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**THE EFFECTIVENESS OF A FISCAL TRANSFER
MECHANISM IN A MONETARY UNION:
A DSGE MODEL FOR THE EURO AREA**

By

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The Effectiveness of a Fiscal Transfer Mechanism in a Monetary Union:

A DSGE Model for the Euro Area*

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Abstract

In this paper, we incorporate a transfer mechanism into a DSGE model with a rich fiscal sector to assess the effectiveness of fiscal transfers for a monetary union, in particular for the Economic and Monetary Union. Using a heterogeneous setup, the model is estimated for the North and the South of Europe using Bayesian methods. The results show that the transfer mechanism is effective in stabilizing the economy of the southern block of countries during the financial crisis, although the total welfare effect for the EMU is negative, though small. Ex ante, a transfer mechanism would be beneficial for both the North and the South in terms of welfare and stabilization purposes.

JEL classification: E62, E63, F42, F45

Keywords: Two-Country DSGE, Fiscal Federalism, Monetary Union, Fiscal Policy

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1 Introduction

The recent lively discussion about the need to complement the Economic and Monetary Union (EMU) in Europe with common fiscal institutions, is one that has revived after the great economic turbulence in Greece and other European countries related to the sovereign debt crisis. Fiscal transfers appeared necessary to help some of the countries that were in trouble by means of the European Financial Stability Facility (EFSF) and the European Stability Mechanism (ESM). The events of the past few years have led many politicians and economists to weigh the pros and cons of deeper fiscal integration and even a common fiscal authority.

Existing federations have a system of federal fiscal transfers, that acts as an automatic insurance against asymmetric macroeconomic shocks in the federation. The EMU does not have an explicit mechanism either to absorb asymmetric shocks or to insure against the asymmetric effects of symmetric shocks within the currency union. An automatic fiscal transfer mechanism that stabilizes asymmetric shocks to member countries of the EMU might be a first step towards federal fiscal integration.

Two important policy issues concerning the design of a fiscal transfer scheme arise with regard to the implementability of this scheme in the Euro area. First, the system should be susceptible to moral hazard as little as possible. The member countries of the EMU will not approve a system of fiscal transfers if the rules may induce more risk-taking behavior of other members, for example by continuously increasing their burden of debt or by underreporting tax revenues and other fiscal variables in order to receive a (higher) transfer. Our design of the transfer rule will take this into account, by conditioning the transfer on GDP rather than fiscal policy variables, and by conditioning on the change in this measure rather than the level of GDP. Secondly, there will be political resistance in member countries to allow for a fiscal transfer scheme if policymakers and the public believe that their country is likely to be a net contributor to this system. As our simulations for the future will show, the probabilities of becoming a net contributor or net recipient will ex ante be approximately equal for each member, regardless of whether this country is a northern or southern country of Europe.

There is a responsibility for the science of economics to analyze the effects of implementing such a transfer mechanism in the Euro area, as these results will affect the willingness of Euro

area countries to strive for fiscal integration. The purpose of this paper is to carry part of this responsibility and to contribute to the existing literature on this topic, in the first place, by incorporating an automatic fiscal transfer mechanism into a dynamic stochastic general equilibrium (DSGE) model. Our model contains several standard features of DSGE models, such as nominal rigidities, but there are also non-standard elements in the model, such as a common monetary authority together with a very rich regional fiscal sector. Besides, the transfer mechanism that is built into the model is less prone to moral hazard issues than other transfer schemes proposed in the literature. Secondly, the model presented in this paper is estimated for the northern and southern region of Europe to analyze the effectiveness of a transfer scheme within the Euro area. So far, models that included a transfer scheme were only symmetrically calibrated, whereas our method is the first, to the best of our knowledge, to take the heterogeneity between European regions into account. The Bayesian estimation of the model allows us to apply federal fiscal transfers to the specific case of the EMU, rather than the general case of two countries or regions within a monetary union. In the third place, we focus on two main questions to give a comprehensive overview of the implications of a fiscal transfer scheme in the Euro area. On the one hand, ex post, what would the transfer mechanism have meant for the regions of the Euro area in the recent crisis? On the other hand, would the transfer mechanism be ex ante beneficial for both the northern and southern region of the EMU? Fourthly, the model allows us to quantify the exact size of the net transfers, as well as the effects of these transfer on the main macroeconomic variables, welfare and risk sharing.

We will show that the results from the Bayesian estimation differ quite substantially between countries, which underlines the need to take the heterogeneity among countries in a monetary union into account. Simulations of the estimated model for the main macroeconomic variables are quite close to the corresponding observed time series. We find that an automatic fiscal transfer mechanism can help stabilizing the economy in periods of recession, as a transfer mechanism introduced in 2007 would have led to higher GDP and consumption for the southern block of countries. The country paying the transfer, in this case the North, has to give in some of its GDP and consumption, leading to a welfare loss for the North that is larger than the gain for the South (in terms of steady state consumption). The timing of the introduction of the transfer scheme is fairly important, since the transfer depends on the growth of GDP from that

moment onwards. If the mechanism would have been introduced at the start of the EMU, then transfer would have been coming from the South in the direction of the North, implying that this type of transfer scheme is by no means one-way traffic. Furthermore, we will show that the transfer scheme is beneficial *ex ante* for both the North and the South, which will increase the willingness to implement such a scheme in the future.

The rest of the paper is organized as follows. Section 2 describes the relevant literature related to this topic and section 3 gives an overview of the two-region model of the monetary union. Section 4 explains the data and the priors needed for the Bayesian estimation and gives the estimation results for the parameters of the model. Section 5 shows the main results, including simulations on the effectiveness of a transfer mechanism, welfare analysis for the introduction of a transfer mechanism in 2007, and results on the introduction of a transfer mechanism at the start of the EMU as well as the implications for the future of introducing a transfer scheme. Section 6 explains how the various channels for interregional risk sharing work within the EMU and shows the effect of a transfer mechanism on these. Section 7 concludes.

2 Related Literature

The necessity of a risk sharing mechanism in the European monetary union has been a vivid debate in the last years. President of the European Council Herman van Rompuy explained his views on how the stability in the EMU can be guaranteed in the long run in a report (van Rompuy (2012)). He pleads, amongst others, for an integrated budgetary framework with common debt issuance or other forms of fiscal solidarity. The idea to introduce a scheme of federal fiscal payments in the EMU is far from new and was already stated in the MacDougall report (European Commission (1977)). The report recommended a system of transfers between member states of the EMU to stabilize the effects of asymmetric shocks. Later in 1989, Jacques Delors pointed out that the coordination of national fiscal policies in terms of a federal adjustment mechanism would be needed for the Economic and Monetary Union to survive (Delors (1989)).

The basic argument for a system of federal fiscal transfers comes from the Optimum Currency Area (OCA) literature, that started with the pioneering articles by Mundell (1961), McKinnon (1963) and Kenen (1969). Next to labor mobility, capital mobility, price and wage flexibility and similarity in business cycles, a successful currency area should also have a risk sharing system. In a monetary union there is no possibility to use monetary policy for national policy purposes, and since there is also no exchange rate flexibility, fiscal transfers are needed if these other adjustment mechanisms fail to stabilize asymmetric shocks. This paper relates to the OCA literature as it focuses on the necessity of having a risk sharing mechanism in the form of a fiscal transfer mechanism in the EMU.

There are broadly two strands in the literature for macroeconomic aspects of federal fiscal arrangements in monetary unions. The empirical literature for fiscal federalism is plentiful. Many papers have tried to estimate the effect of a central fiscal authority or federal arrangement in stabilizing idiosyncratic shocks to regions of an existing monetary union or a federalist country. The pioneering contribution is by Sala-i-Martin & Sachs (1992) who estimate that around 40% of the initial effect of a shock to income is absorbed by the federal taxes and transfers in the US. The most influential study is by Asdrubali et al. (1996) that identifies the importance of the channels of risk sharing across US states. The results show that federal fiscal transfers smooth

around 13% of a shock to GDP. Similar studies have been performed for Canada, Germany, the UK and Italy, amongst others by Melitz & Zumer (1999), Antia et al. (1999) and Hepp & von Hagen (2013). In the past it has been shown by Sorensen & Yosha (1998), and also recently by Furceri & Zdzienicka (2013), that risk sharing mechanisms are less effective in the Euro area than in other federalist countries such as the US and Germany.

Considering the extensive literature on the empirical aspects of federal fiscal policy in monetary unions, one might view it as surprising that the theoretical side of this literature is relatively unexplored, especially for the EMU. There are few papers that provide a sound theoretical analysis for their policy recommendation to coordinate some aspects of fiscal policy on a union-wide level. An example of such a paper is Bargain et al. (2012), that incorporates the element of an EU-wide tax and transfer system into EUROMOD, which is a European tax-benefit calculator used by the Institute for Social and Economic Research (ISER). Their result is that the majority of the countries in the Euro area would profit from it. They also analyze the effects of a fiscal equalization mechanism in this calculator and find that this mechanism has clear redistributing effects between countries, but the effects on macroeconomic stability are ambiguous.

Recently, Farhi & Werning (2012) gave a boost to the theoretical literature that is using DSGE models to examine the effectiveness of federal fiscal transfers. They show analytically that there is a role for the government providing insurance, contingent on the shock that a country experiences, as private insurance and risk-sharing across countries is too low. Evers (2012) provides a quantitative analysis of federal transfer rules that could target regional differences in nominal GDP, consumption, labor income or fiscal deficits. Evers (2015) is so far the most extensive study in this field. The author shows that a fiscal equalization system, in which nominal tax revenues are shared, acts destabilizing due to the responses of consumption, and leads to welfare losses compared to decentralized fiscal authorities. A common fiscal authority with a unitary tax system does stabilize output and consumption. In contrast to Evers (2015), this paper focuses on a rule for the transfers between countries that depend on the differences in economic growth between countries. The model that Evers (2015) uses, incorporates symmetric tax rates in both countries to facilitate nominal tax revenue sharing, but although this type of federal arrangement could be observed in existing federations, it is hard to picture this type of

stabilization mechanism within the EMU due to several moral hazard issues.

A deficiency of the existing literature is that in the theoretical models are calibrated for symmetric countries. Since EMU countries are very different in many respects, it is important to analyze asymmetric countries in order to be able to deduct any policy implications from the research in this field. This paper uses a DSGE model for asymmetric regions that is based on Pytlarczyk (2005) and Kolasa (2009). The two-country DSGE model in this paper also incorporates several established features such as tradables and non-tradables, capital adjustment costs and Calvo price and wage setting. In this sense, this paper builds on a larger history of DSGE models for currency unions, or specifically for the Euro area, by Smets & Wouters (2003), Gali & Monacelli (2008), Jondeau & Sahuc (2008). Instead of calibrating the model with asymmetric countries, we use Bayesian methods to estimate the model for the North and the South of Europe. For calibrated parameters and priors, we rely on parameters and priors chosen in the existing literature, as in Smets & Wouters (2003), Pytlarczyk (2005), Jondeau & Sahuc (2008), Kolasa (2009) and Hollmayr (2012), as well as on data from the OECD database.

The contribution of this paper to the existing literature is fourfold. First, we have extended the theoretical literature on monetary unions by implementing a rich fiscal sector with an automatic fiscal transfer mechanism into a DSGE model for a currency area. The transfer rule that we impose is least susceptible to moral hazard as possible since it conditions on the growth in GDP, and not on the level of fiscal policy variables that are easier to influence. Secondly, this paper adds to the existing empirical literature on federal fiscal policy by using an estimated DSGE model for the Euro area, hence allowing for heterogeneity between regions. Therefore, we can analyze the effectiveness of a fiscal transfer mechanism specifically for the case of the EMU. Thirdly, we provide a comprehensive overview of the implications of a fiscal transfer scheme in the EMU by our dual analysis. On the one hand, we analyze how the transfer mechanism would have changed the situation in the EMU, had it been introduced before the recent crisis. On the other hand, we study whether the transfer mechanism would *ex ante* be beneficial for both regions of the Euro area. Finally, the setup of the transfer scheme within the model allows us to quantify the size of the transfers between the northern and southern region of the Euro area, as well as the exact effect on the main macroeconomic variables and welfare in regions, and on risk sharing between regions. Taking into account the heterogeneity of the countries within

the EMU and quantifying precisely the impact on the real economy is important in order to genuinely assess the implications of a transfer mechanism for the EMU.

3 Model of a Two-Region Monetary Union

The theoretical model that is described in this section builds on a wide range of DSGE models for monetary unions. We will present a model that contains several features that are established in the papers by Obstfeld & Rogoff (1995), Christiano et al. (2005), Smets & Wouters (2003) and Gali & Monacelli (2008), amongst others. The paper by Jondeau & Sahuc (2008) also uses this type of DSGE models, but focuses on the heterogeneity of countries within the currency union. More closely related to the model in this paper is the model by Pytlarczyk (2005) and Kolasa (2009). Both use the same model that has a very detailed structure.

In the model, there are two regions¹, Home and Foreign, and they are assumed to be member of a monetary union. Hence, the regions use the same currency and there is a common central bank that conducts monetary policy. As links between regions within the monetary union are stronger than with the rest of the world, trade with the rest of the world is neglected.² Every region gives shelter to households, firms and a government. Within a region, households are identical, but preferences with respect to consumption and labor are allowed to be heterogeneous across regions. Households provide labor to firms, rent capital to domestic firms and can consume both domestically produced goods and foreign goods. Firms in every region produce a continuum of differentiated tradable goods for the international market, as well as nontradable goods. Prices and wages are set according to a Calvo mechanism with partial indexation to past inflation. It is assumed that there is no migration between regions and that the production factor labor is immobile. There is free international trade, so that transaction costs to trade do not exist. A central monetary authority of the union sets its policy according to a feedback rule that takes into account the union-wide inflation and output gap. Fiscal authorities in every region collect taxes on consumption, capital income and labor income to finance their expenditures. The automatic transfer mechanism is introduced by setting a rule that automatically directs resources from one region's government budget to the budget of the other when that region

¹In the next section, we will estimate the model for the northern and southern region of the Euro area, so with regard to terminology we will use region instead of country.

²According to Masson & Taylor (1993), the European Union as a whole is a relatively closed economy, although all countries within the European Union are very open. Therefore, the countries are modeled as open economies within the monetary union, but at the same time, their paper justifies neglecting the rest of the world.

faces a larger negative (or smaller positive) output gap compared to the other region. In this way, the transfer rule is not very prone to moral hazard issues, as we will explain in this section. Furthermore, by conditioning on the change in GDP this setup avoids a continuous redistribution between regions and the political resistance that would come with this.

In what follows we will describe the model for the home economy only, since the general setup for the foreign economy is similar. Variables referring to the home region are indexed by i and foreign variables are indexed by i^* . Parameters are allowed to be heterogeneous across regions, and for that reason, foreign parameters are marked with an asterisk. The size of the home region H in terms of population is $[0, n]$ and the size of the foreign region is $[n, 1]$, so that the size of the union is normalized to 1.

3.1 Households

3.1.1 Consumption

Both regions are populated by infinitely-lived households. Households may be hit by idiosyncratic shocks, but households have access to complete markets for state-contingent bonds and therefore all households in a given region behave in the same manner and we will consider the optimization problem of the representative household. The preferences of the representative household j are reflected by the following lifetime utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^i(j), L_t^i(j)) \quad (1)$$

The household derives negative utility from providing labor, but private consumption affects utility positively. The instantaneous utility function is assumed to be of the standard CES form:

$$U_t^i(C_t^i(j), L_t^i(j)) = \varepsilon_{C,t}^i \frac{(C_t^i(j) - hC_{t-1}^i)^{1-\sigma}}{1-\sigma} - \varepsilon_{L,t}^i \frac{(L_t^i(j))^{1+\phi}}{1+\phi} \quad (2)$$

with habit formation that is related to past consumption. The shocks processes $\varepsilon_{C,t}^i$ and $\varepsilon_{L,t}^i$ reflect exogenous shocks to consumption and labor supply preferences, and are assumed to follow an AR(1) process.

In line with the existing strand of DSGE models, the consumption bundle of a household in region i , C_t^i , is defined as:

$$C_t^i = \frac{(C_{T,t}^i)^{\gamma_c} (C_{N,t}^i)^{1-\gamma_c}}{\gamma_c \gamma_c (1-\gamma_c)^{1-\gamma_c}} \quad (3)$$

where $C_{T,t}^i$ is the consumption of tradable goods and $C_{N,t}^i$ the consumption of nontradable goods. The share of tradables in consumption is denoted by γ_c . The bundle of tradable goods is composed of both home goods and imported foreign goods:

$$C_{T,t}^i = \frac{(C_{D,t}^i)^\alpha (M_{C,t}^i)^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \quad (4)$$

The weight of the imported goods $M_{C,t}^i$ in the consumption bundle is given by $1-\alpha$, such that $\alpha > 0.5$ implies a home bias in consumption.

The household maximizes lifetime utility subject to a sequence of intertemporal budget constraints:

$$\begin{aligned} (1 + \tau_{C,t}^i) \left(P_{N,t}^i C_{N,t}^i(j) + P_{D,t}^i C_{D,t}^i(j) + P_{M,t}^i M_{C,t}^i(j) \right) + \left(P_{N,t}^i I_{N,t}^i(j) + P_{D,t}^i I_{D,t}^i(j) + P_{M,t}^i M_{I,t}^i(j) \right) \\ + E_t \left\{ Q_{t,t+1}, D_{t+1}^i(j) \right\} \leq D_t^i(j) + (1 - \tau_{L,t}^i) W_t^i(j) L_t^i(j) + (1 - \tau_{K,t}^i) R_{K,t}^i K_t^i(j) + \Pi_t^i(j) \end{aligned} \quad (5)$$

where $P_{N,t}^i(j)$ and $P_{D,t}^i(j)$ denote the prices of the consumed nontradable and tradable domestic varieties of goods, and $P_{M,t}^i(j)$ denotes the price of the consumed imported variety of goods produced the foreign region. D_{t+1}^i is the nominal payoff in period $t+1$ of the government bonds held at the end of period t and $Q_{t,t+1}$ is the stochastic discount factor, which is assumed to be common across regions, since $E_t \{ Q_{t,t+1} \} = R_t^{-1}$, which is common for the monetary union. The nominal wage is given by W_t^i and households receive an income $R_{K,t}^i$ on renting capital K_t . Both capital and labor income are taxed by $\tau_{K,t}^i$ and $\tau_{L,t}^i$, respectively, and consumption is taxed by $\tau_{C,t}^i$. The households own the firms and hence they receive dividends Π_t^i .

The indexes for consumption of domestically produced goods and imported foreign goods are respectively given by the following aggregators:

$$C_{D,t}^i = \left(\left(\frac{1}{n} \right)^{\frac{1}{\phi_D}} \int_0^n C_{D,t}^i(z)^{\frac{\phi_D-1}{\phi_D}} dz \right)^{\frac{\phi_D}{\phi_D-1}} \quad (6)$$

$$M_{C,t}^i = \left(\left(\frac{1}{1-n} \right)^{\frac{1}{\phi_M}} \int_n^1 M_{C,t}^i(z)^{\frac{\phi_M-1}{\phi_M}} dz \right)^{\frac{\phi_M}{\phi_M-1}} \quad (7)$$

Moreover, the consumption bundle of nontradable goods is given by³:

$$C_{N,t}^i = \left(\left(\frac{1}{n} \right)^{\frac{1}{\phi_N}} \int_0^n C_{N,t}^i(z)^{\frac{\phi_N-1}{\phi_N}} dz \right)^{\frac{\phi_N}{\phi_N-1}} \quad (8)$$

The elasticities of substitution between varieties of domestic tradable goods (ϕ_D), between varieties of imported goods from the foreign region (ϕ_M) and between varieties of nontradable domestic goods (ϕ_N) are all allowed to vary across regions.

The consumption demand functions for each variety of goods can be derived from intratemporal optimization:

$$C_{D,t}^i(z) = \frac{1}{n} \gamma_C \alpha \left(\frac{P_{D,t}^i(z)}{P_{D,t}^i} \right)^{-\phi_D} \left(\frac{P_{D,t}^i}{P_{C,t}^i} \right)^{-1} C_t^i \quad (9)$$

$$M_{C,t}^i(z) = \frac{1}{1-n} \gamma_C (1-\alpha) \left(\frac{P_{M,t}^i(z)}{P_{M,t}^i} \right)^{-\phi_M} \left(\frac{P_{M,t}^i}{P_{C,t}^i} \right)^{-1} C_t^i \quad (10)$$

$$C_{N,t}^i(z) = \frac{1}{n} (1-\gamma_C) \left(\frac{P_{N,t}^i(z)}{P_{N,t}^i} \right)^{-\phi_N} \left(\frac{P_{N,t}^i}{P_{C,t}^i} \right)^{-1} C_t^i \quad (11)$$

The price indexes are defined as follows:

$$P_{D,t}^i = \left(\frac{1}{n} \int_0^n P_{D,t}^i(z)^{1-\phi_D} dz \right)^{\frac{1}{1-\phi_D}} \quad (12)$$

$$P_{M,t}^i = \left(\frac{1}{1-n} \int_n^1 P_{M,t}^i(z)^{1-\phi_M} dz \right)^{\frac{1}{1-\phi_M}} \quad (13)$$

$$P_{N,t}^i = \left(\frac{1}{n} \int_0^n P_{N,t}^i(z)^{1-\phi_N} dz \right)^{\frac{1}{1-\phi_N}} \quad (14)$$

where $P_{D,t}^i$ is the price index of domestically produced goods in region i which are traded, $P_{N,t}^i$ is the price index of domestically produced goods in region i which are not traded, and $P_{M,t}^i$ is the price index of imported goods, produced in region i^* and imported into i . The price index of the tradable goods (consumed in region i) is given by:

$$P_{T,t}^i = (P_{D,t}^i)^\alpha (P_{M,t}^i)^{1-\alpha} \quad (15)$$

and the consumer price index is given by:

$$P_{C,t}^i = (P_{T,t}^i)^{\gamma_C} (P_{N,t}^i)^{1-\gamma_C} \quad (16)$$

³For the foreign economy, the term $\frac{1}{n}$ in equation (6) and (8) is replaced by $\frac{1}{1-n}$ and in equation (7) the term $\frac{1}{1-n}$ is replaced by $\frac{1}{n}$.

3.1.2 Investment

Households rent out capital which is assumed to be invested in a homogeneous investment good.

The law of motion for capital accumulation is given by:

$$K_{t+1}^i(j) = (1 - \delta)K_t^i(j) + \varepsilon_{I,t}^i \left(1 - S \left(\frac{I_t^i(j)}{I_{t-1}^i(j)} \right) \right) I_t^i(j) \quad (17)$$

where $\varepsilon_{I,t}^i$ represents the technological progress that is specific for investment, and $S(\cdot)$ represents the adjustment cost function. The value of $S(\cdot)$ is equal to zero if $\frac{I_t^i(j)}{I_{t-1}^i(j)} = 1$, hence if investment stays at the same level, there are no adjustment costs.

The investment bundle of a household in region i is defined as:

$$I_t^i = \frac{(I_{T,t}^i)^{\gamma_I} (I_{N,t}^i)^{1-\gamma_I}}{\gamma_I^{\gamma_I} (1 - \gamma_I)^{1-\gamma_I}} \quad (18)$$

with γ_I as the share of tradables in investment. Moreover, investment in the tradables sector is given by:

$$I_{T,t}^i = \frac{(I_{D,t}^i)^\alpha (M_{I,t}^i)^{1-\alpha}}{\alpha^\alpha (1 - \alpha)^{1-\alpha}} \quad (19)$$

where

$$I_{D,t}^i = \left(\left(\frac{1}{n} \right)^{\frac{1}{\phi_D}} \int_0^n I_{D,t}^i(z)^{\frac{\phi_D-1}{\phi_D}} dz \right)^{\frac{\phi_D}{\phi_D-1}} \quad (20)$$

$$M_{I,t}^i = \left(\left(\frac{1}{1-n} \right)^{\frac{1}{\phi_M}} \int_n^1 M_{I,t}^i(z)^{\frac{\phi_M-1}{\phi_M}} dz \right)^{\frac{\phi_M}{\phi_M-1}} \quad (21)$$

$$I_{N,t}^i = \left(\left(\frac{1}{n} \right)^{\frac{1}{\phi_N}} \int_0^n I_{N,t}^i(z)^{\frac{\phi_N-1}{\phi_N}} dz \right)^{\frac{\phi_N}{\phi_N-1}} \quad (22)$$

The investment demand functions for each variety of goods are as follows:

$$I_{D,t}^i(z) = \frac{1}{n} \gamma_I \alpha \left(\frac{P_{D,t}^i(z)}{P_{D,t}^i} \right)^{-\phi_D} \left(\frac{P_{D,t}^i}{P_{I,t}^i} \right)^{-1} I_t^i \quad (23)$$

$$M_{I,t}^i(z) = \frac{1}{1-n} \gamma_I (1 - \alpha) \left(\frac{P_{M,t}^i(z)}{P_{M,t}^i} \right)^{-\phi_M} \left(\frac{P_{M,t}^i}{P_{I,t}^i} \right)^{-1} I_t^i \quad (24)$$

$$I_{N,t}^i(z) = \frac{1}{n} (1 - \gamma_I) \left(\frac{P_{N,t}^i(z)}{P_{N,t}^i} \right)^{-\phi_N} \left(\frac{P_{N,t}^i}{P_{I,t}^i} \right)^{-1} I_t^i \quad (25)$$

The price index for investment is defined as:

$$P_{I,t}^i = (P_{T,t}^i)^{\gamma_I} (P_{N,t}^i)^{1-\gamma_I} \quad (26)$$

3.1.3 Household optimization

The household maximizes its utility function, taking into account the budget constraint and the law of capital accumulation. The first-order conditions of this optimization problem lead to the Euler equation⁴:

$$\frac{1}{R_t} = \beta \mathbb{E}_t \left[\frac{\varepsilon_{C,t+1}^i (C_{t+1}^i(j) - hC_t^i)^{-\sigma} P_{C,t}^i (1 + \tau_{C,t}^i)}{\varepsilon_{C,t}^i (C_t^i(j) - hC_{t-1}^i)^{-\sigma} P_{C,t+1}^i (1 + \tau_{C,t+1}^i)} \right] \quad (27)$$

Moreover, the first-order conditions imply the following equations for investment demand and the relative price of installed capital, which is also known as Tobin's Q:

$$\begin{aligned} \frac{P_{I,t}^i}{P_{C,t}^i} &= Q_{T,t}^i(j) \varepsilon_{I,t}^i \left(1 - S \left(\frac{I_t^i(j)}{I_{t-1}^i(j)} \right) - \frac{I_t^i(j)}{I_{t-1}^i(j)} S' \left(\frac{I_t^i(j)}{I_{t-1}^i(j)} \right) \right) \\ &+ \mathbb{E}_t \left[Q_{T,t+1}^i(j) \frac{P_{C,t+1}^i}{P_{C,t}^i R_t} \varepsilon_{I,t+1}^i \left(\frac{I_{t+1}^i(j)}{I_t^i(j)} \right)^2 S' \left(\frac{I_{t+1}^i(j)}{I_t^i(j)} \right) \right] \end{aligned} \quad (28)$$

$$Q_{T,t}^i(j) = \mathbb{E}_t \left[\frac{(1 - \tau_{K,t}^i) R_{K,t+1}^i}{P_{C,t}^i R_t} \right] + (1 - \delta) \mathbb{E}_t \left[Q_{T,t+1}^i(j) \frac{P_{C,t+1}^i}{P_{C,t}^i R_t} \right] \quad (29)$$

3.1.4 Wage setting

Labor services of households are aggregated into a homogeneous labor input:

$$L_t^i = \left(\left(\frac{1}{n} \right)^{\frac{1}{\phi_W}} \int_0^n L_t^i(j)^{\frac{\phi_W - 1}{\phi_W}} dj \right)^{\frac{\phi_W}{\phi_W - 1}} \quad (30)$$

The aggregate wage index W_t^i is given by:

$$W_t^i = \left(\frac{1}{n} \int_0^n W_t^i(j)^{1 - \phi_W} dj \right)^{\frac{1}{1 - \phi_W}} \quad (31)$$

where ϕ_W is the elasticity of substitution between labor inputs for different production varieties.

There is Calvo adjustment in the wage setting, which means that only a fraction $1 - \theta_W$ can renegotiate the wage contracts in each period, whereas the wages of the remaining households are partially indexed to past inflation in consumer prices. With δ_W being the degree of partial indexation, the wage of these households in period t is given by:

$$W_t^i(j) = W_{t-1}^i(j) \left(\frac{P_{C,t-1}}{P_{C,t-2}} \right)^{\delta_W} \quad (32)$$

⁴The derivation of this and other first-order conditions can be found in the appendix.

Households that are able to renegotiate their wages maximize the expected present discounted value of future utility, because they know that they may not be able to change their wage for a while. Households do take into account that if wages are not re-optimized, they are still partially indexed to past CPI inflation. The optimal wage is then derived⁵ as:

$$\tilde{W}_t^i(j) = \frac{\phi_W}{\phi_W - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_W \beta)^k MRS_{t+k}^i(j) \varepsilon_{C,t}^i L_{t+k}^i(j) C_{t+k}^i(j)^{-\sigma}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_W \beta)^k \frac{(1-\tau_{L,t+k}^i)}{P_{C,t+k}^i} \varepsilon_{C,t}^i L_{t+k}^i(j) C_{t+k}^i(j)^{-\sigma}} \quad (33)$$

where $MRS_t^i(j) = \frac{\varepsilon_{L,t}^i L_t^i(j)^\phi}{\varepsilon_{C,t}^i (C_t^i(j) - h C_{t-1}^i)^{-\sigma}}$ is the marginal rate of substitution of consumption for leisure.

Given this optimization process, the evolution of the aggregate wage index is:

$$W_t^i = \left(\theta_W \left(W_{t-1}^i \left(\frac{P_{C,t-1}^i}{P_{C,t-2}^i} \right)^{\delta_W} \right)^{1-\phi_W} + (1 - \theta_W) (\tilde{W}_t^i)^{1-\phi_W} \right)^{\frac{1}{1-\phi_W}} \quad (34)$$

The structural parameters for the degree of partial indexation of wages δ_W and for the fraction of households that cannot renegotiate their contracts θ_W are allowed to be different across regions.

3.2 Firms

3.2.1 Production

For the production side of the economy, it is assumed that there is monopolistic competition in both the tradables and nontradables sector. There is a Cobb-Douglas production function in both sectors:

$$Y_{N,t}^i(z) = a_{N,t}^i (K_{N,t}^i(z))^\eta (L_{N,t}^i(z))^{1-\eta} \quad (35)$$

$$Y_{D,t}^i(z) = a_{D,t}^i (K_{D,t}^i(z))^\eta (L_{D,t}^i(z))^{1-\eta} \quad (36)$$

where both $a_{N,t}^i$ and $a_{D,t}^i$ follow an AR(1) process in the log-linearized model that is specific for each region. Then the output indexes in both sectors are as follows:

$$Y_{N,t}^i = \left(\left(\frac{1}{n} \right)^{\frac{1}{\phi_N}} \int_0^n Y_{N,t}^i(z)^{\frac{\phi_N-1}{\phi_N}} dz \right)^{\frac{\phi_N}{\phi_N-1}} \quad (37)$$

⁵The derivation of the optimal wage can be found in the appendix.

$$Y_{D,t}^i = \left(\left(\frac{1}{n} \right)^{\frac{1}{\phi_D}} \int_0^n Y_{D,t}^i(z)^{\frac{\phi_D-1}{\phi_D}} dz \right)^{\frac{\phi_D}{\phi_D-1}} \quad (38)$$

Cost minimization of the firms implies that the capital-labor ratio, which implicitly determines labor demand, is given by:

$$\frac{W_t^i L_t^i}{R_{K,t}^i K_t^i} = \frac{1-\eta}{\eta} \quad (39)$$

where η is the output elasticity with respect to capital. The real marginal cost $MC_{D,t}^i$ is defined as

$$MC_{D,t}^i = \frac{1}{P_{D,t}^i a_{D,t}^i} \left(\frac{W_t^i}{1-\eta} \right)^{1-\eta} \left(\frac{R_{K,t}^i}{\eta} \right)^\eta \quad (40)$$

Similarly, the real marginal cost for the nontradables is defined as:

$$MC_{N,t}^i = \frac{1}{P_{N,t}^i a_{N,t}^i} \left(\frac{W_t^i}{1-\eta} \right)^{1-\eta} \left(\frac{R_{K,t}^i}{\eta} \right)^\eta \quad (41)$$

Factor markets are assumed to be perfectly competitive, such that the rental price of capital $R_{K,t}$ and the aggregate wage index W_t are taken as given by producers. Therefore, the marginal cost for the production of one unit is the same for all firms.

3.2.2 Price setting

Similar to wages, prices are set according to Calvo staggered price setting. A fraction $1 - \theta_D$ of firms that produce for the domestic market can optimally set its prices, while the other firms see their prices partially indexed to past inflation. Inflation is measured here by price levels for products in the same sector. Therefore, for the tradables sector, prices of firms that cannot optimally set prices are given by:

$$P_{D,t}^i(z) = P_{D,t-1}^i(z) \left(\frac{P_{D,t-1}^i}{P_{D,t-2}^i} \right)^{\delta_D} \quad (42)$$

where δ_D is the degree of partial indexation.

The firms that can change their prices will maximize the expected present discounted value of future profits. In doing this, they also take into account that if prices are not re-optimized they

will still be partially indexed to past inflation. The derivation⁶ shows that the optimal price for firms is given by:

$$\tilde{P}_{D,t}^i = \frac{\phi_D}{\phi_D - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_D \beta)^k \frac{U_C^i(C_{t+k})}{U_C^i(C_t)} Y_{D,t+k}^i MC_{D,t+k}^i \left(\frac{P_{D,t+k}^i}{\left(\frac{P_{D,t+k-1}^i}{P_{D,t-1}^i} \right)^{\delta_D}} \right)^{\phi_D}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_D \beta)^k \frac{U_C^i(C_{t+k})}{U_C^i(C_t)} Y_{D,t+k}^i \left(\frac{P_{D,t+k}^i}{\left(\frac{P_{D,t+k-1}^i}{P_{D,t-1}^i} \right)^{\delta_D}} \right)^{\phi_D - 1}} \quad (43)$$

All firms that are allowed to adjust their prices will choose the same optimal price since there are no firm-specific shocks, therefore the firm-specific index j is left out. Given the optimal price that will be set by forward-looking firms if they are allowed to, the evolution of the price index for the domestic tradable goods is given by:

$$P_{D,t}^i = \left(\theta_D \left(P_{D,t-1}^i \left(\frac{P_{D,t-1}^i}{P_{D,t-2}^i} \right)^{\delta_D} \right)^{1-\phi_D} + (1 - \theta_D) (\tilde{P}_{D,t}^i)^{1-\phi_D} \right)^{\frac{1}{1-\phi_D}} \quad (44)$$

The optimization problem solved by firms in the nontradables sector is similar and therefore the optimal price and the evolution of the price index are analogous to equations (43) and (44). In the foreign region, there is a similar process for $P_{D,t}^{i*}$ and $P_{N,t}^{i*}$. Again, the parameters for the pricing behavior, $\delta_D, \delta_N, \theta_D$ and θ_N are allowed to vary across regions.

Since there are no trading frictions, the law of one price holds. As a result, the price of domestically produced goods sold in region i equals the price of these goods sold in region i^* when converted into the same currency:

$$P_{D,t}^i(z) = E_{i^*,t}^i P_{M,t}^{i*}(z) \quad (45)$$

Similarly, the price of goods imported by region i , so produced by region i^* , equals the price of goods produced and sold in region i^* when converted to the same currency:

$$P_{M,t}^i(z) = E_{i^*,t}^i P_{D,t}^{i*}(z) \quad (46)$$

where $E_{i^*,t}^i$ is the nominal exchange rate denoting the currency of region i in terms of the foreign currency of region i^* . An increase in $E_{i^*,t}^i$ thus implies that the home currency depreciates. Of

⁶The optimal price is derived in the appendix.

course, in a currency union the nominal exchange rate between the two regions equals one, hence the general expressions in (45) and (46) can be replaced by $P_{D,t}^i(z) = P_{M,t}^{i*}(z)$ and $P_{M,t}^i(z) = P_{D,t}^{i*}(z)$. Because of home-biased preferences, purchasing power parity (PPP) does not need to hold even though the law of one price holds, as we will later show.

3.3 Common monetary authority and national fiscal authorities

A new feature of this model compared to the model by Kolasa (2009) is the common monetary authority that conducts a single policy for the whole currency area, since in that paper both countries had their own monetary policies. The monetary authority in the currency union responds to the economic conditions on the union-level, as the interest-rate feedback rule takes into account the union-wide inflation and output gap:

$$R_t = (R_{t-1})^\rho \left[\left(\frac{Y_t^{EMU}}{\bar{Y}^{EMU}} \right)^{\psi_y} \left(\frac{\pi_{C,t}^{EMU}}{\bar{\pi}^{EMU}} \right)^{\psi_\pi} \right]^{1-\rho} u_{R,t} \quad (47)$$

where \bar{Y}^{EMU} is the steady state level of total output in the union, $\bar{\pi}^{EMU}$ is the steady state level of inflation in the union, and $u_{R,t}$ is an i.i.d. monetary policy shock.

In this model, there is a rich fiscal sector in order to allow for a more realistic setting to assess the effectiveness of a fiscal transfer mechanism. National fiscal authorities collect consumption, capital income and labor income taxes. The government spends resources on government consumption, which is directed at nontradable goods, on interest on government debt, and on lump-sum transfers to households. The policy rules concerning taxes and expenditures allow for responses to output, which implies fiscal policy has a role as automatic stabilizer, and for responses to government debt in order to ensure fiscal solvency. Debt is accumulated by the government through the issue of bonds, whenever the expenditures by the government are larger than the revenues. The government budget constraint is given by:

$$B_t^i = (1 + R_t)B_{t-1}^i + G_t^i + Z_t^i - \tau_{C,t}^i C_t^i - \tau_{K,t}^i R_{K,t}^i K_t^i - \tau_{L,t}^i W_t^i L_t^i \quad (48)$$

where B_t^i is government debt, G_t^i is government consumption, Z_t^i are lump-sum transfers from the government to households, and $\tau_{C,t}^i$, $\tau_{K,t}^i$ and $\tau_{L,t}^i$ are the tax rates on consumption, capital income and labor income, respectively. Government consumption is determined by the following

policy rule:

$$\log G_t^i = \rho_G \log G_{t-1}^i + \gamma_G \log B_{t-1}^i + u_{G,t}^i \quad (49)$$

There is habit formation in government consumption if $\rho_G \neq 0$. Consumption by the government is assumed to respond to government debt, whereas lump-sum transfers Z_t will respond to the business cycle:

$$\log Z_t^i = \rho_Z \log Z_{t-1}^i + \phi_Z \log Y_{t-1}^i + u_{Z,t}^i \quad (50)$$

The response of the transfer to the state of the economy is expected to be negative, during recessions transfers, such as unemployment insurance, will automatically go up. The reasoning behind the division of government expenditures in government consumption and lump-sum transfers is that the welfare or social security part of expenditures will not be very responsive to government debt and will not add to the GDP of a country. On the other hand, the consumption part of government expenditures is included in GDP, such as investment in infrastructure. Since this type of expenditures is easier to defer and is not as fixed as social security, it is reasonable to assume that government consumption will be decided upon by looking at the level of government debt. The higher the debt level already is, the more likely it is that consumption by the government is lower.

Tax rates will depend on government debt:

$$\log \tau_{C,t}^i = \rho_{\tau_C} \log \tau_{C,t-1}^i + \gamma_{\tau_C} \log B_{t-1}^i + \phi_{\tau_C \tau_K} u_{\tau_K,t}^i + \phi_{\tau_C \tau_L} u_{\tau_L,t}^i + u_{\tau_C,t}^i \quad (51)$$

$$\log \tau_{K,t}^i = \rho_{\tau_K} \log \tau_{K,t-1}^i + \gamma_{\tau_K} \log B_{t-1}^i + \phi_{\tau_K \tau_C} u_{\tau_C,t}^i + \phi_{\tau_K \tau_L} u_{\tau_L,t}^i + u_{\tau_K,t}^i \quad (52)$$

$$\log \tau_{L,t}^i = \rho_{\tau_L} \log \tau_{L,t-1}^i + \gamma_{\tau_L} \log B_{t-1}^i + \phi_{\tau_L \tau_C} u_{\tau_C,t}^i + \phi_{\tau_L \tau_K} u_{\tau_K,t}^i + u_{\tau_L,t}^i \quad (53)$$

Shocks affecting one tax rate are also allowed to affect other tax rates contemporaneously.

The transfer mechanism is introduced by setting a rule that automatically directs resources from one region to the other if that region is facing a large negative output gap or a positive but smaller output gap relative to the other region. Every period, each region pays a certain amount to a common pool, an amount that is proportional to GDP in that region. The payments of both regions come together in the common pool, from which resources are distributed to each region, based on the steady state ratio of that region's GDP to the union-wide GDP level. In

this way, the region with a larger output gap compared to the steady state will receive a transfer in net terms, and in the steady state the transfer equals zero. The payment to the common pool is given by:

$$PAY_t^i = \psi_{PAY} Y_{t-1}^i \quad (54)$$

and

$$PAY_t^{i*} = \psi_{PAY} Y_{t-1}^{i*} \quad (55)$$

The contribution to the common pool in percentages, determined by ψ_{PAY} , is the same for both regions. The common pool is then determined by:

$$CP_t = PAY_t^i + PAY_t^{i*} = \psi_{PAY} Y_{t-1}^i + \psi_{PAY} Y_{t-1}^{i*} = \psi_{PAY} Y_{t-1} \quad (56)$$

where Y_t is the union-wide GDP level. The receipts from the common pool are:

$$REC_t^i = \frac{\bar{Y}}{\bar{Y}^{EMU}} CP_t \quad (57)$$

and

$$REC_t^{i*} = \frac{\bar{Y}^*}{\bar{Y}^{EMU}} CP_t \quad (58)$$

Hence, the net transfer received by region i , which is negative in case the region effectively pays a transfer, is given by:

$$TR_t^i = REC_t^i - PAY_t^i \quad (59)$$

Country i^* receives the amount that region i is paying, or vice versa⁷:

$$TR_t^{i*} = REC_t^{i*} - PAY_t^{i*} = PAY_t^i - REC_t^i = -TR_t^i \quad (60)$$

In the steady state, the payment to and receipt from the common pool by each region are equal and hence the net transfer equals zero.⁸

⁷This transfer mechanism is by definition a zero-sum game, as $PAY_t^i + PAY_t^{i*} = CP_t = \frac{\bar{Y}}{\bar{Y}^{EMU}} CP_t + \frac{\bar{Y}^*}{\bar{Y}^{EMU}} CP_t = REC_t^i + REC_t^{i*}$.

⁸In the steady state, the payment to the common pool by the home region is given by $P\bar{A}Y = \psi_{PAY} \bar{Y}$. The receipt from the common pool is given by $R\bar{E}C = \frac{\bar{Y}}{\bar{Y}^{EMU}} \bar{C}P$, and the steady state of the common pool is $\bar{C}P = \psi_{PAY} \bar{Y}^{EMU}$. As a result, $R\bar{E}C = \frac{\bar{Y}}{\bar{Y}^{EMU}} \psi_{PAY} \bar{Y}^{EMU} = P\bar{A}Y$. The same analysis holds for the foreign region.

The automatic fiscal transfer mechanism will ease the budget constraint of the government with TR_t^i , the difference between receipts from and payments to the common pool:

$$B_t^i = (1 + R_t)B_{t-1}^i + G_t^i + Z_t^i - \tau_{C,t}^i C_t^i - \tau_{K,t}^i R_{K,t}^i K_t^i - \tau_{L,t}^i W_t^i L_t^i - (REC_{t-1}^i - PAY_{t-1}^i) \quad (61)$$

The idea behind the transfer scheme is to provide temporary relief to a region that is hit by a negative business cycle shock, without starting off a continuous redistribution from rich to poor regions. By conditioning on the change in GDP relative to the starting point of the transfer system rather than the level of GDP, it is just as likely that the transfer will go from the rich to the poor region as that it will go in the opposite direction. In this way, this transfer rule avoids the political resistance that would arise if the system would have clear winners and losers upfront. In the long run, however, the year of introduction of the transfer mechanism might not be an appropriate base year to condition the transfer on, as structural trend growth could govern the direction of the transfer, and then a recalculation could be deemed necessary.

Basing the transfer mechanism on the change in GDP, rather than on fiscal policy variables, has clear advantages concerning moral hazard issues. First of all, it is important to base the transfer on a number that cannot be influenced easily, as could be done either by influencing the actual number itself or by under- or overreporting the numbers in your region. A government will find it more difficult to influence its GDP number than the tax revenues for example, which is the variable on which the transfer is based in the study by Evers (2015). Secondly, the susceptibility to fraud will be less if it is not in the region's own interest to influence the variables on which the transfer is based. Having a lower GDP in order to receive the transfer will never yield a region as much income as the income it has lost by this practice. On the other hand, if the transfer is based on the amount of tax revenues, a region might be incentivized to decrease tax rates that, besides the receipt of the transfer, will lead to higher utility of their inhabitants.

3.4 Market clearing conditions

3.4.1 International risk sharing

Financial markets are assumed to be complete and households in all regions can trade in state contingent claims denominated in the home currency. Therefore, the perfect risk sharing con-

dition holds, which states that the ratio of consumer price levels (or the real exchange rate) equals the ratio of marginal utilities of consumption across regions in every state of the world.

The real exchange rate is defined as:

$$Q_{i^*,t}^i = \frac{E_{i^*,t}^i P_{C,t}^{i^*}}{P_{C,t}^i} \quad (62)$$

The perfect risk-sharing condition for the monetary union is then⁹:

$$Q_{i^*,t}^i = \kappa \frac{U'(C_t^{i^*})}{U'(C_t^i)} = \kappa \frac{\varepsilon_{C,t}^{i^*} (C_t^{i^*} - h^* C_{t-1}^{i^*})^{-\sigma^*}}{\varepsilon_{C,t}^i (C_t^i - h C_{t-1}^i)^{-\sigma}} \quad (63)$$

where κ is a constant that depends on the initial conditions and is defined as $\kappa = \frac{E_{i^*,0}^i P_{C,0}^{i^*} U'(C_0^i)}{P_{C,0}^i U'(C_0^{i^*})}$.

The terms of trade is defined as the ratio of producer prices of both economies, or put differently, the ratio of the import prices of a region relative to the export prices of that region:

$$T_{i^*,t}^i = \frac{E_{i^*,t}^i P_{D,t}^{i^*}}{P_{D,t}^i} = \frac{P_{M,t}^i}{P_{D,t}^i} \quad (64)$$

The internal exchange rates are defined as $X_t^i = \frac{P_{N,t}^i}{P_{T,t}^i}$ and $X_t^{i^*} = \frac{P_{N,t}^{i^*}}{P_{T,t}^{i^*}}$. Since both regions are in a monetary union with a single currency, the nominal exchange rate equals one which simplifies the notation to $Q_{i^*,t}^i = \frac{P_{C,t}^{i^*}}{P_{C,t}^i}$ and $T_{i^*,t}^i = \frac{P_{D,t}^{i^*}}{P_{D,t}^i}$.

The law of one price holds on the national level if we assume that the elasticity of substitution among goods is the same within a given region, which implies that:

$$P_{D,t}^i = E_{i^*,t}^i P_{M,t}^{i^*} \quad P_{M,t}^i = E_{i^*,t}^i P_{D,t}^{i^*} \quad (65)$$

Using these conditions, the price indexes derived above and the fact that with a shared currency the nominal exchange rate equals one, allows us to show a relationship between the real exchange rate and the terms of trade¹⁰:

$$Q_{i^*,t}^i = (T_{i^*,t}^i)^{\alpha + \alpha^* - 1} \frac{(X_t^{i^*})^{1 - \gamma_c^*}}{(X_t^i)^{1 - \gamma_c}} \quad (66)$$

The real exchange rate is thus allowed to deviate from purchasing power parity, because of changes in relative prices of tradables versus nontradables in both regions and changes in the terms of trade, as long as there is a home bias. When there is no home bias, the terms of trade cannot be determined by the perfect risk sharing condition anymore.

⁹For the full derivation of the perfect risk-sharing condition, see Chari, Kehoe & McGratten (2002).

¹⁰The derivation of this relationship can be found in the appendix.

3.4.2 Goods market

The goods market clears when total output in the economy equals total demand. Therefore, the output of nontradable goods has to be equal to the sum of consumption, investment and government purchases of nontradables in Home:

$$Y_{N,t}^i = (1 - \gamma_C) \frac{P_{C,t}^i}{P_{N,t}^i} C_t^i + (1 - \gamma_I) \frac{P_{I,t}^i}{P_{N,t}^i} I_t^i + G_t^i \quad (67)$$

Goods market clearance also implies that the output of tradable goods must be equal to the sum of domestic consumption and investment of tradables and tradables exported for consumption and investment abroad. As the export of home produced consumption and investment goods to region i^* is defined as $M_{C,t}^{i^*}$ and $M_{I,t}^{i^*}$, the goods market clearance condition for tradables is given by:

$$Y_{D,t}^i = \gamma_C \alpha \frac{P_{C,t}^i}{P_{D,t}^i} C_t^i + \gamma_I \alpha \frac{P_{I,t}^i}{P_{D,t}^i} I_t^i + \frac{1-n}{n} \gamma_C^* (1 - \alpha^*) \frac{P_{C,t}^{i^*}}{P_{M,t}^{i^*}} C_t^{i^*} + \frac{1-n}{n} \gamma_I^* (1 - \alpha^*) \frac{P_{I,t}^{i^*}}{P_{M,t}^{i^*}} I_t^{i^*} \quad (68)$$

Total output is given by:

$$Y_t^i = Y_{N,t}^i + Y_{D,t}^i \quad (69)$$

3.4.3 Factor markets

Equilibrium in the factor markets requires that the labor and capital market both clear. For the labor market, this implies that:

$$L_t^i = L_{D,t}^i + L_{N,t}^i = \int_0^n L_{D,t}^i(j) dj + \int_0^n L_{N,t}^i(j) dj \quad (70)$$

Similarly, capital market clearing is defined as:

$$K_t^i = K_{D,t}^i + K_{N,t}^i = \int_0^n K_{D,t}^i(j) dj + \int_0^n K_{N,t}^i(j) dj \quad (71)$$

3.5 Solving the model

Because of the size of the model, there is no closed-form solution. Therefore, we will log-linearize the model around the steady state. The complete model in log-linearized form can be found in

the appendix. The model is then solved numerically by running Dynare in Matlab using the log-linearized model¹¹.

The two-region model of the monetary union is captured in a system of 72 equations¹² and 21 exogenous shocks. The shocks to productivity $(u_{a,D,t}^i, u_{a,N,t}^i, u_{a,D,t}^{i*}, u_{a,N,t}^{i*})$, the shocks to preferences in consumption and labor supply $(u_{C,t}^i, u_{C,t}^{i*}, u_{L,t}^i, u_{L,t}^{i*})$ and the shocks to investment efficiency $(u_{I,t}^i, u_{I,t}^{i*})$ are assumed to follow an AR(1) process in the log-linearized model. The shocks to government spending and tax rates $(u_{G,t}^i, u_{G,t}^{i*}, u_{Z,t}^i, u_{Z,t}^{i*}, u_{\tau_C,t}^i, u_{\tau_C,t}^{i*}, u_{\tau_K,t}^i, u_{\tau_K,t}^{i*}, u_{\tau_L,t}^i, u_{\tau_L,t}^{i*})$ and the monetary policy shock to the interest rate $(u_{R,t})$ are assumed to be i.i.d. shocks.

3.6 Welfare measure

To evaluate the normative aspects of a fiscal transfer mechanism, we will use a welfare measure based on a second-order Taylor series expansion of the utility function around the steady state, following Woodford & Benigno (2004) and Jondeau & Sahuc (2005,2008). Welfare is based on the expected discounted value of the sum of utilities, which gives the following second-order approximation¹³:

$$\begin{aligned}
U_0^i = & \sum_{t=0}^{\infty} \beta^t \left(\bar{U}^i + ((1-h)\bar{C}^i)^{-\sigma} \bar{C}^i \left[(\mathbf{E}_0(\hat{c}_t^i) - h\mathbf{E}_0(\hat{c}_{t-1}^i)) + \frac{1}{2} \left(\mathbf{E}_0((\hat{c}_t^i)^2) - h\mathbf{E}_0((\hat{c}_{t-1}^i)^2) \right) \right. \right. \\
& \left. \left. - \frac{\sigma}{2(1-h)} \left(\mathbf{E}_0((\hat{c}_t^i)^2) + h^2\mathbf{E}_0((\hat{c}_{t-1}^i)^2) \right) + \frac{\sigma h}{1-h} \mathbf{E}_0(\hat{c}_t^i \hat{c}_{t-1}^i) \right] \right. \\
& \left. - (\bar{L}^i)^{1+\phi} \left[\mathbf{E}_0(\hat{l}_t^i) + \frac{1}{2}(1+\phi)\mathbf{E}_0((\hat{l}_t^i)^2) \right] + \mathcal{O}(\|\zeta\|^3) \right)
\end{aligned}$$

The welfare effect of a fiscal transfer mechanism is evaluated using the consumption equivalent welfare measure in the tradition of Lucas (2003). The welfare compensation is measured as the permanent relative change in consumption compared to the steady state that will make the representative household indifferent between the current situation and the steady state, indicated by λ . The welfare loss or gain associated with the introduction of a transfer mechanism is then measured by the increase or decrease in the welfare compensation relative to the situation

¹¹The Matlab codes for running the Dynare program are available upon request.

¹²Equations for the transfer mechanism are not included in the estimated version of the model. If these equations were to be included the model entails a system of 77 equations.

¹³Derivations can be found in the appendix.

before introduction of the transfer scheme. Therefore, an increase in λ due to the introduction of the transfer mechanism implies a welfare gain. In this situation the representative household would require a higher steady state consumption in order to be equally well off in the steady state, as compared to the required steady state consumption level before introduction of the transfer mechanism.

The consumption equivalence λ is defined such that:

$$\begin{aligned} U_0^i &= \sum_{t=0}^{\infty} \beta^t \mathbf{E}_0 U(C_t^i, C_{t-1}^i, L_t^i) = \sum_{t=0}^{\infty} \beta^t U((1 + \lambda)\bar{C}^i, \bar{L}^i) \\ &= \sum_{t=0}^{\infty} \beta^t \left[\frac{((1 + \lambda)(1 - h)\bar{C}^i)^{1-\sigma}}{1 - \sigma} - \frac{(\bar{L}^i)^{1+\phi}}{1 + \phi} \right] \end{aligned}$$

Using a first-order Taylor approximation in λ^i we find the consumption equivalent welfare compensation measure:

$$\begin{aligned} \lambda^i &= \frac{1}{1 - h} \left[(\mathbf{E}_0(\hat{c}_t^i) - h\mathbf{E}_0(\hat{c}_{t-1}^i)) + \frac{1}{2}(\mathbf{E}_0((\hat{c}_t^i)^2) - h\mathbf{E}_0((\hat{c}_{t-1}^i)^2)) - \frac{\sigma}{2(1 - h)} (\mathbf{E}_0((\hat{c}_t^i)^2) + h^2\mathbf{E}_0((\hat{c}_{t-1}^i)^2)) \right. \\ &\quad \left. + \frac{\sigma h}{1 - h} \mathbf{E}_0(\hat{c}_t^i \hat{c}_{t-1}^i) \right] - (\bar{L}^i)^{1+\phi} ((1 - h)\bar{C}^i)^{\sigma-1} \left[\mathbf{E}_0(\hat{l}_t^i) + \frac{1}{2}(1 + \phi)\mathbf{E}_0((\hat{l}_t^i)^2) \right] \end{aligned}$$

The welfare compensation λ^i can be further decomposed into components reflecting the means as well as the variances of consumption and labor, denoted respectively by λ_M^i and λ_V^i . The overall welfare compensation is given by $(1 + \lambda^i) = (1 + \lambda_M^i)(1 + \lambda_V^i)$. Moreover, the change in the means can be attributed to both consumption and labor, which leads to the decomposition $(1 + \lambda_M^i) = (1 + \lambda_{M,c}^i)(1 + \lambda_{M,l}^i)$. Likewise, the change in the variance can be decomposed into its consumption and labor component, such that $(1 + \lambda_V^i) = (1 + \lambda_{V,c}^i)(1 + \lambda_{V,l}^i)$. The second-order approximation of welfare as well as the formula for the consumption equivalent welfare measure are similar for the foreign region denoted with i^* . The parameter values and steady state values used for welfare analysis are heterogeneous across regions.

4 Bayesian Estimation

4.1 Bayesian estimation

The model presented in the previous section is estimated with Bayesian estimation techniques using 21 macroeconomic time series as observable variables. Data on GDP, consumption, investment, internal exchange rate, CPI, real wage rate, government debt, total government expenditures, consumption tax revenues and capital income tax revenues is gathered for both a northern and a southern block of countries, as well as data on the nominal interest rate set by the ECB. South represents the so-called PIGS-countries that have experienced dramatic economic troubles during the recent European sovereign debt crisis, i.e. Portugal, Italy, Greece and Spain. The North block contains the other Eurozone countries that are a member of the EMU from the start of the currency union, these are Austria, Belgium, Finland, France, Germany, Ireland, Luxembourg and the Netherlands¹⁴. All variables are seasonally adjusted and first-differenced prior to estimation. For most variables, the data comes from Eurostat, except for the data on CPI, internal exchange rate and real wage rate, for which the source is the OECD database. A complete description of the data used is given in the appendix.

The Metropolis-Hastings algorithm is used in order to obtain estimates of the complete posterior distribution. Estimation is done on a quarterly basis, with an estimation period from the second quarter of 2000 until the fourth quarter of 2013. Lack of data availability, especially for Greece, limits the estimation sample to 55 periods.

4.2 Calibrated parameters

In this section, we present some parameters that are calibrated in the Bayesian estimation of the model. Some of the structural parameters of the model are directly related to the steady state values of variables, such as β , and therefore, these need to be estimated from the means of observable variables. The data used in the estimation is in first differences, however, and

¹⁴Ireland could also have been included in the South-block of countries, then representing the PIIGS-countries. Since Ireland is relatively small in both blocks, it is most likely that the parameter estimates would not have been very different.

hence these structural parameters cannot be pinned down in the estimation. Therefore, some of the structural parameters as well as the steady state values are calibrated, at values that can be found in table 1. For these parameter values, we follow mostly Smets and Wouters (2003), Kolasa (2009), Hollmayr (2012).

Table 1: Calibrated parameters for symmetric regions

Parameter	North	South
<i>Structural parameters</i>		
n	0.66	0.34
β	0.99	0.99
σ	2	2
h	0.75	0.75
γ_C	0.51	0.57
γ_I	0.51	0.57
α	0.96	0.88
δ	0.025	0.025
η	0.33	0.33
<i>Monetary policy parameters</i>		
ρ		0.7
ψ_Y		0.3
ψ_π		0.8
<i>Steady state values</i>		
$\frac{\bar{C}}{\bar{Y}}$	0.54	0.58
$\frac{\bar{I}}{\bar{Y}}$	0.24	0.24
$\frac{\bar{G}}{\bar{Y}}$	0.22	0.18
$\frac{\bar{R}}{\bar{D}}$	0.01	0.01
$\frac{T\bar{A}X_C}{\bar{D}}$	0.0462	0.0353
$\frac{T\bar{A}X_K}{\bar{D}}$	0.015	0.015
$\frac{T\bar{A}X_L}{\bar{D}}$	0.0435	0.0338
$\frac{\bar{C}}{\bar{D}}$	0.0404	0.0321
$\frac{\bar{Z}}{\bar{D}}$	0.0372	0.0214
$\frac{R\bar{E}C}{\bar{D}} = \psi_{PAY} \cdot \frac{\bar{Y}}{\bar{D}}$	$\psi_{PAY} \cdot 0.35$	$\psi_{PAY} \cdot 0.27$

$$\begin{array}{ccc}
\frac{P\bar{A}Y}{D} = \psi_{PAY} \cdot \frac{\bar{Y}}{D} & \psi_{PAY} \cdot 0.35 & \psi_{PAY} \cdot 0.27 \\
\frac{\bar{Y}}{\bar{Y}^{EMU}} & 0.66 & 0.34
\end{array}$$

According to OECD Statistics on population, the relative size of the northern block is 66%. The discount factor β , the intertemporal elasticity of substitution σ and the output elasticity with respect to capital η are parameterized at values that are commonly used in the DSGE literature. The depreciation rate of capital δ equals 2.5% which implies a yearly capital depreciation of approximately 10%. The parameter of habit formation is given by 0.75 for both blocks of countries, which is close to the estimate of Kolasa (2009) for Europe. From OECD Statistics, we calculate the share of tradable goods and services in GDP to approximate the share of tradables in consumption and investment. Here, tradables consists of agriculture, industry, construction, transport, distributive trade and communications. Nontradables are financial activities, real estate activities, scientific and administrative activities, public administration and other service activities. Hollmayr (2012) estimates the home bias of each member country of the EMU, and when converting this into the home bias of the North and the South, we find that the North has a larger home bias than the South, mostly because the northern countries trade a lot with Germany. The southern countries also trade a lot with Germany, but for these Germany is the foreign block.

For monetary policy, a Taylor-type rule is used for the interest rate in which the AR(1) coefficient for smoothing is the same as the priors for the persistence parameters in the estimation. Furthermore, the weight on inflation is larger than the weight on the output gap, which is reasonable for the monetary policy of the European Central Bank.

Some of the log-linearized equations contain steady state values of variables due to additive terms in the original equations. These steady state values are approximated using long-run averages of the variables provided by the OECD Stats Database. Private consumption is assumed to be around 54% of national GDP in North and 58% of GDP in South, private investment 24% in both regions and therefore government expenditures are approximated at 22% and 18% in North and South. The steady state values for the fiscal variables are long-run averages on government debt, tax revenues and government expenditures taken from Eurostat.

4.3 Priors and parameter estimates

The first columns of table 2 show our assumptions regarding the prior distribution of the structural parameters that will be estimated. Prior means are typically set close to values that are found in other studies in this literature. The priors of the standard errors are chosen in such a way that the domain covers reasonable values of the parameter. The parameter for the inverse elasticity of labor supply ϕ is assumed to have a gamma distribution, which guarantees a positive range. The prior mean is set equal to two as is also done by Smets & Wouters (2003). The parameter for capital adjustment costs has a normal prior distribution with mean 4 as in Kolasa (2009). The parameters for the degree of indexation of wages and prices, which is expressed in percentages, are assumed to have a beta distribution, which covers the range between 0 and 1. Moreover, the Calvo probability parameters have a prior mean of either 0.7 and 0.8 and are assumed to have a beta distribution, for the same reason as the indexation parameters.

Table 2: Estimation results: Structural parameters

	Prior distribution			Posterior distribution		
	Type	Mean	St. Error	Mean	10%	90%
ϕ	gamma	2.0	0.4	2.4725	1.8039	3.2210
ϕ^*	gamma	2.0	0.4	1.4489	1.2118	1.6466
S''	normal	4.0	1.0	9.1392	7.6770	10.3141
S''^*	normal	4.0	1.0	9.5634	8.9370	10.2580
δ_D	beta	0.5	0.1	0.3143	0.2482	0.3909
δ_D^*	beta	0.5	0.1	0.7268	0.6277	0.7941
δ_N	beta	0.5	0.1	0.3940	0.3260	0.4973
δ_N^*	beta	0.5	0.1	0.4731	0.3851	0.5533
δ_W	beta	0.5	0.1	0.1902	0.1335	0.2392
δ_W^*	beta	0.5	0.1	0.4192	0.3104	0.4934
θ_D	beta	0.7	0.1	0.8184	0.7910	0.8473
θ_D^*	beta	0.7	0.1	0.8746	0.8516	0.9071
θ_N	beta	0.7	0.1	0.5832	0.5290	0.6580
θ_N^*	beta	0.7	0.1	0.6716	0.6402	0.6953
θ_W	beta	0.8	0.1	0.9894	0.9870	0.9916

θ_W^*	beta	0.8	0.1	0.9788	0.9769	0.9808
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Setting the prior distribution for the regional fiscal policy parameters is more difficult, as the largest part of the fiscal policy sector in our model is an extension to the common models in which these parameters do not exist. Hence, economic intuition is key in providing the parameters with reasonable prior means. As can be seen in table 3, the persistence parameters in the policy rules for tax rates and expenditures are beta distributed, since we assume they are naturally between zero and one. A reasonable prior for the mean of these AR(1) coefficients is 0.7, a value also used for the shock persistence parameters as well as the monetary policy parameter. Economic theory would suggest that the parameter for the responsiveness of government transfers to the state of the economy is negative, since transfers such as unemployment insurance usually increase whenever the economy is in a bad state. Hence, we assume that ϕ_Z is negative such that it is normally distributed with prior mean -0.1. In a similar fashion, the parameter for the responsiveness of government consumption to debt γ_G is normally distributed with mean -0.2, as it seems likely that the government tightens its belt when government debt is increasing. Tax rates move positively with government debt, therefore we assume that the γ -parameters for the tax rates have a normal distribution with a positive prior. Furthermore, we allow for correlation between tax rates by assuming a normal distribution with prior zero, in order not to impose any thought on whether the correlation should be positive or negative.

Table 3: Estimation results: Regional fiscal policy parameters

	Prior distribution			Posterior distribution		
	Type	Mean	St. Error	Mean	10%	90%
ρ_G	beta	0.7	0.1	0.7895	0.7020	0.8501
ρ_G^*	beta	0.7	0.1	0.7892	0.7343	0.8246
ρ_{τ_C}	beta	0.7	0.1	0.7292	0.6746	0.7858
$\rho_{\tau_C}^*$	beta	0.7	0.1	0.7887	0.7418	0.8550
ρ_{τ_K}	beta	0.7	0.1	0.5151	0.4446	0.5706
$\rho_{\tau_K}^*$	beta	0.7	0.1	0.5738	0.4850	0.6402
ρ_{τ_L}	beta	0.7	0.1	0.8330	0.7878	0.8901
$\rho_{\tau_L}^*$	beta	0.7	0.1	0.7914	0.7387	0.8373

ρ_Z	beta	0.7	0.1	0.8604	0.7810	0.9500
ρ_Z^*	beta	0.7	0.1	0.9132	0.8954	0.9297
ϕ_Z	normal	-0.1	0.1	-0.0451	-0.0696	-0.0312
ϕ_Z^*	normal	-0.1	0.1	-0.0592	-0.1236	-0.0164
γ_G	normal	-0.2	0.1	0.0845	0.0576	0.1174
γ_G^*	normal	-0.2	0.1	0.0787	0.0336	0.1265
γ_{τ_C}	normal	0.2	0.1	0.0960	0.0675	0.1309
$\gamma_{\tau_C}^*$	normal	0.2	0.1	0.0687	0.0359	0.1005
γ_{τ_K}	normal	0.2	0.1	0.2064	0.1027	0.3081
$\gamma_{\tau_K}^*$	normal	0.2	0.1	0.2416	0.1406	0.3488
γ_{τ_L}	normal	0.2	0.1	0.0763	0.0439	0.1154
$\gamma_{\tau_L}^*$	normal	0.2	0.1	0.1286	0.1013	0.1589
$\phi_{\tau_C\tau_K}$	normal	0	0.1	0.0026	-0.0000	0.0072
$\phi_{\tau_C\tau_K}^*$	normal	0	0.1	0.0021	-0.0032	0.0089
$\phi_{\tau_C\tau_L}$	normal	0	0.1	-0.0305	-0.0898	0.0558
$\phi_{\tau_C\tau_L}^*$	normal	0	0.1	-0.0511	-0.1259	0.0499
$\phi_{\tau_K\tau_C}$	normal	0	0.1	-0.0590	-0.1884	0.0718
$\phi_{\tau_K\tau_C}^*$	normal	0	0.1	-0.1578	-0.2515	-0.0462
$\phi_{\tau_K\tau_L}$	normal	0	0.1	-0.0185	-0.1102	0.1405
$\phi_{\tau_K\tau_L}^*$	normal	0	0.1	-0.0660	-0.1648	0.0584
$\phi_{\tau_L\tau_C}$	normal	0	0.1	-0.0111	-0.2098	0.1492
$\phi_{\tau_L\tau_C}^*$	normal	0	0.1	-0.0649	-0.1600	0.0416
$\phi_{\tau_L\tau_K}$	normal	0	0.1	-0.0002	-0.0070	0.0066
$\phi_{\tau_L\tau_K}^*$	normal	0	0.1	-0.0286	-0.0408	-0.0165

The variances of the 21 shocks are assumed to follow an inverted gamma distribution with prior means mainly equal to 5, as reported in table 4. Exceptions are the variance of consumer preference and labor preference shocks as well as shocks in capital income tax rates, investment efficiency and the interest rate. Moreover, shocks might be correlated across the two blocks of countries, therefore we estimate this correlation for all shocks, except for a shock to consumer preferences and investment efficiency for which we assume no correlation. The prior on this correlation is set equal to 0.5, as we expect shocks between the two regions in the Euro area to

be positively correlated.

Table 4: Estimation results: Shock parameters

	Prior distribution			Posterior distribution		
	Type	Mean	St. Error	Mean	10%	90%
ρ_D	beta	0.7	0.1	0.9706	0.9588	0.9789
ρ_D^*	beta	0.7	0.1	0.9818	0.9741	0.9904
ρ_N	beta	0.8	0.1	0.9199	0.8991	0.9495
ρ_N^*	beta	0.8	0.1	0.8647	0.8199	0.9004
ρ_C	beta	0.7	0.1	0.8072	0.7782	0.8340
ρ_C^*	beta	0.7	0.1	0.7687	0.7281	0.7990
ρ_L	beta	0.7	0.1	0.5541	0.4758	0.6321
ρ_L^*	beta	0.7	0.1	0.2545	0.2058	0.3081
ρ_I	beta	0.7	0.1	0.7755	0.7479	0.7945
ρ_I^*	beta	0.7	0.1	0.8049	0.7887	0.8203
σ_D	inv. gamma	5	inf	2.6231	2.3019	3.0484
σ_D^*	inv. gamma	5	inf	2.9002	2.4211	3.3976
σ_N	inv. gamma	5	inf	2.1465	1.6224	2.6255
σ_N^*	inv. gamma	5	inf	2.6567	2.2713	3.4427
σ_C	inv. gamma	50	inf	48.6332	38.8198	59.6492
σ_C^*	inv. gamma	50	inf	53.1573	48.2783	56.4603
σ_L	inv. gamma	150	inf	172.4156	130.1887	206.3316
σ_L^*	inv. gamma	150	inf	347.4467	291.9749	409.5791
σ_G	inv. gamma	5	inf	0.9943	0.9083	1.0911
σ_G^*	inv. gamma	5	inf	1.2500	1.0816	1.4728
σ_I	inv. gamma	50	inf	46.3276	40.5957	51.7512
σ_I^*	inv. gamma	50	inf	43.4307	41.2986	45.8492
σ_R	inv. gamma	15	inf	16.9330	15.5673	19.1302
σ_{TC}	inv. gamma	5	inf	0.8158	0.7567	0.8912
σ_{TC}^*	inv. gamma	5	inf	1.1631	1.0601	1.2686
σ_{TK}	inv. gamma	30	inf	35.8199	30.6624	40.4097
σ_{TK}^*	inv. gamma	30	inf	28.5387	23.9674	32.7126
σ_{TL}	inv. gamma	5	inf	1.1178	1.0049	1.2343
σ_{TL}^*	inv. gamma	5	inf	1.4885	1.2497	1.7540
σ_Z	inv. gamma	5	inf	1.1839	0.998305	1.3717

σ_Z^*	inv. gamma	5	inf	1.5146	1.3826	1.6596
$corr_{D,D}^*$	normal	0.5	0.4	0.7236	0.6643	0.7997
$corr_{N,N}^*$	normal	0.5	0.4	0.9021	0.8839	0.9311
$corr_{L,L}^*$	normal	0.5	0.4	0.0140	-0.1471	0.2064
$corr_{G,G}^*$	normal	0.5	0.4	0.3610	0.0861	0.5046
$corr_{TC,TC}^*$	normal	0.5	0.4	0.4889	0.3051	0.6329
$corr_{TK,TK}^*$	normal	0.5	0.4	0.5617	0.4715	0.6832
$corr_{TL,TL}^*$	normal	0.5	0.4	0.4100	0.2713	0.5456
$corr_{Z,Z}^*$	normal	0.5	0.4	0.3879	0.0810	0.5736

In addition to the prior distribution, tables 2, 3 and 4 reports the parameter estimates resulting from the Bayesian estimation with the Metropolis-Hastings algorithm. The results stress the importance to consider the two blocks of countries, North and South, as heterogeneous blocks, since the parameter estimates can differ substantially. The inverse elasticity of labor supply is estimated to be higher in South, in contrast to the indexation of prices and wages. These estimates are in line with the estimates of Jondeau & Sahuc (2008). The Calvo parameters do not differ as much. Capital adjustment costs are lower in North than in South, which is in line with evidence that these costs are low in Germany, the biggest country of the northern block (Pytlarczyk (2005)).

The most interesting results for the purpose of this paper are the estimates on the regional fiscal policy parameters. The smoothing parameters for the policy rules do not show very large discrepancies between the two blocks. Except for capital income tax rates, the other fiscal policy variables are quite persistent. Surprisingly, the coefficient γ_G is significantly positive, suggesting that government expenditures react positively to an increase in government debt, in both the North as well as the South. Moreover, ϕ_Z is estimated to be negative, hence government transfers to households increase during recessions, and decrease during economic booms, suggesting that fiscal policy does act as an automatic stabilizer. Tax rates respond positively to government debt, meaning that tax rates increase whenever government debt increases. Since the expenditures side of the government fiscal policy is increasing with government debt, it is necessary to have a positive response of tax rates to debt for fiscal solvency. The correlations of the tax rates are

not often significantly different from zero.

The posterior estimates on the AR(1) parameters of the shocks in table 4 suggest that the persistence of most shocks is larger in the North than in the South, except for a shock on domestic tradables and investment efficiency. Especially a shock to labor supply preferences is not as lasting in the South, a fact that might be related to the higher indexation of wages in region. Estimates on the volatilities of shocks differ in size, indicating that shocks to consumer preferences, labor supply preferences and investment efficiency have much larger standard deviations than other shocks. In particular, the shock to labor supply preferences is significantly more volatile in South. Evidence is found that there are positive cross-region correlations of shocks, especially in the case of productivity shocks.

4.4 Robustness analysis

In order to establish the robustness of the posterior estimates of the model, we have run robustness checks on the priors chosen for the estimation. The priors for both parameters for the North and the South are altered at the same time, to keep a symmetric setting for the prior distribution in the estimation of the model.

Most of the parameters stay within or close to the confidence interval of the benchmark estimation, even for quite distant prior values. The general impression from the structural parameters is that the estimate is quite robust even to large changes in the prior, especially for the Calvo parameters of prices and wages. Similarly, the estimates of the AR(1) parameters of the shocks are also close to the benchmark estimate for any prior between 0 and 1.

With regards to the regional fiscal policy parameters, the response of government lump-sum transfers to households ϕ_Z as well as the responsiveness of the capital income tax rate to debt γ_{τ_K} are less robust to changing prior values of the mean. The sign of ϕ_Z is not necessarily negative in all robustness checks, which means that government transfers to households might respond positively to an increase in output, which is not in line with our expectations. However, for most priors this parameter will be negative for both the North and the South. Furthermore, we find that the parameter γ_G , which turned out to be surprisingly positive in the benchmark estimation, is also positive for different negative and positive priors. Hence, we are confident

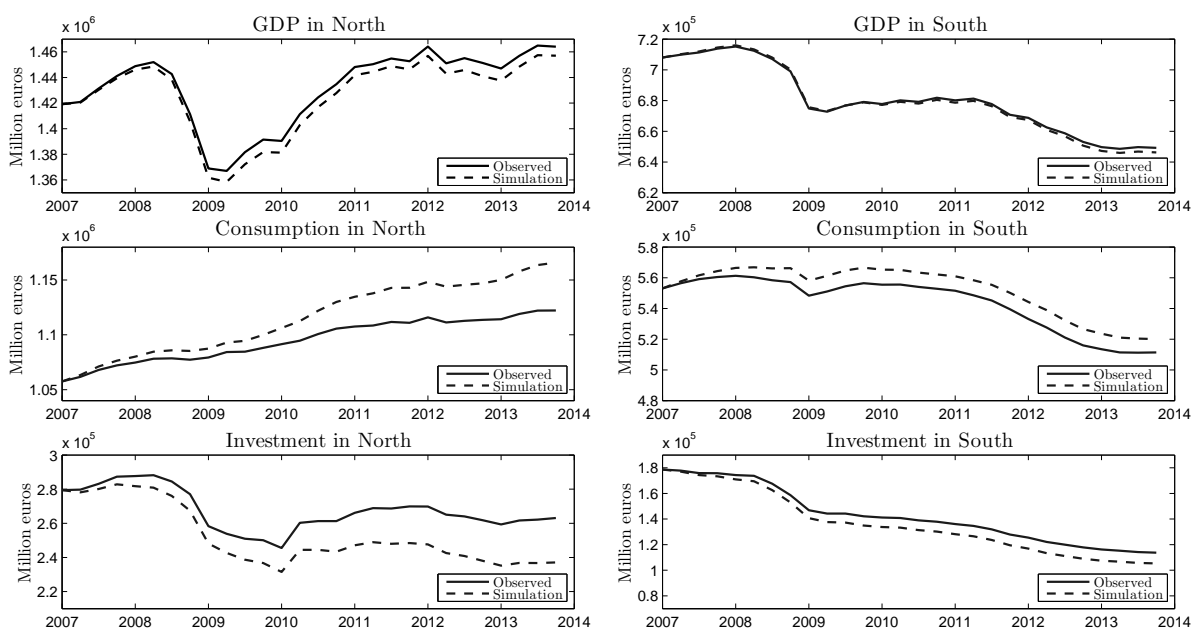
to claim that government consumption responds positively to the debt level, in both the North and the South.

4.5 Fitting the model to the data

In order to say something valuable about the effectiveness of a transfer scheme in the Euro area using our model, we need to make sure that the estimated model fits the data quite well. Figure 1 plots the observed time series and the simulations for real GDP, real consumption and real investment from the first quarter of 2007 until the last quarter of 2013.

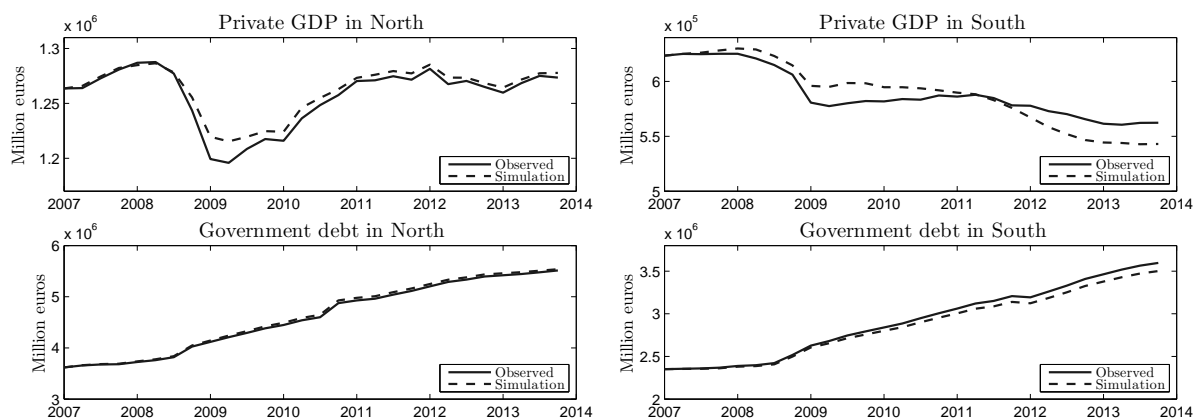
The simulations of the main macroeconomic variables are based on the policy functions and the exogenous shocks estimated by the model. The Bayesian estimation method reports the processes of the 21 shocks which we will use as the basis of the simulations. The economies start in the steady state and from then the economy responds to the exogenous shocks according to the policy functions that were estimated in the Bayesian estimation, using the estimated parameters for the North and the South. The model can reproduce the main behavior of the observed time series quite well. Although the levels do differ for some variables, the main peaks and drops are reflected well by the estimated model.

Figure 1: Model simulations of GDP, consumption and investment



Moreover, the model simulations of private GDP and the government debt level are shown in figure 2. Private GDP contains both private consumption and private investment, and is effectively GDP minus government consumption. The model is doing quite well in reproducing the observed behavior of private GDP in the North, but is doing slightly worse for private GDP in the South. The new aspect of this model is the extensive fiscal policy sector, and hence one would expect that this model does well in reproducing the time series of fiscal variables. As figure 2 shows, the simulated series of government debt closely resembles the observed time series for the North and the South.

Figure 2: Model simulations of private GDP and government debt



5 Policy experiments

The Bayesian estimation of our DSGE model for the northern and southern regions of the EMU has revealed significant heterogeneity in the structural parameters, as well as in the fiscal policy functions. The parameter estimates from the Bayesian estimation are used to simulate the behavior of the main macroeconomic variables for multiple purposes. First, we will address the first main research question of this paper, which is to what extent a fiscal transfer mechanism would have helped to stabilize the economies of the North and the South during the last years of the crisis. Welfare analysis is performed to show how welfare is affected in case a transfer mechanism is introduced. Moreover, we will have a look into the optimal size of the transfer for this period considering multiple objectives. Secondly, we will analyze the introduction of a transfer mechanism at the start of the Economic and Monetary Union in Europe, to give an answer to the question how the transfer scheme would work in that situation. Lastly, we will answer the second research question of this paper, namely whether the transfer mechanism would *ex ante* be beneficial for both regions.

5.1 Would a transfer mechanism have helped the South during the crisis?

In this section, we use the estimated DSGE model for the Euro area to analyze the effectiveness of the introduction of an automatic fiscal transfer mechanism just before the start of the most recent financial crisis. The automatic fiscal transfer mechanism is introduced in 2007, which is assumed to be the starting point for the determination of the percentage that a block receives from the common pool, implying that the net transfer equals zero in the first quarter of 2007. From then onwards, the amount received from the common pool is larger than the amount paid to the common pool for South, as we will see, so the South receives a net transfer from the North.

The fiscal transfer enters the economy through the fiscal policy rules, as it lightens the government budget by the amount difference between the receipt from the common pool and the payment to the common pool. Let us first have a look at the effect of the transfer on the fiscal policy variables. As the coefficient γ_G is estimated to be positive for both the North and the

South, we observe that government consumption increases in the North and decreases in the South as a result of the transfer. The lump-sum transfers from the government to households will decrease in the South and increase in the North, as these respond negatively to the output gap. The transfer makes the tax rates in North increase, whereas the tax rates on consumption, labor income as well as capital income will decrease in South. Government debt in North increases due to the payment of the transfer, an effect that is reinforced by the positive response of the government expenditures on the debt. In South, the government is receiving a net transfer and can therefore lower the amount of debt outstanding, which will lead them to lower their consumption as well with a further decreasing debt level as a result.

In North, the higher consumption tax rate will lead to a decrease in consumption, and the increase in the labor income tax rate causes a decrease in the net real wage, hence labor input goes down since the substitution effect of a drop in the real wage dominates the income effect. The opposite happens in South, where the drop in the tax rate on labor income leads to an increase in labor input. The lower tax rate on consumption leads to an increase in consumption in the South.

The output in the domestic tradables sector goes down in North because of the decrease in both consumption and investment. In South, the output in domestic tradables increases which comes from the increase in consumption and investment, although for the latter only until the beginning in 2012. In South, the nontradables output goes up, in spite of the decrease in government consumption, that is part of nontradables output since the government is assumed to only spend resources in the domestic nontradables sector. A striking result from the Bayesian estimation is that government expenditures increase with government debt, so the effect of the transfer on consumption and investment is reduced by the decrease in government consumption in South. Hence, if we would look at the effect of the introduction of a fiscal transfer mechanism on GDP when corrected for government expenditures¹⁵, see table 5, then the transfer would have a significantly larger impact on both the North and the South.

The fiscal transfer mechanism is designed in such a way that the region with a relatively worse position compared to the starting point or steady state will receive a net transfer from the other

¹⁵From now on we will use the term 'private GDP' for output or GDP corrected for government consumption, as this reflects private consumption and private investment in the economy.

region. More specifically, if the southern GDP is lower compared to its steady state than the ratio of last year's GDP to the steady state GDP of the North, then the South will receive a net transfer from the North. The size of the transfer will depend on the difference in the stance of the economy in both regions. If the South is experiencing a severe downturn and the North is doing rather well, then the transfer will be bigger than in the case where both regions are suffering from a recession or are experiencing an economic boom.

Countries pay 10% of their real GDP of last year to the common pool, from which the money will be redistributed according to the steady state levels of their share of GDP in the union's GDP.¹⁶ Even though the percentage paid to the common pool is quite high, the transfer is at maximum 5642 million euro in the last quarter of 2013, which is 0.39% of the GDP of the North. The South will receive this transfer which is 0.87% of southern GDP. The average size of the transfer over the period 2007-2013, being paid or received every quarter, is 1990 million euro which is 0.14% of GDP for the North and 0.31% of the South. However, the effect of the transfer is only slightly smaller in terms of the main macroeconomic variables, as can be seen in table 5.

Table 5: Effectiveness of transfer mechanism on main variables

Variable	Average effect	Maximum effect
GDP in North	-0.09%	-0.26%
GDP in South	+0.16%	+0.50%
Consumption in North	-0.19%	-0.27%
Consumption in South	+0.31%	+0.49%
Investment in North	-0.18%	-0.45%
Investment in South	+0.22%	+0.80%
Private GDP in North	-0.12%	-0.31%
Private GDP in South	+0.22%	+0.58%

¹⁶In fact, we use the starting values of GDP when the transfer mechanism is introduced as steady state values in order to make sure that the mechanism starts with zero net transfers.

The introduction of an automatic fiscal transfer mechanism in 2007 would have helped the southern countries to dampen the effects of the financial crisis on their economies. Consumption would have been 0.31% higher on average due to a transfer from the North which is on average 0.31% of GDP in the South. The northern countries would have had lower levels of GDP, consumption, investment and private GDP due to the transfer, however, the negative effect for the North is smaller than the positive effect for the South if we look at these main macroeconomic variables.¹⁷

5.1.1 Welfare

In order to assess the impact of the fiscal transfer mechanism on the consumers in both regions, we will use the consumption-equivalent welfare measure that can tell us how consumption should be affected in order to have the same utility level as before the introduction. This welfare measure is based on the utility function in which consumption affects utility positively and for which we assume that consumers dislike labor input. From the simulation results, it becomes clear that welfare in South would have increased if a transfer mechanism was introduced in 2007, since consumption has increased, and the variance of both consumption and labor have decreased. In North, consumption has decreased and the variance of consumption and labor have increased, leading to a welfare loss for the consumers in North. These welfare changes can be expressed in terms of steady state consumption, where a negative value means that a lower level of steady state consumption would be needed to make the North indifferent between the situation with transfer scheme compared to the steady state. For the South, the opposite reasoning applies, so that consumers would like to receive more steady state consumption than in the situation without the transfer mechanism in order to be equally well off as in the steady state. The results for the change in the consumption equivalent welfare measure λ , as well as for the decomposition of λ , are presented in table 6 in both percentages and in terms of steady state consumption level.

¹⁷We realize that these quantitative results depend on the assumption that the values for the fiscal policy parameters do not change when a fiscal transfer mechanism is introduced. However, the results on the effectiveness of this transfer scheme are qualitatively robust and respond quantitatively only very little to changes in values of these parameters within their confidence interval. If a decrease in the responsiveness of one tax rate to government debt is compensated with the increase of this parameter for another tax rate, the results are particularly robust.

Table 6: Consumption-equivalent welfare measure (2007-2013)

	North		South	
	In percentages	In steady state consumption	In percentages	In steady state consumption
$\Delta\lambda$	-3.02%	-29296 million euro	+3.48%	+16521 million euro
$\Delta\lambda_M$	+0.02%	-194 million euro	-0.04%	+190 million euro
$\Delta\lambda_V$	-3.04%	-29490 million euro	+3.51%	+16663 million euro
$\Delta\lambda_{M,c}$	-0.04%	-388 million euro	+0.08%	+380 million euro
$\Delta\lambda_{M,l}$	+0.07%	+679 million euro	-0.11%	-522 million euro
$\Delta\lambda_{V,c}$	-0.19%	-1843 million euro	+0.27%	+1282 million euro
$\Delta\lambda_{V,l}$	-2.85%	-27647 million euro	+3.25%	+15429 million euro

The table reports the λ over the period 2007 until 2013 for both the North and the South, which has the expected sign. The second column shows that consumption in the North should have been 3.02% higher in order to experience the same welfare level as without the transfer mechanism. The minus sign thus indicates that the North would experience a welfare loss if the mechanism would be introduced. Since South benefits from this mechanism, consumption should have been 3.48% lower in order for South to attain the level of welfare that it would have had without the transfers, hence the welfare gain is 3.48%. If we would express the welfare gains and losses in terms of steady state consumption, we find that the North should have had 29296 million euro in order to have the same welfare as without the transfers, which is the welfare loss. On the other hand, for the South, the welfare gain is equal to 16521 million euro. The welfare gain is lower than the welfare loss indicating that for the union as a whole the effect of a transfer scheme on welfare would be negative.

5.1.2 Optimal size of the transfer

Depending on the objective, we could determine the optimal size of the transfer if such a mechanism would have been introduced just before the start of the crisis. The size of the

transfer would then be determined by the amount paid by each region to the common pool, i.e. ψ_{PAY} ¹⁸. If policymakers take into consideration only the absolute welfare gains and losses, the transfer mechanism would not have been introduced. However, if the objective would to minimize the weighted sum of the variance of output, the optimal ψ_{PAY} would be 0.2. Each region should pay 40% of its GDP to the common pool to minimize the weighted variance of consumption, whereas ψ_{PAY} should be equal to 0.1 in order to minimize the weighted variance of private GDP. Hence, a high transfer might work better for stabilizing consumption fluctuations. However, taking into account the considerable welfare loss for the North, a transfer of smaller size might be more attainable.

5.2 Who will receive the transfer?

Rather than introducing the fiscal transfer mechanism in 2007, one could also imagine the implementation of such a transfer mechanism from the start of the EMU. When we introduce the fiscal transfer mechanism at the start of our simulation period¹⁹, the simulations depict an interesting result. Based on the share of GDP of both North and South in the second quarter of 2000, the net transfer will go from South to North, hence the South will actually be paying to the North until 2010.

The design of the transfer mechanism implemented in our model, with payments proportional to GDP and receipts on the basis of the share of GDP in the GDP of the union at the introduction of the mechanism, implies that the North as well as the South can become the one profiting from the transfer mechanism. The South has grown faster than the North from 2000 until 2004, which is the reason that the North has become a net receiver. Afterwards, it still takes time for the North to catch up with the South, which is the reason why the transfer goes in the direction of the northern countries until 2010.

¹⁸Here, the maximum amount of transfer that is still feasible within the model is constructed with $\psi_{PAY} = 0.475$.

¹⁹Please note that the start of our simulation period, i.e. the second quarter of 2000, is not the same as the official start of the EMU, or the effective start with the introduction of the euro. However, this is the closest one could get to this, since we would like to have the transfer mechanism starting with a zero net transfer in the first period to make sure that the system is able to get political support.

In terms of political feasibility, this aspect of the fiscal transfer mechanism can help in convincing countries that the system will not be a one-way street. Depending on the starting year, which is also an important factor to play with in the public debate, the transfers could go in either a northern or a southern direction. But most importantly, upfront there are no clear winners or losers, since the transfer will be based on the growth of GDP that economies experience after the start. As the above mentioned aspects are reasons for mainly the northern countries not to head towards deeper fiscal integration, this result might increase the support for an automatic fiscal transfer mechanism or any other type of fiscal integration.

5.3 Would a transfer mechanism be beneficial for the future?

In order to assess whether an automatic fiscal transfer scheme could actually be implemented in the Euro area, it is important to gain knowledge about the ex ante implications of such a mechanism. Without clear ex ante benefits to both regions, there would be no common ground for the North and the South of Europe to take off with such an initiative. For this purpose, the model has been simulated for 200 periods in the future, equal to 50 years, using random shocks²⁰ occurring every period in both the North and the South.²¹ A transfer scheme would be deemed valuable to regions involved if the transfer would be expected to lead to increased welfare²². The average results over 10.000 simulations show us what countries could expect of a transfer system in terms of long run welfare effects.

²⁰We only take into account the main shocks of the model, i.e. shocks to productivity and investment efficiency. Shocks to consumer and labor supply preferences affect both the behavior of the economy as well as the welfare function itself, which makes identification of the welfare effects of the transfer hardly possible. Fiscal policy shocks are ignored else these shocks might interfere with the working of the transfer scheme, and identification of the effect of the transfer scheme would be hindered.

²¹The distribution of shocks is given by the estimated mean and variance of the shocks, estimated over the period 2000-2013 by the model. Both the North and the South can experience a shock every period. However, there is also the possibility that a region is not affected by a shock in a certain period.

²²For welfare, we use the consumption equivalent measure λ as explained in section 3.6. In the table we report the percentage welfare change due to the transfer scheme in terms of steady state consumption.

Table 7: Simulation for the future: welfare effect of transfer mechanism

	North		South	
	In percentages	In steady state consumption	In percentages	In steady state consumption
$\Delta\lambda$	+2.37%	+22990 million euro	+3.08%	+14622 million euro
$\Delta\lambda_M$	+0.02%	+194 million euro	-0.06%	-285 million euro
$\Delta\lambda_V$	+2.35%	+22796 million euro	+3.14%	+14907 million euro
$\Delta\lambda_{M,c}$	-0.08%	-776 million euro	+0.14%	+665 million euro
$\Delta\lambda_{M,l}$	+0.10%	+970 million euro	-0.20%	-950 million euro
$\Delta\lambda_{V,c}$	+0.29%	+2813 million euro	+0.03%	+142 million euro
$\Delta\lambda_{V,l}$	+2.06%	+19983 million euro	+3.11%	+14765 million euro

Table 7 depicts the change in welfare, and the decomposed welfare components, caused by the presence of a fiscal transfer mechanism. The transfer mechanism is expected to be beneficial for both North and South, as there are substantial welfare gains for both regions. On average over the simulations, the welfare benefit would be equivalent to 22990 million euro in the North and 14622 million euro in the South.

The welfare decomposition shows that the transfer mechanism is expected to do well in terms of stabilizing the economy, as both the welfare component related to the variance of labor input and the variance of consumption are positive for both regions. Although there is a negative level effect for the mean of consumption in the North and for the mean of labor input in the South, these numbers are negligible compared to the positive welfare components. Hence, a transfer mechanism would ex ante be beneficial for both the North and the South in terms of welfare projections.

The direction of the transfer during the simulated future period of 50 years will also matter for the attainability of a fiscal transfer scheme, since a region is not willing to participate in such a scheme if there is a possibility that the transfer will always go in the direction of the other region. In our 10.000 simulations, it does not happen once that the transfer only goes into

one direction during the course of 200 periods. Hence, ex ante, regions can expect that in the future the transfer will go both from the North to South and vice versa. Moreover, it is almost as likely that the North is receiving a transfer from the South as the other way around. In the simulations, on average the North is receiving the transfer in 48.1% of the future periods and the South is receiving it 51.9% of the periods. Therefore, we can conclude that the design of the transfer mechanism does not have clear winners and losers ex ante.

6 Risk sharing

6.1 Variance decomposition of shocks to output

To relate the analysis in this paper to the empirical literature on the channels of risk sharing, we will decompose the variance of GDP into three different channels of risk sharing and compare these with the empirical estimates for existing monetary unions. The most influential empirical paper on this topic is Asdrubali et al. (1996), who estimate that around 10% of a shock to regional GDP on the state level is insured by the US federal fiscal system. Sorensen & Yosha (1998) use the same methods to estimate the regional income insurance also in European countries, and find that there is hardly any risk sharing through the federal government. We will use these methods to measure the fraction of shocks to output that are smoothed via the transfer mechanism.

The approach by Asdrubali et al. (1996) decomposes cross-country variance in GDP into several components to identify the amount of capital market smoothing, credit market smoothing and federal government smoothing. The following identity for GDP is used²³:

$$gdp_t = \frac{gdp_t}{ni_t} \cdot \frac{ni_t}{dni_t} \cdot \frac{dni_t}{c_t} \cdot c_t \quad (72)$$

where gdp_t is the output level in either one of the regions at time t , ni_t denotes real national income, dni_t denotes real disposable national income and c_t is the sum of private consumption expenditures. The difference between GDP and national income consists of factor payments across regions as well as capital depreciation, but we do not observe any factor payments across the border in our model. Smoothing via factor payments or capital depreciation is called capital market smoothing. Disposable national income is national income corrected for the payment or receipt of federal fiscal transfers, and thus measures the scope of risk sharing through the federal budget. The third channel of inter-country risk sharing is the savings channel, as the difference between disposable national income and the private consumption expenditures is given by the savings in a country.

²³Instead of GDP and the sum of private and public consumption, we will use private GDP and private consumption in order to identify the risk sharing channels beyond the disturbing behavior of government consumption.

Asdrubali et al. (1996) express the identity in equation (72) in terms of log-deviations from the steady state, multiply both sides by the log-deviation of GDP and then take expectations of both sides to get the following expression:

$$\begin{aligned} \text{var}(\hat{gdp}_t) &= \text{cov}(\hat{gdp}_t - \hat{n}i_t, \hat{gdp}_t) + \text{cov}(\hat{n}i_t - \hat{d}n i_t, \hat{gdp}_t) \\ &\quad + \text{cov}(\hat{d}n i_t - \hat{c}_t, \hat{gdp}_t) + \text{cov}(\hat{c}_t, \hat{gdp}_t) \end{aligned} \quad (73)$$

If we now divide the equation by the variance of GDP, we find the coefficients that indicate the size of the risk sharing channels:

$$1 = \beta_K + \beta_F + \beta_S + \beta_U \quad (74)$$

where for example $\beta_F = \frac{\text{cov}(\hat{n}i_t - \hat{d}n i_t, \hat{gdp}_t)}{\text{var}(\hat{gdp}_t)}$ is the amount of a shock that is smoothed by the federal budget. The same holds for β_K with capital market smoothing and β_S with smoothing by savings, and β_U represents the amount that is unsmoothed. Full risk sharing between the two regions in the monetary union would thus mean that $\beta_U = 0$, and the larger β_F , the more smoothing of asymmetric shocks to GDP is achieved by federal fiscal arrangements. The estimation of the model in Dynare will give us information on the variance and covariance of these variables, and the results will be presented in the next section.

6.2 Channels of risk sharing between regions

Using the method explained in section 6.1, we will quantitatively analyze the amount of risk sharing by the fiscal transfer mechanism for shocks to GDP. The results of the variance decomposition of output are presented for the situation without and with a transfer mechanism²⁴. Table 8 shows the results of the variance decomposition for the estimated model for productivity shocks that affect GDP²⁵.

When there is no transfer mechanism, there is no risk sharing through the federal channel in the North and the South since only the transfer would affect disposable national income in the model. In reality, the federal government in the EU also has regional programs for structural inequalities that would affect disposable national income, but these programs are not

²⁴The parameter ψ_{PAY} for the size of the transfer is set to $\psi_{PAY} = 0.1$.

²⁵In the analysis here, we disregard government expenditures in order to have a clear idea of what a transfer mechanism could mean in terms of risk sharing for consumers in the North and the South of the EMU.

included in our model. The transfer mechanism in this monetary union will smooth 29.9% of a productivity shock in North and 28.0% of a shock in South. This means that ex ante the transfer mechanism has larger benefits in terms of interregional risk sharing for North than for South, as the simulations for the future in section 5.3 also showed. The amount of smoothing via the capital market is reduced, and the savings channel becomes less important due to the fiscal transfer mechanism. Apparently, the fiscal transfers crowd out private savings such that savings do not co-move as much with shocks to GDP. The benefit of the introduction of the transfer mechanism is clear from the last column in table 8, which shows that the amount of risk that is unsmoothed decreases in both North and South when a transfer scheme is introduced. Hence, a consumer is less affected by a shock to the GDP of its country in case of fiscal transfers. This quantitative assessment of the channels of interregional risk sharing in our two-region model of the Euro area is relatively close to the estimates of Asdrubali et al. (1996) for the United States. The main difference is that the capital market channel appears to be much more important in the United States, whereas the effect that federal fiscal arrangements, and in our model the fiscal transfers, have on interregional risk sharing is potentially larger for the EMU.

Table 8: Channels of risk sharing for productivity shocks

Scenario	Capital market β_K	Transfer mechanism β_F	Savings β_S	Unsmoothed β_U
North without transfer	32.3%	0.0%	32.3%	35.4%
South without transfer	30.9%	0.0%	35.3%	33.8%
North with transfer	22.6%	29.9%	21.3%	26.3%
South with transfer	22.0%	28.0%	24.3%	25.7%

7 Conclusion

This paper provides both a qualitative and a quantitative analysis of the effectiveness of a transfer mechanism in the Economic and Monetary Union, for the past and the future. A DSGE model with two asymmetric regions is estimated using Bayesian methods for a northern and southern block of countries within the Euro area, and the simulations of this model follow the observed data quite closely. The estimated model is used to simulate the hypothetical introduction of an automatic fiscal transfer mechanism in 2007 in the EMU. Moreover, the ex ante implications for the future are studied, by running 10.000 simulations over 50 years in order to get a clear picture on stabilization and welfare consequences.

The design of the transfer scheme is innovative with regard to the implementability of the mechanism in the Euro area. Due to the dependence of the transfer on the relative growth rate in GDP, moral hazard is eliminated as much as possible. Moreover, there are no clear winners or losers upfront, which will make policymakers more eager to implement such a scheme. When introduced in 2007, the transfer scheme would have resulted in a transfer from the North to the South. However, if such a mechanism was introduced in 2000, the South would have been paying a transfer to the North for almost 10 years. The future simulations with random shocks show that over all simulations with 200 periods, the transfer will never go in one way and both regions are equally likely to become net recipient, underlining the political feasibility of this specific transfer mechanism compared to other schemes proposed in the literature.

The analysis of the estimated model and simulations suggests that a transfer mechanism would have been effective in stabilizing the southern economy in the financial crisis. Over the period between 2007 and 2013, consumption and private GDP would have been, respectively, 0.31% and 0.22% higher, achieved by a transfer of 0.31% of southern GDP. This mechanism would have led to a welfare loss for the North that is larger than the welfare gain for the South in absolute terms. The decision for policymakers on whether to introduce a transfer scheme, and if so, then to what extent, depends largely on the objective chosen. If the objective would be to maximize the weighted sum of welfare (in absolute terms) no transfer would have been introduced in 2007, whereas a large transfer scheme would have been introduced if the objective function would entail the weighted variance of consumption.

The discussion about the implementation of a fiscal transfer scheme will depend on policymakers' expectations for the future. The simulations for the future using random shocks show that the transfer scheme would be ex ante beneficial for both the North and the South, as the introduction would imply an average welfare gain of 2.37% for the North and 3.08% for the South. These positive welfare expectations for the future might increase the support for such a way of deeper fiscal integration within the Euro area.

There are several interesting directions along which future research could be extended. More countries could be introduced in the model in order to give a clear representation of the whole EMU and what a transfer mechanism could mean for all countries involved. Furthermore, as there are also discussions about whether there should be even more integration in the form of a fiscal union, one could compare the transfer mechanism as designed here with a common fiscal authority that decides on government expenditures and taxes for all member countries. Finally, the question on how the optimal fiscal transfer rule would look like is one that is stimulated to be answered in the future by the findings in this paper.

References

- Antia, Z., Djoudad, R., and St-Amant, P. (1999). Canada's Exchange Rate Regime and North American Economic Integration: The Role of Risk-Sharing Mechanisms. *Working Papers 99-17, Bank of Canada*.
- Asdrubali, P., Sorensen, B. E., and Yosha, O. (1996). Channels of Interstate Risk Sharing: United States 1963-1990. *The Quarterly Journal of Economics*, 111(4):1081–1110.
- Bargain, O., Dolls, M., Fuest, C., Neumann, D., Peichl, A., Siegloch, S., and Pestel, N. (2012). Fiscal Union in Europe? Redistributive and Stabilising Effects of a European Tax-Benefit System and Fiscal Equalisation Mechanism Fiscal Union in Europe? *Oxford University Centre for Business Taxation Working Papers*, 12/22(September).
- Beetsma, R. M. and Jensen, H. (2005). Monetary and fiscal policy interactions in a micro-founded model of a monetary union. *Journal of International Economics*, 67(2):320–352.
- Bordo, M. D., Markiewicz, A., and Jonung, L. (2011). A Fiscal Union for the Euro: Some Lessons from History. *NBER Working Paper Series*, 17380(September).
- Chari, V. V., Kehoe, P. J., and Grattan, E. R. M. C. (2002). Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates? *Review of Economic Studies*, 69(3):533–563.
- Christiano, L. J., Eichenbaum, M., and Evans, C. L. (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*, 113(1):1–45.
- Delors, J. (1989). Regional Implications of Economic and Monetary Integration. *in Committee for the Study of Economic and Monetary Union, eds., Report on Economic and Monetary Union in the European Community, Luxembourg: Office for Official Publications of the EU*.
- Enderlein, H., Guttenberg, L., and Spiess, J. (2013). Blueprint for a cyclical shock insurance in the euro area. *Jacques Delors Institute Studies & Reports*, (September).
- Engler, P., Ganelli, G., Tervala, J., and Voigts, S. (2013). Fiscal devaluation in a Monetary Union. *Discussion Papers 2013/18, Free University of Berlin, School of Business & Economics*.

- Engler, P. and Voigts, S. (2013). A Transfer Mechanism for a Monetary Union. *Discussion Papers 2013/2, Free University of Berlin, School of Business & Economics*.
- European Commission (1977). Report of the Study Group on the Role of Public Finance in European Integration. Technical report, Studies: Economic and Fiscal Series A 13 (Vol. I), Brussels.
- Evers, M. P. (2006). Federal fiscal transfers in monetary unions: A NOEM approach. *International Tax and Public Finance*, 13(4):463–488.
- Evers, M. P. (2012). Federal fiscal transfer rules in monetary unions. *European Economic Review*, 56(3):507–525.
- Evers, M. P. (2015). Fiscal Federalism and Monetary Unions: A Quantitative Assessment. *Journal of International Economics*, 97(1):59–75.
- Farhi, E. and Werning, I. (2012a). Fiscal Multipliers: Liquidity Traps and Currency Unions. *NBER Working Papers 18381, National Bureau of Economic Research, Inc.*
- Farhi, E. and Werning, I. (2012b). Fiscal Unions. *NBER Working Paper Series*, 18280(August).
- Forni, L., Monteforte, L., and Sessa, L. (2009). The general equilibrium effects of fiscal policy: Estimates for the Euro area. *Journal of Public Economics*, 93(3-4):559–585.
- Furceri, D. and Zdzienicka, A. (2013). The Euro Area Crisis: Need for a Supranational Fiscal Risk Sharing Mechanism? *IMF Working Papers 13/198, International Monetary Fund*.
- Galí, J. and Monacelli, T. (2008). Optimal monetary and fiscal policy in a currency union. *Journal of International Economics*, 76(1):116–132.
- Grudkowska, S. (2011). Demetra+ User Manual. *National Bank of Poland*, October.
- Hepp, R. and von Hagen, J. (2013). Interstate risk sharing in Germany: 1970-2006. *Oxford Economic Papers*, 65(1):1–24.
- Hollmayr, J. (2012). Fiscal Spillovers and Monetary Policy Transmission in the Euro Area. *Working Paper*.

- Iwata, Y. (2009). Fiscal Policy in an Estimated DSGE Model of the Japanese Economy: Do Non-Ricardian Households Explain All ? *ESRI Discussion Paper Series*, (216).
- Jondeau, E. and Sahuc, J.-G. (2008). Optimal Monetary Policy in an Estimated DSGE Model of the Euro Area with. *International Journal of Central Banking*, 4(2):23–72.
- Kenen, P. B. (1969). The Theory of Optimum Currency Areas: An Eclectic View. In *Monetary Problems of the International Economy* by R.A. Mundell and A.K. Swoboda, pages 41–60.
- Kolasa, M. (2009). Structural heterogeneity or asymmetric shocks? Poland and the euro area through the lens of a two-country DSGE model. *Economic Modelling*, 26(6):1245–1269.
- Lucas, R. E. (2003). Macroeconomic Priorities. *American Economic Review*, 93(1):1–14.
- Masson, P. R. and Taylor, M. P. (1993). Fiscal Policy within Common Currency Areas. *Journal of Common Market Studies*, 31(1):29–44.
- McKinnon, R. I. (1963). Optimum currency areas. *The American Economic Review*, 53(4):717–725.
- Melitz, J. and Zumer, F. (1999). Interregional and international risk-sharing and lessons for EMU. *Carnegie-Rochester Conference Series on Public Policy*, 51(1):149–188.
- Mundell, R. A. (1961). A Theory of Optimum Currency Areas. *The American Economic Review*, 51(4):657–665.
- Obstfeld, M. and Rogoff, K. (1995). Exchange Rate Dynamics Redux. *The Journal of Political Economy*, 103(3):624–660.
- Philippon, T. and Martin, P. (2014). Inspecting the Mechanism: Leverage and the Great Recession in the Eurozone. *NBER Working Paper Series*, 20572(October).
- Pytlarczyk, E. (2005). An estimated DSGE model for the German economy within the euro area. *Discussion Paper Series 1: Economic Studies 2005, 33, Deutsche Bundesbank, Research Centre*.
- Ratto, M., Roeger, W., and In 't Veld, J. (2006). Fiscal policy in an estimated open-economy model for the Euro area. *DG ECFIN Economic Papers*, (266).

- Rohe, O. and Jerger, J. (2012). Testing for Parameter Stability in DSGE Models. The Cases of France, Germany, Italy, and Spain. *Working Papers 118, Bavarian Graduate Program in Economics (BGPE)*.
- Sala-i Martin, X. and Sachs, J. (1992). Fiscal Federalism and Optimum Currency Areas: Evidence for Europe from the United States. *CEPR Discussion Papers 632*.
- Smets, F. and Wouters, R. (2003). An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association*, 1(5):1123–1175.
- Sorensen, B. E. and Yosha, O. (1998). International risk sharing and European monetary unification. *Journal of International Economics*, 45(2):211–238.
- Stähler, N. and Thomas, C. (2012). FiMod - A DSGE model for fiscal policy simulations. *Economic Modelling*, 29(2):239–261.
- van Rompuy, H. (2012). Towards a Genuine Economic and Monetary Union: Report by President of the European Council. Technical Report June.

Appendix 1: Log-linearized model

This section contains the log-linearized equations of the model. The variables in small letters with a hat denote the log deviations from the steady state. The constant terms with bar and without time subscript are the steady state values of the corresponding variables.

Market-clearing conditions

$$\hat{y}_{N,t}^i = \frac{\bar{C}}{\bar{Y}_N}(1 - \gamma_C) \left(\hat{c}_t^i - \gamma_C \hat{x}_t^i \right) + \frac{\bar{I}}{\bar{Y}_N}(1 - \gamma_I) \left(\hat{i}_t^i - \gamma_I \hat{x}_t^i \right) + \frac{\bar{G}}{\bar{Y}_N} \hat{g}_t^i \quad (\text{A.1})$$

$$\hat{y}_{N,t}^{i*} = \frac{\bar{C}^*}{\bar{Y}_N^*}(1 - \gamma_C^*) \left(\hat{c}_t^{i*} - \gamma_C^* \hat{x}_t^{i*} \right) + \frac{\bar{I}^*}{\bar{Y}_N^*}(1 - \gamma_I^*) \left(\hat{i}_t^{i*} - \gamma_I^* \hat{x}_t^{i*} \right) + \frac{\bar{G}^*}{\bar{Y}_N^*} \hat{g}_t^{i*} \quad (\text{A.2})$$

$$\begin{aligned} \hat{y}_{D,t}^i &= \frac{\bar{C}}{\bar{Y}_D} \gamma_C \alpha \left(\hat{c}_t^i + (1 - \gamma_C) \hat{x}_t^i + (1 - \alpha) \hat{t}_{i^*,t}^i \right) \\ &\quad + \frac{\bar{I}}{\bar{Y}_D} \gamma_I \alpha \left(\hat{i}_t^i + (1 - \gamma_I) \hat{x}_t^i + (1 - \alpha) \hat{t}_{i^*,t}^i \right) \\ &\quad + \frac{\bar{C}^*}{\bar{Y}_D} \frac{1 - n}{n} \gamma_C^* (1 - \alpha^*) \left(\hat{c}_t^{i*} + (1 - \gamma_C^*) \hat{x}_t^{i*} + \alpha^* \hat{t}_{i^*,t}^i \right) \\ &\quad + \frac{\bar{I}^*}{\bar{Y}_D} \frac{1 - n}{n} \gamma_I^* (1 - \alpha^*) \left(\hat{i}_t^{i*} + (1 - \gamma_I^*) \hat{x}_t^{i*} + \alpha^* \hat{t}_{i^*,t}^i \right) \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} \hat{y}_{D,t}^{i*} &= \frac{\bar{C}^*}{\bar{Y}_D^*} \gamma_C^* \alpha^* \left(\hat{c}_t^{i*} + (1 - \gamma_C^*) \hat{x}_t^{i*} - (1 - \alpha^*) \hat{t}_{i^*,t}^i \right) \\ &\quad + \frac{\bar{I}^*}{\bar{Y}_D^*} \gamma_I^* \alpha^* \left(\hat{i}_t^{i*} + (1 - \gamma_I^*) \hat{x}_t^{i*} - (1 - \alpha^*) \hat{t}_{i^*,t}^i \right) \\ &\quad + \frac{\bar{C}}{\bar{Y}_D^*} \frac{n}{1 - n} \gamma_C (1 - \alpha) \left(\hat{c}_t^i + (1 - \gamma_C) \hat{x}_t^i - \alpha \hat{t}_{i^*,t}^i \right) \\ &\quad + \frac{\bar{I}}{\bar{Y}_D^*} \frac{n}{1 - n} \gamma_I (1 - \alpha) \left(\hat{i}_t^i + (1 - \gamma_I) \hat{x}_t^i - \alpha \hat{t}_{i^*,t}^i \right) \end{aligned} \quad (\text{A.4})$$

$$\hat{y}_t^i = \frac{\bar{Y}_D}{\bar{Y}} \hat{y}_{D,t}^i + \frac{\bar{Y}_N}{\bar{Y}} \hat{y}_{N,t}^i \quad (\text{A.5})$$

$$\hat{y}_t^{i*} = \frac{\bar{Y}_D^*}{\bar{Y}^*} \hat{y}_{D,t}^{i*} + \frac{\bar{Y}_N^*}{\bar{Y}^*} \hat{y}_{N,t}^{i*} \quad (\text{A.6})$$

Consumption (Euler) equations

$$\begin{aligned} \hat{c}_t^i - h \hat{c}_{t-1}^i &= \text{E}_t \left(\hat{c}_{t+1}^i - h \hat{c}_t^i \right) - \frac{1 - h}{\sigma} \left(\hat{r}_t - \text{E}_t \hat{\pi}_{C,t+1}^i \right) + \frac{1 - h}{\sigma} \text{E}_t (\varepsilon_{C,t}^i - \varepsilon_{C,t+1}^i) \\ &\quad - \frac{1 - h}{\sigma} \left(\hat{r}_{C,t}^i - \hat{r}_{C,t+1}^i \right) \end{aligned} \quad (\text{A.7})$$

$$\begin{aligned}\hat{c}_t^{i*} - h^* \hat{c}_{t-1}^{i*} &= \text{E}_t \left(\hat{c}_{t+1}^{i*} - h^* \hat{c}_t^{i*} \right) - \frac{1-h^*}{\sigma^*} \left(\hat{r}_t - \text{E}_t \hat{\pi}_{C,t+1}^{i*} \right) + \frac{1-h^*}{\sigma^*} \text{E}_t (\varepsilon_{C,t}^{i*} - \varepsilon_{C,t+1}^{i*}) \\ &\quad - \frac{1-h^*}{\sigma^*} \left(\hat{r}_{C,t}^{i*} - \hat{r}_{C,t+1}^{i*} \right)\end{aligned}\quad (\text{A.8})$$

International risk sharing condition

$$\hat{q}_{i^*,t}^i = \varepsilon_{C,t}^{i*} - \varepsilon_{C,t}^i - \frac{\sigma^*}{1-h^*} \cdot \left(\hat{c}_t^{i*} - h^* \hat{c}_{t-1}^{i*} \right) + \frac{\sigma}{1-h} \cdot \left(\hat{c}_t^i - h \hat{c}_{t-1}^i \right)\quad (\text{A.9})$$

Capital accumulation

$$\hat{k}_{t+1}^i = (1-\delta) \hat{k}_t^i + \delta (\varepsilon_{I,t}^i + \hat{i}_t^i)\quad (\text{A.10})$$

$$\hat{k}_{t+1}^{i*} = (1-\delta^*) \hat{k}_t^{i*} + \delta^* (\varepsilon_{I,t}^{i*} + \hat{i}_t^{i*})\quad (\text{A.11})$$

Real cost of capital

$$\hat{r}_{K,t}^i = \hat{w}_t^i + \hat{l}_t^i - \hat{k}_t^i\quad (\text{A.12})$$

$$\hat{r}_{K,t}^{i*} = \hat{w}_t^{i*} + \hat{l}_t^{i*} - \hat{k}_t^{i*}\quad (\text{A.13})$$

Investment demand

$$\hat{i}_t^i - \hat{i}_{t-1}^i = \beta \text{E}_t \left(\hat{i}_{t+1}^i - \hat{i}_t^i \right) + \frac{1}{S''} \left(\hat{q}_{T,t}^i + \varepsilon_{I,t}^i \right) + \frac{\gamma_I - \gamma_C}{S''} \hat{x}_t^i\quad (\text{A.14})$$

$$\hat{i}_t^{i*} - \hat{i}_{t-1}^{i*} = \beta^* \text{E}_t \left(\hat{i}_{t+1}^{i*} - \hat{i}_t^{i*} \right) + \frac{1}{S''^*} \left(\hat{q}_{T,t}^{i*} + \varepsilon_{I,t}^{i*} \right) + \frac{\gamma_I^* - \gamma_C^*}{S''^*} \hat{x}_t^{i*}\quad (\text{A.15})$$

Price of installed capital

$$\hat{q}_{T,t}^i = \beta(1-\delta) \text{E}_t \hat{q}_{T,t+1}^i - \left(\hat{r}_t - \text{E}_t \hat{\pi}_{C,t+1}^i \right) + (1-\beta(1-\delta)) \text{E}_t \left(\hat{r}_{K,t+1}^i - \hat{r}_{K,t}^i \right)\quad (\text{A.16})$$

$$\hat{q}_{T,t}^{i*} = \beta^*(1-\delta^*) \text{E}_t \hat{q}_{T,t+1}^{i*} - \left(\hat{r}_t - \text{E}_t \hat{\pi}_{C,t+1}^{i*} \right) + (1-\beta^*(1-\delta^*)) \text{E}_t \left(\hat{r}_{K,t+1}^{i*} - \hat{r}_{K,t}^{i*} \right)\quad (\text{A.17})$$

Labor input

$$\hat{l}_t^i = \eta \left(\hat{r}_{K,t}^i - \hat{w}_t^i \right) + \frac{\bar{Y}_D}{\bar{Y}} \left(\hat{y}_{D,t}^i - a_{D,t}^i \right) + \frac{\bar{Y}_N}{\bar{Y}} \left(\hat{y}_{N,t}^i - a_{N,t}^i \right)\quad (\text{A.18})$$

$$\hat{l}_t^{i*} = \eta^* \left(\hat{r}_{K,t}^{i*} - \hat{w}_t^{i*} \right) + \frac{\bar{Y}_D^*}{\bar{Y}^*} \left(\hat{y}_{D,t}^{i*} - a_{D,t}^{i*} \right) + \frac{\bar{Y}_N^*}{\bar{Y}^*} \left(\hat{y}_{N,t}^{i*} - a_{N,t}^{i*} \right)\quad (\text{A.19})$$

Real wage rate

$$\begin{aligned} \hat{w}_t^i = & \frac{\beta}{1+\beta} E_t \hat{w}_{t+1}^i + \frac{1}{1+\beta} \hat{w}_{t-1}^i + \frac{\beta}{1+\beta} E_t \hat{\pi}_{C,t+1}^i - \frac{1+\beta\delta_W}{1+\beta} \hat{\pi}_{C,t}^i + \frac{\delta_W}{1+\beta} \hat{\pi}_{C,t-1}^i \\ & - \frac{(1-\theta_W)(1-\beta\theta_W)}{\theta_W(1+\beta)} (M\hat{R}S_t^i - \tau_{L,t}^i) \end{aligned} \quad (\text{A.20})$$

$$\begin{aligned} \hat{w}_t^{i*} = & \frac{\beta^*}{1+\beta^*} E_t \hat{w}_{t+1}^{i*} + \frac{1}{1+\beta^*} \hat{w}_{t-1}^{i*} + \frac{\beta^*}{1+\beta^*} E_t \hat{\pi}_{C,t+1}^{i*} - \frac{1+\beta^*\delta_W^*}{1+\beta^*} \hat{\pi}_{C,t}^{i*} + \frac{\delta_W^*}{1+\beta^*} \hat{\pi}_{C,t-1}^{i*} \\ & - \frac{(1-\theta_W^*)(1-\beta^*\theta_W^*)}{\theta_W^*(1+\beta^*)} (M\hat{R}S_t^{i*} - \tau_{L,t}^{i*}) \end{aligned} \quad (\text{A.21})$$

Marginal rate of substitution between consumption and labor

$$M\hat{R}S_t^i = \varepsilon_{L,t}^i - \varepsilon_{C,t}^i + \phi \cdot \hat{l}_t^i + \frac{\sigma}{1-h} (\hat{c}_t^i - h\hat{c}_{t-1}^i) \quad (\text{A.22})$$

$$M\hat{R}S_t^{i*} = \varepsilon_{L,t}^{i*} - \varepsilon_{C,t}^{i*} + \phi^* \cdot \hat{l}_t^{i*} + \frac{\sigma^*}{1-h^*} (\hat{c}_t^{i*} - h^*\hat{c}_{t-1}^{i*}) \quad (\text{A.23})$$

Domestic tradable goods Phillips curves

$$\hat{\pi}_{D,t}^i = \frac{\beta}{1+\beta\delta_D} E_t \hat{\pi}_{D,t+1}^i + \frac{\delta_D}{1+\beta\delta_D} \hat{\pi}_{D,t-1}^i + \frac{(1-\theta_D)(1-\theta_D\beta)}{\theta_D(1+\beta\delta_D)} \hat{M}C_{D,t}^i \quad (\text{A.24})$$

$$\hat{\pi}_{D,t}^{i*} = \frac{\beta^*}{1+\beta^*\delta_D^*} E_t \hat{\pi}_{D,t+1}^{i*} + \frac{\delta_D^*}{1+\beta^*\delta_D^*} \hat{\pi}_{D,t-1}^{i*} + \frac{(1-\theta_D^*)(1-\theta_D^*\beta^*)}{\theta_D^*(1+\beta^*\delta_D^*)} \hat{M}C_{D,t}^{i*} \quad (\text{A.25})$$

Nontradable goods Phillips curves

$$\hat{\pi}_{N,t}^i = \frac{\beta}{1+\beta\delta_N} E_t \hat{\pi}_{N,t+1}^i + \frac{\delta_N}{1+\beta\delta_N} \hat{\pi}_{N,t-1}^i + \frac{(1-\theta_N)(1-\theta_N\beta)}{\theta_N(1+\beta\delta_N)} \hat{M}C_{N,t}^i \quad (\text{A.26})$$

$$\hat{\pi}_{N,t}^{i*} = \frac{\beta^*}{1+\beta^*\delta_N^*} E_t \hat{\pi}_{N,t+1}^{i*} + \frac{\delta_N^*}{1+\beta^*\delta_N^*} \hat{\pi}_{N,t-1}^{i*} + \frac{(1-\theta_N^*)(1-\theta_N^*\beta^*)}{\theta_N^*(1+\beta^*\delta_N^*)} \hat{M}C_{N,t}^{i*} \quad (\text{A.27})$$

Real marginal cost for domestic tradable goods

$$\hat{M}C_{D,t}^i = (1-\eta)\hat{w}_t^i + \eta\hat{r}_{K,t}^i - a_{D,t}^i + (1-\gamma_C)\hat{x}_t^i + (1-\alpha)\hat{t}_{i*,t}^i \quad (\text{A.28})$$

$$\hat{M}C_{D,t}^{i*} = (1-\eta^*)\hat{w}_t^{i*} + \eta^*\hat{r}_{K,t}^{i*} - a_{D,t}^{i*} + (1-\gamma_C^*)\hat{x}_t^{i*} - (1-\alpha^*)\hat{t}_{i*,t}^{i*} \quad (\text{A.29})$$

Real marginal cost for nontradable goods

$$\hat{M}C_{N,t}^i = (1-\eta)\hat{w}_t^i + \eta\hat{r}_{K,t}^i - a_{N,t}^i - \gamma_C\hat{x}_t^i \quad (\text{A.30})$$

$$\hat{M}C_{N,t}^{i*} = (1-\eta^*)\hat{w}_t^{i*} + \eta^*\hat{r}_{K,t}^{i*} - a_{N,t}^{i*} - \gamma_C^*\hat{x}_t^{i*} \quad (\text{A.31})$$

Relative inflation of nontradable consumption goods

$$\Delta \hat{x}_t^i = \hat{\pi}_{N,t}^i - \hat{\pi}_{T,t}^i \quad (\text{A.32})$$

$$\Delta \hat{x}_t^{i*} = \hat{\pi}_{N,t}^{i*} - \hat{\pi}_{T,t}^{i*} \quad (\text{A.33})$$

Inflation of tradable consumption goods

$$\hat{\pi}_{T,t}^i = \hat{\pi}_{D,t}^i + (1 - \alpha) \Delta \hat{t}_{i*,t}^i \quad (\text{A.34})$$

$$\hat{\pi}_{T,t}^{i*} = \hat{\pi}_{D,t}^{i*} - (1 - \alpha^*) \Delta \hat{t}_{i*,t}^i \quad (\text{A.35})$$

CPI inflation

$$\hat{\pi}_{C,t}^i = \gamma_C \hat{\pi}_{T,t}^i + (1 - \gamma_C) \hat{\pi}_{N,t}^i \quad (\text{A.36})$$

$$\hat{\pi}_{C,t}^{i*} = \gamma_C^* \hat{\pi}_{T,t}^{i*} + (1 - \gamma_C^*) \hat{\pi}_{N,t}^{i*} \quad (\text{A.37})$$

Real exchange rate

$$\hat{q}_{i*,t}^i = (\alpha + \alpha^* - 1) \cdot \hat{t}_{i*,t}^i + (1 - \gamma_C^*) \hat{x}_t^{i*} - (1 - \gamma_C) \hat{x}_t^i \quad (\text{A.38})$$

Monetary policy rule

$$\hat{i}_t = \rho \cdot \hat{r}_{t-1}^{EMU} + (1 - \rho) \cdot \left[\psi_y \cdot \hat{y}_{t-1}^{EMU} + \psi_\pi \hat{\pi}_{C,t-1}^{EMU} \right] + u_{R,t} \quad (\text{A.39})$$

Real interest rate

$$\hat{r}_t^i = \hat{i}_t - \text{E}_t \hat{\pi}_{C,t+1}^i \quad (\text{A.40})$$

$$\hat{r}_t^{i*} = \hat{i}_t - \text{E}_t \hat{\pi}_{C,t+1}^{i*} \quad (\text{A.41})$$

Union-wide real interest rate, output and inflation

$$\hat{r}_t^{EMU} = n \cdot \hat{r}_t^i + (1 - n) \cdot \hat{r}_t^{i*} \quad (\text{A.42})$$

$$\hat{y}_t^{EMU} = n \cdot \hat{y}_t^i + (1 - n) \cdot \hat{y}_t^{i*} \quad (\text{A.43})$$

$$\hat{\pi}_t^{EMU} = n \cdot \hat{\pi}_t^i + (1 - n) \cdot \hat{\pi}_t^{i*} \quad (\text{A.44})$$

Government consumption

$$\hat{g}_t^i = \rho_G \cdot \hat{g}_{t-1}^i + \gamma_G \cdot \hat{d}_{t-1}^i + u_{G,t}^i \quad (\text{A.45})$$

$$\hat{g}_t^{i*} = \rho_G^* \cdot \hat{g}_{t-1}^{i*} + \gamma_G^* \cdot \hat{d}_{t-1}^{i*} + u_{G,t}^{i*} \quad (\text{A.46})$$

Lump-sum transfers from government to households

$$\hat{z}_t^i = \rho_Z \cdot \hat{z}_{t-1}^i + \phi_Z \cdot \hat{y}_{t-1}^i + u_{Z,t}^i \quad (\text{A.47})$$

$$\hat{z}_t^{i*} = \rho_Z^* \cdot \hat{z}_{t-1}^{i*} + \phi_Z^* \cdot \hat{y}_{t-1}^{i*} + u_{Z,t}^{i*} \quad (\text{A.48})$$

Consumption tax rate

$$\hat{\tau}_{C,t}^i = \rho_{\tau_C} \cdot \hat{\tau}_{C,t-1}^i + \gamma_{\tau_C} \cdot \hat{d}_{t-1}^i + \phi_{\tau_C \tau_K} \cdot u_{\tau_K,t}^i + \phi_{\tau_C \tau_L} \cdot u_{\tau_L,t}^i + u_{\tau_C,t}^i \quad (\text{A.49})$$

$$\hat{\tau}_{C,t}^{i*} = \rho_{\tau_C}^* \cdot \hat{\tau}_{C,t-1}^{i*} + \gamma_{\tau_C}^* \cdot \hat{d}_{t-1}^{i*} + \phi_{\tau_C \tau_K}^* \cdot u_{\tau_K,t}^{i*} + \phi_{\tau_C \tau_L}^* \cdot u_{\tau_L,t}^{i*} + u_{\tau_C,t}^{i*} \quad (\text{A.50})$$

Capital income tax rate

$$\hat{\tau}_{K,t}^i = \rho_{\tau_K} \cdot \hat{\tau}_{K,t-1}^i + \gamma_{\tau_K} \cdot \hat{d}_{t-1}^i + \phi_{\tau_K \tau_C} \cdot u_{\tau_C,t}^i + \phi_{\tau_K \tau_L} \cdot u_{\tau_L,t}^i + u_{\tau_K,t}^i \quad (\text{A.51})$$

$$\hat{\tau}_{K,t}^{i*} = \rho_{\tau_K}^* \cdot \hat{\tau}_{K,t-1}^{i*} + \gamma_{\tau_K}^* \cdot \hat{d}_{t-1}^{i*} + \phi_{\tau_K \tau_C}^* \cdot u_{\tau_C,t}^{i*} + \phi_{\tau_K \tau_L}^* \cdot u_{\tau_L,t}^{i*} + u_{\tau_K,t}^{i*} \quad (\text{A.52})$$

Labor income tax rate

$$\hat{\tau}_{L,t}^i = \rho_{\tau_L} \cdot \hat{\tau}_{L,t-1}^i + \gamma_{\tau_L} \cdot \hat{d}_{t-1}^i + \phi_{\tau_L \tau_C} \cdot u_{\tau_C,t}^i + \phi_{\tau_L \tau_K} \cdot u_{\tau_K,t}^i + u_{\tau_L,t}^i \quad (\text{A.53})$$

$$\hat{\tau}_{L,t}^{i*} = \rho_{\tau_L}^* \cdot \hat{\tau}_{L,t-1}^{i*} + \gamma_{\tau_L}^* \cdot \hat{d}_{t-1}^{i*} + \phi_{\tau_L \tau_C}^* \cdot u_{\tau_C,t}^{i*} + \phi_{\tau_L \tau_K}^* \cdot u_{\tau_K,t}^{i*} + u_{\tau_L,t}^{i*} \quad (\text{A.54})$$

Consumption tax revenues

$$taxC_t^i = \hat{\tau}_{C,t}^i + \hat{c}_t^i \quad (\text{A.55})$$

$$taxC_t^{i*} = \hat{\tau}_{C,t}^{i*} + \hat{c}_t^{i*} \quad (\text{A.56})$$

Capital income tax revenues

$$taxK_t^i = \hat{\tau}_{K,t}^i + \hat{r}_{K,t}^i + \hat{k}_t^i \quad (\text{A.57})$$

$$ta\hat{x}K_t^{i*} = \hat{\tau}_{K,t}^{i*} + \hat{r}_{K,t}^{i*} + \hat{k}_t^{i*} \quad (\text{A.58})$$

Labor income tax revenues

$$ta\hat{x}L_t^i = \hat{\tau}_{L,t}^i + \hat{w}_t^i + \hat{l}_t^i \quad (\text{A.59})$$

$$ta\hat{x}L_t^{i*} = \hat{\tau}_{L,t}^{i*} + \hat{w}_t^{i*} + \hat{l}_t^{i*} \quad (\text{A.60})$$

Government debt

$$\begin{aligned} \hat{d}_t^i = & \frac{\bar{R}}{\bar{D}} \cdot \hat{r}_t^i + \hat{d}_{t-1}^i + \frac{\bar{G}}{\bar{D}} \cdot \hat{g}_t^i + \frac{\bar{Z}}{\bar{D}} \cdot \hat{z}_t^i - \frac{T\bar{A}\bar{X}C}{\bar{D}} \cdot ta\hat{x}_{C,t}^i \\ & - \frac{T\bar{A}\bar{X}K}{\bar{D}} \cdot ta\hat{x}_{K,t}^i - \frac{T\bar{A}\bar{X}L}{\bar{D}} \cdot ta\hat{x}_{L,t}^i - \left(\frac{R\bar{E}C}{\bar{D}} \cdot r\hat{e}c_t^i - \frac{P\bar{A}Y}{\bar{D}} \cdot p\hat{a}y_t^i \right) \end{aligned} \quad (\text{A.61})$$

$$\begin{aligned} \hat{d}_t^{i*} = & \frac{\bar{R}^*}{\bar{D}^*} \cdot \hat{r}_t^{i*} + \hat{d}_{t-1}^{i*} + \frac{\bar{G}^*}{\bar{D}^*} \cdot \hat{g}_t^{i*} + \frac{\bar{Z}^*}{\bar{D}^*} \cdot \hat{z}_t^{i*} - \frac{T\bar{A}\bar{X}C^*}{\bar{D}^*} \cdot ta\hat{x}_{C,t}^{i*} \\ & - \frac{T\bar{A}\bar{X}K^*}{\bar{D}^*} \cdot ta\hat{x}_{K,t}^{i*} - \frac{T\bar{A}\bar{X}L^*}{\bar{D}^*} \cdot ta\hat{x}_{L,t}^{i*} - \left(\frac{R\bar{E}C^*}{\bar{D}^*} \cdot r\hat{e}c_t^{i*} - \frac{P\bar{A}Y^*}{\bar{D}^*} \cdot p\hat{a}y_t^{i*} \right) \end{aligned} \quad (\text{A.62})$$

Transfer mechanism

$$p\hat{a}y_t^i = \hat{y}_t^i \quad (\text{A.63})$$

$$p\hat{a}y_t^{i*} = \hat{y}_t^{i*} \quad (\text{A.64})$$

$$r\hat{e}c_t^i = \hat{c}p_t \quad (\text{A.65})$$

$$r\hat{e}c_t^{i*} = \hat{c}p_t \quad (\text{A.66})$$

$$\hat{c}p_t = \frac{\bar{Y}}{\bar{Y}_{EMU}} \cdot p\hat{a}y_t^i + \frac{\bar{Y}^*}{\bar{Y}_{EMU}} \cdot p\hat{a}y_t^{i*} \quad (\text{A.67})$$

Productivity shocks in tradable sectors

$$\hat{a}_{D,t}^i = \rho_{a,D} \cdot \hat{a}_{D,t-1}^i + u_{a,D,t}^i \quad (\text{A.68})$$

$$\hat{a}_{D,t}^{i*} = \rho_{a,D}^* \cdot \hat{a}_{D,t-1}^{i*} + u_{a,D,t}^{i*} \quad (\text{A.69})$$

Productivity shocks in nontradable sectors

$$\hat{a}_{N,t}^i = \rho_{a,N} \cdot \hat{a}_{N,t-1}^i + u_{a,N,t}^i \quad (\text{A.70})$$

$$\hat{a}_{N,t}^{i*} = \rho_{a,N}^* \cdot \hat{a}_{N,t-1}^{i*} + u_{a,N,t}^{i*} \quad (\text{A.71})$$

Consumption preference shocks

$$\hat{\varepsilon}_{C,t}^i = \rho_C \cdot \hat{\varepsilon}_{C,t-1}^i + u_{C,t}^i \quad (\text{A.72})$$

$$\hat{\varepsilon}_{C,t}^{i*} = \rho_C^* \cdot \hat{\varepsilon}_{C,t-1}^{i*} + u_{C,t}^{i*} \quad (\text{A.73})$$

Labor supply shocks

$$\hat{\varepsilon}_{L,t}^i = \rho_L \cdot \hat{\varepsilon}_{L,t-1}^i + u_{L,t}^i \quad (\text{A.74})$$

$$\hat{\varepsilon}_{L,t}^{i*} = \rho_L^* \cdot \hat{\varepsilon}_{L,t-1}^{i*} + u_{L,t}^{i*} \quad (\text{A.75})$$

Investment efficiency shocks

$$\hat{\varepsilon}_{I,t}^i = \rho_I \cdot \hat{\varepsilon}_{I,t-1}^i + u_{I,t}^i \quad (\text{A.76})$$

$$\hat{\varepsilon}_{I,t}^{i*} = \rho_I^* \cdot \hat{\varepsilon}_{I,t-1}^{i*} + u_{I,t}^{i*} \quad (\text{A.77})$$

Monetary shocks

$$u_{R,t} \quad (\text{A.78})$$

Government consumption shocks

$$u_{G,t}^i, u_{G,t}^{i*} \quad (\text{A.79})$$

Government transfer shocks

$$u_{Z,t}^i, u_{Z,t}^{i*} \quad (\text{A.80})$$

Consumption tax rate shocks

$$u_{\tau_C,t}^i, u_{\tau_C,t}^{i*} \quad (\text{A.81})$$

Capital income tax rate shocks

$$u_{\tau_K,t}^i, u_{\tau_K,t}^{i*} \quad (\text{A.82})$$

Labor income tax rate shocks

$$u_{\tau_L,t}^i, u_{\tau_L,t}^{i*} \quad (\text{A.83})$$

Appendix 2: Model Derivations

Household optimization: Derivation of first-order conditions

The optimization problem of the household j is to maximize lifetime utility subject to the budget constraint and the capital accumulation function. The corresponding Lagrangian is given by:

$$\begin{aligned} \mathcal{L}_t(j) = & \varepsilon_{C,t}^i \frac{(C_t^i(j) - hC_{t-1}^i)^{1-\sigma}}{1-\sigma} - \varepsilon_{L,t}^i \frac{(L_t^i(j))^{1+\phi}}{1+\phi} \\ & + \lambda_{C,t}(j) \left[D_t^i(j) + (1 - \tau_{L,t}^i)W_t^i(j)L_t^i(j) + (1 - \tau_{K,t}^i)R_{K,t}^i K_t^i(j) + \Pi_t^i(j) - (1 + \tau_{C,t}^i)P_{C,t}C_t^i(j) \right. \\ & \left. - P_{I,t}^i I_t^i(j) - \frac{E_t D_{t+1}^i(j)}{R_t} \right] + \lambda_{K,t}(j) \left[-K_{t+1}^i(j) + (1 - \delta)K_t^i(j) + \varepsilon_{I,t}^i \left(1 - S \left(\frac{I_t^i(j)}{I_{t-1}^i(j)} \right) \right) I_t^i(j) \right] \end{aligned}$$

where $\lambda_{C,t}(j)$ is the Lagrange multiplier on the household's budget constraint and $\lambda_{K,t}(j)$ is the Lagrange multiplier on the capital accumulation function. The first-order conditions are as follows:

$$\frac{\partial \mathcal{L}_t(j)}{\partial C_t(j)} = 0 \Rightarrow \varepsilon_{C,t}^i (C_t^i(j) - hC_{t-1}^i)^{-\sigma} = \lambda_{C,t}(j) P_{C,t}^i$$

$$\frac{\partial \mathcal{L}_t(j)}{\partial D_{t+1}(j)} = 0 \Rightarrow \lambda_{C,t}(j) = \lambda_{C,t+1}(j) \beta R_t$$

which together give the well-known Euler equation:

$$\frac{1}{R_t} = \beta E_t \left[\frac{\varepsilon_{C,t+1}^i (C_{t+1}^i(j) - hC_t^i)^{-\sigma}}{\varepsilon_{C,t}^i (C_t^i(j) - hC_{t-1}^i)^{-\sigma}} \frac{P_{C,t}^i}{P_{C,t+1}^i} \frac{(1 + \tau_{C,t}^i)}{(1 + \tau_{C,t+1}^i)} \right]$$

The first-order conditions with respect to $I_t^i(j)$ is given by:

$$\begin{aligned} \frac{\partial \mathcal{L}_t(j)}{\partial I_t(j)} = 0 \Rightarrow & \lambda_{C,t}(j) P_{I,t}^i = \lambda_{K,t}(j) \varepsilon_{I,t}^i \left(1 - S \left(\frac{I_t^i(j)}{I_{t-1}^i(j)} \right) - \frac{I_t^i(j)}{I_{t-1}^i(j)} S' \left(\frac{I_t^i(j)}{I_{t-1}^i(j)} \right) \right) \\ & + \beta E_t \lambda_{K,t+1}(j) \varepsilon_{I,t+1}^i \left(\frac{I_{t+1}^i(j)}{I_t^i(j)} \right)^2 S' \left(\frac{I_{t+1}^i(j)}{I_t^i(j)} \right) \end{aligned}$$

The relative price of installed capital is defined as $Q_{T,t}(j) \equiv \frac{\lambda_{K,t}(j)}{\lambda_{C,t}(j) P_{C,t}^i}$ (with subscript T for Tobin). If we substitute for $\lambda_{K,t}(j) = Q_{T,t}^i(j) \lambda_{C,t}(j) P_{C,t}^i$, then the first-order condition with respect to $I_t(j)$ results in:

$$\begin{aligned} \lambda_{C,t}(j) P_{I,t}^i = & Q_{T,t}^i(j) \lambda_{C,t}(j) P_{C,t}^i \varepsilon_{I,t}^i \left(1 - S \left(\frac{I_t^i(j)}{I_{t-1}^i(j)} \right) - \frac{I_t^i(j)}{I_{t-1}^i(j)} S' \left(\frac{I_t^i(j)}{I_{t-1}^i(j)} \right) \right) \\ & + \beta E_t Q_{T,t+1}^i(j) \lambda_{C,t+1}(j) P_{C,t+1}^i \varepsilon_{I,t+1}^i \left(\frac{I_{t+1}^i(j)}{I_t^i(j)} \right)^2 S' \left(\frac{I_{t+1}^i(j)}{I_t^i(j)} \right) \end{aligned}$$

$$\Leftrightarrow \frac{P_{I,t}^i}{P_{C,t}^i} = Q_{T,t}^i(j) \varepsilon_{I,t}^i \left(1 - S \left(\frac{I_t^i(j)}{I_{t-1}^i(j)} \right) - \frac{I_t^i(j)}{I_{t-1}^i(j)} S' \left(\frac{I_t^i(j)}{I_{t-1}^i(j)} \right) \right) \\ + \beta \mathbf{E}_t Q_{T,t+1}^i(j) \frac{\lambda_{C,t+1}(j)}{\lambda_{C,t}(j)} \frac{P_{C,t+1}^i}{P_{C,t}^i} \varepsilon_{I,t+1}^i \left(\frac{I_{t+1}^i(j)}{I_t^i(j)} \right)^2 S' \left(\frac{I_{t+1}^i(j)}{I_t^i(j)} \right)$$

Using from the first order condition with respect to $D_{t+1}(j)$ that $\frac{\lambda_{C,t+1}(j)}{\lambda_{C,t}(j)} = \frac{1}{\beta R_t}$ defines investment demand:

$$\frac{P_{I,t}^i}{P_{C,t}^i} = Q_{T,t}^i(j) \varepsilon_{I,t}^i \left(1 - S \left(\frac{I_t^i(j)}{I_{t-1}^i(j)} \right) - \frac{I_t^i(j)}{I_{t-1}^i(j)} S' \left(\frac{I_t^i(j)}{I_{t-1}^i(j)} \right) \right) \\ + \mathbf{E}_t \left[Q_{T,t+1}^i(j) \frac{P_{C,t+1}^i}{P_{C,t}^i R_t} \varepsilon_{I,t+1}^i \left(\frac{I_{t+1}^i(j)}{I_t^i(j)} \right)^2 S' \left(\frac{I_{t+1}^i(j)}{I_t^i(j)} \right) \right]$$

The first-order condition with respect to $K_{t+1}^i(j)$ is given by:

$$\frac{\partial \mathcal{L}_t(j)}{\partial K_{t+1}^i(j)} = 0 \Rightarrow \lambda_{K,t}(j) = \beta \mathbf{E}_t [\lambda_{C,t+1}(j) (1 - \tau_{K,t}^i) R_{K,t+1}^i] + \beta (1 - \delta) \mathbf{E}_t [\lambda_{K,t+1}(j)]$$

Using again the definition of Tobin's Q gives us:

$$Q_{T,t}^i(j) \lambda_{C,t}(j) P_{C,t}^i = \beta \mathbf{E} [\lambda_{C,t+1}(j) (1 - \tau_{K,t}^i) R_{K,t+1}^i] + \beta (1 - \delta) \mathbf{E}_t [Q_{T,t+1}^i(j) \lambda_{C,t+1}(j) P_{C,t+1}^i] \\ \Leftrightarrow Q_{T,t}^i(j) = \beta \mathbf{E}_t \left[\frac{\lambda_{C,t+1}(j)}{\lambda_{C,t}(j)} \frac{(1 - \tau_{K,t}^i) R_{K,t+1}^i}{P_{C,t}^i} \right] + \beta (1 - \delta) \mathbf{E}_t \left[Q_{T,t+1}^i(j) \frac{\lambda_{C,t+1}(j)}{\lambda_{C,t}(j)} \frac{P_{C,t+1}^i}{P_{C,t}^i} \right] \\ \Leftrightarrow Q_{T,t}^i(j) = \mathbf{E}_t \left[\frac{(1 - \tau_{K,t}^i) R_{K,t+1}^i}{P_{C,t}^i R_t} \right] + (1 - \delta) \mathbf{E}_t \left[Q_{T,t+1}^i(j) \frac{P_{C,t+1}^i}{P_{C,t}^i R_t} \right]$$

Household optimization: Wage setting

The household maximizes the following equation to find the optimal wage:

$$\mathbf{E}_t \sum_{k=0}^{\infty} (\theta_W \beta)^k \left[\frac{\varepsilon_{C,t+k}^i}{1 - \sigma} \left(\frac{(1 - \tau_{L,t+k}^i) \tilde{W}_t^i(j) L_{t+k}^i(j)}{P_{C,t+k}^i} \right)^{1-\sigma} - \frac{\varepsilon_{L,t+k}^i}{1 + \phi} (L_{t+k}^i(j))^{\phi} \right] \quad (\text{A.84})$$

subject to

$$L_{t+k}^i(j) = \frac{1}{n} \left[\frac{\tilde{W}_t^i(j)}{W_{t+k}^i} \left(\frac{P_{C,t+k-1}^i}{P_{C,t-1}^i} \right)^{\delta_W} \right]^{-\phi_W} L_{t+k}^i \quad (\text{A.85})$$

Substituting the constraint in the expression to maximize and defining $x_{t,t+k}^i = \left(\frac{P_{C,t+k-1}^i}{P_{C,t-1}^i} \right)^{\delta_W}$ gives:

$$\begin{aligned} & \mathbf{E}_t \sum_{k=0}^{\infty} (\theta_W \beta)^k \left[\frac{\varepsilon_{C,t+k}^i}{1-\sigma} \left(\frac{(1-\tau_{L,t+k}^i) \tilde{W}_t^i(j) \left(\frac{\tilde{W}_t^i(j)}{W_{t+k}^i} x_{t,t+k}^i \right)^{-\phi_W} L_{t+k}^i}{P_{C,t+k}^i} \right)^{1-\sigma} \right. \\ & \left. - \frac{\varepsilon_{L,t+k}^i}{1+\phi} \left(\left(\frac{\tilde{W}_t^i(j)}{W_{t+k}^i} x_{t,t+k}^i \right)^{-\phi_W} L_{t+k}^i \right)^{1+\phi} \right] \end{aligned}$$

The first-order condition with respect to $\tilde{W}_t(j)$ is then derived:

$$\begin{aligned} & \mathbf{E}_t \sum_{k=0}^{\infty} (\theta_W \beta)^k \left[\varepsilon_{C,t+k}^i (1-\phi_W) \left(\frac{(1-\tau_{L,t+k}^i) \tilde{W}_t^i(j) \left(\frac{\tilde{W}_t^i(j)}{W_{t+k}^i} x_{t,t+k}^i \right)^{-\phi_W} L_{t+k}^i}{P_{C,t+k}^i} \right)^{1-\sigma} \frac{1}{\tilde{W}_t^i(j)} \right. \\ & \left. + \varepsilon_{L,t+k}^i \cdot \phi_W \left(\left(\frac{\tilde{W}_t^i(j)}{W_{t+k}^i} x_{t,t+k}^i \right)^{-\phi_W} L_{t+k}^i \right)^{1+\phi} \frac{1}{\tilde{W}_t^i(j)} \right] = 0 \\ \Leftrightarrow & \mathbf{E}_t \sum_{k=0}^{\infty} (\theta_W \beta)^k \left[\varepsilon_{C,t+k}^i (1-\phi_W) \frac{(1-\tau_{L,t+k}^i) \tilde{W}_t^i(j) \left(\frac{\tilde{W}_t^i(j)}{W_{t+k}^i} x_{t,t+k}^i \right)^{-\phi_W} L_{t+k}^i}{P_{C,t+k}^i} \frac{1}{\tilde{W}_t^i(j)} C_{t+k}^i(j)^{-\sigma} \right. \\ & \left. + \varepsilon_{L,t+k}^i \cdot \phi_W \left(\left(\frac{\tilde{W}_t^i(j)}{W_{t+k}^i} x_{t,t+k}^i \right)^{-\phi_W} L_{t+k}^i \right)^{1+\phi} \frac{1}{\tilde{W}_t^i(j)} \right] = 0 \\ \Leftrightarrow & \mathbf{E}_t \sum_{k=0}^{\infty} (\theta_W \beta)^k \\ & \left[\varepsilon_{C,t+k}^i (1-\phi_W) \frac{(1-\tau_{L,t+k}^i) \tilde{W}_t^i(j) L_{t+k}^i(j)}{P_{C,t+k}^i} \frac{1}{\tilde{W}_t^i(j)} C_{t+k}^i(j)^{-\sigma} + \varepsilon_{L,t+k}^i \cdot \phi_W \left(L_{t+k}^i(j) \right)^{1+\phi} \frac{1}{\tilde{W}_t^i(j)} \right] = 0 \\ \Leftrightarrow & \mathbf{E}_t \sum_{k=0}^{\infty} (\theta_W \beta)^k \\ & \left[(1-\phi_W) \frac{(1-\tau_{L,t+k}^i) \tilde{W}_t^i(j)}{P_{C,t+k}^i} \frac{1}{\tilde{W}_t^i(j)} + \phi_W \cdot \frac{\varepsilon_{L,t+k}^i}{\varepsilon_{C,t+k}^i} \frac{L_{t+k}^i(j)^\phi}{C_{t+k}^i(j)^{-\sigma} \tilde{W}_t^i(j)} \right] \cdot \varepsilon_{C,t+k}^i \cdot L_{t+k}^i(j) \cdot C_{t+k}^i(j)^{-\sigma} = 0 \\ \Leftrightarrow & \mathbf{E}_t \sum_{k=0}^{\infty} (\theta_W \beta)^k \left[(1-\phi_W) \frac{(1-\tau_{L,t+k}^i)}{P_{C,t+k}^i} + \phi_W MRS_{t+k}^i(j) \frac{1}{\tilde{W}_t^i(j)} \right] \cdot \varepsilon_{C,t+k}^i \cdot L_{t+k}^i(j) \cdot C_{t+k}^i(j)^{-\sigma} = 0 \end{aligned}$$

$$\begin{aligned}
&\Leftrightarrow (1 - \phi_W) \mathbb{E}_t \sum_{k=0}^{\infty} (\theta_W \beta)^k \frac{(1 - \tau_{L,t+k}^i)}{P_{C,t+k}^i} \cdot \varepsilon_{C,t+k}^i \cdot L_{t+k}^i(j) \cdot C_{t+k}^i(j)^{-\sigma} \\
&= -\phi_W \mathbb{E}_t \sum_{k=0}^{\infty} (\theta_W \beta)^k MRS_{t+k}^i(j) \frac{1}{\tilde{W}_t^i(j)} \cdot \varepsilon_{C,t+k}^i \cdot L_{t+k}^i(j) \cdot C_{t+k}^i(j)^{-\sigma}
\end{aligned}$$

where the marginal rate of substitution of consumption for leisure is:

$$MRS_t^i(j) = \frac{\varepsilon_{L,t}^i L_t^i(j)^\phi}{\varepsilon_{C,t}^i (C_t^i(j) - hC_{t-1})^{-\sigma}} = -\frac{U_{L,t}}{U_{C,t}}$$

As a result, we have:

$$\tilde{W}_t^i(j) = \frac{\phi_W}{\phi_W - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_W \beta)^k MRS_{t+k}^i(j) \varepsilon_{C,t+k}^i L_{t+k}^i(j) C_{t+k}^i(j)^{-\sigma}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_W \beta)^k \frac{(1 - \tau_{L,t+k}^i)}{P_{C,t+k}^i} \varepsilon_{C,t+k}^i L_{t+k}^i(j) C_{t+k}^i(j)^{-\sigma}}$$

Firm optimization: Price setting

To find the optimal price, firms maximize the following expression with respect to $\tilde{P}_{D,t}^i$:

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_D \beta)^k \Lambda_{t,t+k}^i Y_{D,t+k}^i(z) \left[\frac{\tilde{P}_{D,t}^i}{P_{D,t+k}^i} \left(\frac{P_{D,t+k-1}^i}{P_{D,t-1}^i} \right)^{\delta_D} - MC_{D,t+k}^i \right] \quad (\text{A.86})$$

subject to

$$Y_{D,t+k}^i(z) = \frac{1}{n} \left[\frac{\tilde{P}_{D,t}^i}{P_{D,t+k}^i} \left(\frac{P_{D,t+k-1}^i}{P_{D,t-1}^i} \right)^{\delta_D} \right]^{-\phi_D} Y_{D,t+k}^i = \frac{1}{n} \left[\frac{\tilde{P}_{D,t}^i}{P_{D,t+k}^i} X_{t,t+k}^i \right]^{-\phi_D} Y_{D,t+k}^i \quad (\text{A.87})$$

where we define $X_{t,t+k}^i = \left(\frac{P_{D,t+k-1}^i}{P_{D,t-1}^i} \right)^{\delta_D}$. The term $\Lambda_{t,t+k} = \frac{U_C^i(C_{t+k})}{U_C^i(C_t)}$ is the ratio of marginal utilities of consumption as part of the firm's stochastic discount factor.

Substituting the constraint in the expression to maximize gives:

$$\begin{aligned}
&\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_D \beta)^k \Lambda_{t,t+k}^i Y_{D,t+k}^i \left[\frac{\tilde{P}_{D,t}^i}{P_{D,t+k}^i} X_{t,t+k}^i - MC_{D,t+k}^i \right] \left[\frac{\tilde{P}_{D,t}^i}{P_{D,t+k}^i} X_{t,t+k}^i \right]^{-\phi_D} \\
&\Leftrightarrow \mathbb{E}_t \sum_{k=0}^{\infty} (\theta_D \beta)^k \Lambda_{t,t+k}^i Y_{D,t+k}^i \left[\left(\frac{\tilde{P}_{D,t}^i}{P_{D,t+k}^i} X_{t,t+k}^i \right)^{1-\phi_D} - MC_{D,t+k}^i \left(\frac{\tilde{P}_{D,t}^i}{P_{D,t+k}^i} X_{t,t+k}^i \right)^{-\phi_D} \right]
\end{aligned}$$

The first-order condition related to the profit-maximization problem of the firms that are re-optimizing their prices is given by:

$$\begin{aligned}
&\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_D \beta)^k \Lambda_{t,t+k}^i Y_{D,t+k}^i \left[(1 - \phi_D) \left(\frac{\tilde{P}_{D,t}^i}{P_{D,t+k}^i} X_{t,t+k}^i \right)^{1-\phi_D} \frac{1}{\tilde{P}_{D,t}^i} \right. \\
&\quad \left. + \phi_D MC_{D,t+k}^i \left(\frac{\tilde{P}_{D,t}^i}{P_{D,t+k}^i} X_{t,t+k}^i \right)^{-\phi_D} \frac{1}{\tilde{P}_{D,t}^i} \right] = 0
\end{aligned}$$

$$\begin{aligned}
&\Leftrightarrow \mathbb{E}_t \sum_{k=0}^{\infty} (\theta_D \beta)^k \Lambda_{t,t+k}^i Y_{D,t+k}^i \frac{1}{\tilde{P}_{D,t}^i} (1 - \phi_D) \left(\frac{\tilde{P}_{D,t}^i}{P_{D,t+k}^i} X_{t,t+k}^i \right)^{1-\phi_D} \\
&= -\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_D \beta)^k \Lambda_{t,t+k}^i Y_{D,t+k}^i \frac{1}{\tilde{P}_{D,t}^i} \phi_D MC_{D,t+k}^i \left(\frac{\tilde{P}_{D,t}^i}{P_{D,t+k}^i} X_{t,t+k}^i \right)^{-\phi_D} \\
&\Leftrightarrow (1 - \phi_D) \mathbb{E}_t \sum_{k=0}^{\infty} (\theta_D \beta)^k \Lambda_{t,t+k}^i Y_{D,t+k}^i (\tilde{P}_{D,t}^i)^{-\phi_D} \left(\frac{X_{t,t+k}^i}{P_{D,t+k}^i} \right)^{1-\phi_D} \\
&= -\phi_D \mathbb{E}_t \sum_{k=0}^{\infty} (\theta_D \beta)^k \Lambda_{t,t+k}^i Y_{D,t+k}^i (\tilde{P}_{D,t}^i)^{-1-\phi_D} MC_{D,t+k}^i \left(\frac{X_{t,t+k}^i}{P_{D,t+k}^i} \right)^{-\phi_D} \\
&\Leftrightarrow (1 - \phi_D) \tilde{P}_{D,t}^i \mathbb{E}_t \sum_{k=0}^{\infty} (\theta_D \beta)^k \Lambda_{t,t+k}^i Y_{D,t+k}^i \left(\frac{X_{t,t+k}^i}{P_{D,t+k}^i} \right)^{1-\phi_D} \\
&= -\phi_D \mathbb{E}_t \sum_{k=0}^{\infty} (\theta_D \beta)^k \Lambda_{t,t+k}^i Y_{D,t+k}^i MC_{D,t+k}^i \left(\frac{X_{t,t+k}^i}{P_{D,t+k}^i} \right)^{-\phi_D} \\
&\Leftrightarrow \tilde{P}_{D,t}^i = \frac{\phi_D}{\phi_D - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_D \beta)^k \Lambda_{t,t+k}^i Y_{D,t+k}^i MC_{D,t+k}^i \left(\frac{X_{t,t+k}^i}{P_{D,t+k}^i} \right)^{-\phi_D}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_D \beta)^k \Lambda_{t,t+k}^i Y_{D,t+k}^i \left(\frac{X_{t,t+k}^i}{P_{D,t+k}^i} \right)^{1-\phi_D}}
\end{aligned}$$

Using the definitions of $X_{t,t+k}$ and $\Lambda_{t,t+k}$ gives us the optimal price for the re-optimizing firms:

$$\tilde{P}_{D,t}^i = \frac{\phi_D}{\phi_D - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_D \beta)^k \frac{U_C^i(C_{t+k})}{U_C^i(C_t)} Y_{D,t+k}^i MC_{D,t+k}^i \left(\frac{P_{D,t+k}^i}{\left(\frac{P_{D,t+k-1}^i}{P_{D,t-1}^i} \right)^{\delta_D}} \right)^{\phi_D}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_D \beta)^k \frac{U_C^i(C_{t+k})}{U_C^i(C_t)} Y_{D,t+k}^i \left(\frac{P_{D,t+k}^i}{\left(\frac{P_{D,t+k-1}^i}{P_{D,t-1}^i} \right)^{\delta_D}} \right)^{\phi_D - 1}}$$

Given the optimization process for the firms that are forward-looking, the evolution of the price index for the domestic tradable goods is given by:

$$P_{D,t}^i = \left(\theta_D \left(P_{D,t-1}^i \left(\frac{P_{D,t-1}^i}{P_{D,t-2}^i} \right)^{\delta_D} \right)^{1-\phi_D} + (1 - \theta_D) (\tilde{P}_{D,t}^i)^{1-\phi_D} \right)^{\frac{1}{1-\phi_D}}$$

Similarly, for the nontradables sector, the optimal price for the forward-looking firms and the

evolution of the price index are respectively:

$$\tilde{P}_{N,t}^i = \frac{\phi_N}{\phi_N - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_N \beta)^k \frac{U_C^i(C_{t+k})}{U_C^i(C_t)} Y_{N,t+k}^i M C_{N,t+k}^i \left(\frac{P_{N,t+k}^i}{\left(\frac{P_{N,t+k-1}^i}{P_{N,t-1}^i} \right)^{\delta_N}} \right)^{\phi_N}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_N \beta)^k \frac{U_C^i(C_{t+k})}{U_C^i(C_t)} Y_{N,t+k}^i \left(\frac{P_{N,t+k}^i}{\left(\frac{P_{N,t+k-1}^i}{P_{N,t-1}^i} \right)^{\delta_N}} \right)^{\phi_N - 1}}$$

$$P_{N,t}^i = \left(\theta_N \left(P_{N,t-1}^i \left(\frac{P_{N,t-1}^i}{P_{N,t-2}^i} \right)^{\delta_N} \right)^{1-\phi_N} + (1 - \theta_N) (\tilde{P}_{N,t}^i)^{1-\phi_N} \right)^{\frac{1}{1-\phi_N}}$$

Derivation of the relationship between real exchange rate and terms of trade

The relationship between the real exchange rate and the terms of trade can be derived as follows:

$$Q_{i^*,t}^i = \frac{E_{i^*,t}^i P_{C,t}^{i^*}}{P_{C,t}^i} = \frac{P_{C,t}^{i^*}}{P_{C,t}^i} = \frac{(P_{T,t}^{i^*})^{\gamma_c^*} (P_{N,t}^{i^*})^{1-\gamma_c^*}}{(P_{T,t}^i)^{\gamma_c} (P_{N,t}^i)^{1-\gamma_c}}$$

$$\Leftrightarrow Q_{i^*,t}^i = \frac{\left(\frac{P_{N,t}^{i^*}}{P_{T,t}^{i^*}} \right)^{1-\gamma_c^*} P_{T,t}^{i^*}}{\left(\frac{P_{N,t}^i}{P_{T,t}^i} \right)^{1-\gamma_c} P_{T,t}^i} = \frac{P_{T,t}^{i^*} \left(\frac{P_{N,t}^{i^*}}{P_{T,t}^{i^*}} \right)^{1-\gamma_c^*}}{P_{T,t}^i \left(\frac{P_{N,t}^i}{P_{T,t}^i} \right)^{1-\gamma_c}}$$

$$\Leftrightarrow Q_{i^*,t}^i = \frac{(P_{D,t}^{i^*})^{\alpha^*} (P_{M,t}^{i^*})^{1-\alpha^*} \left(\frac{P_{N,t}^{i^*}}{P_{T,t}^{i^*}} \right)^{1-\gamma_c^*}}{(P_{D,t}^i)^{\alpha} (P_{M,t}^i)^{1-\alpha} \left(\frac{P_{N,t}^i}{P_{T,t}^i} \right)^{1-\gamma_c}} = \frac{(P_{D,t}^{i^*})^{\alpha^*} (P_{D,t}^i)^{1-\alpha^*} \left(\frac{P_{N,t}^{i^*}}{P_{T,t}^{i^*}} \right)^{1-\gamma_c^*}}{(P_{D,t}^i)^{\alpha} (P_{D,t}^i)^{1-\alpha} \left(\frac{P_{N,t}^i}{P_{T,t}^i} \right)^{1-\gamma_c}}$$

$$\Leftrightarrow Q_{i^*,t}^i = \frac{(P_{D,t}^{i^*})^{\alpha+\alpha^*-1} \left(\frac{P_{N,t}^{i^*}}{P_{T,t}^{i^*}} \right)^{1-\gamma_c^*}}{(P_{D,t}^i)^{\alpha+\alpha^*-1} \left(\frac{P_{N,t}^i}{P_{T,t}^i} \right)^{1-\gamma_c}} = (T_{i^*,t}^i)^{\alpha+\alpha^*-1} \frac{\left(\frac{P_{N,t}^{i^*}}{P_{T,t}^{i^*}} \right)^{1-\gamma_c^*}}{\left(\frac{P_{N,t}^i}{P_{T,t}^i} \right)^{1-\gamma_c}}$$

$$\Leftrightarrow Q_{i^*,t}^i = (T_{i^*,t}^i)^{\alpha+\alpha^*-1} \frac{(X_t^{i^*})^{1-\gamma_c^*}}{(X_t^i)^{1-\gamma_c}}$$

Appendix 3: Welfare measure

Welfare measure

Welfare is measured using a second-order Taylor series expansion around the steady state. The expected discounted value of the sum of the utilities of the households is approximated by

$$U_0 = \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 U(C_t, C_{t-1}, L_t, \varepsilon_{C,t}, \varepsilon_{L,t})$$

for both blocks of countries²⁶. The period utility function is given by²⁷

$$U(C_t, C_{t-1}, L_t) = \frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\phi}}{1+\phi}$$

The second-order Taylor expansion of the period utility function around the steady state is defined as

$$\begin{aligned} U(C_t, C_{t-1}, L_t) &\approx \bar{U} + \bar{U}_C \tilde{C}_t + \bar{U}_C \tilde{C}_{t-1} + \bar{U}_L \tilde{L}_t + \frac{1}{2} \bar{U}_{CC} \tilde{C}_t^2 + \frac{1}{2} \bar{U}_{CC} \tilde{C}_{t-1}^2 + \frac{1}{2} \bar{U}_{LL} \tilde{L}_t^2 \\ &\quad + \bar{U}_{CC} \tilde{C}_t \tilde{C}_{t-1} + \bar{U}_{CL} \tilde{C}_t \tilde{L}_t + \bar{U}_{CL} \tilde{C}_{t-1} \tilde{L}_t + \mathcal{O}(\|\zeta\|^3) \\ &= \bar{U} + ((1-h)\bar{C})^{-\sigma} \tilde{C}_t - h((1-h)\bar{C})^{-\sigma} \tilde{C}_{t-1} - \bar{L}^{\phi} \tilde{L}_t - \frac{\sigma}{2} ((1-h)\bar{C})^{-\sigma-1} \tilde{C}_t^2 \\ &\quad - \frac{\sigma}{2} h^2 ((1-h)\bar{C})^{-\sigma-1} \tilde{C}_{t-1}^2 - \frac{\phi}{2} \bar{L}^{\phi-1} \tilde{L}_t^2 + \sigma h ((1-h)\bar{C})^{-\sigma-1} \tilde{C}_t \tilde{C}_{t-1} + \mathcal{O}(\|\zeta\|^3) \end{aligned}$$

where the last term denotes the higher order exogenous disturbances. Here, a tilde refers to the deviation of the variable from the steady state, i.e. $\tilde{C}_t = C_t - \bar{C}$. However, our log-linearized model requires log deviations of variables from their steady state value. Using the Taylor expansion

$$\frac{C_t}{\bar{C}} = 1 + \hat{c}_t + \frac{1}{2} \hat{c}_t^2 + \mathcal{O}(\|\zeta\|^3)$$

which in fact gives us

$$\tilde{C}_t = C_t - \bar{C} = \bar{C} \left(\hat{c}_t + \frac{1}{2} \hat{c}_t^2 \right) + \mathcal{O}(\|\zeta\|^3)$$

²⁶We suppress the region index since the welfare measure is the same for the two regions, except for the estimated parameters and steady state values in the function. Union-wide welfare is given by the weighted average of welfare in both regions: $U_t^{EMU} = n \cdot U_t^i + (1-n) \cdot U_t^{i^*}$.

²⁷Here we leave out the ε -terms for simplicity. In fact, leaving both $\varepsilon_{C,t}$ and $\varepsilon_{L,t}$ out of the second-order Taylor series approximation would not affect the numerical values of the welfare measure, since the mean and the covariance with both consumption and labor are almost equal to zero.

we obtain the following approximation of utility around the steady state

$$\begin{aligned}
U(C_t, C_{t-1}, L_t) &\approx \bar{U} + ((1-h)\bar{C})^{-\sigma} \bar{C} \left(\hat{c}_t + \frac{1}{2} \hat{c}_t^2 \right) - h((1-h)\bar{C})^{-\sigma} \bar{C} \left(\hat{c}_{t-1} + \frac{1}{2} \hat{c}_{t-1}^2 \right) \\
&\quad - \bar{L}^\phi \bar{L} \left(\hat{l}_t + \frac{1}{2} \hat{l}_t^2 \right) - \frac{\sigma}{2} ((1-h)\bar{C})^{-\sigma-1} \bar{C}^2 \hat{c}_t^2 - \frac{\sigma}{2} h^2 ((1-h)\bar{C})^{-\sigma-1} \bar{C}^2 \hat{c}_{t-1}^2 \\
&\quad - \frac{\phi}{2} \bar{L}^{\phi-1} \bar{L}^2 \hat{l}_t^2 + \sigma h ((1-h)\bar{C})^{-\sigma-1} \bar{C}^2 \hat{c}_t \hat{c}_{t-1} + \mathcal{O}(\|\zeta\|^3) \\
&= \bar{U} + ((1-h)\bar{C})^\sigma \bar{C} \left[(\hat{c}_t - h\hat{c}_{t-1}) + \frac{1}{2} (\hat{c}_t^2 - h\hat{c}_{t-1}^2) - \frac{\sigma}{2(1-h)} (\hat{c}_t^2 + h^2 \hat{c}_{t-1}^2) \right. \\
&\quad \left. + \frac{\sigma h}{1-h} (\hat{c}_t \hat{c}_{t-1}) \right] - \bar{L}^{1+\phi} \left[\hat{l}_t + \frac{1}{2} (1+\phi) \hat{l}_t^2 \right] + \mathcal{O}(\|\zeta\|^3)
\end{aligned}$$

Expected lifetime utility at time zero is hence given by

$$\begin{aligned}
U_0 &= \sum_{t=0}^{\infty} \beta^t \mathbf{E}_0 \left(\bar{U} + ((1-h)\bar{C})^{-\sigma} \bar{C} \left[(\hat{c}_t - h\hat{c}_{t-1}) + \frac{1}{2} (\hat{c}_t^2 - h\hat{c}_{t-1}^2) - \frac{\sigma}{2(1-h)} (\hat{c}_t^2 + h^2 \hat{c}_{t-1}^2) \right. \right. \\
&\quad \left. \left. + \frac{\sigma h}{1-h} (\hat{c}_t \hat{c}_{t-1}) \right] - \bar{L}^{1+\phi} \left[\hat{l}_t + \frac{1}{2} (1+\phi) \hat{l}_t^2 \right] + \mathcal{O}(\|\zeta\|^3) \right) \\
&= \sum_{t=0}^{\infty} \beta^t \left(\bar{U} + ((1-h)\bar{C})^{-\sigma} \bar{C} \left[(\mathbf{E}_0(\hat{c}_t) - h\mathbf{E}_0(\hat{c}_{t-1})) + \frac{1}{2} (\mathbf{E}_0(\hat{c}_t^2) - h\mathbf{E}_0(\hat{c}_{t-1}^2)) \right. \right. \\
&\quad \left. \left. - \frac{\sigma}{2(1-h)} (\mathbf{E}_0(\hat{c}_t^2) + h^2 \mathbf{E}_0(\hat{c}_{t-1}^2)) + \frac{\sigma h}{1-h} \mathbf{E}_0(\hat{c}_t \hat{c}_{t-1}) \right] - \bar{L}^{1+\phi} \left[\mathbf{E}_0(\hat{l}_t) + \frac{1}{2} (1+\phi) \mathbf{E}_0(\hat{l}_t^2) \right] + \mathcal{O}(\|\zeta\|^3) \right)
\end{aligned}$$

The welfare effect of a fiscal transfer mechanism is evaluated using the consumption equivalent welfare measure in the tradition of Lucas (2003). This measure defines the welfare gain as the permanent percentage change in steady state consumption that will make the representative household indifferent between situation A and the steady state:

$$U^A = \sum_{t=0}^{\infty} \beta^t \mathbf{E}_0 U(C_t^A, C_{t-1}^A, L_t^A) = \sum_{t=0}^{\infty} \beta^t U((1+\lambda)\bar{C}, \bar{L}) = \sum_{t=0}^{\infty} \beta^t \left[\frac{((1-h)(1+\lambda)\bar{C})^{1-\sigma}}{1-\sigma} - \frac{\bar{L}^{1+\phi}}{1+\phi} \right]$$

Using a first-order Taylor approximation in consumption equivalence

$$\frac{((1-h)(1+\lambda)\bar{C})^{1-\sigma}}{1-\sigma} - \frac{\bar{L}^{1+\phi}}{1+\phi} \approx \frac{((1-h)\bar{C})^{1-\sigma}}{1-\sigma} - \frac{\bar{L}^{1+\phi}}{1+\phi} + ((1-h)\bar{C})^{1-\sigma} \cdot \lambda = \bar{U} + ((1-h)\bar{C})^{1-\sigma} \cdot \lambda$$

we obtain a solution for λ :

$$\begin{aligned}
& \sum_{t=0}^{\infty} \beta^t U((1+\lambda)\bar{C}, \bar{L}) = \sum_{t=0}^{\infty} \beta^t \mathbf{E}_0 U(C_t, C_{t-1}, L_t) \\
\Rightarrow & \sum_{t=0}^{\infty} \beta^t \left(\bar{U} + ((1-h)\bar{C})^{1-\sigma} \cdot \lambda \right) = \sum_{t=0}^{\infty} \beta^t \left(\bar{U} + ((1-h)\bar{C})^{-\sigma} \bar{C} [(\mathbf{E}_0(\hat{c}_t) - h\mathbf{E}_0(\hat{c}_{t-1})) \right. \\
& \left. + \frac{1}{2}(\mathbf{E}_0(\hat{c}_t^2) - h\mathbf{E}_0(\hat{c}_{t-1}^2)) - \frac{\sigma}{2(1-h)}(\mathbf{E}_0(\hat{c}_t^2) + h^2\mathbf{E}_0(\hat{c}_{t-1}^2)) + \frac{\sigma h}{1-h}\mathbf{E}_0(\hat{c}_t\hat{c}_{t-1}) \right] \\
& \left. - \bar{L}^{1+\phi} \left[\mathbf{E}_0(\hat{l}_t) + \frac{1}{2}(1+\phi)\mathbf{E}_0(\hat{l}_t^2) \right] \right) \\
\Rightarrow & ((1-h)\bar{C})^{1-\sigma} \cdot \lambda = ((1-h)\bar{C})^{-\sigma} \bar{C} \left[(\mathbf{E}_0(\hat{c}_t) - h\mathbf{E}_0(\hat{c}_{t-1})) + \frac{1}{2}(\mathbf{E}_0(\hat{c}_t^2) - h\mathbf{E}_0(\hat{c}_{t-1}^2)) \right. \\
& \left. - \frac{\sigma}{2(1-h)}(\mathbf{E}_0(\hat{c}_t^2) + h^2\mathbf{E}_0(\hat{c}_{t-1}^2)) + \frac{\sigma h}{1-h}(\mathbf{E}_0(\hat{c}_t\hat{c}_{t-1})) \right] - \bar{L}^{1+\phi} \left[\mathbf{E}_0(\hat{l}_t) + \frac{1}{2}(1+\phi)\mathbf{E}_0(\hat{l}_t^2) \right] \\
\Rightarrow & \lambda = \frac{1}{1-h} \left[(\mathbf{E}_0(\hat{c}_t) - h\mathbf{E}_0(\hat{c}_{t-1})) + \frac{1}{2}(\mathbf{E}_0(\hat{c}_t^2) - h\mathbf{E}_0(\hat{c}_{t-1}^2)) - \frac{\sigma}{2(1-h)}(\mathbf{E}_0(\hat{c}_t^2) + h^2\mathbf{E}_0(\hat{c}_{t-1}^2)) \right. \\
& \left. + \frac{\sigma h}{1-h}\mathbf{E}_0(\hat{c}_t\hat{c}_{t-1}) \right] - \bar{L}^{1+\phi} ((1-h)\bar{C})^{\sigma-1} \left[\mathbf{E}_0(\hat{l}_t) + \frac{1}{2}(1+\phi)\mathbf{E}_0(\hat{l}_t^2) \right]
\end{aligned}$$

Hence, the consumption equivalent welfare measure is given by:

$$\begin{aligned}
\lambda = & \frac{1}{1-h} \left[(\mathbf{E}_0(\hat{c}_t) - h\mathbf{E}_0(\hat{c}_{t-1})) + \frac{1}{2}(\mathbf{E}_0(\hat{c}_t^2) - h\mathbf{E}_0(\hat{c}_{t-1}^2)) - \frac{\sigma}{2(1-h)}(\mathbf{E}_0(\hat{c}_t^2) + h^2\mathbf{E}_0(\hat{c}_{t-1}^2)) \right. \\
& \left. + \frac{\sigma h}{1-h}\mathbf{E}_0(\hat{c}_t\hat{c}_{t-1}) \right] - \bar{L}^{1+\phi} ((1-h)\bar{C})^{\sigma-1} \left[\mathbf{E}_0(\hat{l}_t) + \frac{1}{2}(1+\phi)\mathbf{E}_0(\hat{l}_t^2) \right]
\end{aligned}$$

Steady state

The relative weight of the means and variances of consumption and labor within this consumption equivalent welfare measure are determined by the steady state values of variables in the utility function. The steady state values of $\varepsilon_{C,t}$ and $\varepsilon_{L,t}$ are given by $\bar{\varepsilon}_C = 1$ and $\bar{\varepsilon}_L = 1$. The steady state values of consumption and labor have to be calculated throughout the theoretical model²⁸ and are approximated with numerical methods given the estimated parameters of the model. Steady state consumption is calculated to be $\bar{C} = 2.6232$, $\bar{C}^* = 2.8107$, $\bar{L} = 1.2840$ and $\bar{L}^* = 1.2990$. The parameters in this welfare measure are the parameters for the different regions as they are estimated using Bayesian methods.

²⁸The steady state equations can be found in the appendix.

Decomposition

A decomposition of the welfare measure yields the following welfare compensations for the means and variances of labor and consumption, and on an overall basis:

$$\lambda_M = \frac{1}{1-h} [\mathbf{E}_0(\hat{c}_t) - h\mathbf{E}_0(\hat{c}_{t-1})] - \bar{L}^{1+\phi}((1-h)\bar{C})^{\sigma-1}\mathbf{E}_0(\hat{l}_t)$$

$$\lambda_V = \frac{1}{1-h} \left[\frac{1}{2}(\mathbf{E}_0(\hat{c}_t^2) - h\mathbf{E}_0(\hat{c}_{t-1}^2)) - \frac{\sigma}{2(1-h)}(\mathbf{E}_0(\hat{c}_t^2) + h^2\mathbf{E}_0(\hat{c}_{t-1}^2)) + \frac{\sigma h}{1-h}\mathbf{E}_0(\hat{c}_t\hat{c}_{t-1}) \right] \\ - \bar{L}^{1+\phi}((1-h)\bar{C})^{\sigma-1} \cdot \frac{1}{2}(1+\phi)\mathbf{E}_0(\hat{l}_t^2)$$

$$\lambda_{M,c} = \frac{1}{1-h} [\mathbf{E}_0(\hat{c}_t) - h\mathbf{E}_0(\hat{c}_{t-1})]$$

$$\lambda_{M,l} = -\bar{L}^{1+\phi}((1-h)\bar{C})^{\sigma-1}\mathbf{E}_0(\hat{l}_t)$$

$$\lambda_{V,c} = \frac{1}{1-h} \left[\frac{1}{2}(\mathbf{E}_0(\hat{c}_t^2) - h\mathbf{E}_0(\hat{c}_{t-1}^2)) - \frac{\sigma}{2(1-h)}(\mathbf{E}_0(\hat{c}_t^2) + h^2\mathbf{E}_0(\hat{c}_{t-1}^2)) + \frac{\sigma h}{1-h}\mathbf{E}_0(\hat{c}_t\hat{c}_{t-1}) \right]$$

$$\lambda_{V,l} = -\bar{L}^{1+\phi}((1-h)\bar{C})^{\sigma-1} \cdot \frac{1}{2}(1+\phi)\mathbf{E}_0(\hat{l}_t^2)$$

Appendix 4: Steady state

In this appendix, the steady state equations of the model are presented. Given the non-linearity of the equations, the solution is not explicitly stated, but can be solved for numerically. As before, steady state variables are denoted with a bar.

Real interest rate

$$\bar{R}^i = \bar{R}^{i*} = \frac{1}{\beta} \quad (\text{A.88})$$

Real wage rate

$$\bar{W}^i = \bar{\bar{W}}^i = \frac{\phi_W}{\phi_W - 1} \frac{(\bar{L}^i)^\phi}{((1-h)\bar{C}^i)^{-\sigma}} \quad (\text{A.89})$$

$$\bar{W}^{i*} = \bar{\bar{W}}^{i*} = \frac{\phi_W^*}{\phi_W^* - 1} \frac{(\bar{L}^{i*})^{\phi^*}}{((1-h^*)\bar{C}^{i*})^{-\sigma^*}} \quad (\text{A.90})$$

Labor demand

$$\frac{\bar{W}^i \bar{L}^i}{\bar{R}_K^i \bar{K}^i} = \frac{1 - \eta}{\eta} \quad (\text{A.91})$$

$$\frac{\bar{W}^{i*} \bar{L}^{i*}}{\bar{R}_K^{i*} \bar{K}^{i*}} = \frac{1 - \eta^*}{\eta^*} \quad (\text{A.92})$$

Marginal cost

$$\left(\frac{\bar{W}^i}{1 - \eta} \right)^{1-\eta} = \frac{\phi_D - 1}{\phi_D} \left(\frac{\bar{R}_K^i}{\eta} \right)^{-\eta} \quad (\text{A.93})$$

$$\left(\frac{\bar{W}^{i*}}{1 - \eta^*} \right)^{1-\eta^*} = \frac{\phi_D^* - 1}{\phi_D^*} \left(\frac{\bar{R}_K^{i*}}{\eta^*} \right)^{-\eta^*} \quad (\text{A.94})$$

Capital accumulation

$$\bar{I}^i = \delta \bar{K}^i \quad (\text{A.95})$$

$$\bar{I}^{i*} = \delta^* \bar{K}^{i*} \quad (\text{A.96})$$

Production function

$$\bar{Y}^i = (\bar{K}^i)^\eta (\bar{L}^i)^{1-\eta} \quad (\text{A.97})$$

$$\bar{Y}^{i*} = (\bar{K}^{i*})^\eta (\bar{L}^{i*})^{1-\eta} \quad (\text{A.98})$$

Market equilibrium

$$\begin{aligned} \bar{Y}^i = \bar{Y}_N^i + \bar{Y}_D^i &= (1 - \gamma_C)\bar{C}^i + (1 - \gamma_I)\bar{I}^i + \gamma_C\alpha\bar{C}^i + \gamma_I\alpha\bar{I}^i \\ &+ \frac{1-n}{n}\gamma_C^*(1-\alpha^*)\bar{C}^{i*} + \frac{1-n}{n}\gamma_I^*(1-\alpha^*)\bar{I}^{i*} \end{aligned} \quad (\text{A.99})$$

$$\begin{aligned} \bar{Y}^{i*} = \bar{Y}_N^{i*} + \bar{Y}_D^{i*} &= (1 - \gamma_C^*)\bar{C}^{i*} + (1 - \gamma_I^*)\bar{I}^{i*} + \gamma_C^*\alpha^*\bar{C}^{i*} + \gamma_I^*\alpha^*\bar{I}^{i*} \\ &+ \frac{n}{1-n}\gamma_C(1-\alpha)\bar{C}^i + \frac{n}{1-n}\gamma_I(1-\alpha)\bar{I}^i \end{aligned} \quad (\text{A.100})$$

Appendix 5: Data description

The structural parameters of the model and the processes that govern the 21 shocks of the model are estimated using Bayesian estimation. Data on 21 key macroeconomic and fiscal policy variables in the two blocks of the Euro area is used. Only first generation members of the Economic and Monetary Union are taken into account because of data availability. The northern block consists of Austria, Belgium, Finland, France, Germany, Ireland, Luxembourg and the Netherlands. Greece, Italy, Portugal and Spain constitute the South. The data is quarterly and taken from 2000:Q2 until 2013:Q4, due to a lack of data availability before 2000 for certain variables for Greece.

The key macroeconomic variables are real GDP, real consumption, real investment, consumer price index, real wage rate, the internal exchange rate and the nominal interest rate set in the Euro area by the ECB. Furthermore, we observe data on fiscal variables as government debt, government expenditures and the revenues from consumption taxes and capital income taxes. The source of the time series is explained in more detail below:

- **Real GDP:** Data on 'GDP at market prices' is taken from the Eurostat database. The series is expressed in chain-linked volumes, where the reference year is 2005. The series of all countries are expressed in the same currency, in euros.
- **Real consumption:** Data on 'Final consumption expenditure' is taken from the Eurostat database. The series is expressed in chain-linked volumes, where the reference year is 2005.
- **Real investment:** Investment is approximated by 'Gross fixed capital formation', taken from the Eurostat database. The series is expressed in chain-linked volumes, where the reference year is 2005.
- **Consumer price index:** The 'Harmonised Consumer Price Index' (HICP) is calculated by the OECD with index 2010 = 100.
- **Real wages:** The nominal wage is given by 'Labour compensation per employed person' with index 2010 = 100, as recorded in the OECD.Stat database. To account for price changes, the real wage is obtained by dividing the index for labor compensation by the consumer price index.

- **Internal exchange rate:** The internal exchange rate is determined using data on prices from the OECD.Stat database. The internal exchange rate is defined as the price of non-tradables over the price of tradables. Here the consumer price index of nontradables is used which is the average of the CPI in 'Services' and 'Energy', and the CPI of tradables consists of the CPI of 'Food' and 'Non-food non-energy'. Data on the CPI in services is missing for Germany, Greece and the Netherlands, hence this variable is not taken into account when calculating the internal exchange rate for these countries.
- **Nominal interest rate:** The nominal interest rate in this model is given by 3-month money market rate, documented in the Eurostat database. Since the two blocks in our model have been in the monetary union since the start, the nominal interest rate is the same for both.
- **Consumption taxes:** The tax revenues on consumption taxes are approximated by 'Taxes on production and imports' from the Eurostat database. The series is expressed in millions of euros.
- **Capital income taxes:** The tax revenues on capital income taxes are approximated by 'Capital taxes' from the Eurostat database. The series is expressed in millions of euros.
- **Government debt:** Government debt is given by 'Gross consolidated government debt', which is available from the Eurostat database. The series is expressed in millions of euros.
- **Wage income taxes:** The tax revenues on wage income taxes are approximated by 'Current taxes on income, wealth, etc.', which is available from the Eurostat database. The series is expressed in millions of euros.
- **Government consumption:** Government consumption (both expenditures and transfer) are approximated by 'Final consumption expenditure' by the general government, which is available from the Eurostat database. The series is expressed in millions of euros. Data is available from 2000 onwards for all countries.

Furthermore, we use data on labor input for the welfare analysis, however, this is not used in the estimation process. To approximate labor input, we take actual weekly hours worked from the Eurostat database. Data points are missing for Germany, France and Luxembourg at the

beginning of the time series, here we use interpolation as yearly data are available.

The data is summed up to get series for North and South. For inflation, internal exchange rate and the real wage rate, the series for North and South are averaged using as a weight the share of GDP of the country in total GDP of the block of countries. Seasonal adjustment of the time series is done by the program Demetra+, which is often used by official statistical offices. The TramoSeats method²⁹ is used with specification RSA3. This specification tests for the log/level specification, it automatically detects outliers and it identifies and estimates the best ARIMA model for the seasonal adjustment³⁰. Then, the variables are expressed in 100*log differences to match with the measurement equations of the model.

An important remark has to be made about the data on government debt of Greece. "Greece benefited from a debt relief that reduced its public debt by around 50% in 2012, and from a reduction in interest rates and an extension in the repayment period for the EU and IMF rescue package in 2011" (Martin & Philippon (2014)). In the time series for Greece, there is a significant drop in government debt from the last quarter of 2011 to the first quarter of 2012. However, because the data is summed for South, there is no significant drop in the data for South as a whole, as Greece is relatively small within this block of countries.

The data is mapped onto the model via the observation equations, which is as follows for output:

$$y_t^{obs} = \hat{y}_t^i - \hat{y}_{t-1}^i \quad (\text{A.101})$$

where y_t^{obs} is the observable time series for output, adjusted in the manner described above. The model can give us a simulation of \hat{y}_t^i , based on the estimated parameters, which is the log deviation of output from the steady state. The measurement equations for c_t^{obs} , i_t^{obs} , x_t^{obs} , w_t^{obs} , $taxC_t^{obs}$, $taxL_t^{obs}$, d_t^{obs} , $(g+z)_t^{obs}$ and r_t^{obs} ³¹ are of the same type, and these are similar for both regions. The only exception is the measurement equation for the observable of inflation:

$$\pi_t^{obs} = \hat{\pi}_t^i \quad (\text{A.102})$$

²⁹The TramoSeats method is also used by the OECD to seasonally adjust the data series.

³⁰More information can be found in the Demetra+ User Manual, Sylwia Grudkowska (2011).

³¹The observable time series for the nominal interest rate (set by the ECB) is r_t^{obs} only because the notation i_t^{obs} was already taken for investment. However, the observable r_t^{obs} does represent the nominal interest rate.

For inflation, data on the level of CPI is used. In the model, π_C^i is the deviation of the growth rate of log CPI from the steady state growth rate of log CPI. Hence, the observation equation does not have a lagged term in it as well, since the data is in CPI levels rather than inflation rates.