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UNCERTAIN COMMITMENT POWER IN DURABLE
GOOD MONOPOLY

by
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Understanding Society

Uncertain Commitment Power in a Durable Good Monopoly[*]

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Abstract

This paper considers dynamic pricing strategies in a durable good monopoly model with uncertain commitment power to set price paths. The type of the monopolist is private information of the firm and not observable to consumers. If commitment to future prices is not possible, the initial price is high in equilibrium, but the firm falls prey to the Coase conjecture later to capture the residual demand. The relative price cut is increasing in the probability of commitment as buyers anticipate that a steady price is likely and purchase early. Pooling in prices may occur for perpetuity if commitment is sufficiently weak. Polling for infinity is also preserved if committing to a high price is endogenously chosen by the firm.

JEL Codes: D42, L12, D61, D82
Keywords: Monopoly; Commitment; Information asymmetry

1 Introduction

Sharp price cuts belong to the standard toolset of dynamic pricing. Revenue management operates through lowering prices with various timing patterns

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and methods. Board and Skrzypacz (2016) argue that the idea lies in the forward-looking behavior of consumers as changing the price enables screening. Buyers with low valuation suffer less from delaying purchase, hence, they buy later. In contrast, high willingness-to-pay dictates early purchase at high price. This reasoning has strong explanatory power as many retail markets operate with planned discounts. For example, in the fashion industry, end-of-season sales occur at given times of the year and their timing is commonly known. However, this argument fails to explain uncertain and sharp price cuts, a phenomenon observed in other markets.

Unlike fashion companies, producers of consumer electronic goods and software do not always choose the same calendar dates every year to make discount offers. The uncertain timing of cuts is confirmed by mainstream media outlets that frequently publish articles speculating about future price trends of consumer electronics markets.[1] Stochastic sharp price cuts by firms with strong market power are not unusual. The fifth generation of video game consoles was dominated by the Sony PlayStation between 1995-96. Its original Japanese retail price, 39,800 Yen, was cut to 24,800 Yen in two steps, with the second cut carried out six months before the launch of its main competitor, the Nintendo 64 (Drysdale, 2004). In 2007, Apple unexpectedly cut the price of the 8-gigabyte iPhone from 599 to 399 USD despite having the reputation of not using this strategy and the lack of market shock (Mickalowski et al., 2008).[2] These pricing patterns are not restricted to the domain of consumer electronics. Microsoft undertook a similar step when it cut the price of its operational system MS Vista Ultimate from 299 to 219 USD (Fried, 2008).

In dynamic pricing, commitment refers to the ability of irreversibly choosing a price before a sale period. As common in the literature, this is assumed to be known by the buyers. In the model of Coase (1972) without commitment, price offers are made sequentially and the prices are not restricted by the domain or history of prices and this is commonly known. Ellingsen and Miettinen (2008) assume that bargaining players are able to commit to prices, nevertheless, this ability is observable. This paper relaxes the standard model by assuming that the ability of price commitment is stochastic and it is private information of the seller. The corresponding equilibrium

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[1]The list of most important online outlets include CNET, PC World, TechRadar, etc.

[2]The cut seemed so unlikely, that Apple responded to the overwhelming negative reactions of early buyers by offering them a 100 USD credit they could spend at the company's online and retail stores.

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analysis demonstrates that sharp price cuts, that cannot be perfectly predicted by buyers, are consistent with forward-looking rational agents. Our analysis reveals further patterns. More pronounced cuts occur in durable good markets if commitment is likely and buyers are less patient. Moreover, uncertainty is only present if commitment is sufficiently strong. If this does not hold, there exists a pooling equilibria in which firms with and without the ability of price commitment follow the same pricing strategy.

In actual markets, firms can maintain high price levels in various ways including bilateral contracts with retailers or publicly disclosed inventory. It is not reasonable to assume that buyers are familiar with this information in all cases. In this paper, we abstract away from the source of commitment and introduce a model which assumes that the firm's ability to commit to a price is private information and may be one of two types. A strong type (ST) is able to fully commit to the initially set price. In a dynamic setting with any discount rate, this firm is able to achieve the monopoly payoff. In contrast, a weak type (WT) may set any price. As Coase (1972) shows, a WT falls prey to competitive effects coined as Coase conjecture. As there is unserved demand, the firm is tempted to capture it by reducing price in subsequent periods. However, this creates competitive pressure and incentives to reduce it in earlier periods as well. In the limit with sufficiently patient buyers, the prices reach marginal cost levels. This does not happen if price cuts can be prevented, in other words, if committing to a fixed price is possible.

We show that pooling occurs in equilibrium but may only be sustained initially and collapses in finite time horizon, resulting in a sharp decrease in price. We extend the analysis to firms with imperfect commitment technology (IT). Suppose the firm with commitment cannot keep a steady price, but rather reduces it with an exogenously given parameter. There can be two types of equilibria in this setting. If the commitment technology is sufficiently strong, the trajectory is similar to that of the baseline model, and pooling occurs only initially. However, if it is weak, there is an equilibrium in which pooling happens in each period for perpetuity.

Our primary focus is the possession of commitment power rather than its source. In order to maintain this, we omit the explicit modeling of commitment, and represent it as an exogenous restriction on the strategy space of a player. This approach has two key advantages. First, it enables focusing on the beliefs about the existence of the commitment device which is relevant to the consumers. Second, the particular model can be straightforwardly made payoff-equivalent to standard models of dynamic commitment games.

This paper contributes to two strands of literature: price commitment and dynamic pricing. The role of price commitment in exercising market power has been a cornerstone of an extensive literature on durable good monopolies. Seminal articles focus chiefly on the effect of commitment on prices. Coase (1972) emphasizes that the lack of commitment results in a competitive price in continuous time. Theoretical works challenge this notion by arguing that commitment power can be restored in several ways so that monopoly profit is achievable (Ausubel et al., 2002; McAfee and Wiseman, 2008). The claim of the Pacman conjecture, that discrete time pricing enables profit, is proved by Bagnoli et al. (1989). The cited models have in common that the price trajectory is smooth. Another line of research characterizes price cuts as a strategic move anticipated by rational consumers. The idea behind this is that producers use an observable capacity constraint. The fashion industry delivers empirical evidence of the phenomena that price cuts are widespread and well-known (Gallego and Van Ryzin, 1994). An important feature of this pricing strategy is that this conjecture does not necessarily depend on the information set of buyers on capacities, as lower offers occur periodically.

The celebrated result of Stokey (1979) argues that committing to a high price is favored by a monopolist over screening by lowering the initial price. Conlisk et al. (1984) and Besanko and Winston (1990) take the arrival of new consumers into consideration and show that cyclical patterns may appear in equilibrium with interchangeably decreasing and increasing prices. Another line of research we follow more closely assume that sellers are able to form long-term commitments, as assumed by Elmaghraby et al. (2008), Besbes and Lobel (2015) and Board and Skrzypacz (2016). This framework can be extended to multi-product monopolists, as evidenced by Rochet and Thanassoulis (2017). Ortner (2017) shows that stochastic costs can also restore commitment power. Ellingsen and Miettinen (2008) enrich the setting by introducing costly stochastic commitment. Abreu and Gul (2000) introduce behavioral types. In their bargaining model, players may be irrational with a certain probability in the sense that they accept an offer if and only if it reaches a threshold value. Abreu et al. (2015) explore the effect of stochastic behavioral types on bargaining equilibria. Their concept is substantially different, as counter-proposals are possible.

The paper is structured as follows. Section 2 introduces the baseline model with perfect commitment and shows that any arbitrarily large price cut can be supported in equilibrium. Section 3 extends this by introducing imperfect commitment and shows that a pooling equilibrium may exist if

commitment of the strong type monopolist is sufficiently weak. The model is augmented with a model in which a costly commitment technology is endogenously and privately chosen. Section 4 concludes this article with a discussion.

2 Model with perfect commitment

Perfect commitment is one of the most fundamental approaches to dynamic pricing. In the limit it is equivalent to a static monopoly game in which a single firm sets a take-it-or-leave-it offer. As a well-known result in mechanism design, this also yields an optimal feasible mechanism for the seller (Myerson, 1981). Hence, perfect commitment is an appropriate term to describe it.

Subsection 2.1 defines the model setup of a durable good monopoly. Subsection 2.2 entails an equilibrium analysis.

2.1 Model

A risk-neutral monopolist produces an indivisible good of infinite durability at zero marginal cost. A unit mass of consumers is characterized by valuations distributed according to a uniform density function with support $x \in [\varepsilon, 1]$, where ε is positive and arbitrarily small.[3] Consumers have a single-unit demand for the good.[4] All players face discount rate $\delta < 1$. We further assume that all players are expected revenue maximizers.

The game assumes a dynamic form with one-sided price offers. The firm sets price $p_1 \geq 0$ as the offer in time $t = 1$. In the second part of the period, purchasing buyers obtain a payoff $x - p_1$ and the seller gets p_1. Rejection is followed by a new offer in the subsequent period $t = 2$. In all later periods, purchasing buyers obtain payoff $\delta^{t-1}(x - p_t)$ and the seller receives $\delta^{t-1}p_t$. The process is repeated infinitely many times if there is a residual market. In the absence of acceptance, players obtain their outside option 0. We use the tie-breaking rule that buyers weakly prefer buying earlier.

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[3]We apply this technical assumption such that the type space is normalized and the model falls in the "gap case", using the terminology of Ausubel et al. (2002), that is, valuations are bounded away from zero.

[4]Another, mathematically equivalent way to think about this model is that there is a single buyer whose type is private information, similarly to the cited literature on bargaining. Here, we frame the setting as a durable good monopoly and use the appropriate terminology.

We distinguish two types of monopolists which differ in their strategy space. The *weak type* (WT) may set any price path $p_{c,t}$. The *strong type* (ST) has a constrained strategy space and is only able to set $p_{b,1}$, for all later periods $p_{b,t} = p_{b,1}$, corresponding to the first-period price. The type is observed privately and drawn randomly, being ST with probability $\alpha \in [0,1]$ and WT with probability $1 - \alpha$. All this is common knowledge.

2.2 Equilibrium

Gul et al. (1986) and Fudenberg et al. (1985) show that the model without commitment, $\alpha = 0$, has a unique price path in perfect Bayesian equilibrium. Applying the more general result to our case, $p_t = c^t$ where $c = \frac{\sqrt{1-\delta}}{1+\sqrt{1-\delta}}$. Consumers buy in t if $c^t \leq x \leq c^{t-1}$. That is, the Coasian payoff amounts to

$$\Pi_c = \sum_t \delta^{t-1} c^t (c^t - c^{t-1}) = \frac{c(1-c)}{1 - \delta c^2} = \frac{\frac{\sqrt{1-\delta}}{1+\sqrt{1-\delta}}(1 - \frac{\sqrt{1-\delta}}{1+\sqrt{1-\delta}})}{1 - \delta(\frac{\sqrt{1-\delta}}{1+\sqrt{1-\delta}})^2} = \frac{\sqrt{1-\delta}}{(1+\sqrt{1-\delta})^2 - \delta\sqrt{1-\delta}}.$$

(1)

Corresponding to the Coase conjecture, $\lim_{\delta \to 1} \pi_c = 0$, which means that the seller is forced to offer the competitive price 0 in the limit. Sequential rationality dictates that the seller cuts price in subsequent periods in order to capture residual demand. Hence, some buyer types postpone purchase, putting pressure on earlier periods. As a result, the first-period price $p_1 = 0$ drops to competitive levels if buyers are patient. In terms of the skimming property, higher type buyers purchase earlier. This result can be generalized to any α.

Lemma 1. *In equilibrium, buyer type x' buys no later than x if $x' > x$.*

In what follows we consider ε in the limit. If $0 < \alpha < 1$, buyers are unable to identify the seller's type and beliefs remain unaltered until the two types choose different prices in a period.

First, we derive that $p_{b,1} = p_{c,1} = p_1$ in equilibrium. Suppose this is not the case. If $p_{b,1} \neq p_{c,1}$, the buyers learn the seller's type in $t = 1$ and its payoff in this subgame is the same as if $\alpha = 0$. ST chooses p_m and WT with a lower initial price obtains Π_c as expressed in Equation (1). WT can profitably deviate by charging p_m. That is, the two types pool in $t = 1$ and charge the same price.

Using the skimming property from Lemma 1, there exists a critical level x^* such that a buyer accepts in $t = 1$ if $x \geq x^*$. In equilibrium, in order to capture the residual demand, WT chooses $p_{c,2} < p_1$ and obtains $\delta(x^*)^2 \Pi_c$ in the remaining periods. This subgame is a truncated version of the Coasian game with $\alpha = 0$ as the buyer learns the type. Critical types and prices curtailed by a reduced type space by x^* and the discount parameter δ. In comparison with Coasian pricing:

$$p_1(1 - x^*) + \delta(x^*)^2 \Pi_c > \Pi_c. \tag{2}$$

The critical type satisfies

$$x^* - p_1 = \alpha(\delta x^* - \delta p_1) + (1 - \alpha)\delta x^*(1 - c)$$
$$\iff p_1 A = x^* \tag{3}$$

where $A = \frac{1 - \alpha\delta}{1 - \alpha\delta - (1-\alpha)\delta(1-c)}$. Substituting in (2) yields

$$\frac{p_1(1 - Ap_1)}{1 - \delta A^2(p_1)^2} > \Pi_c \tag{4}$$

If (4) holds, from (3) and $p_1 = p_{b,1} = p_{c,1}$, ST's objective function becomes

$$\max_{p_1} \left[1 - Ap_1 + \delta(Ap_1 - p_1)\right] p_1. \tag{5}$$

which is the sum of the payoff of the first period and the second, where residual buyers with $x \geq p_1$ purchase. That is, $p_1 = \frac{1}{2(A+\delta-A\delta)}$. Substituting in the left-hand-side of (19) yields

$$\frac{2(A+\delta-A\delta) - 2A\delta}{4(A+\delta-A\delta)^2 - A^2\delta} > \frac{1}{4}\frac{8(A+\delta-A\delta) - 4A\delta}{4(A+\delta-A\delta)^2 - A^2\delta} > \frac{1}{4}\frac{4(A+\delta-A\delta)}{4(A+\delta-A\delta)^2 - A^2\delta} > \frac{1}{4} > \Pi_c. \tag{6}$$

That is, in equilibrium, WT finds it profitable in the first round to mimic ST, but cuts price later.[5] This result is formally summarized in Proposition 1.

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[5]Profit $\Pi_c = \frac{\sqrt{1-\delta}}{(1+\sqrt{1-\delta})^2 - \delta\sqrt{1-\delta}}$ attains its maximum $\frac{1}{4}$ on the relevant range at $\delta = 0$.

Proposition 1. *In equilibrium, $p_{b,1} = p_{c,1} > 0$ and $p_{c,t}$ is strictly decreasing. All buyers accept an offer in a finite number of periods if the seller is CT. There is sale only in $t = 1, 2$ if the seller is ST. The probability of first-period acceptance is a decreasing, and the share of unserved buyers is an increasing function of α.*

The probability of facing a steady price is increasing in α, creating a greater residual market. The seller prefers high commitment levels by which ST achieves higher profit. Price is an increasing function of α and the mass of buyers delaying purchase decreases. WT also profits from the higher p_1.

Proposition 2. *In equilibrium, both seller types' expected profit is an increasing and social welfare is a decreasing function of α.*

The above characterization of equilibrium does not imply existence. One also need to establish that WT monopolist does not charge p_1 in $t = 2$.

Lemma 2. *There is an equilibrium if and only if α is above a critical value $\alpha^*(\delta) > 0$.*

The Coase as well as the Pacman conjecture imply that the monopolist does not change the price drastically, the predictions differ chiefly in profit and welfare. Our model is able to explain drastic price changes. A WT monopolist starts with a high price but reduces it subsequently to capture the residual market. The initial price is higher since the firm is able to pretend to commit to a high level. The resulting difference can be drastic in the sense that the relative price cut with patient players can reach 100% in the limit.

Comparative statics reveal that the probability of commitment has a dramatic effect on price cuts. In particular, consider a situation when α is high. The consumers' objective function puts a high weight on the event when the price will not decrease. That is, the critical type x^* as well as the initial price p_1 are high. The critical type is close to the ST's price since price cu t is less likely. Since the second-period price of WT monopolist is proportional to the critical value, this means that the price cut increases with α.

The second-period price cut by the WT monopolist is primarily driven by the discount parameter as dictated by the Coase conjecture. That is, price $p_{c,2}$ can be arbitrarily low in the limit as δ converges to 1.

Proposition 3. *In equilibrium, the ratio of prices of a WT monopolist in $t = 1$ and $t = 2$ is an increasing function of α and it converges to 0 as players become more patient.*

Proposition 3 shows the remarkable feature of this market that a monopolist may apply an arbitrarily strong price cut in the limit. That is, if the consumers believe that the price remains steady, and they exhibit patience. If we keep δ fixed, the new price converges to the Coasian result, $\lim_{\alpha \to 1} \frac{p_{c,2}}{p_1} = 0$. In other words, in the limit a firm charging monopoly price initially may revert to competitive pricing.

2.3 Acquired Stubbornness

The main results do not depend on the particular belief structure. Commitment as private knowledge is a reasonable assumption, but the loss of commitment power can be *interim*. In this alternative model, the seller learns its type privately only after the first period. Kambe (1999) coins the term acquired stubbornness to describe this behavior in which initial pricing occurs before learning type.

Proposition 4. *If commitment type is only revealed after $t = 1$, the equilibrium prices are lower.*

The monopolist's objective function compared to ST in the main model takes the possible loss of market power into consideration. Hence, incentives are stronger to reduce p_1 and capture a bigger market share early if the seller exhibits acquired stubbornness. All other results straightforwardly extend to this alternative setup.

3 Imperfect Commitment

Section 2 shows that a monopolist with no commitment power can profit from asymmetric information and is able to pretend to have power. The case is degenerate in the sense that *ex ante* the firm may only commit to the static monopoly price. This assumption yields the sharp result that in equilibrium the two monopolist types pool in the first period but offer different prices later. We relax this setting by considering a wider range of commitment technologies which do not allow for keeping the initial price

steady. The monopolist is either weak type or has limited commitment. We introduce this model in Subsection 3.1 and characterize the equilibrium in 3.2. Another track is to endogenize commitment and consider a setting in which commitment is voluntary and improperly observed by the buyer. This consideration is explored in Subsection 3.3.

3.1 Model

As before, WT is unconstrained and may set any price in each period. The other, imperfect type (IT) has an exogenously given commitment technology that allows for setting an initial price $p_{1,b}$. In each subsequent period t that price depreciates by an exogenous parameter $\underline{\beta} \leq \beta < 1$, such that $p_{t,b} = \beta^{t-1} p_{t,b}$. A lower bound of is naturally given by the previous section as $\underline{\beta} = \frac{\sqrt{1-\delta}}{1+\sqrt{1-\delta}}$ which corresponds to the price path of the WT if $\alpha = 0$. Parameter β is exogenous and common knowledge. Timing of the game is unaltered.

3.2 Equilibrium

Similarly to perfect commitment, the idea of imperfect commitment is that the monopolist keeps future prices high so that buyers with high willingness-to-pay purchase earlier. We can still use the skimming property.

Lemma 3. *In equilibrium, buyer type x' buys no later than x if $x' \geq x$.*

Analogous to that of Lemma 1 and therefore omitted.

Since some buyers purchase early, profit in the later periods diminishes. WT wishes to increase its profit in these periods and tempted to cut price. A crucial feature of this conjecture is that profit remains positive in later periods. Deviating from the price path of IT provides the same opportunity as in the model with perfect commitment. The WT monopolist is able to reach the Coasian profit in the residual market. Hence, the incentives for price cut are weaker if β is smaller.

Lemma 4. *For any δ there exists a critical value $\beta^*(\delta)$ such that pooling in all periods is an equilibrium of the game if and only if $\beta \leq \beta^*(\delta)$.*

With low β, a WT monopolist does not suffer from high prices in later periods and pooling is subgame-perfect. It is worth to note that lower β

also means that the commitment technology is closer to the price path of the Coase conjecture. The incentive compatibility constraints are less stringent and pretending to be able to commit is easier.

For higher β, better commitment technology means that the IT monopolist is able to attract consumers to buy early by keeping price high, hence, WT is better off with price cutting. If pooling is sustained for $k \geq 0$ periods, buyers keep their *ex ante* beliefs and learn the seller's type in $t = k+1$. The price path splits there and CT, again, falls prey to the Coase conjecture.

Proposition 5. *Suppose $\beta > \beta^*$. In equilibrium, $p_{b,1} = p_{c,1} > 0$ and $p_{c,t}$ is strictly decreasing. All buyers accept an offer in a finite number of periods. The probability of first-period acceptance is a decreasing, and the share of unserved buyers is an increasing function of α.*

The comparative statics show that the probability of price commitment α has a monotonic role in establishing power of the monopolist. Even if it is CT, the firm is able to credibly keep the price high, even in perpetuity if β is sufficiently low.

3.3   Endogenous Commitment

A firm, subject to exogenous restrictions, may decide to sign long-term contracts with suppliers or set inventory. Such a commitment is costly as it entails effort and additional costs. Buyers are aware of this possibility, but they are not able to observe it and form Bayesian beliefs. Suppose the seller is able to randomize between the two options. In a Perfect Bayesian Equilibrium, buyers correctly anticipate the probability of commitment.

We focus on the changes from the baseline model. The monopolist is able to make a binary decision between two options. Option A is imperfect perfect commitment with $\beta < 1$ for a cost c which is drawn from a twice continuously differentiable distribution $F(c)$ with support $[\underline{c}, \overline{c}]$. Option B means that the firm opts for being WT for no additional cost. The choice as well as cost c are not observable by the buyer. At $t = 1$, the choice has already been made and it is not reversible.

At $t = 1$, the choice has already been made and it is not reversible. The ensuring subgame is identical to that of the baseline model, but the conditional probability of commitment is endogenous. The buyer correctly anticipates that she faces IT with probability α. In what follows we derive the range of α that is supported in equilibrium.

As Proposition 5 shows, there is an equilibrium for a given α of the subgame after the commitment choice. Condition (7) defines cost levels by which the monopolist decides to invest,

$$\Pi_b(\alpha, \beta, \delta) + c \geq \Pi_c(\alpha\beta, \delta), \tag{7}$$

in which α expresses the consumers' belief about the probability of commitment. If a critical type c^* exists, it satisfies that investment occurs if and only if $c \leq c^*$. It satisfies that

$$\Pi_b(F(c^*), \beta, \delta) + c^* = \Pi_c(F(c^*), \beta, \delta). \tag{8}$$

Proposition 6. *All seller types opt for no commitment if $\beta < \beta^*(\delta)$. If $\beta \geq \beta^*(\delta)$, there is a unique c^* in equilibrium such that a seller commits if and only if $c \leq c^*$ and the probability of commitment is $\alpha = F(c^*)$.*

That is, the dynamic pricing game with supports endogenous commitment in equilibrium if and only if the commitment technology is sufficiently strong. Weak commitments serve, if costly, does not happen equilibrium as commitment can be imitated.

4 Discussion

Drastic price cuts are sometimes observed in monopoly and oligopoly markets. This belongs to the toolset of pricing strategies and it can be explained by a number of factors including strategic interaction or commitment to capacities. Such cuts may happen randomly, to the surprise of customers. However, the literature offers models that are only able to explain anticipated price cuts. This paper offers a possible answer to the puzzle by building up a simple durable good monopoly model in which commitment to future pricing is private information of the firm.

Price cuts can be explained by uncertain commitment. We show that if it is private knowledge whether the firm can stick to a fix price, a firm that is unable to commit to a certain price sets a high price initially. Capturing the residual market means that buyers learn the commitment type of the monopolist and the one with no commitment power falls prey to the Coase

conjecture. This price cut can be arbitrarily big if buyers are sufficiently patient.

We model the strength of price commitment by that price of the strong type monopolist is discounted by a given parameter $\beta < 1$. The conjecture does not substantially change if the commitment technology is strong enough. For lower values commitment does not deviate sufficiently enough from the Coase conjecture and the weak type monopolist is able to mimic it, there is a pooling equilibrium. This trajectory also holds if commitment is endogenous. In that case, as it can be imitated, the sunk cost of commitment never pays off. However, there is heterogeneity and the share of behavior types may be between 0 and 1 if the commitment technology is strong enough.

This paper contributes to our understanding of pricing strategies of firms with strong market power. Considering imperfect commitment provides a plausible explanation of extreme price changes and uncertain timing. The model is novel in the sense that it links commitment power and the information set of consumers. A straightforward path leading from this point is to incorporate behavioral models of consumer behavior. It is likely that consumer behavior is strongly linked to their level of sophistication as being informed about pricing strategies is still a strong assumption as evidenced by Gabaix and Laibson (2006). Using the same idea about consumers, one may show that acquiring information about firms' commitment decisions can change the optimal pricing of firms and help customers to form higher-quality expectations.

Appendix

Proof of Lemma 1. Suppose that the opposite holds, type x buys in t and x' buys in $t' > t$. There are two possible price trajectories, p_b with probability α and p_c with $1 - \alpha$.

$$\delta^{t'-1}(\alpha(x' - p_{b,t}) + (1 - \alpha)(x' - p_{c,t})) - \delta^{t-1}(\alpha(x' - p_{b,t}) + (1 - \alpha)(x' - p_{c,t})) >$$
$$\delta^{t'-1}(\alpha(x - p_{b,t}) + (1 - \alpha)(x - p_{c,t})) - \delta^{t-1}(\alpha(x - p_{b,t}) + (1 - \alpha)(x - p_{c,t})) \implies$$
$$(\delta^{t'-1} - \delta^{t-1})(x' - x) > 0$$
$$(9)$$

which is contradiction given $\delta < 1$. □

Proof of Proposition 1. Objective function (5) yields the first-order condition

$$0 = 1 - 2Ap_1 + 2\delta Ap_1 - 2\delta \iff p_1 = \frac{1}{2}\frac{1}{A - \delta A + \delta} \tag{10}$$

Since $\frac{\partial A}{\partial \alpha} = \frac{\partial}{\partial \alpha}\frac{1-\alpha\delta}{1-\alpha\delta-(1-\alpha)\delta(1-c)} < 0$ for any δ, we have $\frac{\partial x^*}{\partial \alpha} > 0$.

All buyers are served if the monopolist is CT. Consumers that face ST do not accept p_1 in $t = 2$ if $x < p_1$. The share of unserved buyers is $x^*(1-\alpha) = Ap_1(1-\alpha) = \frac{1}{2}\frac{A}{A-\delta A+\delta}(1-\alpha)$. We have $\frac{\partial x^*}{\partial \alpha} = \frac{\partial}{\partial \alpha}\frac{Ap_1(1-\alpha)}{(1-\alpha)} > 0$, so that commitment increases the probability that a buyer does not accept any offer. □

Proof of Proposition 2. Profit, ST: Taking (5),

$$\frac{\partial \Pi}{\partial \alpha} = \frac{\partial\left[1 - Ap_1 + \delta(Ap_1 - p_1)\right]p_1}{\partial \alpha} = \frac{\partial \frac{3}{4}\frac{1}{A-\delta A+\delta}}{\partial \alpha} > 0. \tag{11}$$

Profit, CT: From the profit $(1 - Ap_1)p_1 + \delta(Ap_1)^2\Pi_c$ we get (12).[6]

$$\frac{\partial \Pi}{\partial \alpha} = \frac{\partial(1 - Ap_1 + \delta A^2 p_1 \Pi_c)p_1}{\partial \alpha} > 0 \tag{12}$$

Welfare: Total surplus equals

$$TS = (1 - x^*)\frac{1 + x^{*2}}{2} + \delta(\alpha(x^{*2} - p_1)\frac{x^{*2} + p_1}{2} + (1 - \alpha)x^{*2}TS_c) \tag{13}$$

Proposition 1 shows that the share of people who buy in the $t = 1$ is decreasing with α.

Suppose $\alpha_1 > \alpha_2$. It is sufficient to show that, if $\alpha = \alpha_1$, no buyer accepts an earlier offer in equilibrium if $\alpha = \alpha_2$ and there is a measurable subset of buyers that postpone it. Case 1: p_1 is accepted if $\alpha = \alpha_2$. Since x^* is an increasing function of α, there is a measurable mass of buyer types that postpone purchase. Case 2: p_1 is accepted if $\alpha = \alpha_2$. Facing ST, $p_{b,2} = p_1$ is accepted. If commitment is not possible, the equilibrium price path is proportional to x^*, so the ranking of periods according to payoff does not change. The buyer either purchases in the same period, or does not accept any offer. Case 3: Offer is not accepted if $\alpha = \alpha_2$. This case follows from Proposition 1. □

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[6]See the web appendix.

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Proof of Lemma 2. The only deviation from the equilibrium we need to examine is that WT sets $p_{b,2} = p_1$, given that ST never makes sales after the second period in equilibrium.

In this case the consumers believe in $t = 2$ that they face ST. The WT monopolist captures the residual market obtaining the discounted the Coasian payoff starting in $t = 3$. Hence, there is an equilibrium if

$$(x^* - p_1)p_1 + \delta(p_1)^2\pi_c \leq (x^*)^2\pi_c \tag{14}$$

which is equivalent to

$$(A - 1 + \delta\pi_c)p_1^2 \leq A^2p^2\pi_c \Leftrightarrow \frac{A-1}{A^2 - \delta} \leq \pi_c. \tag{15}$$

Since $\lim_{\alpha \to 1} A = 1$ for any δ, the left-hand-side of (15) converges to 0 as α approaches 1., so that the critical value exists for any δ. Numerically, an equilibrium exists if and only if

$$\alpha \geq \frac{3\delta^4 - 2\sqrt{1-\delta}\delta^3 - 5\delta^3 + 2\sqrt{1-d}\delta^2 + 2\delta^2}{2\left(-\sqrt{1-\delta}\delta^4 + \delta^4 + \sqrt{1-\delta}\delta^3 - \delta^3\right)} -$$

$$\frac{\sqrt{\delta^8 - 4\sqrt{1-\delta}\delta^7 - 6\delta^7 + 12\sqrt{1-\delta}\delta^6 + 13\delta^6 - 12\sqrt{1-d}\delta^5 - 12\delta^5 + 4\sqrt{1-\delta}\delta^4 + 4\delta^4}}{2\left(-\sqrt{1-\delta}\delta^4 + \delta^4 + \sqrt{1-\delta}\delta^3 - \delta^3\right)}.$$

$$\tag{16}$$

The right-hand side of (16) is increasing and converges to 1 as δ approaches 1, that is, the critical value is always smaller than 1 for any δ on the relevant range. $\qquad\square$

Proof of Proposition 3. Prices $p_{c,t}$ are proportional to x^* as $p_{c,t} = x^*c^{t-1}$ if $t \geq 2$. Since $Ap_1 = x^*$, we have that $\frac{p_{c,2}}{p_1} = c \cdot A$. As $\frac{\partial A}{\partial \alpha} < 0$, the result holds. For the second part we need that $\lim_{\delta \to 1} p_{c,2} = 0$, proved by Coase (1972). Also, A is bounded from below for any δ, which establishes the result. $\qquad\square$

Proof of Proposition 4. The seller sets p_1 and in $t = 2$ learns its type. If it is CT, the subgame is analogous to that of the main model and the Coase conjecture ensures. The buyer's problem remains the same as above,

$$x - p_1 = \alpha(\delta x - \delta p_1) + (1 - \alpha)\delta x(1 - p_1) \Longleftrightarrow Ap_1 = x^*. \tag{17}$$

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The seller solves

$$\max_{p_1}(1 - F(x^*))p_1 + \alpha\delta(F(x^*) - F(p_1)p_1 + (1-\alpha)\delta F(x^*)\Pi_c =$$

$$[1 - (1-\delta\alpha)F(Ap_1) - \delta F(p_1)]\,p_1 + (1-\alpha)\delta F(Ap_1)\Pi_c$$

yielding the first-order condition

$$0 = 1 - (1 - \delta\alpha)\left[\frac{1}{\bar{x}}Ap_1 - \frac{1}{\bar{x}}\right] +$$

$$\delta\frac{1}{\bar{x}}p_1 + F(p_1)) + (1-\alpha)\delta\frac{1}{\bar{x}}A\Pi_c. \tag{18}$$

The RHS of (18) is lower than the RHS of (10) and both are decreasing functions of p_1, which means that the equilibrium p_1 is lower. Since $Ap_1 = x^*$, the residual market and the subsequent prices of an WT monopolist are proportionally lower than in the main model. Since this holds for any given α, Proposition 3 also holds. $\square$

Proof of Lemma 4. We show that pooling for the entire game is not supported in equilibrium. Suppose IT sets $p_{b,1} = p_b$ in $t = 1$ which means that its pricing strategy satisfies $p_{b,t} = \beta^{t-1}p_b$. Assume that $p_{c,t} = p_{b,t}$ and define the critical buyer types by x_t which means that a consumer purchases in t if and only if $x_{t-1} < x \le x_t$, except for period 1, in which $1 \le x \le x_t$. The critical types is

$$x_t - \beta^{t-1}p = \delta(x_t - \beta^t p) \iff x_t = \frac{p(1-\delta\beta)}{1-\delta}\beta^{t-1} \tag{19}$$

which means that the profit in a certain period $t > 1$ equals

$$\delta^{t-1}\beta^{t-1}p(x_{t-1} - x_t) = \delta^{t-1}\beta^{t-1}p(\frac{p(1-\delta\beta)}{1-\delta}\beta^{t-2} - \frac{p(1-\delta\beta)}{1-\delta}\beta^{t-1}) = \frac{\delta^{t-1}p^2(1-\delta\beta)}{1-\delta}(\beta^{2t-3} - \beta^{2t-2})$$
$$\tag{20}$$

and the overall profit starting is

$$(1 - \frac{p(1-\delta\beta)}{1-\delta})p + \sum_{s=2}^{\infty}\frac{\delta^{s-1}p^2(1-\delta\beta)}{1-\delta}(\beta^{2s-3} - \beta^{2s-2}) =$$

$$p + \left[-\frac{(1-\delta\beta)}{1-\delta} + \frac{\delta(1-\beta)(1-\delta\beta)\beta(1-\delta\beta^2)}{1-\delta}\right]p^2 \tag{21}$$

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From the first-order condition, the optimal initial price p is

$$1/\left[2 - \frac{(1-\delta\beta)}{1-\delta} + 2\frac{\delta(1-\beta)(1-\delta\beta)\beta(1-\delta\beta^2)}{1-\delta}\right] = p \qquad (22)$$

A WT monopolist keeps mimicking the IT as long as this price path provides higher payoff than price cutting. If they pool in $t = 1$, pooling is supported in the ensuring subgame if (23) is satisfied.

$$\frac{\delta^{t-1}p^2(1-\beta)(1-\delta\beta)\beta^{2t-3}(1-\delta\beta^2)}{1-\delta} \geq \delta^{t-1}x_t^2\Pi_c = \delta^{t-1}\frac{p^2(1-\delta\beta)^2}{(1-\delta)}\beta^{2t-4}\Pi_c$$

$$\Longleftrightarrow \frac{(1-\delta)(1-\beta)(1-\delta\beta^2)}{(1-\delta\beta)\beta} - \frac{\sqrt{1-\delta}}{(1+\sqrt{1-\delta})^2 - \delta\sqrt{1-\delta}} \geq 0$$

$$(23)$$

For any δ, the left-hand-side of the latter inequality is increasing at β_{min}. We can state that there exists a critical value $\beta^*(\delta)$ such that pooling in all periods is an equilibrium of the game if and only if $\beta \geq \beta^*(\delta)$. $\square$

Proof of Proposition 5. Suppose pooling is sustained for exactly $1 \leq k < \infty$ periods. As in (19), the critical value satisfies

$$x_t - \beta^{t-1}p = \delta(x_t - \beta^t p) \Longleftrightarrow x_t = \frac{(1-\delta\beta)\beta^{t-1}p}{1-\delta} \qquad (24)$$

for $t < k$. For k, we have

$$x_k - \beta^{k-1}p = \delta\left[\alpha(x_k - \beta^k p) + (1-\alpha)(x_k - x_k c)\right] \Longleftrightarrow x_k = \frac{(1-\alpha\delta\beta)\beta^{k-1}p}{1-\delta(1-\alpha c - c)}.$$

$$(25)$$

For $t > k$ the critical value x_t satisfies (24). The residual game always boils down to the situation in Lemma 4, which means, from (23) and the assumption $\beta < \beta^*(\delta)$, that the WT monopolist executes a price cut in period $t = k+1$. Since this is satisfied for any $t > 1$, pooling can only occur in $t = 1$.

Foreseeing the resulting demand implied by the possible price-cut, IT maximizes payoff according to $p_1 = p_{b,1}$

$$\Pi_b = p_1 \left[1 - \frac{(1-\alpha\delta\beta)p}{1-\delta(1-\alpha c - c)} + \delta\beta(\frac{(1-\alpha\delta\beta)p}{1-\delta(1-\alpha c - c)} - \frac{(1-\delta\beta)\beta p_1}{1-\delta}) + \right.$$
$$\left. \sum_{t=3}^{\infty} \delta^{t-1}\beta^{t-1}p_1(\frac{(1-\delta\beta)\beta^{t-1}p_1}{1-\delta} - \frac{(1-\delta\beta)\beta^t p_1}{1-\delta}) \right]$$

$$(26)$$

The first-order condition gives

$$p_1^* = \frac{1}{2} \frac{1}{\frac{(1-\delta\beta)\alpha\delta\beta}{1-\delta(\alpha+(1-\alpha)(1-c))} + \frac{(1-\delta\beta)\beta}{1-\delta} - (\frac{\delta^2(1-\delta\beta)\beta^{t+1}(1-\beta)}{(1-\delta)(1-\delta\beta)})} \qquad (27)$$

The initial price gives the following price trajectory for CT: $p_{c,1} = p_{b,1}$, $p_{c,t} = x_1 c^{t-1}$, where $x_1 = \frac{\alpha\delta\beta p_1}{1-\delta(\alpha+(1-\alpha)(1-c))}$. For CT, pooling in $t = 1$ and price-cutting in $t = 1$ yields

$$\Pi = p_1 \left[1 - \frac{\alpha\delta\beta p_1}{1-\delta(\alpha+(1-\alpha)(1-c))} \right] + \delta \frac{(\alpha\delta\beta p_1)^2}{(1-\delta(\alpha+(1-\alpha)(1-c)))^2}\Pi_c$$

$$(28)$$

which is greater than the Coasian payoff Π_c. The proof of that WT does not try to deviate from the equilibrium and wish to set $p_{c,2} = \beta p_1$ is equivalent to that of Lemma 2. $\qquad\square$

Proof of Proposition 6. Part I: $\beta < \beta^*(\delta)$. In this case, types pool in equilibrium and the left-hand side of Equation (7) is always larger. Hence, no type makes a costly commitment.

Part II: $\beta \geq \beta^*(\delta)$. From (28), we get

$$\Pi_b - \Pi_c = p_1 \left[1 - \frac{\alpha\delta\beta p_1}{1-\delta(\alpha+(1-\alpha)(1-c))} \right] + (\delta\frac{(\alpha\delta\beta p_1)^2}{(1-\delta(\alpha+(1-\alpha)(1-c)))^2} - 1)\Pi_c.$$

$$(29)$$

using Equation (27), the right-hand side of (29) is a decreasing function of α, we can establish that Equation (7) has a unique solution. $\qquad\square$

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