Optimal dynamic taxation with respect to firms

Gradus, R.H.J.M.

RSITY
LIEKE
UNIVERSITEIT
BRABANT

POSTBOX 90153
5000 LE TILBURG
THE NETHERLANDS

[redacted]

DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

[redacted]

OPTIMAL DYNAMIC TAXATION WITH RESPECT TO FIRMS

Raymond Gradus

FEW 369

OPTIMAL DYNAMIC TAXATION WITH RESPECT TO FIRMS

Raymond GRADUS

Tilburg University, P.O.Box 90153 5000 LE Tilburg, The Netherlands

In this paper we develop a framework for determining optimal profit taxation in a market economy with value-maximising firms, which face costs of adjustment for investment. The government chooses this tax rate in such a way that the utility of the consumer, which depends on public and private consumption will be maximised. The private consumption is financed by wage income, while public consumption is financed by tax revenues. We show that there is a dynamic trade-off between public consumption now and in the future. Two possible solutions are derived. The first solution, which is the formal outcome of an open-loop Stackelberg equilibrium of a game between government and firms, is time-inconsistent and is only credible, if there is commitment or if there are reputational forces. The second solution, which corresponds to a feedback Stackelberg equilibrium, is time-consistent, but yields a lower value of steady-state utility.

1. Introduction

In this paper we focus on the problem of the trade-off between investment behaviour of the firm and the tax policy of a 'rational' government. The government may announce a relatively low corporate tax rate and because of that a lower level of public consumption than wanted by consumers. But this relatively low tax rate also implies a higher level of investment, which generates a higher level of total consumption in the future. In this paper we model this dynamic trade-off between corporation taxation now and in the future within a macro-economic framework. We are concerned with two possible solutions. However, which solutions happens, depends on the credibility and reputation of the government. The first solution is time-inconsistent, i.e. becomes suboptimal for the government over time and is only credible, if the firm is expected to believe so. The second solution is time-consistent and results, if the firm expects rational behaviour of the government at all times, but yields a lower value of steady-state utility.

The question of optimal taxation is a very broad one and many strands of literature can be identified (see for example Ramsey (1927), Sandmo

(1976), Atkinson and Stiglitz (1980), Laffer (1981)). The first distinction can be made between papers, which deal with this problem in a static way (e.g. Ramsey (1927) and Laffer (1980)) and in a dynamic way (e.g. Kydland and Prescott (1980), Turnovsky and Brock (1980)). In this paper we deal with the problem of optimal dynamic taxation. We can also distinguish between different kinds of tax rates, e.g. sales tax, wage or income tax and profit tax. Most interest in the literature has been paid to the problem of optimal static income tax, because of their impact on the supply and demand for labour (e.g. Laffer (1981)). Relatively little interest has been paid to the problem above. An example can be found in Fischer (1980), in which a two period problem is treated. However, there is no separation between the decision of the firm and the consumer. We believe, that profit taxation has a greater impact on the outcome of the economic process, because its impact on the capital accumulation and the investment decision, than the place in the literature suggests. In this paper we treat the problem of optimal profit taxation.

The problem of profit taxation can be analysed from a different number of perspectives. First, it can be analysed from the viewpoint of government revenue maximisation (e.g. Gradus (1988a, 1988b)), secondly profit taxation can be used as an instrument for employment policy (e.g. Gradus (1988c)). However as mentioned above, in this paper we choose as objective maximisation of an intertemporal utility function, which depends on private and public consumption.

With respect to the behavioural assumptions we develop a game-theoretic framework. Firms and consumers take the decisions of the others as given, but the government takes into account the way in which the agents will take their decisions. So, the solution corresponds to a Stackelberg game with the government as leader and the firms and the consumers playing Nash against each other (see Başar and Olsder (1982, chapter 7)). Attention is paid to the problems of time-inconsistency, reputation and credibility, which arise by using this framework. For reasons of analytical tractability we assume that there is only one type of consumer and one type of firm.

In section 2 we describe the model for the firm, which is based on neo-classical theory (e.g. Lucas (1967)), while in section 3 the model for the government and consumers is given. In section 4 we compare the open-loop and feedback solutions by applying a numerical example. Special attention is paid to the problem of time-inconsistency. In section 5 we extend the model by incorporating a dynamic wealth constraint for the consumer (e.g. Abel and Blanchard (1983) and Van de Klundert and Peters (1986)), so that there is a solid description of saving behaviour by agents and of investment behaviour by firms. Section 6 concludes and gives some suggestions for future research.

2. The firm's decision problem

Consider a firm operating in an environment without exogenous uncertainty. The firm decides on its demand for labour and investment, which are conditional on its expectations, present and future profit tax rates, present and future interest rates. The firm maximises its discounted stream of net profits (see Van der Ploeg (1987))

$$\max_{i,l} \int_0^\infty ((f(k(t),l(t))-wl(t))(1-\tau(t))-i(t)-\varphi(i(t)))e^{-\int_0^t r(v)dv}\,dt, \tag{1}$$

where: k: the level of the capital stock,

l: the number of employed workers,

i: the rate of investment,

w: the real wage rate (=constant),

τ: the level of corporate tax rate,

r: the rate of interest,

f(k,l): production function,

φ(i): internal adjustment costs,

$$\varphi(0)=0,\ \text{sign}(\varphi')=\text{sign}(i),\ \varphi''>0.$$

With respect to the production function we assume that capital and labour are substitutes and production is characterised by constant returns to scale (so that $f_{ll}f_{kk} - f_{kl}^2 = 0$). The planning horizon is infinite. The strictly convex function $\varphi(.)$ captures that internal adjustment costs increase and are zero only if gross investment is zero. It ensures that capital adjusts in a sluggish manner to changes in interest rate and corporate tax rate. The firm will maximise (1) subject to the capital accumulation equation

$$\dot{k}(t) = i(t) - \delta k(t), \tag{2}$$

where: δ: rate of depreciation.

The necessary conditions for the firm's optimal control problem are:

$$\dot{q}(t) = (r(t)+\delta)q(t) - f_k(1-\tau(t)), \ \lim_{s\to\infty} e^{-\int_t^s r(v)dv} q(s)k(s) = 0, \tag{3}$$

$$\varphi'(i(t)) = q(t) - 1, \tag{4}$$

$$f_l = w, \tag{5}$$

$$\dot{k}(t) = i(t) - \delta k(t), \tag{6}$$

in which: q: the (undiscounted) shadow price of capital.

If we assume that f(k,l) is a Cobb-Douglas production function and that wages are constant, then labour is a linear function of capital and the partial derivative with respect to capital is a constant. So (3)-(6) can be rewritten as follows:

$$\dot{q} = (r+\delta)q - a(1-\tau), \tag{7}$$

$$i = \Phi(q), \ \Phi'>0, \ \Phi(1)=0, \tag{8}$$

$$l = hk, \tag{9}$$

$$\dot{k} = i - \delta k, \tag{10}$$

where a and h are positive constants.

With respect to fixed wages we can assume that there is some union power, that ensures wages to be equal to some fixed level w (e.g. Oswald (1985)). It is also possible to model a labour market, where w is determined by supply and demand for labour (e.g. Abel and Blanchard (1983)). In that case there can be full employment.

The steady-state investment level is just sufficient to provide for depreciation, $i^{*}=\delta k^{*}$, so that the shadow price of capital exceeds one, $q^{*}=1+\varphi'(\delta k^{*})$. This means, that the shadow price of a unit of capital equals the costs of purchasing investment goods plus the marginal costs of adjusting the capital stock. The steady-state capital follows from (7)-(10) and can be expressed as

$$k^{*}=\frac{1}{\delta}\Phi\left(\frac{a(1-\tau^{*})}{r+\delta}\right),\ k^{*}_{\tau}<0,\ k^{*}_{r}<0. \qquad (11)$$

So if the corporate tax rate raises, capital formation decreases and there will be less employment.

3. The government's decision problem

The government maximises a concave utility function, which depends on private and public consumption. We assume that the government has the same utility function as the consumer (e.g. Turnovsky and Brock (1980)), that public consumption will be financed from profit taxation and that there is no debt. As already noted in section 1, an important difference between government and firm is that the government takes account of the manner in which the firms reacts on its taxation decisions, while the firm takes the corporate tax rate as given.

In this section we assume an arbitrary simple model for the consumer. It consumes all its earnings and equilibrium personal savings will be zero at every point in time. So the firm finances investment by retained earnings. In section 5 we will relax this assumption. However, in this section we want to focus on the government's taxation problem.

Given a sequence of interest rates $\{r(t)\}_{t=[0,\infty)}$ the problem for the government and the consumer can be conveniently formulated as the control problem:

$$\max_{\tau} \int_0^{\infty} u(c(t),g(t))e^{-\beta t}\, dt\ , \tag{12}$$

subject to:
$$\dot{q}(t) = (r(t)+\delta)q(t) - a(1-\tau(t)), \tag{13}$$
$$c(t) = (1-\tau(t))(f(k(t),l(t))-wl(t))-i(t)-\varphi(i(t))+wl(t), \tag{14}$$
$$g(t) = \tau(t)(f(k(t),l(t))-wl(t)), \tag{15}$$
$$\dot{k}(t) = \Phi(q(t)) - \delta k(t), \tag{16}$$

where: β: social discount rate,
c: private consumption,
g: public consumption.

Note that equation (15) stands for the fact, that there is no debt, because at every time-point government's spendings, i.e. $g(t)$, are equal to the revenue from taxation.

Furthermore, we assume that there are Cobb-Douglas preferences and that labour is a linear function of capital:

$$u(c(t),g(t)) = \alpha \ln c(t) + (1-\alpha)\ln g(t),\ 0<\alpha<1, \tag{17}$$
$$c(t) = (1-\tau(t))ak(t)+ whk(t)- \Phi(q(t))- \varphi(\Phi(q(t))), \tag{18}$$
$$g(t) = \tau(t)ak(t). \tag{19}$$

The maximisation of (12) with respect to (13)-(19) yields, by assuming an interior solution, the following necessary conditions:

$$\frac{\alpha}{c}\cdot\frac{\partial c}{\partial \tau} + \frac{1-\alpha}{g}\cdot\frac{\partial g}{\partial \tau} - \nu a = 0, \tag{20}$$

$$\begin{aligned}\dot{\nu}(t) &= \beta\nu(t) - (r(t)+\delta)\nu(t) - \lambda(t)\Phi'(q(t)) \\ &\quad - \frac{\alpha}{c(t)}\{\varphi'[\Phi(q(t))]+1\}\ \Phi'(q(t)),\ \nu(0)=0,\end{aligned} \tag{21}$$

$$\dot{\lambda}(t) = (\beta+\delta)\lambda(t) - \alpha[(1-\tau(t))a+wh]/c(t) - (1-\alpha)/k(t),$$
$$\lim_{t\to\infty} e^{-\beta t}\lambda(t)k(t) = 0, \tag{22}$$

where: λ: the government's undiscounted marginal value of capital stock,
ν: the government's undiscounted marginal value of the shadow price of the capital stock to the firm (=q).

Given equation (18), (19) and (20) we can derive:

$$\tau = T(k,\nu,q),\ T_k>0,\ T_q<0,\ T_\nu>0. \tag{23}$$

It should be noted, that the optimal tax rate will be chosen in such a direction, that the following equations holds, along the equilibrium path:

$$\frac{g(t)}{c(t)} = \frac{1-\alpha}{\alpha}\left(1 + \frac{\nu(t)g(t)}{(1-\alpha)k(t)}\right). \tag{24}$$

The steady-state follows from eqs. (21) and (22) and can be expressed as:

$$\nu^{*} = - \frac{\lambda^{*}\Phi^{'}(q^{*})+(\alpha/c^{*})\{\varphi^{'}(\Phi(q^{*}))+1\}\Phi^{'}(q^{*})}{r^{*}+\delta-\beta} < 0^{1)}, \tag{25}$$

$$\lambda^{*} = \{\alpha[(1-\tau^{*})a+wh]/c^{*} + (1-\alpha)/k^{*}\}/(\beta+\delta) > 0. \tag{26}$$

So in the steady-state the amount of public consumption in total consumption is less than $1-\alpha$ (cf. (24), (25)). With equation (11), (18), (25) and (26) the optimal tax rate in the steady-state can be derived:

$$\tau = T(k^{*},\nu^{*},q^{*}). \tag{27}$$

4. About time-inconsistency of optimal plans

In section 3 we have described an optimal profit taxation plan for the government. However, this optimal plan is time-inconsistent (e.g. Kydland and Prescott (1977), Calvo (1978)), because there is an incentive for the government to reoptimize and reconsider its tax strategy at some later date. Once the capital is installed, the government has an incentive to renege on its announcement and ask a higher tax rate. Note, that the marginal value to the government of the firm's shadow price must equal zero at the start of the planning period, because the firm's shadow price is free to jump at that point of time and therefore becomes effectively an additional policy instrument for the government. So, if the government has the possibility at some later point of time to make a new initial plan, this shadowprice becomes zero again. However, one of the basics of open-loop information structure (e.g. Başar and Olsder (1982)) is that the players stick to their announced plan. So if the firm has no reason to believe that the government will stick to its initial plan, the concept used in section 3, which corresponds to a open-loop equilibrium of a Stackelberg game, is no longer a useful concept. In this case the feedback-Stackelberg concept can be used.

Because of the state-separability the open-loop Nash equilibrium is also a candidate for the feedback Nash and Stackelberg equilibrium (e.g. Dockner et al. (1985)), where the Nash equilibrium effectively sets $\{\nu(t)=0,\ \forall t \geq 0\}$ and ignores (19). This equilibrium is time-consistent, because time-consistency implies $\{\nu(t)=0,\ \forall t \geq 0\}$ (e.g. Pohjola (1986)). The open-loop Nash solution is easy to calculate and it turns out that the optimal tax rate is given by

$$\tau = T(k,0,q),\ T_k>0,\ T_q<0. \tag{28}$$

Along the equilibrium path the following equation holds: $\frac{g}{c} = \frac{1-\alpha}{\alpha}$. So, given a certain level of capital, the tax rate in the feedback Stackelberg equilibrium is higher than in the open-loop Stackelberg equilibrium. From equations (7)-(10) it follows that the marginal productivity and the shadow price of capital, i.e. q^*, is lower in the feedback Stackelberg solution. Hence, less capital is accumulated and unemployment is higher.

In this regime there is a reduction in utility for the government and a reduction in the stream of cash-flow for the firm compared with the open-loop Stackelberg solution (see table 1).

The shadow price ν can be interpreted as a price of time-inconsistency. At a later point of time, if capital is installed, there is an incentive for the government to ask a higher tax rate, such that $\frac{g}{c} = \frac{1-\alpha}{\alpha}$. The extra gain of increasing the tax rate, such that q increases by 1, is equal to $-\nu$. So, $-\nu$ equals the marginal value of cheating the firm by suddingly raising the tax rate. In this way $-\nu$ can be interpreted as the government's cost for sticking to its announced plan.

Table 1

A comparison of the open-loop and the feedback solution

FEEDBACK STACKELBERG		OPEN-LOOP STACKELBERG
NO BINDING CONTRACTS		BINDING CONTRACTS
TIME-CONSISTENT		TIME-INCONSISTENT
$\frac{g^*}{c^*} = \frac{1-\alpha}{\alpha}$		$\frac{g^*}{c^*} = \frac{1-\alpha}{\alpha}\left(1 + \frac{\nu^* g^*}{k^*(1-\alpha)}\right)$
τ^*_{fbs}	$>$	τ^*_{ols}
k^*_{fbs}	$<$	k^*_{ols}
u^*_{fbs}	$<$	u^*_{ols}

So for both players it is better that open-loop is played, but at the moment that the capital stock is build up, there is an incentive for the government to reoptimise and ask a higher tax rate. The outcome for the firm is of course lower, if the government cheats the firm by suddenly asking the high rate in stead of sticking to its announced plan. So a time-inconsistent plan requires binding commitments to force the government to stick to its announced tax strategy. However, it should be noted, that reputational forces can also be important to prevent the government from cheating (e.g. Kreps and Wilson (1982)).

The nature of the solutions examined may be further clarified by a numerical example, which is based on the following three assumptions:

(i) quadratic adjustment costs: $\varphi(i)=bi^2$,

(ii) Cobb-Douglas production function: $f(k,l)=k^{\sigma}l^{1-\sigma}$, $0<\sigma<1$,

(iii) $\lim_{t\to\infty} r_t=\beta$,

and the following parameter values: $\sigma=0.375$, $b=4$, $\delta=0.05$, $\beta=0.03$ and $\alpha=\frac{4}{5}$.

Table 2
A numerical example

FEEDBACK STACKELBERG	OPEN-LOOP STACKELBERG
$\tau^* = 0.4711$	$\tau^* = 0.2165$
$q^* = 1.133$	$q^* = 1.678$
$\lambda^* = 42.65$	$\lambda^* = 8.494$
$k^* = 0.3318$	$k^* = 1.695$
$c^* = 0.1071$	$c^* = 0.5980$
$g^* = 0.0268$	$g^* = 0.0628$
$i^* = 0.0166$	$i^* = 0.0848$
$\varphi(i^*) = 0.0011$	$\varphi(i^*) = 0.0287$
$f(k^*,l^*) = 0.1516$	$f(k^*,l^*) = 0.7746$
$u^* = 0.0812$ $(=c^{*\alpha}g^{*(1-\alpha)})$	$u^* = 0.3810$

In the steady-state the following optimal tax rule can be derived:

$$\tau^*_{fbs}=\frac{(1-\alpha)((a+wh)\ -\ \frac{\delta}{2}(1+\frac{a}{r+\delta}))}{(1\ -\ \frac{\delta(1-\alpha)}{2(r+\delta)})a}. \tag{29}$$

The open-loop solution, τ^*_{ols}, is found by a numerical procedure.

This example makes clear the difference between the open-loop and the feedback solution. The feedback solution yields a higher value of steady-state tax rate and because of that a lower level of capital stock than the open-loop solution (see table 2). This lower level of capital stock in the feedback case yields a lower level of steady-state utility. In the open-loop case the share of public consumption goods in total output is lower, but private consumption and total utility will be higher because there is more capital. However, this open-loop solution is time-inconsistent and is only reasonable if there is commitment or there are reputational forces.

5. A dynamic wealth constraint for the consumer

So far, the analysis has assumed an arbitrary model for the consumer, because of the fact that the consumer consumes all its earnings. In this section we will model the saving-investment decision, such as Abel and Blanchard (1983) for example. The consumer can choose between consumption now or in the future. In this way consumption is an increasing function of total wealth in the spirit of Metzler (1951) and an equilibrium between aggregate demand and supply is also now achieved by the endogeneous adjustment of the sequence of current and future interest rates.

In this section we present the model for the consumer. After that we make some remarks regarding the behaviour of the firm, which is under some additional assumptions the same as in section 2. Finally we describe the behaviour of the government, which becomes quite complicated now. With respect to the behavioural assumptions the consumer and the firm take the decision of the other as given, while the government takes into account the way the firm and consumer make their decisions. So the formal outcome of the game corresponds to a three person Stackelberg game with the government as leader and firm and consumer playing Nash against each other.

The consumer choose a sequence of consumption, which maximises the present value of utility

$$\max_{c(t)} \int_0^\infty u(c(t),g(t))e^{-\beta t}\,dt. \tag{30}$$

The wealth constraint can be expressed as

$$\dot{b}(t) = r(t)b(t) + \pi(t) + wl(t) - c(t), \tag{31}$$

where: b: amount of bonds hold by consumer,

π: dividends,

j: total investment expenditures $(i+\varphi(i))$.

So income is the sum of wages, interest on savings and dividends. The optimality conditions are:

$$\frac{\partial u(c(t),g(t))}{\partial c(t)} = x(t), \tag{32}$$

$$\dot{x}(t) = (\beta-r(t))x(t), \ \lim_{t\to\infty} e^{-\beta t}x(t)b(t) = 0, \tag{33}$$

in which: x(t): the costate variable associated with the dynamic budget constraint.

The underlying finance structure in section 2 was that the firm finances investment by retained earnings and never issues new shares or bonds. In this section the firm finances investment by retained earnings or by issuing shares or bonds. However, because of the fact that the interest rate on bonds is also r and the Modigliani-Miller theorem holds, all financing schemes are equivalent in the sense that they lead to the same path of total consumption and investment; they differ, however, in terms of institutional arrangements (for a proof of this see Abel and Blanchard (1983, pp. 680-681)). Note that we still assume that the government has a zero deficit.

The problem for the government can be formulated as the following control problem:

$$\max_{\tau} \int_0^{\infty} u(c(t),g(t))e^{-\beta t}\,dt, \tag{34}$$

subject to: $\dot{q}(t) = (r(t)+\delta)q(t) - a(1-\tau(t)),$ (35)

$$u_c = x, \tag{36}$$

$$g(t) = \tau(t)ak(t), \tag{37}$$

$$\dot{k}(t) = \Phi(q(t)) - \delta k(t), \tag{38}$$

$$\dot{x}(t) = (\beta-r(t))x(t), \tag{39}$$

$$\dot{b}(t) = r(t)b(t) + \pi(t) + wl(t)-c(t). \tag{40}$$

If there are Cobb-Douglas prefences this leads to the following Hamiltonian with the necessary conditions:

$$\begin{aligned} H = {} & \alpha\ln\alpha - x(t)\ln\alpha + (1-\alpha)\ln(a\tau(t)k(t)) + \lambda(t)(\Phi(q(t))-\delta k(t)) \\ & + \eta(t)(\beta-r(t))x(t) + \nu(t)((r(t)+\delta)q(t)-a(1-\tau(t))) \\ & + y(t)(rb(t)+\pi(t)+whk(t)-\alpha/x(t)), \end{aligned} \tag{41}$$

$$\begin{aligned} \dot{\nu}(t) = {} & \beta\nu(t) - (r(t)+\delta)\nu(t)- \lambda(t)\Phi'(q(t)) \\ & - \frac{\alpha}{c(t)}\{\varphi'(\Phi(q(t)))+1\}\Phi'(q(t)), \quad \nu(0)=0, \end{aligned} \tag{42}$$

$$\dot{y}(t) = (\beta-r(t))y(t), \ \lim_{t\to\infty} e^{-\beta t}y(t)b(t) = 0, \tag{43}$$

$$\begin{aligned} \dot{\eta}(t) = {} & \ln\alpha - r(t)b(t) - \pi(t) - whk((t) + \alpha/x(t) + r(t)\eta(t), \\ & \eta(0)=0, \end{aligned} \tag{44}$$

$$\dot{\lambda}(t) = (\beta+\delta)\lambda(t) - \frac{1-\alpha}{k(t)} - (\frac{\partial\pi(t)}{\partial k(t)}+wh)y(t), \ \lim_{t\to\infty}e^{-\beta t}\lambda(t)k(t) = 0, \tag{45}$$

$$\frac{1-\alpha}{\tau(t)} - y(t)\frac{\partial\pi(t)}{\partial\tau(t)} + \nu(t)a = 0. \tag{46}$$

From equation (46) the optimal tax rate can be derived. Note that the optimal corporate tax rate depends on the financing scheme of the firm and

$x(t)=y(t)$. If we have, for example, the financing scheme of section 2, which says that investment is financed by retained earnings:

$$\pi(t) = (1-\tau(t))ak(t) - j(t), \; \frac{\partial\pi}{\partial\tau} = -\frac{\partial c}{\partial\tau}, \tag{47}$$

than equation (46) becomes:

$$\frac{g(t)}{c(t)} = \frac{1-\alpha}{\alpha}\left(1 + \frac{\nu(t)g(t)}{(1-\alpha)k(t)}\right). \tag{48}$$

A different financing scheme is, that firms finances replacement investment by retained earnings and net investment by bonds. In that case (47) and (48) become:

$$\pi(t) = (1-\tau(t))ak(t)-\delta k(t)-\varphi(\delta k(t))-r(t)b(t), \; \frac{\partial\pi}{\partial\tau} = -\frac{\partial c}{\partial\tau}, \tag{49}$$

$$\frac{g(t)}{c(t)} = \frac{1-\alpha}{\alpha}\left(1 + \frac{\nu(t)g(t)}{(1-\alpha)k(t)}\right). \tag{50}$$

Now also equation (24), which says that the share of government's consumption in total output is less than $1-\alpha$, holds at every time-point. There is only one reason, for the fact that equation (24), even in a model with a dynamic budget constraint, no longer holds and that is that the way that the firms finances their investment depends on the level of corporate taxation. Together with the condition for the equilibrium in the goods market:

$$f(k(t),l(t)) = c(t) + g(t) + i(t) + \varphi(i(t)) \tag{51}$$

we have total macro-economic model, which is repeated in the appendix. Themodel has 14 equations and 14 unknown variables and can be solved by the method of multiple shooting as explained in Lipton et al. (1982). Note that the condition for the equilibrium in the goods market, together with the anticipation that this condition will hold at future times, determines at any instant the complete term structure of interest rates.

In the steady-state, where the rate of interest equals the social discount rate and personal savings are zero, we have the same tax rate as in the case without the dynamic budget constraint. So from this point of view we can draw the conclusion that the main features mentioned in section 4 still remain. Although the adjustment process in the case of personal savings differs, the long run results are the same.

6. Conclusions

In this paper we have developed a macro-economic dynamic model with value-maximising firms, infinitely utility-optimising long-lived consumers and a government, which tries to choose its tax instrument in such a direction that the utility of the consumer is maximised. The formal structure of the interaction between government and firms or consumers corresponds to a open-loop Stackelberg game with the government as leader. By doing this we are concerned with the problem of optimal taxation over time. However, the introduction of optimising government in our framework induces that its optimal plan is intertemporally time-inconsistent. So, if there is no reason to believe that the government will stick to its announced plan, this open-loop concept is no longer useful. In that case the solution can correspond to the equilibrium of a feedback Stackelberg game, which is by definition time-consistent. However, this solution yields a lower value of steady-state utility. In this respect it should be mentioned that if the announced policy is credible, because there is commitment or there are reputational forces, the time-inconsistent policy can be chosen and there is a Pareto improvement of steady-state utility. So the credibilty of the government's policy can play an important role in the effectivity of its policy (see also Gradus (1988 b,c)). In this paper we deal with the two possible solutions mentioned above and present an example, which shows the importance of agreement and consistency in economic theory.

In future work, there are many avenues to explore. Firstly, other tax instruments, like wage or sales tax, can be analysed. Secondly, we can analyse what will happen under the assumption of perfect competition in the labour market. Thirdly, a thorough analysis of reputational equilibria is required (e.g. Kreps and Wilson (1982)). Fourthly, it is important to perform an empirical investigation to establisch in 'which' regime the economy has been at various times. For a first and interesting attempt see Weber (1988). Finally, the framework can be used to characterize the dynamic effects of shocks or policies.

1) assuming that $r+\delta-\beta>0$, which is quite reasonable

ACKNOWLEDGEMENTS

Financial support by the Netherlands organization for scientific research is gratefully acknowledged. The author likes to thank Theo van de Klundert, Peter Kort, Rick van der Ploeg and Piet Verheyen for helpful comments.

References

Abel, A.B. and O.J. Blanchard, 1983, An intertemporal model of saving and investment, Econometrica 51, 675-692.

Atkinson, A.B. and J.E. Stiglitz, 1980, Lectures on public economics (McGraw-Hill, London).

Başar, T. and G.J. Olsder, 1982, Dynamic non-cooperative game theory (Academic Press, New York).

Calvo, G.A., 1978, On the time-inconsistency of optimal policy in a monetary economy, Econometrica 46, 1411-1428.

Dockner, E., Feichtinger, G. and S. Jørgensen, 1985, Tractable classes of nonzero-sum open-loop Nash differential games: theory and examples, Journal of Optimization Theory and Applications 45, 179-191.

Fischer, S., 1980, Dynamic inconsistency, cooperation and the benevolent dissembling government, Journal of Economic Dynamics and Control 2, 93-107.

Gradus, R.H.J.M., 1988a, The reaction of the firm on governmental policy: a game-theoretical approach, in G. Feichtinger (ed.), Optimal control theory and Economic Analysis 3, (North-Holland, Amsterdam), 265-290.

Gradus, R.H.J.M., 1988b, A differential game between government and firms: a non-cooperative approach, Research Memorandum, FEW 303 (Tilburg University).

Gradus, R.H.J.M., 1988c, The employment policy of government and firms: to create jobs or to let them create ?, Research Memorandum, FEW 329 (Tilburg University).

Klundert,Th. van de and P. Peters, 1986, Tax incidence in a model with perfect foresight of agents and rationing in markets, Journal of Public Economics 30, 37-59.
Kreps, D.M. and R. Wilson, 1982, Reputation and imperfect information, Journal of Economic Theory 27, 253-279.
Kydland, F. and E. Prescott, 1977, Rules rather than discretion: the inconsistency of optimal plans, Journal of Political Economy 85, 473-492
Laffer, A., 1981, Government exactions and revenue, Canadian Journal of Economics 2, 12-25.
Lipton, D., J. Poterba, J. Sachs and L. Summers, 1982, Multiple shooting in rational expectations models, Econometrica 50, 1329-1333.
Lucas, R., 1967, Adjustment costs and the theory of supply, Journal of Political Economy 75, 321-334.
Metzler, L., 1951, Wealth, Saving and the rate of interest, Journal of Poltical Economy 49, 93-116.
Oswald, A., 1985, The economic theory of trade unions: an introductory survey, Scandinavian Journal of Economics 87, 160-193
Ploeg, F. van der, 1987, Trade unions, investment, and employment: a non-cooperative approach, European Economic Review 39, 1465-1492.
Pohjola, M., 1986, Applications of dynamic game theory to macro-economics, in T. Başar (ed.), Dynamic games and applications in economics, Lecture Notes in Economics and Mathematical Systems, vol. 265 (Springer-Verlag, Berlin) 26-46.
Ramsey, F., 1927, A contribution to the theory of taxation, Economic Journal 37, 47-61.
Sandmo, A., 1976, Optimal taxation: an introduction to the literaure, Journal of Public Economics 6, 37-54.
Turnovsky, S.J. and W.A. Brock, 1980, Time consistency and optimal government policies in perfect foresight equilibrium, Journal of Public Economics 13, 183-212.
Weber, A., 1988, The credibility of monetary policies, policymakers' reputation and the EMS-hypothesis: empirical evidendence from 13 countries, Discussion paper 88.03, CentER for Economic Research, Tilburg.

Appendix: The total macro-economic model

Given the financing scheme that the firm finances replacement investment by retained earnings and net investment by issuing new bonds.

$$q:\ \dot{q} = (r+\delta)q-a(1-\tau),\ \lim_{s\to\infty} e^{-\int_t^s r(v)dv} q(s)k(s) = 0, \tag{A.1}$$

$$c:\ xc = \alpha, \tag{A.2}$$

$$\tau:\ g = \tau ak, \tag{A.3}$$

$$k: \dot{k} = \Phi(q)-\delta k, \; k(0)=k_0, \tag{A.4}$$

$$x: \dot{x} = (\beta-r)x, \; \lim_{t\to\infty} e^{-\beta t}x(t)b(t) = 0, \tag{A.5}$$

$$b: \dot{b} = i+\varphi(i)-\delta k-\varphi(\delta k), \; b(0)=b_0, \tag{A.6}$$

$$\nu: \dot{\nu} = \beta\nu - (r+\delta)\nu-\lambda\Phi'(q)- \frac{\alpha}{c}\{\varphi'(\Phi(q)+1)\}\Phi'(q), \; \nu(0)=0, \tag{A.7}$$

$$\eta: \dot{\eta} = r\eta+\ln\alpha-i-\varphi(i)+\delta k+\varphi(\delta k), \; \eta(0)=0, \tag{A.8}$$

$$\lambda: \dot{\lambda}=(\beta+\delta)\lambda-\frac{1-\alpha}{k}-(a(1-\tau)+wh)x, \; \lim_{t\to\infty} e^{-\beta t}\lambda(t)k(t) = 0, \tag{A.9}$$

$$i: i=\Phi(q), \tag{A1.10}$$

$$g: \frac{g}{c} = \frac{1-\alpha}{\alpha}(1+\frac{\nu g}{k(1-\alpha)}), \tag{A1.11}$$

$$l: l = hk, \tag{A1.12}$$

$$r: f(k,l) = c+g+i+\varphi(i), \tag{A1.13}$$

$$\pi: \pi = (f(k,l)-wl)(1-\tau)-\delta k-\varphi(\delta k)-rb. \tag{A1.14}$$

IN 1988 REEDS VERSCHENEN

297 Bert Bettonvil
Factor screening by sequential bifurcation

298 Robert P. Gilles
On perfect competition in an economy with a coalitional structure

299 Willem Selen, Ruud M. Heuts
Capacitated Lot-Size Production Planning in Process Industry

300 J. Kriens, J.Th. van Lieshout
Notes on the Markowitz portfolio selection method

301 Bert Bettonvil, Jack P.C. Kleijnen
Measurement scales and resolution IV designs: a note

302 Theo Nijman, Marno Verbeek
Estimation of time dependent parameters in lineair models using cross sections, panels or both

303 Raymond H.J.M. Gradus
A differential game between government and firms: a non-cooperative approach

304 Leo W.G. Strijbosch, Ronald J.M.M. Does
Comparison of bias-reducing methods for estimating the parameter in dilution series

305 Drs. W.J. Reijnders, Drs. W.F. Verstappen
Strategische bespiegelingen betreffende het Nederlandse kwaliteits-concept

306 J.P.C. Kleijnen, J. Kriens, H. Timmermans and H. Van den Wildenberg
Regression sampling in statistical auditing

307 Isolde Woittiez, Arie Kapteyn
A Model of Job Choice, Labour Supply and Wages

308 Jack P.C. Kleijnen
Simulation and optimization in production planning: A case study

309 Robert P. Gilles and Pieter H.M. Ruys
Relational constraints in coalition formation

310 Drs. H. Leo Theuns
Determinanten van de vraag naar vakantiereizen: een verkenning van materiële en immateriële factoren

311 Peter M. Kort
Dynamic Firm Behaviour within an Uncertain Environment

312 J.P.C. Blanc
A numerical approach to cyclic-service queueing models

313 Drs. N.J. de Beer, Drs. A.M. van Nunen, Drs. M.O. Nijkamp
Does Morkmon Matter?

314 Th. van de Klundert
Wage differentials and employment in a two-sector model with a dual labour market

315 Aart de Zeeuw, Fons Groot, Cees Withagen
On Credible Optimal Tax Rate Policies

316 Christian B. Mulder
Wage moderating effects of corporatism
Decentralized versus centralized wage setting in a union, firm, government context

317 Jörg Glombowski, Michael Krüger
A short-period Goodwin growth cycle

318 Theo Nijman, Marno Verbeek, Arthur van Soest
The optimal design of rotating panels in a simple analysis of variance model

319 Drs. S.V. Hannema, Drs. P.A.M. Versteijne
De toepassing en toekomst van public private partnership's bij de grote en middelgrote Nederlandse gemeenten

320 Th. van de Klundert
Wage Rigidity, Capital Accumulation and Unemployment in a Small Open Economy

321 M.H.C. Paardekooper
An upper and a lower bound for the distance of a manifold to a nearby point

322 Th. ten Raa, F. van der Ploeg
A statistical approach to the problem of negatives in input-output analysis

323 P. Kooreman
Household Labor Force Participation as a Cooperative Game; an Empirical Model

324 A.B.T.M. van Schaik
Persistent Unemployment and Long Run Growth

325 Dr. F.W.M. Boekema, Drs. L.A.G. Oerlemans
De lokale produktiestructuur doorgelicht.
Bedrijfstakverkenningen ten behoeve van regionaal-economisch onderzoek

326 J.P.C. Kleijnen, J. Kriens, M.C.H.M. Lafleur, J.H.F. Pardoel
Sampling for quality inspection and correction: AOQL performance criteria

357 R.M. Heuts, M. Bronckers
A modified coordinated reorder procedure under aggregate investment and service constraints using optimal policy surfaces

358 B.B. van der Genugten
Linear time-invariant filters of infinite order for non-stationary processes

359 J.C. Engwerda
LQ-problem: the discrete-time time-varying case

360 Shan-Hwei Nienhuys-Cheng
Constraints in binary semantical networks

361 A.B.T.M. van Schaik
Interregional Propagation of Inflationary Shocks

362 F.C. Drost
How to define UMVU

363 Rommert J. Casimir
Infogame users manual
Rev 1.2 December 1988

364 M.H.C. Paardekooper
A quadratically convergent parallel Jacobi-process for diagonal dominant matrices with nondistinct eigenvalues

365 Robert P. Gilles, Pieter H.M. Ruys
Characterization of Economic Agents in Arbitrary Communication Structures

366 Harry H. Tigelaar
Informative sampling in a multivariate linear system disturbed by moving average noise

367 Jörg Glombowski
Cyclical interactions of politics and economics in an abstract capitalist economy

IN 1989 REEDS VERSCHENEN

368 Ed Nijssen, Will Reijnders
"Macht als strategisch en tactisch marketinginstrument binnen de distributieketen"