$, and noting that $\overline{p}$ and $\overline{q}_3$ are related through the equation $\overline{q}_3=D\left(\overline{p}\right)$, then y is the solution of the following equation

$$0=F(S_2,\overline{p},c_2)=\int_0^y D[c_2+\left(\overline{p}-c_2\right)e^{r(\tau-y)}]d\tau-yD(\overline{p})-S_2 \tag{3}$$

where $S_2<S_2^{\max}(\infty)$ as stated in Condition 2.

Remark: It is clear that an increase in S_2 will increase y. The proof is as follows. Keeping $\overline{p}$ and c_2 constant, and differentiating the previous equation totally, we obtain

$$\left\{[D(c_2)-D(\overline{p})]-r\left(\overline{p}-c_2\right)\int_0^y(e^{r(\tau-y)})D'[c_2+\left(\overline{p}-c_2\right)e^{r(\tau-y)}]d\tau\right\}dy=dS_2.$$

Thus

$$\frac{\partial y}{\partial S_2}>0. \tag{4}$$

Having solved for y, we can determine the price at time T, when the high-cost deposit begins being extracted, as

$$p(T)=c_2+\left(\overline{p}-c_2\right)e^{-ry}\equiv p_2. \tag{5}$$

Next, we can determine the length of the time interval $[t_3,T)$ over which energy demand is met by both extraction from the lowest cost deposit and via production of renewable energy at capacity level. We denote this length by $z\equiv T-t_3$. Then, since $p(t_3)=c_3$ by definition, the Hotelling rule gives

$$z=\frac{1}{r}\ln\left[\frac{p(T)-c_1}{c_3-c_1}\right].$$

Substituting for $p(T)$, we obtain

$$0=G(y,c_1,c_2,c_3,\overline{p})=(c_3-c_1)e^{rz}-(c_2-c_1)-\left(\overline{p}-c_2\right)e^{-ry}. \tag{6}$$

[7] Analogous to the Appendix, $p(t)$ can be derived from the condition $(p(t)-c_2)e^{-rt}=(p(T)-c_2)e^{-rT}=(p(\overline{T})-c_2)e^{-r\overline{T}}$.

It is easy to see that

$$\frac{\partial z}{\partial y} < 0. \tag{7}$$

From Equations (4) and (7), we conclude that an increase in S_2 will reduce z. Specifically, as S_2 approaches $S_2^{\max}$, z approaches zero. Moreover, analogously to the determination of y in Equation (1), since, over the period $[0, T]$ the total demand for energy must equal total supply that comes from deposit 1 and from renewable energy produced at capacity after time t_3, we know that T must satisfy the equation

$$\int_0^T D[p(t)]dt = S_1 + [T - t_3]\,\overline{q}_3, \tag{8}$$

where, since deposit 1 is extracted over the interval $[0, T)$, the Hotelling rule applies to this deposit over that period such that

$$p(t) = c_1 + (c_2 - c_1)e^{r(t-T)} + (\overline{p} - c_2)\,e^{r(t-T-y)}. \tag{9}$$

Finally, from inserting Equation (9) into (8), the following equation determines T as

$$0 = H(y, z, T, c_1, c_2, \overline{p}) = \int_0^T D\left[c_1 + \left(c_2 + (\overline{p} - c_2)\,e^{-ry} - c_1\right)e^{r(t-T)}\right]dt - S_1 - zD(\overline{p}). \tag{10}$$

3.2 Comparative statics

In this section, different policy scenarios aimed at reducing anthropogenic carbon emissions are analyzed with regard to their effects on supply-side extraction and production behavior. We also differentiate between a weak green paradox (as introduced by Gerlagh 2011) and an overall green paradox. A weak green paradox is said to arise when an apparently green-oriented policy results in a short-run increase in emissions. In our analysis, a weak green paradox can be identified as a decrease of $p(0)$, which indicates higher initial resource extraction and/or a decrease in T. An overall green paradox occurs when the overall extraction duration of both resources (which is represented by $\overline{T}$ in our paper) decreases due to the policy measure. Moreover, later in the welfare analysis, we introduce the concept of a strong green paradox (see Gerlagh 2011), which occurs when the policy is environmentally harmful over the long run (e.g., when the present value of the stream of future damages increases due to greater accumulated emissions at all times up to time of exhaustion).

To assess the possibility of a green paradox, we apply the implicit function theorem to the system of Equations (3), (6), and (10) to determine the response of the endogenous variables (y, z, T) as well as of price behavior, to changes in the exogenous parameters c_1, c_2, c_3, and $\overline{p}$. The changes in the exogenous parameters are assumed to result from four different policy measures (two taxes on the exhaustible resources, subsidization of the renewable resource, and an exogenous increase in capacity) intended to slow down carbon extraction.

3.2.1 Effect of a subsidy for renewable energy

In the first part of our comparative static analysis, we investigate how subsidizing clean energy affects the extraction speed of the exhaustible resources. From the literature, we know that a subsidy can have detrimental effects on the environment if the clean energy is available at a constant cost without capacity constraint (see, e.g., Hoel 2008). But does a green paradox also arise in the presence of a capacity-constrained green energy source or can this capacity-constrained source alleviate pressure on exhaustible resource use? Examples of such subsidy systems include the renewable energy feed-in tariffs in Germany and Sweden or, analogously, the exemption of biofuels from taxation. In the following, subsidization of green energy is modeled as a decrease of the constant marginal production cost, c_3. The effect of a change in c_3 on the endogenous variables (y, z, T) can be computed from the following matrix equation

$$\begin{bmatrix} F_y & F_z & F_T \\ G_y & G_z & G_T \\ H_y & H_z & H_T \end{bmatrix} \begin{bmatrix} dy \\ dz \\ dT \end{bmatrix} = \begin{bmatrix} -F_{c_3} \\ -G_{c_3} \\ -H_{c_3} \end{bmatrix} dc_3 \tag{11}$$

where

$$F_y = -r\left(\overline{p} - c_2\right) \int_0^y D'[p(\tau)] e^{r(\tau - y)} d\tau > 0$$

$$G_y = r\left(\overline{p} - c_2\right) e^{-ry} > 0$$

$$G_z = r(c_3 - c_1) e^{rz} > 0$$

$$G_{c_3} = e^{rz} > 0$$

$$H_y = \int_0^T D'[p(t)] \left[-r \left(\overline{p} - c_2 \right) e^{-ry} e^{r(t-T)} \right] dt > 0$$

$$H_z = -D(\overline{p}) < 0$$

$$H_T = D[p(T)] + \int_0^T D'[p(t)] \left[-r e^{r(t-T)} \right] \left(c_2 + \left(\overline{p} - c_2 \right) e^{-ry} - c_1 \right) dt > 0$$

$$F_z, F_T, F_{c_3}, G_T, H_{c_3} = 0.$$

Let J denote the determinant of the 3×3 matrix on the left-hand side of Equation (11). Calculation shows that

$$J = F_y G_z H_T > 0. \tag{12}$$

Then, using Cramer's rule, we obtain the effect of an increase in c_3 on the variables y, z, and T :

$$\frac{dy}{dc_3} = 0 \tag{13}$$

$$\frac{dz}{dc_3} = \frac{-e^{rz}}{J} \left[F_y H_T \right] < 0 \tag{14}$$

$$\frac{dT}{dc_3} = \frac{e^{rz}}{J} \left[F_y H_z \right] < 0. \tag{15}$$

Thus, we see from Equations (13)-(15) that an increase in the clean energy producer's unit cost, c_3, has no effect on the length of time over which deposit 2 is extracted ($dy/dc_3 = 0$), but will shorten the life of the low-cost deposit 1 ($dT/dc_3 < 0$) and will also shorten the interval of time over which both q_1 and q_3 are positive ($dz/dc_3 < 0$). The initial price $p(0)$ will be higher, as can be derived from Equation (9):

$$\frac{dp(0)}{dc_3} = -r \left(c_2 + \left(\overline{p} - c_2 \right) e^{-ry} - c_1 \right) e^{-rT} \frac{dT}{dc_3} > 0. \tag{16}$$

Since $\overline{p}$ and y are not affected by the increase in c_3, we can deduce that the price at which the high cost deposits begins to be extracted will be unaffected, see Equation (5):

$$\frac{dp_2}{dc_3} = 0.$$

The effect of an increase in c_3 on t_3 (i.e., on the time interval over which all energy is supplied from deposit 1 alone) can also be computed. Since $t_3 + z = T$,

$$\frac{dt_3}{dc_3} = \frac{dT}{dc_3} - \frac{dz}{dc_3} = \frac{e^{rz}F_y}{J}\left[H_z + H_T\right] > 0. \tag{17}$$

The analytical results are summarized in Proposition 1.

Proposition 1: *Subsidizing the clean energy product results in a lower initial price of energy. This leads to a faster extraction of the lowest-cost exhaustible resource during the initial phase* $[0, t_3)$ *. However, this phase itself is shortened (*t_3 *is brought closer to time* 0*), and thus clean energy production will begin earlier. This effect allows deposit 1 to be extracted over a longer period. Therefore, in total and contrary to Hoel's (2008) model where subsidization of clean energy (a fall in* c_3*) results in earlier exhaustion of the exhaustible resource, subsidizing clean energy lengthens the life of the aggregate resource stock (i.e., an increase in* $y + T$ *in our model). Thus, there is a weak green paradox effect, but no overall green paradox effect.*

This first result can be understood as follows (see also the illustrated price path in Section 4.2): Subsidization of the renewable energy is equivalent to a decrease in c_3. From $dy/dc_3 = 0$ (Equation (13)), we know that subsidizing the renewable backstop has no effect on how long it will take to exhaust S_2. For illustration purposes, let T^* denote the time of exhaustion of S_1 when the renewable technology is subsidized. Let the equilibrium price path that results from the subsidy be denoted by $\widetilde{p}(t)$. From the invariance of y, it follows that $\widetilde{p}(T^*) = p(T)$. This in turn ensures that the aggregated supply of energy over the length of time y equals the demand.

Moreover, from Equation (15) follows that subsidization of the renewable resource increases the time span of extraction of S_1 by $(T^* - T)$. This means that resource stock S_1 is available for longer and the price level $p(T) = \widetilde{p}(T^*)$ is reached later.

Additionally, an intuitive explanation of the effect of a green-energy subsidy on the extraction q_1 at the production start date of the renewable energy and, therefore, on z is as follows. If the price path were not affected, subsidizing the backstop would lead to earlier production of the renewable energy, implying that, given the unchanged time path of price, the supply of energy is greater than demand. Since this situation would be a disequilibrium, the price path must change. In consequence, $p(0)$ (see Section 4.2, Figure 1, p_0) declines to $p^*(0)$ (Section 4.2, Figure 1, p_0^*), as seen in Equation (16). This decrease moderates the decline in t_3, restoring the balance between supply and demand; still, the analytical results show that $t_3^* < t_3$ (Equation (17)).

These considerations show that two opposed effects work on T^* and z. (1) Due to the decrease of c_3, t_3 decreases (Equation (17)), which increases T since, as q_3 is available earlier, it can alleviate the demand for q_1 sooner. This effect tends to increase z. (2) To equalize demand and supply at t_3, $p(0)$ decreases, as explained previously (see Equation (16)). This second effect works in a direction opposite to the first effect and tends to postpone t_3 and also to shorten z. Moreover, due to a lower initial price level, the demand for energy increases and is satisfied by an increase in q_1 in period $[0, t_3)$. Which of the two effects dominates depends on their relative strength, which has been analyzed analytically. From $dT/dc_3 < 0$ and $dz/dc_3 < 0$ (Equations (15) and (14)), we find that the first effect is stronger than the second. This means that the exhaustible-resource-saving effect (of the subsidy on renewable energy) on S_1 dominates the demand-increasing effect of the price decrease (the effect of $dT/dc_3 + dy/dc_3$ is unambiguous).

Therefore, when there is a subsidization of the renewable backstop under capacity constraint, there is no overall green paradox effect in the long run, but there is a weak green paradox effect over the time interval $[0, t_3)$.

3.2.2 Effect of an increase in capacity

We now investigate the effect of an increase in capacity $\overline{q}_3$, which could occur due to a technological innovation such as, for example, the repowering of wind mills or a change from first-generation to second-generation biofuels. An increase in capacity is equivalent to a decrease in the capacity-induced choke price ($\overline{p}$).[8]

The general case

The effect of a change in $\overline{q}_3$ on the endogenous variables (y, z, T), which is identical to a change in $\overline{p}$ since $D(\overline{p}) = \overline{q}_3$, can be computed, analogously to the previous section, from the following matrix equation as

$$\begin{bmatrix} F_y & F_z & F_T \\ G_y & G_z & G_T \\ H_y & H_z & H_T \end{bmatrix} \begin{bmatrix} dy \\ dz \\ dT \end{bmatrix} = \begin{bmatrix} -F_{\overline{p}} \\ -G_{\overline{p}} \\ -H_{\overline{p}} \end{bmatrix} d\overline{p}$$

where

$$F_{\overline{p}} = -yD'\left(\overline{p}\right) + \int_0^y D'[p(\tau)]e^{r(\tau-y)}d\tau \gtrless 0$$

$$G_{\overline{p}} = -e^{-ry} < 0$$

$$H_{\overline{p}} = -zD'\left(\overline{p}\right) + \int_0^T D'[p(t)]e^{r(t-T-y)}dt \gtrless 0$$

and the determinant J has been determined in Equation (12).

The comparative static results are ambiguous:

$$\frac{dy}{d\overline{p}} = \frac{-F_{\overline{p}}}{J}\left[G_z H_T\right] \text{ has the sign of } -F_{\overline{p}}$$

$$\frac{dz}{d\overline{p}} = \frac{1}{J}\left\{F_{\overline{p}}G_y H_T - G_{\overline{p}}F_y H_T\right\} \gtrless 0$$

$$\frac{dT}{d\overline{p}} = \frac{1}{J}\left\{F_y\left[e^{-ry}D(\overline{p}) - H_{\overline{p}}G_z\right] - F_{\overline{p}}\left[-D(\overline{p})G_y - H_y G_z\right]\right\} \gtrless 0.$$

[8]Modeling a dynamic capacity constraint would complicate the analysis and potentially induces additional extraction and production phases. For the sake of simplicity, the present paper abstracts from any kind of dynamic transition process in the supply of renewable energy.

The effect on the life of the aggregate resource stock is also ambiguous:

$$\frac{d(T+y)}{d\overline{p}} = \frac{1}{J}\left\{F_y\left[e^{-ry}D(\overline{p}) - H_{\overline{p}}G_z\right] - F_{\overline{p}}\left[G_zH_T - D(\overline{p})G_y - H_yG_z\right]\right\} \gtrless 0. \tag{18}$$

However, the effects on the price path are unambiguous (see Equation (9)). First, an increase in capacity (a fall in $\overline{p}$) necessarily leads to a lower initial price:

$$\frac{dp(0)}{d\overline{p}} > 0. \tag{19}$$

Second, a fall in $\overline{p}$ lowers the price at which deposit S_2 begins to be exploited:

$$\frac{dp(T)}{d\overline{p}} > 0. \tag{20}$$

Proposition 2: *An increase in the capacity of the clean energy sector has an ambiguous effect on the life of the aggregate resource stock, and it lowers the scarcity rent of both exhaustible resource stocks.*

To obtain clearer results, let us consider the case of linear demand.

The special case of linear demand

In the following, we assume that demand is linear with the functional form

$$D[p(t)] = A - p(t). \tag{21}$$

Then, taking into account Equation (21), Equation (3) becomes

$$\int_0^y \left[A - \left(c_2 + (\overline{p} - c_2)\, e^{r(\tau - y)}\right)\right] d\tau = y\,(A - \overline{p}) + S_2.$$

Differentiating totally, we obtain after some rearrangement,

$$\frac{dy}{d\overline{p}} = -\frac{S_2}{\left(1 - e^{-ry}\right)\left(\overline{p} - c_2\right)^2} < 0. \tag{22}$$

Thus, an expansion in capacity $\overline{q}_3$, which leads to a fall in $\overline{p}$, lengthens the life of deposit 2. Moreover, from Equations (6) and (22), we can derive the effect of an increase in $\overline{p}$ on z as

$$\frac{dz}{d\overline{p}} = \frac{1}{r}\left(\frac{1}{c_2 - c_1 + (\overline{p} - c_2)\, e^{-ry}}\right)\left[e^{-ry} - r\,(\overline{p} - c_2)\, e^{-ry}\frac{dy}{d\overline{p}}\right] > 0. \tag{23}$$

Thus, a fall in $\overline{p}$ shortens the phase during which both q_1 and q_3 are supplied to the market. To find the effect of an increase in $\overline{p}$ on T, insert the linear demand function (21) into Equation (10), leading to

$$\int_0^T \left[A - c_1 - \left(c_2 + (\overline{p} - c_2)\, e^{-ry} - c_1\right) e^{r(t-T)}\right] dt = S_1 + z\,(A - \overline{p}),$$

where y and z are both functions of $\overline{p}$, with derivatives given by Equations (22) and (23).

Rearranging terms and totally differentiating leads to

$$\begin{aligned} &\left[A - c_1 - \left(c_2 + (\overline{p} - c_2)\, e^{-ry} - c_1\right) e^{-rT}\right] \frac{dT}{d\overline{p}} \\ &= \left\{-\left(\frac{1 - e^{-rT}}{r}\right) r\,(\overline{p} - c_2)\, e^{-ry} \frac{dy}{d\overline{p}} + (A - \overline{p})\frac{dz}{d\overline{p}} + \left(\frac{1 - e^{-rT}}{r}\right) e^{-ry}\right\} - z. \end{aligned} \tag{24}$$

Consider the right-hand side (RHS) of Equation (24). The sum of the terms inside the curly brackets {...} is positive. However, because z is positive, the sign of the RHS seems ambiguous. On the left-hand side, the expression inside the square brackets [...] is ambiguous, though it is positive if A is sufficiently large.

The effect of an increase in $\overline{p}$ on the life of the aggregate resource stock, $y + T$, is also ambiguous. The results shown in Equations (22), (23), and (24) can be summarized in the following proposition.

Proposition 3: *Under linear demand, an increase in the capacity of the clean energy sector (i.e., a decrease in $\overline{p}$) will lengthen the life of deposit 2, shorten the interval of simultaneous supply of q_1 and q_3, and has an ambiguous effect on the life of deposit 1 and of the aggregate resource stock. In the special case where A is large and z is very small (i.e., S_2 approaches $S_2^{\max}$ from below), an increase in capacity will shorten the life of deposit 1:*

$$\frac{dT}{d\overline{p}} > 0. \tag{25}$$

An increase in the capacity of the renewable resource increases the extraction duration of the second exhaustible resource: $dy/d\overline{p} < 0$ (Equation (22)). This indicates that a capacity expansion of the renewable resource sector permits the stock of higher-cost resource S_2 to be spread over a longer period. In contrast, if z is small and A is large, we can state that $dT/d\overline{p} > 0$ (Equation (25)), and the effect of a capacity increase on the extraction duration of the low-cost stock S_1 is negative. This case is especially plausible since we know that a capacity expansion reduces the energy price at the exhaustion point of S_1 (Equation (20)), which indicates a faster extraction of q_1. Additionally, as with the subsidy, the capacity increase induces a reduction in the initial energy price, which also accelerates exhaustion (Equation (19)).

Moreover, increased capacity shortens the period of parallel supply of q_1 and q_3: $dz/d\overline{p} > 0$ (Equation (23)). Therefore, the capacity increase cannot alleviate the demand for S_1 and, consequently, weakening the capacity constraints leads to at least a weak green paradox with regard to the cheaper exhaustible resource. This holds irrespective of the effect on t_3, which is ambiguous. Nevertheless, it is not clear whether there will be an overall green paradox: $(d(T+y)/d\overline{p})$ is ambiguous and further evaluation is necessary.

3.2.3 Effect of a tax on the low-cost exhaustible resource

The effect of different tax schemes on exhaustible resources are evaluated in the present and next subsection. We first consider a tax on the low-cost exhaustible resource that causes an increase in the (tax-inclusive) constant marginal extraction costs of deposit 1.[9] The effect of a tax on the low-cost resource on the endogenous variables (y, z, T) as well as on the price path, which is analogously modeled as a marginal increase in the extraction costs c_1, can be computed from the following matrix equation

$$\begin{bmatrix} F_y & F_z & F_T \\ G_y & G_z & G_T \\ H_y & H_z & H_T \end{bmatrix} \begin{bmatrix} dy \\ dz \\ dT \end{bmatrix} = \begin{bmatrix} -F_{c_1} \\ -G_{c_1} \\ -H_{c_1} \end{bmatrix} dc_1$$

[9]Many authors, including Sinn (2008) or Sinclair (1992,1994), show that a credible commitment to a tax rate that is high today but decreases over time is the best strategy for slowing down extraction of fossil fuels. Therefore, we examine that situation here with the simplification that we assume a tax on the low-cost resource but not on the high-cost resource that will be extracted later.

where

$$F_{c_1} = 0$$

$$G_{c_1} = 1 - e^{rz} < 0$$

$$H_{c_1} = \int_0^T D'[p(t)] \left(1 - e^{r(t-T)}\right) dt < 0$$

and J has been determined in Equation (12).

In contrast to the calculations in Section 3.2.1, the results are unambiguous. First, the tax on deposit 1 does not change the length of the extraction period for deposit 2:

$$\frac{dy}{dc_1} = 0 \tag{26}$$

This result in turn implies that the price at which extraction of deposit 2 begins is unaffected from changes in c_1; see Equation (5). Second, the tax lengthens the interval over which q_1 and q_3 are simultaneously supplied:

$$\frac{dz}{dc_1} = \frac{1}{J}\left[F_y(-G_{c_1})H_T\right] > 0. \tag{27}$$

The extraction of the low cost deposit will be spread out over a longer period:

$$\frac{dT}{dc_1} = \frac{1}{J}\left[F_y G_z(-H_{c_1}) - F_y(-G_{c_1})H_z\right] > 0. \tag{28}$$

Moreover, from Equation (9), the initial price of the extracted resource will be raised, though by a smaller amount than the increase in tax:

$$1 > \frac{dp(0)}{dc_1} = 1 - e^{-rT} > 0. \tag{29}$$

Only the effect on the time at which the renewable energy is made available, t_3, is ambiguous:

$$\frac{dt_3}{dc_1} = \frac{dT}{dc_1} - \frac{dz}{dc_1} = \frac{1}{J}\left[F_y G_z(-H_{c_1}) + F_y G_{c_1} H_z\right] - \frac{1}{J}\left[F_y(-G_{c_1} H_T\right] \gtrless 0. \tag{30}$$

But since the exhaustion time for the cheaper resource, T, is delayed, the ambiguous sign of Equation (30) is of no consequence with regard to the green paradox.[10] The results are summarized in Proposition 4.

Proposition 4: *While a tax on the low-cost resource has no effect on the extraction duration of the high-cost resource, it does result in a higher initial price of energy. Moreover, the overall period of extraction from the cheaper resource lengthens, leading to slower extraction of the cheaper exhaustible resource during the initial phase* $[0, t_3)$ *both due to an increase in* T *and an increase in* $p(0)$*. Thus, there is neither a weak nor an overall green paradox. (The ambiguous effect on the interval* $[0, t_3)$ *is of no consequence for the green paradox results.)*

This result can be understood as follows (see also the price path of this scenario illustrated in Section 4.2): A tax on the cheaper exhaustible resource is equivalent to an increase in c_1. From $dy/dc_1 = 0$ (Equation (26)), we know that a tax on the low-cost resource has no effect on how long it will take to exhaust S_2. Parallel to the case of subsidizing the renewable resource, from the invariance of y it follows that p_2 is unchanged, see Equation (5). Moreover, $dT/dc_1 > 0$ (Equation (28)) implies that the tax increases the time span of extraction of S_1 by $(T^* - T)$. Together with $p^*(0) > p(0)$ (Equation (29)), this means that the price level p_2 (at which the second deposit begins to be exploited) is reached later and the exhaustible resource S_1 is available longer. The price path during $[0, T)$ is flatter and the price level is higher such that extraction of S_1 is spread over a longer period of time. Therefore, the old and the new price path during the extraction of S_1 must intersect.[11] Nevertheless, the effect on t_3 is not clear. Even though we know from $dz/dc_1 > 0$ (Equation (27)) that the time span of simultaneous use of q_1 and q_3 increases, we do not know whether the production of clean energy will begin earlier or later, as the sign of dt_3/dc_1 (Equation (30)) is ambiguous. In

[10] Thus, our result for the multi-resource case supports Sinn's (2008) proposition that "high tax now and low tax later is good for the environment."

[11] The "intersection" is easily explained by the fact that the two price paths correspond to different values of c_1.

conclusion, the imposition of a constant unit tax on the low-cost exhaustible resource gives rise to neither a weak green paradox (since $dp(0)/dc_1 > 0$) nor an overall green paradox (since $dT/dc_1 + dy/dc_1 > 0$).

3.2.4 Effect of a tax on the extraction of the high-cost exhaustible resource

We now examine how a tax on (an increase in) c_2 affects the endogenous variables.[12] This can be computed from the following matrix equation

$$\begin{bmatrix} F_y & F_z & F_T \\ G_y & G_z & G_T \\ H_y & H_z & H_T \end{bmatrix} \begin{bmatrix} dy \\ dz \\ dT \end{bmatrix} = \begin{bmatrix} -F_{c_2} \\ -G_{c_2} \\ -H_{c_2} \end{bmatrix} dc_2$$

where

$$F_{c_2} = \int_0^y D'[p(t)] \left(1 - e^{r(\tau - y)}\right) dt < 0$$

$$G_{c_2} = -(1 - e^{-ry}) < 0$$

$$H_{c_2} = \int_0^T D'[p(t)](1 - e^{-ry})e^{r(t-T)} dt < 0$$

and J has been determined in Equation (12).

The general case

Even though the signs of the above partial derivatives are unambiguous, some results of the comparative statics are ambiguous. The tax on the high cost exhaustible resource will lead to a lengthening of its extraction period:

$$\frac{dy}{dc_2} = \frac{1}{J}\left[-F_{c_2} G_z H_T\right] > 0. \tag{31}$$

However, the effect on the period of simultaneous use of green energy and the low cost resource is not clear:

$$\frac{dz}{dc_2} = \frac{1}{J}\left[F_y(-G_{c_2})H_T - (-F_{c_2})G_y H_T\right] \gtrless 0,$$

[12] Increasing taxes on fossil fuels is common practice throughout the world, not only for fiscal reasons, but due to growing awareness of the consequences of climate change and the exhaustibility of fossil fuels. However, according to Sinn (2008) and others, this practice causes detrimental green paradox effects.

and the effect on the period of exploitation of the low cost deposit is also ambiguous:

$$\frac{dT}{dc_2} = \frac{1}{J}\left[F_y G_z(-H_{c_2}) + (-F_{c_2})G_y H_z - (-F_{c_2})G_z H_y - F_y(-G_{c_2})H_z\right] \gtrless 0.$$

We summarize the results in Proposition 5.

Proposition 5: *A tax on the high-cost exhaustible resource deposit lengthens the exploitation period of this deposit, but has an ambiguous effect on the life of the lower cost resource and of the aggregate resource stock.*

Therefore, to obtain sharper results, we consider the case of linear demand in the following.

The special case of linear demand

In case of a linear demand function as formulated in Equation (21), the partial derivatives have the following signs:

$$\frac{dz}{dc_2} > 0 \tag{32}$$

$$\frac{dT}{dc_2} > 0 \tag{33}$$

$$\frac{dp(0)}{dc_2} > 0 \tag{34}$$

$$\frac{dt_3}{dc_2} < 0. \tag{35}$$

From Equations (31)-(35), we can now state Proposition 6.

Proposition 6: *Under linear demand, a tax on the high-cost resource extraction (an increase of c_2) will lengthen the life of both deposits 1 and 2, lengthen the interval of simultaneous supply of q_1 and q_3, and therefore increase the life of the aggregate resource stock.*

The effects can be understood as follows: A change in the marginal extraction costs has no effect on the price ceiling determined by $D(\overline{p}) = \overline{q}_3$. Therefore, when $\overline{p}$ and $\overline{q}_3$ are given, a longer (slower) extraction of deposit 2, as indicated by $dy/dc_2 > 0$ (Equation (31)), is possible only when demand is reduced during the considered time span. This can be reached by an overall price level increase. From

$$\frac{dp(T)}{dc_2} > 0, \tag{36}$$

we know that $p(T^*) > p(T)$. This means that extraction from the high cost deposit starts from a higher price level and $q_2(t)$ is already initially lower. Therefore, to have S_2 exhausted at $\overline{T}^* > \overline{T}$, the price path is flatter such that $y^* > y$. The changes in the depletion path of deposit 1 which are a flatter price path and a longer extraction period ($dT/dc_2 > 0$; see Equation (33)) with a higher initial price ($dp(0)/dc_2 > 0$; see Equation (34)) can be explained analogously. This means that S_1 is more valuable to the resource owner (higher price and higher scarcity rent). Moreover, even though the tax on the high-cost resource postpones production of green energy ($dt_3/dc_2 > 0$; see Equation (35)), the length of simultaneous production of q_1 and q_3 increases ($dz/dc_2 > 0$; see Equation (32)). In the end, neither a weak nor an overall green paradox is found.

In the following section, the comparative static policy analysis is complemented by a numerical analysis, which allows us to link the theoretical model to a concrete example of the fossil fuel market and derive precise results, which are missing from the analytical part. Moreover, we conduct a welfare analysis to discover the social consequences of the different scenarios. In this context, we introduce two different explicit damage functions as well as a situation where the various deposits have different carbon contents. This extended welfare analysis allows us to draw further conclusions regarding the strong green paradox effect defined by Gerlagh (2011).

4 Numerical analysis

In the following, we provide a numerical illustration of the previous model. In this section, we use for illustration the concrete example of an oil market. The parameter values are chosen to reflect, in a stylized manner, real-world relations for the different oil market parameters. Our numerical exercise allows not only the derivation of unambiguous results, but also a concrete illustration of the relevant effects. We derive the accumulation paths of anthropogenic carbon in the atmosphere and compare their resulting social consequences. In addition to the situation of zero decay we also evaluate the climate effect for a positive depreciation of anthropogenic carbon. The analysis begins by describing the stylized oil market example in Section 4.1. The numerical results are derived in Section 4.2, followed by a welfare analysis in Section 4.3.

4.1 The oil market example

The parameters are chosen so as to reflect, in a stylized manner, the relations between marginal extraction costs for conventional oil, unconventional oil, and advanced biofuel (see, e.g., IEA 2012). Therefore, we set $c_1 = 0.75, c_2 = 1.75$, and $c_3 = 4$. This reflects the cost structure observed in oil markets: Biofuel has the highest, unconventional oil has medium, and conventional oil the lowest production costs. Moreover, we continue to assume the case of linear demand, $D[p(t)] = A - p(t)$ (see Equation (21)), and that $\overline{p} > c_3$. We choose $A = 20, \overline{p} = 15$, $r = 0.01$. Then $\overline{q}_3 = A - \overline{p} = 5$. To compute the pollution stock, we specify the stock sizes S_1 and S_2. Let us assume that $S_2 = 900$ and $S_1 = 700$, which reflects the fact that there is more unconventional than conventional oil available. First, we need to make sure that $S_2 < S_2^{\max}$. This means that we first have to compute the value $S_2^{\max}$ from our specifications of the cost parameters c_1, c_2, and c_3 and of capacity $\overline{q}_3$ (which is equal to $A - \overline{p}$). From Condition 2 with $S_2^{\max}$ equals approximately 1249, it follows that $S_2 = 900$ does indeed satisfy the condition $S_2 < S_2^{\max}$.[13]

[13] This condition is also fulfilled for all following model specifications.

4.2 Derivation of numerical results

We now show how numerical results can be derived for the base case.[14] We first calculate the length of Phase 3, which is $y = \overline{T} - T = 144.30$. Second, we solve for the length of Phase 2, $z \equiv T - t_3$, which is the time interval over which the lowest-cost deposit and the renewable energy are available simultaneously. From Equation (6) follows that $z = 23.96$. Next, we solve for T (the time at which deposit 1 is exhausted) from Equation (10) such that we have $T = 51.18$. Moreover, the length of Phase 1, t_3, and the total length of Phases 1-3, $\overline{T}$, can be calculated as $t_3 = T - z = 27.22$ and $\overline{T} = T + y = 195.48$.

From Equation (9), the equilibrium price at time t (for $0 \leq t \leq T$) is $p(t) = 1.75 + e^{0.01(t-51.18)} + 13.25e^{0.01(t-195.48)}$; specifically, $p(0) = 3.23$, $p(T) = 4.88$, and, as expected, $p(t_3) = 4 = c_3$. Moreover, from Equation (2), the equilibrium price path for $T \leq t \leq \overline{T}$ is $p(t) = 1.75 + 13.25e^{0.01(t-195.48)}$. Finally, we have $p(\overline{T}) = 15$.

Table 1 sets out the results of the numerical analysis for the different policy scenarios and the base case in the chosen numerical example. This allows comparing the effects of the respective policy measures on extraction speed and duration of the fossil fuel extraction. In our first policy scenario, there is a subsidy on the green energy at the rate 1 per unit (e.g., one Euro per kilowatt-hour). Consequently, the (marginal) production costs of the green energy decrease from 4 to 3 per unit. In an alternative policy scenario, there is a capacity expansion from 5 to 6; therefore, $\overline{p}$ decreases from 15 to 14. In a third policy scenario, the tax on the low-cost exhaustible resource increases from 0 to 1, such that the (marginal) production cost of the green energy increases from 0.75 to 1.75 per unit. Finally, in the fourth scenario, the tax on the high-cost exhaustible resource is increased from 0 to 1; therefore, the (marginal) production cost of the green energy increases from 1.75 to 2.75 per unit.

[14] The results of the different policy scenarios can be derived analogously.

	base scenario	subsidy on green energy[1]	capacity increase of green energy[2]	tax on low cost resource[3]	tax on high cost resource[4]
y	144.30	144.30	151.49	144.30	151.49
z	23.96	60.73	12.78	33.00	36.74
T	51.18	64.95	47.61	54.62	56.42
t_3	27.22	4.23	34.83	21.61	19.68
$\overline{T}$	195.48	209.26	199.10	198.92	207.94
p(0)	3.23	2.91	3.04	3.56	3.42
p(T)	4.88	4.88	4.44	4.88	5.44

Table 1: Numerical results for the different policy scenarios
[1] *new green energy production costs:* $c_3^{new} = c_3 - 1$; [2] *new maximum capacity:* $\overline{q}_3^{new} = \overline{q}_3 + 1$; [3] *extraction costs of* S_1 *increase to:* $c_1^{new} = c_1 + 1$; [4] *extraction costs of* S_2 *increase to:* $c_2^{new} = c_2 + 1$

Recall that in Proposition 3, an increase in capacity will increase y, may reduce T, and the effect on $\overline{T} \equiv T + y$ is ambiguous (see Equation 18). Our numerical results show that an increase in the capacity of the green substitute in deed decreases T but does not reduce $\overline{T}$ indicating that there is no overall green paradox. This is because numerically $d\overline{T}/d\overline{p} < 0$ (which means $\overline{T}$ increases). Moreover, we find that $dt_3/d\overline{p} < 0$. The reason for the positive effect on t_3 is that $p(0)$ is lower than before; therefore, it takes longer for $p(t)$ to reach c_3. However, with $dy/d\overline{p} < 0$, it also takes longer to exhaust the aggregate resource stock than is the case in the base scenario.

With regard to the effect of a tax on the low-cost resource on t_3, the numerical analysis shows that $dt_3/dc_2 > 0$ (Table 1, fifth column). Parallel to the previous explanation, the slight increase in t_3 is mostly explained by the increase in $p(0)$ that flattens the price path. This can be summarized in Proposition 7.

Proposition 7: *In our numerical simulation, a capacity increase of the green energy substitute leads to earlier production of green energy (t_3 decreases) and does not produce an* *overall green paradox effect since it induces an increase of* $\overline{T}$.

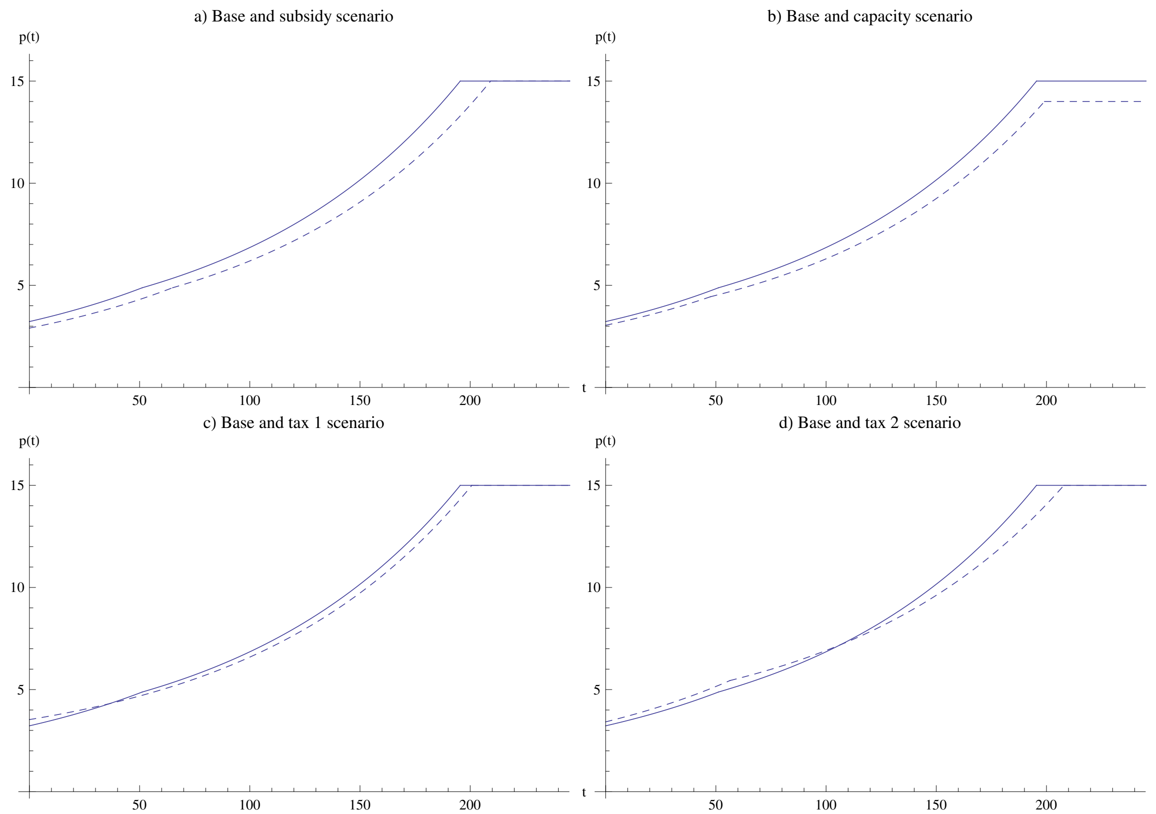

Figure 1: Price paths of the policy scenarios compared with the base scenario

Figure 1 illustrates how the different policy measures of the chosen numerical example affect the price paths. In the figure, the price paths of the different policy scenarios shown in dashed lines are compared with the base case price path in solid lines. The upper left graph named "a) Subsidy and base case" illustrates the effect of subsidizing the renewable energy good on the price path compared to the base case, the upper right graph named "b) Capacity and base case" describes a capacity increase of the renewable energy good compared to the base case, the lower left graph named "c) Tax 1 and base case" shows the effects of a tax on the low-cost and the lower right graph "d) Tax 2 and base case" those of a tax on the high-cost exhaustible resource on the price path compared to the base case. The policy measures reduce the price level for most time periods. However, we know from Table 1 that this does not lead to a decrease in the overall extraction duration of the exhaustible resources.

Concerning the price path, we know that in the standard model, price paths do not cross. However, in the case where there exists a capacity constraint on the green energy, we have shown that, for example, in Proposition 4 a tax on the extraction of the low cost resource leads to a new price path that crosses the old one from above (section *c*) of Figure 1). The price path behavior is striking in the case of a capacity increase of green energy (section *b*) of Figure 1). First, the capacity increase reduces the capacity-constrained choke price. Second, the increase in capacity can overcompensate the higher demand resulting from the lower price path such that the overall extraction duration of the exhaustible resources increases (see also Table 1) even though an overall higher demand needs to be satisfied.

In the following, the emission paths and the resulting welfare effects in terms of damages from accumulated anthropogenic carbon pollution in the atmosphere will be determined. We calculate accumulated emissions in the situation where the various deposits have different carbon contents, evaluate the welfare effects for both a zero and positive decay rate for the atmospheric carbon, and compare the effects of various policies on the social cost, under two alternative specifications of a damage function. This permits us to derive explicit social consequences resulting from carbon use under the analyzed policy scenarios, allowing for both flow and stock damages.

4.3 Welfare analysis

In the following , we complement our analytical and numerical investigation by a welfare analysis that evaluates the effects of the different policies, using specific functional forms modeling social damage from anthropogenic carbon emissions. Thereby, the evolution of the stock of atmospheric carbon is modeled under alternative assumptions about the accumulation of anthropogenic carbon in the atmosphere. First, the decay rate of atmospheric carbon is assumed to be zero. Second, the zero depreciation rate assumption is relaxed and a more realistic model is introduced in which the atmospheric carbon stock partially decays from the atmosphere over time. The latter is modeled based on considerations of Archer (2005) and others who analyze the accumulated stock of emissions in the atmosphere. We study both the cases of linear and convex social damage functions, and compare the present value damages of the different policy scenarios. Accumulated emissions depend not only on the speed of extraction. A faster accumulating stock brings higher damages closer to the present. With a positive decay rate, different mechanisms influence the welfare effects of climate policy. Later in the analysis, these are identified and discussed in more detail with regard to their implications for the analysis as well as for policymakers.[15]

4.3.1 Emission Paths

For our welfare analysis, we must first compute the emission paths of the different policy scenarios for the chosen numerical example. To calculate them, we have to specify the emission parameters of the extracted exhaustible resources. In the following, we assume that $\eta_1 = 1$ is the emission parameter of the low-cost exhaustible resource and $\eta_2 = 2$ is the one for the high-cost exhaustible resource. For our fuel market example, this reflects that conventional oil is not only cheaper, but also has lower emissions during extraction and production, than unconventional oil.

During the first extraction phase, i.e., the time interval $[0, t_3)$, all energy comes from deposit 1. Since extraction from this deposit must equal energy demand, emissions from the consumption of $q_1(t)$ are $\varepsilon_1(t) = \eta_1 q_1(t)$. Over the time interval $[t_3, T)$, Phase 2, energy demand is met by extraction from deposit 1 and by renewable energy supply $\overline{q}_3 = A - \overline{p} = 2$ such that emissions at any time t in the interval $[t_3, T)$ are $\varepsilon(t) = \eta_1(Q(t) - \overline{q}_3)$. Over the time interval $[T, \overline{T})$, Phase 3, energy demand is met by extraction from deposit 2 and by

[15] In this paper, we analyze a partial equilibrium resource model. We focus on social damages from accumulated anthropogenic carbon pollution in the atmosphere. We do not take into account further effects on production or consumption. Therefore, a welfare effect of a policy measure is positive in the present analysis if it decreases pollution damages compared to the baseline scenario without policy intervention.

renewable energy supply $\overline{q}_3 = 2$. Thus extraction from deposit 2 at any time t during the interval $[T, \overline{T})$ is $q_2(t) = D[p(t)] - \overline{q}_3$ and emissions from consumption of q_2 at any point of time in $\left[T, \overline{T}\right)$ are $\varepsilon_2(t) = \eta_2(Q(t) - \overline{q}_3)$.

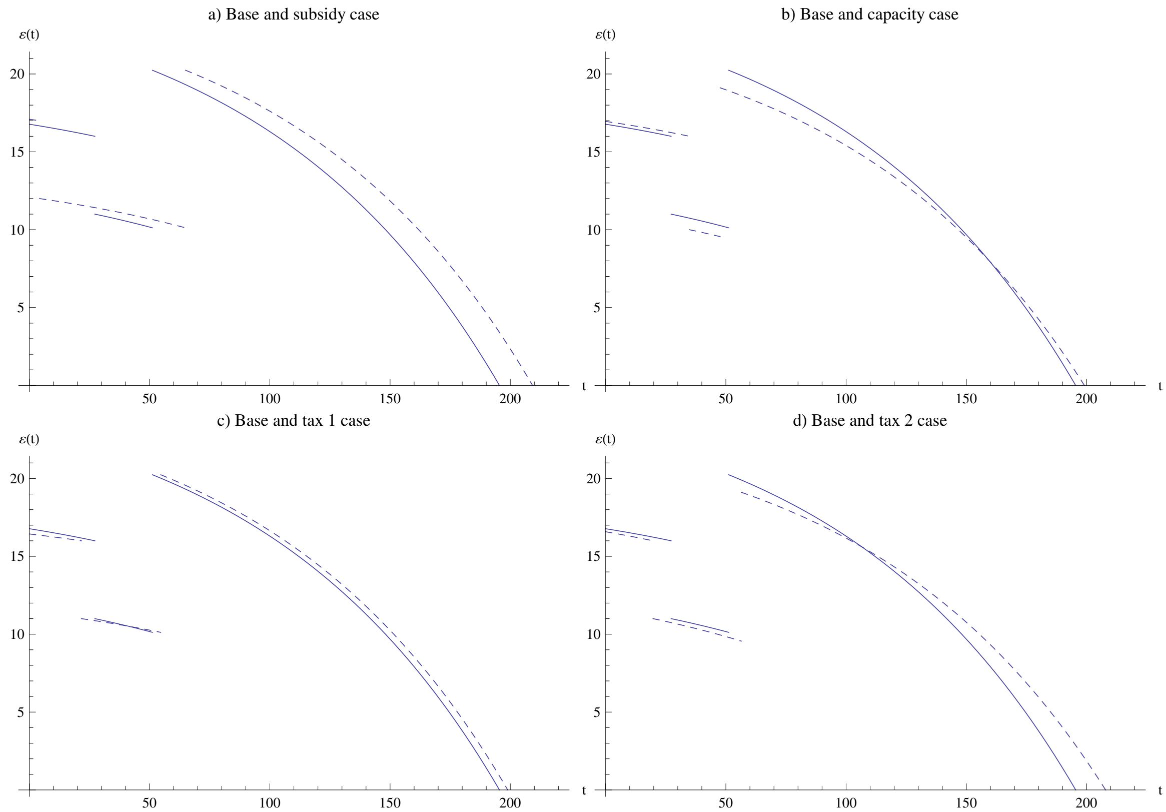

Figure 2: Emission paths of policy scenarios compared with base scenario

Figure 2 shows how the different policy measures illustrated in dashed lines affect CO_2 emission streams compared to the base case illustrated in solid line.[16] The graphs are named analogous to Figure **??**. We can see immediately how the policy measures extend the overall emitting period. In the subsidy scenario, emissions tend to be higher than in the base case (since the resource price is lower), except that z is larger and t_3 occurs sooner. This compensates for higher emissions in the beginning, such that the total extraction period

[16]The marginal analysis conducted in the previous section has determined the direction of a policy effect. Here, the calibration of the model determines the quantitative effect of a policy measure. For example, the chosen emission intensities influence the extent of the changes in the emission levels associated with changes in the fuel mix but not their direction. The discussion of the qualitative and quantitative effects is continued in the following sections.

of the cheaper resource increases (the exhaustion duration of the expensive resource being constant, the emission path shifts to the right). In the capacity expansion scenario, a slightly lower price path leads to slightly higher emissions during the extraction period of the low-cost deposit. Moreover, z is shorter compared to the base case, but emissions are lower due to the increased capacity. Nevertheless, the increased capacity cannot stretch the first extraction period until T; however, emissions from the dirtier resource can be slowed down, at least until the emission paths cross. In the first tax scenario, due to the higher initial price, emissions can be reduced initially and their path can be flattened with only slight changes in t_3 and a slight increase in z. Since there are no changes in the emission flows from the dirty energy good (the emission path shifts to the right), the overall effect on $\overline{T}$ is positive. Finally, taxing the dirty energy good has effects similar to those found when taxing the cheap exhaustible energy good and therefore lowers the periodical emissions during and increases the first extraction phase (including z). In the second phase, which consequently starts later, emissions start lower but since the path is flatter, end up higher. Also in this policy scenario, the overall effect on $\overline{T}$ is positive.

4.3.2 Pollution and damages with a zero decay rate

In this section, a decay rate of zero is assumed, which means that once anthropogenic CO_2 has been emitted into the atmosphere, it remains there forever. $\delta = 0$ can more broadly be interpreted as an approximation of a δ close to 0 meaning that the potentially existing decay of atmospheric carbon is just not relevant in the considered period of time and as a first approximation can be ignored (see, e.g. Sinclair 1994 and van der Ploeg and Withagen 2011). When there is no decay, the volume of pollution, here, $V(t)$, is identical to accumulated emissions, here, $E(t)$. In the following, the welfare analysis is conducted for the baseline scenario and can be performed analogously for the other policy scenarios.

Over the time interval $[0, t_3)$ in Phase 1, accumulated stock pollution (which is the accumulated stock of anthropogenic carbon in the atmosphere) is

$$V^{\text{phase 1}}(t') = \eta_1 \left[A - c_1\right] t' - \eta_1 \left(c_2 - c_1 + (\overline{p} - c_2)e^{-ry}\right) e^{-rT} \left(\frac{e^{rt'} - 1}{r}\right) \tag{37}$$

at time t' (for $0 \leq t' \leq t_3$).[17]

Analogously, over the time interval $[t_3, T)$ in Phase 2, the stock of pollution at any time $t' \in [t_3, T)$ is

$$V^{\text{phase 2}}(t') = \eta_1 \left(A - c_1 - \overline{q}_3\right) t' + \eta_1 \overline{q}_3 t_3 - \eta_1 \left(c_2 - c_1 + (\overline{p} - c_2)e^{-ry}\right) e^{-rT} \left(\frac{e^{rt'} - 1}{r}\right) \tag{38}$$

and from the results presented in Table 1, we can calculate $V(T) = 700$, which is, as expected, the size of S_1 multiplied with $\eta_1 = 1$.

Over the time interval $[T, \overline{T})$ in Phase 3, the accumulated stock pollution at time t' for $T \leq t' \leq \overline{T}$ is

$$V^{\text{phase 3}}(t') = V(T) + \eta_2 \left(A - c_2 - \overline{q}_3\right)(t' - T) - \eta_2(\overline{p} - c_2)e^{-r(T+y)} \left(\frac{e^{rt'} - e^{rT}}{r}\right) \tag{39}$$

with the decay rate of pollution stock still being zero. Again, from Table 1, we can calculate $V(\overline{T}) = 2.500$, which is obviously once S_1 plus twice the size of S_2 since $\eta_2 = 2$.

Finally, in Phase 4, which lasts from $t = \overline{T}$ until infinity, the accumulated pollution stays in the atmosphere forever as

$$V^{\text{phase 4}}(t) = V^{\text{phase 4}}(\overline{T}) = \eta_1 S_1 + \eta_2 S_2 = 2.500. \tag{40}$$

In the following, the damages from the accumulated atmospheric pollution are analyzed. If the damage function $C[V(t)]$ is linear, say $C[V(t)] = \theta V(t)$ (and is also equal to $\theta E(t)$ in

[17] The accumulated stock of carbon in the atmosphere can be derived from $V^{\text{phase 1}}(t') = \int_0^{t'} \eta_1 \left[A - c_1 - (c_2 - c_1 + (\overline{p} - c_2)e^{-ry})\, e^{r(t-T)}\right] dt$. Rearranging this term leads to Equation (37). Inserting the numerical results derived in Table 1 gives $V^{\text{phase 1}}(t') = \int_0^{t'} 1.75 \left(17.25 + e^{0.01(t-40.37)} + 14.25e^{0.01(t-187.60)}\right) dt$ for $t \leq t_3$.

this section with $\delta = 0$, see above), then damage at time $0 \leq t' \leq t_3$ is $C[V(t)] = \theta\eta_1 q_1(t)$ and analogously for the other extraction phases. Together with Equations (37) - (40), the resulting total discounted stream of damages from $t = 0$ to ∞ is

$$D(t_0) = \int_0^{t_3} e^{-rt}\theta V^{\text{phase 1}}(t)dt + \int_{t_3}^{T} e^{-rt}\theta V^{\text{phase 2}}(t)dt + \int_{T}^{\overline{T}} e^{-rt}\theta V^{\text{phase 3}}(t)dt$$
$$+\theta V^{\text{phase 4}}(\overline{T})\left(\frac{e^{-r\overline{T}}-1}{r}\right). \tag{41}$$

With this simple linear damage function, we can calculate the discounted damages for the different policy scenarios without any further specification of θ. Inserting the values of the numerical analysis into Equation (41) allows us to directly compare the welfare effects of the different policy scenarios with the business as usual case. Comparing the discounted damages for the period between 0 and infinity for the different policy scenarios based on the numerical example chosen here gives

$$D^{\text{tax 2}}(t_0) \leq D^{\text{subsidy}}(t_0) \leq D^{\text{tax 1}}(t_0) \leq D^{\text{base case}}(t_0) \leq D^{\text{capacity}}(t_0) \tag{42}$$

where $D^{tax2}(t_0)$ stands for damages in the policy scenario where the high-cost exhaustible resource is taxed, $D^{\text{subsidy}}(t_0)$ for the scenario with subsidization of the renewable substitute, $D^{\text{tax 1}}(t_0)$ for the scenario where the low-cost exhaustible resource is taxed, $D^{\text{base case}}(t_0)$ for the baseline scenario, and $D^{\text{capacity}}(t_0)$ for the scenario where there is a capacity increase of the renewable substitute.

What happens now when the damage function is convex? For example, if the damage function is quadratic, say

$$D[V(t)] = a\frac{V(t)^2}{b}, \tag{43}$$

as in van der Ploeg and Withagen (2011), with $a = 0.00012$ and $b = 2$, we can also compute a similar integral of discounted damages. If we continue to assume that the decay is zero ($\delta = 0$), the volume of pollution $V(t)$ continues to be equal to accumulated emissions $E(t)$. Calculating and comparing the present value of damages, we receive qualitatively the same results, the same order, as in (42).

The welfare order derived under the assumption that the decay rate is zero (see inequality (42)) applies to both the linear and the quadratic damage functions, indicating that all policy measures, except the capacity increase, reduce the damages of carbon emissions compared to the base case situation. The damages in the scenario with a capacity expansion are higher than in all other scenarios because the capacity expansion, which comes into affect in the future, at time t_3, lowers the initial price of energy, $p(0)$, leading to increased demand for energy for the period $[0, t3]$, and hence greater pollution damages earlier on. Since $r > 0$, near-term emissions are more important for the welfare and this first green paradox effect cannot be compensated by the resource-saving effect of a capacity increase on S_2 (relatively high y).

Regarding the tax on the high-cost exhaustible resource, we find that it induces a general reduction of the extraction speed (leading to higher $\overline{T}$), and both T and y become larger. This results in lower damages compared to the baseline scenario. Subsidizing green energy also has positive welfare effects: For t_3, T, and $\overline{T}$, subsidization performs even better than the tax on the high-cost exhaustible resource. However, the main reason why the overall positive effect is smaller is that emissions in the beginning are higher for the subsidy case (since $p(0)$ is smaller). A tax on the low-cost exhaustible resource also reduces damages, but since z is relatively short and t_3 relatively high, the positive effects are not very strong.

To this point, for both types of damage function, the welfare analysis implies that green energy policy measures can be either welfare increasing or detrimental, depending on how they affect the extraction behavior of the resource owners. Of course, the results depend on the model's underlying assumptions and parameter specifications. One strong assumption is the decay rate of zero. Therefore, in the next section, a welfare analysis employing a positive decay rate of atmospheric emissions is conducted.

4.3.3 Pollution and damages with a positive decay rate

In this section, we assume a positive depreciation of the carbon stock in the atmosphere, in accordance with Archer (2005). This is arguably a more plausible scenario. Indeed, Archer (2005), or also Houghton et al. (1990, 1992), explain (though in a highly simplified way) that a fraction of the anthropogenic carbon emissions that are in the atmosphere re-enter the carbon cycle again and are absorbed by different carbon sinks, mostly the oceans.[18] This means that although a fraction of the anthropogenic atmospheric carbon stock (let's call it α) will stay in the atmosphere forever, the other part $(1-\alpha)$ will depreciate slowly over time at a positive rate δ. Modeling anthropogenic carbon is a widely discussed issue in resource economics literature (see, e.g., Hoel 2011; Hoel and Kverndokk 1998; Farzin and Tahvonen 1996). For the sake of simplicity, the rate of decay δ is assumed to be constant over time. Based on these considerations, for each emitted ton of CO_2 at time t, the resulting amount of CO_2 in the atmosphere at time $\tau > t$ is approximated by $\alpha + (1-\alpha)e^{-\delta(\tau - t)}$ (Hoel 2011).[19]

Since there is positive decay, accumulated emissions $E(t)$ always exceed the volume $V(t)$ of atmospheric pollution for all $t > 0$. We can calculate now total pollution in the atmosphere in the first phase as

$$V^{\text{phase 1}}(t) = \alpha\eta_1 \int_0^t q_1(\tau)d\tau + (1-\alpha)\eta_1 e^{-\delta t}(\int_0^t q_1(\tau)e^{\delta\tau}d\tau) \tag{44}$$

with $t \in [0, t_3)$. For $t \geq t_3$, the term $\alpha\eta_1[...]$, from now on $a_1(t)$, with $t = t_3$ $(a_1(t_3))$ stays constant and only the term $(1-\alpha)\eta_1 e^{-\delta t}[...]$, henceforth, $b_1(t_3)$, with $t = t_3$ further depreciates, resulting in $b_1(t_3)e^{-\delta(t-t_3)}$.

[18] For simplicity, we abstract from any lags between emission production, pollution accumulation, and damages as described, for example, in Houghton et al. (1990, 1992).

[19] As the findings of, e.g., Houghton et al. (1990, 1992) imply, the decay rate might not be constant over time. They report that the decay rate of atmospheric carbon declines over time depending on the saturation of the oceans. However, at least as an approximation, this effect is also captured in our model since we assume that a share of anthropogenic carbon stays in the atmosphere forever.

Total pollution in the second phase is

$$V^{\text{phase 2}}(t) = a_1(t_3) + b_1(t_3)e^{-\delta(t-t_3)}$$
$$+\alpha\eta_1 \int_{t_3}^{t} (q_1(\tau) - \overline{q}_3)d\tau + (1-\alpha)\eta_1 e^{-\delta(t-t_3)}(\int_{t_3}^{t} (q_1(\tau) - \overline{q}_3)e^{\delta(\tau-t_3)}d\tau) \tag{45}$$

with $t \in [t_3, T)$. Analogous to the case of $t \in (0, t_3)$ for $t \geq T$, the term $\alpha\eta_1[...]$ for $t = T$, now $a_2(T)$, stays constant and only the term $(1-\alpha)\eta_1 e^{-\delta(t-t_3)}[...]$ with $t = T$ (henceforth, $b_2(T)$) further depreciates, resulting in $b_2(T)e^{-\delta(t-T)}$ for $t > T$.

Total pollution during the third phase is

$$V^{\text{phase 3}}(t) = a_1(t_3) + a_2(T) + b_1(t_3)e^{-\delta(t-t_3)} + b_2(T)e^{-\delta(t-T)} + \alpha\eta_2 \int_{T}^{t} (q_2(\tau) - \overline{q}_3)d\tau$$
$$+(1-\alpha)\eta_2 e^{-\delta(t-T)}(\int_{T}^{t} (q_2(\tau) - \overline{q}_3)e^{\delta(\tau-T)}d\tau) \tag{46}$$

with $t \in [T, \overline{T})$. As before, for $t \geq \overline{T}$, the term $\alpha\eta_2[...]$ with $t = \overline{T}$, which we will call $a_3(\overline{T})$ in the following, stays constant and only the term $(1-\alpha)\eta_2 e^{-\delta(t-T)}[...]$ with $t = \overline{T}$ (henceforth, $b_3(\overline{T})$) further depreciates, resulting in $b_3(\overline{T})e^{-\delta(t-\overline{T})}$ for $t > \overline{T}$.

Moreover, from $t = \overline{T}$ on (Phase 4), there is no further anthropogenic CO_2 emitted in the atmosphere. Therefore, total pollution remains constant at

$$V^{\text{phase 4}}(t) = a_1(t_3) + a_2(T) + a_3(\overline{T}) + b_1(t_3)e^{-\delta(t-t_3)} + b_2(T)e^{-\delta(t-T)} + b_3(\overline{T})e^{-\delta(t-\overline{T})} \tag{47}$$

for all t with $t \in [\overline{T}, \infty)$.

Figure 3 shows atmospheric pollution over time that results from the different extraction scenarios, under the assumption that $\alpha = 0.25$. The different policy scenarios in comparison with the base case are presented analogous to Figure 1. As t approaches infinity, the atmospheric polluting stock of anthropogenic carbon converges to $V(t \to \infty) = 625$. This is because a fraction $(1-\alpha)$ of the anthropogenic carbon stock depreciates from the atmosphere over time. Moreover, in case of a capacity expansion (an increase in $\overline{q}_3$) and taxing the high-cost exhaustible resource, the peak of accumulated pollution is clearly lower than in the base case, while for subsidization of the green energy and for a tax on the low-cost exhaustible resource, it is quite similar to the base case. Regarding the tax on the high-cost exhaustible resource, this is because emissions are postponed ($\overline{T}$ is very large) and therefore the time path of the pollution stock is flatter. Regarding the capacity expansion scenario, pollution is slightly higher in the beginning such that due to the constant decay rate, more carbon has already depreciated from the atmosphere when the peak of the atmospheric pollution stock is reached.

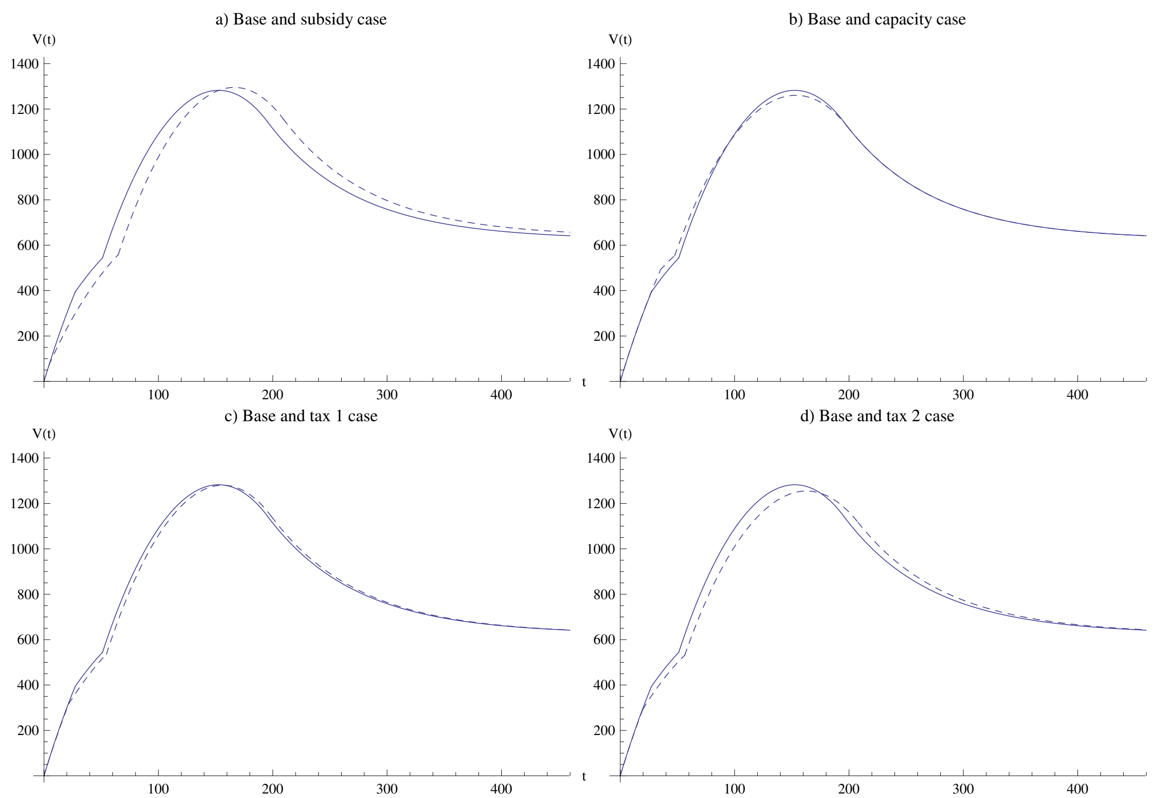

Figure 3: Accumulated pollution of policy scenarios compared with base scenario and positive decay

Based on the pollution paths, the welfare effects of the different policy measures are calculated and compared. Analogous to the previous subsection, if damage function $C[V(t)]$ is linear, say $C[V(t)] = \theta V(t)$, and using the above notation together with Equations (44) -

(47), the integral of the stream of discounted damages over all phases is

$$
\begin{aligned}
D(t_0) = & \int_0^{t_3} e^{-rt}\theta[a_1(t)+b_1(t)]dt + \int_{t_3}^{\infty} e^{-rt}\theta[a_1(t_3)+e^{-\delta(t-t_3)}b_1(t_3)]dt \\
& + \int_{t_3}^{T} e^{-rt}\theta[a_2(t)+b_2(t)]dt + \int_{T}^{\infty} e^{-rt}\theta[a_2(T)+e^{-\delta(t-T)}b_2(T)]dt \\
& + \int_{T}^{\overline{T}} e^{-rt}\theta[a_3(t)+b_3(t)]dt + \int_{\overline{T}}^{\infty} e^{-rt}\theta[a_3(\overline{T})+e^{-\delta(t-\overline{T})}b_3(\overline{T})]dt
\end{aligned} \tag{48}
$$

where the first line describes the streams of discounted damages for first period emissions, the second line those for second period emissions and the third line those for third period emissions.

Inserting the values of the numerical analysis into Equation (48) allows us to directly compare welfare effects of the different policy scenarios with the business as usual case. Under the linear damage function, comparing the discounted damages of the period between 0 and infinity for the different policy scenarios gives

$$
D^{\text{tax 2}}(t_0) \leq D^{\text{subsidy}}(t_0) \leq D^{\text{tax 1}}(t_0) \leq D^{\text{base case}}(t_0) \leq D^{\text{capacity}}(t_0) \tag{49}
$$

which is the same welfare order as in the previous subsection with a zero decay rate (see (42)).

In contrast, with the convex damage function (43), we obtain a somewhat different ranking of the discounted stream of damages for the different policy scenarios:

$$
D^{\text{tax 2}}(t_0) \leq D^{\text{tax 1}}(t_0) \leq D^{\text{subsidy}}(t_0) \leq D^{\text{base case}}(t_0) \leq D^{\text{capacity}}(t_0). \tag{50}
$$

Comparing (42) with (49) and (50), we see that three of the four policy measures are welfare increasing compared to the laissez-faire situation. Again, only a capacity increase of the renewable backstop leads to higher damages compared to the base case. However, in the latter situation of positive decay rate and a convex damage function, the welfare order changes slightly compared to the scenario with a positive decay rate and a linear damage function, as well as compared to the zero decay situation previously analyzed. Here, a tax on the high-cost exhaustible resource still reduces damages the most compared to the base case, but now a tax on the low-cost exhaustible resource is the second and a subsidy on the renewable substitute is the third effective instrument. This is because in the subsidy scenario, emissions are higher in the beginning, and therefore damages, due to the underlying convex damage function, are relatively higher than with the linear damage function.[20]

The generally poor performance of the capacity increase scenario that has been found can be seen in the light of Gerlagh (2011)'s definition of a strong green paradox. A relaxation of the capacity constraint of the green substitute leads not only to an increase in the near-term emissions but also to an overall welfare loss for society and, therefore, a strong green paradox occurs. This green paradox result is summarized in Proposition 8.

Proposition 8: *Numerical simulations show that a capacity expansion of the green energy substitute leads to a strong green paradox since it reduces social welfare compared to the laissez-faire case.*

A capacity increase might result from technological progress (e.g., second-generation biofuels), but can also be induced by the respective policy measures (e.g., when the government allows import of a biofuel previously banned from the market or introduces a biofuel quota). The following subsection sheds some further light on the policy relevance of the presented model.

[20] While the direction of an effect of a policy measure (its marginal effect) is a general result and (qualitatively) independent of the underlying parameter choice, the quantitative effect in terms of the resulting welfare order is not. This is especially the case if the analyzed policy measures are not marginal. In practice, policy measures are not marginal. To illustrate the effect of considering this in a policy instrument evaluation, this numerical analysis is also not. A further discussion on this can be found in the last section of this paper.

5 Policy relevance

The model presented in this paper exhibits a considerable degree of flexibility and is able to capture various current problems. To illustrate this broad applicability, this section provides (stylized) evidence that supports this paper's approach, showing that it is highly relevant. In addition to the oil market application presented in Section 4, this section shows, by way of illustration, how the model can also be used to analyze the transformation of the electricity sector.

As already explained in Section 4, a natural application of our model is an oil market with conventional and unconventional oil as well as biofuels as a clean substitute. The cost structure and environmental impacts can be described as it is captured in the parameter choice in Section 4. The consideration of two rather than one "dirty" resource is supported by the recent emergence of unconventional carbon resources such as extra heavy oil, oil sands, and oil shale (Gordon 2012).[21] Extracting oil from unconventional sites is more costly as well as more energy intensive and, thus, unconventional oil has a higher CO_2 emission intensity and extraction cost level than conventional oil. The modeling framework applied here is well suitable for capturing this issue. Specifically, beside different technological problems, biofuel production raises concerns land use since there might not be enough (suitable) land available for biofuel production and, even if there were, using it for that purpose might seriously compromise food production and raise sustainability concerns (see, e.g., Sinn 2012). Thus, it seems to be the case that there is a constraint imposed on the share of biofuels production. The share of biomass from global primary energy supply is currently about 15%. This, however, is to a very large extent attributable to so-called "traditional biomass" - the use of firewood, charcoal as well as agricultural residues (IEA 2012). The share of biofuels in global road transport, however, is merely 3% and several problems indicate that it is more than reasonable to assume that biomass is not a backstop technology that can be

[21]This might also be seen as an approximation of an increasing (instead of flat) marginal cost curve.

used without constraints (IEA 2011).[22] A core result of the theoretical as well as numerical analysis is the negative welfare effect of the capacity expansion scenario. Therefore, in the context of the present analysis, the global biomass potential that actually exists can be seen as a considerable problem. In light of our findings, transport sector policies such as blending mandates must be analyzed carefully regarding possible green paradox effects.

The already mentioned electricity sector is another possible application for our model. There is a similar situation present as in the oil market example: Electricity is generated from both different "dirty" and exhaustible conventional resources as well as green ones simultaneously - despite the fact that renewable energy is considerably more expensive than electricity conventionally produced. Widely discussed topics like climate change, energy security or resource scarcity increase the attractiveness of using renewable energies such as, for example, wind or solar power rather than (or at least in addition to) coal or gas. In consequence, policy instruments such as feed-in-tariffs or green energy quotas are in place in many countries. For example, Germany today generates 20% of total electricity from renewable sources such as wind and solar and the European Union aims at reaching this share at the European level fro 2020. Clearly, there are limits to increasing this share.[23] In other words, assuming that a backstop resource for electricity generation is unrestrictedly available is problematic. The results of our model indicate that policy instruments intending

[22] Even though projections certainly indicate that there is a vast potential for biomass (for example, unused and surplus land, has the potential of about 550-1,500 EJ biomass production in 2050 (IEA 2011)), the way to exploit this potential is nevertheless long and stony. To mention just a few of the challenges, crop yields need to increase considerably, and substantial parts of land needs to be converted. In addition to that, IEA (2011) points to regulatory requirements and stresses the importance of ensuring that food security is not compromised (see also Sinn 2012).

[23] For example, substantial adjustments of the electricity transmission and distribution network are required. What is more, finding solutions for the related problems of intermittent renewable energies and the considerable lack of storage facilities is anything but easy. In addition to these technological challenges, there are also important regulatory ones. The requirement of backup power plants to guarantee network stability sparked the debate on an entire redesign of electricity market - see the discussion on so-called capacity markets (IEA 2012). Moreover, the development of renewable energies in electricity production must be seen in the context of the energy political triangle which poses a restriction on the increase of green electricity production, see Preface and the last chapter of this thesis. Since the present paper only focuses on the supply side production decisions of energy goods, a further consideration of those topics would be beyond the scope of this paper.

to increase the capacity constraint of a renewable substitute are not without problems but bringing their market entry forward may have positive long-term effects. However, a detailed analysis of possible green paradox effects in the electricity market requires a corresponding calibration of the numerical model.

There are even more ways of interpreting our model. An example is nuclear energy - a "conventional," but carbon-free energy source, which is capacity-constrained by regulatory, political, and maybe even (safety-related) technological restrictions. More generally, in contrast to the case where the renewable energy is clean, the case where the backstop technology is dirty, however, is also of interest (see, e.g., van der Ploeg and Withagen 2012a). Regarding a dirty backstop, one might think, for example, of liquid fuels produced with coal-to-liquids technologies. Of course, also for this cases, a detailed analysis of possible green paradox effects requires a corresponding calibration of the numerical model.

These reflections bear witness to the broad applicability of this paper's model. It is fairly obvious that applications of this model make an important contribution to current energy policy debates. In a nutshell, the model applied in this paper can capture different situations that are currently present in the discussion about energy markets. At the same time, the results obtained in this paper clearly indicate that neglecting the important feature of capacity-constrained backstop technologies can lead to wrong policy decisions.

6 Summary and conclusions

This paper addresses the considerable difficulty of decarbonizing an economy and analyzes the behavior of agents, especially regarding the supply side of energy production, to obtain a clearer understanding of how various policies may affect energy markets. The model applied in this paper has two important features. First, it encompasses three different resources with different extraction costs. One of these resources is assumed to be "green" and capacity-constrained as it has been demonstrated in the previous section. Second, the model allows resources with different extraction costs to be used simultaneously. These two features distinguish this paper from the majority of recent work on climate policies.

Based on a partial equilibrium model of Holland (2003) and with particular reference to a concrete oil market example, we analyzed the effects of different climate policies on an energy market characterized by two cheap but dirty fuels and a green but expensive and capacity-constrained substitute. After an implicit determination of the endogenous variables, we analyzed the effects of four different policy scenarios on supply-side extraction and production behavior, as well as the resulting energy price path, using a comparative static approach.

The analysis was complemented by a numerical section in which the model and its results were illustrated based in the context of a concrete oil market example. Additionally, an extensive welfare analysis was conducted using various specifications for the amount and development of anthropogenic carbon in the atmosphere as well as alternative specifications of the environmental damage function.

We tested our comparative static results for three different types of green paradox; the weak green paradox of Gerlagh (2011), which involves a short-term increase of carbon emissions, the overall green paradox, which occurs when the overall extraction duration of all available fossil fuels is shortened, and the strong green paradox (Gerlagh 2011), which arises when overall welfare decreases as a consequence of a policy measure. We found a weak green paradox for subsidization of the green energy, and both a weak and a strong green paradox for capacity enhancement.

The basic point of a green paradox can simply be summed up as "good intentions do not always breed good deeds" (Sinn 2008). Or, more specifically for our paper, a green paradox arises when a policy measure intended to slow down resource extraction so as to increase overall social welfare achieves the exact opposite effect (which is here increased (short-term) extraction speed and/ or increasing overall damages). This basic effect can occur via various channels, including intertemporal arbitrage, spatial, technological, or extraction order effects (for more details, see, e.g., van der Werf and Di Maria 2011). Intertemporal effects were pointed out in Sinn (2008), referring earlier analysis of firm's extraction decisions in anticipation of future tax changes (Long and Sinn 1985). A technology-induced green paradox was pointed out in Strand (2007) and can also be found in the next chapter of this thesis. Hwang and Mai (2004) showed a green paradox result in a spatial model.[24] For our analysis, the intertemporal as well as the extraction order effect are important. The intertemporal effect can be found both for subsidization and capacity enhancement of green energy goods. In either case, the policy measure decreases future resource rents and therefore increases the (short-term) extraction speed of fossil fuels. We find an extraction order effect by using Holland (2003)'s basic conditions in the present model framework for a cost reversal to occur. Moreover, as illustrated by our oil market example, a policy that delays production of the green substitute can also be seen as a green paradox in the extraction order sense, which is exactly what occurs in the capacity enhancement scenario.

Even though our green backstop was pared down to its most simple form and did not include, for example, the possibility of a gradual relaxation of the capacity constraint or any uncertainty about its success, we found that a renewable energy sector subject to a capacity constraint, a characteristic of green energy we actually observe, casts doubts on the welfare effects of some policy measures that intend to reduce carbon emissions (more concretely,

[24] While the model of Hwang and Mai (2004) does not deal with open economies, by endogenizing the choice of a firm's location, it is pointing out to a root of the Green Paradox: Policy makers quite often fail to take into account the full ability of firms or individuals to make spatial or intertemporal adjustments to their plans in response to policy measures. The literature on "carbon leakage" is based on the same insight. For example, the carbon-leakage model by Babiker (2005) assumes spatial competition among Cournot oligopolists, a feature that has been exploited in modeling firms' locational choice (Markusen and Venables 1988, Markusen et al. 1993, 1994).

to transfer them into the future) by, directly or indirectly, promoting green energies. This feature differentiates our results from the general conclusions of the existing green paradox literature and is of significant consequence for policy advice. Thereby, the model allows further differentiation between different green paradox effects compared (both qualitatively and quantitatively) to those with an unrestricted backstop technology and only one exhaustible resource as it is usually discussed in the literature. Moreover, we showed that the capacity constraint itself on the renewable substitute may reduce at least to some extent the reliance on exhaustible resources and thereby helps policymakers to implement effective climate policies. We found in our extensive welfare analysis that while a policy measure might induce adverse short-term effects (weak green paradox), the welfare effects can nevertheless be those intended. This feature further differentiates our results from the general conclusions of the existing green paradox literature.

More concretely, for a policymaker who wants to support green energy to reduce anthropogenic carbon emissions, the welfare analysis implies that a tax on the high-cost exhaustible resource has the best welfare effects. A subsidy for the green energy or a tax on the low-cost exhaustible resource seem also to be useful instruments. All the three measures reduce, directly or indirectly, the costs of production without crowding out the exhaustible resources. This is the case due to the existence of a capacity constraint on the backstop technology. Here, it breaks the neutrality of a constant unit tax and due to the upper price floor provided by the capacity-constrained choke price, assures effectiveness of the respective policy instrument. Even if a policy measure leads to a weak green paradox in the short run as it is the case for the subsidy, the overall welfare effect is positive. In contrast, increasing production capacity of the green substitute produces welfare decreasing effects since anticipation of reduction in costs of this technology can induce rapid extractions of the exhaustible resources, as shown by Strand (2007) and Hoel (2008).[25]

[25] The more the capacity constraint is weakened, the more the green substitute turns into a "classical" backstop technology.

However, when conducting a welfare analysis and making recommendations to policy-makers, the underlying welfare effects must be considered carefully in the context of the respective energy market situation. The sign of the marginal effects are generally valid and therefore also the direction of effects for the analyzed policy measures. However, since we did not analyze marginal effects in the numerical analysis, the result of the welfare analysis depends, at least to some extent, on the underlying parameter choice (at least when effects are not proportional and work in different directions). While the direction of effect is a general result and independent of the underlying parameter choice, the size of the effect is not. However, this does not limit the value of the conducted welfare analysis, in contrast, it illustrates the importance of a precise evaluation of the concrete market situation.

In this context, a special trade-off when it comes to policy implications is worth emphasizing. In the event of partial depreciation of the atmospheric stock of carbon (which is the real-world situation), the two parameters δ and r work in the opposite direction: A higher discount rate r may tend to imply policy action that postpones carbon extraction so as to push the damages far into the future, whereas a higher decay rate δ might even induce a shortening of the extraction period.[26] Consequently, with $\delta > 0$ and $r > 0$, we have a trade-off between these two parameters. The longer we can postpone extraction, the lower tend to be the damages due to the positive social discount rate. On the other hand, the higher the current emissions, the higher the future absolute depreciation in the atmosphere.[27]

A potential limitation of our approach is that we modelled the cost structures as well

[26]This is because δ is a constant rate of decay: The higher the emissions per period of time, the higher the absolute decay in the following periods (thereby, the consequences in the future for today's behavior decrease) and the faster it converges toward $C[\alpha V(t)]$.

[27]For example, changing the time horizon or time preference rate might change the welfare order. One might argue, for example, that politicians have a relatively high time preference rate, or short time horizons. Moreover, to avoid the specific problems associated with finite time horizons, in the present model we chose an infinite horizon so as to capture all socially relevant effects.

as the capacity constraint in a very simple way. However, both simplifications are widely used in literature and accepted as approximations for more elaborated cost and capacity structures actually observed in energy markets. The two exhaustible resources with different cost structures can be understood as a single energy good which becomes more difficult to produce (both more costly as well as more carbon intensive) with increasing scarcity. Beside we need two exhaustible energy goods to illustrate our oil market example, this is another reason for introducing two exhaustible resources instead of only one in a broader energy market context as it has been explained in the previous section. Moreover, we abstained from introducing a more realistic, maybe variable or endogenous, capacity constraint of the renewable energy good. A reason for this is that our simply constructed capacity constraint is sufficient to demonstrate the mechanisms and implications we were interested in. A more realistic formulation for a capacity constraint may be possible, based on opportunity cost considerations (the higher the price for the exhaustible resource, the more of the renewable is available, see next chapter of this thesis). However, depending on the underlying model specifications, the results would not differ, only the calculations would be more complex.

Our approach should thus be viewed as a first step toward analyzing the complexity of energy markets comprised of a variety of energy goods with a special focus on the integration of capacity-constrained (green) backstop technologies. To our knowledge, this very important aspect of energy markets is mostly ignored in the literature. Since the present analysis is of partial nature, a consequent next research step would be to discover first-best energy policies as well as effects and trade-offs of different policy measures in a general equilibrium model with capacity-constrained energy sources. Another resulting research approach is the derivation of socially optimal investments in green capacity technology with an endogenous capacity constraint. This is especially important to evaluate the welfare effects in the context of the politically determined development plans of green energy we observe in many countries (see, e.g. Preface and the last chapter of this thesis). A further important research topic is on the impact of uncertainty about capacity potentials of green energy for policy action. Moreover, a closer look at the green substitute is needed to understand the extraction and production decisions of the energy suppliers and to find the resulting implications for climate and climate policy. A first step into this direction is done in the next chapter of this thesis. There, an analysis of the effects of increasing substitutability (both exogenous and endogenous) between an exhaustible and a renewable resource on resource extraction, climate, and the respective climate policy implications is conducted.

APPENDIX

In this Appendix, we identify conditions for the parameter values such that T is exactly equal to t_3, such that Phase 2 collapses to a single point. If $T = t_3$, then from time t_3, energy supply comes both from deposit 2 and from the clean energy sector (deposit 1 having been exhausted, we have identical starting-times of clean energy production and extraction from the high-cost deposit with $T = t_3$). As defined before, the time at which deposit 2 is exhausted is called $\overline{T}$. At $\overline{T}$ and from then on, the price of energy must equal $\overline{p} \equiv U'(\overline{q}_3) \equiv \phi(\overline{q}_3)$. During the time interval $t \in \left[t_3, \overline{T}\right)$, the Hotelling rule must hold for deposit 2:

$$(p(t) - c_2)\, e^{-rt} = (p(t_3) - c_2)\, e^{-rt_3} = \left(p\left(\overline{T}\right) - c_2\right) e^{-r\overline{T}} \equiv (\overline{p} - c_2)\, e^{-r\overline{T}}.$$

From this equation, the explicit price path between t_3 and $\overline{T}$ as well as the extraction duration can be determined. With $p(t_3) = c_3$, it follows that the length of time it takes for the price to rise from c_3 to $\overline{p}$ is

$$x = \frac{1}{r} \ln \left[\frac{\overline{p} - c_2}{c_3 - c_2} \right]$$

where x is defined as

$$x \equiv \overline{T} - t_3.$$

Moreover, for all $t \in \left[t_3, \overline{T}\right)$, the price path is

$$p(t) = c_2 + \frac{(p(t_3) - c_2)\, e^{-rt_3}}{e^{-rt}} = c_2 + (c_3 - c_2)\, e^{r(t - t_3)}.$$

From this, total demand for energy over the time interval $\left[t_3, \overline{T}\right)$ can be determined as

$$\int_{t_3}^{\overline{T}} D\left[p(t)\right] dt = \int_{t_3}^{\overline{T}} D\left[c_2 + (c_3 - c_2)\, e^{r(t-t_3)}\right] dt.$$

Then, we use $x \equiv \overline{T} - t_3$ and the substitution $\tau = t - t_3$ to obtain

$$\int_0^{\overline{T}-t_3} D\left[c_2 + (c_3 - c_2)\, e^{r\tau}\right] d\tau \equiv \int_0^x D\left[c_2 + (c_3 - c_2)\, e^{r\tau}\right] d\tau.$$

Total demand must be met by total supply, which is the output of the clean energy sector and extractions from deposit 2:

$$\int_0^x D\left[c_2 + (c_3 - c_2)\, e^{r\tau}\right] d\tau = x\overline{q}_3 + \int_0^x q_2(\tau) d\tau \text{ (recall } \tau = t - t_3).$$

It follows that if S_2 is just equal to a threshold value $S_2^{\max}(\infty)$ defined by

$$S_2^{\max}(\infty) \equiv \int_0^x D\left[c_2 + (c_3 - c_2)\, e^{r\tau}\right] d\tau - \frac{\overline{q}_3}{r} \ln\left[\frac{\overline{p} - c_2}{c_3 - c_2}\right],$$

then t_3 is indeed the time at which deposit 2 begins to be extracted (and sold at price $p(t_3) = c_3$ at that moment), and the time at which deposit 1 has just been exhausted.

Can we determine time t_3 in this case? Analogous to the above, since over the time interval $[0, t_3)$ deposit 1 is being exploited, the Hotelling rule applied to deposit 1 must hold with equality for all $t \leq t_3$:

$$(p(t) - c_1)e^{-rt} = p(0) - c_1 = (c_3 - c_1)e^{-rt_3}.$$

Rearranging gives us the price path between $t = 0$ and $t = t_3$ and, under the consideration that total demand must be met by total supply, we obtain

$$\int_0^{t_3} D\left[c_1 + (c_3 - c_1)\, e^{-r(t_3 - t)}\right] dt = S_1.$$

This equation determines t_3 and hence $p(0)$ as functions of S_1 (given the assumption that $S_2 = S_2^{\max}(\infty)$). We summarize the results for this razor's edge case in the following proposition.

Proposition: (Razor's edge case) *If the size of deposit 2 is equal to the threshold value* $S_2^{\max}$ *defined by*

$$S_2^{\max} \equiv \int_0^x D\left[c_2 + (c_3 - c_2)\, e^{r\tau}\right] d\tau - \frac{\overline{q}_3}{r} \ln\left[\frac{\overline{p} - c_2}{c_3 - c_2}\right],$$

with

$$x \equiv \frac{1}{r} \ln\left[\frac{\overline{p} - c_2}{c_3 - c_2}\right],$$

then the equilibrium time path of extraction is continuous and consists of ***three*** *phases:*

Phase 1 $($*the time interval* $[0, t_3))$*: The whole market is supplied from deposit 1 only:* $Q = q_1$*. This deposit will be exhausted at time* t_3*, where* t_3 *is the solution of*

$$\int_0^{t_3} D\left[c_1 + (c_3 - c_1)\, e^{-r(t_3 - t)}\right] dt = S_1.$$

At time t_3*, the price of energy is* $p(t_3) = c_3$*.*

Phase 2 $($*the time interval* $\left[t_3, \overline{T}\right))$*: The whole market is supplied from both the high cost deposit* $($*deposit 2*$)$ *and the clean energy sector:* $Q = q_2 + \overline{q}_3$ *where* $q_2(t) > 0$ *for all* t *in* $\left[t_3, \overline{T}\right)$*. The length of this phase is equal to* x*. At time* $\overline{T}$*, the price of energy is* $\overline{p}$*, and deposit 2 is exhausted.*

Phase 3: After time $\overline{T}$*, the whole energy market is satisfied by the clean energy sector:* $Q = \overline{q}_3$*.*

References

[1] Archer, D., 2005, Fate of fossil fuel CO_2 in geologic time. *Journal of Geophysical Research* 110, C09S05, doi:10.1029/2004JC002624.

[2] Amigues, J.-P., Favard, P., Gaudet, G., Moreaux, M., 1998, On the optimal order of natural resource use when the capacity of the inexhaustible substitute is limited,

Journal of Economic Theory 80 (1), 153-170.

[3] Babiker, M. H., 2005, Climate Change Policy, Market Structure, and Carbon Leakage, *Journal of International Economics*, 65(2): 421-445.

[4] Chakravorty, U., M. Moreaux, and M. Tidball, 2008, Ordering the Extraction of Polluting Non-renewable Resources, *American Economic Review* 98(3), pp. 1128-44.

[5] Dasgupta, P., and Heal, G., 1974, The optimal depletion of exhaustible resources, *Review of Economic Studies* 41, Symposium on the economics of exhaustible resources, 3-28.

[6] Farzin, Y. H., and Tahvonen, O., 1996, Global carbon cycle and the optimal time path of carbon tax, *Oxford Economic Papers* 48 (4).

[7] Gerlagh, R., 2011, Too much Oil, *CESifo Economic Studies* 57 (1), 79-102.

[8] Gordon, D., 2012, Understanding unconventional oil, *The Carnegie Papers*.

[9] Grafton, Q., Kompas, T., and Long, N. V., 2012, Substitution between biofuels and fossil fuels: Is there a Green Paradox?, *Journal of Environmental Economics and Management* 64 (3), 328-341.

[10] Herfindahl, O.C., 1967, Depletion and economic theory, in: Mason Gaffney (Ed.), *Extractive Resources and Taxation: Proceedings*, University of Wisconsin Press, Madison.

[11] Hoel, M., 2008, Bush meets Hotelling: Effects of improved renewable energy technology on greenhouse gas emissions, *CESifo Working Paper* No. 2492.

[12] Hoel, M., 2011, The supply side of CO_2 with country heterogeneity, *Scandinavian Journal of Economics* 113 (4).

[13] Hoel, M., and Kverndokk, S., 1998, Depletion of fossil fuels and the impacts of global warming, *Resource and Energy Economics* 18 (2), 115-136.

[14] Holland, S., 2003, Extraction capacity and the optimal order of extraction, *Journal of Environmental Economics and Management* 45 (3), 569-588.

[15] Houghton, J. T., Callander, B. A., and Varney, S. K., (Eds.), 1992, Climate Change: Supplementary Report to the IPCC Scientific Assessment (Report from Working Group I, Intergovernmental Panel on Climate Change). WMO, UNEP, IPCC. Cambridge: Cambridge University Press.

[16] Houghton, J. T., Jenkins, G. J., and Ephraums, J., (Eds.), 1990, Climate Change: IPCC Scientific Assessment (Report from Working Group I, Intergovernmental Panel on Climate Change). WMO, UNEP, IPCC. Cambridge: Cambridge University Press.

[17] Hwang, H, and C.C. Mai (2005), "The Effects of Pollution Taxes on Urban Areas with Endogenous Plant Location," *Environmental & Resource Economics 2*9: 57-65.

[18] IEA, 2011, Technology roadmap - Biofuels for transport, Paris.

[19] IEA, 2012, World Energy Outlook 2012, Paris.

[20] Kemp, M. C., and Long, N. V., 1980, On two folk theorems concerning the extractions of exhaustible resources, *Econometrica* 48 (3), 663-673.

[21] Long, N. V. and Sinn, H.-W., 1985, Surprise Price Shift, Tax Changes and the Supply Behaviour of Resource Extracting Firms, *Australian Economic Papers*, 24(45): 278-289.

[22] Markusen, J. and A. Venables (1988), "Trade Policy with Increasing Returns and Imperfect Competition: Contradictory Results from Competing Assumptions," *Journal of International Economics* 24: 299-316.

[23] Markusen, J., E. Morey, and N. Olewiler (1993), "Environmental Policy when Market Structure and Plant Location are Endogenous," *Journal of Environmental Economics and Management* 24: 299-316.

[24] Markusen, J., E. Morey, and N. Olewiler (1995), "Competition in Regional Environmental Policies when Plant Location are Endogenous," *Journal of Public Economics* 56: 55-77.

[25] Nordhaus, W.D. (2007), *The Challenge of Global Warming: Economic Models and Environmental Policy,* Yale University.

[26] Sinclair, P.J.N., 1992, High Does Nothing and Rising is Worse: Carbon Taxes Should Keep Falling to Cut Harmful Emissions, *Manchester School* 60: 41-52.

[27] Sinclair, P.J.N., 1994, On the optimum trend of fossil fuel taxation, *Oxford Economic Papers* 46, 869-877.

[28] Sinn, H.-W., 2008, Public policies against global warming: a supply side approach, *International Tax and Public Finance* 15 (4), 360-394.

[29] Sinn, H.-W., 2012, The green paradox: A supply-side approach to global warming. Cambridge, Massachusetts.

[30] Strand, J. 2007, Technology Treaties and Fossil Fuels Extraction, *The Energy Journal* 28: 129-142.

[31] van der Ploeg, R., and Withagen, C., 2011, Growth, Renewables and the Optimal Carbon Tax, *OxCarre Research Paper* 55.

[32] van der Ploeg, R., and Withagen, C., 2012a, Too much coal, too little oil, *Journal of Public Economics* 96(1), 62-77.

[33] van der Ploeg, R., and Withagen, C., 2012b, Is there really a green paradox? , *Journal of Environmental Economics and Management* 64 (3), 342-363.

[34] van der Werf, E., and Di Maria, C., 2011, Unintended Detrimental Effects of Environmental Policy: The Green Paradox and Beyond, *CESifo Working Paper Series* No. 3466.