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Veröffentlichungsversion / Published Version

Zeitschriftenartikel / journal article

Empfohlene Zitierung / Suggested Citation:

Legéndi, R. O., Gulyás, L., & Mansury, Y. (2013). Model Replication in the Context of Agent-Based Simulation: Lessons Learnt from Two Case Studies. *European Quarterly of Political Attitudes and Mentalities*, 2(1), 74-85. <https://nbn-resolving.org/urn:nbn:de:0168-ssoar-341826>

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Model Replication in the Context of Agent-Based Simulation. Lessons Learnt from Two Case Studies¹

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*Submission for ECPAM'2012: Jun 14th, 2012 Accepted for ECPAM'2012: July 12th, 2012
Accepted for EQPAM: October 25th, 2012*

Abstract

This paper examines model replication in the context of agent-based simulation through two case studies. Replication of a computational model and validation of its results is an essential tool for scientific researchers, but it is rarely used by modelers. In our work we address the question of validating and verifying simulations in general, and summarize our experience in approaching different models through replication with different motivations. Two models are discussed in details. The first one is an agent-based spatial adaptation of a numerical model, while the second experiment addresses the exact replication of an existing economic model.

Keywords: agent-based simulation, model replication

1. Introduction

Agent-based modeling and simulation are getting more and more attention recently. It is an approach that has been widely employed across fields that are as diverse as sociology (Gilbert and Troitzsch, 2005), economics (Tesfatsion, 2002; Farmer & Foley, 2009), biology (Politopoulos, 2007), epidemiology (Kampis & Gulyás, 2010), regional (Mansury et al., 2011) and political sciences (Cederman, 2001; Kollman & Page, 2006) - it is a computational model that can bridge over multiple disciplines (Axelrod, 2005).

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European Quarterly of Political Attitudes and Mentalities - EQPAM, Volume2, No.1, January 2013, pp. 74-85.

ISSN 2285 – 4916

ISSN-L 2285 – 4916

¹ This paper has been previously published in the Proceedings of the 1st *European Conference on Political Attitudes and Mentalities*, ECPAM'2012, September 2-4, 2012, University of Bucharest, Romania, in: Voinea C.F. (ed.), "Political Attitudes and Mentalities. The Historical Heritage of Europe: A Challenge for the Future of Political Analysis", University of Bucharest-Ars Docendi Press, 2012, Bucharest, pp.149-163.

Despite the popularity of agent-based computational models, model replication, validation and verification are surprisingly hard tasks, and the simulation results have rarely been replicated, let alone implemented by anyone else except the original author. Only a few select “classic” models are the exceptions to this observation (Wilensky & Rand, 2007).

Reproduction of results and validation of experiments via replication are essential in science. This tradition originates in the natural sciences. If we consider simulations as computer experiments, then we must see reproducibility as a crucial issue. In models created to describe real-world phenomena or to support policymakers, emphasis must be put on the reproducibility of experiments in order to validate the scientific claim. If different implementations of a model fail to generate the same output under exactly the same conditions, then the scientific value is in question.

1. Replication, Docking and Validation

In the context of computer simulations, there is a growing list of publications covering the topics of docking (the alignment of different models to demonstrate that they produce similar results) and model replications (the reimplementing of an original model, typically in a different environment). An extensive review of these approaches can be found in (Miodownik et al., 2008).

Alignment of computational models, or docking, as referred commonly (analogous to orbital docking of dissimilar spacecrafts), is a process where we pit a series of models against each other in the attempt to verify if they produce the same results – crucial for equivalence testing of models. The ability to verify whether two different models targeting the same phenomena can (or cannot) generate the same results is essential. A few examples of successful docking experiments include (Epstein & Axtell, 1996), (Grimm et al., 2005) or (Miodownik, 2006).

Replication is another critical issue, which is an important method of science. These models were designed to describe, explain, or predict real-world phenomena. Without the ability to replicate results, it would be impossible for artificial models to target real-world systems or to develop valid theories for them. The issue of model replication and how to evaluate the replication results are often raised at several forums.

One of the major problems is ambiguity: there are several concepts about how a model should be defined in order to minimize uncertainty. Various approaches exist, from textual frameworks like the Overview, Design concepts, and Design details, or ODD (Grimm et al., 2006), to extensions on top of standardized modeling languages like AgentUML (Bauer et al., 2001). But there is still no consensus on best practice. Until these frameworks become widely accepted and applied, the methodology of replication remains an issue. A good example of a successful replication of a classic agent-based model is (Bigbee et al., 2007), and several notable examples can be found in (Miodownik et al., 2008).

Replication helps us to get a deeper understanding of the relevant properties, to identify the key issues of the analyzed phenomena, and to deploy simulation as a research tool successfully. With our work, we would like to contribute to the growing list of qualitative analysis of models through replication. Our motivation is to solidify results, and to increase the scientific value of both the models and the agent-based methodology.

2. Case Studies

In the next sections, we introduce the models we targeted with replication. The first one is the spatial agent-based adaptation of Fujita-Krugman-Venables (FKV)'s core-periphery model (Fujita et al., 1999), which originally is an analytically-intractable model. Simulation is therefore the only resort. In our work, we frame the model within an agent-based context, and then replicate the key results of the original model. The second one is an exact replication of Delli Gatti et al.'s Bottom-up Adaptive Macroeconomics (BAM) model (Delli Gatti et al., 2011). In the next sections, we give an overview of these models.

3.1. An Agent-Based Adaptation of the New Economic Geography

In this section, we review the work described in (Mansury et al., 2011a, 2011b). Empirical results indicate that Zipf's Law can describe spatial patterns. It may be interpreted as, when ranking cities, the frequency f of the population of a given city exponentially decreases as its rank r increases: $f(r) \sim r^{-\alpha}$. Related to this is the Pareto principle which states that for many events, roughly 80% of the effects comes from 20% of the causes. These laws are often found in physical and social systems. Zipf's Law implies that the n^{th} -ranked city has $1/n^{\text{th}}$ the population of the 1st ranked city.

Previous work (Gulyas & Mansury, 2007) successfully explained the Zipf's Law through a spatial agent-based model, but lacks micro-foundations. In a previous work, we extended the FKV model, which is a general-equilibrium model of core-periphery spatial patterns with excellent micro-foundations, but is not designed to generate a hierarchical system of cities.

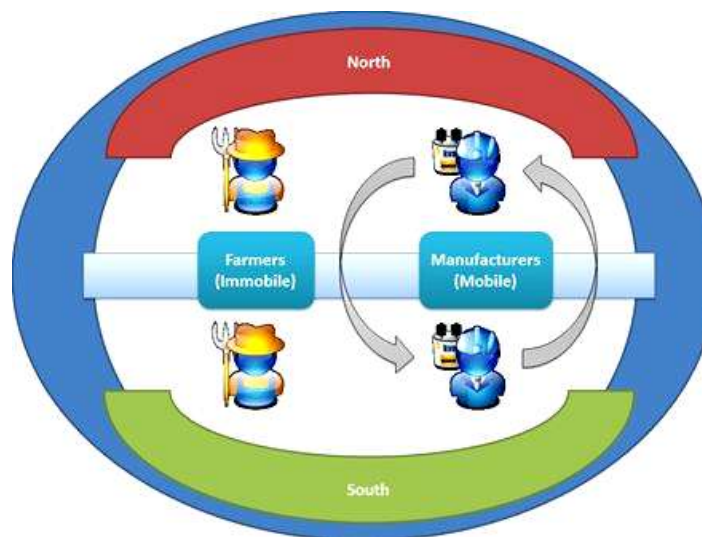


Figure 1.
 Overview of the FKV model

In a work-in-progress (Mansury et al., 2011a), we proposed a spatial agent-based version of the FKV's core-periphery model. The model retains the key features of the original model, including consumers' love for varieties, increasing returns in production, and the tension between centripetal (agglomeration) and centrifugal (dispersion) forces. The agent-based model successfully produces catastrophic agglomeration, bifurcation, and the tomahawk diagram – hallmarks of the New Economic Geography. There are two novel

outcomes. First, the agent-based framework allows us to move away from the representative agent paradigm into a model of heterogeneous agents. Second, the bottom-up approach enables migration to proceed in a non ad-hoc way. Both contributions rely on the mutual complementarity between FKV's analytical model and our computational platform.

Figure 1 depicts an overview of the model which basically contains two regions (North and South) and two sectors (Agriculture and Manufacture). The Farmers' locations are permanent, Workers are mobile. Agent-level analysis considers only workers, while aggregate analysis includes both farmers and workers. In each time step, agents evaluate the „attractiveness” of a location based on a utility function that incorporates both the effects of agglomeration and dispersion. The model simulation sequence consists of (1) determining the northern and southern varieties, output, and prices, (2) updating wages, (3) allowing agents to decide whether to migrate, (4) updating the spatial distribution of workers, and (5) repeating step (1) in the next iteration.

3.2. Results

One of the most important results of the original model is the Tomahawk-diagram (Figure 2) which describes the population migration λ with respect to "freeness" of trade φ .

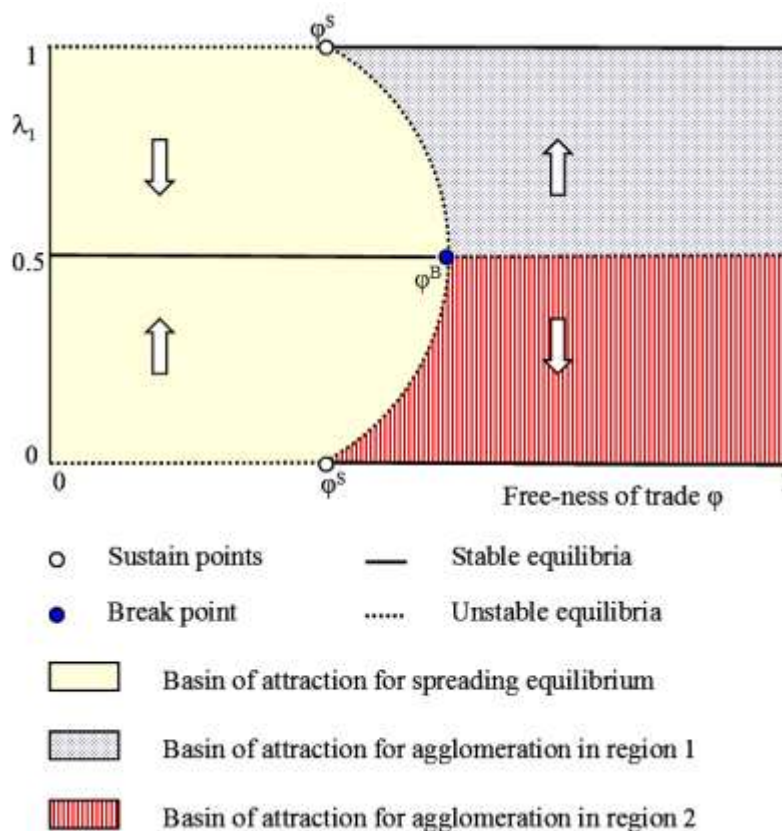


Figure 2.

The Tomahawk diagram (Fujita et al., 1999)

We validated our replication results against this diagram (Mansury et al., 2011b). The original Tomahawk diagram has a *break point* where the transportation cost symmetry switches from stable to unstable. It has a closed form solution (1) which is confirmed by the agent-based model (in the demonstrated example, for the given parameters result in $\tau_B \sim 1.6265$, or equivalently $\varphi_B \sim 0.1428$).

$$\tau_B = \left[\frac{\{\sigma(1+\mu)-1\}(1+\mu)}{\{\sigma(1-\mu)-1\}(1-\mu)} \right]^{\frac{1}{\sigma-1}} \quad (1)$$

Additionally, we also found identical results for the *sustain points* where transport cost for the core-periphery becomes stable for the first time. It has an implicit function (2), whose value is also confirmed by our implementation ($\varphi_B \sim 0.0937$, or equivalently $\tau_s \sim 1.8074$).

$$1 = \Psi^{-1/\sigma} \varphi^{\mu/(1-\sigma)} \text{ where } \Psi \equiv \frac{\varphi^2(1+\mu) + (1-\mu)}{2\varphi} \quad (2)$$

A sample of our results can be seen below in a specific case, which verifies the results mentioned above. The charts contains data acquired from ten different values of the initial population distribution λ (noted as *manInitRate*) at ten different measure points of φ (values noted on the charts), for the parameters of $\mu=0.4$ and $\sigma=5.0$, with 2000 agents in each run. We included an initial and the final state as a demonstration in Figure 3 and Figure 4, respectively.

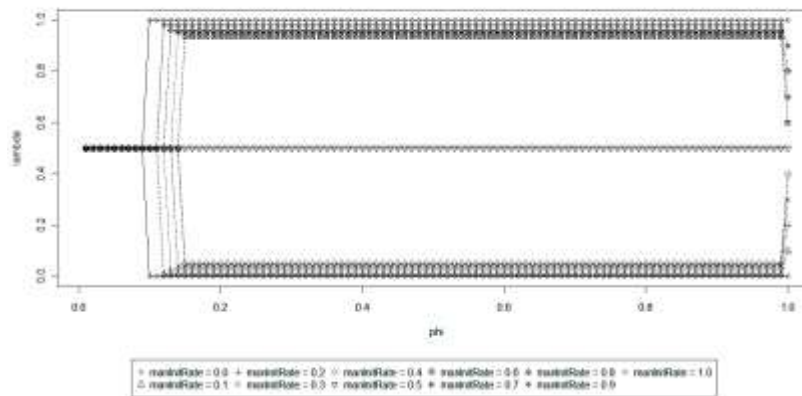


Figure 3.
Simulation state at time step 2000.

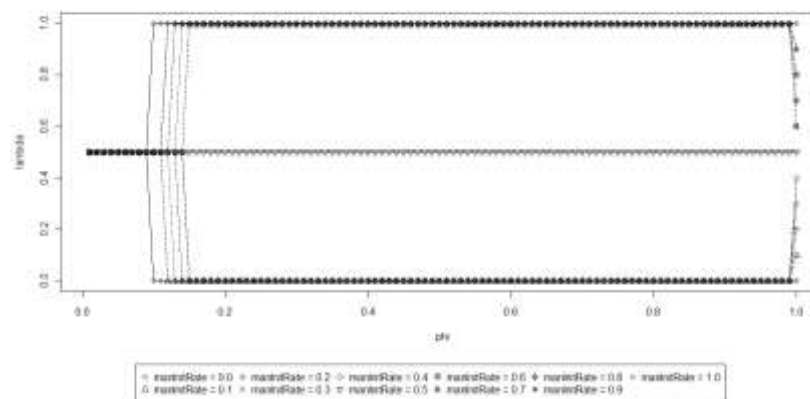


Figure 4.
Simulation state at time step 5000.

4. Replication of the Bottom-up Adaptive Macroeconomics

In this section we turn our attention to our second case study. The authors of the Bottom-up Adaptive Macroeconomics (BAM) model (Delli Gatti, 2011) applied empirical external validation based on real-world data. In our current work, we validate the simulation through replication. Our goal is to create an alternative implementation in a different environment and verify whether the models produce the same results – in other words, we test their identity.

In another recent work, we successfully created an alternative implementation of the original BAM model. The replicated model resembles the results of the original one. We shall briefly describe the structure of the original model, and discuss the reasons for replication. In the following section, we provide an extract of our results published in (Legendi & Gulyas, 2012).

4.1. Overview of the Original Model

The original model considers three different actors, Households, Firms and Banks: Households supply labor, buy consumption goods and hold deposits, Firms demand labor, produce and sell consumption goods, give shares to their respective owners, and Banks receive deposits from households and extend loans to Firms.

There are three different markets in the model between these participants: the labor market (in which workers are homogeneous), the product market (which is also homogeneous, and contains non-durable goods) and the credit market. An overview of the model can be seen on Figure 5.

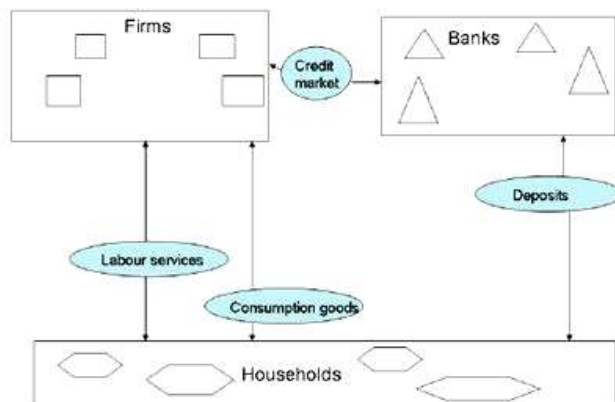


Figure 5.
 Structure of the BAM model (Delli Gatti, 2012)

In the model, the behavior of the agents is not (necessarily) the outcome of an optimization process: generally behavior changes adaptively according to rules of thumb. The markets are fully decentralized and characterized by a continuous search and matching process.

4.2. Reasons for Replication

In the introduction of this paper we described that model replication might help us validate the scientific results of an agent-based model. However, there were some additional issues we considered.

We created the first prototypes of this replicated model in the framework of the CRISIS project²: *“The CRISIS project addresses building a next generation macroeconomic and financial system policymaking model: a bottom-up agent-based simulation that fully accounts for the heterogeneity of households, firms, and government actors. The model will incorporate the latest evidence from behavioral economics in portraying agent behavior, and the CRISIS team will also collect new data on agent decision making using experimental economics techniques. While any model must make simplifying assumptions about human behavior, the CRISIS model will be significantly more realistic in its portrayal of relevant agent behavior than the current generation of policymaking models.”* (CRISIS project description, 2012).

The replication of the BAM model was created as a first attempt to create the basics of a unified CRISIS programming library and the structure of the integrated simulator. The CRISIS model will also have a gaming interface to allow decision makers and other stakeholders to directly interact with the model, and to connect to leading edge visualization and scenario analysis tools. The replicated model described in this paper will serve as the first version of the engine for the gaming interface to be created.

4.3. Efficiency

In our work, we tried to focus our efforts not just on creating a validated replication for the Mark 1 model, but tried to make it as efficient as it is possible on the Java platform. The reason why we chose Java as the basis of our work is its ease of use and support of various simulation tools. That is also a reason why we chose Mason, *“a fast discrete-event multiagent simulation library core in Java, designed to be the foundation for large custom-purpose Java simulations [...]”* (Luke et al., 2005) as our simulation

² <https://www.crisis-economics.eu/>

platform, whose main design goal and philosophy focuses on supporting the creation of efficient computer simulations.

4.4. Tool Support: Model Exploration Module

In our work we were also relying on a set of tools to enable and help the evaluation of replicated models. Among these, the Model Exploration Module (MEME) was the most important. MEME is an automated, generic tool (supporting various simulation platforms like NetLogo, Repast, Fables and Mason) designed to help modelers dealing with enormous parameter spaces in a reasonable time (Iványi et al., 2007).

One of the approaches to handle such amount of required simulation runs is to exploit a private grid or distributed cloud computing resources in order to execute large-scale parameter space explorations or sensitivity analyses (Máhr et al., 2010). MEME also offers statistical tools to reduce the number of simulation runs needed for collecting enough information (i.e., samples) from a given parameter space region to judge the behavior of the simulation. These statistical methods are based on the *Design of Experiments* (Box et al., 2005) literature that was developed for the experimental sciences with the goal to reduce the number of costly experiments without jeopardizing the amount of knowledge that can be collected from the experiments.

MEME has a growing list of classic designs of experiments, including (fractional) factorial design(s), the *Box-Behnken* design, the central composite design and Latin hypercube designs, as well as it offers a number of more advanced, dynamic exploration strategies, based on heuristic optimization methods. These latter include *Genetic Algorithms* and *Iterative Interpolation*. A good example how these techniques can be exploited in computer simulations is (Szabó et al., 2009).

4.5. Results

During the implementation, we paid special attention to efficiency. We tried to use one of the most efficient modeling libraries available for Java, Mason, and we tried to use the most efficient language constructs (until it was against readability). Note that this was not a design concept with the original Matlab model.

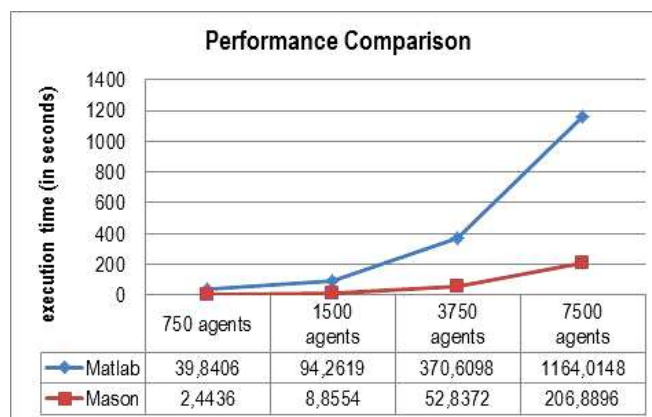


Figure 6.

Averaged execution time comparison for both the original and replicated model

The execution time of 40 runs is averaged on Figure 6 where each measurement point shows the average of 5 different runs. As a result we got that the Java model is faster by several magnitudes (usually between a factor of 6 to 10) compared to the original model.

We were also able to create the replication in a way that is absolutely identical to the original model: it generates the exact same output for the same parameter vectors (including random number generator seeds). Unfortunately, this approach involves the non-idiomatic usage of language constructs in Java, rendering it quite inefficient. (Generating the exact same random sequences as in the original implementation yields a vast amount of temporal data structures). If we sacrifice the absolute identity of behavior and create similar functions using proper building blocks of the language, the results are still very similar (and the execution is much faster).

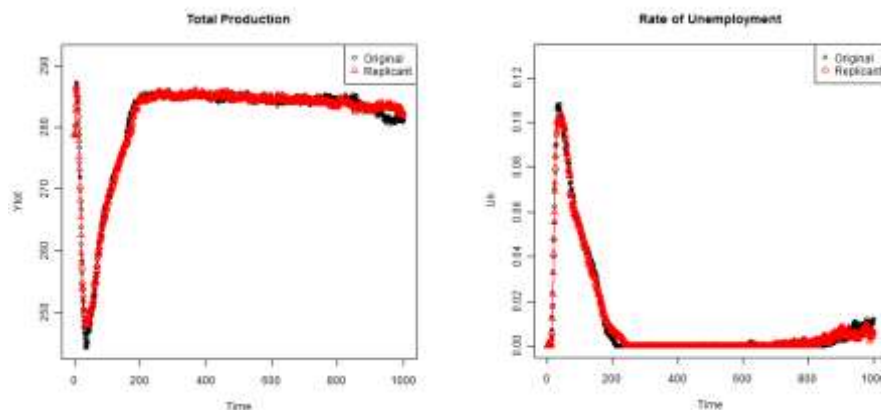
During the experiments, we performed a parameter sweep by manipulating the parameters having the most significant influence on the output of the simulation according to the original work (Delli Gatti, 2011). A rigorous evaluation of these results can be found in (Legéndi & Gulyas, 2012). Here we include only a brief overview (see Figure 7) due to space limitations. The reported results illustrate how various aggregate measures of the simulated system change over time for the original model (noted by black circles) and for the optimized version of the replicated (noted by red triangles) model. Note that the less efficient version creates results that are absolutely identical to the black circles.

The shown statistics report the aggregate production, the unemployment rate, the global interest rate, the leverage ratio, and the number of defaults in the simulated economy. It is clear that the two series show the same general trends. Minor oscillations and deviations are present that can be ascribed to the minor differences in the random number sequences.

5. Conclusions and Future Work

In this paper we discussed the issue of replication of computer simulations, providing two examples from our research practice.

In the core-periphery model, our replication efforts were extended in several ways. The new computational approach, i.e., agent-based modeling, allows us to introduce multiple levels of heterogeneity, as discussed in details in (Mansury et al., 2011b).



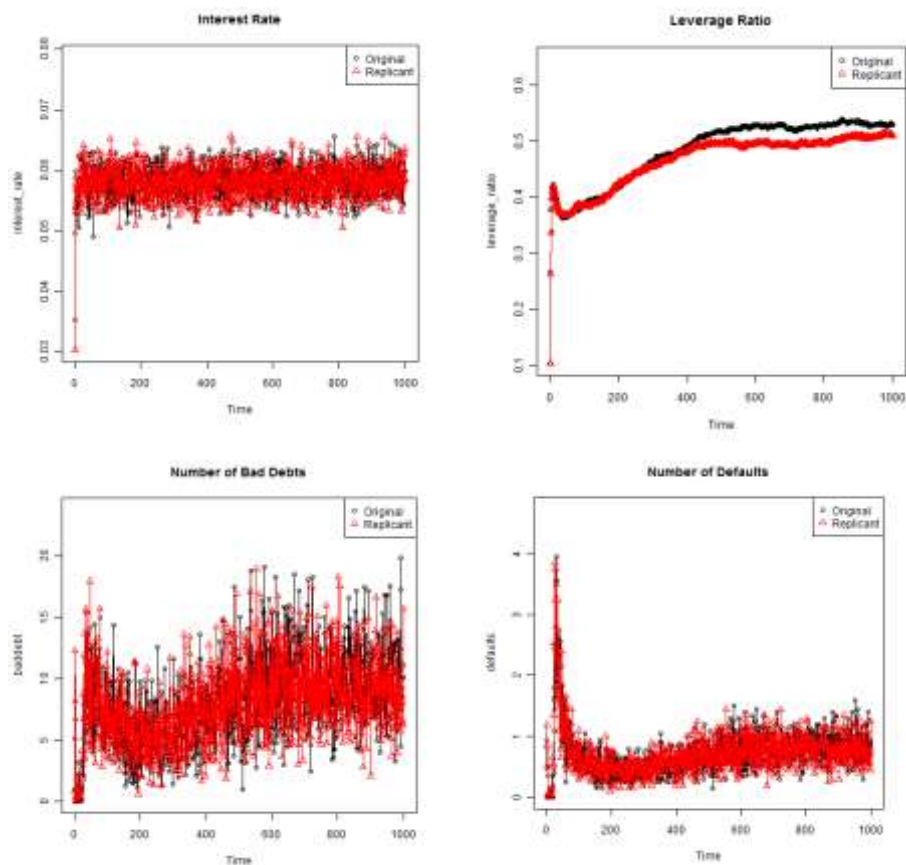


Figure 7.

A brief overview of the results of the original and replicated model, an average of 40 runs included

Agents individually observe the wage of both the local and the remote location (i.e., we added local and remote noise to the system). Furthermore, we introduced agent-specific migration thresholds, which captures different attachments to the home region (i.e. some of the agents prefer migrating, some of them never migrate). We found that for specific activation regimes, the stable equilibria of the original homogeneous system become fragile and breakable.

We are currently working on extending the model to an N-cities setup. With small cities, monopolistic competition is no longer tenable, where an oligopoly may be a result. Our current work includes analysis of different activation regimes – it seems to have a major effect on the simulation results.

In the second, bottom-up macroeconomics example, we were able to successfully replicate the results of an agent-based economic model. In this paper, we provided a basic exploration of our replication results, showing that they are comparable to the data generated by the original implementation.

The benefits we gained by an alternative implementation is, on one hand, technical. The speedup made possible by the new implementation is significant (the performance gain is over 5 orders of magnitude). Our version also opens up the window of standardized simulation tools, allowing us to perform more extensive parameter space explorations by exploiting cloud systems or by using sophisticated *Design*

of *Experiments* techniques. On the other hand, the successful replication result solidifies the base for further research and developments, discussed above, that are to be built on top of the replicated model.

Acknowledgements

This work was partially supported by the Hungarian Government (KMOP-1.1.2-08/1-2008-0002), the European Union Seventh Framework Programme FP7/2007-2013 under grant agreement CRISIS-ICT-2011-288501 (CRISIS – Complexity Research Initiative for Systemic InstabilitieS) and mOSAIC 2011-256910 (Open-Source API and Platform for Multiple Clouds). These supports are gratefully acknowledged.

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