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RESEARCH MEMORANDUM

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A QUALIFICATION OF THE DEPENDENCE IN THE GENERALIZED EXTREME VALUE CHOICE MODEL

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A Qualification of the Dependence in the Generalized Extreme Value Choice Model

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Abstract

The Generalized Extreme Value model (GEV) of discrete choice theory is shown to be observationally equivalent to a random utility maximization model with independent utilities and type-1 extreme value distributions. The observational equivalence is not only in terms of choice probabilities, but in terms of the entire joint distribution of choice and achieved utility.

1 Introduction

In the random utility maximization model of discrete choice (RUM) a finite number of alternatives is indexed by $i \in \mathcal{A} = \{1, \ldots, m\}$ and the indirect utility of alternative i is given by a random variable, V_i. The joint-distribution of $V = (V_1, \ldots, V_m)$ summarizes the frequencies of observed utilities and reflects the unobserved attributes of the alternatives and the taste variations among the choice makers (McFadden (1981), Ben Akiva and Lerman (1985)). The choice maker is rational: (s)he selects the alternative with the highest utility. The so achieved utility as well as the selected alternative itself constitute the apparent variables to the observer. These data are regarded as a sample derived from the distribution of the utility levels. The mathematical form of the latter defines the structural characteristics of the model and

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generates the distributions of the observed variables. It is of practical as well as of theoretical interest to know whether the distribution of the observed variables could be generated by another structural model. If so, it would be impossible to discriminate between the alternative models on the basis of the observed variables and the models are said to be observationally equivalent (Koopmans and Reiersøl (1950)). The main result of this paper is that a prominent random utility model with dependence is observationally equivalent to a simple model with independence.

The most widely used RUM model in empirical work is the Multinomial Logit model (MNL). It is computational feasible, but has a very restrictive pattern of interalternative substitution and is characterized by the Independence of Irrelevant Alternatives axiom (IIA). This axiom states that the relative odds for any two alternatives are independent of the attributes or even the availability of a third alternative and has been subject to serious criticism (Debreu (1960), McFadden (1981)). The MNL model features independent utility levels with type 1 extreme value distributions.

The Generalized Extreme Value model (GEV) has been introduced as an extension of the MNL model (McFadden (1978)). The motivation was to retain the computational feasibility, but to permit more flexible pattern of substitution and to relax the IIA axiom. The utility levels follow a more general multivariate extreme value distribution.

This paper provides an MNL representation of the GEV model. The GEV model is shown to be observationally equivalent to a RUM model with independently distributed utility levels and type 1 extreme value distributions, just as in the MNL model. Since the parameters of these distributions depend on the GEV model, the representation does not satisfy the IIA axiom. The proof is by construction of two utilities' vectors generating the GEV model and the MNL representation, respectively, such that with probability one the observed variables are equal. This equality is, of course, much stronger that the equality of the probabilities that some component of the utility vector is maximum in either case. Thus the observational equivalence occurs in the strong sense that there is a perfect match of the achieved utility realizations in the two models.

2 The MNL and GEV models

The MNL and GEV models belong to the family of RUM models in which the utility levels are assumed to have the additively separable form $V_i - c_i$ with the first term random and the second deterministic. In the MNL model, the random terms are independent and follow type 1 extreme value distributions with parameters (A_i, μ):

$$P\{V_i < u\} = \exp(-A_i\, e^{-\mu u}).$$

It follows that $V_i - c_i$, $i = 1, \ldots, m$, are independent and have type 1 extreme value distributions with parameters $(A_i e^{-\mu c_i}, \mu)$, respectively, and generate the choice probabilities

$$p(i, c) = \frac{A_i\, e^{-\mu c_i}}{\sum_{j=1}^{m} A_j\, e^{-\mu c_j}}, \quad i \in \mathcal{A}. \tag{1}$$

In the GEV model, the random vector $V = (V_1, \ldots, V_m)$ has the more general multivariate extreme value distribution, with p.d.f.

$$F_0(u_1, \ldots, u_m) = \exp(-H(e^{-\mu u_1}, \ldots, e^{-\mu u_m}))$$

where $\mu > 0$ is a parameter and where H is a non-negative, linearly homogeneous function with continuous mixed partial derivatives (non-positive even and non-negative odd mixed partial derivatives) such that $\lim_{x_j \to \infty} H(x_1, \ldots, x_m) = \infty$ for all j. It follows that $(V_1 - c_1, \ldots, V_m - c_m)$ has the multivariate extreme value distribution

$$F(u_1, \ldots, u_m) = \exp(-H(e^{-\mu c_1} e^{-\mu u_1}, \ldots, e^{-\mu c_m} e^{-\mu u_m})).$$

which generates the logit-like choice probabilities

$$p(i, c) = \frac{e^{-\mu c_i} H_i(e^{-\mu c_1}, \ldots, e^{-\mu c_m})}{H(e^{-\mu c_1}, \ldots, e^{-\mu c_m})}, \quad i \in \mathcal{A}. \tag{2}$$

Here H_i is the i-th partial derivative of H. The GEV model reduces to the MNL model when $H(x_1, \ldots, x_m) = \sum_{j=1}^{m} A_j\, x_j$. It reduces to the Nested Multinomial Logit model (McFadden (1978), Börsch-Supan (1990)) when

$$H(x_1, \ldots, x_m) = \sum_{l=1}^{n} \left(\sum_{j \in \mathcal{A}_l} A_j x_j^{\theta_l^{-1}} \right)^{\theta_l}.$$

Here $(\mathcal{A}_l)_{l=1,\dots,n}$ is a partition of $\mathcal{A}$ and each parameter θ_l not equal to one introduces a correlation among the alternatives within $\mathcal{A}_l$. More generally, the GEV model accommodates patterns of dependence between the unobserved attributes of the alternatives.

3 MNL representation of a GEV model

Let $\mathcal{M}$ refer to a RUM model generated by a random utilities' vector $V = (V_1, \dots, V_m)$, which now incorporates the deterministic terms (c_i), without loss of generality. Associated with V are maximum utility M and best alternative I defined by

$$\begin{aligned} M &= \max_j V_j \\ I &= i \ \text{ if } \ V_i = M. \end{aligned}$$

The probability of ties is assumed to be zero so that I is well defined up to a negligible event. M and I constitute the observed variables. Let $\mathcal{M}^*$ refer to a second RUM model, generated by $V^* = (V_1^*, \dots, V_m^*)$ with observed variables M^* and I^*.

Definition (Koopmans and Reiersøl). *The models $\mathcal{M}$ and $\mathcal{M}^*$ are said to be observationally equivalent if they generate the same joint distribution of the observed variables, that is*

$$(M, I) \stackrel{d}{=} (M^*, I^*).$$

Remark. The observational equivalence is a strong representational concept for RUM models. Besides the choice probabilities, it compares the distributions of achieved utility. When it holds, it is not possible to discriminate between the alternative models on the basis of the observed variables.

Consider now a GEV model, generated by the multivariate extreme value distribution F. The following spectral representation of F is due to de Haan (1984).

Theorem (de Haan). *There exist m measurable functions g_i taking values in $\mathbb{R} \cup \{-\infty\}$, and a finite measure λ on $[0,1]$ such that, if $(T_n, R_n)_n$ is* *an enumeration of points of the Poisson process on $[0,1] \times \mathbb{R}$ with intensity measure $\lambda(dt)\, e^{-r}\, dr$, then $V = (V_1, \ldots, V_m)$ defined by*

$$V_k = \sup_n \left(g_k(T_n) + \mu^{-1} R_n \right), \quad k = 1, \ldots, m,$$

has the distribution F.

Remark. In fact, the measure λ is the Lebesgue measure on $[0,1]$ restricted to a σ-field of Borel sets with respect to which the functions g_i are measurable.

The previous representation defines a vector $V = (V_1, \ldots, V_m)$ which generates the GEV model. Let it represent the utility levels. Alternative i is chosen on the event

$$\{V_i = \max_j V_j\} \quad = \quad \{ \sup_n (g_i(T_n) + \mu^{-1} R_n) = \max_j \; \sup_n (g_j(T_n) + \mu^{-1} R_n)\}.$$

It is clear that the points of the Poisson process with low $g_i(T_n)$ do not contribute to the realization of this event. If we throw them out, the dependence between the V_i's is eliminated. More precisely, for each i define the set E_i^* and the random variable V_i^* by

$$\begin{aligned} E_i^* &= \{t \in [0,1] : g_i(t) > g_j(t) \quad for\ all\ j \neq i\}, && (3) \\ V_i^* &= \sup_{n: T_n \in E_i^*} (g_i(T_n) + \mu^{-1} R_n). && (4) \end{aligned}$$

The following lemma is crucial. (Recall that two sets are almost surely equal ($\overset{a.s.}{=}$) if their symmetric difference has probability zero and that two random variables are almost surely equal if they are equal with probability one.)

Lemma. *The random variables V_i^*, $i = 1, \ldots, m$, are independent and have the type 1 extreme value distributions with parameters (A_i^*, μ), respectively, with*

$$A_i^* = \int_{\{t \in [0,1] : g_i(t) > g_j(t) \quad for\ all\ j \neq i\}} e^{\mu g_i(t)} \lambda(dt). \tag{5}$$

Here the functions g_i are defined by the spectral representation of the distribution F and where λ is the Lebesgue measure on $[0,1]$. They are such that

$$\{V_i^* = \max_j V_j^*\} \overset{a.s.}{=} \{V_i = \max_j V_j\} \quad , \quad i \in \mathcal{A}, \tag{6}$$

and

$$\max_j V_j^* \stackrel{a.s.}{=} \max_j V_j. \tag{7}$$

Proof: See the Appendix.

The main result is the MNL representation of the GEV model:

Theorem. *The GEV model is observationally equivalent to a RUM model in which the utilities are independent random variables and have type 1 extreme value distributions. The parameters of the distributions are obtained by (5) from the spectral representation of the multivariate extreme value distribution generating the GEV model.*

Proof. Let the GEV model be generated by $V = V_1, \ldots, V_m)$ and let $\mathcal{M}^*$ be the RUM model generated by $V^* = (V_1^*, \ldots, V_m^*)$ as defined by (4). By Lemma 1, V^* has independent components with type 1 extreme value distributions. The parameters A_i^* of these distributions are given by (5) from the spectral representation of F. By (6), with probability one the choices in the GEV model and in $\mathcal{M}^*$ coincide. By (7), the maximum utilities are equal with probability one. Thus the observed variables are equal with probability one . Hence they have the same distribution. □

Remarks. 1. The MNL representation is based on the stochastic structure of the GEV model. The representation is strong, as discussed in the remark following the definition of observational equivalence. Much weaker is the representation provided by the "universal" logit model. The latter expresses choice probabilities in a "logit form" by an algebraic transformation which does not take into account the stochastic structure and, therefore, may even be inconsistent with the RUM hypothesis (McFadden (1981),p. 227, Train (1986), p.21).

2. The IIA property need not hold for the MNL representation generated by V^*. For example, suppose that alternative m is removed from the choice set. In the GEV model, the utility vector is now $\tilde{V} = (V_1, \ldots, V_{m-1})$. Its distribution $\tilde{F}$ is still multivariate extreme value, and admits the spectral representation defined by the functions $(g_1, \ldots, g_{m-1})$. Therefore the MNL

representation is generated by $\tilde{V}^* = (\tilde{V}_1^*, \ldots, \tilde{V}_{m-1}^*)$, where

$$\begin{aligned} \tilde{E}_i^* &= \{t \in [0,1] : g_i(t) > g_j(t) \quad \textit{for all } j \neq i,\ j < m\}, \\ \tilde{V}_i^* &= \sup_{n: T_n \in \tilde{E}_i^*} (g_i(T_n) + \mu^{-1} R_n). \end{aligned}$$

Because the function g_m, associated with alternative m, does not intervene any more, the stochastic structure is changed and the relative odds of the remaining alternatives are affected. The removal of alternative m is formally equivalent to putting $c_m = \infty$. The same break-down of the IIA property holds for more general changes of the systematic costs. Thus, let the utilities be endowed with the additively separable form. The systematic parts of the utilities are considered exogenous. They will enter the MNL representation as follows. The GEV model is now generated by $(V_1 - c_1, \ldots, V_m - c_m)$ defined by the functions $g_i - c_i$ of the spectral representation. The MNL representation is therefore generated by

$$V_{c,i}^* = \sup_{n: T_n \in E_{c,i}} \left(g_i(T_n) - c_i + \mu^{-1} R_n \right)$$

where

$$E_{c,i} = \{t \in [0,1] : \ g_i(t) - c_i > g_j(t) - c_j \text{ for all } j \neq i\}.$$

The random variables $V_{c,i}^*$ are independent and follow type 1 extreme value distribution with parameters $(A_{c,i}^*, \mu)$, where

$$A_{c,i}^* = e^{-\mu c_i} \int_{t \in E_{c,i}} e^{\mu g_i(t)} \lambda(dt).$$

The IIA axiom is violated since the costs influence the region of integration.

4 Conclusion

The dependence of utilities across alternatives accommodated by the GEV model of discrete choice theory has been qualified. More precisely the model is observationally equivalent to an MNL representation. The observational equivalence is not limited to choice probabilities, but holds for the entire distributions of choice and of achieved utility in the two models.

Appendix

Proof of the Lemma. Our proof relies on the spectral representation for the distribution F (see also Dagsvik (1989)). Let $(T_n^i, R_n^i)_n$ be an enumeration of the points of the Poisson process which are in $E_i \times \mathbb{R}$. For each i, $(T_n^i, R_n^i)_n$ constitutes a Poisson process with intensity measure $I_{E_i}(t)\lambda(dt)\, e^{-r}\, dr$. Because the sets E_i are disjoint, these i Poisson processes are independent. Thus the random variables $V_1^*, \ldots, V_m^*$, are independent. On the other hand

$$
\begin{aligned}
P\{V_i^* \le y\} &= P\left\{\sup_{n:\, T_n \in E_i^*} g_i(T_n) + \mu^{-1} R_n < y\right\} \\
&= P\left\{\forall n\, (T_n, R_n) \notin \{(t,r) : t \in E_i^*\ ,\ g_i(t) + \mu^{-1}\, r > y\}\right\} \\
&= \exp\left(-\int_{t,r:\, t\in E_i^*,\, g_i(t)+\mu^{-1}r>y} \lambda(dt)\, e^{-r}\, dr\right) \\
&= \exp\left(-e^{-\mu y}\int_{t\in E_i^*} e^{\mu g_i(t)}\lambda(dt)\right).
\end{aligned}
$$

after straightforward integration. Thus V_i^* follows the type 1 extreme value distribution with parameters μ and A_i^*, with

$$A_i^* = \int_{t\in E_i^*} e^{\mu g_i(t)}\lambda(dt).$$

Here, according to the remark following de Haan's theorem, the measure λ can be taken to be the Lebesgue measure on $[0,1]$. For ease of notation, define

$$h_i(T_n, R_n) = g_i(T_n) + \mu^{-1}\, R_n.$$

Let

$$V_i^\circ = \sup_{n: T_n \notin E_i} h_i(T_n, R_n)$$

so that

$$V_i = \max\,(V_i^*, V_i^\circ)$$

By definition, if $T_n \notin E_i$ then

$$h_i(T_n, R_n, c) \le \max_{j\neq i} h_j(T_n, R_n).$$

Hence

$$\begin{aligned} V_i^\circ &\le \sup_{n:T_n \notin E_i} \max_{j\neq i} h_j(T_n, R_n) \\ &= \max_{j\neq i} \sup_{n:T_n \notin E_i} h_j(T_n, R_n) \\ &\le \max_{j\neq i} \sup_{n} h_j(T_n, R_n) \\ &= \max_{j\neq i} V_j. \end{aligned}$$

It follows that

$$\begin{aligned} \{\, V_i > \max_{j\neq i} V_j \,\} &= \{\, V_i^* > \max_{j\neq i} V_j \,\} \\ &\subseteq \{V_i^* > \max_{j\neq i} V_j^*\} \end{aligned}$$

because $V_i^* \le V_j$. On the other hand

$$\cup_i \{\, V_i > \max_{j\neq i} V_j \,\} \overset{a.s.}{=} \Omega.$$

It follows that

$$\{V_i > \max_{j\neq i} V_j\} \overset{a.s.}{=} \{V_i^* > \max_{j\neq i} V_j^*\}$$

because the sets $\{V_i^* > \max_{j\neq i} V_j^*\}$ are disjoint. Finally, (6) follows because ties are negligible. Furthermore, $\max_i V_i^* \le \max_i V_i$. Since

$$\begin{aligned} P\left\{\max_i V_i > \max_i V_i^*\right\} &\le \sum_j P\{\max_i V_i = V_j^\circ,\ V_j^\circ > V_i^*\} \\ &\le \sum_j P\{\max_{i\neq j} V_i = V_j^\circ,\ V_j = V_j^\circ\} \\ &\le \sum_j P\{\max_{i\neq j} V_i = V_j\} \\ &= 0, \end{aligned}$$

strict inequality occurs with probability zero. Consequently, $\max_i V_i \overset{a.s.}{=} \max_i V_i^*$. □

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