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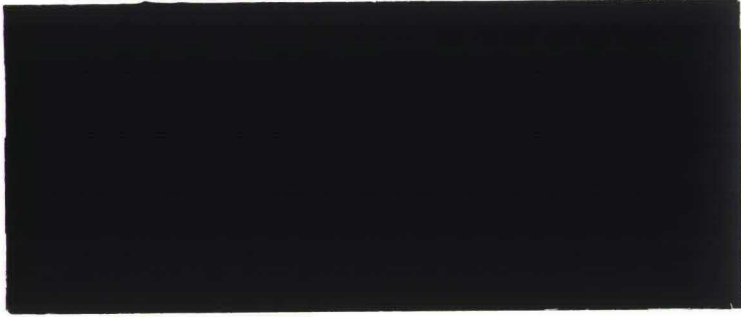
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**OPTIMAL DYNAMIC TAXATION, SAVING AND  
INVESTMENT**

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## OPTIMAL DYNAMIC TAXATION, SAVING AND INVESTMENT\*

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In this paper we develop a framework for determining optimal dynamic taxation based on maximizing welfare in a macro-economic market economy with value-maximizing firms, which face costs of adjustment for investment and utility optimizing consumers. We derive welfare rankings of often used tax rates such as profit tax, sales tax, consumption and wage tax. However, such an optimal tax policy may be time-inconsistent, and therefore, also the implications of time-inconsistency for welfare rankings of tax rates are given. It is shown that the wage and consumption tax rate cannot be a source for time-inconsistency, which gives an incentive to implement these taxes.

### 1. Introduction

Many recent papers have developed macro-economic models to study the dynamic evolution of the economy in order to analyze dynamic effects of fiscal policy (e.g. Hall (1971), Brock and Turnovsky (1981), Abel and Blanchard (1983), Judd (1985) and Van de Klundert and Peters (1986)). Aim of these papers is to investigate the incidence of different tax rates such as a tax on profits, a sales tax, a wage tax or a consumption tax. However, in these kinds of models the tax rates are given exogenously.

Recently, there was a policy discussion in the United States (e.g. Andrew Kupfer in the Fortune of August 1988) about which tax rates should be raised.

"They (Bush or Dukakis) will raise your taxes, after they are safely elected. But not all the pain is created equal. You don't necessarily have to sell out 70% of your salary in income taxes to cure the budget deficit. There are other taxes that nibble at your wallet a little bit at a time: consumption taxes, which tax you when spend." (p.24)

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In this paper we deal with this problem by a more normative approach. We are concerned with optimal dynamic taxation, where the government wants to maximize the utility of a representative consumer (e.g. Turnovsky and Brock (1980)) and tries to choose its tax rate in such a way that this objective is maximized. Moreover, we assume that the government takes into account the way that the firm and consumer will react on its tax policy. So, the government behaves as the leader in an open-loop Stackelberg equilibrium (e.g. Başar and Olsder (1982)). However, as pointed out by Kydland and Prescott (1977) and Calvo (1978) such an optimal policy may be time-inconsistent. Once the current is history, the effect of policies on behavior in that period are of little or no interest. Therefore, we are concerned with two possible solutions. The first solution, which is the formal outcome of an open-loop Stackelberg equilibrium of game between government, consumers and firms, is only credible, if there is commitment or if there are reputational forces. The second solution, which corresponds to a Nash-Cournot equilibrium is time-consistent, but yields a lower value of steady-state utility. In this paper we show that this problem of time-consistency depends on the kind of tax rates the government chooses.

Our approach builds on Abel and Blanchard's (1983) macro-economic market model, which describes intertemporal choice of consumers and firms in a free market economy. Abel and Blanchard use this framework to analyze the dynamic effects of fiscal policy. We extend the Abel and Blanchard model by modelling optimal government's behavior and give some rules for the optimal choice of tax policy.

Much attention has been paid in the literature to the problem of optimal government's behavior, especially in tax problems (see for a survey Atkinson and Stiglitz (1980)). However, most papers deal with this problem in a micro-economic way. For instance, they treat taxation on different commodities (e.g. Diamond and Mirlees (1971)) or optimal income taxation (e.g. Mirlees (1971), Dixit and Sandmo (1976)). A macro-economic example of optimal taxation is the paper by Turnovsky and Brock (1980), but in this model there is no capital accumulation. A model with capital accumulation and optimal capital and labor taxation is the paper by Fischer (1980). Since this is a two-period model with no separation between consumers and firms, no accumulation in the second period, only labor and

capital taxes and no taxes in the first period, this paper disregards some important issues. Nevertheless, it is a nice example illustrating the time-inconsistency in dynamic optimal taxation.

The paper is organized as follows. In section 2 we model the firms' and consumers' decision problem. Furthermore, the equilibrium on the goods and labor market is described. In section 3 we describe optimal government's behavior, if it takes into account the way that the agents will make their decisions. In section 4 we interpret the calculations of section 3. Finally, in section 5 we make some remarks and suggestions for future research.

## 2. The firm's and consumer's decision problem

For reasons of analytical tractability we assume that there is only one representative type of consumer and firm. The consumer and the firm are confronted with the following types of tax rates: a profit tax rate, a wage tax rate, a consumption tax rate and a sales tax rate.

### 2.1. The firm

Consider a firm operating in an environment without exogenous uncertainty. The firm decides on its demand for labor and investment, which are conditional on its expectations, present and future profit tax rates and present and future interest rates. The firm maximizes its discounted stream of net profits

$$\max_{i, l} \int_0^{\infty} [\{f(k, l)(1-\tau_y) - wl\}(1-\tau_z) - i - \varphi(i)] e^{-\int_0^t r(v) dv} dt, \quad (1)$$

$$\varphi(0) = 0, \text{ sign}(\varphi') = \text{sign}(i), \varphi'' > 0,$$

where  $k$ ,  $l$ ,  $i$ ,  $w$ ,  $r$ ,  $f(k, l)$ ,  $\varphi(i)$ ,  $\tau_z$  and  $\tau_y$  denote respectively the level of the capital stock, the number of employed workers, the rate of investment, the real wage rate, the rate of interest, the production function, the internal adjustment costs, the proportional tax rate on profits and the proportional tax rate on sales.

With respect to the production function we assume that capital and labor are substitutes and production is characterised by constant returns to scale Cobb-Douglas function (so that  $f_{ll}f_{kk} - f_{kl}^2 = 0$ ) with  $\sigma$  the parameter of this function. The planning horizon is infinite. The strictly convex function  $\varphi(\cdot)$  captures that internal adjustment costs increase and are zero only if gross investment is zero. It ensures that capital adjusts in a sluggish manner to changes in interest rate and tax rates. The firm will maximize (1) subject to the capital accumulation equation

$$\dot{k} = i - \delta k, \quad (2)$$

where  $\delta$  is the rate of depreciation.

The necessary conditions for the firm's optimal control problem are:

$$\dot{q} = (r+\delta)q - f_k(1-\tau_z)(1-\tau_y), \quad \lim_{t \rightarrow \infty} e^{-\int_0^t r(v)dv} q(t)k(t) = 0, \quad (3)$$

$$\varphi'(i) = q - 1, \quad (4)$$

$$(1-\tau_y)f_l = w, \quad (5)$$

$$\dot{k} = i - \delta k, \quad (6)$$

in which  $q$  is the (undiscounted) shadow price of capital, and  $\varphi'$  denotes the derivative of  $\varphi$  with respect to  $i$  and we add subscripts to  $f$  to denote partial derivatives.

The investment demand depends on  $q$  (cf. (4)):

$$i = \Phi(q), \quad \Phi' > 0, \quad \Phi(1) = 0. \quad (7)$$

An expression for  $q$ , which is equal to the present value of marginal profits, can be obtained by solving the differential equation (3)



$$q(t) = \int_t^{\infty} f_k(k, \ell) (1-\tau_z)(1-\tau_y) e^{-\int_t^s (r(v)+\delta) dv} ds. \quad (8)$$

After having paid wages to the workers, the firm has to decide how to distribute profit and to finance investment. It may finance investment by retained earnings or by issuing new shares or bonds. We will assume that there is fiscal equality between equity or bonds and shares, so that all financing schemes are equivalent in the sense that they lead to the same path of consumption and investment (for a proof of this see Abel and Blanchard (1983, pp. 680-681)).

## 2.2. The Consumer

With respect to the consumer we will assume that he takes present and future wage rates, interest rates, tax rates and dividend paid by the firm as given. Furthermore, we assume that the welfare of a consumer depends on private ( $c$ ) and public consumption ( $g$ ). Given a constant rate of time preference  $\beta$ , the decision problem is to choose a time path of consumption, that maximizes the present value of utility

$$\max_c \int_0^{\infty} u(c, g) e^{-\beta t} dt \quad (9)$$

subject to the wealth constraint, which can be expressed as

$$\dot{b} = rb + \pi + w\ell(1-\tau_\ell) - c(1+\tau_c), \quad b(0) = b_0, \quad (10)$$

where  $b$ ,  $\pi$ ,  $\tau_\ell$  and  $\tau_c$  denote respectively the amount of bonds hold by the consumer, dividends, the proportional tax rate on wages and the proportional tax rate on consumption. The utility function is assumed to be concave in its arguments, such that the marginal utility from the consumption of both private and public goods is positive. Furthermore, income is

the sum of wages, interest on savings and dividends. Furthermore, we assume that there are No-Ponzi games, i.e.,

$$\lim_{t \rightarrow \infty} e^{-\int_0^t r(v) dv} b(t) = 0. \quad (11)$$

The optimality conditions are:

$$u_c / (1 + \tau_c) = x, \quad (12)$$

$$\dot{x} = (\beta - r)x, \quad \lim_{t \rightarrow \infty} e^{-\beta t} x(t) = 0, \quad (13)$$

in which  $x$  is the costate variable associated with the dynamic budget constraint and we add subscripts to the utility function to indicate partial derivatives. As shown by Abel and Blanchard total consumption is the sum of human wealth and non-human wealth. It is assumed that the supply of labor is inelastic.

### 2.3. Market Clearing

Assume that there is an equilibrium on the goods market, so that spendings in this economy are equal to production:

$$f(k, \ell) = c + g + i + \varphi(i). \quad (14)$$

This condition determines at any instant of time the interest rates, while the wage rate is determined by equilibrium in the labor market. Because the supply of labor ( $\ell_s$ ) is assumed to be inelastic it is not difficult to derive the wage rates, which are given by equality of demand and supply of labor

$$\ell_s = \ell. \quad (15)$$

### 3. Optimal government's policies

The question we are now addressing ourselves at is: how will the government choose its tax rates when confronted with the behavior of the consumer and firm as described in section 2. It is reasonable to assume that the government takes into account the manner in which the firm and consumer react on its taxation decisions. We therefore formulate our problem in a game-theoretic framework. The formal outcome of our model corresponds to a three persons open-loop Stackelberg game with the government as leader and the firm and consumer playing Nash against each other (cf. Başar and Olsder (1982)). An important question in this respect is: will the government take into account, that the market prices,  $r$  and  $w$ , also depend on its tax policy? We will assume that this is not the case. Moreover, it is our guess that this effects are relatively small. As mentioned before the government maximizes the same utility function as the consumer subject to (3)-(6) and (10)-(13). Furthermore, we assume that the government's budget is balanced. It should be noted, that outstanding government's debt can easily be introduced into the model. As noted by for example Barro (1979), under assuming Ricardo debt neutrality, meaning that the interest rate on the government's bond is also  $r$ , this will not lead to other results.

The government's problem can be formulated as follows:

$$\max_{\tau_l, \tau_y, \tau_z, \tau_c} \int_0^{\infty} u(c, g) e^{-\beta t} dt \quad (16)$$

$$\text{s.t. } \dot{q} = (r+\delta)q - f_k(1-\tau_z)(1-\tau_y), \quad (17)$$

$$\dot{k} = \Phi(q) - \delta k, \quad (18)$$

$$\dot{x} = (\beta-r)x, \quad (19)$$

$$\dot{b} = rb + \pi + wl(1-\tau_l) - c(1+\tau_c), \quad (20)$$

$$u_c = x/(1+\tau_c), \quad (21)$$

$$c = \{(1-\tau_z)(1-\tau_y)f(k, l) - i - \varphi(i) + wl(\tau_z - \tau_l)\}/(1+\tau_c), \quad (22)$$

$$g = c\tau_c + \tau_y f(k, \ell) + \tau_z [(1-\tau_y)f(k, \ell) - w\ell] + w\ell\tau_\ell. \quad (23)$$

$$\ell = h(\tau_y)k, \quad (24)$$

where equation (24) can be obtained from (5) and  $h(\tau_y)$  denotes the capital to labor ratio. It should be noted that we can eliminate  $b$  and  $x$ . Substituting (22) into (21) gives us a value for  $x$ . As already stated the distribution of investment and consumption will not be influenced by financial streams.

Furthermore, we assume that the tax rates have exogenously given upperbounds:

$$0 \leq \tau_\ell \leq \bar{\tau}_\ell < 1, \quad (25)$$

$$0 \leq \tau_c \leq \bar{\tau}_c < 1, \quad (26)$$

$$0 \leq \tau_y \leq \bar{\tau}_y < 1, \quad (27)$$

$$0 \leq \tau_z \leq \bar{\tau}_z < 1. \quad (28)$$

There could be several reasons for imposing upperbounds. If the tax rates are high there is a great resistance of the tax payer to pay, high costs of administration and compliance (cf. Alt (1983)). We try to formalize these ideas by assuming upperbound on the tax rates. So, profits, sales, consumption and labor are never taxed away completely. Furthermore, we assume for reasons of analytical tractability Cobb-Douglas preferences

$$u(c, g) = \alpha \ln c + (1-\alpha) \ln g, \quad 0 < \alpha < 1, \quad (29)$$

and quadratic adjustment costs

$$\varphi(i) = bi^2, \quad b > 0. \quad (30)$$

The maximisation of (16) with respect to (17) through (30) yields the following necessary conditions:

$$\dot{\nu} = \beta\nu - \frac{\partial H}{\partial q} = (\beta - r - \delta)\nu - \frac{\lambda}{2b} - \frac{\alpha q}{2bc} \left( \frac{1}{1+\tau_c} \right) + \frac{(1-\alpha)q}{2bg} \left( \frac{\tau_c}{1+\tau_c} \right), \quad \nu(0)=0, \quad (31)$$

$$\begin{aligned} \dot{\lambda} &= \beta\lambda - \frac{\partial H}{\partial k} = (\beta + \delta)\lambda - \frac{1-\alpha}{g} (1 - (1-\tau_y)(1-\tau_z)/(1+\tau_c)) \sigma h^{1-\sigma} \nu (1-\tau_y) \\ &\quad (1-\tau_z) \sigma h^{1-\sigma} \frac{\alpha}{c} \{ (1-\tau_y)(1-\tau_z)/(1+\tau_c) \} \sigma h^{1-\sigma} + wh(\tau_z - \tau_\ell) \}, \\ \lim_{t \rightarrow \infty} e^{-\beta t} \lambda(t) k(t) &= 0, \end{aligned} \quad (32)$$

$$\begin{aligned} &-\frac{\alpha}{c} [ \{ (1-\tau_z)(1-\tau_y) f(k, hk) - i - \varphi(i) + whk(\tau_z - \tau_\ell) \} / (1+\tau_c)^2 ] + \frac{1-\alpha}{g} [ \{ (1-\tau_z) \\ &(1-\tau_y) f(k, hk) - i - \varphi(i) + whk(\tau_z - \tau_\ell) \} / (1+\tau_c)^2 ] \stackrel{>}{\lessdot} 0 \leftrightarrow \tau_c = \begin{cases} \bar{\tau}_c \\ 0 \end{cases} \tau_c \in [0, \bar{\tau}_c], \end{aligned} \quad (33)$$

$$-\frac{\alpha}{c} whk / (1+\tau_c) + \frac{1-\alpha}{g} whk / (1+\tau_c) \stackrel{>}{\lessdot} 0 \leftrightarrow \tau_\ell = \begin{cases} \bar{\tau}_\ell \\ 0 \end{cases} \tau_\ell \in [0, \bar{\tau}_\ell], \quad (34)$$

$$\begin{aligned} &-\frac{\alpha}{c} [ \{ (1-\tau_y) f(k, hk) + whk \} / (1+\tau_c) ] + \frac{1-\alpha}{g} [ \{ (1-\tau_y) f(k, hk) + whk \} / (1+\tau_c) ] + \\ &\nu (1-\tau_y) \sigma h^{1-\sigma} \stackrel{>}{\lessdot} 0 \leftrightarrow \tau_z = \begin{cases} \bar{\tau}_z \\ 0 \end{cases} \tau_z \in [0, \bar{\tau}_z], \end{aligned} \quad (35)$$

$$\begin{aligned} &-\frac{\alpha}{c} [ (1-\tau_z) \{ f(k, hk) + \frac{1-\sigma}{\sigma} h \tau_y k \} / (1+\tau_c) + \frac{h \tau_y w}{\sigma (1-\tau_y)} k ] + \frac{1-\alpha}{g} [ (1-\tau_z) \\ &\{ f(k, hk) + \frac{1-\sigma}{\sigma} h \tau_y k \} / (1+\tau_c) + \frac{h \tau_y w}{\sigma (1-\tau_y)} k ] + \nu (1-\tau_z) h^{1-\sigma} \\ &\stackrel{>}{\lessdot} 0 \leftrightarrow \tau_y = \begin{cases} \bar{\tau}_y \\ 0 \end{cases} \tau_y \in [0, \bar{\tau}_y], \end{aligned} \quad (36)$$

where the Hamiltonian is defined by

$$\begin{aligned} H &= \alpha \ln [ \{ (1-\tau_z)(1-\tau_y) f(k, h(\tau_y)k) - i - \varphi(i) + wh(\tau_y)k(\tau_z - \tau_\ell) \} / (1+\tau_c) ] + \\ &(1-\alpha) \ln [ f(k, h(\tau_y)k) - i - \varphi(i) - \{ (1-\tau_z)(1-\tau_y) f(k, h(\tau_y)k) - i - \varphi(i) + \end{aligned}$$

$$\text{whk}(\tau_z - \tau_\ell) / (1 + \tau_c) + \nu((r + \delta)q - \sigma h(\tau_y)^{1-\sigma}(1 - \tau_z)(1 - \tau_y)) + \lambda(\Phi(q) - \delta k). \quad (37)$$

The costate variabile  $\lambda$  which can be interpreted as the government's shadow price of capital, will have a positive value. The costate variabile  $\nu$ , which is the marginal value to the government of the firm's investment rate, is typically negative, because the government and the firm have conflicting objectives. In other words the government can influence the firm's investment decisions  $q$  by announcing a certain tax policy. The value by which the government's objective is affected is given by  $\nu$ .

Define the switching functions for the different tax rates as follows:

$$B_{\tau_\ell}(t) = -\frac{\alpha}{c} \text{whk} / (1 + \tau_c) + \frac{1-\alpha}{g} \text{whk} / (1 + \tau_c), \quad (38)$$

$$B_{\tau_c}(t) = -\frac{\alpha}{c} [ \{ (1 - \tau_z)(1 - \tau_y) f(k, hk) - i - \rho(i) + \text{whk}(\tau_z - \tau_\ell) \} / (1 + \tau_c)^2 ] + \frac{1-\alpha}{g} [ \{ (1 - \tau_z)(1 - \tau_y) f(k, hk) - i - \rho(i) + \text{whk}(\tau_z - \tau_\ell) \} / (1 + \tau_c)^2 ], \quad (39)$$

$$B_{\tau_z}(t) = -\frac{\alpha}{c} [ \{ (1 - \tau_y) f(k, hk) + \text{whk} \} / (1 + \tau_c) ] + \frac{1-\alpha}{g} [ \{ (1 - \tau_y) f(k, hk) + \text{whk} \} / (1 + \tau_c) ] + \nu(1 - \tau_y) \sigma h^{1-\sigma}, \quad (40)$$

$$B_{\tau_y}(t) = -\frac{\alpha}{c} [ (1 - \tau_z) \{ f(k, hk) + \frac{1-\sigma}{\sigma} h \tau_y k \} / (1 + \tau_c) + \frac{h \tau_y w}{\sigma(1 - \tau_y)} k ] + \frac{1-\alpha}{g} [ (1 - \tau_z) \{ f(k, hk) + \frac{1-\sigma}{\sigma} h \tau_y k \} / (1 + \tau_c) + \frac{h \tau_y w}{\sigma(1 - \tau_y)} k ] + \nu(1 - \tau_z) h^{1-\sigma}. \quad (41)$$

From (38) we obtain

$$\tau_\ell = \begin{cases} \bar{\tau}_\ell & \text{if } B_{\tau_\ell}(t) > 0 \\ \tau_\ell \in [0, \bar{\tau}_\ell] & \text{if } B_{\tau_\ell}(t) = 0. \\ 0 & \text{if } B_{\tau_\ell}(t) < 0 \end{cases} \quad (42)$$

Consider the case that  $\alpha$  is relatively high (cf. (30)), then the weight of government's consumption in total utility is relatively small and we can derive the following result

$$B_{\tau_{\lambda}}(t) = B_{\tau_c}(t) = 0 \geq B_{\tau_z}(t) \geq B_{\tau_y}(t), \quad (43)$$

which can be interpreted as follows: the government will impose wage and consumption tax, such that the values of the switching functions  $B_{\tau_{\lambda}}$  and

$B_{\tau_c}$  are equal to zero. This is the case for a whole class of linear combinations between  $\tau_{\lambda}$  and  $\tau_c$ . Because in that case the values of the switching functions  $B_{\tau_y}$  and  $B_{\tau_z}$  are less than zero, these taxes will be on their lowerbounds, i.e. zero. If  $\alpha$  is relatively small, government's consumption is high, so it must collect a lot of money to keep the budget balanced. Therefore the government sets  $\tau_{\lambda}$  and  $\tau_c$  on their upperbounds and it will also raise the profit tax to finance its expenditures. The last tax rate which can be raised is the sales tax rate. It should be noted that the government can raise this tax rate before the upperbound on the sales tax rate has been reached.

So if the government raises taxes there is a classification of tax rates, which it wants to raise first

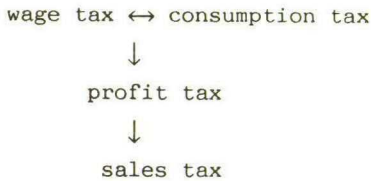


Fig. 1 Classification of the tax rates

The reason for this is that profit and sales taxation are more distortionary than the other tax rates, because of their influence on the capital accumulation.

The nature of the solution examined may be further clarified by a numerical example, which is based on the following parameter values:

$$\beta=0.03, \delta=0.05, \sigma=0.5, \bar{\tau}_\ell=0.2, \bar{\tau}_z=0.3, \ell_s=0.044, b=10. \quad (44)$$

In this example we assume that there are only two taxes, a sales and a profit tax, which have the following upperbounds  $\bar{\tau}_\ell=0.2$  and  $\bar{\tau}_z=0.3$ . In table 1 the steady-state solutions for different values of  $\alpha$  are given.

[insert table 1]

For a large value of  $\alpha$ , i.e. 0.9, private consumption is preferred more than public consumption and the government asks a relatively low amount of wage tax and no profit tax. If  $\alpha$  decreases, the amount of wage tax increases until the moment that the upperbound on wage tax has been reached. After this point that the upperbound is reached, the government will not immediately increase its spendings by increasing the profit tax rate, because of its negative effect on the capital accumulation. Only at the moment that  $\alpha$  is small, i.e. 0.25, the government uses profit taxation to finance its expenditures (see table 1).

#### 4. About the time-inconsistency of optimal plans

In section 3 we have described an optimal taxation plan for the government. However, this optimal plan can be time-inconsistent. Once the capital stock is installed, there is an incentive for the government to renege on its announcement and ask a higher sales or profit tax rate. The problem of a time-inconsistent optimizing government in a dynamic economic environment was first stated by Kydland and Prescott (1977). Obviously, a time-inconsistent plan requires binding commitments to force the government to stick to its announced tax strategy. However, it should be noted that when reputational forces are important, binding contracts may not be necessary to prevent the government from cheating (e.g. Kreps and Wilson (1982)). So, if there are no binding contracts or no reputational forces the open-loop Stackelberg equilibrium is no longer a rational expectations



equilibrium and is no longer a useful concept. In that case the Nash-Cournot equilibrium can be used in which case  $\nu(t)=0$ ,  $\forall t \geq 0$  and the government ignores (31). Substituting  $\nu(t)=0$  into (38)-(41) gives that the switching functions of consumption, wage and profit tax become equal<sup>1)</sup>:

$$B_{\tau_l}(t) = B_{\tau_c}(t) = B_{\tau_z}(t) = \frac{1-\alpha}{g} - \frac{\alpha}{c} = 0 > B_{\tau_y}(t). \quad (45)$$

This means that unless the upperbounds on these taxes are not reached the government finances their expenditures by wage, consumption and sales taxes, where it is indifferent about which tax rates should be chosen. Although it takes into account the effect of its taxation on capital accumulation, it cannot deal with it, because every announced policy of low capital taxation becomes suboptimal once the capital is installed and is therefore incredible. So in general in the consistent case the capital accumulation will be lower and because of that the consumer's life-time utility will be lower.

Let us now clarify the difference between the consistent and inconsistent case with a numerical example, where we assume the same parameter values as in table 1. Furthermore, assume in the consistent case that the upperbound on wage tax is first reached<sup>2)</sup>. In table 2 the steady-state solution is given for the case that  $\alpha=0.75$ .

[insert table 2]

The consistent solution yields a higher value of steady-state profit tax rate and because of that a lower level of capital stock than in the inconsistent solution (see table 2). This lower level of capital stock in the consistent case yields a lower level level of steady-state utility. In the inconsistent case the share of public consumption goods in the total output is lower, but private consumption and total utility will be higher, because there is more capital in this economy.

So the outcome in the consistent case is bad for society. Therefore, as pointed out by, e.g. Kydland (1989, p. 268), there would be attempts,

like institutional arrangements, laws or reputational forces to prevent the government from cheating. In practice, the firm makes its investment decision based on its expectations of current and future profit and sales tax rates. In general there are reasons to believe that the expected profit tax rate is higher than what follows from optimal but time-inconsistent behavior, but lower than one should expect from purely time-consistent behavior. A more detailed description of these expectations based on modelling of threats and incentives could be an interesting area of future research (cf. Pohjola (1984)).

## 5. Conclusions

In this paper we have developed a macro-economic dynamic model with value-maximizing firms, infinitely long-lived utility-optimizing consumers and a government, which tries to choose its tax instruments in such a way that the utility of the consumer is maximized. The formal structure of the interaction between government and firms or consumers corresponds to an open-loop Stackelberg game with the government as leader. Moreover, there was market clearing on the labor and output market.

Recently, there was a discussion in the United States about which tax rate should be raised confronted with a too large deficit. From the framework we have presented we can derive that there is a strong incentive to raise consumption or wage tax. The reason for this is quite clear, because both taxes cannot be a source for time-inconsistency and they have no influence on the capital accumulation. The answer to the question whether the consumption or wage tax rate should be chosen is not given by our model. Moreover, in the literature (e.g. Atkinson and Stiglitz (1980), Laffer (1981)) there has been given some arguments why consumption tax should be preferred. Incorporating such effects into our model should be an interesting field of future research.

As already noted before, the introduction of optimizing government behavior in dynamic economic models may lead to the problem of time-inconsistency. However, this problem of time-inconsistency only occurs, if the government chooses profit or sales taxation to finance its expenditures. So by choosing the right instruments the government can deal with the problem of time-inconsistency. Also reputational forces could be

important to prevent the government from cheating (cf. Kreps and Wilson (1982)).

Of course, the model presented here is simple. Nevertheless, it points out some important issues of normative tax behavior of the government. In future work, there are many avenues to explore. Firstly, we can bring money into our model. Especially the question how the optimal monetary policies will influence capital accumulation is of interest. Secondly, a more general utility framework with leisure and money can be analysed. Thirdly, a more game-theoretic framework with threats, incentives and feedback and closed-loop information could be an interesting feature of future research. Finally, the introduction of distributive goals can be of interest.

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#### Notes

- 1) Unless the wage, the consumption and the profit tax rates are not on their upperbound, this equality between the switching functions hold.
- 2) This not necessarily follows from (45). In fact all kind of linear combinations between  $\tau_l$  and  $\tau_z$  are possible. However, the combinations with low  $\tau_z$  yields more capital accumulation. So, from this point of view there is a reason, even for the consistent case, for first raising the wage tax before the profit tax is raised. However, it should be noted that directly after the upperbound on wage tax have been reached the government increases the profit tax, so that  $g/c=(1-\alpha)/\alpha$ . This is contrary to the inconsistent case.

TABLE 1. The steady-state by different  $\alpha$

	$\alpha=0.25$	$\alpha=0.50$	$\alpha=0.75$	$\alpha=0.90$
$\tau_\rho$	0.200	0.200	0.200	0.150
$\tau_z$	0.024	0.000	0.000	0.000
q	1.619	1.636	1.636	1.636
k	0.619	0.636	0.636	0.636
c	0.105	0.108	0.108	0.112
g	0.018	0.017	0.017	0.012
$\nu$	-18.308	-15.888	-14.352	-13.131
$\lambda$	22.154	23.466	25.921	26.262
i	0.031	0.032	0.032	0.032
$\varphi(i)$	0.010	0.010	0.010	0.010
f	0.164	0.167	0.167	0.167
g/c	0.174	0.154	0.154	0.111
w	1.886	1.910	1.910	1.910

TABLE 2. The outcome by different solution-concept

	Consistent	Inconsistent
$\tau_l$	0.200	0.200
$\tau_z$	0.195	0.000
q	1.497	1.636
k	0.497	0.636
c	0.087	0.108
g	0.029	0.017
$\nu$	0	-14.352
$\lambda$	31.893	25.921
i	0.025	0.032
$\mu(i)$	0.006	0.010
f	0.148	0.167
u	-2.655	-2.595
g/c	0.333	0.154
w	1.680	1.910

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