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By

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Abstract

A model of global oil production is applied to study cartelization by OPEC countries. Writing out the shadow price on quota allocations so as to draw correspondence to coefficients of cooperation (Cyert et al. 1973), we examine the incentives that different OPEC members to collude. We find that heterogeneity in OPEC and the supplies of the non-OPEC fringe create strong incentives against OPEC cooperation. OPEC's optimal supply strategy although observed to be substantially more restrictive than that of a Cournot-Nash oligopoly, is found to still be more accommodative than that of a perfect cartel. The strategy involves allocating larger than proportionate quotas to smaller and relatively costlier producers as if to bribe their participation in the cartel. This is contrary to predictions of the standard cartel model that such producers should be allocated relatively more stringent quotas. Furthermore, we find that cartel collusion is likely to be sustained for elastic than inelastic demand. Since global oil demand is well known to be inelastic, this observation provides another structural explanation for why OPEC behavior is inconsistent with that of a perfect cartel. Our study points to multiple headwinds that limit OPECs ability to raise long-run global oil prices.

Keywords: Keywords: Imperfect cartels, Oil, OPEC, Nash bargaining, Collusion strategies
JEL: C61, C7, L13, L22, L71, Q31

1. Introduction

OPEC's longevity, given predictions of its demise by experts and the textbook cartelization model has come as a surprise to many. A growing body of literature (Smith, 2005; Kaufmann et al., 2008), however, suggests that OPEC is not and should not be regarded as a perfectly colluding (i.e., standard) cartel. Indeed, concessions made when bargaining for quotas may engender quota allocations that vastly diverge from those of a perfect cartel. While economic theory prescribes that perfect cartels must assign quotas so that marginal revenues (alternatively, full marginal costs) are equalized across members (Schmalensee, 1987), OPEC's actual quota allocation scheme plausibly diverges from this rule.

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Technically, equalization of marginal revenues requires that the least efficient (i.e., high cost and low reserve) producers cut their production, so as to accommodate for relatively higher production shares from more efficient (i.e., low cost and large reserve) producers. For OPEC, this means that Saudi Arabia would front-load its production, while high cost producers such as Venezuela would postpone theirs to a time when their (full) marginal costs of extraction are in line with those of Saudi Arabia.[1] The reverse, however, has been noted for OPEC and some other cartels such as the Railroad Commission of Texas in the 1920s-1950s, where the less efficient (i.e., the small and generally high cost) producers tend to acquire larger than proportionate production shares (Griffin and Xiong, 1997; Libecap, 1989). The less efficient producers are given unproportionally larger quotas as if to bribe their participation in the cartel. As shown in Polasky (1992), such a quota allocation scheme conforms more to non-cooperative oligopoly behavior than perfect cartelization.

The objective of this paper is to introduce a formal model of quota negotiation in OPEC, and use it to investigate optimal production allocations amongst its members. Our main argument is that the noted production scheme where smaller producers in cartels, get unproportionally larger quotas, can be explained by concessions at the bargaining stage. We propose a two-stage model of global oil production where, in the first stage OPEC producers bargain over production allocations, i.e., quotas. We assume these quotas are enforceable. In the second stage, each OPEC member then chooses its optimal production plan, subject to its quota restriction, while making independent judgments about optimal investments in capacity and resource development. Non-OPEC decision making, on the other hand, is confined to the second stage where optimal levels for production, investments in capacity, and resource development are all chosen. We assume that OPEC producers know the form of the demand function and therefore act as prices setters. Non-OPEC producers, on other hand, know only the time path for the global oil price. They act as a competitive fringe a là Salant (1976).

We find that OPEC has a substantial but not outright ability to raise global prices. Analyzing compensating changes in output between OPEC members, Smith (2005) arrives at the same conclusion as ours. Our means to the conclusion is, however, different than his in the following respects: (i) methodologically, we employ a numerical optimization model calibrated to empirical world oil market data, which is in contrast to his econometric approach, and (ii) our approach permits us to explicitly asses how the different attributes (e.g., reserve holdings, extraction cost, and production capacity) uniquely affect a members' bargaining power during quota allocation, and hence the overall effectiveness of the cartel at raising prices. Our simulation based analysis bridges the gap between econometric studies on OPEC that are often short of data points to appropriately analyze relationships between OPEC members, leading to low powers of such tests; and analytic studies that use general models and producer characteristics in order to derive closed form solutions, while offering little to no (quantitative) insight into OPECs actual behavior and ability to influence global oil prices prices.

In the literature, studies that estimate OPEC's payoffs to cartelization with the aim of assessing its ability to raise the global oil price include Pindyck (1978); Griffin and Xiong (1997); Berg et al. (1997).[2] These authors

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[1]Full marginal costs constitute the marginal cost of lifting the resource out of the ground, plus the scarcity rent of depleting the resource.

[2]Other studies such as Salant (1976); Ulph and Folie (1980) use analytic approaches to investigate the benefits to producers when a segment of the market colludes. While Salant (1976) shows that the fringe benefits more, Ulph and Folie (1980) by contrast show that the cartel gains the most if it has a significant cost advantage over the fringe. These analytic studies, however, rely on changes in the in situ value of the resource (i.e., the Hotelling rent) as result of cartel formation to draw their conclusions, rather than on present value profits as is done with numerical optimization models.

achieve this by comparing long-term net present values for a monolithic OPEC against those that would be obtained if the cartel were to dissolve and act competitively. They find that OPEC enjoys moderate to substantial gains from cartelization. More specifically, Pindyck (1978) finds gains of 50% to 100%, Griffin and Xiong (1997) finds 24%, whereas Berg et al. (1997) finds gains of 18%. While these aggregate estimates indicate that adequate incentives exist for OPEC to cartelize, these unfortunately do not tell us much about OPECs actual ability to raise oil prices. Understanding the distribution of collusion gains across OPEC members and over the different collusion possibilities, is key to understanding the effectiveness of the OPEC collusion arrangement and hence the extent of OPECs eventual impact in raising the global oil price.

Our analysis indicates that OPEC collectively has positive gains from perfect cartelization (estimated to be 25%), and thus has positive incentives to cartelize. Heterogeneity within the cartel is, however, an important factor that impedes full cooperation since for plausible demand elasticity estimates, most members' profits are observed to be non-monotonic in the degree of cartelization. For most producers, individual profits initially increase because of collusion, but then begin to decline as cooperation approaches perfect cartelization. This decline is strengthened by the presence of a non-OPEC fringe that increases its production whenever the cartel further withholds. In fact, non-OPEC producers are generally the biggest gainers from OPEC's attempts at stronger collusion. Because of the non-OPEC "free rider" problem and heterogeneity between OPEC members, the perfect cartelization approach seems inadequate for capturing the intricacies of OPEC behavior. Instead, OPEC plausibly sets production where it can ensure the highest gains for most of its members, while at the same time crowding out non-OPEC production. Such an equilibrium point does not have to correspond with perfect cartelization.

We point out that the more elastic the demand curve, the more likely OPEC producers are to perfectly cartelize. This result is not specific to OPEC, but is a general result. While Lofaro (1999); Selten (1973) show that the number of producers in the cartel, Salant et al. (1983) the size of the cartel relative to the fringe, and Hyndman (2008); Rotemberg and Saloner (1986) the level of demand will influence cooperation in a cartel, to the best of our knowledge, no study shows how demand elasticity influences cooperation in a cartel. It is generally perceived that cartels are more likely to form in cases where demand is inelastic. While this may appear to be the case because of the high profits that low (absolute) elasticities induce when producers collude, it is not necessarily the case that the cartel will be a perfectly colluding one. The intuition behind this result is that since gains from collusion are more (less) substantial with inelastic (elastic) demand, cartel members need to make minimal (deep) cuts in production, thus colluding less (more) stringently, in order to raise prices and hence profits. Another interpretation is that although there are substantial gains to be obtained from collusion when demand is inelastic, the incentive to cheat on allocations is also high. A cartel that issues quotas with the aim of minimizing or eliminating cheating will therefore be forced to issue less stringent quotas.

The rest of the article is organized as follows. In the next section, we describe the structure and features of the global oil market model used, including how coefficients of cooperation can be used to interpret the stringency of quota allocations. We also set up scenarios to asses how varying degrees of OPEC collusion affects OPEC profits. This same section, details the data used to parametrize the model. Section 3 presents initial results from analyzing OPEC's cartelization gains and elaborates upon the impacts of demand elasticity on cartel collusion. The full model for OPEC cooperation with bargaining modeled explicitly is described in section 4 and the corresponding simulation results presented. Section five discusses and concludes the article.

3

2. The Model

The model proposed here extends the crude oil reserve additions model of Okullo and Reynès (2011) where the global conventional crude oil market is modeled using a simple Hotelling model for resource depletion, with one state variable to track the depletion of reserves and additions to reserves are given exogenously. Firstly, we highlight the notable features of the current model, and then provide a full mathematical representation and description, including the equilibria under which solutions are calculated.

2.1. Highlighted model features:

1. The model accounts for the increasingly important role for unconventional resources in meeting future oil demand. That is, in addition to conventional crude oil, the model accounts for oil supply from tar sands and natural gas liquids.

2. Production is constrained to capacity that is accumulated through investments. Capacity grows slowly because of (i) the positive marginal price for installed capacity that avoids its wasteful installation, and (ii) because of exogenously given, history based physical limits on its periodical expansion.

3. Depletion rates that account for natural decline in reservoir productivity are represented in the model (Adelman, 1990; Cairns and Davis, 2001; Nystad, 1987; Okullo et al., 2015). These impose reasonable upper bounds, as dictated by geological constraints, on the share of reserves that are extracted in any given period.

4. Reserve development is endogenous to the model. It is assumed that producers know with full certainty the size of their initial reserves and resource endowments. Producers must, however, convert resources into reserves through costly development to facilitate extraction.

5. The model has a sufficiently detailed representation of the global oil market. Eighteen oil producers are represented. Each OPEC producer is accounted for individually except for Venezuela and Ecuador that are modeled as one. In the case of non-OPEC, there are seven producing regions that are modeled: Asia and the Pacific, Brazil, Europe, Former Soviet Union, North America, South and Central America, and Rest of the world. The grouping "Rest of the World" consists of producers from Africa and the Middle East that are not in OPEC. On the demand side, the model accounts for two demand regions, OECD and non-OECD.

2.2. Model description

There are two stages for the OPEC decision tree and one stage for non-OPEC. In stage one, OPEC producers negotiate over production allocations. In stage two, they choose production subject to negotiated quota restrictions, while non-cooperatively adjusting levels for investments in capacity and resource development. Non-OPEC decision making is confined to the second stage, where choices for production, capacity investments, and resource development are made. We provide more detail on the second-stage problem next but, defer the discussion of the first-stage negotiation process to section 4.

The producers' objective is to choose allocations for production, investments in capacity, and additions to reserves in order to maximize the discounted sum of net profits. Those choices are made subject to dynamic changes in developed reserves, installed capacity, and the depletion of undeveloped resources. In each period, a set of (instantaneous) constraints ensure that production neither exceeds installed capacity, nor the geologically extractable reserves base, and that capacity can only be expanded gradually inline with historical limits. For OPEC

producers, the prenegotiated level of output, i.e., the quota, restricts output choice. We assume that quotas are enforceable. That is, the production target issued by the cartel is never exceeded although, a producer may choose to under produce its quota allocation.[3]

The mathematical representation for the producer i's second stage problem over a foreseeable future, $t \in [\tau, \infty)$, is:

$$max_{\{q_{it}, I_{it}, x_{it},\}} \pi_{i\tau} = \int_{t=\tau}^{t=\infty} (P(Y_t, Q_t) q_{it} - C(q_{it}, R_{it}, S_{it}) - W(I_{it}) - Z(x_{it})) e^{-\delta t} dt \quad (1)$$

s.t.

$$\dot{R}_{it} = x_{it} - q_{it} \quad (2)$$

$$\dot{K}_{it} = I_{it} - \Delta K_{it} \quad (3)$$

$$\dot{S}_{it} = -x_{it} \quad (4)$$

$$K_{it} \geq q_{it}, \gamma R_{it} \geq q_{it}, b K_{it} \geq I_{it} - \Delta K_{it} \quad (5)$$

$$\tilde{q}_{it} = q_{it} + s_{it} \ i \in c \quad (6)$$

$$R_{i\tau}, S_{i\tau} K_{i\tau} > 0, R_{it}, S_{it}, I_{it}, K_{it}, q_{it}, x_{it}, s_{it} \geq 0; \ i = 1, ..., n; \ t \in [\tau, \infty) \quad (7)$$

where the initial reserve size, $R_{i\tau}$, initial capital stock, $K_{i\tau}$, and initial resource stock, $S_{i\tau}$, are known, given, and positive. Non-negativity constraints govern the levels for reserves, R_{it}, capacity, K_{it}, resources, S_{it}, production, q_{it}, investments, I_{it}, and additions to reserves, x_{it}. c denotes the OPEC cartel, $\tilde{q}_{it}$ the quota allocation to a member of the OPEC cartel, and s_{it} a slack variable. Dropping indices where no confusion arises, $P(\bullet)$ is the oil price which is a function of aggregate production, Q, and autonomous demand, Y. $C(\bullet)$ is the cost of extracting oil, $W(\bullet)$ the cost of investing in new capacity, and $Z(\bullet)$ the cost of converting undeveloped resources into extractable reserves. The discount rate, depreciation rate for capital, and the intensity of geological constraints[4] are denoted by δ, Δ, and γ respectively, while b gives in percentage, the per period limit to capacity expansion.

Equation (2) says that reserves decline through extraction, but are augmented through additions[5]. Capacity, in contrast, expands through investment, but declines because of depreciation[6] (Equation 3). The dynamics for the resource base are given by Equation (4) which says that resources decrease by the amount that is developed, that is, the amount that is added to proven reserves. The instantaneous constraints represented in (5), respectively, require that production per period neither exceeds installed capacity nor the geologically extractable reserve base, γR_{it},

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[3]While this assumption may appear restrictive, data for OPEC countries presented in Table 1 of Dibooglu and AlGudhea (2007) shows that deviations from allocated quotas happen only occasionally and are acceptably small when considered over the duration of more than a year.

[4]For an extended discussion on how γ influences production, please see Okullo et al. (2015).

[5]As demonstrated in Okullo et al. (2015), modeling both the extraction and the reserve augmentation process, combined with the constraint $\gamma R_{it} \geq q_{it}$ in Equation (5) allows us to account for the impacts that geological constraints can have on production and price profiles. In particular, over the interval where $q_{it} = \gamma R_{it}$, production traces a geologically constrained extraction path, such that empirically observed bell-shaped production paths and U-shaped paths for price and rent are explainable by the model.

[6]Note that investments in capacity expansion cease before extraction ceases because as extraction draws to a close, the marginal cost installing new capacity exceeds the discounted future marginal value of this capacity.

and that capacity expansion is bounded by the periodical physical limit, bK_{it}. 6 applies only to OPEC producers. It says that production is at most as great as the assigned quota.

Salant's (1982) dominant cartel versus competitive fringe model is similar to our second stage model. OPEC producers know the form of the demand function; they each perceive price to be a function of their individual output and therefore act as price setters. Given the quota constraint, an OPEC producer chooses its actions, taking as given other OPEC members choices and those of the non-OPEC fringe. The seven non-OPEC producers by contrast, know only the time path for price. They form the competitive fringe, choosing their actions (i.e., production, investment, and reserve development), conditional on the given crude oil price path.

We assume that only the initial states (i.e., reserves, capacity, and resources) and time are relevant for the formation of a producer's strategy. This means producers' strategies are open-loop. Such strategies are well-known to be computationally tractable and impose reasonable informational constraints on the producer since knowledge of states at every possible instant is not required.[7] However, as we discuss later in section 4, the two stage structure of our model combined with the open loop information structure may result in time inconsistent solutions. We address this issue in greater detail in that section.

Formulating the Lagrangian for the producers problem and taking derivatives (refer to AppendixA), we obtain the condition that the OPEC producer chooses production such that marginal revenues[8] are equal to the full marginal cost of production:

$$P_t(\bullet)\left(1+\frac{q_{it}}{\varepsilon_t^o q_t^c}\right)=C_{q_{it}}(\bullet)+\lambda_{it}+\mu_{it}+\kappa_{it}+\omega_{it} \tag{8}$$

where $\lambda_{it}, \mu_{it}, \kappa_{it}$, and ω_{it} are the shadow prices associated with the reserve stock, extractable reserves base, installed capacity and quota constraint, respectively. $\varepsilon_t^o = \frac{(\varepsilon^w - \varepsilon^n)}{q_t^c}Q_t + \varepsilon^n$ is the residual demand elasticity that OPEC producers face, where $\varepsilon^w (<0)$ denotes the global price elasticity of demand for oil[9], and $\varepsilon^n (>0)$ is the price elasticity of supply for non-OPEC oil. q_t^c is total production by the cartel, and $C_{q_{it}}(\bullet)$ is the marginal cost of extraction. When the shadow price on the quota constraint is positive, $\omega_{it} > 0$, then $s_{it} = 0$ and producers extract at their assigned quotas. On the other hand, if the quota constraint is inactive, i.e., $s_{it} \neq 0$, then $\omega_{it} = 0$ meaning that the quota does not impose any economic cost to the producer.

(8) can be rewritten to provide an evaluation of the degree of cartelization, interpreted relative to oligopoly or monopoly behavior. Taking ω_{it} into the brackets on the left hand side of (8) gives, after some algebra:

$$P_t(\bullet)\left(1+\varphi_{it}\frac{1}{\varepsilon_t^o}+(1-\varphi_{it})\frac{q_{it}}{\varepsilon_t^o q_t^c}\right)=C_{q_{it}}(\bullet)+\lambda_{it}+\mu_{it}+\kappa_{it} \tag{9}$$

where $\varphi_{it} = -\frac{\omega_{it}\varepsilon_t^o q_t^c}{P_t q_t^{c-i}}$, for q_t^{c-i} the production of the cartel excluding member i's production. In the literature (see e.g., Symeonidis, 2000; Lofaro, 1999; Geroski et al., 1987; Cyert and DeGroot, 1973), φ_{it} is referred to as the coefficient of cooperation or the market conduct parameter. It provides a concise interpretation of the

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[7]See Dockner et al. (2000) for a discussion of open-loop and Markov or feedback strategies.

[8]Non-OPEC producers equate price rather than marginal revenues, to their full marginal cost. That is, $P_t(\bullet) = C_{q_{it}}(\bullet)+\lambda_{it}+\mu_{it}+\kappa_{it}$. Note that non-OPEC producers are not bound to quota restrictions.

[9]Since two demand regions are modeled, ε^w is computed by weighting the demand elasticity of the two regions, OECD and non OECD, by their optimal consumption levels.

degree of cartelization. Values close to zero indicate low degrees of collusion, i.e., less stringent quota allocations, whereas values close to one indicate highly collusive behavior. $\varphi_{it} = 0$ for all members for every t corresponds to independent non-cooperative behavior while $\varphi_{it} = 1$ corresponds to the fully cooperative outcome, i.e., joint profit maximization. In the former, producers act as Cournot-Nash oligopolies while, in the latter, the different cartel producers act as though they belong to a multi-national monopoly.

Because of the one-to-one correspondence between the allocated quota and the coefficient of cooperation, bargaining over quota allocations is interchangeable with bargaining over φ_{it}. For the rest of the analysis, we shall focus on φ_{it} rather than on explicit quota allocations to evaluate the degree of collusion. In section 3, we shall exogenously vary the coefficient of cooperation and examine how OPEC members' profits change. In section 4, we shall determine the optimal φ_{it} using the Nash Bargaining Solution concept. To keep the analysis of section 3 and part of 4 tractable, we shall fix φ_{it} to be the same for all producers but, will also in section 4 discuss and present results when φ_{it} is allowed to be different. Restricting $\varphi_{it} = \varphi_{jt}$ amounts to imposing the extra condition that the opportunity cost of collusion, i.e., the profits foregone due to the imposed quota restriction, is the same for all colluding producers. This can help narrow the strategy space, which in turn allows for faster numerical computation of the solution to the negotiation stage problem.

2.3. Additional model attributes

To complete the model, the following are defined:

1. Let k denote the two modeled demand regions OECD and non-OECD. The demand function used in the model is of an isoelastic form: $Q_{kt} = A_k P_t^{\varepsilon} Y_{kt}^{(\eta_k + \eta_{1k} log Y_{kt})}$, where Q_{kt} is consumption in region k at time t, A_k is the autonomous demand, Y_{kt} is the Gross Domestic Product (GDP) used to calibrate the time dependent shift in the demand for oil. ε_k ($\varepsilon_k < 0$) is the elasticity of demand for oil, η_k ($\eta_k > 0$) the income elasticity, and η_{1k} ($\eta_{1k} < 0$) a coefficient for energy efficiency. For details on the suitability of this specification, see Medlock and Soligo (2001).

2. For each oil producer (eighteen in total), the following cost functions are used: $C(q_{it}, \Phi_{it}) = \bar{c}_i \times q_{it}^2 \times \left(\alpha_i + \Phi_{it} (1 - \Phi_{it})^{1/\beta_i} \right)$ for production costs, $W(I_{it}) = \bar{w}_i \times I_{it}$ for investment costs, and $Z(x_{it}) = \bar{z}_i \times x_{it}^2$ for finding and development costs. $\bar{c}_i$ is the coefficient used to (iteratively) calibrate the simulation model, so as to ensure that observed production in the base year, is reproduced by the model. Φ_{it} tracks the state of depletion of the producer's reserve base[10], and α_i ($\alpha_i > 0$) and β_i ($\beta_i < 0$) are the coefficients that set the producer's initial costs of production and the speed with which production costs rise with depletion, respectively. $\bar{w}_i$ is the producer's marginal cost of investment which is also equivalent to the average investment costs, whereas $\bar{z}_i$ is a coefficient used to calibrate the producer's discovery costs.

2.4. Scenarios, data and algorithm

For the initial investigation into the effect of cooperation on OPEC profits, the following market structures are specified on the premise that non-OPEC producers are always acting competitively: (i) Competitive (COM); in

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[10]$\Phi_{it} = \frac{R_{io} + S_{io} - (R_{it} + S_{it})}{R_{io} + S_{io} - S_{it}}$

this market OPEC producers supply competitively without any market power. This structure is used as a benchmark against which OPEC's gains to cartelization are computed. (ii) Oligopoly (OLI); here OPEC producers act independently, i.e., $\varphi_{it} = 0$, $\forall i \in c$ and t. As a consequence, there are no gains from cooperation, and the strategies used are Cournot-Nash. (iii) Imperfect collusion 1 (ICOL1); OPEC producers in this instance partially collude, but with a low level of cooperation ($\varphi_{it} = 0.2$, $\forall i \in c$ and t). This implies that while quotas are closer to Cournot-Nash quantities, their may be moderate gains to members from agreeing to coordinate production strategies. (iv) Imperfect collusion 2 (ICOL2): here the coefficient of cooperation is set at a much higher level than in ICOL1 but is still smaller than that implied by joint profits maximization. We set $\varphi_{it} = 0.8$, $\forall i \in c$ and t. In this scenario, while the cartel may be effective at constraining output and raising prices, total profits of the group are not at maximum yet. However, it may turn out that for some members coordinating strategies at this level may be more profitable than at joint profit maximization. (v) Perfect collusion (COL); i.e., $\varphi_{it} = 1$, $\forall i \in c$ and t. OPEC producers in this instance extract oil subject to the same marginal revenue curve. This implies that marginal costs of extraction are equalized across members. Efficiency in this case implies that the least cost producers attain relatively larger quotas while production form higher cost periods is differed to later periods.

Data used in the simulations are as follows. Elasticities are taken from the literature (Krichene, 2002; Gately and Huntington, 2002; Dahl and Sterner, 1991): long-term OECD demand and income elasticities are set to -0.7 and 0.56, respectively, whereas non-OECD demand and income elasticities are set to -0.4 and 0.53, respectively. Non-OPEC's elasticity of oil supply is set to 0.1(Horn, 2004). For energy efficiency, we set η_1 to -0.2 and -0.1 for OECD and non-OECD respectively. These energy efficiency estimates have been chosen to correspond with estimates from Medlock and Soligo (2001). GDP projections used are taken from IIASA 2009; they correspond to the medium growth estimates. Base year production and proven reserves estimates are taken from BP (2009); OPEC (2011), while, remaining resources are computed from USGS (2000) mean estimates of ultimately recoverable reserves. Data on production, investment, and exploration costs are collected from Aguilera et al. (2009); Brandt (2011); EIA (2011).[11] Following Salant (1982); Griffin and Xiong (1997); Pindyck (1978) the same discount rate of 5% is used for all producers; assuming equal discount rates is standard in such analysis.

The model is solved as a Mixed Complementarity Problem (MCP) using the GAMS PATH complementarity programming solver[12] (Ferris and Munson, 2000). For a range of plausible elasticity estimates, the programmed algorithm is first checked for validity, robustness, and consistency by (i) assigning any two players the same initial conditions and (ii) altering the order of players in the model. In the first case, two producers with similar initial conditions are observed to attain the same extraction, investment, and reserve additions profiles. Then, by changing the order of producers when solving for the optimized profiles, the algorithm always converges to the same profile for each producer irrespective of its position in the order. These exercises confirm the validity, robustness and consistency of our algorithm. To validate the uniqueness of the solution, widely diverging initial values are assigned to the decision variables, each time the algorithm iteratively converges to the same solution.

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[11]A detailed description of how the production cost function is calibrated for each region is given in Okullo and Reynès (2011). Because of difficulty in acquiring investment costs data, the same average investment costs per barrel of capacity per year is used for all producers in each respective oil resource category; this data on average investment costs is from Brandt (2011) Table 3.10. The data for exploration costs is obtained from EIA (2011) Table 11 where it is given as finding costs. This data is at regional levels. For any producers that fall in the same region, the same exploration cost profile is assumed.

[12]The optimality (first order and transversality) conditions used to implement the model's algorithm are given in AppendixA.

Although the algorithm is solved for the period 2005 (the base year) to 2100, in order to minimize distortions to profiles as a result of using a finite (rather than an infinite) planning horizon, the reporting period is limited to 2065. Additionally, to reduce computational time, each model period is set equal to ten years; thus, the model solves for production every 10th year, starting with the base year. Models such as ours are designed to capture long-term trends, thus the simulated results cannot explain short term movements mainly characterized by erratic random-walk like fluctuations empirically observed in the oil price for example. Moreover, as is standard in such analysis, our results should be seen as indicative scenarios given the best available data collected and model specification, but not actual real world predictions.
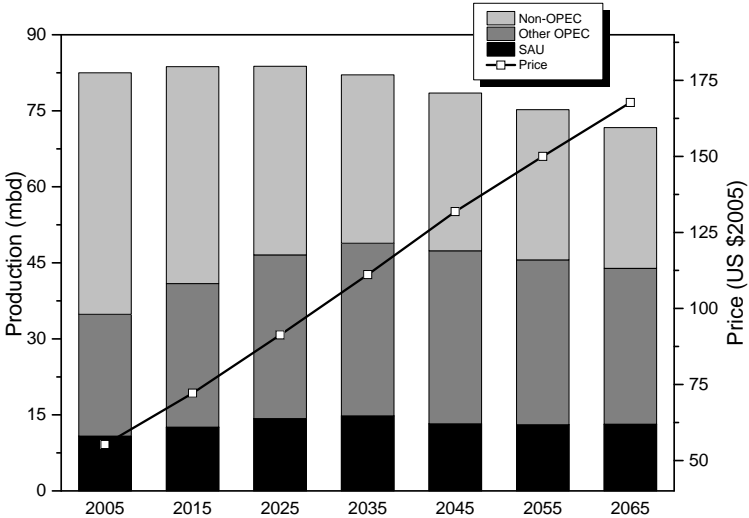
3. Results: gains to cartelization

Figure 1: Global crude oil production profile and global crude oil prices, 2005 to 2065.

Figure 1 shows the global oil price and the global oil production profile when OPEC producers are Cournot-Nash oligopolies (OLI) in the residual demand market. Our projections for production for match well with observed 2015 data and also track well IEA (2014) projections to 2025. Beyond 2025, however, our projections are more conservative than IEA (2014) projections. The reason for the divergence is that we use a more conservative assumption for available resources and we do not model production from newer unconventional resources, such as shale oil that has experienced a recent surge from near negligent levels back in 2005. Our model predicts 86.07 mbd of global oil production in 2015, which compares well with (preliminary) estimates of 86.03 mbd, having subtracted 4.2 mbd of 2015 estimated shale oil supply (EIA, 2015).

In the initial years, a steadfast increase in OPEC production is observed; this increase is primarily driven by a strong demand for oil and declining production in non-OPEC countries. More specifically, Saudi Arabia is seen to initially follow an expansionary production policy: it increases production from about 10.8 mbd in 2005

9

to 14.8 mbd in 2035. However, due to geophysical[13] constraints and a slower growth in global oil demand[14], its production declines thereafter to 13.2 mbd in 2065. Other OPEC producers are seen to follow an even more expansionary policy as compared to Saudi Arabia: by 2035 (other OPEC peak year), their production is observed to increase by 10 mbd relative to 2005 levels before declining by 3.3 mbd to a production level of 30.7 mbd in 2065, due to the impacts of geophysical restrictions and resource limitation in smaller countries such as Algeria and Libya. Production in non-OPEC countries, but the Former Soviet Union, Brazil, and the group Rest of the World, monotonically declines from its 2005 levels due to geophysical constraints and resource limitations.
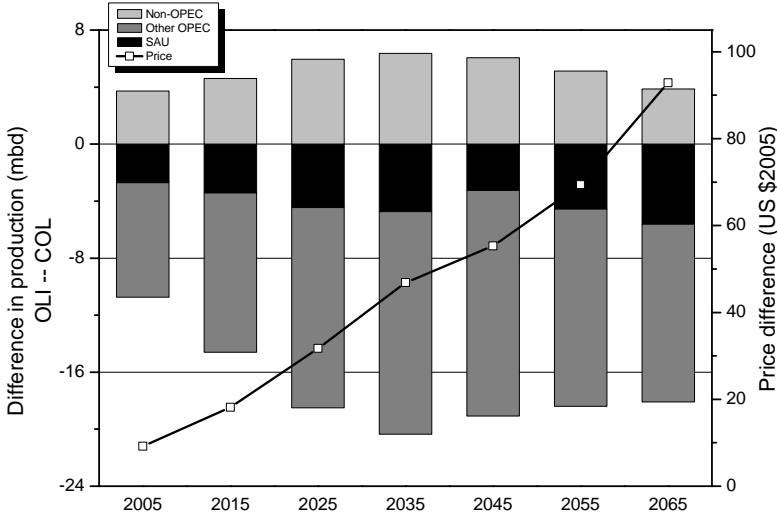

Figure 2: The difference in production, and price, between the full collusion (COL) and oligopoly (OLI) scenarios.

How does the global oil production and price profile change if OPEC acts as a perfect cartel instead? The impact of switching behavior from Cournot-Nash is substantial. We see from Figure 2 that perfect cartelization (full collusion) by OPEC leads to significant reductions in output by Saudi Arabia, but even more for the other OPEC countries. These reductions engender an increase in oil prices which in turn induces non-OPEC countries to increase their production. Nevertheless, because non-OPEC countries have meager resources, the cuts by OPEC countries more than outweigh their increase in production, ultimately leading to substantially higher prices for the OPEC full collusion outcome as compared to the OPEC oligopoly outcome. Indeed, the global oil price in the COL outcome is 9 dollars higher than OLI prices in 2005 and close to 93 dollars higher in 2065.

3.1. Incentives for perfect collusion

The reductions that OPEC makes to its extraction bring positive gains to the cartel. As Table 1 shows, full cartelization increases OPEC gains by 25% relative to the competitive outcome. This gain of 25% is in line with estimates by Griffin and Xiong (1997); Berg et al. (1997). A possible explanation for this congruence, despite the higher prices that are observed in the oil market since the early 2000's is an increase in extraction, development, and

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[13]Geophysical refers to the interaction of geological and capacity constraints.

[14] A declining global oil intensity leads to a slower growth in global oil demand.

Table 1: OPEC and non-OPEC Net Present Values (NPV) in 2005 trillion US$ and resource extraction under alternative scenarios of OPEC cohesion*

		COM	OLI	COL
NPVs (Tn. $)	Total	28.69	31.14 (9%)	41.37 (44%)
	OPEC	14.86	16.15 (9%)	18.61 (25 %)
	non-OPEC	13.83	14.98 (8%)	22.76 (65 %)
Cum. Extraction (bbls)	Total	1838.67	1783.20	1510.72
	OPEC share	58%	56%	40%

*In brackets is the percentage increase in OLI, COL relative to COM

exploration costs that may have limited the increase in OPEC gains. Indeed, Energy Information Administration data indicates that over the period 1999 to 2007, average crude oil lifting costs (and finding costs) increased steadily to nearly double (respectively, triple) levels (EIA, 2011). More importantly, however, by comparing OPEC's full cartelization gains of 25% to the 9% that is obtained if the cartel is simply a Cournot oligopoly, it is apparent that OPEC members have strong incentives to collude.

OPECs cooperation generates substantial benefits for non-OPEC producers as well. Table 1 shows that non-OPEC producers profits increase by 65% compared to the case were the cartel acts competitively. This increase in non-OPEC oil wealth indicates the challenge that OPEC faces in the real world. That is, although collusion allows OPEC to increase its gains, the fact that they also increase non-OPEC gains could entice some members to overproduce their quota allocations so as to reap some of the benefits that would otherwise go to non-OPEC producers. This tendency, has in the literature been referred to as cheating and is thoroughly investigated by Griffin and Xiong (1997). In this paper, we argue that the increase in non-OPEC profits, due to OPEC collusion, has broader implications that influences OPEC's actual structure and the way quotas are allocated. OPEC will structure itself as an imperfect cartel so as: (i) to more evenly distribute among members the burden of holding back production and (ii) to reap comparatively more of the gains from their own attempt at collusion, that would otherwise go to non-OPEC producers. These explanations are particularly credible since recent econometric evidence (Smith, 2005; Kaufmann et al., 2008) indicates that OPEC fits neither Cournot oligopoly nor perfect cartelization models. We investigate this issue next.

3.2. Incentives for partial collusion

Table 2 shows OPEC's net present values, by producer, in the competitive outcome — the case in which the OPEC cartel is dissolved and its members have no influence on price at all — and the percentage increase in gains when collusion is at $\varphi = 0.2$ (ICOL1), $\varphi = 0.8$ (ICOL2), and $\varphi = 1$ (COL). Notably, by moving from COM to ICOL1, all OPEC producers gain, and in turn the cartel. Moving even further to ICOL2, all members still gain and so does the cartel. But on moving further to COL, all OPEC members, with the exception of Saudi Arabia (indicated in bold), loose relative to the ICOL2 outcome, meanwhile non-OPEC gains continue to rise. Because of the general losses within OPEC ranks, the cartel as a whole looses.

Table 2: OPEC members' and Non-OPEC Net Present Values (NPV) in 2005 trillion US$ under alternative scenarios of OPEC cohesion*

		Percentage gain relative to COM		
	COM	**ICOL1**	**ICOL2**	**COL**
Algeria	0.50	23%	45%	44%
Angola	0.38	23%	47%	45%
Iran	1.86	19%	26%	22%
Iraq	0.88	20%	21%	14%
Kuwait	1.16	21%	27%	22%
Libya	0.67	23%	35%	30%
Nigeria	0.87	22%	35%	31%
Qatar	0.42	23%	31%	25%
Saudi Arabia	**5.21**	**10%**	**20%**	**23%**
United Arab Emirates	1.15	21%	27%	22%
Venezuela	1.76	21%	33%	30%
Total OPEC	**14.86**	**17%**	**27%**	**25%**
Non-OPEC	**13.83**	**18%**	**53%**	**65%**

*In our calculation of net present values, earnings follow output. There are no transfers between colluding producers.

What these observations indicate is that heterogeneity[15] within the OPEC cartel greatly influences the benefits OPEC members individually earn from cooperation, which then influences OPEC's likely choice for φ and hence the way quotas are allocated to members. Clearly, members collectively gain over a part of the cooperation values. As the sacrifices from cooperation become greater to some members, however, these members start to loose. Considering that OPEC quotas are determined through negotiation, it is more logical that OPEC producers would settle for ICOL2, instead of COL; first, since more members gain, and second, because for the parameterized supply and demand elasticities, the cartel as a whole gains by staying at ICOL2. This suggests that OPEC will not necessarily assign quotas so as to equalize marginal revenues (as a perfect cartel would do), but will inherently recognize differences in marginal revenues curves between members when assigning quotas. This is in fact a plausible reason why econometric testing for OPEC behavior as a perfect cartel has been in vain.

In support of the notion that members could find it hard to commit to a full cooperation outcome, we also see from Table 2 that the existence of the non-OPEC fringe, and of course their level of oil supply, further limits the gains that OPEC producers attain from increased cooperation. To allow for higher profits, the cartel members have to cut their production far below the level that the fringe can offset; this is the only way OPEC can induce high prices in the oil market. The more OPEC cuts production, however, the more profitable it becomes for the fringe to increase production. Indeed, by OPEC moving from ICOL1 to ICOL2, then to COL, non-OPEC becomes

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[15]Note that heterogeneity between producers in our model is captured through reserve and resource endowments, production level, initial level of marginal costs, and steepness of marginal costs as reserves and resources get increasingly depleted.

the bigger beneficiary. Why would OPEC attempt stronger collusion when in effect most of the gains are being eaten away by non-OPEC countries? Instead, OPEC will most likely choose a level of cooperation lower than that implied by COL, so as to crowd out more of non-OPEC's price-dependent production and retain relatively more profit for its members. Simply put, OPEC will assign quotas not as a perfect cartel, but instead as an imperfect cartel. Next, we show that OPEC's optimal cooperation level (choice for φ) is substantially influenced by demand elasticity in the oil market.

3.3. Changes in demand elasticity

The impact of changes in demand elasticity are not investigated in the studies of Griffin and Xiong (1997) and Berg et al. (1997). Yet, as Dahl and Sterner's survey on elasticities indicates, the uncertainty about demand elasticities is rather large. Therefore, to see how OPEC's gains might be influenced if elasticities are incorrectly specified, we double[16] demand elasticities for OECD and non-OECD to -1.4 and -0.8, respectively. This is equivalent to availing consumers with more substitutes to which they can easily turn to given a unit increment in the oil price, and implies that the price path obtained using these larger elasticities should be lower than that implied by the reference elasticities. The impact of these new elasticities on net present values is reported in Table 3.

We see that with larger (absolute) demand elasticities, more OPEC producers benefit from full collusion than with lower (absolute) demand elasticities. As indicated by the producers in bold, the number of OPEC members who would favor (or become indifferent about) full collusion now increases from one to six. The cartel as a whole marginally gains by moving from ICOL2 (11.32%) to COL (11.36%). Under the threat of ready substitutes, therefore, the model indicates that OPEC producers are more likely to adapt a more cooperative outcome. Nonetheless, since half of the cartel still looses by moving from ICOL2 to COL, it follows that even under these circumstances of larger (absolute) elasticities, full cooperation will not be the naturally prevailing strategy.

Increasing the elasticity of supply of non-OPEC oil from 0.1 to 0.4 also makes the elasticity of demand for OPEC oil more elastic. For base year production levels, this is equivalent to increasing elasticity of demand for OPEC oil from -1.51 to -1.92. We test the implications of these changes; we rerun the simulations with the reference OECD and non-OECD demand elasticities, but increase the supply elasticity for non-OPEC oil as indicated. We find that the cartel as a whole increases gains by moving from ICOL2 (26%) to COL (27%). Individually, the same members that do not loose from full collusion in Table 3, are also found not loose in this instance. Moreover, Iran is now included in this group, bringing the number of members who could favor full collusion to seven. Given that four out of the eleven OPEC members still loose from full collusion, we still reach the same conclusion: full cooperation will not naturally follow as the prevailing strategy.

The mechanism through which OPEC members become indifferent about full cooperation, for an increasingly

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[16]Other than doubling, we can halve these long-run elasticities. Using an isoleastic demand curve in Equation 9 combined with OPECs market share, however, constrains from using combinations of OECD and non-OECD elasticities that yield a consumption weighted elasticity of absolute value lower than 0.42, if we are to obtain solutions to the OPEC perfect cartelization problem. The behavioral effects of knowing what happens when absolute elasticity is lower can nonetheless be observed by comparing the results in the doubled elasticity case to the results in the reference elasticities case. As regards to the profits that the perfect cartel may earn relative to the competition outcome, we know that the lower the elasticity, the higher the percentage gains from perfect collusion since the market approaches limit pricing. Though limit pricing as argued by de Sa et al. (2012) could be relevant for OPEC, for computational and practical reasons the approach is not considered here. And although using a linear demand function can allow us to use lower elasticities, its price elasticity of demand can change with price in a manner that may not be reflective of future market conditions, thus limiting its use for long-term analysis such as ours.

Table 3: OPEC members' and non-OPEC Net Present Values (NPV) in 2005 trillion US$ under alternative scenarios about OPEC cohesion, doubled demand elasticities case*

		Percentage gain relative to COM		
	COM	ICOL1	ICOL2	COL
Algeria	**0.42**	**9%**	**19%**	**20%**
Angola	**0.32**	**9%**	**19%**	**20%**
Iran	1.41	7%	11%	10%
Iraq	0.67	7%	9%	7%
Kuwait	0.89	7%	11%	10%
Libya	**0.54**	**8%**	**14%**	**14%**
Nigeria	**0.70**	**8%**	**14%**	**14%**
Qatar	0.33	8%	13%	12%
Saudi Arabia	**3.80**	**5%**	**9%**	**10%**
United Arab Emirates	0.88	7%	11%	10%
Venezuela	**1.38**	**8%**	**13%**	**13%**
Total OPEC	**11.34**	**6%**	**11%**	**11%**
Non-OPEC	**11.77**	**6%**	**15%**	**19%**

*In our calculation of net present values, earnings follow output. There are not transfers between colluding producers.

elastic demand curve, is as follows. The large (absolute) demand elasticities induce a more elastic marginal revenue curve. In such circumstances, scaling back production by a small amount does not significantly raise prices and hence profits. To ably do so, deeper cuts in production are necessary, implying higher degrees of collusion by the cartel. Of the past studies on OPEC cartelization — Griffin and Xiong (1997); Berg et al. (1997); Pindyck (1978) — no study highlights the possibility for the elasticity of demand to influence OPEC cooperation as seen above. One reason for this is that given collusion, OPEC is modeled as a perfect cartel, which then reveals only one side of the story: gains from cooperation are high (low) when demand is inelastic (elastic). It says nothing about the degree of collusion required to sufficiently raise prices. Our analysis indicates that cartels will assign less (more) stringent quotas when market demand is inelastic (elastic). This result is generalized analytically in AppendixB for case of a non-exhaustible resource. In the literature on industrial organization, it has been shown that the size of the cartel versus that of the fringe (Salant et al., 1983), the number of members in the cartel (Lofaro, 1999, and the references therein), and the level of prices (Rotemberg and Saloner, 1986), will influence the ability to collude. No study that we are aware of shows how demand elasticity influences collusion. Our result draws similarities to Rotemberg and Saloner's where it is shown that a cartel will behave less (resp. more) collusively in periods of high (resp. low) demand.

4. Endogenous cartelization

The preceding section relies upon exogenous changes in the coefficient of cooperation to draw the conclusion that there is a strong incentive for OPEC to structure itself as an imperfect cartel because of: (i) heterogeneities within the cartel, (ii) the presence of the non-OPEC fringe, and (iii) an inelastic demand curve that makes it unlikely for OPEC to negotiate stringent production allocations. This section expands on those results using an endogenously chosen degree of cartelization. We continue to use the coefficient of cooperation as the negotiation variable in place of an explicit quota allocation. As mentioned earlier, there is one-to-one correspondence between the two and can therefore be used interchangeably. We first describe the methodology used to choose φ_{it}, then present some numerical results.

4.1. Methodology

We use the Nash bargaining solution (Nash, 1950, 1953) to select the optimal φ_{it}, although, other bargaining models such as the Kalai-Smorodinksy (Kalai and Smorodinsky, 1975) solution may be used. To save on notation, we describe the model set-up for the case where φ_{it} is restricted to be identical across OPEC producers, i.e., $\varphi_{it} = \varphi_{jt}, \forall i \neq j; i, j \in c$, but shall later present simulation results for the case where φ_{it} differ. Mathematically, the Nash bargaining problem[17] for the OPEC cartel is:

$$max_{\{\varphi_t\}} G_\tau (\bullet) = \prod_{i \in c} \pi_i \left(\{\varphi_t\}_{t=\tau}^{t=\infty} \right) \tag{10}$$

s.t.

$$\pi_i (\varphi_t) \in \Omega, \quad \varphi_t \in [0,1] \tag{11}$$

where $\pi_i \left(\{\varphi_t\}_{t=\tau}^{t=\infty} \right)$ is the net present value profit that accrues to the OPEC member i at a time τ for $\{\varphi_t\}_{t=\tau}^{t=\infty}$ levels of cooperation[18]. Ω denotes a compact set of possible profit realizations. As earlier mentioned, the model is set up as a two stage problem. Objective (10) is solved in the first stage to maximize the product of individual OPEC members' net present value profits by selecting $\varphi_t \in [0,1]$ subject to the optimality conditions of the second stage problem presented in (1)-(7). Since φ_t is concave in individual profits and bounded, it is possible to find an optimal solution. Note that by virtue of φ_t being bounded at zero from below, negotiating producers always earn at least their disagreement (i.e., Cournot-Nash) profits. Thus, the disagreement point is implicitly embedded into the objective function (10) negating the need to introduce it explicitly, as is usually the case when setting up the Nash Bargaining Problem.

Because the model (10)-(11) is hierarchical, solutions obtained using open-loop strategies can be time inconsistent. This is the property that when agents reconsider their solutions after some time has passed, they may have the

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[17]Although not introduced in the current model because of their dependence on the past observations, it is nonetheless, possible to add bargaining weights that are consistent with exogenous factors such as a member country's GDP, population, external debt and share of oil revenues in GDP. Alsalem et al. (1997) reports that a combination of these variables are sometimes used by OPEC when deciding how to distribute quotas.

[18] Recall that φ_t simply summarizes the extent to which producers scale back output, given that the optimal quota and optimal (individual) production coincide.

incentive to deviate from their original plans. Such solutions, therefore, cannot be credible unless binding agreements are assumed (Zaccour, 2008; Yeung and Petrosyan, 2012; Haurie, 1976). Our global oil model, (10)-(11) is time inconsistent for two reasons. Firstly, when the OPEC cartel reconsiders its quota decisions after a period of commitment, it has the incentive to revise them downwards, that is, issue more stringent quotas. The reason for this being that at the time it reconsiders its decision, it perceives its members' costs of collusion from that point onward as being lower than before. The second effect works in the opposite direction. In this case, the cartel would like to announce ambitious output targets so as to pre-empt supply by the non-OPEC fringe, even when such a strategy would not be credible ex-post (Groot et al., 2000, 2003).

We adopt two approaches for dealing with the time inconsistency issue. The first, is that in a model such as ours, the act of forming the cartel can itself be regarded as a commitment device that binds producers to their initially agreed plans. This negates the need to reconsider plans after some time has elapsed. We shall, therefore, present our open-loop results but, they should strictly be regarded as binding commitment solutions.

The often used approach for dealing with time inconsistency in non-cooperative games is to compute feedback equilibria as these strategies are known to be robust to off-equilibrium outcomes. However, considering the hierarchical play, the non-linear functional forms, the multiple players, and multiple state variables[19] in our model, it is a non-trivial undertaking to try and find sub-game perfect solutions, unless of course the strategies employed by the producers are severely restricted. Dockner et al. (2000) illustrates one of these restrictions and shows that while the restriction does indeed generate a sub-game perfect equilibrium, there is no way of knowing whether the imposed policy functions are indeed optimal. Moreover, although such an approach may generate time consistent solutions, it severely restricts strategic interaction in the model.

Our chosen and second approach of dealing with time inconsistency is to compute a renegotiated closed-loop sequential equilibrium (Yang, 2003). In such an equilibrium, the original plan is reconsidered after short-periods of commitment, where if the new plans deviate from the original plan, earlier plans are discarded and the new plans taken up. The solution obtained here is akin to what is observed in practice where contracts are drawn up but are renegotiated after some time of commitment. The algorithm for such an equilibrium requires solving the open-loop model (10)-(11) sequentially through time, while each time picking the solutions (i.e., quotas, production, and investments) of the initial periods from each sequence and concatenating these to create the equilibrium solution. Jørgensen et al. (2010) advocates for this strategy in cases when the model under consideration is intractable for computing feedback strategies and negotiation permits for periodical reevaluation of strategies.

For our calculations, we use the GAMS NLPEC solver, with CONOPT 3 as the subsolver (Ferris et al., 2005). The stability of our solutions are verified by solving the model several times, each time using diverging starting values for φ_t. For each solve, the algorithm is found to converge to the same unique time path for φ (including other decision variables). In the next section we present some results from our simulation model. Note that our intention here is to quantitatively examine how quotas are likely to be allocated to members given their divergent attributes. This is especially important since the joint profit maximizing approach is unappealing because of its assumption that members implement some form of revenue sharing scheme. Our premise is that OPEC members

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[19]The second-stage decision problem has eighteen producers each with three state variables. This implies that the first-stage OPEC negotiation is based on double the number of second-stage states, since co-states to each second stage state become state variables in the first stage. This makes for one hundred and eight ($18 \times 3 \times 2$) state variables over each we must approximate the subgame perfect equilibrium.

make concessions on the size of the quota that any particular member receives, so as to give the various participants the incentive to remain a part of the organization.

4.2. Results

We compare four equilibrium strategies for OPEC: (i) Cournot-Nash oligopoly ($\varphi = 0$), (ii) binding commitments Nash bargaining with optimal and identical φ_t, (iii) sequential commitments Nash bargaining with optimal and identical φ_t, and (iii) perfect cartelization ($\varphi = 1$). We label these, OLI, IC-BC, IC-SC, and PC, respectively. The results presented next are based on the calibration of each model so that it reproduces base year production data. As indicated in subsection 2.3, this is accomplished by iteratively recalibrating the coefficient $\bar{c}$ in the producers cost function. As a result of this recalibration, net present values are not a good indicator of the incentive to cooperate, but φ from the bargaining solution is. The ensuing analysis is therefore not concerned with how cooperation affects OPEC producers' profit, but rather the extent to which the projected equilibrium for global oil production with optimal cooperation coefficients differs from the traditional Cournot-Nash and perfect cartelization equilibria.
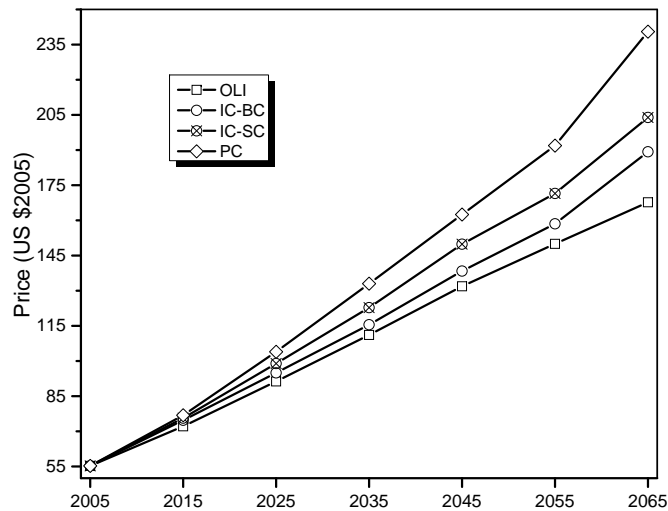

Figure 3: Global crude oil prices, 2005 to 2065, in three states of OPEC cartelization: oligopoly (OLI), imperfect cartelization (IC-BC, IC-SC), and perfect cartelization (PC),.

Figure 3 shows the global crude oil price in the four scenarios of OPEC cartelization. As expected, price in the perfect collusion (PC) scenario is higher than price in the oligopoly (OL) scenario. Prices implied by the bargaining outcomes are between the two extremes. The bargaining solutions give rise to OPEC's optimal structural arrangement, and thus provides insight into how OPEC might actually behave in the global oil market. We see that OPECs preferred arrangement is to issue production allocations such that global oil prices are not so high so as to crowd-in non-OPEC production, but not so low as to forego profits from coordinating strategies.

Price in the bargaining with sequential commitments scenario is higher than price in the binding commitments scenario. The reason for this is that every time the OPEC cartel reconsiders its decision as spelled out in the

definition of the equilibrium, the incentive to issue more stringent quotas outweighs the incentive to pre-empt the non-OPEC fringe leading to a delayed path for extraction and hence higher global oil prices.
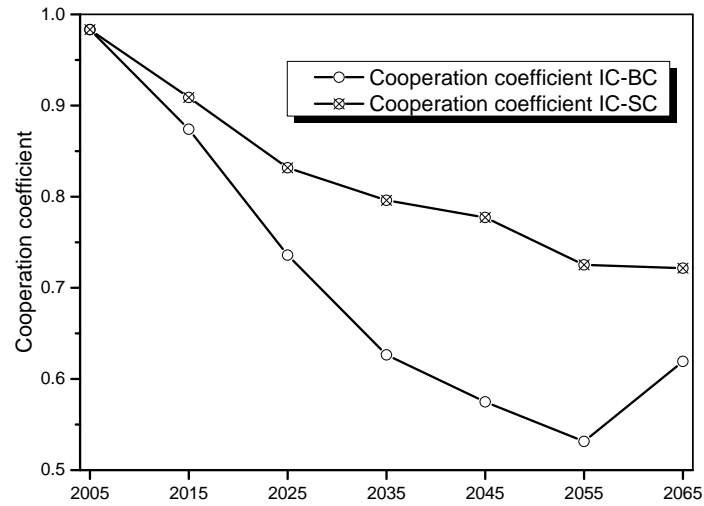

Figure 4: Optimal OPEC cooperation levels, 2005 to 2065.

The coefficients of cooperation resulting from OPECs optimal strategies, IC-BC and IC-SC, are shown in Figure 4. Production allocations are most stringent in the initial years, but then become gradually less stringent overtime. As can be seen by comparing IC-BC to IC-SC, part of this change can be explained by the perceived change in the costs of commitment. The time paths highlight two issues that are in contrast to what is captured in the traditional cartel model. Firstly, that the relative stringency of OPEC's quota allocations can optimally vary from period to period as indicated by the changing coefficient of cooperation, and secondly, OPEC strategies while plausibly highly collusive, are insufficient to classify it as a perfect cartel. To check the robustness of our results, we reran the model with doubled demand elasticities. Basically, we obtain nearly the same time path as in Figure 4, but with generally higher levels of cooperation — in the IC-BC (IC-SC) scenario the cooperation coefficient is at 1.00 (1.00) in 2005, falling to 0.51 (0.70) in 2065. This indicates that under a wide range of elasticities, OPEC will still prefer imperfect perfect collusion, and, as earlier indicated, that OPEC is likely to attain higher levels of cooperation when its residual demand curve is more elastic.

Imperfect collusion has clear benefits: it accrues relatively more of the gains from cooperation to OPEC than to non-OPEC. To see this, we compare relative differences in OPEC and non-OPEC net present values for the IC-BC, IC-SC and PC structures. When OPEC acts as a perfect cartel (PC scenario), until 2065, it earns US$ 24.02 trillion. In case it acts as an imperfect cartel with binding commitments (IC-BC), it earns US$ 24.83 trillion, and as an imperfect cartel with sequential commitments it earns US$ 24.78 trillion. The difference for IC-BC and IC-SC relative to PC is 3.4 and 3.2 percent, respectively. By contrast, non-OPEC earns US$ 20.20 trillion in the PC structure, and US$ 17.54 trillion in the IC-BC and US$ 18.52 trillion in the IC-SC structure, a relative difference of -13.12 and -8.3 percent, respectively. Clearly, non-OPEC looses while OPEC accrues relatively more gains to itself from the choice to imperfectly cartelize.

Figure 5 shows by how much OPEC (non-OPEC) production decreases (increases) when OPEC acts as an
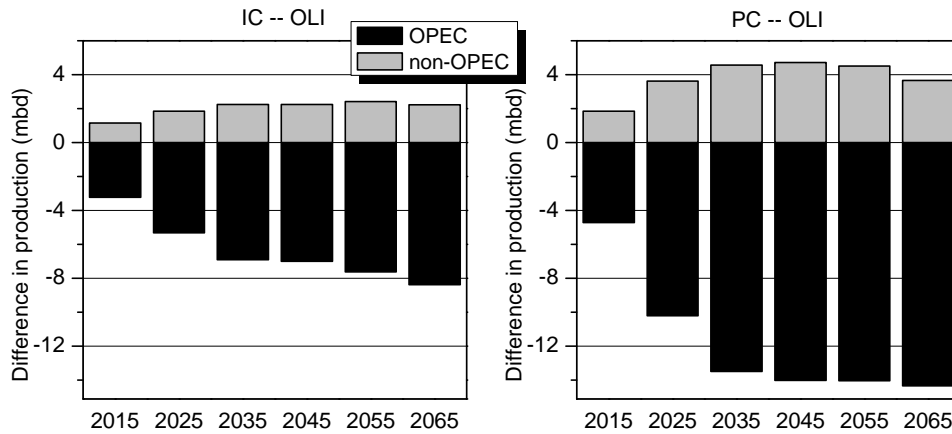
Figure 5: Difference in production for IC relative to OLI (IC less OLI), and PC relative to OLI (PC less OLI), 2015 to 2065.

imperfect cartel with binding commitments, and also when OPEC acts as a perfect cartel, both relative to the OLI outcome.[20] In the PC scenario, OPEC reduces its production by nearly twice as much, as compared to the IC scenario. Clearly the deep cuts required for perfect collusion are far from optimal from a negotiation perspective, otherwise the negotiating producers would set their all their cooperation coefficients at one. So rather than increase profits through further cuts in production, OPEC finds it more suitable to supply more relative to perfect cooperation so as to ensure that every member is Pareto indifferent about cooperation. The increased level of supply, more than crowds out non-OPEC production allowing OPEC to receive a larger relative share of gains in the oil market from its own attempts at collusion.

So far we have restricted our attention to the case where φ_{it} is identical amongst OPEC producers. What happens when φ_{it} are allowed to differ across OPEC producers? Our results indicate that the difference in the production profiles for most producers is marginal (not shown for brevity), and the difference in the price profile moderate (see: Table C.4 and Table C.5 in AppendixC). Concerning the coefficient of cooperation, we see in Table C.4 and Table C.5 that it is optimal for smaller OPEC producers (e.g., Qatar, Angola, Algeria) to behave less collusively as compared to the larger producers (e.g., Saudi Arabia, Iran, Venezuela, Kuwait). This allows smaller producers to follow an even more more expansionary production approach, increasing their production while the larger producers hold back. This observation is particularly interesting. It indicates that during the negotiation process, the power to acquire larger than proportionate production shares is biased towards smaller OPEC producers. Empirical support for such behavior within cartels is reported in Libecap (1989); Griffin and Xiong (1997).

5. Discussion

The question of whether OPEC is a cartel or not has been the subject of several econometric studies. Unfortunately, no consistent answer arises from these exercises. Moreover, of the few numerical simulation studies

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[20]The graph for sequential commitments is not presented as it tells more or less the same story.

that have been used to study OPEC, none investigates the likely structure of OPEC at a level of detail that sheds light on individual members incentives to cartelize. In fact, the only study that comes close is that of Griffin and Xiong (1997). The authors' focus, however, is on OPEC members' incentive to cheat and not OPEC's cartelization structure itself. In this paper, we have tried to answer the question of what OPEC's preferred degree of collusion is, using a tractable empirically calibrated global oil market model. Of course, in reality OPEC behavior is influenced by several political and economic uncertainties that can be difficult incorporate in simulation models such as ours. Nonetheless, this should not prevent us from trying to understand OPEC behavior on the basis of the best available data. Our results overwhelmingly lead us to conclude that OPEC is a cartel that is characterized by imperfect collusion. In the words of Adelman (2002), OPEC is clumsy cartel or in the words of Smith (2005) a bureaucratic syndicate.

As our results indicate, imperfect collusion arises because the cartel has to balance both internal and external interests. Balancing internal interests means that the cartel has to assign quotas in a manner that would not cause some members to exit the arrangement. This as indicated requires giving larger production quotas than would be implied by effective collusion, more especially to smaller producers. To balance external interests, OPEC has to ensure that its production is large enough so as to crowd out non-OPEC supply that thrives under situations of high prices. As was seen, effective collusion generates high prices. To achieve these prices, OPEC must cut output far beyond levels the fringe can offset; this increases the cartels gains. Nonetheless, because non-OPEC production also thrives under situations of high prices, perfect collusion of OPEC benefits the non-OPEC fringe more than it does OPEC. The optimal and most plausible strategy for OPEC therefore, is one of keeping prices fairly low so as to discourage production, investment in capacity, and exploration in non-OPEC countries, but still high enough to provide positive benefits to its members from collusion. This strategy is captured in the imperfect cartelization arrangement.

Effects of changes in the discount rate, income elasticity, changes in the growth rate of GDP, and energy efficiency on OPEC cooperation were investigated. These do not affect our conclusion that OPEC is an imperfect cartel. They nonetheless influence the time path of the cooperation coefficient. A higher (lower) discount rate leads to lower (higher) cooperation levels, a higher (lower) level of income elasticity leads to a higher (lower) levels of cooperation, a faster (slower) GDP growth rate implies higher (lower) levels of cooperation, and finally a higher (lower) level of energy efficiency results in lower (higher) levels of cooperation. Results are also checked for robustness by assuming that the coefficients, $\bar{w}$ and $\bar{z}$, on marginal exploration and marginal investments are both 50% lower (resp. higher) than used in the main simulations. Our conclusions about OPEC being an imperfect cartel are upheld. The effects of on the cooperation level, however, are ambiguous.

Given the above results, an important and yet general contribution of our paper is the introduction of a tractable model that can be used to study cartelization in cases where there is imperfect collusion between parties. Implied in our coefficient of cooperation is the extent to which producers scale back output in order to raise prices. Although we do not explicitly model the likelihood for different producers to cheat on their quotas, the inclusion of this dimension would likely make our conclusions on quota allocations even stronger. This, in addition to extending the model to Markov perfect strategies will be the focus of future work.

Acknowledgments

AppendixA. Necessary conditions and transversilty of the OPEC Nash bargaining problem

The Lagrangian to the problem (1)-(7) is:

$$
\begin{aligned}
\mathcal{L}_{it} = P\left(Y_t, Q_t\right) q_{it} &- C\left(q_{it}, R_{it}, S_{it}\right) - W\left(I_{it}\right) - Z\left(x_{it}\right) + \upsilon_{it}\left(x_{it} - q_{it}\right) \\
&+ \chi_{it}\left(I_{it} - \Delta K_{it}\right) - \lambda_{it} x_{it} + \mu_{it}\left(\gamma R_{it} - q_{it}\right) \\
&+ \kappa_{it}\left(K_{it} - q_{it}\right) + \xi_{it}\left(b K_{it} - I_{it} + \Delta \cdot K_{it}\right)
\end{aligned}
$$

In the above definition of the Lagrangian, the non-negativity constraints are excluded as these make no difference in our simulation algorithm. The necessary and transversality conditions for the lower l evel game are given as:

$$
\frac{d\mathcal{L}}{dq_{it}} \equiv P\left(Y_t, Q_t\right) + q_{it}\frac{q_t^c}{q_t^c}\frac{\partial P\left(Y_t, Q_t\right)}{\partial q_{it}}\frac{\partial q_t^c}{\partial q_t^c}\frac{P\left(Y_t, Q_t\right)}{P\left(Y_t, Q_t\right)} - C_q\left(q_{it}, R_{it}, S_{it}\right) - \upsilon_{it} - \mu_{it} - \kappa_{it} - \omega_{it} = 0 \quad \text{(A.1)}
$$

$$
\frac{d\mathcal{L}}{dI} \equiv -W_I\left(I_{it}\right) + \chi_{it} - \xi_{it} = 0 \quad \text{(A.2)}
$$

$$
\frac{d\mathcal{L}}{dx} \equiv -Z_x\left(x_{it}\right) + \upsilon_{it} - \lambda_{it} = 0 \quad \text{(A.3)}
$$

$$
\dot{\upsilon}_{it} = \delta \cdot \upsilon_{it} + C_R\left(q_{it}, R_{it}, S_{it}\right) - \gamma \cdot \mu_{it} \quad \text{(A.4)}
$$

$$
\dot{\chi}_{it} = \left(\delta + \Delta\right) \cdot \chi_{it} - \kappa_{it} - \xi_{it} \cdot \left(b + \Delta\right) \quad \text{(A.5)}
$$

$$
\dot{\lambda}_{it} = \delta \cdot \lambda_{it} + C_S\left(q_{it}, R_{it}, S_{it}\right) \quad \text{(A.6)}
$$

$$
\mu_{it}\left(\gamma R_{it} - q_{it}\right) = 0; \mu_{it} \geq 0, \left(\gamma R_{it} - q_{it}\right) \geq 0 \quad \text{(A.7)}
$$

$$
\kappa_{it}\left(K_{it} - q_{it}\right) = 0; \kappa_{it} \geq 0, \left(K_{it} - q_{it}\right) \geq 0 \quad \text{(A.8)}
$$

$$
\xi_{it}\left(b K_{it} - I_{it} + d \cdot K_{it}\right) = 0; \xi_{it} \geq 0; \left(b K_{it} - I_{it} + \Delta \cdot K_{it}\right) \geq 0 \quad \text{(A.9)}
$$

$$
\omega_{it}\left(\tilde{q}_{it} - q_{it}\right) = 0; \omega_{it} \geq 0, \left(\tilde{q}_{it} - q_{it}\right) \geq 0 \quad \text{(A.10)}
$$

$$
\text{including Equations (2)-(4)} \quad \text{(A.11)}
$$

$$
\lim_{t\to\infty} e^{-\delta t}\lambda_{it} \geq 0, \lim_{t\to\infty} e^{-\delta t}\lambda_{it}S_{it} = 0 \quad \text{(A.12)}
$$

$$
\lim_{t\to\infty} e^{-\delta t}\upsilon_{it} \geq 0, \lim_{t\to\infty} e^{-\delta t}\upsilon_{it}R_{it} = 0 \quad \text{(A.13)}
$$

$$
\lim_{t\to\infty} e^{-\delta t}\chi_{it} \geq 0, \lim_{t\to\infty} e^{-\delta t}\chi_{it}K_{it} = 0 \quad \text{(A.14)}
$$

By carefully selecting functional forms, we can ensure the concavity of the Lagrangian and hence the existence of a solution for the defined game (Léonard and Long, 1992, pp. 210-214, 288-289). Note that for non-OPEC producers, $\frac{\partial P(Y_t, Q_t)}{\partial q_{it}} = 0$.

The Nash bargaining scheme with negotiation and production set up as a hierarchical play can now be explicitly written as:

$$max_{\{\varphi_{it}\}} G_\tau (\bullet) = \prod_{i \in c} \left(\pi_i \left(\{\varphi_{it}\}_{t=\tau}^{t=\infty} \right) \right)$$

where π_i are net present value profits, subject to Equation (9), (A.2)-(A.14) for both OPEC and non-OPEC. In addition to the first order conditions to be derived for this defined bi-level game, we also have to impose the following $e^{-\delta t} \zeta_t \nu_t \to 0$, $e^{-\delta t} \theta_t \chi_t \to 0$, $e^{-\delta t} \rho_t \lambda_t \to 0$ (Xie, 1997), where ζ_t, θ_t and ρ_t are shadow prices on the respective lower level co-state variables, turned state variables in the upper level game.

AppendixB. Elasticity of demand and the level of cooperation

Here, we analytically show that increasing the absolute value of the elasticity of demand (making the demand curve more elastic) increases the incentive to cooperate among colluding producers. For analytical convenience and to illustrate the generality of the result, we concentrate on the case of a non-depleting resource and simplify the relations of model (1)-(7) so as to concentrate only on extraction decisions. In addition, we assume all players, i, $i \in n$ $(n > 1)$, are involved in the cooperation game and that these players are identical.

With these simplifications, the equivalent of Equation (8), for a producer i can be written as:

$$P(Q) + P_Q(Q) \cdot Q \cdot \Phi - C_q(q) = 0 \tag{B.1}$$

where Φ incorporates the coefficient of cooperation that is defined in the main text, and is positively related to it[21], $Q = n \cdot q$ is the total marketed output, $P(Q)$, is the market price, and $C_q(q)$ and q are the marginal costs of production, and the production level for a producer i, respectively. The marginal cost function is restricted such that $C_q > 0, C_{qq} > 0$. Denoting the marginal revenue function as $MR(Q, \Phi, \varepsilon) = P(Q, \varepsilon) + P_Q(Q, \varepsilon) \cdot Q \cdot \Phi$, we can find how elasticity influences cooperation by taking the total derivative of $MR(Q, \Phi, \varepsilon) - C_q(q) = 0$. This gives:

$$MR_Q(Q, \Phi, \varepsilon) dQ + MR_\Phi(Q, \Phi, \varepsilon) d\Phi + MR_\varepsilon(Q, \Phi, \varepsilon) d\varepsilon - C_{qq}(q) dq = 0 \tag{B.2}$$

Since $\Phi = \frac{dQ}{dq} \frac{q}{Q}$, Equation B.2 can be re-written as:

$$MR_Q(Q, \Phi, \varepsilon) dQ + MR_\Phi(Q, \Phi, \varepsilon) d\Phi + MR_\varepsilon(Q, \Phi, \varepsilon) d\varepsilon - \frac{1}{\Phi} C_{qq}(q) \frac{q}{Q} dQ = 0 \tag{B.3}$$

First considering the case where, Φ, changes but ε remains constant, we have that:

---

[21]To see this, notice that $\Phi_i = \frac{q_i + \varphi q^{c-i}}{Q}$. Thus $\frac{\partial \Phi}{\partial \varphi} = n - 1$ where $n - 1 > 0$.

$$\frac{dQ}{d\Phi} = -\frac{MR_\Phi(Q,\Phi,\varepsilon)}{\left(MR_Q(Q,\Phi,\varepsilon) - \frac{1}{\Phi}C_{qq}(q)\frac{q}{Q}\right)} \tag{B.4}$$

And when, ε, changes, but Φ remains constant, we have:

$$\frac{dQ}{d\varepsilon} = -\frac{MR_\varepsilon(Q,\Phi,\varepsilon)}{\left(MR_Q(Q,\Phi,\varepsilon) - \frac{1}{\Phi}C_{qq}(q)\frac{q}{Q}\right)} \tag{B.5}$$

For a linear demand function, we know that
$\left(MR_Q(Q,\Phi,\varepsilon) - \frac{1}{\Phi}C_{qq}(q)\frac{q}{Q}\right)$, $MR_\Phi(Q,\Phi,\varepsilon) < 0$ and that $MR_\varepsilon(Q,\Phi,\varepsilon) > 0$.
This implies that $\frac{d\Phi}{d\varepsilon} < 0$.

For an isoelastic demand function we have that $MR_\Phi(Q,\Phi,\varepsilon) < 0$ and that
$\left(MR_Q(Q,\Phi,\varepsilon) - \frac{1}{\Phi}C_{qq}(q)\frac{q}{Q}\right)$, $MR_\varepsilon(Q,\Phi,\varepsilon) \lesseqgtr 0$, implying $\frac{d\Phi}{d\varepsilon} \lesseqgtr 0$.

For the linear demand function, it can therefore be definitively concluded that increasing elasticity lowers the level of cooperation in the cartel. For an isoelastic demand function, the sign will depend on the model parameters. Nonetheless, for a large range of parameters tested we find $\frac{d\Phi}{d\varepsilon} < 0$.

AppendixC. Results when we solve for individual φ's

Table C.4:   OPEC's coefficients of cooperation and global crude oil prices in the case where bargaining is on individual φ, with binding commitments

	2005	2015	2025	2035	2045	2055	2065
Algeria	0.91	0.71	0.41	0.50	0.57	0.56	0.51
Angola	0.81	0.54	0.26	0.34	0.52	0.57	0.52
Iran	1.00	1.00	0.98	1.00	0.87	0.45	0.72
Iraq	0.70	0.65	0.58	0.59	0.51	0.29	0.26
Kuwait	0.88	0.82	0.75	0.77	0.53	0.34	0.50
Libya	0.65	0.60	0.48	0.12	0.15	0.29	0.38
Nigeria	0.79	0.74	0.22	0.22	0.25	0.42	0.61
Qatar	0.64	0.58	0.53	0.39	0.31	0.40	0.29
Saudi Arabia	1.00	1.00	1.00	1.00	1.00	1.00	1.00
United Arab Emirates	0.94	0.85	0.77	0.80	0.58	0.41	0.46
Venezuela (Includes Ecuador)	1.00	1.00	0.93	0.95	0.77	0.65	0.70
Production weighted average	0.91	0.86	0.74	0.72	0.63	0.49	0.56
Simple weighted average	0.85	0.77	0.63	0.61	0.55	0.49	0.54
Cooperation coefficient – identical φ_i*	0.98	0.87	0.74	0.63	0.57	0.53	0.62
Price (US \$2005) –individual φ_i	55.24	75.59	99.89	127.90	149.49	171.50	209.64
Price (US \$2005) – identical φ_i*	55.24	74.74	94.93	115.48	138.38	158.53	189.27

* For ease of comparison, we include these cases for bargaining over an identical φ.

Table C.5: OPEC's coefficients of cooperation and global crude oil prices in the case where bargaining is on individual φ, with sequential commitments

	2005	2015	2025	2035	2045	2055	2065
Algeria	0.91	0.66	0.41	0.49	0.55	0.63	0.47
Angola	0.81	0.53	0.22	0.35	0.52	0.64	0.52
Iran	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Iraq	0.70	0.72	0.75	0.91	0.98	1.00	1.00
Kuwait	0.88	0.89	0.89	1.00	1.00	1.00	1.00
Libya	0.65	0.62	0.58	0.32	0.31	0.42	0.34
Nigeria	0.79	0.76	0.33	0.45	0.39	0.44	0.72
Qatar	0.64	0.59	0.55	0.46	0.33	0.47	0.29
Saudi Arabia	1.00	1.00	1.00	1.00	1.00	1.00	1.00
United Arab Emirates	0.94	0.91	0.91	1.00	1.00	1.00	1.00
Venezuela (Includes Ecuador)	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Production weighted average	0.91	0.87	0.81	0.83	0.84	0.88	0.89
Simple weighted average	0.85	0.79	0.70	0.73	0.74	0.78	0.76
Cooperation coefficient – identical φ_i*	0.98	0.91	0.83	0.80	0.78	0.73	0.72
Price (US $2005) –individual φ_i	55.24	75.98	101.83	132.19	161.70	189.65	239.42
Price (US $2005) – identical φ_i*	55.24	75.63	98.94	122.79	149.84	171.49	203.92

* For ease of comparison, we include these cases for bargaining over an identical φ.

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