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Tuijp, Patrick

Publication date:
2016

Document Version
Publisher's PDF, also known as Version of record

Link to publication in Tilburg University Research Portal

Citation for published version (APA):
Tuijp, P. (2016). *The pricing of illiquidity and illiquid assets: Essays on empirical asset pricing*. CentER, Center for Economic Research.

The Pricing of Liquidity and Illiquid Assets: Essays on Empirical Asset Pricing

Proefschrift

ter verkrijging van de graad van doctor
aan Tilburg University
op gezag van de rector magnificus,
prof. dr. E.H.L. Aarts,
in het openbaar te verdedigen
ten overstaan van een door het college voor promoties aangewezen commissie
in de aula van de Universiteit
op dinsdag 28 juni 2016 om 14.15 uur

door

Patrick Frederik Albert Tuijp

geboren op 8 februari 1985 te Amsterdam

Promotores:

prof. dr. J.J.A.G. Driessen

prof. dr. A. Beber

Overige commissieleden:

dr. D.G.J. Bongaerts

prof. dr. F.C.J.M. de Jong

prof. dr. F.A. de Roon

Acknowledgements

This thesis is the product of a journey during which I have met a great many people in various places. In this section, I aim to express my gratitute to a number of people who have made the journey more pleasant with their support, academic or otherwise.

First and foremost, I would like to thank my supervisors, Joost Driessen and Alessandro Beber. Their insights, comments, and criticism have been invaluable and have greatly improved the quality of this thesis. It has been a pleasure to work with them and I feel priviledged to have them as my supervisors. I also thank my thesis committee, Dion Bongaerts, Frank de Jong, and Frans de Roon, for reading the different papers thoroughly and providing many insightful suggestions and comments. While working on the papers that make up this dissertation, I had the pleasure to collaborate with Erasmo Giambona and benefited from the expertise of Esther Eiling and Anthony Neuberger.

In addition, I would like to thank a number of people for having directly stimulated me to pursue a PhD and showing me the way to get there. Years ago, while I was working as a student assistant at VU University Amsterdam, Roman Kräussl and André Lucas suggested to me that I should apply for the research master program at the Tinbergen Institute. In that process, the recommendation by Jaap Abbring to prepare for the research master by "taking as many econometrics courses as is humanly possible" has resulted in my bachelor degree in econometrics and operations research.

During the bachelor in econometrics and operations research, I was inspired by many of the lecturers. Two of them I would like to mention specifically. Rein Nobel has taught me how to think rigorously about probability and mathematics. His comments regarding the Dutch educational system, and basically anything that made the news or happened during his lectures, made it even more worthwhile to attend his courses. At the same time, Henk Tijms has shown me how to think creatively and intuitively, while maintaining the rigor. In addition to this, I was most fortunate to have Henk Tijms and Drew Creal as my bachelor thesis supervisors.

My years at the Tinbergen Institute, the University of Amsterdam, and Tilburg University have been shaped by the people that I have met there. Thank you for all the discussions and good times. It was a pleasure to share offices with Hao Liang, Natalya Martynova, Ran Xing, and Yang Zhou. Also, I would like to thank Anton van Boxtel, Vincent van Kervel, and Christiaan van der Kwaak for reading my work and giving valuable advice.

Over the years, I have actively participated in a number of associations. During my studies of econometrics, I have made many friends through study association Kraket. It was particularly inspiring to be part of the founding editorial board of the magazine Sector, which still exists today. I have also been involved with the PhD candidates Network of the Netherlands (PNN), where I ended up being the organization's president. This has been a great experience, and I have had many good times with my fellow board members.

It goes without saying that I have found a lot of support in my friends. Thank you for all the good times that we have shared. Learning and playing jazz guitar with a number of you provided a welcome distraction from my academic work.

Finally, I would like to thank my parents. Throughout my life, they have stimulated me to make the most of myself, to always aim high, and to not forget to have a good time along the way. Fred, Nelly, thank you for your continued support in all my endeavours. Without you I would not have made it this far.

Contents

Introduction

This PhD thesis studies the impact of liquidity on the way stock prices are formed, as well as the effect of limited diversification on residential real estate prices. In this introduction, I will first discuss the concept of liquidity and give an example. Next, I will discuss limited diversification and how it could affect stock and real estate prices. Finally, I will summarize the contents of each chapter of this dissertation.

In financial economics, the liquidity of an asset is generally defined as the ease with which it can be traded. This ease can be in the form of being able to trade quickly, with little effort, or against a low cost. The first and third chapter of this PhD thesis study stock market liquidity. For the purposes of this introduction, I will start with an example that I do not study in this dissertation, but most people are familiar with: foreign currency. When you change your money into foreign currency and back, you typically lose a small amount in the process, even after fees. Suppose that you travel from the Netherlands to the U.K. and back, and the exchange rate remains the same, then the prices at which you buy and sell your British pounds will still be different. At an exchange office, you will for instance find that you have to pay 1.37 euro to buy one British pound, while you get back only 1.35 euro for every British pound that you sell. So, for every euro that you change to pounds and back, you will lose 2 euro cents. These two cents will, among other things, compensate the exchange office for holding large amounts of different currencies, and running the risk that the currencies that they hold decline in value.

The foreign currency example can be applied directly to the stock market, where there is a similar difference between the price at which you buy (the ask price) and the price at which you sell (the bid price). In general, we say that the market liquidity of an asset is high when at a single point in time – so that the value of the asset itself does not change – the difference between the price at which you buy and the price at which you sell is small. There are many other aspects to market liquidity, but this example should at least provide a useful way to think about the concept.

To see why liquidity would matter to investors in general, we need only consider what happens when it disappears from the market. This occurred, for instance, during the crash of October 1987, the Asian financial crisis, the Russian default and LTCM collapse in 1998, and the 2007–2009 financial crisis (Liu, 2006; Nagel, 2012). Such periods of illiquidity typically coincide with asset price declines (Chordia, Roll, and Subrahmanyam, 2001) and may happen suddenly (Brunnermeier and Pedersen, 2009). This can be very costly to financial institutions that are forced to sell their illiquid asset holdings at firesale prices following outflows, or to cover losses (Brunnermeier and Pedersen, 2009; Coval and Stafford, 2007).

Liquidity has not always played a prominent part in the academic literature. Traditional stock valuation models, such as the well-known Sharpe (1964), Lintner (1965), and Black (1972) CAPM, assume that markets are perfectly liquid – meaning that there is always someone to trade with and the price at which you can buy equals the price at which you can sell – and hence liquidity plays no role in the pricing results. Since then, many empirical studies have shown that liquidity does in fact matter for asset prices in many different markets. In the first chapter of this

PhD thesis, I focus on the way the liquidity of stocks and the investment horizon interact, and in the third chapter I investigate the evolution of liquidity over time.

Another part of this dissertation considers how limited diversification affects residential real estate prices. Diversification is the technical term for reducing risk by not putting all one's eggs in one basket, and has been applied often in the context of the stock market. When investing in stocks, the loss on one stock can be offset by another, especially when there is a large number of stocks in the investment portfolio. Generally not all risk can be eliminated in this way, and the risk that remains when holding all stocks in the market is called market risk. Prices are linked to risk because expected stock returns are viewed a compensation for the willingness of investors to accept a certain level of risk. The main result of the CAPM mentioned above is that because stock-specific risk (or idiosyncratic risk) can be diversified away by including many different stocks in a single portfolio, only market-level risk should matter for stock prices. If there is limited diversification, both market risk and stock-specific risk matter for stock prices. It turns out, however, that it is difficult to test whether this effect occurs in the stock market, because it is hard to measure the degree of diversification. Residential real estate is a natural asset class to test for such effects, as home owners tend to own only a single house and are thus severely under-diversified (Tracy, Schneider, and Chan, 1999). In the second chapter of this PhD thesis, I will therefore look into the residential real estate market, to obtain evidence on the effects of limited diversification on prices.

The first chapter of this PhD thesis is joint work with with Alessandro Beber and Joost Driessen and investigates what happens when short-term investors and long-term investors interact in a market with both liquid and illiquid stocks. This is a natural topic: liquidity only matters when trading actually takes place. Long-term investors are thus less concerned about liquidity, simply because they trade less often. When a stock is deemed unattractive by investors due to illiquidity and hence is sold at a lower price than justified by the value of the future dividends, we say that it commands a liquidity premium. Clearly, assets that command such a liquidity premium are attractive to long-term investors. As David Swensen, the Chief Investment Officer of the Yale Endowment Fund, has noted: “Accepting illiquidity pays outsize dividends to the patient long-term investor,” (Swensen, 2000).

In the first chapter, we explicitly model an economy with short-term and long-term investors, who can invest in a range of assets with different liquidity. By incorporating liquidity risk – the risk that an asset becomes more or less liquid over time – as well as heterogeneous investment horizons, we bridge the seminal papers by Amihud and Mendelson (1986) and Acharya and Pedersen (2005). In addition, we find that it can be optimal for short-term investors not to invest in the least liquid assets, which results in a segmentation where the least liquid assets are held only by long-term investors.

Our model features a liquidity premium that can be decomposed into three parts. The first part reflects the basic return premium that investors demand to be compensated for holding an illiquid asset. In equilibrium, however, the least liquid securities are held by long-term investors who trade infrequently and therefore are less concerned with illiquidity. Consequently, we actually find a smaller liquidity premium for the least liquid assets. We call this reduction a segmentation premium, and it forms the second part of our decomposition.

The third part is a liquidity spillover premium that arises due to the correlation between returns on the liquid and illiquid securities. If there were no liquidity premium on the illiquid assets, then we could set up a near-arbitrage as follows. By buying liquid securities and selling illiquid securities, we would earn the liquidity premium on the liquid assets while most of the return risk would cancel out. Trading on this strategy would change prices until it is no longer profitable to do so. By selling the illiquid securities, their prices decline until they also reflect the liquidity premium on the liquid assets to some extent. Hence, in equilibrium, we still find a small liquidity premium on the illiquid assets.

We test the empirical relevance of the model on the cross-section of U.S. stocks over the period 1964 to 2009. Our results show that by accounting for heterogenous investment horizons and segmentation, we can better explain price differences between stocks that differ in liquidity, and the decomposition of the liquidity premium into the three parts discussed above allows us to show the source of these differences.

The second chapter of this dissertation is joint work with Erasmo Giambona and concerns residential real estate market. The chapter studies the consequences of limited diversification for residential real estate prices. Homeowners who are investing in the residential real estate market typically have a highly leveraged position in one, or a few properties (Tracy, Schneider, and Chan, 1999). The leverage consists of the mortgage, with typical loan-to-value ratios of 75% (Green and Wachter, 2005). The individual investors cannot easily hold a well-diversified portfolio of small positions in many houses because housing is a lumpy investment – it is difficult, if not impossible to buy any desired fraction of a single property.

It is a well-known theoretical result in financial economics that asset-specific risk should matter for prices when investors are underdiversified (Merton, 1987; Levy, 1978; Malkiel and Xu, 2004), yet for stocks, empirical evidence on the pricing of idiosyncratic risk has been mixed. As there is a clear indication that homeowners are underdiversified, the residential real estate market provides a good opportunity to study the pricing of idiosyncratic risk. A key issue is measuring the extent to which homeowners are underdiversified. We suggest to measure this through the fraction of people in a certain region who are homeowners. If many people own the house in which they live, then the housing stock is strongly dispersed and hence there is little potential for any single investor to hold a large, well-diversified portfolio. This implies that if homeownership is high in certain regions, then idiosyncratic risk should matter more strongly for house prices in those regions.

To test this relation, we use house price index data from the Federal Housing Finance Agency (FHFA) for the period 1980 until 2012, and we measure homeownership using IPUMS census data. In our analysis, we include homeownership as an interaction effect with idiosyncratic risk. In that way, we can see whether increased homeownership indeed leads to a stronger impact of idiosyncratic risk on residential real estate prices beyond a certain base level. Our results show that this indeed is the case.

The third chapter returns to the pricing of liquidity in the stock market. In contrast to the first chapter, which concerns the differences between liquid and illiquid stocks, this chapter focuses on the impact of changes in liquidity over time. In this chapter, I show that liquidity risk matters for stock prices only in relation to an overall deterioration in liquidity, but not to a deterioration in liquidity that occurs only for the least liquid stocks.

To motivate my analysis, I start from empirical evidence by Næs, Skjeltorp, and Ødegaard (2011), who show that there are two ways in which investors shift their portfolios towards more liquid assets in response to changing expectations about the real economy. The first is an across-asset-class flight to liquidity, where investors exit the stock market altogether. One example of this is a shift to the bond market, such as in Goyenko and Ukhov (2009). This coincindes with an overall decrease in liquidity in the stock market. The second is a within-asset-class flight to liquidity, where investors shift their holdings from illiquid stocks to more liquid stocks. The least liquid stocks then become even less liquid, while liquid stocks are not affected as much. These shifts in investor holdings of course need not be the only events related to changes to the shape of the cross-section of liquidity, and I specifically do not rule out other mechanisms.

Earlier research by Pástor and Stambaugh (2003) shows that the risk of an overall deterioration in liquidity matters for stock prices. The risk of only the least liquid stocks becoming less liquid could also be quite relevant to large institutional investors. This can be understood best from a trading strategy put forward by Duffie and Ziegler (2003). They show that under certain conditions financial institutions will choose to sell liquid assets first if they need to close out positions. This happens for instance in response to an unwanted increase in risk run by the institutions. Given that the institutions end up holding mostly illiquid securities if they follow this strategy, even the case where only the least liquid assets become less liquid poses a significant risk and can lead to large losses, or even insolvency.

To investigate the different ways in which liquidity can change over time, I statistically disentangle the case of an overall deterioration in liquidity and the case where only the least liquid assets become even less liquid. It turns out that an overall deterioration in liquidity is associated most strongly with market downturns, while the deterioration in the least liquid segment is related to active trading in the most liquid segment. The latter is in line with the within-asset-class flight to liquidity of Næs, Skjeltorp, and Ødegaard (2011).

By combining these two ways in which liquidity can change over time with the pricing model of Acharya and Pedersen (2005), I am able to test which of these effects is relevant for stock prices. The results show that only an overall deterioration in liquidity matters for stock prices statistically and economically, while there is no such effect for a deterioration that occurs only for the least liquid stocks.

Summarizing, this dissertation shows that the investment horizon matters for the impact liquidity has on stock prices, that only the risk of an overall deterioration in liquidity has an impact on stock prices, and that underdiversification is likely to play a role in the pricing of house-specific risk for U.S. residential real estate.

Chapter 1

Pricing Liquidity Risk with Heterogeneous Investment Horizons[1]

1.1 Introduction

The investment horizon is a key feature distinguishing different categories of investors, with high-frequency traders and long-term investors such as pension funds at the two extremes of the investment horizon spectrum. Most of the literature on horizon effects in portfolio choice and asset pricing builds on the theoretical insight of Merton's (1971) hedging demands and demonstrates that long-horizon decisions can differ substantially from single-period decisions for various model specifications.

Surprisingly, the interaction between investment horizons and liquidity has attracted much less attention. Even in the absence of hedging demands, heterogeneous investment horizons can have important asset pricing effects for the simple

[1]This chapter is based on joint work with Alessandro Beber and Joost Driessen. We thank Ken Singleton and two anonymous referees for very useful comments and suggestions. We also thank Jack Bao (the WFA discussant), Bart Diris, Darrell Duffie, Frank de Jong, Pete Kyle, Marco Pagano, Richard Payne, Dimitri Vayanos, and seminar participants at University of Essex, University of Maryland, University of North-Carolina, Tilburg University, CSEF-IGIER Symposium on Economics and Institutions, the Duisenberg School of Finance, the Erasmus Liquidity conference, the Financial Risks International Forum, the SoFie conference at Tinbergen Institute, and the WFA conference for very useful comments.

reason that different horizons imply different trading frequencies. More specifically, liquidity plays a distinct role for investors with diverse horizons because trading costs only matter when trading actually takes place. The investment horizon then becomes a key element in the asset pricing effects of liquidity.

We explicitly take this standpoint and derive a new liquidity-based asset pricing model featuring risk-averse investors with heterogeneous investment horizons and stochastic transaction costs. Investors with longer investment horizons are clearly less concerned about trading costs, because they do not necessarily trade every period. Our model generates a number of new implications on the pricing of liquidity that are strongly supported empirically when we test them on the cross-section of U.S. stock returns.

Previous theories of liquidity and asset prices have largely ignored heterogeneity in investor horizons, with the exception of the seminal paper of Amihud and Mendelson (1986), who study a setting where risk-neutral investors have heterogenous horizons. Their model generates *clientele* effects: short-term investors hold the liquid assets and long-term investors hold the illiquid assets, which leads to a concave relation between transaction costs and expected returns.[2] Besides risk-neutrality, Amihud and Mendelson (1986) assume that transaction costs are constant. However, there is large empirical evidence that liquidity is time-varying. Assuming stochastic transaction costs, Acharya and Pedersen (2005) set out one of the most influential asset pricing models with liquidity risk, where various liquidity risk premiums are generated. However, this model includes homogeneous investors with a one-period horizon and thus implies a linear (as opposed to concave) relation between (expected) transaction costs and expected returns. Our paper bridges these two seminal papers, because our model entails heterogeneous

[2] Hopenhayn and Werner (1996) propose a similar set-up featuring risk-neutral investors with heterogeneity in impatience and endogenously determined liquidity effects.

horizons, as in Amihud and Mendelson (1986), with stochastic illiquidity and risk aversion, as in Acharya and Pedersen (2005). This leads to a number of novel and important implications for the impact of both expected liquidity and liquidity risk on asset prices.

Our model setup is easily described. We have multiple assets with i.i.d. dividends and stochastic transaction costs, and many investor types with mean-variance utility over terminal wealth but different investment horizons. We obtain a stationary equilibrium in an overlapping generation setting and we solve for expected returns in closed form.

This theoretical setup implies the existence of an intriguing equilibrium with partial segmentation. Short-term investors optimally choose not to invest in the most illiquid assets, intuitively because their expected returns are not sufficient to cover expected transaction costs. In contrast, long-term investors trade less frequently and can afford to invest in illiquid assets. This clientele partition is different from Amihud and Mendelson (1986), because our risk-averse long-horizon investors also buy liquid assets for diversification purposes.

The partial segmentation equilibrium implies different expressions for the expected returns of liquid and segmented assets. For liquid assets, expected returns contain the familiar compensation for expected transaction costs and a mixture of a liquidity premium and standard-CAPM risk premium. The presence of investors with longer investment horizons, however, reduces the importance of liquidity risk relative to a homogeneous investor setting. Furthermore, the effect of expected liquidity is relatively smaller, given that long-horizon investors do not trade every period, and it varies in the cross-section of stocks as a function of the covariance between returns and illiquidity costs. Interestingly, we identify cases in the cross-section of stocks where high liquidity risk may actually lead to a lower premium on expected liquidity because of a greater presence of long-term investors.

The expected returns of segmented assets contain additional terms, both for risk premia and in expected liquidity effects. More specifically, there are *segmentation* and *spillover* risk premia. The segmentation risk premium is positive and is caused by imperfect risk sharing, as only long-term investors hold these illiquid assets. The spillover risk premium can be positive or negative, depending on the correlation between illiquid (segmented) and liquid (non-segmented) asset returns. For example, if a segmented asset is highly correlated with non-segmented assets, the spillover effect is negative and neutralizes the segmentation risk premium, because in this case the segmented asset can be replicated (almost exactly) by a portfolio of non-segmented assets.

The expected liquidity term also contains a segmentation effect, in that expected liquidity matters less for segmented assets that are held only by long-term investors. Along the same lines as the risk premium, it also contains an expected liquidity spillover term, with a sign that is a function of the correlation between liquid and illiquid assets. In sum, the presence of these additional effects implies that the total expected liquidity premium can be larger for liquid assets relative to segmented assets. Hence, in contrast to Amihud and Mendelson (1986) and Acharya and Pedersen (2005), the relation between expected returns and expected liquidity in our model is not necessarily strictly increasing.

In summary, our model demonstrates that incorporating heterogeneous investment horizons has a considerable impact on the way liquidity affects asset prices. It changes the relative size of liquidity and market risk premia, leads to cross-sectional differences in liquidity effects, and generates segmentation and spillover effects.

Armed with this array of novel theoretical predictions, we take the model to the data to test its empirical relevance. Specifically, we analyze the cross-section of U.S. stocks over the period 1964 to 2009 and use the illiquidity measure of Amihud (2002) to proxy for liquidity costs, as in Acharya and Pedersen (2005).

We estimate our asset pricing model using the Generalized Method of Moments (GMM) and find that a version with two horizons (one month and ten years) generates a remarkable cross-sectional fit of expected stock returns. Specifically, for 25 liquidity-sorted portfolios, the heterogeneous-horizon model generates a cross-sectional R^2 of 82.2% compared to 62.2% for the single-horizon model, with similar improvements when using other portfolio sorting criteria. Our model achieves this substantial increase in explanatory power using the same degrees of freedom and imposing more economic structure on the composition of the risk premium and on the loading of expected returns on expected liquidity. As an upshot of our richer model, the empirical estimates can also be used to make inferences about the risk-bearing capacity of investors in each horizon class.

We also estimate a version of our heterogeneous horizon model without liquidity risk, thus incorporating only the effects of expected liquidity and the associated segmentation and spillover effects. As explained above, this model setup deviates from Amihud and Mendelson (1986) in that investors are risk-averse, rather than risk-neutral. Interestingly, the fit of this version of the model is as good as the fit of a model with liquidity risk. For our empirical application to the cross-section of U.S. stocks, what matters is the combination of expected liquidity and partial segmentation. While the cost of the homogenous horizon assumption is about 20% in terms of R^2, in the end the cost of assuming constant transaction costs seems negligible.

The final important implication of the empirical estimates of our model is the more prominent role of the effect of expected liquidity on expected returns compared to the homogeneous horizon case. Averaged across the 25 liquidity-sorted portfolios, the expected liquidity premium generates about 2.40% in annual returns in our model, as compared with 0.36% in the homogeneous-horizon model. The presence of partial segmentation is thus crucial to understand the effect of expected liquidity on asset prices.

The remainder of the paper is organized as follows. Section 1.2 reviews the relevant literature. Section 1.3 presents the general liquidity asset pricing model that allows for arbitrarily many investment horizons and assets. We describe our estimation methodology in Section 1.4. Section 1.5 illustrates the data and Section 1.6 presents our empirical findings. We conclude with a summary of our results in Section 1.7.

1.2 Related Literature

Our paper contributes to the existing literature on liquidity and asset pricing along several dimensions. First, our model is related to theoretical work on portfolio choice and illiquidity (see Amihud, Mendelson, and Pedersen (2005) for an overview). Starting with the work of Constantinides (1986), several researchers have examined multi-period portfolio choice in the presence of transaction costs. In contrast to these papers, we focus on a general equilibrium setting with heterogenous investment horizons in the presence of liquidity risk. We obtain a tractable asset pricing model by simplifying the analysis in some other dimensions. In particular, we assume no intermediate rebalancing for long-term investors.

Second, our empirical results contribute to a rich literature that has empirically studied the asset pricing implications of liquidity and liquidity risk. Amihud (2002) finds that stock returns are increasing in the level of illiquidity both in the cross-section (consistent with Amihud and Mendelson (1986)) and in the time-series. Pástor and Stambaugh (2003) show that the sensitivity of stock returns to aggregate liquidity is priced. Acharya and Pedersen (2005) integrate these effects into a liquidity-adjusted CAPM that performs better empirically than the standard CAPM. The liquidity-adjusted CAPM is such that, in addition to the standard CAPM effects, the expected return on a security increases with the level of illiquidity and is influenced by three different liquidity risk covariances. Several articles build on these seminal papers and document the pricing of liquidity and liquidity risk in various asset classes.[3] However, none of these papers study the liquidity effects of heterogenous investment horizons.

Third, our paper is also related to empirical research showing the relation between liquidity and investors' holding periods. For example, Chalmers and Kadlec (1998) find evidence that it is not the spread, but the amortized spread that is more relevant as a measure of transaction costs, as it takes into account the length of investors' holding periods. Cremers and Pareek (2009) study how investment horizons of institutional investors affect market efficiency. Cella, Ellul, and Giannetti (2013) demonstrate that investors' short horizons amplify the effects of market-wide negative shocks. All of these articles use turnover data for stocks and investors to capture investment horizons. In contrast, we estimate the degree of heterogeneity in investment horizons by fitting our asset pricing model to the cross-section of U.S. stock returns.

Finally, our modeling approach is somewhat related to recent theories where some investors do not trade every period, although there is no explicit role for transaction costs and illiquidity. For example, Duffie (2010) studies an equilibrium pricing model in a setting where some "inattentive" investors do not trade every period. He uses this setup to study how supply shocks affect price dynamics in a single-asset model. In contrast, besides incorporating transaction costs, our focus is cross-sectional as we study a market with multiple assets in a setting where dividends, transaction costs, and returns are all i.i.d. Similarly, Brennan and Zhang (2013) develop an asset pricing model where a representative agent has

[3] For example, Bekaert, Harvey, and Lundblad (2007) focus on emerging markets, Sadka (2010) studies hedge funds, Franzoni, Nowak, and Phalippou (2012) focus on private equity, Bao, Pan, and Wang (2011) study corporate bonds, and Bongaerts, De Jong, and Driessen (2011) focus on credit default swaps.

a stochastic horizon.[4] However, liquidity effects are neglected and investors are homogeneous, in that they hold the same assets and those assets are liquidated simultaneously.

1.3 The Model

In this section, we first lay down the foundations of our liquidity asset pricing model with multiple assets and horizons. We then analyze the main equilibrium implications of the model. Finally, we explore a number of special cases of the model to obtain additional interesting insights.

1.3.1 Model Setup and Assumptions

Our liquidity asset pricing model features both stochastic liquidity and heterogenous investment horizons in a setting with multiple assets. We develop a theoretical framework that is also suitable for empirical estimation. Our model is built on the following assumptions:

- There are K assets, with asset i paying each period a dividend $D_{i,t}$.[5] Selling the asset costs $C_{i,t}$. Transaction costs and dividends are *i.i.d.* in order to obtain a stationary equilibrium. There is a fixed supply of each asset, equal to S_i shares, and a risk-free asset with exogenous and constant return R_f.
- We model N classes of investors with horizons h_j, where $j = 1,..,N$. It turns out that empirically it is sufficient to take $N = 2$, so we will impose this condition from here onwards to simplify the expressions. We thus have short-term

[4] Using a similar motivation, Kamara, Korajczyk, Lou, and Sadka (2015) study empirically how the horizon that is used to calculate returns matters for the pricing of various risk factors.

[5] We assume that the proceeds of the dividends at all times are added to the risk-free deposit.

investors with horizon of h_1 periods and long-term investors with horizon h_2. Appendix 1.A.1 solves the model for any N.

- Investors have mean-variance utility over terminal wealth with risk aversion A_j for investor type j.
- We have an overlapping generations (OLG) setup. Each period, a fixed quantity $Q_j > 0$ of type j investors enters the market and invests in some or all of the K assets.
- Investors with horizon h_j only trade when they enter the market and at their terminal date, hence they do not rebalance their portfolio at intermediate dates.

Most assumptions follow Acharya and Pedersen (2005).[6] The key extension is that we allow for heterogenous horizons, while Acharya and Pedersen (2005) only have one-period investors. We make two simplifying assumptions to obtain tractable solutions. First, we assume i.i.d. dividends and transaction costs so as to obtain a stationary equilibrium. In reality transaction costs are relatively persistent over time. In the empirical section of the paper, we show that the i.i.d. assumption does not have a major impact on our empirical results.

The second simplifying assumption is that investors do not rebalance at intermediate dates. This assumption is important mainly for the long-term investors. As long as rebalancing trades are small relative to the total positions, we do not expect that relaxing this assumption would generate very different results. Also note that, in presence of transaction costs, investors only rebalance their portfolio infrequently (see, for example, Constantinides (1986)). In addition, positions in some categories of investment assets, such as private equity, may be hard to rebalance.

[6] Acharya and Pedersen (2005) start with investors with exponential utility and normally distributed dividends and costs, which amounts to assuming mean-variance preferences.

1.3.2 Equilibrium Expected Returns

In this subsection we describe how we obtain the equilibrium expected returns given our model setup. First, note that, at time t, investors with horizon h_j solve a maximization problem where they choose the quantity of stocks purchased $y_{j,t}$ (a vector with one element for each asset) to maximize utility over their holding period return, taking into account the incurred transaction costs. The utility maximization problem is given by

$$\max_{y_{j,t}} \mathbb{E}\left[W_{j,t+h_j}\right] - \frac{1}{2}A_j \mathrm{Var}\left(W_{j,t+h_j}\right) \tag{1.1}$$
$$W_{j,t+h_j} = \left(P_{t+h_j} + \sum_{k=1}^{h_j} R_f^{h_j-k} D_{t+k} - C_{t+h_j}\right)' y_{j,t} + R_f^{h_j}\left(e_j - P_t' y_{j,t}\right),$$

where R_f is the gross risk-free rate, $W_{j,t+1}$ is wealth of the h_j investors at time $t+1$, P_{t+1} is the $K \times 1$ vector of prices, and e_j is the endowment of the h_j investors.

In the remainder of the text of the paper, we set $R_f = 1$ to simplify the exposition. Appendix 1.A.1 derives the model for $R_f \geq 1$, which leads to very similar expressions. In the empirical analysis, we obviously estimate the version of the asset pricing model with R_f equal to the historical average of the risk-free rate.

The optimal portfolio choice may reflect endogenous segmentation, which is the possibility that some classes of investors do not hold some assets in equilibrium because the associated trading costs are too high relative to the expected return over the investment horizon. To this end, we introduce sets B_j ($j = 1,2$) that are subsets of $\{1,\ldots,K\}$, where K is the number of tradable assets. The set B_j represents the set of assets that investors j will buy in equilibrium. We find that a short-horizon investor (with horizon h_1) will endogenously avoid investing in assets for which the associated transaction costs are too large. The sets B_j thus depend on the level of transaction costs of the assets. Note that, for markets to clear, long-term investors will hold all assets in equilibrium, so that $B_2 = \{1, \ldots, K\}$. In Appendix 1.A.2, we describe in more detail under which conditions endogenous segmentation arises.

The solution to this utility maximization problem is the usual mean-variance solution, corrected for transaction costs and the possibility of segmentation. As shown in Appendix 1.A.1, the solution can be written as

$$y_{j,t} = \frac{1}{A_j} \operatorname{diag}(P_t)^{-1} \operatorname{Var}\left(\sum_{k=1}^{h_j} R_{t+k} - c_{t+h_j}\right)^{-1}_{B_j,p} \times \left(h_j \mathbb{E}[R_{t+1} - 1] - \mathbb{E}[c_{t+1}]\right), \tag{1.2}$$

where R_{t+1} denotes the $K \times 1$ vector of gross asset returns, with $R_{i,t+1} = (D_{i,t+1} + P_{i,t+1})/P_{i,t}$, and c_{t+1} the $K \times 1$ vector of percentage costs, with $c_{i,t} = C_{i,t}/P_{i,t}$. For a generic matrix M, the notation M_{B_j} is used to indicate the $|B_j| \times |B_j|$ matrix containing only the rows and columns of M that are in B_j. We write $M^{-1}_{B_j,p}$ for the inverse of M_{B_j} with zeros inserted at the locations where rows and columns of M were removed. With this convention, $\operatorname{Var}\left(\sum_{k=1}^{h_j} R_{t+k} - c_{t+h_j}\right)^{-1}_{B_j,p}$ corresponds to the $K \times K$ matrix containing the inverse of the covariance matrix of the set of assets that investors j invest in, with zeros inserted for the rows and columns that were not included (the assets that investors j do not invest in). The optimal demand vector $y_{j,t}$ thus contains zeros for those assets in which investor j does not invest.[7]

With i.i.d. dividends and costs, given a fixed asset supply, a wealth-independent optimal demand, and with the same type of investors entering the market each period, we obtain a stationary equilibrium where the price of each asset $P_{i,t}$ is constant over time. At any point in time, the market clears with new investors buying

[7] We compute the long-term covariance matrices using the i.i.d. assumption. Appendix 1.A.3 provides further details.

the supply of stocks minus the amount still held by the investors that entered the market at an earlier point in time,

$$Q_1 y_{1,t} + Q_2 y_{2,t} = S - \sum_{k=1}^{h_1-1} Q_1 y_{1,t-k} - \sum_{k=1}^{h_2-1} Q_2 y_{2,t-k}, \tag{1.3}$$

where S is the vector with supply of assets (in number of shares of each of the assets). Given the i.i.d. setting, we have constant demand over time, $y_{j,t} = y_{j,t-k}$ for all j and k.

We let $R_t^m = \widetilde{S}_t' R_t / \widetilde{S}_t' \iota$ and $c_t^m = \widetilde{S}_t' c_t / \widetilde{S}_t' \iota$, where $\widetilde{S}_t = \mathrm{diag}(P_t) S$ denotes the dollar supply of assets. Appendix 1.A.1 shows that under the stated assumptions we obtain the following result.

PROPOSITION 1: *A stationary equilibrium exists with the following equilibrium expected returns*

$$\begin{aligned}\mathbb{E}\left[R_{t+1} - 1\right] = {} & (\gamma_1 h_1 V_1 + \gamma_2 h_2 V_2)^{-1} (\gamma_1 V_1 + \gamma_2 V_2)\, \mathbb{E}\left[c_{t+1}\right] \\ & + (\gamma_1 h_1 V_1 + \gamma_2 h_2 V_2)^{-1} \operatorname{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right),\end{aligned} \tag{1.4}$$

where

$$V_j = h_j \operatorname{Var}\left(R_{t+1} - c_{t+1}\right) \operatorname{Var}\left(\sum_{k=1}^{h_j} R_{t+k} - c_{t+h_j}\right)^{-1}_{B_j,p}, \tag{1.5}$$

and $\gamma_j = Q_j / (A_j \widetilde{S}' \iota)$.[8] R_f *is set equal to 1 for ease of exposition.*

Proposition 1 shows that the equilibrium expected returns contain two components. The first component is a compensation for the expected transaction costs. The second component is a compensation for market risk and liquidity risk. Note that the loadings on expected costs and return covariances are matrices. This is in

[8] The time subscript for supply $\widetilde{S}_t$ is omitted, as supply is constant over time.

contrast to standard linear asset pricing models, where these loadings are scalars and therefore all assets have the same exposure to expected costs and the return covariance.

In the equilibrium equation (1.4), the parameter γ_j has an interesting interpretation as *risk-bearing capacity*. Specifically, the OLG setup implies that in every period the total number of h_j-investors in the market is equal to $h_j Q_j$. This total number is important because it determines among how many h_j-investors the risky assets can be shared. Their risk aversion A_j is also important, because it determines the size of the position these investors are willing to take in the risky assets. Therefore, we can indeed interpret the quantity

$$h_j \gamma_j = \frac{h_j Q_j}{A_j} \frac{1}{\widetilde{S}' \iota} \tag{1.6}$$

as the *risk-bearing capacity* of the h_j-investors (scaled by the total market capitalization).

1.3.3 Interpreting the Equilibrium: Special Cases

We now consider several special cases to gain intuition for the different effects that the general equilibrium model generates. It is important to note that, in the empirical analysis, we estimate the general model in equation (1.4). Hence, these special cases are only used here to better understand the new implications of our equilibrium model.

We begin with an integration setting where both short-term and long-term investors hold all assets. In this setting, we consider the following special cases:

- the liquidity CAPM of Acharya and Pedersen (2005);
- the expected liquidity effect without liquidity risk;
- the expected liquidity effect with liquidity risk;

- the market and liquidity risk premia with two assets.

We then consider a special case within the endogenous segmentation setting, where the short-term investors do not invest in assets that are very illiquid. Finally, we summarize and discuss the array of novel predictions of our model.

Liquidity CAPM of Acharya and Pedersen (2005)

If we have only one investor type with a one-period horizon, we obtain a model similar to the liquidity CAPM of Acharya and Pedersen (2005). Specifically, consider the case where $N=1$ (or $\gamma_2=0$), $h_1=1$, and $B_1=\{1,\dots,K\}$, so that there is just one class of one-period investors. For ease of comparison, we write the equilibrium equation in beta form. In this case, the equilibrium expected returns simplify to

$$\begin{aligned}\mathbb{E}\left[R_{t+1}-1\right] &= \mathbb{E}\left[c_{t+1}\right] \\ &\quad + \frac{\operatorname{Var}\left(R^m_{t+1}-c^m_{t+1}\right)}{\gamma_1}\frac{\operatorname{Cov}\left(R_{t+1}-c_{t+1},R^m_{t+1}-c^m_{t+1}\right)}{\operatorname{Var}\left(R^m_{t+1}-c^m_{t+1}\right)},\end{aligned} \tag{1.7}$$

which is an i.i.d. version of the equilibrium relation of Acharya and Pedersen (2005).

Expected liquidity effect without liquidity risk

We now allow for two distinct investor horizons, but assume constant transaction costs (i.e. $\operatorname{Var}(c_{t+1})=0$). In the integration setting ($B_1=B_2=\{1,\dots,K\}$), we obtain a linear asset pricing model with scalar loadings on expected liquidity and risk

$$\mathbb{E}\left[R_{t+1}-1\right]=\frac{\gamma_1+\gamma_2}{\gamma_1 h_1+\gamma_2 h_2}\mathbb{E}\left[c_{t+1}\right]+\frac{1}{\gamma_1 h_1+\gamma_2 h_2}\operatorname{Cov}\left(R_{t+1},R^m_{t+1}\right). \tag{1.8}$$

We immediately see that the loading on expected liquidity equals $1/h_1$ if $\gamma_2 = 0$ and $1/h_2$ if $\gamma_1 = 0$. As the horizon h_j increases, it follows that the impact of expected liquidity on returns decreases with the investor horizon.

To illustrate the difference with the single-horizon case in equation (1.7), where the loading on expected liquidity is equal to 1, let us use a simple example with $h_1 = 1$, $h_2 = 2$, $\gamma_1 = 2$, and $\gamma_2 = 1$. In this simple example, the loading on expected liquidity is equal to

$$\frac{\gamma_1 + \gamma_2}{\gamma_1 h_1 + \gamma_2 h_2} = \frac{3}{4}, \tag{1.9}$$

which is exactly halfway between the expected liquidity coefficient with only one-period investors ($1/h_1 = 1$) and the loading when there are only two-period investors ($1/h_2 = 1/2$). More generally, we observe that the introduction of long-term investors in the model decreases the impact of expected liquidity on expected returns.

Expected liquidity effect with liquidity risk

We now extend the previous special case C.2 to a setting with stochastic transaction costs. For simplicity, we take $\mathrm{Var}(c_{t+1})$ and $\mathrm{Var}(R_{t+1} - c_{t+1})$ to be diagonal matrices (in this example only), we set $h_1 = 1$, and still consider the integration setting ($B_1 = B_2 = \{1, \ldots, K\}$). In this case, we obtain

$$\begin{aligned} \mathbb{E}\left[R_{i,t+1} - 1\right] = {} & \frac{\gamma_1 + \gamma_2 V_{2,i}}{\gamma_1 h_1 + \gamma_2 h_2 V_{2,i}} \mathbb{E}\left[c_{i,t+1}\right] \\ & + \frac{1}{\gamma_1 h_1 + \gamma_2 h_2 V_{2,i}} \mathrm{Cov}\left(R_{i,t+1} - c_{i,t+1}, R^m_{t+1} - c^m_{t+1}\right), \end{aligned} \tag{1.10}$$

where $V_{2,i}$ denotes the i-th diagonal element of V_2. In this case, we can write $V_{2,i}$ as

$$V_{2,i} = \frac{h_2 \mathrm{Var}\left(R_{i,t+1} - c_{i,t+1}\right)}{(h_2 - 1)\,\mathrm{Var}\left(R_{i,t+1}\right) + \mathrm{Var}\left(R_{i,t+1} - c_{i,t+1}\right)}. \tag{1.11}$$

Now consider the following ratio:

$$\frac{\text{Var}\left(R_{i,t+1} - c_{i,t+1}\right)}{\text{Var}\left(R_{i,t+1}\right)}. \tag{1.12}$$

This ratio is a good measure of the amount of liquidity risk, as it increases with Var($c_{i,t+1}$) and with Cov($R_{i,t+1}, c_{i,t+1}$). We can show that the expected liquidity coefficient in (1.10) decreases with this "liquidity risk" ratio. That is, higher liquidity risk leads to a smaller expected liquidity premium. This result might seem counterintuitive at first sight, but it has a natural interpretation. If an asset has higher liquidity risk, it will be held in equilibrium mostly by long-term investors. Long-term investors care less about liquidity and this leads to the smaller expected liquidity effect.

Market and liquidity risk premia with two assets

We now focus on interpreting the risk premia that the model generates in equilibrium. In the general model of equation (1.4), expected returns are determined by a mix of market and liquidity risk premia. This mix becomes especially clear when we consider the two-asset case ($K = 2$), $h_1 = 1$, again in the integration setting. Formally:

PROPOSITION 2: *In the two-asset case ($K = 2$), with two horizons ($N = 2$), $h_1 = 1$, $R_f = 1$, and no segmentation ($B_1 = B_2 = \{1, \ldots, K\}$), the equilibrium expected returns are*

$$\begin{aligned}\mathbb{E}\left[R_{t+1} - 1\right] = {} & (\gamma_1 h_1 V_1 + \gamma_2 h_2 V_2)^{-1} (\gamma_1 V_1 + \gamma_2 V_2)\, \mathbb{E}\left[c_{t+1}\right] \\ & + (\gamma_1 \lambda_1 + \gamma_2 \lambda_2) \text{Cov}\left(R_{t+1} - c_{t+1}, R^m_{t+1} - c^m_{t+1}\right) \\ & + \gamma_2 \lambda_2 (h_2 - 1) \text{Cov}\left(R_{t+1}, R^m_{t+1}\right),\end{aligned} \tag{1.13}$$

where $\lambda_j = h_j^2/d_0 d_j$ is a scalar parameter. The definitions of the determinants d_0 and d_j are given by equations (1.48) *and* (1.50) *in Appendix 1.A.4.*

In this equilibrium, the total risk premium is a weighted sum of market and liquidity risk premia. Holding everything else constant, we can show that liquidity risk becomes less important relative to market risk when the long-term investors become less risk averse or more numerous (formally, as γ_2 increases). As γ_2 increases, long-term investors hold a larger fraction of the total supply in equilibrium and these investors care less about liquidity risk compared to short-term investors.

Segmentation effects

The special cases discussed above show the expected liquidity and risk premia effects when all investors have positive holdings of all assets. Now we show what happens to expected returns when some assets are only held by long-term investors (endogenous segmentation).

To obtain tractable theoretical expressions, we focus on the special case where V_2 equals the identity matrix and set $h_1 = 1$. The simplification $V_2 = I$ is appropriate when the variability of returns is much higher than the variability of transaction costs. As we show later in the empirical section, this is indeed the case in our data and we can thus rely on these theoretical simplified expressions. Of course, our benchmark empirical estimation focuses on the unrestricted equilibrium in equation (1.4).

Without loss of generality, we order the assets by liquidity, with the most liquid assets first. The returns on the assets that are in B_1 are denoted by R_t^{liq}, and the returns on the assets that are not in B_1 are denoted by R_t^{illiq}. We use this notation also for the costs. Appendix 1.A.5 shows that in this setting we obtain the following proposition.

PROPOSITION 3: *If $N = 2$, $h_1 = 1$, $V_2 = I$, $R_f = 1$, and B_1 contains only those assets that the short-term investors hold, then for these "liquid" assets the expected returns are*

$$\begin{aligned}\mathbb{E}\left[R^{liq}_{t+1} - 1\right] = & \frac{\gamma_1 + \gamma_2}{\gamma_1 h_1 + \gamma_2 h_2}\mathbb{E}\left[c^{liq}_{t+1}\right] \\ & + \frac{1}{\gamma_1 h_1 + \gamma_2 h_2}\mathrm{Cov}\left(R^{liq}_{t+1} - c^{liq}_{t+1}, R^{m}_{t+1} - c^{m}_{t+1}\right).\end{aligned} \tag{1.14}$$

The expected returns on "illiquid" assets only held by long-term investors are

$$\begin{aligned}\mathbb{E}\left[R^{illiq}_{t+1} - 1\right] = & \frac{1}{h_2}\mathbb{E}\left[c^{illiq}_{t+1}\right] + \frac{h_2 - h_1}{h_2}\frac{\gamma_1}{\gamma_1 h_1 + \gamma_2 h_2}\beta\mathbb{E}\left[c^{liq}_{t+1}\right] \\ & + \frac{1}{\gamma_1 h_1 + \gamma_2 h_2}\mathrm{Cov}\left(R^{illiq}_{t+1} - c^{illiq}_{t+1}, R^{m}_{t+1} - c^{m}_{t+1}\right) \\ & + \left(\frac{1}{\gamma_2 h_2} - \frac{1}{\gamma_1 h_1 + \gamma_2 h_2}\right)\mathrm{Cov}\left(R^{illiq}_{t+1} - c^{illiq}_{t+1}, R^{m}_{t+1} - c^{m}_{t+1}\right) \\ & - \left(\frac{1}{\gamma_2 h_2} - \frac{1}{\gamma_1 h_1 + \gamma_2 h_2}\right)\beta\mathrm{Cov}\left(R^{liq}_{t+1} - c^{liq}_{t+1}, R^{m}_{t+1} - c^{m}_{t+1}\right),\end{aligned} \tag{1.15}$$

where the matrix β *denotes the* liquidity spillover beta*, defined as*

$$\beta = \mathrm{Cov}\left(R^{illiq}_{t+1} - c^{illiq}_{t+1}, R^{liq}_{t+1} - c^{liq}_{t+1}\right)\mathrm{Var}\left(R^{liq}_{t+1} - c^{liq}_{t+1}\right)^{-1}. \tag{1.16}$$

First, we note that the equilibrium expected returns for liquid assets are similar to the special cases discussed previously, since these assets are held by both short-term and long-term investors. For the "illiquid" assets, the pricing is more complex. In what follows, we thus discuss separately the different components that make up expected excess returns for illiquid assets.

We start by analyzing the expected liquidity effect that we can decompose into three parts:

$$\begin{aligned}
&\frac{\gamma_1+\gamma_2}{\gamma_1 h_1+\gamma_2 h_2}\mathbb{E}\left[c_{t+1}^{illiq}\right] \\
&\quad+\left(\frac{1}{h_2}-\frac{\gamma_1+\gamma_2}{\gamma_1 h_1+\gamma_2 h_2}\right)\mathbb{E}\left[c_{t+1}^{illiq}\right] \\
&\quad+\frac{h_2-h_1}{h_2}\frac{\gamma_1}{\gamma_1 h_1+\gamma_2 h_2}\beta\mathbb{E}\left[c_{t+1}^{liq}\right].
\end{aligned} \tag{1.17}$$

The first component, which we denote *full risk-sharing expected liquidity premium,* is the expected liquidity effect that one would obtain if these assets were held by both short-term and long-term investors. The second term (*segmentation expected liquidity premium*) reflects that, in fact, only long-term investors hold the illiquid assets and this term dampens the effect of expected liquidity since $\frac{1}{h_2}-\frac{\gamma_1+\gamma_2}{\gamma_1 h_1+\gamma_2 h_2}<0$. The third component (*spillover expected liquidity premium*) arises from the exposure (as given by β) of the illiquid assets to the liquid assets. If this exposure is positive, this increases the expected liquidity effect for the illiquid assets since $\frac{h_2-h_1}{h_2}\frac{\gamma_1}{\gamma_1 h_1+\gamma_2 h_2}>0$. In other words, if liquid and illiquid assets are positively correlated, the expected liquidity effect on illiquid assets cannot be much lower than the effect for liquid assets, because long-term investors would take advantage by shorting the illiquid assets and buying the liquid assets.

We now turn to the risk premia, where we have a natural interpretation for each of the various covariance terms in the equilibrium relation for the illiquid assets. The term

$$\frac{1}{\gamma_1 h_1+\gamma_2 h_2}\mathrm{Cov}\left(R_{t+1}^{illiq}-c_{t+1}^{illiq},R_{t+1}^{m}-c_{t+1}^{m}\right) \tag{1.18}$$

gives the *full risk-sharing risk premium* that would arise if both types of investors would hold the asset. The second term,

$$\left(\frac{1}{\gamma_2 h_2} - \frac{1}{\gamma_1 h_1 + \gamma_2 h_2}\right) \mathrm{Cov}\left(R_{t+1}^{illiq} - c_{t+1}^{illiq}, R_{t+1}^{m} - c_{t+1}^{m}\right), \tag{1.19}$$

gives the *segmentation risk premium*, which shows the impact of the lower risk sharing due to long-term investors only holding the illiquid assets. Since $\frac{1}{\gamma_2 h_2} - \frac{1}{\gamma_1 h_1 + \gamma_2 h_2} > 0$, this segmentation premium increases expected returns in case of positive return covariance. The third term,

$$-\left(\frac{1}{\gamma_2 h_2} - \frac{1}{\gamma_1 h_1 + \gamma_2 h_2}\right) \beta \mathrm{Cov}\left(R_{t+1}^{liq} - c_{t+1}^{liq}, R_{t+1}^{m} - c_{t+1}^{m}\right), \tag{1.20}$$

defines a *spillover risk premium*. Along the lines of the discussion above for the expected liquidity effect, this term concerns the relative pricing of the illiquid versus liquid assets. If these two assets are positively correlated (high elements of β), their expected returns cannot be too far apart. This term reduces the effect of segmentation when the elements of β are nonzero. Specifically, if

$$\mathrm{Cov}\left(R_{t+1}^{illiq} - c_{t+1}^{illiq}, R_{t+1}^{m} - c_{t+1}^{m}\right) = \beta \mathrm{Cov}\left(R_{t+1}^{liq} - c_{t+1}^{liq}, R_{t+1}^{m} - c_{t+1}^{m}\right), \tag{1.21}$$

the net effect of segmentation is equal to zero.

We can also rewrite the expected returns on segmented assets in Proposition 3 in a more compact form:

$$\begin{aligned}\mathbb{E}\left[R_{t+1}^{illiq} - 1\right] = {} & \frac{1}{h_2}\mathbb{E}\left[c_{t+1}^{illiq}\right] + \beta\left(\mathbb{E}\left[R_{t+1}^{liq}\right] - \frac{1}{h_2}\mathbb{E}\left[c_{t+1}^{liq}\right]\right) \\ & + \frac{1}{\gamma_2 h_2}\mathrm{Cov}\left(R_{t+1}^{illiq} - c_{t+1}^{illiq} - \beta\left(R_{t+1}^{liq} - c_{t+1}^{liq}\right), R_{t+1}^{m} - c_{t+1}^{m}\right),\end{aligned} \tag{1.22}$$

which can provide some additional intuition. In particular, this expression shows in a different way how segmentation matters. The expected returns on segmented assets are driven by the exposure to net-of-cost returns of the liquid assets, plus an additional effect that comes from the systematic exposure of the residual return on segmented assets, $R_{t+1}^{illiq} - c_{t+1}^{illiq} - \beta(R_{t+1}^{liq} - c_{t+1}^{liq})$.

The total segmentation risk premium, as expressed in equation (1.22), is in the spirit of the international asset pricing literature (e.g., De Jong and De Roon (2005)), where segmentation also leads to additional effects on expected returns.

To better illustrate how segmentation influences the impact of expected liquidity on expected returns, we consider again the simple example earlier in this section, where $h_1 = 1$, $h_2 = 2$, $\gamma_1 = 2$, and $\gamma_2 = 1$. We also impose $\mathrm{Var}(c_{t+1}) = 0$ and $\beta = 0$. In this segmentation setting, we find that the loading on expected liquidity is

$$\frac{\gamma_1 + \gamma_2}{\gamma_1 h_1 + \gamma_2 h_2} = \frac{3}{4} \tag{1.23}$$

for the liquid assets, and

$$\frac{1}{h_2} = \frac{1}{2} \tag{1.24}$$

for the illiquid assets. This example shows that the effect of expected liquidity is smaller for the illiquid assets, because these assets are only held by long-term investors in equilibrium. Note that in this case the total expected liquidity component of expected returns for liquid assets ($\frac{3}{4}\mathbb{E}\left[c_{t+1}^{liq}\right]$) is not necessarily smaller than the premium for illiquid assets ($\frac{1}{2}\mathbb{E}\left[c_{t+1}^{illiq}\right]$).

Summary and Discussion

Our model shows that the asset pricing effects of liquidity are much more complex once we allow for heterogenous horizons and segmentation. In summary, the main theoretical implications are:

(i) the expected liquidity effect is decreasing with investor horizons;

(ii) the expected liquidity effect is decreasing with the amount of liquidity risk;

(iii) for “segmented” assets the expected liquidity effect is dampened because of the exclusive ownership of long-term investors;

(iv) for “segmented” assets the expected liquidity effect also contains a spillover term due to the correlation between segmented and non-segmented assets;

(v) the total risk premium is a mix of a market risk premium and a liquidity risk premium. The liquidity risk premium becomes relatively more important when short-term investors are more numerous or less risk-averse;

(vi) for “segmented” assets there is an additional segmentation risk premium due to limited risk sharing;

(vii) for “segmented” assets there is an additional spillover risk premium due to the correlation between segmented and non-segmented assets.

Note that the sign of the various effects listed above is not always unambiguous. For example, the spillover effects clearly depend on the sign of the correlation between segmented and non-segmented assets. The model thus predicts a more complex relation between liquidity and expected returns compared to Acharya and Pedersen (2005) and Amihud and Mendelson (1986). For example, one of the most interesting predictions of Amihud and Mendelson (1986) is the concave relationship between expected liquidity and expected returns. In our model, the effect that drives this concave relation is also present (a smaller expected liquidity coefficient for segmented assets, point 3 above). However, there are other segmentation and spillover effects that also play a role. These additional effects are not present in Amihud and Mendelson (1986), because they assume risk-neutral investors. In their model long-term investors only invest in illiquid assets and not in the liquid assets. In contrast, in our model with risk averse agents, long-term investors will diversify and invest in liquid assets as well, leading to spillover and segmentation effects.

We thus observe that the introduction of heterogenous investment horizons into a liquidity asset pricing model has strong implications for the pricing of liquid versus illiquid assets. Specifically, we find various and potentially contrasting effects on the liquidity (risk) premia. It then becomes an empirical question to understand the relevance of these additional effects. We take on this task in the next Sections of the paper.

1.4 Empirical Methodology

In this section, we explain how our liquidity asset pricing model can be estimated. We also explore the economic mechanism that allows the identification of the parameters. We then discuss alternative approaches for a robust computation of standard errors.

1.4.1 GMM Estimation

We use a Generalized Method of Moments (GMM) methodology to estimate the equilibrium condition given by equation (1.4), but without imposing $R_f = 1$. The key estimated parameters are $\gamma_j = Q_j/(A_j\widetilde{S}'\iota)$, that is, the *risk-bearing capacity* of the different classes of investors. We define the vector of pricing errors of all assets, denoted by $g(\psi,\gamma)$, as

$$\begin{aligned} g(\psi,\gamma) = {} & \mathbb{E}\left[R_{t+1}-1\right] - \left(\gamma_1 h_1 V_1 + \gamma_2 h_2 V_2\right)^{-1}\left(\gamma_1 V_1 + \gamma_2 V_2\right)\mathbb{E}\left[c_{t+1}\right] \\ & - \left(\gamma_1 h_1 V_1 + \gamma_2 h_2 V_2\right)^{-1}\operatorname{Cov}\left(R_{t+1}-c_{t+1}, R^m_{t+1}-c^m_{t+1}\right), \end{aligned} \tag{1.25}$$

where γ is the vector of parameters, and ψ is a vector containing the underlying expectations and covariances that enter the pricing errors. Specifically, ψ contains all expected returns, expected costs, covariances entering the V_j matrices, and the covariances with the market return. In a first step, we estimate all elements of ψ

by their sample moments. In a second step, we perform a GMM estimation of γ, using an identity weighting matrix across all assets. We thus minimize the sum of squared pricing errors over γ,

$$\min_{\gamma} g(\widehat{\psi},\gamma)' g(\widehat{\psi},\gamma). \tag{1.26}$$

In Appendix 1.A.6, we derive the asymptotic covariance matrix of this GMM estimator, taking into account the estimation error in all sample moments in ψ, in line with the approach of Shanken (1992).

1.4.2 Identification

To gain insight into the economic mechanism that allows the identification of the parameters, it is useful to illustrate some comparative statics results. Specifically, a change in γ_j means that the horizon h_j investors become either more numerous, or less risk averse, or both. Appendix 1.A.7 shows that the effect of such a change on expected returns is given by

$$\frac{\partial \mathbb{E}\left[R_{t+1}-1\right]}{\partial \gamma_j} = \left(\gamma_1 h_1 V_1 + \gamma_2 h_2 V_2\right)^{-1} V_j \left(\mathbb{E}\left[c_{t+1}\right] - h_j \mathbb{E}\left[R_{t+1}-1\right]\right). \tag{1.27}$$

We observe two contrasting effects of an increase in γ_j. The first effect is an increase in the risk premium due to the impact of expected liquidity. The second effect is the increased amount of risk sharing, which leads to a decrease in the risk premium proportional to the original risk premium. For long-term investors, the latter effect dominates and an increase in γ implies lower expected returns for all assets. For short-term investors, however, the expected costs may exceed the expected return $h_j \mathbb{E}\left[R_{t+1}-1\right]$ for the more illiquid assets. This is exactly what we observe in the data for some more illiquid stocks. Hence, an increase in γ_1, which corresponds to the short-term investors, may increase the expected return of illiquid assets and decrease the expected return of liquid assets. We also observe that hedging considerations could play a different role for short-term versus long-term investors, because the matrix pre-multiplying the difference between the liquidity cost and the scaled risk premium can reverse the sign of the partial derivatives in equation (1.27).

In summary, this comparative statics exercise shows that the estimated parameters for short-term versus long-term investors may have opposing effects on equilibrium expected returns for different assets and, as such, can be properly identified.

1.4.3 Bootstrap Standard Errors

We use a bootstrap test to check the robustness of the asymptotic standard errors. We generate bootstrap samples by re-sampling the data and then carrying out the first step of the estimation procedure to obtain estimates for the different moments that enter the vector of pricing errors.

The test is a bootstrap t-test based on the bootstrap estimate of the standard error. The test does not provide asymptotic refinements, but has the advantage that it does not require direct computation of asymptotically consistent standard errors and thus provides a check on the asymptotic standard errors. Overall, we find that the bootstrap standard errors are close to the asymptotic standard errors.

1.5 Data

We largely follow Acharya and Pedersen (2005) in our data selection and construction. We use daily stock return and volume data from CRSP from 1964 until 2009 for all common shares listed on NYSE and AMEX. As our empirical measures of liquidity rely on volume, we do not include Nasdaq since the volume data includes interdealer trades (and only starts in 1982). Overall, we consider a number of stocks ranging from 1,056 to 3,358, depending on the month. To correct for survivorship bias, we adjust the returns for stock delisting (see Shumway (1997) and Acharya and Pedersen (2005)).

The relative illiquidity cost is computed as in Acharya and Pedersen (2005). The starting point is the Amihud (2002) illiquidity measure, which is defined as

$$ILLIQ_{i,t} = \frac{1}{Days_{i,t}} \sum_{d=1}^{Days_{i,t}} \frac{\left|R_{i,t,d}\right|}{Vol_{i,t,d}} \tag{1.28}$$

for stock i in month t, where $Days_{i,t}$ denotes the number of observations available for stock i in month t, and $R_{i,t,d}$ and $Vol_{i,t,d}$ denote the trading volume in millions of dollars for stock i on day d in month t, respectively.

We follow Acharya and Pedersen (2005) and define a normalized measure of illiquidity that deals with non-stationarity and is a direct measure of trading costs, consistent with the model specification. The normalized illiquidity measure can be interpreted as the dollar cost per dollar invested and is defined for asset i by

$$c_{i,t} = \min\left\{0.25 + 0.30 ILLIQ_{i,t} P^m_{t-1}, 30.00\right\}, \tag{1.29}$$

where P^m_{t-1} is equal to the market capitalization of the market portfolio at the end of month $t-1$ divided by the value at the end of July 1962. The product with P^m_{t-1} makes the cost series $c_{i,t}$ relatively stationary and the coefficients 0.30 and 0.25 are chosen as in Acharya and Pedersen (2005) to match approximately the level and variance of $c_{i,t}$ for the size portfolios to those of the effective half spread reported by Chalmers and Kadlec (1998). The value of normalized liquidity $c_{i,t}$ is capped at 30% to make sure the empirical results are not driven by outliers.

We obtain the book-to-market ratio using fiscal year-end balance sheet data from COMPUSTAT in the same manner as Ang and Chen (2002). They follow

Fama and French (1993) in defining the book value of a firm as the sum of common stockholders' equity, deferred taxes, and investment credit minus the book value of preferred stocks. The ratio is obtained by dividing the book value by the fiscal year-end market value.

We construct the market portfolio on a monthly basis and only use stocks that have a price on the first trading day of the corresponding month between $5 and $1000. We include only stocks that have at least 15 observations of return and volume during the month. Following Acharya and Pedersen (2005), we use equal weights to compute the return on the market portfolio.

We construct 25 illiquidity portfolios, 25 illiquidity variation portfolios, and 25 book-to-market and size portfolios, as in Acharya and Pedersen (2005). The portfolios are formed on an annual basis. For these portfolios, we require again for the stock price on the first trading day of the corresponding month to be between $5 and $1000. For the illiquidity and illiquidity variation portfolios, we require to have at least 100 observations of the illiquidity measure in the previous year.

Table 1 shows the estimated average costs and average returns across the 25 illiquidity portfolios. The values correspond closely to those found in Table 1 of Acharya and Pedersen (2005). Most importantly, we see that average returns tend to be higher for illiquid assets. Also, the table shows that returns on more illiquid portfolios are more volatile. This finding holds for returns net of costs as well. The returns (net of costs) on more illiquid portfolios tend to co-move more strongly with market returns (also net of costs).

1.6 Empirical Results

In this section, we take the model to the data. First, we estimate the parameters of the model for the segmented case and compare it with single-horizon models (e.g., Acharya and Pedersen (2005)). We also explore the implications of the estimates for the importance of the different components of expected returns. We then study the robustness of our results to the choice of the investor horizon, to the extent of segmentation, and to pricing different sets of portfolios.

1.6.1 Estimation Setup

We estimate the parameters of the equilibrium relation given by equation (1.4) for the sample period 1964–2009 using the GMM methodology described in Section 1.4.1. We first estimate the model on 25 portfolios of stocks listed on NYSE and AMEX, sorted on illiquidity. In the next subsection, we also estimate the model for 25 illiquidity-variation portfolios and 25 Book/Market-by-Size portfolios.

Our benchmark estimation is based on two classes of investors.[9] The first class (short horizon) has an investment horizon h_1 of one month, the second class (long horizon) has an investment horizon h_2 of 120 months (10 years). The choice of the length of the long horizon can be related to the results of using the methodology of Atkins and Dyl (1997) for our sample.[10] Over the 1964-2009 period, we find an average holding period of 5.59 years. The robustness tests later in Section 1.6.3 show that the empirical results are virtually unchanged with the long horizon set at five years or longer.

Long-term investors tend to hold more illiquid assets. Consistent with this idea, Table 1.1 shows that turnover tends to be much lower and has a smaller standard deviation for the least liquid portfolios. We thus impose a segmentation cutoff, where the one-month investors invest only in the 19 most liquid portfolios. We choose this threshold based on the empirical evidence in Table 1.1. While monthly

[9] Adding a third class of investors does not yield substantial empirical improvement. The corresponding coefficient does not necessarily go to zero, but the R^2 remains essentially unchanged, with little gain in terms of explanatory power.

[10] Atkins and Dyl (1997) find that the mean investor holding period for NYSE stocks during the period 1975–1989 is roughly equal to 4.01 years.

expected excess returns are larger or similar to expected costs for most portfolios, for the six least liquid portfolios, the costs become roughly 2 to 9 times higher than the monthly average return. As the one-month investors incur the costs each period, these assets can be seen as prohibitively costly.[11]

This simple rule for the one-month investor (hold the asset if the expected monthly return exceeds the expected transaction costs and have a zero position otherwise) would also be the optimal rule with a diagonal covariance matrix of returns, as equation (1.2) shows.[12] Furthermore, Figure 1.6 shows that this threshold maximizes the cross-sectional R^2 across all possible cutoffs, including the model without any segmentation.

Having set horizons and the segmentation cutoff, we now estimate the model parameters $\gamma_j = Q_j/(A_j\widetilde{S}'\iota)$ and, in some cases, a constant term in the expected return equation (α). We denote the models with and without a constant term as specifications (SEG+α) and (SEG), respectively. The role of the constant term is basically to provide a specification check, because it should equal zero under the null hypothesis. Recall that we can interpret $h_j\gamma_j$ as the risk-bearing capacity of h_j-investors. The risk-bearing capacity is determined by the risk aversion (A_j) and size (Q_j) of the h_j-investor group. Hence, the interpretation of the estimated parameters can offer interesting insights on the risk aversion or size of the short-term versus long-term investor groups.

We compare our model with a baseline one-period horizon model as in equation (1.7), with $N = 1$ and $h_1 = 1$. Here, we follow Acharya and Pedersen (2005) and allow for a slope coefficient κ on the expected liquidity term $\mathbb{E}[c_{t+1}]$, although

[11] A portfolio-level analysis along the lines of Atkins and Dyl (1997) shows that the first 19 portfolios have average holding periods between 2.49 and 7.91 years, while portfolios 20 through 25 have average holding periods between 10.67 and 30.12 years, suggesting that short-term investors are unlikely to trade these illiquid stocks.

[12] To determine endogenously what are the portfolios held by the one-month investors, we can cast the problem as a mean-variance optimization exercise for the one-month investors. However, with this exercise, we run into the often-encountered issue of extreme positions in some portfolios due to close-to-singular covariance matrices.

formally the Acharya and Pedersen (2005) model implies a coefficient on expected liquidity equal to one. This coefficient is used by Acharya and Pedersen (2005) to correct for the fact that the typical holding period does not equal the estimation period of one month. We denote these single-horizon specifications as (AP) and (AP+α) if we add the constant term. These single-horizon specifications provide a very useful baseline case to understand the empirical improvement of having multiple horizons and segmentation, because they have the same degrees of freedom of the segmented models. For both categories, there are two estimated parameters and, possibly, a constant. Specifically, the single horizon case contains one horizon parameter and one expected liquidity coefficient, while the multiple horizon case has one parameter for each horizon.

1.6.2 Benchmark Estimation Results

Table 1.2 shows the results for the illiquidity portfolios. We find that the first specification of the segmented model (SEG), without a constant term, improves the R^2 of the Acharya-Pedersen model by about 20%, from 62% to 82%. Importantly, this improvement is achieved retaining the parsimony of the original model – both models depend on two parameters. The fit is graphically displayed in Figure 1.2. The graphs indicate that accounting for segmentation and heterogeneous horizons leads to smaller pricing errors in the upper-right end of the plot, i.e., for the more illiquid portfolios (as Table 1.1 shows that illiquid portfolios tend to have higher excess returns). Since the more illiquid portfolios are also characterized by segmentation, this is first suggestive evidence that the economic source of the improved fit of our model is obtained by effectively constraining the clientele of the illiquid assets to the long-term investors. Table 1.2 also shows that the segmented model still outperforms the AP model when we allow for a constant term α in the asset pricing equation.

We then investigate the sources of this improved fit in more detail and use the empirical estimates to decompose expected returns into an expected liquidity component and risk premium component, according to equation (1.4). We depict this decomposition for the single-horizon and two-horizon case with segmentation in Figure 1.3. We notice that in the single horizon (AP) case, the impact of the expected liquidity term is relatively modest. This is because the expected costs increase exponentially when moving from liquid to illiquid portfolios, while the expected returns do not exhibit such an exponentially increasing pattern (see Table 1.1 as well as Figure 1.1). If anything, the expected returns increase with illiquidity at a lower rate for the more illiquid portfolios: the expected return levels off after portfolio 19, but the expected expected liquidity term keeps rising. The (AP) specification implies a linear relation between expected costs and expected returns, and thus has difficulty fitting the cross-section of liquid versus illiquid portfolios. As a result, the expected liquidity effect is rather small for the (AP) specification (a few basis points per month for most portfolios).

Our model with segmentation reduces the impact of the expected liquidity term on the illiquid portfolios relative to the impact on the liquid portfolios. Hence, our model allows for a much larger overall expected liquidity premium (between 10 and 40 basis points per month) and this improves the fit substantially as shown by Figure 1.2 and Figure 1.3. The average expected liquidity premium across portfolios is about 20 basis points per month for the (SEG) specification, compared to an average effect of 3 basis points for the (AP) specification. Since only long-term investors hold the most illiquid assets, the expected liquidity premium is relatively limited for these assets. This explains the drop in the impact of the cost term around portfolios 19 and 20. Figure 1.3 also shows that the covariance term provides the largest overall contribution to the expected excess returns.

To gain further insight into the impact of segmentation, we make use of Proposition 3 to decompose both the expected liquidity effect and the covariance effect into a full risk-sharing component, a segmentation component, and a spillover component. We show these components in Figure 1.4. The decomposition indicates clearly how the impact of segmentation on the total expected return builds up. For the expected liquidity premium given in equation (1.17) (upper panel in Figure 1.4), the full risk-sharing effect increases sharply for the least liquid portfolios since expected costs increase exponentially when moving to illiquid assets. This effect is mostly canceled out by the negative segmentation effect, which arises because the long-term investors care less about liquidity. There is still a modest liquidity spillover premium. Hence, the liquidity spillover effect drives most of the expected liquidity effect for the least liquid assets. This is also what causes the drop in the model-implied expected return going from portfolio 19 to 20, as depicted in Figure 1.3.

For the covariance component of expected returns (lower panel of Figure 1.4), we observe that the segmentation premium and the spillover risk premium in equations (1.19) and (1.20) mostly cancel out because the returns on illiquid portfolios are strongly related to liquid portfolio returns. Hence the risk premia of liquid and illiquid assets are quite similar. This is evidence showing that the effect of segmentation is almost entirely driven by the expected liquidity term.

The estimates in Table 1.2 can be used to obtain insight into the structural parameters in the asset pricing model. For example, if we assume for simplicity that risk aversion is constant across investor classes (i.e., $A_1 = A_2$), we can make inferences about the number of investors in each class. More specifically, we examine the ratio $(h_2\gamma_2)/(h_1\gamma_1) = (h_2Q_2)/(h_1Q_1)$.[13] The results for specifications (SEG) and (SEG+α) show that the estimates imply that there are respectively 2.1 and 2.6 times as many long horizon investors as there are short horizon investors.

[13]As Q_j investors with horizon h_j enter each period, at each point in time the total number of type-j investors equals h_jQ_j. Also note that including $\widetilde{S}'\iota$ in the γ_j does not influence our comparison.

We show some comparative statics results for each model parameter in Figure 1.5 (see equation (1.27) for the analytical expression). The graphs illustrate the impact on the risk premium of an increase in the γ_j, that is, an increase in the quantity of class j investors, a decrease in their risk aversion, or both. The top panel shows the baseline case with one-period homogeneous investors. Here, the larger risk-sharing (with more numerous or less risk averse investors) is all that matters. Looking now at long-term investors in the heterogeneous horizon model (Figure 1.5, bottom right panel), we see that the effect of an increase in γ_2 on the risk premium is always negative. This is consistent with the theoretical analysis of Section 1.4.2, where we show that for long-term investors the risk sharing effect dominates the liquidity effect (absent hedging considerations). In other words, this finding confirms empirically that long term investors are less concerned about liquidity. For the short term investors (Figure 1.5, bottom left panel), we see that the effect of γ_1 on expected returns is positive for the most illiquid portfolios and negative for the more liquid portfolios, again in line with our intuition in Section 1.4.2. These comparative statics results show that γ_1 and γ_2 have quite different effects on expected returns, which implies that these parameters are well identified empirically.

1.6.3 Robustness Across Horizons and Portfolios

In this subsection, we check the robustness of our empirical findings to different modeling assumptions. We first test the sensitivity of model performance to the choice of the long term investor horizon and we compute the R^2 for $h_2 = 30, 60, 120, 240, 480$ months. The results are given in Figure 1.6, and show that the explanatory power of the model is relatively insensitive to the choice of horizon. In addition, the coefficients do not vary much across the different choices. The performance is also robust to varying h_1, the short-term investor horizon, as long as it does not grow too large. More specifically, with $h_1 = 6$ months we still obtain a substantial improvement over the single-horizon model.

The second robustness check concerns the assumption of i.i.d. transaction costs, which is required to obtain a tractable solution for the asset pricing model. Empirically, transaction costs are persistent over time. For example, Acharya and Pedersen (2005) estimate an AR(2) model for their monthly measure of transaction costs. For our empirical application, the i.i.d. assumption is not a major concern for two reasons. First, as shown above, the model generates a good fit even when the short-term investors have a six-month horizon ($h_1 = 6$). The persistence of transaction costs is obviously lower at a semi-annual frequency compared to the monthly frequency in Acharya and Pedersen (2005). Second, and more importantly, we estimate a version of the model without liquidity risk (hence with constant c_{t+1}). The results in Table 1.3 show that the model fit is virtually unchanged. This shows that the good fit of the heterogenous-horizon model is not obtained through the liquidity risk channel, but rather via the expected liquidity effect and the associated segmentation and spillover effects. In addition, it follows from the result in Appendix 1.A.3 that without liquidity risk, $V_2 = I$ (assuming $R_f = 1$). The results for the model without liquidity risk thus indicate that the assumption that $V_2 = I$ does not seem to be very restrictive, validating the analysis of Section 1.3.3.

Another robustness test is related to the specific choice of the baseline model. Equation (1.7) is an i.i.d. version of the Acharya and Pedersen (2005) model, which is a conditional model. To obtain an unconditional version, they take expectations on both sides and apply a standard result regarding the expectation of a conditional covariance. This means that the covariance component in their specification is actually a covariance between *residuals* of $R_{t+1} - c_{t+1}$ and *residuals* of $R^m_{t+1} - c^m_{t+1}$, obtained with an AR(2) model for returns and liquidity. Unreported estimation results show that the conditional model with AR(2) residuals yields very similar results as the unconditional specification of equation (1.7). Hence, the comparison of the explanatory power between the results of the models in Table 1.2 does not depend on the specific version of the one-period single horizon model that is used.

As a final robustness check, we also estimate our model for two different portfolio sorts. As before, the segmentation cutoff is set by comparing the average monthly return to the average transaction costs, with the one-month investors only investing in the portfolios where the monthly return exceeds the costs. Table 1.4, Panel A, shows even larger improvements in the cross-sectional fit of our model for the σ(illiquidity) portfolios: the R^2 equals 64.1% in the AP model versus 86.5% in the heterogenous horizon model for the case without a constant term. This shows that the model captures well both the pricing of the level of liquidity and liquidity risk. For the B/M-by-size portfolios, the improvement is also very substantial (see Table 1.4, Panel B): here the cross-sectional R^2 equals 35.0% in the AP model versus 54.4% in the heterogenous horizon model (without constant term).[14] In summary, for any portfolio sorting criteria, our heterogeneous investment horizon model with segmentation provides at least a 20% R^2 improvement in the cross-sectional fit.

1.7 Conclusions

Heterogeneous investment horizons can have important asset pricing effects through the distinct role of liquidity. Different horizons imply different trading

[14] If we include a constant term in the asset pricing model, the improvement in R^2 is even larger. In this case, the estimate for γ_1 in the heterogenous-horizon model tends to infinity, implying a zero risk premium for the non-segmented portfolios. For these portfolios the returns are best explained by the constant term plus the expected liquidity effect.

frequencies and therefore trading costs can have a varying impact for the expected returns of assets held by short-term versus long-term investors.

We present a new liquidity-based asset pricing model with heterogeneous investment horizon investors and stochastic transaction costs. Our model contributes to the literature by effectively bridging the clientele of investors in the seminal Amihud and Mendelson (1986) paper with the risk-averse agents and stochastic illiquidity of the Acharya and Pedersen (2005) model. The increased generality of our model delivers a number of new theoretical insights. It also provides a useful metric to understand the empirical cost of restrictive assumptions, such as horizon homogeneity, in fitting the cross-section of U.S. stock returns.

The most intriguing theoretical result is the existence of an equilibrium with partial segmentation. Short-term investors optimally choose not to invest in the most illiquid assets, intuitively because their expected returns are not sufficient to cover expected transaction costs. In contrast, long-term investors trade less frequently and can afford to invest in illiquid assets. In this equilibrium, the expected returns of segmented assets contain additional terms, both for risk premia and in expected liquidity effects. These additional terms depend partly on the segmented ownership and partly on the correlation between liquid and illiquid assets.

The additional structure imposed by our model delivers a substantial increase in the cross-sectional explanatory power for U.S. stock returns. For a number of portfolio sorting criteria, we find that our heterogeneous horizon model increases the R^2 by at least 20% compared to an homogeneous-horizon liquidity asset pricing model. With the same degrees of freedom, we obtain this large empirical improvement through a suitable characterization of the relation between excess returns and different features of expected liquidity and the liquidity risk premium. This characterization depends crucially on the presence of partial segmentation and agents' risk aversion.

1.A Derivations

1.A.1 Main result

To derive the main result, we consider N classes of investors, as this shows the generality of the result and shortens the proof. We start by introducing sets B_j ($j = 1, \ldots, N$) that represent the assets that investor j optimally holds in his or her portfolio. In Appendix 1.A.2 we describe the conditions that are required for these optimal portfolios. We let the B_j be subsets of $\{1, \ldots, K\}$, where K is the number of assets. Without loss of generality we assume that for some j it holds that $B_j = \{1, \ldots, K\}$.

Proof of Proposition 1: To derive the equilibrium, we first consider each investor's optimization problem. For the investors with horizon h_j it is given by

$$\max_{y_{j,t}} \mathbb{E}\left[W_{j,t+h_j}\right] - \frac{1}{2} A_j \mathrm{Var}\left(W_{j,t+h_j}\right) \tag{1.30}$$

$$W_{j,t+h_j} = \left(P_{t+h_j} + \sum_{k=1}^{h_j} R_f^{h_j-k} D_{t+k} - C_{t+h_j}\right)' y_{j,t} + R_f^{h_j}\left(e_j - P_t' y_{j,t}\right).$$

We first introduce notation that will allow us to derive the equilibrium in the case where investor j holds only assets that are in B_j. For a $K \times K$ matrix M, we denote by M_{B_j} the $|B_j| \times |B_j|$ matrix (with $|\cdot|$ the cardinality of a set) with the rows and columns that are not elements of B_j removed. As it will be used frequently, we also introduce the notation $M_{B_j,p}^{-1}$ for the inverse of M_{B_j} with zeros inserted at the locations where rows and columns of M were removed, so that $M_{B_j,p}^{-1}$ is a $K \times K$ matrix.

For example, let

$$M = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 2 & 4 \\ 3 & 5 & 7 \end{bmatrix}$$

and let $B_j = \{1, 3\}$. Then

$$M_{B_j} = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix},$$

so that

$$M_{B_j}^{-1} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}.$$

We then have

$$M_{B_j,p}^{-1} = \begin{bmatrix} 7 & 0 & -2 \\ 0 & 0 & 0 \\ -3 & 0 & 1 \end{bmatrix}.$$

If we apply this operation to the covariance matrix in the optimization problem of investor j, this yields the solution considering only the assets in B_j padded with zeros, so that it is a $K \times 1$ vector. The benefit is that it makes the solution vectors $y_{j,t}$ $(j = 1, \ldots, N)$ conformable to addition, which allows us to derive the equilibrium.

Thus, given that the optimal portfolio of the investor consists only of assets that are elements of B_j, the solution is

$$\begin{aligned} y_{j,t} = \frac{1}{A_j} \mathrm{Var_t} \left(P_{t+h_j} + \sum_{k=1}^{h_j} R_f^{h_j-k} D_{t+k} - C_{t+h_j} - R_f^{h_j} P_t \right)_{B_j,p}^{-1} \\ \times \mathbb{E}_t \left[P_{t+h_j} + \sum_{k=1}^{h_j} R_f^{h_j-k} D_{t+k} - C_{t+h_j} - R_f^{h_j} P_t \right]. \end{aligned} \tag{1.31}$$

Using the i.i.d. assumption for dividends and costs, we obtain a stationary equilibrium with constant prices and i.i.d. returns. It is then straightforward to derive that $y_{j,t}$ can be written as (derivation available on request)

$$\begin{aligned} y_{j,t} = \frac{1}{A_j} \mathrm{diag}\left(P_t\right)^{-1} \mathrm{Var} \left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+h_j} \right)_{B_j,p}^{-1} \\ \times \left(\mathbb{E} \left[\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+h_j} \right] - \sum_{k=0}^{h_j-1} R_f^{h_j-k} \right). \end{aligned} \tag{1.32}$$

Similarly, it is also straightforward to show that

$$\mathbb{E} \left[\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} \right] - \sum_{k=0}^{h_j-1} R_f^{h_j-k} = \rho_j \left(\mathbb{E}\left[R_{t+1}\right] - R_f \right), \tag{1.33}$$

where $\rho_j = \sum_{k=1}^{h_j} R_f^{h_j-k}$. Making further use of the i.i.d. assumption by which $\mathbb{E}(c_{t+h_j}) = \mathbb{E}(c_{t+k})$ for all j and k, the allocations can thus be written as

$$y_{j,t} = \frac{1}{A_j}\operatorname{diag}(P_t)^{-1}\operatorname{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+h_j}\right)_{B_j,p}^{-1} \times \left(\rho_j\left(\mathbb{E}\left[R_{t+1}\right] - R_f\right) - \mathbb{E}\left[c_{t+1}\right]\right). \tag{1.34}$$

Each period a fixed quantity $Q_j > 0$ of type j investors enters the market. The equilibrium condition at time t is

$$\sum_{j=1}^{N} Q_j y_{j,t} = S - \sum_{j=1}^{N}\sum_{k=1}^{h_j-1} Q_j y_{j,t-k}, \tag{1.35}$$

which is equivalent to

$$\sum_{j=1}^{N}\sum_{k=0}^{h_j-1} Q_j y_{j,t-k} = S. \tag{1.36}$$

Under the i.i.d. assumption we have $y_{j,t-k} = y_{j,t}$ for all k, so that

$$\sum_{j=1}^{N} h_j Q_j y_{j,t} = S. \tag{1.37}$$

Scaling by price we obtain

$$\sum_{j=1}^{N} h_j Q_j \operatorname{diag}(P_t)\, y_{j,t} = \widetilde{S}_t, \tag{1.38}$$

where $\widetilde{S}_t = \operatorname{diag}(P_t)S$. At this point it is useful to introduce the notation $R_{t+1}^m = \widetilde{S}_t' R_{t+1}/\widetilde{S}_t'\iota$, and $c_{t+1}^m = \widetilde{S}_t' c_{t+1}/\widetilde{S}_t'\iota$. We note that in the i.i.d. setting with constant prices, $\widetilde{S}_t$ is constant over time, hence we omit the time subscript and write $\widetilde{S}$ in what follows. This allows us to write

$$\operatorname{Var}(R_{t+1} - c_{t+1})\,\widetilde{S} = \widetilde{S}'\iota \operatorname{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right).$$

Then, multiplying both sides of (1.38) by $(1/\widetilde{S}'\iota)\operatorname{Var}(R_{t+1} - c_{t+1})$, and filling in the expression for the optimal allocations gives

$$\sum_{j=1}^{N} h_j \frac{Q_j}{A_j \widetilde{S}'\iota} \operatorname{Var}(R_{t+1} - c_{t+1}) \operatorname{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+1}\right)_{B_j,p}^{-1} \times \left(\rho_j\left(\mathbb{E}\left[R_{t+1}\right] - R_f\right) - \mathbb{E}\left[c_{t+1}\right]\right) = \operatorname{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right). \tag{1.39}$$

We define $\gamma_j = Q_j/(A_j\widetilde{S}'\iota)$ and

$$V_j = h_j \mathrm{Var}\left(R_{t+1} - c_{t+1}\right) \mathrm{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+h_j}\right)_{B_j,p}^{-1}. \tag{1.40}$$

This allows us to write

$$\sum_{j=1}^{N} \gamma_j V_j \left(\rho_j \left(\mathbb{E}\left[R_{t+1}\right] - R_f\right) - \mathbb{E}\left[c_{t+1}\right]\right) = \mathrm{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right). \tag{1.41}$$

We can rewrite this equilibrium condition as

$$\begin{aligned}\mathbb{E}\left[R_{t+1}\right] - R_f = & \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j\right)^{-1} \sum_{j=1}^{N} \gamma_j V_j \mathbb{E}\left[c_{t+1}\right] \\ & + \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j\right)^{-1} \mathrm{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right).\end{aligned} \tag{1.42}$$

Q.E.D.

1.A.2 Endogenous Segmentation

In this Appendix we describe under which conditions endogenous segmentation arises. Consider the usual (non-segmented) mean-variance solution for the short-term investors

$$\begin{aligned}y_{1,t} = & \frac{1}{A_1} \mathrm{diag}\left(P_t\right)^{-1} \mathrm{Var}\left(\sum_{k=1}^{h_1} R_f^{h_1-k} R_{t+k} - c_{t+h_1}\right)^{-1} \\ & \times \left(\rho_1 \left(\mathbb{E}\left[R_{t+1}\right] - R_f\right) - \mathbb{E}\left[c_{t+1}\right]\right),\end{aligned} \tag{1.43}$$

Suppose that the costs on some illiquid assets are so high that, in equilibrium, some elements of $y_{1,t}$ are negative. Without loss of generality, order the assets such that $y_{1,t} = \left(y_{liq,1,t}, y_{illiq,1,t}\right)$ with $y_{liq,1,t}$ having only positive (or non-negative) elements and $y_{illiq,1,t}$ having only negative elements. In this case, these investors do not want to buy the more illiquid assets. Of course, it is still possible that the investor wants to short these illiquid assets, but this is unlikely given the high transaction costs. To see this formally, we note that if the optimal position in the illiquid assets were negative (and positive for the liquid assets), the optimal portfolio would be

$$z_{1,t} = \frac{1}{A_1} \operatorname{diag}(P_t)^{-1} \operatorname{Var}\left(\sum_{k=1}^{h_1} R_f^{h_1-k} R_{t+k} - \delta_1 c_{t+h_1}\right)^{-1} \\ \times \left(\rho_1 \left(\mathbb{E}\left[R_{t+1}\right] - R_f\right) - \delta_1 \mathbb{E}\left[c_{t+1}\right]\right), \tag{1.44}$$

where δ_1 is a diagonal matrix with elements equal to 1 if the investor is long in the respective asset, and -1 if the investor is short (see Bongaerts, De Jong, and Driessen (2011)). Consider the i-th asset. If $z_{illiq,1,i,t} < 0$, this is indeed the solution to the optimal portfolio rule, but this is unlikely if costs are high for this asset. In turn, if $z_{illiq,1,i,t} > 0$ and the corresponding element of $y_{illiq,1,t}$ is negative, it is optimal for the short-term investors to have a zero position in the illiquid assets. We thus focus here on the case in which costs are high enough so that the short-term investors optimally have a zero position in the illiquid assets. Hence, the set B_1 contains only those assets that are liquid enough for the short-term investors to invest in them.

1.A.3 Computing the long-term covariance matrix

We use the i.i.d. assumption to rewrite part of the moment conditions as follows

$$\operatorname{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+h_j}\right)^{-1} = \left(\left(\sum_{k=1}^{h_j-1} R_f^{2(h_j-k)}\right) \operatorname{Var}\left(R_{t+1}\right) + \operatorname{Var}\left(R_{t+1} - c_{t+1}\right)\right)^{-1}. \tag{1.45}$$

This allows us to compute the covariance terms using only one-period covariances.

1.A.4 Market and liquidity risk premia with two assets

Proof of Proposition 2: We consider the two-asset case ($K = 2$), with two horizons ($N = 2$), $h_1 = 1$, and no segmentation. We start from (1.38), multiply both sides by $1/\widetilde{S}_t'\iota$, and use the expression for the allocations to obtain

$$\sum_{j=1}^{N} \frac{Q_j}{A_j \widetilde{S}'\iota} h_j \operatorname{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+1}\right)^{-1} \left(\rho_j \left(\mathbb{E}\left[R_{t+1}\right] - R_f\right) - \mathbb{E}\left[c_{t+1}\right]\right) = \frac{\widetilde{S}}{\widetilde{S}'\iota}. \tag{1.46}$$

This yields

$$\begin{aligned}\mathbb{E}\left[R_{t+1}\right]-R_f = &\left(\sum_{j=1}^{N}\gamma_j\rho_j h_j \mathrm{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k}-c_{t+1}\right)^{-1}\right)^{-1} \\ &\times\left(\sum_{j=1}^{N}\gamma_j\rho_j h_j \mathrm{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k}-c_{t+1}\right)^{-1}\right)\mathbb{E}\left[c_{t+1}\right] \\ &+\left(\sum_{j=1}^{N}\gamma_j\rho_j h_j \mathrm{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k}-c_{t+1}\right)^{-1}\right)^{-1}\frac{\widetilde{S}}{\widetilde{S}'\iota}.\end{aligned} \tag{1.47}$$

Next, we introduce for $j=1,2$ the determinants

$$d_j = \det\left(\mathrm{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k}-c_{t+h_j}\right)\right), \tag{1.48}$$

we note that $K=2$ implies that the $\mathrm{adj}(\cdot)$ operator is additive, and we apply (1.45) to write

$$\begin{aligned}\mathrm{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k}-c_{t+h_j}\right)^{-1} &= \frac{1}{d_j}\,\mathrm{adj}\,\mathrm{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k}-c_{t+h_j}\right) \\ &= \frac{1}{d_j}\left(\left(\sum_{k=1}^{h_j-1} R_f^{2(h_j-k)}\right)\mathrm{adj}\,\mathrm{Var}\left(R_{t+1}\right)+\mathrm{adj}\,\mathrm{Var}\left(R_{t+1}-c_{t+1}\right)\right).\end{aligned} \tag{1.49}$$

We now let

$$d_0 = \det\left(\sum_{j=1}^{N}\gamma_j\rho_j h_j \mathrm{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k}-c_{t+1}\right)^{-1}\right) \tag{1.50}$$

and

$$\sigma_j = \sum_{k=1}^{h_j-1} R_f^{2(h_j-k)}. \tag{1.51}$$

Making use of the fact that the adj$(\cdot)$ operator is equal to its own inverse (as $K = 2$), we find

$$\begin{aligned}
&\left(\sum_{j=1}^{N}\gamma_j\rho_j h_j \mathrm{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k}R_{t+k}-c_{t+1}\right)^{-1}\right)^{-1} \\
&= \frac{1}{d_0}\,\mathrm{adj}\left(\sum_{j=1}^{N}\gamma_j\frac{\rho_j h_j}{d_j}\left(\sigma_j\,\mathrm{adj}\,\mathrm{Var}\left(R_{t+1}\right)+\mathrm{adj}\,\mathrm{Var}\left(R_{t+1}-c_{t+1}\right)\right)\right) \\
&= \sum_{j=1}^{N}\gamma_j\lambda_j\left(\sigma_j\mathrm{Var}\left(R_{t+1}\right)+\mathrm{Var}\left(R_{t+1}-c_{t+1}\right)\right),
\end{aligned} \tag{1.52}$$

where $\lambda_j = \rho_j h_j/d_0 d_j$. It now follows from (1.39) that

$$\begin{aligned}
&\left(\sum_{j=1}^{N}\gamma_j\rho_j h_j \mathrm{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k}R_{t+k}-c_{t+1}\right)^{-1}_{B_j,p}\right)^{-1}\frac{\widetilde{S}}{\widetilde{S}'\iota} \\
&= \sum_{j=1}^{N}\gamma_j\lambda_j\left(\sigma_j\mathrm{Cov}\left(R_{t+1},R^m_{t+1}\right)+\mathrm{Cov}\left(R_{t+1}-c_{t+1},R^m_{t+1}-c^m_{t+1}\right)\right) \\
&= \left(\sum_{j=1}^{N}\gamma_j\lambda_j\right)\mathrm{Cov}\left(R_{t+1}-c_{t+1},R^m_{t+1}-c^m_{t+1}\right)+\left(\sum_{j=1}^{N}\gamma_j\lambda_j\sigma_j\right)\mathrm{Cov}\left(R_{t+1},R^m_{t+1}\right).
\end{aligned} \tag{1.53}$$

The result now follows by applying (1.53) to (1.47) with $N = 2$. Q.E.D.

1.A.5 Segmentation effects

For this part, we specialize to $N = 2$, and $h_1 = 1$. To derive the result below, we assume that $V_2 = I$, that the h_1-investors invest only in the most liquid assets, and that the h_2-investors invest in all assets.

Proof of Proposition 3: If we sort the assets by liquidity with the most liquid assets first, writing

$$\mathrm{Var}\left(R_{t+1}-c_{t+1}\right) = \begin{bmatrix} V_{liq} & V_{liq,illiq} \\ V_{illiq,liq} & V_{illiq} \end{bmatrix}, \tag{1.54}$$

we have

$$
\begin{aligned}
V_1 &= h_1 \operatorname{Var}(R_{t+1} - c_{t+1}) \operatorname{Var}\left(\sum_{k=1}^{h_1} R_f^{h_1-k} R_{t+k} - c_{t+h_1}\right)_{B_1,p}^{-1} \\
&= \begin{bmatrix} V_{liq} & 0 \\ 0 & V_{illiq} \end{bmatrix} \begin{bmatrix} V_{liq}^{-1} & 0 \\ 0 & 0 \end{bmatrix} \\
&= \begin{bmatrix} I & 0 \\ V_{illiq,liq} V_{liq}^{-1} & 0 \end{bmatrix}.
\end{aligned} \tag{1.55}
$$

Using $N = 2$ and $V_2 = I$ in (1.42) leads to the equilibrium relation

$$
\begin{aligned}
\mathbb{E}[R_{t+1}] - R_f = {} & (\gamma_1 \rho_1 V_1 + \gamma_2 \rho_2 I)^{-1} (\gamma_1 V_1 + \gamma_2 I) \mathbb{E}[c_{t+1}] \\
& + (\gamma_1 \rho_1 V_1 + \gamma_2 \rho_2 I)^{-1} \operatorname{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right).
\end{aligned}
$$

To find the liquidity risk effect, we focus on the factor

$$
\begin{aligned}
(\gamma_1 \rho_1 V_1 + \gamma_2 \rho_2 I)^{-1} &= \begin{bmatrix} (\gamma_1 \rho_1 + \gamma_2 \rho_2) I & 0 \\ \gamma_1 \rho_1 V_{illiq,liq} V_{liq}^{-1} & \gamma_2 \rho_2 I \end{bmatrix}^{-1} \\
&= \begin{bmatrix} (\gamma_1 \rho_1 + \gamma_2 \rho_2)^{-1} I & 0 \\ -\gamma_1 \rho_1 (\gamma_2 \rho_2)^{-1} (\gamma_1 \rho_1 + \gamma_2 \rho_2)^{-1} V_{illiq,liq} V_{liq}^{-1} & (\gamma_2 \rho_2)^{-1} I \end{bmatrix}.
\end{aligned}
$$

In what follows, we will use the *liquidity spillover beta*, defined by

$$
\begin{aligned}
\beta &= V_{illiq,liq} V_{liq}^{-1} \\
&= \operatorname{Cov}\left(R_{t+1}^{illiq} - c_{t+1}^{illiq}, R_{t+1}^{liq} - c_{t+1}^{liq}\right) \operatorname{Var}\left(R_{t+1}^{liq} - c_{t+1}^{liq}\right)^{-1}.
\end{aligned} \tag{1.56}
$$

For the impact of the level of liquidity we write

$$
\begin{aligned}
& (\gamma_1 \rho_1 V_1 + \gamma_2 \rho_2 I)^{-1} (\gamma_1 V_1 + \gamma_2 I) \\
&= \begin{bmatrix} (\gamma_1 \rho_1 + \gamma_2 \rho_2)^{-1} I & 0 \\ -\gamma_1 \rho_1 (\gamma_2 \rho_2)^{-1} (\gamma_1 \rho_1 + \gamma_2 \rho_2)^{-1} \beta & (\gamma_2 \rho_2)^{-1} I \end{bmatrix} \begin{bmatrix} (\gamma_1 + \gamma_2) I & 0 \\ \gamma_1 \beta & \gamma_2 I \end{bmatrix} \\
&= \begin{bmatrix} (\gamma_1 + \gamma_2)(\gamma_1 \rho_1 + \gamma_2 \rho_2)^{-1} I & 0 \\ \left(\gamma_1 (\gamma_2 \rho_2)^{-1} - \gamma_1 \rho_1 (\gamma_2 \rho_2)^{-1} (\gamma_1 + \gamma_2)(\gamma_1 \rho_1 + \gamma_2 \rho_2)^{-1}\right) \beta & \rho_2^{-1} I \end{bmatrix}.
\end{aligned} \tag{1.57}
$$

We rewrite the scalar part of the spillover coefficient using the identity

$$\frac{\gamma_1}{\gamma_2\rho_2}-\frac{\gamma_1\rho_1\left(\gamma_1+\gamma_2\right)}{\gamma_2\rho_2\left(\gamma_1\rho_1+\gamma_2\rho_2\right)}=\frac{\rho_2-\rho_1}{\rho_2}\frac{\gamma_1}{\gamma_1\rho_1+\gamma_2\rho_2}. \tag{1.58}$$

Combining the results above, we can write the equilibrium relation for the liquid assets as

$$\mathbb{E}\left[R_{t+1}^{liq}\right]-R_f=\frac{\gamma_1+\gamma_2}{\gamma_1\rho_1+\gamma_2\rho_2}\mathbb{E}\left[c_{t+1}^{liq}\right]+\frac{1}{\gamma_1\rho_1+\gamma_2\rho_2}\mathrm{Cov}\left(R_{t+1}^{liq}-c_{t+1}^{liq},R_{t+1}^{m}-c_{t+1}^{m}\right). \tag{1.59}$$

and the equilibrium relation for the illiquid assets as

$$\begin{aligned}\mathbb{E}\left[R_{t+1}^{illiq}\right]-R_f=&\frac{1}{\rho_2}\mathbb{E}\left[c_{t+1}^{illiq}\right]+\frac{\rho_2-\rho_1}{\rho_2}\frac{\gamma_1}{\gamma_1\rho_1+\gamma_2\rho_2}\beta\mathbb{E}\left[c_{t+1}^{liq}\right]\\&+\frac{1}{\gamma_2\rho_2}\mathrm{Cov}\left(R_{t+1}^{illiq}-c_{t+1}^{illiq},R_{t+1}^{m}-c_{t+1}^{m}\right)\\&-\frac{\gamma_1\rho_1}{\gamma_2\rho_2\left(\gamma_1\rho_1+\gamma_2\rho_2\right)}\beta\mathrm{Cov}\left(R_{t+1}^{liq}-c_{t+1}^{liq},R_{t+1}^{m}-c_{t+1}^{m}\right).\end{aligned} \tag{1.60}$$

The desired expressions now follow directly. Q.E.D.

1.A.6 Estimation Methodology – Obtaining Standard Errors

We denote the required moments that enter the asset pricing model by the vector ψ. This vector contains expected returns, expected costs, and all required covariances of returns and costs. It is straightforward to derive the asymptotic covariance matrix of the sample estimator of these moments (since covariances can be written as second moments plus products of first moments),

$$\sqrt{T}\left(\widehat{\psi}-\psi\right)\xrightarrow{d}\mathcal{N}\left(0,S_\psi\right). \tag{1.61}$$

We can now use the delta method to find the standard errors for $\widehat{\gamma}$.

Consider the GMM minimization problem given by

$$\min_{\gamma}\ g(\widehat{\psi},\gamma)'g(\widehat{\psi},\gamma), \tag{1.62}$$

for which the solution is implicitly given by

$$2G_\gamma(\widehat{\psi},\gamma)'g(\widehat{\psi},\gamma)=0, \tag{1.63}$$

where

$$G_\gamma(\psi,\gamma) = \frac{\partial g(\psi,\gamma)}{\partial \gamma}. \tag{1.64}$$

Dividing both sides of (1.63) by 2 and evaluating at $\widehat{\gamma}$, we may write

$$G_\gamma(\widehat{\psi},\widehat{\gamma})' g(\widehat{\psi},\gamma_0) + G_\gamma(\widehat{\psi},\widehat{\gamma})' \left(g(\widehat{\psi},\widehat{\gamma}) - g(\widehat{\psi},\gamma_0)\right) = 0. \tag{1.65}$$

Next, we expand $g(\widehat{\psi},\widehat{\gamma})$ around γ_0 to obtain

$$g(\widehat{\psi},\widehat{\gamma}) - g(\widehat{\psi},\gamma_0) \approx G_\gamma(\widehat{\psi},\widehat{\gamma})\left(\widehat{\gamma} - \gamma_0\right). \tag{1.66}$$

It follows that

$$G_\gamma(\widehat{\psi},\widehat{\gamma})' g(\widehat{\psi},\gamma_0) + G_\gamma(\widehat{\psi},\widehat{\gamma})' G_\gamma(\widehat{\psi},\widehat{\gamma})\left(\widehat{\gamma} - \gamma_0\right) = 0. \tag{1.67}$$

We now expand $g(\widehat{\psi},\gamma_0)$ around ψ_0 and use the fact that $g(\psi_0,\gamma_0) = 0$ to find that

$$g(\widehat{\psi},\gamma_0) \approx G_\psi(\widehat{\psi},\widehat{\gamma})\left(\widehat{\psi} - \psi_0\right), \tag{1.68}$$

where

$$G_\psi(\psi,\gamma) = \frac{\partial g(\psi,\gamma)}{\partial \psi}. \tag{1.69}$$

Hence

$$G_\gamma(\widehat{\psi},\widehat{\gamma})' G_\gamma(\widehat{\psi},\widehat{\gamma})\left(\widehat{\gamma} - \gamma_0\right) = -G_\gamma(\widehat{\psi},\widehat{\gamma})' G_\psi(\widehat{\psi},\widehat{\gamma})\left(\widehat{\psi} - \psi_0\right). \tag{1.70}$$

Using this result we obtain

$$\sqrt{T}\left(\widehat{\gamma} - \gamma_0\right) \approx -\left(G_\gamma(\widehat{\psi},\widehat{\gamma})' G_\gamma(\widehat{\psi},\widehat{\gamma})\right)^{-1} G_\gamma(\widehat{\psi},\widehat{\gamma})' G_\psi(\widehat{\psi},\widehat{\gamma}) \sqrt{T}\left(\widehat{\psi} - \psi_0\right). \tag{1.71}$$

It follows that

$$\sqrt{T}\left(\widehat{\gamma} - \gamma_0\right) \xrightarrow{d} \mathcal{N}\left(0, \left(G_\gamma' G_\gamma\right)^{-1} G_\gamma' G_\psi S_\psi G_\psi' G_\gamma \left(G_\gamma' G_\gamma\right)^{-1}\right). \tag{1.72}$$

This result allows us to compute standard errors for the γ estimates taking into account the pre-estimation of the various moments ψ. For the final estimation procedure, we restrict the γ_j pertaining to the horizons h_j to be positive by estimating the logs. We use the usual, additional, delta method correction for the computation of the standard errors.

1.A.7 Comparative statics

We consider an increase in γ_k, so that the horizon h_k investors become either more numerous, or less risk averse, or both. We find

$$\begin{aligned}\frac{\partial\left(\mathbb{E}\left[R_{t+1}\right]-R_f\right)}{\partial\gamma_k} = &-\left(\sum_{j=1}^{N}\gamma_j\rho_j V_j\right)^{-1}\rho_k V_k\left(\sum_{j=1}^{N}\gamma_j\rho_j V_j\right)^{-1}\sum_{j=1}^{N}\gamma_j V_j\mathbb{E}\left[c_{t+1}\right] \\ &+\left(\sum_{j=1}^{N}\gamma_j\rho_j V_j\right)^{-1}V_k\mathbb{E}\left[c_{t+1}\right] \\ &-\left(\sum_{j=1}^{N}\gamma_j\rho_j V_j\right)^{-1}\rho_k V_k\left(\sum_{j=1}^{N}\gamma_j\rho_j V_j\right)^{-1} \\ &\times\mathrm{Cov}\left(R_{t+1}-c_{t+1},R_{t+1}^m-c_{t+1}^m\right).\end{aligned} \tag{1.73}$$

Rearranging gives

$$\frac{\partial\left(\mathbb{E}\left[R_{t+1}\right]-R_f\right)}{\partial\gamma_k} = \left(\sum_{j=1}^{N}\gamma_j\rho_j V_j\right)^{-1}V_k\left(\mathbb{E}\left[c_{t+1}\right]-\rho_k\left(\mathbb{E}\left[R_{t+1}\right]-R_f\right)\right). \tag{1.74}$$

Table 1.1. Descriptive statistics

This table shows descriptive statistics for the data used to estimate the model. The CRSP data used are monthly data corresponding to 25 value-weighted US stock portfolios sorted on illiquidity with sample period 1964–2009. The average excess return $\mathbb{E}[R_{t+1}] - R_f$, standard deviation of returns $\sigma(R_{t+1})$, standard deviation of returns net of costs $\sigma(R_{t+1} - c_{t+1})$, and covariance between portfolio and market level returns net of costs $\mathrm{Cov}\left(R_{t+1} - c_{t+1}, R^m_{t+1} - c^m_{t+1}\right)$ are computed from the time-series observations.

Portfolio	$\mathbb{E}[R_{t+1}] - R_f$ (%)	$\mathbb{E}[c_{t+1}]$ (%)	$\sigma(R_{t+1})$ (%)	$\sigma(c_{t+1})$ (%)	$\sigma(R_{t+1} - c_{t+1})$ (%)	$\mathrm{Cov}(\ldots,\ldots)$ $(\cdot 100)$	trn (%)	$\sigma(\text{trn})$
1	0.3264	0.2518	4.2022	0.0017	4.2024	0.1767	5.9158	6.2783
2	0.3755	0.2568	4.6114	0.0060	4.6123	0.2135	6.9554	6.2195
3	0.4354	0.2611	4.6451	0.0093	4.6465	0.2212	6.9501	6.0284
4	0.5600	0.2664	4.7600	0.0133	4.7621	0.2286	7.1681	6.8244
5	0.5953	0.2732	5.0470	0.0192	5.0506	0.2442	6.8267	6.0381
6	0.5495	0.2827	4.8503	0.0261	4.8549	0.2371	6.8506	6.4248
7	0.5571	0.2928	4.7960	0.0329	4.8009	0.2377	6.4540	5.5812
8	0.5431	0.3083	4.9547	0.0470	4.9637	0.2441	6.2103	5.4767
9	0.6055	0.3253	4.9567	0.0608	4.9678	0.2473	6.0450	5.0996
10	0.6309	0.3448	4.8916	0.0699	4.9049	0.2442	5.6554	4.8098
11	0.6734	0.3725	5.1438	0.0941	5.1611	0.2572	5.4089	4.2691
12	0.5630	0.4037	4.8935	0.1072	4.9144	0.2445	4.7951	3.7103
13	0.6529	0.4376	4.8893	0.1240	4.9165	0.2463	4.7662	3.6871
14	0.7400	0.4758	5.0515	0.1529	5.0836	0.2535	4.4090	3.0553
15	0.6506	0.5453	5.0894	0.1981	5.1365	0.2565	4.2004	3.0606
16	0.6331	0.6210	5.0289	0.2298	5.0830	0.2524	4.0327	2.5514
17	0.8010	0.7238	5.0751	0.3034	5.1522	0.2531	4.2676	4.4986
18	0.6807	0.8283	5.0265	0.3259	5.1102	0.2504	4.2420	5.3765
19	0.8664	0.9742	5.1872	0.4031	5.2873	0.2563	3.7374	2.6377
20	0.6789	1.2769	5.3153	0.5771	5.4804	0.2594	3.4632	2.3135
21	0.8253	1.5531	5.4042	0.6395	5.5692	0.2597	2.9857	1.7014
22	0.8381	1.9864	5.3899	0.8188	5.5838	0.2556	3.8557	7.9582
23	0.8183	2.8059	5.4890	1.2157	5.9041	0.2685	3.1721	2.6555
24	0.7511	4.4659	5.5545	2.0282	6.3574	0.2732	2.6859	1.9129
25	0.8712	8.0759	6.0417	4.0085	7.8284	0.2780	2.4010	2.1144

Table 1.2. GMM estimation results: Illiquidity portfolios

This table shows the results from estimation of the various specifications of the model. The estimates are based on monthly data corresponding to 25 value-weighted US stock portfolios sorted on illiquidity with sample period 1964–2009. An equal-weighted market portfolio is used. The specifications are special cases of the relation

$$\begin{aligned}\mathbb{E}\left[R_{t+1}\right]-R_f=\alpha+\kappa\left(\gamma_1\rho_1V_1+\gamma_2\rho_2V_2\right)^{-1}\left(\gamma_1V_1+\gamma_2V_2\right)\mathbb{E}\left[c_{t+1}\right]\\+\left(\gamma_1\rho_1V_1+\gamma_2\rho_2V_2\right)^{-1}\operatorname{Cov}\left(R_{t+1}-c_{t+1},R^m_{t+1}-c^m_{t+1}\right),\end{aligned} \tag{1.75}$$

where $\gamma_j=Q_j/(A_j\widetilde{S}'\iota)$, $\rho_j=\sum_{k=1}^{h_j}R_f^{h_j-k}$, and

$$V_j=h_j\operatorname{Var}\left(R_{t+1}-c_{t+1}\right)\operatorname{Var}\left(\sum_{k=1}^{h_j}R_f^{h_j-k}R_{t+k}-c_{t+h_j}\right)^{-1}_{B_j,p}. \tag{1.76}$$

We set $h_1=1$, and $h_2=120$. The parameters are estimated using GMM. For each coefficient the t-value is given in parentheses. The cross-sectional R^2 is reported in the rightmost column. Estimates for the heterogeneous horizon model, where short term investors invest only in the 19 most liquid portfolios, are denoted by SEG. AP indicates that the specification corresponds to a variant of the Acharya and Pedersen (2005) specification (1.7). Where the value of κ is unreported, it is set to 1.

	γ_1	γ_2	α	κ	R^2
(SEG)	0.2080	0.0036			0.8224
	(0.5922)	(1.9400)			
(SEG+α)	0.0830	0.0018	-0.0050		0.8722
	(0.2763)	(1.1285)	(-0.5358)		
(AP)	0.3973			0.0287	0.6215
	(2.2428)			(0.1672)	
(AP+α)	0.1737		-0.0078	0.0088	0.7660
	(0.8451)		(-0.5432)	(0.0213)	

Table 1.3. GMM estimation results: Illiquidity portfolios, without liquidity risk

This table shows the results from estimation of the various specifications of the model without liquidity risk. The setup is the same as in Table 1.2, but with c_{t+1} taken to be constant and equal to its estimated mean.

	γ_1	γ_2	α	κ	R^2
(SEG)	0.2008	0.0038			0.8243
	(0.3237)	(1.4004)			
(SEG+α)	0.0802	0.0019	-0.0049		0.8725
	(0.1172)	(0.9545)	(-0.2375)		
(AP)	0.3922			0.0425	0.6136
	(2.2360)			(0.2500)	
(AP+α)	0.1675		-0.0081	0.0402	0.7621
	(0.8949)		(-0.5933)	(0.0994)	

Table 1.4. GMM estimation results: σ(illiquidity) and B/M-by-size portfolios

This table shows the results from estimation of the various specifications of the model for different portfolio types. The setup is the same as in Table 1.2. Panel A shows the results for 25 portfolios sorted on illiquidity variation. For Panel B 25 value-weighted portfolios sorted on book-to-market value and size are used. In both cases the same rule for the segmentation threshold is used as in Table 1.2: the one-month investors only invest in assets for which the monthly average return exceeds the average transaction cost.

Panel A: σ(illiquidity) portfolios					
	γ_1	γ_2	α	κ	R^2
(SEG)	0.2030	0.0037			0.8650
	(0.5557)	(1.9920)			
(SEG+α)	0.0796	0.0019	-0.0046		0.9078
	(0.2854)	(1.4201)	(-0.7796)		
(AP)	0.3993			0.0278	0.6407
	(2.2445)			(0.1718)	
(AP+α)	0.1755		-0.0076	0.0014	0.7867
	(0.9360)		(-0.6027)	(0.0037Z)	
Panel B: B/M-by-size portfolios					
	γ_1	γ_2	α	κ	R^2
(SEG)	0.7721	0.0027			0.5442
	(0.3613)	(0.5327)			
(SEG+α)	$1.8 \cdot 10^{14}$	0.0030	0.0018		0.7579
	(0.0000)	(0.9388)	(0.5242)		
(AP)	0.4630			0.0424	0.3498
	(2.0839)			(0.2590)	
(AP+α)	0.8201		0.0025	0.0540	0.3923
	(0.8285)		(0.8093)	(0.5891)	

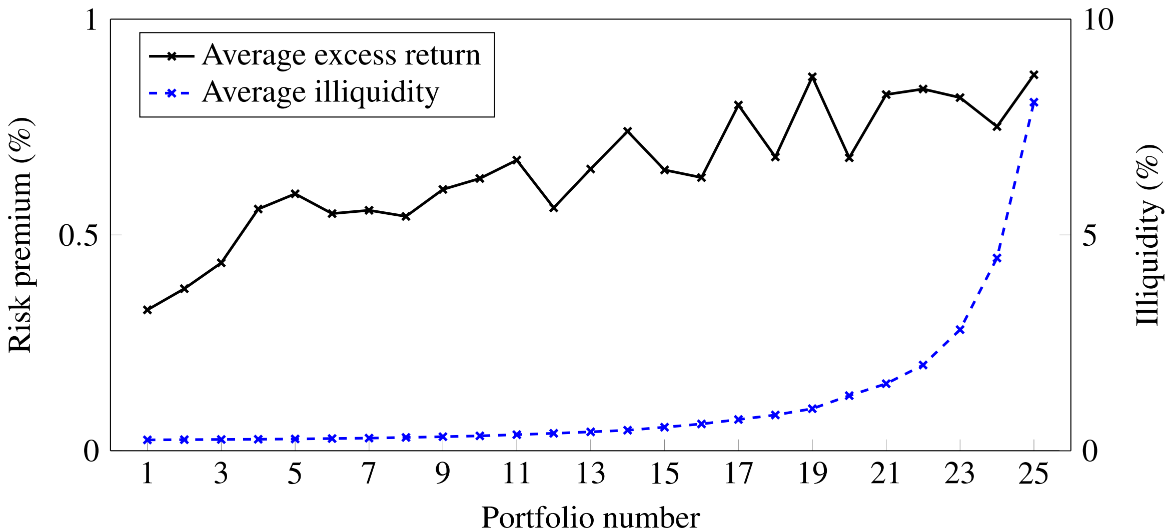

Figure 1.1. Expected returns and level of illiquidity. This figure illustrates the average monthly return (left axis) and average transaction costs (right axis) for the 25 US stock portfolios sorted on illiquidity. Portfolio 1 is the most liquid portfolio, while portfolio 25 is the least liquid portfolio.

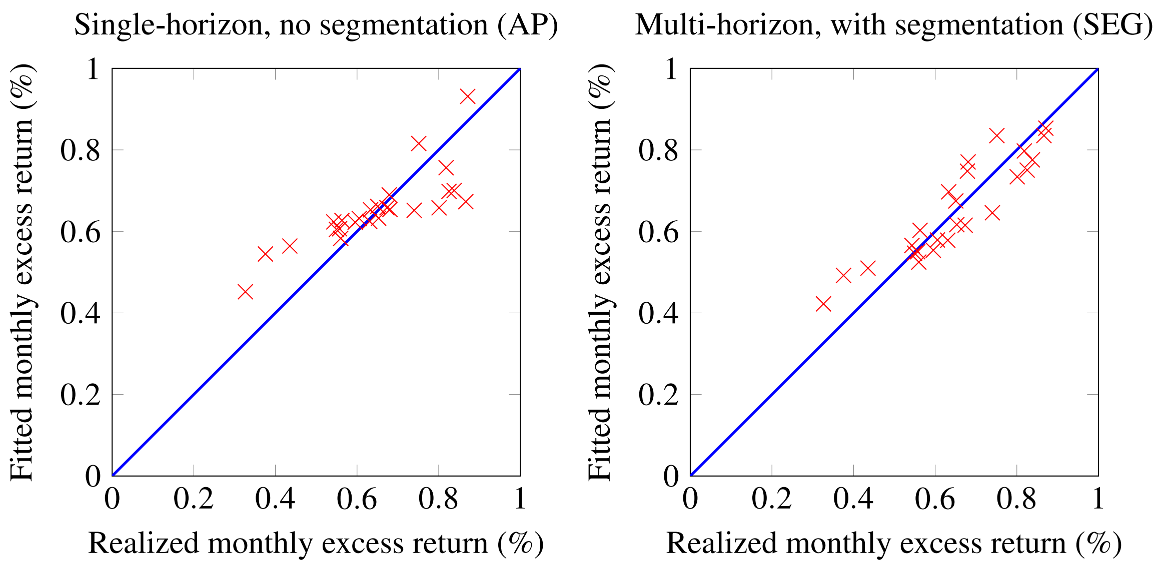

Figure 1.2. Fitted excess returns vs. realized excess returns. The left panel shows the goodness of fit for the Acharya and Pedersen (2005) specification (AP). The right panel shows the fit for the heterogeneous horizon specification (SEG). The graphs correspond to the estimation results as given in Table 1.2.

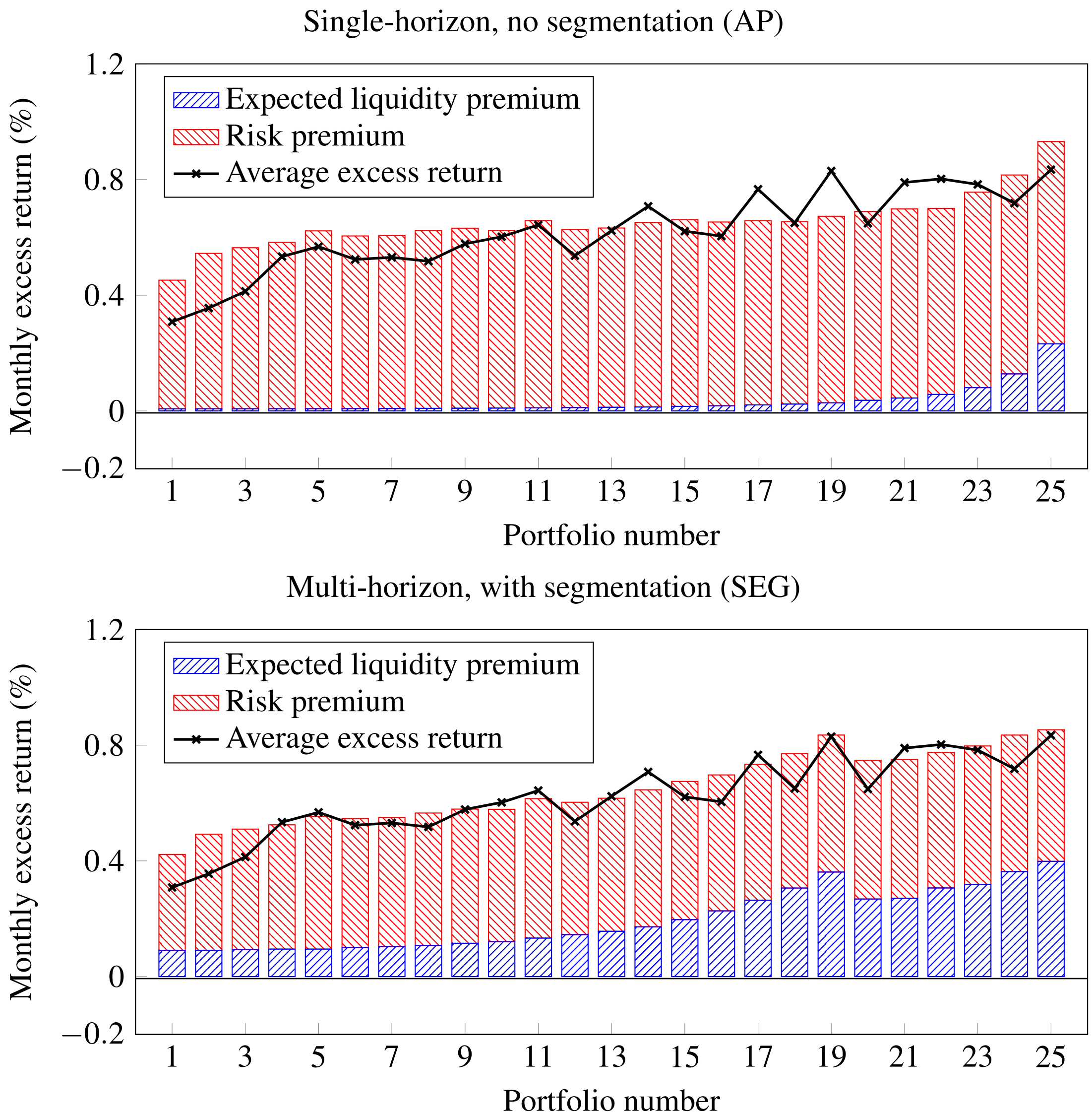

Figure 1.3. Decomposition of predicted excess returns in the expected liquidity premium and the risk premium. In each panel the lower part shows the expected liquidity premium and the upper part the risk premium. The line indicates the actual excess return. The upper panel shows the decomposition for the Acharya and Pedersen (2005) specification (AP). The lower panel shows the decomposition for the heterogeneous horizon specification (SEG). The graphs correspond to the estimation results as given in Table 1.2.

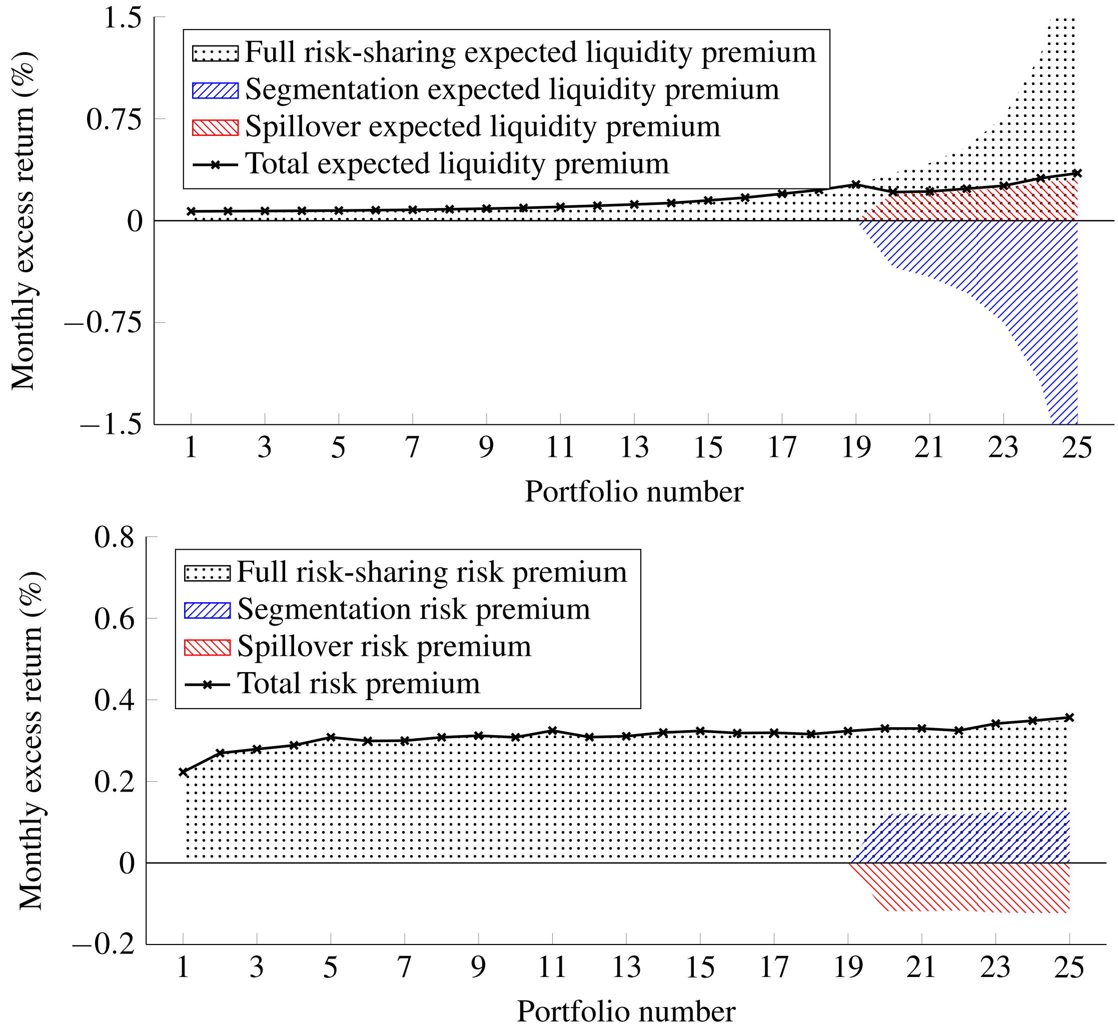

Figure 1.4. Segmentation effects. The top panel shows the decomposition of the expected liquidity premium into three components: the full risk-sharing component, the segmentation component, and the spillover component. The bottom panel shows a similar decomposition for the risk premium. In all cases the heterogeneous horizon specification (SEG) is used. The coefficient values correspond to the estimation results as given in Table 1.2.

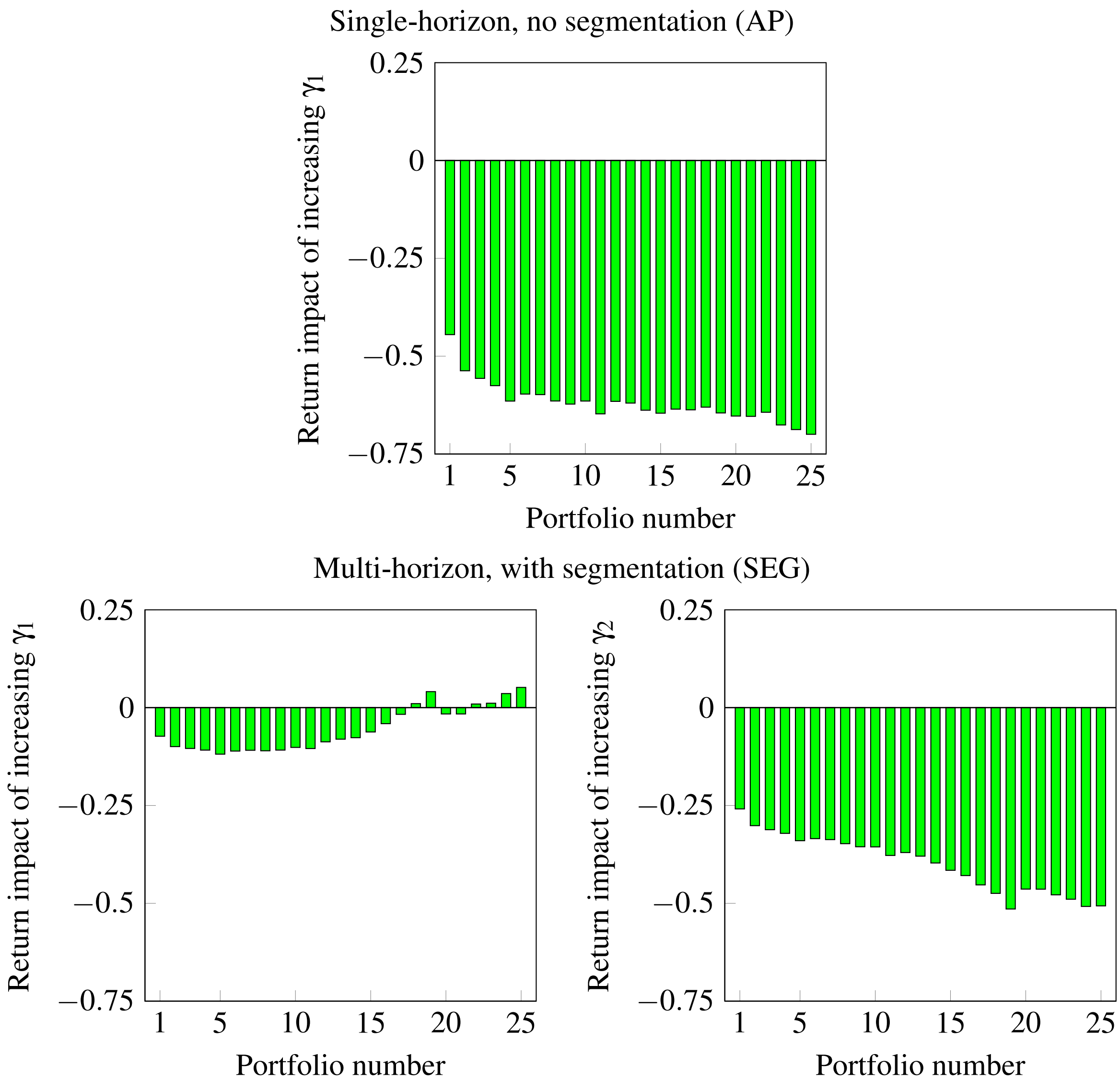

Figure 1.5. Comparative statics. The comparative statics are computed according to equation (1.27), and give the sensitivity of the expected return to the parameter γ_j. The top panel shows the comparative statics for the Acharya and Pedersen (2005) specification (AP). The bottom panel shows the comparative statics for the heterogeneous horizon specification (SEG). The graphs correspond to the estimation results as given in Table 1.2.

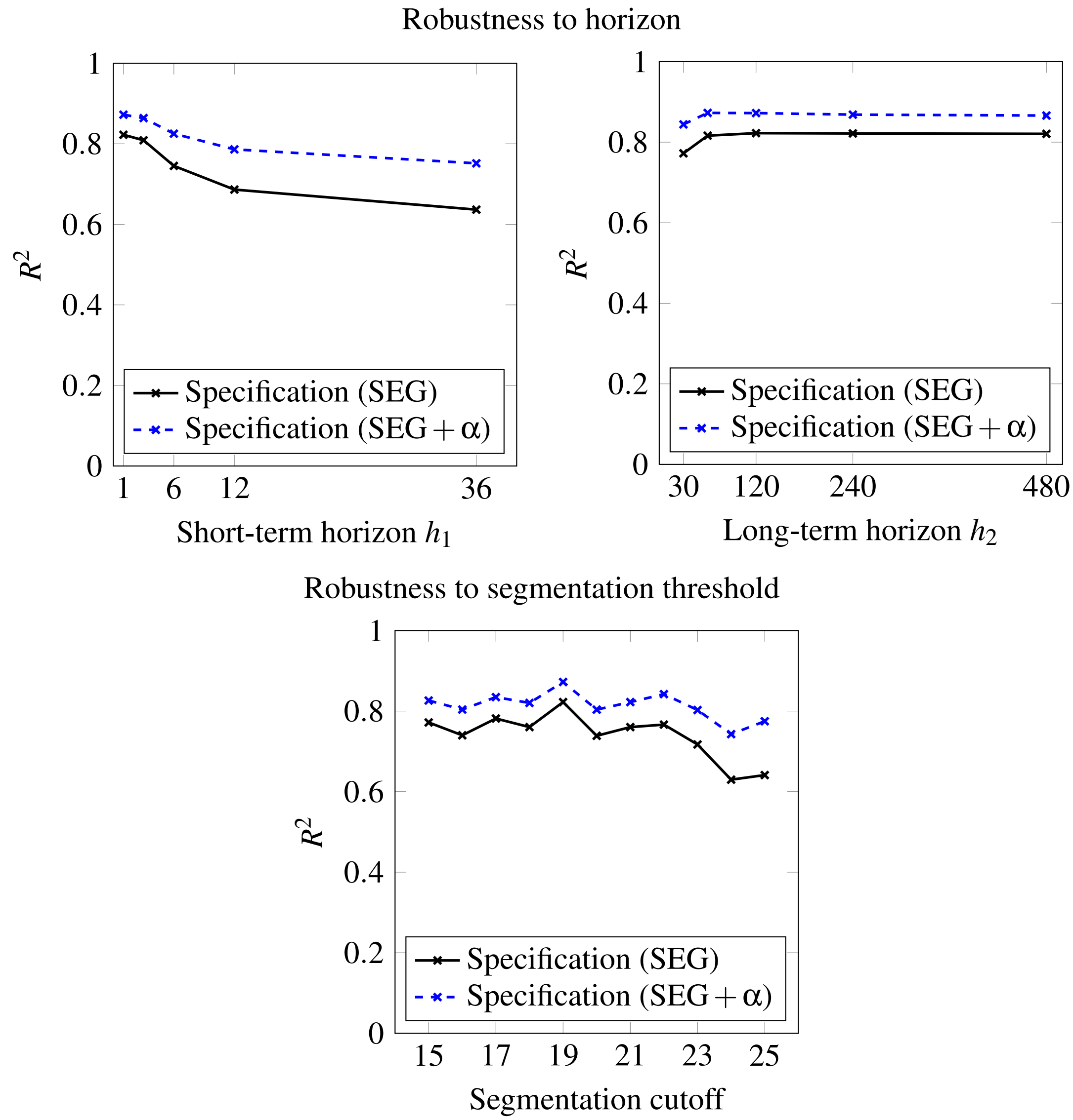

Figure 1.6. Robustness to horizons and segmentation. This graph shows the sensitivity of the cross-sectional R^2 to varying the horizons and to varying the segmentation threshold. The data and the specifications are the same as in Table 1.2. Setting $h_1 = 1$, we let $h_2 = 30, 60, 120, 240, 480$. Alternatively, we fix $h_2 = 120$ and let $h_1 = 1, 3, 6, 12, 36$. For the segmentation level we take $h_1 = 1$, $h_2 = 120$ and let the short-term investors invest in the $16, \ldots, 25$ most liquid portfolios. The case of 25 corresponds to integration.

Chapter 2

The Effect of Homeownership on the Idiosyncratic Housing Risk Premium[1]

2.1 Introduction

Diversification is at the core of modern portfolio theory. Yet, housing makes up about two-thirds of the typical U.S. household portfolio. In the aftermath of WW I, the U.S. Department of Labor together with several real-estate industry groups launched a nationwide Own Your Own Home campaign to promote homeownership as a way to mitigate social unrest and favor economic growth. Over the years, homeownership has been central in the political agenda of both Democrats and Republicans. In 1995, the Clinton administration amended the Community Reinvestment Act of 1977 to promote lending in low-income neighborhoods. In 2003, President Bush signed into law the American Dream Downpayment Assistance Act to help families with their down payments and closing costs. Homeownership reached a record high 69% in the years that lead to the subprime crisis of 2007.

[1]This chapter is based on joint work with Erasmo Giambona. We thank Joost Driessen, Stuart Gabriel, Frank de Jong, Stephen A. Ross (our discussant at the UConn Center for Real Estate 50th Anniversary Symposium) for their insightful comments and suggestions.

Research has shown that hedging incentives encourage homeownership and decrease expected returns on housing. Sinai and Souleles (2005) show that homeownership risk is reduced if the household tends to move across highly correlated housing markets (or if mobility is low). If house prices increase in the current household market, high correlation across housing markets implies that house prices increase also in the housing market that the household intends to move to. Hence, homeownership works as a hedge against future housing consumption across housing markets. Han (2010) shows that hedging incentives are strong, in particular, for households that tend to move within the same local housing markets. A classic example is the case of younger homeowners that plan to move up the housing ladder within the same market. Because house prices tend to be highly correlated within housing markets, owning a home is a hedge against the risk that house prices will have increased when the household intends to move to a larger house. In support of this prediction, Han (2013) finds that expected housing returns decrease when hedging incentives are strong.

In this study, we focus on the portfolio implications of homeownership.[2] Cocco (2005) shows that because housing is lumpy, homeownership undermines portfolio diversification, especially for younger and poorer households. We argue that the under-diversification caused by homeownership is risky because housing is mostly financed with mortgage debt (especially, for younger and poorer households). If house values fall below outstanding mortgage debt, then moving across housing markets (even highly correlated markets) is difficult because it requires that households are able (and willing) to cover the capital loss on the home they currently own.[3] Hence, homeownership can hinder labor mobility in down housing mar-

[2]Theoretically, Ortalo-Magné and Prat (2010) are among the first to combine portfolio and hedging considerations in the context of housing.

[3]Head and Lloyd-Ellis (2012) show that housing liquidity (how quickly households can sell their houses) affects the decision of home owners to accept job offers from other cities.

kets (which is perhaps when mobility is most needed), especially for younger and poorer households (who are perhaps those that need mobility the most).[4] Therefore, we predict that idiosyncratic risk is priced in the housing market and the premium for idiosyncratic risk increases with homeownership.

To test our predictions, we follow Ang, Hodrick, Xing, and Zhang (2006) by formulating a conditional factor model for housing returns that includes housing market and stock market factors, as well as an idiosyncratic risk factor. As we focus on individual volatility rather than aggregate volatility, we test the model using the approach of Goyal and Santa-Clara (2003), who include idiosyncratic risk as a characteristic, and estimate the price of risk. To investigate the relation between the price of idiosyncratic risk and homeownership, we use the approach of Shanken (1990) to estimate a time-varying risk premium that depends not only on homeownership, but also on the unemployment rate and the rent level. These variables feature in our main specification as interactions with the idiosyncratic risk level. As in Han (2013), we measure idiosyncratic risk as the ARCH(1) estimate of residual volatility for a standard CAPM that includes both housing and stock market factors.

We use the Federal Housing Finance Agency (FHFA) all transactions House Price Index (HPI) to measure housing returns at the MSA level. We have annual observations for 296 MSAs over the period 1980–2012. As special dynamics due to the subprime crisis, such as the limited diversification potential documented by Cotter, Gabriel, and Roll (2015), could influence our results, we follow Han (2013) and initially focus on the period until 2007. We apply an AR(1) filter to unsmooth the series of housing returns. To estimate the level of idiosyncratic risk,

[4] Kahl, Liu, and Longstaff (2003) argue that entrepreneurs who hold illiquid and undiversified portfolios due to selling restrictions on their company stocks incur considerable costs. In our context, their argument suggests that when a household has a large fraction of its wealth invested in housing, he or she might reduce consumption below optimal and use the proceeds from reduced consumption to invest in other assets (diversifying excessive exposure to housing).

we estimate an ARCH(1) model for the residuals of a standard CAPM regression of MSA-level housing returns on a national-housing index and the S&P500 stock market index. The variables that we use to explain time-variation in the premium for idiosyncratic risk are obtained from the U.S. census through the Integrated Public Use Microdata Series (IPUMS).

We find that idiosyncratic risk is priced for U.S. housing. In the cross section of decile portfolios of MSAs sorted by idiosyncratic risk, we observe that idiosyncratic risk commands a premium of 0.57% per annum. The premium commanded by idiosyncratic housing risk is positively related to the homeownership rate. For our main regression model, we find that the coefficient on the idiosyncratic risk factor is 0.1463, the coefficient on the idiosyncratic risk/homeownership interaction is 0.0438, and both are statistically significant at the 1% level. This combined evidence suggests that when homeownership increases by one standard deviation, the price for idiosyncratic risk increases by one third of the original price.

Our analysis thus far suggests that idiosyncratic risk is priced and the premium for idiosyncratic risk increases with homeownership. As discussed, previous literature has documented that the risk of homeownership decreases when hedging incentives are strong. Therefore, it is important that we assess the robustness of our findings when households have strong incentives to own a house for hedging. Following the literature, we use the percentage of the population that moves within the same MSA (or across highly correlated MSAs)and rental volatility at the MSA level to proxy for hedging incentives. If a large fraction of the population in a certain MSA traditionally moves within the same MSA, then it is less likely that households from this MSA will move to other MSAs (especially MSAs with uncorrelated housing markets) for job related reasons. Hence, homeownership as a hedge against house price increases is more likely to be important when within MSA migration is high. Similarly, when rent volatility is high, homeownership becomes important to insure against rent risk (Sinai and Souleles, 2005). We find that the interaction of idiosyncratic risk with homeownerships remains positively significant in our housing return regressions when we control for hedging incentives.

To recap, we find strong evidence that idiosyncratic risk is priced in the housing market and that the premium for idiosyncratic risk increases with homeownership. Our analysis indicates that the idiosyncratic housing risk premium remains economically sizable after we control for hedging incentives. Further, our results are robust to including year fixed effects, MSA fixed effects, and using the Fama-MacBeth estimation.

Our paper relates to three streams of literature. As discussed, Sinai and Souleles (2005) and Han (2010, 2013) show that hedging incentives encourage homeownership and reduce expected returns on housing. We complement these studies by showing that idiosyncratic risk is priced in the housing market. Idiosyncratic risk matters in housing because homeownership hinders portfolio diversification, which, in turn, reduces labor mobility.

Second, we relate to the literature that emphasizes how homeownership can introduce portfolio distortions. Cocco (2005) shows that housing crowds out stock holdings. This is true especially for younger and poorer households who will have limited wealth to invest in other assets once they become home owners. Similarly, Brueckner (1997) argues that the consumption benefits of housing may motivate consumers to overinvest in housing. This overinvestment will lead to a mean-variance inefficient portfolio with a reduction in diversification benefits that needs to be balanced against the increase in consumption benefits. Flavin and Yamashita (2002) argue that young households, for which housing is a particularly large fraction of their wealth, tend also to be highly leveraged, and are thus inclined to reduce the risk of their portfolio by using excess wealth to pay down their mortgage, or buy bonds rather than stocks. We contribute to this literature by showing that beyond these effects, the lack of diversification due to the overinvestment in housing has pricing consequences for housing itself. Not only do (young) homeowners tend to have an inefficient position in the stock market, they are also exposed to idiosyncratic risk in the housing market.

Third, we relate to research on the pricing of idiosyncratic risk. Merton (1987) argues that people tend to hold familiar stocks and hence have under-diversified portfolios. If the practice of holding familiar stocks is widespread, then idiosyncratic risk should be priced in equilibrium. Similar arguments are put forward by Levy (1978) and Malkiel and Xu (2004). The empirical evidence for stocks is mixed. Ang, Hodrick, Xing, and Zhang (2006) show that the cross-sectional price of market volatility risk is negative for the stock market, consistent with past research in option pricing (e.g., Bakshi and Kapadia, 2003; Carr and Wu, 2009). In addition, they find a strong negative correlation between idiosyncratic volatility and average returns. By contrast, Goyal and Santa-Clara (2003), provide time-series evidence that the price for idiosyncratic risk is positive for the stock market.

Housing is an ideal laboratory to study the pricing of idiosyncratic risk. First, housing, unlike stocks, is lumpy, which implies that home owners are likely to be under-diversified (Tracy, Schneider, and Chan, 1999). Second, we can use variation in homeownership rates across regions and time to proxy for the degree of under-diversification. Third, it is possible to identify conceptually the frictions (e.g., reduced labor mobility) through which idiosyncratic risk is priced in housing. As Case, Cotter, and Gabriel (2011) and Cannon, Miller, and Pandher (2006), we show that idiosyncratic risk is priced in the housing market. But we go further. We propose that one channel through which the under-diversification caused by homeownership increases risk for households is reduced job mobility. We then show empirically that idiosyncratic risk affects housing returns through homeownership. To our knowledge, our study is the first to document a positive relation between idiosyncratic risk and returns through low diversification (i.e., homeownership).

The paper is organized as follows. Section 2.2 describes the data. We describe the empirical approach in Section 2.3 and present the results in Section 2.4. Section 2.5 summarizes the findings and concludes the paper.

2.2 Data

In this section, we describe the data that we use for our analyses. First, we present the data on housing returns and the various controls and interaction effects that we use. Then, we proceed by discussing our measure of idiosyncratic risk and its empirical properties.

2.2.1 Real Estate and Macroeconomic Data

We use the Federal Housing Finance Agency (FHFA) all transactions House Price Index (HPI) to measure housing returns at the MSA level. In addition, we have data on the homeownership rate, the unemployment rate, and the rent level. These are obtained from the U.S. census through the Integrated Public Use Microdata Series (IPUMS). Our dataset consists of annual observations for 296 MSAs and our sample runs from 1980 until 2012. Using MSA-level data implies that we are averaging out some of the idiosyncratic risk. We follow Han (2013) and winsorize the housing returns at the 1st and 99th percentiles. In the cases where we impose that the market price of risk equals the market risk premium, we also winsorize the beta times the market risk premium at the 1st and 99th percentiles. To correct for smoothing that occurs as a result of using appraisal-based values and temporal aggregation, we apply an AR(1) filter to the housing returns (Fisher, Geltner, and Webb, 1994). Specifically, we construct our unsmoothed series as the mean HPI return plus the AR(1) residual.

Table 2.1 presents some descriptive statistics. Most notably, the mean house price index shows an annual real excess return of -1.47% relative to an average annual risk-free rate of 5.04%, while the stock market index produced a positive excess return of 7.54%. The volatility of the housing market, 4.06%, is lower than that of the stock market, which is 16.99%. The homeownership rate was on average 71.84%, but it has varied substantially during the sample period. Figure 2.1 shows the strong increase in homeownership during the early 2000s and the decline afterwards.

2.2.2 Idiosyncratic Risk

To measure idiosyncratic risk, we start by running a standard CAPM of the form

$$R_{i,t} - R_f = \alpha_i + \beta_{FHFA,i}\left(R_{m,FHFA,t} - R_f\right) + \beta_{S\&P,i}\left(R_{m,S\&P,t} - R_f\right) + \varepsilon_{i,t}. \quad (2.1)$$

on housing returns. For the individual returns we take the FHFA all transactions house price index at the MSA level. The market returns are based on the FHFA all transactions house price index at the national level. We also include as a factor the return on the S&P500 index, obtained from Kenneth French's website.

We take the time-varying residual standard deviation $\sigma_{\varepsilon,i,t}$ of (2.1) as our measure of idiosyncratic risk for MSA i. Our estimate of $\sigma_{\varepsilon,i,t}$ derives from the following ARCH(1) model.

$$\sigma^2_{\varepsilon,i,t} = \alpha_{0,i} + \alpha_{1,i}\varepsilon^2_{i,t-1}. \quad (2.2)$$

We use all available observations to estimate the $\sigma_{\varepsilon,i,t}$. For robustness, we also consider $\sigma_{\varepsilon,i,t}$ estimated as the RMSE of (2.1).

The descriptive statistics in Table 2.1 show that the level of idiosyncratic risk varies substantially in our sample. It is on average 4.09% with a standard deviation of 2.73%. The largest value in our sample is 50.90%. Our mean estimate is comparable to the 4.59% found by Case, Cotter, and Gabriel (2011). To gain insight into the relation between idiosyncratic risk and other characteristics, we sort the MSAs into idiosyncratic risk deciles and compute averages of all variables for each decile. The corresponding averages are given in Table 2.2. Moving from the bottom to the top decile of the idiosyncratic risk distribution, we see that housing returns increase from 4.25% to 5.09% per annum, while idiosyncratic risk increases from 1.52% to 9.33%. Figure 2.2 shows the housing market return as well as the average idiosyncratic risk level. Table 2.3 shows correlations between the different variables that are used in this study.

2.3 Empirical Methodology

In this section we set out the empirical strategy. Our analysis consists of two stages. First, we show that idiosyncratic risk is indeed priced for the US housing market. Second, we analyze the determinants of the idiosyncratic risk premium.

2.3.1 Pricing of Idiosyncratic Risk

For our analysis, we consider idiosyncratic risk as a stock characteristic that commands a premium. Our asset pricing model is given by

$$\begin{aligned}\mathbb{E}_{t-1}\left[R_{i,t}-R_f\right] = \lambda_{0,t-1} + \lambda_{FHFA,t-1}\beta_{FHFA,i} + \lambda_{S\&P,t-1}\beta_{S\&P,i} \\ + \lambda_{\sigma,t-1}\sigma_{\varepsilon,i,t},\end{aligned} \tag{2.3}$$

where $\lambda_{FHFA,t-1}$, $\lambda_{S\&P,t-1}$, and $\lambda_{\sigma,t-1}$ denote the prices of risk for our pricing factors. Note that, given the ARCH parameters, $\sigma_{\varepsilon,i,t}$ is known at time $t-1$.

To test whether idiosyncratic risk is priced, we consider panel regressions of the form

$$\begin{aligned} R_{i,t} - R_f = {} & \lambda_{0,i} + \xi_t + \lambda_{FHFA,t-1}\beta_{FHFA,i,t-1} + \lambda_{S\&P,t-1}\beta_{S\&P,i,t-1} \\ & + \lambda_{\sigma,t-1}\sigma_{\varepsilon,i,t} + \varepsilon_{i,t}, \end{aligned} \tag{2.4}$$

where we use rolling OLS regressions with a time window of 10 years to estimate $\beta_{FHFA,i}$, and $\beta_{S\&P,i}$. In our basic specification, we include year dummies to control for unobserved time-varying macro shocks. We cluster the standard errors by year to adjust for cross-sectional correlation. In other specifications, we include MSA fixed effects to control for, e.g., geographical constraints and local amenities (Han, 2013). The importance of local amenities and other local parameters is supported by Hwang and Quigley (2006) and Goetzmann, Spiegel, and Wachter (1998), who find that housing markets cluster and that local aspects matter for house price fluctuations. We also cluster standard errors by MSA, and include two-dimensional clustering by year and MSA as well.

In addition to the panel regressions, we run Fama and MacBeth (1973) regressions to establish our pricing relation (as in Cotter, Gabriel, and Roll, 2015). To control for geographical constraints and local amenities, we include MSA fixed effects by demeaning the housing returns for each MSA. We also apply the Petersen (2009) correction, which adjusts the standard errors for the presence of an unobserved MSA effect (see, e.g., Rubin and Smith, 2009, for a similar application in the context of stock returns).

To identify the channel through which idiosyncratic risk is priced, we use homeownership as a proxy for underdiversification. Homeowners are typically underdiversified as they tend to own a single house, rather than a well-diversified portfolio of housing (Tracy, Schneider, and Chan, 1999). In addition, when the homeownership rate is high, it is more difficult for individual investors to attain higher degrees of diversification. We use the approach of Shanken (1990) to model the price of risk for idiosyncratic risk as a function of homeownership (see also Ferson and Harvey, 1999; Petkova and Zhang, 2005).

We model the time-varying coefficient on idiosyncratic risk as a function of the homeownership rate. That is, we model $\lambda_{\sigma,i,t-1}$ as

$$\lambda_{\sigma,i,t-1} = \lambda_{\sigma,0} + \lambda_{\sigma,1}\text{HOMEOWN}_{i,t-1}, \tag{2.5}$$

where $\text{HOMEOWN}_{i,t}$ denotes the rate of homeownership at time t. At the return level, we control for outmigration, unemployment, population, population growth, median income, income growth, rent risk, and the fraction of the population that is between 20 to 45 years. For the outmigration variable, as well as the fraction between 20 and 45 years we use IPUMS data and follow Han (2013). We control for outmigration using a dummy variable, denoted by SAMEMSAMED, that equals 1 if the outmigration rate is less than 14%. Rent risk is computed as the rolling standard deviation of the rent level, divided by the rolling average of median income. Effectively, this setup means that we model $\lambda_{0,i}$ to be a function of these variables. As an additional robustness check, we also run our regressions with these control variables included in (2.5) as well.

Combining (2.5) with specification (2.4) and adding the controls, we obtain the main specifications that we use. We also include specifications where we control directly for macroeconomic shocks by including both the housing market beta and the stock market beta. Omitting the additional controls, his leads to the following empirical model.

$$\begin{aligned} R_{i,t} - R_f = {} & \lambda_{0,i} + \xi_t + \lambda_{FHFA,t-1}\beta_{FHFA,i} + \lambda_{S\&P,t-1}\beta_{S\&P,i} \\ & + \lambda_{\sigma,0}\sigma_{\varepsilon,i,t} + \lambda_{\sigma,1}\sigma_{\varepsilon,i,t} \times \text{HOMEOWN}_{i,t-1} + \varepsilon_{i,t} \end{aligned} \tag{2.6}$$

We impose $\lambda_{FHFA,t} = R_{m,FHFA,t} - R_f$ and $\lambda_{S\&P,t} = R_{m,S\&P,t} - R_f$ to improve the power of our test. Using the interactions in (2.5) leads to strongly correlated regressors. Hence, we standardize within each MSA the variables that we interact idiosyncratic risk with (i.e. homeownership as well as the additional controls).

2.4 Empirical Results

In Table 2.4 we present the estimation results for (2.4) and (2.6). The results show that idiosyncratic risk is priced in this panel setting when including the relevant control variables and clustering the standard errors by year. The idiosyncratic risk coefficient value of 0.20 is close to the value of 0.18 found by Cannon, Miller, and Pandher (2006). The price of risk for idiosyncratic housing risk is positively related to the homeownership rate, which is an indication that underdiversification is indeed the mechanism through which idiosyncratic risk is priced.

Table 2.5 shows the results when including MSA fixed effects and clustering the standard errors by MSA, while Table 2.6 presents the results when including fixed effects and clustering both by year and by MSA. Table 2.6 also presents Fama and MacBeth (1973) estimates. We implement MSA fixed effects in the Fama and MacBeth (1973) setting by taking deviations from the mean return within each MSA. Across all specifications we find similar results showing the idiosyncratic risk is priced, and that its price of risk depends positively on the homeownership rate. The results in Table 2.6 show that the effect is indeed present using the Fama and MacBeth (1973) approach. Together with the results of in Table 2.4, these results show that idiosyncratic risk is indeed priced, and that the idiosyncratic risk premium increases with homeownership.

Across the various specifications, the significant control variables have the expected signs. The population level and growth, as well as income growth have positive signs. For hedging variables that measure a decrease in risk, for instance because homeownership hedges against a future increase in rents, such as rent risk and the fraction of the population aged 20 to 45 years we see a negative sign.

The results of specification with additional interactions of control variables with idiosyncratic risk given in Table 2.10 show that our results are robust to including these additional interactions. In addition, Table 2.7 and Table 2.8 respectively show that our results are also robust to using the RMSE of a full-sample and a rolling window CAPM regression to estimate idiosyncratic risk. Moreover, Table 2.8 shows that the results are also robust to including the stock and housing market betas as characteristics, rather than imposing that the market price of risk equals the market risk premium.

2.5 Conclusions

Although housing makes up two-thirds of a typical U.S. household's investment portfolio, the risk-return relationship for housing has received little attention. A notable difference with the stock market is that the typical real estate investor's portfolio displays limited diversification. U.S. home-owners tend to have a highly levered position in a single house, rather than own a large, diversified portfolio.

Due to this limited diversification, a risk premium for idiosyncratic risk arises naturally. Viewing the homeownership rate as a proxy for the degree of underdiversification, we can say more about the extent to which idiosyncratic risk is priced. When the homeownership rate is high, investors tend to be more underdiversified. Hence, we would expect to see a higher premium for idiosyncratic risk in that case.

Earlier research has shown cross-sectionally that a premium for idiosyncratic risk exists (Cannon, Miller, and Pandher, 2006). We contribute to this literature by showing that this premium is also significant in the time-series dimension, and that its time-variation is important in understanding the pricing mechanism. Specifically, the time-series dimension allows us to identify the interaction with the homeownership rate.

This paper establishes that idiosyncratic risk is indeed priced for U.S. real estate. We analyze returns to housing measured by the FHFA all transactions House Price Index (HPI). We measure idiosyncratic volatility by estimating an ARCH(1) model for housing returns, while controlling for housing market and stock market factors. Following the asset pricing approach of Ang, Hodrick, Xing, and Zhang (2006), we find a significant impact of idiosyncratic volatility on housing returns. The premium commanded by idiosyncratic housing risk is positively related to the homeownership rate.

Table 2.1. Descriptive statistics

This table shows descriptive statistics for the data used to estimate the model. The sample consists of annual observations at the MSA level during the period 1980–2012. $R_{i,t} - R_f$ denotes the real excess return on the MSA-level FHFA all transactions House Price Index (HPI), $R_{m,FHFA,t} - R_f$ the excess return on the national FHFA all transactions HPI, $R_{m,S\&P,t} - R_f$ the return on the S&P500 index, and R_f denotes the risk-free rate. $\beta_{FHFA,i}$ denotes the beta with respect to the housing market, and $\beta_{S\&P,i}$ the beta with respect to the stock market, and both are obtained from estimation of (2.1). $\sigma_{\varepsilon,i,t}$ denotes the idiosyncratic risk level estimated by ARCH(1) specification (2.2). HOMEOWN denotes the homeownership rate, UNEMP the unemployment rate, RENT the rent level, HEDGEWITHIN the Han (2013) within-MSA heding measure, MEDINCOME median income, and RENTRISK rent risk.

	Observations	Mean	Standard Deviation	Minimum	Maximum
$R_{i,t} - R_f$ (%)	8,402	-0.9798	5.3745	-29.4695	22.4880
$R_{m,FHFA,t} - R_f$ (%)	9,472	-1.4702	4.0633	-10.4975	6.3792
$R_{m,S\&P,t} - R_f$ (%)	9,768	7.5424	16.9866	-38.3404	31.1993
R_f (%)	9,768	5.0439	3.4724	0.0400	14.7175
$\beta_{FHFA,i}$	9,768	0.8135	0.3372	0.1632	1.8554
$\beta_{FHFA,i,t}$ (rolling)	6,951	0.8711	0.8889	-10.6995	16.0752
$\beta_{S\&P,i}$	9,768	-0.0236	0.0369	-0.2040	0.0701
$\beta_{S\&P,i,t}$ (rolling)	6,951	-0.0173	0.1517	-4.3484	3.1495
$\sigma_{\varepsilon,i,t}$ (CAPM)	9,768	3.8287	1.6213	1.5284	12.2708
$\sigma_{\varepsilon,i,t}$ (ARCH)	5,679	4.0889	2.7285	0.0962	50.9009
$\sigma_{\varepsilon,i,t}$ (rolling)	6,951	3.2162	2.1867	0.0000	22.7505
HOMEOWN (%)	7,272	71.8320	6.6746	38.3191	89.6937
UNEMP (%)	7,272	6.4786	2.1917	1.8762	25.3859
RENT (USD)	7,262	145.0379	72.8086	34.4915	668.6465
HEDGEWITHIN	6,996	0.5242	0.4995	0.0000	1.0000
MEDINCOME (USD)	7,272	23799.5275	57666.7127	5910.0000	2.512e+06
RENTRISK	5,921	0.0008	0.0006	0.0000	0.0040

Table 2.2. Descriptive statistics for portfolios sorted by idiosyncratic risk

This table shows descriptive statistics for excess return on the decile portfolios sorted by idiosyncratic risk. Idiosyncratic risk, denoted by $\sigma_{\varepsilon,i,t}$, is measured as the ARCH(1) estimate based on (2.2). The returns are in excess of the risk-free rate. The sample consists of annual observations of excess housing returns at the MSA level during the period 1980–2012.

	$R_{i,t}-R_f$	CAPM α	$\beta_{FHFA,i,t}$	$\beta_{S\&P,i,t}$	$\sigma_{\varepsilon,i,t}$	Home Ownership	Unemployment Rate	Rent Level
1								
Mean	-0.79	-0.69	0.74	-0.01	1.52	74.21	5.87	152.10
Std. Dev.	2.60	0.12	0.17	0.01	0.11	2.47	1.23	23.95
2								
Mean	-0.36	-0.19	0.84	-0.01	2.16	73.48	6.10	154.85
Std. Dev.	2.85	0.12	0.18	0.01	0.10	2.73	1.15	21.71
3								
Mean	-0.43	-0.26	0.81	-0.01	2.59	72.63	6.14	161.81
Std. Dev.	2.42	0.16	0.18	0.01	0.15	2.35	1.28	30.91
4								
Mean	-0.21	0.03	0.86	-0.02	3.05	72.30	6.13	165.99
Std. Dev.	2.99	0.19	0.17	0.02	0.18	2.87	1.27	25.26
5								
Mean	-0.07	0.09	0.94	-0.03	3.46	71.80	6.63	164.65
Std. Dev.	3.17	0.17	0.17	0.03	0.24	2.95	1.31	32.46
6								
Mean	-0.19	0.29	0.93	-0.03	3.89	71.59	6.67	171.23
Std. Dev.	3.25	0.15	0.19	0.03	0.28	2.72	1.11	32.72
7								
Mean	-0.05	0.32	0.88	-0.02	4.33	71.74	6.71	173.69
Std. Dev.	3.25	0.17	0.10	0.02	0.37	2.44	1.47	34.59
8								
Mean	0.23	0.50	0.98	-0.02	4.94	70.00	7.15	191.52
Std. Dev.	3.70	0.14	0.24	0.03	0.69	2.62	1.50	39.46
9								
Mean	-0.01	0.35	0.93	-0.01	5.93	71.90	6.96	171.05
Std. Dev.	3.53	0.17	0.32	0.03	1.51	2.67	1.53	35.13
10								
Mean	0.05	0.42	0.97	-0.02	9.33	73.04	7.01	171.46
Std. Dev.	3.99	0.17	0.24	0.04	4.53	2.36	1.36	38.61

Table 2.3. Correlations

This table shows correlations for the data used to estimate the model. The sample consists of annual observations for 296 MSAs during the period 1980–2012. The definitions of the variables are as in Table 2.1.

	$R_{i,t}-R_f$	$R_{m,FHFA,t}-R_f$	$R_{m,S\&P,t}-R_f$	R_f	$\sigma_{\varepsilon,i,t}$ (CAPM)	$\sigma_{\varepsilon,i,t}$ (ARCH)	$\sigma_{\varepsilon,i,t}$ (rolling)	HOMEOWN	UNEMP
$R_{i,t}-R_f$	1.0000								
$R_{m,FHFA,t}-R_f$	0.6089***	1.0000							
$R_{m,S\&P,t}-R_f$	0.1162***	0.3279***	1.0000						
R_f	-0.3825***	-0.6437***	-0.0076	1.0000					
$\sigma_{\varepsilon,i,t}$ (CAPM)	0.0504***	-0.0087	-0.0043	0.0123	1.0000				
$\sigma_{\varepsilon,i,t}$ (ARCH)	0.0040	-0.0575***	0.0336**	0.0671***	0.4295***	1.0000			
$\sigma_{\varepsilon,i,t}$ (rolling)	-0.0947***	-0.1844***	0.0308*	0.1883***	0.5566***	0.4219***	1.0000		
HOMEOWN	0.0241	0.0442***	-0.0499***	-0.1616***	-0.2039***	-0.0242	-0.2395***	1.0000	
UNEMP	0.0676***	0.2517***	0.1097***	-0.3979***	0.1829***	0.1064***	0.1116***	-0.1983***	1.0000
RENT	0.0721***	0.1017***	-0.0299*	-0.2813***	0.2640***	0.0131	0.1011***	-0.6312***	0.2518***
HEDGEWITHIN	-0.0922***	-0.1172***	0.0681***	0.3975***	-0.0578***	-0.0698***	0.0472***	-0.1712***	-0.1965***
MEDINCOME	0.1288***	0.1593***	-0.1118***	-0.4154***	0.0507***	-0.0858***	-0.1897***	0.1992***	-0.0982***
RENTRISK	-0.1406***	-0.1149***	0.0499***	0.0792***	0.1490***	0.0729***	0.3514***	-0.6956***	0.2427***

	RENT	HEDGEWITHIN	MEDINCOME	RENTRISK
$R_{i,t}-R_f$				
$R_{m,FHFA,t}-R_f$				
$R_{m,S\&P,t}-R_f$				
R_f				
$\sigma_{\varepsilon,i,t}$ (CAPM)				
$\sigma_{\varepsilon,i,t}$ (ARCH)				
$\sigma_{\varepsilon,i,t}$ (rolling)				
HOMEOWN				
UNEMP				
RENT	1.0000			
HEDGEWITHIN	-0.0756***	1.0000		
MEDINCOME	0.4658***	-0.1869***	1.0000	
RENTRISK	0.4863***	0.0904***	-0.3096***	1.0000

Table 2.4. Pricing of idiosyncratic risk and premium determinants (year clustering).

This table shows the estimation results for the pricing regressions for idiosyncratic risk and the determinants of its premium. The data used are annual data at the MSA level for the period 1990–2006. The definitions of the variables are as in Table 2.1. SAMEMSAMED, is a dummy variable that equals 1 if the outmigration rate is less than 14% (see Han, 2013). Standard errors are given in parentheses.

	$R_{i,t} - R_f$	$R_{i,t} - R_f$	$R_{i,t} - R_f$	$R_{i,t} - R_f$	$R_{i,t} - R_f$
$\sigma_{\varepsilon,i,t} \times$ HOMEOWN				0.3329***	0.3007***
				(0.0654)	(0.0708)
$\sigma_{\varepsilon,i,t}$	0.0606		0.0933	0.2071***	0.2040***
	(0.0677)		(0.0688)	(0.0399)	(0.0425)
HOMEOWN		0.0032	0.0035	-0.0097**	-0.0108*
		(0.0028)	(0.0028)	(0.0038)	(0.0058)
SAMEMSAMED					-0.0042*
					(0.0024)
UNEMP					-0.0024
					(0.0025)
POPULATION					-0.0018
					(0.0037)
POPGROWTH					0.0025***
					(0.0007)
MEDINCOME					-0.0113*
					(0.0057)
INCOMEGR					0.0056
					(0.0039)
RENTRISK					-0.0029
					(0.0031)
FRAC20TO45YRS					-0.0048
					(0.0056)
Constant	-0.0160***	-0.0010	-0.0045	-0.0083**	-0.0099*
	(0.0030)	(0.0019)	(0.0044)	(0.0029)	(0.0049)
Fixed Effects	Year	Year	Year	Year	Year
Clustering	Year	Year	Year	Year	Year
R^2	0.0363	0.0390	0.0421	0.0630	0.0734
R^2_{adj}	0.0324	0.0347	0.0367	0.0575	0.0646
RSS	9.0763	7.3755	6.2804	6.1433	5.4661
Observations	4,161	3,842	3,218	3,218	2,859

Table 2.5. Pricing of idiosyncratic risk and premium determinants (MSA clustering).

This table shows the estimation results for the pricing regressions for idiosyncratic risk and the determinants of its premium. The data used are annual data at the MSA level for the period 1990–2006. The definitions of the variables are as in Table 2.1. SAMEMSAMED, is a dummy variable that equals 1 if the outmigration rate is less than 14% (see Han, 2013). Standard errors are given in parentheses.

	$R_{i,t}-R_f$	$R_{i,t}-R_f$	$R_{i,t}-R_f$	$R_{i,t}-R_f$	$R_{i,t}-R_f$
$\sigma_{\varepsilon,i,t}$ × HOMEOWN				0.3284***	0.2899***
				(0.0524)	(0.0535)
$\sigma_{\varepsilon,i,t}$	0.0338		0.0531	0.2059***	0.1816***
	(0.0361)		(0.0492)	(0.0575)	(0.0569)
HOMEOWN		0.0040	0.0054*	-0.0080**	-0.0148***
		(0.0025)	(0.0028)	(0.0032)	(0.0044)
SAMEMSAMED					-0.0046
					(0.0042)
UNEMP					-0.0033
					(0.0024)
POPULATION					0.0002
					(0.0038)
POPGROWTH					0.0026**
					(0.0011)
MEDINCOME					-0.0342***
					(0.0106)
INCOMEGR					0.0107**
					(0.0044)
RENTRISK					-0.0082**
					(0.0041)
FRAC20TO45YRS					-0.0111*
					(0.0058)
Constant	-0.0152**	-0.0010	-0.0018	-0.0077	-0.0174
	(0.0068)	(0.0066)	(0.0080)	(0.0080)	(0.0146)
Fixed Effects	Year & MSA	Year & MSA	Year & MSA	Year & MSA	Year & MSA
Clustering	MSA	MSA	MSA	MSA	MSA
R^2	0.0402	0.0442	0.0462	0.0665	0.0808
R^2_{adj}	0.0363	0.0399	0.0409	0.0610	0.0720
RSS	8.0032	6.4638	5.5410	5.4231	4.7902
Observations	4,161	3,842	3,218	3,218	2,859

Table 2.6. Pricing of idiosyncratic risk and premium determinants (year and MSA clustering).

This table shows the estimation results for the pricing regressions for idiosyncratic risk and the determinants of its premium. The data used are annual data at the MSA level for the period 1990–2006. The definitions of the variables are as in Table 2.1. SAMEMSAMED, is a dummy variable that equals 1 if the outmigration rate is less than 14% (see Han, 2013). Standard errors are given in parentheses. FMB indicates Fama and MacBeth (1973) standard errors are used with 2 lags. For this case, we apply the Petersen (2009) correction, which adjusts for the presence of an unobserved MSA effect.

	$R_{i,t} - R_f$	$R_{i,t} - R_f$	$R_{i,t} - R_f$	$R_{i,t} - R_f$
$\sigma_{\varepsilon,i,t} \times$ HOMEOWN	0.3007***	0.2899***	0.3139**	0.2899***
	(0.0708)	(0.0535)	(0.1453)	(0.0783)
$\sigma_{\varepsilon,i,t}$	0.2040***	0.1816***	0.0917	0.1816***
	(0.0425)	(0.0569)	(0.1938)	(0.0675)
HOMEOWN	-0.0108*	-0.0148***	-0.0104**	-0.0148**
	(0.0058)	(0.0044)	(0.0039)	(0.0070)
SAMEMSAMED	-0.0042*	-0.0046	-0.0017	-0.0046
	(0.0024)	(0.0042)	(0.0017)	(0.0044)
UNEMP	-0.0024	-0.0033	-0.0003	-0.0033
	(0.0025)	(0.0024)	(0.0014)	(0.0035)
POPULATION	-0.0018	0.0002	-0.0057	0.0002
	(0.0037)	(0.0038)	(0.0041)	(0.0039)
POPGROWTH	0.0025***	0.0026**	0.0002	0.0026***
	(0.0007)	(0.0011)	(0.0068)	(0.0008)
MEDINCOME	-0.0113*	-0.0342***	-0.0023	-0.0342**
	(0.0057)	(0.0106)	(0.0085)	(0.0138)
INCOMEGR	0.0056	0.0107**	0.0175	0.0107**
	(0.0039)	(0.0044)	(0.0150)	(0.0054)
RENTRISK	-0.0029	-0.0082**	-0.0059	-0.0082**
	(0.0031)	(0.0041)	(0.0110)	(0.0039)
FRAC20TO45YRS	-0.0048	-0.0111*	-0.0034	-0.0111
	(0.0056)	(0.0058)	(0.0079)	(0.0084)
Constant	-0.0099*	-0.0174	0.0052	0.0034
	(0.0049)	(0.0146)	(0.0057)	(0.0084)
Fixed Effects	Year	Year & MSA	FMB & MSA	Year & MSA
Clustering	Year	MSA	FMB	Year & MSA
R^2	0.0734	0.0808	0.1755	0.1880
R^2_{adj}	0.0646	0.0720		0.1229
RSS	5.4661	4.7902		4.7902
Observations	2,859	2,859	2,859	2,859

Table 2.7. Pricing of idiosyncratic risk and premium determinants (robustness to idiosyncratic risk estimation method).

This table shows the estimation results for the pricing regressions for idiosyncratic risk and the determinants of its premium. The data used are annual data at the MSA level for the period 1990–2006. The definitions of the variables are as in Table 2.1. For these regressions, idiosyncratic risk is estimated as the RMSE of a CAPM regression. Similarly to the early tables, SAMEMSAMED is a dummy variable that equals 1 if the outmigration rate is less than 14% (see Han, 2013). Standard errors are given in parentheses. FMB indicates Fama and MacBeth (1973) standard errors are used with 2 lags. For this case, we apply the Petersen (2009) correction, which adjusts for the presence of an unobserved MSA effect.

	$R_{i,t}-R_f$	$R_{i,t}-R_f$	$R_{i,t}-R_f$
$\sigma_{\varepsilon,i,t}$ × HOMEOWN	0.4911***	0.4777***	0.2680**
	(0.0584)	(0.0531)	(0.0928)
$\sigma_{\varepsilon,i,t}$	0.2369***	0.2501***	0.4187***
	(0.0758)	(0.0726)	(0.0698)
HOMEOWN	-0.0161***	-0.0176***	-0.0068**
	(0.0026)	(0.0039)	(0.0029)
SAMEMSAMED		-0.0037	-0.0043**
		(0.0026)	(0.0018)
UNEMP		-0.0018	-0.0028*
		(0.0025)	(0.0014)
POPULATION		-0.0023	-0.0101*
		(0.0031)	(0.0051)
POPGROWTH		0.0017***	0.0035
		(0.0006)	(0.0036)
MEDINCOME		-0.0131**	-0.0208***
		(0.0050)	(0.0057)
INCOMEGR		0.0031	0.0126
		(0.0037)	(0.0126)
RENTRISK		-0.0017	0.0002
		(0.0028)	(0.0033)
FRAC20TO45YRS		-0.0061	-0.0027
		(0.0047)	(0.0063)
Constant	-0.0098**	-0.0118**	-0.0033
	(0.0039)	(0.0045)	(0.0035)
Fixed Effects	Year	Year	FMB
Clustering	Year	Year	FMB(2L,P)
R^2	0.0683	0.0780	0.1652
R^2_{adj}	0.0636	0.0706	
RSS	7.1506	6.3852	
Observations	3,842	3,403	3,403

Table 2.8. Pricing of idiosyncratic risk and premium determinants (robustness to idiosyncratic risk estimation method).

This table shows the estimation results for the pricing regressions for idiosyncratic risk and the determinants of its premium. The data used are annual data at the MSA level for the period 1990–2006. The definitions of the variables are as in Table 2.1. For these regressions, idiosyncratic risk is estimated as the RMSE of a rolling CAPM regression with a time window of 10 years. Similarly to the early tables, SAMEMSAMED is a dummy variable that equals 1 if the outmigration rate is less than 14% (see Han, 2013). Standard errors are given in parentheses.

	$R_{i,t} - R_f$	$R_{i,t} - R_f$	$R_{i,t} - R_f$
$\sigma_{\varepsilon,i,t} \times$ HOMEOWN	0.3893***	0.3549***	0.3549***
	(0.0704)	(0.1032)	(0.0821)
$\sigma_{\varepsilon,i,t}$	0.3558***	0.3182***	0.3182**
	(0.0853)	(0.1099)	(0.1541)
HOMEOWN	-0.0110**	-0.0140***	-0.0140**
	(0.0050)	(0.0043)	(0.0069)
SAMEMSAMED	-0.0041	-0.0077**	-0.0077**
	(0.0026)	(0.0038)	(0.0031)
UNEMP	-0.0028	-0.0021	-0.0021
	(0.0027)	(0.0022)	(0.0034)
POPULATION	-0.0029	-0.0012	-0.0012
	(0.0033)	(0.0036)	(0.0039)
POPGROWTH	0.0014**	0.0017	0.0017**
	(0.0006)	(0.0011)	(0.0008)
MEDINCOME	-0.0162***	-0.0350***	-0.0350**
	(0.0045)	(0.0100)	(0.0161)
INCOMEGR	0.0066	0.0122***	0.0122**
	(0.0043)	(0.0039)	(0.0059)
RENTRISK	-0.0028	-0.0062*	-0.0062*
	(0.0028)	(0.0036)	(0.0034)
FRAC20TO45YRS	-0.0067	-0.0130**	-0.0130
	(0.0049)	(0.0052)	(0.0080)
Constant	-0.0161*	-0.0205	-0.0057
	(0.0082)	(0.0134)	(0.0100)
Fixed Effects	Year	Year & MSA	Year & MSA
Clustering	Year	MSA	Year & MSA
R^2	0.0734	0.0765	0.1869
R^2_{adj}	0.0660	0.0691	0.1229
RSS	6.4168	5.6310	5.6310
Observations	3,403	3,403	3,403

Table 2.9. Pricing of idiosyncratic risk and premium determinants (robustness to including beta).

This table shows the estimation results for the pricing regressions for idiosyncratic risk and the determinants of its premium. The data used are annual data at the MSA level for the period 1990–2006. The definitions of the variables are as in Table 2.1. For these regressions, idiosyncratic risk is estimated as in the original specifications, but here we include the housing market and stock market betas as characteristic, rather than imposing that the market price of risk equals the market risk premium. Similarly to the early tables, SAMEMSAMED is a dummy variable that equals 1 if the outmigration rate is less than 14% (see Han, 2013). Standard errors are given in parentheses.

	$R_{i,t} - R_f$	$R_{i,t} - R_f$	$R_{i,t} - R_f$
$\sigma_{\varepsilon,i,t} \times$ HOMEOWN	0.2036***	0.1739***	0.1739***
	(0.0372)	(0.0449)	(0.0360)
$\sigma_{\varepsilon,i,t}$	0.1962***	0.1264***	0.1264***
	(0.0373)	(0.0342)	(0.0230)
HOMEOWN	-0.0158***	-0.0165***	-0.0165***
	(0.0025)	(0.0041)	(0.0053)
$\beta_{FHFA,i,t}$ (rolling)	0.0075	0.0092***	0.0092
	(0.0062)	(0.0014)	(0.0071)
$\beta_{S\&P,i,t}$ (rolling)	0.0154	0.0256***	0.0256
	(0.0274)	(0.0086)	(0.0274)
SAMEMSAMED	-0.0023	-0.0019	-0.0019
	(0.0020)	(0.0031)	(0.0045)
UNEMP	0.0012	-0.0001	-0.0001
	(0.0020)	(0.0018)	(0.0017)
POPULATION	-0.0046	-0.0070**	-0.0070*
	(0.0028)	(0.0032)	(0.0038)
POPGROWTH	0.0012*	0.0014	0.0014**
	(0.0006)	(0.0012)	(0.0005)
MEDINCOME	-0.0181**	-0.0292***	-0.0292**
	(0.0063)	(0.0108)	(0.0139)
INCOMEGR	0.0044	0.0078**	0.0078***
	(0.0035)	(0.0033)	(0.0025)
RENTRISK	-0.0053**	-0.0088***	-0.0088**
	(0.0022)	(0.0033)	(0.0034)
FRAC20TO45YRS	-0.0179***	-0.0219***	-0.0219***
	(0.0038)	(0.0040)	(0.0041)
Constant	-0.0672***	-0.0667***	-0.0675***
	(0.0034)	(0.0086)	(0.0044)
Fixed Effects	Year	Year & MSA	Year & MSA
Clustering	Year	MSA	Year & MSA
R^2	0.4767	0.4869	0.5172
R^2_{adj}	0.4714	0.4816	0.4781
RSS	3.1158	2.8748	2.8748
Observations	2,859	2,859	2,859

Table 2.10. Pricing of idiosyncratic risk and premium determinants (idiosyncratic risk interactions).

This table shows the estimation results for the pricing regressions for idiosyncratic risk and the determinants of its premium. The data used are annual data at the MSA level for the period 1990–2006. The definitions of the variables are as in Table 2.1. SAMEMSAMED, is a dummy variable that equals 1 if the outmigration rate is less than 14% (see Han, 2013). Standard errors are given in parentheses.

	$R_{i,t} - R_f$	$R_{i,t} - R_f$	$R_{i,t} - R_f$	$R_{i,t} - R_f$	$R_{i,t} - R_f$	$R_{i,t} - R_f$
$\sigma_{\varepsilon,i,t}$ × HOMEOWN	0.3007***	0.2278*	0.2899***	0.2657***	0.2899***	0.2657**
	(0.0708)	(0.1300)	(0.0535)	(0.0925)	(0.0783)	(0.1289)
$\sigma_{\varepsilon,i,t}$	0.2040***	0.2676**	0.1816***	0.1470	0.1816***	0.1470*
	(0.0425)	(0.0981)	(0.0569)	(0.0960)	(0.0675)	(0.0830)
HOMEOWN	-0.0108*	-0.0076*	-0.0148***	-0.0137***	-0.0148**	-0.0137***
	(0.0058)	(0.0042)	(0.0044)	(0.0046)	(0.0070)	(0.0050)
SAMEMSAMED	-0.0042*	-0.0017	-0.0046	-0.0080	-0.0046	-0.0080
	(0.0024)	(0.0075)	(0.0042)	(0.0055)	(0.0044)	(0.0094)
UNEMP	-0.0024	-0.0021	-0.0033	-0.0058	-0.0033	-0.0058
	(0.0025)	(0.0030)	(0.0024)	(0.0036)	(0.0035)	(0.0037)
POPULATION	-0.0018	0.0054	0.0002	0.0097*	0.0002	0.0097**
	(0.0037)	(0.0033)	(0.0038)	(0.0056)	(0.0039)	(0.0048)
POPGROWTH	0.0025***	0.0004	0.0026**	-0.0004	0.0026***	-0.0004
	(0.0007)	(0.0031)	(0.0011)	(0.0026)	(0.0008)	(0.0030)
MEDINCOME	-0.0113*	-0.0160**	-0.0342***	-0.0444***	-0.0342**	-0.0444**
	(0.0057)	(0.0068)	(0.0106)	(0.0123)	(0.0138)	(0.0182)
INCOMEGR	0.0056	0.0038	0.0107**	0.0073	0.0107**	0.0073
	(0.0039)	(0.0073)	(0.0044)	(0.0123)	(0.0054)	(0.0117)
RENTRISK	-0.0029	-0.0027	-0.0082**	-0.0087**	-0.0082**	-0.0087
	(0.0031)	(0.0049)	(0.0041)	(0.0043)	(0.0039)	(0.0059)
FRAC20TO45YRS	-0.0048	-0.0057	-0.0111*	-0.0182**	-0.0111	-0.0182***
	(0.0056)	(0.0034)	(0.0058)	(0.0088)	(0.0084)	(0.0058)
$\sigma_{\varepsilon,i,t}$ × SAMEMSAMED		-0.0597		0.0910		0.0910
		(0.1970)		(0.1249)		(0.1631)
$\sigma_{\varepsilon,i,t}$ × UNEMP		-0.0039		0.0689		0.0689
		(0.1007)		(0.0799)		(0.1041)
Fixed Effects	Year	Year	Year & MSA	Year & MSA	Year & MSA	Year & MSA
Clustering	Year	Year	MSA	MSA	Year & MSA	Year & MSA
R^2	0.0734	0.0763	0.0808	0.0847	0.1880	0.1914
R^2_{adj}	0.0646	0.0649	0.0720	0.0733	0.1229	0.1240
RSS	5.4661	5.4487	4.7902	4.7700	4.7902	4.7700
Observations	2,859	2,859	2,859	2,859	2,859	2,859

Table 2.10. Pricing of idiosyncratic risk and premium determinants (idiosyncratic risk interactions, continued).

This table shows the estimation results for the pricing regressions for idiosyncratic risk and the determinants of its premium. The data used are annual data at the MSA level for the period 1990–2006. The definitions of the variables are as in Table 2.1. SAMEMSAMED, is a dummy variable that equals 1 if the outmigration rate is less than 14% (see Han, 2013). Standard errors are given in parentheses.

	$R_{i,t}-R_f$	$R_{i,t}-R_f$	$R_{i,t}-R_f$	$R_{i,t}-R_f$	$R_{i,t}-R_f$	$R_{i,t}-R_f$
$\sigma_{\varepsilon,i,t}\times$ POPULATION		-0.1756*		-0.2235**		-0.2235**
		(0.0958)		(0.1078)		(0.1075)
$\sigma_{\varepsilon,i,t}\times$ POPGROWTH		0.0545		0.0746		0.0746
		(0.0794)		(0.0659)		(0.0787)
$\sigma_{\varepsilon,i,t}\times$ MEDINCOME		0.1243		0.2242		0.2242
		(0.2078)		(0.1776)		(0.2400)
$\sigma_{\varepsilon,i,t}\times$ INCOMEGR		0.0392		0.0820		0.0820
		(0.1339)		(0.3473)		(0.3106)
$\sigma_{\varepsilon,i,t}\times$ RENTRISK		-0.0001		0.0124		0.0124
		(0.0885)		(0.0760)		(0.1033)
$\sigma_{\varepsilon,i,t}\times$ FRAC20TO45YRS		0.0269		0.1621		0.1621
		(0.0994)		(0.1455)		(0.1148)
Constant	-0.0099*	-0.0126*	-0.0174	-0.0165	0.0034	0.0041
	(0.0049)	(0.0066)	(0.0146)	(0.0162)	(0.0084)	(0.0118)
Fixed Effects	Year	Year	Year & MSA	Year & MSA	Year & MSA	Year & MSA
Clustering	Year	Year	MSA	MSA	Year & MSA	Year & MSA
R^2	0.0734	0.0763	0.0808	0.0847	0.1880	0.1914
R^2_{adj}	0.0646	0.0649	0.0720	0.0733	0.1229	0.1240
RSS	5.4661	5.4487	4.7902	4.7700	4.7902	4.7700
Observations	2,859	2,859	2,859	2,859	2,859	2,859

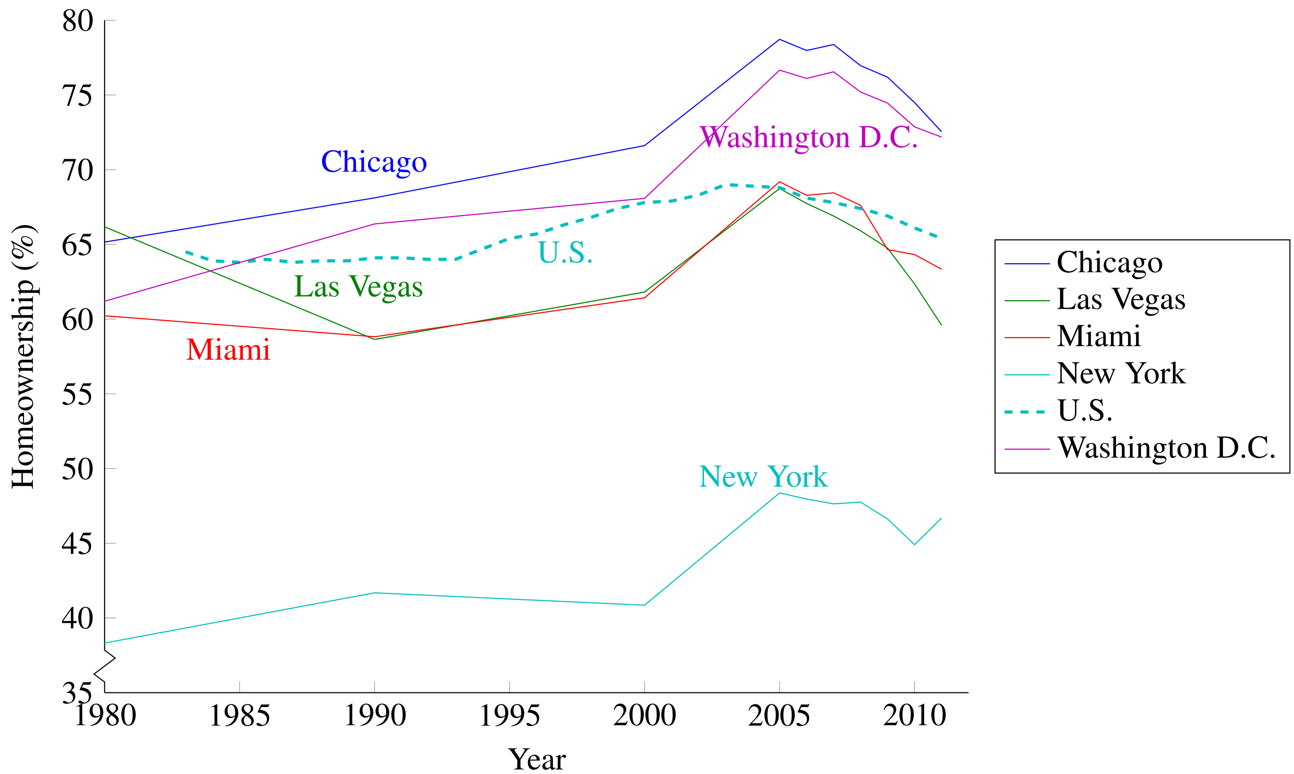

Figure 2.1. Homeownership per year. This figure shows the evolution of homeownership in the U.S. and various MSAs over time.

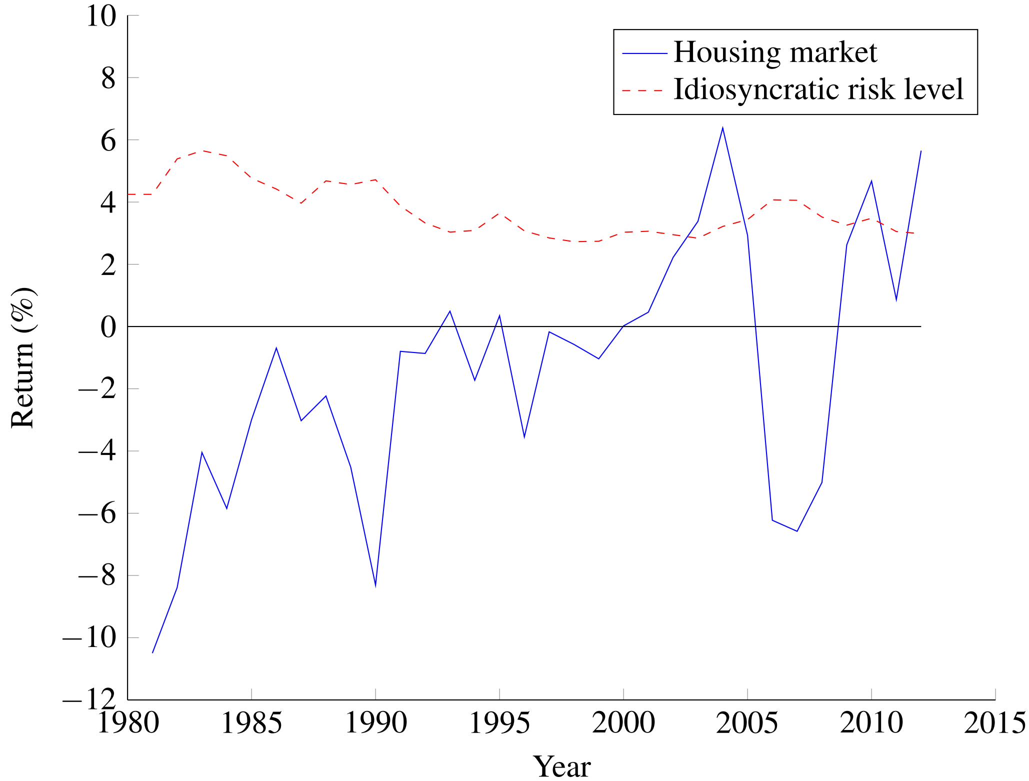

Figure 2.2. Market-level housing return and the average idiosyncratic risk level per year. This figure shows the evolution of the market-level housing return and the average idiosyncratic risk level over time.

Chapter 3

Pricing Effects of Time-Series Variation in Liquidity[1]

3.1 Introduction

Stock market liquidity has well-established pricing implications for the cross-section of stock returns (e.g., Acharya and Pedersen, 2005). Even though there is evidence that liquidity betas are time-varying, and that liquidity premia are varying over time (Kamara, Lou, and Sadka, 2008; Vayanos, 2004), little is known about the sources of this variation (Comerton-Forde, Hendershott, Jones, Moulton, and Seasholes, 2010; Karolyi, Lee, and van Dijk, 2012). A better understanding is not only relevant for asset pricing, but also for portfolio choice (e.g., Ang, Papanikolaou, and Westerfield, 2014).

Recent studies focus on liquidity commonality and flights to quality (Rösch and Kaserer, 2013). Næs, Skjeltorp, and Ødegaard (2011) empirically show that investors respond to changing expectations about the real economy by shifting their portfolios towards more liquid assets in two ways. The first type of shift

[1]I thank Alessandro Beber, Dion Bongaerts, Anton van Boxtel, Joost Driessen, Björn Hagströmer, Frank de Jong, Vincent van Kervel, Frans de Roon, and seminar participants at the 8th Financial Risks International Forum, the 32nd International Conference of the French Finance Association, the 2014 Netspar Pension Day, the University of Amsterdam, and the VU University Amsterdam for their insightful comments and suggestions.

happens when investors decide to exit the stock market, for instance to the bond market, as in Goyenko and Ukhov (2009). I refer to this as an *across-asset-class flight to liquidity*. The second type of shift occurs when investors move from illiquid stocks into liquid stocks. I call this a *within-asset-class flight to liquidity* (see also, e.g., Beber, Brandt, and Kavajecz, 2009; Baele, Bekaert, Inghelbrecht, and Wei, 2015; Vayanos, 2004). In this paper, I will use these mechanisms to motivate my analysis, but empirically I will consider the stock market only.

This paper considers the changes in the cross-section of liquidity that coincide with such portfolio shifts, as well as the impact of these changes in liquidity on asset prices. In the case of reduced overall stock market participation, there is a decrease in liquidity across all stocks, resulting in a level shift (Næs, Skjeltorp, and Ødegaard, 2011). When investors shift their holdings to more liquid stocks, the illiquid stocks become even less liquid, while liquid stocks are not affected as strongly (Næs, Skjeltorp, and Ødegaard, 2011). This increase in liquidity dispersion across stocks is essentially a liquidity slope effect.

Of course, the across and within asset class flights to liquidity need not be the only drivers of changes in the level or the slope of the cross-section of liquidity. The level component should pick up commonality in liquidity (see, e.g., Chordia, Roll, and Subrahmanyam, 2000; Rösch and Kaserer, 2013). For instance, if there is a funding liquidity freeze, all assets will become less liquid at the same time (Brunnermeier and Pedersen, 2009; Chordia, Sarkar, and Subrahmanyam, 2005). For the slope component, we note that empirically, the market liquidity of high-volatility stocks is more sensitive to inventory shocks (Comerton-Forde, Hendershott, Jones, Moulton, and Seasholes, 2010). The reason is that high-volatility stocks are less liquid (Chordia, Sarkar, and Subrahmanyam, 2005; Ho and Stoll, 1983), and that funding liquidity deterioration hits volatile, illiquid stocks hardest (Brunnermeier and Pedersen, 2009).

The level component should be priced, as it is well-known that liquidity commonality is priced in the cross-section (Pástor and Stambaugh, 2003; Acharya and Pedersen, 2005). It is less straightforward why the slope component would command a premium. For a possible motivation, consider a fund manager facing a large outflow. A potential strategy is to sell liquid assets first to minimize transactions costs (Duffie and Ziegler, 2003; Scholes, 2000). Empirical evidence suggests that this is indeed the strategy that is followed by various institutional investors (Ben-David, Franzoni, and Moussawi, 2012; Manconi, Massa, and Yasuda, 2012; Xing, 2016). However, after initially selling most of the liquid assets, this may force the fund to sell illiquid assets at great cost after a large negative return. In this way, the strategy increases tail losses and the probability of insolvency (Duffie and Ziegler, 2003). As this is compounded by exposure to slope risk, it seems not unlikely for the slope component to be priced.

The characterization in terms of level and slope effects naturally leads to a principal components analysis. Studies that focus on principal components of liquidity (e.g., Hagströmer, Anderson, Binner, and Nilsson, 2009; Korajczyk and Sadka, 2008) tend to consider the level of liquidity, while I use innovations. The advantage of using innovations is that they are roughly i.i.d and therefore allow a factor model interpretation for the principal components (see e.g., Basilevsky, 1994). This is not possible when using the level of liquidity, as liquidity is highly serially correlated (see e.g., Acharya and Pedersen, 2005, but also Table 3.2). It turns out that the first two principal components of illiquidity innovations are indeed the empirically relevant ones. I study what drives these components, as well as their impact on asset prices.

I explore the pricing implications by rephrasing the liquidity beta structure of Acharya and Pedersen (2005) in terms of the principal component exposures. This results in a parsimonious specification with a market return beta, a liquidity level beta, a liquidity slope beta, and a principal component residual beta. As the level and slope component are the empirically relevant ones, the residual beta should not be priced. An advantage of this formulation is that the principal component betas are much less correlated than the Acharya and Pedersen (2005) betas, so they allow for improved identification of different liquidity effects.

To isolate the two components of liquidity, I use 25 liquidity-sorted portfolios of U.S. stocks over the period 1964 to 2013, with market liquidity measured by the illiquidity measure of Amihud (2002) (adjusted as in Acharya and Pedersen, 2005). The principal components analysis reveals that 66% of the variation of illiquidity innovations can be explained by these first two principal components. The level component, which explains roughly 57% of the variation, is negatively related with market returns, negatively related with funding risk, and positively related with changes in the risk-free rate. This indicates that the level component reflects adverse selection, inventory risk, and margin requirements. The slope component, which explains about 9% of the variation, is negatively related with market returns in times of capital constraints on the funding market, positively related with the default spread, positively related with liquid portfolio turnover, negatively related with illiquid portfolio turnover, and negatively related with investor sentiment. Hence, the slope component reflects inventory risk, trading in the liquid segment of the market, and investor sentiment.

When we consider liquidity-related crises such as those discussed in Pástor and Stambaugh (2003), we can see that the level component, which reflects adverse selection, inventory risk, and margin requirements, displays the largest shocks during the 1970 domestic unrest, Silver Thursday (March 1980) and the 1987 crash. The latter is consistent with evidence that market making collapsed at during that year's stock market crisis (Brady, Cotting, Kirby, Opel, and Stein (1988) in Hameed, Kang, and Viswanathan, 2010). The slope component, which reflects inventory risk, trading in the liquid segment of the market, and sentiment, experienced relatively larger shocks during the 1990 crisis, the LTCM crisis (September

1998), and the dotcom bubble burst (May 2000). During the Russian default (August 1998) and the 2008 subprime crisis, both channels were active.

The results of the asset pricing analysis show that only the level beta is statistically significant. Moreover, the economic impact of the three liquidity betas is markedly different. While the liquidity level beta has a significant economic impact of about 1.5% p.a., there is no meaningful economic impact of the other liquidity betas. In summary, the pricing analysis indicates that market liquidity is economically relevant to risk premia only through level rather than slope effects. A subsample analysis reveals that the liquidity betas vary over time. Hence, I also consider time-varying betas which are estimated using rolling time windows of 120 months. The results of the analysis using time-varying betas are similar to the static results.

The pricing result that only the level component is relevant for stock prices has implications for management of liquidity crises. The risk that overall market liquidity deteriorates because liquidity disappears from part of the market (slope effect) does not lower prices of unaffected stocks. On the other hand, the risk of an overall liquidity freeze (level effect) has a price impact beyond what is implied by a drop in liquidity of the individual assets. This makes economic sense, as an institution that sells liquid assets first when under stress to meet capital requirements (see Duffie and Ziegler, 2003, for a discussion of strategies) will not be affected by the slope effect, but will certainly be harmed by the level effect.

The paper is organized as follows. Section 3.2 gives an overview of relevant literature. Section 3.3 describes the data. I describe the empirical approach and present the results in Section 3.4. Section 3.5 summarizes the findings and concludes the paper.

3.2 Related Literature

This paper builds on previous research on liquidity commonality, determinants of liquidity, principal components of liquidity, liquidity and asset pricing, and flights to liquidity. Most directly related is the paper by Acharya and Pedersen (2005), who develop a liquidity CAPM and show that both the level of liquidity and liquidity risk are priced in the cross-section of stocks. They derive three liquidity betas that are related to specific liquidity risks. On the basis of their model, it is possible to identify the pricing effects of the level and slope of the cross-section of liquidity by deriving betas for the respective effects.

To motivate why the level and the slope of liquidity should be relevant, I build on empirical work by Næs, Skjeltorp, and Ødegaard (2011), who use individual investor holdings data for all stocks on the Oslo Stock Exchange to show that changes to the cross-section of liquidity coincide with portfolio shifts by investors. Specifically, a decrease in overall liquidity coincides with investors exiting the stock market altogether (*across-asset-class flight to liquidity*), for instance to move into the bond market as in Goyenko and Ukhov (2009), while a decrease in the liquidity of only the least liquid stocks coincides with a shift towards more liquid stocks (*within-asset-class flight to liquidity*).

Earlier work has also studied the principal components of liquidity. For instance, Korajczyk and Sadka (2008) analyze liquidity commonality through a principal components analysis. They look at principal components of liquidity itself (measure-specific) and principal components of various measures of liquidity (across-measure). In addition, they construct liquidity factors to see whether there is a liquidity premium associated with the various principal components. Hagströmer, Anderson, Binner, and Nilsson (2009) estimate systematic liquidity using a principal components analysis based on the level of liquidity. They find that the market average liquidity yields the same degree of commonality as the principal components-based estimates, but the latter are better able to explain stock returns. In addition, they find that for some estimators of liquidity the liquidity covariance matrix changes over time, so that it is necessary to use a rolling window principal component estimator. This finding, however, does not apply to the Amihud (2002) measure that is used in this study.

A similar analysis is performed by Kim and Lee (2014), who use an across-measure principal components analysis of liquidity to investigate the pricing implications using the Acharya and Pedersen (2005) liquidity CAPM. The setup of this paper is different, as I use a principal components analysis on the cross-section of liquidity, rather than on different liquidity measures. Also, in contrast with Hagströmer, Anderson, Binner, and Nilsson (2009) and Korajczyk and Sadka (2008), I consider liquidity innovations, rather the level of liquidity. This setup allows for a factor interpretation of the principal components (e.g., Basilevsky, 1994), and it allows me to leverage the Acharya and Pedersen (2005) liquidity CAPM to establish pricing implications.

The liquidity level component that I use is strongly related to the market average liquidity, and therefore to liquidity commonality Chordia, Roll, and Subrahmanyam (e.g., 2000). This liquidity commonality is one of the liquidity risks shown by Acharya and Pedersen (2005) to be priced in the cross-section of stocks. There are many recent papers on liquidity commonality. For instance, Rösch and Kaserer (2013) find that liquidity commonality itself varies significantly over time, and that it is higher during market crises. Brockman, Chung, and Pérignon (2009) and Lee (2011) show that liquidity commonality occurs internationally, while Mancini, Ranaldo, and Wrampelmeyer (2013) show that it also occurs in the foreign exchange market.

Although it is straightforward that liquidity commonality should be priced, it being a nondiversifiable risk, it is less clear why the liquidity slope component should command a premium. For a possible motivation, consider a fund manager facing a large outflow. A potential strategy is to sell liquid assets first to minimize transactions costs (Duffie and Ziegler, 2003; Scholes, 2000). Empirical evidence suggests that this is indeed the strategy that is followed by various institutional investors (Ben-David, Franzoni, and Moussawi, 2012; Manconi, Massa, and Yasuda, 2012; Xing, 2016). However, after initially selling most of the liquid assets, this may force the fund to sell illiquid assets at great cost after a large negative return. In this way, the strategy increases tail losses and the probability of insolvency (Duffie and Ziegler, 2003).

To facilitate the interpretation of the principal components of liquidity, I relate them to a number determinants of liquidity. In the literature, the effect of inventory turnover rates, inventory risks, and frictions such as margin requirements on liquidity is well-documented (Demsetz, 1968; Ho and Stoll, 1981; Stoll, 1978). For instance, high turnover should lead to high liquidity, as it implies lower inventory risk for market makers. A low default spread will lead to high liquidity for the same reason. The market return should be negatively associated with liquidity, especially when declines occur, as these declines indicate increased future volatility (Brunnermeier and Pedersen, 2009; Comerton-Forde, Hendershott, Jones, Moulton, and Seasholes, 2010; Kyle and Xiong, 2001), and thus lead to higher inventory risk. A higher cost of margin trading will lead to lower liquidity, as it increases the cost of liquidity provision.

An increase in volatility of market returns should lead to an increase in illiquidity due to increased adverse selection and inventory risk (Comerton-Forde, Hendershott, Jones, Moulton, and Seasholes, 2010; Stoll, 1978). Empirically, the market liquidity of high-volatility stocks is more sensitive to inventory shocks (Comerton-Forde, Hendershott, Jones, Moulton, and Seasholes, 2010). This result arises because high-volatility stocks are less liquid (Chordia, Sarkar, and Subrahmanyam, 2005; Ho and Stoll, 1983), and funding liquidity deterioration hits volatile, illiquid stocks hardest (Brunnermeier and Pedersen, 2009).

Changes in the commercial paper spread are positively related to capital constraints on the funding market. When these capital constraints become more binding, the negative impact of a market decline on liquidity will be amplified (Gatev and Strahan, 2006; Hameed, Kang, and Viswanathan, 2010; Krishnamurthy, 2002). Chordia, Roll, and Subrahmanyam (2001) suggest that a high term spread will lead to lower stock market liquidity, as investors could move their wealth from equity into longer-term Treasury bonds in response to the high spread. As Baker and Stein (2004) argue that positive investor sentiment leads to higher liquidity, I also consider the Baker and Wurgler (2006) sentiment measure.

3.3 Identifying and understanding the level and slope of liquidity

In this section, I describe the data that are used, and the construction of the variables that I include in my regressions.

3.3.1 Data

The portfolio return and illiquidity data are constructed following Acharya and Pedersen (2005). I use CRSP daily stock return and volume data from 1964 until 2013 for all common shares listed on NYSE and AMEX. Due to the inclusion of interdealer trades and the shorter history, I exclude Nasdaq. The returns are adjusted for stock delisting following Shumway (1997). In addition to these stock-level data, I use data on the commercial paper spread, the default spread, and the term spread, obtained from the Federal Reserve Bank of St. Louis, and the Baker and Wurgler (2006) sentiment measure.

Illiquidity is measured by the Amihud (2002) illiquidity measure, which is defined as

$$ILLIQ_{i,t} = \frac{1}{Days_{i,t}} \sum_{d=1}^{Days_{i,t}} \frac{\left|R_{i,t,d}\right|}{Vol_{i,t,d}} \tag{3.1}$$

for stock i in month t, where $Days_{i,t}$ denotes the number of observations available for stock i in month t, and $R_{i,t,d}$ and $Vol_{i,t,d}$ denote the trading volume in millions of dollars for stock i on day d in month t, respectively.

To deal with non-stationarity and for comparability with returns, I follow Acharya and Pedersen (2005) and use a normalized illiquidity measure

$$c_{i,t} = \min\left\{0.25 + 0.30 ILLIQ_{i,t} P^m_{t-1}, 30.00\right\}, \tag{3.2}$$

where P^m_{t-1} is equal to the market capitalization of the market portfolio at the end of month $t-1$ divided by the value at the end of July 1962. The coefficients 0.30 and 0.25 are such that the series for size-sorted portfolios matches approximately the level and variance of the effective half spread reported by Chalmers and Kadlec (1998). Outliers are removed by setting the maximum value of $c_{i,t}$ to 30% (Acharya and Pedersen, 2005).

Using this measure, I construct 25 illiquidity-sorted portfolios, following Acharya and Pedersen (2005). The portfolios are formed annually, and include stocks for which the price on the first trading day of the formation month is between \$5 and \$1000, and which have at least 100 observations of illiquidity during that month. For the market portfolio, the same criteria apply, but only 15 observations of illiquidity during the formation month are required. The illiquidity portfolios are value-weighted, while the market portfolio is equally-weighted. The time-series of illiquidity at the portfolio level is given in Figure 3.2, and the corresponding time-series for the market portfolio is shown in Figure 3.1.

Table 3.1 and Table 3.2 show descriptive statistics at the portfolio and the market level, respectively. Table 3.1 shows that, on average, less liquid portfolios command a higher risk premium, have more volatile returns, higher liquidity risk, and lower percentage turnover. These facts are consistent with Acharya and Pedersen (2005). Table 3.2 shows that the time-series of market liquidity is highly persistent. The first order autocorrelation is equal to 89%.

3.3.2 Principal Components of Liquidity

To disentangle the liquidity level and slope effects, I use a principal components analsis. For a factor model interpretation of the components, we need the dependent variable to be i.i.d. (see, e.g., Basilevsky, 1994). Due to the persistence of c_t^i (see Table 3.2), I follow Acharya and Pedersen (2005) and use an AR(2) specification to obtain illiquidity innovations that should be roughly i.i.d. I run the PCA on these illiquidity innovations. As the variance of the illiquidity innovations varies substantially across portfolios, I base the PCA on the correlation matrix. This ensures that the principal components are not biased towards the least liquid portfolios, which have the most volatile innovations. To estimate the principal components and the loadings, I use the full sample period. In the appendix to this chapter, I analyze the robustness to using a rolling estimation window.

Following Acharya and Pedersen (2005), I start by computing un-normalized liquidity, truncated for outliers, as

$$\overline{ILLIQ}_{i,t} = \sum_{j=1}^{N_i} w_{j,i,t} \min\left\{ ILLIQ_{i,t},\ \frac{30.00 - 0.25}{0.30 P_{t-1}^m} \right\}, \tag{3.3}$$

where N_i is the number of stocks in portfolio i, and $w_{j,i,t}$ denotes the portfolio weight. The AR(2) model takes the form

$$\begin{aligned}\left(0.25+0.30\overline{ILLIQ}_t^m P_{t-1}^m\right) = a_0 + a_1\left(0.25+0.30\overline{ILLIQ}_{t-1}^m P_{t-1}^m\right) \\ + a_2\left(\left(0.25+0.30\overline{ILLIQ}_{t-2}^m P_{t-1}^m\right)\right) \\ + u_t.\end{aligned} \tag{3.4}$$

We take the residual u, interpret it as the illiquidity innovation according to

$$c_t^i - \mathbb{E}_{t-1}\left[c_t^i\right] = u_t, \tag{3.5}$$

and use it as the input for the principal components analysis. The residual is shown for the 25 liquidity-sorted portfolios in Figure 3.3. The principal components analysis yields a factor decomposition of the form

$$c_t^i - \mathbb{E}_{t-1}\left[c_t^i\right] = \gamma_{LL,i}F_{LL,t} + \gamma_{LS,i}F_{LS,t} + \varepsilon_{i,t}. \tag{3.6}$$

I use this decomposition, including the loadings (factor exposures) for the subsequent analysis of pricing effects.

The loadings obtained from the Principal Components Analysis on the time-series of illiquidity for the 25 liquidity-sorted portfolios, and the illiquidity of the equal-weighted market portfolio are given in Table 3.3. The first principal component, which explains about 57% of the variation, has positive loadings for all portfolios, capturing the level of liquidity innovations. The second principal component, which explains about 9% of the variation, has negative loadings on the most liquid portfolios and positive loadings on the least liquid portfolios, capturing the slope of liquidity innovations. The third and fourth principal component respectively explain 4% and 3% of the variation. Using the scree plot in Figure 3.4, we find that the first two principal components are indeed the pertinent ones. The percentages of the variation in illiquidity explained by the components already show an interesting finding. It seems that the impact of level effects on market liquidity is much stronger than that of slope effects.

To validate the interpretation of the first two principal components of liquidity innovations, Table 3.4 presents the correlations between the components, innovations of market liquidity, and the cross-sectional dispersion of liquidity innovations. The table shows that, while both components are correlated with market liquidity innovations, the level component shows a much stronger correlation. Only the second component is significantly correlated with liquidity dispersion. This suggests that the level component represents market liquidity commonality (liquidity co-movement), while the slope component captures the extent to which liquidity varies in the cross-section at a certain point in time.

3.3.3 Determinants of the Level and Slope of Liquidity

To economically interpret the first two principal components of liquidity, I consider the determinants of liquidity used by Chordia, Roll, and Subrahmanyam (2001) and Hameed, Kang, and Viswanathan (2010). These include proxies for trading activity, inventory turnover rates, inventory risks, margin requirements, capital constraints on the funding market, and investor sentiment. The aim of this analysis is to establish what the principal components are related to by computing multivariate correlations. Hence I use a regression framework where all variables are contemporaneous, and I am not trying to show causality here.

Following Chordia, Roll, and Subrahmanyam (2001), I include the risk-free rate as a short-term interest rate to proxy for margin requirements, the term spread to measure the relative attractiveness of the bond market, and the default spread to proxy for inventory risk. Based on Hameed, Kang, and Viswanathan (2010), I also consider the volatility of monthly market returns as an inventory risk proxy, market percentage turnover as an inventory risk proxy, the impact of market downturns, the spread in commercial paper to proxy for capital constraints on the funding market, and the Baker and Wurgler (2006) index for investor sentiment.

A possible interpretation of the slope component is that it coincides with the portfolio shift from illiquid to liquid stocks that is documented by Næs, Skjeltorp, and Ødegaard (2011). To investigate this channel, I follow Beber, Driessen, and Tuijp (2012) and classify the liquidity-sorted portfolios into a liquid segment (the 19 most liquid portfolios) and an illiquid segment (the 6 least liquid portfolios). Next, I compute the percentage turnover separately for each segment. Using this decomposition, we may see whether the principal components of liquidity are related to overall changes in turnover or only to turnover in one segment of the market. Hence we should expect the slope component to be positively related to turnover in the liquid segment, but negatively related to turnover in the illiquid segment.

The effect of market turnover on overall illiquidity should be negative, as high turnover should lead to lower inventory risk. As short rates reflect the cost of margin trading, the risk-free rate should be positively associated with overal illiquidity. Chordia, Roll, and Subrahmanyam (2001) suggest that investors could move their wealth from equity into bonds after an increase in longer-term Treasury bond yields, causing increased trading activity, but also increased illiquidity in the stock market, leading to a positive relation with the term spread. An increase in the default spread could increase inventory risk, and hence overal illiquidity. Changes in the commercial paper spread are positively related to capital constraints on the funding market, and hence augment the negative impact of a market decline on liquidity (Gatev and Strahan, 2006; Hameed, Kang, and Viswanathan, 2010; Krishnamurthy, 2002). An increase in volatility of market returns should lead to an increase in illiquidity due to increased adverse selection and inventory risk (Comerton-Forde, Hendershott, Jones, Moulton, and Seasholes, 2010; Stoll, 1978). The market return should also be negatively associated with liquidity, especially when declines occur, which for instance lead to increased future volatility (Brunnermeier and Pedersen, 2009; Comerton-Forde, Hendershott, Jones, Moulton, and Seasholes, 2010; Kyle and Xiong, 2001).

As the principal components are formed according to (3.6), using innovations of illiquidity, I include first differences of most of the determinants in my regressions. This results in

$$\begin{aligned} F_{k,t} = {} & \beta_0 + \beta_1 \mathrm{MKTDOWN}_t + \beta_2 \mathrm{MKTDOWN}_t \times \mathrm{CAP}_t + \beta_3 r_t^m \\ & + \beta_4 \Delta\mathrm{DEF}_t + \beta_5 \Delta\mathrm{CPSPREAD}_t + \beta_6 \Delta\sigma_t^m + \beta_7 \Delta\mathrm{TRN}_t^{\mathrm{liq}} \\ & + \beta_8 \Delta\mathrm{TRN}_t^{\mathrm{illiq}} + \beta_9 \Delta\mathrm{TERM}_t + \beta_{10} \Delta r_t^f + \beta_{11} R_{trn,t}^2 \\ & + \beta_{12} \Delta\mathrm{SENT}_t^{\perp} + \varepsilon_t, \end{aligned} \tag{3.7}$$

where $k = LL, LS$. In this equation, MKTDOWN represents the market return when it is negative and zero otherwise, CAP is a dummy variable that measures capital constraints on the funding market and equals 1 when the non-financial commercial paper spread has increased and 0 otherwise, DEF denotes the default spread, CPSPREAD the non-financial commercial paper spread, σ_t^m the market volatility measured as the monthly standard deviation of daily market returns, $\mathrm{TRN}^{\mathrm{liq}}$ is the percentage turnover in the in the liquid segment of the market (portfolios 1 through 19), $\mathrm{TRN}_t^{\mathrm{illiq}}$ is the turnover in the illiquid segment of the market (portfolios 20 through 25), TERM denotes the term spread, r_t^f the risk-free rate, R_{trn}^2 denotes the Karolyi, Lee, and van Dijk (2012) turnover commonality measure, and $\mathrm{SENT}^{\perp}$ denotes the orthogonalized Baker and Wurgler (2006) sentiment index.[2] All variables are standardized, so that the regression coefficients indicate

[2] The orthogonalization follows Baker and Wurgler (2006) and is with respect to several macroeconomic conditions. The reason to orthogonalize the sentiment measure is to reduce the likelihood that the sentiment index is connected to systematic risk.

by how many standard deviations each component changes given a one standard deviation change in the respective variables.

Table 3.5 presents estimation results for regressions (3.7). As expected, the market return has a negative impact on liquidity level component. The market return also has a negative impact on the slope component, but only when there are capital constraints on the funding market (measured through the commerical paper spread). The commercial paper spread itself has a positive impact on the level component, indicating a more direct impact of funding risk. The level component increases with changes in the the risk-free rate, indicating that margin requirements have a positive impact on the level component. The fact that the default spread is not significant for the level component, but does have a positive impact on the slope component, could be explained by the results of Comerton-Forde, Hendershott, Jones, Moulton, and Seasholes (2010), who find that inventory risk affects high volatility stocks much more strongly than low volatility stocks. As high volatility stocks are also the least liquid ones, a slope effect seems natural.

The percentage turnover for the liquid portfolios has a positive effect on the slope component. There is a negative effect for the turnover of the illiquid portfolios. This could possibly indicate trading shifting to the liquid part of the market (as in Næs, Skjeltorp, and Ødegaard, 2011) when the slope component is active. Only the level component is associated with turnover commonality. Lower turnover commonality leads to a decrease in the level of liquidity. The slope component is decreasing in the Baker and Wurgler (2006) index for investor sentiment, implying that illiquid assets will become even less liquid when sentiment decreases. These results are in line with the findings of Karolyi, Lee, and van Dijk (2012), and with Baker and Stein (2004) who argue that positive sentiment leads to higher liquidity.

Comparing the results for both components, we see that the level component reacts more strongly to market returns. The level component is also associated with increases in the risk-free rate. These are related to adverse selection, inventory risk, and margin requirements. The slope component is associated with market returns in times of capital constraints on the funding market, increases in the default spread and percentage turnover for liquid portfolios, and decreases in the Baker and Wurgler (2006) index. These are related to inventory risk, the amount of trading in the liquid segment of the market, and investor sentiment.

If we look at the plots of the standardized first two principal component series, given in Figure 3.5, we can focus on a number of liquidity-related crises, such as those mentioned by Pástor and Stambaugh (2003). We see that the level component displayes the largest shocks during the 1970 domestic unrest, Silver Thursday (March 1980) and the 1987 crash. That the level component was active during the 1987 crash is consistent with evidence that market making collapsed at during that year's stock market crisis (Brady, Cotting, Kirby, Opel, and Stein (1988) in Hameed, Kang, and Viswanathan, 2010). The slope component, was most active during the 1990 crisis, the LTCM crisis (September 1998), and the dotcom bubble burst (May 2000). During the Russian default (August 1998) and the 2008 subprime crisis, both channels experienced large shocks.

3.4 Asset Pricing Tests

In this section I set out the asset pricing tests. The approach consists of two stages. First, I obtain static pricing results through an Acharya and Pedersen (2005) approach. Second, I estimate time-varying factor exposure to the two liquidity components and investigate time-varying risk premia.

3.4.1 Static Pricing of the Level and Slope of Liquidity

To determine the pricing implications for the level and the slope of liquidity, we consider the model by Acharya and Pedersen (2005),

$$\mathbb{E}_t\left[r^i_{t+1}\right] - r^f = \kappa\mathbb{E}_t\left[c^i_{t+1}\right] + \lambda_t \frac{\operatorname{Cov}_t\left(r^i_{t+1} - c^i_{t+1}, r^m_{t+1} - c^m_{t+1}\right)}{\operatorname{Var}_t\left(r^m_{t+1} - c^m_{t+1}\right)}. \tag{3.8}$$

They obtain an unconditional specification given by

$$\mathbb{E}\left[r^i_{t+1}\right] - r^f = \kappa\mathbb{E}\left[c^i_{t+1}\right] + \lambda\beta^{1i} + \lambda\beta^{2i} - \lambda\beta^{3i} - \lambda\beta^{4i}, \tag{3.9}$$

where

$$\beta^{1i} = \frac{\operatorname{Cov}\left(r^i_{t+1}, r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right]\right)}{\operatorname{Var}\left(r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right] - \left(c^m_{t+1} - \mathbb{E}_t\left[c^m_{t+1}\right]\right)\right)}, \tag{3.10}$$

$$\beta^{2i} = \frac{\operatorname{Cov}\left(c^i_{t+1} - \mathbb{E}_t\left[c^i_{t+1}\right], c^m_{t+1} - \mathbb{E}_t\left[c^m_{t+1}\right]\right)}{\operatorname{Var}\left(r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right] - \left(c^m_{t+1} - \mathbb{E}_t\left[c^m_{t+1}\right]\right)\right)}, \tag{3.11}$$

$$\beta^{3i} = \frac{\operatorname{Cov}\left(r^i_{t+1}, c^m_{t+1} - \mathbb{E}_t\left[c^m_{t+1}\right]\right)}{\operatorname{Var}\left(r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right] - \left(c^m_{t+1} - \mathbb{E}_t\left[c^m_{t+1}\right]\right)\right)}, \tag{3.12}$$

$$\beta^{4i} = \frac{\operatorname{Cov}\left(c^i_{t+1} - \mathbb{E}_t\left[c^i_{t+1}\right], r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right]\right)}{\operatorname{Var}\left(r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right] - \left(c^m_{t+1} - \mathbb{E}_t\left[c^m_{t+1}\right]\right)\right)}, \tag{3.13}$$

and the market price of risk is given by

$$\lambda = \mathbb{E}\left[r^i_{t+1} - r^f - c^m_{t+1}\right]. \tag{3.14}$$

In this setup, the three liquidity betas β^{2i}, β^{3i}, and β^{4i} have natural interpretations. Acharya and Pedersen (2005) argue that β^{2i} reflects commonality in liquidity, β^{3i} is high for securities that are desirable because they have high returns when the market is illiquid (see also Pástor and Stambaugh, 2003), and β^{4i} is high for securities that are desirable because they are liquid when the market return is low.

Taking this model to the data, Acharya and Pedersen (2005) find that the return premium for β^{4i} is highest.

Using the factor exposure decomposition from (3.6), we can reformulate the Acharya and Pedersen (2005) beta structure as in the following proposition (the proof is given in Appendix 3.A.1).

PROPOSITION 4: *The factor decomposition given in* (3.6) *allows us to rewrite* (3.9) *as*

$$\mathbb{E}\left[r^i_{t+1}\right] - r^f = \kappa\mathbb{E}\left[c^i_{t+1}\right] + \lambda\beta^{Ri} + \lambda\beta^{LLi} + \lambda\beta^{LSi} + \lambda\beta^{LRi}, \tag{3.15}$$

with

$$\beta^{Ri} = \frac{\mathrm{Cov}\left(r^i_{t+1}, r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right]\right)}{\mathrm{Var}\left(r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right] - \left(c^m_{t+1} - \mathbb{E}_t\left[c^m_{t+1}\right]\right)\right)}, \tag{3.16}$$

$$\beta^{LLi} = \frac{\mathrm{Cov}\left(\gamma_{LL,i}F_{LL,t}, \gamma_{LL,m}F_{LL,t}\right) - \mathrm{Cov}\left(r^i_{t+1}, \gamma_{LL,m}F_{LL,t}\right) - \mathrm{Cov}\left(\gamma_{LL,i}F_{LL,t}, r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right]\right)}{\mathrm{Var}\left(r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right] - \left(c^m_{t+1} - \mathbb{E}_t\left[c^m_{t+1}\right]\right)\right)}, \tag{3.17}$$

$$\beta^{LSi} = \frac{\mathrm{Cov}\left(\gamma_{LS,i}F_{LS,t}, \gamma_{LS,m}F_{LS,t}\right) - \mathrm{Cov}\left(r^i_{t+1}, \gamma_{LS,m}F_{LS,t}\right) - \mathrm{Cov}\left(\gamma_{LS,i}F_{LS,t}, r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right]\right)}{\mathrm{Var}\left(r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right] - \left(c^m_{t+1} - \mathbb{E}_t\left[c^m_{t+1}\right]\right)\right)}. \tag{3.18}$$

$$\beta^{LRi} = \frac{\mathrm{Cov}\left(\varepsilon_{i,t}, \varepsilon_{m,t}\right) - \mathrm{Cov}\left(r^i_{t+1}, \varepsilon_{m,t}\right) - \mathrm{Cov}\left(\varepsilon_{i,t}, r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right]\right)}{\mathrm{Var}\left(r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right] - \left(c^m_{t+1} - \mathbb{E}_t\left[c^m_{t+1}\right]\right)\right)}. \tag{3.19}$$

The four betas have natural interpretations as sensitivities to the market return β^{Ri}, the level component of liquidity β^{LLi}, the slope component of liquidity β^{LSi}, and the other components of liquidity β^{LRi}. For each of the three liquidity betas in Proposition 4, we see that they are combinations of the three Acharya and Pedersen (2005) liquidity betas, using the principal component exposures rather than illiquidity c^i_{t+1} itself. Hence, the interpretation of the components of the betas follows from that of the Acharya and Pedersen (2005) betas.

For the level component beta β^{LLi}, the first part reflects the premium that investors demand for commonality in level shifts of liquidity, the second part reflects the premium that investors are willing to pay for a security that provides high returns when level shifts make the market less liquid, and the third part reflects the premium that investors are willing to pay for a security that is liquid due to level shift exposure when market returns are low. The interpretations for the slope component beta β^{LSi} and the residual component beta β^{LRi} are similar.

Not only does this reformulation allow us to look into the pricing consequences of level shifts and slope changes, it also results in betas that should be not as strongly correlated. As a result, we are able to obtain better identification of the level and slope effects, while retaining a clear economic interpretation. In addition, this allows us to check whether the residual component beta indeed is not priced, as we would expect from the PCA scree plot. As the Acharya and Pedersen (2005) betas cannot be rewritten in terms of the principal component betas, the improved identification for the latter cannot be used to provide inference regarding the original beta structure. For this reason, it would be interesting to estimate the model for all parts of the PCA betas separately, yielding nine liquidity betas instead of three, with

$$\beta_2^{LLi} = \frac{\mathrm{Cov}\left(\gamma_{LL,i} F_{LL,t}, \gamma_{LL,m} F_{LL,t}\right)}{\mathrm{Var}\left(r_{t+1}^m - \mathbb{E}_t\left[r_{t+1}^m\right] - \left(c_{t+1}^m - \mathbb{E}_t\left[c_{t+1}^m\right]\right)\right)}, \tag{3.20}$$

$$\beta_3^{LLi} = \frac{\mathrm{Cov}\left(r_{t+1}^i, \gamma_{LL,m} F_{LL,t}\right)}{\mathrm{Var}\left(r_{t+1}^m - \mathbb{E}_t\left[r_{t+1}^m\right] - \left(c_{t+1}^m - \mathbb{E}_t\left[c_{t+1}^m\right]\right)\right)}, \tag{3.21}$$

$$\beta_4^{LLi} = \frac{\mathrm{Cov}\left(\gamma_{LL,i} F_{LL,t}, r_{t+1}^m - \mathbb{E}_t\left[r_{t+1}^m\right]\right)}{\mathrm{Var}\left(r_{t+1}^m - \mathbb{E}_t\left[r_{t+1}^m\right] - \left(c_{t+1}^m - \mathbb{E}_t\left[c_{t+1}^m\right]\right)\right)}, \tag{3.22}$$

and similarly for the other liquidity betas. The numbering $2,3,4$ corresponds to the related Acharya and Pedersen (2005) liquidity betas. In the appendix to this chapter, I show that we cannot identify all nine parts separately in the static setup, as for $k = LL, LS$, it holds that $\mathrm{Cov}\left(\gamma_{k,i} F_{k,t}, \gamma_{k,m} F_{k,t}\right)$ and $\mathrm{Cov}\left(\gamma_{k,i} F_{k,t}, r_{t+1}^m - \mathbb{E}_t\left[r_{t+1}^m\right]\right)$ are perfectly correlated in the cross-section by construction. Therefore, I include β_2^{LLi} and β_4^{LLi} as they appear in the full level beta: $\beta_2^{LLi} - \beta_4^{LLi}$, and similarly for the slope beta.

Every regression is run with κ free, and also with its value fixed to $\kappa = 0.034$ to improve estimation efficiency following Acharya and Pedersen (2005). The κ should roughly equal the average monthly turnover in the sample, to proxy for the inverse of the holding period.[3] In the sample used by Acharya and Pedersen (2005) the average monthly turnover is 0.034. In my sample, it is 0.0827. In the appendix to this chapter I provide results where I fix $\kappa = 0.0827$. Another way to improve efficiency, is to impose that the market price of risk λ equals the observed average market risk premium net of transaction costs, as given in (3.14). I provide the results for this case in the appendix to this chapter.

To compute the economic impact of each term, we use the λ coefficient estimate obtained from (3.9) or (3.15) and fix κ at 0.034 for reasons of efficiency, following Acharya and Pedersen (2005). The annualized economic impact of the liquidity level term is computed as $\kappa\left(\mathbb{E}_t\left[c^{25}_{t+1}\right] - \mathbb{E}_t\left[c^{1}_{t+1}\right]\right) \cdot 12$, and similarly for the other terms.

To check the stability of the results, I run a subsample analysis. The analysis is done for the first half of the sample (January 1964 until December 1988) and the second half of the sample (January 1989 until December 2013). In addition, I run the analysis without the most liquid portfolio, and without the least liquid portfolio, to see whether these extreme portfolios influence the results.

3.4.2 Static Pricing of the Level and Slope of Liquidity Results

The variables that appear in (3.15) are given in Table 3.6. Most notably, the level beta is increasing in illiquidity, while the slope beta is smaller for the very liquid assets, and roughly similar in size across the other assets.

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[3]The authors argue that when the estimation period is κ times the typical investor's holding period, the risk premium and the betas will scale with κ, but the $\mathbb{E}\left[c^{i}_{t+1}\right]$ term will not. Hence, they scale the latter by κ to correct for this.

Compared to the Acharya and Pedersen (2005) betas, the level-slope formulation yields less multicollinearity. See Table 3.7 for a comparison. Where the correlation between β^{2i} and β^{4i} is equal to -0.9957, the strongest correlation for the principal component betas is 0.8270. The reduced correlation allows for improved identification of the pricing impact of different aspects of liquidity, although we still cannot identify the individual impact of the original betas. What we can see from Table 3.7, however, is that the level beta is most strongly related to β^{2i} (commonality in liquidity) and β^{4i} (liquid when the market return is low), while the slope beta is most strongly related to β^{3i} (high returns when the market is illiquid).

Table 3.8 gives the estimation results for (3.15). The pricing analysis reveals that statistically, of the liquidity betas only the level component is significant. In terms of the economic impact only the liquidity level channel has a significant contribution of about 1.5% p.a. The total effect of liquidity risk found by Acharya and Pedersen (2005) is equal to 1.1% per annum. Hence, this result indicates that the largest part of the economic contribution of liquidity risk to the risk premium is driven through the level channel. This is confirmed by the economic contributions of the slope channel and the other principal components, which are equal to 0.01% p.a. and 0.18% p.a., respectively.

Looking at Table 3.8, we see in the full regression that the impact of the level component is driven by $\beta_2^{LLi} - \beta_4^{LLi}$, which represents the level aspect of commonality in liquidity and being liquid when the market return is low. For the slope component, the corresponding term is the only significant one, even though the slope component had no effect in the results given in Table 3.8. In addition, the level of liquidity $\mathbb{E}\left[c_{t+1}^i\right]$ also has a significant impact, in line with Acharya and Pedersen (2005).

The subsample results, given in the appendix to this chapter, show that the results are absent for the first half of the sample (January 1964 until December 1988), but are strongly present during the second half of the sample (January 1989 until December 2013). Also, the betas vary substantially between the two halves of the sample. This motivates the time-varying beta analysis. The appendix to this chapter shows that the results do still hold for the analysis without the most liquid portfolio, and the analysis without the least liquid portfolio.

3.4.3 Time-varying Pricing of the Level and Slope of Liquidity

Motivated by the subsample analysis, I consider time-varying versions of (3.15). I start by running a constant coefficient time-varying beta regression of the form

$$\begin{aligned} r_t^i - r^f - \kappa \mathbb{E}_{t-1}\left[c_t^i\right] = \lambda_0 + \lambda_R \beta_{R,i,t-1} + \lambda_{LL} \beta_{LL,i,t-1} + \lambda_{LS} \beta_{LS,i,t-1} \\ + \lambda_{LR} \beta_{LR,i,t-1} + \varepsilon_t. \end{aligned} \tag{3.23}$$

To obtain $\beta_{R,i,t}$, $\beta_{LL,i,t}$, and $\beta_{LS,i,t}$, I compute rolling estimates of (3.16)–(3.18) with a time window of 120 months. For the time-periods where insufficient observations are available, I require that there are at least 60 observations to compute the rolling estimates. The factor premium estimates and the corresponding standard errors are obtained using the Fama and MacBeth (1973) procedure.

For the rolling estimates of the betas, I use time-varying factor exposures to ensure proper conditioning. These time-varying factor exposures result from rolling window estimates for the conditional factor model

$$c_t^i - \mathbb{E}_{t-1}\left[c_t^i\right] = \gamma_{LL,i,t} F_{LL,t} + \gamma_{LS,i,t} F_{LS,t} + \varepsilon_{i,t}, \tag{3.24}$$

where $F_{LL,t}$ denotes the liquidity level factor and $F_{LS,t}$ the liquidity slope factor, both obtained from the PCA on illiquidity innovations. The window size is set to 120 months for all portfolios. As for the betas, I require that there are at least 60 observations to compute the rolling estimates if insufficient observations are available.

As the variance of the liquidity innovations differs considerably across the portfolios, it is not immediately obvious that the optimal time window size should be equal across the different portfolios. In the appendix to this chapter, I use the method of Ang and Kristensen (2012) to estimate the optimal window size through a kernel-based method. It turns out that the optimal bandwidth is roughly 120 observations for all portfolios. This validates the window size chosen for the rolling window estimates.

To further investigate how the pricing of liquidity varies over time, I estimate time-varying risk premia using the cross-sectional regression

$$
\begin{aligned}
r_t^i - r^f - \kappa \mathbb{E}_{t-1}\left[c_t^i\right] = {} & \lambda_{0,t-1} + \lambda_{R,t-1}\beta_{R,i,t-1} + \lambda_{LL,t-1}\beta_{LL,i,t-1} \\
& + \lambda_{LS,t-1}\beta_{LS,i,t-1} + \lambda_{LR,t-1}\beta_{LR,i,t-1} + \varepsilon_t
\end{aligned} \tag{3.25}
$$

for each period t. This specification is motivated by Vayanos (2004), who shows that liquidity premia increase during more volatile times, and by Watanabe and Watanabe (2008), who show that the price of liquidity risk varies over time, even when taking into account a time-varying liquidity beta.

For the time-varying versions, I also use the different setups for κ and λ as discussed in the static analysis. Similarly, I also consider the nine different parts of the liquidity betas. In this setting, there are no inherent multicollinearity issues, but β_2^{LLi} and β_4^{LLi} are still nearly perfectly correlated in the data. Therefore I include them as $\beta_2^{LLi} - \beta_4^{LLi}$.

3.4.4 Time-varying Pricing of the Level and Slope of Liquidity Results

The time-varying factor exposure of the illiquidity innovations for the 25 liquidity-sorted portfolios to the level and slope component is given in Figure 3.7. Both effects have remained relatively stable over time, although there is substantial variation in exposure for the least liquid portfolio. The rolling window estimates of the level and slope beta are shown in Figure 3.8. As for the exposure of the illiquidity innovations to the principal components, we see that the betas are relatively stable for the liquid portfolios, while they vary more strongly for the least liquid portfolio.

When we keep the prices of risk fixed and use the Fama and MacBeth (1973) procedure, we find the results given in Table 3.10. The level beta is significant in the specification with all the variables and κ free, while the slope beta is significant across all specifications. Looking at the results for the decomposition of the betas in Table 3.11, we see that β_3^{LLi} is significant across a number of specifications and has the correct sign, indicating that investors value having high returns when the market is illiquid. For the slope component, we see no significant terms in the decomposition, contrasting with the results in Table 3.10. The estimation results for (3.25), where the prices of risk are time-varying, are shown in Figure 3.9. This results in the liquidity risk premia given in Figure 3.10 (level beta) and Figure 3.11 (slope beta).

3.5 Conclusions

This paper identifies two liquidity components and analyzes their pricing implications. A principal components analysis of 25 liquidity-sorted portfolios of U.S. stocks over the period 1964 to 2013, shows that 66% of the variation of illiquidity can be explained by the first two principal components. The loadings on the first factor, which explains about 57%, are consistent with a level component interpretation. The loadings on the second component, which explains about 9%, show that it is a slope component. Although both components are correlated with market liquidity innovations, the correlation is much stronger for the level component. The slope component is related to liquidity dispersion, while the level component is not.

To investigate the economic meaning of the principal components, I run regressions on various determinants of liquidity and look at which component was active during various liquidity crises. The level component, which reflects adverse selection, inventory risk, and margin requirements, has the largest shocks during the 1970 domestic unrest, Silver Thursday (March 1980) and the 1987 crash. The slope component, which reflects inventory risk, trading in the liquid segment of the market, and investor sentiment, experienced large shocks during the 1990 crisis, the LTCM crisis (September 1998), and the dotcom bubble burst (May 2000). During the Russian default (August 1998) and the 2008 subprime crisis, both components experienced large shocks.

The pricing analysis shows that only the liquidity level beta is statistically significant in a liquidity CAPM. Additionally, only the liquidity level beta has a significant economic impact of about 1.5% p.a. This is roughly equal to the total economic impact of liquidity risk in our setting, and to the total economic impact found by Acharya and Pedersen (2005). Hence, it seems that only the level of market liquidity is relevant to risk premia, while the slope has no significant effect. In addition, the betas appear to vary over time, which motivates the analysis of time-varying betas and risk premia. It appears, however, that liquidity betas are relatively stable over time. Not surprisingly, the time-varying beta analysis produces results that are similar to the static analysis, although it does provide more support for a slope effect.

Combining the results above, we see that the liquidity level component captures most of the variation in liquidity, and is the only part of liquidity risk with a meaningful economic impact. The liquidity slope component does explain part of the time-variation in liquidity, but is not statistically significant in a pricing context, and does not carry a sizeable economic impact. This has implications for management of liquidity crises. In the case of a slope effect, where liquidity only deteriorates for part of the market, there will be no price impact beyond what is implied by the drop in liquidity of the individual securities. For a level effect, however, there will be an additional impact through the level beta. A possible economic motivation is that institutions that sell liquid assets first when under stress to meet capital requirements (see Duffie and Ziegler, 2003, for a discussion of strategies) will not be strongly affected by the slope effect, but will certainly be harmed by the level effect.

3.A Derivations Factor Pricing

3.A.1 Main result

Proof of Proposition 4: We use the factor exposure decomposition given in (3.6) and the orthogonality of the factors to obtain

$$\begin{aligned}
&\operatorname{Cov}\left(c^i_{t+1}-\mathbb{E}_t\left[c^i_{t+1}\right],c^m_{t+1}-\mathbb{E}_t\left[c^m_{t+1}\right]\right) \\
&=\operatorname{Cov}\left(\gamma_{LL,i}F_{LL,t}+\gamma_{LS,i}F_{LS,t}+\varepsilon_{i,t},\gamma_{LL,m}F_{LL,t}+\gamma_{LS,m}F_{LS,t}+\varepsilon_{m,t}\right) \\
&=\operatorname{Cov}\left(\gamma_{LL,i}F_{LL,t},\gamma_{LL,m}F_{LL,t}\right)+\operatorname{Cov}\left(\gamma_{LS,i}F_{LS,t},\gamma_{LS,m}F_{LS,t}\right)+\operatorname{Cov}\left(\varepsilon_{i,t},\varepsilon_{m,t}\right).
\end{aligned}\tag{3.26}$$

This gives

$$\beta^{2i}=\frac{\operatorname{Cov}\left(\gamma_{LL,i}F_{LL,t},\gamma_{LL,m}F_{LL,t}\right)+\operatorname{Cov}\left(\gamma_{LS,i}F_{LS,t},\gamma_{LS,m}F_{LS,t}\right)+\operatorname{Cov}\left(\varepsilon_{i,t},\varepsilon_{m,t}\right)}{\operatorname{Var}\left(r^m_{t+1}-\mathbb{E}_t\left[r^m_{t+1}\right]-\left(c^m_{t+1}-\mathbb{E}_t\left[c^m_{t+1}\right]\right)\right)}.\tag{3.27}$$

Similarly, we find

$$\beta^{3i}=\frac{\operatorname{Cov}\left(r^i_{t+1},\gamma_{LL,m}F_{LL,t}\right)+\operatorname{Cov}\left(r^i_{t+1},\gamma_{LS,m}F_{LS,t}\right)+\operatorname{Cov}\left(r^i_{t+1},\varepsilon_{m,t}\right)}{\operatorname{Var}\left(r^m_{t+1}-\mathbb{E}_t\left[r^m_{t+1}\right]-\left(c^m_{t+1}-\mathbb{E}_t\left[c^m_{t+1}\right]\right)\right)},\tag{3.28}$$

and

$$\begin{aligned}
\beta^{4i}=&\frac{\operatorname{Cov}\left(\gamma_{LL,i}F_{LL,t},r^m_{t+1}-\mathbb{E}_t\left[r^m_{t+1}\right]\right)+\operatorname{Cov}\left(\gamma_{LS,i}F_{LS,t},r^m_{t+1}-\mathbb{E}_t\left[r^m_{t+1}\right]\right)}{\operatorname{Var}\left(r^m_{t+1}-\mathbb{E}_t\left[r^m_{t+1}\right]-\left(c^m_{t+1}-\mathbb{E}_t\left[c^m_{t+1}\right]\right)\right)} \\
&+\frac{\operatorname{Cov}\left(\varepsilon_{i,t},r^m_{t+1}-\mathbb{E}_t\left[r^m_{t+1}\right]\right)}{\operatorname{Var}\left(r^m_{t+1}-\mathbb{E}_t\left[r^m_{t+1}\right]-\left(c^m_{t+1}-\mathbb{E}_t\left[c^m_{t+1}\right]\right)\right)},
\end{aligned}\tag{3.29}$$

Collecting the commonality and dispersion effects, we obtain a new beta structure that allows us to disentangle the risk premia for the commonality and dispersion components:

$$\mathbb{E}\left[r^i_{t+1}\right]-r^f=\kappa\mathbb{E}\left[c^i_{t+1}\right]+\lambda\beta^{Ri}+\lambda\beta^{LLi}+\lambda\beta^{LSi}+\lambda\beta^{LRi},\tag{3.30}$$

with

$$\beta^{Ri} = \frac{\mathrm{Cov}\left(r^i_{t+1}, r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right]\right)}{\mathrm{Var}\left(r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right] - \left(c^m_{t+1} - \mathbb{E}_t\left[c^m_{t+1}\right]\right)\right)}, \tag{3.31}$$

$$\beta^{LLi} = \frac{\mathrm{Cov}\left(\gamma_{LL,i}F_{LL,t}, \gamma_{LL,m}F_{LL,t}\right) - \mathrm{Cov}\left(r^i_{t+1}, \gamma_{LL,m}F_{LL,t}\right) - \mathrm{Cov}\left(\gamma_{LL,i}F_{LL,t}, r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right]\right)}{\mathrm{Var}\left(r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right] - \left(c^m_{t+1} - \mathbb{E}_t\left[c^m_{t+1}\right]\right)\right)}, \tag{3.32}$$

$$\beta^{LSi} = \frac{\mathrm{Cov}\left(\gamma_{LS,i}F_{LS,t}, \gamma_{LS,m}F_{LS,t}\right) - \mathrm{Cov}\left(r^i_{t+1}, \gamma_{LS,m}F_{LS,t}\right) - \mathrm{Cov}\left(\gamma_{LS,i}F_{LS,t}, r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right]\right)}{\mathrm{Var}\left(r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right] - \left(c^m_{t+1} - \mathbb{E}_t\left[c^m_{t+1}\right]\right)\right)}. \tag{3.33}$$

$$\beta^{LRi} = \frac{\mathrm{Cov}\left(\varepsilon_{i,t}, \varepsilon_{m,t}\right) - \mathrm{Cov}\left(r^i_{t+1}, \varepsilon_{m,t}\right) - \mathrm{Cov}\left(\varepsilon_{i,t}, r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right]\right)}{\mathrm{Var}\left(r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right] - \left(c^m_{t+1} - \mathbb{E}_t\left[c^m_{t+1}\right]\right)\right)}. \tag{3.34}$$

Q.E.D.

Table 3.1. Descriptive statistics (portfolio level)

This table shows descriptive statistics for the data used to estimate the model. The CRSP data used are monthly data corresponding to 25 value-weighted US stock portfolios sorted on illiquidity with sample period 1964–2013. The average excess return $\mathbb{E}[R_{t+1}] - R_f$, standard deviation of returns $\sigma(R_{t+1})$, standard deviation of returns net of costs $\sigma(R_{t+1} - c_{t+1})$, and covariance between portfolio and market level returns net of costs $\mathrm{Cov}\left(R_{t+1} - c_{t+1}, R^m_{t+1} - c^m_{t+1}\right)$ are computed from the time-series observations.

Portfolio	$\mathbb{E}[R_{t+1}] - R_f$ (%)	$\mathbb{E}[c_{t+1}]$ (%)	$\sigma(R_{t+1})$ (%)	$\sigma(c_{t+1})$ (%)	$\sigma(R_{t+1} - c_{t+1})$ (%)	$\mathrm{Cov}(\ldots,\ldots)$ $(\cdot 100)$	trn (%)	$\sigma(\mathrm{trn})$
1	0.3264	0.2518	4.2022	0.0017	4.2024	0.1767	5.9158	6.2783
2	0.3755	0.2568	4.6114	0.0060	4.6123	0.2135	6.9554	6.2195
3	0.4354	0.2611	4.6451	0.0093	4.6465	0.2212	6.9501	6.0284
4	0.5600	0.2664	4.7600	0.0133	4.7621	0.2286	7.1681	6.8244
5	0.5953	0.2732	5.0470	0.0192	5.0506	0.2442	6.8267	6.0381
6	0.5495	0.2827	4.8503	0.0261	4.8549	0.2371	6.8506	6.4248
7	0.5571	0.2928	4.7960	0.0329	4.8009	0.2377	6.4540	5.5812
8	0.5431	0.3083	4.9547	0.0470	4.9637	0.2441	6.2103	5.4767
9	0.6055	0.3253	4.9567	0.0608	4.9678	0.2473	6.0450	5.0996
10	0.6309	0.3448	4.8916	0.0699	4.9049	0.2442	5.6554	4.8098
11	0.6734	0.3725	5.1438	0.0941	5.1611	0.2572	5.4089	4.2691
12	0.5630	0.4037	4.8935	0.1072	4.9144	0.2445	4.7951	3.7103
13	0.6529	0.4376	4.8893	0.1240	4.9165	0.2463	4.7662	3.6871
14	0.7400	0.4758	5.0515	0.1529	5.0836	0.2535	4.4090	3.0553
15	0.6506	0.5453	5.0894	0.1981	5.1365	0.2565	4.2004	3.0606
16	0.6331	0.6210	5.0289	0.2298	5.0830	0.2524	4.0327	2.5514
17	0.8010	0.7238	5.0751	0.3034	5.1522	0.2531	4.2676	4.4986
18	0.6807	0.8283	5.0265	0.3259	5.1102	0.2504	4.2420	5.3765
19	0.8664	0.9742	5.1872	0.4031	5.2873	0.2563	3.7374	2.6377
20	0.6789	1.2769	5.3153	0.5771	5.4804	0.2594	3.4632	2.3135
21	0.8253	1.5531	5.4042	0.6395	5.5692	0.2597	2.9857	1.7014
22	0.8381	1.9864	5.3899	0.8188	5.5838	0.2556	3.8557	7.9582
23	0.8183	2.8059	5.4890	1.2157	5.9041	0.2685	3.1721	2.6555
24	0.7511	4.4659	5.5545	2.0282	6.3574	0.2732	2.6859	1.9129
25	0.8712	8.0759	6.0417	4.0085	7.8284	0.2780	2.4010	2.1144

Table 3.2. Descriptive statistics (market level)

This table shows market level descriptives. The CRSP data used are monthly data with sample period 1964–2013. In this table, R^f denotes the risk-free rate, R^m denotes the market return, c^m the market illiquidity, $\sigma(R^m)$ the volatility of market returns, Liquidity Dispersion is the cross-sectional standard deviation of illiquidity, R^2_{liq} and R^2_{trn} respectively denote the Karolyi, Lee, and van Dijk (2012) liquidity and turnover commonality measures, CP Spread is the non-financial commercial paper spread, SENT$^\perp$ denotes the Baker and Wurgler (2006) sentiment index, $F_{LL,t}$ denotes the liquidity level factor, and $F_{LS,t}$ denotes the liquidity slope factor.

	Mean	Stdev	Skewness	Kurtosis	Min	Max	ACF (1st lag)
R^f	0.0045	0.0026	0.6658	3.9211	0.0000	0.0135	0.9585
R^m	0.0113	0.0504	-0.5734	6.7533	-0.2634	0.2354	0.1675
c^m	0.0115	0.0029	1.3385	4.7736	0.0073	0.0229	0.8671
$R^m - c^m$	-0.0002	0.0514	-0.6067	6.7374	-0.2795	0.2235	0.1851
$c^m_t - \mathbb{E}_{t-1}[c^m_t]$	0.0000	0.0014	0.9703	6.6921	-0.0045	0.0088	-0.0790
$\sigma(R^m)$	0.0071	0.0050	4.1588	28.6524	0.0017	0.0469	0.6610
Turnover	0.0827	0.0669	1.9567	7.3358	0.0161	0.4383	0.9571
Term Spread	0.0169	0.0132	-0.5569	2.7849	-0.0265	0.0442	0.9511
Default Spread	0.0111	0.0047	1.6887	6.4478	0.0055	0.0338	0.9629
Liquidity Dispersion	0.0195	0.0078	0.6111	3.1340	0.0045	0.0460	0.8816
R^2_{liq}	0.2466	0.1054	1.8217	7.2781	0.0684	0.7665	0.4481
R^2_{trn}	0.4002	0.1277	0.6026	3.2085	0.1622	0.8062	0.3842
CP Spread	0.5874	0.5213	2.5052	12.9236	0.0400	4.3800	0.8594
ΔSENT$^\perp$	-0.0052	0.9763	0.3093	5.4497	-3.5268	4.3673	-0.0834
$F_{LL,t}$	0.0236	3.7261	0.6882	9.1136	-19.1966	18.6432	0.1916
$F_{LS,t}$	0.1109	1.5016	0.6166	5.4568	-5.9776	7.3365	-0.0321

Table 3.3. Principal Components Analysis results: portfolio loadings

This table shows the loadings on the first and second principal component of illiquidity for 25 liquidity-sorted portfolios. Portfolio 1 is the most liquid, portfolio 25 the least liquid. The CRSP data used are monthly data corresponding to 25 value-weighted US stock portfolios sorted on illiquidity with sample period 1964–2013.

Portfolio	First	Second
1	0.0746	-0.0800
2	0.2172	-0.2476
3	0.2267	-0.2133
4	0.2258	-0.1895
5	0.2332	-0.1943
6	0.2250	-0.1852
7	0.2310	-0.1698
8	0.2191	-0.1532
9	0.2302	-0.1519
10	0.2192	-0.1056
11	0.2288	-0.1104
12	0.2106	-0.0115
13	0.2117	-0.0222
14	0.2218	-0.0312
15	0.2011	0.0942
16	0.2070	0.0930
17	0.2052	0.1090
18	0.1904	0.1510
19	0.1862	0.1834
20	0.1583	0.2328
21	0.1649	0.2833
22	0.1363	0.2873
23	0.1460	0.3372
24	0.1359	0.3143
25	0.1007	0.3502
Mkt	0.1723	0.2015

Table 3.4. Correlations of principal components

This table shows correlations between the first two principal components of liquidity innovations, the first difference of market liquidity, and liquidity dispersion.

	First ($F_{LL,t}$)	Second ($F_{LS,t}$)	$c_t^m - \mathbb{E}_{t-1}[c_t^m]$	LIQDISP$_t$
First ($F_{LL,t}$)	1.0000			
Second ($F_{LS,t}$)	0.0000	1.0000		
$c_t^m - \mathbb{E}_{t-1}[c_t^m]$	0.6611***	0.2974***	1.0000	
LIQDISP$_t$	0.0194	0.2243***	0.0909**	1.0000

Table 3.5. Principal component regressions.

This table shows the estimation results for the regressions of the first (level) and second (slope) principal component of illiquidity innovations on several determinants of liquidity. That is,

$$\begin{aligned} F_{k,t} = {} & \beta_0 + \beta_1 \mathrm{MKTDOWN}_t + \beta_2 \mathrm{MKTDOWN}_t \times \mathrm{CAP}_t + \beta_3 r_t^m + \beta_4 \Delta\mathrm{DEF}_t + \beta_5 \Delta\mathrm{CPSPREAD}_t + \beta_6 \Delta\sigma_t^m \\ & + \beta_7 \Delta\mathrm{TRN}_t^{\mathrm{liq}} + \beta_8 \Delta\mathrm{TRN}_t^{\mathrm{illiq}} + \beta_9 \Delta\mathrm{TERM}_t + \beta_{10} \Delta r_t^f + \beta_{11} R^2_{trn,t} + \beta_{12} \Delta\mathrm{SENT}_t^{\perp} + \varepsilon_t, \end{aligned} \tag{3.35}$$

for $k = LL, LS$. In this equation, MKTDOWN represents the market return when it is negative and zero otherwise, CAP is a dummy variable that measures capital constraints on the funding market, DEF denotes the default spread, CPSPREAD the non-financial commercial paper spread, σ_t^m the market volatility, $\mathrm{TRN}^{\mathrm{liq}}$ is the percentage turnover in the in the liquid segment of the market (portfolios 1 through 19), $\mathrm{TRN}_t^{\mathrm{illiq}}$ is the turnover in the illiquid segment of the market (portfolios 20 through 25), TERM denotes the term spread, r_t^f the risk-free rate, R^2_{trn} denotes the Karolyi, Lee, and van Dijk (2012) turnover commonality measure, and $\mathrm{SENT}^{\perp}$ denotes the orthogonalized Baker and Wurgler (2006) sentiment index. All variables are standardized to facilitate the interpretation. The CRSP data used are monthly data corresponding to 25 value-weighted US stock portfolios sorted on illiquidity with sample period 1964–2013. Standard errors are given in parentheses.

	First ($F_{LL,t}$)	Second ($F_{LS,t}$)
$\mathrm{MKTDOWN}_t$	-0.2721***	0.0916
	(0.0938)	(0.1117)
$\mathrm{MKTDOWN}_t \times \mathrm{CAP}_t$	0.1936**	-0.2681***
	(0.0775)	(0.0922)
r_t^m	-0.4269***	-0.1409
	(0.0718)	(0.0855)
$\Delta\mathrm{DEF}_t$	-0.0049	0.2035***
	(0.0345)	(0.0411)
$\Delta\mathrm{CPSPREAD}_t$	0.1171***	0.0272
	(0.0403)	(0.0480)
$\Delta\sigma_t^m$	0.0717	-0.2633***
	(0.0504)	(0.0600)
$\Delta\mathrm{TRN}_t^{\mathrm{liq}}$	-0.0402	0.2824***
	(0.0634)	(0.0755)
$\Delta\mathrm{TRN}_t^{\mathrm{illiq}}$	-0.0149	-0.2516***
	(0.0609)	(0.0725)
$\Delta\mathrm{TERM}_t$	-0.0078	-0.0262
	(0.0357)	(0.0425)
Δr_t^f	0.1087***	-0.1024**
	(0.0381)	(0.0454)
$R^2_{trn,t}$	-0.0698*	-0.0029
	(0.0411)	(0.0490)
$\Delta\mathrm{SENT}_t^{\perp}$	-0.0361	-0.0957**
	(0.0381)	(0.0453)
Constant	-0.0096	0.0109
	(0.0354)	(0.0421)
R^2	0.4236	0.2020
Observations	437	437

Table 3.6. Regression variables

This table shows the variables used for the regression

$$\mathbb{E}\left[r^i_{t+1}\right] - r^f = \alpha + \kappa \mathbb{E}\left[c^i_{t+1}\right] + \lambda_1 \beta^{Ri} + \lambda_2 \beta^{LLi} + \lambda_3 \beta^{LSi} + \lambda_4 \beta^{LRi}, \tag{3.36}$$

for 25 liquidity-sorted portfolios, where $\beta^{net} = \beta^{1i} + \beta^{2i} - \beta^{3i} - \beta^{4i}$, with β^{ki} as in (3.10)–(3.13). Portfolio 1 is the most liquid, portfolio 25 the least liquid. The CRSP data used are monthly data corresponding to 25 value-weighted US stock portfolios sorted on illiquidity with sample period 1964–2013.

Portfolio	$\mathbb{E}\left[r^i_{t+1}\right] - r^f$ (%)	$\mathbb{E}\left[c^i_{t+1}\right]$ (%)	$\beta^{net,i}$ $(\cdot 100)$	β^{Ri} $(\cdot 100)$	β^{LLi} $(\cdot 100)$	β^{LSi} $(\cdot 100)$	β^{LRi} $(\cdot 100)$
1	0.39	0.25	67.08	66.26	0.67	0.02	0.13
2	0.44	0.26	80.25	79.19	0.83	0.02	0.21
3	0.50	0.26	83.86	82.74	0.87	0.03	0.21
4	0.64	0.27	86.22	85.05	0.92	0.04	0.21
5	0.64	0.27	92.14	90.78	1.03	0.06	0.27
6	0.61	0.28	89.57	88.28	0.99	0.04	0.25
7	0.62	0.29	88.83	87.55	1.05	0.06	0.18
8	0.60	0.30	90.99	89.59	1.14	0.06	0.20
9	0.66	0.32	91.78	90.30	1.19	0.05	0.24
10	0.67	0.34	91.35	89.85	1.19	0.05	0.26
11	0.74	0.36	95.83	94.20	1.35	0.05	0.23
12	0.65	0.39	91.62	90.01	1.33	0.06	0.23
13	0.72	0.42	91.03	89.35	1.44	0.04	0.20
14	0.79	0.46	94.54	92.68	1.60	0.06	0.20
15	0.72	0.53	94.38	92.31	1.76	0.06	0.25
16	0.69	0.60	92.36	90.17	1.86	0.07	0.26
17	0.84	0.69	92.70	90.27	2.18	0.05	0.21
18	0.73	0.79	90.60	88.05	2.20	0.08	0.28
19	0.88	0.93	93.33	90.63	2.50	0.07	0.14
20	0.73	1.21	94.44	91.24	3.13	0.08	-0.01
21	0.79	1.48	94.56	91.14	3.33	0.06	0.04
22	0.88	1.88	91.99	87.92	3.63	0.07	0.36
23	0.87	2.66	96.46	90.86	5.17	0.06	0.37
24	0.81	4.24	95.26	87.62	7.01	0.07	0.56
25	0.91	8.15	95.07	85.22	8.68	0.09	1.08

Table 3.7. Beta correlations

This table shows the correlations between the Acharya and Pedersen (2005) betas (Panel A), as well as the correlation between β^{Ri}, β^{LLi}, β^{LSi}, and β^{LRi} (Panel B) and the correlation across the two types of betas (Panel C). The CRSP data used are monthly data corresponding to 25 value-weighted US stock portfolios sorted on illiquidity with sample period 1964–2013.

Panel A (AP)	β^{1i}	β^{2i}	β^{3i}	β^{4i}
β^{1i}	1.0000			
β^{2i}	0.0196	1.0000		
β^{3i}	-0.8291***	-0.4308**	1.0000	
β^{4i}	-0.0485	-0.9957***	0.4761**	1.0000
Panel B (PCA)	β^{Ri}	β^{LLi}	β^{LSi}	β^{LRi}
β^{Ri}	1.0000			
β^{LLi}	0.1102	1.0000		
β^{LSi}	0.5619***	0.6467***	1.0000	
β^{LRi}	-0.0460	0.7679***	0.4169**	1.0000
Panel C (AP and PCA)	β^{1i}	β^{2i}	β^{3i}	β^{4i}
β^{Ri}	1.0000	0.0196	-0.8291***	-0.0485
β^{LLi}	0.1102	0.9865***	-0.5412***	-0.9956***
β^{LSi}	0.5619***	0.5807***	-0.7649***	-0.6129***
β^{LRi}	-0.0460	0.8342***	-0.2255	-0.8156***

Table 3.8. Pricing regression results.

This table shows the estimation results for

$$\mathbb{E}\left[r_{t+1}^{i}\right]-r^{f}=\alpha+\kappa\mathbb{E}\left[c_{t+1}^{i}\right]+\lambda\beta^{net,i}, \tag{3.37}$$

where κ is restricted to 0.034 and $\beta^{net}=\beta^{1i}+\beta^{2i}-\beta^{3i}-\beta^{4i}$, with β^{ki} as in (3.10)–(3.13). The table also shows estimation results for

$$\mathbb{E}\left[r_{t+1}^{i}\right]-r^{f}=\alpha+\kappa\mathbb{E}\left[c_{t+1}^{i}\right]+\lambda_1\beta^{Ri}+\lambda_2\beta^{LLi}+\lambda_3\beta^{LSi}+\lambda_4\beta^{LRi}, \tag{3.38}$$

with κ both restricted to 0.034 and unrestricted and where β^{Ri}, β^{LLi}, β^{LSi}, and β^{LRi} as in (3.16)–(3.19). The CRSP data used are monthly data corresponding to 25 value-weighted US stock portfolios sorted on illiquidity with sample period 1964–2013. Newey and West (1987) standard errors are given in parentheses.

β^{net}	0.0146***				0.0154***			
	(0.0022)				(0.0026)			
β^{Ri}		0.0151***	0.0133***	0.0126***		0.0128***	0.0106***	0.0106***
		(0.0026)	(0.0031)	(0.0027)		(0.0024)	(0.0027)	(0.0028)
β^{LLi}		0.0132*	0.0071	0.0218**		0.0647**	0.0592**	0.0604
		(0.0069)	(0.0086)	(0.0104)		(0.0267)	(0.0249)	(0.0350)
β^{LSi}			1.2464	1.1504			1.4243	1.4378
			(1.3008)	(1.4064)			(1.2978)	(1.2402)
β^{LRi}				-0.1792				0.0116
				(0.1221)				(0.1941)
$\mathbb{E}\left[c_{t+1}^{i}\right]$	0.034	0.034	0.034	0.034	0.0257***	-0.0272	-0.0289	-0.0315
	(—)	(—)	(—)	(—)	(0.0058)	(0.0257)	(0.0248)	(0.0495)
α	-0.6621***	-0.6980***	-0.5920**	-0.5098**	-0.7221***	-0.5426**	-0.4170**	-0.4152*
	(0.2043)	(0.2349)	(0.2412)	(0.2060)	(0.2384)	(0.2030)	(0.1948)	(0.1999)
R^2	0.6468	0.6562	0.6675	0.7083	0.7634	0.8037	0.8140	0.8140
Observations	25	25	25	25	25	25	25	25

Table 3.9. Pricing regression results for decomposed betas.

This table shows the estimation results for

$$\mathbb{E}\left[r_{t+1}^{i}\right] - r^{f} = \alpha + \kappa\mathbb{E}\left[c_{t+1}^{i}\right] + \lambda\beta^{net,i}, \tag{3.39}$$

where κ is restricted to 0.034 and $\beta^{net} = \beta^{1i} + \beta^{2i} - \beta^{3i} - \beta^{4i}$, with β^{ki} as in (3.10)–(3.13). The table also shows estimation results for

$$\mathbb{E}\left[r_{t+1}^{i}\right] - r^{f} = \alpha + \kappa\mathbb{E}\left[c_{t+1}^{i}\right] + \lambda_1\beta^{Ri} + \lambda_2\beta^{LLi} + \lambda_3\beta^{LSi} + \lambda_4\beta^{LRi}, \tag{3.40}$$

with κ both restricted to 0.034 and unrestricted and where β^{Ri}, β^{LLi}, β^{LSi}, and β^{LRi} as in (3.16)–(3.19). The CRSP data used are monthly data corresponding to 25 value-weighted US stock portfolios sorted on illiquidity with sample period 1964–2013. Newey and West (1987) standard errors are given in parentheses.

β^{net}	0.0146***				0.0154***			
	(0.0022)				(0.0026)			
β^{Ri}		0.0028	0.0069	0.0028		0.0046	0.0097	0.0175**
		(0.0060)	(0.0059)	(0.0066)		(0.0055)	(0.0075)	(0.0062)
$\beta_2^{LLi} - \beta_4^{LLi}$		-0.0074	0.0564	0.0208		0.0353	0.0766	0.1091**
		(0.0117)	(0.0401)	(0.0396)		(0.0287)	(0.0574)	(0.0431)
β_3^{LLi}		-0.8472*	-0.3787	-0.5490		-0.6477	-0.1457	0.5212
		(0.4080)	(0.4673)	(0.4773)		(0.3949)	(0.5766)	(0.4551)
$\beta_2^{LSi} - \beta_4^{LSi}$			59.1359*	33.1134			167.5353	501.5881***
			(32.0279)	(36.4540)			(185.8307)	(146.0623)
β_3^{LSi}			-0.5102	-0.5961			-0.2599	1.4332
			(1.4235)	(1.2447)			(1.5934)	(1.2456)
β_2^{LRi}				-3.6846***				-8.6162***
				(1.2609)				(1.5017)
β_3^{LRi}				-0.4618				-0.2002
				(0.3883)				(0.2685)
β_4^{LRi}				-0.0757				-0.1074
				(0.1138)				(0.1032)
$\mathbb{E}\left[c_{t+1}^{i}\right]$	0.034	0.034	0.034	0.034	0.0257***	-0.0097	0.1365	0.4836***
	(—)	(—)	(—)	(—)	(0.0058)	(0.0239)	(0.1717)	(0.1269)
α	-0.6621***	-0.4142*	-0.3763*	-0.2786	-0.7221***	-0.3835*	-0.4245*	-0.5838***
	(0.2043)	(0.2133)	(0.1899)	(0.2100)	(0.2384)	(0.1938)	(0.2192)	(0.1967)
R^2	0.6468	0.7194	0.7569	0.8218	0.7634	0.8228	0.8351	0.9297
Observations	25	25	25	25	25	25	25	25

Table 3.10. Pricing regression results (rolling beta analysis).

This table shows the Fama and MacBeth (1973) estimation results for

$$r_t^i - r^f = \alpha + \kappa c_{t-1}^i + \lambda \beta_{t-1}^{net,i}, \tag{3.41}$$

where κ is restricted to 0.034 and $\beta_{t-1}^{net} = \beta_{t-1}^{1i} + \beta_{t-1}^{2i} - \beta_{t-1}^{3i} - \beta_{t-1}^{4i}$, with β_{t-1}^{ki} as in (3.10)–(3.13). The table also shows estimation results for

$$r_t^i - r^f = \lambda_0 + \kappa c_{t-1}^i + \lambda_1 \beta_{R,i,t-1} + \lambda_2 \beta_{LL,i,t-1} + \lambda_3 \beta_{LS,i,t-1} + \lambda_4 \beta_{LR,i,t-1} + \varepsilon_t, \tag{3.42}$$

with κ both restricted to 0.034 and unrestricted, and where β^{Ri}, β^{LLi}, β^{LSi}, and β^{LRi} as in (3.16)–(3.19). The betas have been estimated with a rolling time window of 120 months. The CRSP data used are monthly data corresponding to 25 value-weighted US stock portfolios sorted on illiquidity with sample period 1964–2013. Newey and West (1987) standard errors are given in parentheses.

β^{net}	0.0082**				0.0078**			
	(0.0040)				(0.0039)			
β^{Ri}		0.0078**	0.0058	0.0088**		0.0056	0.0035	0.0037
		(0.0038)	(0.0039)	(0.0040)		(0.0038)	(0.0038)	(0.0040)
β^{LLi}		-0.0287	0.0432	0.0023		0.0212	0.1118*	0.1516**
		(0.0326)	(0.0508)	(0.0558)		(0.0482)	(0.0660)	(0.0745)
β^{LSi}			0.3148**	0.3450**			0.5664***	0.5930***
			(0.1367)	(0.1527)			(0.1753)	(0.1878)
β^{LRi}				-0.0234				-0.0279
				(0.0571)				(0.0671)
c_{t-1}^i	0.034	0.034	0.034	0.034	0.0190	-0.0158	-0.1204	-0.2008**
	(—)	(—)	(—)	(—)	(0.0259)	(0.0614)	(0.0769)	(0.0920)
α	0.2643	0.3518	0.4462	0.1637	0.3378	0.4925*	0.5499*	0.4752
	(0.3244)	(0.2963)	(0.2920)	(0.3032)	(0.3077)	(0.2985)	(0.2842)	(0.3041)
R^2	0.1382	0.2635	0.3293	0.3914	0.2530	0.3301	0.3908	0.4503
Observations	13,500	13,500	13,500	13,500	13,500	13,500	13,500	13,500

Table 3.11. Pricing regression results for decomposed betas (rolling beta analysis).

This table shows the Fama and MacBeth (1973) estimation results for

$$r_t^i - r^f = \alpha + \kappa c_{t-1}^i + \lambda \beta_{t-1}^{net,i}, \tag{3.43}$$

where κ is restricted to 0.034 and $\beta_{t-1}^{net} = \beta_{t-1}^{1i} + \beta_{t-1}^{2i} - \beta_{t-1}^{3i} - \beta_{t-1}^{4i}$, with β_{t-1}^{ki} as in (3.10)–(3.13). The table also shows estimation results for

$$r_t^i - r^f = \lambda_0 + \kappa c_{t-1}^i + \lambda_1 \beta_{R,i,t-1} + \lambda_2 \beta_{LL,i,t-1} + \lambda_3 \beta_{LS,i,t-1} + \lambda_4 \beta_{LR,i,t-1} + \varepsilon_t, \tag{3.44}$$

with κ both restricted to 0.034 and unrestricted, and where β^{Ri}, β^{LLi}, β^{LSi}, and β^{LRi} as in (3.16)–(3.19). The betas have been estimated with a rolling time window of 120 months. The CRSP data used are monthly data corresponding to 25 value-weighted US stock portfolios sorted on illiquidity with sample period 1964–2013. Newey and West (1987) standard errors are given in parentheses.

β^{net}	0.0082**				0.0078**			
	(0.0040)				(0.0039)			
β^{Ri}		0.0027	0.0053	-0.0081		0.0025	0.0023	-0.0092
		(0.0061)	(0.0067)	(0.0070)		(0.0062)	(0.0066)	(0.0072)
$\beta_2^{LLi} - \beta_4^{LLi}$		-0.0311	-0.0123	-0.0667		0.0042	0.1130	0.2475**
		(0.0424)	(0.0649)	(0.0895)		(0.0567)	(0.0854)	(0.1219)
β_3^{LLi}		-0.6088	-0.8370*	-1.6100***		-0.5475	-0.9379**	-1.2224**
		(0.4136)	(0.4939)	(0.5494)		(0.4049)	(0.4693)	(0.5415)
β_2^{LSi}			-638.9801	1000.4931			-697.7494	1180.0652
			(984.7313)	(646.1290)			(968.2381)	(901.5105)
β_3^{LSi}			-3.3394	-2.9894			-2.2176	-0.5787
			(2.1240)	(2.1683)			(2.1243)	(2.1826)
β_4^{LSi}			5.2283	-6.7267			6.3501	-6.9160
			(6.9178)	(4.8732)			(6.9107)	(6.7775)
β_2^{LRi}				-5.8393				-4.9727
				(6.3010)				(7.5155)
β_3^{LRi}				-0.6608				-0.5042
				(0.4336)				(0.4273)
β_4^{LRi}				-0.0881				-0.0801
				(0.0743)				(0.1075)
c_{t-1}^i	0.034	0.034	0.034	0.034	0.0190	-0.0171	-0.2676**	-0.4688**
	(—)	(—)	(—)	(—)	(0.0259)	(0.0625)	(0.1343)	(0.1951)
α	0.2643	0.2722	0.5538*	0.4210	0.3378	0.4241	0.5459*	0.4753
	(0.3244)	(0.2962)	(0.2947)	(0.3328)	(0.3077)	(0.2963)	(0.2928)	(0.3362)
R^2	0.1382	0.3254	0.4858	0.6222	0.2530	0.3876	0.5356	0.6626
Observations	13,500	13,500	13,500	13,500	13,500	13,500	13,500	13,500

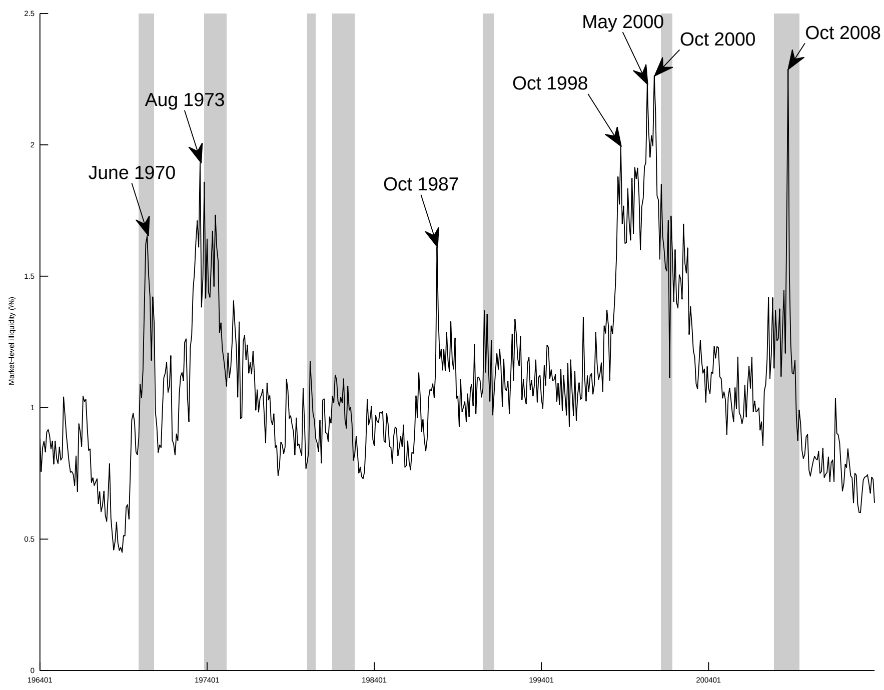

Figure 3.1. Market illiquidity. This graph shows the time-series of market-level illiquidity.

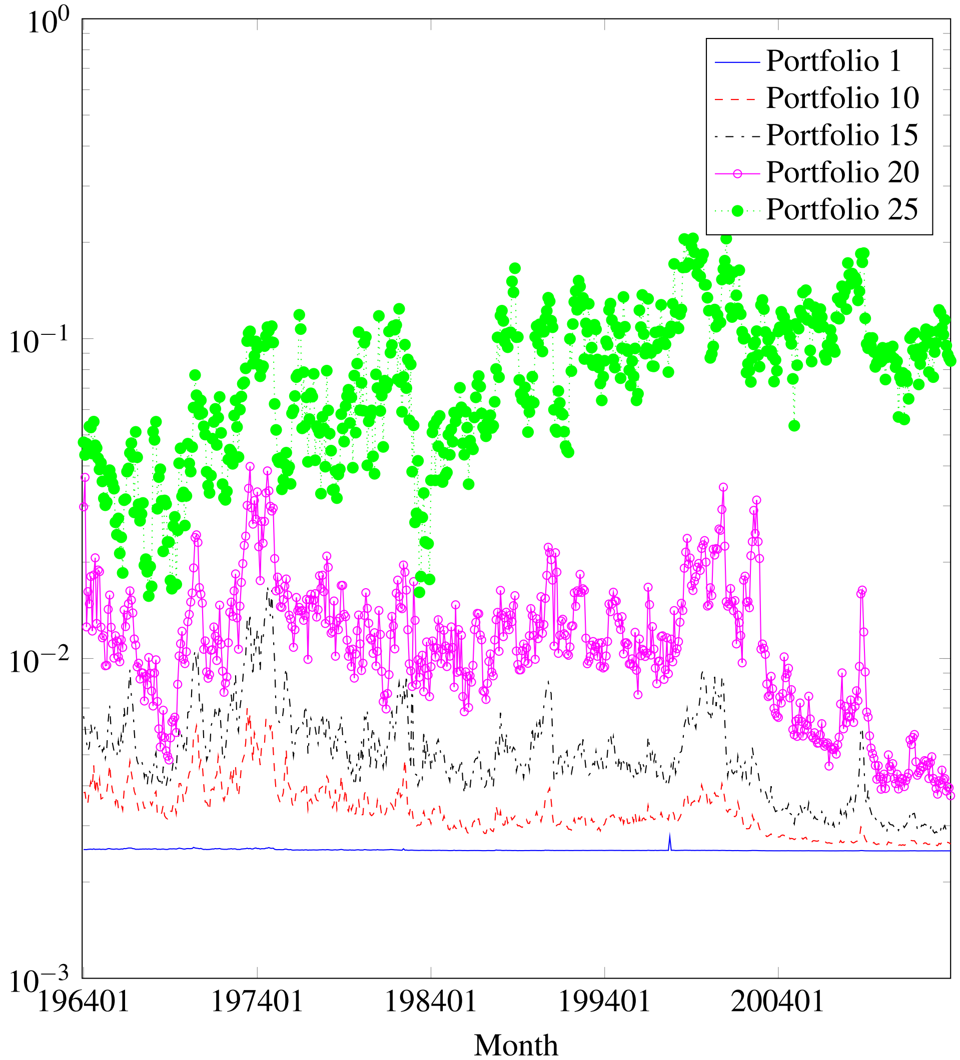

Figure 3.2. Portfolio illiquidity. This graph shows the time-series of illiquidity for four of the 25 liquidity-sorted portfolios, with the first portfolio being the most liquid and the 25th portfolio being the least liquid.

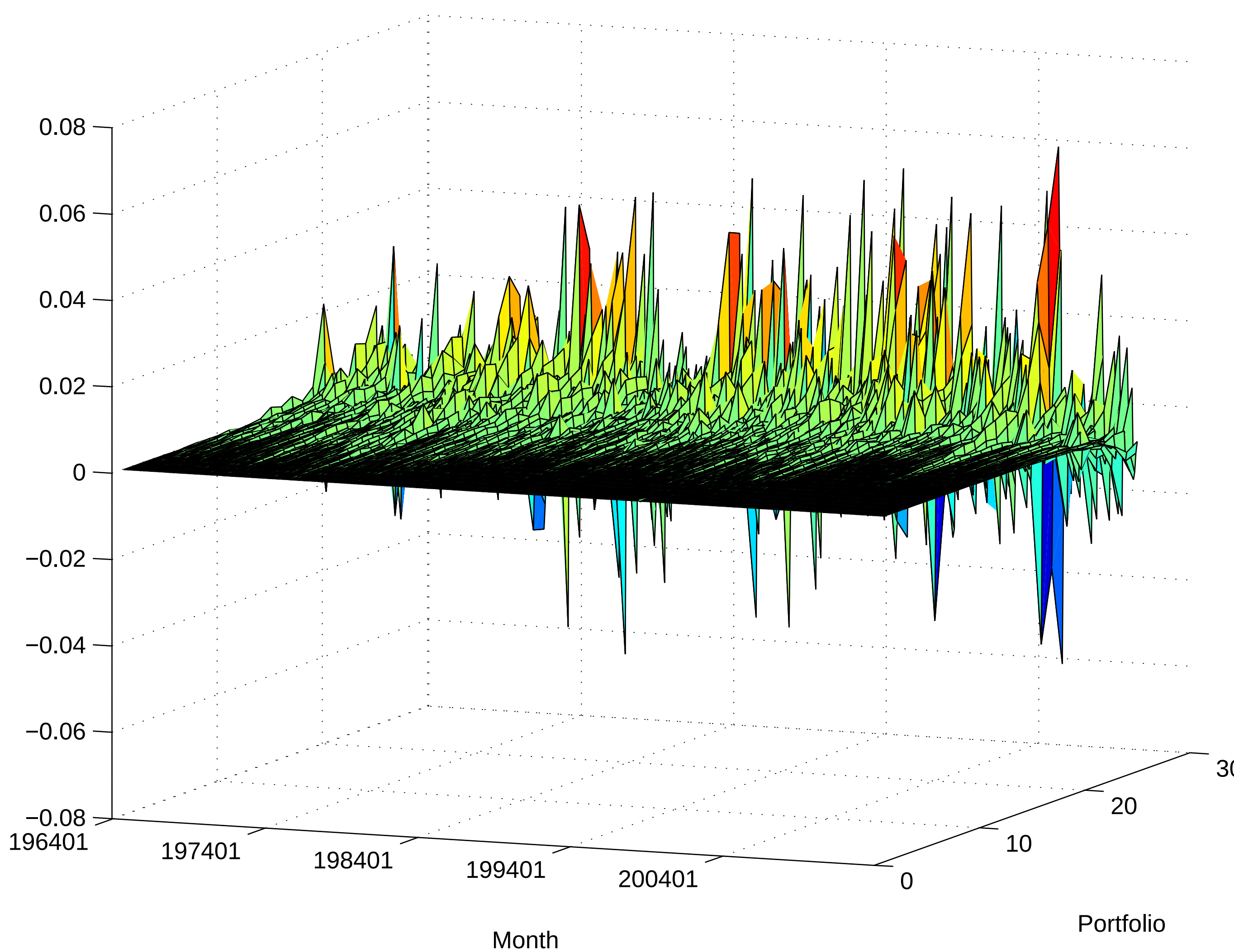

Figure 3.3. Portfolio illiquidity innovations. This graph shows the time-series of illiquidity innovations (AR(2) residuals) for 25 liquidity-sorted portfolios, with the first portfolio being the most liquid and the 25th portfolio being the least liquid.

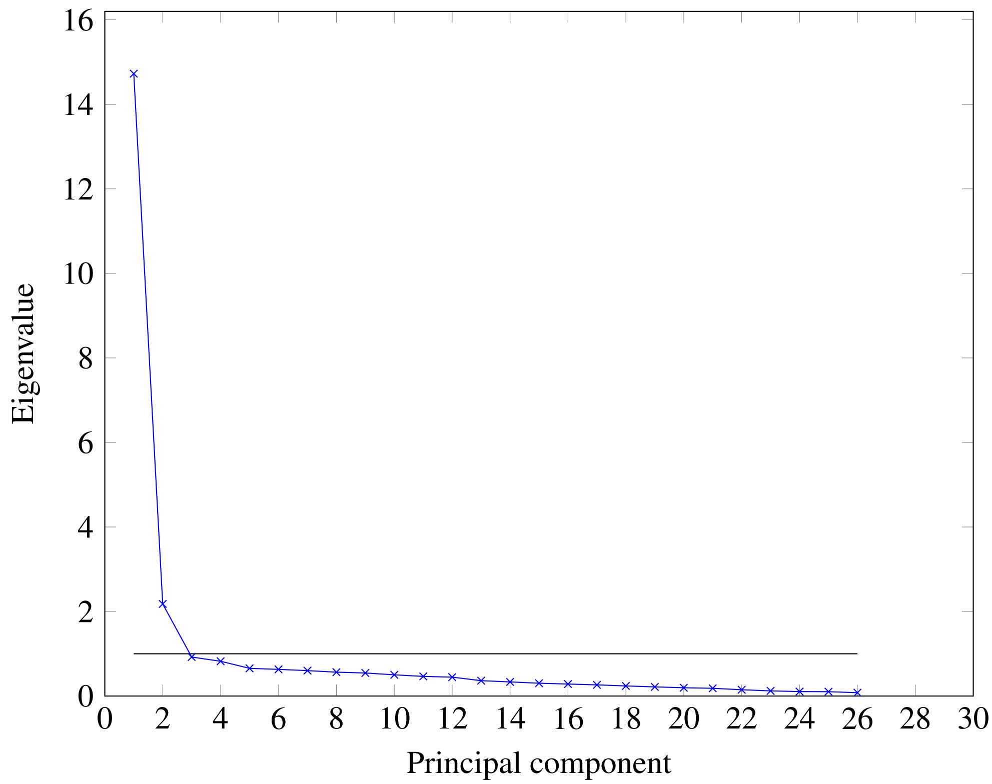

Figure 3.4. Scree plot for the principal components of illiquidity innovations. This graph shows the eigenvalues corresponding to the principal components of the illiquidity innovations.

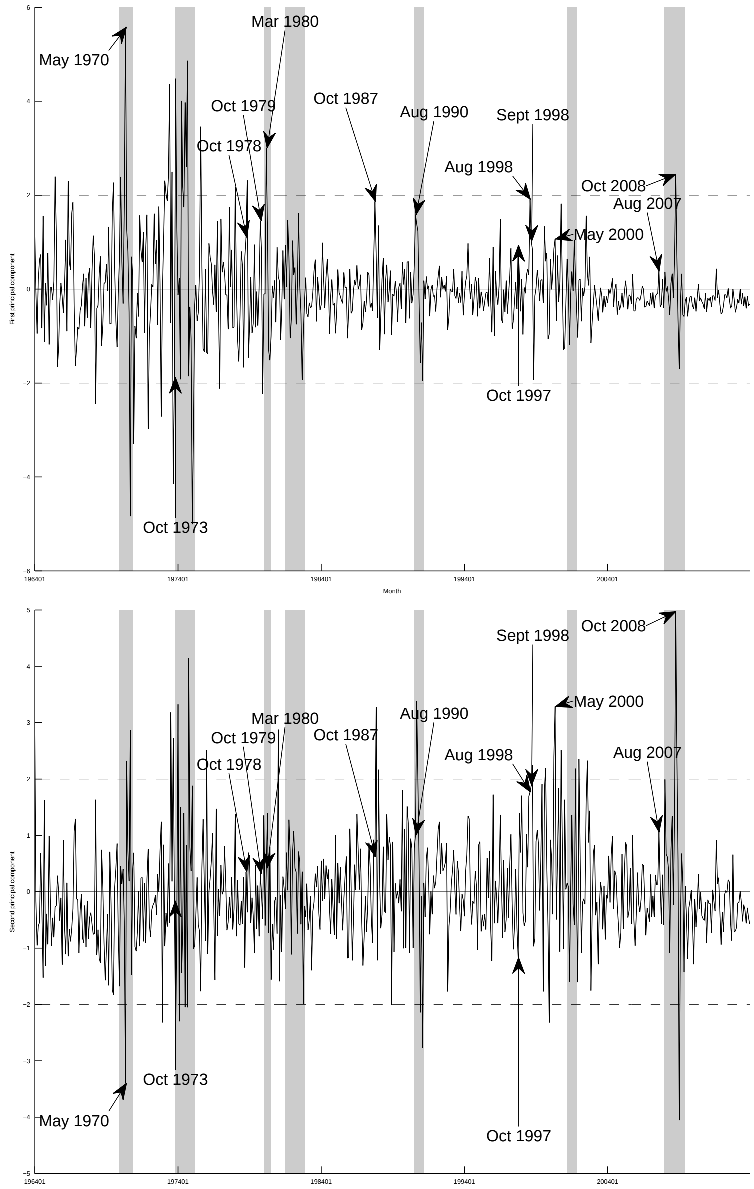

Figure 3.5. Principal components of market illiquidity. The top panel shows the first principal component of market illiquidity (liquidity commonality effect). The bottom panel shows the second principal component of market illiquidity (liquidity dispersion effect). Both principal components are standardized, and the two-standard-deviation bound is indicated for each series.

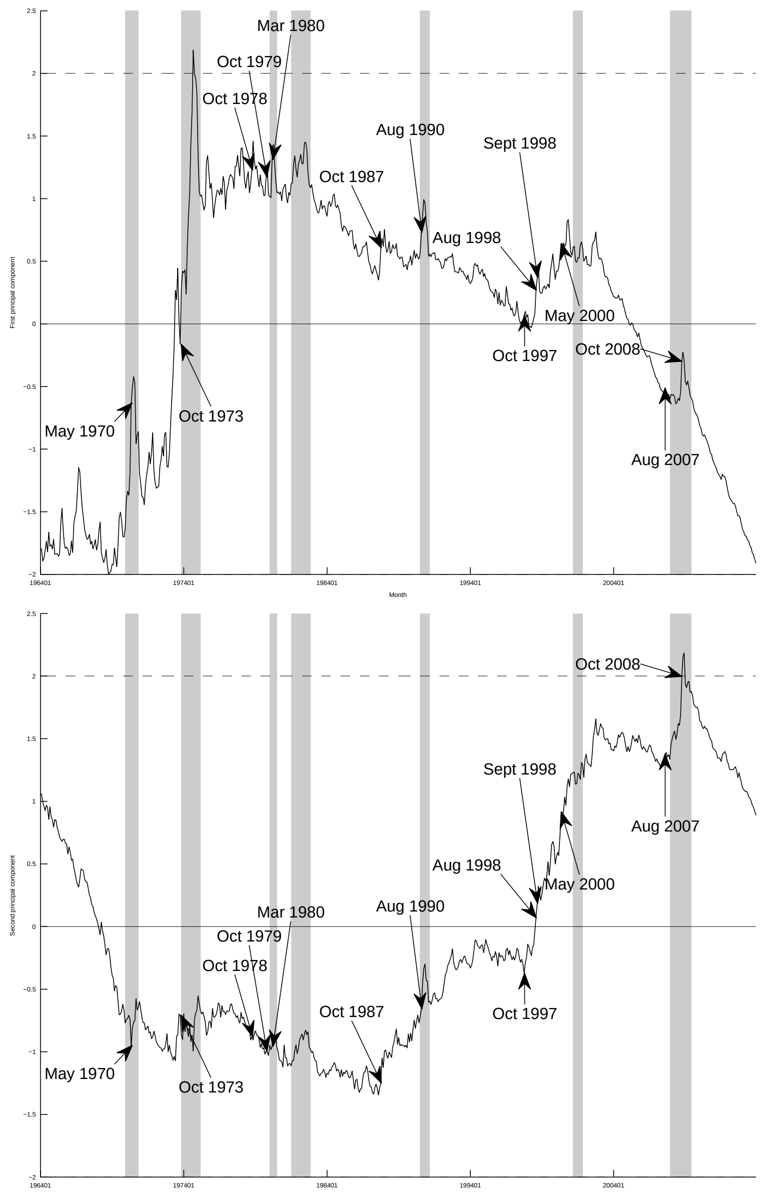

Figure 3.6. Cumulative principal components of market illiquidity. The top panel shows the first principal component of market illiquidity (liquidity commonality effect). The bottom panel shows the second principal component of market illiquidity (liquidity dispersion effect). Both principal components are standardized and cumulated over time to show the impact on the level of liquidity, rather than on the innovations. The two-standard-deviation bound is indicated for each series.

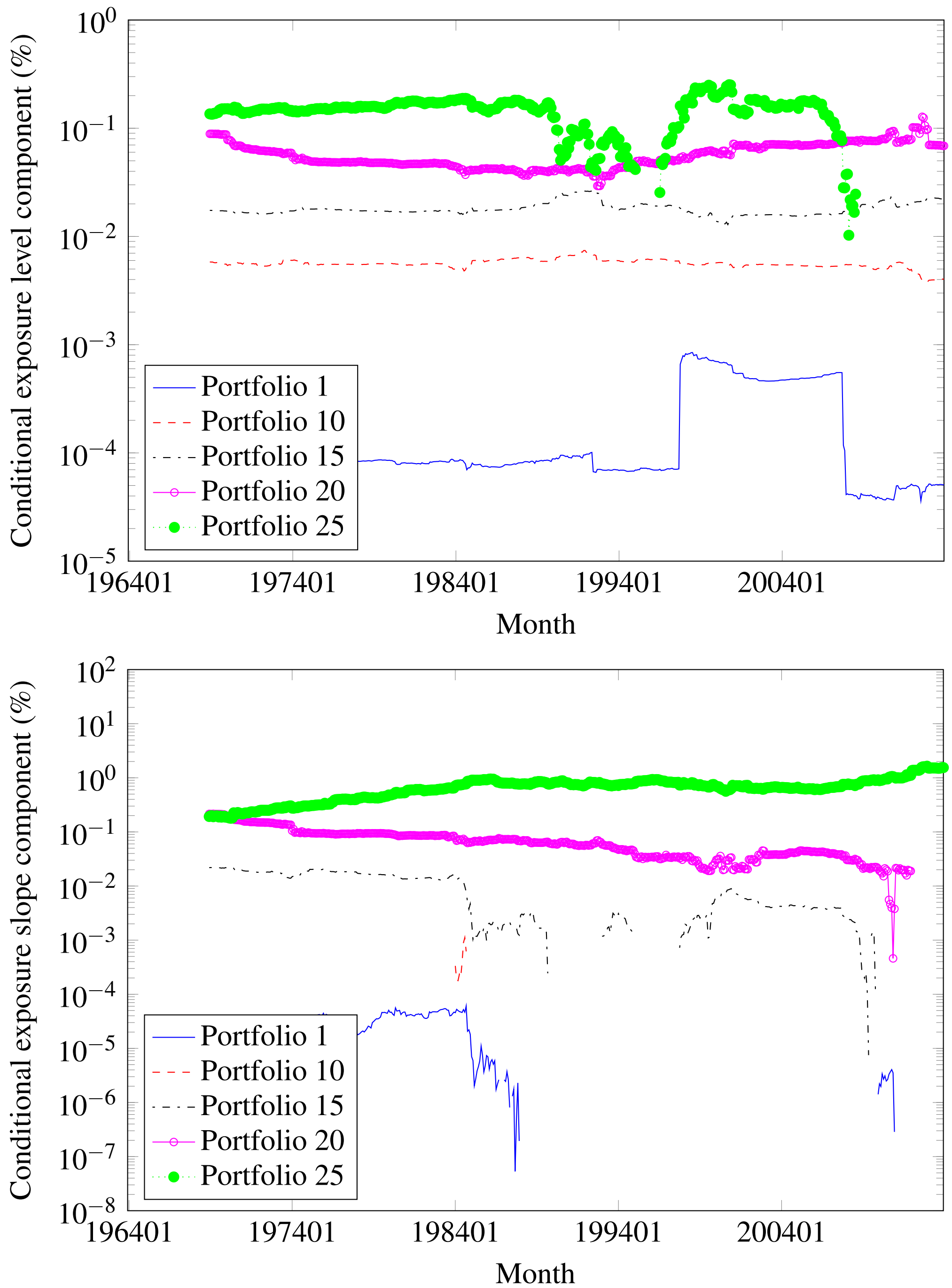

Figure 3.7. Conditional exposure of illiquidity to the liquidity level and slope component. This graph shows the conditional exposure to the liquidity level (top panel) and slope (bottom panel) component for four of the 25 liquidity-sorted portfolios, with the first portfolio being the most liquid and the 25th portfolio being the least liquid.

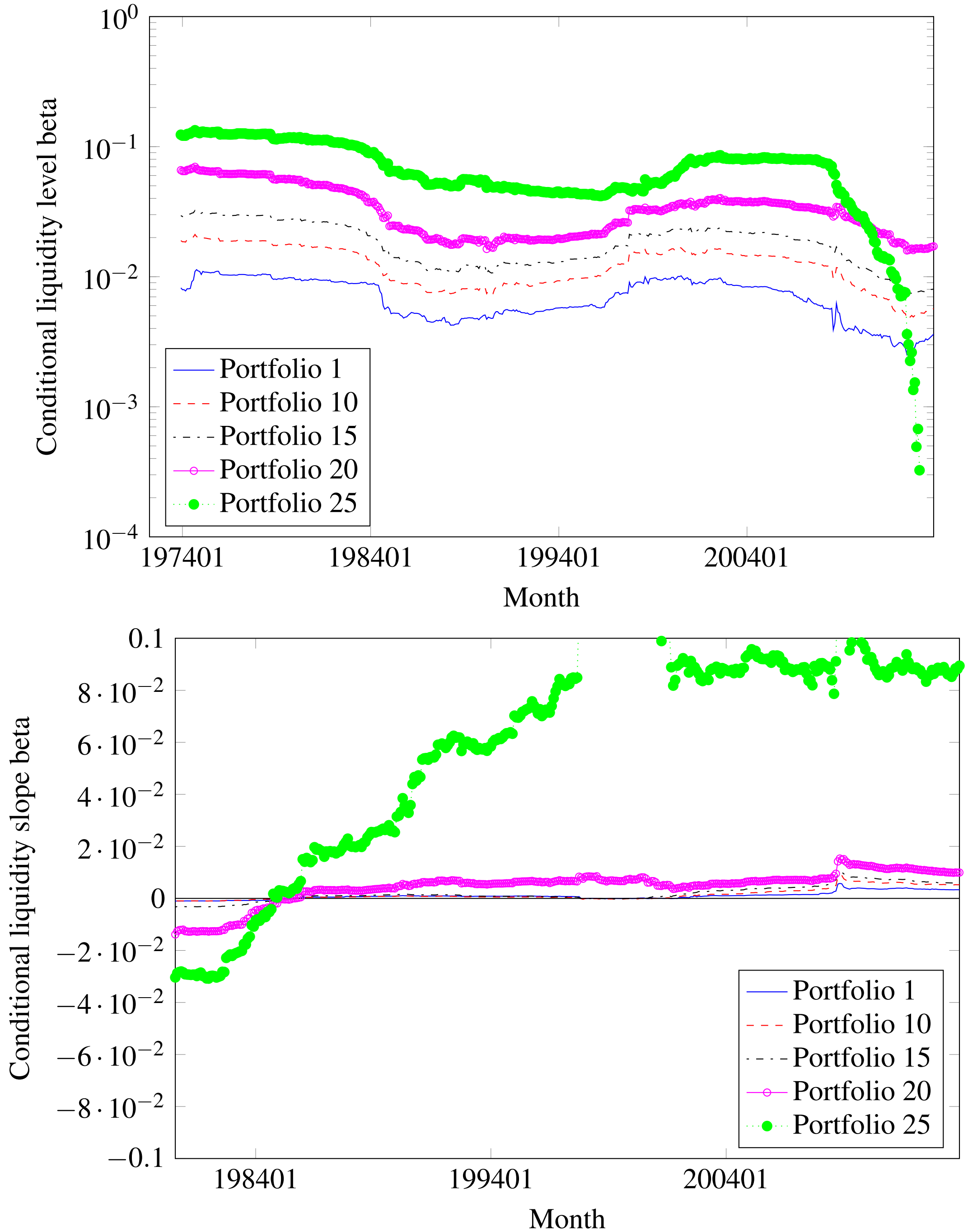

Figure 3.8. Conditional beta for the liquidity level and slope component. This graph shows the conditional beta for the liquidity level (top panel) and slope (bottom panel) component for four of the 25 liquidity-sorted portfolios. The first portfolio is the most liquid and the 25th portfolio is the least liquid.

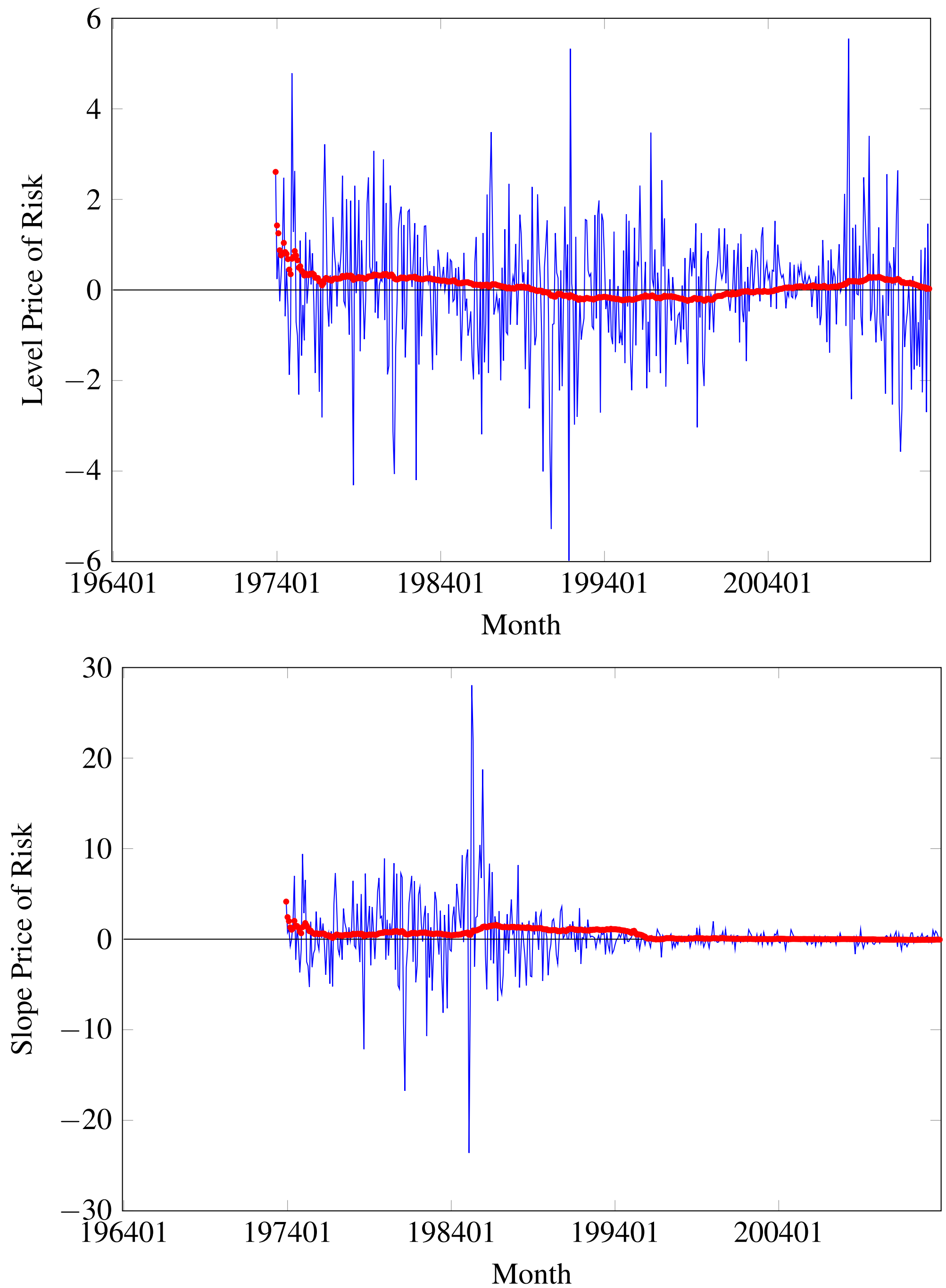

Figure 3.9. Time-varying prices of risk. The time-varying market prices of risk pertaining to the rolling beta estimates of the first (top panel) and second (bottom panel) principal component. The dotted line indicates the 120-month moving average.

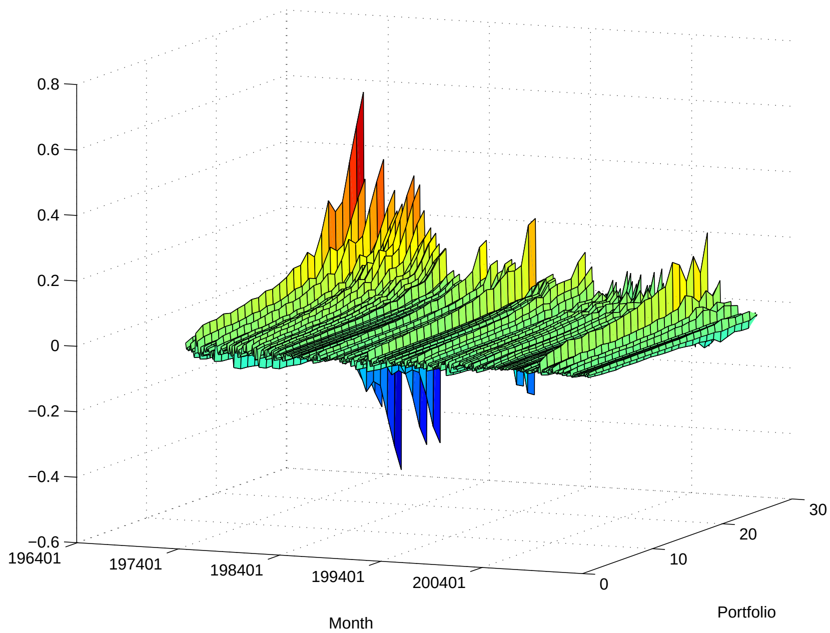

Figure 3.10. Conditional risk premium to the liquidity level component. This graph shows the conditional risk premium to the liquidity level component for 25 liquidity-sorted portfolios. The first portfolio is the most liquid and the 25th portfolio is the least liquid.

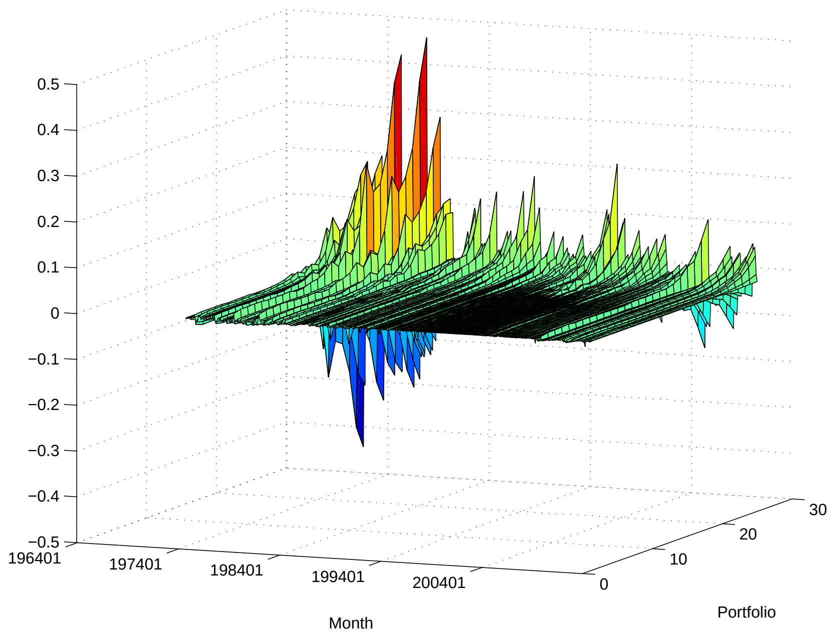

Figure 3.11. Conditional risk premium to the liquidity slope component. This graph shows the conditional risk premium to the liquidity slope component for 25 liquidity-sorted portfolios. The first portfolio is the most liquid and the 25th portfolio is the least liquid.

3.B Additional Proofs

3.B.1 Principal Components Beta Decomposition

In this section, I show that in the static setting, two of the components of the level and slope betas are cross-sectionally perfectly correlated. Let $k = LL$ for the level beta, and $k = LS$ for the slope beta. Introduce

$$\sigma = \text{Var}\left(r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right] - \left(c^m_{t+1} - \mathbb{E}_t\left[c^m_{t+1}\right]\right)\right). \tag{3.45}$$

The decomposition of $\beta^{k,i}$ is given by

$$\begin{aligned}\beta^{k,i} = {} & \frac{1}{\sigma}\text{Cov}\left(\gamma_{k,i}F_{k,t}, \gamma_{k,m}F_{k,t}\right) - \frac{1}{\sigma}\text{Cov}\left(r^i_{t+1}, \gamma_{k,m}F_{k,t}\right) \\ & - \frac{1}{\sigma}\text{Cov}\left(\gamma_{k,i}F_{k,t}, r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right]\right).\end{aligned} \tag{3.46}$$

I will now show that the first term and the third term are cross-sectionally perfectly correlated. To see this, write

$$\text{Cov}\left(\gamma_{k,i}F_{k,t}, \gamma_{k,m}F_{k,t}\right) = \gamma_{k,i}\gamma_{k,m}\text{Var}\left(F_{k,t}\right) \tag{3.47}$$

and

$$\text{Cov}\left(\gamma_{k,i}F_{k,t}, r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right]\right) = \gamma_{k,i}\text{Cov}\left(F_{k,t}, r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right]\right). \tag{3.48}$$

Taking the cross-sectional correlation (across i), we obtain

$$\begin{aligned}\text{Corr}\left(\gamma_{k,i}\gamma_{k,m}\text{Var}\left(F_{k,t}\right), \gamma_{k,i}\text{Cov}\left(F_{k,t}, r^m_{t+1} - \mathbb{E}_t\left[r^m_{t+1}\right]\right)\right) &= \text{Corr}\left(\gamma_{k,i}, \gamma_{k,i}\right) \\ &= 1.\end{aligned} \tag{3.49}$$

A consequence of this is that it is not possible in the static setting to separately identify the prices of risk for these two components in the case of the level beta and the slope beta.

3.B.2 Ang and Kristensen (2012) Optimal Bandwidth

In this section, I give some details regarding the Ang and Kristensen (2012) estimation method. We start from the bias and variance, which are given by

$$\mathbb{E}\left[\widehat{\gamma}(t)-\gamma(t)\right] \approx \frac{1}{2}h^2\gamma^{(2)}(t)\mu_2, \tag{3.50}$$

$$\operatorname{Var}\left(\widehat{\gamma}(t)\right) \approx \frac{\kappa_2}{nh}\Lambda_{FF}^{-1}(t)\otimes\Sigma(t), \tag{3.51}$$

where

$$\mu_2 = \int_{-\infty}^{\infty} z^2 K(z)\,\mathrm{d}z, \tag{3.52}$$

which has $\mu_2 = 1$ for the Gaussian kernel, and

$$\kappa_2 = \int_{-\infty}^{\infty} K^2(z)\,\mathrm{d}z, \tag{3.53}$$

which has $\kappa_2 \approx 0.2821$ for the Gaussian kernel, to obtain the approximate mean squared error (AMSE)

$$\begin{aligned}
\operatorname{MSE}\left(\widehat{\gamma}(t)\right) &= \operatorname{tr}\left(\operatorname{Var}\left(\widehat{\gamma}(t)\right)\right) + \mathbb{E}\left[\widehat{\gamma}(t)-\gamma(t)\right]'\mathbb{E}\left[\widehat{\gamma}(t)-\gamma(t)\right] && (3.54)\\
&\approx \frac{\kappa_2}{nh}\operatorname{tr}\left(\Lambda_{FF}^{-1}(t)\otimes\Sigma(t)\right) + \frac{1}{4}h^4\|\gamma^{(2)}(t)\|^2 \\
&= \operatorname{AMSE}\left(\widehat{\gamma}(t)\right). && (3.55)
\end{aligned}$$

This yields the following FOC for the minimum

$$\frac{\partial \operatorname{AMSE}\left(\widehat{\gamma}(t)\right)}{\partial h} = -\frac{\kappa_2}{nh^2}\operatorname{tr}\left(\Lambda_{FF}^{-1}(t)\otimes\Sigma(t)\right) + h^3\|\gamma^{(2)}(t)\|^2 = 0. \tag{3.56}$$

It follows that

$$
\begin{aligned}
\kappa_2 \mathrm{tr}\left(\Lambda_{FF}^{-1}(t)\otimes\Sigma(t)\right) &= nh^5\|\gamma^{(2)}(t)\|^2 \\
\Leftrightarrow h^5 &= \frac{\kappa_2 \mathrm{tr}\left(\Lambda_{FF}^{-1}(t)\otimes\Sigma(t)\right)}{n\|\gamma^{(2)}(t)\|^2} \\
\Leftrightarrow h &= \left(\frac{\kappa_2 \mathrm{tr}\left(\Lambda_{FF}^{-1}(t)\otimes\Sigma(t)\right)}{\|\gamma^{(2)}(t)\|^2}\right)^{\frac{1}{5}} \times n^{-1/5}.
\end{aligned}
\tag{3.57}
$$

For the approximate mean integrated squared error (AMISE), we find

$$
\begin{aligned}
\mathrm{AMISE}\left(\widehat{\gamma}(t)\right) &= \int_0^T \mathrm{AMSE}\left(\widehat{\gamma}(t)\right)\,\mathrm{d}t \\
&= \int_0^T \frac{\kappa_2}{nh}\mathrm{tr}\left(\Lambda_{FF}^{-1}(t)\otimes\Sigma(t)\right)\,\mathrm{d}t + \int_0^T \frac{1}{4}h^4\|\gamma^{(2)}(t)\|^2\,\mathrm{d}t \\
&= \frac{1}{nh}\int_0^T \kappa_2 \mathrm{tr}\left(\Lambda_{FF}^{-1}(t)\otimes\Sigma(t)\right)\,\mathrm{d}t + \frac{1}{4}h^4\int_0^T \|\gamma^{(2)}(t)\|^2\,\mathrm{d}t \\
&= \frac{1}{nh}V(\gamma) + \frac{1}{4}h^4 B(\gamma),
\end{aligned}
\tag{3.58}
$$

so that we obtain the following FOC for the minimum

$$
\frac{\partial \mathrm{AMISE}\left(\widehat{\gamma}(t)\right)}{\partial h} = -\frac{1}{nh^2}V(\gamma) + hB(\gamma) = 0. \tag{3.59}
$$

It follows that

$$
\begin{aligned}
\frac{1}{nh^2}V(\gamma) &= h^3 B(\gamma) \\
\Leftrightarrow h^5 &= \frac{V(\gamma)}{nB(\gamma)} \\
\Leftrightarrow h &= \left(\frac{V(\gamma)}{B(\gamma)}\right)^{\frac{1}{5}} \times n^{-1/5},
\end{aligned}
\tag{3.60}
$$

where

$$V(\gamma) = \int_0^T \kappa_2 \text{tr}\left(\Lambda_{FF}^{-1}(t) \otimes \Sigma(t)\right) \mathrm{d}t \tag{3.61}$$

$$B(\gamma) = \int_0^T \|\gamma^{(2)}(t)\|^2 \mathrm{d}t.$$

3.B.3 Ang and Kristensen (2012) Optimal Bandwidth Estimation

To estimate the optimal bandwidth, we need to find $\gamma^{(2)}(t)$. We start from

$$\gamma(t) = \left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i'\right)^{-1} \left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i R_i\right) \tag{3.62}$$

and take the first derivative, which is given by

$$\begin{aligned}
\gamma^{(1)}(t) &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i'\right)^{-1}\left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i R_i\right)\right) \qquad (3.63)\\
&= \frac{\mathrm{d}}{\mathrm{d}t}\left(\left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i'\right)^{-1}\right)\left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i R_i\right)\\
&\quad - \left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i'\right)^{-1}\left(\sum_{i=1}^{n} \frac{\mathrm{d}K_{hT}(t_i - t)}{\mathrm{d}t} X_i R_i\right)\\
&= -\left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i'\right)^{-1}\frac{\mathrm{d}}{\mathrm{d}t}\left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i'\right)\\
&\quad \times \left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i'\right)^{-1}\left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i R_i\right)\\
&\quad - \left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i'\right)^{-1}\left(\sum_{i=1}^{n} \frac{\mathrm{d}K_{hT}(t_i - t)}{\mathrm{d}t} X_i R_i\right)\\
&= \left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i'\right)^{-1}\left(\sum_{i=1}^{n} \frac{\mathrm{d}K_{hT}(t_i - t)}{\mathrm{d}t} X_i X_i'\right)\gamma(t)\\
&\quad - \left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i'\right)^{-1}\left(\sum_{i=1}^{n} \frac{\mathrm{d}K_{hT}(t_i - t)}{\mathrm{d}t} X_i R_i\right).
\end{aligned}$$

This gives the following expression for the second derivative.

$$\begin{aligned}
\gamma^{(2)}(t) &= \frac{\mathrm{d}}{\mathrm{d}t}\Bigg(\left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i'\right)^{-1}\left(\sum_{i=1}^{n} \frac{\mathrm{d}K_{hT}(t_i - t)}{\mathrm{d}t} X_i X_i'\right)\gamma(t) \qquad (3.64)\\
&\quad - \left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i'\right)^{-1}\left(\sum_{i=1}^{n} \frac{\mathrm{d}K_{hT}(t_i - t)}{\mathrm{d}t} X_i R_i\right)\Bigg),
\end{aligned}$$

or

$$
\begin{aligned}
\gamma^{(2)}(t) = & \left(\frac{\mathrm{d}}{\mathrm{d}t}\left(\sum_{i=1}^{n} K_{hT}\left(t_i - t\right) X_i X_i'\right)^{-1}\right)\left(\sum_{i=1}^{n} \frac{\mathrm{d}K_{hT}\left(t_i - t\right)}{\mathrm{d}t} X_i X_i'\right)\gamma(t) \\
& - \left(\sum_{i=1}^{n} K_{hT}\left(t_i - t\right) X_i X_i'\right)^{-1}\left(\sum_{i=1}^{n} \frac{\mathrm{d}^2 K_{hT}\left(t_i - t\right)}{\mathrm{d}t^2} X_i X_i'\right)\gamma(t) \\
& + \left(\sum_{i=1}^{n} K_{hT}\left(t_i - t\right) X_i X_i'\right)^{-1}\left(\sum_{i=1}^{n} \frac{\mathrm{d}K_{hT}\left(t_i - t\right)}{\mathrm{d}t} X_i X_i'\right)\gamma^{(1)}(t) \\
& - \left(\frac{\mathrm{d}}{\mathrm{d}t}\left(\sum_{i=1}^{n} K_{hT}\left(t_i - t\right) X_i X_i'\right)^{-1}\right)\left(\sum_{i=1}^{n} \frac{\mathrm{d}K_{hT}\left(t_i - t\right)}{\mathrm{d}t} X_i R_i\right) \\
& + \left(\sum_{i=1}^{n} K_{hT}\left(t_i - t\right) X_i X_i'\right)^{-1}\left(\sum_{i=1}^{n} \frac{\mathrm{d}^2 K_{hT}\left(t_i - t\right)}{\mathrm{d}t^2} X_i R_i\right),
\end{aligned}
\tag{3.65}
$$

or

$$
\begin{aligned}
\gamma^{(2)}(t) = &- \left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i' \right)^{-1} \left(\sum_{i=1}^{n} \frac{\mathrm{d}K_{hT}(t_i - t)}{\mathrm{d}t} X_i X_i' \right) \\
&\times \left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i' \right)^{-1} \left(\sum_{i=1}^{n} \frac{\mathrm{d}K_{hT}(t_i - t)}{\mathrm{d}t} X_i X_i' \right) \gamma(t) \\
&- \left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i' \right)^{-1} \left(\sum_{i=1}^{n} \frac{\mathrm{d}^2 K_{hT}(t_i - t)}{\mathrm{d}t^2} X_i X_i' \right) \gamma(t) \\
&+ \left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i' \right)^{-1} \left(\sum_{i=1}^{n} \frac{\mathrm{d}K_{hT}(t_i - t)}{\mathrm{d}t} X_i X_i' \right) \gamma^{(1)}(t) \\
&+ \left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i' \right)^{-1} \left(\sum_{i=1}^{n} \frac{\mathrm{d}K_{hT}(t_i - t)}{\mathrm{d}t} X_i X_i' \right) \\
&\times \left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i' \right)^{-1} \left(\sum_{i=1}^{n} \frac{\mathrm{d}K_{hT}(t_i - t)}{\mathrm{d}t} X_i R_i \right) \\
&+ \left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i' \right)^{-1} \left(\sum_{i=1}^{n} \frac{\mathrm{d}^2 K_{hT}(t_i - t)}{\mathrm{d}t^2} X_i R_i \right),
\end{aligned}
\tag{3.66}
$$

so that

$$
\begin{aligned}
\gamma^{(2)}(t) = &- \left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i' \right)^{-1} \left(\sum_{i=1}^{n} \frac{\mathrm{d}K_{hT}(t_i - t)}{\mathrm{d}t} X_i X_i' \right) \gamma^{(1)}(t) \\
&- \left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i' \right)^{-1} \left(\sum_{i=1}^{n} \frac{\mathrm{d}^2 K_{hT}(t_i - t)}{\mathrm{d}t^2} X_i X_i' \right) \gamma(t) \\
&+ \left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i' \right)^{-1} \left(\sum_{i=1}^{n} \frac{\mathrm{d}K_{hT}(t_i - t)}{\mathrm{d}t} X_i X_i' \right) \gamma^{(1)}(t) \\
&+ \left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i' \right)^{-1} \left(\sum_{i=1}^{n} \frac{\mathrm{d}^2 K_{hT}(t_i - t)}{\mathrm{d}t^2} X_i R_i \right),
\end{aligned}
\tag{3.67}
$$

and therefore

$$\gamma^{(2)}(t) = -\left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i'\right)^{-1} \left(\sum_{i=1}^{n} \frac{\mathrm{d}^2 K_{hT}(t_i - t)}{\mathrm{d}t^2} X_i X_i'\right) \gamma(t) \\ + \left(\sum_{i=1}^{n} K_{hT}(t_i - t) X_i X_i'\right)^{-1} \left(\sum_{i=1}^{n} \frac{\mathrm{d}^2 K_{hT}(t_i - t)}{\mathrm{d}t^2} X_i R_i\right). \tag{3.68}$$

We have

$$K_{h_k T}(z) = \frac{K\left(\frac{z}{h_k T}\right)}{h_k T} \tag{3.69}$$

and use the Gaussian kernel

$$K(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right). \tag{3.70}$$

Hence,

$$\frac{\mathrm{d}K_{h_k T}(z)}{\mathrm{d}z} = \frac{K'\left(\frac{z}{h_k T}\right)}{(h_k T)^2}, \tag{3.71}$$

$$\frac{\mathrm{d}^2 K_{h_k T}(z)}{\mathrm{d}z^2} = \frac{K''\left(\frac{z}{h_k T}\right)}{(h_k T)^3}, \tag{3.72}$$

and

$$\frac{\mathrm{d}K(z)}{\mathrm{d}z} = -\frac{z}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right), \tag{3.73}$$

$$\begin{aligned} \frac{\mathrm{d}^2 K(z)}{\mathrm{d}z^2} &= \frac{z^2}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) - \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) \\ &= \frac{z^2 - 1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right). \end{aligned} \tag{3.74}$$

Summarizing the results, we have obtained the following expression.

$$\begin{aligned}\gamma^{(2)}(t) = &-\left(\sum_{i=1}^{n} K_{hT}\left(t_i - t\right) X_i X_i'\right)^{-1}\left(\sum_{i=1}^{n} \frac{\mathrm{d}^2 K_{hT}\left(t_i - t\right)}{\mathrm{d}t^2} X_i X_i'\right)\gamma(t) \\ &+\left(\sum_{i=1}^{n} K_{hT}\left(t_i - t\right) X_i X_i'\right)^{-1}\left(\sum_{i=1}^{n} \frac{\mathrm{d}^2 K_{hT}\left(t_i - t\right)}{\mathrm{d}t^2} X_i R_i\right),\end{aligned} \tag{3.75}$$

where

$$\gamma(t) = \left(\sum_{i=1}^{n} K_{hT}\left(t_i - t\right) X_i X_i'\right)^{-1}\left(\sum_{i=1}^{n} K_{hT}\left(t_i - t\right) X_i R_i\right), \tag{3.76}$$

$$\frac{\mathrm{d}^2 K_{h_k T}(z)}{\mathrm{d}z^2} = \frac{K''\left(\frac{z}{h_k T}\right)}{\left(h_k T\right)^3}, \tag{3.77}$$

$$\frac{\mathrm{d}^2 K(z)}{\mathrm{d}z^2} = \frac{z^2 - 1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right). \tag{3.78}$$

3.C Additional Empirical Results

3.C.1 Time-varying Factor Exposure

To check whether the rolling window time-varying factor exposure estimates are appropriate, I use the method of Ang and Kristensen (2012). Given the liquidity level factor $F_{LL,t}$ and the liquidity slope factor $F_{LS,t}$ obtained from the PCA on illiquidity innovations, we can estimate the conditional factor model

$$c_t^i - \mathbb{E}_{t-1}\left[c_t^i\right] = \gamma_{0,i,t} + \gamma_{LL,i,t} F_{LL,t} + \gamma_{LS,i,t} F_{LS,t} + \varepsilon_{i,t}, \tag{3.79}$$

where $\varepsilon_{i,t}$ can accommodate both heteroskedasticity and time-varying cross-sectional correlations. We write $\Sigma(t)$ for the covariance structure of the full ε_t. Ang and Kristensen (2012) provide conditional kernel-based estimators with standard errors that take into account the underlying factor dynamics. Note that the variance of the liquidity innovations differs considerably across the portfolios. Consequently, the optimal time window size may also vary across the portfolios. The method of Ang and Kristensen (2012) is similar to that of Lewellen and Nagel (2006), who use a fixed-length time window to estimate time-varying exposure. In the framework of Ang and Kristensen (2012), a fixed time window can be implemented by using a backward-looking uniform kernel.

We arrange the factors in a vector $F_t = \left(F_{LL,t}, F_{LS,t}\right)'$ and assume that they follow the discretized diffusion model

$$\Delta F_t = \mu_F\left(t\right)\Delta + \Lambda_{FF}^{1/2}\left(t\right)\sqrt{\Delta}u_t, \tag{3.80}$$

where $u_t \sim II\mathcal{D}(0,I)$, and the possibly random functions $\mu_F\left(t\right)$ and $\Lambda_{FF}^{1/2}\left(t\right)$ are twice differentiable.

To estimate the $\gamma_{\ell,k,t}$, Ang and Kristensen (2012) use a Gaussian kernel, with a different bandwidth for each asset. They have

$$K_{h_kT}(z) = \frac{K\left(\frac{z}{h_kT}\right)}{h_kT} \tag{3.81}$$

where

$$K(z) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right). \tag{3.82}$$

Their estimator for $\gamma(t)$ in (3.79) is a weighted multivariate OLS estimator,

$$\widehat{\gamma}(t) = \left(\sum_{s=1}^{T} K_{h_kT}(s-t)X_iX_i'\right)^{-1}\left(\sum_{s=1}^{T} K_{h_kT}(s-t)X_ic_i\right), \tag{3.83}$$

where $X_i = (1, F_t')'$. Ang and Kristensen (2012) show (see Section 3.B.2 for the details) that the optimal bandwidth h_k^* is given by

$$h_k^* = \left(\frac{\kappa_2 \text{tr}\left(\Lambda_{FF}^{-1}(t) \otimes \Sigma(t) \right)}{\|\gamma^{(2)}(t)\|^2} \right)^{\frac{1}{5}} \times n^{-1/5}, \tag{3.84}$$

where $\gamma^{(2)}(t)$ is the second derivative of $\gamma(t)$, and

$$\kappa_2 = \int_{-\infty}^{\infty} K^2(z)\, \mathrm{d}z, \tag{3.85}$$

which gives $\kappa_2 \approx 0.2821$ for the Gaussian kernel. Feasible estimation is possible given estimates of $\Lambda_{FF}(t)$, $\Sigma(t)$, and $\gamma^{(2)}(t)$. Further details, including a two-step estimation procedure, are provided in Ang and Kristensen (2012).

The results of this procedure are given in Table 3.12. The average optimal bandwidth h across portfolios equals 105 months, or 8.75 years. The choice of 120 months for the estimation window for the factor coefficients corresponds to the 80th percentile of the bandwidths for all portfolios, and is close to the mode of 119 months.

3.C.2 Robustness to Principal Components Estimation Sample

In this section, I provide the results for the case where a rolling estimation window is used for the principal components.

Table 3.12. Ang and Kristensen (2012) bandwidths for time-varying factor exposure estimation

This table shows the Ang and Kristensen (2012) optimal estimation bandwidths for the estimation of (3.79) on 25 liquidity-sorted portfolios, as well as the equally-weighted market portfolio. Portfolio 1 is the most liquid, portfolio 25 the least liquid. The CRSP data used are monthly data corresponding to 25 value-weighted US stock portfolios sorted on illiquidity with sample period 1964–2013.

Portfolio	Bandwidth (fraction of sample)	Bandwidth (months)
1	0.0959	57.56
2	0.1467	88.01
3	0.1980	118.83
4	0.1968	118.08
5	0.1862	111.72
6	0.1707	102.40
7	0.1977	118.62
8	0.1636	98.14
9	0.2561	153.65
10	0.2307	138.44
11	0.2350	140.97
12	0.2127	127.60
13	0.1714	102.86
14	0.1683	100.97
15	0.1651	99.09
16	0.1670	100.22
17	0.1850	111.00
18	0.2010	120.57
19	0.1617	97.04
20	0.1350	81.03
21	0.1339	80.32
22	0.2060	123.59
23	0.1384	83.02
24	0.1602	96.13
25	0.1426	85.58
Mkt	0.1252	75.12
Average	0.1750	105.02
Median	0.1695	101.68
Min	0.0959	57.56
Max	0.2561	153.65

3.C.3 Pricing regression with κ equal to average monthly turnover

Acharya and Pedersen (2005) argue that when the estimation period is κ times the typical investor's holding period, the risk premium and the betas will scale with κ, but the $\mathbb{E}\left[c^i_{t+1}\right]$ term will not. Hence, they scale the latter by the average monthly turnover κ in the sample (a proxy for the inverse of the holding period). In the sample used by Acharya and Pedersen (2005) the average monthly turnover is 0.034. In Table 3.13, I present the results where I fix $\kappa = 0.0827$, the average

Table 3.13. Pricing regression with κ equal to average monthly turnover.

This table shows the estimation results for

$$\mathbb{E}\left[r_{t+1}^{i}\right]-r^{f}=\alpha+\kappa\mathbb{E}\left[c_{t+1}^{i}\right]+\lambda\beta^{net,i}, \tag{3.86}$$

where κ is restricted to 0.0827 and $\beta^{net}=\beta^{1i}+\beta^{2i}-\beta^{3i}-\beta^{4i}$, with β^{ki} as defined in the paper. The table also shows estimation results for

$$\mathbb{E}\left[r_{t+1}^{i}\right]-r^{f}=\alpha+\kappa\mathbb{E}\left[c_{t+1}^{i}\right]+\lambda_1\beta^{Ri}+\lambda_2\beta^{LLi}+\lambda_3\beta^{LSi}+\lambda_4\beta^{LRi}, \tag{3.87}$$

with κ unrestricted and with β^{Ri}, β^{LLi}, β^{LSi}, and β^{LRi} as defined in the paper. The CRSP data used are monthly data corresponding to 25 value-weighted US stock portfolios sorted on illiquidity with sample period 1964–2013. Newey and West (1987) standard errors are given in parentheses.

β^{net}	0.0102***				0.0154***			
	(0.0027)				(0.0026)			
β^{Ri}		0.0170***	0.0153***	0.0141***		0.0128***	0.0106***	0.0106***
		(0.0032)	(0.0040)	(0.0030)		(0.0024)	(0.0027)	(0.0028)
β^{LLi}		-0.0278**	-0.0332**	-0.0069		0.0647**	0.0592**	0.0604
		(0.0117)	(0.0129)	(0.0119)		(0.0267)	(0.0249)	(0.0350)
β^{LSi}			1.1087	0.9365			1.4243	1.4378
			(1.5406)	(1.6287)			(1.2978)	(1.2402)
β^{LRi}				-0.3211**				0.0116
				(0.1498)				(0.1941)
$\mathbb{E}\left[c_{t+1}^{i}\right]$	0.0827	0.0827	0.0827	0.0827	0.0257***	-0.0272	-0.0289	-0.0315
	(—)	(—)	(—)	(—)	(0.0058)	(0.0257)	(0.0248)	(0.0495)
α	-0.3122	-0.8218***	-0.7275**	-0.5802**	-0.7221***	-0.5426**	-0.4170**	-0.4152*
	(0.2318)	(0.2833)	(0.3126)	(0.2250)	(0.2384)	(0.2030)	(0.1948)	(0.1999)
R^2	0.2324	0.6400	0.6467	0.7441	0.7634	0.8037	0.8140	0.8140
Observations	25	25	25	25	25	25	25	25

monthly turnover in my estimation sample. The results show that, although the level beta remains significant, setting $\kappa = 0.0827$ results in a negative sign for the level beta, rather than the expected positive sign.

3.C.4 Pricing regression with fixed market price of risk

In this subsection, I impose the market price of risk for the CAPM beta to be equal to

$$\lambda=\mathbb{E}\left[r_{t+1}^{m}-r^{f}-0.034c_{t+1}^{m}\right], \tag{3.88}$$

following the theoretical result by Acharya and Pedersen (2005), rather than estimating it in the regression framework. The 0.034 corrects for the holding period

Table 3.14. Pricing regression results with fixed market price of risk for the CAPM beta.

This table shows the estimation results for

$$\mathbb{E}\left[r^i_{t+1}\right] - r^f = \alpha + \kappa\mathbb{E}\left[c^i_{t+1}\right] + \lambda\beta^{net,i}, \tag{3.89}$$

where κ is restricted to 0.034 and $\beta^{liq,net} = \beta^{2i} - \beta^{3i} - \beta^{4i}$, with β^{ki} as defined in the paper. The table also shows estimation results for

$$\mathbb{E}\left[r^i_{t+1}\right] - r^f = \alpha + \kappa\mathbb{E}\left[c^i_{t+1}\right] + \lambda_1\beta^{Ri} + \lambda_2\beta^{LLi} + \lambda_3\beta^{LSi} + \lambda_4\beta^{LRi}, \tag{3.90}$$

with κ unrestricted and with β^{LLi}, β^{LSi}, and β^{LRi} as defined in the paper. For these analyses, I have fixed

$$\lambda = \mathbb{E}\left[r^m_{t+1} - r^f - 0.034c^m_{t+1}\right]. \tag{3.91}$$

The CRSP data used are monthly data corresponding to 25 value-weighted US stock portfolios sorted on illiquidity with sample period 1964–2013. Newey and West (1987) standard errors are given in parentheses.

$\beta^{liq,net}$	0.0136				0.0956***			
	(0.0089)				(0.0325)			
β^{LLi}		0.0157	-0.0002	0.0173		0.0874***	0.0643**	0.0696*
		(0.0095)	(0.0103)	(0.0115)		(0.0286)	(0.0244)	(0.0337)
β^{LSi}			2.9792***	2.6389**			2.3016**	2.2902**
			(1.0251)	(1.0447)			(1.0299)	(1.0696)
β^{LRi}				-0.2026				0.0564
				(0.1329)				(0.1893)
$\mathbb{E}\left[c^i_{t+1}\right]$	0.034	0.034	0.034	0.034	-0.0719*	-0.0525*	-0.0395*	-0.0510
	(—)	(—)	(—)	(—)	(0.0349)	(0.0276)	(0.0223)	(0.0452)
α	-0.0067	-0.0070	-0.1360***	-0.1040**	-0.1029**	-0.0752**	-0.1646***	-0.1780***
	(0.0264)	(0.0251)	(0.0474)	(0.0458)	(0.0439)	(0.0339)	(0.0462)	(0.0491)
R^2	0.1208	0.1354	0.3293	0.4219	0.6668	0.6599	0.7222	0.7238
Observations	25	25	25	25	25	25	25	25

in a similar manner as the κ coefficient in the model. In my sample, the value of the unconditional market price of risk estimate equals $\lambda = 0.7200$. The results in Table 3.14 show that imposing this restriction produces results similar to the benchmark regression.

3.C.5 Pricing regression subsample analysis

In this subsection, I investigate the stability of the model by estimating it for the subsamples running from 1964 until 1988 and from 1989 until 2013. The results are given in Table 3.15 and Table 3.16, respectively. From the results, we see that

Table 3.15. Pricing regression results (first subsample).

This table shows the estimation results for

$$\mathbb{E}\left[r^i_{t+1}\right] - r^f = \alpha + \kappa \mathbb{E}\left[c^i_{t+1}\right] + \lambda \beta^{net,i}, \tag{3.92}$$

where κ is restricted to 0.034 and $\beta^{net} = \beta^{1i} + \beta^{2i} - \beta^{3i} - \beta^{4i}$, with β^{ki} as defined in the paper. The table also shows estimation results for

$$\mathbb{E}\left[r^i_{t+1}\right] - r^f = \alpha + \kappa \mathbb{E}\left[c^i_{t+1}\right] + \lambda_1 \beta^{Ri} + \lambda_2 \beta^{LLi} + \lambda_3 \beta^{LSi} + \lambda_4 \beta^{LRi}, \tag{3.93}$$

with κ unrestricted and with β^{Ri}, β^{LLi}, β^{LSi}, and β^{LRi} as defined in the paper. The CRSP data used are monthly data corresponding to 25 value-weighted US stock portfolios sorted on illiquidity with sample period 1964–1988. Newey and West (1987) standard errors are given in parentheses.

β^{net}	0.0211***				0.0215***			
	(0.0027)				(0.0034)			
β^{Ri}		0.0218***	0.0146***	0.0125***		0.0189***	0.0139***	0.0156***
		(0.0040)	(0.0029)	(0.0025)		(0.0041)	(0.0016)	(0.0022)
β^{LLi}		0.0131	0.1229***	0.1661***		0.1002**	-0.0399	-0.1533
		(0.0099)	(0.0349)	(0.0428)		(0.0476)	(0.0326)	(0.1215)
β^{LSi}			0.3108***	0.4169***			0.8476***	0.9908***
			(0.0886)	(0.1104)			(0.1857)	(0.2706)
β^{LRi}				-0.1416*				0.1313
				(0.0726)				(0.1130)
$\mathbb{E}\left[c^i_{t+1}\right]$	0.034	0.034	0.034	0.034	0.0288	-0.1295	0.6952***	0.9928**
	(—)	(—)	(—)	(—)	(0.0169)	(0.0829)	(0.1766)	(0.3867)
α	-1.2054***	-1.2439***	-0.8006***	-0.6629***	-1.2346***	-1.0747***	-0.7190***	-0.8099***
	(0.2297)	(0.3203)	(0.2104)	(0.1699)	(0.2876)	(0.3146)	(0.1173)	(0.1498)
R^2	0.8211	0.8119	0.8808	0.9008	0.8558	0.8607	0.9478	0.9527
Observations	25	25	25	25	25	25	25	25

the that the liquidity betas vary over time. This motivates the time-varying beta analysis.

Table 3.16. Pricing regression results (second subsample).

This table shows the estimation results for

$$\mathbb{E}\left[r_{t+1}^i\right] - r^f = \alpha + \kappa \mathbb{E}\left[c_{t+1}^i\right] + \lambda \beta^{net,i}, \tag{3.94}$$

where κ is restricted to 0.034 and $\beta^{net} = \beta^{1i} + \beta^{2i} - \beta^{3i} - \beta^{4i}$, with β^{ki} as defined in the paper. The table also shows estimation results for

$$\mathbb{E}\left[r_{t+1}^i\right] - r^f = \alpha + \kappa \mathbb{E}\left[c_{t+1}^i\right] + \lambda_1 \beta^{Ri} + \lambda_2 \beta^{LLi} + \lambda_3 \beta^{LSi} + \lambda_4 \beta^{LRi}, \tag{3.95}$$

with κ unrestricted and with β^{Ri}, β^{LLi}, β^{LSi}, and β^{LRi} as defined in the paper. The CRSP data used are monthly data corresponding to 25 value-weighted US stock portfolios sorted on illiquidity with sample period 1989–2013. Newey and West (1987) standard errors are given in parentheses.

β^{net}	0.0078***				0.0069***			
	(0.0021)				(0.0019)			
β^{Ri}		0.0070***	0.0073***	0.0065***		0.0073***	0.0073***	0.0059***
		(0.0018)	(0.0016)	(0.0016)		(0.0016)	(0.0016)	(0.0016)
β^{LLi}		-0.0010	0.0343	0.1280*		0.0258	0.0202	0.0925
		(0.0154)	(0.0632)	(0.0724)		(0.0490)	(0.1018)	(0.0889)
β^{LSi}			-0.0256	-0.1088*			0.0128	0.0755
			(0.0387)	(0.0622)			(0.2178)	(0.1791)
β^{LRi}				0.0716				0.1076***
				(0.0472)				(0.0348)
$\mathbb{E}\left[c_{t+1}^i\right]$	0.034	0.034	0.034	0.034	0.0217***	0.0197	0.0133	-0.0878
	(—)	(—)	(—)	(—)	(0.0046)	(0.0203)	(0.1128)	(0.1011)
α	-0.0658	0.0286	-0.0178	-0.0115	0.0345	-0.0147	-0.0110	0.0314
	(0.2118)	(0.1903)	(0.1628)	(0.1416)	(0.1958)	(0.1697)	(0.1683)	(0.1491)
R^2	0.3548	0.4093	0.4213	0.4954	0.4160	0.4082	0.4084	0.5324
Observations	25	25	25	25	25	25	25	25

3.C.6 Pricing regression robustness to portfolios 1 and 25

In this section, I provide results for the benchmark regression where I leave out portfolio 1 or portfolio 25. The motivation for this check is that portfolios 1 and 25 seem to be special both in terms of their liquidity and the PCA loadings. This is to be expected, as these are the most extreme portfolios. The results in the section show that the results are robust to excluding either of these portfolios.

Table 3.17. Pricing regression results excluding portfolio 1.

This table shows the estimation results for

$$\mathbb{E}\left[r_{t+1}^{i}\right]-r^{f}=\alpha+\kappa\mathbb{E}\left[c_{t+1}^{i}\right]+\lambda\beta^{net,i}, \tag{3.96}$$

where κ is restricted to 0.034 and $\beta^{net}=\beta^{1i}+\beta^{2i}-\beta^{3i}-\beta^{4i}$, with β^{ki} as defined in the paper. The table also shows estimation results for

$$\mathbb{E}\left[r_{t+1}^{i}\right]-r^{f}=\alpha+\kappa\mathbb{E}\left[c_{t+1}^{i}\right]+\lambda_{1}\beta^{Ri}+\lambda_{2}\beta^{LLi}+\lambda_{3}\beta^{LSi}+\lambda_{4}\beta^{LRi}, \tag{3.97}$$

with κ unrestricted and with β^{Ri}, β^{LLi}, β^{LSi}, and β^{LRi} as defined in the paper. The CRSP data used are monthly data corresponding to 25 value-weighted US stock portfolios sorted on illiquidity with sample period 1964–2013. Newey and West (1987) standard errors are given in parentheses.

β^{net}	0.0193***				0.0217***			
	(0.0024)				(0.0022)			
β^{Ri}		0.0216***	0.0200***	0.0180***		0.0182***	0.0159***	0.0159***
		(0.0025)	(0.0036)	(0.0038)		(0.0031)	(0.0037)	(0.0038)
β^{LSi}		0.0150**	0.0109	0.0211**		0.0569**	0.0535**	0.0580
		(0.0057)	(0.0075)	(0.0100)		(0.0261)	(0.0243)	(0.0343)
β^{LSi}			0.8209	0.8405			1.0867	1.1292
			(1.2871)	(1.4019)			(1.2685)	(1.2055)
β^{LRi}				-0.1344				0.0457
				(0.1210)				(0.1876)
$\mathbb{E}\left[c_{t+1}^{i}\right]$	0.034	0.034	0.034	0.034	0.0210***	-0.0163	-0.0187	-0.0285
	(—)	(—)	(—)	(—)	(0.0050)	(0.0256)	(0.0254)	(0.0490)
α	-1.0933***	-1.2791***	-1.1792***	-0.9909***	-1.3003***	-1.0166***	-0.8715***	-0.8785***
	(0.2172)	(0.2171)	(0.2771)	(0.2991)	(0.1950)	(0.2645)	(0.2827)	(0.2818)
R^2	0.5683	0.5994	0.6060	0.6346	0.7441	0.7708	0.7783	0.7791
Observations	24	24	24	24	24	24	24	24

Table 3.18. Pricing regression results excluding portfolio 25.

This table shows the estimation results for

$$\mathbb{E}\left[r_{t+1}^i\right] - r^f = \alpha + \kappa \mathbb{E}\left[c_{t+1}^i\right] + \lambda \beta^{net,i}, \tag{3.98}$$

where κ is restricted to 0.034 and $\beta^{net} = \beta^{1i} + \beta^{2i} - \beta^{3i} - \beta^{4i}$, with β^{ki} as defined in the paper. The table also shows estimation results for

$$\mathbb{E}\left[r_{t+1}^i\right] - r^f = \alpha + \kappa \mathbb{E}\left[c_{t+1}^i\right] + \lambda_1 \beta^{Ri} + \lambda_2 \beta^{LLi} + \lambda_3 \beta^{LSi} + \lambda_4 \beta^{LRi}, \tag{3.99}$$

with κ unrestricted and with β^{Ri}, β^{LLi}, β^{LSi}, and β^{LRi} as in defined in the paper. The CRSP data used are monthly data corresponding to 25 value-weighted US stock portfolios sorted on illiquidity with sample period 1964–2013. Newey and West (1987) standard errors are given in parentheses.

α	-0.6621***	-0.6980***	-0.5920**	-0.5098**	-0.7221***	-0.5426**	-0.4170**	-0.4152*
	(0.2043)	(0.2349)	(0.2412)	(0.2060)	(0.2384)	(0.2030)	(0.1948)	(0.1999)
β^{net}	0.0151***				0.0147***			
	(0.0024)				(0.0025)			
β^{Ri}		0.0142***	0.0117***	0.0119***		0.0093***	0.0089***	0.0068**
		(0.0025)	(0.0027)	(0.0028)		(0.0023)	(0.0029)	(0.0028)
β^{LLi}		0.0232*	0.0164	0.0210*		0.1947**	0.1849**	0.2652***
		(0.0122)	(0.0125)	(0.0111)		(0.0802)	(0.0845)	(0.0769)
β^{LSi}			1.6334	1.4276			0.4144	0.2385
			(1.2679)	(1.3153)			(1.5068)	(1.5106)
β^{LRi}				-0.1013				0.2381
				(0.1837)				(0.1934)
$\mathbb{E}\left[c_{t+1}^i\right]$	0.034	0.034	0.034	0.034	0.0416**	-0.2387*	-0.2259	-0.3665***
	(—)	(—)	(—)	(—)	(0.0197)	(0.1249)	(0.1324)	(0.1255)
α	-0.7024***	-0.6320***	-0.4851**	-0.4805**	-0.6691***	-0.3245*	-0.3017	-0.2132
	(0.2187)	(0.2180)	(0.2071)	(0.2038)	(0.2244)	(0.1673)	(0.1903)	(0.1881)
R^2	0.6776	0.6897	0.7087	0.7153	0.7514	0.8208	0.8216	0.8359
Observations	24	24	24	24	24	24	24	24

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Nederlandse Samenvatting

Dit proefschrift bestudeert de invloed van liquiditeit op de totstandkoming van koersen van aandelen en daarnaast wat de gevolgen zijn van beperkte diversificatie voor de waarde van woningen. In deze inleiding zal ik eerst het concept liquiditeit bespreken aan de hand van een voorbeeld. Vervolgens zal ik ingaan op beperkte diversificatie en wat daar de consequenties van kunnen zijn voor de waarde van aandelen en vastgoed. Tot slot zal ik de inhoud van elk hoofdstuk van het proefschrift samenvatten.

In de financiële economie wordt de liquiditeit van een eigendom in het algemeen gedefinieerd als het gemak waarmee het eigendom kan worden verhandeld. Dat gemak kan de volgende vormen aannemen: het snel kunnen verhandelen, het met weinig moeite kunnen verhandelen, of kunnen handelen tegen lage kosten. Het eerste en derde hoofdstuk van dit proefschrift bestuderen de liquiditeit van aandelen. Voor deze introductie zal ik beginnen met een voorbeeld dat ik niet behandel in dit proefschrift, maar dat voor de meesten herkenbaar zal zijn: vreemde valuta. Bij het in- en terugwisselen van geld naar een andere munteenheid verlies je meestal een kleine hoeveelheid, zelfs na wisselkosten. Stel dat je heen en weer reist tussen Nederland en het Verenigd Koninkrijk, terwijl de wisselkoers gelijk blijft. Dan zullen de prijzen waartegen je Britse ponden koopt en verkoopt toch anders zijn. Bij een wisselkantoor zul je bijvoorbeeld 1,37 euro per Britse pond betalen, terwijl je per verkochte Britse pond slechts 1,35 euro terug zult krijgen. Dit betekent dat je voor elke euro die je in- en terugwisselt uiteindelijk 2 eurocent zult verliezen. Deze twee cent compenseren het wisselkantoor bijvoorbeeld voor het aanhouden van grote hoeveelheden van verschillende valuta en het daaraan verbonden risico dat de waarde van die valuta zal dalen.

Het voorbeeld aangaande vreemde valuta kan direct worden toegepast op de markt voor aandelen, waar een soortgelijk verschil bestaat tussen de koers waartegen je koopt (de laatkoers) en de koers waartegen je verkoopt (de biedkoers). In het algemeen zeggen we dat de marktliquiditeit van een eigendom hoog is wanneer op een bepaald moment – opdat de waarde van het eigendom zelf niet verandert – het verschil tussen de prijs waartegen je koopt en de prijs waartegen je verkoopt klein is. Er zijn vele andere aspecten van marktliquiditeit, maar dit voorbeeld zou ten minste een nuttige manier moeten bieden om over liquiditeit na te denken.

Als we willen begrijpen waarom liquiditeit van belang kan zijn voor beleggers in het algemeen, dan hoeven we slechts te beschouwen wat er gebeurt wanneer de liquiditeit verdwijnt uit de markt. Dit gebeurde bijvoorbeeld tijdens de beurskrach van oktober 1987, de Azië-crisis, de Roebelcrisis, de instorting van LTCM in 1998, en de financiële crisis van 2007–2009 (Liu, 2006; Nagel, 2012). Dergelijke illiquide periodes vallen regelmatig samen met sterke waardedalingen (Chordia, Roll, en Subrahmanyam, 2001) en kunnen plotseling optreden (Brunnermeier en Pedersen, 2009). Dit kan zeer kostbaar zijn voor financiële instellingen die gedwongen zijn hun illiquide activa te verkopen tegen zeer lage prijzen in reactie op uitstromen, of om verliezen te dekken (Brunnermeier en Pedersen, 2009; Coval en Stafford, 2007).

Liquiditeit heeft niet altijd een prominente rol gespeeld in de academische literatuur. Traditionele modellen voor de waardering van aandelen, zoals het bekende Sharpe (1964), Lintner (1965) en Black (1972) CAPM, nemen aan dat markten perfect liquide zijn – wat betekent dat er altijd een tegenpartij is om mee te handelen en dat de prijs waartegen je kunt kopen gelijk is aan de prijs waartegen je kunt verkopen – en daarom speelt liquiditeit geen rol in de waardering. Sindsdien heeft een groot aantal empirische studies laten zien dat liquiditeit er wel degelijk toe doet in de waardering van verschillende soorten eigendommen. In het eerste hoofdstuk van dit proefschrift richt ik me op het samenspel tussen de liquiditeit van aandelen en de beleggingshorizon bij de totstandkoming van de koers, terwijl het derde hoofdstuk gaat over de invloed van de verandering van liquiditeit in de loop der tijd.

Een ander deel van dit proefschrift gaat over de consequenties van beperkte diversificatie voor de waarde van woningen. Diversificatie is de technische term voor het spreiden van risico over veel verschillende beleggingen. Bij het beleggen in bijvoorbeeld aandelen kan het verlies op het ene aandeel worden gecompenseerd door de winst op een ander aandeel, zeker wanneer men een groot aantal aandelen in portefeuille heeft. In het algemeen kan niet al het risico op deze manier worden weggenomen en het risico dat overblijft wanneer men alle aandelen die worden verhandeld in portefeuille heeft wordt dan ook marktrisico genoemd. Koersen zijn gekoppeld aan risico omdat verwachte aandelenrendementen worden gezien als vergoeding voor de bereidheid van beleggers om een zekere hoeveelheid risico te aanvaarden. Het hoofdresultaat van het bovengenoemde CAPM is dat aandeelspecifiek (of idiosyncratisch) risico er niet toe doet voor de waardering, aangezien dit vermeden kan worden door veel verschillende aandelen op te nemen in de beleggingsportefeuille. Daarom is in de context van dit model alleen marktrisico relevant voor de waardering van aandelen. Als echter sprake is van beperkte diversificatie, dan zouden zowel marktrisico als aandeelspecifiek risico relevant moeten zijn voor de waardering van aandelen. Het blijkt echter dat het lastig is om dit effect aan te tonen voor de aandelenmarkt, aangezien het moeilijk is te meten in hoeverre daar sprake is van beperkte diversificatie. De woningmarkt biedt hier uitkomst, aangezien veel huiseigenaren slechts één huis bezitten en daardoor hun beleggingen zeer beperkt hebben gediversificeerd (Tracy, Schneider, en Chan, 1999). In het tweede hoofdstuk van dit proefschrift zal ik dan ook de markt voor woonhuizen beschouwen om te analyseren wat de effecten zijn van beperkte diversificatie op woningwaardes.

Het eerste hoofdstuk van dit proefschrift is gebaseerd op onderzoek dat is uitgevoerd in samenwerking met Alessandro Beber en Joost Driessen. Het onderzoek gaat over de interactie tussen korte- en lange-termijnbeleggers die in dezelfde markt handelen in liquide en illiquide aandelen. Het is logisch dit te analyseren, liquiditeit doet er immers enkel toe wanneer men daadwerkelijk handelt. Lange-termijnbeleggers zijn dan ook minder bezorgd over liquiditeit, aangezien zij minder vaak handelen. Als beleggers een illiquide aandeel onaantrekkelijk vinden en het wordt verhandeld tegen een waarde die lager is dan wordt gerechtvaardigd door de waarde van de toekomstige dividenden, dan zeggen we dat het aandeel een liquiditeitspremie biedt. Het mag duidelijk zijn dat aandelen die een dergelijke liquiditeitspremie bieden aantrekkelijk zijn voor lange-termijnbeleggers. Zoals werd opgemerkt door David Swensen, de Chief Investment Officer van het Yale Endowment Fund: "Het accepteren van illiquiditeit betaalt zich dubbel en dwars uit voor de geduldige lange-termijnbelegger," (Swensen, 2000).[4]

In het eerste hoofdstuk modelleren we een economie waarin zowel korte- als lange-termijnbeleggers aanwezig zijn. Deze beleggers kunnen beleggen in een spectrum van aandelen met verschillende liquiditeit. Door zowel liquiditeitsrisico – het risico dat een eigendom meer of minder liquide wordt in de loop der tijd – als heterogene beleggingshorizonten in ons model op te nemen, zijn we in staat aspecten van de bekende modellen van Amihud en Mendelson (1986) en Acharya en Pedersen (2005) te combineren. Daarnaast laten we zien dat het optimaal kan zijn voor korte-termijnbeleggers om in het geheel niet te beleggen in de minst liquide aandelen, wat resulteert in een segmentatie waar de minst liquide aandelen volledig in handen zijn van lange-termijnbeleggers.

[4]Het oorspronkelijke citaat luidt: "Accepting illiquidity pays outsize dividends to the patient long-term investor."

Ons model bevat een liquiditeitspremie die kan worden ontbonden in drie delen. Het eerste deel bevat de basale premie die beleggers verlangen ter compensatie voor het aanhouden van een illiquide aandeel. In economisch evenwicht is het echter zo dat de minst liquide aandelen worden aangehouden door langetermijnbeleggers die zelden handelen en daarom minder bezorgd zijn om illiquiditeit. Als gevolg daarvan vinden we dat er juist voor de minst liquide aandelen een kleinere liquiditeitspremie is. We noemen deze afname die het tweede deel van onze ontbinding vormt dan ook een segmentatiepremie.

Het derde deel is een liquiditeitsoverlooppremie die ontstaat door de correlatie tussen rendementen op de liquide en illiquide aandelen. Als er in het geheel geen liquiditeitspremie op de minst liquide aandelen zou zitten, dan zouden we als volgt een zogenoemde bijna-arbitragestrategie op kunnen zetten. Door liquide aandelen te kopen en illiquide aandelen te verkopen, zouden we de liquiditeitspremie op de liquide aandelen kunnen verdienen, terwijl het grootste deel van het rendementsrisico opgeheven zou worden door de tegengestelde posities. Het uitvoeren van deze strategie zou de prijzen veranderen tot het niet langer winstgevend is om dit te doen. Door de illiquide aandelen te verkopen, zou hun koers dalen tot er voor die aandelen ook een zekere liquiditeitspremie aanwezig is, die gerelateerd is aan de liquiditeitspremie op de liquide aandelen.

We onderzoeken de empirische relevantie van het model door het te schatten op in de V.S. genoteerde aandelen gedurende de periode 1964 tot 2009. Onze resultaten laten zien dat we door rekening te houden met heterogene beleggingshorizonten en segmentatie de koersverschillen tussen liquide en illiquide aandelen beter kunnen verklaren. De ontbinding van de liquiditeitspremie in de drie hierboven genoemde delen laat zien waar deze verschillen vandaan komen.

Het tweede hoofdstuk van dit proefschrift is gebaseerd op onderzoek dat is uitgevoerd in samenwerking met Erasmo Giambona en betreft de markt voor koopwoningen. Het hoofdstuk bestudeert de gevolgen van beperkte diversificatie voor de prijzen van woningen. Huiseigenaren die beleggen in de woningmarkt hebben over het algemeen een positie met een grote hefboom in één of een paar woningen (Tracy, Schneider, en Chan, 1999). De hefboom bestaat uit de hypotheek, waarbij in de V.S. de hypotheek over het algemeen zo'n 75% van de woningwaarde vertegenwoordigt (Green en Wachter, 2005). De individuele beleggers kunnen niet gemakkelijk een goed-gediversificeerde portefeuille van kleine posities in een groot aantal huizen aanhouden omdat het vaak lastig, zo niet onmogelijk is om elk willekeurig deel van een woning te kopen.

Het is een bekend theoretisch resultaat in de financiële economie dat eigendomspecifiek risico alleen relevant is voor de waardering wanneer sprake is van beperkte diversificatie (Merton, 1987; Levy, 1978; Malkiel en Xu, 2004). Voor aandelen zijn de empirische resultaten hieromtrent echter niet eenduidig. Aangezien er een duidelijke indicatie is dat bij huiseigenaren sprake is van beperkte diversificatie, biedt de woningmarkt een goede kans om de gevolgen van eigendomspecifiekrisico voor de waardering te analyseren. Het is hier van belang om een maatstaf te hebben die aangeeft in hoeverre bij de huiseigenaren sprake is van beperkte diversificatie. Wij stellen voor om dit te meten via het percentage eigenaar-bewoners in een zeker gebied. Als veel inwoners het huis waarin zij wonen bezitten, dan is de voorraad huizen sterk gespreid over de inwoners en is er slechts een beperkte mogelijkheid om een grote, goed gediversificeerde portefeuille woningen te bezitten. Dit betekent dat woningspecifiek risico een grotere rol zou moeten spelen in de woningwaarde in gebieden waar het percentage eigenaar-bewoners hoog is.

We toetsen of dit verband bestaat aan de hand van de huizenprijsindex van het Federal Housing Finance Agency (FHFA) en statistische data van IPUMS omtrent huizenbezit. In onze analyse nemen we het percentage huizenbezit op als interactie met woningspecifiek risico. Hierdoor kunnen we vaststellen of een hoger percentage huizenbezit inderdaad samengaat met een sterkere invloed van woningspecifiek risico op de waardering. Uit onze resultaten volgt dat dit inderdaad het geval is.

Het derde hoofdstuk keert terug naar de invloed van liquiditeit op de waardering van aandelen. In tegenstelling tot het eerste hoofdstuk, dat gaat over de verschillen tussen liquide en illiquide aandelen, gaat dit hoofdstuk over het effect van verschillen in liquiditeit in de loop der tijd. In dit hoofdstuk toon ik aan dat liquiditeitsrisico alleen relevant is voor de waarde van aandelen in de context van een algemene daling in liquiditeit in de hele markt, maar niet in de situatie waarin alleen de minst liquide aandelen nog minder liquide worden.

Voor mijn analyse ga ik uit van empirisch bewijs van Næs, Skjeltorp, en Ødegaard (2011), die laten zien dat er twee manieren zijn waarop beleggers meer liquide beleggingen in hun portefeuilles opnemen in reactie op gewijzigde verwachtingen aangaande de reële economie. De eerste manier is een verschuiving naar bijvoorbeeld de obligatiemarkt (Goyenko en Ukhov, 2009, zoals in). Dit gaat samen met een algemene daling in liquiditeit in de hele aandelenmarkt. De tweede manier is een verschuiving van minder liquide naar meer liquide aandelen. In dat geval worden de minst liquide aandelen nog minder liquide, terwijl de liquiditeit van de relatief liquide aandelen niet sterk verandert. Deze portefeuilleverschuivingen hoeven natuurlijk niet de enige gebeurtenissen te zijn die gerelateerd zijn aan veranderingen in de relatieve liquiditeit en ik sluit andere mogelijkheden dan ook niet uit.

Eerder onderzoek door Pástor en Stambaugh (2003) laat zien dat het risico op een algemene daling in liquiditeit relevant is voor de waardering van aandelen. Het risico dat alleen de minst liquide aandelen nog minder liquide worden kan ook zeer relevant zijn voor grote institutionele beleggers. Dit kan het beste geïllustreerd worden aan de hand van een handelsstrategie die wordt voorgesteld door Duffie en Ziegler (2003). Zij laten zien dat financiële instellingen er onder bepaalde voorwaarden voor zullen kiezen om eerst hun liquide beleggingen te verkopen als zij posities moeten sluiten. Dat laatste kunnen zij doen bijvoorbeeld in reactie op een ongewenste toename in het risico dat zij lopen op hun beleggingen. Aangezien de financiële instellingen door die strategie uiteindelijk vooral illiquide beleggingen zullen aanhouden, kan zelfs het geval waarin alleen de minst liquide aandelen nog minder liquide worden een groot risico vormen. Die situatie kan dan leiden tot grote verliezen, of zelfs tot insolventie.

Met behulp van een statistische methode kan ik de twee verschillende manieren waarop liquiditeit in de loop der tijd kan variëren onderscheiden. Het blijkt dat een algemene daling in liquiditeit het sterkst is geassocieerd met negatieve rendementen op marktniveau, terwijl de afname van liquiditeit in het minst liquide segment gerelateerd is aan een hoog handelsvolume in het meest liquide segment. Het laatste is in overeenstemming met de portefeuilleverschuivingen van illiquide aandelen naar liquide aandelen zoals beschreven door Næs, Skjeltorp, en Ødegaard (2011).

Door deze twee manieren waarop liquiditeit kan variëren in de loop der tijd te combineren met het waarderingsmodel van Acharya en Pedersen (2005) ben ik in staat om te toetsen welke van deze twee manieren relevant is voor de waardering van aandelen. De resultaten laten zien dat alleen een algemene daling in liquiditeit statistisch en economisch relevant is voor de waardering, terwijl een dergelijk effect niet optreedt voor het geval waarin alleen de minst liquide aandelen nog minder liquide worden.

Samenvattend laat dit proefschrift zien dat de beleggingshorizon relevant is voor de mate waarin liquiditeit een rol speelt in de waardering van aandelen, dat alleen het risico van een algemene daling in liquiditeit van invloed is op de waardering van aandelen, en dat beperkte diversificatie er waarschijnlijk voor zorgt dat woningspecifiek risico een rol speelt in de waardering van woonhuizen in de V.S.