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**MEASURING IMPACT OF UNCERTAINTY IN A STYLIZED
MACRO-ECONOMIC CLIMATE MODEL WITHIN A DYNAMIC
GAME PERSPECTIVE**

By

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Measuring impact of uncertainty in a stylized macro-economic climate model within a dynamic game perspective

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February, 2018

Abstract: In this paper we try to quantify/measure the main factors that influence the equilibrium outcome and pursued strategies in a simplistic model for the use of fossil versus green energy over time. The model is derived using the standard Solow macro-economic growth model in a two-country setting within a dynamic game perspective. After calibrating the model for a setting of OECD versus non-OECD countries we study what kind of uncertainties affect the outcomes of the linearized model most, assuming both countries use Nash strategies to cope with shocks that impact the model. The main outcome of this study is that the parameters that occur in the objective of both players seem to carry the most uncertainty for both the outcome of the model and strategies.

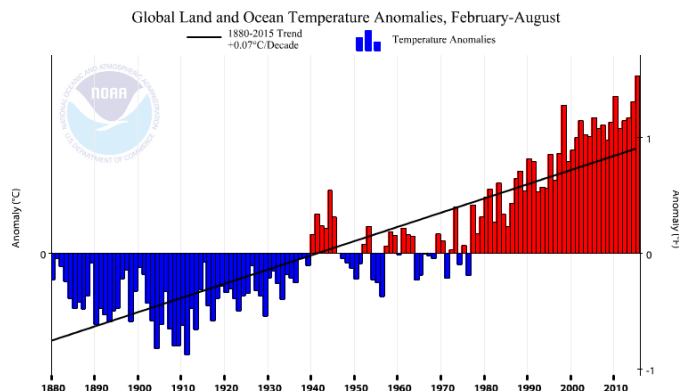
Keywords: differential games; environmental engineering; uncertain dynamic systems; linearization; economic systems; open-loop control systems.

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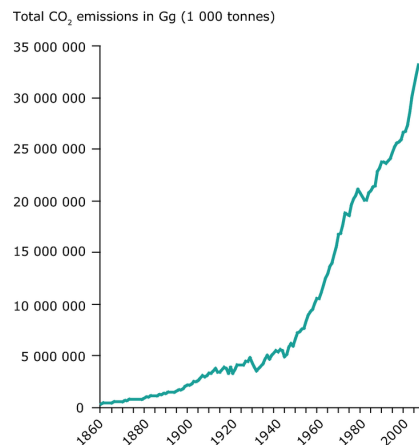
1 Introduction

Climate change is a key topic on the agenda of most of the world's leading presidents. From reports of the European Environment Agency (EEA) and the Intergovernmental Panel on Climate Change (IPCC) the average global temperature is rising. This can be seen in Figure 1a. Here the global land and ocean temperature anomalies are plotted, with 1940 as a base year. From this plot we can see that the average change in temperature per decade is about $+0.07^\circ\text{C}$. According to data from the National Centers for Environmental Information (NCEI), the total CO_2 emission has been increasing exponentially over time, see Figure 1b.

According to the IPCC reports, with 90% probability, a doubling (compared to its value in the year 2000) of CO_2 concentration will lead to an increase of the average world temperature by 1.5°C . This increase will affect all countries. The broadly accepted consensus is therefore that actions are needed to reduce the level of CO_2 emission all over the world. For instance, by using more *green energy* instead of *fossil energy*. However, nowadays fossil fuel reserves are abundant. This means that it is not easy to convince countries to restrict their use of fossil energy and begin to expand their green energy use. This comes with the fact that currently, using green energy is more expensive than using fossil energy. In particular countries which experience a period of economic growth could be rather sceptic about changing their climate policy to a more green policy. They have to invest in green energy resources, which costs money and could deteriorate their economic growth. There are some policies



(a) Global temperature anomalies (EEA)



(b) Global CO₂ emission (NOAA)

Figure 1: Climate facts

that try to mitigate this problem. For instance, introducing a carbon tax, subsidizing the use of green energy and forming coalitions of countries to get cooperation gains. Each of these policies has its advantages and disadvantages. For instance, the disadvantage of a carbon tax is that it will only work well, if it is implemented over the whole world. Next to this comes the difficulty to price this tax for legally emitting CO₂? Another policy is to introduce tradable permits that give companies the right to emit a certain amount of CO₂ per year. But how should these rights be distributed over the world?

With rapid advances in computing power over the last decade, large-scale models have become essential to decision-making in public policy. However there are also risks in using these models. A central issue in the economics of climate change is understanding and dealing with the vast array of uncertainties. These range from those regarding economic and population growth, emission intensities and new technologies, the carbon cycle and climate response, to the costs and benefits of different policy objectives. Most of the time policy makers must make their decisions based on the outcome of a model that assumes a lot of (possibly) uncertain parameters. Typically, some sensitivity analyses on particular parameters are executed to give the policymaker an indication of the uncertainty of the model he deals with. Of course this does not give a good representation of the uncertainties involved into the model. What we often want is to give a measure of uncertainty and to provide a kind of probability distribution for the outcome of the model. This is hardly possible to realize. A more down-to-earth approach is performing an elaborate uncertainty analysis consisting of (see, e.g., [13]): i) **Stochastic parameters**, where parameters are assumed to belong to a set of values and corresponding probability distribution; ii) **Stochastic relations**, where relations are assumed to contain a stochastic element; iii) **Deterministic, worst-case scenario**, where a new variable is added to the system which can be viewed as *nature* that is always counteracting the objective(s); iv) **Scenario analyses**, where scenarios consisting of combinations of different assumptions about possible states of the world are considered. Scenario analysis involves performing model runs for different combinations of assumptions and comparing the results; and v) **Extending**

the model: this means that some parts of the model are reconsidered and extended where necessary.

The main focus of this paper is to see how (a number of) above uncertainty modeling procedures impact the results in a linear quadratic differential game model that analyzes the ratio between fossil energy and green energy use in a macro-economic growth model. The analysis is limited to a two player setting, where we consider the two players to be the OECD and non-OECD countries. Our main object of study are the corresponding steady state equilibrium values of the involved economic target variables and instruments. The benchmark model we use is obtained along the lines of a similar model used by [5] to analyze the impact of pollution over time on the fossil fuel/green energy ratio in a dynamic world characterized by four players that have different interests.

Already several studies exist that try to incorporate uncertainty into energy system models. E.g., [20] presents a framework for determining optimal climate change policy under uncertainty. They use econometric estimates for some parameters, which are then used to solve the model. They compare the results with those derived from an analysis with best-guess parameter values. Their aim is to show that incorporating uncertainty within a climate model can significantly change the optimal policy recommendations. In particular they suggest that analyses which ignore uncertainty can lead to inefficient policy recommendations. [8] look at model and parametric uncertainties for population, total factor productivity, and climate sensitivity. They estimate the pdfs of key output variables, including CO₂ concentrations, temperature, damages, and the social cost of carbon (SCC). They investigate uncertainty of outcomes for climate change using multiple integrated assessment models (IAMs)¹. This multi-model inter-comparison approach is also preferred by [3] and many other research teams. Furthermore, [7] introduce PROMETHEUS, a stochastic model of the world energy system that is designed to produce joint empirical distributions of future outcomes. They represent causal chains for all important variables, with time series analysis for providing patterns of variation over time. [23] investigates the question whether uncertainty about climate change is too large for running an expected cost benefit analysis. The approach he takes is to test whether the uncertainties about climate change are infinite. This is done by calculating the expectation and variance of the marginal costs of CO₂ emissions. In short, he concludes that climate change is an area that tests decision analytic tools to the extreme.

In this paper we differ from the above mentioned papers by several aspects. All models above are trying to quantify uncertainty within an IAM that does not incorporate interrelations between players. The models are developed to optimize a policy for a country, without incorporating the interrelations between countries. We want to investigate which parts of the model (parameters, relations, scenarios etc.) carry the most uncertainty in the model outcomes. That is, we want to get a broad overview of the uncertainty involved, by applying and evaluating multiple approaches as described above. The results can be used to conclude which parts need special attention when calibrating.

The outline of the rest of the paper is as follows. In Section 2, we create our simple dynamic linear two country growth model along the lines of [5] based on the standard Solow growth model introduced by [21]. We integrate the impact of CO₂ emission on economic growth in this model

¹Combination of the scientific and economic aspects of climate change used to assess policy options for climate change. Details can be found in [14].

to get a world energy model. Using an extensive model calibration, we arrive at our benchmark model. In Section 3 we perform some experiments with our benchmark model. This to illustrate the basic operation of the model and explain the outcome of the model by investigating the use of the different forms of energy for both players under different scenarios. Next, in Section 4, we perform an extensive uncertainty analysis of this model. Approaches i), ii) and iv) are used to analyze this impact. Section 5 concludes. The Appendix contains elaborations on several issues.

2 The model

In this section we formulate our benchmark endogenous growth model. The model is based on the standard Solow exogenous growth model introduced by [21]. The model can be obtained along the lines of [5]. Therefore, details are omitted. Also a more extensive discussion on the ins and outs of some equations used can be found there. We consider two countries, identified as the OECD and non-OECD countries from now on. With Y_i denoting the output, F_i the production/use of fossil energy, G_i the production/use of green energy; K_i the amount of capital, L_i the total population, T_i the state of technology, E_i the total CO₂ emission, and A_i measuring the total factor productivity, all in country i , $i = 1, 2$, the basic model equations are as follows,

$$Y_i = A_i K_i^{\alpha_i} L_i^{\beta_i} E_i^{\gamma_i} T_i^{\kappa_i}, \quad \alpha_i, \beta_i, \kappa_i \geq 0, \quad (2.1)$$

$$\dot{K}_i = s_i Y_i + s_{ij} Y_j - \delta_i K_i + \tau_i T_i, \quad (2.2)$$

$$\dot{T}_i = g_i T_i + g_{ij} T_j + \epsilon_i K_i, \quad (2.3)$$

$$\dot{E}_i = \zeta_i F_i + \zeta_{ij} F_j - \xi_i E_i, \quad (2.4)$$

$$\dot{L}_i = \eta_i L_i, \text{ with } j = 2 \text{ if } i = 1, \text{ and } j = 1 \text{ if } i = 2. \quad (2.5)$$

That is, we assume that production is provided by a Cobb-Douglas function (2.1); the change in capital is endogenous and depends on domestic and foreign investment, depreciation of the current capital and domestic technology (2.2); technological progress depends on both domestic and foreign technology and the amount of domestic capital (2.3); the change in CO₂ emission is endogenous too and increases due to domestic and foreign use of fossil fuels and depreciation of the current stock of CO₂ emission (2.4); and labor supply grows at a constant rate η_i (2.5).

Under the assumption that the Cobb-Douglas production functions satisfy constant returns to scale (i.e. $\alpha Y(K, L) = Y(\alpha K, \alpha L)$, or, in this specific case the production function parameters satisfy $\alpha_i + \beta_i + \kappa_i + \gamma_i = 1$), above equations can be rescaled in terms of effective labor. Taking natural logarithms, with $y_i := \log(\frac{Y_i}{L_i})$, $k_i := \log(\frac{K_i}{L_i})$, $t_i := \log(\frac{T_i}{L_i})$, $e_i := \log(\frac{E_i}{L_i})$, $f_i := \log(\frac{F_i}{L_i})$ and $g_i := \log(\frac{G_i}{L_i})$, equations (2.1-2.5) can be rewritten as,

$$\begin{aligned} y_i(t) &= \log(A_i) + \kappa_i t_i(t) + \alpha_i k_i(t) + \gamma_i e_i(t) \\ \dot{k}_i(t) &= -(\eta_i + \delta_i) + e^{-k_i(t)} (s_i e^{y_i(t)} + s_{ij} e^{y_j(t)+t(\eta_j-\eta_i)} + \tau_i e^{t_i(t)}) \\ \dot{t}_i(t) &= -\eta_i + g_i + e^{-t_i(t)} (g_{ij} e^{t_j(t)+t(\eta_j-\eta_i)} + \epsilon_i e^{k_i(t)}) \\ \dot{e}_i(t) &= -(\xi_i + \eta_i) + e^{-e_i(t)} (\zeta_i e^{f_i(t)} + \zeta_{ij} e^{f_j(t)+t(\eta_j-\eta_i)}). \end{aligned} \quad (2.6)$$

Furthermore, with $U := \mu_i Y_i - (F_i + G_i)$ and $E := E_1 + E_2$, we assume both countries like to maximize next total discounted welfare

$$W_i = \int_0^{\infty} e^{-\theta_i t} (-U^2(t) - \pi_i E^2(t) - \rho_i G_i^2(t)) dt. \quad (2.7)$$

Here, μ_i is the proportion of output in country i that can only be produced with the use of energy. In this welfare function the weight of meeting the energy requirements is set equal to 1 in order to emphasize the need for realizing this objective. Factor ρ_i represents the actual disadvantages of using green energy for country i . For instance, when the price of using green energy is higher than the price of using fossil energy. Furthermore, each country has its own availability of resources. It might be difficult to use green energy, because there are no resources in the neighborhood. Factor π_i expresses that the higher the CO₂ emission, the more it is disliked. For instance, emitting lots of CO₂ will entail costs.

Next we calibrate our parameters in the above model (2.6,2.7). We choose to concentrate on the OECD countries and the non-OECD countries as our two parties involved, since information about these groups of countries is widely available. There are two databases where most of the parameters are calibrated from. <http://data.oecd.org> from the OECD and <http://data.worldbank.org> from the World Bank, respectively. For the OECD countries there is a lot of data available. For the non-OECD members this is more problematic. Very small countries often do not have any data available. Since these countries are very small in all aspects concerning the variables involved compared to more developed (higher-income) non-OECD countries, we exclude them from our analysis. So for calibration purposes, we only use information from the higher-income non-OECD countries. A detailed account for the calibrations of key parameters, initial variables and policy parameters can be found in Appendix A. Tables 1-3, below, report the results for the OECD (first row in each table) and non-OECD (second row in each table) countries. Furthermore we used the acronym O (n-O) for the OECD (non-OECD) countries, respectively.

Table 1: Non-spillover parameters

	A	α	β	γ	κ	η	δ	τ	ϵ	ξ
O	2,043	.23	.76	-.021	.027	.0073	.062	.018	.114	.025
n-O	499	.35	.69	-.050	.011	.0073	.075	.031	.031	.025

Table 2: Initial variables calibration

	y	k	t	e	f	g
O	10.55	12.30	5.45	2.29	8.13	6.67
n-O	9.85	10.58	5.18	2.46	8.38	5.89

Assuming that both countries play a non-cooperative open-loop strategy, we next determine the corresponding necessary conditions. This gives rise to a set of non-linear differential equations. Based on our initial variable calibration we determine the steady-state values of the variables satisfying these conditions. These equilibrium values are tabulated (again row-wise for both countries) in Table 4.

Table 3: Policy parameter calibration

	θ	μ	π	ρ
O	.04	1.40	.15	.01
n-O	.06	1.45	.075	.05

Table 4: Equilibrium variables

	y^e	k^e	t^e	e^e	f^e	g^e
O	15.07	29.98	31.47	13.94	9.11	11.89
n-O	16.94	31.64	32.74	14.15	11.79	12.17

Assuming that both countries operate within the neighborhood of above steady-state values, we can approximate the dynamics around this nonlinear model by the next linear model (see Appendix B)

$$\begin{aligned}
y_{li}(t) &= \kappa_i t_{li}(t) + \alpha_i k_{li}(t) + \gamma_i e_{li}(t) \\
\dot{k}_{li}(t) &= \tilde{s}_i(y_{li}(t) - k_{li}(t)) + \tilde{s}_{ij}(y_{lj}(t) - k_{li}(t)) + \tilde{\tau}_i(t_{li}(t) - k_{li}(t)) \\
\dot{t}_{li}(t) &= \tilde{g}_{ij}(t_{lj}(t) - t_{li}(t)) + \tilde{\epsilon}_i(k_{li}(t) - t_{li}(t)) \\
\dot{e}_{li}(t) &= \tilde{\zeta}_i(f_{li}(t) - e_{li}(t)) + \tilde{\zeta}_{ij}(f_{lj}(t) - e_{li}(t)).
\end{aligned} \tag{2.8}$$

The corresponding parameters are provided, row-wise again, for both countries in Tables 5 and 6.

Table 5: Parameter calibration for non-spillover parameters, linearized model

	α	γ	κ	$\tilde{\tau}$	$\tilde{\epsilon}$	ξ
O	0.23	-0.021	0.027	0.0802	0.0256	0.025
n-O	0.35	-0.050	0.011	0.0929	0.0103	0.025

Table 6: Parameter calibration for spillover parameter: \tilde{s} and \tilde{g} , linearized model

$\tilde{s} (*10^{-6})$	O	n-O	\tilde{g}	O	n-O
O	0.0689	0.0196		0.0170	0.0178
n-O	0.0035	0.1210		0.0497	-0.0050

Table 7: Parameter calibration for spillover parameter: $\tilde{\zeta}$, linearized model

$\tilde{\zeta}$	O	n-O
O	0.0068	0.1165
n-O	0.0055	0.0943

The corresponding objective function for both players is obtained then by carrying out a second-order Taylor expansion of the welfare function W_i (2.7). This results in a quadratic cost criterion (see Appendix C for details)

$$\tilde{J}_i := \frac{1}{2} \int_0^\infty [x^T(t) \ u^T(t)] H_i'' \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} dt, \quad i = 1, 2, \tag{2.9}$$

where $x^T = [k_{l1} \ k_{l2} \ t_{l1} \ t_{l2} \ e_{l1} \ e_{l2}]$ is the state variable of our model (2.8); $u^T = [f_{l1} \ f_{l2} \ g_{l1} \ g_{l2}]$ the corresponding control variable and matrix H_i'' is as reported in Appendix C.

3 Benchmark model simulations

In this section we illustrate, by considering a couple of scenarios, how the model (2.6,2.7) will approximately respond if it is out of equilibrium. To that end we will perform two different kind of shocks to the equilibrium, *symmetric shocks* and *asymmetric shocks*. Symmetric shocks are shocks that hit all countries at the same time. Asymmetric shocks are shocks that occur to just one of both countries. Furthermore, we distinguish between two forms of cooperation. We have a *cooperative situation* and a *non-cooperative situation*. In the cooperative situation we discuss a regime where both countries form a coalition. In the non-cooperative situation we discuss the regime where both countries play actions in the Nash sense. We only analyze the impact of an emission shock, as such a shock is mostly related to our control variables.

To perform the simulations, we used the numerical toolbox developed by [16] to solve N -player affine linear-quadratic open-loop differential games. Clearly, the use of open-loop strategies is made to simplify the analysis. A discussion of pros and cons using this setting can be found in, e.g., [5]. In particular we recall some observations from literature suggesting that the difference between open-loop and feedback policies in practice might not be that large (see, e.g., [15],[1]).

3.1 Asymmetric emission shock

We start with an asymmetric positive CO₂ emission shock which hits the non-OECD countries.

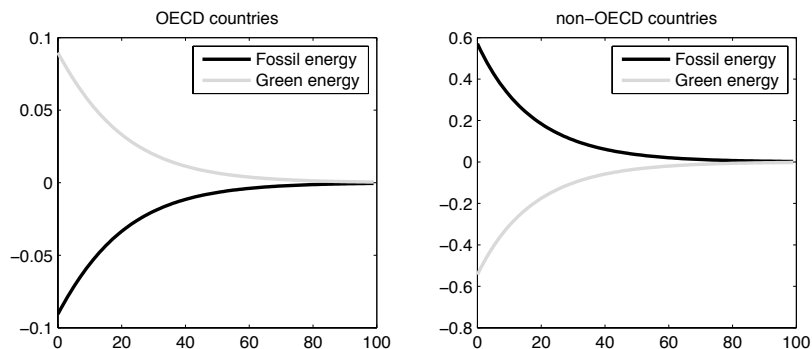


Figure 2: Control variables

Figure 2 shows the response of both countries in a non-cooperative setting in terms of energy consumption, f and g . We see that the non-OECD countries immediately start to use more fossil energy and less green energy. The reason for this is that fossil energy is less expensive than using green energy for them. On the other hand, the OECD countries start to increase their use of green energy and reduce their fossil energy consumption due to the high cost involved on CO₂ emissions for them.

Figure 3 shows the corresponding evolution of capital, technology and stock of emissions for both countries. Note that the line corresponding to capital coincides with the line for technology for both the OECD and non-OECD countries. We see that for both the non-OECD and OECD capital and

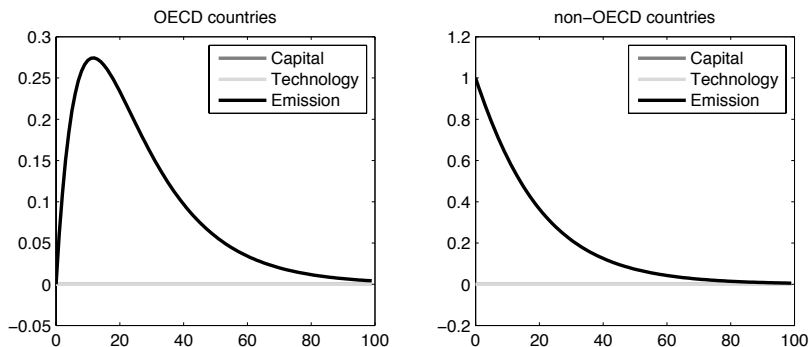


Figure 3: State variables

technology are almost not affected and that the emission shock lags behind in the OECD countries, reaching a peak after approximately 10 years.

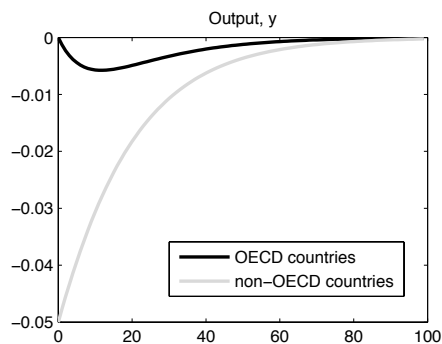


Figure 4: Output variables

From Figure 4 we see that for the non-OECD countries output initially drops significantly. Furthermore, the shock has less impact on output in the OECD countries. Also clear again is the lagged reaction by the OECD countries. The output drop in the OECD countries is caused by the increased use of green energy, needed to compensate the large CO₂ emission in the non-OECD countries. Since this impact on capital and technology growth is rather small, output is affected not too much in the OECD countries.

Next we consider the case how both countries respond if they decide to fight the shock collectively. This is modeled by assuming that control instruments by both countries are determined such that the weighted sum of both welfare functions is collectively minimized. We assumed weights to be equal, i.e. $\frac{1}{2}$. The simulation results are similar to the figures presented above for the non-cooperative case. The only major difference is that all curves reach their equilibrium earlier than in the non-cooperative setting. So we omit the figures and focus on the losses of both countries incurred under the two cooperation scenarios. Table 8 reports them. Here we use the acronym NC (C) for the non-cooperative (cooperative) setting.

We see that cooperation for OECD countries would be profitable. However non-OECD countries do not profit from it. Probably this can be explained by the fact that in a cooperative mode of play

Table 8: Losses under asymmetric shock

	NC	C	Loss reduction (%)
O	4.1684	0.72411	82.6
n-O	1.9403	2.5121	-22.8

more green energy is used to compensate the shock. As this instrument is more costly than using fossil fuels for the non-OECD countries their losses increase.

3.2 Symmetric emission shock

Next we consider a symmetric emission shock. First, we consider a non-cooperative setting again.

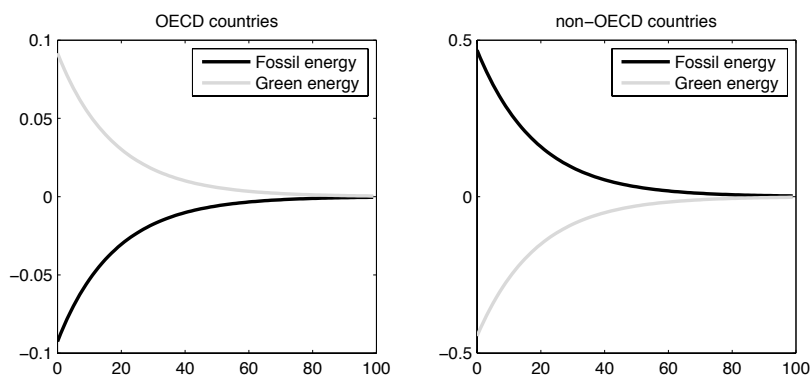


Figure 5: Control variables

Figure 5 illustrates the response of both countries in terms of energy consumption. We see that countries react in the opposite way. The OECD countries increase their green energy use in favor of the fossil energy use, and the non-OECD countries do exactly the opposite. This can be explained by the fact that the OECD countries worry more about climate change, resulting in the fact that it is for them more profitable to use green energy than fossils.

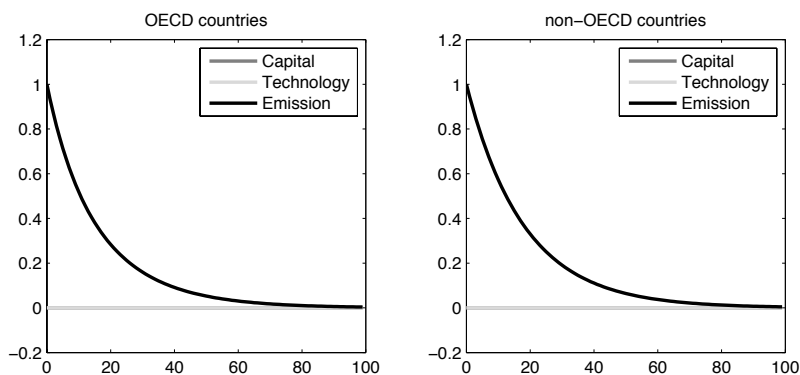


Figure 6: State variables

In Figure 6, we again show the results for capital, technology and stock of emissions. The graphs are similar to the asymmetric shock case. The only difference is that the OECD countries experience

now from the outset on an increase in the stock of emissions.

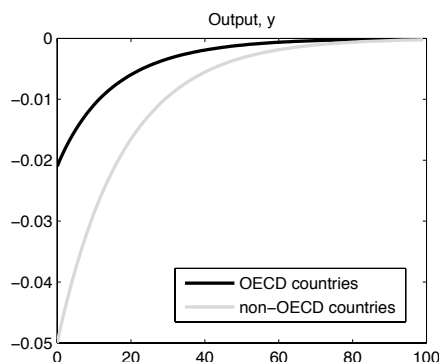


Figure 7: Output variables

Concerning output, we see from Figure 7 that the emission shock hits the non-OECD countries most. Their output decreases more than twice as much as that for the OECD countries. This is likely due to the fact that production in the non-OECD is much more vulnerable to large CO₂ emissions than the production in the OECD countries.

In a cooperative setting we observe the same change in response occurring as in the asymmetric shock case. Again figures are similar to the non-cooperative setting, with again a faster convergence towards the equilibrium values. Table 9 reports losses for both countries and the corresponding cost reduction under both cooperation regimes.

Table 9: Losses under symmetric shock

	NC	C	Loss reduction (%)
O	10.2873	7.7841	24.3
n-O	3.0024	3.4179	-13.8

The first observation is that the losses are higher for both countries now than in the asymmetric shock scenario. This makes sense, as both countries have to deal with an extra shock now. For the rest we see, similar as in the asymmetric scenario, that the OECD countries are the only one to profit from cooperation.

4 Uncertainty analysis

Clearly in arriving at our linear adjustment model (2.8,2.9) several approximations were made and the question is how sensitive results obtained in this linear model are to inaccuracies in the original model specification (2.1-2.5,2.7). In Sections 4.1-4.5 below we consider some potential inaccuracies and analyze how they impact the results presented in the previous section. To that end we distinguish two kinds of impact. The *impact on the equilibrium values* and the *impact on the optimal strategies*.

In Section 4.1, we address the consequences of our assumption that our production function satisfies constant returns to scale. Section 4.2 considers the effects if we assume that some parameters

are stochastically determined. Whereas Section 4.4 assumes realization of capital in (2.2) is stochastic. Finally, in Section 4.5 we consider a scenario where both the initial use of green energy and parameter ρ , which represents the disadvantage of using green energy, are correlated.

4.1 The production function

In this section we reconsider the assumption that the production function (2.1) satisfies constant returns to scale. After calibration it turns out that the sum of the involved parameters, $\alpha_i + \beta_i + \gamma_i + \kappa_i$, equals 0.905 for OECD countries and 1.03 for non-OECD countries. The corresponding tabulated numbers in Table 1 were obtained by normalizing these parameters for both countries. In this section we consider how equilibrium values of (2.6) change if we fix all but one of these parameters to their calibrated value, and estimate the remaining parameter as the difference between one and the sum of the calibrated parameters. That is, if e.g. we calibrated $\alpha_i = \bar{\alpha}_i$, $\beta_i = \bar{\beta}_i$, $\gamma_i = \bar{\gamma}_i$, we fix κ_i at $1 - \bar{\alpha}_i - \bar{\beta}_i - \bar{\gamma}_i$. We calculated for all four possible combinations corresponding equilibrium values of (2.6). Table 10 reports the average of all equilibrium variables for all these four possibilities.

Table 10: Weighted equilibrium variables

	y	k	t	e	f	g
O	16.29	30.18	31.68	14.00	9.38	13.31
n-O	16.82	31.85	32.95	14.22	11.84	11.96

For the non-OECD countries we see that the differences are marginal. For the OECD we see some more substantial differences in output (8%) and energy variables ($f \approx 3\%$, $g \approx 12\%$). The average of all differences for non-OECD variables turns out to be 0.79% and for the OECD countries 4.14%. So, the equilibrium values lie, on average, for the OECD countries within a range of 5% of the variables we used and within a range of 1% of the variables we used for the non-OECD countries.

4.2 Stochastic parameters

Next we consider the case that two of the key parameters in the model, ξ and θ , are in fact only approximately known. More precisely, we add a distribution function to these parameters and simulate 100 realizations based on this distribution.

First, we look at the parameter that indicates the natural decrease of CO₂ emission over time, ξ . We initially assumed, based on Inman [10], a CO₂ lifetime of 30 years for 50% of the CO₂ emission today. The IPCC, on the other hand, estimates a CO₂ lifetime of 50 years for 50% of the CO₂ emission today. This results in a 20 year difference between the two studies. We estimate a distribution to the lifetime of CO₂ emission as shown in Figure 8.

After some lengthy simulations it turns out that apparently this assumption has not a large impact on the resulting value of the objective function. Complementary to this approach we also calculated the equilibrium values for the complete, specified range of CO₂ lifetimes. It turns out that the CO₂ lifetime is not affecting the equilibrium values much. Figure 9, shows the corresponding plot of the equilibrium values for OECD and non-OECD countries.

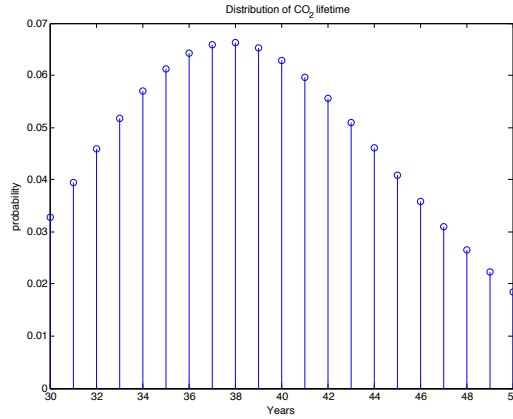
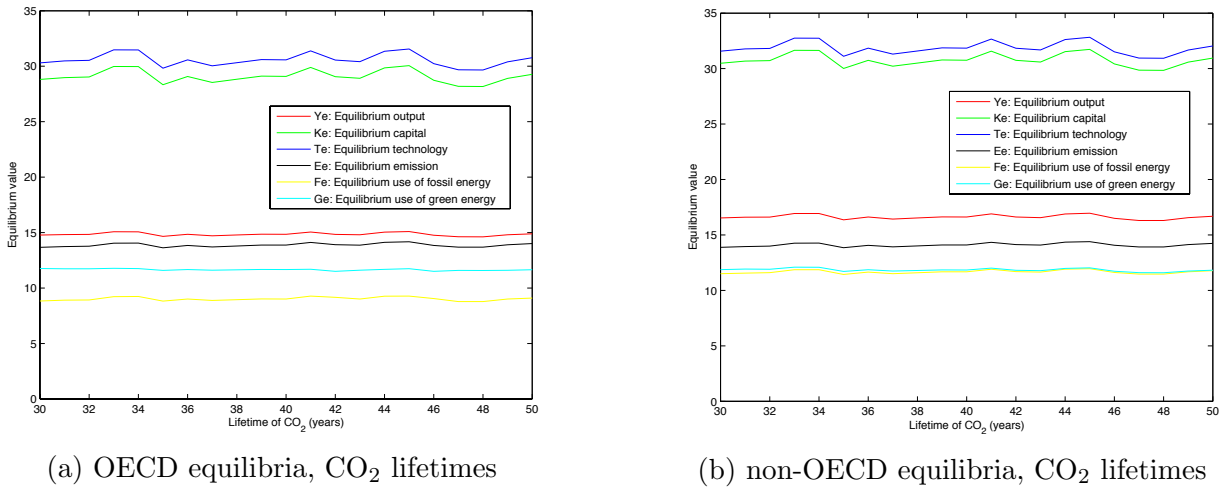


Figure 8: Estimated probability distribution ξ



(a) OECD equilibria, CO₂ lifetimes

(b) non-OECD equilibria, CO₂ lifetimes

Figure 9: Equilibrium values, CO₂ lifetimes

As we can see Figure 9, the equilibrium values are rather constant for the specified range of CO₂ lifetimes. By comparing equilibrium outcomes just for extremal choices of this parameter we get the percentage changes tabulated in Table 11.

y	k	t	e	f	g	Average
3.04%	6.02%	5.74%	2.21%	3.61%	3.26%	3.98%
3.77%	5.69%	5.52%	2.13%	2.97%	4.78%	4.14%

Table 11: %-differences from original equilibrium

We see that the average percentage difference of both countries is around 4%.

Finally, we also determined the effect on the optimal strategies a change in this parameter has. As an example, we plotted in Figure 10 the effect on the optimal control variables if the lifetime of CO₂ is increased from 30 years (*Old*) to 50 years (*New*) in the non-cooperative asymmetric shock benchmark case. We see that both trajectories do not change significantly.

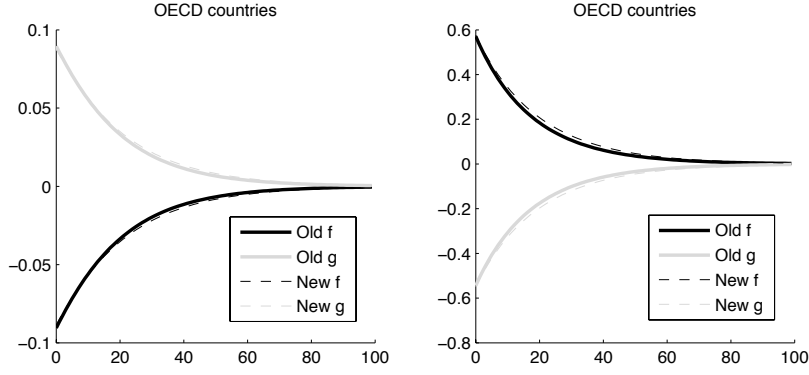


Figure 10: Control variables

Secondly, we investigate the impact of uncertainty with respect to the discount rate, θ . Note that we set the discount rate equal to 4% for the OECD countries and 6% for non-OECD countries. According to data from the Impact Data Source [9], there is a variability with a spread of 1/2 in these numbers. Therefore, we consider the case that the discount rate for OECD (non-OECD) countries is normally distributed with a mean of 4% (6%), and a variance of 0.2%. Simulations using these different discount rates turn out to be cumbersome. Even after quite some iterations there is still a large variability in the outcome of a number of equilibrium variables and value of the objective function. So convergence of the variables is too time consuming to establish in this case. Similar to the previous case we also calculated equilibrium values on a grid where θ_1 ranges between 3.5 and 4.5 and θ_2 ranges between 5.5 and 6.5. Figure 14 shows the corresponding equilibrium values for the green energy consumption. More detailed results are visualized in Figures 17,18 of Appendix D, where we fitted a plane through the equilibrium values for every variable.

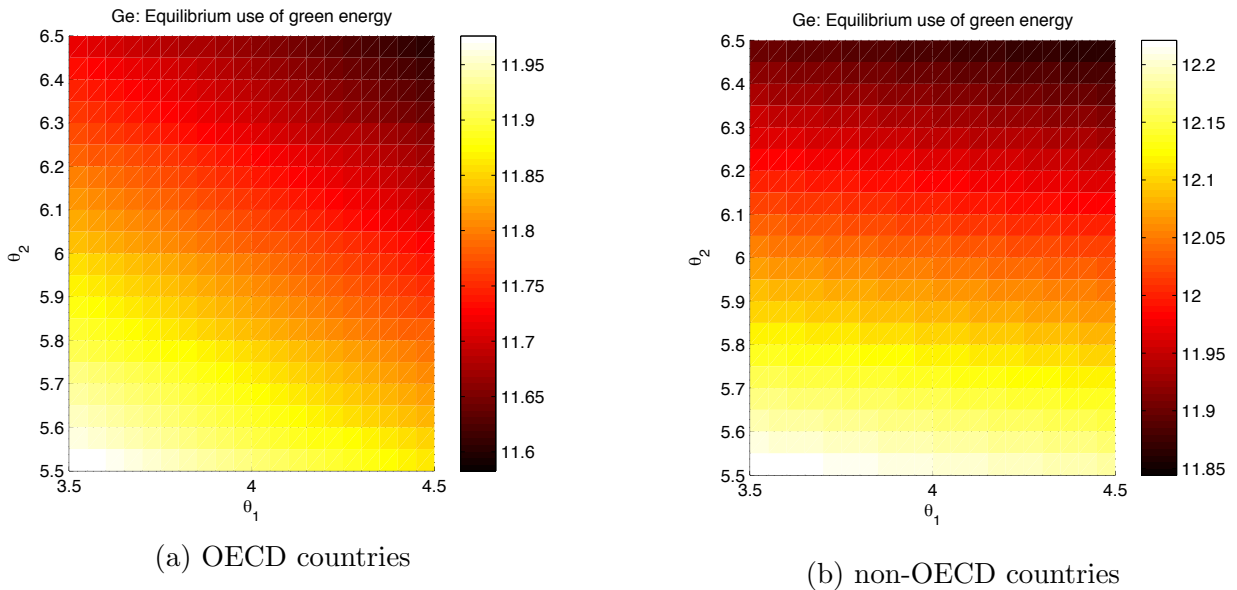


Figure 11: Equilibrium values, green energy use

From this figure we also see that the OECD countries are more influenced for their green energy use, by the discount rate of the non-OECD countries, than vice versa. Similar simulations as for the ξ variable show that changes in the discount factor do not significantly influence the optimal strategies of both players. Finally, Table 12 below shows results of comparing equilibrium outcomes just for extremal choices (within our distributional setting) of the discount factor,

y	k	t	e	f	g	Average
3.02%	6.05%	5.75%	4.50%	6.33%	3.47%	4.85%
3.70%	5.69%	5.51%	4.45%	5.35%	4.43%	4.85%

Table 12: %-differences from original equilibrium

From this we see that the average percentage difference in both countries is just below 5%.

4.3 Stochastic policy parameters

A parameter which turns out to affect results more significantly than those considered in the previous subsection is the parameter π_i in the objective function which measures the *social cost of carbon* emissions (SCC). That is the (monetized) damages associated with excess carbon emissions in a given year (relative to its equilibrium value). We use a distribution for this parameter π_i , based upon the results of Tol [24]. In this paper, 28 studies are listed with all their own estimates of the social cost of carbon. We exclude some of these studies because they present very extreme results and, according to this paper, the used methods to arrive at these results are questionable. Excluding these questionable results and taking a weighted average of the remaining study results in a histogram we plotted in Figure 12. We have divided the results in sectors with a \$15 range. The marginal damages are represented per metric ton of carbon dioxide emission (/tC).

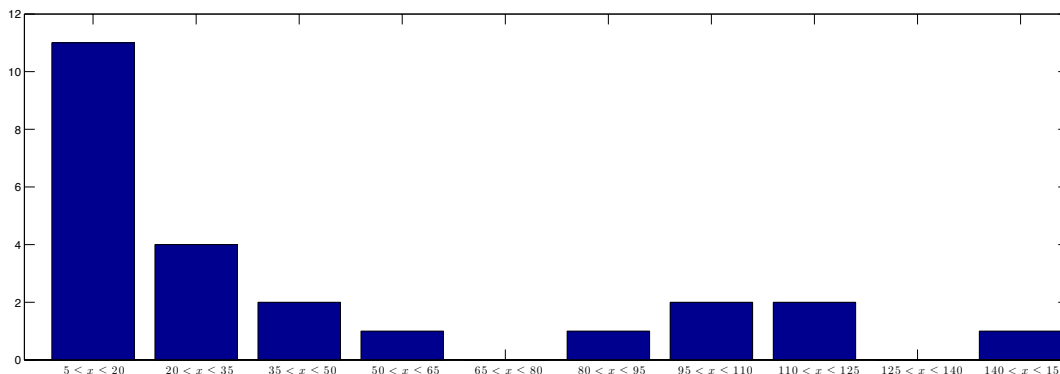


Figure 12: Distribution of the social cost of carbon, according to the studies discussed (\$)

Our initial estimate for OECD countries, $\pi_{\text{oecd}} = 0.15$, is chosen in line with the most occurring range of social costs of carbon, i.e. 5-20\$/tC. Figure 12 shows that all remaining studies estimate this cost to be higher. This means that it could be that we are underestimating this cost by our choice of π_i . We assumed that with every sector, π_i increases by .05 points and sampled 100 values from the distribution implied by the histogram in Figure 12. After some extensive calculations, we obtain the

histograms of equilibrium values plotted in Figure 19. From these histograms, we see that changing this parameter has a major impact on the equilibrium values of e , f and g . To investigate the impact of π_i in more detail we plotted in Figure 20 and Figure 21 equilibrium values for both countries if these parameters are chosen from a grid, where π_{oecd} ranges from 0.15 till 0.65, and $\pi_{\text{non-oecd}}$ from 0.075 till 0.575. The graphs show also some outliers for the variables y , k and t . Some further simulations show that this is due to slow convergence of parameters. Figures 22 and 23 visualize this impact from a different perspective. Here for the e , f and g variables we plotted equilibrium outcomes for a fixed $\pi_{\text{non-oecd}}$ and varying π_{oecd} . We observe a clear non-linear (probably, quadratic, but we did not test for this in detail) response. Furthermore, they illustrate that for a fixed $\pi_{\text{non-oecd}}$, these equilibrium variables change much more for OECD countries than for non-OECD countries if π_{oecd} changes (as to be expected).

Remark 4.1 The EU has set itself a long-term goal of reducing greenhouse gas emissions by 80-95% when compared to 1990 levels by 2050. If we assume that we must reach the average of this long-term goal (87.5% reduction), we can reverse our calculations to obtain that this level is reached when $f < 6.05$. Iteratively calculating the equilibrium values using different values for π , we conclude that we reach this f if $\pi = [0.2, 0.125]$ (see also Figure 22). From Figure 12 it follows that this corresponds to an SCC of approximately 20\$/tC. So, if a country is planning to tax CO₂ emission, then according to this model, they should consider a carbon tax of approximately 20\$/tC to reach the European Climate targets.

To visualize the impact on strategies and state trajectories, we plotted in Figure 13,14a and 14b for the asymmetric shock these trajectories in case $\pi = [0.65 \ 0.575]$. For comparison reasons we also included the corresponding benchmark plots. From 14a we see that the emission peak has drastically decreased, whereas the capital and technology variables are still negligible in comparison to the emission variable. Figure 14b shows that output is stabilized much faster towards its equilibrium value. From Figure 13 we see that this is due to the fact that the non-OECD countries react on the shock now by substituting less green energy for fossils. This makes that the shock is less severe for OECD countries who, therefore, need less control policies to mitigate its impact.

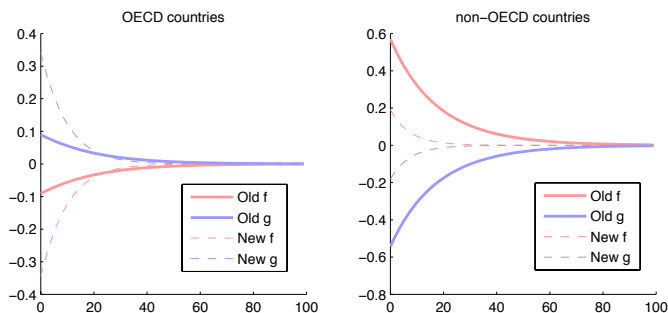


Figure 13: Control variables, simulation with π

4.4 Stochastic relations

One of the equations which might be oversimplified is the relation of the accumulation of capital. Therefore it seems reasonable to include some uncertainty in the proposed equation. As we do not

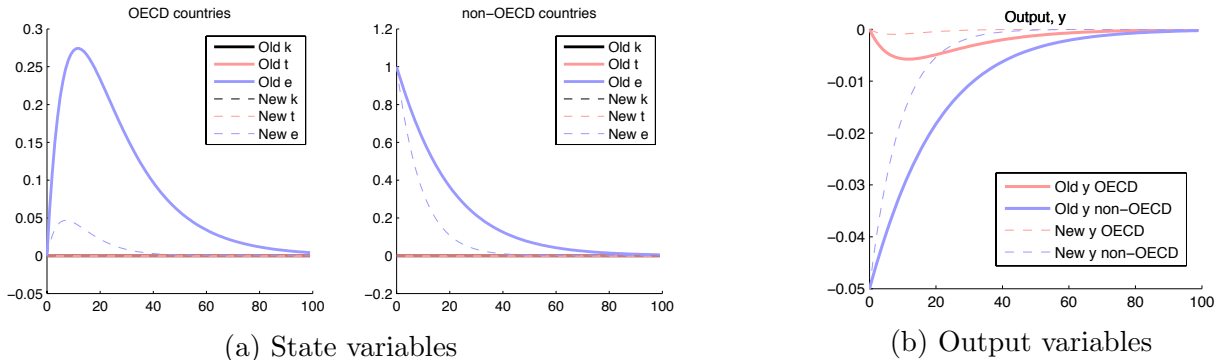


Figure 14: State and output variables, simulation with π

know much about the involved uncertainties we assume that these are normally distributed, with mean zero and variance 0.2, and lie within a 1% range from the initial capital values. In our case, 1% of initial capital is about 0.123. Basically, we use then this “restricted” normal distribution for sampling. So, capital accumulation, \dot{k}_i , is assumed to be generated by the next equation

$$\dot{k}_i = -(\eta_i + \delta_i) + e^{-k_i(t)} (s_i e^{y_i(t)} + s_{ij} e^{y_j(t) + t(\eta_j - \eta_i)} + \tau_i e^{t_i(t)}) + \Lambda,$$

where for every simulation constant Λ is drawn once from the *restricted* normal $N(0, 0.2)$ distribution. Roughly spoken we observe that under this assumption equilibrium values, values for the objective function and strategies are affected similarly as in the considered stochastic parameter context. See, Figures 24,25 in Appendix D. Analogously to the stochastic parameter case, we also considered the realization of the equilibrium values if we vary λ between ± 0.123 . Figure 15 shows the results.

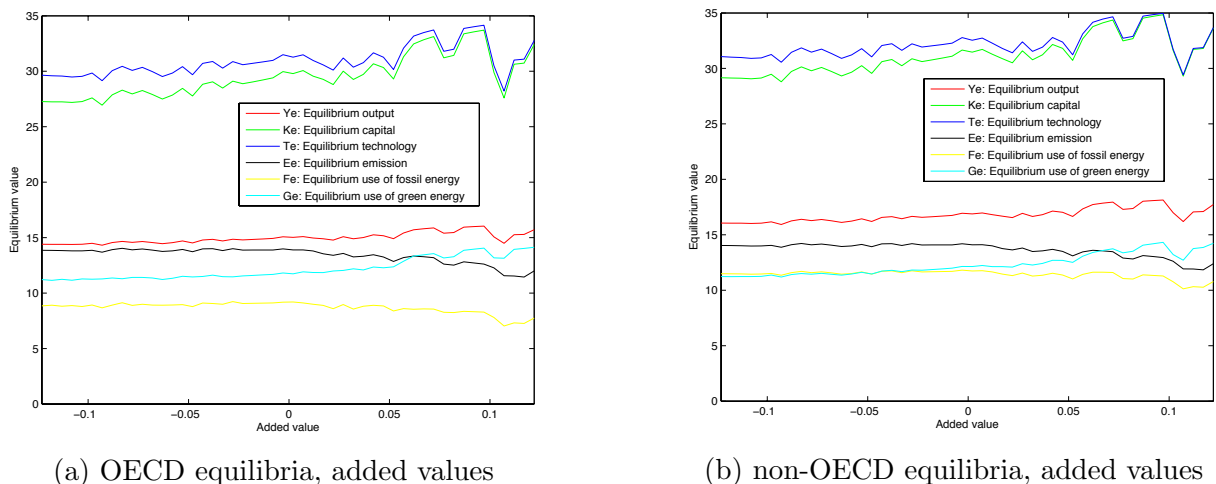


Figure 15: Equilibrium values, added values

We see that if we increase Λ , equilibrium capital value increases. This is reasonable, since capital accumulation is increased at any point in time with a constant. Therefore, equilibrium output increases too (see production function). If capital grows faster, we need less energy to increase our capital and technology. This causes fossil energy use to decrease and green energy use to increase if we increase Λ . Furthermore, we see that at the very extreme high added values ($\Lambda > 0.05$), the

equilibrium values start to be very volatile. The reason for this is probably that the equilibrium is not yet converged. Again, by considering a worst-case scenario from a noise realization perspective, we like to quantify the involved uncertainty. Therefore, we look at the maximal, absolute percentage difference of the equilibrium values of the simulated relation and the original equilibrium. We restrict this simulation to the interval $[-0.05, 0.05]$. The reason to restrict to this interval we just discussed above. The results are tabulated in Table 13.

y	k	t	e	f	g	Average
5.01%	10.12%	7.36%	4.87%	6.04%	6.27%	6.61%
5.95%	9.01%	6.67%	4.61%	4.34%	8.05%	6.44%

Table 13: %-differences from original equilibrium

We conclude that the average percentage difference in both countries is around 6.5%.

4.5 Scenario analysis

In this section we want to investigate the impact of considering a larger value for the initial use of green energy in both countries. However, using more green energy, in percentage, will typically be an outcome of good availability of resources and a smaller price for it. This can be modeled by assuming that the parameter that represents the disadvantages of using green energy, ρ_i , becomes smaller. Based on some calibrations, we estimated that to investigate a 5% increase in the use of green energy this should be accompanied by a decrease of parameter ρ_i by 5%. We calculated the new equilibrium variables under this scenario and found that this adjustment has no large impact on them. What attracts attention is that all equilibrium values changed in a negative direction. The interpretation of this is that if we change our initial green energy use towards its original equilibrium value and decrease, for instance, the difficulty of accessing the green energy market, all variables are converging to an equilibrium value that is lower than the one we had in the beginning.

Furthermore, this scenario has no impact on the optimal strategies. Also for this scenario analysis we calculated, for all possible combinations of ratio's between 0 and 5% for both countries, corresponding equilibrium outcomes. This means that we look at the equilibrium results where the initial f and g are changed. We determined all equilibrium values when initial values of fossils varies between 76.1% – 81.1% for OECD countries and between 87.3% – 92.3% for non-OECD countries, respectively. Next we fitted a plane through these values. These planes are shown in Figures 26,27 of Appendix D. For the worst-case scenario the results are tabulated in Table 14.

y	k	t	e	f	g	Average
3.08%	6.19%	5.88%	4.70%	7.06%	2.26%	4.86%
3.77%	5.83%	5.62%	4.58%	5.63%	3.48%	4.82%

Table 14: %-differences from orig. equilibrium

From this table we see that the maximal percentage difference of both countries is, on average, just below 5%.

5 Concluding remarks

In this paper we consider a simplistic model that analyzes the ratio between fossil energy use and green energy use within a context of OECD and non-OECD countries. For that purpose we developed, starting from some basic economic relationships, a growth model including both these factors. As this model is highly non-linear, we determined for this model its equilibrium points, under the assumption that both players want to maximize their welfare. To see how both players will react to distortions, we derived the corresponding linear dynamics around the equilibrium. Some shock simulations with this benchmark model turn out to provide results that are not too unrealistic. We also considered the question if a coalition of OECD countries and non-OECD countries could be profitable for both countries. It turns out that this is not the case. The non-OECD countries will in general not profit from this, where the OECD countries will.

Given the large number of uncertainties involved in modeling this kind of problems, the next step was to perform an extensive uncertainty analysis. We found that small changes to the parameters used in the dynamics of the model do not affect the outcome of the model much. Adding, for instance, stochastics to such a particular parameter carries in the worst-case, on average, 5% uncertainty in the equilibrium values. If we add stochastics to a complete state equation, we get on average an extra 6.5% uncertainty in the equilibrium values. This means that changing the set-up of one of the state equations in our model with a small amount, has a larger impact on the outcome of the model than changing the parameters within this state equation with a small amount.

So far, the uncertainty involved seems to have no direct effect on the optimal strategies of both players after an emission shock. However, we also investigated the uncertainty involved in the parameters that occur in the objective function of both players. In particular, we investigated the effect on the outcome of the model by changing the preference rate for emitting CO₂. This parameter seems to have a larger effect on the optimal strategies than we just discussed. This effect is most visible in getting faster/slower back to the equilibrium of variables. This is likely due to a changed use in as well amounts of energy as the ratio of used fossil/green energy. The structure of the paths of the variables stays approximately the same, only the time it takes to get back to the equilibrium decreases/increases.

In Table 15, a short overview is given where the approximate uncertainty is tabulated for each analysis. This uncertainty is divided in uncertainty on the equilibrium value and uncertainty on the optimal strategies of both players.

	Equilibrium	Strategies
Shares of income	$\approx 5\%$	$\approx 0\%$
Parameter (dynamics)	$\approx 5\%$	$\approx 0\%$
Parameter (objective)	$\approx 50\%^2$	$\approx 75\%^3$
Relation	$\approx 6.5\%$	$\approx 0\%$
Scenario	$\approx 5\%$	$\approx 0\%$

Table 15: Overview of the uncertainties

What we can conclude is that the calibration of the parameters that occur in the objective of the players need special attention. These parameters carry the most uncertainty for the outcome of the model. Both in the equilibrium and in the optimal strategies. Secondly, we see that the structure of the optimal strategies after an emission shock occurred, does not vary much if we perform some uncertainty analyses. Changing the parameters of the objective does not affect the path of the variables much. It only changes the size of the reaction of both players. The direction seems to be very stable against the uncertainty analyses performed.

Potential lines for further research could be to extend the uncertainty analysis with a worst-case scenario analysis. This will give some extra insights in the total uncertainty involved. Furthermore, we now have performed several uncertainty analyses separate from each other. This can be extended to analyses, where different uncertainty analyses are combined.

Appendix A: Calibrations of parameters and initial values

Non-spillover parameter calibration

A: Total factor productivity, A , is in fact the last parameter we calibrate. First all other initial values of the variables in our model are calibrated. Finally A is taken such that the production function applies.

α : For the OECD-countries the GDP in 2014 was about 49.289.717 million dollars. In the same year, the total investment in capital was about 10.111.756 million dollars. If we divide those numbers, we get the fraction of GDP that is invested in capital, which gives us a good estimate for α_{oecd} . Similarly, for non-OECD-countries total GDP in 2014 was about 33.721.083 million dollars and total investment in capital was about 12.093.681 million dollars. Again the quotient gives us an estimate for $\alpha_{\text{non-oecd}}$.

β : The labor share in income, β_i , is estimated in the same way. For the OECD-countries the gross national income per capita in 2013 was about 38.213 US dollars. In the same year, the disposable income per capita was about 26.500 US dollars. If we divide those numbers, we get the fraction of labor income to total income, which gives us a good estimate for β_{oecd} . Similarly, for the non-OECD-countries the gross national income per capita in 2013 was about 21.082 US dollars and disposable income per capita was about 15.000 US dollars. The quotient gives us the estimate for β_{oecd} .

γ : This calibration is based upon Tol [22]. He estimates the climate change damage for several regions of the world. To get the results for our two countries, we use appropriate weights and calculate the weighted emission share of income.

κ : For the OECD-countries the percentage of the GDP that is spent on research and development (R&D) in 2014 was about 2.4%. This is used as an estimate for κ_{oecd} . For the non-OECD-countries

²Based upon the average difference of the most extreme value of π . For details we refer to Section 4.3.

³Based upon the maximal *absolute* change between strategies. As discussed earlier the change in the structure is approximately 0%.

this percentage was about 1.1%.

η : In this paper we restrict our analysis to the so-called, high-income non-OECD members. As low-income countries have a (relative) small impact on global CO₂ emission. Figure 16 shows for both countries the population growth. From this we observe that the assumption that both growth rates coincide is reasonable. We choose η_i equal to the data of 2014, i.e., $\eta_i = 0.73\%$.



Figure 16: Population growth rate for OECD and (high-income) non-OECD members

δ : There is a lot of variety in the service life of different forms of capital. So it is difficult to catch this depreciation of capital in one number. We use the percentage of 6.24%, as obtained by Oulton et al. [19]. This number is based upon a weighted average for OECD countries. Based on data from the World Bank [25], we assumed depreciation rate of natural capital for non-OECD countries to be 20% higher than the rate of the OECD countries.

τ : According to Claassen’s logarithmic law of usefulness [4], $Y_i = \log_{10}(T_i)$. As α_i is the capital share in income we have $K_i = \alpha_i \cdot \log_{10}(T_i)$. So the slope of this function at T_i is an estimate of τ_i . From the literature we recall that $T = 5$ is approximately an average starting point of technology. Using this number, yields then the tabulated estimate of τ_i .

ϵ : From data from the World Bank [25], we obtain that the expenditures on research and development as a percentage of GDP are 2.4% (1.1%) for OECD countries (non-OECD countries). So, $T_{\text{oecd}} = 0.024 \cdot Y_{\text{oecd}} = 0.024 \cdot \frac{K_{\text{oecd}}}{\alpha_{\text{oecd}}} = \epsilon_{\text{oecd}} K_{\text{oecd}}$. And similarly for $\epsilon_{\text{non-oecd}}$.

ξ : Until now there is no consensus about the exact lifetime of CO₂ in our atmosphere. For instance, the IPCC estimates it around hundred years, where other studies start at 30 years. Based upon Inman [10], we use a CO₂ lifetime of 30 years for 50% of the CO₂ emission today. The rest

of the CO₂ emission remains more than several hundreds of years in the atmosphere. Including this will not affect the model significantly, so we omit it. This means that each year about 2.5% leaves the atmosphere without human intervention.

Spillover parameter calibration

s: Direct saving rates of capital in the country itself, s_{ii} , are estimated based on data of the World Bank on gross national savings as a percentage of GDP. In 2013 these numbers are 20.4% (29.4%) for OECD countries (high-income non-OECD countries). For the cross-terms we use the corresponding data for Foreign Direct Investment (Net inflows) as a percentage of GDP for both countries. The results are shown in Table 16.

Table 16: Parameter calibration for spillover parameter s and g

s	O	n-O	g	O	n-O
O	0.204	0.009		0.017	0.005
n-O	0.055	0.294		0.177	-0.005

g: We estimate domestic technological progress due to the domestic state of technology by the growth of the number of researchers in R&D from the World Bank [25]. The increase in domestic technological progress due to foreign technology use is estimated by dividing the amount of high-technology exports by the domestic country's GDP (see Table 16).

ζ: The increase of CO₂ emission due to the domestic use of fossil fuels is assumed to be proportional to the amount of used fossil fuels. We set the proportion of CO₂ emission due to fossil energy use for non-OECD countries to 1.00 and for OECD countries to 0.85. We base these numbers on the engagement and implementation of CO₂ emission reducing production techniques. Furthermore, we base this on the number of international climate partnerships⁴. The cross-terms are obtained by the fact that CO₂ emission affects all countries at the same time. So the increase of CO₂ emission in the OECD countries from the used fossil fuels in non-OECD countries is just the same number as for the non-OECD countries themselves, so $\zeta_{\text{oecd,non-oecd}} = 1.00$. Similar for $\zeta_{\text{non-oecd,oecd}}$. The results are shown in Table 17.

Table 17: Parameter calibration for spillover parameter: ζ

ζ	OECD	non-OECD
OECD	0.85	1.00
non-OECD	0.85	1.00

⁴For instance, UN Framework Convention on Climate Change, International Convention on the Prevention of Pollution from Ships, Global Methane Initiative, Global Data Center Energy Efficiency Task Force, Carbon Sequestration Leadership Forum etc.

Initial variable calibration

All initial values for the variables we use in our model are expressed per (working) capita. For the number of working people in OECD (non-OECD) we used the number 837,816,057 (227,833,932).

y: This is based on GDP per capita for 2014 data from the World Bank [25], see Table 18. Our initial value is the logarithm of these numbers, as shown in Table 2.

k: Capital per capita for both countries is based on the capital intensities of both countries. We use the results of Berlemann et al. [2]. However, these results are calibrated for the year 2000. So we have to multiply these numbers with the average price increase in the period 2000-2014. In this time period, the prices increased with about 37.5%. The result is shown in Table 18. Again, by taking the logarithm of these numbers we obtain our initial estimate of k , as shown in Table 2.

t: t is calibrated using the total number of researchers (FTE)⁵ in 2013. For the OECD countries there were about 4,403,168 FTE researchers and for non-OECD, there were about 2,111,638 FTE researchers. We multiply these numbers with the gross average wage in the corresponding country. For OECD countries this wage is equal to \$44,290 and for the non-OECD countries the gross average wage is equal to \$19,077 (see the World Bank [25]). The numbers (per capita) are provided again in Table 18. Again, by taking the logarithm of these numbers we find our initial estimate of t , shown in Table 2.

e: The initial CO₂ emission per capita is calibrated using information from the World Bank [25]. In the last column of Table 18, the total CO₂ emission in metric tons per capita in 2014 is represented. This is based on data from 2011 and on the CO₂ emission accumulation from the data bank of the OECD [18]. According to this database the CO₂ emission from 2011 till 2014 has not changed significantly. Therefore the results of the 2011 database are used as an initial estimate for E/L. taking the natural logarithm gives us the estimates for e , stated in Table 2.

Table 18: Variable calibration

	GDP/L	K/L	T/L	E/L
OECD	\$38,35	219,56	232.77	9.9
non-OECD	\$19,04	39,42	176.81	11.7

f, g: To calibrate the variable f we need an estimate of the total energy used in each country, and the percentage of fossil energy from this total energy consumption. We assume that the remainder is green energy used. We use data from 2013 of the World Bank [25]. The corresponding estimates are given in Table 19. Again taking the natural logarithm gives us the estimates for f and g .

Policy parameter calibration

θ : Calibrations of the discount factors are based upon suggestions from the Impact Data Source [9]. We assume that the OECD countries are more interested in future developments than the non-OECD

⁵Full Time Equivalent: somebody who spends 40% of his time on for instance R&D is counted as 0.4 FTE.

Table 19: Energy consumption data

	Energy/L	% Fossil	F/L	G/L
OECD	4,17	81.1%	3,39	789
non-OECD	4,71	92.3%	4,35	363

countries, see Table 3.

μ : We calculate this number from our base year, that is: $\mu_i = \frac{f_i + g_i}{y_i(2014)}$.

ρ : We assume that it is five times more difficult to use green energy in non-OECD countries than in OECD countries (see Table 3). This is inspired by next observations. The price of using green energy in non-OECD countries is higher than the price of using fossil energy. Each country has its own availability of resources, it may be difficult to use green energy, because there might be no resources in the neighbourhood. This is confirmed by the International Institute for Environment and Development (IIED) [12], who concluded that a lot of non-OECD countries have little access to the green energy market. Furthermore, adapting green energy into their system seems still very difficult to achieve. According to data from the World Bank [25], about 92% of the energy used in non-OECD countries is fossil energy, where this percentage for OECD members lies just above 80%. This strengthens our decision to give a higher weight to the non-OECD countries. Finally, we keep this factor rather low, compared to the weight of meeting the energy requirements (which is 1), in order to give these energy requirements the highest priority.

π : In Table 3 one can find the weight of the costs of CO₂ emission we choose. Again, since we want that the energy requirements are (almost) certainly met, we give weights that are small compared to the weight of meeting the energy demand (which is 1). Furthermore, we express the fact that OECD countries have a higher preference for using green energy by giving them a weight which is twice as high as for the non-OECD countries.

Appendix B: Linearization of the model

Consider next deviations of variables from their equilibrium value: $y_{li}(t) := y_i(t) - y_i^e$, $k_{li}(t) := k_i(t) - k_i^e$, $t_{li}(t) := t_i(t) - t_i^e$ and $e_{li}(t) := e_i(t) - e_i^e$. Using these variables we rewrite the model equations (2.6) as follows.

- $$y_{li}(t) = y_i(t) - y_i^e = \log(A_i) + \kappa_i t_i(t) + \alpha_i k_i(t) + \gamma_i e_i(t) - (\log(A_i) + \kappa_i t_i^e + \alpha_i k_i^e + \gamma_i e_i^e)$$

$$= \kappa_i t_{li}(t) + \alpha_i k_{li}(t) + \gamma_i e_{li}(t)$$
- $$\dot{k}_{li}(t) = \frac{d}{dt}(k_i(t) - k_i^e) = \dot{k}_i(t) = \frac{L_i(t)}{K_i(t)} \cdot \frac{\dot{K}_i(t)}{L_i(t)} - \eta_i$$

$$= \frac{L_i(t)}{K_i(t)} \frac{1}{L_i(t)} \left(s_i Y_i(t) + \sum_{j \neq i} s_{ij} Y_j(t) - \delta_i K_i(t) + \tau_i T_i(t) \right) - \eta_i$$

$$= -(\delta_i + \eta_i) + s_i \frac{L_i Y_i}{K_i L_i} + \sum_{j \neq i} s_{ij} e^{t(\eta_j - \eta_i)} \frac{L_i Y_j}{K_i L_j} + \tau_i \frac{L_i T_i}{K_i \cdot L_i}$$

Next we use the MacLaurin series expansion of $\log(x)$ around x^e : $x \approx x^e + x^e(\log(x) - \log(x^e))$, to approximate above expression,

$$\begin{aligned}
&\approx -(\delta_i + \eta_i) + s_i \left(\frac{L_i^e Y_i^e}{K_i^e L_i^e} + \frac{L_i^e Y_i^e}{K_i^e L_i^e} (\log(\frac{L_i Y_i}{K_i L_i}) - \log(\frac{L_i^e Y_i^e}{K_i^e L_i^e})) \right) + \\
&\quad \sum_{j \neq i} s_{ij} e^{t(\eta_j - \eta_i)} \left(\frac{L_i^e Y_j^e}{K_i^e L_j^e} + \frac{L_i^e Y_j^e}{K_i^e L_j^e} (\log(\frac{L_i Y_j}{K_i L_j}) - \log(\frac{L_i^e Y_j^e}{K_i^e L_j^e})) \right) + \\
&\quad \tau_i \left(\frac{L_i^e T_i^e}{K_i^e L_i^e} + \frac{L_i^e T_i^e}{K_i^e L_i^e} (\log(\frac{L_i T_i}{K_i L_i}) - \log(\frac{L_i^e T_i^e}{K_i^e L_i^e})) \right) \\
&= -(\delta_i + \eta_i) + s_i \left(e^{y_i^e - k_i^e} + e^{y_i^e - k_i^e} (\log(\frac{L_i Y_i}{K_i L_i}) - \log(\frac{L_i^e Y_i^e}{K_i^e L_i^e})) \right) + \\
&\quad \sum_{j \neq i} s_{ij} e^{t(\eta_j - \eta_i)} \left(e^{y_j^e - k_j^e} + e^{y_j^e - k_j^e} (\log(\frac{L_i Y_j}{K_i L_j}) - \log(\frac{L_i^e Y_j^e}{K_i^e L_j^e})) \right) + \\
&\quad \tau_i \left(e^{t_i^e - k_i^e} + e^{t_i^e - k_i^e} (\log(\frac{L_i T_i}{K_i L_i}) - \log(\frac{L_i^e T_i^e}{K_i^e L_i^e})) \right) \\
&= \tilde{s}_i \left(\log(\frac{L_i Y_i}{K_i L_i}) - \log(\frac{L_i^e Y_i^e}{K_i^e L_i^e}) \right) + \sum_{j \neq i} \tilde{s}_{ij} e^{t(\eta_j - \eta_i)} \left(\log(\frac{L_i Y_j}{K_i L_j}) - \log(\frac{L_i^e Y_j^e}{K_i^e L_j^e}) \right) + \\
&\quad \tilde{\tau}_i \left(\log(\frac{L_i T_i}{K_i L_i}) - \log(\frac{L_i^e T_i^e}{K_i^e L_i^e}) \right) + \underbrace{\left(-(\delta_i + \eta_i) + e^{-k_i^e} (s_i e^{y_i^e} + \sum_{j \neq i} s_{ij} e^{y_j^e + t(\eta_j - \eta_i)} + \tau_i e^{t_i^e}) \right)}_{= 0, \text{ see the conditions from which the equilibrium values are obtained.}}
\end{aligned}$$

$$\begin{aligned}
&= \tilde{s}_i \left(\log(\frac{Y_i}{L_i}) - \log(\frac{Y_i^e}{L_i^e}) - (\log(\frac{K_i}{L_i}) - \log(\frac{K_i^e}{L_i^e})) \right) + \\
&\quad \sum_{j \neq i} \tilde{s}_{ij} e^{t(\eta_j - \eta_i)} \left(\log(\frac{Y_j}{L_j}) - \log(\frac{Y_j^e}{L_j^e}) - (\log(\frac{K_i}{L_i}) - \log(\frac{K_i^e}{L_i^e})) \right) + \\
&\quad \tilde{\tau}_i \left(\log(\frac{T_i}{L_i}) - \log(\frac{T_i^e}{L_i^e}) - (\log(\frac{K_i}{L_i}) - \log(\frac{K_i^e}{L_i^e})) \right) \\
&= \tilde{s}_i (y_{li} - k_{li}) + \sum_{j \neq i} \tilde{s}_{ij} e^{t(\eta_j - \eta_i)} (y_{lj} - k_{lj}) + \tilde{\tau}_i (t_{li} - k_{li})
\end{aligned}$$

where, $\tilde{s}_i = s_i e^{y_i^e - k_i^e}$ and $\tilde{s}_{ij} = s_{ij} e^{y_j^e - k_i^e}$ and $\tilde{\tau}_i = \tau_i e^{t_i^e - k_i^e}$.

Similarly, using the MacLaurin series expansion of $\log(x)$ around x^e again, we obtain next approximations.

$$\begin{aligned}
\bullet \quad &\dot{t}_i(t) = \frac{d}{dt}(t_i(t) - t_i^e) = \dot{t}_i(t) = \frac{L_i(t)}{T_i(t)} \frac{\dot{T}_i(t)}{L_i(t)} - \eta_i = \frac{L_i(t)}{T_i(t)} \frac{1}{L_i(t)} \left(g_i T_i + \sum_{j \neq i} g_{ij} T_j + \epsilon_i K_i \right) - \eta_i \\
&= -\eta_i + g_i + \sum_{j \neq i} g_{ij} \frac{L_i T_j L_j}{T_i L_j L_i} + \epsilon_i \frac{L_i K_i}{T_i L_i} = -\eta_i + g_i + \sum_{j \neq i} g_{ij} e^{t(\eta_j - \eta_i)} \frac{L_i T_j}{T_i L_j} + \epsilon_i \frac{L_i K_i}{T_i L_i} \\
&\approx -\eta_i + g_i + \sum_{j \neq i} g_{ij} e^{t(\eta_j - \eta_i)} \left(\frac{L_i^e T_j^e}{T_i^e L_j^e} + \frac{L_i^e T_j^e}{T_i^e L_j^e} (\log(\frac{L_i T_j}{T_i L_j}) - \log(\frac{L_i^e T_j^e}{T_i^e L_j^e})) \right) + \\
&\quad \epsilon_i \left(\frac{L_i^e K_i^e}{T_i^e L_i^e} + \frac{L_i^e K_i^e}{T_i^e L_i^e} (\log(\frac{L_i K_i}{T_i L_i}) - \log(\frac{L_i^e K_i^e}{T_i^e L_i^e})) \right) \\
&= -\eta_i + g_i + \sum_{j \neq i} g_{ij} e^{t(\eta_j - \eta_i)} \left(e^{t_j^e - t_i^e} + e^{t_j^e - t_i^e} (\log(\frac{L_i T_j}{T_i L_j}) - \log(\frac{L_i^e T_j^e}{T_i^e L_j^e})) \right) + \\
&\quad \epsilon_i \left(e^{k_i^e - t_i^e} + e^{k_i^e - t_i^e} (\log(\frac{L_i K_i}{T_i L_i}) - \log(\frac{L_i^e K_i^e}{T_i^e L_i^e})) \right)
\end{aligned}$$

$$\begin{aligned}
&= \sum_{j \neq i} \tilde{g}_{ij} e^{t(\eta_j - \eta_i)} \left(\log\left(\frac{L_i T_j}{T_i L_j}\right) - \log\left(\frac{L_i^e T_j^e}{T_i^e L_j^e}\right) \right) + \\
&\quad \tilde{\epsilon}_i \left(\log\left(\frac{L_i K_i}{T_i L_i}\right) - \log\left(\frac{L_i^e K_i^e}{T_i^e L_i^e}\right) \right) + \underbrace{\left(-\eta_i + g_i + e^{-t_i} \left(\sum_{j \neq i} g_{ij} e^{t_j^e + t(\eta_j - \eta_i)} + \epsilon_i e^{k_i^e} \right) \right)}_{= 0, \text{ see the conditions from which the equilibrium values are obtained.}}
\end{aligned}$$

$$\begin{aligned}
&= \sum_{j \neq i} \tilde{g}_{ij} e^{t(\eta_j - \eta_i)} \left(\log\left(\frac{T_j}{L_j}\right) - \log\left(\frac{T_j^e}{L_j^e}\right) - \left(\log\left(\frac{T_i}{L_i}\right) - \log\left(\frac{T_i^e}{L_i^e}\right) \right) \right) + \\
&\quad \tilde{\epsilon}_i \left(\log\left(\frac{K_i}{L_i}\right) - \log\left(\frac{K_i^e}{L_i^e}\right) - \left(\log\left(\frac{T_i}{L_i}\right) - \log\left(\frac{T_i^e}{L_i^e}\right) \right) \right) \\
&= \sum_{j \neq i} \tilde{g}_{ij} e^{t(\eta_j - \eta_i)} (t_{lj} - t_{li}) + \tilde{\epsilon}_i (k_{li} - t_{li})
\end{aligned}$$

where, $\tilde{g}_{ij} = g_{ij} e^{t_j^e - t_i^e}$ and $\tilde{\epsilon}_i = \epsilon_i e^{k_i^e - t_i^e}$.

- $$\begin{aligned}
\dot{e}_{li} &= \frac{d}{dt}(e_i(t) - e_i^e) = \dot{e}_i(t) = \frac{L_i(t)}{E_i(t)} \frac{\dot{E}_i(t)}{L_i(t)} - \eta_i = \frac{L_i(t)}{E_i(t)} \frac{1}{L_i(t)} \left(\zeta_i F_i(t) + \sum_{j \neq i} \zeta_{ij} F_j(t) - \xi_i E_i(t) \right) - \eta_i \\
&= -(\xi_i + \eta_i) + \zeta_i \frac{L_i F_i}{E_i L_i} + \sum_{j \neq i} \zeta_{ij} \frac{L_i F_j L_j}{E_i L_j L_i} = -(\xi_i + \eta_i) + \zeta_i \frac{L_i F_i}{E_i L_i} + \sum_{j \neq i} \zeta_{ij} e^{t(\eta_j - \eta_i)} \frac{L_i F_j}{E_i L_j} \\
&\approx -(\xi_i + \eta_i) + \zeta_i \left(\frac{L_i^e F_i^e}{E_i^e L_i^e} + \frac{L_i^e F_i^e}{E_i^e L_i^e} \left(\log\left(\frac{L_i F_i}{E_i L_i}\right) - \log\left(\frac{L_i^e F_i^e}{E_i^e L_i^e}\right) \right) \right) + \\
&\quad \sum_{j \neq i} \zeta_{ij} e^{t(\eta_j - \eta_i)} \left(\frac{L_i^e F_j^e}{E_i^e L_j^e} + \frac{L_i^e F_j^e}{E_i^e L_j^e} \left(\log\left(\frac{L_i F_j}{E_i L_j}\right) - \log\left(\frac{L_i^e F_j^e}{E_i^e L_j^e}\right) \right) \right) \\
&= -(\xi_i + \eta_i) + \zeta_i \left(e^{f_i^e - e_i^e} + e^{f_i^e - e_i^e} \left(\log\left(\frac{L_i F_i}{E_i L_i}\right) - \log\left(\frac{L_i^e F_i^e}{E_i^e L_i^e}\right) \right) \right) + \\
&\quad \sum_{j \neq i} \zeta_{ij} e^{t(\eta_j - \eta_i)} \left(e^{f_j^e - e_i^e} + e^{f_j^e - e_i^e} \left(\log\left(\frac{L_i F_j}{E_i L_j}\right) - \log\left(\frac{L_i^e F_j^e}{E_i^e L_j^e}\right) \right) \right) \\
&= \tilde{\zeta}_i \left(\log\left(\frac{L_i F_i}{E_i L_i}\right) - \log\left(\frac{L_i^e F_i^e}{E_i^e L_i^e}\right) \right) + \sum_{j \neq i} \tilde{\zeta}_{ij} e^{t(\eta_j - \eta_i)} \left(\log\left(\frac{L_i F_j}{E_i L_j}\right) - \log\left(\frac{L_i^e F_j^e}{E_i^e L_j^e}\right) \right) + \\
&\quad \underbrace{\left(-(\xi_i + \eta_i) + e^{-e_i^e} \left(\zeta_i e^{f_i^e} + \sum_{j \neq i} \zeta_{ij} e^{f_j^e + t(\eta_j - \eta_i)} \right) \right)}_{= 0, \text{ see the conditions from which the equilibrium values are obtained.}}
\end{aligned}$$

$$\begin{aligned}
&= \tilde{\zeta}_i \left(\log\left(\frac{F_i}{L_i}\right) - \log\left(\frac{F_i^e}{L_i^e}\right) - \left(\log\left(\frac{E_i}{L_i}\right) - \log\left(\frac{E_i^e}{L_i^e}\right) \right) \right) + \\
&\quad \sum_{j \neq i} \tilde{\zeta}_{ij} e^{t(\eta_j - \eta_i)} \left(\log\left(\frac{F_j}{L_j}\right) - \log\left(\frac{F_j^e}{L_j^e}\right) - \left(\log\left(\frac{E_i}{L_i}\right) - \log\left(\frac{E_i^e}{L_i^e}\right) \right) \right) \\
&= \tilde{\zeta}_i (f_{li} - e_{li}) + \sum_{j \neq i} \tilde{\zeta}_{ij} e^{t(\eta_j - \eta_i)} (f_{lj} - e_{li})
\end{aligned}$$

where, $\tilde{\zeta}_i = \zeta_i e^{f_i^e - e_i^e}$ and $\tilde{\zeta}_{ij} = \zeta_{ij} e^{f_j^e - e_i^e}$.

Appendix C: Objectives linearized model

Under the assumption that the players reached an equilibrium in the non-linear model (2.6,2.7), it follows that if this equilibrium is perturbed by a small disturbance, the dynamics of the corresponding disturbed system are obtained by linearizing the non-linear system (2.6) around this equilibrium. Furthermore, without going into detail (see for more details, e.g., [6][p.177]), assuming this disturbance is measured by “ ϵ ” one can make a second-order Taylor expansion of the cost function $J(\epsilon)$ around $\epsilon = 0$, yielding:

$$J_i(\epsilon) = J_i^* + \epsilon \frac{dJ_i(\epsilon)}{d\epsilon}(0) + \frac{1}{2}\epsilon^2 \frac{d^2J_i(\epsilon)}{d\epsilon^2}(0) + O(\epsilon^2).$$

Since for $\epsilon = 0$ we are at the equilibrium of the optimized non-linear model, it follows that $\frac{dJ_i(\epsilon)}{d\epsilon}(0) = 0$. Therefore, if the system is out of equilibrium, the consistent optimal response of players is approximately obtained by minimizing $\frac{d^2J_i(\epsilon)}{d\epsilon^2}(0)$ subject to the linearized model.

Note that, with $x^T(t) = [k_{l1} \ k_{l2} \ t_{l1} \ t_{l2} \ e_{l1} \ e_{l2}]$, $u^T(t) = [f_{l1} \ f_{l2} \ g_{l1} \ g_{l2}]$ and $z^T := [x^T \ u^T]$, (2.7) equals

$$\int_0^\infty g(t, z(t))dt, \text{ where } g(t, z(t)) = e^{-\theta_i t} \left((\mu_i y_i(t) - (f_i(t) + g_i(t)))^2 + \pi_i \left(\sum_{j=1}^2 e_j(t) \right)^2 + \rho_i g_i^2(t) \right).$$

Therefore, $\frac{\partial^2 J_i(\epsilon)}{\partial \epsilon^2}(0) = \int_0^\infty z^T(t) H_i'' z(t) dt$, where $H_i'' = \left(\frac{\partial^2 g(z)}{\partial z_i \partial z_j} \right)$ evaluated at the equilibrium values. After some calculations we obtain then next matrices

$$H_1'' = \begin{bmatrix} 0.0907 & 0 & -0.0907 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0907 & 0 & 0.1078 & -0.0172 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0172 & 0.0172 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3.4494 & 0.3000 & -0.1740 & -2.9754 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.3000 & 0.3000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.1740 & 0 & 2.1740 & 0 & 2.0000 & 0 \\ 0 & 0 & 0 & 0 & -2.9754 & 0 & 0 & 2.9754 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2.0000 & 0 & 2.0200 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$H_2'' = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.4261 & 0 & -0.4261 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1683 & -0.1683 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.4261 & -0.1683 & 0.5945 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1500 & 0.1500 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1500 & 1.4185 & -0.0701 & -1.1984 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.0701 & 0.0701 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1.1984 & 0 & 3.1984 & 0 & 2.0000 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.0000 & 0 & 2.1000 \end{bmatrix}.$$

Appendix D: Simulations

Stochastic parameter θ

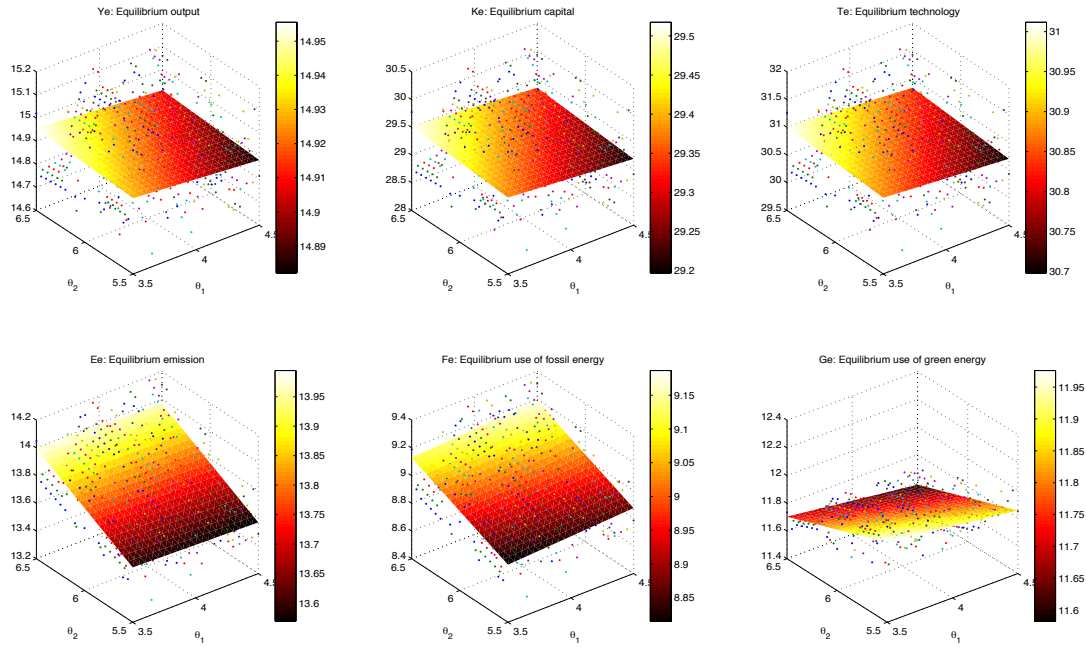


Figure 17: Equilibrium values on grid, OECD countries: fitted plane

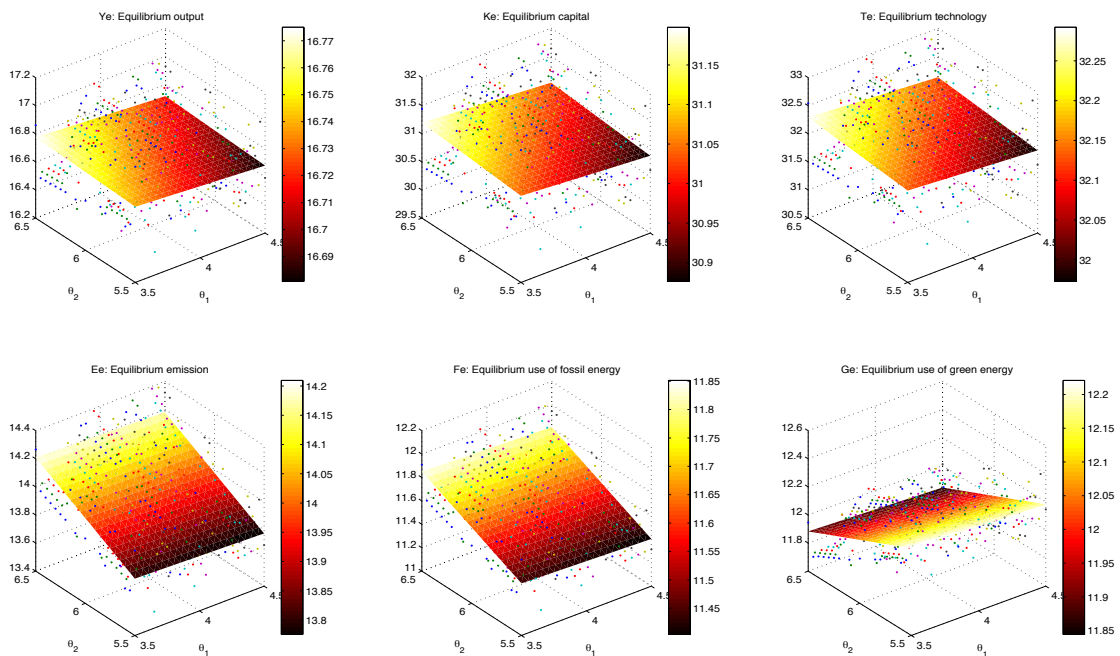


Figure 18: Equilibrium values on grid, non-OECD countries: fitted plane

Stochastic policy parameter π

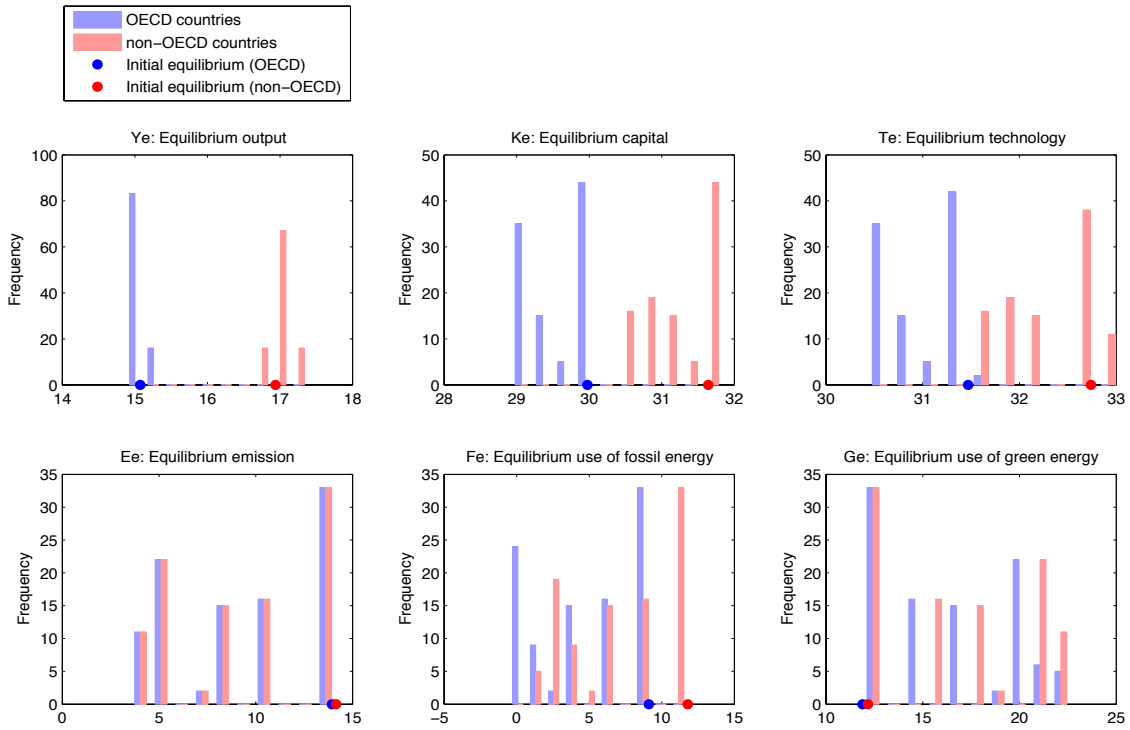


Figure 19: Equilibrium values with stochastic π_i

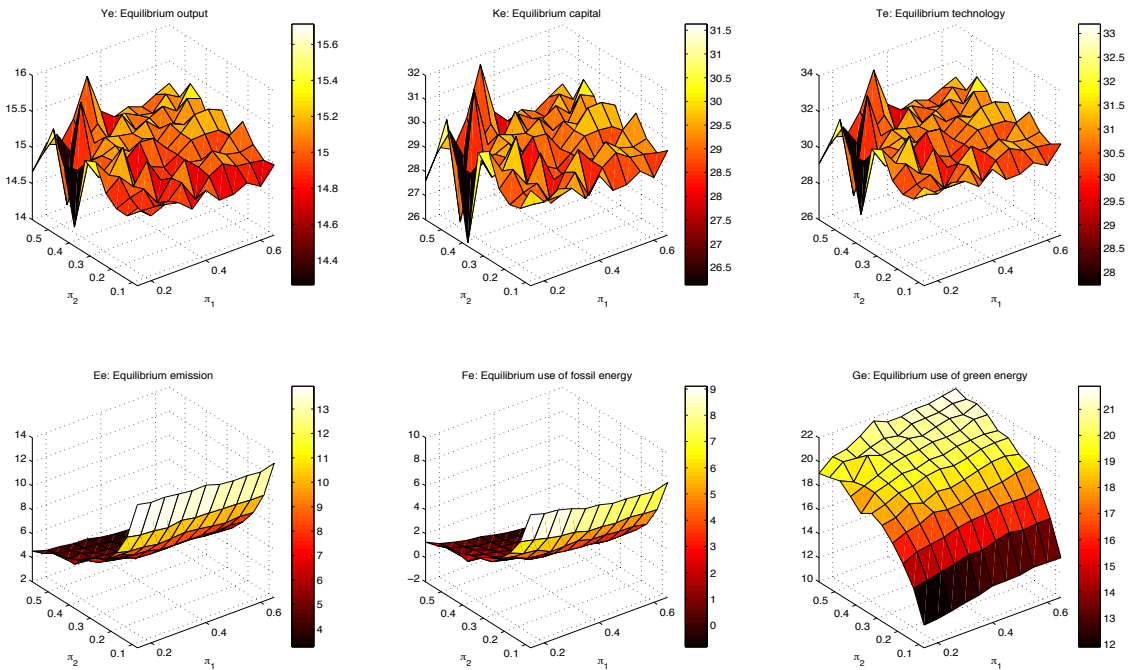


Figure 20: Equilibrium values OECD countries with different π_i

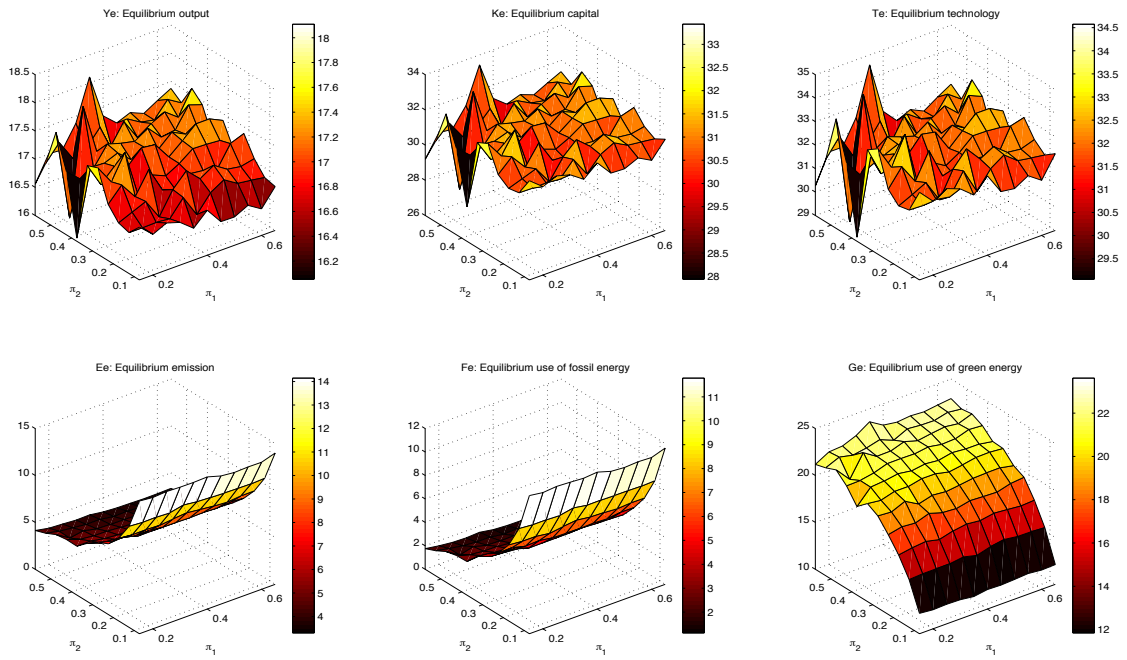


Figure 21: Equilibrium values non-OECD countries with different π_i

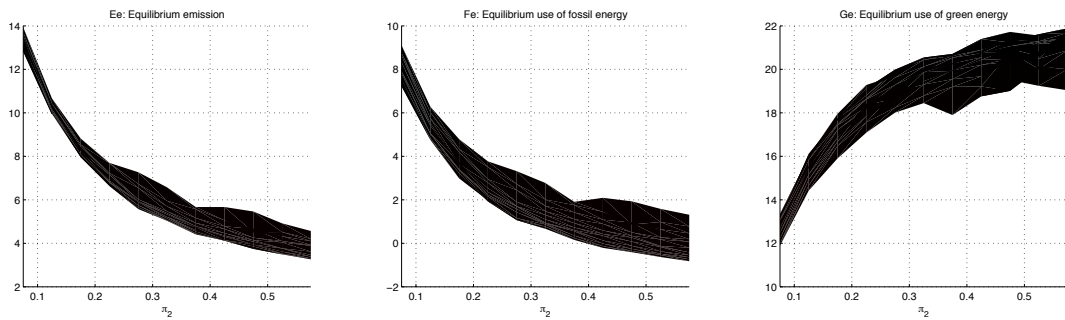


Figure 22: e , f and g for OECD countries

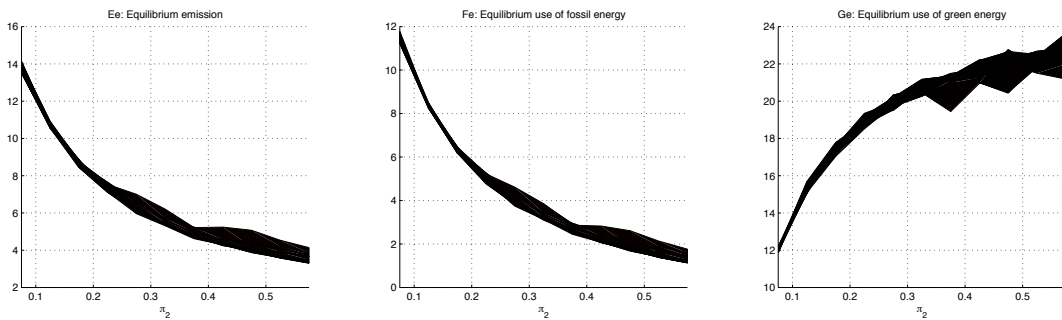


Figure 23: e , f and g for non-OECD countries

Stochastic relation \dot{k}

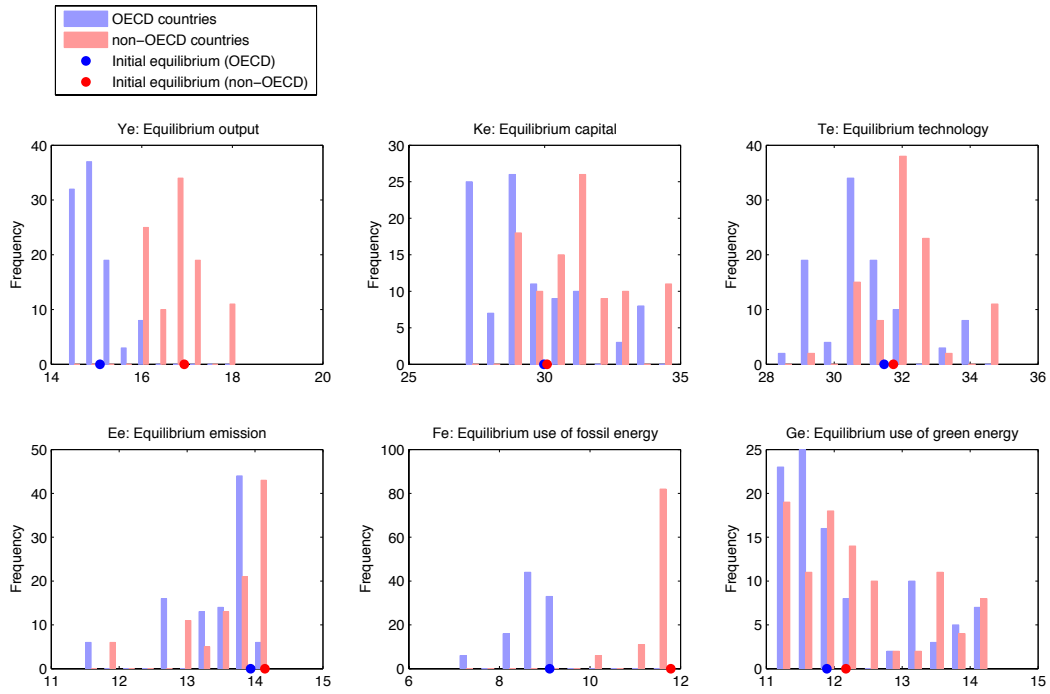


Figure 24: Simulation with \dot{k} , Λ stochastic: equilibrium values

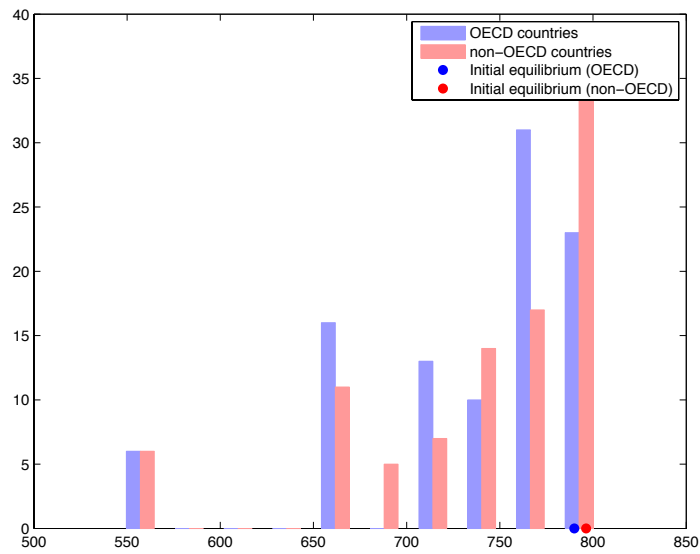


Figure 25: Simulation with \dot{k} , Λ stochastic: objective function values

Scenario analysis ρ, g

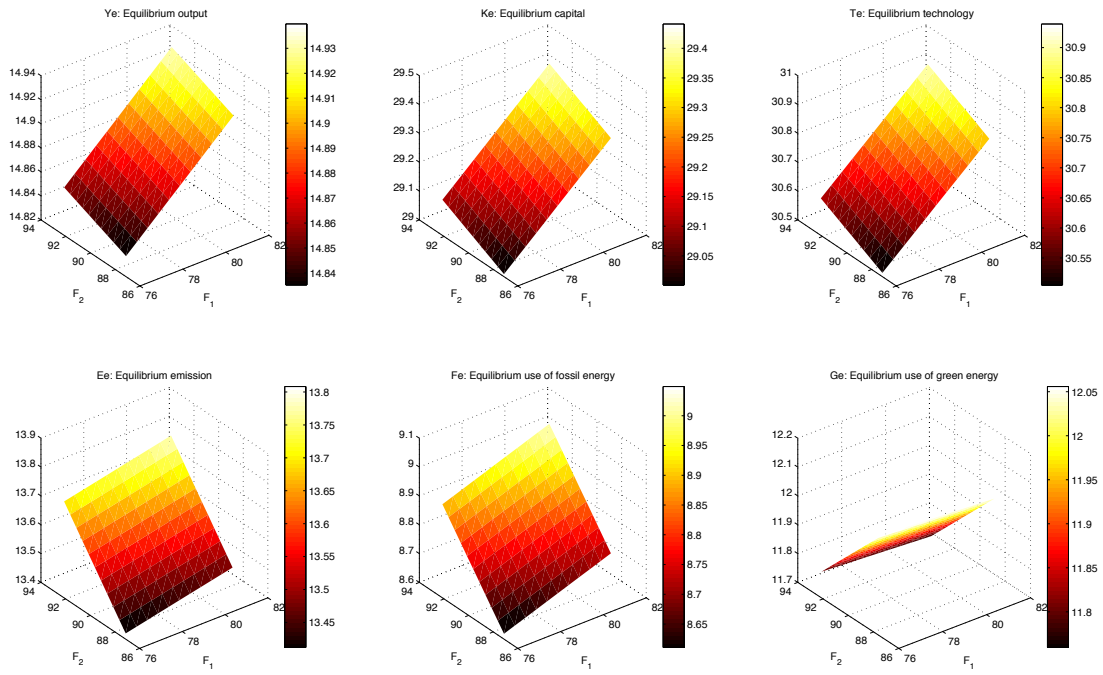


Figure 26: Equilibrium values on grid, OECD countries: fitted planes

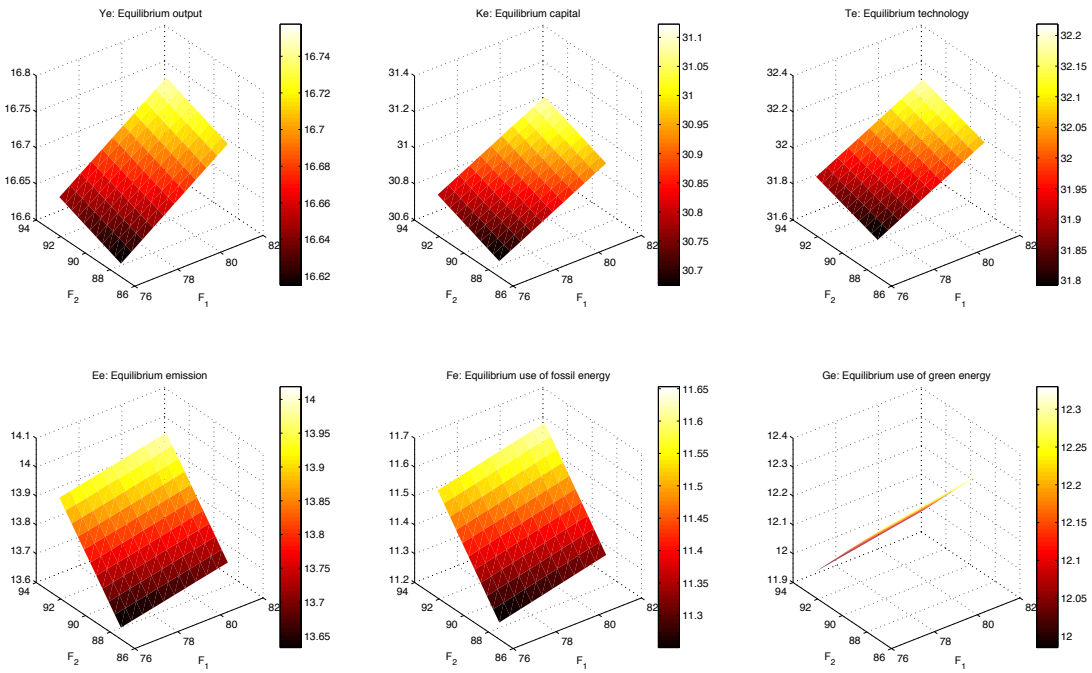


Figure 27: Equilibrium values on grid, non-OECD countries: fitted planes

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