

Tilburg University

Information matrix test, parameter heterogeneity and ARCH

Bera, A.K.; Lee, S.

Publication date: 1993

Document Version Publisher's PDF, also known as Version of record

Link to publication in Tilburg University Research Portal

Citation for published version (APA): Bera, A. K., & Lee, S. (1993). *Information matrix test, parameter heterogeneity and ARCH: A synthesis*. (Reprint Series). CentER for Economic Research.

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
 You may not further distribute the material or use it for any profit-making activity or commercial gain
 You may freely distribute the URL identifying the publication in the public portal

Take down policy If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.





Information Matrix Test, Parameter Heterogeneity and ARCH: A Synthesis

> by Anil K. Bera and Sangkyu Lee

Reprinted from Review of Economic Studies, 60, 1993

Reprint Series no. 135

CENTER FOR ECONOMIC RESEARCH

Board

Harry Barkema Helmut Bester Eric van Damme, chairman Frank van der Duyn Schouten Jeffrey James

Management

Eric van Damme (graduate education) Arie Kapteyn (scientific director) Marie-Louise Kemperman (administration)

Scientific Council

Anton Barten Eduard Bomhoff Willem Buiter Jacques Drèze Theo van de Klundert Simon Kuipers Jean-Jacques Laffont Merton Miller Stephen Nickell Pieter Ruys Jacques Sijben

Residential Fellows

Lans Bovenberg Werner Güth Jan Magnus Shigeo Muto Theodore To Karl Wärneryd Karl-Erik Wärneryd

Research Coordinators

Eric van Damme Frank van der Duyn Schouten Harry Huizinga Arie Kapteyn Université Catholique de Louvain Erasmus University Rotterdam Yale University Université Catholique de Louvain Tilburg University Groningen University Université des Sciences Sociales de Toulouse University of Chicago University of Oxford Tilburg University Tilburg University

CentER, Erasmus University Rotterdam University of Frankfurt CentER, LSE Tohoku University University of Pittsburgh Stockholm School of Economics Stockholm School of Economics

Address	: Warandelaan 2, P.O. Box 90153, 5000 LE Tilburg, The Netherlands
Phone	: +31 13 663050
Telex	: 52426 kub nl
Telefax	: +31 13 663066
E-mail	: "center@htikub5.bitnet"

ISSN 0924-7874



Information Matrix Test, Parameter Heterogeneity and ARCH: A Synthesis

> by Anil K. Bera and Sangkyu Lee

RYI

Estimation

Reprinted from Review of Economic Studies, 60, 1993

> Reprint Series no. 135



Review of Economic Studies (1993) 60, 229-240 © 1993 The Review of Economic Studies Limited

Information Matrix Test, Parameter Heterogeneity and ARCH: A Synthesis

ANIL K. BERA

University of Illinois at Urbana-Champaign and CentER, Tilburg University

and

SANGKYU LEE CNB Economic Research Institute at Seoul

First version received May 1989; final version accepted October 1991 (Eds.)

We apply the White information matrix (IM) test to the linear regression model with autocorrelated errors. A special case of one component of the test is found to be identical to the Engle Lagrange multiplier (LM) test for autoregressive conditional heteroskedasticity (ARCH). Given Chesher's interpretation of the IM test as a test for parameter heterogeneity, this establishes a connection among the IM test, ARCH and parameter variation. This also enables us to specify conditional heteroskedasticity in a more general and convenient way. Other interesting by-products of our analysis are tests for the variation in conditional and static skewness which we call tests for "heterocliticity".

1. INTRODUCTION

In a pioneering article, White (1982) suggested the information matrix (IM) test as a general test for model specification. In recent years, this test has received a lot of attention. In particular, Chesher (1984) demonstrated that it can be viewed as a Lagrange multiplier (LM) test for specification error against the alternative of parameter heterogeneity. As a by-product of this analysis, Chesher (1983) and Lancaster (1984) provided an " nR^{2n} version of the IM test. An application of the IM test to the linear regression model by Hall (1987) led to the very interesting result that the test decomposed asymptotically into three components, one testing heteroskedasticity and the other two testing some forms of normality. Engle (1982), in an apparently unrelated influential paper, introduced the autoregressive conditional heteroskedasticity (ARCH) model which characterizes explicitly the conditional variance of the regression disturbances. He also suggested an LM test for ARCH. The purpose of this paper is to establish a connection among the IM test, parameter heterogeneity and ARCH and, as far as the IM test is concerned, we examine only the algebraic structure of the test.

An important finding by Hall (1987) was that the components of the IM test are insensitive to serial correlation. Hall also commented "had our original specification included first-order autoregressive errors, then the IM test does not decompose asymptotically into the sum of our original three component test... plus the LM test against first-order serial correlation. In this more general framework the indicator vector no longer has a block diagonal covariance matrix due to the inclusion of the autoregressive coefficient in the parameter vector." (p. 262). In the next section, we start with a linear regression model with autoregressive (AR) errors and apply the IM test to it. The indicator vector is found to have a block diagonal covariance matrix. And as the null model now has more parameters, naturally we get a few extra components in the IM test. From the additional components of the statistic, we can also obtain Engle's LM test for ARCH as a special case. The implication of this result is discussed in detail in Section 3. Given Chesher's interpretation of the IM test as a test for parameter heterogeneity or random coefficients, it is now easy to give a random coefficient interpretation to ARCH. This fact has been noted recently by several authors (see, e.g., Tsay (1987)). This provides us with a convenient framework to extend ARCH so that the interaction factor between past residuals could also be considered and as a consequence we suggest an augmented ARCH (AARCH) model. The last section of the paper contains some concluding remarks.

2. THE IM TEST FOR THE LINEAR REGRESSION MODEL WITH AR ERRORS

We consider the linear regression model

$$y_t = x_t'\beta + \varepsilon_t,\tag{1}$$

where y, is the t-th observation on the dependent variable, x, is a $k \times 1$ vector of fixed regressors and the ε_i are assumed to follow a stationary AR(p) process

$$\varepsilon_{i} = \sum_{j=1}^{p} \phi_{j} \varepsilon_{i-j} + u_{i}, \qquad (2)$$

with $u_t \sim \text{NIID}(0, \sigma_u^2)$. We will write this AR(p) process as $\varepsilon_i = \underline{\varepsilon}_i' \phi + u_t$ where $\underline{\varepsilon}_i = (\varepsilon_{t-1}, \ldots, \varepsilon_{t-p})'$ and $\phi = (\phi_1, \ldots, \phi_p)'$. Assuming that $\underline{\varepsilon}_1$ is given, the log-likelihood function for this model can be written as

$$L(\theta) = \sum_{i=1}^{n} l_i(\theta) = -\frac{n}{2} \log 2\pi - \frac{n}{2} \log \sigma_u^2 - \frac{1}{2\sigma_u^2} \sum_{i=1}^{n} (\varepsilon_i - \underline{\varepsilon}_i' \phi)^2,$$

where $\theta = (\beta', \phi', \sigma_u^2)'$ is a $q \times 1$ vector of parameters with q = k + p + 1. Note that $(\varepsilon_t - \varepsilon_t'\phi)$ involves β since $\varepsilon_t - \varepsilon_t'\phi = (y_t - y_t'\phi) - (x_t - x_t'\phi)'\beta$, where $y_t = (y_{t-1}, \dots, y_{t-p})'$ and $\underline{x}_t = (\underline{x}_{t-1}, \dots, \underline{x}_{t-p})'$.

Let $\hat{\theta}$ denote the maximum likelihood estimate (MLE) of θ . Then White's IM test is constructed based on

$$d(\hat{\theta}) = \operatorname{vech} C(\hat{\theta}) = \frac{1}{n} \sum_{t=1}^{n} d_t(\hat{\theta}) \quad (\operatorname{say}),$$

where

230

$$C(\hat{\theta}) = \frac{1}{n} \sum_{l=1}^{n} \left[\frac{\partial^2 l_l(\hat{\theta})}{\partial \theta \partial \theta'} + \left(\frac{\partial l_l(\hat{\theta})}{\partial \theta} \right) \left(\frac{\partial l_l(\hat{\theta})}{\partial \theta} \right)' \right] = A(\hat{\theta}) + B(\hat{\theta}) \quad (\text{say}).$$

Note that $-A(\hat{\theta})^{-1}$ and $B(\hat{\theta})^{-1}$ are the two different estimators for the asymptotic variance of $\sqrt{n} \hat{\theta}$ using the Hessian matrix and the outer product form, respectively. Therefore, the IM test principles can also be viewed as a test based on the difference of two estimators.

A consistent estimator of the variance matrix of $\sqrt{n} d(\hat{\theta})$ is (see White (1982, p. 11))

$$\hat{V}(\hat{\theta}) = \frac{1}{n} \sum_{i=1}^{n} a_i(\hat{\theta}) a_i'(\hat{\theta}), \qquad (3)$$

where $a_t(\hat{\theta}) = d_t(\hat{\theta}) - \nabla d(\hat{\theta}) A(\hat{\theta})^{-1} \nabla l_t(\hat{\theta})$ with

$$\nabla d(\hat{\theta}) = \frac{1}{n} \sum_{i=1}^{n} \frac{\partial d_i(\hat{\theta})}{\partial \theta} \text{ and } \nabla l_i(\hat{\theta}) = \frac{\partial l_i(\hat{\theta})}{\partial \theta}.$$

Then the White IM test takes the form of

$$T_w = nd'(\hat{\theta})\hat{V}(\hat{\theta})^{-1}d(\hat{\theta}).$$
(4)

When the model (1) is correct, T_w follows an asymptotic χ^2 distribution with q(q+1)/2 degrees of freedom. If there is an intercept term in the regression model (1), the χ^2 degrees of freedom should be reduced by one. Similar adjustments are necessary if the regressors contain some polynomial terms and a constant, or if some of the exogenous variables are binary (see White (1980, p. 825)). It should also be noted that White (1982) derived the IM test for IID observations. However, as shown in White (1987), the IM equality holds under fairly general conditions. For our autoregressive case, mixing conditions stated in White (1987) are satisfied, and therefore the IM test remains valid.

After some algebra and rearranging the terms in $d(\hat{\theta})$, we can write (for algebraic derivations, see Appendices A and B), suppressing θ such that writing \hat{d} for $d(\hat{\theta})$,

$$\hat{d} = (\hat{d}'_1, \hat{d}'_2, \hat{d}_3, \hat{d}'_4, \hat{d}'_5, \hat{d}'_6)', \tag{5}$$

where

$$\hat{d}_{1}: \left[\frac{1}{n\hat{\sigma}_{u}^{4}}\sum_{i=1}^{n}(\hat{u}_{i}^{2}-\hat{\sigma}_{u}^{2})(x_{ii}-\underline{x}_{ii}'\hat{\phi})(x_{ij}-\underline{x}_{ij}'\hat{\phi})\right], \quad i,j=1,2,\ldots,k; i \leq j$$

$$\hat{d}_2: \quad \left[\frac{1}{n\hat{\sigma}_u^4}\sum_{i=1}^n (\hat{u}_i^2 - \hat{\sigma}_u^2)\hat{\varepsilon}_{i-i}\hat{\varepsilon}_{i-j}\right], \qquad i, j = 1, 2, \dots, p; \ i \leq j$$

$$\hat{d}_{3}: \left[\frac{1}{4n\hat{\sigma}_{u}^{8}}\sum_{t=1}^{n}(\hat{u}_{t}^{4}-3\hat{\sigma}_{u}^{4})\right]$$
$$\hat{d}_{4}: \left[\frac{1}{n\hat{\sigma}_{u}^{4}}\sum_{t=1}^{n}(\hat{u}_{t}^{2}-\hat{\sigma}_{u}^{2})(x_{ii}-x_{ii}'\hat{\phi})\hat{\varepsilon}_{t-j}-\frac{1}{n\hat{\sigma}_{u}^{2}}\sum_{t=1}^{n}\hat{u}_{t}x_{t-ji}\right], \quad i=1,2,\ldots,k; j=1,2,\ldots,p$$

$$\hat{d}_{5}: \left[\frac{1}{2n\hat{\sigma}_{u}^{6}} \sum_{t=1}^{n} \hat{u}_{t}^{3}(x_{ti} - \underline{x}_{ti}'\hat{\phi}) \right], \quad i = 1, 2, ..., k$$

$$\hat{d}_6: \quad \left[\frac{1}{2n\hat{\sigma}_u^6}\sum_{t=1}^n \hat{u}_t^3 \hat{\varepsilon}_{t-i}\right], \qquad i=1,2,\ldots,p.$$

Our expressions for \hat{d}_1 , \hat{d}_3 and \hat{d}_5 are identical to those of Δ_1 , Δ_3 and Δ_2 of Hall (1987, pp. 259-260) if we put $\hat{\phi} = 0$. If it is desirable to test only in a certain direction, we can pre-multiply \hat{d} by a selection matrix whose elements are either zero or unity (see White (1982, pp. 9-10) and Hall (1987, p. 258)).

Now, to obtain the IM test statistic, all we need is to derive the variance matrix of \hat{d} . We find that the variance matrix is block diagonal (for detailed derivation, see Appendix B). Denote the estimator of the variance of $\sqrt{n} \hat{d}_i$ as $\hat{V}(\hat{d}_i) = \hat{V}_i$, i = 1, 2, ..., 6. To express

 \hat{V}_i 's succinctly, we define the vectors whose typical elements are described as

$$\begin{split} \underline{\chi}_{i} &: \left[(x_{ii} - \underline{x}'_{ii}\hat{\phi})(x_{ij} - \underline{x}'_{ij}\hat{\phi}) - \frac{1}{n} \sum_{i=1}^{n} (x_{ii} - \underline{x}_{ii}\hat{\phi})(x_{ij} - \underline{x}_{ij}\hat{\phi}) \right], \qquad i, j = 1, 2, \dots, k; i \leq j \\ \underline{\xi}_{i} &: \left[(\hat{\epsilon}_{i-i}\hat{\epsilon}_{i-j} - \frac{1}{n} \sum_{i=1}^{n} \hat{\epsilon}_{i-i}\hat{\epsilon}_{i-j} \right], \qquad i, j = 1, 2, \dots, p; i \leq j \\ \underline{s}_{i} &: \left[(x_{ii} - \underline{x}'_{ii}\hat{\phi})\hat{\epsilon}_{i-j} \right], \qquad i = 1, 2, \dots, k; j = 1, 2, \dots, p \\ \underline{z}_{i} &: \left[(x_{i-j}, \frac{1}{n}) \hat{\epsilon}_{i-j} \right], \qquad i = 1, 2, \dots, k; j = 1, 2, \dots, p \\ \underline{z}_{i} &: \left[x_{i-ji} \right], \qquad i = 1, 2, \dots, k; j = 1, 2, \dots, p \\ \underline{z}_{i} &: \left[x_{i} - \underline{x}'_{ii}\hat{\phi} \right], \qquad i = 1, 2, \dots, k. \end{split}$$
We also denote

$$\hat{W} = \nabla \hat{d}_{41} \hat{A}_{11}^{-1} \nabla' \hat{d}_{41} + \frac{1}{n \hat{\sigma}_{\mu}^2} \sum_{i=1}^{n} \underline{z}_i \underline{z}_i',$$

where ∇d_{41} is the (4, 1) block of $\nabla d(\theta)$ and A_{11} is the upper left-hand corner block of $A(\theta)$ (see Appendix B). Then we have very concise forms of \hat{V}_i 's as follows:

$$\hat{V}_{1} = \frac{2}{n\hat{\sigma}_{u}^{4}} \sum_{i=1}^{n} \underline{\chi}_{i} \underline{\chi}_{i}', \qquad \hat{V}_{2} = \frac{2}{n\hat{\sigma}_{u}^{4}} \sum_{i=1}^{n} \underline{\xi}_{i} \underline{\xi}_{i}', \qquad \hat{V}_{3} = \frac{3}{2\hat{\sigma}_{u}^{8}}$$
$$\hat{V}_{4} = \frac{2}{n\hat{\sigma}_{u}^{4}} \sum_{i=1}^{n} \underline{\xi}_{i} \underline{\xi}_{i}' + \hat{W}, \qquad \hat{V}_{5} = \frac{3}{2n\hat{\sigma}_{u}^{6}} \sum_{i=1}^{n} \underline{t}_{i} \underline{t}_{i}', \qquad \hat{V}_{6} = \frac{3}{2n\hat{\sigma}_{u}^{6}} \sum_{i=1}^{n} \underline{\xi}_{i} \hat{\underline{\xi}}_{i}'.$$

Given the block diagonality of the variance matrix of \hat{d} , we can write the IM test as

$$T_{W} = \sum_{i=1}^{6} T_{i} = n \sum_{i=1}^{6} \hat{d}'_{i} \hat{V}_{i}^{-1} \hat{d}_{i}, \qquad (6)$$

that is, the derived IM test statistic is found to be decomposed as the sum of six quadratic forms. In the next section, we analyse these components of T_w in detail.

3. INTERPRETATION OF THE COMPONENTS OF THE IM TEST

Using Chesher's analysis, we can say the statistic T_1 is a test for randomness of the regression parameters in the presence of autocorrelation. If we put $\hat{\phi} = 0$, then this reduces to the White (1980) test for heteroskedasticity (and T_{1n} in Hall (1987, p. 261)). Recently, there have been some robustness studies of various tests for heteroskedasticity in the presence of autocorrelation (see, e.g., Epps and Epps (1977), Bera and Jarque (1982), Godfrey and Wickens (1982), Bumb and Kelejian (1983), Bera and McKenzie (1986)) and their general conclusion is that various tests for heteroskedasticity are sensitive to the presence of autocorrelation. A by-product of our analysis is that we have a simple test for heteroskedasticity in the presence of autocorrelation. All we need to do is to modify the White test slightly. Instead of regressing the squares of the least-squares residuals on the squares and cross-products of x,'s, we should regress \hat{u}_i^2 on the squares and cross-products of $(x_t - x'_t \hat{\phi})$ after estimating the model with an appropriate AR process. For example, if there is AR(1) error, then the regressors should be the squares and cross-products of $(x_t - \hat{\phi}_1 x_{t-1})$. Similarly, the modification of T_{2n} in Hall (1987), which is our T_5 , requires that we should replace x_i by $(x_i - x'_i \hat{\phi})$. Our T_3 is a (kurtosis) test for normality, and it utilizes the conditional mean corrected residuals rather than the OLS residuals.

Let us now concentrate on the new test statistics we obtain by including ϕ in our model. The statistic T_2 tests the randomness of $\phi = (\phi_1, \phi_2, \dots, \phi_p)'$. Suppose that the parameters of autoregressive errors are varying around a mean value with finite variances.

This can be formulated as $\phi_t \sim (\phi, \Omega)$, where $\phi_t = (\phi_{1t}, \phi_{2t}, \dots, \phi_{pt})'$. Then T_2 is the LM statistic for testing H_0 : $\Omega = 0$. Let us first consider a very special case in which $\phi = 0$ and Ω is diagonal. Therefore, we have $\hat{\phi}_1 = \hat{\phi}_2 = \cdots = \hat{\phi}_p = 0$, and $\hat{u}_{t-i} = \hat{\varepsilon}_{t-i}$ $(i = 1, 2, \dots, p)$, where the $\hat{\varepsilon}_t$ are the OLS residuals. Consequently, T_2 reduces to

$$T_{2} = \frac{1}{2} \left[\sum_{t=1}^{n} \hat{\underline{u}}_{t}^{2} \left(\frac{\hat{u}_{t}^{2}}{\hat{\sigma}_{u}^{2}} - 1 \right) \right]' \left[\sum_{t=1}^{n} \underline{\xi}_{t} \underline{\xi}_{t}' \right]^{-1} \left[\sum_{t=1}^{n} \hat{\underline{\mu}}_{t}^{2} \left(\frac{\hat{u}_{t}^{2}}{\hat{\sigma}_{u}^{2}} - 1 \right) \right], \tag{7}$$

where $\hat{u}_i^2 = (\hat{u}_{i-1}^2, \hat{u}_{i-2}^2, \dots, \hat{u}_{i-p}^2)'$ and a typical element of ξ_i is now $(\hat{u}_{i-i}^2 - 1/n \sum_{i=1}^n \hat{u}_{i-i}^2)$, for $i = 1, 2, \dots, p$. This is identical to the Engle (1982) LM statistic for testing the *p*-th-order linear ARCH disturbances, i.e., testing $H_0: \alpha_1 = \alpha_2 = \dots = \alpha_p = 0$ in the ARCH process specified as

$$\operatorname{var}\left(u_{i} \mid \underline{u}_{i}\right) = \sigma_{u}^{2} + \alpha_{1} u_{i-1}^{2} + \cdots + \alpha_{p} u_{i-p}^{2},$$

where $u_t = (u_{t-1}, u_{t-2}, \dots, u_{t-p})'$. An asymptotically equivalent form of this statistic is nR^2 where R^2 is the coefficient of multiple determination from the regression of \hat{u}_t^2 on a unit term and $(\hat{u}_{t-1}^2, \hat{u}_{t-2}^2, \dots, \hat{u}_{t-p}^2)$.

From our representation of the test for ARCH as a test for randomness of ϕ parameters and its equivalence to one component of the IM test, the consequence of the presence of ARCH is that the "usual" estimators for variance of $\hat{\phi}$ will be inconsistent if ARCH is ignored. This is similar to the case that the standard variance estimator for $\hat{\beta}$ is inconsistent in the presence of static heteroskedasticity. Therefore, the standard tests for autocorrelation are not valid in the presence of ARCH (see, e.g., Diebold (1986) and Bera *et al.* (1990)). This result is not entirely obvious since under ARCH, the disturbances are still unconditionally homoskedastic. Although the above point could be made without an IM test interpretation, the IM test framework provides an easy guide for checking whether the standard inference procedures fail.

We now relax the assumption of the diagonality of Ω . The structure of the test statistic will remain the same except R^2 will be obtained by regressing \hat{u}_i^2 on a constant and the squares and *cross-products* of the lagged residuals. T_2 will then be a LM statistic for testing H_0 : $\alpha_{ij} = 0$ ($i \ge j = 1, 2, ..., p$) in

$$\operatorname{var}(u_{t}|u_{t}) = \sigma_{u}^{2} + \sum_{i=1}^{p} \sum_{i=1}^{p} \alpha_{ij} u_{t-i} u_{t-j}, \quad i \ge j.$$
(8)

The above specification of conditional variance generalizes the Engle ARCH model. This will be called the augmented ARCH (AARCH) process. Properties and testing of this model are discussed in Bera *et al.* (1990). Lastly, if we additionally relax the assumption of $\phi = 0$, \hat{u}_t will no longer be equal to $\hat{\varepsilon}_t$ and T_2 will have to be calculated from the regression of \hat{u}_t^2 on a constant and the squares and cross products of $\hat{\varepsilon}_{t-i}$ (i = 1, 2, ..., p). This will give us the LM statistics for testing ARCH or AARCH in the presence of autocorrelation.

From the above discussion, it is clear that the Engle ARCH model can be viewed as a special case of random coefficient autoregressive (RCAR) model. To see this more clearly, let us write equation (2) as

$$\varepsilon_i = \sum_{i=1}^p \phi_{ji} \varepsilon_{i-j} + u_i.$$

If it is assumed that $\phi_{ji} \sim (0, \alpha_j)$ and $\operatorname{cov}(\phi_{ji}, \phi_{j'i}) = 0$, for $j \neq j'$, then the conditional variance is given by

var
$$(\varepsilon_i | \varepsilon_i) = \sigma_u^2 + \sum_{i=1}^p \alpha_i \varepsilon_{i-i}^2$$
.

Here we observe that ARCH and the above RCAR models have the same first two conditional moments as mentioned in Tsay (1987) where it is called as second-order equivalence. If we further assume that the ϕ_{ji} are normally distributed, then all the moments of ARCH and RCAR processes will be the same, e.g., for p = 1, the first four moments are

$$\mu_1 = 0, \qquad \mu_2 = \frac{\sigma_u^2}{1 - \alpha_1}, \qquad \mu_3 = 0 \quad \text{and} \quad \mu_4 = \frac{3\sigma_u^4(1 + \alpha_1)}{(1 - \alpha_1)(1 - 3\alpha_1^2)}$$

(see Engle (1982, p. 992)). Here we should note that calculation of moments are much easier under the RCAR scheme.

By comparing T_1 and T_2 , we note that they test for static and conditional heteroskedasticity, respectively. Given the block diagonality of the covariance matrix of the IM test in our case, we can test for static and conditional heteroskedasticity simultaneously simply by adding up these two statistics. The statistic T_4 is also related to T_1 and T_2 . From the expression of \hat{d}_4 , we note that T_4 has two components. The second component is based on $(n\hat{\sigma}_u^2)^{-1} \sum_i \hat{u}_i \chi'_i$ and this can be viewed as some form of a test for exogeneity. In certain practical applications when the x_i are known to be exogenous, this part can be ignored. Then T_4 will be based only on the first component of \hat{d}_4 and the resulting test will be a test for conditional heteroskedasticity caused by the interaction between the disturbance term and the regressors. We should, however, note that by excluding the second component of \hat{d}_4 we do not get exactly an IM test but something that is very close to an IM test. For the special case,

$$\hat{V}_4 = \frac{2}{n\hat{\sigma}_u^2} \sum_{i=1}^n \underline{s}_i \underline{s}_i'$$

and therefore, for obtaining T_4 , we run the regression of \hat{u}_t^2 on a constant and crossproducts of lagged innovations $\hat{\underline{\varepsilon}}_t$, and transformed exogenous variables $x_t - \underline{x}_t'\hat{\phi}$. As a natural consequence, a general test statistic for heteroskedasticity would be $T_1 + T_2 + T_4$ which under the null hypothesis will have an asymptotic χ^2 distribution with $(k+p) \times (k+p+1)/2$ degrees of freedom. To get reasonable power, we will have to make a judicious selection of the regressors from the set of squares and cross-products of $x_t - \underline{x}_t'\hat{\phi}$ and $\hat{\underline{\varepsilon}}_t$, or make some adjustment to the test statistic (see Bera (1986)).

The last two statistics T_5 and T_6 can be viewed as the statistics for testing variation in the third moment of u_i . In T_5 , the variation is assumed to depend on the exogenous variables x_i and in T_6 , on the lagged innovation process. In some sense, we could say that T_5 and T_6 test for static and conditional *heterocliticity*, respectively. The term heterocliticity is used since, when the skewness coefficient is plotted against x_i or ε_{i-i} , we obtain the *clitic* curve (see Kendall and Stuart (1973, p. 362)). As noted in Hall (1987), the test for normality (skewness part) proposed by Bowman and Shenton (1975) and Jarque and Bera (1987) is a special case of T_5 while T_3 which tests for the variation of σ_u^2 is a pure test for kurtosis. In this connection, let us mention that if the IM test is applied to an ARCH model, that leads to a test for *heterokurticity* (for details see Bera and Zuo (1991)). This provides a specification test for an estimated ARCH model.

4. CONCLUSION

Our application of the White IM test to the linear regression model with autoregressive errors provides many interesting results. The most important result is that a special case of one component of this test is identical to the Engle LM test for ARCH. Chesher's interpretation of the IM test as the test for parameter heterogeneity leads us natually to specify the ARCH processes as a random coefficient autoregressive (RCAR) model. From both theoretical and practical points of view, this representation of ARCH is convenient and useful. As discussed in Bera *et al.* (1990), we can now easily verify the stationarity condition for ARCH as a special case of the RCAR model, study the robustness of the test for the AR process in the presence of ARCH and vice versa, and generalize the ARCH process to take account of interaction between the disturbance terms.

The difference between the static and conditional heteroskedasticity is now clear. The former could be related to the variation of the regression coefficients and the latter to the variation of the autoregressive parameters. A mixture of them is possible when the heteroskedasticity is caused by the interaction between exogenous variables and disturbances. We have also discussed the possibilities of static and conditional variations in skewness—what we call heterocliticity.

APPENDIX A: THE DERIVATIVES OF THE LOG-LIKELIHOOD FUNCTION

For our model, the vector of parameter is $\theta = (\beta', \phi', \sigma_u^2)'$ and the log-density function for the *t*-th observation conditional on the information set Ψ_{t-1} , in which $\underline{\varepsilon}_t = (\varepsilon_{t-1}, \dots, \varepsilon_{t-p})'$ is contained, is given by

$$l_t(\theta) = -\frac{1}{2}\log 2\pi - \frac{1}{2}\log \sigma_u^2 - \frac{1}{2\sigma_u^2}(\varepsilon_t - \underline{\varepsilon}_t'\phi)^2.$$

Note that $u_t = \varepsilon_t - \varepsilon_t' \phi = (y_t - y_t' \phi) - (x_t - x_t' \phi)' \beta$ where $y_t = (y_{t-1}, \dots, y_{t-p})'$ and $x_t = (x_{t-1}, \dots, x_{t-p})'$. Then the first and second partial derivatives of $l_t(\theta)$ with respect to θ are easily obtained. The first derivatives are

$$\frac{\partial l_i(\theta)}{\partial \beta} = \frac{1}{\sigma_u^2} u_i(x_i - x_i'\phi), \qquad \frac{\partial l_i(\theta)}{\partial \phi} = \frac{1}{\sigma_u^2} u_i \xi_i \text{ and } \frac{\partial l_i(\theta)}{\partial \sigma_u^2} = -\frac{1}{2\sigma_u^2} + \frac{1}{2\sigma_u^4} u_i^2.$$

The second derivatives are

$$\begin{split} \frac{\partial^2 l_i(\theta)}{\partial \beta \partial \beta'} &= -\frac{1}{\sigma_u^2} \left(x_i - y_i' \phi \right) (x_i - y_i' \phi)', \qquad \frac{\partial^2 l_i(\theta)}{\partial \phi \partial \phi'} = \frac{1}{\sigma_u^2} g_i g_i'', \\ \frac{\partial^2 l_i(\theta)}{\partial (\sigma_u^2)^2} &= \frac{1}{2\sigma_u^4} - \frac{1}{\sigma_u^6} u_i^2, \qquad \frac{\partial^2 l_i(\theta)}{\partial \beta \partial \phi'} = -\frac{1}{\sigma_u^2} \left(x_i - y_i' \phi \right) g_i' - \frac{1}{\sigma_u^2} u_i g_i', \\ \frac{\partial^2 l_i(\theta)}{\partial \beta \partial \sigma_u^2} &= -\frac{1}{\sigma_u^4} u_i (x_i - y_i' \phi)' \quad \text{and} \quad \frac{\partial^2 l_i(\theta)}{\partial \phi \partial \sigma_u^2} = -\frac{1}{\sigma_u^4} u_i g_i'. \end{split}$$

APPENDIX B: A CONSISTENT COVARIANCE MATRIX ESTIMATOR FOR THE INFORMATION MATRIX TEST

A consistent covariance estimator for the IM test proposed by White (1982) is stated as

$$\hat{V}(\hat{\theta}) = \frac{1}{n} \sum_{i=1}^{n} a_i(\hat{\theta}) a_i'(\hat{\theta}), \tag{B.1}$$

where $a_i(\hat{\theta}) = d_i(\hat{\theta}) - \nabla d(\hat{\theta}) A(\hat{\theta})^{-1} \nabla l_i(\hat{\theta})$. Let us begin with the indicator vector $d(\hat{\theta})$ which is defined as

$$d(\hat{\theta}) = \operatorname{vech} [C(\hat{\theta})] = \operatorname{vech} [A(\hat{\theta}) + B(\hat{\theta})],$$

where

$$A(\hat{\theta}) = \frac{1}{n} \sum_{i=1}^{n} \left[\frac{\hat{\sigma}^{2} l_{i}(\theta)}{\hat{\theta} \hat{\theta} \hat{\theta}'} \right]_{\theta = \hat{\theta}}$$

$$= \frac{1}{n} \sum_{i=1}^{n}$$

$$\begin{bmatrix} -\frac{1}{\sigma_{u}^{2}} (x_{i} - \underline{x}'_{i} \phi)(x_{i} - \underline{x}'_{i} \phi)' & -\frac{1}{\sigma_{u}^{2}} (x_{i} - \underline{x}'_{i} \phi)\underline{\varepsilon}_{i} - \frac{1}{\sigma_{u}^{2}} u_{i} \underline{x}'_{i} & -\frac{1}{\sigma_{u}^{4}} (x_{i} - \underline{x}'_{i} \phi)u_{i} \\ -\frac{1}{\sigma_{u}^{2}} \underline{\varepsilon}_{i} (x_{i} - \underline{x}'_{i} \phi)' - \frac{1}{\sigma_{u}^{2}} u_{i} \underline{x}_{i} & -\frac{1}{\sigma_{u}^{2}} \underline{\varepsilon}_{i} \underline{\varepsilon}_{i}' & -\frac{1}{\sigma_{u}^{4}} \underline{\varepsilon}_{i} u_{i} \\ -\frac{1}{\sigma_{u}^{4}} u_{i} (x_{i} - \underline{x}'_{i} \phi)' & -\frac{1}{\sigma_{u}^{4}} u_{i} \underline{\varepsilon}_{i}' & \frac{1}{\sigma_{u}^{4}} - \frac{1}{\sigma_{u}^{4}} u_{i}^{2} \end{bmatrix} \quad \boldsymbol{e}_{i} = \hat{\boldsymbol{\theta}}_{i}$$

and

$$B(\hat{\theta}) = \frac{1}{n} \sum_{r=1}^{n} \left[\left(\frac{\partial l_{i}(\theta)}{\partial \theta} \right) \left(\frac{\partial l_{i}(\theta)}{\partial \theta} \right)^{r} \right]_{\theta=\hat{\theta}}$$

$$= \frac{1}{n} \sum_{r=1}^{n}$$

$$\begin{bmatrix} \frac{1}{\sigma_{w}^{2}} u_{i}^{2} (x_{i} - \underline{x}_{i}^{\prime} \phi) (x_{i} - \underline{x}_{i}^{\prime} \phi)^{\prime} & \frac{1}{\sigma_{w}^{4}} u_{i}^{2} (x_{i} - \underline{x}_{i}^{\prime} \phi) \underline{\varepsilon}_{i}^{\prime} & \frac{1}{2\sigma_{w}^{6}} (x_{i} - \underline{x}_{i}^{\prime} \phi) (u_{i}^{3} - \sigma_{w}^{2} u_{i}) \\ \frac{1}{\sigma_{w}^{4}} u_{i}^{2} \underline{\varepsilon}_{i} (x_{r} - \underline{x}_{i}^{\prime} \phi)^{\prime} & \frac{1}{\sigma_{w}^{4}} u_{i}^{2} \underline{\varepsilon}_{i} \underline{\varepsilon}_{i}^{\prime} & \frac{1}{2\sigma_{w}^{6}} \underline{\varepsilon}_{i} (u_{i}^{3} - \sigma_{w}^{2} u_{i}) \\ \frac{1}{2\sigma_{w}^{6}} (u_{i}^{3} - \sigma_{w}^{2} u_{i}) (x_{r} - \underline{x}_{i}^{\prime} \phi)^{\prime} & \frac{1}{2\sigma_{w}^{6}} (u_{i}^{3} - \sigma_{w}^{2} u_{i}) \underline{\varepsilon}_{i}^{\prime} & \frac{1}{4\sigma_{w}^{4}} - \frac{1}{2\sigma_{w}^{6}} u_{i}^{2} + \frac{1}{4\sigma_{w}^{4}} u_{i}^{4} \end{bmatrix}$$

From $A(\hat{\theta})$ and $B(\hat{\theta})$, $C(\hat{\theta})$ is easily derived as

 $C(\hat{\theta}) = A(\hat{\theta}) + B(\hat{\theta})$

$$= \frac{1}{n} \sum_{r=1}^{n} \left[\frac{1}{\sigma_{u}^{4}} (u_{t}^{2} - \sigma_{u}^{2})(x_{t} - y_{t}^{\prime}\phi)(x_{t} - y_{t}^{\prime}\phi)' \frac{1}{\sigma_{u}^{4}} (u_{t}^{2} - \sigma_{u}^{2})(x_{t} - y_{t}^{\prime}\phi)y_{t}^{\prime} - \frac{1}{\sigma_{u}^{2}} u_{t}y_{t}^{\prime} \frac{1}{2\sigma_{u}^{6}} (x_{t} - y_{t}^{\prime}\phi)(u_{t}^{3} - 3\sigma_{u}^{2}u_{t}) \right] \\ \frac{1}{\sigma_{u}^{4}} (u_{t}^{2} - \sigma_{u}^{2})y_{t}(x_{t} - y_{t}^{\prime}\phi)' - \frac{1}{\sigma_{u}^{2}} u_{t}y_{t} \frac{1}{\sigma_{u}^{4}} (u_{t}^{2} - \sigma_{u}^{2})y_{t}y_{t}^{\prime} \frac{1}{2\sigma_{u}^{6}} x_{t}^{\prime} \frac{1}{2\sigma_{u}^{6}} x_{t}^{\prime} \frac{1}{2\sigma_{u}^{6}} x_{t}^{\prime} \frac{1}{2\sigma_{u}^{6}} x_{t}^{\prime} \frac{1}{2\sigma_{u}^{6}} x_{t}^{\prime} \frac{1}{2\sigma_{u}^{6}} (u_{t}^{3} - 3\sigma_{u}^{2}u_{t}) \right] \\ \frac{1}{2\sigma_{u}^{6}} (u_{t}^{3} - 3\sigma_{u}^{2}u_{t})(x_{t} - y_{t}^{\prime}\phi)' \frac{1}{2\sigma_{u}^{6}} (u_{t}^{3} - 3\sigma_{u}^{2}u_{t})y_{t}^{\prime} \frac{1}{2\sigma_{u}^{6}} (u_{t}^{4} - 6\sigma_{u}^{2}u_{t}^{2} + 3\sigma_{u}^{4}) \right] = -\phi$$

Now it is straightforward to obtain $d(\hat{\theta})$. For analytical convenience, we rearrange $d(\hat{\theta})$ as described in the paper. Then the first part of $a_i(\hat{\theta})$ defined from $d(\hat{\theta}) = 1/n \sum_{i=1}^n d_i(\hat{\theta})$ can be written as

$$d_{t}(\hat{\theta}) = (\hat{d}'_{11}, \hat{d}'_{12}, \hat{d}_{13}, \hat{d}'_{14}, \hat{d}'_{15}, \hat{d}'_{16})', \tag{B.2}$$

236

where

$$\begin{split} \hat{d}_{t1} &= [\hat{\sigma}_{u}^{-4}(\hat{u}_{t}^{2} - \hat{\sigma}_{u}^{2})(x_{t1} - \underline{x}_{t1}'\hat{\phi})^{2}, \dots, \hat{\sigma}_{u}^{-4}(\hat{u}_{t}^{2} - \hat{\sigma}_{u}^{2})(x_{tk} - \underline{x}_{tk}'\hat{\phi})^{2}, \\ \hat{\sigma}_{u}^{-4}(\hat{u}_{t}^{2} - \hat{\sigma}_{u}^{2})(x_{t1} - \underline{x}_{t1}'\hat{\phi})(x_{t2} - \underline{x}_{t2}'\hat{\phi}), \dots, \\ \hat{\sigma}_{u}^{-4}(\hat{u}_{t}^{2} - \hat{\sigma}_{u}^{2})(x_{t(k-1)} - \underline{x}_{t(k-1)}\hat{\phi})(x_{tk} - \underline{x}_{tk}'\hat{\phi})]' \end{split}$$

is a $[k(k+1)/2] \times 1$ vector,

$$\hat{d}_{t2} = [\hat{\sigma}_{u}^{-4}(\hat{u}_{t}^{2} - \hat{\sigma}_{u}^{2})\hat{\varepsilon}_{t-1}^{2}, \dots, \hat{\sigma}_{u}^{-4}(\hat{u}_{t}^{2} - \hat{\sigma}_{u}^{2})\hat{\varepsilon}_{t-p}^{2}, \hat{\sigma}_{u}^{-4}(\hat{u}_{t}^{2} - \hat{\sigma}_{u}^{2})\hat{\varepsilon}_{t-1}\hat{\varepsilon}_{t-2}, \dots, \\ \hat{\sigma}_{u}^{-4}(\hat{u}_{t}^{2} - \hat{\sigma}_{u}^{2})\hat{\varepsilon}_{t-p+1}\hat{\varepsilon}_{t-p}]'$$

is a $[p(p+1)/2] \times 1$ vector,

$$\hat{d}_{13} = (4\hat{\sigma}_{u}^{8})^{-1}(\hat{u}_{1}^{4} - 6\hat{\sigma}_{u}^{2}\hat{u}_{1}^{2} + 3\hat{\sigma}_{u}^{4})$$

is a scalar,

$$\hat{d}_{i4} = [\hat{\sigma}_{u}^{-4}(\hat{u}_{i}^{2} - \hat{\sigma}_{u}^{2})(x_{i} - \underline{x}'_{i}\hat{\phi})'\hat{\varepsilon}_{i-1}, \dots, \hat{\sigma}_{u}^{-4}(\hat{u}_{i}^{2} - \hat{\sigma}_{u}^{2})(x_{i} - \underline{x}'_{i}\hat{\phi})'\hat{\varepsilon}_{i-p}]' - [\hat{\sigma}_{u}^{-2}x'_{i-1}, \dots, \hat{\sigma}_{u}^{-2}x'_{i-p}]'$$

is a $kp \times 1$ vector,

$$\hat{d}_{15} = (2\hat{\sigma}_{u}^{6})^{-1}(\hat{u}_{1}^{3} - 3\hat{\sigma}_{u}^{2}\hat{u}_{l})(x_{l} - x_{l}^{\prime}\hat{\phi})$$

is a $k \times 1$ vector, and finally,

$$\hat{d}_{t6} = [(2\hat{\sigma}_{u}^{6})^{-1}(\hat{u}_{1}^{3} - 3\hat{\sigma}_{u}^{2}\hat{u}_{t})\hat{\epsilon}_{t-1}, \dots, (2\hat{\sigma}_{u}^{6})^{-1}(\hat{u}_{t}^{3} - 3\hat{\sigma}_{u}^{2}\hat{u}_{t})\hat{\epsilon}_{t-p}]$$

is a $p \times 1$ vector.

Next we consider

$$\nabla d(\theta_0) = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^n E\left[\frac{\partial d_i(\theta_0)}{\partial \theta}\right].$$

Using the normality assumption of the u_i and taking expectation conditional on the information set Ψ_{i-1} iteratively, after some algebra we can get the following simple form of $\nabla d(\theta_0)$

$$\nabla d(\theta_0) = \begin{bmatrix} 0 & 0 & \nabla d_{13} \\ 0 & 0 & \nabla d_{23} \\ 0 & 0 & 0 \\ \nabla d_{41} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where $\nabla d_{13} = (mx_{11}, \dots, mx_{kk}, mx_{12}, \dots, mx_{(k-1)k})'$ is a $[k(k+1)/2] \times 1$ vector with

$$mx_{ij} = -\frac{1}{\sigma_u^4} \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^n (x_{ii} - x'_{ii}\phi_0)(x_{ij} - x'_{ij}\phi_0), \qquad i, j = 1, 2, \dots, k; i \leq j,$$

 $\nabla d_{23} = (m\epsilon_{11}, \dots, m\epsilon_{pp}, m\epsilon_{12}, \dots, m\epsilon_{(p-1)p})'$ is a $[p(p+1)/2] \times 1$ vector with

$$m\varepsilon_{ij} = -\frac{1}{\sigma_u^4} \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^n \varepsilon_{i-i} \varepsilon_{i-j}, \qquad i, j = 1, 2, \ldots, k; i \leq j,$$

and $\nabla d_{41} = (w_{11}, w_{12}, \dots, w_{kp})'$ is a $kp \times k$ matrix with

$$w_{ij} = \frac{1}{\sigma_w^2} \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^n (x_i - x_i' \phi_0)' x_{i-j,i}, \qquad i = 1, 2, \dots; k, j = 1, 2, \dots, p.$$

This implies that $\nabla d(\theta_0)$ can be estimated consistently by the $\nabla d(\hat{\theta})$ which is

$$\nabla d(\hat{\theta}) = \begin{bmatrix} 0 & 0 & \nabla \hat{d}_{13} \\ 0 & 0 & \nabla \hat{d}_{23} \\ 0 & 0 & 0 \\ \nabla \hat{d}_{41} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
(B.3)

where for example, $\nabla \hat{d}_{13} = (\overline{mx}_{11}, \dots, \overline{mx}_{kk}, \overline{mx}_{12}, \dots, \overline{mx}_{(k-1)k})'$ is a $[k(k+1)/2] \times 1$ vector with

$$\overline{mx}_{ij} = -\frac{1}{n\hat{\sigma}_{u}^{4}} \sum_{i=1}^{n} (x_{ii} - y_{ii}'\hat{\phi})(x_{ij} - y_{ij}'\hat{\phi}), \qquad i, j = 1, 2, \dots, k; \ i \leq j.$$

Similarly, we can simplify $A(\hat{\theta})$ as follows:

$$\mathcal{A}(\hat{\theta}) = \begin{bmatrix} -\frac{1}{n\hat{\sigma}_{w}^{2}}\sum_{i=1}^{n}(x_{i}-x_{i}'\hat{\phi})(x_{i}-x_{i}'\hat{\phi})' & 0 & 0\\ 0 & -\frac{1}{n\hat{\sigma}_{w}^{2}}\sum_{i=1}^{n}\hat{\varepsilon}_{i}\hat{\varepsilon}', & 0\\ 0 & 0 & -\frac{1}{2\hat{\sigma}_{w}^{4}} \end{bmatrix}$$
(B.4)

For future use, let us denote the upper left-hand corner block of $A(\hat{\theta})$ as $A_{11}(\hat{\theta}) = \hat{A}_{11}$. We can simplify the expression for $A(\hat{\theta})$ further by using analytic expectation of $g_i g'_i$. For example, when p = 1, $n^{-1} \sum_i \hat{g}_{i-1}^2$ can be replaced by $\hat{\sigma}_i^2/(1 - \hat{\phi}_1)$. This might provide better finite sample performance. However, then we will lose the nR^2 interpretation of our test statistics. Also, no general expression for the analytic expectation can be given for all values of p_i .

Finally, $\nabla l_i(\hat{\theta}) = \partial l_i(\hat{\theta}) / \partial \theta$ is easily given from Appendix A by

$$\nabla l_{t}(\hat{\theta}) = \begin{bmatrix} \frac{1}{\hat{\sigma}_{u}^{2}} \hat{u}_{t}(x_{t} - y_{t}'\hat{\phi}) \\ \frac{1}{\hat{\sigma}_{u}^{2}} \hat{u}_{t}\hat{\ell}_{t} \\ -\frac{1}{2\hat{\sigma}_{u}^{2}} + \frac{1}{2\hat{\sigma}_{u}^{4}} \hat{u}_{t}^{2} \end{bmatrix}$$
(B.5)

and we denote the first $(k \times 1)$ vector of $\nabla l_i(\hat{\theta})$ as $\nabla l_i(\hat{\theta}) = \nabla \hat{l}_i$.

For the following discussion, recall the definitions of \underline{x}_i , $\underline{\xi}_i$, \underline{s}_i and \underline{r}_i , provided in the main text. From (B.2)-(B.5), $a_i(\hat{\theta})$ can be easily derived as

$$a_{i}(\hat{\theta}) = d_{i}(\hat{\theta}) - \nabla d(\hat{\theta}) A(\hat{\theta})^{-1} \nabla l_{i}(\hat{\theta}) = (\hat{a}'_{11}, \hat{a}'_{12}, \hat{a}'_{13}, \hat{a}'_{14}, \hat{a}'_{15}, \hat{a}'_{16})'$$
(B.6)

238

where

$$\begin{split} \hat{a}_{i1} &= \frac{1}{\hat{\sigma}_{u}^{4}} \left(\hat{u}_{i}^{2} - \hat{\sigma}_{u}^{2} \right) \underline{\chi}_{i}, \qquad \hat{a}_{i2} &= \frac{1}{\hat{\sigma}_{u}^{4}} \left(\hat{u}_{i}^{2} - \hat{\sigma}_{u}^{2} \right) \underline{\xi}_{i}, \qquad \hat{a}_{i3} &= \hat{d}_{i3} \\ \hat{a}_{i4} &= \hat{d}_{i4} - \nabla \hat{d}_{41} \hat{A}_{11}^{-1} \nabla \hat{l}_{i1}, \qquad \hat{a}_{i5} &= \hat{d}_{i5} \quad \text{and} \quad \hat{a}_{i6} &= \hat{d}_{i6}. \end{split}$$

Now we establish the block diagonality of the covariance matrix of the 1M test, say $V(\theta_0)$. It is assumed that all conditions stated in White (1982) are satisfied. Given (B.2)-(B.6) with the normality assumption of the u_i , $V(\theta_0)$ takes the form of

$$V(\theta_0) = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} E \begin{bmatrix} \frac{2}{\sigma_u^4} \underline{x}_i \underline{x}_i' & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{\sigma_u^4} \underline{\xi}_i \underline{\xi}_i' & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{2\sigma_u^8} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{\sigma_u^4} \underline{s}_i \underline{s}_i' + W & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{3\sigma_u^6} \underline{s}_i \underline{s}_i' & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{3}{2\sigma_u^6} \underline{\varepsilon}_i \underline{\varepsilon}_i' \\ 0 & 0 & 0 & 0 & 0 & \frac{3}{2\sigma_u^6} \underline{\varepsilon}_i \underline{\varepsilon}_i' \end{bmatrix}_{\theta = \theta_0}$$

where $W = \nabla d_{41}A_{11}^{-1}\nabla d'_{41} + n^{-1}\sigma_{u}^{-2}\sum_{i} z_{i}'$, and the diagonal elements are consistently estimated by \hat{V}_{i} , i = 1, 2, ..., 6, stated in the main text. To prove this result, let us consider

$$V(\theta_0) = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} E[a_i(\theta_0) a_i'(\theta_0)].$$
(B.7)

In the first stage, we evaluate $E[a_i(\theta_0)a'_i(\theta_0)]$ conditional on the information set Ψ_{t-1} using the normality assumption of the u_t and taking the expectation iteratively. In the next stage, we use the facts that at $\theta = \theta_0$, $E(\xi_t) = 0$, and $E(\xi_t) = 0$ for all t. Then we have the result.

Acknowledgement. We are grateful to two referees for their very useful comments. Both of them pointed out some errors in our earlier derivation and made many helpful suggestions which improved the exposition of the paper. We also wish to express our appreciation to the participants of the 1988 North American Econometric Society Summer Meeting, Rob Engle, Bruce Hansen, Hal White, Jan Kmenta, Pravin Trivedi, Xiao-Lei Zuo and, in particular, Alastair Hall for constructive comments on an earlier draft of the paper. All errors, of course, remain our own. Financial support from the Research Board and the Bureau of Economic and Business Research of the University of Illinois is gratefully acknowledged.

REFERENCES

BERA, A. K. (1986), "Model Specification Test through Eigenvalues", (Paper presented at the 1986 North-American Summer Meeting of the Econometric Society, Duke University, Durham, N.C.).

BERA, A. K. and JARQUE, C. M. (1982). "Model Specification Tests: A Simultaneous Approach", Journal of Econometrics, 20, 59-82.

BERA, A. K. and MCKENZIE, C. R. (1986), "Alternative Forms and Properties of the Score Test", Journal of Applied Statistics, 13, 13-25.

BERA, A. K. and ZUO, X.-L. (1991), "Specification Tests for ARCH Models" (mimeo, Department of Economics, University of Illinois at Urbana-Champaign).

BERA, A. K., LEE, S. and HIGGINS, M. L. (1990), "Interaction between Autocorrelation and Conditional Heteroskedasticity: A Random Coefficient Approach", Journal of Business and Economic Statistics (forthcoming).

- BOWMAN, K. O. and SHENTON, L. R. (1975), "Omnibus Contours for Departure from Normality Based on b₁ and b₂", Biometrika, 62, 243-250.
- BUMB, B. L. and KELEJIAN, H. H. (1983), "Autocorrelated and Heteroscedastic Disturbances in Linear Regression Analysis: A Monte Carlo Study", Sankhyā (Series B), 45, 257-270.
- CHESHER, A. D. (1983), "The Information Matrix Test: Simplified Calculation via a Score Test", Economics Letters, 13, 45-48.

CHESHER, A. D. (1984), "Testing for Neglected Heterogeneity", Econometrica, 52, 865-872.

- DIEBOLD, F. X. (1986), "Testing for Serial Correlation in the Presence of ARCH", Proceedings of the American Statistical Association, Business and Economic Statistics Section, 323-328.
- ENGLE, R. F. (1982), "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation", Econometrica, 50, 987-1007.
- EPPS, T. W. and EPPS, M. L. (1977), "The Robustness of Some Standard Tests for Autocorrelation and Heteroscedasticity When Both Problems are Present", *Econometrica*, 45, 745-753.
- GODFREY, L. G. and WICKENS, M. R. (1982), "Tests of Misspecification Using Locally Equivalent Alternative Models", in Chow, G. C. and Corsi, P. (eds.), Evaluating the Reliability of Macro-Economic Models (New York: John Wiley & Sons), 71-103.

HALL, A. (1987), "The Information Matrix Test for the Linear Model", Review of Economic Studies, 54, 257-263.

JARQUE, C. M. and BERA, A. K. (1987), "An Efficient Large-Sample Test for Normality of Observations and Regression Residuals", International Statistical Review, 55, 163-172.

- KENDALL, M. G. and STUART, A. (1973) The Advanced Theory of Statistics, Volume 2 (London: Charles Griffin).
- LANCASTER, T. (1984), "The Covariance Matrix of the Information Matrix Test", Econometrica, 52, 1051-1053.

TSAY, R. S. (1987), "Conditional Heteroscedastic Time Series Models", Journal of the American Statistical Association, 82, 590-604.

WHITE, H. (1980), "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity", Econometrica, 48, 817-838.

WHITE, H. (1982), "Maximum Likelihood Estimation of Misspecified Models", Econometrica. 50, 1-25.

WHITE, H. (1987), "Specification Testing in Dynamic Models", in Bewley, T. F. (ed.), Advances in Econometrics: Fifth World Congress, Volume I (Cambridge: Cambridge University Press), 1-58.

Reprint Series, CentER, Tilburg University, The Netherlands:

- No. 1 G. Marini and F. van der Ploeg, Monetary and fiscal policy in an optimising model with capital accumulation and finite lives, *The Economic Journal*, vol. 98, no. 392, 1988, pp. 772 - 786.
- No. 2 F. van der Ploeg, International policy coordination in interdependent monetary economies, *Journal of International Economics*, vol. 25, 1988, pp. 1 - 23.
- No. 3 A.P. Barten, The history of Dutch macroeconomic modelling (1936-1986), in W. Driehuis, M.M.G. Fase and H. den Hartog (eds.), *Challenges for Macroeconomic Modelling*, Contributions to Economic Analysis 178, Amsterdam: North-Holland, 1988, pp. 39 - 88.
- No. 4 F. van der Ploeg, Disposable income, unemployment, inflation and state spending in a dynamic political-economic model, *Public Choice*, vol. 60, 1989, pp. 211 - 239.
- No. 5 Th. ten Raa and F. van der Ploeg, A statistical approach to the problem of negatives in input-output analysis, *Economic Modelling*, vol. 6, no. 1, 1989, pp. 2 19.
- No. 6 E. van Damme, Renegotiation-proof equilibria in repeated prisoners' dilemma, Journal of Economic Theory, vol. 47, no. 1, 1989, pp. 206 - 217.
- No. 7 C. Mulder and F. van der Ploeg, Trade unions, investment and employment in a small open economy: a Dutch perspective, in J. Muysken and C. de Neubourg (eds.), Unemployment in Europe, London: The Macmillan Press Ltd, 1989, pp. 200 - 229.
- No. 8 Th. van de Klundert and F. van der Ploeg, Wage rigidity and capital mobility in an optimizing model of a small open economy, *De Economist*, vol. 137, nr. 1, 1989, pp. 47 75.
- No. 9 G. Dhaene and A.P. Barten, When it all began: the 1936 Tinbergen model revisited, *Economic Modelling*, vol. 6, no. 2, 1989, pp. 203 - 219.
- No. 10 F. van der Ploeg and A.J. de Zeeuw, Conflict over arms accumulation in market and command economies, in F. van der Ploeg and A.J. de Zeeuw (eds.), *Dynamic Policy Games in Economics*, Contributions to Economic Analysis 181, Amster- dam: Elsevier Science Publishers B.V. (North-Holland), 1989, pp. 91 - 119.
- No. 11 J. Driffill, Macroeconomic policy games with incomplete information: some extensions, in F. van der Ploeg and A.J. de Zeeuw (eds.), *Dynamic Policy Games in Economics*, Contributions to Economic Analysis 181, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1989, pp. 289 - 322.
- No. 12 F. van der Ploeg, Towards monetary integration in Europe, in P. De Grauwe et al., De Europese Monetaire Integratie: vier visies, Wetenschappelijke Raad voor het Regeringsbeleid V 66, 's-Gravenhage: SDU uitgeverij, 1989, pp. 81 - 106.

- No. 13 R.J.M. Alessie and A. Kapteyn, Consumption, savings and demography, in A. Wenig, K.F. Zimmermann (eds.), *Demographic Change and Economic Development*, Berlin/Heidelberg: Springer-Verlag, 1989, pp. 272 - 305.
- No. 14 A. Hoque, J.R. Magnus and B. Pesaran, The exact multi-period mean-square forecast error for the first-order autoregressive model, *Journal of Econometrics*, vol. 39, no. 3, 1988, pp. 327 - 346.
- No. 15 R. Alessie, A. Kapteyn and B. Melenberg, The effects of liquidity constraints on consumption: estimation from household panel data, *European Economic Review*, vol. 33, no. 2/3, 1989, pp. 547 - 555.
- No. 16 A. Holly and J.R. Magnus, A note on instrumental variables and maximum likelihood estimation procedures, Annales d'Économie et de Statistique, no. 10, April-June, 1988, pp. 121 - 138.
- No. 17 P. ten Hacken, A. Kapteyn and I. Woittiez, Unemployment benefits and the labor market, a micro/macro approach, in B.A. Gustafsson and N. Anders Klevmarken (eds.). The Political Economy of Social Security, Contributions to Economic Analysis 179, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1989, pp. 143 164.
- No. 18 T. Wansbeek and A. Kapteyn, Estimation of the error-components model with incomplete panels, *Journal of Econometrics*, vol. 41, no. 3, 1989, pp. 341 - 361.
- No. 19 A. Kapteyn, P. Kooreman and R. Willemse, Some methodological issues in the implementation of subjective poverty definitions, *The Journal of Human Resources*, vol. 23, no. 2, 1988, pp. 222 - 242.
- No. 20 Th. van de Klundert and F. van der Ploeg, Fiscal policy and finite lives in interdependent economies with real and nominal wage rigidity, Oxford Economic Papers, vol. 41, no. 3, 1989, pp. 459 - 489.
- No. 21 J.R. Magnus and B. Pesaran, The exact multi-period mean-square forecast error for the first-order autoregressive model with an intercept, *Journal of Econometrics*, vol. 42, no. 2, 1989, pp. 157 - 179.
- No. 22 F. van der Ploeg, Two essays on political economy: (i) The political economy of overvaluation, *The Economic Journal*, vol. 99, no. 397, 1989, pp. 850 - 855; (ii) Election outcomes and the stockmarket, *European Journal of Political Economy*, vol. 5, no. 1, 1989, pp. 21 - 30.
- No. 23 J.R. Magnus and A.D. Woodland, On the maximum likelihood estimation of multivariate regression models containing serially correlated error components, *International Economic Review*, vol. 29, no. 4, 1988, pp. 707 - 725.
- No. 24 A.J.J. Talman and Y. Yamamoto, A simplicial algorithm for stationary point problems on polytopes, *Mathematics of Operations Research*, vol. 14, no. 3, 1989, pp. 383 - 399.
- No. 25 E. van Damme, Stable equilibria and forward induction, Journal of Economic Theory, vol. 48, no. 2, 1989, pp. 476 - 496.

- No. 26 A.P. Barten and L.J. Bettendorf, Price formation of fish: An application of an inverse demand system, *European Economic Review*, vol. 33, no. 8, 1989, pp. 1509 - 1525.
- No. 27 G. Noldeke and E. van Damme, Signalling in a dynamic labour market, Review of Economic Studies, vol. 57 (1), no. 189, 1990, pp. 1 - 23.
- No. 28 P. Kop Jansen and Th. ten Raa, The choice of model in the construction of input-output coefficients matrices, *International Economic Review*, vol. 31, no. 1, 1990, pp. 213 - 227.
- No. 29 F. van der Ploeg and A.J. de Zeeuw, Perfect equilibrium in a model of competitive arms accumulation, *International Economic Review*, vol. 31, no. 1, 1990, pp. 131 -146.
- No. 30 J.R. Magnus and A.D. Woodland, Separability and aggregation, *Economica*, vol. 57, no. 226, 1990, pp. 239 247.
- No. 31 F. van der Ploeg, International interdependence and policy coordination in economies with real and nominal wage rigidity, *Greek Economic Review*, vol. 10, no. 1, June 1988, pp. 1 - 48.
- No. 32 E. van Damme, Signaling and forward induction in a market entry context, Operations Research Proceedings 1989, Berlin-Heidelberg: Springer-Verlag, 1990, pp. 45 - 59.
- No. 33 A.P. Barten, Toward a levels version of the Rotterdam and related demand systems, Contributions to Operations Research and Economics, Cambridge: MIT Press, 1989, pp. 441 - 465.
- No. 34 F. van der Ploeg, International coordination of monetary policies under alternative exchange-rate regimes, in F. van der Ploeg (ed.), Advanced Lectures in Quantitative Economics, London-Orlando: Academic Press Ltd., 1990, pp. 91 - 121.
- No. 35 Th. van de Klundert, On socioeconomic causes of 'wait unemployment', *European Economic Review*, vol. 34, no. 5, 1990, pp. 1011 1022.
- No. 36 R.J.M. Alessie, A. Kapteyn, J.B. van Lochem and T.J. Wansbeek, Individual effects in utility consistent models of demand, in J. Hartog, G. Ridder and J. Theeuwes (eds.), *Panel Data and Labor Market Studies*, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1990, pp. 253 - 278.
- No. 37 F. van der Ploeg, Capital accumulation, inflation and long-run conflict in international objectives, Oxford Economic Papers, vol. 42, no. 3, 1990, pp. 501-525.
- No. 38 Th. Nijman and F. Palm, Parameter identification in ARMA Processes in the presence of regular but incomplete sampling, *Journal of Time Series Analysis*, vol. 11, no. 3, 1990, pp. 239 - 248.
- No. 39 Th. van de Klundert, Wage differentials and employment in a two-sector model with a dual labour market, *Metroeconomica*, vol. 40, no. 3, 1989, pp. 235 256.

- No. 40 Th. Nijman and M.F.J. Steel, Exclusion restrictions in instrumental variables equations, *Econometric Reviews*, vol. 9, no. 1, 1990, pp. 37 55.
- No. 41 A. van Soest, I. Woittiez and A. Kapteyn, Labor supply, income taxes, and hours restrictions in the Netherlands, *Journal of Human Resources*, vol. 25, no. 3, 1990, pp. 517 - 558.
- No. 42 Th.C.M.J. van de Klundert and A.B.T.M. van Schaik, Unemployment persistence and loss of productive capacity: a Keynesian approach, *Journal of Macro- economics*, vol. 12, no. 3, 1990, pp. 363 - 380.
- No. 43 Th. Nijman and M. Verbeek, Estimation of time-dependent parameters in linear models using cross-sections, panels, or both, *Journal of Econometrics*, vol. 46, no. 3, 1990, pp. 333 - 346.
- No. 44 E. van Damme, R. Selten and E. Winter, Alternating bid bargaining with a smallest money unit, Games and Economic Behavior, vol. 2, no. 2, 1990, pp. 188 - 201.
- No. 45 C. Dang, The D₁-triangulation of Rⁿ for simplicial algorithms for computing solutions of nonlinear equations, *Mathematics of Operations Research*, vol. 16, no. 1, 1991, pp. 148 - 161.
- No. 46 Th. Nijman and F. Palm, Predictive accuracy gain from disaggregate sampling in ARIMA models, *Journal of Business & Economic Statistics*, vol. 8, no. 4, 1990, pp. 405 - 415.
- No. 47 J.R. Magnus, On certain moments relating to ratios of quadratic forms in normal variables: further results, Sankhya: The Indian Journal of Statistics, vol. 52, series B, part. 1, 1990, pp. 1 - 13.
- No. 48 M.F.J. Steel, A Bayesian analysis of simultaneous equation models by combining recursive analytical and numerical approaches, *Journal of Econometrics*, vol. 48, no. 1/2, 1991, pp. 83 - 117.
- No. 49 F. van der Ploeg and C. Withagen, Pollution control and the ramsey problem, Environmental and Resource Economics, vol. 1, no. 2, 1991, pp. 215 - 236.
- No. 50 F. van der Ploeg, Money and capital in interdependent economies with overlapping generations, *Economica*, vol. 58, no. 230, 1991, pp. 233 - 256.
- No. 51 A. Kapteyn and A. de Zeeuw, Changing incentives for economic research in the Netherlands, European Economic Review, vol. 35, no. 2/3, 1991, pp. 603 - 611.
- No. 52 C.G. de Vries, On the relation between GARCH and stable processes, *Journal of Econometrics*, vol. 48, no. 3, 1991, pp. 313 324.
- No. 53 R. Alessie and A. Kapteyn, Habit formation, interdependent preferences and demographic effects in the almost ideal demand system, The Economic Journal, vol. 101, no. 406, 1991, pp. 404 - 419.
- No. 54 W. van Groenendaal and A. de Zeeuw, Control, coordination and conflict on international commodity markets, Economic Modelling, vol. 8, no. 1, 1991, pp. 90 - 101.

- No. 55 F. van der Ploeg and A.J. Markink, Dynamic policy in linear models with rational expectations of future events: A computer package, Computer Science in Economics and Management, vol. 4, no. 3, 1991, pp. 175 - 199.
- No. 56 H.A. Keuzenkamp and F. van der Ploeg, Savings, investment, government finance, and the current account: The Dutch experience, in G. Alogoskoufis, L. Papademos and R. Portes (eds.), External Constraints on Macroeconomic Policy: The European Experience, Cambridge: Cambridge University Press, 1991, pp. 219 - 263.
- No. 57 Th. Nijman, M. Verbeek and A. van Soest, The efficiency of rotating-panel designs in an analysis-of-variance model, Journal of Econometrics, vol. 49, no. 3, 1991, pp. 373 - 399.
- No. 58 M.F.J. Steel and J.-F. Richard, Bayesian multivariate exogeneity analysis an application to a UK money demand equation, Journal of Econometrics, vol. 49, no. 1/2, 1991, pp. 239 - 274.
- No. 59 Th. Nijman and F. Palm, Generalized least squares estimation of linear models containing rational future expectations, International Economic Review, vol. 32, no. 2, 1991, pp. 383 - 389.
- No. 60 E. van Damme, Equilibrium selection in 2 x 2 games, Revista Espanola de Economia, vol. 8, no. 1, 1991, pp. 37 52.
- No. 61 E. Bennett and E. van Damme, Demand commitment bargaining: the case of apex games, in R. Selten (ed.), Game Equilibrium Models III - Strategic Bargaining, Berlin: Springer-Verlag, 1991, pp. 118 - 140.
- No. 62
 W. Güth and E. van Damme, Gorby games a game theoretic analysis of disarmament campaigns and the defense efficiency hypothesis -, in R. Avenhaus, H. Karkar and M. Rudnianski (eds.), Defense Decision Making Analytical Support and Crisis Management, Berlin: Springer-Verlag, 1991, pp. 215 240.
- No. 63 A. Roell, Dual-capacity trading and the quality of the market, *Journal of Financial Intermediation*, vol. 1, no. 2, 1990, pp. 105 124.
- No. 64 Y. Dai, G. van der Laan, A.J.J. Talman and Y. Yamamoto, A simplicial algorithm for the nonlinear stationary point problem on an unbounded polyhedron, *Siam Journal* of Optimization, vol. 1, no. 2, 1991, pp. 151 - 165.
- No. 65 M. McAleer and C.R. McKenzie, Keynesian and new classical models of unemployment revisited, *The Economic Journal*, vol. 101, no. 406, 1991, pp. 359 - 381.
- No. 66 A.J.J. Talman, General equilibrium programming, Nieuw Archief voor Wiskunde, vol. 8, no. 3, 1990, pp. 387 - 397.
- No. 67 J.R. Magnus and B. Pesaran, The bias of forecasts from a first-order autoregression, *Econometric Theory*, vol. 7, no. 2, 1991, pp. 222 - 235.

- No. 68 F. van der Ploeg, Macroeconomic policy coordination issues during the various phases of economic and monetary integration in Europe, European Economy - The Economics of EMU, Commission of the European Communities, special edition no. 1, 1991, pp. 136 - 164.
- No. 69 H. Keuzenkamp, A precursor to Muth: Tinbergen's 1932 model of rational expectations, *The Economic Journal*, vol. 101, no. 408, 1991, pp. 1245 - 1253.
- No. 70 L. Zou, The target-incentive system vs. the price-incentive system under adverse selection and the ratchet effect, *Journal of Public Economics*, vol. 46, no. 1, 1991, pp. 51 - 89.
- No. 71 E. Bomhoff, Between price reform and privatization: Eastern Europe in transition, Finanzmarkt und Portfolio Management, vol. 5, no. 3, 1991, pp. 241 - 251.
- No. 72 E. Bomhoff, Stability of velocity in the major industrial countries: a Kalman filter approach, International Monetary Fund Staff Papers, vol. 38, no. 3, 1991, pp. 626 - 642.
- No. 73 E. Bomhoff, Currency convertibility: when and how? A contribution to the Bulgarian debate, Kredit und Kapital, vol. 24, no. 3, 1991, pp. 412 - 431.
- No. 74 H. Keuzenkamp and F. van der Ploeg, Perceived constraints for Dutch unemployment policy, in C. de Neubourg (ed.), *The Art of Full Employment - Unemployment Policy* in Open Economies, Contributions to Economic Analysis 203, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1991, pp. 7 - 37.
- No. 75 H. Peters and E. van Damme, Characterizing the Nash and Raiffa bargaining solutions by disagreement point axions, *Mathematics of Operations Research*, vol. 16, no. 3, 1991, pp. 447 - 461.
- No. 76 P.J. Deschamps, On the estimated variances of regression coefficients in misspecified error components models, *Econometric Theory*, vol. 7, no. 3, 1991, pp. 369 - 384.
- No. 77 A. de Zeeuw, Note on 'Nash and Stackelberg solutions in a differential game model of capitalism', *Journal of Economic Dynamics and Control*, vol. 16, no. 1, 1992, pp. 139 - 145.
- No. 78 J.R. Magnus, On the fundamental bordered matrix of linear estimation, in F. van der Ploeg (ed.), Advanced Lectures in Quantitative Economics, London-Orlando: Academic Press Ltd., 1990, pp. 583 - 604.
- No. 79 F. van der Ploeg and A. de Zeeuw, A differential game of international pollution control, Systems and Control Letters, vol. 17, no. 6, 1991, pp. 409 414.
- No. 80 Th. Nijman and M. Verbeek, The optimal choice of controls and pre-experimen- tal observations, *Journal of Econometrics*, vol. 51, no. 1/2, 1992, pp. 183 - 189.
- No. 81 M. Verbeek and Th. Nijman, Can cohort data be treated as genuine panel data?, Empirical Economics, vol. 17, no. 1, 1992, pp. 9 - 23.

No. 82	E. van Damme and W. Güth, Equilibrium selection in the Spence signaling game, in R. Selten (ed.), <i>Game Equilibrium Models II - Methods, Morals, and Markets</i> , Berlin: Springer-Verlag, 1991, pp. 263 - 288.
No. 83	R.P. Gilles and P.H.M. Ruys, Characterization of economic agents in arbitrary communication structures, <i>Nieuw Archief voor Wiskunde</i> , vol. 8, no. 3, 1990, pp. 325 - 345.
No. 84	A. de Zeeuw and F. van der Ploeg, Difference games and policy evaluation: a conceptual framework, <i>Oxford Economic Papers</i> , vol. 43, no. 4, 1991, pp. 612 - 636.
No. 85	E. van Damme, Fair division under asymmetric information, in R. Selten (ed.), Rational Interaction - Essays in Honor of John C. Harsanyi, Berlin/Heidelberg: Springer-Verlag, 1992, pp. 121 - 144.
No. 86	F. de Jong, A. Kemna and T. Kloek, A contribution to event study methodology with an application to the Dutch stock market, <i>Journal of Banking and Finance</i> , vol. 16, no. 1, 1992, pp. 11 - 36.
No. 87	A.P. Barten, The estimation of mixed demand systems, in R. Bewley and T. Van Hoa (eds.), Contributions to Consumer Demand and Econometrics, Essays in Honour of Henri Theil, Basingstoke: The Macmillan Press Ltd., 1992, pp. 31 - 57.
No. 88	T. Wansbeek and A. Kapteyn, Simple estimators for dynamic panel data models with errors in variables, in R. Bewley and T. Van Hoa (eds.), <i>Contributions to Consumer Demand and Econometrics, Essays in Honour of Henri Theil</i> , Basingstoke: The Macmillan Press Ltd., 1992, pp. 238 - 251.
No. 89	S. Chib, J. Osiewalski and M. Steel, Posterior inference on the degrees of freedom parameter in multivariate- <i>t</i> regression models, <i>Economics Letters</i> , vol. 37, no. 4, 1991, pp. 391 - 397.
No. 90	H. Peters and P. Wakker, Independence of irrelevant alternatives and revealed group preferences, <i>Econometrica</i> , vol. 59, no. 6, 1991, pp. 1787 - 1801.
No. 91	G. Alogoskoufis and F. van der Ploeg, On budgetary policies, growth, and external deficits in an interdependent world, <i>Journal of the Japanese and International Economies</i> , vol. 5, no. 4, 1991, pp. 305 - 324.
No. 92	R.P. Gilles, G. Owen and R. van den Brink, Games with permission structures: The conjunctive approach, <i>International Journal of Game Theory</i> , vol. 20, no. 3, 1992, pp. 277 - 293.
No. 93	J.A.M. Potters, I.J. Curiel and S.H. Tijs, Traveling salesman games, Mathematical Programming, vol. 53, no. 2, 1992, pp. 199 - 211.
No. 94	A.P. Jurg, M.J.M. Jansen, J.A.M. Potters and S.H. Tijs, A symmetrization for finite two-person games, Zeitschrift für Operations Research - Methods and Models of Operations Research, vol. 36, no. 2, 1992, pp. 111 - 123.

- No. 95 A. van den Nouweland, P. Borm and S. Tijs, Allocation rules for hypergraph communication situations, *International Journal of Game Theory*, vol. 20, no. 3, 1992, pp. 255 - 268.
- No. 96 E.J. Bomhoff, Monetary reform in Eastern Europe, European Economic Review, vol. 36, no. 2/3, 1992, pp. 454 458.
- No. 97 F. van der Ploeg and A. de Zeeuw, International aspects of pollution control, Environmental and Resource Economics, vol. 2, no. 2, 1992, pp. 117 - 139.
- No. 98 P.E.M. Borm and S.H. Tijs, Strategic claim games corresponding to an NTU-game, Games and Economic Behavior, vol. 4, no. 1, 1992, pp. 58 - 71.
- No. 99 A. van Soest and P. Kooreman, Coherency of the indirect translog demand system with binding nonnegativity constraints, *Journal of Econometrics*, vol. 44, no. 3, 1990, pp. 391 - 400.
- No. 100 Th. ten Raa and E.N. Wolff, Secondary products and the measurement of productivity growth, *Regional Science and Urban Economics*, vol. 21, no. 4, 1991, pp. 581 - 615.
- No. 101 P. Kooreman and A. Kapteyn, On the empirical implementation of some game theoretic models of household labor supply, *The Journal of Human Resources*, vol. 25, no. 4, 1990, pp. 584 - 598.
- No. 102 H. Bester, Bertrand equilibrium in a differentiated duopoly, International Economic Review, vol. 33, no. 2, 1992, pp. 433 - 448.
- No. 103 J.A.M. Potters and S.H. Tijs. The nucleolus of a matrix game and other nucleoli, Mathematics of Operations Research, vol. 17, no. 1, 1992, pp. 164 - 174.
- No. 104 A. Kapteyn, P. Kooreman and A. van Soest, Quantity rationing and concavity in a flexible household labor supply model, *Review of Economics and Statistics*, vol. 72, no. 1, 1990, pp. 55 - 62.
- No. 105 A. Kapteyn and P. Kooreman, Household labor supply: What kind of data can tell us how many decision makers there are?, *European Economic Review*, vol. 36, no. 2/3, 1992, pp. 365 - 371.
- No. 106 Th. van de Klundert and S. Smulders, Reconstructing growth theory: A survey, De Economist, vol. 140, no. 2, 1992, pp. 177 - 203.
- No. 107 N. Rankin, Imperfect competition, expectations and the multiple effects of monetary growth, *The Economic Journal*, vol. 102, no. 413, 1992, pp. 743 - 753.
- No. 108 J. Greenberg, On the sensitivity of von Neumann and Morgenstern abstract stable sets: The stable and the individual stable bargaining set, International Journal of Game Theory, vol. 21, no. 1, 1992, pp. 41 - 55.
- No. 109 S. van Wijnbergen, Trade reform, policy uncertainty, and the current account: A non-expected-utility approach, *American Economic Review*, vol. 82, no. 3, 1992, pp. 626 - 633.

No. 110	M. Verbeek and Th. Nijman, Testing for selectivity bias in panel data models, International Economic Review, vol. 33, no. 3, 1992, pp. 681 - 703.
No. 111	Th. Nijman and M. Verbeek, Nonresponse in panel data: The impact on estimates of a life cycle consumption function, <i>Journal of Applied Econometrics</i> , vol. 7, no. 3, 1992, pp. 243 - 257.
No. 112	I. Bomze and E. van Damme, A dynamical characterization of evolutionarily stable states, <i>Annals of Operations Research</i> , vol. 37, 1992, pp. 229 - 244.
No. 113	P.J. Deschamps, Expectations and intertemporal separability in an empirical model of consumption and investment under uncertainty, <i>Empirical Economics</i> , vol. 17, no. 3, 1992, pp. 419 - 450.
No. 114	K. Kamiya and D. Talman, Simplicial algorithm for computing a core element in a balanced game, <i>Journal of the Operations Research</i> , vol. 34, no. 2, 1991, pp. 222 - 228.
No. 115	G.W. Imbens, An efficient method of moments estimator for discrete choice models with choice-based sampling, <i>Econometrica</i> , vol. 60, no. 5, 1992, pp. 1187 -1214.
No. 116	P. Borm, On perfectness concepts for bimatrix games, OR Spektrum, vol. 14, no. 1, 1992, pp. 33 - 42.
No. 117	A.P. Jurg, I. Garcia Jurado and P.E.M. Borm, On modifications of the concepts of perfect and proper equilibria, OR Spektrum, vol. 14, no. 2, 1992, pp. 85 - 90.
No. 118	P. Borm, H. Keiding, R.P. McLean, S. Oortwijn and S. Tijs, The compromise value for NTU-games, <i>International Journal of Game Theory</i> , vol. 21, no. 2, 1992, pp. 175 - 189.
No. 119	M. Maschler, J.A.M. Potters and S.H. Tijs, The general nucleolus and the reduced game property, <i>International Journal of Game Theory</i> , vol. 21, no. 1, 1992, pp. 85 - 106.
No. 120	K. Wärneryd, Communication, correlation and symmetry in bargaining, <i>Economics Letters</i> , vol. 39, no. 3, 1992, pp. 295 - 300.
No. 121	M.R. Baye, D. Kovenock and C.G. de Vries. It takes two to tango: equilibria in a model of sales, <i>Games and Economic Behavior</i> , vol. 4, no. 4, 1992, pp. 493 - 510.
No. 122	M. Verbeek, Pseudo panel data, in L. Mátyás and P. Sevestre (eds.), <i>The Econometrics of Panel Data</i> , Dordrecht: Kluwer Academic Publishers, 1992, pp. 303 - 315.
No. 123	S. van Wijnbergen, Intertemporal speculation, shortages and the political economy of price reform, <i>The Economic Journal</i> , vol. 102, no. 415, 1992, pp. 1395 - 1406.
No. 124	M. Verbeek and Th. Nijman, Incomplete panels and selection bias, in L. Mátyás and P. Sevestre (eds.), <i>The Econometrics of Panel Data</i> , Dordrecht: Kluwer Academic Publishers, 1992, pp. 262 - 302.

- No. 125 J.J. Sijben, Monetary policy in a game-theoretic framework, Jahrbücher für Nationalökonomie und Statistik, vol. 210, no. 3/4, 1992, pp. 233 - 253.
- No. 126 H.A.A. Verbon and M.J.M. Verhoeven, Decision making on pension schemes under rational expectations, *Journal of Economics*, vol. 56, no. 1, 1992, pp. 71 - 97.
- No. 127 L. Zou, Ownership structure and efficiency: An incentive mechanism approach, Journal of Comparative Economics, vol. 16, no. 3, 1993, pp. 399 - 431.
- No. 128 C. Fershtman and A. de Zeeuw, Capital accumulation and entry deterrence: A clarifying note, in G. Feichtinger (ed.), Dynamic Economic MOdels and Optimal Control, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1992, pp. 281 296.
- No. 129 L. Bovenberg and C. Petersen, Public debt and pension policy, *Fiscal Studies*, vol. 13, no. 3, 1992, pp. 1 14.
- No. 130 R. Gradus and A. de Zeeuw, An employment game between government and firms, Optimal Control Applications & Methods, vol. 13, no. 1, 1992, pp. 55 - 71.
- No. 131 Th. Nijman and R. Beetsma, Empirical tests of a simple pricing model for sugar futures, Annales d'Économie et de Statistique, no. 24, 1991, pp. 121 - 131.
- No. 132 F. Groot, C. Withagen and A. de Zeeuw, Note on the open-loop Von Stackelberg equilibrium in the Cartel versus Fringe model, *The Economic Journal*, vol. 102, no. 415, 1992, pp. 1478 - 1484.
- No. 133 S. Eijffinger and N. Gruijters, On the effectiveness of daily intervention by the Deutsche Bundesbank and the Federal Reserve System in the US dollar - deutsche mark exchange market, in Baltensperger/Sinn (eds), Exchange-Rate Regimes and Currency Unions, Basingstoke: The Macmillan Press Ltd., 1992, pp. 131 - 156.
- No. 134 M. R. Baye, D. Kovenock and C. G. de Vries, It takes two to tango: equilibria in a model of sales, *Games and Economic Behavior*, vol 4, 1992, pp. 493 510.
- No. 135 A. K. Bera and S. Lee, Information matrix test, parameter heterogeneity and ARCH: a synthesis, *Review of Economic Studies*, 60, 1993, pp. 229 - 240.

P.O. BOX 90153, 5000 LE TILBURG, THE NETHERLANDS

