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Fractal Modeling of Historical Demographic Processes

Dmitry Zhukov, Valery Kanishchev & Sergey Lyamin *

Abstract: *»Fraktale Modellierung historisch-demografischer Prozess«*. The article presents several results of the computer modeling of demographic processes in the late traditional rural communities by means of fractal geometry. A team of contributors developed a model and software for it, then carried out its verification, data processing, computer modeling and interpretation of results. The analysis of modeling outcomes allowed to build a holistic picture of the demographic behavior in rural communities of the Tambov province – one of the typical agrarian regions of 19th and 20th century Russia. Authors describe the degree and ways of how demographic behavior of the society was influenced by such factors as famine, war, epidemics, a level of health care infrastructure development, etc. Besides, it was possible to trace some non-linear effects in demographic strategies agrarian communities followed during modernization processes in Russia in the second half of the 19th and 20th centuries.

Keywords: fractal modeling, computer modeling, fractal geometry, modernization, traditional society.

1. Introduction

Our task is to present several results of the computer modeling of demographic processes in the late traditional rural communities by means of fractal geometry.

The study of demographic behavior of the 19th and 20th century Russian rural communities is attended with difficulties of searching primary data on birth rate and mortality. From one side, the first obstacle is the existing lacunas in historical sources. From the other one, the processing of available array of information leads to enormous expenses of resources. In this situation the application of demographic behavior modeling methods is an efficient instrument to extrapolate estimation results for one group of communities to the others that are similar in their essential characteristics but not provided with correct data concerning mortality and birth rate.

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In this article authors present one of the fractal models created in the Center for fractal modeling of social and political processes (<http://www.ineternum.ru>).

A team of contributors developed a model and software for it, then carried out its verification, data processing, computer modeling and interpretation of results. The analysis of modeling outcomes allowed to build a holistic picture of the demographic behavior in rural communities in the Tambov province – one of the typical agrarian regions of 19th and 20th century Russia.

2. Fractal Model of Demographic Behavior

The pivot of our model of demographic behavior is human behavior. We simulate two intersubjective intentions that are inherent to social organism and determine the demographic strategy in its dialectical unity and inconsistency – a demand for children and a demand for personal survival. The former intention should be understood as a seeking for collective (species) immortality and the latter one – for personal immortality. The general indicator of the first intention is birth rate and that of second intention is survival rate taken as a rate inversely proportional to mortality.

Fractal geometry opens possibilities to create heuristically productive models. These models offer interesting properties: they allow discovering and simulating not only linear effects but also non-linear ones that are results of interaction of a series of driving factors.

The non-linear behavior of the object can be particularly described, according to our hypothesis, by means of general fractal model of transit (GFMT). The GFMT mathematical apparatus has already been described by authors in relation to other fractal models demonstrating dynamics of systems on transit from one qualitative state to another (Zhukov and Lyamin 2010). For the purposes of the present study the "Demofractal" model and software for its implementation were developed on GFMT basis (programmer – Yulia Movchko).

The program carries out procedures of building algebraic fractal: it generates images of system attractors and each attractor's basins. Together these instruments allow to understand the system's dynamics, and moreover – to forecast this dynamics. GFMT demonstrates main objects and events of the chaos theory and synergetics.

In GMFT various states of the systems are displayed by means of representation point in 2-D phase space. According to the model's conditions different fields of the phase space may qualitatively have a certain sense. The phase space allows to reflect a state of the system at any specific time with one point (which coordinates correspond with performance of key characteristics of the system plotted along the phase space axes. And a change of system conditions in time is displayed with a sequence of points, i.e. a trajectory in the phase space.

In GMFT a system is considered in a context of transition from one ideal state (“zero pole”) to another ideal state (“pole of endlessness”). In reality, in most cases this transition starts, continues and ends somewhere between these poles – in an area that is called TOMH for research purposes (see, for example, Picture 1).

In a plane a point is conceivably a qualitative condition of the system risen from a combination of different performance of two key values of the system characteristics (binary characteristics). More specifically, H_x is a value of one characteristic (birth rate) plotted along x-axis; and H_y is a value of another one (survival rate) plotted along y-axis. The lowest values for each characteristic are $x=0$, $y=0$. The maximum values for each characteristic (according to the principles of marking model’s phase space) are $x=|2|$, $y=|2|$.

The model’s mathematical apparatus contains an iterated formula

$$Z_{n+1} = Z_n^2 A + C \tag{1}$$

(where Z and C – complex numbers: $Z\,(d_{zn}\,;\,k_{zn})$, $C\,(d_c\,;\,k_c)$), as well as a set of mathematical conditions that allow to identify a geometrical sense of operations on complex numbers with results of nuclear interactions of the model’s factors. The iterated formula generates number sequence that sets a trajectory in the 2-D phase space or, in other words, describes a system’s evolution on its two key parameters. The point’s trajectory is directly impacted by factors, described by formula (1).

Let us assume that:

Z^2 – inner “inertia” of the system, demographic behavioral inertia (demographic sociocultural influence of society on itself, a natural tendency to reproduction and sameness);

A – natural factor; not only “externally” natural (ecological), but “internally” natural (physiological) ones. A determines both univocal limitation of survival and demand for children as well as stimulation of these two basic intentions. Indicators for calculating A may be various, and the main ones among them are a degree of occupancy of ecological and technological niche (availability of resources), an index for ecological (environmental) favor (non-favor), and some others;

C – external impulses in regard of the system – may be considered as a dual unity;

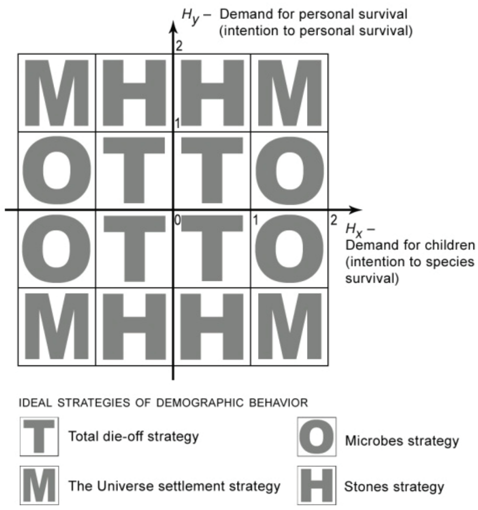
D_c – factors of control over demand for children (can be both approving and limiting); the main indicator here is an index of means for population control;

K_c – factors of control over a demand for personal survival (they can also be both approving and limiting); the main indicator is an index of means for mortality control, or a level of development of living facilities.

Working with the fractal-building program, a researcher gives an opportunity to watch points’ trajectories, tracing starting points (initial states of the system) and end stable points (if there are any) – attractors of the system evolution. Thus, the fractal-building computer program can generate images of system attractors (conditionally these images are called "space of perspectives") and basins ("space of potentials") Basins (understood to be a set of initial states from which the system then runs into a particular attractor) point on the potential scenarios of evolution. Attractors provide guidance on most probable and comfortable outcomes of the system development under the impact of model's factors. Of course, in real and particularly changeable historical circumstances these perspectives had not always taken place. But Demofractal is useful exactly with the possibility to catch changes in intentions, goals and strategies of a community, that are not always evident from the study of available statistical data.

Different zones of the Demofractal complex plane denote different demographic strategies. We should notice specifics of graphic outputs of Demofractal work. The imagery is symmetrical about x- and y-axes. That is why, strictly speaking, the qualitative sense of imagery is placed in one quarter (any one quarter) of a construction limited by semi-axes.

Figure 1: Marking of the Demofractal Phase Space



Ideal strategies are considered as a correlation of two intentions. Moreover, an "ideal" quantitative meaning of each intension's value is taken concerning its mean level. This mean level is a purely mathematical value and may not coincide with real historical transitional level from intentions' traditional values to modernized ones.

T – "Total die-off strategy": a combination of low survival with low demand for children.

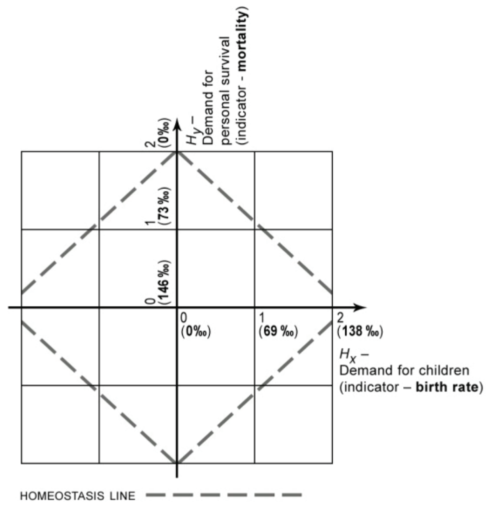
O – “Microbes strategy”: a combination of high demand for children with low survival.

M – “the Universe settlement strategy”: a combination of high demand for children with high survival.

H – “Stones strategy”: a combination of low demand for children with high survival.

In order to interpret modeling outcomes we should define maxima of general indicators of both intensions, i.e. birth rate and mortality rate. Let us assume that hypothetical maximum and minimum values of birth rate and mortality rate are those indicated on the Figure 2.

Figure 2: The Homeostasis Line (with Respect to Layout Lines of the Demofractal Space)



In order to increase heuristic productiveness of resultant images analysis we constructed an additional instrumental figure in the Demofractal space – the homeostasis line, on which intensions’ values are balanced (see Figure 2). (It means that on the homeostasis line an intention to births and a value reciprocal of an intention to survival, i.e. mortality, are equal). This instrument (the homeostasis line) allows to introduce a respective notion into the model and to carry out analysis of communities’ demographic behavior, taking into account their position with respect to this line.

It is exactly the homeostasis, i.e. simple reproduction, balancing births and deaths, that different communities tend to (or, anyway, “must tend to” in theory). The homeostasis line crosses various types of demographic strategy. It is not surprising, because homeostasis may be both at high (many children – many deaths) and low levels.

The model was hitherto subject to verification in order to figure out feasibility and interpretability of results in a certain range of factors' values (impact of environment, control over intention to survival, control over demand for children)

Model's scale calibration was a detection of factors' entry values for the known attractors.

Thus, a preliminary verification and calibration of the Demofractal model make it possible to apply fractal modeling to the studies of a big number of settlements for which combinations of entry factors are known. Such a research would allow, firstly, to distinguish types of settlements (to figure out long- and short-term combinations of intentions for each community and as well as for their groups). Secondly, introduction of factors' calculated values into the program would help us judge birth and mortality rates. This indirect way to data acquisition is meaningful if a direct study of historical realities is either impossible (because of lack of sources) or implies resource-cost procedures of processing a huge array of sources by hand.

We remind that the program shows virtual, model strategies that expressed orientations and aspirations of a community and carried out into practice only under favorable (for this strategy) conditions.

3. Results of Modeling and Data Interpretation

Experiments were conducted for three temporal periods: 1862-1917, 1917-1920 and 1920-1926.

The data on 253 randomly chosen rural communities of the Tambov province for a period of 1862-1917 was taken as a first object for modeling demographic behavior. Testing every settlement separately we observe that almost in all cases attractors (possible prospects of a community) are stable and concentrated in one point, and potential fields are completely filled in with one color. It all indicates that a community faces a quite realistic but the only one, non-variable prospect for survival.

Figure 3: Example from the 1862-1917 Sample: Stable, Gathered in one Point Attractors

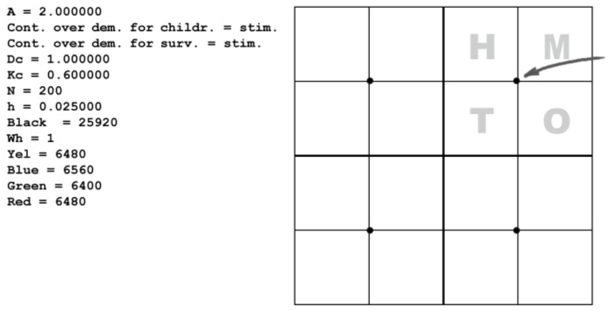
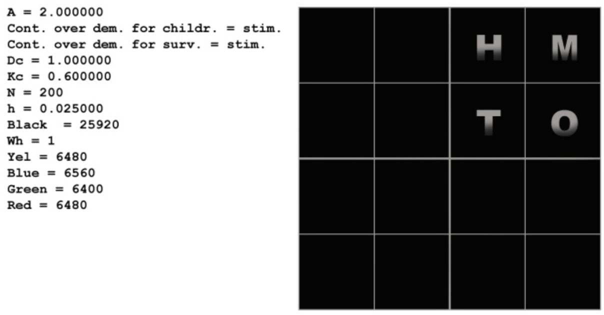
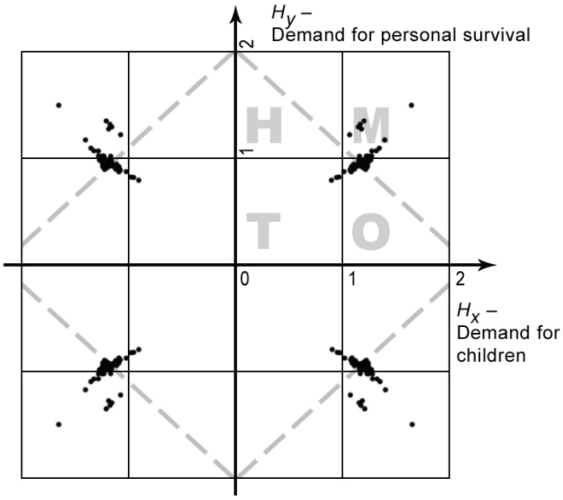


Figure 4: Example from the 1862-1917 Sample: Potential Fields Filled in with One Color



The model study showed that rural communities in the present sample had held (more correctly – tried to hold) to traditional type of reproduction (relatively high mortality rate along with relatively high birth rate). Attractors' dispersion of demographic intentions for different settlements in the model's phase space is quite small. Nevertheless we can mark four relative groups (clouds) of attractors. The rural societies were in transit. The differentiation of sub-types of demographic behavior began to take shape within the "grand" demographic strategy (see Table 1).

Figure 5: A Consolidated Space of Communities' Attractors of the 1862-1917 Sample (Every Point is a Set of All Attractors of the Single Settlement)



There is the following point that stands out particularly. Experiments demonstrate that in many cases we deal with communities on the brink (or, if you will, on the threshold) of the demographic catastrophe. The communities under study are close to the absolute dissipation of attractors and, consequently, basins. The attractors disappear and so do prospects for survival.

A term "dissipation" – diffusion – is used here not in a strict sense, but as a metaphor, and implies that when some factor values increase or decrease, the image in the potential field and the attractor field disappears. There emerges an effect of dissipation of attractor clouds and basins in the respective Demofractal areas that is accompanied with the growth of chaos in the basins' outlines as well as the attractors' appearance in the physically almost unreal areas. Such dissipation immediately antecedes a complete disappearing of attractors from the area of physically possible values (in the case of negative dynamics of model's basic entry parameters to be kept). That means that attractors are going to infinity or, rarely, are concentrating in zero. In a quantitative sense it should be interpreted as social catastrophe, i.e. transition from physically possible attractors of the community to the physically impossible ones (in the long run).

The absolute dissipation of attractors and, consequently, basins occurs when value of *A* slightly increases. (Figure 6 represents a consequent phases of transient dissipation with natural factor getting better for one of the settlements under study; the dissipation is almost the same for other settlements with insignificant variations).

If natural factor becomes a less aggressive (even not positive) towards intensions, the types of communities being studied simply cease to exist in any long run at factors' values given (attractors go to infinity). In other words, the improvement of natural environment as over birth still takes place leads society to catastrophe very rapidly – evidently, as a result of ecological and technological niche overloading.

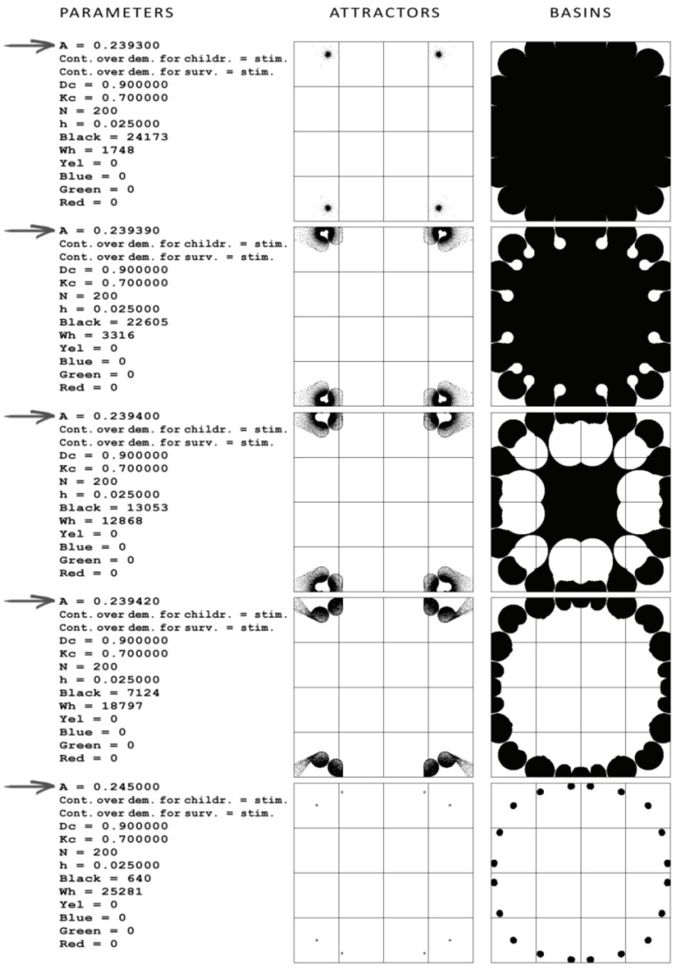
Depopulation crises in general, as S. A. Nefedov and P. V. Turchin demonstrate, are distinctive for the medieval traditional society (Nefedov and Turchin 2010). "The fall of consumption, S. A. Nefedov states, leads to the deceleration of the population growth, and population stabilizes near asymptote of K, corresponding its maximum possible size during minimum consumption. This state of 'famine homeostasis' actually turns out to be unstable; with lack of food reserves a large-scale crop failure sooner or later causes 'demographic catastrophe', a severe famine accompanied with epidemics. A catastrophe means a drastic fall of the population size; then the period of growth begins within a new demographic cycle" (Nefedov 2002).

What draws attention is the rapidity of the catastrophe scenario we demonstrated in conducted experiments.

The first exit from the "pitfall" is a jumping switching to the regime of modernized development. This scenario implies a low value of limiting control over demand for children (D_c *(limiting) = 0,4*), a continuation of the previously low-level stimulating control over survival (K_c *(positive) = 0,6*) and relatively low reduction of the natural factor aggression ($A = 0,5$). In this case a community gets attractors in a relatively modernized area. Noteworthy that this scenario contemplates a change of direction as well as a significant change of one of the factors' value.

Another variation is a return to the supra-traditional scenario. This exit implied a maintenance of the stimulating control over demand for children on the previous level (D_c *(positive) = 1,1*); a small improvement of natural factor ($A=0,25$) as well as a significant reduction of the quite low level of stimulating control over survival – up to zero level (K_c *(positive) = 0,1*). In this case a community does not practically provide any support to personal survival, in fact getting rid of 'unnecessary' people. This leads to the interesting non-linear results: attractors move to the area of over birth and middle-level mortality. A community goes into a regime close to the "Universe settlement" strategy. However, attractors' cloud in this scenario is not a point; that is why it should be a subject of a separate study.

Figure 6: Example of Basins' and Attractors' Dissipation

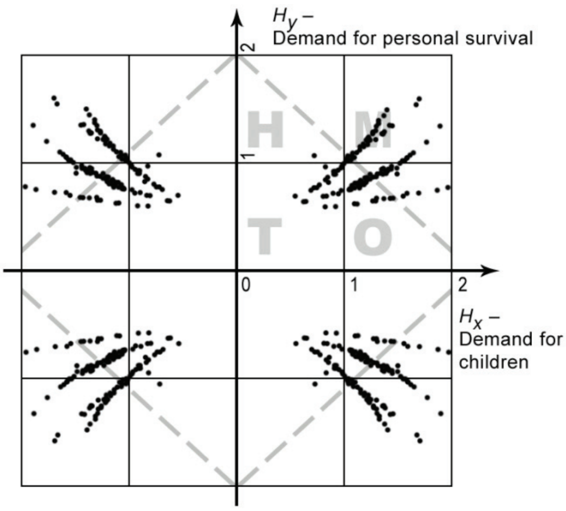


Summarizing the description of two scenarios of exit from the "pitfall" we can notice that a community directly faced the immediate necessity to take a crucial step forward (in the modernized state) or equally fundamental step back. Of course, by taking historical context into account, we can assert that there was no any real choice ("forward" or "back") for that type of communities, because modernization that pushed them "ahead" was conditioned by factors far beyond the scope of this model. The only choice that might take place is that of size of social losses as a result of late transition to the modernization scenario. The effect of this delay should have been a community's existence for some time in the situation that we figuratively call dissipation, i.e. the situation when a community did not have real prospects for the development and sharply fell to physically impossible (in the long run) demographic indicators. This is a situation of demographic overheat, with payoff directly proportional to its time duration.

In our experiments a transition from the traditionalist "pitfall" to any exit scenario was a consequence of precipitous change of one or several factors. As it stood during experiments we did not find cases of gradual change (in the process of gradual transformation of factors' values) of attractors' image from traditional to modernized one. The continuation of the previous tendencies under factors' gradual change led to dissipation.

The issue of modeling of 1917-1920 demographic intentions is a continuation of study in another time duration. While adjustments stemmed from the historical context are inevitable (and natural), our main focus was a cross-temporal comparison. The random sample of 249 villages was prepared for analysis.

Figure 7: A Consolidated Space of Communities' Attractors of the 1917-1920 Sample



Modeling demonstrates that in this sample rural communities, as previously, followed (more correctly – tried to follow) traditional pattern of reproduction – relatively high birth rates and relatively high mortality (see Table 1).

As opposed to the previous period, the attractors' dispersion of demographic intentions of different settlements in the model's phase space is quite large. The increase of dispersion within the suggested hypothesis can be explained with "environmental shock" that stimulated accentuation of particular, sub-regional (highly personal) strategies of survival. In other words, the Civil War combined with War communism caused more intentions to personal survival ("everyone dies alone" effect).

The society was generally diversified in its response to the environmental shock, but intentions to increase birth rate and to personal survival prevailed. That is why we argue that a particular rural community, with a little direct impact from the Civil War and thus with a greater number of men of reproductive age inside, placed its stakes on collective survival (increase of births). This observation fits quite well to the concept of the "communal revolution", that had spread out in rural Russia in 1917-1918.

In essence, this was a proper response of the traditional society to the environmental calls. This type of social entities was aimed on survival and therefore should have responded to diminishing capabilities for that with a firm eagerness to the extensive population growth. Whereas a modernized society, aimed to achieve optimum between population size and environmental resources (this balance is "quality of life" indicator), more often tends to react on the degradation of environment with optimization (reduction) of its population.

Special mention should be made of "external" communities, i.e. settlements without attractors within realistic values area in this period. They lost theoretical prospects for survival and converged to die-off. (We should pay attention that in reality these communities might not have just persisted but even had not had demographic problems within a studied triennial; nonetheless, under existed internal and external conditions they should have had to disappear). There is a quite significant share of such communities – 16,5% – in the sample.

For sure, the non-linear reaction of the studied traditional communities on the environmental shock did not eliminate the presence of several groups of them that had reacted in the linear manner, i.e. – they were dying off. Moreover, we find a significant dispersion of survival strategies, with some of them more correctly to be called "strategies of simple die-off" (rapid or gradual) under unfavorable conditions. This is also a specific "response" to external calls – death as a way to solve all the problems.

It is necessary to make a reservation – we did not study a long-term pressing of unfavorable external circumstances on communities, we were interested in the impact of a rapid shock. Maybe this is the long-term pressing that would have made social entities become less "ambitious" regarding their birth rate, but the response of the traditional society to the short-term environmental catastrophe had been quite optimistic and productive. Of course, we mean only the growth of intensions to increase number of births and to survive that could lead to real shifts in population size if only they have been implemented for a long period of time and under constant conditions as well.

We must also pay attention that attractors of demographic behavior of different communities are located logically – along the lines fanned from a certain point in the "total die-off" zone (zone of low birth and high mortality rates). These conglomerations of attractors may point on some "lines of force" within the objective mechanisms of crisis overcoming. We called these lines "vectors of salvation", because they designate directions which communities unconsciously tried to follow in order to protect themselves from the increasing aggressiveness of environment. Vectors of salvation are perpendicular to the homeostasis line on which rates of births and deaths are balanced along the entire length. It means that the environmental shock causes the reaction of the traditional society directed against the line of homeostatic evolution (i.e. against gradual balancing of environmental resources and main intensions).

The 1920-1926 database contains information on 1179 settlements of the Tambov province, and this is a source of empirical data for the third phase of modeling. The random sample for estimation consists of 55 settlements.

At this time (1920-1926) communities faced not just the environmental shock but a serious deterioration of environment – famine, epidemics and a large rebellion (Antonovshchina) had had a drastic impact on traditional society.

Figure 8: A Consolidated Space of Communities' Attractors of the 1920-1926 Sample

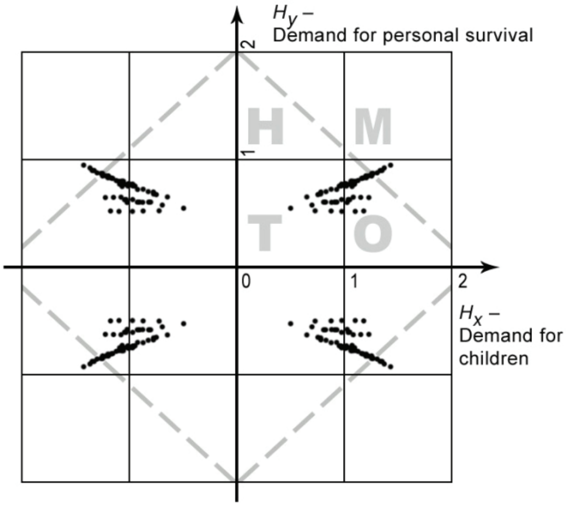


Table 1: Types of Demographic Behavior in 1860-1917, 1917-1920 and 1920-1926

Type of demographic behavior	Percentage of communities (settlements) of this type in the sample		
	1860-1917	1917-1920	1920-1926
O – high birth rate and high mortality ("microbes strategy")	69.7	32	56.4
M – high birth rate and relatively low (for traditional society) mortality ("Universe settlement strategy")	26.3	37	0
T – relatively low (for traditional society) birth rate and high mortality ("total die-off strategy")	4	14	43.4
H – relatively low (for traditional society) birth rate and relatively low (for traditional society) mortality ("stones strategy")	0	0.5	0
– ("external")	0	16.5	0

It was discovered that traditional society had been much more "prepared" to the catastrophic deterioration of living conditions than to minor improvements. The latter (in computer experiments carried out) reduced high mortality pressing and stimulated a demographic explosion, but then an overflow of resource niche and eventually a depopulation downfall. On contrary, socio-economic and military crises did not trigger crisis of demographic strategy – when conditions had become worse, communities evidently realigned desirable demographic priorities towards a stable survival. To some extent, this indirectly supports a basic hypothesis that traditional (in demographic sense) society is adaptive to extreme (from the present-day point of view) parameters of environment. Therefore a crisis situation simply checked intersubjective prerequisites for the next demographic outbreak, but did not lead to the demographic catastrophe.

We can also see vectors of salvation to have existed in the studied period. It is evident that as before a community either was living in mode of response to the present catastrophe, or was waiting for that in the future or was remembering the past one.

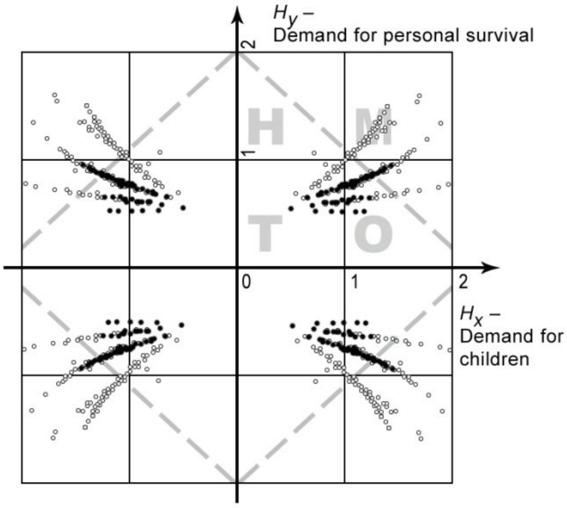
The studied society does not aspire to "settle the Universe" (zone of low mortality and high birth rates), but it is still under transformation not along the homeostasis line but contrary to (vectors of salvation are perpendicular to the homeostasis line). Society appears on the homeostasis line only at its intersection point with the present line of evolution and only as an effect of unfavorable conditions. That is how a particular situation of balance emerges while the general development is unbalanced.

It is evident that neither environmental shock nor major disaster can make traditional society proceed to the homeostatic modernization, i.e. to maintain balance between population size and resources for the sake of personal survival. It is clear that such radical changes in demographic behavior need inner qualitative transformation of the format of the community's existence (its socio-economic and cultural bases) and not the quantitative modification of living conditions.

Comparing vectors of salvation of the studied period with those of the previous one, we can discover that one of them has completely disappeared. It is exactly the vector that led to the zone of "the Universe settlement". Generally vectors are retained more against x-axis, i.e. a demand for personal survival is much less explicit. It is clear that society has already failed to see personal survival as a means of that of system.

The intention to reproduction decreased a little (but by modern standards it is still a huge birth rate). A tendency to compensatory reproduction of the previous period was not supported with improvement of environment and growth of resource base. That is why against the background of the deepening crisis society moved to "wilder" forms of traditional survival (unloading "excessive" population, etc.).

Figure 9: A Consolidated Space of Communities' Attractors of the 1917-1920 (Circles) and 1920-1926 (Black Points) Periods



We can tentatively qualify this complex of effects as the "centripetal reaction" because it is directed to the central zone of the Demofractal phase space – to the zone of "total die-off". Of course, this is the reaction that is not a product of social reflection and goal-setting, but an "involuntary" (determined by dominating norms and practices of life-sustaining activity) response to objective call and circumstances. We can suggest a hypothesis that the centripetal reaction is related with the presence of vectors of salvation and is distinctive to society with "catastrophic thinking" that develops its own strategies (not only demographic ones) as those of escape from catastrophe or adaptation to catastrophe. Thus it is correct to refer almost all traditional social entities to that type of society, because they permanently exist on the brink of famine, expecting the enemies' invasion, "plague", etc. Let us suppose that the centripetal reaction (adaptation to catastrophe) appears as a trend to optimize population size and resources, but by very archaic and not modernized means, as well as a result of a longtime extraordinary deterioration of the basic parameters of living.

Depending on favorable/unfavorable conditions of environment, strategies of the society with "catastrophic thinking" pulse along vectors of salvation and not along the homeostatic line.

4. Conclusion

In conclusion it should be mentioned that modeling by means of the Demofractal neither makes available empirical facts (strictly speaking) for the historian nor claims to replace historical sources in this part. Because, as a rule, we do not have opportunities to voluntarily experiment with social and political phenomena, their virtual models may be used as a sort of "heuristic machine" for hypothesizing.

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