

Tilburg University

Block-tridiagonal linear systems and branched continued fractions

Cuyt, A.; Verdonk, B.

Publication date:
1987

Document Version
Publisher's PDF, also known as Version of record

[Link to publication in Tilburg University Research Portal](#)

Citation for published version (APA):
Cuyt, A., & Verdonk, B. (1987). *Block-tridiagonal linear systems and branched continued fractions*. (Research Memorandum FEW). Faculteit der Economische Wetenschappen.

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

CBM

R

1987 243
7626

1987

243



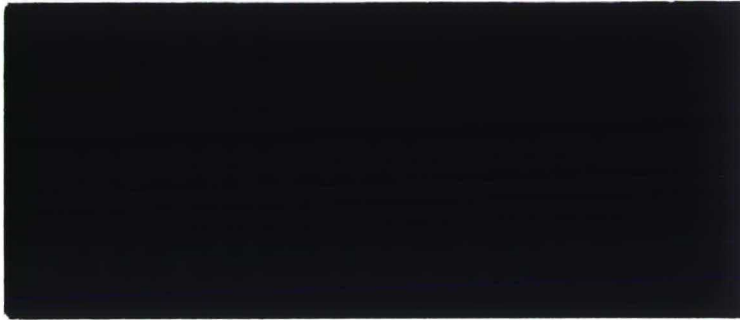
UNIVERSITY

LEIEN
UNIVERSITEIT
BRABANT

POSTBOX 90153
5000 LE TILBURG
THE NETHERLANDS



* C I N O O 3 4 5 *



DEPARTMENT OF ECONOMICS
RESEARCH MEMORANDUM



 K.U.B.
BIBLIOTHEEK
TILBURG

**Block-tridiagonal linear systems and
branched continued fractions**

Annie Cuyt

Brigitte Verdonk

FEW 243

**Block-tridiagonal linear systems
and branched continued fractions.**

Annie Cuyt — Brigitte Verdonk***

* Department of econometrics
University of Tilburg
Postbus 90153
NL-5000 LE Tilburg
The Netherlands

◦ Senior Research Assistant
NFWO

• Department of mathematics and computer science
Universiteit Antwerpen (UIA)
Universiteitsplein 1
B-2610 Wilrijk
Belgium

Abstract.

The convergent of an ordinary continued fraction can be computed by solving a tri-diagonal linear system for its first unknown. In this paper this approach is generalized to branched continued fractions and it is shown how the convergent of a branched continued fraction can be considered as the first unknown of a block-tridiagonal linear system. Hence algorithms for the solution of such systems of equations can be used for the computation of convergents of branched continued fractions, which have applications in approximation theory, systems theory, . . . In future research special attention will be paid to the use of parallel algorithms.

Block-tridiagonal linear systems and branched continued fractions.

In the case of ordinary continued fractions

$$B_i = b_0^{(i)} + \frac{a_1^{(i)}}{b_1^{(i)}} + \frac{a_2^{(i)}}{b_2^{(i)}} + \dots \quad i = 0, 1, 2, \dots \quad (1)$$

forward evaluation of and determinant formulas for

$$C_n^{(i)} = b_0^{(i)} + \sum_{j=1}^n \frac{a_j^{(i)}}{b_j^{(i)}}$$

are well-known. If we denote $C_n^{(i)} = P_n^{(i)}/Q_n^{(i)}$ then $P_n^{(i)}$ and $Q_n^{(i)}$ can be computed by the following three-term recurrence relation [5]

$$\begin{cases} P_k^{(i)} = b_k^{(i)} P_{k-1}^{(i)} + a_k^{(i)} P_{k-2}^{(i)} \\ Q_k^{(i)} = b_k^{(i)} Q_{k-1}^{(i)} + a_k^{(i)} Q_{k-2}^{(i)} \end{cases} \quad k = 1, \dots, n \quad (2)$$

with $P_{-1}^{(i)} = 1 = Q_0^{(i)}$, $P_0^{(i)} = b_0^{(i)}$ and $Q_{-1}^{(i)} = 0$. Using this three-term recurrence relation one can prove that $P_n^{(i)}$ and $Q_n^{(i)}$ are also given by the following determinant formulas [4]

$$P_n^{(i)} = \begin{vmatrix} b_0^{(i)} & -1 & & & & \\ a_1^{(i)} & b_1^{(i)} & -1 & & & \\ & a_2^{(i)} & \ddots & \ddots & & \\ & & \ddots & & -1 & \\ & & & & & a_n^{(i)} & b_n^{(i)} \end{vmatrix} \quad Q_n^{(i)} = \begin{vmatrix} b_1^{(i)} & -1 & & & & \\ a_2^{(i)} & b_2^{(i)} & -1 & & & \\ & a_3^{(i)} & \ddots & \ddots & & \\ & & \ddots & & -1 & \\ & & & & & a_n^{(i)} & b_n^{(i)} \end{vmatrix} \quad (3)$$

and hence that, if $Q_n^{(i)} \neq 0$, $C_n^{(i)} = b_0^{(i)} + x_1^{(i)}$ where $x_1^{(i)}$ is the first unknown of the tridiagonal system

$$\begin{pmatrix} b_1^{(i)} & -1 & & & & \\ a_2^{(i)} & b_2^{(i)} & -1 & & & \\ & a_3^{(i)} & \ddots & \ddots & & \\ & & \ddots & & -1 & \\ & & & & & a_n^{(i)} & b_n^{(i)} \end{pmatrix} \begin{pmatrix} x_1^{(i)} \\ \vdots \\ x_n^{(i)} \end{pmatrix} = \begin{pmatrix} a_1^{(i)} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad (4)$$

Let us now generalize (3) and (4) for branched continued fractions [3, 6]

$$B_0 + \left| \frac{a_1}{B_1} \right| + \left| \frac{a_2}{B_2} \right| + \dots \quad (5)$$

where each of the B_i is an ordinary continued fraction as in (1). A convergent of (3) is denoted by

$$C_{n,m_0,\dots,m_n} = C_{m_0}^{(0)} + \sum_{j=1}^n \left| \frac{a_j}{C_{m_j}^{(j)}} \right| \quad (6)$$

where

$$C_{m_j}^{(j)} = b_0^{(j)} + \sum_{k=1}^{m_j} \left| \frac{a_k^{(j)}}{b_k^{(j)}} \right|$$

If we denote C_{n,m_0,\dots,m_n} as $P_{n,m_0,\dots,m_n}/Q_{n,m_0,\dots,m_n}$ then clearly P_{n,m_0,\dots,m_n} and Q_{n,m_0,\dots,m_n} can be computed by applying the three-term recurrence relation (2) to the expression (6) :

$$\begin{cases} P_{k,m_0,\dots,m_k} = C_{m_k}^{(k)} P_{k-1,m_0,\dots,m_{k-1}} + a_k P_{k-2,m_0,\dots,m_{k-2}} & k = 1, \dots, n \\ Q_{k,m_0,\dots,m_k} = C_{m_k}^{(k)} Q_{k-1,m_0,\dots,m_{k-1}} + a_k Q_{k-2,m_0,\dots,m_{k-2}} \end{cases} \quad (7)$$

with $P_{-1} = 1 = Q_{0,m_0}$, $P_{0,m_0} = C_{m_0}^{(0)}$ and $Q_{-1} = 0$. As an immediate consequence

$$P_{n,m_0,\dots,m_n} = \begin{vmatrix} C_{m_0}^{(0)} & -1 & & & \\ a_1 & C_{m_1}^{(1)} & -1 & & \\ & a_2 & \ddots & \ddots & \\ & & \ddots & & -1 \\ & & & a_n & C_{m_n}^{(n)} \end{vmatrix}$$

$$Q_{n,m_0,\dots,m_n} = \begin{vmatrix} C_{m_1}^{(1)} & -1 & & & \\ a_2 & C_{m_2}^{(2)} & -1 & & \\ & a_3 & \ddots & \ddots & \\ & & \ddots & & -1 \\ & & & a_n & C_{m_n}^{(n)} \end{vmatrix}$$

and $C_{n,m_0,\dots,m_n} = C_{m_0}^{(0)} + x_1$ where x_1 is the first unknown of the tridiagonal system

$$\begin{pmatrix} C_{m_1}^{(1)} & -1 & & & \\ a_2 & C_{m_2}^{(2)} & -1 & & \\ & a_3 & \ddots & \ddots & \\ & & \ddots & & -1 \\ & & & a_n & C_{m_n}^{(n)} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} a_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Note that in the coefficient matrix of this linear system each $C_{m_i}^{(i)}$ is itself a quotient of determinants. We shall prove in the next theorem that C_{n, m_0, \dots, m_n} is also the first unknown of a block-tridiagonal linear system where now the partial numerators and denominators $a_j^{(i)}$ and $b_j^{(i)}$ for $j = 0, \dots, m_i$ and $i = 0, \dots, n$ of the branched continued fraction (5) appear in the coefficient matrix of the system instead of the $C_{m_i}^{(i)}$. To this end we introduce the notations

$$B_{m_j}^{(j)} = \begin{pmatrix} b_0^{(j)} & -1 & & & \\ a_1^{(j)} & b_1^{(j)} & -1 & & \\ & a_2^{(j)} & \ddots & \ddots & \\ & & \ddots & & -1 \\ & & & a_{m_j}^{(j)} & b_{m_j}^{(j)} \end{pmatrix} \quad (m_j + 1) \times (m_j + 1)$$

$$A_j = \begin{pmatrix} a_j & 0 & \dots & 0 \\ 0 & & & \\ \vdots & & 0 & \\ 0 & & & \end{pmatrix} \quad (m_j + 1) \times (m_{j-1} + 1)$$

$$I_j = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & & & \\ \vdots & & 0 & \\ 0 & & & \end{pmatrix} \quad (m_j + 1) \times (m_{j+1} + 1)$$

so that $P_{m_j}^{(j)} = \det B_{m_j}^{(j)}$.

Theorem.

If $Q_{n,m_0,\dots,m_n} \neq 0$ then $C_{n,m_0,\dots,m_n} = C_{m_0}^{(0)} + x_0^{(1)}$ where $x_0^{(1)}$ is the first unknown of the block-tridiagonal linear system

$$\begin{pmatrix} B_{m_1}^{(1)} & -I_1 & & & & \\ A_2 & B_{m_2}^{(2)} & -I_2 & & & \\ & A_3 & \ddots & \ddots & & \\ & & \ddots & & -I_{n-1} & \\ & & & & & A_n & B_{m_n}^{(n)} \end{pmatrix} \begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix} = \begin{pmatrix} a_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad (8)$$

with $X_j = (x_0^{(j)}, \dots, x_{m_j}^{(j)})^t$.

For the proof we need the following two lemmas.

Lemma 1.

$$\begin{vmatrix} B_{m_1}^{(1)} & -I_1 & & & \\ A_2 & B_{m_2}^{(2)} & -I_2 & & \\ & A_3 & \ddots & \ddots & \\ & & \ddots & & -I_{n-1} \\ & & & & A_n & B_{m_n}^{(n)} \end{vmatrix} = Q_{n,m_0,\dots,m_n} Q_{m_1}^{(1)} \dots Q_{m_n}^{(n)}$$

Proof. For $n = 1$ the left hand side reduces to

$$\det B_{m_1}^{(1)} = P_{m_1}^{(1)}$$

We also know from (7) that for $n = 1$

$$Q_{1,m_0,m_1} = C_{m_1}^{(1)} = \frac{P_{m_1}^{(1)}}{Q_{m_1}^{(1)}}$$

and hence that

$$Q_{1,m_0,m_1} Q_{m_1}^{(1)} = P_{m_1}^{(1)} = \det B_{m_1}^{(1)}$$

Suppose the lemma is valid for Q_{k,m_0,\dots,m_k} ($k = 1, \dots, n$). We shall prove it then for $Q_{n+1,m_0,\dots,m_{n+1}}$. A Laplacian expansion [1] of

$$\begin{vmatrix} B_{m_1}^{(1)} & -I_1 & & & & \\ A_2 & B_{m_2}^{(2)} & -I_2 & & & \\ & A_3 & \ddots & \ddots & & \\ & & \ddots & & -I_n & \\ & & & & & A_{n+1} & B_{m_{n+1}}^{(n+1)} \end{vmatrix}$$

along the last $(m_{n+1} + 1)$ rows reveals that the above determinant equals

$$\det \mathcal{B}_{m_{n+1}}^{(n+1)} \cdot \begin{vmatrix} \mathcal{B}_{m_1}^{(1)} & -I_1 & & & \\ \mathcal{A}_2 & \mathcal{B}_{m_2}^{(2)} & -I_2 & & \\ & \mathcal{A}_3 & \ddots & \ddots & \\ & & \ddots & & -I_{n-1} \\ & & & \mathcal{A}_n & \mathcal{B}_{m_n}^{(n)} \end{vmatrix} +$$

$$(-1)^{1+m_n} \begin{vmatrix} a_{n+1} & -1 & & & \\ 0 & b_1^{(n+1)} & \ddots & & \\ \vdots & a_2^{(n+1)} & \ddots & & \\ & & \ddots & & -1 \\ 0 & & & a_{m_{n+1}}^{(n+1)} & b_{m_{n+1}}^{(n+1)} \end{vmatrix} \cdot \begin{vmatrix} \mathcal{B}_{m_1}^{(1)} & -I_1 & & & \\ \mathcal{A}_2 & \mathcal{B}_{m_2}^{(2)} & \ddots & & \\ & \ddots & \ddots & & \\ & & & \mathcal{B}_{m_{n-1}}^{(n-1)} & 0 \\ & & & \mathcal{A}_n & Z \end{vmatrix}$$

where

$$Z = \begin{pmatrix} -1 & & & -1 \\ b_1^{(n)} & \ddots & & 0 \\ a_2^{(n)} & \ddots & & \vdots \\ & \ddots & & -1 \\ & & a_{m_n}^{(n)} & b_{m_n}^{(n)} & 0 \end{pmatrix}$$

This expression can immediately be simplified as

$$P_{m_{n+1}}^{(n+1)} Q_{n, m_0, \dots, m_n} Q_{m_1}^{(1)} \dots Q_{m_n}^{(n)} + (-1)^{1+m_n} a_{n+1} Q_{m_{n+1}}^{(n+1)} \begin{vmatrix} \mathcal{B}_{m_1}^{(1)} & -I_1 & & & \\ \mathcal{A}_2 & \mathcal{B}_{m_2}^{(2)} & \ddots & & \\ & \ddots & \ddots & & \\ & & & \mathcal{B}_{m_{n-1}}^{(n-1)} & 0 \\ & & & \mathcal{A}_n & Z \end{vmatrix}$$

By making a Laplacian expansion along the columns of Z and using the fact that $\det Z = (-1)^{1+m_n} Q_{m_n}^{(n)}$ it can further be simplified as

$$P_{m_{n+1}}^{(n+1)} Q_{n, m_0, \dots, m_n} Q_{m_1}^{(1)} \dots Q_{m_n}^{(n)} + a_{n+1} Q_{m_{n+1}}^{(n+1)} Q_{m_n}^{(n)} Q_{n-1, m_0, \dots, m_{n-1}} Q_{m_1}^{(1)} \dots Q_{m_{n-1}}^{(n-1)}$$

On the other hand we can write from (7)

$$Q_{n+1, m_0, \dots, m_{n+1}} = \frac{P_{m_{n+1}}^{(n+1)}}{Q_{m_{n+1}}^{(n+1)}} Q_{n, m_0, \dots, m_n} + a_{n+1} Q_{n-1, m_0, \dots, m_{n-1}}$$

from which we obtain

$$Q_{n+1, m_0, \dots, m_{n+1}} Q_{m_1}^{(1)} \dots Q_{m_{n+1}}^{(n+1)} = P_{m_{n+1}}^{(n+1)} Q_{n, m_0, \dots, m_n} Q_{m_1}^{(1)} \dots Q_{m_n}^{(n)} + a_{n+1} Q_{m_{n+1}}^{(n+1)} Q_{n-1, m_0, \dots, m_{n-1}} Q_{m_1}^{(1)} \dots Q_{m_n}^{(n)}$$

Since this right hand side coincides with a Laplacian expansion for

$$\begin{vmatrix} B_{m_1}^{(1)} & -I_1 & & & & \\ A_2 & B_{m_2}^{(2)} & -I_2 & & & \\ & A_3 & \ddots & \ddots & & \\ & & \ddots & & & -I_n \\ & & & & A_{n+1} & B_{m_{n+1}}^{(n+1)} \end{vmatrix}$$

our lemma is proved. ■

Lemma 2.

$$\begin{vmatrix} B_{m_0}^{(0)} & -I_0 & & & & \\ A_1 & B_{m_1}^{(1)} & -I_1 & & & \\ & A_2 & \ddots & \ddots & & \\ & & \ddots & & & -I_{n-1} \\ & & & & A_n & B_{m_n}^{(n)} \end{vmatrix} = P_{n, m_0, \dots, m_n} Q_{m_0}^{(0)} \dots Q_{m_n}^{(n)}$$

Proof. For $n = 0$ we know from (7) that

$$P_{0, m_0} = C_{m_0}^{(0)} = \frac{P_{m_0}^{(0)}}{Q_{m_0}^{(0)}}$$

and hence

$$P_{0, m_0} Q_{m_0}^{(0)} = P_{m_0}^{(0)} = \det B_{m_0}^{(0)}$$

The rest of the inductive proof is completely analogous to that of lemma 1 and is left to the reader. ■

Let us now try to prove our main result.

Proof of the theorem. Remark that for $n = 1$ (6) reduces to

$$C_{1,m_0,m_1} = C_{m_0}^{(0)} + \frac{a_1}{b_0^{(1)} + \sum_{k=1}^{m_1} \frac{a_k^{(1)}}{b_k^{(1)}}}$$

where $C_{1,m_0,m_1} - C_{m_0}^{(0)}$ is the first unknown $x_0^{(1)}$ of the tridiagonal linear system

$$\begin{pmatrix} b_0^{(1)} & -1 & & & \\ a_1^{(1)} & b_1^{(1)} & -1 & & \\ & a_2^{(1)} & \ddots & \ddots & \\ & & \ddots & & -1 \\ & & & a_n^{(1)} & b_n^{(1)} \end{pmatrix} \begin{pmatrix} x_0^{(1)} \\ \vdots \\ x_{m_1}^{(1)} \end{pmatrix} = \mathcal{B}_{m_1}^{(1)} \begin{pmatrix} x_0^{(1)} \\ \vdots \\ x_{m_1}^{(1)} \end{pmatrix} = \begin{pmatrix} a_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

More generally, a Laplacian expansion of

$$P_{n,m_0,\dots,m_n} Q_{m_0}^{(0)} \dots Q_{m_n}^{(n)} = \begin{vmatrix} \mathcal{B}_{m_0}^{(0)} & -I_0 & & & \\ \mathcal{A}_1 & \mathcal{B}_{m_1}^{(1)} & -I_1 & & \\ & \mathcal{A}_2 & \ddots & \ddots & \\ & & \ddots & & -I_{n-1} \\ & & & \mathcal{A}_n & \mathcal{B}_{m_n}^{(n)} \end{vmatrix}$$

along the first $(m_0 + 1)$ rows learns us that this determinant also equals

$$(\det \mathcal{B}_{m_0}^{(0)}) \cdot Q_{n,m_0,\dots,m_n} Q_{m_1}^{(1)} \dots Q_{m_n}^{(n)} +$$

$$(-1)^{1+m_0} \begin{vmatrix} -1 & & & & & & \\ b_1^{(0)} & \ddots & & & & & \\ a_2^{(0)} & \ddots & & & & & \\ & \ddots & & & & & \\ & & & -1 & & & \\ & & & a_{m_0}^{(0)} & b_{m_0}^{(0)} & 0 & \\ & & & & & 0 & \end{vmatrix} \begin{vmatrix} Y & -I_1 \\ 0 & \mathcal{B}_{m_2}^{(2)} \\ \vdots & \mathcal{A}_3 \\ \vdots & \ddots \\ 0 & \mathcal{A}_n & \mathcal{B}_{m_n}^{(n)} \end{vmatrix} \begin{vmatrix} -I_{n-1} \\ -I_{n-1} \\ \vdots \\ -I_{n-1} \end{vmatrix}$$

where

$$Y = \begin{pmatrix} a_1 & -1 & & & \\ 0 & b_1^{(1)} & \ddots & & \\ \vdots & a_2^{(1)} & \ddots & & \\ & & \ddots & & -1 \\ 0 & & & a_{m_1}^{(1)} & b_{m_1}^{(1)} \end{pmatrix}$$

This expression can immediately be simplified as

$$P_{n,m_0,\dots,m_n} Q_{m_0}^{(0)} \dots Q_{m_n}^{(n)} =$$

$$P_{m_0}^{(0)} Q_{n,m_0,\dots,m_n} Q_{m_1}^{(1)} \dots Q_{m_n}^{(n)} + Q_{m_0}^{(0)} \begin{vmatrix} Y & -I_1 & & & \\ 0 & B_{m_2}^{(2)} & \cdots & & \\ \vdots & A_3 & \ddots & & \\ & & \ddots & & -I_{n-1} \\ 0 & & & A_n & B_{m_n}^{(n)} \end{vmatrix}$$

The value C_{n,m_0,\dots,m_n} we are interested in is thus given by

$$\begin{aligned} C_{n,m_0,\dots,m_n} &= \frac{P_{n,m_0,\dots,m_n}}{Q_{n,m_0,\dots,m_n}} \\ &= \frac{P_{n,m_0,\dots,m_n} Q_{m_0}^{(0)} \dots Q_{m_n}^{(n)}}{Q_{n,m_0,\dots,m_n} Q_{m_0}^{(0)} \dots Q_{m_n}^{(0)}} \end{aligned}$$

From lemma 2 and the last Laplacian expansion we know that this quotient equals

$$\frac{P_{m_0}^{(0)}}{Q_{m_0}^{(0)}} + \frac{\begin{vmatrix} Y & -I_1 & & & \\ 0 & B_{m_2}^{(2)} & \cdots & & \\ \vdots & A_3 & \ddots & & \\ & & \ddots & & -I_{n-1} \\ 0 & & & A_n & B_{m_n}^{(n)} \end{vmatrix}}{Q_{n,m_0,\dots,m_n} Q_{m_1}^{(1)} \dots Q_{m_n}^{(n)}}$$

Using lemma 1 the second term in this expression is apparently the first unknown $x_0^{(1)}$ of our block-tridiagonal linear system. ■

If we try to describe the result of the theorem we can look upon it as follows. Formula (4) for ordinary continued fractions (1) generalizes to formula (8) for branched continued fractions (5) by replacing

$$\begin{aligned} b_j^{(i)} &\rightarrow B_{m_j}^{(j)} \\ a_j^{(i)} &\rightarrow A_j \\ -1 &\rightarrow -I_j \end{aligned}$$

Continuing this idea it is easy to see that for two-branched continued fractions

$$B_0^{(0)} + \sum_{j=1}^{\infty} \left| \frac{a_j^{(0)}}{B_j^{(0)}} \right| + \sum_{i=1}^{\infty} \left| \frac{a_i}{B_0^{(i)} + \sum_{j=1}^{\infty} \left| \frac{a_j^{(i)}}{B_j^{(i)}} \right|} \right|$$

with

$$B_j^{(i)} = b_{j0}^{(i)} + \sum_{k=1}^{\infty} \frac{a_{jk}^{(i)}}{b_{jk}^{(i)}}$$

which result by inserting an ordinary continued fraction for each denominator $b_j^{(i)}$ in (5), a formula similar to (8) can be proved where now within $\beta_{m_i}^{(i)}$ each $b_j^{(i)}$ is in its turn replaced by a block of the form

$$\begin{pmatrix} b_{j0}^{(i)} & -1 & & \\ a_{j1}^{(i)} & b_{j1}^{(i)} & \ddots & \\ & \ddots & \ddots & \end{pmatrix}$$

This procedure can be repeated k times and so a general determinant representation can be given for the convergent of a k -branched continued fraction. It is our purpose to discuss parallel algorithms for the computation of (6) by introducing parallel algorithms for the solution of block-tridiagonal linear systems like (8). The computation of this type of convergents arises in approximation theory [2], systems theory, and other applications which are under investigation [3].

REFERENCES

1. A. C. Aitken, "Determinants and matrices", Oliver & Boyd, Edinburgh, 1967.
2. A. Cuyt, *A recursive computation scheme for multivariate rational interpolants*, SIAM J. Num. Anal. (to appear).
3. A. Cuyt and B. Verdonk, *A review of branched continued fraction theory for the construction of multivariate rational approximants*, J. Appl. Numer. Math., submitted.
4. J. Mikloško, *Investigation of algorithms for numerical computation of continued fractions*, USSR Comp. Math. and Math. Phys. **16** (1976), 1-12.
5. O. Perron, "Die Lehre von den Kettenbrüchen I", Teubner, Stuttgart, 1977.
6. V. Skorobogatko, "Branched continued fractions and their applications in mathematics", (in Russian), Nauka, Moscow, 1983.

IN 1986 REEDS VERSCHENEN

- 202 J.H.F. Schilderincx
Interregional Structure of the European Community. Part III
- 203 Antoon van den Elzen and Dolf Talman
A new strategy-adjustment process for computing a Nash equilibrium in a noncooperative more-person game
- 204 Jan Vingerhoets
Fabrication of copper and copper semis in developing countries. A review of evidence and opportunities
- 205 R. Heuts, J. van Lieshout, K. Baken
An inventory model: what is the influence of the shape of the lead time demand distribution?
- 206 A. van Soest, P. Kooreman
A Microeconomic Analysis of Vacation Behavior
- 207 F. Boekema, A. Nagelkerke
Labour Relations, Networks, Job-creation and Regional Development. A view to the consequences of technological change
- 208 R. Alessie, A. Kapteyn
Habit Formation and Interdependent Preferences in the Almost Ideal Demand System
- 209 T. Wansbeek, A. Kapteyn
Estimation of the error components model with incomplete panels
- 210 A.L. Hempenius
The relation between dividends and profits
- 211 J. Kriens, J.Th. van Lieshout
A generalisation and some properties of Markowitz' portfolio selection method
- 212 Jack P.C. Kleijnen and Charles R. Standridge
Experimental design and regression analysis in simulation: an FMS case study
- 213 T.M. Doup, A.H. van den Elzen and A.J.J. Talman
Simplicial algorithms for solving the non-linear complementarity problem on the simplotope
- 214 A.J.W. van de Gevel
The theory of wage differentials: a correction
- 215 J.P.C. Kleijnen, W. van Groenendaal
Regression analysis of factorial designs with sequential replication
- 216 T.E. Nijman and F.C. Palm
Consistent estimation of rational expectations models

- 217 P.M. Kort
The firm's investment policy under a concave adjustment cost function
- 218 J.P.C. Kleijnen
Decision Support Systems (DSS), en de kleren van de keizer ...
- 219 T.M. Doup and A.J.J. Talman
A continuous deformation algorithm on the product space of unit simplices
- 220 T.M. Doup and A.J.J. Talman
The 2-ray algorithm for solving equilibrium problems on the unit simplex
- 221 Th. van de Klundert, P. Peters
Price Inertia in a Macroeconomic Model of Monopolistic Competition
- 222 Christian Mulder
Testing Korteweg's rational expectations model for a small open economy
- 223 A.C. Meijdam, J.E.J. Plasmans
Maximum Likelihood Estimation of Econometric Models with Rational Expectations of Current Endogenous Variables
- 224 Arie Kapteyn, Peter Kooreman, Arthur van Soest
Non-convex budget sets, institutional constraints and imposition of concavity in a flexible household labor supply model
- 225 R.J. de Groof
Internationale coördinatie van economische politiek in een twee-regio-twee-sectoren model
- 226 Arthur van Soest, Peter Kooreman
Comment on 'Microeconomic Demand Systems with Binding Non-Negativity Constraints: The Dual Approach'
- 227 A.J.J. Talman and Y. Yamamoto
A globally convergent simplicial algorithm for stationary point problems on polytopes
- 228 Jack P.C. Kleijnen, Peter C.A. Karremans, Wim K. Oortwijn, Willem J.H. van Groenendaal
Jackknifing estimated weighted least squares
- 229 A.H. van den Elzen and G. van der Laan
A price adjustment for an economy with a block-diagonal pattern
- 230 M.H.C. Paardekooper
Jacobi-type algorithms for eigenvalues on vector- and parallel computer
- 231 J.P.C. Kleijnen
Analyzing simulation experiments with common random numbers

- 232 A.B.T.M. van Schaik, R.J. Mulder
On Superimposed Recurrent Cycles
- 233 M.H.C. Paardekooper
Sameh's parallel eigenvalue algorithm revisited
- 234 Pieter H.M. Ruys and Ton J.A. Storcken
Preferences revealed by the choice of friends
- 235 C.J.J. Huys en E.N. Kertzman
Effectieve belastingtarieven en kapitaalkosten
- 236 A.M.H. Gerards
An extension of König's theorem to graphs with no odd- K_4
- 237 A.M.H. Gerards and A. Schrijver
Signed Graphs - Regular Matroids - Grafts
- 238 Rob J.M. Alessie and Arie Kapteyn
Consumption, Savings and Demography
- 239 A.J. van Reeken
Begrippen rondom "kwaliteit"
- 240 Th.E. Nijman and F.C. Palmer
Efficiency gains due to using missing data. Procedures in regression models
- 241 Dr. S.C.W. Eijffinger
The determinants of the currencies within the European Monetary System

IN 1987 REEDS VERSCHENEN

- 242 Gerard van den Berg
Nonstationarity in job search theory

Bibliotheek K. U. Brabant



17 000 01059978 6