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Nonstationarity in job search theory

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Abstract

Generally, structural job search models are taken to be stationary, which is unrealistic in most cases. In this paper we examine models in which every exogenous variable can cause nonstationarity, for instance because its value is dependent on unemployment duration. A general differential equation that describes the evolution of the reservation wage over time, is derived. We present comparative dynamics for the reservation wage and the unemployment duration distribution. Some numerical examples show the restrictiveness of the stationarity assumption. Finally, it is outlined how the results can be used for empirical analysis.

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1. Introduction

This paper examines the movement of a job seeking individual's reservation wage over time, in a general non-stationary job model. Also, results concerning comparative dynamics and the distribution of the duration of unemployment are derived.

Recently, the use of job search models for the analysis of unemployment duration has become widespread. The reduced form approach in empirical studies (see e.g. Lancaster (1979) and Kooreman & Ridder (1983)), in which only hazards of the duration distribution are estimated, seems to be replaced gradually by a more structural approach. The latter way of modeling is characterized by the explicit use of a reservation wage equation in empirical analysis. Lancaster & Chesher (1983) and Narendranathan & Nickell (1985) use the complete theoretical framework of job search theory to make inferences about search behaviour.

However, the structural models used in these two studies are stationary. This implies that variables like the unemployment benefit or the rate of arrival of job offers are assumed to be constant over the spell of unemployment which is often at variance with reality. What's more, various reduced form empirical studies indicate a significant duration dependence of the reemployment probability (see e.g. Blau & Robins (1986), Kooreman & Ridder (1983), Lancaster (1979) and Narendranathan, Nickell & Stern (1985)). Consequently, there is a need to model reservation wage movements over time based on a nonstationary theoretical framework.

In the last fifteen years, a few papers have been published that pay some attention to nonstationarity in job search theory (see e.g. Burdett (1979), Gronau (1971), Heckman & Singer (1982), Lippman & McCall (1976b) and Mortensen (1984)). Although these articles draw important qualitative conclusions concerning the movement of the reservation wage over time, generally no attention is paid to a rigorous derivation of formulas for the time dependence of the reservation wage. Furthermore, only very specific departures from stationarity are examined, like finite lifetimes or shifting wage offer distributions. Most models are specified in discrete time.

In this paper we examine the consequences of nonstationarity in continuous-time job search models in a rather general setting. Section 2 gives a brief overview of job search theory. Various causes of nonstationarity that may arise are discussed, like macro-economic events and changes in the personal situation of individuals during the spell of unemployment. In section 3 and 4 we present the main theorems concerning the movement of the reservation wage over time in nonstationary models. The exogenous variables like unemployment benefits and wage offer distributions are allowed to vary over time in a very general way. The more specific the assumptions about the time paths of the exogenous variables, the more detailed our inferences about the time path of the reservation wage are. In section 3, we also give some comparative dynamics results. These results concern the shift in the optimal reservation wage path if we replace some particular time path of an exogenous variable by another. We also examine the unemployment duration density in case of nonstationarity.

In section 5 we illustrate by means of numerical examples the importance of allowing for nonstationarity. Section 6 concludes. It is outlined how the results of this paper can be used for estimation purposes and for policy analysis.

2. Job search theory and the introduction of nonstationarity

Job search theory tries to describe the behaviour of unemployed individuals in a dynamic and uncertain world. Job offers arrive at random intervals folowing a (semi-)Poisson process with arrival rate λ. Such job offers are random drawings (without recall) from a wage offer distribution with distribution function $F(w)$. Once a job is accepted, it will be kept forever at the same wage. It is assumed that individuals know λ and $F(w)$. During the spell of unemployment, a benefit b is being received. One can think of b as unemployment benefits minus the costs of searching for a job. Unemployed individuals aim at maximization of their own expected present value of income (over an infinite horizon).

The job search model described here contains three exogenous variables (λ, b and $F(w)$) and one constant parameter ρ, which is the subjective rate of discount ($\rho > 0$). We now discuss the concepts of stationarity and nonstationarity in this model. Let time T_0 denote the point of time at which an individual becomes unemployed. We call the job search model that describes the search behaviour of this individual stationary, if the exogenous variables, λ, b and F(w) are constant on the time interval $[T_0, \infty\rangle$. In combination with the infinite horizon assumption, this means that in case of stationarity, the unemployed individual's perception of the future is independent of time or unemployment duration. Consequently, the optimal strategy is constant during the spell of unemployment.

Let us assume that F(w) is continuous in w, that this distribution has a finite first moment and that $0 \leq \lambda < \infty$ and $|b| < \infty$. For a stationary job search model satisfying these conditions, it has been shown many times that the optimal strategy can be characterized by a reservation wage property (see e.g. Lancaster & Chesher (1983)) Jobs will be accepted if their wages exceed the reservation wage φ while a wage below φ induces one to reject the offer and search for a better one. The reservation wage is determined by

$$(1) \qquad \varphi = b + \frac{\lambda}{\rho} \cdot \int_{\varphi}^{\infty} \bar{F}(w)dw \qquad \bar{F} := 1-F$$

Nonstationarity arises if one or more of the exogenous variables change after T_0. Such a change may be due to business cycle effects. For instance, an increase in the aggregate unemployment level may induce a fall in λ. Changes may also occur because of policy changes like a reduction of all unemployment benefits. Finally, for a job searcher the exogenous variables may change because of changes in his personal situation. Unemployment benefits and F(w) may be dependent on the elapsed unemployment duration. Sooner or later these features of the labour market and personal characteristics of job searchers are recognized and used in determining the optimal strategy. So, generally, the optimal strategy is not constant in case of a nonstationry model.

In this paper we consider nonstationarity as a result of time dependence and duration dependence of exogenous variables. Dependencies of exogenous variables on the number of rejected offers or the levels of wages associated with rejected offers are ruled out. Further, throughout the paper we will be concerned with rational job searchers; they are assumed to correctly anticipate changes in the values of the exogenous variables. For instance, we expect people to know how unemployment benefits are related to unemployment duration. Unanticipated changes are trivial to model, using (1).

3. The reservation wage in nonstationary job search models

3.1. Assumptions

In order to be able to derive an expression for the optimal search strategy in a nonstationary model, we have to examine the exogenous variables more closely. For ease of exposition we let calender time start at the moment that one becomes unemployed, so that calender time and unemployment duration coincide. In this way we can consider duration dependence and other forms of nonstationarity simultaneously. If the job offer arrival rate λ depends on time, the waiting time until the next job offer does not have an exponential distribution. Consider someone whose elapsed unemployment duration equals t. The job offer probability in a small interval (t+u, t+u+du) conditional on not having received an offer between t and t+u, is $\lambda(t+u)du$. Defining $g(u;t)$ to be the density of waiting time u for someone whose elapsed duration equals t, we have

$$(2) \qquad \lambda(t+u)du = \frac{g(u;t)du}{1-G(u;t)}$$

in which $G(u;t)$ represents the distribution function of u. By integration,

$$(3) \qquad g(u;t) = \lambda(t+u).\ \exp\{-\int_0^u \lambda(t+w)dw\} \qquad u \geq 0.$$

In order to obtain properly defined present values and in order to restrict attention to economically meaningful cases, we impose the following weak conditions concerning the exogenous variables in our model.

1. *Wage offers at time t are drawn randomly from a distribution with a distribution function* $F(w;t)$, *which is a continuous function of* w *and monotonically increasing in* w *on some interval* $\langle\alpha(t), \beta(t)\rangle$ *with* $0 \leq \alpha(t) < \beta(t) \leq \infty$, $F(\alpha(t);t) = 0$ *and* $\lim_{w\to\beta(t)} F(w;t) = 1$. *This holds*

for every $t \geq 0$. *The mean of the distribution is a uniformly bounded function of* t.

2. *For every* $t \geq 0$, $0 < \lambda(t) \leq \Lambda < \infty$ *and* $|b(t)| \leq B < \infty$; Λ *and* B *being fixed numbers.*
3. $F(w;t)$, $\lambda(t)$ *and* $b(t)$ *are continuous functions of* t *on* $[0,\infty\rangle$ *except possibly for a finite number of points. If an exogenous variable is discontinuous in t at some point, say* t^*, *then it is right-continuous, and the left-hand limit of this variable at* t^* *does exist (e.g., in case of b:*

$$\lim_{t \downarrow t^*} b(t) = b(t^*) \text{ and } \lim_{t \uparrow t^*} b(t) \text{ exists)}$$

4. *There exists some number* T *such that all exogenous variables are constant on* $[T,\infty\rangle$.
5. $0 < \rho < \infty$

We allow for negative b, in order to capture the case in which search costs exceed unemployment benefits. Further, from assumptions 2 and 4, we infer that $g(u;t)$ as defined in equation (3) is a properly defined density for every $t \geq 0$.
Note that a model which satisfies assumptions 1-5 allows for quite general patterns of movement of the exogenous variables over time, comprising virtually every nonstationary situation that may arise in practice.

3.2. The optimal path of the reservation wage

Let $R(t)$ denote the expected discounted lifetime income at t, if from t onwards the optimal search strategy is followed. It can be proven that under assumptions 1-5, $R(t)$ is a bounded continuous function of t. We now present a characterization of the time path of the optimal strategy.

Theorem 1

Let assumptions 1-5 be satisfied. Then the optimal strategy of a job searcher at time t *can be characterized by a reservation wage* $\varphi(t)$.

$\varphi(t)$ is a bounded and continuous function of t *and it satisfies the following differential equation for every point in time at which* $b(t)$, $\lambda(t)$ *and* $F(w;t)$ *are continuous in* t.

$$(4) \qquad \varphi'(t) = \rho.\varphi(t) - \rho.b(t) - \lambda(t).Q(\varphi(t);t),$$

where $Q(\varphi(t);t)$ *is defined as*

$$Q(\varphi(t);t) := {}_{\varphi(t)}\!\int^{\infty} \bar{F}(w;t)dw$$

If one or more of the exogenous variables are discontinuous in t *at some point, then the right-hand side of* (4) *gives the right-hand derivative of* φ *with respect to* t *at that point. The left-hand derivative can be calculated by replacing the values of the exogenous variables at* t *in the right-hand side of* (4) *by their left-hand limits at that discontinuity point.*

The proof is given in the appendix. It also contains a discussion of the uniqueness of the reservation wage.

The differential equation (4) is also given by Mortensen (1984). However, in Mortensen's model the exogenous variables are forced to have very simple functional forms; in fact the only departure from stationarity is (in terms of our model) a simultaneous discrete change in λ and b when the unemployment duration equals T time-units. This change is interpreted to be a consequence of liquidity constraints.

Let us now try to get an intuitive feeling for equation (4). First notice that if $\varphi'(t) = 0$, equation (4) reduces to equation (1). This was to be expected as $\varphi'(t) = 0$ is a necessary condition for stationarity. Let $\varphi_0(t)$ be the optimal reservation wage at time t if the environment remains stationary after t, i.e., from equation (1),

$$(5) \qquad \varphi_0(t) = b(t) + \frac{\lambda(t)}{\rho}.Q(\varphi_0(t);t) \qquad t \geq 0$$

We want to compare $\varphi(t)$ and $\varphi_0(t)$. Of course, $\varphi(t) = \varphi_0(t)$ implies $\varphi_R'(t) = 0$. (Right-hand derivatives are indicates by a subscript R.)

Further, from equation (4), for every $\varphi(t)$,

$$(6) \qquad \frac{\partial\ \varphi_R'(t)}{\partial\ \varphi(t)} = \rho + \lambda(t).\bar{F}(\varphi(t);t) = \rho + \vartheta(t) > 0$$

in which $\vartheta(t)$ denotes the rate of escape from unemployment at time t (see subsection 3.4). Consequently, if $\varphi(t) \gtrless \varphi_0(t)$ then $\varphi_R'(t) \gtrless 0$. Let $R_0(t)$ and $R(t)$ denote the value of search at t for the cases in which the optimal reservation wages are given by $\varphi_0(t)$ and $\varphi(t)$, respectively. It is clear that $\varphi_0(t) = \rho.R_0(t)$ and $\varphi(t) = \rho.R(t)$ (see appendix 1). Using these eqations we can show that the relationship between φ and φ_0 that we found above is perfectly plausible. If for example $\varphi(t) > \varphi_0(t)$ then $R(t) > R_0(t)$ which means that there are future changes in the values of the exogenous variables that altogether benefit the value of search $R(t)$ as compared to the "stationary state" value of search $R_0(t)$. As time proceeds, these future changes come nearer. Both because future income is discounted by a positive rate ρ and because the probability of not finding a job before the changes take place (following the optimal strategy) increases as time proceeds, this implies that R will rise at t (compare equation (6)). So the right-hand derivative of R with respect to time at t is positive and consequently $\varphi_R'(t) > 0$. Note that the argument applies to every two possible reservation wages at t, in the sense that it makes clear that given the values of the exogenous variables at t, $\varphi_1(t) > \varphi_2(t)$ implies $\varphi_{1R}'(t) > \varphi_{2R}'(t)$. This is exactly what equation (6) says. In section 4, where we make an additional assumption concerning the exogenous variables, we return to the interrelations between φ , φ' and φ_0.

Theorem 1 can be used in order to determine φ as a function of time. First solve for φ at the point T after which all exogenous variables are constant (this is easily done using equation (1)). $\varphi(t)$ is a continuous function of t. Therefore $\varphi(T)$ serves as an initial condition for the differential equation (4) in the time interval ending at T within which the exogenous variables are continuous. Thus $\varphi(t)$ can be solved for every t in this interval. Backward reduction leads to the solution $\varphi(t)$ for every $t \geq 0$. In most cases solving the differential equation can only be done numerically. Generally not even $Q(\varphi(t);t)$ cannot be expressed in closed form, for instance if wage offers are lognormally distributed.

If we place restrictions on the way that exogenous variables may vary over time, then we can sometimes draw qualitative conclusions concerning the time path of φ. As an example, consider models in which $b(t)$, $\lambda(t)$ and the mean and variance of $F(w;t)$ do not increase as a function of t on the time interval $[0,T\rangle$. Then, from a simple revealed preference argument, it follows that $\varphi(t)$ will never increase on $[0,T]$. Sufficient conditions for a strictly decreasing reservation wage are, however, less simple. In appendix 2 a result is presented.

3.3. Comparative dynamics

In this subsection we examine the consequences for the optimal reservation wage path when replacing some particular time path of an exogenous variable by a different (higher) path. For sake of convenience we will be using the term "reference model" in case every exogenous variable follows the reference path, while the term "alternative model" denotes cases in which one exogenous variable does not follow its reference path while the others do. Variables in the reference model will be labelled with a subscript r. Consider two arbitrary points in time t_1 and t_2, such that $0 \leq t_1 < t_2 \leq \infty$. We consider four different departures from the reference model:

C_1) $\forall t \in [t_1, t_2\rangle$ $\qquad b(t) > b_r(t)$

C_2) $\forall t \in [t_1, t_2\rangle$ $\qquad \lambda(t) > \lambda_r(t)$

C_3) $\forall t \in [t_1, t_2\rangle$ $\qquad F(w;t)$ first order stochastically dominates $F_r(w;t)$, that is, $\forall w \in \langle\alpha_r(t), \beta(t)\rangle$, $\qquad \bar{F}(w;t) > \bar{F}_r(w;t)$.

C_4) $\forall t \in [t_1,t_2\rangle$ $\qquad F(w;t)$ is a mean preserving spread of $F_r(w;t)$, that is, $E(w;t) = E_r(w;t)$ and
$\forall x \in \langle\alpha(t), \beta(t)\rangle$ $\qquad \int_{\alpha(t)}^{x} F(w;t)dw > \int_{\alpha(t)}^{x} F_r(w;t)dw$

It is important to remark that in every case C_i, for every exogenous variable the time paths in the reference model and the alternative model are equivalent outside the interval $[t_1,t_2\rangle$. Notice that changing location and scale of the wage offer distribution are special cases of C_3 and C_4, respectively.

Theorem 2

Consider one of the deviations C_1, C_2, C_3 or C_4 from a reference model. Let the exogenous variables of both the reference model and the alternative model satisfy assumptions 1-5. In addition we assume that in cases C_2, C_3 and C_4 there is a $t_3 \in [0,t_2\rangle$ such that $\forall t \in [t_3,t_2\rangle$ $\varphi_r(t) < \beta(t)$, while in case C_4 also $\forall t \in [t_3,t_2\rangle$ $\varphi_r(t) > \alpha(t)$. Then, as a result,

(i) $\forall t \in [0,t_2\rangle$ $\qquad \varphi(t) > \varphi_r(t)$

(ii) $\forall t \in [t_2,\infty\rangle$ $\qquad \varphi(t) = \varphi_r(t)$

(iii) $\forall t \in [0,t_1\rangle$ $\qquad \varphi'(t) > \varphi_r'(t)$ *if t is a point at which φ and φ_r are differentiable with respect to time. If they are not differentiable at some point $t \in [0,t_1\rangle$ then the inequality still holds in that point for the left- and right-hand derivatives. Further, $\varphi_L'(t_1) > \varphi_{rL}'(t_1)$. (A subscript L denotes left-hand derivatives)*

(iv) $\varphi_L'(t_2) \leq \varphi_{rL}'(t_2)$.

The proof is given in appendix 3. By reversing the reference model and the alternative model, we obtain the results in case of "downward" shifting exogenous variables. Simultaneous occurrance of some C_1, C_2, C_3, C_4 can be examined by sequential application of theorem 2. In theorem 2, the inequality restrictions concerning $\varphi_r(t)$ are imposed only for expositional elegance; they rule out uninteresting cases in which changing exogenous variables do not influence the reservation wage path. Sufficient conditions in terms of the exogenous variables are given in appendix 3. Note that if we take for every $t \geq 0$ that $\alpha(t) = 0$, $\beta(t) = \infty$ (which holds for example in case of lognormally districtured wages) and $b(t) > 0$, then the restrictions are always satisfied.

The intuition behind (i) and (ii) is straightforward. Any future shift in the time path of exogenous variables that benefits the expected discounted lifetime income, induces job searchers to be more selective in their search process. As for the period up to t_1, the shift in exogenous variables after point t_1 becomes more important when going forward in time. This implies that $\varphi(t)$ will shift away from $\varphi_r(t)$ when t comes closer to t_1. However, it is not always true that $\forall t \in \langle t_1,t_2\rangle\ \varphi'(t) < \varphi_r'(t)$, if properly defined. It is easy to find time paths of the exogenous variables in the alternative model that cause $\varphi'(t) > \varphi_r'(t)$ for some $t \in \langle t_1,t_2\rangle$.

Mortensen (1984) gives the signs of the derivatives of the reservation wage with respect to exogenous variables in a stationary model. Those results are in accordance with theorem 2 (take the reference model and the alternative model to be stationary, so $t_1 = 0$, $t_2 = \infty$).

3.4. The unemployment duration distribution

Given our results concerning the time path of the reservation wage, we can construct the unemployment duration distribution in a nonstationary job search model and extend the comparative dynamics analysis using this distribution. Define the hazard $\theta(t)$ of leaving unemployment at time t as

$$(7) \qquad \theta(t) = \lambda(t).\bar{F}(\varphi(t);t)$$

By virtue of assumption 1, $\theta(t)$ is a continuous function of $\varphi(t)$. From theorem 1 and assumption 3 then, $\theta(t)$ is a continuous function of t except for points of time at which $\lambda(t)$ or $F(w;t)$ are discontinuous functions of t.

The unemployment duration density is given by the well known equation

$$(8) \qquad h(t) = \theta(t).\exp\{-{}_0\int^t \theta(u)du\}$$

From the continuity of $\varphi(t)$ and the piecewise continuity of $\bar{F}$ and λ as a function of t and from the boundedness of $\bar{F}$ and λ, it is clear that the integral in equation (7) exists for every $t \geq 0$. For points of time at which $\theta(t)$ is a continuous function of t, h(t) is continuous as well, and vice versa. Note that though h(t) is discontinuous at points where $\lambda(t)$ or

$F(w;t)$ are discontinuous functions of t, the distribution function associated with $h(t)$ is a continuous function of t on the whole interval $[0,\infty\rangle$.

If it exists, the expected unemployment duration can be written as

$$(9) \qquad E(t) = {}_0\int^{\infty} \exp\{-{}_0\int^{t}\vartheta(u)du\}dt$$

From (7) we infer that if for some t $\alpha(t) < \varphi(t) < \beta(t)$, then shifts in net benefits that cause a rise of $\varphi(t)$ will also cause a fall of $\vartheta(t)$. Because of the continuity of $\varphi(t)$ and $F(w;t)$ as functions of t and w respectively, $\vartheta(t)$ will fall in at least a neighbourhood of t. Consequently, we have as a corollary from theorem 2,

Corollary

Let assumptions 1-5 be satisfied. If we raise $b(t)$ *for every* $t \in [t_1,t_2\rangle$ *with* $0 \leq t_1 < t_2 \leq \infty$, *and if there is a point* t_3 *with* $0 \leq t_3 \leq t_2$ *at which* $\alpha(t_3) < \varphi(t_3) < \beta(t_3)$, *then the expected unemployment duration rises, if it exists.*

In appendix 3 sufficient conditions for $\alpha(t_3) < \varphi(t_3) < \beta(t_3)$ are given.

4. Exogenous variables as step functions of time

In the sequel we adopt an additional assumption, namely:

6. $F(w;t)$, $\lambda(t)$ *and* $b(t)$ *are step functions of* t *on* $[0,\infty\rangle$.

For $b(t)$ in particular this is what we often see in practice. Further, every continuously changing exogenous variable can be approximated by a step function.

We can now split the positive real axis on which time is measured into intervals within which the exogenous variables are constant. On such an interval, equation (4) reduces to a constant coefficient differential equation. Moreover, this differential equation has a stationary solution, (i.e. the solution for which $\varphi'(t) = 0$) which is constant on that interval. This solution corresponds to φ_0 as it is defined by equation (5) in a more general setting. In subsection 3.2 we showed that $\varphi_R'(t)$ can be considered to be a monotonically increasing function of $\varphi(t)$. This also holds for $\varphi_L'(t)$. Further, we infer that if in a model that satisfies assumptions 1-6 for some t $\varphi'(t)$ exists, then so does $\varphi''(t)$. By differentiating the constant coefficient differential equation with respect to t we find that $\varphi'(t)$ and $\varphi''(t)$ have equal sign. Thus we have the following information about the shape of $\varphi(t)$ within intervals on which the exogenous variables are constant:

Theorem 3

Let assumptions 1-6 be satisfied. Let the exogenous variables be constant on an interval $[t_*, t^*\rangle$, $0 \leq t_* < t^* \leq \infty$. *Then for every* $t \in \langle t_*, t^*\rangle$ *we have*

$$\varphi(t) \gtrless \varphi_0 \quad \Leftrightarrow \quad \varphi'(t) \gtrless 0 \quad \Leftrightarrow \quad \varphi''(t) \gtrless 0 \quad \Leftrightarrow$$

$$\varphi(t_*) \gtrless \varphi_0 \quad \Leftrightarrow \quad \varphi_R'(t_*) \gtrless 0 \quad \Leftrightarrow \quad \varphi(t^*) \gtrless \varphi_0 \quad \Leftrightarrow \quad \varphi_L'(t^*) \gtrless 0$$

Deviations of $\varphi(t)$ from φ_0 arise because of anticipations of future changes of the values of exogenous variables. As time proceeds, these changes come nearer. Now the rate of discount is positive and the probability of finding a job before the end of the present interval when following the optimal strategy, decreases when t rises. Therefore anticipations become stronger and φ shifts away further from φ_0. As φ is the only variable that changes within the interval, this in turn implies that φ' increases in absolute value, which explains the sign of φ''. (Note the differences and similarities with the analysis of equation (6) in subsection 3.2; in this section $\varphi''(t) = \partial\varphi'(t)/\partial\varphi(t) \cdot \varphi'(t)$ holds.) Note that the sign of $\varphi - \varphi_0$ at the end point of an interval can be thought of as determining the sign of the slope of φ within the interval.

Now suppose there is only one point in time, T, at which exogenous variables are allowed to change values. In addition, suppose that only one exogenous variable changes in value at T, according to one of the following four rules: (if necessary, values of the exogenous variables before and after T will be distinguished by subscripts 1 and 2, respectively)

D_1) $b_1 > b_2$
D_2) $\lambda_1 > \lambda_2$ while $\varphi(T) < \beta$
D_3) F_1 first order stochastically dominates while $\varphi(T) < \beta_1$
D_4) F_1 is a mean preserving spread of F_2 while $\alpha_1 < \varphi(T) < \beta_1$.

Then we have two intervals, $[0,T\rangle$ and $[T,\infty\rangle$ with each its own stationary solution which will be denoted by φ_1 and φ_2, respectively. Whether $\varphi_1 \gtrless \varphi_2$ can be examined by calculating the well;known derivative of φ_1 with respect to the exogenous variable that changes at T. If the derivative is negative, then $\varphi_2 < \varphi_1$ and consequently $\varphi(T) < \varphi_1$, as $\varphi(t) = \varphi_2$ for $t \geq T$. We can then apply theorem 3 in order to obtain the following

<u>Corollary</u>

Let assumptions 1-6 be satisfied. Let T be the only one point in time at which exogenous variables are allowed to change values, according to D_1, D_2, D_3 or D_4. Then $\varphi_1 > \varphi_2$ and
(i) *for every* $t \in [0,T\rangle$ $\varphi_2 < \varphi(t) < \varphi_1$, $\varphi'(t) < 0$, $\varphi''(t) < 0$
(ii) $\varphi(T) = \varphi_2$, $\varphi_L'(T) < 0$, $\varphi_R'(T) = 0$.

Note that a part of this corollary can also be proven using theorem 2. Burdett (1979) and Mortensen (1977) proved that in case D_1, for every $t \in [0,T\rangle$ $\varphi'(t) < 0$, in a model in which time devoted to search is endogenous. Mortensen (1984) also proved that for every $t \in [0,T\rangle$ $\varphi'(t) < 0$ if, in terms of our model, both λ and b decrease at T.

5. <u>Numerical examples</u>

In order to illustrate the results so far, we give some numerical examples. We consider two different models. In both models wage offers are assumed to be distributed uniformly between 1500 and 3000 units of money per month. The first model (I) is nonstationary because of a decrease in benefits from 2400 to 1200 when current unemployment duration equals 24 months. ρ and λ are taken to be 0.01 and 0.1, respectively. In model II both b and λ change when duration equals 24 months: for $t < 24$, $b = 2475$ and $\lambda = 0.5$, while for $t \geq 24$ $b = 750$ and $\lambda = 0.05$. Again, ρ equals 0.01. Both models satisfy assumptions 1-6 and we can apply the results from sections 1-4.

Figures 1 and 3 give the optimal reservation wage paths. The changes in exogenous variables at $t = 24$ cause the job searcher to lower his reservation wage long before he reaches that point. Note that the results are in accordance with the corollary in section 4. Figure 2 shows several duration densities which follow from model I. The solid line represents the density of the job searcher who uses the optimal reservation wage function $\varphi(t)$ as his strategy. The dotted line is the density function of a so-called naive searcher. Such a person does not anticipate changes in the values of exogenous variables at all, so if $t < 24$, his reservation wage equals the stationary solution of the differential equation that holds on $t < 24$, while if $t \geq 24$, it equals the optimal $\varphi(t)$. Therefore his density is discontinuous at $t = 24$. Now if a search model is stationary, then $\varphi(t)$ and $\theta(t)$ are time-invariant and, from equation (8), the duration is distributed exponentially. In order to examine whether the nonstationary model I has a distribution which is radically different from an exponential one, we drew the dashed line in figure 2. This line represents the (exponential) density in a model, in which exogenous variables are set at the average value they have at the moment of getting employed according to the nonstationary model. So in our case we set $b = 2400 \cdot P(t < 24) + 1200 \cdot P(t \geq 24) = 1859$. Note that on the interval $[24,\infty\rangle$ both the density of the naive searcher and the density of the rational searcher have an exponential shape (i.e. a constant hazard). Of course, the naive searcher has an exponential-like density on $[0,24\rangle$ too.

Figure 5 gives duration densities in case of model II. The solid, dashed and dotted lines have the same interpretation as in figure 2. Note that the decrease in λ at $t = 24$ causes the duration density of the job searcher who uses the optimal strategy, to be discontinuous at $t = 24$. This is also shown in figure 4, which gives the hazard rate of the searcher who uses the optimal strategy. As time proceeds, $\varphi(t)$ decreases and

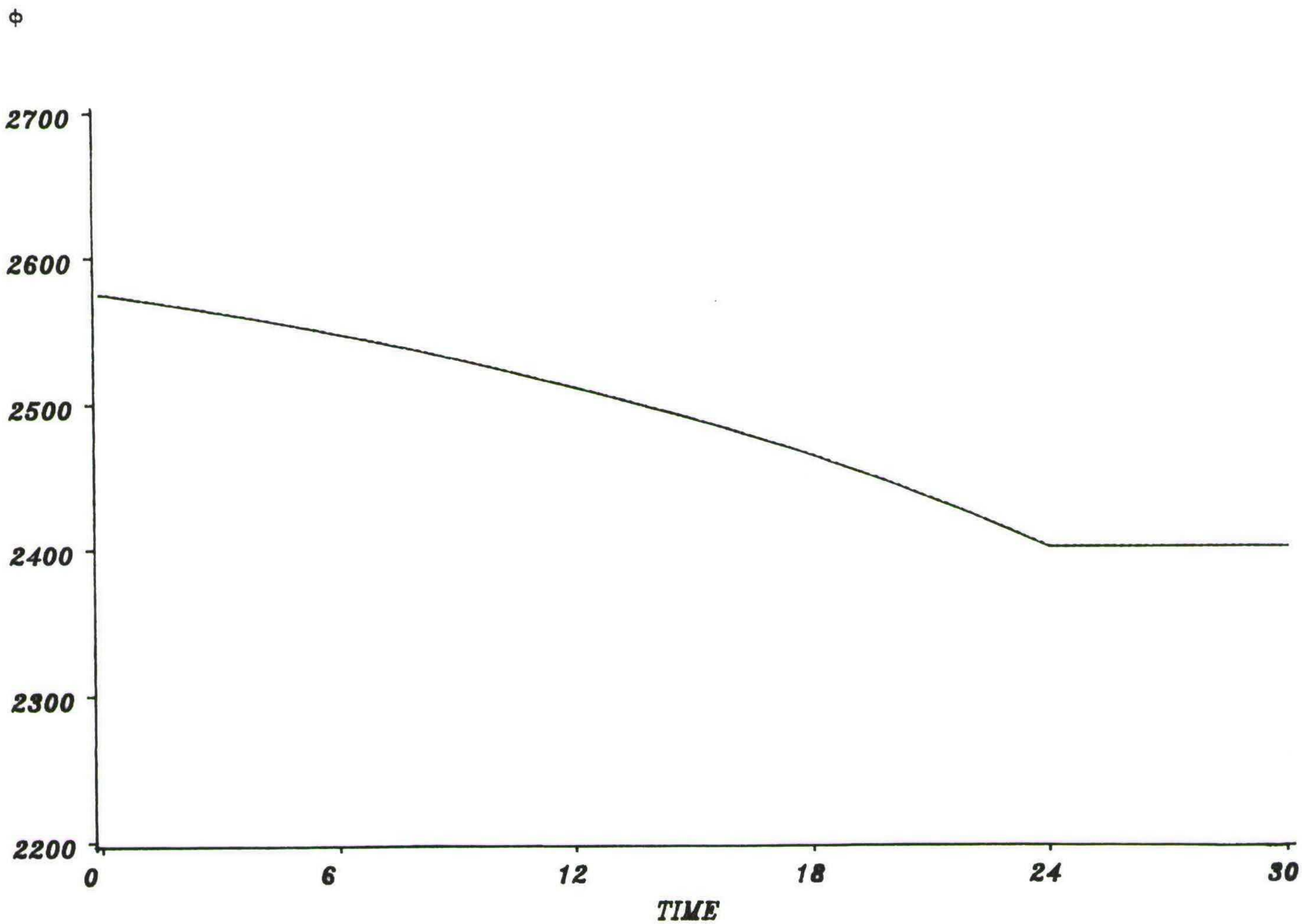

FIG. 1: The reservation wage in model I

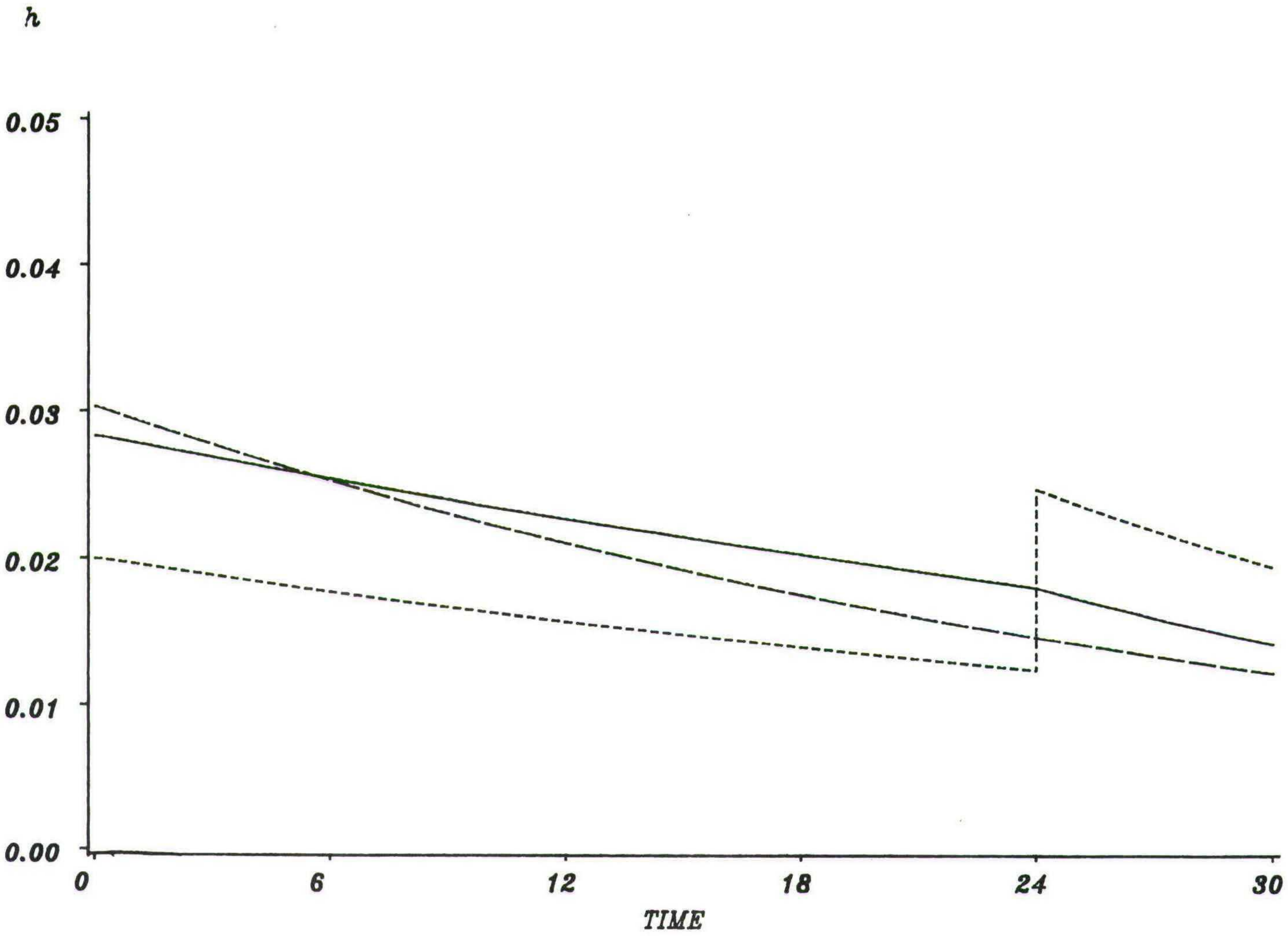

FIG. 2:The duration density in model I

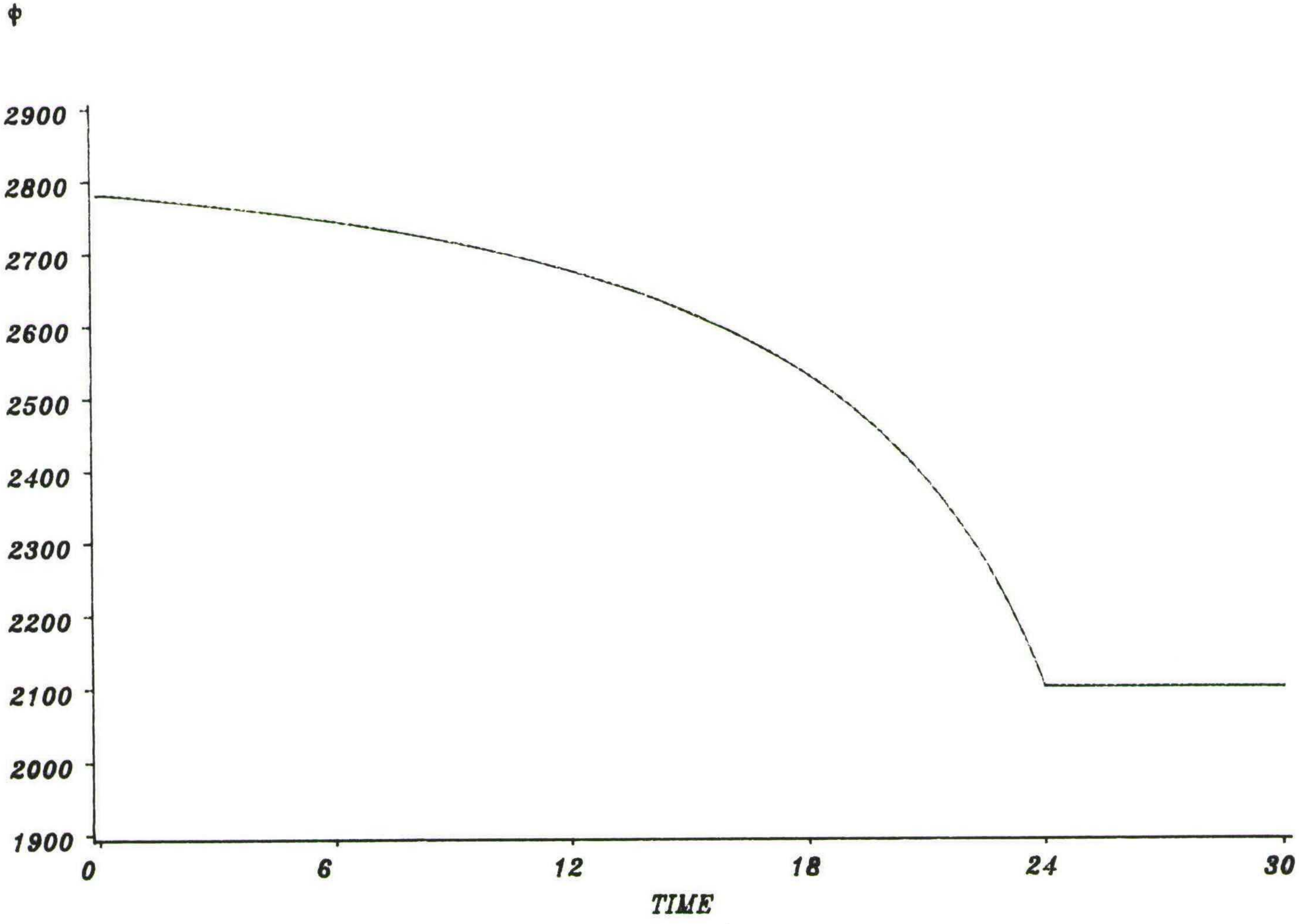

FIG. 3: The reservation wage in model II

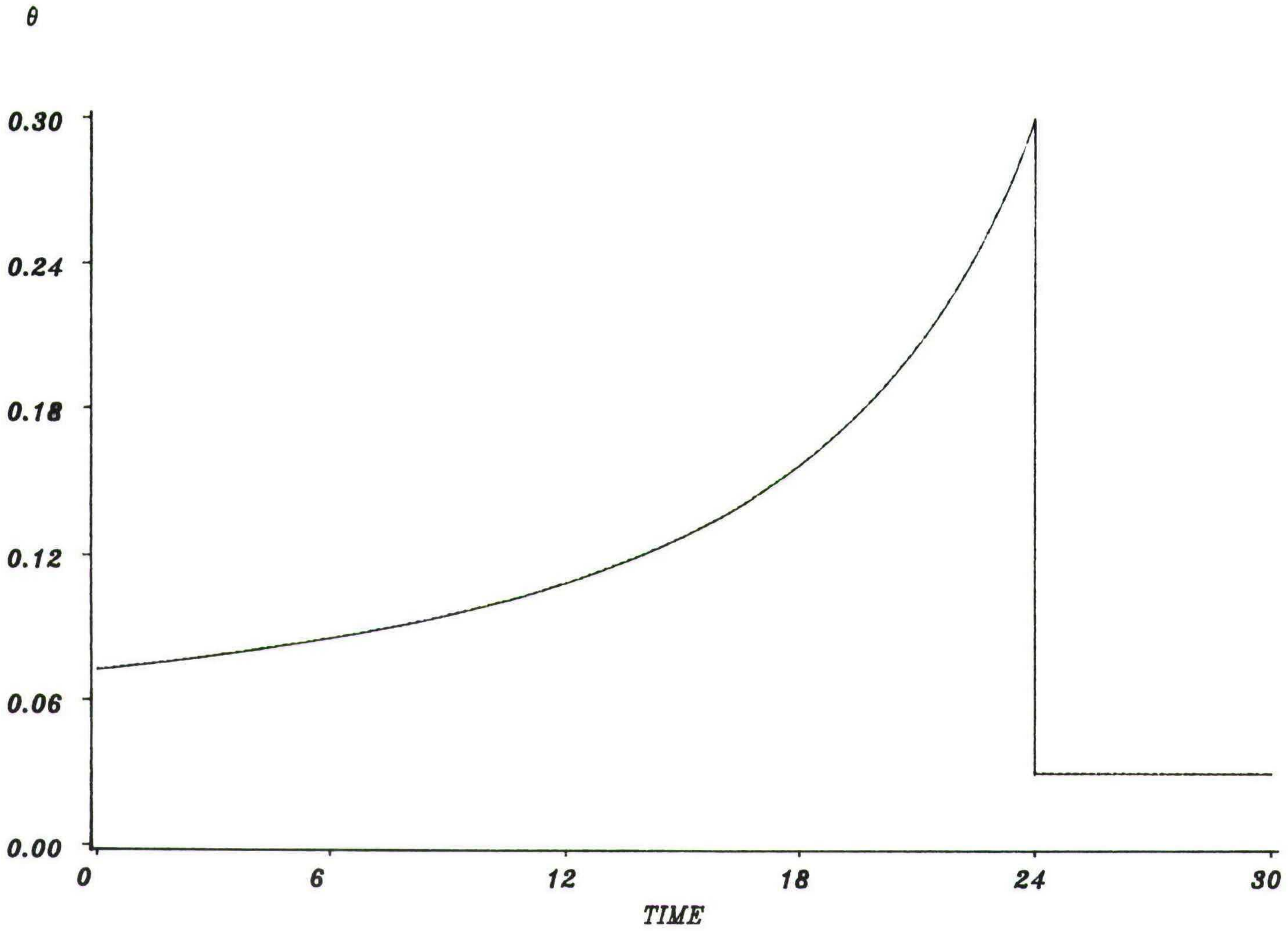

FIG. 4: The hazard in model II

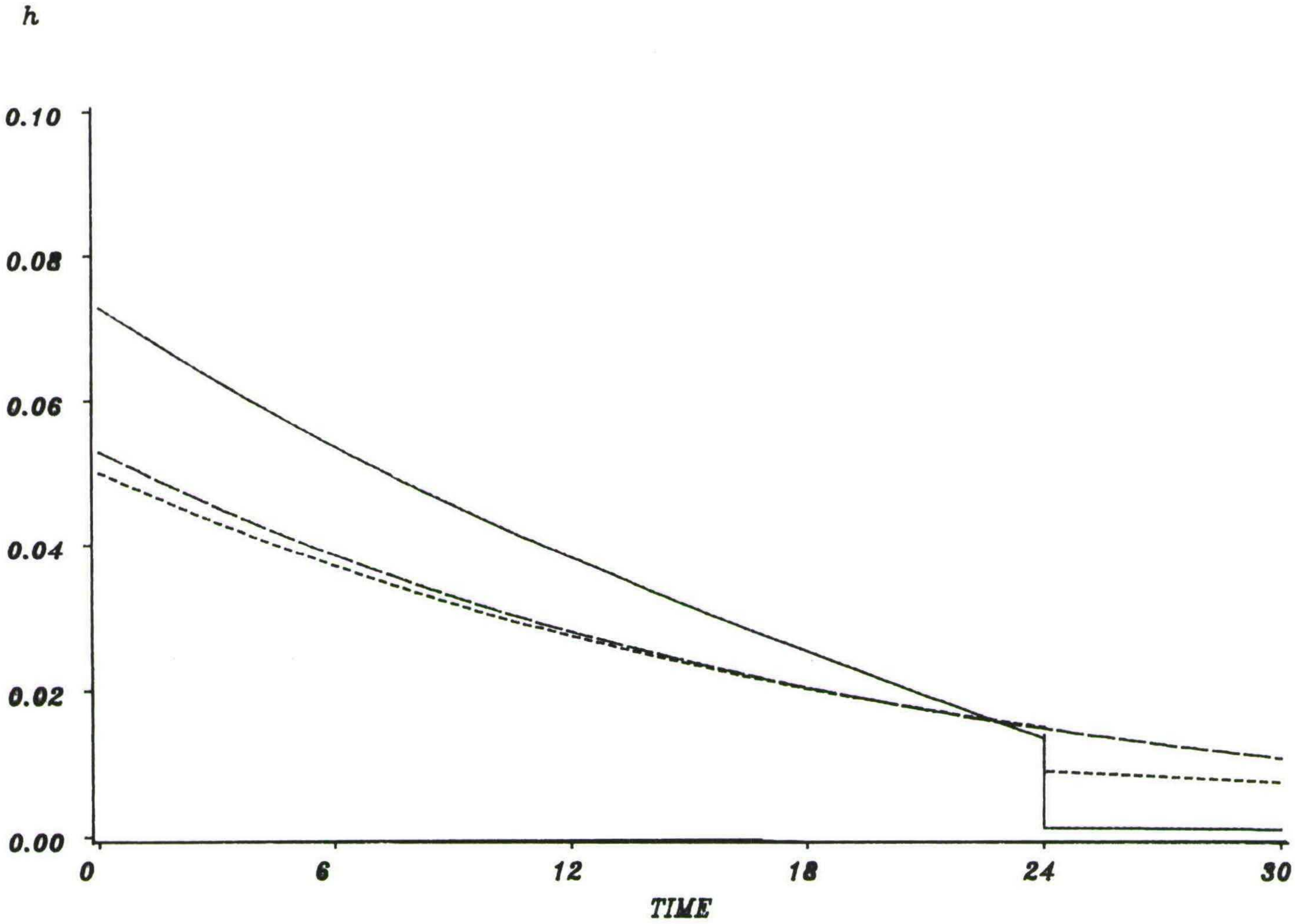

FIG. 5: The duration density in model II

$\vartheta(t)$ increases until $t = 24$. At that point, the value of ϑ drops to one tenth of its left limit at this point, because λ decreases from 0.5 to 0.05 while $\varphi(t)$ is continuous at $t = 24$.

Using these special examples as a guideline, we can try to derive somewhat informally some general properties of $\varphi(t)$ and $h(t)$, in a model in which exogenous variables only change value at $t = 24$. First, if b is the only variable that changes value at $t = 24$, then the duration density will look more like an exponential density than if λ or $F(w)$ change value at $t = 24$. This is because changing λ or $F(w)$ results in a discontinuous density.

Now let us call anticipation (of changes of the values of exogenous variables) strong if $\varphi(t)$ is close to $\varphi(24)$ for even very small t and $\varphi(t)$ is much smaller than the stationary solution on $[0,24\rangle$. (Clearly, then, anticipation is strong if e.g. λ and β are small, $\beta-\alpha$ is large and exogenous variables decrease substantially.) If anticipation is strong or if the exogenous variables do not change substantially at $t = 24$, then $\varphi(t)$ lies close to $\varphi(24)$ for all values of t and consequently $\vartheta(t)$ does not show much variation in time. Therefore $h(t)$ will not differ very much from an exponential density in such cases. If, on the other hand, anticipation is very weak, then of course $h(t)$ will look like the duration density of a naive searcher.

It is clear that assuming stationarity in the model while it is not present in reality gives rise to errors when estimating the model. We saw that sometimes a nonstationary model has a density $h(t)$ that does not differ much from an exponential density, e.g. if b decreases at T while λ and $F(w)$ are constant and anticipation is strong. Even in such cases substantial errors can be made if one assumes stationarity and uses duration data to estimate structural coefficients.

6. Conclusion

In this paper we have examined nonstationarity in job search theory. The optimal reservation wage path over time has been derived under some weak assumptions concerning the exogenous variables. We also have given comparative dynamics results. Furthermore, by assuming the exogenous variables to be step functions of time we were able to derive additional properties of the reservation wage path. Generally these properties are in accordance with economic intuition. In section 5 we considered somewhat informally the restrictiveness of the stationarity assumption.

The results of this paper can be used for estimating stuctural nonstationarity job search models, in order to make inferences about search behaviour. For such purposes, data on durations and post-unemployment wages, and (interview) data on reservation wages can be used. As for the exogenous variables, b can be taken to represent observed official benefits. It is well known that in the estimation of structural job search models, one easily runs into identification problems (see Flinn & Heckman (1982) and Ridder & Gorter (1986)). Generally it is necessary to have a parametrized $F(w)$ and to have λ or the parameters of $F(w)$ be dependent on some observables.

Once a nonstationary structural job search model is estimated, it can be used for policy analysis. By means of simulations, the consequences of alternative unemployment benefit policies on search behaviour can be examined.

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Appendix

1. Proof of theorem 1

Consider a moment t at which an offer is pending. Let $I(t)$ denote the expected present value of income when following the optimal strategy at t. An acceptance policy at time t can be characterized by $p(w;t)$, $0 \leq p \leq 1$, giving the probability that a wage offer w will be accepted. We define $R(t)$ to be the return of the optimal strategy, if the offer at t is rejected.

$$\text{(A1)} \qquad I(t) = \sup_{p(.;t)} {}_0\!\int^{\infty} [p(w;t)\ \frac{w}{\rho} + (1-p(w;t)).R(t)]dF(w;t)$$

Assumption 1 guarantees the existence of the integral in (A1). From (A1) it follows that the optimal acceptance policy $p^*(.;t)$ is given by

$$\text{(A2)} \qquad p^*(w;t) = 1 \qquad \text{if } w \geq \rho.R(t)$$

$$p^*(w;t) = 0 \qquad \text{otherwise}$$

thus (A1) can be written as

$$\text{(A3)} \qquad I(t) = R(t) + \frac{1}{\rho}.Q(\rho R(t);t)$$

From assumptions 2, 3, 4 and 5 and from the boundedness and continuity of $R(t)$ as a function of t, $R(t)$ is properly defined by

$$\text{(A4)} \qquad R(t) = {}_0\!\int^{\infty} g(u;t)[{}_0\!\int^{u} b(t+s)e^{-\rho s}ds + e^{-\rho u}I(t+u)]du$$

The optimal policy (A2) can be characterized by a reservation wage $\varphi(t)$, which is continuous in t and bounded,

$$\text{(A5)} \qquad \varphi(t) = \rho.R(t)$$

Substitution of (A3) and (A5) in (A4) gives

$$\varphi(t) = {}_0\int^{\infty} g(u;t)[\rho \cdot {}_0\int^{u} b(t+s)e^{-\rho s}ds$$

$$+ e^{-\rho u}\{\varphi(t+u) + Q(\varphi(t+u);\ t+u)\}]du$$

Using (3), this can be transformed into

$$\text{(A6)} \qquad \varphi(t) = \exp\{{}_0\int^{t}\lambda(v)dv + \rho t\} \cdot {}_t\int^{\infty} g(y;0)[\rho \cdot {}_0\int^{y} b(v)e^{-\rho v}dv$$

$$+ e^{-\rho y}(Q(\varphi(y);y) + \varphi(y))]dy$$

$$- \rho e^{\rho t} \cdot {}_0\int^{t} b(v)e^{-\rho v}dv$$

If we differentiate (A6) with respect to t we obtain

$$\text{(A7)} \qquad \varphi'(t) = \rho.\varphi(t) - \rho b(t) - \lambda(t).Q(\varphi(t);t)$$

As can be seen from (A6), differentiation is only allowed in points at which $\lambda(t)$, $b(t)$ and $F(w;t)$ are continuous in t. However, because these variables are always right-continuous in t, the right-hand side of (A7) gives the right-hand derivative of $\varphi(t)$ with respect to t at points at which exogenous variables are discontinuous. Similarly, because the left-hand limits of these variables exist, the left-hand derivative of $\varphi(t)$ with respect to t at such discontinuity points is defined by

$$\text{(A8)} \qquad \varphi_L'(t) = \rho.\varphi(t) - \rho.\lim_{\tau\uparrow t} b(\tau) - \lim_{\tau\uparrow t} \lambda(\tau).\lim_{\tau\uparrow t} Q(\varphi(t);\tau)$$

This completes the proof of theorem 1.

It can be proven that the results in theorem 1 remain valid if we replace $\lambda(t) > 0$ in assumption 2 by $\lambda(t) \geq 0$, that is, if we allow for situations that people remain unemployed forever in the sense that ${}_0\int^{\infty} g(u;t)du < 1$.

If $\varphi(t)$ (as defined by (A5)) does not lie within the upper and lower bound of the interval on which the wage offer density is positive, then there are generally many other reservation wages that describe the optimal policy at t. Still, the reservation wage as defined by (A5) can be used any time to describe optimal behaviour of job searchers during their spell of unemployment.

2. Strictly decreasing exogenous variables

We consider models in which one exogenous variables is time dependent in a way that is described by one of the following four cases, while the others are constant on the interval $[0,\infty\rangle$

K_1) $\forall t \in [0,T\rangle$ $\forall\tau > 0$ $\quad b(t) > b(t+\tau)$

K_2) $\forall t \in [0,T\rangle$ $\forall\tau > 0$ $\quad \lambda(t) > \lambda(t+\tau)$

K_3) $\forall t \in [0,T\rangle$ $\forall\tau > 0$ $\quad F(w;t)$ first order stochastically dominates $F(w;t+\tau)$, that is, $\forall w \in \langle\alpha(t+\tau), \beta(t)\rangle$ $\quad \bar{F}(w;t) > \bar{F}(w;t+\tau)$

K_4) $\forall t \in [0,T\rangle$ $\forall\tau > 0$ $\quad F(w;t)$ is a mean preserving spread of $F(w;t+\tau)$, that is, $E(w;t) = E(w;t+\tau)$ and

$$\forall x \in \langle\alpha(t), \beta(t)\rangle \quad \int_{\alpha(t)}^{x} F(w;t)dw > \int_{\alpha(t)}^{x} F(w;t+\tau)dw$$

Note that in all cases we allow the exogenous variable to be discontinuous in a finite number of points. In order to rule out uninteresting situations in which decreasing exogenous variables do not make the reservation wage time dependent, we impose some restrictions on $\varphi(t)$. In cases K_2 and K_3 we impose for every $t \in [0,T\rangle$ that $\varphi(t) < \beta(t)$, while in case K_4 for every $t \in [0,T\rangle$ $\quad \alpha(t) < \varphi(t) < \beta(t)$ has to hold. Let $f_L(a)$ denote the left-hand limit of $f(x)$ at $x=a$, if it exists. The restrictions can be characterized by the following restrictions on the exogenous variables.

K_2) $b < \beta$

K_3) $b < \beta_L(T)$

K_4) $\alpha_L(T).(1+\frac{\lambda}{\rho}) - \frac{\lambda}{\rho}.E(w;T) < b < \beta_L(T)$.

Theorem A1

Let assumptions 1-5 be satisfied. Let in addition one exogenous variable be time dependent according to K_1, K_2, K_3 or K_4 while the others are constant on the time interval $[0,\infty\rangle$. Then

(i) $\forall t \in [0,T\rangle \quad \varphi(t) < \varphi_0(t)$

(ii) $\forall t \in [0,T\rangle \quad \varphi'(t) < 0$ *if this derivative exists. At points* t *where* $\varphi'(t)$ *does not exist (i.e. points at which one of the exogenous variables is discontinuous), both* $\varphi'_L(t) < 0$ *and* $\varphi'_R(t) < 0$ *hold. If an exogenous variable is discontinuous at T, then* $\varphi'_L(T) < 0$, *otherwise* $\varphi'(T) = 0$.

Clearly, these results make economic sense. Any future decrease in b, λ or the mean or variance of F will make the value of search in the present smaller than it would have been if the exogenous variables were constants. From the discussion of equations (4), (5) and (6) in subsection 3.2, this means that $\varphi(t) < \varphi_0(t)$ for every $t \in [0,T\rangle$ and that φ decreases as lower values of the exogenous variables come nearer.

<u>Proof of theorem A1</u>

The structure of the proof is as follows. First we restrict attention to an unspecified time interval within which the exogenous variables are continuous. In lemma A1 we show that sufficient for (i) and (ii) to hold in the interval is that, loosely speaking, $\varphi_0(t)$ is strictly decreasing within that interval. The remainder of the proof is concerned with finding conditions that impose the required property to $\varphi_0(t)$ for every interval, using backward induction.

We split the time axis into a finite number of intervals, within which every exogenous variable is continuous in time. The intervals are closed to the left side and open to the right. The last interval is $[T,\infty\rangle$. Now consider one such interval, say $[t_*,t^*\rangle$. From theorem 1, φ is a differentiable function of t and φ_0 is a continuous function of t, on $[t_*,t^*\rangle$. Further, $\varphi_L(t^*) = \varphi(t^*)$ but it may be that $\varphi'_L(t^*) \neq \varphi'_R(t^*)$ or $\varphi_{0L}(t^*) \neq \varphi_0(t^*)$.

<u>Lemma A1</u>

Let assumptions 1-5 be satisfied. Consider the interval $[t_*,t^*\rangle$ as defined before. If $\varphi(t^*) \leq \varphi_{0L}(t^*)$ and if

(A9) $\forall t \in [t_*, t^*\rangle\ \forall \tau \in \langle 0, t^* - t\rangle \varphi_0(t+\tau) < \varphi_0(t)$

then $\forall t \in [t_*, t^*\rangle$ $\varphi(t) < \varphi_0(t)$, $\forall t \in \langle t_*, t^*\rangle$ $\varphi'(t) < 0$; $\varphi_R'(t_*) < 0$ and if $\varphi_{0L}(t^*) > \varphi(t^*)$ then $\varphi_L'(t^*) < 0$ while if $\varphi_{0L}(t^*) = \varphi(t^*)$ then $\varphi_L'(t^*) = 0$.

<u>Proof of lemma A1</u>

Suppose that at some $t \in [t_*, t^*\rangle$ $\varphi_0(t) \leq \varphi(t)$ holds. Then, from the discussion of equations (5) and (6) in subsection 3.2, $\varphi'(t) \geq 0$ if $t > t_*$, while $\varphi_R'(t) \geq 0$ if $t = t_*$. On the other hand, $\varphi(t^*) \leq \varphi_{0L}(t^*)$. φ and φ_0 are continuous functions of t on $[t_*, t^*\rangle$ and φ_0 is decreasing in t. Therefore $\varphi_0(t) \leq \varphi(t)$ cannot hold for any $t \in [t_*, t^*\rangle$. If $\varphi_0(t) > \varphi(t)$ for every $t \in [t_*, t^*\rangle$ then, again from subsection 3.2, $\varphi'(t) < 0$ for every $t \in \langle t_*, t^*\rangle$ and $\varphi_R'(t_*) < 0$. Furthermore, if $\varphi_{0L}(t^*) > \varphi(t^*)$ then $\varphi_L'(t^*) < 0$ while if $\varphi_{0L}(t^*) = \varphi(t^*)$ then $\varphi_L'(t^*) = 0$. This completes the proof of lemma A1.

Basically, we now only have to prove that φ_0 is decreasing in t. Consider case K_2. For every $t \geq T$ $\varphi(t) = \varphi_0(t)$ holds, due to the stationarity after T. If λ is discontinuous at T, then $\lambda_L(T) > \lambda(T)$. Because $b < \beta$ holds, we have for every $t \geq 0$ that $\varphi_0(t) < \beta$ holds (see equation (5)). Consequently, $Q(\varphi_0(t) > 0$ and therefore $\lambda_L(T) > \lambda(T)$ implies $\varphi_{0L}(T) > \varphi_0(T)$, as can be seen from equation (5). If $\lambda_L(T) = \lambda(T)$, then $\varphi_{0L} = \varphi_0(T)$. So in any case $\varphi(T) \leq \varphi_{0L}(T)$. Now consider the interval $[t_*, t^*\rangle$ with $t^* = T$. Take a $t \in [t_*, t^*\rangle$ and a $\tau > 0$. Then, because b and F(w) are constant in case K_2,

$$\text{(A10)} \quad \varphi_0(t+\tau) - \varphi_0(t) = \frac{\lambda(t+\tau)}{\rho}\{Q(\varphi_0(t+\tau)) - Q(\varphi_0(t))\}$$

$$+ \frac{1}{\rho}.Q(\varphi_0(t))\ \{\lambda(t+\tau) - \lambda(t)\}$$

Again, $Q(\varphi_0(t)) > 0$. Further, $\lambda(t+\tau) < \lambda(t)$. Inspection of (A10) shows that therefore $\varphi_0(t+\tau) \geq \varphi_0(t)$ cannot hold. Because this is true for every $t \in [t_*, t^*\rangle$ and for every $\tau > 0$, we infer that (A9) holds for the interval ending at T.

So the conditions of lemma A1 are satisfied and we can apply it, noting that $\varphi_{0L}(t^*) > \varphi_0(t^*)$ if $\lambda_L(t^*) > \lambda(t^*)$ while $\varphi_{0L}(t^*) = \varphi_0(t^*)$ if λ is continuous at t^*. In the latter case $\varphi_L'(t^*) = 0$ of course implies $\varphi'(T) = 0$.

As for the interval $[u,t_*\rangle$ before $[t_*,t^*\rangle$ with $t^* = T$, we can go through the same lines of argument. We have seen that $\varphi(t_*) < \varphi_0(t_*)$. Again, λ may be discontinuous at t_*. In that case it follows that $\varphi_{0L}(t_*) > \varphi_0(t_*)$. So $\varphi(t_*) < \varphi_{0L}(t_*)$ holds in any case. Furhter, φ_0 decreases in t on $[u,t_*\rangle$ and lemma A1 can be applied again. Going backwards in time, one thus obtains theorem A1 for case K_2. Proof of the other cases are analogous.

Lippman & McCall (1976b) consider a generalization of case K_3, for which they derive a result similar to theorem A1 in a discrete-time model with the property that in every period exactly one job offer arrives.

3. Proof of theorem 2

We split the time axis into a finite number of intervals, within which all exogenous variables from both models are continuous functions of time. The intervals are closed to the left and open to the right. We let t_1 and t_3 be left-hand bounds of an interval and we let t_2 be the right-hand bound of an interval. Now consider one of the intervals, say, $[t_*,t^*\rangle$. From theorem 1, φ and φ_r are differentiable functions of t on $[t_*,t^*\rangle$. Further, φ and φ_r are continuous at t_* and t^* but they may not be differentiable at those points.

We outline the proof of case C_2. Just like the proof of theorem A1, we work backward in time. First, suppose $t_2 < \infty$. For every $t \geq t_2$ $\varphi(t) = \varphi_r(t)$ holds, due to the equivalence of the exogenous variables of both models on $[t_2,\infty\rangle$. Consider the interval $[u,t_2\rangle$. (By definition $t_3 \leq u$.) From equation (4), we have for every $t \in \langle u,t_2\rangle$

$$\text{(A11)}\quad \varphi'(t) - \varphi_r'(t) = \rho(\varphi(t) - \varphi_r(t)) - \lambda(t)\{Q(\varphi(t);t) - Q(\varphi_r(t);t)\}$$
$$+ \{\lambda_r(t) - \lambda(t)\}.Q(\varphi_r(t);t)$$

If $t = u$, we replace $\varphi'(t) - \varphi_r'(t)$ by $\varphi_R'(u) - \varphi_{rR}'(u)$. As for every $t \in [u,t_2\rangle$ $\varphi_r(t) < \beta(t)$ holds, we have $Q(\varphi_r(t);t) > 0$ on $[u,t_2\rangle$. So if there is a $t \in \langle u,t_2\rangle$ at which $\varphi(t) \leq \varphi_r(t)$ then it follows from (A11) that $\varphi'(t) < \varphi_r'(t)$. Also, if $\varphi(u) \leq \varphi_r(u)$ then $\varphi_R'(u) < \varphi_{rR}'(u)$. But $\varphi(t_2) = \varphi_r(t_2)$ and φ and φ_r are continuous functions of t. Therefore for every $t \in [u,t_2\rangle$ $\varphi(t) > \varphi_r(t)$ has to hold. Further, according to theorem 1,

$$\text{(A12)} \quad \varphi_L'(t_2) - \varphi_{rL}'(t_2) = \{\lambda_{rL}(t_2) - \lambda_L(t_2)\}.Q_L(\varphi_r(t_2);t_2)$$

which is nonpositive.

Now consider the interval $[y,u\rangle$. We just derived that $\varphi(u) > \varphi_r(u)$. Going through the same line of argument, if follows that for every $t \in [y,u\rangle$ $\varphi(t) > \varphi_r(t)$. Whether $t_3 = u$ or $t_3 \leq y$ does not matter for this result. We can proceed this way until we arrive at the interval of which t_1 is the right-hand bound, say $[v,t_1\rangle$. We now have for every $t \in \langle v,t_1\rangle$

$$\text{(A13)} \quad \varphi'(t) - \varphi_r'(t) = \rho(\varphi(t) - \varphi_r(t)) - \lambda(t)\ \{Q(\varphi(t);t) - Q(\varphi_r(t);t)\}$$

For $t = v$ we have to replace $\varphi'(t) - \varphi_r'(t)$ by $\varphi_R'(v) - \varphi_{rR}'(v)$. If there is a $t \in \langle v,t_1\rangle$ at which $\varphi(t) \leq \varphi_r(t)$ holds, then it follows from (A13) that $\varphi'(t) \leq \varphi_r'(t)$, regardless of $t \lessgtr t_3$. Similarly, $\varphi(v) \leq \varphi_r(v)$ implies $\varphi_R'(v) \leq \varphi_{rR}'(v)$. But $\varphi(t_1) > \varphi_r(t_1)$ and φ and φ_r are continuous functions of t. Therefore for every $t \in [v,t_1\rangle$ $\varphi(t) > \varphi_r(t)$ has to hold. Further,

$$\text{(A14)} \quad \varphi_L'(t_1) - \varphi_{rL}'(t_1) = \rho(\varphi(t_1) - \varphi_r(t_1)) - \lambda_L(t_1)\ \{Q_L(\varphi(t_1);t_1) - Q_L(\varphi_r(t_1);t_1)\}$$

which is positive. Also, from (A13) it follows that for every $t \in \langle v,t_1\rangle$ $\varphi(t) > \varphi_r(t)$ implies that $\varphi'(t) > \varphi_r'(t)$ while $\varphi(v) > \varphi_r(v)$ implies that $\varphi'_R(v) > \varphi_{rR}'(v)$. Backward induction leads to the results for $t < v$.

If $t_2 = \infty$ we first examine the interval $[T,\infty\rangle$ on which the exogenous variables are constant. Because $T \geq t_3$ we have $Q(\varphi_r(t);t) > 0$ on this interval. Therefore increasing λ in this interval induces an increasing reservation wage. Now we can go through the same lines of argument as before concerning the intervals that lie to the left of T. This completes the proof in case C_2. Proofs of the other cases are analogous.

We now give sufficient conditions for the inequality restrictions on $\varphi_r(t)$ on the interval $[t_3,t_2\rangle$. Without loss of generality we take $t_3 \geq t_1$. Suppose that for every $t \geq t_3$ it holds that $b_r(t) < \beta_r(t)$, while $\beta_r(t)$ does not increase as a function of t on $[t_3,\infty\rangle$. Using theorem 1, we can then prove that as a result $\varphi_r(t) < \beta_r(t)$ for every $t \in [t_3,t_2\rangle$. In case C_2 $\beta_r(t) \equiv \beta(t)$ while in cases C_3 and C_4 $\beta_r(t) \leq \beta(t)$ on $[t_1,t_2\rangle$. Further, in all three cases $b_r(t) \equiv b(t)$. This gives the sufficient condition for $\varphi_r(t) < \beta(t)$ on $[t_3,t_2\rangle$. Analogously, we can prove that in case C_4 sufficient for $\varphi_r(t) > \alpha(t)$ on $[t_3,t_2\rangle$ is, that $\alpha_r(t)$ does not decrease on $[t_3,\infty\rangle$ and that for every $t \geq t_3$

$$b(t) > \alpha_r(t) - \frac{\lambda(t)}{\rho} \{E(w;t) - \alpha_r(t)\}$$

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