

An Experimental Evaluation of a 3D Visible Light Positioning System in an Industrial Environment with Receiver Tilt and Multipath Reflections

Yousef Almadani^{1*}, Muhammad Ijaz¹, Bamidele Adebisi¹, Sujan Rajbhandari², Sander Bastiaens³, Wout Josphe³, and David Plets³

¹*Manchester Metropolitan University, Faculty of Science and Engineering, Department of Engineering, Manchester, M1 5GD, United Kingdom*

²*Huawei Technologies Sweden AB, 41250 Gothenburg, Sweden*

³*Ghent University, imec-WAVES, Department of Information Technology, iGent-Technologiepark 126, Ghent, 9052, Belgium*

Abstract

In this paper, two different three-dimensional (3D) indoor visible light positioning (VLP) algorithms are experimentally assessed for an industrial environment. The Cayley-Menger determinant (CMD) and linear least square (LLS) trilateration algorithms use the received signal strength (RSS) to estimate the receiver's 3D position without prior knowledge of its height. The unknown 3D position of the receiver is estimated by the trilateration algorithms coupled with a cost function under different realistic scenarios. The performances of the algorithms are experimentally evaluated in terms of positioning error by considering two different light-emitting diode (LED) configurations in the presence of different receiver tilt angles, and with multipath reflections. It is observed that the widespread square LED configuration results in position ambiguities while a star-shaped configuration is much more accurate. Experimental tests performed in a 4 m × 4 m × 4.1 m area with four LEDs reported a median positioning error of 10.6 and 10.5 cm using the LLS and CMD algorithms, respectively, without the presence of receiver tilt or multipath reflections. However, when a receiver tilt of 10° was added, the median error increased to 22.7 cm using the LLS algorithm and 21.6 cm using the CMD algorithm. Overall, the achieved mean and maximum values using the LLS algorithm were 13.1 and 39 cm, respectively, while they were 12.2 and 34 cm using the CMD algorithm.

1
2
3
4
5
6
7
8
9 *Key words:* indoor visible light positioning, localization, industrial, tilt,
10 VLC, VLP
11

14 1. Introduction

15
16 Indoor positioning is a very promising research domain that is gaining
17 wide attention due to its potential in Industry 4.0 and the health sector.
18 Conventional positioning methods that rely on satellites such as global po-
19 sitioning system (GPS) are unreliable for indoor positioning due to the high
20 penetration loss from walls and building materials. Complementary methods
21 such as assisted-GPS and pseudo-satellite have been proposed to address the
22 shortcomings of conventional satellite-based systems. However, the accura-
23 cies of these systems are still inadequate with the added complexity of inte-
24 grating two different systems [1]. Other technologies have also been proposed
25 for indoor positioning and navigation such as Bluetooth, ultrasound, ultra-
26 wideband (UWB), and radio-frequency (RF) based techniques [2]. While
27 encouraging results have been achieved using Bluetooth and UWB, there
28 is also another emerging technology that makes use of the ubiquitous light
29 fixture’s infrastructure.
30

31
32 Visible light positioning (VLP) is one of the most promising technologies
33 being proposed for indoor positioning given the readily available lighting
34 infrastructure and its many advantages such as increased bandwidth, secu-
35 rity and low relative complexity when compared with RF-based positioning.
36 While most of the technologies being researched and proposed for indoor
37 localization are based on the highly congested RF spectrum, VLP systems
38 are not sensitive to electromagnetic interference, which enables them to be
39 used in areas that are sensitive to electromagnetic waves such as hospitals
40 and certain power plants [3].
41
42
43
44
45

46 2. Related Work

47
48 In [4], the researchers proposed a multiple-classifiers fusion localization
49 framework by using received signal strength (RSS) fingerprints. The ex-
50 periment was performed within a 0.7 m \times 0.7 m area with four LEDs and
51 achieved a median square positioning error of less than 5 cm for the majority
52 of the area. In [5], a 3D VLC positioning system based on modified particle
53 swarm optimization (PSO) algorithm is presented and has been experimen-
54 tally tested. The researchers evaluated the system using four LEDs in a cube
55
56
57
58

1
2
3
4
5
6
7
8
9
33 frame measuring $0.9\text{ m} \times 0.9\text{ m} \times 1.5\text{ m}$ and achieved an average error of
34 3.5 cm for a 3D VLP system. In [6], a machine learning (ML) technique with
35 height tolerance was tested using three LEDs within an area of $1.1\text{ m} \times 1\text{ m}$
36 $\times 2.5\text{ m}$. The result shows that over 80% of the results can be under 5 cm
37 with an improved height tolerance range of 15 cm . Researchers in [7] intro-
38 duced and experimentally tested a VLP method based on median shift (MS)
39 algorithm and unscented Kalman filter (UKF) using image sensors. The test
40 area of their experimental setup was $1.9\text{ m} \times 1\text{ m} \times 1.9\text{ m}$ and achieved a
41 positioning accuracy of up to 0.42 cm , with an accuracy of 1.41 cm when half
42 of the LED was shielded. The work in [8] used an RSS-based VLP system
43 combined with a deep neural network based on the Bayesian Regularization
44 (BR-DNN) with a sparse diagonal training data set. The method was tested
45 in a $1.8\text{ m} \times 1.8\text{ m} \times 2.1\text{ m}$ area and achieved a maximum positioning error
46 of 4.58 cm for an even set, and 3.4 cm under a diagonal set of LEDs. In
47 [9], a low-complexity time-difference-of-arrival (TDoA) method with an en-
48 hanced practical localization using cross-correlation is reported and achieved
49 a positioning accuracy of 9.2 cm in a $1.2\text{ m} \times 1.2\text{ m}$ testbed area. A 2D
50 VLP system using differential phase difference of arrival (DPDoA) was ex-
51 perimentally tested in [10] and achieved an average root-mean-square (RMS)
52 positioning error of 1.8 cm and a maximum of 8 cm in a testbed area of
53 $1\text{ m} \times 1.2\text{ m} \times 2\text{ m}$. Researchers in [11] proposed a fusion positioning
54 system based on extended Kalman filters (EKF), which uses an inertial nav-
55 igation unit to improve the performance of the VLP system. An average
56 positioning error of 33.9 cm was achieved based on RSS alone and 14.5 cm
57 when combined with an EKF.

58 Three typical office environments were tested in [12]. Their proposed
59 method locates the receiver using trilateration/multi-lateration if over three
60 light sources are perceived, along with an optimization process. If less than
61 three signals are received, then a fusion method is used with an inertial mea-
62 surement unit (IMU). The achieved 90th percentile positioning errors for the
63 three environments were 0.4 m , 0.7 m , and 0.8 m . When only one transmitter
64 is available, the 90th percentile error increased to 1.1 m . The work in [13]
65 proposed the use of the received light intensity with accelerometer measure-
66 ments to compute distances between the transmitters and the receiver. An
67 error of less than 25 cm was reported in a $5\text{ m} \times 3\text{ m} \times 3\text{ m}$ area. A gain
68 difference positioning method based on the angle of arrival and the received
69 signal strength was proposed in [14]. The method uses multiple tilted re-
70 ceivers to calculate the 3D location with reported average error distances of

Table 1: A summary of the experimental work in indoor VLP systems

Ref.	Method	2D/3D	Test Area (W L H) (m)	Accuracy (cm)	No. of LEDs
[4]	Fingerprints	2D	$0.7 \times 0.7 \times 1.48$	5	4
[6]	RSS w/ ML	3D	$1.1 \times 1 \times 2.5$	3.65	3
[7]	MS-UKF	2D	$1.9 \times 1 \times 1.9$	0.42	4
[8]	RSS w/ BR-DNN	2D	$1.8 \times 1.8 \times 2.1$	4.58	4
[9]	TDoA	2D	$1.2 \times 1.2 \times 2$	9.2	3
[10]	DPDoA	2D	$1 \times 1.2 \times 2$	1.8	3
[11]	RSS	2D	$2.5 \times 2.84 \times 2.5$	33.9	7
[11]	RSS-EKF	2D		14.5	
[13]	RSS w/ Accelerometer	3D	$5 \times 3 \times 3$	25	3
[14]	RSS ratio	3D	$2 \times 2 \times 2.5$	3	1 w/ multiple PDs
[5]	PSO	3D	$0.9 \times 0.9 \times 1.5$	3.492	4
[15]	LED-ID w/ ROS	2D	$1 \times 1 \times 1.5$	0.82	4
[16]	LED-ID w/ ROS & ML	2D	$0.8 \times 0.8 \times 2$	2	5
[12]	RSS w/ IMU	3D	5×8	45	
[12]	RSS w/ IMU	3D	2×12	70	5
[12]	RSS w/ IMU	3D	3.5×6.5	80	

less than 3 cm. Table 1 provides a summary of the discussed experimental work on indoor VLP systems.

In [15], the researchers proposed an indoor robot VLP positioning package based on robot positioning system (ROS) with a efficient LED-ID detection scheme for rolling shutter. The system was experimentally tested in a $1 \text{ m} \times 1 \text{ m} \times 1.5 \text{ m}$ area with 36 uniformly distributed test points. The results reported an average accuracy of 0.82 cm, while 90% of the errors were less than 1.417 cm. The work in [16] proposed a double light positioning algorithm. The system uses LED-ID to determine the position of a receiver as well as a CMOS image sensor combined with machine learning a algorithm to identify the LED-ID. The system was tested in a $0.8 \text{ m} \times 0.8 \text{ m} \times 2 \text{ m}$ area and all of the reported positioning errors were within 3.85 cm with an average accuracy of 2 cm

As can be seen, the majority of the experimental work studied the performance of 2D VLP systems and generally required the use of additional hardware or the use of some complex algorithm for 3D localization. Additionally, most of the experiments analyzed the performance in relatively very small areas. In contrast to some of the previous works by other researchers, this paper examines a purely RSS-based 3D VLP system in a higher and larger area without the need for an additional receiver or complex algorithms.

In this paper, we experimentally assess and compare the performances of two 3D VLP positioning algorithms under different scenarios that are realistic industrial environments. The Cayley-Menger determinant (CMD)

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

94 and linear least square (LLS) algorithms are coupled with a cost function
95 to estimate a true 3D position without prior knowledge of the receiver's
96 height. The algorithms are evaluated for two different LED configurations
97 with different degrees of receiver tilt, and in the presence of a filled storage
98 rack to examine the effect of multipath reflections on the performance. The
99 algorithm could be used for VLP-based unmanned aerial vehicles (UAVs)
100 tracking in industrial warehouses. This is an emerging area where UAVs,
101 or drones, are employed for different sets of application such as stock-taking
102 in warehouses and inspecting hard-to-reach areas [17]. The commonly used
103 RF-based technologies generally suffer from electromagnetic interference or
104 unstable RF signals, deeming it unsuitable in providing high positioning
105 accuracy. It is especially not suitable in environments that have constant
106 sudden changes, e.g. forklifts or automated guided vehicles (AGVs), and
107 movement of people. This paper is partly an extension of our previous work
108 in [17]. However, this work considers an additional LED layout configuration,
109 an additional receiver tilt angle value, considers the presence of storage rack,
110 and examines the performance for 2D positioning as well.

111 The remainder of the paper is organized as follows: Section 3 details the
112 experimental setup. Section 4 presents the system model and the positioning
113 algorithms along with the cost function. Experimental results are presented
114 in Section 5 and is then followed by a discussion of the main findings in
115 Section 6. Finally, the paper concludes in Section 7.

116 3. Experimental Setup

117 The 3D algorithm is analyzed experimentally in a VLP lab that mea-
118 sures 4 m \times 4 m with the height of the LEDs at approximately 4.1 m, as
119 shown in Figure 1 (a). Black curtains are used as a substitute for walls
120 to ensure that uncontrolled reflections from walls and objects are avoided.
121 Four BXRE-50C3001-D-24 LEDs, shown in the inset of Figure 1 (a), are
122 intensity-modulated using transmitting pulse trains with a duty cycle of 0.5
123 with frequencies of 500 Hz, 1 kHz, 2 kHz, and 4 kHz. This ensures that the
124 contributions from the different LEDs can be demultiplexed individually at
125 the receiver's side.

126 The receiver is a commercial photodiode with an integrated electrical
127 amplifier (PDA36A2¹ by Thorlabs) that has an active area A_{pd} of 13 mm².

¹<https://www.thorlabs.com/thorproduct.cfm?partnumber=PDA36A2>

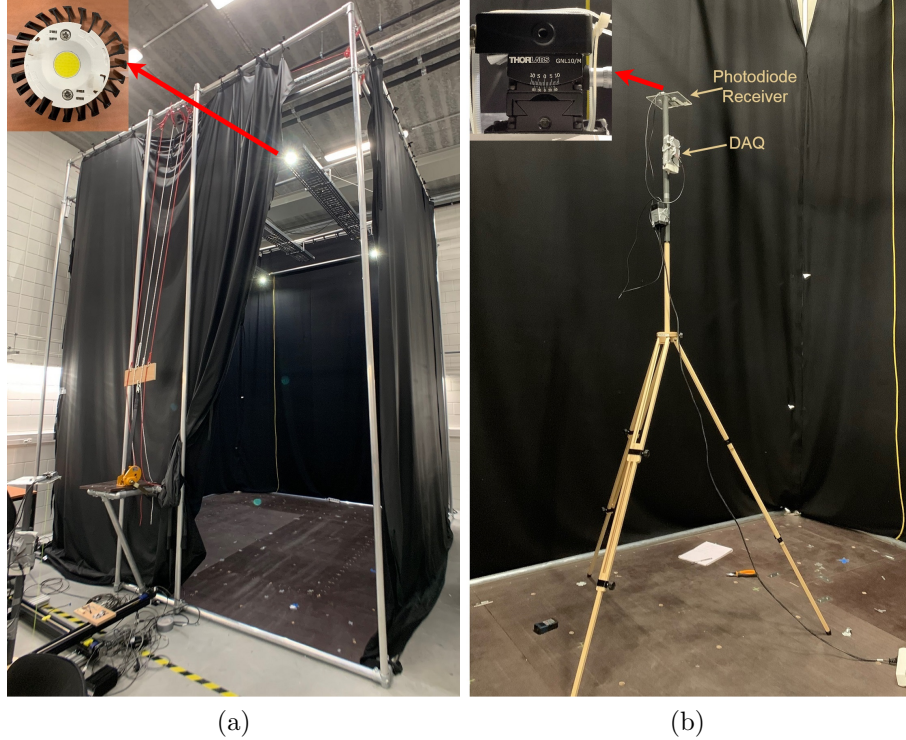


Figure 1: (a) The VLP lab experimental setup with black curtains with a view of the LEDs attached to ceiling rails, and (b) a tripod with the receiver mounted on top.

The photodiode's responsivity was estimated at 0.22 A/W by weighing the photodiode's responsivity spectrum with the LED's spectrum. The receiver is attached to a tripod with a vertical pole that allows adjustment of the receiver's height as shown in Figure 1 (b). The data is acquired using National Instrument's USB-6212 for processing. A fast Fourier transform (FFT)-based demodulation is used to extract the received power values for each LED in MATLAB[®], as specified in [18]. Table 2 shows the main parameters used in the experimental setup.

Figure 2 shows a path consisting of forty-eight points selected to take the receiver around the room at different heights ranging from 0.64 m to 2.55 m. The black line indicates the travel path, the green square denotes the start point, and red denotes the endpoint. The measurements were configured to sample 256 times using the DAQ with a sampling rate of 128 kHz. Twenty-

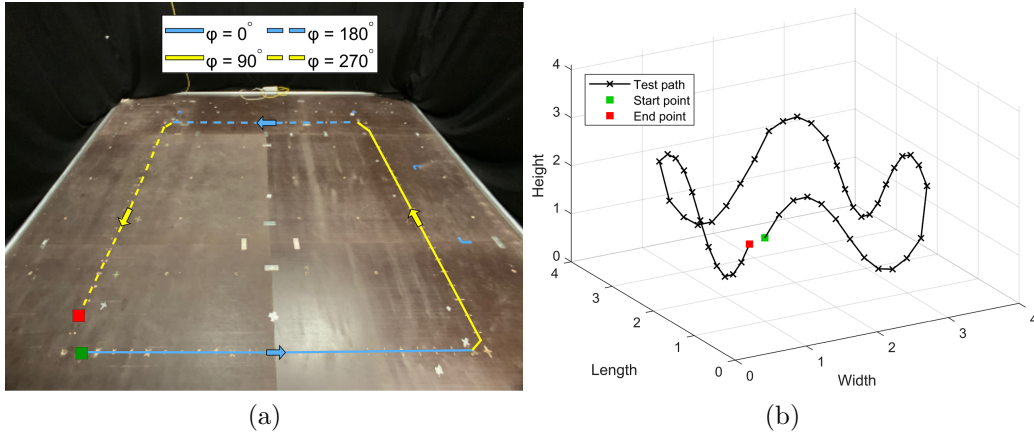


Figure 2: (a) The test path shown inside the VLP lab demonstrating the azimuthal orientation φ of the receiver; (b) A 3D view of the path demonstrating the height variations of the receiver along the specified path.

141 five power value readings were averaged at each location to reduce the impact
 142 of noise.

143 Two LED configurations denoted as ‘Square’ and ‘Star’ are used for the
 144 evaluation of the VLP as shown in Figure 3. The square-shaped is a typical
 145 configuration that is adopted by many researchers while the star configura-
 146 tion has a central LED circularly surrounded by the other three LEDs. Our
 147 previous work in [19] indicates that a classic configuration with four LEDs
 148 mounted in a square-shape is not able to accurately solve the 3D position
 149 ambiguity. Therefore, to counter this problem, a star-shaped configuration
 150 was proposed.

151 4. System Model

152 In this section, the VLC’s system model is outlined and the positioning
 153 algorithms along with the cost function are explained.

154 4.1. VLC System Model

155 The radiation of an LED chip follows a Lambertian pattern. Considering
 156 the line-of-sight (LoS) path between the LED transmitters and the receiver,

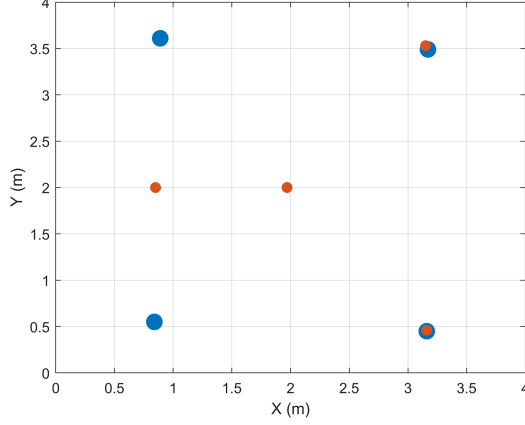


Figure 3: Top view of LEDs' locations in the with the blue dots representing the 'Square' configuration and the red dots representing the 'Star' configuration.

the received power can be modeled as [20]:

$$P_{ri} = P_{ti} \frac{(m+1)A_{pd}}{2\pi d_i^2} \cos^m(\alpha) \cos(\beta) T_{pd}(\beta) G_{pd}(\beta) \quad (1)$$

where P_{ti} is the transmitted power from the i^{th} LED, m is the Lambertian order, d_i is the distance between the i^{th} LED transmitter and the receiver, α is the angle of irradiance, β is the angle of incidence. The parameters are illustrated in Figure 4 (a). The optical filter's gain $T_{pd}(\beta)$, and the optical concentrator's gain $G_{pd}(\beta)$ are assumed to be equal to 1. Additionally, by assuming that the transmitters and the receiver are horizontally parallel, $\cos(\alpha) = \cos(\beta) = \frac{h_{LED}-z}{d_i} = \frac{\Delta h}{d_i}$, then d_i can be estimated as \hat{d}_i using the received signal power, P_{ri} [21]:

$$\hat{d}_i = \sqrt[m+3]{\frac{(m+1)A_{pd}P_{ti}\Delta h^{m+1}}{2\pi P_{ri}}} \quad (2)$$

where $\Delta h = h_{LED} - z$ is the unknown vertical height difference between the LED _{i} transmitter and the receiver. Since Δh is unknown, the estimated distance \hat{d}_i cannot be directly calculated from P_{ri} without knowing Δh , or equivalently, z . Due to this, a set of estimated distances \hat{d}_i is generated for different receiver heights, z , ranging from a minimum height h_{min} to maximum height, $h_{max} \leq h_{LED}$ with 1 mm intervals.

Table 2: Summary of the system parameters

Parameter	Value
Room Width x Length x Height	4 m × 4 m × 4.1 m
Transmitters' Power - P_t	13.3 W, 16.6 W, 16.4 W, 16.1 W
Transmitter's semi-angle - α	60°
Receiver's Height Range - z	0.64 - 2.55 m
Photodetector's Area - A_{pd}	13 mm ²
Receiver's Responsivity	0.22 A/W

The measured power of the LEDs can vary from their advertised values by up to 20%, as demonstrated in [22, 23]. Due to this, we collect one measurement directly under each transmitter as a calibration step ($\alpha = \beta = 0$). Then the estimated transmitted power is calculated using $P_{ti} = \frac{P_{ri} 2\pi d_i^2}{A_{pd}(m+1)}$ [24]. Table 2 lists the transmitted power for each transmitter.

In the case of receiver tilt, the received power will be impacted by an adapted angle of incidence. In this case, the angle of incidence in (1) is replaced with:

$$\cos(\beta_{tilt}) = \frac{(x-x_i) \cos(\varphi) \sin(\theta) + (y-y_i) \sin(\varphi) \sin(\theta) + (z-h_{LED}) \cos(\theta)}{d} \quad (3)$$

where (x, y, z) are the receiver's coordinates, (x_i, y_i, z_i) are the LED's coordinates, θ is the receiver's tilting angle, which is the angle difference between the normal vector of the xy-plane and the normal vector of the receiver. φ is the azimuthal rotation angle, which is the angle difference between the x-axis and the orthogonal projection of the receiver's normal vector on the xy-plane.

4.2. Positioning Algorithms

Two positioning algorithms are used in this paper, CMD and LLS. The performance of the CMD algorithm is compared with LLS as the latter is widely adopted in VLP systems.

4.2.1. Cayley-Menger Determinant

The Cayley-Menger determinant is used in distance geometry for determining the volume of a triangular pyramid (tetrahedron) based on the distances between any two of four vertices [25]. Figure 4 (b) shows the position

1
2
3
4
5
6
7
8
9
194 of three points (transmitters), p_1 , p_2 , and p_3 , with p_4 being the unknown
195 receiver's location.

196 The Cayley-Menger bideterminant of two sequences of n points $[p_1, p_2, \dots, p_n]$
197 and $[q_1, q_2, \dots, q_n]$ is defined as [26]:

$$D(p_1, \dots, p_n; q_1, \dots, q_n) = 2\left(\frac{-1}{2}\right)^n \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & D(p_1, q_1) & D(p_1, q_2) & \cdots & D(p_1, q_n) \\ 1 & D(p_2, q_1) & D(p_2, q_2) & \cdots & D(p_2, q_n) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & D(p_n, q_1) & D(p_n, q_2) & \cdots & D(p_n, q_n) \end{vmatrix} \quad (4)$$

198 where $D(p_i, q_j)$ is the squared distance between points p_i and q_j . When two
199 sequences of points are the same (i.e., $p_i = q_i$), then $D(p_1, \dots, p_n; q_1, \dots, q_n)$
200 is denoted by $D(p_1, \dots, p_n)$ and is simply called CMD [26]. So (4) becomes:

$$D(p_1, p_2, p_3, p_4) = \left(\frac{1}{8}\right) \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & D(p_1, p_2) & D(p_1, p_3) & D(p_1, p_4) \\ 1 & D(p_1, p_2) & 0 & D(p_2, p_3) & D(p_2, p_4) \\ 1 & D(p_1, p_3) & D(p_2, p_3) & 0 & D(p_3, p_4) \\ 1 & D(p_1, p_4) & D(p_2, p_4) & D(p_3, p_4) & 0 \end{vmatrix} \quad (5)$$

201 with p_4 is the unknown location of the drone, $D(p_4, p_1)$, $D(p_4, p_2)$ and
202 $D(p_4, p_3)$ are the distances \hat{d}_1 , \hat{d}_2 and \hat{d}_3 that are computed from the RSS
203 for a given receiver height. It is then possible to calculate the unknown position
204 of the receiver (p_4) with respect to three known transmitter coordinates
205 (p_1, p_2, p_3) using [26]:

$$p_4 = p_1 + k_1 v_1 + k_2 v_2 \pm k_3 (v_1 v_2) \quad (6)$$

where $v_1 = p_2 - p_1$ and $v_2 = p_3 - p_1$, and

$$k_1 = -\frac{D(p_1, p_2, p_3; p_1, p_3, p_4)}{D(p_1, p_2, p_3)}, k_2 = \frac{D(p_1, p_2, p_3; p_1, p_2, p_4)}{D(p_1, p_2, p_3)}, k_3 = \frac{\sqrt{D(p_1, p_2, p_3, p_4)}}{D(p_1, p_2, p_3)}$$

206 The CMD algorithm then outputs $(\hat{x}, \hat{y}, \hat{z})$ for each of the generated possible
207 heights Δh , and then the cost function is used to estimate the receiver's
208 height, h , and its corresponding location.

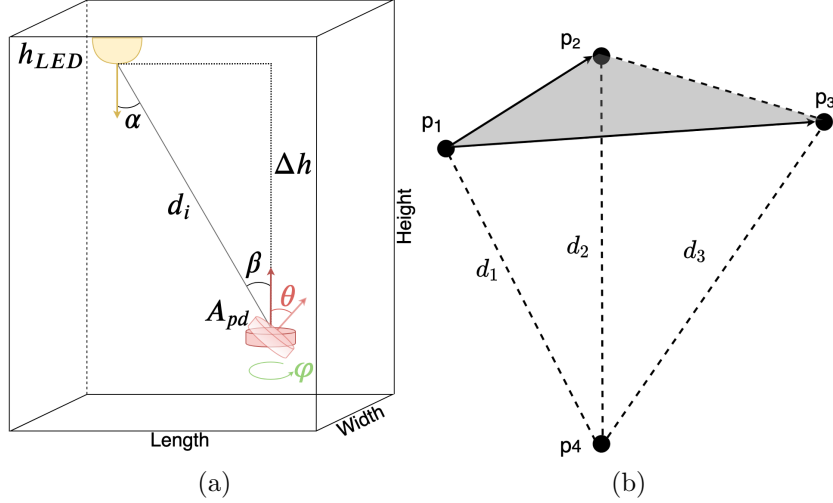


Figure 4: (a) The VLC channel parameters; (b) The parameters of the CMD trilateration algorithm.

209 4.2.2. Linear Least Squares

210 The LLS algorithm is used in this paper as a benchmark for comparison
 211 with the CMD algorithm as it is the most widely adopted trilateration
 212 positioning algorithm in VLP systems [27–29].

213 As the correct distances cannot be estimated directly without knowing
 214 the receiver's height, 2D trilateration using LLS is performed for each of the
 215 generated heights, Δh . The horizontal distance between the LED_i and the
 216 receiver is given by:

$$d_i^2(\Delta h) = (x_i - x)^2 + (y_i - y)^2 = x^2 - 2xx_i + x_i^2 + y^2 - 2yy_i + y_i^2 \quad (7)$$

217 These equations can be expressed in a matrix form as $b=Ax$, where

$$b = \frac{1}{2} \begin{bmatrix} d_1^2(\Delta h) - x_1^2 - y_1^2 - d_N^2(\Delta h) + x_N^2 + y_N^2 \\ d_2^2(\Delta h) - x_2^2 - y_2^2 - d_N^2(\Delta h) + x_N^2 + y_N^2 \\ \vdots \\ d_{N-1}^2(\Delta h) - x_{N-1}^2 - y_{N-1}^2 - d_N^2(\Delta h) + x_N^2 + y_N^2 \end{bmatrix} \quad (8)$$

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

$$A = \begin{bmatrix} x_1 - x_N & y_1 - y_N \\ x_2 - x_N & y_2 - y_N \\ \vdots & \vdots \\ x_{N-1} - x_N & y_{N-1} - y_N \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \end{bmatrix} \quad (9)$$

218 The algorithm then outputs the estimated position $\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix}$ for each of the
219 generated possible heights (Δh) using:

$$x = (A^T A)^{-1} A^T b \quad (10)$$

220 4.2.3. Cost function

221 Once all of the possible receiver locations have been generated using (6)
222 and (10) for both algorithms, the final most probable 3D position of the
223 receiver is found at the minimum of the cost function $C(h)$ as [19]:

$$C(h) = \frac{1}{N} \sum_{i=1}^N [\hat{d}_i(h) - \sqrt{(\hat{x}(h) - x_i)^2 + (\hat{y}(h) - y_i)^2 + (\hat{z}(h) - z_i)^2}]^2 \quad (11)$$

224 where $C(h)$ is the average squared error between the estimated distances \hat{d}_i
225 using (2), and the distances of the estimated 3D location of the unknown
226 receiver calculated using (6) and (10). It should be noted that the cost
227 function minimization described above can be used in conjunction with any
228 2D trilateration algorithm [24].

229 The positioning error, which is the distance difference between the final
230 calculated position and the actual position of the receiver, is calculated using:

$$D_{error} = \sqrt{(\hat{x} - x)^2 + (\hat{y} - y)^2 + (\hat{z} - z)^2} \quad (12)$$

231 where $z = h$. The CMD algorithm only requires three signals to estimate the
232 receiver's position while the LLS generally utilizes all the received signals.
233 In our experiment, the LLS algorithm in (8) is restricted to use only the
234 strongest three signals to ensure a fair comparison. Also, restricting the LLS
235 to use only the strongest signals has been shown to increase the positioning
236 accuracy and lessen the impact of multipath reflection [30, 31]. The cost
237 function on the other hand uses all four signals from the LEDs for the mini-
238 mization, as three LEDs do not suffice for an unambiguous 3D localization.

5. Results

The performance of the algorithms is experimentally evaluated for different parameters in terms of positioning error while considering different realistic factors: (i) different LED configurations, (ii) different receiver tilt angles, and (iii) introduced multipath reflection through the inclusion of a storage rack. Moreover, the results section also examines the performance of the algorithms for a 2D system. In this case, the height of the receiver is assumed to be exactly known through the use of an additional sensor.

5.1. Positioning Accuracy for Untilted Receiver

5.1.1. Square Configuration

Figure 5 (a) shows the CDF using the CMD and LLS algorithms for a 2D and 3D positioning system. The median (p_{50}) and maximal (p_{90}) 2D errors recorded using the LLS algorithm are 11.7 cm and 26.7 cm, while these are 9.9 cm and 15.8 cm using the CMD algorithm. In a 3D system, the measured median error is 17.1 cm and the maximal error is 88.4 cm for the LLS algorithm while the CMD algorithm achieves a median error of 55.9 cm and a maximal error of 177.9 cm. The positioning errors for the 2D estimation are much smaller than the 3D estimation. This is due to the height being known to the receiver, avoiding the need for the cost function and eliminating the 3D positioning ambiguity [19]. In the case of 2D positioning, the CMD outperforms the LLS algorithms slightly while the LLS algorithm outperforms the CMD algorithm in a 3D system. However, the 3D estimation for both algorithms is unreliable due to the high positioning errors under the square configuration. This is caused by the position ambiguity in a square configuration as expected and further analyzed in our previous work [19, 24]. The issue arises because some locations in the room have the same received power values, and distances once converted, as other locations, which occurs due to the radiation pattern's geometrical properties [32].

5.1.2. Star Configuration

Figure 5 (b) shows the CDF of the positioning errors using the star arrangement of LEDs for both 2D and 3D position estimation. The overall error values have decreased noticeably when compared with the square arrangement as the position ambiguity is not present in the star configuration. The performance of the LLS and CMD algorithms are very similar for the

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

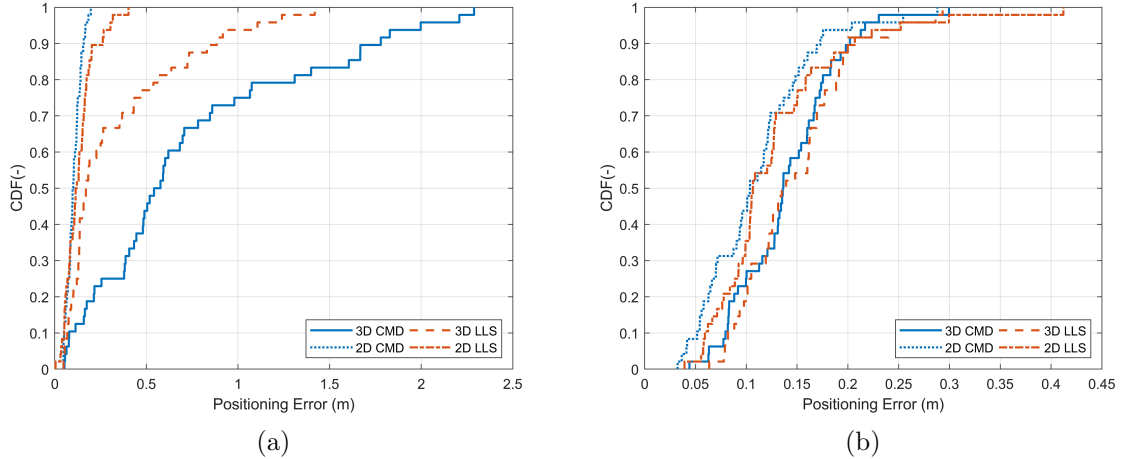


Figure 5: The CDF of the 2D and 3D positioning errors for both algorithms with a parallel receiver. (a) Under a square LED configuration; (b) Under a star LED configuration.

273 3D system with the median and maximal errors achieved using the LLS al-
 274 gorithm are 10.6 cm & 24.9 cm, and 10.5 cm & 21.1 cm using the CMD
 275 algorithm, respectively. In the case of the 2D system, median and maximal
 276 positioning errors of 8 and 25.2 cm were measured using the LLS algorithm
 277 and 6.7 cm & 14.6 cm using the CMD algorithm. Note that most of the
 278 large errors occurred at heights of more than 2 meters as can be seen in
 279 Figure 6, which depicts the estimated 3D paths and shows a deviation when
 280 the receiver is over 2 meters.

281 5.2. Positioning Accuracy for a tilted receiver

282 The errors introduced by the receiver tilt are due to the assumption in
 283 (2) that the transmitters' and receiver's plane are perfectly parallel to each
 284 other. This assumption is widely adopted due to its simplicity. However, it
 285 is unrealistic as it is almost impossible to achieve perfectly parallel planes
 286 in real-life settings, as even a 1° difference can increase the positioning error
 287 [33]. This is especially important when considering the use of a VLP system
 288 with aerial receivers, as they tilt for movement. Therefore, the effect of tilting
 289 on the performance of positioning algorithms is investigated here.

290 To accurately assess the effect of the receiver's tilt, the receiver is mounted

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

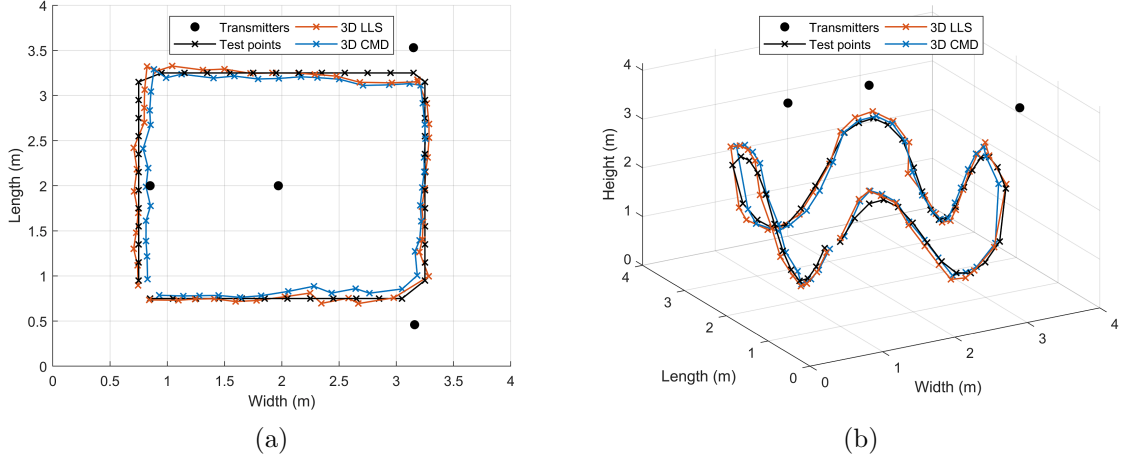


Figure 6: An illustration of the estimated paths under a star configuration when the receiver is parallel. (a) A top-view of the test points and the estimated 3D positions using the LLS and CMD algorithms; (b) a 3D view of the test points and the estimated points.

291 on a Thorlabs GNL10/M² goniometer with a range of $\pm 10^\circ$ and a precision
 292 of 1° as shown in the inset of Figure 1 (b). Two tilt angles of 5° and 10°
 293 are considered and investigated. The tilt of the receiver is set to a forward tilt
 294 angle, meaning that the receiver is always facing the direction of movement
 295 along the path outlined earlier in Section 3 and shown in Figure 2 (a). The
 296 forward tilt is introduced here because UAVs normally tilt forward to move.

297 *5.2.1. Square Configuration*

298 Figure 7 (a) shows the CDF of the positioning errors using the square-
 299 shaped LED configuration for both 2D and 3D estimation with a receiver
 300 tilt angle $\theta = 5^\circ$. The measured median and maximal errors using the LLS
 301 algorithm were 9.5 cm and 17.8 cm, and it is 8.8 cm and 15.3 cm when
 302 the CMD algorithm is used. In a 3D system, the median and maximal
 303 errors for 2D using the LLS algorithm are 17.4 and 76.9 cm, while it is 62.9
 304 and 177.5 cm when the CMD algorithm is used. The results show that LLS
 305 outperforms the CMD algorithm in a square configuration. Figure 7 (c) shows
 306 the performance of the system with a receiver tilt of 10° . For a 2D system,

²<https://www.thorlabs.com/thorproduct.cfm?partnumber=GNL10/M>

Table 3: A summary of the experimentally obtained median and maximal positioning errors for the two LED configurations for 2D and 3D localization when the receiver has a tilt of 0° , 5° , and 10° .

Positioning Error(cm)	2D LLS		2D CMD		3D LLS		3D CMD	
	P_{50}	P_{90}	P_{50}	P_{90}	P_{50}	P_{90}	P_{50}	P_{90}
Square ($\theta = 0^\circ$)	11.7	26.2	9.9	15.8	17.1	88.4	55.9	177.9
Star ($\theta = 0^\circ$)	8	25.2	6.7	14.6	10.6	24.9	10.5	21.1
Square ($\theta = 5^\circ$)	9.5	17.8	8.8	15.3	17.4	76.9	62.9	177.5
Star ($\theta = 5^\circ$)	10.7	20.7	10.4	17.3	13.7	20	13.6	20.2
Square ($\theta = 10^\circ$)	19.4	28.1	15.6	22.8	27.1	186.4	106.7	181.8
Star ($\theta = 10^\circ$)	23.2	36.3	18.8	31.3	22.7	32.2	21.6	34.2

the recorded median errors are 19.4 and 15.6 cm for the LLS and CMD algorithms, respectively. The largest errors recorded are when a 3D system was used with a receiver tilt $\theta = 10^\circ$ with a median of 27.1 cm using LLS, and 106.7 cm using CMD. These results again demonstrate the unreliability of using a square layout when implementing the algorithm. Table 3 lists a summary of the obtained accuracies across all tilt angles for the CMD and LLS algorithms under the two LED configurations.

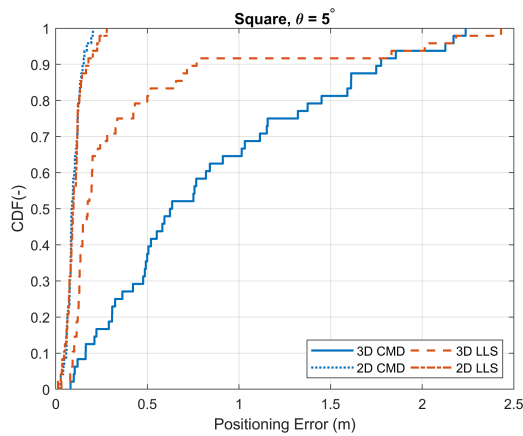
5.2.2. Star Configuration

Figure 7 (b) shows the CDF of the positioning error for the entire path when the receiver is tilted by $\theta = 5^\circ$ under a star configuration. When the LLS algorithm is used for 3D positioning, the median error is 13.7 cm and the maximal error is 20 cm. In the case of 2D positioning, the median error is 10.7 cm and the maximal error is 20.7 cm, which is slightly better than 3D positioning. When the CMD algorithm is used for 2D positioning, the median and maximal errors recorded were 10.4 and 17.3 cm, and in the case of 3D positioning, the median and maximal errors are 13.6 and 20.2 cm.

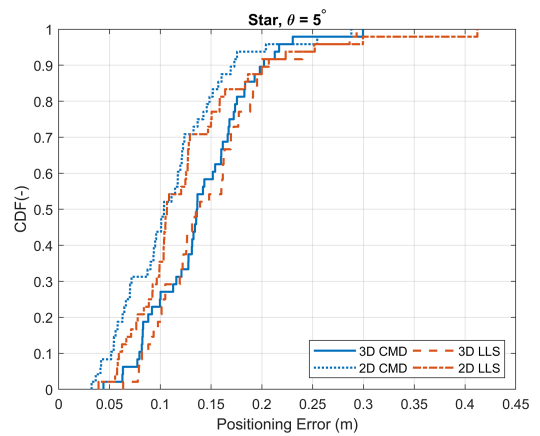
The measured positioning errors with $\theta = 10^\circ$ are shown in Figure 7 (d). Median and maximal errors for the 2D system are 23.2 cm and 36.3 cm for the LLS algorithm, while it is 18.8 cm and 31.3 cm for the CMD algorithm, respectively. In a 3D positioning system, the median and maximal errors were 22.7 cm and 32.2 cm when using the LLS algorithm, and 21.6 cm and 34.2 cm using the CMD algorithm.

It can be noticed that some of the errors are higher under a square setting with an untilted receiver than when the receiver is $\theta = 5^\circ$, see Table 3. The increase is due to some of the measured samples having large errors that have skewed the maximal errors. Note that, the tilt effect could be alleviated

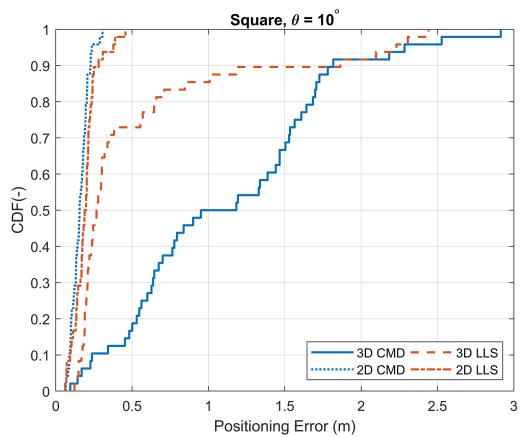
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65



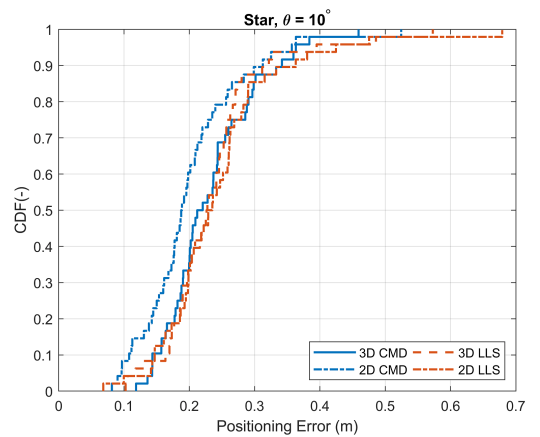
(a)



(b)



(c)



(d)

Figure 7: The CDF of the 2D and 3D positioning errors for both algorithms with receiver tilt, θ . (a) Square LED configuration with a receiver tilt of 5° ; (b) Star LED configuration with a receiver tilt of 5° ; (c) Square LED configuration with a receiver tilt of 10° ; (d) Star LED configuration with a receiver tilt of 10° .

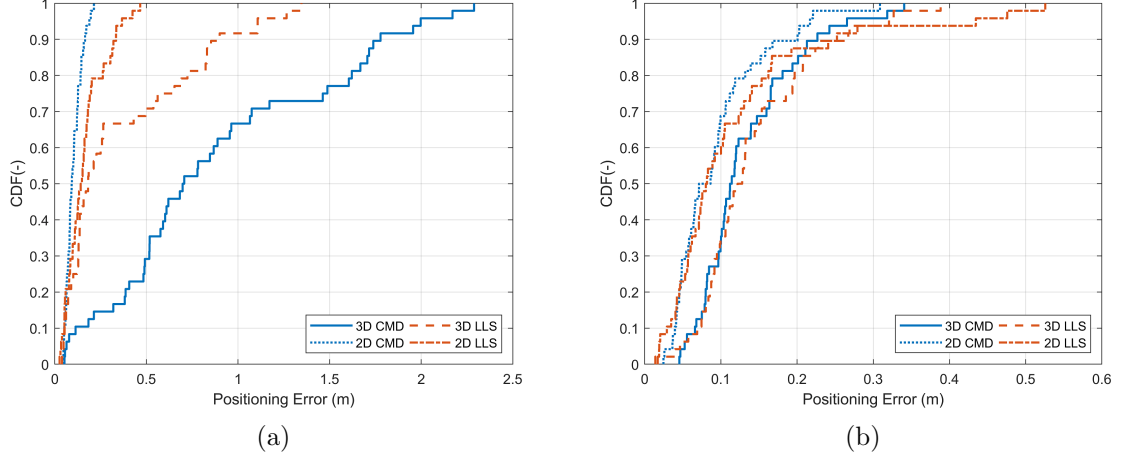


Figure 9: The CDF of the 2D and 3D positioning errors for both algorithms with a parallel receiver in the presence of a storage rack. (a) Under a square LED configuration; (b) and under a star LED configuration.

reflectivity of the boxes ranges between 33-42% depending on the color tone of the cardboard as demonstrated in the inset of Figure 8. These values were obtained using DIALux³. The same measurement procedure and scenarios outlined earlier (two LED configurations with 2D and 3D using the CMD and LLS trilateration algorithms) have been repeated, and then the positioning error was calculated using (12).

5.3.1. Untilted Receiver

Figure 9 (a) shows the CDF of the positioning errors using a square configuration with the inclusion of the storage rack. In the 2D system, the median and maximal errors using the LLS algorithm are 14.5 cm and 33.4 cm, whereas the CMD algorithm achieve a median and maximal value of 9.3 cm and 16.5 cm using the CMD algorithm.

Figure 9 (b) shows the CDF of the positioning errors using the LLS and CMD algorithms under a star LED configuration. The median and maximal 2D errors using the LLS algorithm are 8.1 cm and 25.2 cm, whereas a median error of 7.9 cm and a maximal error of 20.1 cm when the CMD algorithm is

³<https://www.dial.de/en/dialux/>

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

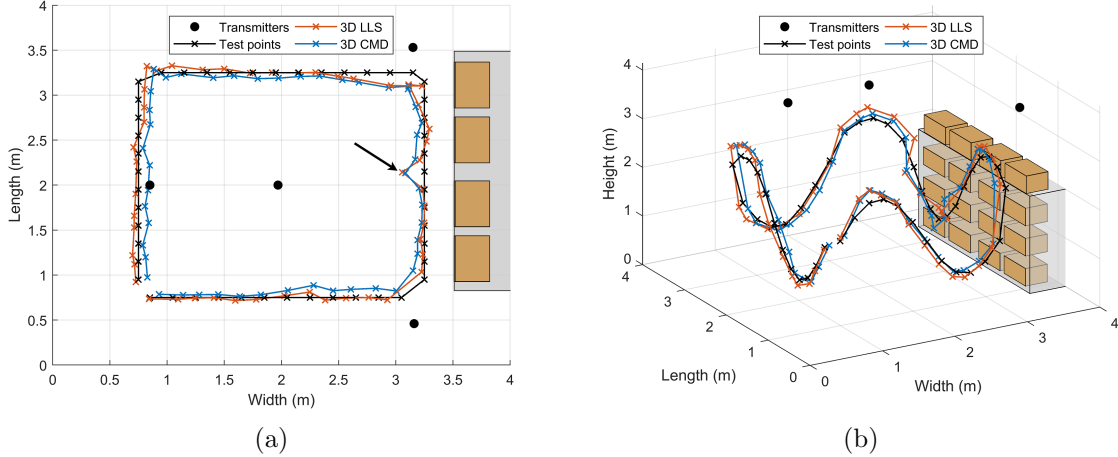


Figure 10: (a) A top-view of the test points and the estimated 3D positions using the LLS and CMD algorithms when the receiver is parallel; (b) a 3D view of the test points and the estimated 3D points.

used. The errors increase slightly in a 3D system with median and maximal errors of 12.5 cm and 26.7 cm using the LLS algorithm. In a 3D system, the CMD algorithm achieved a median and a maximal value of 11.3 cm and 22.7 cm.

Figure 10 illustrates the estimated paths using the CMD and LLS algorithms. The errors on the right side and top-right side near the storage rack are due to reflections from the boxes and the metal rods [37]. The bottom-right path is not particularly affected as some receiver heights are higher than the storage rack. Figure 10 (a) demonstrates the detrimental impact of reflections for the points that run parallel to the storage rack. One particular point directly across the metal rod is heavily affected by the multipath reflection emanating from the central LED and as highlighted in Figure 10 (a). The positioning error for that point in the 3D systems reported an error of 19.7 cm using the LLS algorithm, increasing from 6.7 cm when the point was calculated prior to adding a storage rack. Using the CMD algorithm, that specific point reported an error of 26.6 cm, whereas it was 8.6 cm prior to the addition of the storage rack. Overall, the results do not differ greatly when compared with the results in the absence of the storage rack except for the points that are nearest to storage rack.

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

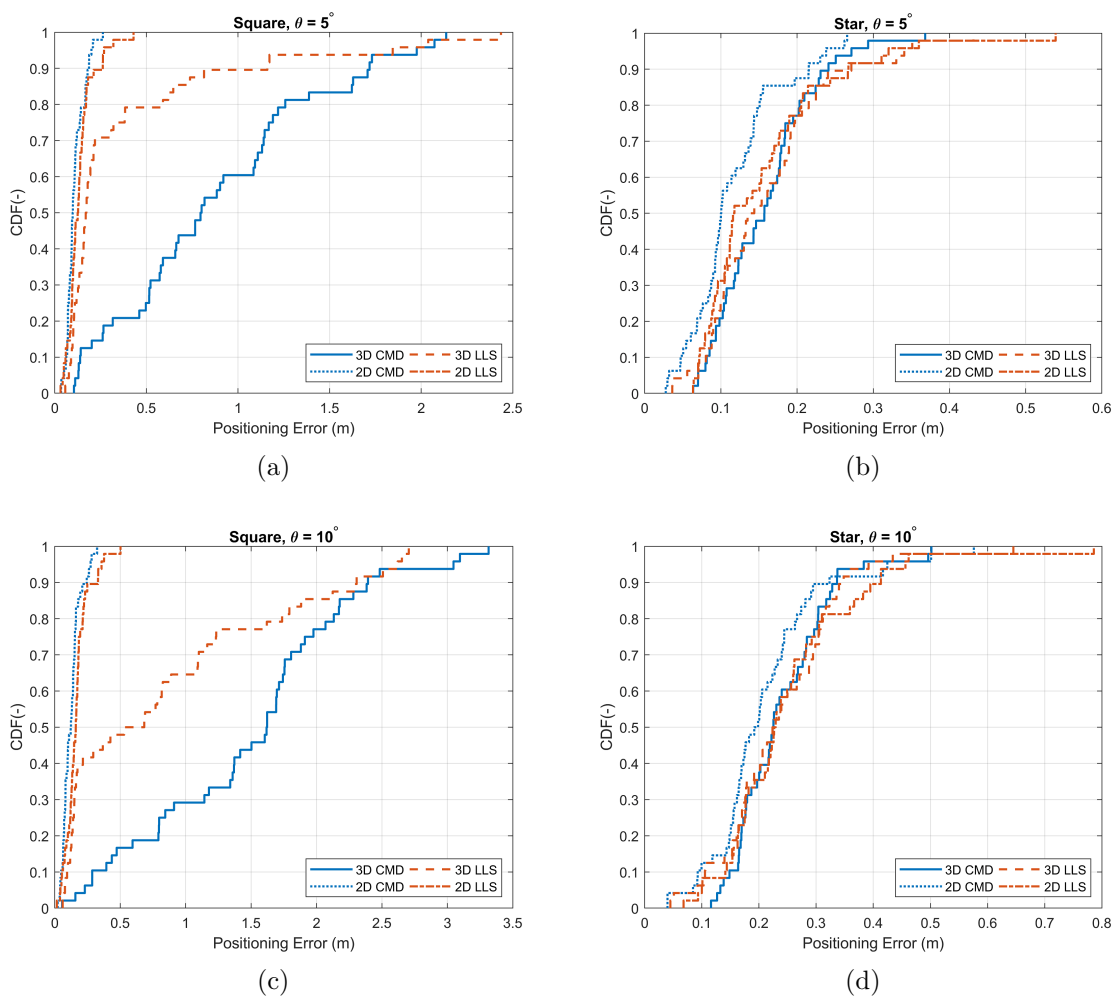


Figure 11: The CDF of the 2D and 3D positioning errors for both algorithms when the receiver is tilted and with the inclusion of a storage rack. (a) Square configuration with a receiver tilt of 5° ; (b) Star configuration with a receiver tilt of 5° ; (c) Square configuration with a tilt of 10° ; (d) Star configuration with a receiver tilted 10° .

Table 4: A summary of the experimentally obtained median and maximal positioning errors for the two LED configurations for 2D and 3D localization when the receiver has a tilt of 0° , 5° , and 10° in the presence of a storage rack.

Positioning Error (cm)	2D LLS		2D CMD		3D LLS		3D CMD	
	P_{50}	P_{90}	P_{50}	P_{90}	P_{50}	P_{90}	P_{50}	P_{90}
Square ($\theta = 0^\circ$)	14.5	33.4	9.3	16.5	18.2	90	70.2	177.8
Star ($\theta = 0^\circ$)	8.1	25.2	7.9	20.1	12.2	26.7	11.3	22.7
Square ($\theta = 5^\circ$)	12.6	26.2	9.8	18.7	17.1	116	79.8	171.8
Star ($\theta = 5^\circ$)	11.7	26.7	10	21.5	13.9	27.1	15.7	24.1
Square ($\theta = 10^\circ$)	16	33	12.3	24.4	60.7	230.7	162	239.3
Star ($\theta = 10^\circ$)	22.8	41.3	19.5	32.3	22.5	34.8	22.5	33.7

5.3.2. Tilted Receiver

Similar to Subsection 5.2, the measurements are repeated with the receiver tilted by 5° and 10° . This means that the system/receiver will suffer from both the effects of tilt and multipath reflections. Figure 11 shows the CDF of the positioning errors when the receiver is tilted 5° and 10° for both LED configurations. Under a square setting and when the receiver is tilted by 5° , the measured median and maximal 2D errors using the LLS algorithm were 12.6 and 26 cm, whereas it is 9.8 cm and 18.7 cm when the CMD algorithm is used, see Figure 11 (a). In the 3D system, the measured median and maximal values are 17.1 cm and 116 cm using the LLS algorithm. Using the CMD algorithm achieved 3D median and maximal values of 79.8 and 171.8 cm. Here, the results show that 70% of the errors in a 3D system using the LLS algorithm are below 22 cm, as shown in Figure 11 (a).

In the 2D system when the receiver is tilted by 10° , the LLS algorithm achieved median and maximal errors of 16 and 33 cm. While the CMD algorithm achieved median and maximal values of 12.3 and 24.4 cm. In the 3D system, the LLS algorithm reported a median of 60.7 cm and using the CMD algorithm reported 1.62 m as shown in Figure 11 (c). As expected, the errors increase when the tilt is increased to 10° .

Figure 11 (b) demonstrates the CDF for a receiver with a tilt of 5° under the star arrangement. Using the LLS algorithm, the achieved 2D median and maximal errors are 11.7 cm and 26.7 cm, whereas they are 10 cm and 21.5 cm when the CMD algorithm is used. For the 3D positioning system, the median error using the LLS algorithm is 13.9 cm, an increase of 13.9% when compared with an untilted receiver. Using the CMD algorithm, the median is 15.7 cm, increasing by 39% to when the receiver was untilted. When

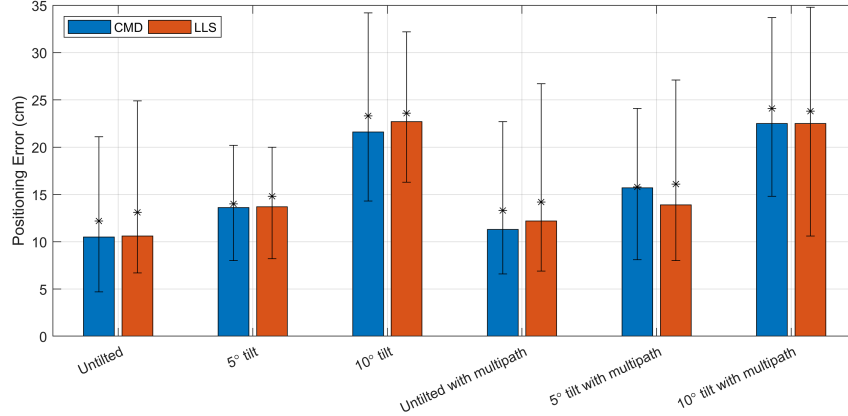


Figure 12: The bars show the achieved 3D median errors using the CMD and LLS trilateration algorithms under a star configuration, the error bars show the 10% and 90% quantiles, and the asterisks represent the mean error.

the tilt is 5° , the CMD algorithm outperforms the LLS algorithm when it comes to 2D positioning. The results, however, are nearly identical in the 3D positioning system.

When the receiver’s tilt is set to 10° under a star arrangement, the performance of the two algorithms in both 2D and 3D positioning system are similar. The median 3D error reported 22.5 cm for both algorithms, see Figure 11. Table 4 lists a summary of the obtained accuracies across all tilt angles in the presence of the storage rack. Compared to when the receiver was untilted, the errors increased by 84% using the LLS algorithm and doubled when using the CMD algorithm.

6. Discussion

We experimentally evaluated and compared two different VLP trilateration algorithms in a $4\text{ m} \times 4\text{ m} \times 4.1\text{ m}$ room under two different LED configurations for both 2D and 3D systems. The performances of the algorithms were also examined in the presence of a storage rack to examine the effects of multipath reflections. Our experiments demonstrated the impracticality of using a square-shaped configuration and showed the higher positioning accuracy of a star-shaped configuration. Previous simulation work identified an issue when the LEDs were placed in a square configuration [19].

1
2
3
4
5
6
7
8
9
431 Therefore, a star-shaped configuration was proposed. This shortcoming was
432 experimentally examined in this paper.

433 The results under a star configuration were highly more accurate com-
434 pared to the square configuration. The 3D median error achieved using LLS
435 and CMD were 10.6 cm and 10.5 cm, respectively. When a tilt of 5° was in-
436 troduced, the 3D median errors increased slightly to 13.7 cm and 13.6 cm for
437 LLS and CMD, an increase of 29.3% and 29.5%. A tilt of 10° increased the
438 3D median errors of LLS and CMD to 22.7 cm and 21.6 cm, corresponding to
439 an increase of 114.2% and 106% when compared with a horizontal receiver.
440 From these results, we can conclude that the positioning error increases by
441 around 30% if the receiver is tilted by 5° , and essentially doubles when the
442 receiver is tilted by 10° . Figure 12 shows the median errors for all of the
443 considered scenarios under a star arrangement, the error bars show the 10%
444 and 90% quantiles, and the asterisks show the mean error. A slight difference
445 in terms of positioning error between the median and mean can be seen for
446 some of the scenarios.

447 The effect of multipath reflections on the performance of VLP systems
448 was also examined. A metallic storage rack filled with boxes was added in
449 the evaluated room and tested with a horizontal receiver with a receiver
450 tilt of 5° and 10° . The results for a 3D system under a star configuration
451 reported a median error of 12.2 cm using LLS, an increase of 15% when com-
452 pared with an empty room. Using the CMD algorithm, the median error was
453 11.3 cm, which represents an increase of 7.6% compared to its performance
454 in an empty room. The storage rack was 26 cm away from the closest points
455 and the impact of reflections on one particular point (pointed out in Figure
456 10) increased the positioning error in a 3D system using the LLS algorithm
457 by 13 cm, and by 18 cm using the CMD algorithm [37]. This points out
458 the severity of multipath reflections from metallic structures. As mentioned
459 before, both algorithms in this paper select the three strongest signals to
460 increase the positioning accuracy and lessen the impact of multipath reflec-
461 tions as noted in [30, 31]. However, while the impact of reflections may have
462 been reduced, it is still not sufficient enough in limiting the degrading effect
463 of reflections.

464 The differences in the performances of the algorithms are because they
465 differ mathematically in how they calculate the receiver's position. The CMD
466 method is an analytic procedure, that calculates a point through geometric
467 interrelations [38]. Whereas the least square method is a numeric procedure
468 that calculates the point at which the distance from three circles intersects.

1
2
3
4
5
6
7
8
9
469 The observation that the CMD trilateration algorithm outperforms the least
10 square quadratic method has also been noted by researchers in [39] when
11 they compared different trilateration algorithms.
12

13 It should be noted that some of the errors observed in the experiments
14 could also be caused by other factors. The experimentally adjusted tilt an-
15 gle can be slightly different from the intended values, the LED having small
16 unknown tilt angles [33], the LED radiation pattern not being perfectly Lam-
17 bertian, and imperfections in the demultiplexing process.
18
19

20 477 **7. Conclusion**

21
22 In this paper, two VLP algorithms were experimentally analyzed and
23 compared. The LLS and CMD algorithms were tested in a $4\text{ m} \times 4\text{ m} \times 4.1\text{ m}$
24 room with four LEDs. Two different LED configurations were compared
25 using two different trilateration algorithms and their 2D and 3D performances
26 were evaluated. The LLS and CMD algorithms achieved an accuracy of
27 10.6 and 10.5 cm in a 3D system, respectively. The performances of the
28 algorithms were also examined in the presence of a storage rack to examine
29 the effects of multipath reflections. We also experimentally demonstrated the
30 impracticality of using a square-shaped configuration and showed the higher
31 positioning accuracy of a star-shaped configuration. The proposed algorithm
32 is suitable for real-time implementation based on our previous work reporting
33 a computation time of 17 ms, which can be further reduced to less than 2 ms
34 using a fast search algorithm [19].
35

36 The presented work highlights the need to take into account the light
37 arrangements to optimize the performance of a 3D VLP system as well as
38 the effect of receiver tilt and multipath reflections on the performance of
39 VLP systems. It also extended the use of a trilateration algorithm that is
40 not widely used into VLP systems. Future work could examine integrating
41 an IMU sensor to compensate for the undesirable effects of tilt. Additional
42 plans could also investigate the performance of the algorithm under a circular
43 LED arrangement as the work in [40] reported that a circular arrangement
44 offers a slightly higher illuminance uniformity than the optimum condition
45 using rectangular LED arrangement.
46
47
48
49
50
51

52 501 **Acknowledgments**

53
54 The authors would like to thank Willem Raes and Nobby Stevens of
55 DRAMCO, ESAT, KU LEUVEN, Belgium for providing the LED modules
56
57
58

1
2
3
4
5
6
7
8
9 used in this work.

10 11 12 **Funding Information**

13
14 This research received no external funding.

15 16 17 **References**

- 18
19 [1] R. Xu, W. Chen, Y. Xu, S. Ji, A new indoor positioning sys-
20 tem architecture using GPS signals, *Sensors* 15 (2015) 10074–10087.
21 doi:10.3390/s150510074.
22
- 23
24 [2] F. Zafari, A. Gkelias, K. K. Leung, A survey of indoor localization
25 systems and technologies, *IEEE Communications Surveys Tutorials* 21
26 (2019) 2568–2599. doi:10.1109/COMST.2019.2911558.
27
- 28
29 [3] H. Burchardt, N. Serafimovski, D. Tsonev, S. Videv, H. Haas, Vlc: Be-
30 yond point-to-point communication, *IEEE Communications Magazine*
31 52 (2014) 98–105.
32
- 33
34 [4] X. Guo, S. Shao, N. Ansari, A. Khreishah, Indoor localization using
35 visible light via fusion of multiple classifiers, *IEEE Photonics Journal* 9
36 (2017) 1–16. doi:10.1109/JPHOT.2017.2767576.
37
- 38
39 [5] Y. Cai, W. Guan, Y. Wu, C. Xie, Y. Chen, L. Fang, Indoor high pre-
40 cision three-dimensional positioning system based on visible light com-
41 munication using particle swarm optimization, *IEEE Photonics Journal*
42 9 (2017) 1–20. doi:10.1109/JPHOT.2017.2771828.
43
- 44
45 [6] C. Hsu, S. Liu, F. Lu, C. Chow, C. Yeh, G. Chang, Accurate indoor
46 visible light positioning system utilizing machine learning technique with
47 height tolerance, in: *2018 Optical Fiber Communications Conference*
48 *and Exposition (OFC)*, 2018, pp. 1–3.
49
- 50
51 [7] Z. Xie, W. Guan, J. Zheng, X. Zhang, S. Chen, B. Chen, A high-
52 precision, real-time, and robust indoor visible light positioning method
53 based on mean shift algorithm and unscented Kalman filter, *Sensors* 19
54 (2019). doi:10.3390/s19051094.
55
56
57
58
59
60
61
62
63
64
65

- 1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65
- 532 [8] H. Zhang, J. Cui, L. Feng, A. Yang, H. Lv, B. Lin, H. Huang, High-
533 precision indoor visible light positioning using deep neural network based
534 on the Bayesian regularization with sparse training point, *IEEE Pho-*
535 *tonics Journal* 11 (2019) 1–10. doi:10.1109/JPHOT.2019.2912156.
- 536 [9] P. Du, S. Zhang, C. Chen, A. Alphones, W. Zhong, Demonstration
537 of a low-complexity indoor visible light positioning system using an
538 enhanced TDOA scheme, *IEEE Photonics Journal* 10 (2018) 1–10.
539 doi:10.1109/JPHOT.2018.2841831.
- 540 [10] S. Zhang, W. Zhong, P. Du, C. Chen, Experimental demonstra-
541 tion of indoor sub-decimeter accuracy VLP system using differential
542 PDOA, *IEEE Photonics Technology Letters* 30 (2018) 1703–1706.
543 doi:10.1109/LPT.2018.2866402.
- 544 [11] Z. Li, L. Feng, A. Yang, Fusion based on visible light positioning and
545 inertial navigation using extended Kalman filters, *Sensors* 17 (2017).
546 doi:10.3390/s17051093.
- 547 [12] L. Li, P. Hu, C. Peng, G. Shen, F. Zhao, Epsilon: A visible light
548 based positioning system, in: *11th USENIX Symposium on Networked*
549 *Systems Design and Implementation (NSDI 14)*, USENIX Association,
550 Seattle, WA, 2014, pp. 331–343.
- 551 [13] M. Yasir, S. Ho, B. N. Vellambi, Indoor positioning system using visible
552 light and accelerometer, *Journal of Lightwave Technology* 32 (2014)
553 3306–3316. doi:10.1109/JLT.2014.2344772.
- 554 [14] S. Yang, H. Kim, Y. Son, S. Han, Three-dimensional visible
555 light indoor localization using aoa and rss with multiple optical
556 receivers, *Journal of Lightwave Technology* 32 (2014) 2480–2485.
557 doi:10.1109/JLT.2014.2327623.
- 558 [15] W. Guan, S. Chen, S. Wen, Z. Tan, H. Song, W. Hou, High-accuracy
559 robot indoor localization scheme based on robot operating system using
560 visible light positioning, *IEEE Photonics Journal* 12 (2020) 1–16.
- 561 [16] W. Guan, X. Zhang, Y. Wu, Z. Xie, J. Li, J. Zheng, High precision
562 indoor visible light positioning algorithm based on double leds using
563 cmos image sensor, *Applied Sciences* 9 (2019). doi:10.3390/app9061238.

- 1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65
- 564 [17] Y. Almadani, M. Ijaz, W. Joseph, S. Bastiaens, S. Rajbhandari, B. Ade-
565 bisi, D. Plets, A novel 3d visible light positioning method using re-
566 ceived signal strength for industrial applications, *Electronics* 8 (2019).
567 doi:10.3390/electronics8111311.
- 568 [18] S. De Lausnay, L. De Strycker, J. Goemaere, N. Stevens, B. Nauwelaers,
569 A visible light positioning system using frequency division multiple ac-
570 cess with square waves, in: 2015 9th International Conference on Sig-
571 nal Processing and Communication Systems (ICSPCS), 2015, pp. 1–7.
572 doi:10.1109/ICSPCS.2015.7391787.
- 573 [19] D. Plets, Y. Almadani, S. Bastiaens, M. Ijaz, L. Martens, W. Joseph,
574 Efficient 3d trilateration algorithm for visible light positioning, *Journal*
575 *of Optics* 21 (2019) 05LT01. doi:10.1088/2040-8986/ab1389.
- 576 [20] J. M. Kahn, J. R. Barry, *Wireless infrared communications*, *Proceedings*
577 *of the IEEE* 85 (1997) 265–298. doi:10.1109/5.554222.
- 578 [21] Y. Almadani, M. Ijaz, S. Rajbhandari, U. Raza, B. Adebisi, Applica-
579 tions of visible light communication for distance estimation: a short sur-
580 vey, in: 2019 IEEE Jordan International Joint Conference on Electrical
581 Engineering and Information Technology (JEEIT), 2019, pp. 261–265.
582 doi:10.1109/JEEIT.2019.8717459.
- 583 [22] D. Plets, S. Bastiaens, L. Martens, W. Joseph, N. Stevens, On the
584 impact of led power uncertainty on the accuracy of 2d and 3d visible light
585 positioning, *Optik* 195 (2019) 163027. doi:10.1016/j.ijleo.2019.163027.
- 586 [23] D. Plets, S. Bastiaens, N. Stevens, L. Martens, W. Joseph, Monte-carlo
587 simulation of the impact of led power uncertainty on visible light posi-
588 tioning accuracy, in: 2018 11th International Symposium on Commu-
589 nication Systems, Networks Digital Signal Processing (CSNDSP), 2018,
590 pp. 1–6. doi:10.1109/CSNDSP.2018.8471838.
- 591 [24] D. Plets, S. Bastiaens, M. Ijaz, Y. Almadani, L. Martens, W. Raes,
592 N. Stevens, W. Joseph, Three-dimensional visible light positioning: an
593 experimental assessment of the importance of the leds’ locations, in:
594 2019 International Conference on Indoor Positioning and Indoor Navi-
595 gation (IPIN), 2019, pp. 1–6. doi:10.1109/IPIN.2019.8911763.

- 1
2
3
4
5
6
7
8
9
596 [25] Z. Jia, C. Wu, Z. Li, Y. Zhang, B. Guan, The indoor local-
597 ization and tracking estimation method of mobile targets in three-
598 dimensional wireless sensor networks, *Sensors* 15 (2015) 29661–29684.
599 doi:10.3390/s151129661.
- 600 [26] F. Thomas, L. Ros, Revisiting trilateration for robot local-
601 ization, *IEEE Transactions on Robotics* 21 (2005) 93–101.
602 doi:10.1109/TRO.2004.833793.
- 603 [27] T.-H. Do, M. Yoo, An in-depth survey of visible light communication
604 based positioning systems, *Sensors* 16 (2016). doi:10.3390/s16050678.
- 605 [28] Y. Zhuang, L. Hua, L. Qi, J. Yang, P. Cao, Y. Cao, Y. Wu, J. Thomp-
606 son, H. Haas, A survey of positioning systems using visible led
607 lights, *IEEE Communications Surveys Tutorials* 20 (2018) 1963–1988.
608 doi:10.1109/COMST.2018.2806558.
- 609 [29] J. Luo, L. Fan, H. Li, Indoor positioning systems based on visible
610 light communication: State of the art, *IEEE Communications Surveys*
611 *Tutorials* 19 (2017) 2871–2893. doi:10.1109/COMST.2017.2743228.
- 612 [30] W. Gu, M. Aminikashani, P. Deng, M. Kavehrad, Impact of multi-
613 path reflections on the performance of indoor visible light position-
614 ing systems, *Journal of Lightwave Technology* 34 (2016) 2578–2587.
615 doi:10.1109/JLT.2016.2541659.
- 616 [31] W. Tang, J. Zhang, B. Chen, Y. Liu, Y. Zuo, S. Liu, Y. Dai, Analysis of
617 indoor VLC positioning system with multiple reflections, in: *2017 16th*
618 *International Conference on Optical Communications and Networks (IC-*
619 *OCN)*, 2017, pp. 1–3. doi:10.1109/ICOCN.2017.8121297.
- 620 [32] J. O. Roa, A. R. Jiménez, F. Seco, J. C. Prieto, J. Ealo, Optimal
621 placement of sensors for trilateration: Regular lattices vs meta-heuristic
622 solutions, in: R. Moreno Díaz, F. Pichler, A. Quesada Arencibia (Eds.),
623 *Computer Aided Systems Theory – EUROCAST 2007*, Springer Berlin
624 Heidelberg, Berlin, Heidelberg, 2007, pp. 780–787.
- 625 [33] D. Plets, S. Bastiaens, L. Martens, W. Joseph, An analysis of the impact
626 of led tilt on visible light positioning accuracy, *Electronics* 8 (2019).
627 doi:10.3390/electronics8040389.

- 1
2
3
4
5
6
7
8
9
628 [34] E. Jeong, S. Yang, H. Kim, S. Han, Tilted receiver angle error compen-
629 sated indoor positioning system based on visible light communication,
630 Electronics Letters 49 (2013) 890–892. doi:10.1049/el.2013.1368.
- 631 [35] J. Kim, S. Yang, Y. Son, S. Han, High-resolution indoor positioning
632 using light emitting diode visible light and camera image sensor, IET
633 Optoelectronics 10 (2016) 184–192. doi:10.1049/iet-opt.2015.0073.
- 634 [36] B. Zhou, V. Lau, Q. Chen, Y. Cao, Simultaneous positioning and orient-
635 ating for visible light communications: Algorithm design and perfor-
636 mance analysis, IEEE Transactions on Vehicular Technology 67 (2018)
637 11790–11804. doi:10.1109/TVT.2018.2875044.
- 638 [37] Y. Almadani, M. Ijaz, S. Bastiaens, S. Rajbhandari, W. Joseph, D. Plets,
639 An experimental analysis of the effect of reflections on the performance
640 of visible light positioning systems in warehouses, in: 2019 IEEE 2nd
641 British and Irish Conference on Optics and Photonics (BICOP), 2019,
642 pp. 1–4.
- 643 [38] B. Neuwinger, U. Witkowski, U. Rückert, Ad-hoc communication
644 and localization system for mobile robots, in: Advances in Robotics,
645 Springer Berlin Heidelberg, Berlin, Heidelberg, 2009, pp. 220–229.
- 646 [39] K.-W. Lee, J.-B. Park, B.-H. Lee, Dynamic localization with hybrid
647 trilateration for mobile robots in intelligent space, Intelligent Service
648 Robotics 1 (2008) 221–235. doi:10.1007/s11370-007-0012-1.
- 649 [40] C. W. Chow, Y. Liu, C. H. Yeh, J. Y. Sung, Y. L. Liu, A practical
650 in-home illumination consideration to reduce data rate fluctuation in
651 visible light communication, IEEE Wireless Communications 22 (2015)
652 17–23. doi:10.1109/MWC.2015.7096280.