Financial stability from a network perspective

Leon Rincon, C.E.

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PROEFSCHRIFT

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CARLOS EDUARDO LEÓN RINCÓN

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PROMOTORES:	prof. dr. Ron Berndsen
	prof. dr. Luc Renneboog
OVERIGE COMMISSIELEDEN:	dr. Olivier De Jonghe
	dr. Ronald Heijmans
	dr. Fabiana Penas
	dr. Kimmo Soramäki
	prof. dr. Wolf Wagner

The prize is the pleasure of finding the thing out, the kick in the discovery, the observation that other people use it [my work]–those are the real things, the honors are unreal to me.

Richard Feynman (1981)
Nobel Prize in Physics, 1965

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Think about it. Most –if not all- aspects and issues in our daily life may be reduced to a network representation. Neurons form the complex network that is our brain; family, friends, and acquaintances form our social network; co-authors and co-citation form our bibliographic network; cars, buses, trains, ships, and airplanes form our transport network; power, water, waste, and communications are the base of our critical infrastructure network and our well-being. Examples of how all kinds of networks surround us or even inhabit within us could go on endlessly.

Likewise, acknowledging those who contributed to this dissertation could be very lengthy. However, as the network approach is nothing but a mapping technique for understanding complex systems, my intention is to be consistent throughout the dissertation: Even the acknowledgements will come as a network.

In this case, the network of contributors to this dissertation is a multi-layer one, in which persons and institutions acting in different moments and circumstances (i.e. the layers) of my life have shaped this research work. At the end, when we add all the layers we will have something with a fancy name: a multiplex, a network of elements interconnected by links of different types. This multiplex will exhibit the clusters that supported or fed this research effort. As in many other real-world networks, some participants may overlap across clusters and layers.

For the sake of keeping this acknowledgements as brief as possible, I will try to focus on the clusters, and not on persons and individuals. However, it will be inevitable to highlight (in alphabetical order) the role of those that I selfishly consider the most central elements in this network. Let's start.

About the academic and scientific layer of this multiplex I would like to thank the support and intellectual contributions of my PhD supervisors, the PhD Committee, my colleagues and supervisors at Banco de la República, my colleagues researching in network analysis around the world, my usual co-authors, and good professors at undergraduate and graduate level. Direct and indirect contributions worth highlighting were made by Joaquín Bernal, Ron Berndsen, Freddy Cepeda, Iftekhar Hasan, Karen Leiton, Clara Machado, Ricardo Mariño, Constanza Martínez, Serafín Martínez-Jaramillo, Silvia Juliana Mera, Fabio Ortega, Jhonatan Pérez, Luc Renneboog, Alejandro Reveiz, Miguel Sarmiento, Kimmo Soramäki, and Wolf Wagner.

Regarding the professional layer, I would like to emphasize how working at Banco de la República allowed me to achieve this academic and scientific honor. Banco de la República granted the funds, time and data necessary to undertake such amusing, interesting and fruitful research work. For this I will always be thankful to Joaquín Bernal, Pamela Cardozo, Clara Machado, Fernando Tenjo, José Tolosa, José Darío Uribe, and Hernando Vargas. Technical support by Carlos Cadena, Jorge Cely, and Santiago Hernández were also key for its completion.

It is also imperative to highlight the importance of the link between Banco de la República and Tilburg School of Economics and Management: The vision and support of both institutions allowed me to complete my PhD in what may be deemed an unusual way. Pursuing a PhD in the standard fashion was too costly in professional and personal terms. As put forward by renowned mathematical physicist Freeman Dyson[1], the standard PhD “was invented as a system for educating German professors in the 19th century, […] takes too long and discourages women from becoming scientists […] it’s good for a very small number of people who are going to spend their lives being professors”. My plan has never been to be a full-time professor –at least not at this age. Yet, research as a way to attain Feynman’s *pleasure of finding the thing out* is all to me, not only in finance and economics, but also in my own life. Thus, instead of spending four to five years abroad, I kept my desk and duties as Research & Development Manager at the Financial Infrastructure Oversight Department, and allowed my wife and kids to keep on with their life as usual. In this sense, the link between Banco de la República and Tilburg School of Economics and Management has been most important for this research project. Both institutions’ executive and administrative staff deserves my sympathy and gratitude for their vision and support throughout the project. I foresee that this link between Tilburg School of Economics and Management and Banco de la República will strengthen for the greater good of sound applied research on finance and economics.

A special layer is dedicated to those who have spurred my intellectual growth at some stage of my life. The intellectual affinity with Alejandro Reveiz, which has yielded (and will always yield) so many rewarding and challenging ideas; the vision and fortitude of Clara Machado; the early work of Freddy Cepeda on local financial networks; and the early academic support and training by Beethoven Herrera, are most appreciated at this stage of my academic and professional career. My most sincere gratitude for the continuous guidance and backing from Luc Renneboog throughout the writing of the articles and during the entire PhD program.

[1] The interview of F. Dyson is available in Quanta Magazine. http://www.quantamagazine.org/20140326-a-rebel-without-a-ph-d/

And last but not least, my family. I am grateful for the time granted; the patience during my trips to Tilburg and the other cities where these articles were put to test; and for the loving support received at home. At the end, all my effort becomes meaningful through the eyes of my lovely daughter and son, Luciana and Juan Felipe; through the love of my beautiful wife, Martha; and through the memories of my life along my mother and brother. To my family, all my mind and soul.

Carlos León

Bogotá, December 2014

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Not much effort has been devoted to describing economic systems in an unbiased, detached way, as one would describe, say, an ant's nest.

Per Bak (1996)

1. Introduction

The map is not the terrain. We all know that. But we keep forgetting it. Financial stability is no exception: Policy and decision making tend to rely on a deceitful belief that our most-celebrated models (i.e. the maps) are fair representations of financial markets (i.e. the terrain). However, as demonstrated in the global financial crisis, the roughness and complexity of the financial terrain exceeded the elegance and simplicity of conventional maps.

The network perspective provides new mapping techniques for financial markets. Instead of taking a reductionist approach to financial markets, the network perspective allows analyzing the financial system as a whole, as a large number of mutually interacting financial firms that organize in a complex manner. Also, instead of making plain assumptions about how a handful of hypothetic representative individual financial firms relate to each other, the network perspective allows working with the enormous amount of factual market data that is produced when a large number of financial firms interact among them.

In this vein, this dissertation presents five articles that make use of network analysis for better understanding financial markets. The goal is to have a better map of financial markets by updating what we know about financial networks' architecture, and to contribute to related literature by linking the findings to financial stability.

1.1. Background

As put forward by Barabási (2003), economic theory has considered agents as interacting not with each other but rather with "the market", a mythical entity that mediates all economic transactions (e.g. Adam Smith's invisible hand). However, the market may be depicted as a weighted and directed network, with economic agents as nodes, and with interactions (i.e. transactions, exposures) as links among them; therefore, *the structure and evolution of this weighted and directed network determine the outcome of all macroeconomic processes* (Barabási, 2003).

Perhaps the first and most well-known approach to financial markets from a network perspective is that of Allen and Gale (2000). Based on the assumption of homogeneous (i.e. similarly interconnected) and equally behaved banks, Allen and Gale (2000) set the standard model for the analysis of financial contagion. Several influential papers that followed (Freixas et al., 2000; Cifuentes et al., 2005; Nier et al., 2008; Gai and Kapadia, 2010; Battiston et al., 2012a) embraced the same assumptions to some extent, and converged –*ceteris paribus*- to the existence of diversification or absorption effects due to the dispersion of shocks within larger or denser financial networks. Accordingly, examining direct linkages generally results in more connections reducing the risk of contagion (Allen and Babus, 2008).

However, Allen and Gale (2000) also demonstrated that contagion effects depend on the pattern of connections between banks. Unfortunately, it seems reasonable that the lack of observed market data for examining the pattern of connections back then, along with the elegance of assuming the existence of representative (i.e. homogeneous and equally behaved) agents, forced the simplest model to be the standard one. As in Miller and Page (2007, p.84), *homogeneity is not a feature we often observe in the world but rather a necessity imposed on us by our modeling techniques*.

Paradoxically, Barabási and Albert (1999) had recently documented that the homogenous networks of random graph theory (Erdös and Rényi, 1960) did not fit real-world data. As a homogeneous network was the mainstay of the standard model for direct financial contagion, the empirical evidence of Barabási and Albert suggested that the standard model might be elegant, yet too simplistic.

Boss et al. (2004) and Inaoka et al. (2004) provided early empirical evidence against the assumption of homogeneous financial networks. Further evidence surfaced shortly (see Renault et al. 2007; Soramäki et al., 2007; Becher et al., 2008; Cepeda, 2008; Iori et al., 2008; May et al., 2008; Pröpper et al., 2008; Haldane, 2009; Martínez-Jaramillo et al., 2012; Craig and von Peter, 2014; Fricke and Lux, 2014; in 't Veld and van Lelyveld, 2014).

As in most real-world networks, financial networks display right-skewed distributions, in which most financial firms have a few connections, and few financial firms concentrate a lot of connections. In those networks the extraction or failure of a participant will have significantly different outcomes depending on how the participant is selected. When randomly selected, the effect will be negligible, and the network may withstand the removal of several randomly selected participants without significant structural changes; however, if selected because of their high connectivity, the effect of extracting a small number of participants may significantly affect the network's structure. Thus, as in Haldane (2009), financial networks have been characterized as *robust-yet-fragile*.

Nowadays the inhomogeneous connective structures of financial networks, and the corresponding robust-yet-fragile nature of financial markets, are well-established stylized facts. Although the traditional random network model has not been abandoned, the inhomogeneous model may be considered as the most updated map for financial markets, and the benchmark for financial networks analysis.

1.2. Research Questions

Although other features may further characterize financial networks (e.g. clustering, small average distance between participants), their most salient trait is an inhomogeneous connective structure, which also concurs with financial institutions' well-documented inhomogeneity in size (Gabaix et al., 2013; León, 2014). Therefore, it is expected that those few heavily linked participants connect to participants across the whole network.

However, network theory has evolved after the identification of real-world networks' inhomogeneous distribution of connections. Two interesting strands have called the attention of network scientists: the networks' hierarchical structure, and the relations between different networks.

Regarding the first strand, network science has identified hierarchical structures in real-world networks. Typically, these hierarchies take the form of modules, which may be described as densely interconnected communities that tend to be sparsely connected to other communities. As highlighted by Anderson (1999), because communities are sparsely connected under a hierarchical structure, most components or subsystems receive inputs from only a few of the other components, making the whole system resilient; this is, change (e.g. shocks, information, contagion) in modular hierarchical systems tends to be limited or isolated to local communities.

Many cases of real-worlds exhibiting a modular hierarchy have been reported. Recent studies about the human connectome (i.e. neural connections in the brain) have identified that our brain displays a modular architecture in which a "rich-club" of regions serve as "brain hubs" for the entire network (see van den Heuvel and Sporns (2011)). In social contexts, modularity has been identified as a byproduct of densely interconnected groups of individuals sharing similar opinions and cultural interests (see Assenza et al. (2011)). Simon (1962) identifies modularity as a characteristic of biological systems and formal organizations (e.g. a firm with formal authority relations). In the management of forests and utility grids, and the design of computers and the World Wide Web, modularity is a salient and intentional feature that protects systemic resilience (Haldane and May, 2011).

About the second strand, network science has acknowledged that networks do not exist in isolation. Networks interconnect to other networks such that there are *networks of networks*. Two main types of such networks of networks have been examined: *Multiplex networks*, consisting of multi-layer networks containing participants of one sort but with several kinds of connections between them (Baxter et al., 2014), and *interacting networks*, consisting of multi-layer networks of distinct types of participants that relate across networks[1]. Preliminary results show that analyzing complex systems as a network of coupled networks may alter the basic assumptions that network theory has relied on for single-layer networks (Kenett et al., 2014). Also, coupling networks allows for modeling contagion as a non-isolated effect that may affect several otherwise independent systems or networks, say how the power network may affect the communications network (and vice versa).

Cases of multi-layer networks have been studied rather recently. For instance, transport systems (Kurant and Thiran, 2006; Cardillo et al., 2013), electrical networks (Pahwa et al., 2014), physiological systems (Ivanov and Bartsch, 2014), critical infrastructures (Martí, 2014; Rome et al., 2014) and cooperation networks (Gómez-Gardenes et al., 2012). About financial networks, Montagna and Kok (2013) model interbank contagion in the Eurozone with a triple-layer multiplex network, whereas Bargigli et al. (2013) examine the Italian interbank multiplex network by transaction type.

[1] Put another way, multiplex networks reveal how single-type participants (e.g. financial institutions) relate to each other in different environments (e.g. markets). On the other hand, interacting networks reveal how distinct layers, corresponding to different types of participants, couple between them. In the interacting networks the layers couple because their participants connect across layers, whereas in the multiplex the layers couple because of participants' overlapping across layers.

Both strands have deep implications for financial stability. If modules or communities make networks resilient due to their ability to isolate contagion, it is of utmost importance to test whether (or not) modularity is a salient feature of financial networks. Also, if financial networks architecture combines scale-free and modular features, well-connected financial institutions should not be connected throughout the whole network, but well-connected to their module and to other well-connected financial institutions.

Likewise, if networks' coupling changes the main properties of financial networks, financial markets' maps should be updated as well. Moreover, the ability to link financial institutions across different layers (i.e. markets) will allow for modeling contagion as a multi-market issue.

Therefore, three main research questions are addressed in this dissertation. First, do financial networks concur with the inhomogeneous connective structure reported as a stylized fact of financial networks? Second, do financial networks display a hierarchical structure in the form of sparsely interconnected communities of dense interaction? Third, do Colombian financial networks organize in networks of networks? Based on the corresponding findings, each article addresses the main implications for understanding financial markets, with emphasis on the implications for financial stability. In this vein, an updated map of financial markets may change policy- and decision-making by financial authorities, and change models used by academics studying financial markets.

1.3. Overview

The five articles in this dissertation address the research questions in an independent yet articulated form. They all share a common methodological framework based on network analysis basics, whereas some specific methods are introduced and implemented according to the goal of each article.

The first article, in Chapter 2, is a first attempt to break down a single market (i.e. the Colombian sovereign securities market) into different layers of interaction corresponding to distinct trading and registering platforms existing in the local financial market. The main objective is to build a network of networks in the Colombian case, and to try to examine how single-layer networks differ from each other and from the resulting multiplex network. Results will be relevant for financial stability because the role of well-connected financial institutions across networks will be revealed, whereas the connective structure of financial institutions' interactions under different economic and operational environments will be examined.

The second article, in Chapter 3, addresses an overlooked issue: How to measure the importance of financial market infrastructures within their corresponding network. Unlike most financial networks, in which the interactions occur between financial institutions, this chapter builds a network with the interactions between those multilateral systems used for the purposes of executing, exchanging, clearing, settling or recording payments, securities, derivatives, or other financial transactions, commonly known as financial market infrastructures.[2] Building and analyzing such network is relevant for financial stability because of financial market infrastructures' role in the safe and efficient functioning of financial markets. Identifying those financial market infrastructures whose failure or impairment could trigger greater disruptions in the financial system and economic activity may serve the purpose of assisting financial authorities in focusing their attention and resources –the intensity of oversight, supervision and regulation- where the infrastructure-related systemic impact is estimated to be the greatest. The article in Chapter 3 is already published in the Journal of Financial Market Infrastructures (Vol. 2 (3), 2014).

[2] Financial institutions correspond to depository institutions (e.g. banks), broker-dealers, investment companies (e.g. mutual funds), insurance companies, and credit unions. Financial market infrastructures correspond to multilateral systems providing trading, clearing, settling, recording, and compressing services for transactions between financial institutions.

The third article, in Chapter 4, examines the Colombian interbank funds market. The main purpose of this chapter is to study the connective and hierarchical structure of the Colombian non-collateralized money market, and to use an information retrieval algorithm for identifying those financial institutions that simultaneously excel at borrowing and lending central bank's liquidity (i.e. super-spreaders). Also, by estimating a probit model, the main features that determine the probability of being a super-spreader are examined. Instead of limiting the participants to financial institutions, our approach includes central bank's monetary policy transactions (i.e. open market operations via repos); despite central banks are the most important participants in interbank markets (Allen et al., 2009; Freixas et al., 2011), most literature on interbank networks exclude them. Results are important for financial stability. Examining the connective structure of the interbank funds network provides the central bank with additional elements for the implementation of monetary policy, whereas identifying the liquidity super-spreaders and their main determinants allows pinpointing which financial institutions are those contributing to liquidity transmission the most, but also those that may distort the distribution of central banks' liquidity the greatest.

The fourth article, in Chapter 5, updates what we know about three different financial networks in the Colombian case. This chapter intends to test whether or not financial networks may be depicted as displaying a modular scale-free network, an architecture that has been identified as ubiquitous in many other types of real-world networks. Accordingly, this article addresses –for the first time- the question regarding the presence of a modular hierarchy in financial networks, and discusses the main implications for financial stability. As stated before, the presence of modularity conveys resilience to systems because contagion tends to be limited or isolated to local communities, and underscores the complexity of dealing with heavily connected financial institutions because they are the source of networks' fragility but also the source of their resilience. Results are most valuable for financial authorities pursuing financial stability as they support the macro-prudential dimension of financial stability. The article in Chapter 5 is already published in the Journal of Financial Stability (Vol.15, 2014).

The fifth article, in Chapter 6, makes an analytical link between Chapter 3 and Chapter 5. By building a two-layer network composed of financial institutions and financial market infrastructures, this article captures –for the first time- how financial transactions do not occur directly (i.e. bilaterally) between financial institutions, but through the settlement services provided by the corresponding financial market infrastructures. In this sense, this chapter explicitly models the role of financial market infrastructures as financial markets' "plumbing", and recognizes that traditional analysis of financial institutions networks is of a virtual or logical nature. Based on the proposed two-layer network, this article examines whether or not the modular scale-free architecture of financial institutions network of Chapter 5 is preserved after considering the role of financial market infrastructures. Examining the main connective and hierarchical features of this two-layer network is relevant for financial stability: It will identify the role of financial market infrastructures in the robustness, resilience and fragility of financial systems, and their role in financial contagion.

1.4. The Datasets

Banco de la República (the Colombian Central Bank) provided all the information used to build the financial networks in this dissertation. In this case the information gathered by Banco de la República is particularly rich in both cross-section and time-series dimensions. It served the purpose of building and analyzing financial institutions' single- and multi-layer networks. Most importantly, the information also allowed building and analyzing for the first time financial market infrastructures' networks, and a network comprising financial institutions and financial market infrastructures. In this sense the datasets used in this dissertation are unique, and allowed contributing to literature on financial networks and financial stability.

Using datasets pertaining to a medium size emerging market may appear restrictive at first. Nonetheless, there are several advantages arising from working with data from Colombia (or other non-industrialized countries). First, in the Colombian case the central bank owns and manages the most important data sources of financial transactions (i.e. the large-value payment system and the sovereign securities settlement system), and it is also responsible for the oversight of all other clearing and settlement systems; therefore, all transactional data is rather centralized, and may be conveniently articulated to build several types of single- and multi-layer financial networks. Second, as the linkages between the Colombian financial system and those abroad are rather limited, working with local financial networks is comprehensive yet parsimonious for analytical purposes.

However, using Colombian datasets raise one key question: How representative are the results attained? As this type of methods and analysis is rather new, it is difficult to test whether or not the results here presented may be considered as stylized facts of financial systems around the globe. As discussed throughout the dissertation, the overlap with some other research works on financial networks would preliminarily suggest that results should not be an isolated case. New research will demonstrate whether our findings are stylized facts or not. Either way, the outcome of this dissertation is just a first step towards an updated map of financial markets, and not the definitive one (if such thing does exist).

1.5. Contributions

As previously argued, this dissertation aims at providing an updated map of interactions among financial institutions and financial market infrastructures.

This updated map will help academics and authorities to better understand financial markets. Several specific contributions are worth discussing.

1.5.1. The Macro-Prudential Dimension of Financial Stability

Traditional (reductionist) understanding of financial systems has relied on the individual understanding of financial firms, which has been known as the *micro-prudential* dimension of financial stability (Crocket, 2000; De Nicolò et al., 2012). In this dimension, as highlighted by Crockett (2000), *financial stability is ensured as long as each and every institution is sound*. Consequently, the policy objective of the micro-prudential dimension of financial stability is –in fact- limiting *idiosyncratic risk*.[3]

In contrast, attaining financial stability by limiting *systemic risk* requires a comprehensive approach to the financial system and the economy as a whole. In this approach systemic risk is a negative externality. Financial market's participants have clear incentives to manage their own risk (e.g. credit, market, legal, operational, etc.), but no incentives exist for them to account the effects of their actions on other institutions or the system as a whole (León et al., 2012). As in Trichet (2009), each individual institution is clearly motivated to prevent its own collapse but not necessarily the collapse of the system as a whole.

One lesson from the crisis is that interconnectedness among banks and other financial institutions can generate externalities with adverse effects on the real economy (De Nicolò et al., 2012). In this vein, the network perspective allows to assess the externalities posed by financial institutions and financial market infrastructures in the corresponding market. First, as it allows identifying the actual connective and hierarchical structure of the financial network, the extent of

[3] However, as stated by De Nicolò et al. (2012), ensuring the stability of each institution individually can at times destabilize the system as a whole.

potential contagion among financial institutions may be examined. Second, as the importance of each financial institution or market infrastructure for the financial network may be estimated, the network perspective provides quantitative tools for assessing the externalities that arise from financial linkages. In this sense, the network perspective allows identifying systemically important financial institutions due to the potential externalities stemming from their interconnectedness.

For instance, centrality measures such as those discussed and implemented in chapters 2, 3, and 4, may serve financial authorities to design system-calibrated capital and/or liquidity surcharges. The most evident case is that of interbank funds network super-spreaders (Chapter 4): Those financial institutions that excel as contributors to global borrowing and lending may severely disrupt the liquidity transmission in case they fail, thus they should be required to maintain higher buffers of liquidity or capital in order to be a source of stability amid adverse market conditions.

Requiring system-calibrated capital or liquidity requirements is a way to internalize the implications arising from financial institutions' role within financial networks. Instead of increasing capital or liquidity requirements uniformly to all financial institutions, using network centrality measures may provide an objective quantitative framework for designing surcharges corresponding to their contribution to systemic risk. This overlaps with recent proposals consisting of imposing a "super-spreader tax" based on network centrality to reduce potential socialized losses (see Markose (2012) and Markose et al. (2012)).

1.5.2. Pursuing Financial Stability in a Modular Scale-Free Financial Network

The financial networks examined in this dissertation may be depicted as approximating the inhomogeneous and hierarchical architectures exhibited by most real-world networks, including actual financial networks. In this sense, the inhomogeneous distribution of links and of their intensity, presumably approximating a power-law or other type of particularly skewed distribution, along with the existence of some sort of hierarchical organization (e.g. core-periphery), matches what has been pinpointed as stylized facts of financial networks (see Fricke and Lux, 2014).

The overlap of our findings with the reported stylized facts of financial networks contributes with additional evidence against the homogeneity assumptions of conventional direct contagion models based on the seminal work of Allen and Gale (2000). Nevertheless, findings depart from the core-periphery hierarchical structure reported by Craig and von Peter (2014), Fricke and Lux (2014), in 't Veld and van Lelyveld (2014), and Wetherilt et al. (2010): Evidence suggests that a modular hierarchy may be a plausible alternate model within the modular scale-free architecture suggested by Barabási (2003).

A modular scale-free architecture of financial networks conveys interesting features for financial stability. As modularity contributes to the resilience of systems by limiting cascades and isolating feedbacks, financial networks exhibiting a modular scale-free architecture may be considered as robust and resilient, yet fragile. This updates the depiction of financial networks as robust-yet-fragile as reported by Haldane (2009).

Moreover, as well-connected financial institutions lead modules, their role in the resilience and robustness of financial network should be highlighted. The evidence of modularity should dissuade financial authorities from dismantling or downsizing systemically important financial institutions, a popular –yet naïvedemand after the crisis that may backfire in the form of a less robust financial network. Instead, systemic-calibrated prudential requirements (e.g. capital, liquidity) should be designed and imposed to strengthen well-connected financial institutions, with the ultimate aim of enhancing the ability of the observed architecture to limit cascades and isolate feedbacks.

1.5.3. Multi-Layer Financial Networks

Evidence reported in this dissertation points out the usefulness of the network perspective of financial stability. However, it is particularly important to acknowledge that network analysis is already heading to additional levels of complexity, such as the coupling of networks. The network of networks strand delivers an unprecedented framework for integrating different markets and economic environments, with several analytical benefits.

As documented in this dissertation, examining and analyzing single-layer networks in isolation may be misleading (see Chapter 2): Non-linear effects and hidden connective features arise when aggregating financial networks. Results also reveal that the importance (i.e. centrality) of financial institutions tends to overlap across networks, thus emphasizing the role of well-connected participants as conduits across different markets or economic environments.

Likewise, amid the unique datasets available, the multi-layer network approach allowed building an interacting network that examines how financial institutions and financial market infrastructures couple (see Chapter 6). Following the early introduction of financial market infrastructures' network as a metaphorical multi-floor warehouse by Berndsen (2011), the resulting interacting network provides a comprehensive and genuine picture of how financial transactions are settled, whilst revealing the role of financial market infrastructures in the safe and efficient functioning of financial markets.

1.5.4. The Role of Financial Market Infrastructures in Financial Stability

Although the importance of financial market infrastructures' well-functioning has been documented before (Bernanke, 2011; CPSS and IOSCO, 2012; Dudley, 2012a,b), financial networks literature does not deal with this type of financial market participants. In contrast, this dissertation examines the network of Colombian financial market infrastructures and assesses their systemic importance (Chapter 3), and also couples this network with financial institutions' network (Chapter 6).

The results highlight the unique features of financial market infrastructures as the "plumbing" of financial systems. Their role in the different stages of the execution of financial transactions is key for understanding their importance for financial stability as critical infrastructures and as potential conduits for contagion across financial markets. Also, the results emphasize how the well-functioning of financial market infrastructures is critical for modularity –and its advantages- to exist in financial networks.

1.5.5. Strengthening Financial Authorities' Supervision and Oversight Functions

One of the main consequences arising from the great financial crisis is the demand for stronger supervision and oversight tools. Network analysis provides a comprehensive toolbox of statistics that may serve the purpose of monitoring financial markets' dynamics from a macro-prudential dimension of financial stability.

Some of the network statistics implemented and analyzed in this dissertation may allow financial authorities assessing the collective behavior and sentiments of financial institutions with a minimal lag. For instance, monitoring the dynamics of the networks' density and the average distance between financial institutions could be variables worth following in order to gauge market participants' willingness to take risk. In this sense, those statistics that capture the overall connective structure of financial networks may provide valuable information regarding how emergent system dynamics arise from individual behavior.

Financial institutions' centrality (i.e. importance) is also worth monitoring. From the simplest (e.g. number of connections) to the most compound centrality measures (e.g. PageRank, hub and authority centrality, betweenness), monitoring how financial institutions' importance within the market evolves is valuable for supervisory and oversight purposes. Sharp and baffling drops in centrality may signal the unwillingness of counterparties to engage in transactions (i.e. taking risk) or the shift of funding counterparties, thus may be regarded as an alternative source of market discipline information –besides market prices. On the other hand, steep increases in centrality may signal unusual risk taking by the market, or may point out the rise of a new heavily connected financial institution worth being followed more closely.

Literature on how to use financial networks' statistics and financial institutions' centrality measures as sources of information for supervisory and oversight purposes is missing. However, it is likely that research work at central banks and other financial authorities will fill this gap in the near future. This will strengthen financial markets supervision and oversight within the macro-prudential dimension of financial stability.

1.6. The Challenges Ahead

Each chapter presents a set of challenges to be addressed. However, there are some joint issues that arise as broad challenges for the network perspective of financial stability. Some of this challenges come in the form of how to profit from the results in order to enhance financial stability. For instance, a key issue is how financial regulation can effectively enhance modularity in order to make financial markets more robust and resilient. As argued in the dissertation, system-calibrated capital and liquidity surcharges may be an obvious strategy imported from epidemic theory that could strengthen central financial institutions. However, the design of such surcharges is still underway: Centrality scores, as those presented in chapters 2, 3, and 4, may be a starting point, but robust testing about their stability and performance is pending.

An interesting research extension results from the overlap between the generating process behind the modular scale-free network architecture and the mechanisms behind the core-periphery hierarchy documented in trading relationships literature. As suggested in Chapter 5, the cost related to maintaining a large number of counterparties is a common feature that may be further examined in order to link both financial networks' and trading relationships' literature in a constructive manner.

Assessing and testing the stability of the networks is also a challenge worth taking. Despite the networks' connective features tend to be stable overtime (see Chapter 5's appendix), the stability and predictability of linkages between financial institutions is still unconfirmed. Testing the stability of connections among financial institutions is valuable for related literature.

From a theoretical point of view, it is interesting to study whether there is an optimal level of inhomogeneity and clustering that balances efficiency and stability for financial networks. This is, network modeling should address the tradeoffs arising from changes in the architecture of financial networks.

The advantages of interconnectedness across countries and markets should be examined in forthcoming research works. As acknowledged by CPSS (2008), the evolution of the global payment and settlement network, which comprises cross-border relations among financial institutions and financial market infrastructures, has increased the potential for disruptions to spread quickly and widely across multiple systems. Therefore, there is a titanic challenge ahead: coupling the several layers that make part of the global payment and settlement network, which presumably will consist of an intricate mix of multiplex and interacting networks. In this case, the objective is to study the global financial network's structure and its link to financial stability.

More evidence is required to confirm whether some of the most important findings in this dissertation may be considered as stylized facts of financial networks or not. The uniqueness of the Colombian datasets used in this dissertation allows exploring new paths on financial network analysis, but it also complicates contrasts and comparisons with other countries' financial markets. New data from other countries, probably from other central banks or financial market infrastructures, will allow testing the universality of the findings here reported.

Finally, it is important to realize that several findings are intricately connected to fractal theory, complex adaptive systems, and self-organizing criticality. For the sake of making the articles fit standard economics and finance literature those connections were strategically concealed. A monumental challenge worth addressing is to study and take advantage of those connections in a comprehensive, robust and meaningful manner. The research works of Simon (1962), Gell-Mann (1994), Bak (1996), Krugman (1996), Holland (1998), Anderson (1999), Schweitzer et al. (2009), and Farmer et al. (2012), are most valuable for this challenge.

1.7. Final Considerations

As usual when writing research articles, two caveats are worth stating for all chapters of this dissertation. First, the opinions and statements are the sole responsibility of the authors and do not represent those of the institutions they belong to (i.e. Banco de la República, De Nederlandsche Bank, and Tilburg University). Second, all remaining errors are the authors' own.

All chapters were written independently. Each chapter is a stand-alone article that does not require a good knowledge of network analysis' basics. Yet, for that same reason, every chapter contains a section on network analysis basics, and it may turn recurrent throughout the dissertation.

The chapters may be read in any order. However, their sequence attempts to making the reading experience the most efficient and enjoyable, with the most thought-provoking at the end.

Regarding the picture in the cover, it is intended to link the traits of financial networks with other structures around and within us. I should acknowledge that the human connectome (i.e. the network of the human brain) image belongs to van den Heuvel and Sporns (2011), and was kindly provided by Martijn P. van den Heuve. I thank my brother, Mauricio, for his creative and technical assistance when composing the final image.

2

Another direction of improvement is to make physical machines three dimensional instead of all on a surface of a chip. That can be done in stages instead of all at once –you can have several layers and then add many more layers as time goes on.

Richard Feynman (1985)
Nobel Prize in Physics, 1965

2. A Multi-Layer Network of the Sovereign Securities Market

Abstract

After merging transactions from three different trading and registering individual platforms into a multi-layer network we study the network of Colombian sovereign securities settlements. Examining this *network of networks* enables us to confirm that (i) studying isolated single-layer trading and registering networks yields a misleading perspective on the relations between and risks induced by participating financial institutions; (ii) a multi-layer approach produces a connective structure consistent with most real-world networks (e.g. sparse, inhomogeneous, and clustered); and (iii) the multi-layer network preserves the main connective features of its constituent layers. The results highlight the importance of mapping and understanding how financial institutions relate to each other across multiple financial environments, and emphasize the critical role of too-connected financial institutions in financial stability due to their overlap across distinct layers.

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2.1. Introduction

Financial market infrastructures are considered the “plumbing” of financial systems (Bernanke, 2011). Financial market infrastructures responsible for settling transactions between financial institutions fulfill a pivotal position within that plumbing, thus they are of particular interest to disentangle and dissect this complex structure of financial systems by means of network analysis. Under this analytical framework, we study the local sovereign securities settlement infrastructure in order to attain a more detailed and actual illustration of how financial institutions organize themselves in the corresponding legal, operational, and economic contexts.

As transactions settled originate in different trading and registering platforms, a single network of settlements should necessarily aggregate networks from other financial market infrastructures. In this vein, the settlement network is a *network of networks*, which entails additional sources of complexity, critical for understanding financial markets. For instance, contagion across different financial layers may yield an intricate but extensive process that may further threaten financial stability by the direct coupling of otherwise distinct and distant financial markets. This is consistent with recent efforts to examine the consequences of considering financial systems as multi-layer networks, in which financial institutions interact across several environments or layers (e.g. markets, asset classes, trading and registering platforms, jurisdictions).

The theoretical development of multi-layer networks (and its empirical validation) is still in scaffolding, and consequently most literature on multi-layer financial networks is still preliminary. Nevertheless, there is a consensus regarding the need to examine how the basic assumptions of real-world networks are altered in a context of multi-layer networks or networks of networks. Most examinations have focused on how real-world networks’ coupling affects their well-documented inhomogeneous connective architecture, in which connections being distributed in an extremely skewed fashion (presumably following a power-law distribution) contribute to their robustness; as most participants are weakly connected, most real-world networks, either biological or man-made, tend to be robust to the random failure of their constituents (see Barabási (2003), Strogatz (2003), Newman (2010)).

In this sense, studying networks of financial networks allows to confirm if their individual well-documented *robust-yet-fragile* feature (Haldane, 2009) is also valid within a financial multi-layer network. Moreover, studying how single-layer financial networks' couple allows understanding the rationale behind the main connective features of financial multi-layer networks.

Consequently, in the light of the above view of the transactions in a settlement market infrastructure as a network of networks, this paper has two main purposes: First, we examine how Colombian sovereign securities' transactions that come from single-layer networks (corresponding to distinct trading and registering platforms) can be aggregated into a *multiplex network* of the sovereign securities settlement system. Second, we verify whether or not viewing a financial system as a multiplex network is a superior view in that it preserves the main connective properties of its constituents, while revealing some additional sources of complexity.

This paper contributes to the financial literature by examining the connective structure of financial institutions' interactions under different economic and operational environments, and by investigating how those interactions aggregate and result in the local sovereign securities' market. Moreover, in the context of the networks of networks literature, our work contributes to the study of financial market infrastructures by revealing that they critically depend on their interaction with other financial market infrastructures; in our case, the local sovereign securities settlement market infrastructure (DCV) depends on its interaction with trading and registering platforms (SEN, MEC and MEC-R). Finally, our work highlights that financial market infrastructures are a vital source of granular and timely data for examining single- and multi-layer networks as a way to better understand financial markets.

2.2. Background

Most efforts to characterize the topology of complex systems by means of network analysis have assumed that each system is isolated (i.e. non-coupled) from other networks. In the case of single-layer networks (i.e. monoplex networks) such efforts have converged to an inhomogeneous connective structure, typically in the form of the approximate power-law distribution of links and their weights of real-world networks. The power-law or Pareto distribution of links is commonly referred as a *scale-free network* (Barabási and Albert, 1999), and it corresponds to the most documented type of network in social, biological and man-made complex systems.[1] In the case of financial networks, most literature confirms their scale-free nature (Boss et al., 2004; Inaoka et al., 2004; Renault et al. 2007; Soramäki et al., 2007; Cepeda, 2008; May et al., 2008; Pröpper et al., 2008; Bech and Atalay, 2010; Bargigli et al., 2013; León et al., 2014; León and Berndsen, 2014), whereas other papers confirm their inhomogeneity but report a divergence from a strict power-law distribution of links (Martínez-Jaramillo et al., 2012; Craig and von Peter, 2014; Fricke and Lux, 2014; in 't Veld and van Lelyveld, 2014).[2]

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[1] In scale-free networks the number of connections is distributed as in a power-law (i.e. extreme heterogeneity and skewness), with a few heavily connected participants and many poorly connected participants. Due to this particular type of inhomogeneity there is no typical participant in the network, thus it has no scale (i.e. it is scale-free or scale-invariant).

[2] As in in 't Veld and van Lelyveld (2014), the power-law distribution of links is an asymptotic property, thus testing the scale-free features of a network is problematic. In this sense, a strict match between observed and expected theoretical properties for determining the scale-free properties of non-large networks –as those examined here- may be impractical.

Yet, as highlighted by Cardillo et al. (2013), *many biological and man-made networked systems are characterized by the simultaneous presence of different sub-networks organized in separate layers, with connections and participants of qualitatively different types* (p.1). This multi-layered nature of networks, also known as network of networks, has been the focus of network scientists rather recently, among which Kurant and Thiran (2006) provided one of the most seminal contributions.

Research on multi-layer financial networks has recently emerged. The standard multi-layer framework in finance corresponds to the so-called multiplex network, which may be described as networks containing participants of one sort but with several kinds of connections between them (Baxter et al., 2014).[3] Figure 1 displays a two-layer network composed by layers X and Y, and the multiplex (Z) resulting from merging X and Y. Vertical lines connecting superimposed vertexes are the participants, whereas each vertex has a function in the corresponding layer.

[3] Several applications of multiplex networks have been documented for non-financial complex systems, such as transport systems (Kurant and Thiran, 2006; Cardillo et al., 2013), electrical networks (Pahwa et al., 2014), physiological systems (Ivanov and Bartsch, 2014), critical infrastructures (Martí, 2014; Rome et al., 2014) and cooperation networks (Gómez-Gardenes et al., 2012). Yet, other types of multi-layer networks are available (e.g. with layers containing participants of different sorts).

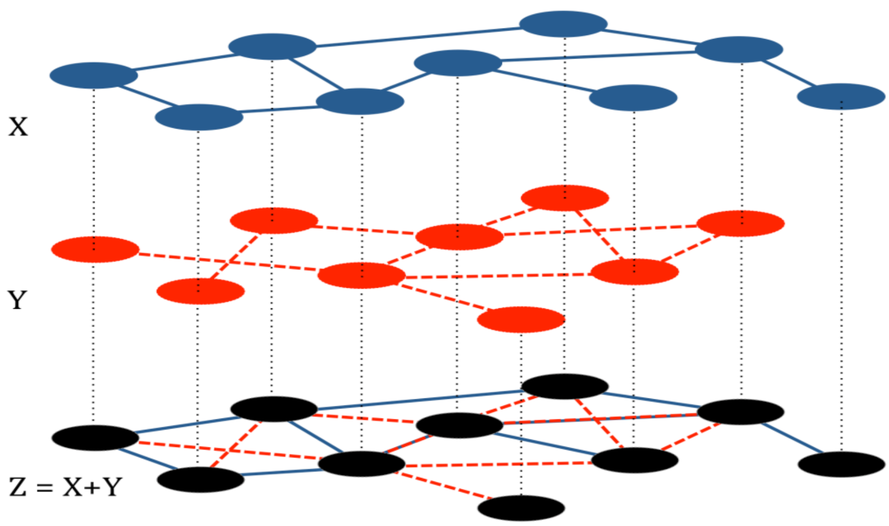

Figure 1. A multiplex network. Two-layer networks, X and Y, and the multiplex (Z) resulting from merging X and Y. Vertical lines connecting superimposed vertexes are the participants, whereas each vertex is a role in the corresponding layer.

Montagna and Kok (2013) model interbank contagion in the Eurozone with a triple-layer multiplex network consisting of long-term direct bilateral exposures, short-term bilateral exposures, and common exposures to financial assets. Bargigli et al. (2013) examine the Italian interbank multiplex network by transaction type (i.e. secured and non-secured) and by maturity (i.e. overnight, short-term, and long-term).

In our case, we examine the Colombian sovereign securities' market. We use a triple-layer multiplex network corresponding to the three Colombian sovereign securities' trading and registering environments (i.e. SEN, MEC, MEC-R), that altogether meet in the settlement market infrastructure for local sovereign securities: DCV.[4] As is the case in other countries, the sovereign securities settlement infrastructure is owned and operated by the central bank (Banco de la

[4] *Sistema Electrónico de Negociación* (SEN, Electronic Trading System); *Mercado Electrónico Colombiano* (MEC, Colombian Electronic Market); *Módulo de Registro del Mercado Electrónico Colombiano* (MEC-R, Colombian Electronic Market Register Module); *Depósito Centralizado de Valores* (DCV, Centralized Securities Depository).

República), and works under a delivery versus payment setting, and it interacts with the large-value payment system within a real-time gross settlement framework.

As our network of networks deals with three different monoplex networks composed by a single type of participant (i.e. financial institutions) but distinct kinds of connections corresponding to different trading and registering platforms with divergent operational and economic features, it may be well defined as a multiplex network. Accordingly, the literature suggests that working on multi-layer networks may alter the basic assumptions that according to network theory lie at the basis of single networks (Kenett et al., 2014).

Contrary to what one would expect, the literature shows that the coupling of scale-free networks may yield a less robust network (Buldyrev et al., 2010; Gao et al., 2012). Exceptions to this finding would occur when the number of links (i.e. the *degree*) of interdependent participants coincides across the layers. This is, scale-free networks' robustness is likely to be preserved if *positively correlated multiplexity* exists, such that a high-degree vertex in one layer likely is high-degree in the other layers as well (Kenett et al., 2014; Lee et al., 2014).

Our results contribute to the related literature in three ways. First, our results help to understand the economics of financial market infrastructures as the collective function of several layers of interaction between financial institutions. In our case, the settlement network is envisaged as a collection of the transactions routed via distinct trading and registering platforms with different economic and operational features. Second, our empirical analysis illustrates how the main features of individual networks aggregate into a multiplex network. Third, the multi-layer network reveals that contagion across different financial layers may yield an intricate but extensive process that may further threaten financial stability by the direct coupling of otherwise distinct and distant financial markets and their participants (i.e. *cross-system risk*[5]). As obtaining empirical data on multi-layer networks is particularly difficult (D'Souza et al., 2014), and given the novelty of multi-layer network analysis, these three contributions could be valuable for financial authorities, market practitioners, and the academics.

2.3. The Dataset

The clearing and settlement of local sovereign securities market takes place in DCV. Colombia's Central Bank owns and operates DCV, a financial market infrastructure that is both the local sovereign securities' clearing and settlement system, and also their central securities depository. Working on a real-time gross settlement system and a delivery-versus-payment mechanism, DCV clears and settles spot market transactions, repos and sell-buy back transactions. DCV is the second most systemically important local financial market infrastructure according to León and Pérez (2014), only surpassed by the large-value payment system.

Most transactions in DCV result from three different Colombian sovereign securities' trading and registering platforms: SEN, MEC and MEC-R.[6] SEN is the main local sovereign securities' trading platform, owned and operated by Colombia's Central Bank, in which a group of 14 market makers trade and settle anonymously, and free of any counterparty limit.[7] MEC is an anonymous trading platform privately owned and operated by the Colombian Stock Exchange (*Bolsa de Valores de Colombia*), which also serves as trading platform for other non-sovereign fixed income securities. SEN and MEC, are both multilateral trading

[5] Cross-system risk corresponds to the potential effects caused by a financial institution experiencing problems across different markets or layers (CPSS, 2008).

[6] Other trading platforms exist (i.e. GFI, Icap and Tradition), but their contribution has always been practically nil, which is why they are excluded from the analysis.

[7] SEN also provides an open trading platform for non-market makers and a registering platform for over-the-counter trades, but it is not used.

platforms, but differ in particular issues. SEN is a "rich-club" of participants brought together by the Ministry of Finance into a market-maker scheme, in which they trade and settle anonymously without counterparty limits, whereas MEC is an open platform that allows participants to manage counterparty risk by imposing limits amid an anonymous trading environment.

Unlike MEC and SEN, MEC-R is not a multilateral trading platform; it is a facility provided by the Colombian Stock Exchange that enables local over-the-counter (OTC) sovereign securities market transactions to be registered for settlement purposes. Transactions registered in MEC-R come from trades agreed outside the electronic trading systems (e.g. by phone), in which counterparty limits play a vital role due to its bilateral (i.e. non anonymous) nature.

DCV's daily transactions corresponding to the third quarter of 2013 (i.e. from July 2 to September 30, 63 days) are used to construct the monoplex and multiplex networks analyzed in this paper. The choice of our data is induced by practical reasons, such as that fact that it is a recent period that does not display seasonal effects (e.g. Easter, Christmas, end of the year, large firms' tax collection deadlines) and that can be considered typical according to the recent local sovereign securities' market dynamics.[8] Our database consists of 35,775 consolidated registers between 159 financial institutions.

[8] The local sovereign securities' market has not experienced structural shifts as a consequence of recent external events such as the Global Financial Crisis. With regard to the local market, the failure of one of the largest and most connected broker dealers in the local financial market in November 2012 is the only recent event worth mentioning. Based on our previous research on the Colombian sovereign securities' network the choice of the sample period for studying its main structural features is rather trivial as long as well-known seasonal effects are avoided.

2.4. Methodological Approach

We examine the main connective features of the monoplex networks and the resulting multiplex network. Particularly, we try to determine whether or not the monoplex and multiplex networks follow the sparse, scale-free, small-world, and clustered patterns that are ubiquitous in real-world networks. Therefore, each network is examined for its main connective features, while each participant is evaluated for its importance (i.e. centrality) within every network.

2.4.1. Main Connective Features of the Network

A network, or graph, represents patterns of connections between the parts of a system. The most common mathematical representation of a network is the *adjacency matrix*. For a directed network, in which the direction of the linkages between n elements or vertexes is relevant, the adjacency matrix (A) is a square matrix of dimensions $n \times n$, potentially non-symmetrical, with elements A_{ij} such that

$$A_{ij} = \begin{cases} 1 \text{ if there is an edge from } i \text{ to } j, \\ 0 \text{ otherwise.} \end{cases} \quad (1)$$

It may be useful to assign real numbers to the edges. These numbers may represent distance, frequency or value, in what is called a weighted network and its corresponding weighted adjacency matrix.

The simplest metric for approximating the connective pattern of a network is *density* (d), a statistic that measures its cohesiveness. Let n be the number of participants or vertexes in the network, the density of a graph with no self-edges is the ratio of the number of actual edges (m) to the maximum possible number of edges (2).

$$d = \frac{m}{n(n-1)} \tag{2}$$

By construction, the density is restricted to the $0 < d \leq 1$ range. Formally, Newman (2010) states that a sufficiently large network for which the density (d) tends to a constant as n tends to infinity is considered as *dense*, whereas a network for which the density tends to zero as n tends to infinity is called *sparse*. However, as one frequently works with non-sufficiently large networks, networks are commonly characterized as sparse when the density is much smaller than the upper limit ($d \ll 1$), and as *dense* when the density approximates the upper limit ($d \cong 1$).

An informative alternative to density is to examine the degree's probability distribution ($\mathcal{P}_k$), with degree (k_i) corresponding to the number of edges or links of the i-vertex. Such distribution provides a natural summary of the connectivity in the graph (Kolaczyk, 2009). Akin to density, the first moment of the distribution of degree (μ_k) measures the cohesion of the network, and is usually restricted to the $0 < \mu_k < n-1$ range. A sparse graph has an average degree that is much smaller than the size of the graph ($\mu_k \ll n-1$).

Most real-world networks display right-skewed distributions, in which the majority of vertexes are of very low degree, and few vertexes are of very high degree, hence inhomogeneous. Such right-skew of real-world networks' degree distributions has been found to approximate a power-law distribution, in what is commonly known as scale-free networks (Barabási and Albert, 1999).

The power-law (or Pareto-law) distribution suggests that the probability of observing a vertex with k edges obeys the potential functional form in (3), where z is a (arbitrary) constant, and γ is known as the *exponent* of the power-law.

$$\mathcal{P}_k \propto z k^{-\gamma} \tag{3}$$

According to Newman (2010), values in the range $2 \leq \gamma \leq 3$ are typical of scale-free networks, although values slightly outside this range are possible and are occasionally observed. On the other hand, values much greater than 3 are considered typical of homogeneous or random networks.[9]

Regarding the small-world feature (i.e. every vertex can be reached in a limited number of steps), the mean geodesic distance (ℓ) reflects the global structure and measures how big the network is. This average distance depends on the way the entire network is connected, and cannot be inferred from any local measurement (Strogatz, 2003). Let g_{ij} be the *geodesic distance* (i.e. the shortest path in terms of number of edges) from vertex i to j, the mean geodesic distance for vertex i (ℓ_i) corresponds to the mean of g_{ij}, averaged over all reachable vertexes j in the network, as in (4). Respectively, the mean geodesic distance or average path length of a network (i.e. for all pairs of vertexes) is denoted as ℓ (without the subscript), and corresponds to the mean of ℓ_i over all vertexes.

$$\ell_i = \frac{1}{(n-1)} \sum_{j(\neq i)} g_{ij} \qquad \ell = \frac{1}{n} \sum_{i} \ell_i \tag{4}$$

The mean geodesic distance (ℓ) of random or Poisson networks is small, and increases slowly with the size of the network. Therefore, as stressed by Albert and Barabási (2002), random graphs are *small-world* because, in spite of their often large size, in most networks there is relatively a short path between any two vertexes. According to Newman et al. (2006), ℓ approximates $\ln n$ for random networks ($\ell \sim \ln n$), where such slow logarithmic increase with the size of the network coincides with the small-world effect (i.e. short average path lengths). In

[9] Random networks correspond to those originally studied by Erdös and Rényi (1960), in which connections are homogeneously distributed between participants due to the assumption of exponentially decaying tail processes for the distribution of links –such as the Poisson distribution. This type of network, also labeled as “random” or “Poisson”, was –explicitly or implicitly- the main assumption of most literature on networks before the seminal work of Barabási and Albert (1999) on scale-free networks.

the case of scale-free networks, the mean geodesic distance has been found to be smaller than $\ell \sim \ln n$, about $\ell \sim \ln\ln n$, which Cohen and Havlin (2003) refer to as *ultra-small-world*

The clustering coefficient (c) corresponds to the property of network transitivity. It measures the average probability that two neighbors of a vertex are themselves neighbors; or, in other words, the frequency with which loops of length three (i.e. triangles) appear in the network. Let a *triangle* be a graph of three vertexes that is fully connected, and a *connected triple* be a graph of three vertexes with at least two connections, the calculation of the network's clustering coefficient is as follows: [10]

$$c = \frac{(\text{number of triangles in the network}) \times 3}{\text{number of connected triples}} \tag{5}$$

Hence, by construction, clustering reflects the local structure. It depends only on the interconnectedness of a typical neighborhood, the inbreeding among vertexes tied to a common center, and thus it measures how incestuous the network is (Strogatz, 2003). Intuitively, the probability of connection of two vertexes in a random or homogeneous graph tends to be the same for all vertexes regardless the existence of a common neighbor. Therefore, the clustering coefficient is expected to be low in the case of random graphs, and tends to zero in the limit for large random networks.

Real-world complex networks tend to exhibit a large degree of clustering. Albert and Barabási (2002) report that in most –if not all- real networks the clustering coefficient is typically much larger then in comparable random networks, and

[10] If three vertexes (a, b, c) exist in a graph, a triangle exists when edges (a,b), (b,c) and (c,a) are present (i.e. the graph is complete), whereas a connected triple exists if at least two of these edges are present. In this sense, a triangle occurs when there is transitivity (i.e. two neighbors of a vertex are themselves neighbors). The factor of three in the numerator arises because each triangle is counted three times when connected triplets are counted (Newman, 2010).

that this factor slowly increases with the number of vertexes. Accordingly, in inhomogeneous graphs, as those resulting from real-world networks, the probability of two neighbors of a vertex being themselves neighbors is reported to be in the 10% and 60% range in most cases (Newman, 2010). In this sense, a particularly low mean geodesic distance combined with high clustering in scale-free networks implies that a few too-connected vertexes play a key role in bringing the other vertexes close to each other. In the context of studies on robustness of and contagion in financial systems, it is important to identify those too-connected players that are central in a network and could threaten financial stability in case they would fail.

2.4.2. Importance within the Network

Regarding the importance of each participant within every network, *centrality* is the most prevalent and widely used concept. Centrality's most common and simple measure is degree centrality (k_i), which assesses how connected a vertex is to the network. For weighted networks, the strength (s_i) measures the total weight of connections for a given vertex and hence provides an assessment of the intensity of the interaction between participants.

Intuitively, the larger the degree or the strength, the more important the vertex is for the network. Nevertheless, the analytical reach of these two metrics as measures of the relative importance of a vertex is limited because they do not take into account the global properties of the network; this is, they are local measures of importance, and they do not capture neighbors' importance.

The simplest global measure of centrality is eigenvector centrality, whereby the centrality of a vertex is proportional to the sum of the centrality of its adjacent vertexes; accordingly, the centrality of a vertex is the weighted sum of centrality at all possible order adjacencies. Hence, in this case centrality arises from (i) being connected to many vertexes; (ii) being connected to central vertexes; (iii) or both.[11] Alternatively, as put forward by Soramäki and Cook (2013), eigenvector centrality may be thought of as the proportion of time spent visiting each participant in an infinite random walk through the network.

Eigenvector centrality is based on the *spectral decomposition* of a matrix. Let Ω be an adjacency matrix (weighted or non-weighted), Λ a diagonal matrix containing the eigenvalues of Ω, and Γ an orthogonal matrix satisfying $\Gamma\Gamma' = \Gamma\Gamma = \mathrm{I}_n$, whose columns are eigenvectors of Ω, such that

$$\Omega = \Gamma\Lambda\Gamma' \tag{6}$$

If the diagonal matrix of eigenvalues (Λ) is ordered so that $\lambda_1 \geq \lambda_2 \cdots \lambda_n$, the first column in Γ corresponds to the principal eigenvector of Ω. The principal eigenvector (Γ_1) may be considered as the leading vector of the system, the one that is able to explain most of the underlying system. The positive n-scaled scores corresponding to each element in the principal eigenvector may be considered as their weights within an index. Because the largest eigenvalue and its corresponding eigenvector provide the highest accuracy (i.e. explanatory power) for reproducing the original matrix and capturing the main features of networks (Straffin, 1980), Bonacich (1972) envisaged $\Gamma_1$as a global measure of popularity or centrality within a social network.

However, eigenvector centrality has some drawbacks. As stated by Bonacich (1972), eigenvector centrality works for symmetric structures only (i.e. undirected graphs); yet, it is possible to work with the right (or left) eigenvector (as in Markose et al. (2012)), but this may entail some information loss. Yet, the most severe inconvenience from estimating eigenvector centrality on asymmetric matrices arises from vertexes with only outgoing or incoming edges, which will

[11] For instance, Markose et al. (2012) use eigenvector centrality to determine the most dominant financial institutions in the US credit default swap market, and to design a super-spreader tax that mitigates potential socialized losses.

may result in zero eigenvector centrality, and may cause some other non-strongly connected vertexes to have zero eigenvector centrality as well (Newman, 2010).

Among some alternatives to surmount the drawbacks of eigenvector centrality (e.g. PageRank, Katz centrality), the HITS (Hypertext Induced Topic Search) algorithm by Kleinberg (1998) has two main advantages that matter for our empirical design. First, it provides two separate centrality measures, *authority centrality* and *hub centrality*, which correspond to the eigenvector centrality as recipient and as originator of links, respectively. Second, it avoids introducing inconvenient stochastic or arbitrary adjustments to the network –as in PageRank or Katz centrality.

The estimation of authority and hub centrality results from estimating standard eigenvector centrality (6) on two modified versions of the adjacency matrix, $\mathcal{A}$ and $\mathcal{H}$, as in (7).

$$\mathcal{A} = \Omega^T\Omega \qquad\qquad \mathcal{H} = \Omega\Omega^T \tag{7}$$

Multiplying the adjacency matrix with a transposed version of itself enables one to identify directed (*in* or *out*) second order adjacencies. Regarding $\mathcal{A}$, multiplying Ω^T with Ω sends weights backwards –against the arrows, towards the pointing vertex-, whereas multiplying Ω with Ω^T (as in $\mathcal{H}$) sends scores forwards –with the arrows, towards the pointed-to vertex (Bjelland et al., 2008). Thus, the HITS algorithm works on a circular thesis: The authority centrality of each participant is defined to be proportional to the sum of the hub centrality of the participants that point to it, and the hub centrality of each participant is defined to be proportional to the sum of the authority centrality of the participant it points to.

Therefore, authority (a) and hub ($\hbar$) centrality provide global measures of participants' centrality that supplements traditional local measures such as degree (k) and strength (s). Together, local and global measures of centrality will be useful to comprehensively test the presence of positively correlated multiplexity in a system.

2.5. Main Results

The multiplex network here analyzed consists of aggregating three different networks: SEN, MEC, MEC-R. In our sample period (third quarter of year 2013), SEN accounted for 52.6% of the value of transactions in DCV, whereas MEC and MEC-R accounted for 18.5% and 28.7%, respectively. The main connective features of SEN, MEC, MEC-R and the resulting sovereign securities market multiplex are presented in Table 1. All statistics in Table 1 correspond to the mean of those estimated on daily networks; no serial aggregation of daily networks was implemented for estimating the statistics.

SEN, the largest contributor to the sovereign securities multiplex by market value of transactions, exhibits a rather odd connective structure. Against most documented real-world networks, SEN appears to be non-sparse, with a density about 0.60 and with participants directly connecting on average with more than half of their counterparties. Moreover, it appears to be homogeneous by degree, with the corresponding power-law exponent (6.05) well above typical values for scale-free networks (i.e. $\gamma_{k} \gg 3$) and inhomogeneous by strength. SEN is also non-clustered because the clustering coefficient is below the expected value for random networks of the same size ($\mu_{k}/n = 0.55$), thus it may be considered trivial. Yet, we could consider it as approximately ultra-small-world network due to the low mean geodesic distance ($\ell = 1.36$). Such a unique connective structure results from its design: It is a 14-vertex "rich-club" network created by the Ministry of Finance to foster sovereign securities market's liquidity, in which financial institutions agree to trade anonymously, free of any counterparty limit, while observing requisites that determine their permanence in the network.[12] Taken all together, these special features of SEN ultimately govern how participants relate to each other in the network.

Table 1
Main connective features of networks [a]

Statistics	SEN	MEC	MEC-R	Sovereign securities multiplex
Contribution (by value of transactions)	0.53	0.18	0.29	1,00
Participants (n)	14	90	91	159
Density (d)	0.60	0.05	0.02	0.02
Mean degree (μ_k)	7.75	4.31	1.45	3.46
Degree power-law exponent (γ_k)	6.05	2.76	2.99	3.66
Strength power-law exponent (s)	2.92	2.43	2.66	2.40
Mean geodesic distance (ℓ)	1.36 [2.63]	2.23 [4.50]	3.11 [4.51]	2.78 [5.07]
Clustering coefficient (c)	0.43 [0.55]	0.10 [0.05]	0.03 [0.02]	0.10 [0.02]

[a] All corresponds to sample means estimated on daily statistics for the analyzed quarter (63 days). Expected values for homogeneous networks of n size appear in brackets for the mean geodesic distance (ℓ) and the clustering coefficient (c). We use the algorithm by Clauset et al. (2009) for estimating the power-law exponents.

Figure 2 presents the graph corresponding to SEN's network, in which the diameter of the vertexes and the width of the arrows are determined by their

[12] The permanence in the SEN network is determined by means of a ranking revised each year. The score obtained by each participant depends on fulfilling requisites such as maximum bid-ask spreads; consistently quoting bid and ask prices; participating in primary auctions of sovereign securities; and maintaining a minimum level of capital.

strength and the market value of the sovereign securities' deliveries, respectively. The number of edges appears to be evenly distributed, but their width and the diameter of the vertexes are inhomogeneous. Likewise, it can be easily observed that participants are well-connected and –therefore- the network is rather dense.

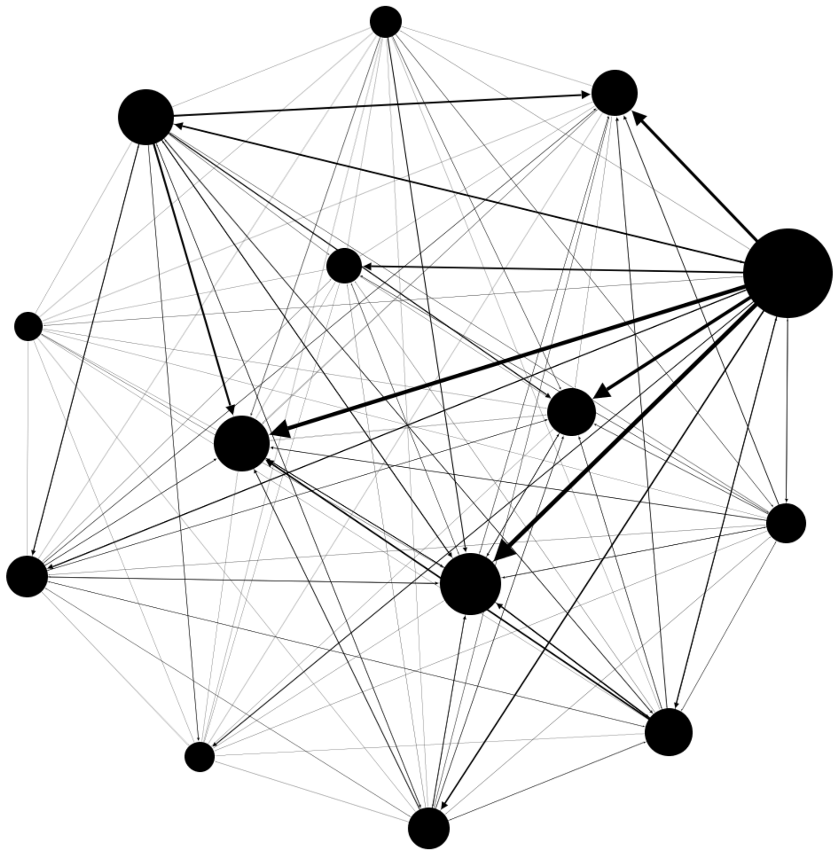

Figure 2. SEN graph. Vertexes correspond to participating financial institutions; the direction and width of each arrow represents the delivery of local sovereign securities and its market value, respectively; vertexes' diameter correspond to their strength.

When we turn to the connective structure of our second network, MEC, we find that its characteristics agree with what the literature considers a typical real-world network: It is sparse (the average density is about 0.05), scale-free (the average power law exponent estimated on degree and strength is around 2.76 and 2.43, respectively), and it is approximately ultra-small-world (the mean geodesic distance averages to 2.23 edges).[13] Although the clustering level is not particularly high ($c = 0.10$), it is still higher than the probability of any two vertexes in MEC being connected ($\mu_{k}/n = 0.05$), such that we could consider it as a clustered network. Figure 3 presents the graph corresponding to MEC's network, in which there is a high degree of correspondence between the participants pertaining to SEN network (in black) and those vertexes at which most edges are concentrated (i.e. with the highest degree) and which display the largest diameters (i.e. with the highest strength).

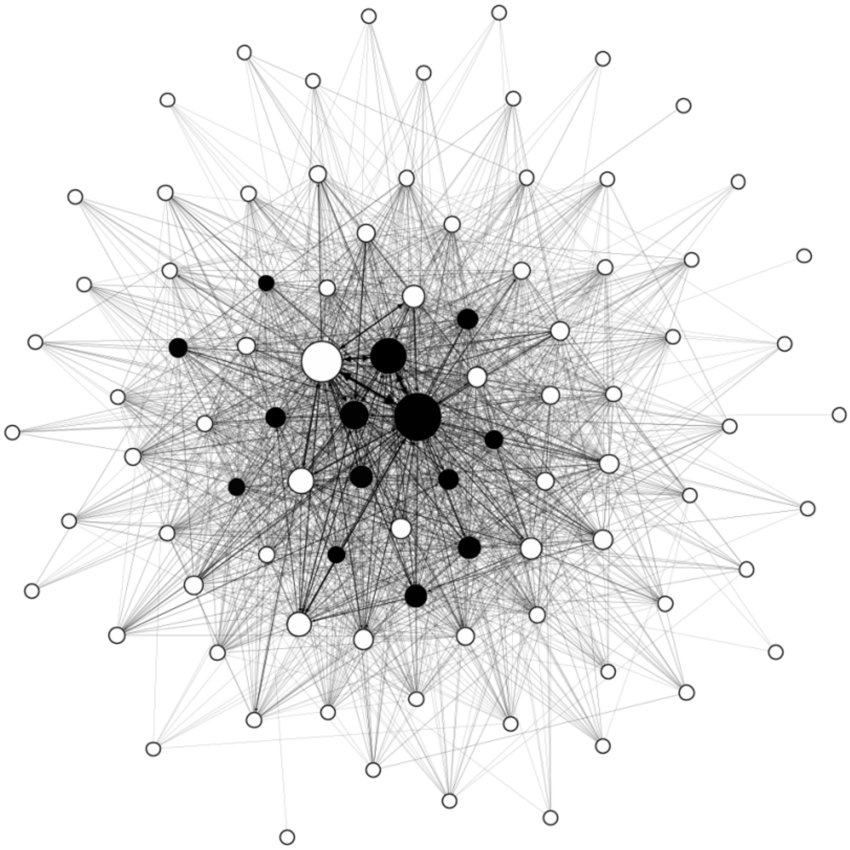

Figure 3. MEC graph. Vertexes correspond to participating financial institutions; the direction and width of each arrow represents the delivery of local sovereign securities and its market value, respectively; vertexes' diameter correspond to their strength; SEN participants are in black.

[13] Some aggregated statistics for MEC and MEC-R reported by Saade (2010) concur with ours regarding their sparseness, inhomogeneity and clustering.

Akin to MEC, the connective structure of MEC-R is very much like what we would expect of real-world networks. The MEC-R network is sparse, with average density about 0.02; scale-free, with the average power-law exponent estimated on degree and strength around 2.99 and 2.66, respectively; and approximately ultra-small-world, with the mean geodesic distance averaging 3.11 edges. The clustering level is not particularly high ($c = 0.03$), but it is slightly higher than the probability of any two vertexes of MEC-R being connected ($\mu_{k}/n = 0.02$), and could thus be considered as somewhat clustered.

Figure 4 presents the graph corresponding to MEC-R's network. As with MEC, there is a high degree of correspondence between the participants pertaining to SEN network (in black) and those vertexes at which most edges are concentrated (i.e. vertexes with the highest degree) and which display the largest diameters (i.e. vertexes with the highest strength).

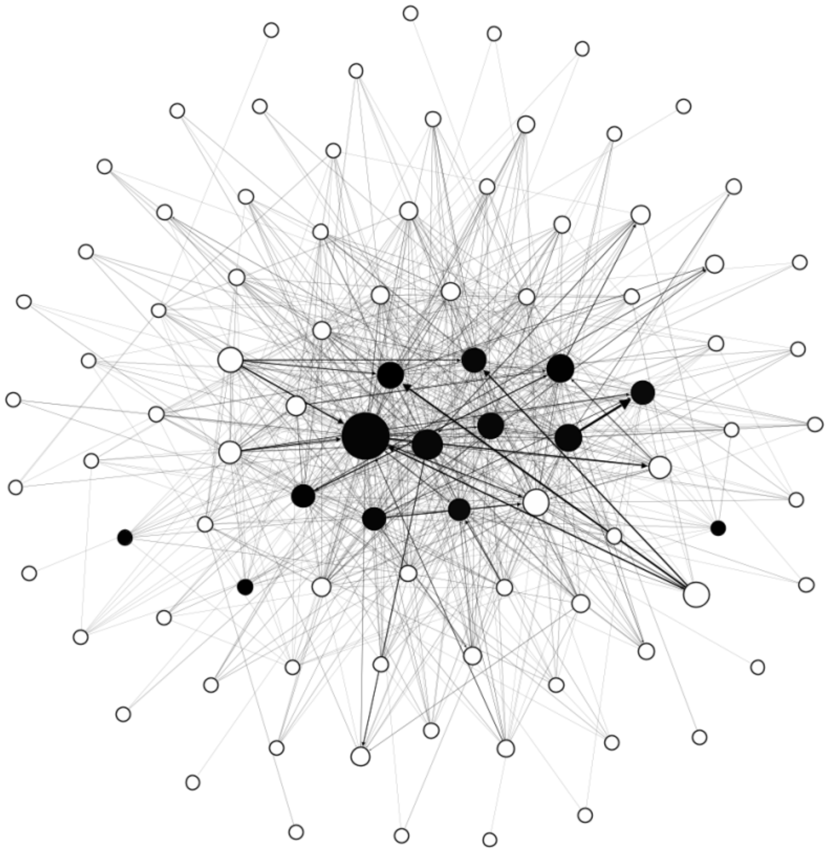

Figure 4. MEC-R graph. Vertexes correspond to participating financial institutions; the direction and width of each arrow represents the delivery of local sovereign securities and its market value, respectively; vertexes' diameter correspond to their strength; SEN participants are filled in black.

According to Table 1, the sovereign securities market multiplex network displays the sparse, inhomogeneous, approximately scale-free[14], approximately ultra-small-world features documented in real-world networks. The clustering coefficient ($c = 0.10$) of the multiplex network is higher than the probability of

[14] As in Newman (2010), exponent values slightly outside the $2 \leq \gamma \leq 3$ range are possible and are observed occasionally. Moreover, the level of the exponent is consistent with the inhomogeneous and skewed distribution of degree, a feature that may be easily verified by visual inspection of Figure 5. As before, the power-law distribution of links is an asymptotic property, thus a strict match between observed and expected theoretical properties for determining the scale-free properties of non-large networks may be impractical.

any two vertexes being connected within ($\mu_{k}/n = 0.02$), which indicates that significant clustering is present. Consequently, the multiplex reproduces the overall connective features of MEC and MEC-R, and ignores those of SEN, which is –paradoxically- its main contributor in terms of the market value of transactions. In this sense, the multiplex network of the Colombian sovereign securities market appears to largely preserve the robust-yet-fragile (Haldane, 2009) features of MEC and MEC-R.

Figure 5 presents the graph corresponding to the sovereign securities market multiplex, with the vertexes filled out in black pertaining to the SEN network. As in MEC and MEC-R, there is a high degree of correspondence between the participants pertaining to SEN network and the participants with the highest concentration of edges (i.e. the highest degree) and with the largest diameters (i.e. the highest strength).

Figure 5 exhibits three rather clear clusters or communities. The most populated and dense comprises all financial institutions pertaining to the SEN network. Because all monoplex graphs (figures 2, 3, 4) do not reveal clear clusters (as the multiplex graph does), visual inspection may suggest that aggregating layers plays a key role in the existence of modules in the multiplex network. Remarkably, multiplex network's greater divergence from expected clustering values concurs with visual inspection. Implementing proper methods for identifying partitions or clusters in networks (e.g. spectral partitioning, dendrograms, minimal spanning trees) is beyond the scope of this paper, but should be used to further confirm (or reject) the role of aggregating layers in network's clustering.

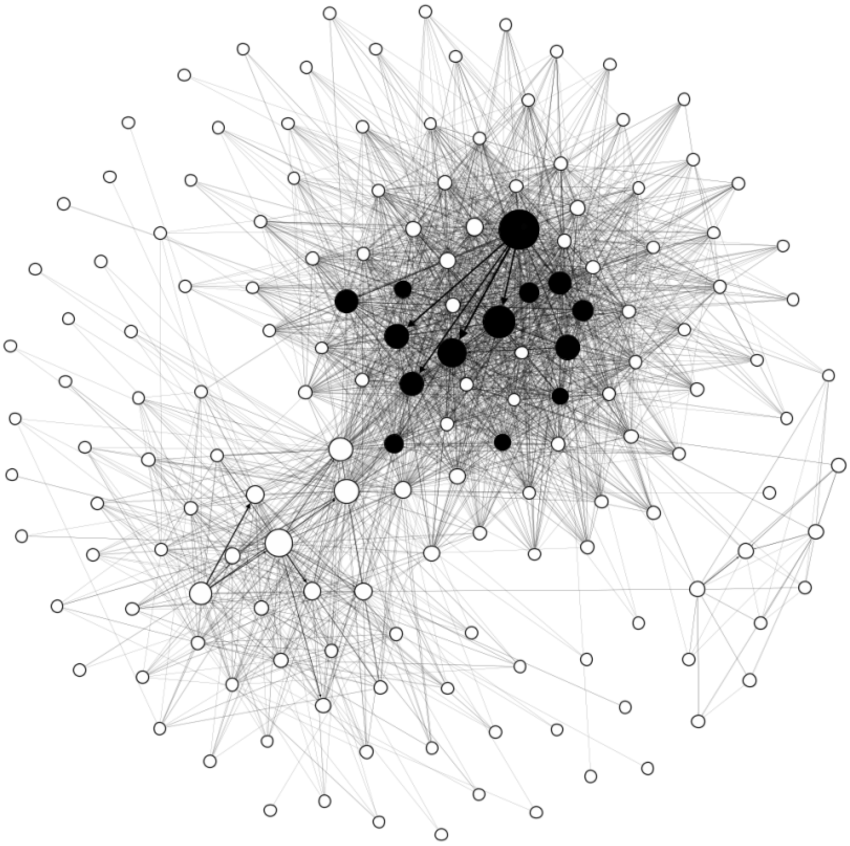

Figure 5. Sovereign securities market graph. Vertexes correspond to participating financial institutions; the direction and width of each arrow represents the delivery of local sovereign securities and its market value in the three monoplex networks, respectively; vertexes' diameter correspond to their strength; SEN participants are filled in black.

The main features of the multiplex of the sovereign securities market point out that it is a network of networks that preserves the main characteristics of the two networks that contribute the least (47%) to the total market value of sovereign securities transactions, whereas the most salient connective features of SEN appear to contribute less. Therefore, using SEN as a benchmark for the local sovereign securities market may be particularly misleading.[15]

[15] This is the case of Estrada and Morales (2008), and Laverde and Gutiérrez (2012).

Interestingly, as the three monoplex networks result from different platforms with distinct economic settings and dissimilar operational frameworks, our multiplex network approach reveals how each of the monoplex networks shapes the multiplex one. First, the statistical and graphical resemblance between the multiplex and the MEC and MEC-R monoplex networks is evident from Table 1 and the corresponding graphs, respectively. Second, when coupling SEN, MEC, and MEC-R, it is evident that SEN is no longer a “rich-club” network of 14-financial institutions densely interconnected, but a network in which there is a core of densely connected vertexes surrounded by a vast periphery of non-linked participants, and with a connective structure approximating that of MEC and MEC-R. Under this comprehensive view revealed by the multi-layer approach, the SEN network is turned into a sparse, inhomogeneous and potentially scale-free network that preserves its ultra-small-world and non-clustered nature.[16]

The connective coincidence across the layers in the sovereign securities multiplex network may suggest the presence of positively correlated multiplexity in the sense of Kenett et al. (2014) and Lee et al. (2014). This means that the main connective features of the constituent monoplex networks may be preserved as a consequence of consistency in the importance of participants across networks (i.e. vertexes’ centrality overlapping across layers). The correlation matrix estimated on financial institutions’ degree ($\mathcal{k}$), strength ($\mathcal{s}$), hub centrality ($\mathcal{h}$) and authority centrality ($\mathcal{a}$) in Table 2 confirms that there is indeed a positive and non-negligible linear dependence in financial institutions’ role and importance across the three different trading and registering platforms, either by local (i.e. degree and strength) or by global (i.e. hub and authority) measures of centrality. Thus, our evidence confirms that the presence of positively correlated multiplexity prompts a multiplex network that tends to preserve the main features of its constituent monoplex networks.

[16] As can be derived from equations 1, 4 and 3, respectively, a large number of non-participating financial institutions will turn the network sparse and less clustered, whereas the small-world characteristic will in this case remain the same because the mean geodesic distance considers reachable vertexes only.

	k_{MEC}	k_{SEN}	k_{MEC-R}	s_{MEC}	s_{SEN}	s_{MEC-R}	h_{MEC}	h_{SEN}	h_{MEC-R}	a_{MEC}	a_{SEN}	a_{MEC-R}
k_{MEC}	1											
k_{SEN}	0.55	1										
k_{MEC-R}	0.81	0.71	1									
s_{MEC}				1								
s_{SEN}				0.48	1							
s_{MEC-R}				0.67	0.70	1						
h_{MEC}							1					
h_{SEN}							0.29	1				
h_{MEC-R}							0.41	0.49	1			
a_{MEC}										1		
a_{SEN}										0.53	1	
a_{MEC-R}										0.63	0.70	1

Figure 6. Monoplex networks' degree, strength, authority and hub centrality correlation matrix. The correlation matrix is estimated based on the contribution of each financial institution to the total degree, strength, hub and authority centrality in the whole sample; non-participating financial institutions are assigned a contribution equal to zero. Correlations across centrality measures are omitted to enhance readability.

2.6. Final Remarks

Financial markets are complex systems that we can understand better by examining how its constituents, the financial institutions, relate to one another. This type of research is particularly important to detect the pivotal players in a system whose robustness is vital for financial stability. Analyses on financial contagion enable us to chart systemic risks. The right approach to gain insight on the interrelated functioning of financial institutions is precisely by means of applying network science to financial data. The main objective is to get a comprehensive view of how financial institutions interact, and to avoid the pitfalls of traditional institution-centric analyses by means of a macro-prudential approach to financial stability.

As has been acknowledged rather recently, it is critical to go beyond single-layer networks to better understand how financial institutions interrelate without assuming that each financial network is isolated from the others. Data recorded in financial market infrastructures is a convenient source of information to take a step towards examining how financial institutions are linked across different economic environments.

We have examined how financial institutions relate to each other in different trading and registering environments of sovereign securities (i.e. the platforms). We have also studied how those environments aggregate into a multiplex network that portrays the local sovereign securities as a network of networks. When we build the multiplex network, we observe strong non-linear effects: The network characteristics of the most important monoplex network (i.e. SEN) in terms of the market value of its transactions do not hand over its characteristics to the multiplex network. SEN appears to be a non-sparse network with its participants directly connecting on average with more than half of their counterparties. It is also homogeneous by degree, inhomogeneous by strength, and non-clustered. In contrast, the multiplex resulting from the three constituting networks conforms to most real-world networks' features, i.e. sparse, inhomogeneous, scale-free, ultra-small-world, and clustered, which are the traits of the lesser important networks MEC and MEC-R.

The unusual connective features of SEN actually reinforce the real-world networks' characteristics displayed by MEC and MEC-R by means of a strong linear dependence between most central participants across all networks. This important finding would have been omitted if one studies monoplex networks in isolation, and could have misled the perspective on the relations between and risks induced by participating financial institutions. Likewise, this finding would have been elusive without the data from the corresponding settlement market infrastructure (i.e. DCV).

Furthermore, the evidence of positively correlated multiplexity reveals some interesting characteristics of the local sovereign securities market. Four characteristics are worth reporting: First, the inhomogeneous and approximate scale-free connective structure, consistent with the robust-yet-fragile nature of financial networks (Haldane, 1999), is the result of structural similarity (i.e. positive correlated multiplexity) across networks within a multi-layer analytical framework. Second, due to the ultra-small-world nature of all networks analyzed in this framework, it is evident that the role of too-connected financial institutions is critical for the efficiency of the market, not only at the single-layer level, but also for the whole market when the coupling of single-layer networks is considered. Third, notwithstanding the economic and operational differences of each monoplex network, financial institutions that are considered "too-connected to fail" tend to overlap across networks. Fourth, as too-connected financial institutions couple otherwise distinct market environments, their role in financial stability is critical due to their contribution to cross-system risk. These four characteristics are also an essential focus in order to better understand the local sovereign securities market and its contribution to financial stability.

This research has contributed to the literature by highlighting the relevance of network analysis on data from financial market infrastructures. First, data gathered by financial market infrastructures have the potential to overcome the main obstacles for working on multi-layer networks highlighted by D'Souza et al. (2014), namely the independent ownership and operation of layers, and the lack of data standardization and sharing. Second, unlike traditional balance sheet data, financial market infrastructures gather granular information from distinct financial environments, which may allow breaking down aggregated financial data into very different layers of complexity in order to reach a deeper understanding of financial markets.

To conclude, we would like to highlight some interesting research extensions. As all financial markets in all countries comprise distinct layers of interactions among financial institutions, the multi-layer analysis here implemented may be reproduced with ease. Reproducing this analysis for other countries and markets (e.g. foreign exchange, non-sovereign fixed income, derivatives, etc.) could confirm whether our main findings may be considered new stylized facts of financial networks –or not.

Also, multi-layer analysis can be done at the level of transactions' or participants' intrinsic characteristics. For instance, some interesting intrinsic characteristics are the maturity of the traded sovereign securities; the volatility of the securities; the ownership structures of the participating financial institutions; institutions' credit risk ratings; the types of institutions (e.g. banking, non-banking); and the final beneficiary of the transactions (e.g. proprietary or non-proprietary). All these potential multi-layer networks may help to understand how financial institutions relate into each other in distinct financial environments, and in the entire financial system.

3

In most systems in nature, it is somewhat arbitrary as to where we leave off the partitioning, and what subsystems we take as elementary.

Herbert Simon (1962)
Nobel Prize in Economics, 1978

3. Assessing Financial Market Infrastructures' Systemic Importance with Authority and Hub Centrality

Abstract

Two information retrieval measures, authority centrality and hub centrality, are implemented to assess the systemic importance of Colombian financial market infrastructures. Unlike standard centrality measures (e.g. degree, strength, eigenvector), authority and hub centrality allow for assessing financial market infrastructures' global importance despite the strictly hierarchical (i.e. directed and acyclic) nature of the networks they comprise, while assessing importance as the mutually reinforcing centrality arising from nodes pointing to other nodes (i.e. hubs) and from nodes being pointed-to by other nodes (i.e. authorities). Results verify the systemic importance of the only large-value payment system (CUD) and the sovereign securities' main settlement system and central depositary (DCV), the foremost authority-central and hub-central financial market infrastructures, respectively. This paper contributes to financial stability and systemic risk literature with a macro-prudential perspective of *infrastructure-related risk*.

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3.1. Introduction

Despite a proper understanding of financial institutions' linkages is vital for assessing systemic risk, there is an additional –mostly neglected- source of connectedness arising from financial market infrastructures (FMIs). This source of financial connectedness results in infrastructure-related systemic risk (Berndsen, 2011; DNB, 2011), which is the component of systemic risk that can be brought about by the improper functioning of financial infrastructure, or where financial infrastructure acts as the conduit for shocks that have arisen elsewhere.

Notwithstanding the recent financial crisis highlighted the importance of FMIs for the safe and efficient functioning of financial markets (CPSS-IOSCO, 2012; Dudley, 2012a,b) and the relevance of infrastructure-related systemic risk for financial stability, most efforts focus on identifying so-called systemically important financial institutions (SIFIs). Therefore, in order to make a contribution to the systemic risk literature, this paper aims at extending the use of network analysis from the identification of SIFIs to the identification of systemically important financial market infrastructures (SIFMIs), where the latter will be broadly defined as those providing trading, payments, clearing and settlement services to financial institutions, whose failure or impairment could trigger greater disruptions in the financial system and economic activity due to their *centrality*.

Despite it is tempting to use standard centrality metrics (e.g. *degree, strength* or *eigenvector centrality*), some particularities of FMIs' networks render these metrics inconvenient or useless to identify SIFMIs. The main particularity of FMIs' networks is their strict hierarchical pattern, which results in a precise type of network to work with: a directed acyclic network.

In order to tackle the particularities resulting from this type of network, this paper addresses the assessment of systemic importance for FMIs by means of the estimation of *authority centrality* and *hub centrality*. These two metrics, first proposed by Kleinberg (1998) for information retrieval purposes, are suitable for FMIs' networks because they (i) are designed for directed networks, even in the case of directed and acyclic networks, and (ii) are capable of simultaneously measuring mutually reinforcing centrality arising from nodes pointing to other nodes (i.e. *hubs*) and from nodes being pointed-to by other nodes (i.e. *authorities*).

The approach here proposed verifies that in the Colombian case systemic importance is strongly dominated by the only large value payment system (CUD) and by the sovereign securities' main settlement system and central depository (DCV). This confirms (i) the preeminence of the sovereign debt market as the most important contributor to local financial system's liquidity; (ii) the supremacy of sovereign securities as sources of liquidity for financial institutions in the local money market; and (iii) the significance of the impact on liquidity arising from Real-Time Gross Settlement (RTGS) systems interrelated continuously with other real-time based systems, as acknowledged by CPSS (1997). It is worth highlighting that both CUD and DCV are owned and operated by the Colombia's Central Bank (*Banco de la República*), as is customary in many other countries, presumably due to the importance of their safe and efficient functioning for financial markets.

It is rather evident from the recent financial crisis that methods aimed at supporting financial authorities' efforts to identify SIFIs and SIFMIs are particularly valuable to enhance their policy-making (e.g. prudential regulation, oversight and supervision) and decision-making (e.g. resolving, restructuring or providing emergency liquidity) capabilities. Hence, this paper contributes to financial stability and systemic risk literature with a macro-prudential perspective of infrastructure-related risk.

3.2. An Overview of Colombian FMIs

Under the definition of FMIs as those multilateral systems among participating financial institutions used for the purposes of executing, exchanging, clearing, settling or recording payments, securities, derivatives, or other financial transactions, Figure 1 provides an overview of Colombian FMIs. Each level relates to a specific type of infrastructure, corresponding to a broad classification of FMIs' duties or functions: (A) trading and registration, (B) clearing and settlement, (C) large-value payment systems, and (D) retail payment systems.

Level A comprises securities and currency trading platforms, and over the counter (OTC) transactions' registration platforms, both types hereafter referred as TPs. Regarding securities' TPs, Colombia's Central Bank owns and operates SEN (*Sistema Electrónico de Negociación*), the main sovereign securities TP. Sovereign securities may also be traded in MEC (*Mercado Electrónico Colombiano*), which is owned and operated by the Colombian Stock Exchange (*Bolsa de Valores de Colombia* - BVC); MEC also provides a trading and registration platform for other types of fixed income securities such as corporate, municipal and commercial papers. The Colombian Stock Exchange also provides TP for equity and financial futures through BVC EQUITY[1] and BVC FUTURES, respectively.

DECEVAL REGISTRATION (DSR) is a TP owned and operated by DECEVAL central securities depository and securities settlement system (CSD+SSS), and provides registration services for fixed income securities. DERIVEX FUTURES provides TP services for the energy futures market. Local branches (subsidiaries) of three international inter-dealer brokerage firms (i.e. ICAP, GFI and Tradition, displayed as IDBROK) allow transactions between participants through hybrid systems (voice and data). Regarding Peso/Dollar trading and registration

[1] Local regulation does not allow OTC equity trading.

platforms, SET-FX and IDBROK provide TP services for foreign exchange market participants.

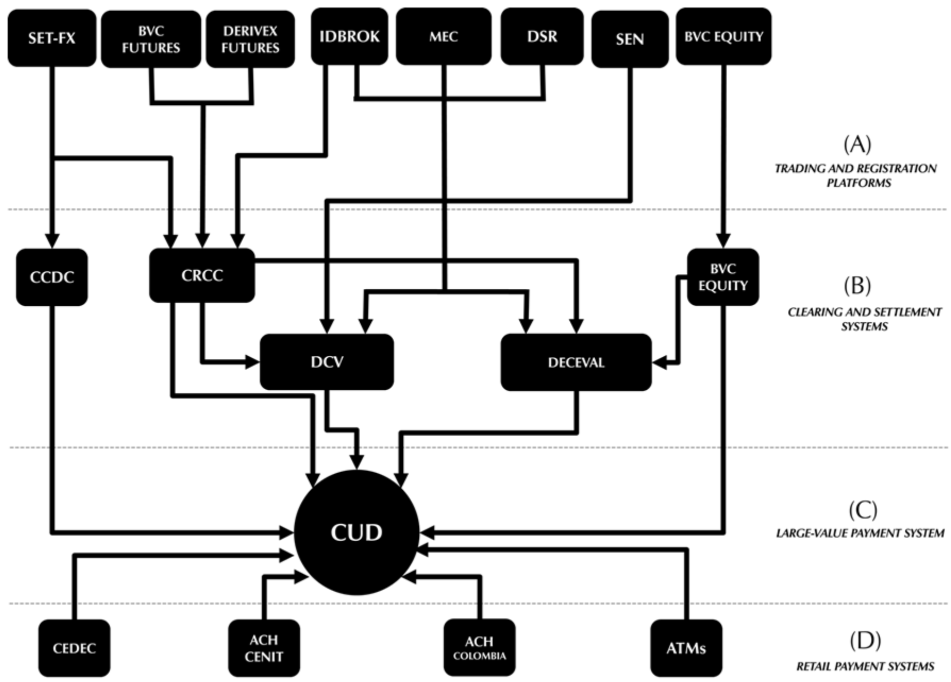

Figure 1. Colombian FMIs. Each level relates to a specific type of infrastructure, corresponding to a broad classification of FMIs' duties or functions: (A) trading and registration, (B) clearing and settlement, (C) large-value payment systems, and (D) retail payment systems.[2]

Level B corresponds to clearing and settlement systems. The central bank owns and operates DCV (*Depósito Central de Valores*), a FMI that is both the securities settlement system (SSS) and the central securities depository (CSD) for

[2] The daily average number of transactions managed by each FMI in Figure 1 during 2012 is the following: CUD, 8,196; DCV, 4,209; DECEVAL, 9,198; BVC EQUITY, 3,595; CCDC, 1,399; CRCC, 159; CEDEC, 120,857; ACH CENIT, 38,504; ACH Colombia, 471,629; SET-FX, 1,399; DERIVEX FUTURES, 145; BROKERS, 113; MEC, 2,756; DSR, 61; SEN, 1,043; BVC EQUITY, 3,595.

sovereign securities exclusively. DCV works under a Real-Time Gross Settlement System (RTGS) and a Delivery-versus-Payment (DvP) mechanism.[3]

Privately owned DECEVAL (*Depósito Centralizado de Valores de Colombia*) provides CSD and SSS services for corporate and non-sovereign government securities (e.g. issued by municipalities), along with CSD services for the equity market. Central counterparty (CCP) services for futures markets are provided by CRCC (*Cámara de Riesgo Central de Contraparte de Colombia*). The Colombian Stock Exchange (BVC) provides SSS services for local equity markets via BVC EQUITY.[4]

Regarding currencies, CCDC (*Cámara de Compensación de Divisas de Colombia*) provides clearing and settlement for the Peso/Dollar spot market[5], whereas CRCC offers clearing and settlement services for Peso/Dollar non-delivery forwards.

Level C comprises the only local large-value payments system (*Cuentas de Depósito* – CUD), where all cash leg's settlement (in local currency) takes place. The large-value payments system (LVPS), owned and operated by Colombia's

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[3] DCV working on a RTGS framework means that there is a continuous (real-time) settlement of securities, where each transfer is processed individually on an order-by-order basis (without netting), conditional on the existence of funds in the LVPS (CUD). However, DCV also includes liquidity saving mechanisms in the form of liquidity optimization algorithms; Banco de la República (2012) describes DCV's functionality.

[4] Based on CPSS (2003) terms, the settlement institutions (i.e. institutions across whose books transfers between participants take place in order to achieve settlement) for the equity market are the local large-value payment system (CUD) for the Peso leg, and DECEVAL for the equity leg. The settlement system (i.e. a system used to facilitate the settlement of transfers of funds or financial instruments) is provided by BVC.

[5] Based on CPSS (2003) terms, the settlement institutions for the foreign exchange market are the local large-value payment system (CUD) for the Peso leg, and Citibank-New York for the Dollar leg. The settlement system is provided by CCDC.

central bank, works under a Real-Time Gross Settlement System (RTGS) framework.[6]

Level D corresponds to retail payment systems. The central bank owns and operates both CENIT Automated Clearing House (ACH) and Cheques Clearing House (CCH), whereas commercial banks own ACH-COLOMBIA. ATM provides clearing and settlement for transactions made through debit cards, credit cards, via POS (point of sale) and automated teller machines.

3.3. Assessing Systemic Importance with Authority Centrality and Hub Centrality

A network, or graph, represents patterns of connections between the parts of a system. Two concepts arise from this definition: parts and connections. The parts of the system correspond to the participants or elements, and are commonly referred to as *vertexes*, whereas the connections correspond to the relations between the elements of the system, and are called *edges*. These concepts tend to have an equivalent when applied to specific networks, such as *nodes* and *links* in computer science, *actors* and *ties* in sociology, *neuron* and *synapse* in neural networks, *web page* and *hyperlink* in the World Wide Web network, or *financial institution* and *payment* (or *exposure*) in financial networks.

The most common mathematical representation of a network is the *adjacency matrix*. Let n represent the number of vertexes or participants, the adjacency matrix A is a square matrix of dimensions $n \times n$ with elements A_{ij} such that

[6] CUD working on a RTGS framework means that there is a continuous (real-time) settlement of funds, where each transfer is processed individually on an order-by-order basis (without netting), conditional on the existence of the corresponding securities or currencies in the related clearing and settlement systems (e.g. DCV, DECEVAL). However, CUD also includes liquidity saving mechanisms in the form of liquidity optimization algorithms; Banco de la República (2012) describes CUD's functionality.

$$A_{ij} = \begin{cases} 1 \text{ if there is an edge between vertexes } i \text{ and } j, \\ 0 \text{ otherwise.} \end{cases} \tag{1}$$

A network defined by the adjacency matrix in (1) is referred as an undirected graph, where the existence of the (i, j) edge makes both vertexes i and j adjacent or connected, and where the direction of the edge or link is unimportant. However, the assumption of a reciprocal relation between vertexes is inconvenient for some networks. For instance, the deliveries of money between financial institutions, or transactions between FMIs, constitute a graph where the character of sender and recipient is a particularly sensitive source of information for analytical purposes, where the assumption of a reciprocal relation between both parties is unwarranted. Thus, the adjacency matrix of a directed network or *digraph* differs from the undirected case, with elements A_{ij} such that

$$A_{ij} = \begin{cases} 1 \text{ if there is an edge from } i \text{ to } j, \\ 0 \text{ otherwise.} \end{cases} \tag{2}$$

Consequently, the undirected adjacency matrix is always symmetrical with respect to the main diagonal, whereas the directed case tends to be non-symmetrical; the direction of the connection is usually displayed with an arrow, and it is common to use the terms *arc* and *directed edge* interchangeably. If self-edges are allowed the main diagonal may have non-zero elements, and the graph may be known as a *multigraph*.

Moreover, networks may require edges to represent more information than that conveyed in a simple binary (0 or 1) relation; in the complex network context, edges are not binary, but are weighted according to the economic interaction under consideration (Schweitzer et al., 2009). Therefore, it may be useful to assign real numbers to the edges, where these numbers may represent distance, frequency or value, in what is called a weighted network and its corresponding weighted adjacency matrix (W_{ij}). For a financial network the weights could be the monetary value of the transaction or of the exposure.

3.3.1. Basic Centrality Measures: Degree Centrality and Eigenvector Centrality.

A large volume of research on networks has been devoted to the centrality concept (Newman, 2010). The preeminence of centrality is also characteristic of research on financial networks, in which central financial institutions are commonly regarded as systemically important. However, centrality is still an elusive concept that may be approximated from different perspectives, where different centrality measures are available.[7]

Centrality's most common and simple measure is degree centrality. Degree centrality assesses how intensely a node is connected to the network, and it corresponds to the number of links or edges attached to the participant under analysis. In directed graphs, where the adjacency matrix is non-symmetrical, *in-degree* (k_i^{in}) and *out-degree* (k_i^{out}) quantify the number of incoming and outgoing edges, respectively (3); for undirected graphs, $k_i = k_i^{in} = k_i^{out}$.

$$k_i^{in} = \sum_{j=1}^{n} A_{ji} \qquad\qquad k_i^{out} = \sum_{j=1}^{n} A_{ij} \tag{3}$$

In the weighted graph case the degree may be informative, yet inadequate for analyzing the network; financial networks are a good case of degree being limited for analytical purposes. Strength (s_i) measures the total weight of connections for a given vertex, which provides an assessment of the intensity of the interaction between participants. Akin to degree, in the directed graph case in-strength (s_i^{in}) and out-strength (s_i^{out}) sum the weight of incoming and outgoing edges, respectively (4); for undirected graphs, $s_i = s_i^{in} = s_i^{out}$.

[7] This section briefly covers basic concepts of network analysis with emphasis on centrality measures used for financial networks. *Closeness centrality* and *betweenness centrality* are not considered because their ability to accurately identify central nodes in payment systems is questionable (see Soramäki and Cook, 2013). For a comprehensive review of centrality measures please refer to Newman (2010).

$$s_i^{in} = \sum_{j=1}^{n} W_{ji} \qquad\qquad s_i^{out} = \sum_{j=1}^{n} W_{ij} \tag{4}$$

Intuitively, the larger the degree or the strength, the more important the vertex is for the network. Nevertheless, the analytical reach of these two metrics as measures of the relative importance of a vertex is limited because they do not take into account the global properties of the network; this is, degree and strength do not capture neighbor's centrality as a source of centrality, thus they are local measures of importance by construction.

The simplest global measure of centrality is eigenvector centrality, whereby the centrality of a vertex is proportional to the sum of the centrality of its adjacent vertexes; accordingly, the centrality of a vertex is the weighted sum of centrality at all possible order adjacencies. Hence, centrality arises from (i) being connected to many vertexes; (ii) being connected to central vertexes; (iii) or both.

Eigenvector centrality is based on the *spectral decomposition* of a matrix. Let Ω be an adjacency matrix (weighted or non-weighted), Λ a diagonal matrix containing the eigenvalues of Ω, and Γ an orthogonal matrix satisfying $\Gamma\Gamma' = \Gamma\Gamma = \mathrm{I}_n$, whose columns are eigenvectors of Ω, such that

$$\Omega = \Gamma\Lambda\Gamma' \tag{5}$$

If the diagonal matrix of eigenvalues (Λ) is ordered so that $\lambda_1 \geq \lambda_2 \cdots \lambda_n$, the first column in Γ corresponds to the principal eigenvector of Ω. The principal eigenvector (Γ_1) may be considered as the leading vector of the system, the one that is able to explain the most of the underlying system, in which the positive n-scaled scores corresponding to each element may be considered as their weights within an index. As stated by Bonacich (1972), each i-centrality score contained in Γ_1 corresponds to weighting i's neighbors' centrality at all adjacency orders.

Because the largest eigenvalue and its corresponding eigenvector provide the highest accuracy (i.e. explanatory power) for reproducing the original matrix and capturing the main features of networks (Straffin, 1980; Boots, 1984), Bonacich (1972) envisaged Γ_1 as a global measure of popularity or centrality within a social network, as was also suggested by Gould (1967) and Tinkler (1972) in Geography and Physics.

As before, eigenvector centrality is convenient because it is a global measure of centrality within a network; it captures all-order adjacencies as contributing to the centrality of each participant. An analogy by Soramäki and Cook (2013) is also illustrative: *Eigenvector centrality can be thought of as the proportion of time spent visiting each node in an infinite random walk through the network.*

However, eigenvector centrality has some drawbacks. First, as stated by Bonacich (1972), eigenvector centrality works for symmetric structures only (i.e. undirected graphs)[8]. Yet, the most severe inconvenience from estimating eigenvector centrality on asymmetric matrices arises from vertexes with only outgoing or incoming edges, which will always result in zero eigenvector centrality, and may cause some other non-strongly connected vertexes to have zero eigenvector centrality as well (Newman, 2010). This issue is especially important for a particular type of directed network: a directed acyclic network. This type of directed network, also known as DAG (directed acyclic graph), where no cycles between nodes exist, and where nodes non-strongly connected to two or more nodes exist, may result in all nodes yielding zero eigenvector centrality.[9] As stressed by Newman (2010), this makes standard eigenvector centrality completely useless for acyclic networks.

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[8] Nevertheless, it is possible to work with one of the two sets of eigenvectors resulting from a non-symmetrical matrix for assessing centrality, as in Markose et al. (2012).

[9] A strongly connected node corresponds to a node that is reachable from and may reach other nodes in a directed network. In the presence of non-strongly connected nodes it is common to find non-zero eigenvector centralities due to the precision or iteration seeds of the algorithms; yet, eigenvector centrality in directed acyclic networks is inconsistent because it usually yields equal centrality to all nodes.

3.3.2. Issues Arising From Measuring Centrality for Colombian FMIs with Degree and Eigenvector Centrality

Figure 1 displayed the functional connections between local FMIs, where links and nodes do not graphically represent their relative importance within the network. Figure 2 corresponds to the topological network of local FMIs, where nodes' diameter is weighted according to the gross monetary value of the transactions managed by each FMI, and where edges' thickness is weighted according to the monetary value of the transactions flowing between FMIs.

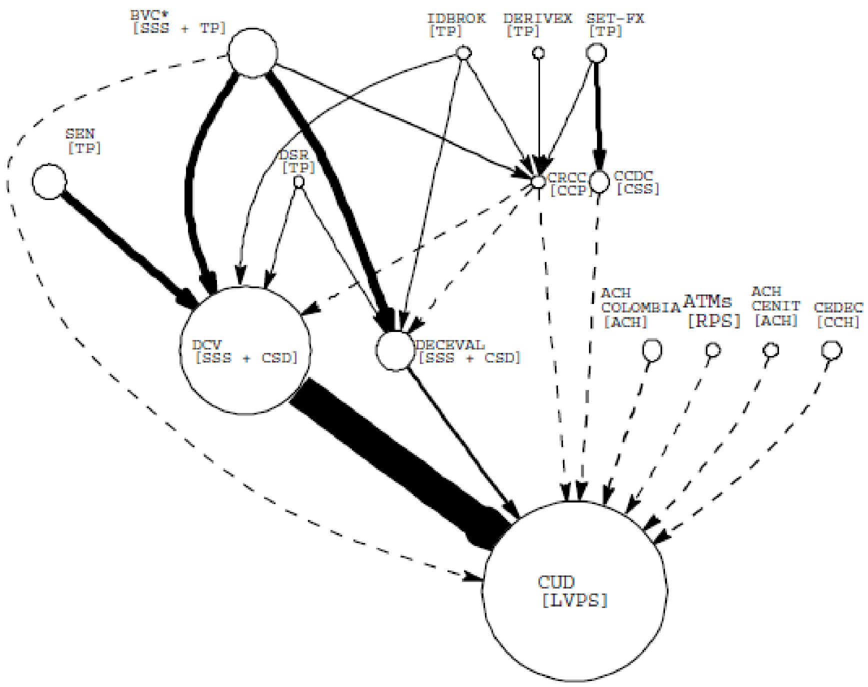

Figure 2. Colombian FMIs. Nodes' diameter and edges' thickness correspond to the monetary value of transactions. Edges representing net (gross) flows are in dashed (solid) lines. BVC performs SSS (equity) and TP (fixed income, equity, futures) duties.

Regarding the weights of the edges, the operational characteristics of the originating FMI determine whether the monetary values underneath the weights are gross or net. In Figure 2 FMIs that work under a netting operational framework (e.g. ACHs, CCP, CSS, CCH) generate net transactions (dashed edges), whereas FMIs that work under a gross settlement operational framework (e.g. SSSs+CSDs) or that merely capture financial firm's transactions (TPs), generate gross transactions (solid edges).

The main topological characteristic evident from Figure 2 is that the local FMIs' network is strictly hierarchical. Securities, foreign exchange, and derivatives transactions, they are captured or registered by TPs; afterwards, those transactions are cleared by the corresponding securities (SSSs and CSDs), currencies (CSS) or derivatives (CCP) infrastructures, which concurrently interact with the RTGS-based LVPS to settle the corresponding leg and the cash leg. Transactions belonging to ACHs, RPS and CCH are settled in the LVPS directly.

The network in Figure 2 has no loops or undirected edges, and all edges are directed downwards, where there is a node that has incoming edges only and no outgoing ones (i.e. the LVPS), and several nodes with outgoing edges only and no incoming ones (i.e. TPs). Due to this strict hierarchical structure, the FMIs' network belongs to the particular case aforementioned: a directed acyclic network.[10] It is important to realize that this strict hierarchical structure is not coincidental, but follows legal and operational considerations that tend to be stable overtime, in which it is unlikely to find settlement systems sending transactions to trading platforms; LVPS sending transactions to ACHs; or CCHs sending transactions to TPs.

As the FMIs' network is weighted, acyclic and directed, the usefulness of basic metrics for centrality (degree, strength and eigenvector) is worth examining. As mentioned before, degree or strength centrality would not be able to capture

[10] The network acyclic property was tested according to the simple procedure described by Newman (2010): (i) find a node with no outgoing edges; (ii) if no such node exists, the network is cyclic; (iii) if such a node does exist, remove it and all its ingoing edges from the network; (iv) if all nodes have been removed, the network is acyclic; otherwise go back to step (i). Moreover, the fact that the graph could be drawn with all edges pointing downward also confirms that the network is acyclic.

neighbor's importance, making all adjacent nodes equally important despite the origin of their preceding connections. This issue is particularly important in the case under analysis because some FMIs function as collectors or concentrators of other FMIs' edges, where this shortcoming is more acute in the case of FMIs that work under netting frameworks. Additionally, as already documented, standard eigenvector centrality for acyclic networks yields undesirable results: All nodes would have equal –zero- centrality. Hence, its application to the herein considered case is inconsistent.

Consequently, non-basic centrality measures should be considered for the Colombian FMIs case, where the network's topology (i.e. weighted directed acyclic network) should determine the choice of metrics. Next section addresses the implementation of the HITS algorithm for simultaneously estimating two metrics: authority centrality and hub centrality.

3.3.3. The HITS Algorithm[11]

Unlike financial institutions' networks, which are composed by a myriad of nodes that interconnect in a rather non-hierarchical fashion[12], FMIs' networks are

[11] Other adjustments to standard eigenvector centrality are available. Two methods are worth mentioning: *Katz centrality* and *PageRank centrality*. Katz centrality avoids the issues regarding eigenvector centrality in directed acyclic networks by giving each node an initial amount of centrality; as this initial amount of centrality is arbitrary, this option is not considered in the herein case. PageRank (Brin and Page, 1998), the algorithm behind Google's search engine, introduces a stochastic adjustment that randomly allows (i.e. creates) connections between nodes. As FMIs connections are strictly hierarchical, where such randomly created connections are implausible, PageRank centrality is discarded; likewise, as it shares such stochastic adjustment, *SinkRank* (Soramäki and Cook, 2013) is also discarded.

[12] This does not mean that connections in financial institutions' networks are completely random (as in a Poisson or Erdos-Rényi graph), or that tiered (e.g. core-periphery) architectures are not observable. In this case this means that each node's functions within financial institutions' networks are not as strict as in the case of FMIs, where each node develops a rather specific duty (e.g. SSS, LVPS, ACH) that conditions its connections to

composed by a non-large set of nodes, with each FMI developing a specialized duty (e.g. LVPS, SSS, ACH, etc.) that clearly discriminates each node and defines its position within the network, and that strictly determines the number and direction of all nodes' connections. In this sense, this strict hierarchy not only results in acyclic and directed networks, but also results in a particular challenge: To recognize that some FMIs are designed to serve as collectors or concentrators of other FMIs' transactions, whereas some others are designed to serve as originators of transactions, whilst some FMIs serve both purposes.

Kleinberg (1998) designed the HITS algorithm (Hypertext Induced Topic Search), which may be useful for simultaneously recognizing such dual role of FMIs (i.e., collecting and/or originating transactions) in a proper manner. The original use of the algorithm was *information retrieval* for *internet link analysis*, where the algorithm's main premise is to recognize that webpages serve two purposes: (i) to provide information on a topic, and (ii) to provide links to other webpages containing information on a topic. Consequently, the intuition behind the algorithm is the existence of a mutually reinforcing relationship between two different types of pages within the Web: (i) authorities, which are commonly cited regarding a certain topic, thus they are informative and tend to exhibit a large in-degree; and (ii) hubs, which cite many related authorities, thus they are a useful resource for finding authorities and tend to exhibit a large out-degree.

Figure 3 depicts both concepts, where nodes A and B strictly correspond to a hub and an authority, respectively. It is worth highlighting that a single node may concurrently display some level of authority centrality and hub centrality; this is the case of nodes C, D and E.

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other FMIs. For instance, León and Berndsen (2014) and León et al. (2014) document the existence of modular and scale-free hierarchies within Colombian financial institutions' networks.

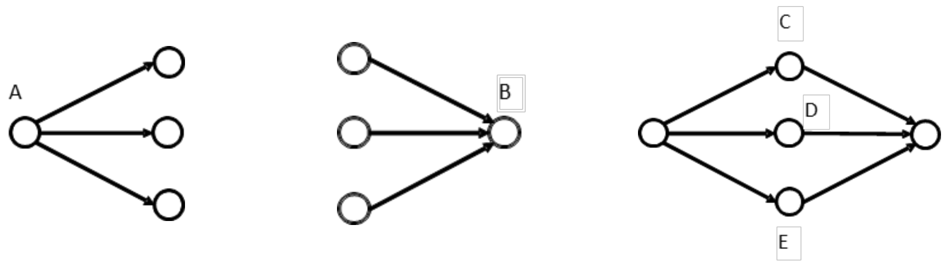

Figure 3. Authority and hub centrality within a network. In panel (a) A is a hub. In (b) B is an authority. In (c) C, D, and E are hubs and authorities.

In this sense, as stressed by Langville and Meyer (2012), Kleinberg's algorithm identifies popularity or importance based on a pair of interdependent circular thesis: (i) A webpage is a good hub if it points to good authorities, and (ii) a webpage is a good authority if it is pointed-to by good hubs. As a result, good authorities are pointed-to by good hubs, and good hubs point to good authorities, where each webpage has some authority score and some hub score.

This may be conveniently reduced for the case at hand as follows: Authority central FMIs receive transactions from hub central FMIs, and hub central FMIs send transactions to authority central FMIs, where each FMI has some authority score and some hub score.

Due to the interdependent circularity aforementioned, the algorithm recognizes that the authority centrality of each node is defined to be proportional to the sum of the hub centrality of the nodes that point to it, and that the hub centrality of each node is defined to be proportional to the sum of the authority centrality of the nodes it points-to. In order to make such recognition the algorithm uses the eigenvector centrality previously presented, but it circumvents the documented issue regarding directed networks by simultaneously and iteratively estimating the authority centrality and hub centrality for each node based on the circular (i.e. mutually reinforcing) premise that a node with zero authority centrality (i.e. not pointed by others) still can have non-zero hub centrality because of pointing to other nodes, and that those nodes it points to have non-zero authority centrality, and so on.

Kleinberg's procedure may be summarized as the estimation of eigenvalue centrality (as in (5)) on two modified versions of the original adjacency matrix, where these two matrices correspond to an authority matrix ($\mathcal{A}$) and a hub matrix ($\mathcal{H}$). Let Ω be the adjacency matrix, the authority and hub matrices ($\mathcal{A}$ and $\mathcal{H}$) are estimated as follows:

$$\mathcal{A} = \Omega^T\Omega \qquad \mathcal{H} = \Omega\Omega^T \tag{6}$$

Both, $\mathcal{A}$ and $\mathcal{H}$ are symmetrical matrices by construction. Moreover, multiplying the adjacency matrix with a transposed version of itself allows identifying directed (in or out) second order adjacencies. Regarding $\mathcal{A}$, multiplying $\Omega^T$with Ω sends weights backwards –against the arrows, towards the pointing node-, whereas multiplying Ω with Ω^T (as in $\mathcal{H}$) sends scores forwards –with the arrows, towards the pointed-to node (Bjelland et al., 2008).

Because $\mathcal{A}$ and $\mathcal{H}$ are symmetrical nonnegative matrices (even if Ω is directed and acyclic), a unique eigenvector centrality for $\mathcal{A}$ and $\mathcal{H}$ may be estimated. Accordingly, the authority centrality of each node is defined to be proportional to the sum of the hub centrality of the nodes that point to it, and the hub centrality of each node is defined to be proportional to the sum of the authority centrality of the nodes it points-to. As the authority ($\mathcal{A}$) and hub matrix ($\mathcal{H}$) share a single set of eigenvalues, let $\lambda_1$be the largest eigenvalue of the matrix $\mathcal{A}$ or $\mathcal{H}$, a financial institution's authority centrality (a_i) and hub centrality ($\hbar_i$) correspond to

$$a_i = \lambda_1^{-1}\sum_j \Omega_{ji}\hbar_j \qquad \hbar_i = \lambda_1^{-1}\sum_j \Omega_{ij}a_j \tag{7}$$

Hence, authority centrality and hub centrality addresses three issues regarding the FMIs depicted in Figure 2. First, it provides a dual centrality score based on

FMIs' role within the –directed and acyclic- network, where their collector and/or originator roles are captured. Second, as it captures the authority centrality and hub centrality of the FMIs connected to each FMI, it provides a dual centrality weighted measure of their global importance within the network. Third, as the authority (hub) centrality of each FMI is defined to be proportional to the sum of the hub (authority) centrality of the nodes that point to it (it points-to), non-substitutable FMIs at any level of the hierarchy will accumulate all hub (authority) centrality from its neighbors, therefore resulting in a potentially high centrality score that captures its lack of substitutes.

3.4. Main Results

This section presents the main results from implementing in/out-degree centrality, in/out strength, authority centrality and hub centrality. As eigenvector centrality yields equal scores to all nodes due to the directed and acyclic features of the network under analysis, results are not reported. Table 1 presents the three metrics of centrality for the adjacency matrix behind the network in Figure 2. Weights used for estimating strength, authority and hub centrality correspond to the daily average gross monetary value of the transactions occurred during year 2011 between the FMIs considered.[13]

As expected, using degree centrality yields less dissimilar results in cross section. Differences between FMIs are less marked under degree centrality because all links are considered as being equally important, while neighbors' importance is overlooked. It is frequent to find several FMIs with exactly the same degree centrality score due to coincidences in the number of links; for instance, all FMIs with one outgoing link (SEN, DERIVEX, DCV, DECEVAL, CCDC, ACH

[13] Data provided by FMIs is consolidated and examined in Banco de la República (2012), which is the main source of the aggregated data used in this paper's calculations. Gross monetary values are used for all calculations because particularly netting-efficient FMIs (e.g. CCP and CSS) resulted in –artificially- rather low centrality figures.

COLOMBIA, ACH CENIT, ATMs and CEDEC) have an out-degree score equal to 0.044, corresponding to the inverse of the total number of outgoing links $(1/23 = 0.044)$.

Table 1
Colombian FMIs' degree, strength, authority and hub centrality [b]

Type	FMI	Degree centrality		Strength centrality [c]		Authority and hub centrality [c]	
		In (k_i^{in})	Out (k_i^{out})	In (s_i^{in})	Out (s_i^{out})	Auth. (a_i)	Hub (h_i)
TP	BVC [a]	0.000	0.174	0.000	0.220	0.000	0.010
	SEN	0.000	0.044	0.000	0.101	0.000	0.000
	IDBROK	0.000	0.130	0.000	0.002	0.000	0.000
	SET-FX	0.000	0.087	0.000	0.043	0.000	0.000
	DERIVEX	0.000	0.044	0.000	0.000	0.000	0.000
	DSR	0.000	0.087	0.000	0.000	0.000	0.000
SSS+ CSD	DCV	0.217	0.044	0.200	0.478	0.003	0.750
	DECEVAL	0.174	0.044	0.115	0.021	0.003	0.034
CCP	CRCC	0.174	0.130	0.008	0.008	0.000	0.007
CSS	CCDC	0.044	0.044	0.041	0.041	0.000	0.065
LVPS	CUD	0.391	0.000	0.636	0.000	0.994	0.000
ACH	ACH COL.	0.000	0.044	0.000	0.035	0.000	0.055
	CENIT	0.000	0.044	0.000	0.010	0.000	0.016
RPS	ATMs	0.000	0.044	0.000	0.010	0.000	0.016
CCH	CEDEC	0.000	0.044	0.000	0.030	0.000	0.047

[a] BVC performs SSS and TP functions, for equities and local sovereign securities, respectively. [b] Scores are normalized (i.e. the sum of each column is 1.0). [c] Weights used for estimating strength, authority and hub centrality correspond to the daily average gross monetary value of the transactions occurred during year 2011.

In-degree centrality regards the LVPS (CUD) as the most important FMI with a 0.391 score, in which the score corresponds to the contribution of CUD to the network's total in-degree $(9/23 = 0.391)$; DCV, DECEVAL and CRCC follow with in-degree centrality scores equal to 0.217, 0.174 and 0.174, respectively.

BVC (TP+SSS) is the most out-degree central FMI with a 0.174 score, followed by CRCC and IDBROK, both with a 0.130 score.

In the case of in-strength centrality the LVPS is the most central FMI with a 0.636 score, followed by both SSSs+CSDs (i.e. DCV and DECEVAL), with 0.200 and 0.115 scores, respectively. As strength centrality corresponds to the contribution to the total network's weights, this means that CUD, DCV and DECEVAL contribute with 63.6%, 20.0% and 11.5% of the FMIs' received gross transactions. Regarding out-strength centrality*,* DCV is assigned a 0.478 score, meaning that about half of the value of FMIs' transactions originate in this SSS+CSD.

Cross-section differences increasing when shifting from degree to strength is intuitive, and follows the value of the transactions between FMIs; thus, as expected, the weighted network may be considered as more informative than the non-weighted for the case at hand. However, as said, the ability of degree and strength centrality to identify the sources of infrastructure-related systemic impact may be limited due to their local nature, in which the importance of a FMI is independent of the FMIs it is linked to.

Regarding authority centrality*,* it is clear that the LVPS (CUD) is the most central node, with a 0.994 score; this score should be interpreted as CUD concentrating 99.4% of all authority scores within the FMIs network. Despite this may seem an extreme result, it is rather intuitive: In the absence of the local currency settlement –by the sole LVPS-, no other market (i.e. securities, foreign exchange, derivatives) or infrastructure (i.e. SSS, CCP, ACHs, RPS, CCH, etc.) would be able to settle its transactions. As evident in Figure 2, the LVPS is the ultimate and sole collector of transactions within the FMIs' network, and it has no substitute. This is congruent with common knowledge of Colombian markets because the settlement of all other FMIs critically depends on CUD's proper functioning.

Two other FMIs have a non-zero authority score: DCV and DECEVAL (SSSs+CSDs). They act as collectors of the transactions captured by the most important local TPs (i.e. SEN and BVC), which both provide trading platforms for the most liquid local market: the local sovereign securities market. As DCV and DECEVAL share several neighbors in their role as collectors of transactions from other FMIs (e.g. BVC, DSR, IDBROK, CRCC) they are regarded as substitutes to some extent; this partially explains why their individual authority scores are much lower than CUD's, the ultimate and non-substitutable collector.

DCV's role as the main collector of SEN and BVC, along with its direct and intense link with the most authority central FMI (i.e. CUD), results in its systemic importance as hub within FMIs' network: DCV's hub centrality score is 0.750. This is intuitive because a critical and circular (i.e. mutually reinforcing) relationship between the local sovereign securities market and the cash settlement of all local financial markets is well-known; for instance, during 2010 and 2011 DCV's contribution to CUD's payments was about 46.9% and 44.1%, respectively (Banco de la República, 2012). In this sense, as it is locally acknowledged, the absence or failure of the most hub central FMI (i.e. DCV) would halt FMIs' network, especially because of its liquidity contribution to the CUD (i.e. the most authoritative FMI), which includes the management of sovereign securities as collaterals for liquidity provision by the central bank (i.e. via intraday or overnight repos) and for collateralized money market transactions between financial institutions (e.g. repos and sell/buy backs).

It is worth highlighting that the strong mutually reinforcing link between DCV and CUD was expected because of (i) the aforementioned preeminence of the local sovereign debt market; (ii) sovereign securities being the most widely required collateral for money market operations, either with the central bank or between financial institutions; and (iii) both FMIs sharing a real-time DvP framework. Regarding the latter, as acknowledged by CPSS (1997), *when a RTGS system is interrelated continuously with other systems as in the case of real-time DvP systems, the impact on RTGS liquidity can be more widespread and significant*.

The second most central hub is CCDC (CSS), with a 0.065 score. This is intuitive because the CCDC is the only FMI that combines the collector and originator roles for the local foreign exchange market. The third and fourth most central hubs are ACH Colombia (0.055) and CEDEC (0.047), which is also expected due to their role as the most important provider of low-value transfer services in Colombia and the sole provider of clearing and settlement for cheques issued by banking institutions, respectively. Other FMIs that have a systemic role as hubs are DECEVAL (0.034), ACH CENIT (0.016), BVC (0.010) and CRCC (0.007). In this sense, as expected, their systemic role within the FMIs' network is rather limited when compared to DCV's or CCDC's due to the monetary value of their transactions.

All in all, authority and hub centrality as metrics of systemic importance yield an intuitive result: Two FMIs could seriously imperil the safe and efficient functioning of the local financial markets: CUD (LVPS) and DCV (SSS+CSD), which are both owned and operated by the central bank. Central banks owning and managing LVPS and CSDs is a rather widespread practice, which may be due to these FMIs' critical roles within financial markets.[14]

Results display that other FMIs may threaten the safe and efficient functioning of local financial markets. The most evident is the CCDC, which is the second most hub central FMI due to the foreign exchange market being the second most important contributor to the CUD (LVPS), and because of its lack of substitutes within the local FMIs network.

[14] The World Bank (2011) reports that in 39% of the 149 countries surveyed the CSD is operated by the central bank, with such involvement of the central bank as the CSD operator being more common for CSDs that handle government securities only. According to the report, central banks are usually heavily involved during early stages of setting up securities markets (as is the case in Colombia); for instance, in lower-middle income and low income countries CSDs operated by the central bank are 49% and 71%, respectively. The same report documents that 96.6% of RTGS based LVPS are operated by central banks.

Other FMIs' threats to the safe and efficient functioning of local financial markets could be contained or mitigated by the existence of a substitute, or could be managed due to the low monetary value of their operations and the corresponding low impact in the functioning of the FMIs' network. The relative low systemic importance of those FMIs follows the degree of development attained by the markets they serve (i.e. derivatives, corporate debt, equity), which is still minor when compared to the local sovereign securities market. However, it is important to highlight that this does not mean that these FMIs are not capable of endangering the markets they serve or that they are not capable of stressing the system as a whole because of financial institutions simultaneously participating in several markets and various FMIs (i.e. cross-system risk).

3.5. Final Remarks

Despite the importance of systemic risk management is a well-known fact, infrastructure-related risk is a rather unmapped source of systemic risk. In this sense, the safe and efficient functioning of FMIs is an important source of financial stability because financial institutions, non-financial firms, and individuals rely on the clearing and settlement of their transactions, even in the middle of a period of financial turmoil. Hence, notwithstanding that FMIs have been a source of strength during the crisis (Dudley, 2012a), financial authorities should not limit their efforts of identifying and managing sources of systemic importance to financial institutions: They should also identify and manage systemically important financial market infrastructures (SIFMIs).

Network analysis' centrality has been a common and interesting metric for identifying systemically important financial institutions (SIFIs) under a macro-prudential approach. However, to the best knowledge of the authors, centrality has not been equally used for identifying SIFMIs. Therefore, concurrent with contemporary emphasis on the identification of sources of systemic importance and on infrastructure-related risk, this paper extended the use of centrality metrics from identifying SIFIs to identifying SIFMIs.

Besides the theoretical and practical advantages of using authority and hub centrality, results are intuitive and match local market's functioning in a convenient manner. Results highlight the systemic importance of the LVPS (CUD) and the most important SSSs+CSDs (DCV), in which their importance arises from their authority and hub centrality, respectively, with CUD's non-substitutability being the main driver of its extreme dominance as the ultimate SIFMI in the Colombian case.

Consequently, under the here proposed approach, both CUD and DCV display the highest systemic importance for the Colombian financial markets; this is, unlike other local FMIs, (i) the malfunctioning of CUD or DCV may halt the entire financial circuit, triggering greater disruptions in the financial system and economic activity, or (ii) they may act as powerful conduits for transmitting shocks with the local financial system.

CUD and DCV being the local SIFMIs is intuitive and concurrent with market's functioning because DCV's sovereign securities settlements contributes with nearly half of the payments processed in CUD, and DCV depends on the existence of funds in CUD for settling its transactions. This is particularly important due to the sovereign securities market being the most liquid and important within the local economy, along with the documented (CPSS, 1997) enhanced interconnection arising from both working on a real-time DvP framework.

It is important to point out that the systemic importance of CUD and DCV may be validated by the fact that they are both owned and operated by the central bank. As documented by The World Bank (2011), this is a widespread practice that may result from the critical nature of their roles, especially for low and middle-income countries.

Results are particularly useful for financial authorities. They may serve the purpose of assisting financial authorities in focusing their attention and resources –the intensity of oversight, supervision and regulation- where the infrastructure-related systemic impact is estimated to be the greatest. They may also serve the purpose of tracking the development of existing FMIs, or even identifying FMIs that are non-substitutable and large and thus may be a potential source of *single-point-of-failure risk*, as is the case of CUD in the Colombian FMIs network.

As it is always the case, it is important to highlight some caveats regarding the herein proposed model and its results. First, systemic importance is a relative (i.e. cross-section) concept, and the preeminence of CUD and DCV does not mean that other FMIs' systemic importance is null or negligible; CUD and DCV being those FMIs capable of critically impairing the financial system as a whole does not mean that other FMIs are not capable of endangering the markets they serve, or even stressing the system as a whole.

Second, results should not be regarded in isolation. They are not intended to substitute sound judgment by financial authorities, or to be regarded as the sole metric to use when deciding the systemic importance of a FMI. Despite the preeminence of CUD and DCV as SIFMIs matches local markets' functioning and common-sense, the chosen algorithm (i.e. based on the mutually reinforcing relationship between authority and hub centrality) and the dominance of sovereign securities market may be underestimating other FMIs' systemic importance.

Third, the model is specifically designed to capture the liquidity transmission channel across FMIs, but may fail to capture the market confidence transmission channel, which is especially relevant because financial institutions tend to simultaneously participate in several markets and various FMIs. Likewise, the model is not designed to consider that financial institutions may be exposed to the failure of FMIs, as would be the case of a CCP failing to fulfill its role as the counterparty of all buyers and sellers in a given market.

Finally, some challenges remain. First, financial institutions' and financial infrastructures' systemic importance has been assessed independent one from the other. As envisaged by León and Berndsen (2014), a truly comprehensive view of financial markets' functioning may require merging both institutions and infrastructures in the same network, especially because financial institutions tend to simultaneously participate in several markets and various FMIs. Second, authority and hub centrality may be useful for identifying SIFIs as well, and may aid financial authorities to identify hubs and authorities within the entire financial system, and within each market (i.e. sovereign securities, corporate securities, foreign exchange, and derivatives).[15] Third, due to the rapid evolution of local markets, the systemic importance of FMIs is also a dynamic concept that requires a periodic assessment and analysis.

[15] Applications of authority and hub centrality for financial institutions' networks may be found in León et al. (2013) and León and Pérez (2014).

4

Studying the individual grains under the microscope doesn't give a clue as to what is going on in the whole sandpile. Nothing in the individual grain of sand suggests the emergent properties of the pile.

Per Bak (1996)

4. Identifying Central Bank Liquidity Super-Spreaders in Interbank Funds Networks

Abstract

Our evidence suggests that the Colombian interbank funds market exhibits an inhomogeneous and hierarchical network structure, akin to a core-periphery organization, in which a few financial institutions fulfill the role of central bank's liquidity *super-spreaders*. Under our analytical framework a super-spreader simultaneously excels at receiving (borrowing) and distributing (lending) central bank's liquidity for the whole network, as measured by financial institutions' *hub centrality* and *authority centrality*, respectively. Our results concur with evidence from other interbank markets and other financial networks regarding the flaws of traditional direct financial contagion models based on homogeneous and non-hierarchical networks. Also, concurrent with literature on lending relationships in interbank markets, our results confirm that the probability of being a super-spreader is mainly determined by financial institutions' size. Therefore, we provide additional elements for the implementation of monetary policy and for safeguarding financial stability.

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4.1. Introduction

The interbank funds market plays a central role in monetary policy transmission: It allows *banks to exchange central bank money in order to share liquidity risks* (Fricke and Lux, 2014). For that reason, *they are the focus of central banks' implementation of monetary policy and have a significant effect on the whole economy* (Allen et al., 2009; p.639), whereas the interbank rate is commonly regarded as central bank's main target for assessing the effectiveness of monetary policy transmission. In addition, as there are powerful incentives for participants to monitor each other, the interbank funds market also plays a key role as a source of market discipline (Rochet and Tirole, 1996; Furfine, 2001).

This paper proposes an alternative approach to the analysis of the interbank funds market and its role for monetary policy transmission and financial stability. The suggested approach consists of using network analysis and an information retrieval algorithm for studying the connective and hierarchical structure of the Colombian interbank funds market. As suggested by Georg and Poschmann (2010), our approach includes central bank's monetary policy transactions (i.e. open market operations via repos) in the interbank funds network. Hence, based on a unique dataset, our approach is also more realistic than traditional network analysis on interbank data.

Our main findings come in the form of the identification of an inhomogeneous and hierarchical connective (core-periphery) structure. A few financial institutions fulfill the role of *super-spreaders* of central bank's money within the interbank funds market because of their *authority centrality* and *hub centrality*, which correspond to their simultaneous importance as global borrowers and lenders, respectively. The main results concur with those of Inaoka et al. (2004), Soramäki et al. (2007), Fricke and Lux (2014), in 't Veld and van Lelyveld (2014) and Craig and von Peter (2014) for the Japanese, U.S., Italian, Dutch and German interbank funds markets, respectively. Hence, we find further evidence against traditional assumptions of homogeneity in interbank direct contagion models (á la Allen and Gale, 2000), whereas the similarities across different interbank funds markets' topology support what Fricke and Lux (2014) allege *might be classified as a new "stylized fact" of modern interbank networks*.

Our research work contributes with new tools to examine and understand the structure and dynamics of interbank funds' networks. The resulting insights are important for the implementation of monetary policy and safeguarding financial stability. For instance, testing that financial institutions' size determines the probability of being a super-spreader in the Colombian case further supports some of the most salient findings of interbank relationships literature, as those reported in Cocco et al. (2009) and Afonso et al. (2013).

These new elements may be useful for analyzing one of the most interesting phenomena marking the Global Financial Crisis (GFC), namely the "freezing" of the interbank funds market (Acharya et al., 2012; Gale and Yorulmazer, 2013), in which money market primary dealers exerted market power and did not fulfill their role as liquidity conduits. In particular, identifying key players in the interbank funds market is important because their behavior contributes to determine the most effective set of policy instruments to achieve an efficient interest rate transmission. For instance, as suggested in Acharya et al. (2012), the presence of liquidity abundant financial institutions with market power could support central bank's virtuous role in the efficiency and stability of the interbank market as credible provider of liquidity to a broad spectrum of financial institutions. Also, characterizing the actual topology of the interbank funds network is essential for policymakers because of the relation between its structure and its resilience, robustness, and efficiency.

Likewise, our results may serve as an empirical input for the study of interbank markets' efficiency. For example, the existence of super-spreaders that provide efficient liquidity short-cuts between financial institutions may alleviate the inefficiencies resulting from the under-provision of liquidity cross-insurance in interbank markets (see Castiglionesi and Wagner (2013)).

This paper is organized in five sections. The second presents the review of existing related literature. The third section introduces the methodological approach, and presents the dataset and its main topological features from the network analysis perspective. The fourth section presents the main results. The fifth presents a probit model that explores the determinants of the probability of being a super-spreader in the Colombian interbank funds market, and the sixth section concludes.

4.2. Literature Review

The recent GFC evidenced a significant reduction in the intermediation of funds in the interbank market in most industrialized economies. In the case of the U.S., the fragile liquidity conditions forced the Federal Reserve (Fed) into a rapid reduction of its policy rate, and to implement several unconventional measures to bring liquidity directly to the money market primary dealers (i.e. the group of financial institutions that help the Fed implement monetary policy) in order to assure the intermediation of funds among financial institutions. However, instead of serving as liquidity conduits, primary dealers avoided counterparty risk and hoarded, thus aggravating the adverse liquidity conditions (Afonso et al., 2011; Gale and Yorulmazer, 2013).[1] Accordingly, the Fed had to implement additional measures to grant liquidity to other participants of the interbank funds market and to participants of other markets as well. A similar strategy was implemented by most central banks from industrialized economies.

[1] Avoiding counterparty risk and hoarding are unrelated (Gale and Yorulmazer, 2013). In the first case not supplying liquidity to other financial institutions follows concerns on the credit quality of its counterparties, whereas hoarding is due to concerns on its own access to liquidity in the future.

Colombia's Central Bank faced a similar stance back in 2002. By mid-2002 a regional market crisis triggered by political stress in Brazil led to the disruption of external credit lines and to a *sudden stop* that weakened the liquidity position of financial institutions, particularly that of brokerage firms (Vargas and Varela, 2008). These financial institutions were confronted with local credit institutions' reluctance to supply liquidity amidst volatile and uncertain market conditions; as was the case during the GFC, by mid-2002 Colombian credit institutions (i.e. banking firms) with access to central bank's liquidity feared counterparty risk and hoarded. Under these circumstances, Colombia's Central Bank decided to move up its standing purchases of local sovereign securities (i.e. TES – *Títulos de Tesorería*) on the secondary market and to authorize brokerage firms and investment funds to conduct temporary expansion operations with the central bank (BDBR, 2003). Thus, after August 2002 credit institutions, brokerage firms and trust companies have been allowed to access central bank's temporary monetary expansion operations (e.g. open market operations via repos) in the Colombian financial market.

One of the main lessons from the GFC is that policy makers have to properly identify the role of the key players in the interbank funds markets. These financial institutions may be considered the driving forces behind the supply and demand for funds in the interbank market, i.e. the liquidity super-spreaders. However, not only super-spreaders may be regarded as those contributing to liquidity transmission the most, but also as those that may distort the distribution of central banks' liquidity the greatest, as was the case of primary dealers in the U.S. interbank funds market, or of credit institutions in the Colombian money market in 2002. As documented by Acharya et al. (2012), the GFC provides evidence on how banks with excess liquidity in the interbank markets (i.e. *surplus banks*) exerted their market power by rationing liquidity to financial institutions in need of liquidity.[2] This underscores the importance of identifying

[2] Acharya et al. (2012) document how the market power of J.P. Morgan may have resulted in the liquidity rationing that affected non-depositary institutions as Bear Sterns

super-spreaders because of their role for financial stability (drivers of contagion risk) and for monetary policy transmission (conduits of central bank money).

Several studies on the topology of interbank funds market networks had been conducted, mainly to identify their properties, such as Inaoka et al. (2004) for Japan (BoJ-NET); Bech and Atalay (2010) and Soramäki et al. (2007) for the U.S. (Fedwire); Boss et al. (2004) for Austria; in ’t Veld and van Lelyveld (2014) and Pröpper et al. (2008) for the Netherlands; Craig and von Peter (2014) for Germany; Fricke and Lux (2014) for Italy; Cajueiro and Tabak (2008) and Tabak et al. (2013) for Brazil; and Martínez-Jaramillo et al. (2012) for Mexico.[3] Some of these studies also implement network metrics (e.g. centrality) for analytical purposes related to financial stability and contagion. Only Boss et al. (2004) includes the central bank as a participant in the interbank funds’ network, but does not address its particular role.

In order to identify the topology of the Colombian interbank funds network, our model implements standard network analysis’ metrics on a network resulting from merging the Colombian interbank funds market and the central bank’s open market operations (i.e. repos). Afterwards, we introduce an information retrieval algorithm to estimate *authority centrality* and *hub centrality* (Kleinberg, 1998), and to identify interbank funds market’s super-spreaders. Under our analytical framework a financial institution may be considered a super-spreader for central bank’s liquidity if it simultaneously excels at distributing liquidity to other participants (i.e. it is a good hub) and it excels at receiving liquidity from good hubs (i.e. it is a good authority), with the central bank being among the best hubs.

amid the GFC. Likewise, Acharya et al. also report that liquidity rationing by super-spreaders may have occurred in several episodes before the GFC, such as the collapse of Long-Term Capital Management in 1998 and of Amaranth Advisors in 2006.

[3] There are few studies worth mentioning in the Colombian case. Cardozo et al. (2011) and González et al. (2013) describe the functioning of the local money market. Estrada and Morales (2008) and Capera-Romero et al. (2013) study the link between the local interbank funds market structure and financial stability; however, both studies’ quantitative and analytical results are limited by their choice of datasets.

To the best of our knowledge, implementing an information retrieval algorithm for identifying super-spreaders in an interbank network that comprises central bank's liquidity provision has not been documented in related literature.

The closest research work is that of Craig and von Peter (2014), Fricke and Lux (2014) and in 't Veld and van Lelyveld (2014), who document the existence of core-periphery structures in the German, Italian and Dutch interbank funds markets, respectively. Such tiered hierarchical structure not only concurs with our results, but also verifies the importance of a limited number of financial institutions for the transmission of liquidity within the money market; in this sense, the so-called *top-tier* or *money center banks* of Craig and von Peter (2014) are analogous to our liquidity super-spreaders. However, because their main objective is different from ours, none of those papers include the direct liquidity provision by the central bank in their models, nor do they implement network analysis metrics and an information retrieval algorithm to pinpoint liquidity super-spreaders. Therefore, our work makes a contribution to the identification of central bank's liquidity super-spreaders in interbank funds.

Identifying central bank's money super-spreaders is not only critical for the implementation of monetary policy, but it also coincides with the *robust-yet-fragile* characterization of financial networks by Haldane (2009). This characterization poses major challenges from the financial stability perspective, including the revision of traditional interbank contagion models of Allen and Gale (2000) and of most interbank direct contagion models that followed (e.g. Cifuentes et al., 2005; Gai and Kapadia, 2010; Battiston et al., 2012a).

Our results concur with recent literature on the inhomogeneous and core-periphery features of interbank funds networks, and further support that these are stylized facts of interbank funds markets, as claimed by Fricke and Lux (2014). Moreover, an overlooked feature common to the U.S., Austrian, Dutch and Colombian interbank funds market is revealed: They are *ultra-small* networks in the sense of Cohen and Havlin (2003). This feature is consistent with the existence of a core that provides an efficient short-cut for most peripheral participants in the network, and points out that the structure of these interbank funds networks favors an efficient spread of liquidity, but also of contagion effects.

As tested by Craig and von Peter (2014) for the German interbank market, the probability of being a super-spreader in the Colombian case is determined by financial institutions' size. This result is robust to other samples, and overlap with alternative measures of importance (i.e. centrality) within the interbank funds network. Accordingly, concurrent with literature on lending relationships in interbank markets (Cocco et al., 2009; Afonso et al., 2013), size may be the main factor behind the interbank funds connective and hierarchical architecture. In this sense, we provide evidence that financial institutions do not connect to each other randomly, but they interact based on a size-related preferential attachment process, presumably driven by too-big-to-fail implicit subsidies or market power.

4.3. Methodological Approach

Three methodological steps are necessary for assessing financial institutions' central bank liquidity spreading capabilities in the local interbank funds market. First, the corresponding network merging interbank funds and monetary policy transactions has to be built from available data. Second, network analysis' basic statistics have to estimated and interpreted. Third, appropriate metrics for assessing the spreading capabilities of financial institutions have to be chosen. These three steps are introduced next.

4.3.1. The Interbank Funds and Central Bank's Repo Network

Data from the local large-value payment system (CUD – *Cuentas de Depósito*) was used to filter two types of transactions: interbank funds and central bank's repos. In the Colombian case the interbank funds market is not limited to credit institutions. As defined by local regulation, it corresponds to funds *provided (acquired) by a financial institution to (from) other financial institution without any agreement to transfer investments or credit portfolios*; this is, the interbank funds market consists of all non-collateralized borrowing/lending between all types of financial institutions.

The interbank funds market is the second contributor to the exchange of liquidity between financial institutions in the Colombian money market. As of 2013, the interbank funds market represents about 15.4% of financial institutions' exchange of liquidity, below sell/buy backs on sovereign local securities (84.4%), but above repos between financial institutions (0.2%).[4] Despite the fact that the use of sell/buy backs between financial institutions exceeds that of the interbank funds market, analyzing the former for monetary purposes may be inconvenient because its interest rate may be affected by the presence of securities-demanding financial institutions (instead of cash-demanding), and by the absence of mobility restrictions on collateral (Cardozo et al., 2011). Hence, as the interbank funds market is the focus of central bank's implementation of monetary policy (Allen et al., 2009), it is also the focus of our analysis.

Central bank's repos correspond to the liquidity granted to financial institutions on behalf of monetary policy considerations by means of standard open market operations, in which the eligible collateral is mainly local sovereign securities.

[4] Only sell/buy backs and repos with sovereign local securities as collateral are considered. Sovereign local securities acting as collaterals for borrowing between financial institutions in the money market usually account for about 80% of the total; if repos with the central bank are included, sovereign local securities represent about 90% of all collateralized liquidity sources.

Access to liquidity by means of central bank's repos is open to different types of financial institutions (i.e. banking and non-banking), but is limited to those that fulfill some financial and legal prerequisites. For instance, as of December 2013, 87 financial institutions were eligible for taking part in central bank's repo auctions: 42 credit institutions (CIs), 20 investment funds (IFs), 18 brokerage firms (BKs), 4 pension funds (PFs) and 3 other financial institutions (Xs). As of 2013, the value of Colombian central bank's repo facilities was about six times that of interbank funds transactions.

Merging the interbank funds market and the central bank's repos into a single network follows several reasons. First, by construction, the central bank is the most important participant of the interbank funds market, in which its intervention determines the efficient allocation of money among financial institutions, as underscored by Allen et al. (2009) and Freixas et al. (2011). Second, as in Acharya et al. (2012), the liquidity provision by the central bank is an important factor that may improve the private allocation of liquidity among banks in presence of frictions in the interbank market (i.e. market power by surplus banks). Third, merging both networks allows for comprehensively assessing how central bank's liquidity spreads across financial institutions in the interbank funds market; therefore, as in Georg and Poschmann (2010; p.2), *a realistic model of interbank markets has to take the central bank into account*. Fourth, as the access to central bank's repos is open to all types of financial institutions, identifying which institutions effectively access the central bank's open market operations facilities and excel as distributors of liquidity may provide useful information for designing liquidity facilities and implementing monetary policy.

In this vein, merging the individual networks of the interbank funds market and the central bank's repos into a single network yields what is commonly known as a *multiplex network*. As in Baxter et al. (2014), a multiplex network may be described as networks containing participants of one sort but with several kinds of connections between them. In our case financial institutions are the participants, whereas accessing liquidity from the central bank or other financial institutions results in their connections.

Figure 1 displays an analytical representation of the corresponding multiplex network. It is a multiplex network (bottom layer) resulting from merging two individual networks, respectively, the central bank repo network (upper layer) and the interbank funds network (intermediate layer). In this case vertical lines connecting superimposed vertexes are the participants, whereas each vertex is a participant fulfilling a role in the corresponding layer. In this analytical representation the role of the central bank is bounded to its repo network, but all other vertexes may overlap across the interbank funds and repo networks.

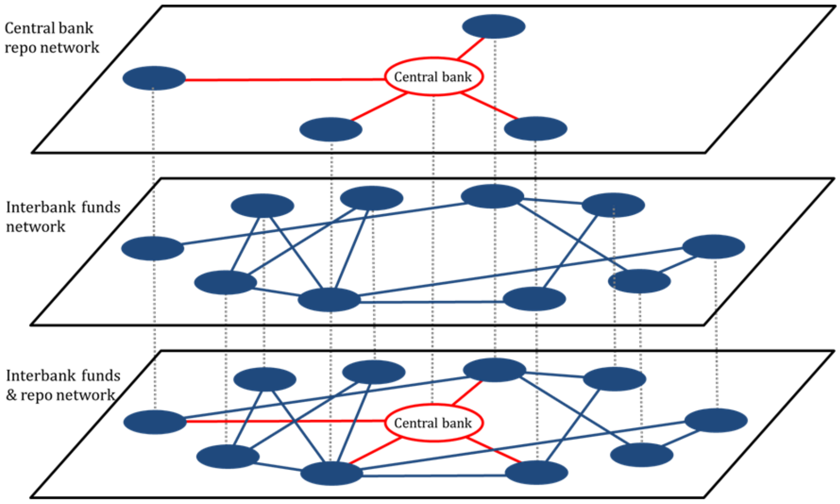


Figure 1. Merging the central bank repo network and the interbank funds network into a multiplex network. Vertical lines connecting superimposed vertexes are the participants, whereas each vertex is a participant fulfilling a role in the corresponding layer.

Accordingly, based on data from January 2 to December 17, 2013, Figure 2 displays the actual multiplex network resulting from merging the interbank funds market and the central bank's repo facilities.[5] The direction of the arrow or arc corresponds to the direction of the funds transfer (i.e. towards the borrower), whereas its width represents its monetary value. Only the original transaction (i.e. from the lender to the borrower) is considered; transactions consisting of borrowers paying back for interbank or repo funds are omitted, as are intraday repos.

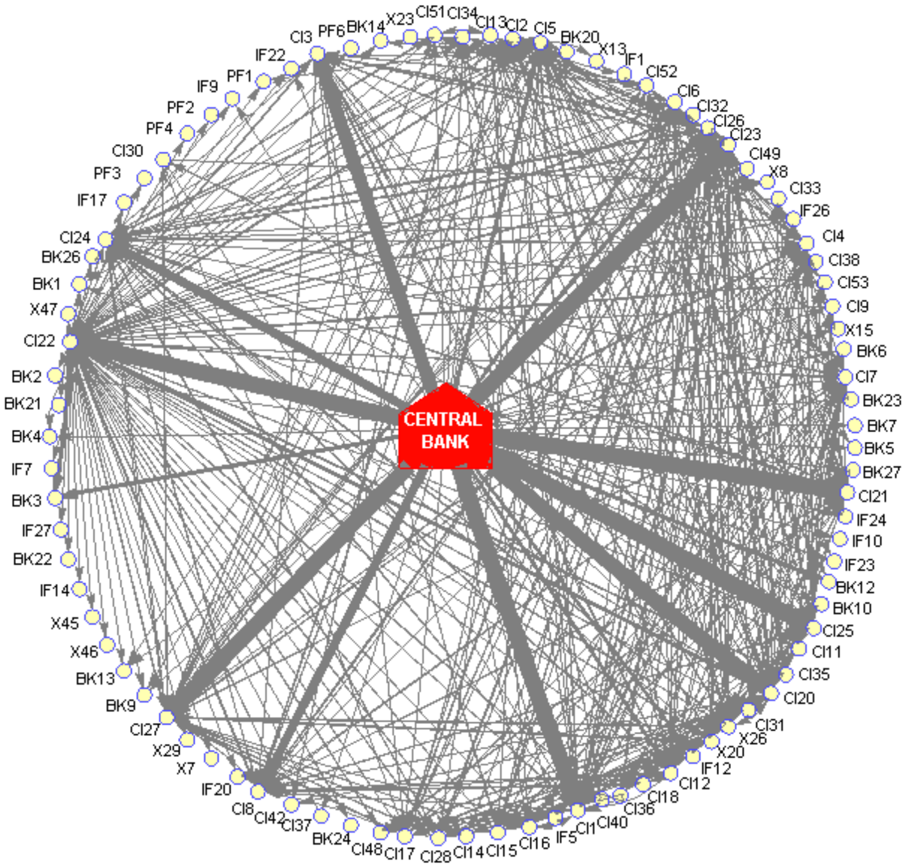


Figure 2. The interbank funds and central bank's repo network. The direction of the arrow corresponds to the direction of the funds transfer (i.e. towards the borrower), whereas its width represents its monetary value. Credit institution (CI); brokerage firm (BK); investment fund (IF); pension fund (PF); other financial institution (X).

[5] The database was extracted from the large-value payment system (CUD) by means of filtering the corresponding transaction codes; the Colombian Central Bank (i.e. the owner and operator of CUD) assigns transaction codes, and financial institutions and financial infrastructures are obliged to use them to report their transactions.

Some salient features of Figure 2 are worth mentioning. First, due to the open (i.e. non-tiered) access to central bank's liquidity, all types of financial institutions are connected to the central bank via repos. Second, the widest links correspond to funds from the central bank to some credit institutions (e.g. CI22, CI21, CI20, CI1, CI8, CI27, CI3, CI23), which corresponds to the role of the central bank as liquidity provider within 2012-2013's expansionary monetary policy framework. Third, there is a noticeable concentration of interbank links in credit institutions receiving funds from the central bank; the estimated correlation coefficient (0.75) provides evidence of the linear dependence between the liquidity granted by the central bank via repos to financial institutions and their number of links. Fourth, most weakly connected institutions correspond to non-credit institutions.

4.3.2. Network Analysis

A network, or graph, represents patterns of connections between the parts of a system. The most common mathematical representation of a network is the *adjacency matrix*. Let n represent the number of vertexes or participants, the adjacency matrix A is a square matrix of dimensions $n \times n$ with elements A_{ij} such that

$$A_{ij} = \begin{cases} 1 \text{ if there is an edge between vertexes } i \text{ and } j, \\ 0 \text{ otherwise.} \end{cases} \tag{1}$$

A network defined by the adjacency matrix in (1) is referred as an undirected graph, where the existence of the (i, j) edge makes both vertexes i and j adjacent or connected, and where the direction of the link or edge is unimportant. However, the assumption of a reciprocal relation between vertexes is inconvenient for some networks. Thus, the adjacency matrix of a directed network or *digraph* differs from the undirected case, with elements A_{ij} being referred as directed edges or arcs, such that

$$A_{ij} = \begin{cases} 1 \text{ if there is an edge from } i \text{ to } j, \\ 0 \text{ otherwise.} \end{cases} \quad (2)$$

It may be useful to assign real numbers to the edges. These numbers may represent distance, frequency or value, in what is called a weighted network and its corresponding weighted adjacency matrix (W_{ij}). For a financial network, the weights could be the monetary value of the transaction or of the exposure.

Regarding the characteristics of the system and its elements, a set of concepts is commonly used. The simplest concept is the vertex *degree* (k_i), which corresponds to the number of edges connected to it. In directed graphs, where the adjacency matrix is non-symmetrical, *in degree* (k_i^{in}) and *out degree* (k_i^{out}) quantifies the number of incoming and outgoing edges, respectively (3).

$$k_i^{in} = \sum_{j=1}^{n} A_{ji} \qquad k_i^{out} = \sum_{j=1}^{n} A_{ij} \quad (3)$$

In the weighted graph case the degree may be informative, yet inadequate for analyzing the network. *Strength* (s_i) measures the total weight of connections for a given vertex, which provides an assessment of the intensity of the interaction between participants. Akin to degree, in case of a directed graph *in strength* (s_i^{in}) and *out strength* (s_i^{out}) sum the weight of incoming and outgoing edges, respectively (4).

$$s_i^{in} = \sum_{j=1}^{n} W_{ji} \qquad s_i^{out} = \sum_{j=1}^{n} W_{ij} \quad (4)$$

Some metrics enable us to determine the connective pattern of the graph. The simplest metric for approximating the connective pattern is *density* (d), which measures the cohesion of the network. The *density* of a graph with no self-edges is the ratio of the number of actual edges (m) to the maximum possible number of edges (5).

$$d = \frac{m}{n(n-1)} \tag{5}$$

By construction, density is restricted to the $0 < d \leq 1$ range. Networks are commonly labelled as sparse when the density is much smaller than the upper limit ($d \ll 1$), and as dense when the density approximates the upper limit ($d \cong 1$). The term *complete network* is used when $d = 1$.

An informative alternative measure for density is the degree probability distribution ($\mathcal{P}_{k}$). This distribution provides a natural summary of the connectivity in the graph (Kolaczyk, 2009). Akin to density, the first moment of the distribution of degree (μ_{k}) measures the cohesion of the network, and is usually restricted to the $0 < \mu_{k} < n - 1$ range. A sparse graph has an average degree that is much smaller than the size of the graph ($\mu_{k} \ll n - 1$).

Most real-world networks display right-skewed distributions, in which the majority of vertexes are of very low degree, and few vertexes are of very high degree, hence the networks are inhomogeneous. Such right-skewness of degree distributions of real-world networks has been documented to approximate a power-law distribution (Barabási and Albert, 1999). In traditional random networks, in contrast, all vertexes have approximately the same number of edges.[6]

The power-law (or Pareto-law) distribution suggests that the probability of observing a vertex with k edges obeys the potential functional form in (6), where z is an arbitrary constant, and γ is known as the *exponent* of the power-law.

[6] Random networks correspond to those originally studied by Erdös and Rényi (1960), in which connections are homogeneously distributed between participants due to the assumption of exponentially decaying tail processes for the distribution of links –such as the Poisson distribution. This type of network, also labeled as “random” or “Poisson”, was –explicitly or implicitly- the main assumption of most literature on networks before the seminal work of Barabási and Albert (1999) on scale-free networks.

$$\mathcal{P}_{k} \propto z k^{-\gamma} \tag{6}$$

Besides degree distributions approximating a power-law, other features have been identified as characteristic of real-world networks: (i) low mean geodesic distances; (ii) high clustering coefficients; and (iii) significant degree correlation, which we explain below.

Let g_{ij} be the *geodesic distance* (i.e. the shortest path in terms of number of edges) from vertex i to j. The mean geodesic distance for vertex i (ℓ_i) corresponds to the mean of g_{ij} averaged over all reachable vertexes j in the network (Newman, 2010), as in (7). Respectively, the mean geodesic distance or average path length of a network (i.e. for all pairs of vertexes) is denoted as ℓ (without the subscript), and corresponds to the mean of ℓ_i over all vertexes. Consequently, the mean geodesic distance (ℓ) reflects the global structure; it measures how big the network is, it depends on the way the entire network is connected, and cannot be inferred from any local measurement (Strogatz, 2003).

$$\ell_i = \frac{1}{(n-1)} \sum_{j(\neq i)} g_{ij} \qquad\qquad \ell = \frac{1}{n} \sum_{i} \ell_i \tag{7}$$

The mean geodesic distance (ℓ) of random or Poisson networks is small, and increases slowly with the size of the network; therefore, as stressed by Albert and Barabási (2002), random graphs are small-world because in spite of their often large size, in most networks there is relatively a short path between any two vertexes. For random networks: $\ell \sim \ln n$ (Newman et al., 2006). This slow logarithmic increase with the size of the network coincides with the small-world effect (i.e. short average path lengths).

However, the mean geodesic distance for scale-free networks is smaller than $\ell \sim \ln n$. As reported by Cohen and Havlin (2003), scale-free networks with $2 < \gamma < 3$ tend to have a mean geodesic distance that behaves as $\ell \sim \ln\ln n$, whereas networks with $\gamma = 3$ yield $\ell \sim \ln n/(\ln \ln n)$, and $\ell \sim \ln n$ when $\gamma > 3$.

For that reason, Cohen and Havlin (2003) state that scale-free networks can be regarded as a generalization of random networks with respect to the mean average geodesic distance, in which scale-free networks with $2 < \gamma < 3$ are "ultra-small".

The clustering coefficient (c) corresponds to the property of network transitivity. It measures the average probability that two neighbors of a vertex are themselves neighbors; the coefficient hence measures the frequency with which loops of length three (i.e. triangles) appear in the network (Newman, 2010). Let a *triangle* be a graph of three vertexes that is fully connected, and a *connected triple* be a graph of three vertexes with at least two connections, the calculation of the network's clustering coefficient is as follows: [7]

$$c = \frac{(\text{number of triangles in the network}) \times 3}{\text{number of connected triples}} \tag{8}$$

Hence, by construction, clustering reflects the local structure. It depends only on the interconnectedness of a typical neighborhood, the inbreeding among vertexes tied to a common center, and thus it measures how incestuous the network is (Strogatz, 2003). Intuitively, in a random graph, the probability of a connection between two vertexes tends to be the same for all vertexes regardless of the existence of a common neighbor. Therefore, in the case of random graphs the clustering coefficient is expected to be low, and tends to zero - in the limit – for large random networks.

Contrarily, real-world complex networks tend to exhibit a large degree of clustering. Albert and Barabási (2002) report that in most –if not all- real networks the clustering coefficient is typically much larger than it is in a

[7] If three vertexes (i.e. a, b, c) exist in a graph, a triangle exists when edges (a,b), (b,c) and (c,a) are present (i.e. the graph is complete), whereas a connected triple exists if at least two of these edges are present. In this sense, a triangle occurs when there is transitivity (i.e. two neighbors of a vertex are themselves neighbors). The factor of three in the numerator arises because each triangle is counted three times when the connected triplets are counted (Newman, 2010).

comparable random network. Accordingly, in inhomogeneous graphs, as those resulting from real-world networks, the probability of two neighbors of a vertex being themselves neighbors is reported to be in the 10% and 60% range (Newman, 2010). In this sense, scale-free networks combining particularly low mean geodesic distance and high clustering implies that the existence of a few too-connected vertexes plays a key role in bringing the other vertexes close to each other. It also indicates that the scale-free topology is more efficient in bringing the vertexes close than is the topology of random graphs (Albert and Barabási, 2002).

Besides displaying low mean geodesic distances and clustering, real-world graphs also display a non-negligible degree correlation between vertexes. They are characterized by either a positive correlation, where high-degree (low-degree) vertexes tend to be connected to other high-degree (low-degree) vertexes, or a negative correlation, where high-degree vertexes tend to be connected to low-degree vertexes. Positive degree correlation, also known as *homophily* or *assortative mixing by degree*, results in the core-periphery structure typical of social networks, whereas negative degree correlation (i.e. *dissortative mixing by degree*) is typical of technological, informational, and biological networks, which display star-like features that do not usually have a core-periphery but have uniform structures (Newman, 2010). In contrast, the degree of random (i.e. homogeneous) networks tends to be uncorrelated.

Degree correlation may be measured by means of estimating the assortativity coefficient (Newman, 2010). As before, let m be the number of edges, the degree assortativity coefficient of a network (r_{k}) is estimated as follows:

$$r_{k} = \frac{\sum_{ij}\left(A_{ij} - k_i k_j / 2m\right) k_i k_j}{\sum_{ij}\left(k_i \delta_{ij} - k_i k_j / 2m\right) k_i k_j} \tag{9}$$

Where

$$\delta_{ij} = \begin{cases} 0 \text{ if } i \neq j \\ 1 \text{ if } i = j \end{cases}$$

However, it should be noted that the assortativity coefficient is not limited to vertexes' degree. Other characteristics of vertexes (e.g. age, income, gender, ethnics, size) may condition their tendency to be connected. In this case, the characteristics of connected vertexes may be correlated, which results in *assortative mixing by scalar characteristics* (Newman, 2010). For financial networks it is important to assess the intensity of the interaction between participants. Based on (9), it is possible to estimate the *assortative mixing by strength* (10).

$$r_s = \frac{\sum_{ij}\left(A_{ij} - k_i k_j / 2m\right) s_i s_j}{\sum_{ij}\left(k_i \delta_{ij} - k_i k_j / 2m\right) s_i s_j} \qquad (10)$$

Network analysis' basic statistics estimated for the interbank funds and central bank's repo network are presented in Table 1.

Table 1
Standard statistics for the interbank funds and central bank's repo network

Statistic	Including the central bank	Excluding the central bank
Participants	92	91
Density	0.07 [a]	0.07
Mean geodesic distance	2.04	2.05
Clustering (non-weighted | weighted)	0.13 | 0.11	0.16 | 0.16
Degree	(In | Out)	(In | Out)
Mean	6.62 | 6.62	6.16 | 6.16
Standard deviation	8.35 | 10.68	8.17 | 10.00
Skewness	1.59 | 2.55	1.59 | 2.64
Kurtosis	4.78 | 11.33	4.81 | 13.11
Power-law exponent	1.60 | 3.50 [b]	1.60 | 1.71
Assortativity index	0.54 | 0.06	0.57 | 0.15
Strength	(In | Out)	(In | Out)
Mean	1.09 | 1.09	1.10 | 1.10
Standard deviation	3.35 | 8.49	3.16 | 3.02
Skewness	5.37 | 9.37	6.40 | 4.29
Kurtosis	37.24 | 89.24	51.32 | 24.99
Power-law exponent	1.43 | 2.00 [b]	3.14 [b]| 1.41
Assortativity index	0.04 | -0.05	0.05 | -0.01

This table shows that the interbank funds and central bank's repo network is an approximate scale-free network, akin to other social networks documented in literature, and it resembles a core-periphery structure. [a] The calculation of density is adjusted for the exclusion of financial institutions' payback for the repo. [b] Based on Clauset et al. (2009) goodness-of-fit tests, there is a strong case for a power-law distribution with the estimated exponent.

Evidence advocates that the network is (i) sparse, with low density resulting from the number of observed links being much smaller than the potential number of links, and with an average degree (i.e. mean of links per institution) much smaller than the number of participants; (ii) ultra-small in the sense of Cohen and Havlin (2003), in which the average minimal number of links required to connect any two financial institutions (i.e. the mean geodesic distance) is particularly low (i.e. ~2) with respect to the number of participants; (iii) somewhat clustered, in which the probability of two counterparties of a financial institution being themselves counterparties is higher than expected in a random network (i.e. ~ 0), and higher than the probability of any two participants being connected in the network under analysis (i.e. $\sim 6/92$); (iv) inhomogeneous, in which the dispersion, asymmetry, kurtosis and the order of the power-law exponent for the distribution of links and their monetary values suggest the presence of a few financial institutions that are heavily connected and large contributors to the system, whereas most institutions are weakly connected and minor contributors, with the distribution of degree and strength presumably approximating a scale-free distribution;[8] (v) *assortative mixing by degree*, which means that heavily (weakly) connected financial institutions tend to be connected with other heavily (weakly) connected, especially for the in-degree case.

Altogether, these features concur with the scale-free and assortative mixing by degree connective structure of social networks reported by Newman (2010), and suggest the presence of a structure similar to a core-periphery within the network under analysis. Moreover, as the interbank funds network is ultra-small in the sense of Cohen and Havlin (2003), with participants being one financial institution away from the others, the process of liquidity spreading within the interbank funds network is highly efficient; likewise, contagion spreads within the network with ease. These main features are robust to the exclusion of the central bank.

A remarkable but overlooked feature in Table 1 is worth noting. A mean geodesic distance around 2 not only agrees with ultra-small networks (Cohen and Havlin,

[8] The estimation of the power-law exponent was based on the maximum likelihood method proposed by Clauset et al. (2009); this method is preferred to the traditional ordinary least-squares due to documented issues regarding the latter (as in Clauset et al., 2009, Stumpf and Porter, 2012). Despite some of the estimated power-law exponents do not make a strong case based on the goodness-of-fit tests of Clauset et al. (2009), the level of the exponent provides enough evidence of the alleged inhomogeneity in the distribution of degree and strength. Moreover, as the power-law distribution of links is an asymptotic property, a strict match between observed and expected theoretical properties for determining the scale-free properties of non-large networks may be impractical.

2003), but also suggests that the bulk of financial institutions require about two links (i.e. circa one financial institution in-between) to connect to any other financial institution in the interbank funds network, meaning that the core provides an efficient short-cut for most peripheral participants in the network; again, the spreading capabilities of the network are particularly high. Interestingly, mean geodesic distances reported by Boss et al. (2004), Soramäki et al. (2007), Pröpper et al. (2008), and Bech and Atalay (2010) for the Austrian, U.S. and Dutch interbank funds networks are about 2, consistent with ultra-small networks and with the role of a core providing an effective short-cut for the network; likewise, mean geodesic distances reported by León and Berndsen (2014) for the Colombian large-value payment system (CUD) and the main local sovereign securities settlement system (DCV – *Depósito Central de Valores*) are also about 2.

All in all, these findings concur with those of Craig and von Peter (2014) about the presence of tiering in the interbank funds market in the German banking system, and with the corresponding *money center banks*. Moreover, as also highlighted by Craig and von Peter (2014), these features verify that the connective structure of financial networks departs from traditional assumptions of homogeneity and representative agents (as in Allen and Gale, 2000; Freixas et al., 2000; Cifuentes et al., 2005; Gai and Kapadia, 2010), and further supports the need to achieve the main goal of this paper: identifying which financial institutions are particularly relevant for the network.

4.3.3. Identifying Super-Spreaders in Financial Networks

Whenever financial networks' observed connectedness structure is inhomogeneous the underlying system's fragility issue arises. In those networks the extraction or failure of a participant will have significantly different outcomes depending on how the participant is selected. When randomly selected, the effect will be negligible, and the network may withstand the removal of several randomly selected participants without significant structural changes. However, if selected because of their high connectivity, extracting a small number of participants may significantly affect the network's structure. In this sense, a rising amount of financial literature is encouraging the usage of network metrics of importance (e.g. centrality) for identifying super-spreaders (Haldane, 2009; Haldane and May, 2011; León et al., 2012; Markose et al., 2012; Markose, 2012).

Most literature on financial super-spreaders seeks to identify those institutions that may lead contagion effects due to their network connectivity, *high-infection individuals* (Haldane, 2009), or those that *dominate in terms of network centrality and connectivity* (Markose et al., 2012). Despite the traditional negative connotation of super-spreaders in financial networks, in the present case the super-spreader financial institution is considered a good conduit for monetary policy as well.

There are many approaches for assessing the importance of individuals or institutions within a network. However, centrality is the most common concept, with many definitions and measures available. The simplest measures are related to local metrics of centrality, such as degree (i.e. number of links, $\mathcal{k}_i$) or strength (i.e. weight of links, $\mathcal{s}_i$), but they fall short to take into account the global properties of the network; this is, the centrality of the counterparties is not taken into account as a source of centrality. Moreover, they do not capture the in-between or intermediation role of vertexes.

An alternative to degree and strength centrality is betweenness centrality. It measures the extent to which a vertex lies on paths of other vertexes (Newman, 2010). It is based on the role of the i-vertex in the geodesic (i.e. the shortest) path between two other (p and q) vertexes ($\mathcal{g}_{pq}$). Accordingly, let $u_{pq,i}$ be the number of geodesic paths from p to q that pass through vertex i, and v_{pq} the total number of geodesic paths from p to q, the betweenness centrality of vertex i ($\mathcal{b}_i$) is

$$\mathscr{b}_i = \sum_{pq} \frac{u_{pq,i}}{v_{pq}} \tag{11}$$

In the case at hand, betweenness centrality is appealing. A central intermediary in the interbank funds market should fulfill an in-between role for the network: It should stand in the interbank funds' path of other financial institutions. Yet, as it is a path-dependent centrality measure, it does not consider linkages' intensity or value, and it does not consider the centrality of adjacent nodes as a source of centrality.

The simplest global and non-path-based measure of centrality is eigenvector centrality, whereby the centrality of a vertex is proportional to the sum of the centrality of its adjacent vertexes; accordingly, the centrality of a vertex is the weighted sum of centrality at all possible order adjacencies. Hence, in this case centrality arises from (i) being connected to many vertexes; (ii) being connected to central vertexes; (iii) or both.[9] Alternatively, as put forward by Soramäki and Cook (2013), eigenvector centrality may be thought of as the proportion of time spent visiting each participant in an infinite random walk through the network.

Eigenvector centrality is based on the *spectral decomposition* of a matrix. Let Ω be an adjacency matrix (weighted or non-weighted), Λ a diagonal matrix containing the eigenvalues of Ω, and Γ an orthogonal matrix satisfying $\Gamma\Gamma' = \Gamma\Gamma = \mathrm{I}_n$, whose columns are eigenvectors of Ω, such that

$$\Omega = \Gamma\Lambda\Gamma' \tag{12}$$

If the diagonal matrix of eigenvalues (Λ) is ordered so that $\lambda_1 \geq \lambda_2 \cdots \lambda_n$, the first column in Γ corresponds to the principal eigenvector of Ω. The principal eigenvector (Γ_1) may be considered as the leading vector of the system, the one that is able to explain the most of the underlying system, in which the positive n-

9 For instance, Markose et al. (2012) use eigenvector centrality to determine the most dominant financial institutions in the U.S. credit default swap market, and to design a super-spreader tax that mitigates potential socialized losses.

scaled scores corresponding to each element may be considered as their weights within an index.

Because the largest eigenvalue and its corresponding eigenvector provide the highest accuracy (i.e. explanatory power) for reproducing the original matrix and capturing the main features of networks (Straffin, 1980), Bonacich (1972) envisaged Γ_1 as a global measure of popularity or centrality within a social network.

However, eigenvector centrality has some drawbacks. As stated by Bonacich (1972), eigenvector centrality works for symmetric structures only (i.e. undirected graphs); however, it is possible to work with the right (or left) eigenvector (as in Markose et al., 2012), but this may entail some information loss. Yet, the most severe inconvenience from estimating eigenvector centrality on asymmetric matrices arises from vertexes with only outgoing or incoming edges, which will always result in zero eigenvector centrality, and may cause some other non-strongly connected vertexes to have zero eigenvector centrality as well (Newman, 2010). In the case of acyclic graphs, such as financial market infrastructures' networks (León and Pérez, 2014), this may turn eigenvector centrality useless; this is also our case because the central bank has no incoming links, and because some peripheral financial institutions are weakly connected.

Among some alternatives to surmount the drawbacks of eigenvector centrality (e.g. PageRank, Katz centrality), the HITS (Hypertext Induced Topic Search) information retrieval algorithm by Kleinberg (1998) is convenient for several reasons. There are four main advantages in our case: (i) Unlike eigenvector centrality, it is designed for directed networks, in which the adjacency matrix may be non-symmetrical; (ii) it provides two separate centrality measures, *authority centrality* and *hub centrality*, which correspond to the eigenvector centrality as recipient and as originator of links, respectively; (iii) when dealing with weakly connected vertexes, it avoids introducing stochastic or arbitrary adjustments (as in PageRank and Katz centrality) that may be undesirable from an analytical point of view, and (iv) because the authority (hub) centrality of each vertex is defined to be proportional to the sum of the hub (authority) centrality of the vertexes that point to it (it points to), the importance of vertexes fulfilling an in-between role for the network tends to be captured.[10]

The estimation of authority centrality (a_i) and hub centrality ($\hbar_i$) results from estimating standard eigenvector centrality (12) on two modified versions of the adjacency matrix, $\mathcal{A}$ and $\mathcal{H}$ (13).

$$\mathcal{A} = \Omega^T\Omega \qquad\qquad \mathcal{H} = \Omega\Omega^T \tag{13}$$

Multiplying the adjacency matrix with a transposed version of itself allows identifying directed (*in* or *out*) second order adjacencies. Regarding $\mathcal{A}$, multiplying Ω^T with Ω sends weights backwards –against the arrows, towards the pointing node-, whereas multiplying Ω with Ω^T (as in $\mathcal{H}$) sends scores forwards – with the arrows, towards the pointed-to node (Bjelland et al., 2008). Thus, the HITS algorithm works on a circular thesis: The authority centrality (a_i) of each participant is defined to be proportional to the sum of the hub centrality ($\hbar_i$) of the participants that point to it, and the hub centrality of each participant is defined to be proportional to the sum of the authority centrality of the participant it points-to.

The circularity of the HITS algorithm is most convenient for identifying super-spreaders of central bank's liquidity. An institution may be considered a good conduit for central bank's liquidity if it simultaneously is a good hub (i.e. it excels at distributing liquidity within the interbank funds market) and a good authority (i.e. it excels at receiving liquidity from good hubs, with the central bank being among the best hubs). On the other hand, if an institution is a good

[10] The relevance of the in-between role of a vertex has an inverse relation with the existence of other vertexes providing the same connective role. Thus, a vertex being the sole provider of a connective role will concentrate all the weighted average centrality of the vertexes it connects. Thus, in this sense, the HITS algorithm captures the in-between role of vertexes.

authority but a meager hub it may be regarded as a poor conduit for central bank's liquidity; likewise, if an institution is a good hub but a modest authority its central bank's liquidity transmission capabilities may be regarded as low.

The eigenvector centrality framework behind the estimation of authority centrality and hub centrality allows both metrics to capture the impact of liquidity on a global scale. Accordingly, all financial institutions that are connected to the central bank and the most important hubs, either directly or indirectly, inherit some degree of authority centrality depending on the intensity of the links to those providers of liquidity. Likewise, all financial institutions that distribute liquidity in the system inherit some degree of hub centrality depending on the intensity of the links to all those receiving liquidity.

In this sense, an institution simultaneously displaying a high score in both authority (a_i) and hub centrality ($\hbar_i$) is expected to be a dominant participant in the transmission of funds from the central bank to the interbank funds market and within the interbank funds market. Therefore, the liquidity spreading index of an i-financial institution (LSI_i) corresponds to the product of both normalized centrality measures, as in (14). The choice of the product operator is consistent with the aim of identifying institutions that simultaneously are a good hub and a good authority.[11]

$$LSI_i = \frac{\left(\frac{a_i}{\sum_{i=1}^{n} a_i} \times \frac{\hbar_i}{\sum_{i=1}^{n} \hbar_i}\right)}{\sum_{i=1}^{n}\left(\frac{a_i}{\sum_{i=1}^{n} a_i} \times \frac{\hbar_i}{\sum_{i=1}^{n} \hbar_i}\right)} \tag{14}$$

Where, by construction

[11] Other conjunction operators may be chosen, such as $min(\cdot)$. Using the average of hub centrality and authority centrality is feasible, but may fail to discard institutions that are good authorities but mediocre hubs, and vice versa.

$$0 \leq LSI_i \leq 1$$

and

$$LSI = \sum_{i=1}^{n} LSI_i = 1$$

As LSI_i is a measure of the contribution of an individual financial institution to the product of all financial institutions' hub and authority centrality, super-spreaders may be defined as those contributing the most to LSI. Super-spreaders are those financial institutions that simultaneously excel as global borrowers and lenders of central bank's money in the interbank funds network. To the best of our knowledge, this is the first attempt to use a global and non-path dependent centrality measure to identify super-spreaders in an interbank network comprising the central bank.

The link between being a super-spreader and systemic importance is straightforward. Hub centrality and authority centrality are measures that capture financial institutions' global interconnectedness-related externalities, as a lender and borrower of interbank funds, respectively. Unlike other alternatives to measuring centrality in networks, these two measures are convenient because of their ability to capture direct and indirect linkages at all order of adjacencies, while considering the direction and weight of the connections. However, despite being a super-spreader of interbank funds should encompass the simultaneous ability to borrow and lend, in the case of systemic importance this may not be the case: A financial institution being a large hub or a large authority may be regarded as a great contributor to systemic risk. Therefore, the design of the LSI in this article follows the main purpose of identifying those financial institutions that simultaneously excel as borrowers and lenders; in the case of pursuing financial stability by limiting systemic risk, the index should not oblige such simultaneity.

4.4. Main Results

Based on the methodological approach described in the previous section, the liquidity-spreading index (LSI_i) was estimated for the interbank funds and central bank's repo network. This network comprises 28,393 lending transactions from January 2 to December 17, 2013. Figure 3 presents the top-30 financial institutions by their estimated LSI_i.[12]

The first 17 are credit institutions (CIs), which together contribute with 99.98% of LSI. The concentration in the top-ranked financial institutions is clear, with the first (CI22) contributing with about 30% of the LSI, and the top-five (CI22, CI20, CI1, CI23, CI8) contributing with about 79%. Hence, results suggest that CIs provide the main conduit for central bank's liquidity within the Colombian financial system. As reported in Appendix 1, CIs providing the main conduit for central bank's liquidity is robust to other samples (i.e. 2011 and 2012). Likewise, the most important super-spreaders (e.g. CI22, CI20, CI1, C23) tend to be stable across samples.

[12] The central bank's LSI_i is neither reported, nor analyzed. After estimating LSI_i as in (14) the central bank's score is excluded, and the remaining scores are standardized accordingly. This follows our focus on identifying super-spreader financial institutions different from the central bank. The same procedure applies for other centrality measures here implemented.

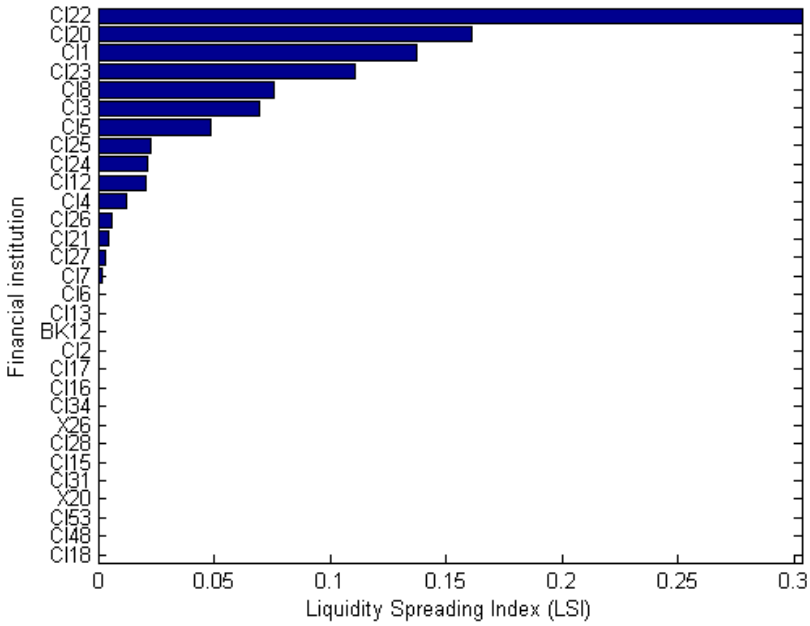


Figure 3. Top-30 financial institutions by estimated LSI_i. Credit institutions (CIs) dominate the contribution to LSI. Other types of contributing institutions are brokerage firms (BKs) and other financial institutions (Xs).

Figure 4 displays a hierarchical visualization of how liquidity spreads from the central bank throughout the interbank funds market. The hierarchies introduced correspond to different levels of contribution to LSI. Two levels were chosen for illustrative purposes: The first layer (i.e. the closest to the central bank, in shadowed boxes) corresponds to those eleven financial institutions in the 99th percentile of LSI, whereas the second layer corresponds to those eighty whose contribution is about 1% of the LSI. The height of the boxes corresponds to the authority centrality (i.e. importance as global borrower, a_i), whereas their width to the hub centrality (i.e. importance as global lender, $\hbar_i$), with those financial institutions receiving liquidity directly from the central bank (i.e. via repos) appearing with a thicker border; the width of the arrows correspond to the monetary value of the transactions, whereas their direction corresponds to the direction of the funds (i.e. towards the borrower).

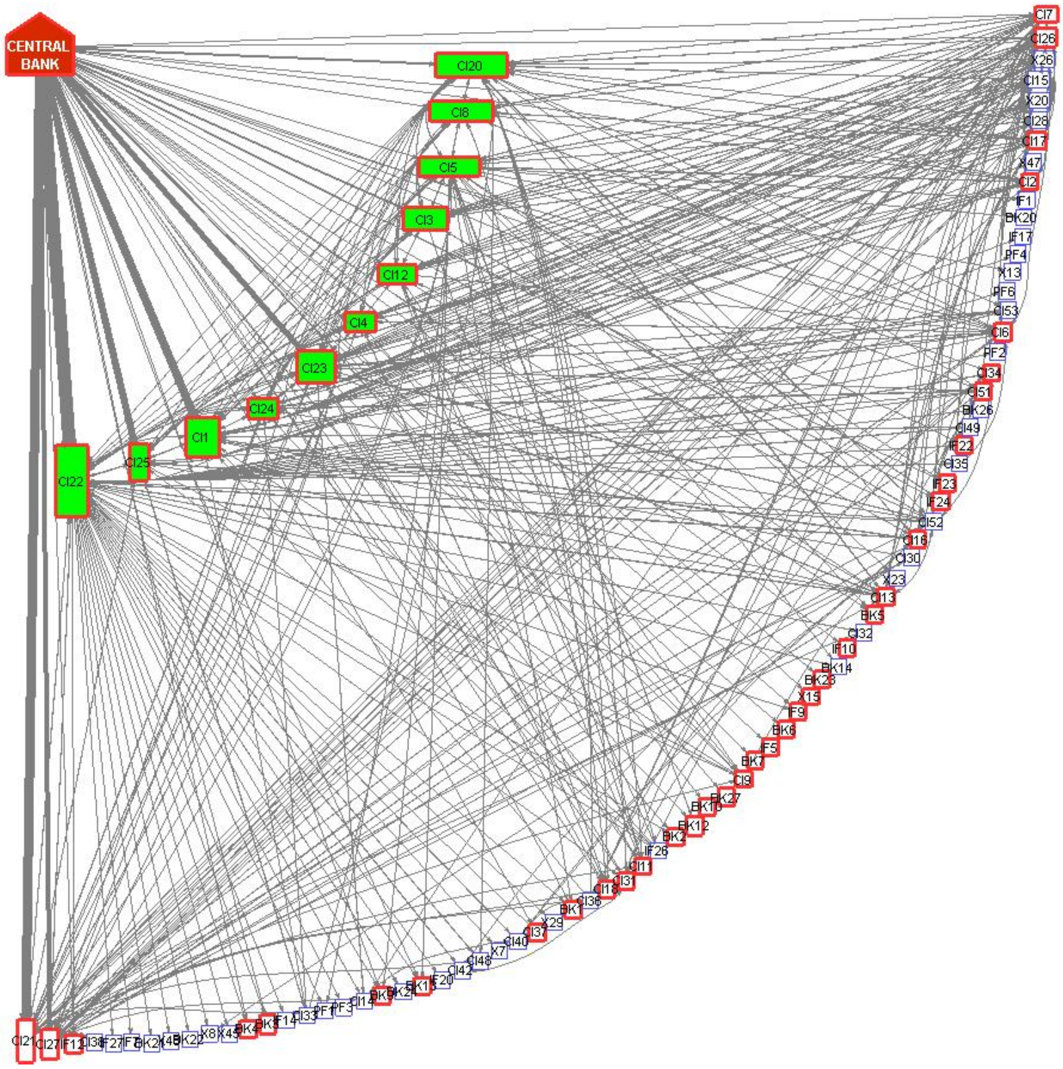


Figure 4. Hierarchical visualization of the interbank funds and central bank's repo network. The height of the boxes corresponds to the authority centrality (a_i), width to the hub centrality ($\hbar_i$); the first layer of institutions (shadowed boxes) corresponds to the 99th percentile of the LSI_i; financial institutions receiving liquidity directly from the central bank are marked with a thicker border; as usual, the width of the arrows corresponds to the monetary value of the transactions, whereas their direction goes towards the borrower. Institutions in the visualization are credit institutions (CIs); brokerage firms (BKs); investment funds (IFs); pension funds (PFs); other financial institutions (Xs).

Visual inspection of Figure 4 yields some interesting remarks. Regarding the first layer, it is unmistakable that it congregates the biggest (i.e. highest and widest) boxes, which signals their superior liquidity spreading capabilities within the network; in this sense, under the arbitrarily chosen percentiles, the first layer gathers what could be considered as central bank's liquidity super-spreaders: CI22, CI25, CI1, CI24, CI23, CI4, CI12, CI3, CI5, CI8, CI20.

It is also visible that the height (i.e. authority centrality) and width (i.e. hub centrality) of financial institutions in the first layer is dissimilar in cross-section: Some boxes (e.g. CI1 and CI23) are squared (i.e. with similar authority and hub centrality), whereas others are vertical (i.e. larger authority centrality, e.g. CI22) or horizontal rectangles (i.e. larger hub centrality, e.g. CI20). Such contrast suggests that super-spreaders are not homogeneous. Moreover, this contrast overlaps with the estimated linear dependence between hub centrality and authority (see Appendix 2), in which correlation is positive, yet not decisively strong (i.e. 0.33).

It is noticeable that the first layer congregates credit institutions (CIs) only, whereas the second displays a mixed composition. Also, financial institutions in the first layer tend to coincide with those that directly receive the most liquidity from the central bank (i.e. by the width of the arrows), and they all have direct linkages to the central bank (i.e. boxes with red borders). However, several financial institutions in the second layer have direct links to the central bank as well, with CI21 receiving a particularly large amount of liquidity from the central bank but failing to distribute it within the network (i.e. a high yet narrow box).

Figure 5 displays the graph corresponding to the interbank funds transactions between the eleven institutions in the first layer (i.e. the core) of Figure 4. The diameter of the circles corresponds to the value of each financial institution's lending within the core network; as usual, the width of the arrows corresponds to the monetary value of the transactions, whereas their direction corresponds to the direction of the funds (i.e. towards the borrower). The sum of transactions' value within the core represents 52.07% of the interbank funds network.

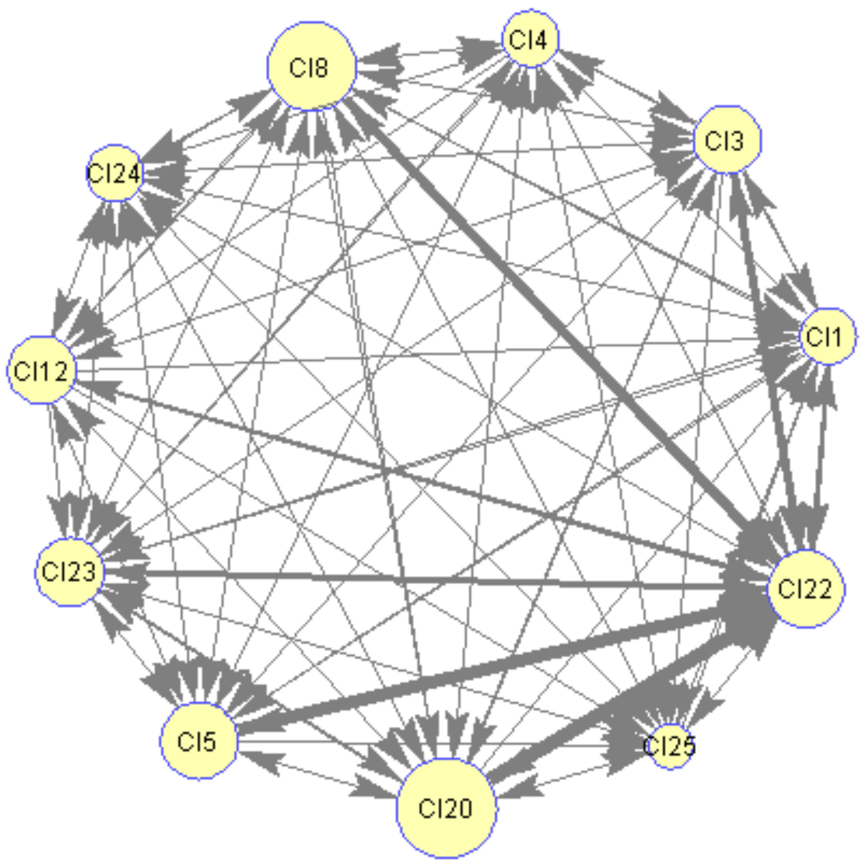


Figure 5. The interbank funds core network. Credit institutions (CIs) here included correspond to those in the first layer of Figure 4. The diameter of the circles corresponds to the value of the lending within the core network; as usual, the width of the arrows corresponds to the monetary value of the transactions, whereas their direction goes towards the borrower. The central bank is not considered.

As expected, the core is a dense graph (i.e. 93.6% of the potential connections is observed), with a mean geodesic distance about 1.06, in which the degree is evenly distributed (i.e. mean degree 9.36; standard deviation about 1.00). Nevertheless, the strength displays inhomogeneity, with the diameter of each financial institution and the width of arrows varying in cross section; for instance, the total lending of CI20 is about 15 times that of CI25, whereas the total borrowing of CI22 is about 177 times that of CI12.

Regarding the periphery, it is evident that most financial institutions in Figure 4 display small boxes (i.e. low authority and hub centrality). This not only concurs with previous evidence of inhomogeneity in the network under analysis, but also with literature on financial networks. It is also noticeable that many financial institutions in the second layer maintain very few connections with the rest of the network, most of them as borrowers, which suggests that during the period under analysis (i.e. almost a yearlong) they had a limited number of counterparties in the interbank funds market, either by choice or by market constraints. On the other hand, all financial institutions in the first layer appear to be heavily connected to the entire network, as borrowers and lenders, as expected from core financial institutions in a core-periphery structure.

Figure 6 displays the graph corresponding to the interbank funds transactions between the institutions in the second layer of Figure 4 (i.e. the periphery). The sum of transactions' value within the periphery represents 10.66% of the whole interbank funds network, whereas transactions between the core and the periphery represent about 37.27%. Such preference of peripheral financial institutions to maintain relationships with the core overlaps with evidence reported by Cocco et al. (2009), Fricke and Lux (2014), and Craig and von Peter (2014).

As expected, the periphery is a sparse graph (i.e. 2.4% of the potential connections is observed), in which degree and strength are unevenly distributed; mean degree is about 1.9, with a standard deviation about 3.5, whereas mean strength is about 1.3% with a 4.0% standard deviation.[13] Most peripheral institutions (48) have no links with other peripheral institutions during the period under analysis (i.e. about one year), which means that their liquidity sources were restricted to borrowing from core financial institutions or the central bank. The residual, comprised by 32 institutions, are rather well-connected between them, and most of them (30) are credit institutions.

[13] As is customary to exclude non-reachable (i.e. unconnected) participants from the calculation of the mean geodesic distance, and because most of the financial institutions are non-reachable, the mean geodesic distance of the periphery may not be informative, thus it is not reported.

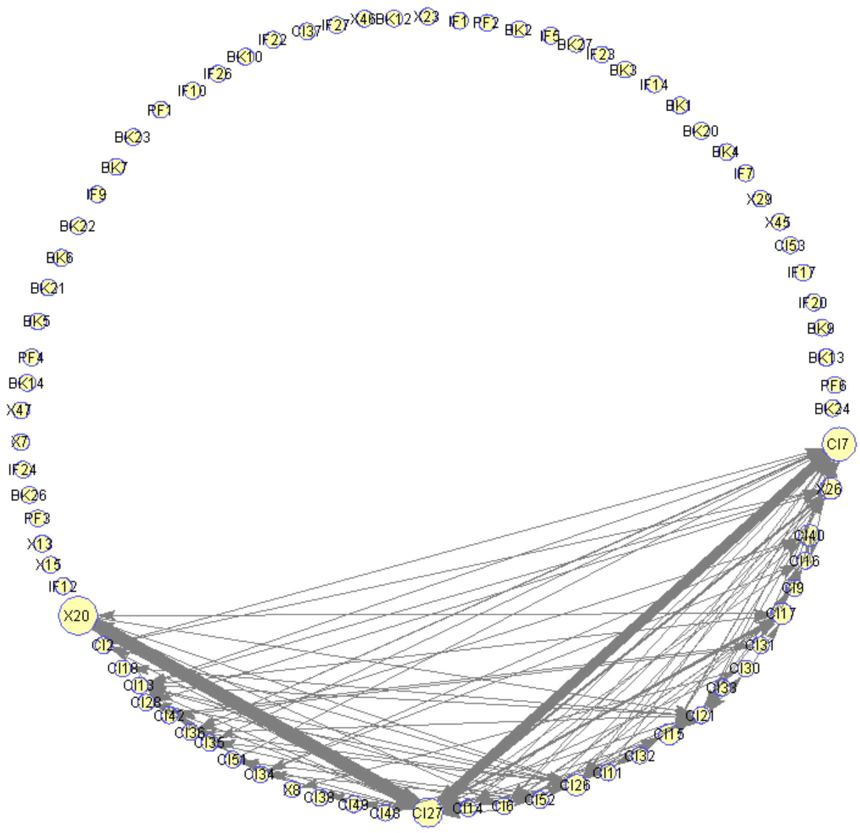


Figure 6. The interbank funds periphery network. Institutions here included correspond to those in the second layer of Figure 4. The diameter of the circles corresponds to the value of the lending within the periphery network; as usual, the width of the arrows corresponds to the monetary value of the transactions, whereas their direction goes towards the borrower. Institutions in the visualization are credit institutions (CIs); brokerage firms (BKs); investment funds (IFs); pension funds (PFs); other institutions (Xs). The central bank is not considered.

4.5. What Makes a Super-Spreader in the Colombian Interbank Funds Market?

The size of institutions in financial markets is known to be inhomogeneous, with a few that may be regarded as "too-large" and many "too-small", presumably approximating a power-law distribution (Gabaix et al., 2003; Fiaschi et al., 2013), even in the Colombian case (León, 2014). Cajueiro and Tabak (2008), Craig and von Peter (2014), Fricke and Lux (2014), and in 't Veld and van Lelyveld (2014) confirm that there is a significant relation between financial institutions' size and their position in the interbank funds' hierarchy in the respective Brazilian, German, Italian, and Dutch interbank markets. In these markets large banks tend to be in the core, whereas small banks are found in the periphery. This is consistent with Cocco et al. (2009), who report that size is an important determinant of interbank lending relationships, with smaller banks being less likely to act as intermediaries.

Regarding the Colombian case the relation between size and the role as super-spreader in the interbank funds market is evident. Figure 7 exhibits the double logarithmic scale plot for Colombian financial institutions' assets value, in which the horizontal axis corresponds to the logarithm of assets value, the vertical axis to the logarithm of the cumulative frequency for each asset value, and each circle represents a single financial institution.

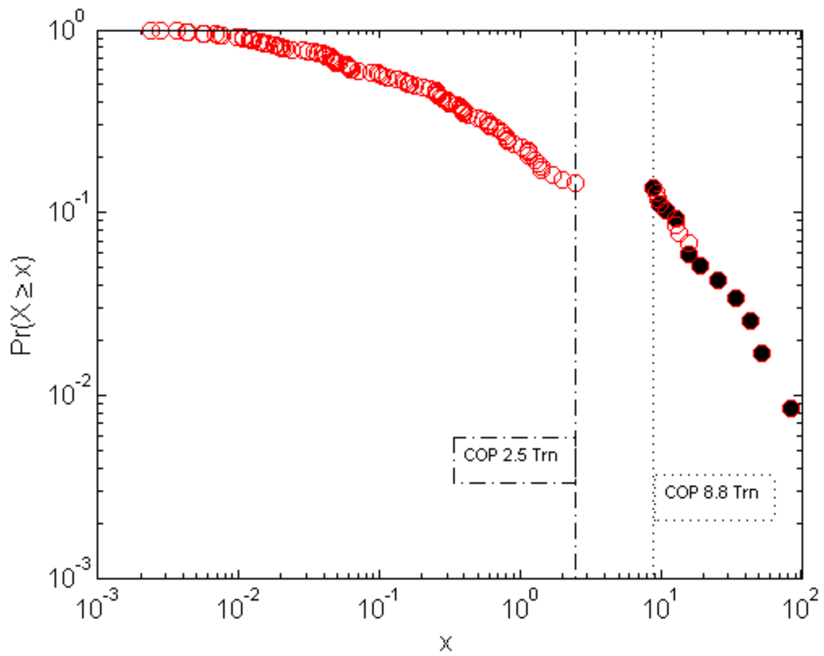


Figure 7. Distribution of Colombian financial institutions' size (double logarithmic scale). There are two different size regimes, in which super-spreaders in Figure 4 (filled circles) correspond to large financial institutions. Size corresponds to the 2013 average asset value reported by the Financial Superintendence of Colombia; filled circles correspond to super-spreaders in Figure 5. Based on León (2014).

As also reported by Fiaschi et al. (2013) for the U.S. financial market, such double logarithmic plot exhibits an interesting feature: It is an “interrupted” plot. Such interruption, also reported for the Colombian case (León, 2014), yields two different size regimes with two different distributional forms. It verifies that in the Colombian financial market there are large (i.e. above COP 8.8 Trillion) and small (i.e. below COP 2.5 Trillion) financial institutions, and that they may be pinpointed rather objectively.

Filling (in black) the circles corresponding to the super-spreaders (i.e. financial institutions in the 99th percentile of LSI, Figure 4) yields an obvious observation: In the Colombian interbank funds market all super-spreaders pertain to the largest financial institutions regime (i.e. assets above COP 8.8 trillion). The average size of super-spreaders is about 33 times that of other financial institutions; this agrees with evidence reported by Craig and von Peter (2014) for the German interbank funds market (i.e. about 51 times). Therefore, two distinctive features may determine super-spreading capabilities of financial institutions in the Colombian interbank funds market, namely being a credit institution and being large.

In order to provide further evidence on the characteristics of the financial institutions that may be considered as super-spreaders, we implement a probit regression model on a set of institution-specific variables that are standard in the literature: size, leverage, financial performance, and the concentration of borrowing and lending counterparties. These variables serve as regressors in the probit model, in which the dependent variable (LSI_i) is binary according to financial institution’s super-spreader features: $LSI_i = 1$ if it pertains to the 99th percentile (i.e. it is a super-spreader), and $LSI_i = 0$ otherwise.

Regarding the choice of the independent variables, not only size ($size$) is a leading determinant of the position within a core-periphery structure for the German, Italian and Dutch interbank markets, but graphical inspection of Figure 7 also points out the relevance of size in the Colombian case. Leverage (lev) corresponds to the traditional debt to assets ratio, which is intended to test whether super-spreaders may be predicted by the capital structure of financial institutions. Financial performance corresponds to the return over assets ratio (roa), which is intended to test whether super-spreaders may be predicted by their profitability. Finally, the borrowing concentration ($borr$) and lending concentration ($lend$) correspond to the calculation of the Herfindahl-Hirschman index (HHI) on the contribution of borrowing and lending counterparties for each financial institution, respectively. Including these two variables aims at examining whether concentrating (or diversifying) counterparties may serve to predict super-spreaders.[14]

Accordingly, based on the choice of percentile for the dataset under analysis, the probit regression model serves as a test of the significance of the selected institution-centric variables for predicting the membership of the eleven super-spreaders in Figure 4. Let X represent the set of institution-specific variables (i.e. $size$, lev, roa, $borr$, $lend$); $\mathcal{P}$ denote probability; and Φ the Cumulative Distribution Function of the standard normal distribution, the probit regression model is as in (15).

$$\mathcal{P}(LSI_i = 1 | X) = \Phi(X'\beta) \qquad (15)$$

Where

$$LSI_i = \begin{cases} 1 \text{ if } i \text{ is a super-spreader} \\ 0 \text{ otherwise} \end{cases}$$

[14] Financial institutions' access to central bank's liquidity –an intuitive variable- would predict super-spreaders perfectly; hence, despite its consideration in the probit model makes its estimation unfeasible, it should be considered for analytical purposes. Some institution-centric variables (e.g. equity, return over equity) were discarded due to their lack of significance or redundancy with those presented, whereas others (e.g. non-perfoming loans) were excluded because they are available for credit institutions only. Initial liquidity balance in central bank's accounts, cash, and proprietary investments, were discarded due to potential multicollinearity with size; the cross-correlation between the three variables is high, and asset size encompasses them all. Likewise, the value of repos with the central bank is also discarded for potential multicollinearity with size.

The independent variables correspond to daily averages during the sample (i.e. January 2 – December 17, 2013). All independent variables are standard scores (i.e. number of standard deviations above the estimated mean). Standard descriptive statistics for the variables are presented in Appendix 3.

However, because the functional form of LSI_i in (14) seeks to filter out those financial institutions that simultaneously display both authority centrality (a_i) and hub centrality (h_i), the same probit regression model is implemented in two alternate models with authority and hub centrality as dependent variables. Using authority centrality and hub centrality as alternative dependent variables helps us to examine if the selected independent variables differ in their explanatory power because of the potentially distinct role of financial institutions as global receivers or distributors of liquidity. Simple local centrality measures, namely degree (i.e. number of links, k_i) and strength (i.e. weight of links, s_i), and betweenness centrality (i.e. role as connector between vertexes, b_i) are also reported for robustness and comparison purposes.

Overall, concurrent with the literature, we expect a strong and positive linear dependence between size and the probability of being a super-spreader. One would expect that the more leveraged a financial institution is, the cheaper its cost of capital, and consequently the cheaper the liquidity it may lend. Therefore, we expect a positive relation between leverage and the probability of being a super-spreader and a good hub, but we do not have a clear expectation on the relation with the probability of being a good authority. Regarding financial performance, as larger banks are reported to be more cost and profit efficient than their smaller peers in the intermediation of funds in the Colombian financial system (Sarmiento and Galán, 2014), we expect a positive and linear dependence between financial performance and the probability of being super-spreader, good hub, and good authority. About the concentration of borrowing and lending, we expect an inverse relation between concentration of counterparties and the probability of being a super-spreader; on the other hand, the probability of not being a super-spreader is expected to be high for peripheral financial institutions, which have been documented to concentrate their borrowing relationships (see Cocco et al. (2009) and Afonso et al. (2013), who analyze small financial institutions in the periphery of the U.S. and Portuguese interbank markets, respectively).

Regarding alternative centrality measures, we expect financial institutions' degree (k_i), strength (s_i), and betweenness (b_i) to coincide with their LSI_i. As LSI_i is a global measure of centrality that incorporates the number of linked neighbors, the intensity of the linkages at all possible order adjacencies, and the in-between role of vertexes, we expect to observe consistency with degree, strength and betweenness. The linear dependence (i.e. correlation) between the selected dependent variables supports such expectation (see Appendix 2).

Table 2 shows the results of estimating the probit regression model in (15)[15]. The overall fit of the probit model is adequate for predicting super-spreaders: The pseudo R-squared is about .74, and the estimated model predicts 93.51% of the observations (96.97% of $LSI_i = 0$ and 75.29% of $LSI_i = 1$). Size is the sole significant determinant of the probability of being a super-spreader. This concurs with Craig and von Peter's (2014) findings of large banks that dominate wholesale activity in money markets (i.e. *money center banks*) in the German interbank market.

[15] An analogous Ordinary Least Squares cross-section model yielded results consistent with those attained with the probit model here reported.

Table 2
Probit regression on selected determinants

Variable [a, b]	LSI_i	h_i	a_i	k_i [h]	s_i	b_i
Size	2.758	2.456	3.848	168.48	3.644	1.585
$(size)$ [c]	(2.40)**	(3.16)***	(3.38)***	(1.80)*	(2.34)**	(2.43)**
Leverage	1.002	0.322	-0.101	0.065	-0.233	0.988
(lev) [d]	(0.41)	(0.85)	(-0.51)	(0.31)	(-1.34)	(0.95)
Financial performance	-0.377	-0.320	0.128	0.157	0.005	-0.458
(roa) [e]	(-0.29)	(-1.26)	(0.77)	(0.72)	(0.03)	(-0.80)
Borrowing concentration	0.010	-0.765	0.324		-0.392	-0.664
$(borr)$ [f]	(0.03)	(-3.43)***	(1.44)		(-2.09)**	(-2.42)**
Lending concentration	-0.069	0.091	-0.009		-0.194	-0.069
$(lend)$ [g]	(-0.11)	(0.42)	(-0.05)		(1.11)	(-0.21)
Constant	-2.144	-0.294	0.282	67.48	0.870	-1.591
	(-1.24)	(-0.89)	(0.77)	(1.80)*	(1.55)	(-2.27)**
Observations	77					
Observations = 1	11	27	25	65	37	16
Pseudo R-squared	.741	.559	.420	.506	.342	.560
% of correctly classified [i]	.935	.883	.844	.870	.779	.896

The probability of being a super-spreader is determined by financial institutions' size. The probability of financial institutions contributing to the 99th percentile of other centrality measures is also determined by size, but some (i.e. h_i, s_i, and b_i) by borrowing concentration as well. [a] All independent variables are standard scores of the original variable (i.e. number of standard deviations above the estimated mean), whereas the dependent variables correspond to 1 when the financial institution contributes to the 99th percentile, and zero otherwise. [b] *t*-statistics in parenthesis, significant at .10*, .05** and .01***. [c] Assets' value, as reported by the Financial Superintendence of Colombia (SFC). [d] Debt to assets ratio, based on balance sheet data reported by SFC. [e] Return over assets. [f] Herfindahl-Hirschman index on weighted borrowing counterparties. [g] Herfindahl-Hirschman index on weighted lending counterparties. [h] Borrowing and lending concentration are not reported because the maximum likelihood estimation was unfeasible (i.e. perfect prediction). [i] Weighted average of correct classifications of the dependent variable, in which the correct classification of a super-spreader consists of a predicted probability above .5, whereas the correct classification of a non-super-spreader consists of a predicted probability lower than or equal to .5

As expected, size is the major determinant of the probability of being a super-spreader, a good hub, and a good authority. Likewise, size is the major determinant of the probability of displaying high degree, strength and betweenness. In the case of hub centrality, strength and betweenness, the probability is also determined by borrowing concentration, in which the negative sign denotes that the less concentrated the borrowing counterparties, the more likely is to be a central financial institution in the interbank funds market. Leverage, financial performance, and lending concentration are neither determinants of the probability of being a super-spreader, nor determinants of the probability of being central to the network. These results are robust to other samples (i.e. 2011 and 2012, in Appendix 4). Also, as expected, there is consistency between LSI_i and the alternative dependent variables.

In this sense, financial institutions do not connect to each other randomly, but they interact based on a size-related preferential attachment process. Such size-related preferential attachment coincides with literature about the role of market power and too-big-to-fail subsidies (e.g. implicit or explicit access to last-resort lending) on the increased likelihood of large financial institutions to appear in both sides (i.e. borrowing and lending) of financial markets, their ability to obtain lower funding rates, and their willingness to engage in riskier activities by means of increasing leverage and risk-taking (see Cocco et al., 2009; Bertay et al., 2013; IMF, 2014). Likewise, the size-related preferential attachment process supports evidence of smaller financial institutions relying on stable borrowing and lending relationships with large counterparties (see Cocco et al., 2009; Afonso et al., 2013).

4.6. Final Remarks

In this paper we find that the Colombian interbank funds market displays an inhomogeneous and hierarchical (akin to a core-periphery) connective structure, in which a few financial institutions fulfill the role of super-spreaders of central bank's money within the interbank funds market. Thus, our research work not only contributes to central banks' efforts to analyze the structure and functioning of interbank funds markets, but also contributes to designing liquidity facilities, implementing monetary policy, and identifying those financial institutions with a systemic role in the corresponding market and other related ones (e.g. sovereign securities, foreign exchange, etc.).

Accordingly, four particular contributions of our research work are worth stating. First, we propose a methodological approach that explores the connective structure of the interbank funds network and identifies those financial institutions that may be considered as the most important conduits for monetary policy transmission and for liquidity spreading among participating financial institutions. In this sense, our approach is able to identify interbank funds' systemically important financial institutions, which should be the focus of financial authorities' efforts for preserving financial stability. Likewise, in the sense of Acharya et al. (2012), the presence of super-spreaders –with market power- could support central bank's virtuous role in the efficiency and stability of the interbank market as credible provider of liquidity to a broad spectrum of financial institutions.

Second, our results support recent findings about the existence of some stylized facts in financial networks, namely an inhomogeneous and hierarchical connective structure that contradicts traditional assumptions in interbank contagion models (i.e. homogeneity, symmetry, linearity, normality, static equilibrium). Confirming the robust-yet-fragile characterization of financial networks by Haldane (2009) entails major challenges for financial authorities contributing to financial stability. For instance, as argued after the crisis (Kambhu et al., 2007; May et al., 2008; Haldane and May, 2011; León and Berndsen, 2014), the most evident challenge comes in the form of focusing financial authorities' preventive actions on super-spreaders, which requires shifting from institution-calibrated to system-calibrated prudential regulation.

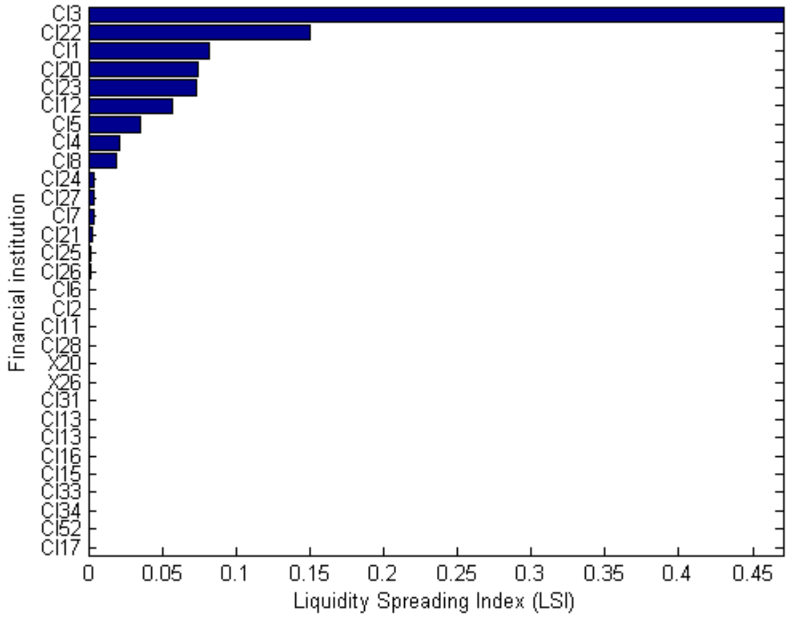
Third, as is the case of interbank funds networks in the U.S., Netherlands and Austria, and consistent with the existence of a core-periphery hierarchy, the Colombian interbank funds network is ultra-small, with an average geodesic distance around two. This not only means that the spreading capabilities of interbank funds network are particularly high, either for liquidity or for contagion effects, but it also suggests that the existence of super-spreaders may alleviate the inefficiencies resulting from the under-provision of liquidity cross-insurance in interbank markets documented by Castiglionesi and Wagner (2013).

Fourth, by means of a probit regression model, we confirm that the probability of being a super-spreader is determined by financial institutions' size in the Colombian case. This concurs with evidence from other countries. Accordingly, size may be the main factor behind the interbank funds network's reported scale-free connective structure and its core-periphery hierarchical organization. Nevertheless, as causality may not be inferred from the probit model, it is uncertain whether size is the driving force (i.e. the cause) behind the connective and hierarchical structure of the interbank funds network, or it is the result (i.e. the effect). Moreover, based on complex adaptive systems literature, it may be the case that size is –simultaneously- the driving force and the result of the interbank funds network dynamics by means of feedback effects.

Further related research work may come in several forms. First, new centrality measures that explicitly assess the extent of disruption in a network based on absorbing Markov chains should be considered; in this vein, implementing *SinkRank* (Soramäki and Cook, 2013) for identifying super-spreaders is an interesting methodological alternative worth exploring. Second, it is imperative to test the robustness of results under stringent financial liquidity conditions, such as a disruption of local or external credit lines, or a contractionary monetary policy; we attempted such test, but available data does not cover periods that could be fair examples of such conditions –for instance, year 2002. Third, the causality in interbank funds networks' dynamics should be explored to understand the role of size and other variables as causes and effects. Fourth, due to its contribution to money market liquidity, collateralized borrowing should also be considered for identifying central bank's liquidity supers-spreaders –as an additional layer in the multiplex network in Figure 1.

4.7. Appendix 1

2011



2012

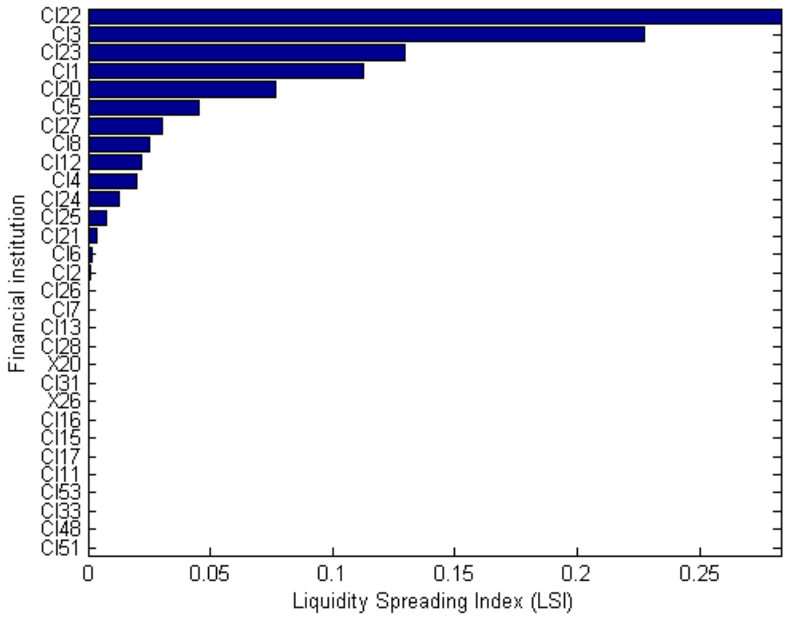


Figure 8. Top-30 financial institutions by estimated LSI_i. Credit institutions (CIs) dominate the contribution to LSI. Other types of contributing institutions are other financial institutions (Xs).

4.8. Appendix 2

2011

	LSI	h	a	k	s	b
LSI	1					
h	0.71	1				
a	0.69	0.41	1			
k	0.41	0.60	0.56	1		
s	0.70	0.48	0.99	0.62	1	
b	0.21	0.34	0.38	0.54	0.43	1

2012

	LSI	h	a	k	s	b
LSI	1					
h	0.74	1				
a	0.89	0.56	1			
k	0.58	0.73	0.65	1		
s	0.90	0.63	0.99	0.72	1	
b	0.54	0.39	0.66	0.57	0.69	1

2013

	LSI	h	a	k	s	b
LSI	1					
h	0.64	1				
a	0.83	0.33	1			
k	0.75	0.64	0.75	1		
s	0.88	0.41	0.99	0.80	1	
b	0.78	0.28	0.78	0.64	0.84	1

Figure 9. Linear dependence (correlation) between LSI and traditional centrality measures. Liquidity Spreading Index (LSI), estimated as in (14); authority (a) and hub centrality (h) are estimated as in (13); degree (k) corresponds to the number of incoming and outgoing links (3); strength (s) corresponds to the value (weight) of the incoming and outgoing links (4); betweenness (b) corresponds to the extent to which a vertex lies on paths between other vertexes (11).

4.9. Appendix 3

Table 3
Standard statistics of variables in the probit model
2013

Variable	Mean	Standard Deviation	Kurtosis	Skewness
LSI	0.013	0.045	26.931	4.636
$\mathcal{h}$	0.013	0.033	13.012	3.187
$\mathcal{a}$	0.013	0.038	27.426	4.595
$\mathcal{k}$	0.013	0.017	7.367	1.860
$\mathcal{s}$	0.013	0.035	32.848	4.979
$\mathcal{b}$	0.013	0.070	63.327	7.676
$size$	5.68×10^6	14.08×10^6	20.858	3.982
lev	0.844	0.205	6.567	-1.977
roa	0.039	0.070	8.599	-0.222
$borr$	0.737	0.263	2.210	-0.621
$lend$	0.270	0.313	3.192	1.099

Liquidity Spreading Index (LSI), estimated as in (14); authority ($\mathcal{a}$) and hub centrality ($\mathcal{h}$) are estimated as in (13); degree ($\mathcal{k}$) corresponds to the number of incoming and outgoing links (3); strength ($\mathcal{s}$) corresponds to the value (weight) of the incoming and outgoing links (4); betweenness ($\mathcal{b}$) corresponds to the extent to which a vertex lies on paths between other vertexes (11); $size$ is the asset value in COP million, as reported by the Financial Superintendence of Colombia (SFC); lev is the debt to assets ratio, based on balance sheet data reported by SFC; roa is the return over assets; $borr$ and $lend$ are the Herfindahl-Hirschman indexes on weighted borrowing and lending counterparties, respectively. All statistics are estimated based on original variables (i.e. they are not standardized).

4.10. Appendix 4

Table 4
Probit regression on selected determinants
2011

Variable [a, b]	LSI_i	$\mathscr{h}_i$	a_i	$\mathscr{k}_i$ [h]	$\mathscr{s}_i$	$\mathscr{b}_i$
Size $(size)$ [c]	4.668 (1.72)*	2.712 (3.18)***	14.373 (1.90)**	8.784 (1.75)*	9.645 (2.03)**	2.806 (2.63)***
Leverage (lev) [d]	5.097 (0.04)	0.732 (1.09)	0.149 (0.22)	-0.147 (-0.74)	0.071 (0.18)	-0.264 (-0.31)
Financial performance (roa) [e]	-7.369 (-0.83)	-0.175 (-0.41)	-0.205 (-0.40)	0.101 (0.59)	-0.250 (-0.76)	0.062 (0.09)
Borrowing concentration $(borr)$ [f]	0.024 (0.04)	-0.910 (-3.42)***	2.013 (1.59)		-0.585 (-2.22)**	-1.188 (2.09)**
Lending concentration $(lend)$ [g]	-1.927 (-1.07)	0.002 (0.01)	-4.994 (-1.29)		-0.858 (-1.84)*	-0.971 (-0.96)
Constant	-8.379 (-1.03)	-0.443 (-1.04)	-1.717 (-1.16)	4.114 (2.09)**	2.492 (1.42)	-1.935 (-2.81)***
Observations	77					
Observations = 1	10	27	18	57	63	13
Pseudo R-squared	.826	.605	.853	.178	.166	.686
% of correctly classified [i]	.961	.883	.948	.829	.818	.922

The probability of being a super-spreader is determined by financial institutions' size. The probability of financial institutions contributing to the 99th percentile of other centrality measures is also determined by size, but some (i.e. $\mathscr{h}_i$, $\mathscr{s}_i$, and $\mathscr{b}_i$) by borrowing concentration as well. [a] All independent variables are standard scores of the original variable (i.e. number of standard deviations above the estimated mean), whereas the dependent variables correspond to 1 when the financial institution contributes to the 99th percentile, and zero otherwise. [b] *t*-statistics in parenthesis, significant at .10*, .05** and .01***. [c] Asset value, as reported by the Financial Superintendence of Colombia (SFC). [d] Debt to assets ratio, based on balance sheet data reported by SFC. [e] Return over assets. [f] Herfindahl-Hirschman index on weighted borrowing counterparties. [g] Herfindahl-Hirschman index on weighted lending counterparties. [h] Borrowing and lending concentration are not reported because the maximum likelihood estimated was unfeasible (i.e. perfect prediction). [i] Weighted average of correct classifications of the dependent variable, in which the correct classification of a super-spreader consists of a predicted probability above .5, whereas the correct classification of a non-super-spreader consists of a predicted probability lower than or equal to .5

Table 5
Probit regression on selected determinants
2012

Variable [a, b]	LSI_i	$\mathscr{h}_i$	a_i	$\mathscr{k}_i$ [h]	$\mathscr{s}_i$	$\mathscr{b}_i$
Size	2.365	10.523	3.474	6.838	11.311	1.107
$(size)$ [c]	(1.73)*	(1.87)*	(3.24)***	(1.33)	(2.23)**	(2.86)***
Leverage	7.503	-0.183	0.125	0.044	0.118	0.446
(lev) [d]	(0.77)	(-0.68)	(0.45)	(0.23)	(0.52)	(1.04)
Financial performance	0.932	0.104	-0.190	0.158	-0.182	-0.338
(roa) [e]	(0.69)	(0.10)	(-0.84)	(0.88)	(-0.84)	(-0.88)
Borrowing concentration	-0.194	-2.061	0.517		-0.205	-0.379
$(borr)$ [f]	(-0.34)	(-2.23)**	(1.91)*		(-0.95)	(-1.75)*
Lending concentration	-0.306	0.298	-0.339		-0.387	-0.257
$(lend)$ [g]	(-0.22)	(0.51)	(-1.08)		(-1.58)	(-0.88)
Constant	-5.986	1.463	0.039	3.431	3.987	-0.919
	(-0.97)	(-0.84)	(0.10)	(1.66)*	(2.00)**	(-3.09)***

Observations	70					
Observations = 1	11	28	22	57	34	18
Pseudo R-squared	.768	.778	.539	.178	.458	.397
% of correctly classified [i]	.928	.943	.914	.829	.829	.857

The probability of being a super-spreader is determined by financial institutions' size. The probability of financial institutions contributing to the 99th percentile of other centrality measures is also determined by size, but some (i.e. $\mathscr{h}_i$, $\mathscr{s}_i$, and $\mathscr{b}_i$) by borrowing concentration as well. [a] All independent variables are standard scores of the original variable (i.e. number of standard deviations above the estimated mean), whereas the dependent variables correspond to 1 when the financial institution contributes to the 99th percentile, and zero otherwise. [b] *t*-statistics in parenthesis, significant at .10*, .05** and .01***. [c] Asset value, as reported by the Financial Superintendence of Colombia (SFC). [d] Debt to assets ratio, based on balance sheet data reported by SFC. [e] Return over assets. [f] Herfindahl-Hirschman index on weighted borrowing counterparties. [g] Herfindahl-Hirschman index on weighted lending counterparties. [h] Borrowing and lending concentration are not reported because the maximum likelihood estimated was unfeasible (i.e. perfect prediction). [i] Weighted average of correct classifications of the dependent variable, in which the correct classification of a super-spreader consists of a predicted probability above .5, whereas the correct classification of a non-super-spreader consists of a predicted probability lower than or equal to .5

Chapter 4: Identifying Central Bank Liquidity Super-Spreaders in Interbank Funds Networks

5

By some accounts, the single most overriding consideration in assessing a system's complexity is its hierarchical organization.

John Casti (1979)

5. Rethinking Financial Stability: Challenges Arising From Financial Networks' Modular Scale-Free Architecture

Abstract

We examine the connective architecture of the main Colombian payment and settlement systems in order to update what we know about local financial networks, and to elaborate on the main consequences for financial stability. Evidence suggests that local financial networks display a modular (i.e. clustered) scale-free (i.e. inhomogeneous) architecture. Results concur with other real-world networks, and propose new insights and challenges for authorities contributing to financial stability. For instance, (i) traditional reductionist assumptions for modeling financial systems (e.g. homogeneity) may be particularly misleading; (ii) the observed modular scale-free architecture favors robustness and resilience; (iii) the generating process of such architecture overlaps with literature on trading relationships; (iv) carelessly reducing inhomogeneity in financial systems may backfire in the form of a less robust and more crisis-prone financial system; and (v) financial authorities should understand and take advantage of the existing architecture by means of designing and implementing macro-prudential regulation and system-calibrated requirements.

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5.1. Introduction

Identifying and examining the connective architecture of financial systems has been pinpointed as a critical factor for understanding financial markets. As is the case of other complex systems, the architecture of financial systems not only reveals how they have evolved, but it also suggests how they may be affected by shocks, and how authorities should intervene in order to pursue its safe and efficient functioning. In this vein, unlike traditional institution-centric (i.e. micro-prudential) approaches to financial markets, a comprehensive or macro-prudential approach that addresses the architecture of linkages between financial institutions may aid authorities to better understand, regulate, supervise, and oversee the financial system.

Network science has been widely used for examining the connective architecture of complex systems. It contrasts, compares and integrates techniques and algorithms developed in several disciplines to increase the understanding of natural and manmade networks (Börner et al., 2007). Under the network analysis approach financial markets are nothing but a weighted and directed network among financial institutions (Barabási, 2003). Thus, financial networks may be studied and analyzed with the aim of identifying, examining and contrasting financial markets' main connective features in order to better understand their structure and evolution.

Accordingly, our work uses network science with two main objectives. First, we identify and examine the connective architecture of transactions from the three main Colombian payment and settlement systems. Second, we contrast their main actual features with those assumed by traditional models of financial systems (e.g. Allen and Gale, 2000) and with those documented for most real-world networks. Such examination and contrast serves the purpose of updating what we know about the connective architecture of local financial networks and to elaborate on how to pursue financial stability under a macro-prudential approach.

In this sense, our work elaborates on the importance of taking into account financial networks' structure when trying to devise policies that enhance the resilience of the financial system (Battiston et al., 2012a).

Our work verifies that local financial networks exhibit a modular (i.e. clustered) scale-free (i.e. inhomogeneous) structure, a ubiquitous architecture well-documented in other social and biological networks. Related literature points out that the inhomogeneity in scale-free networks yields a structure that is robust to random shocks but fragile to targeted attacks, as in the nowadays celebrated *robust-yet-fragile* characterization of financial networks by Haldane (2009). On the other hand, a modular architecture favors resiliency by limiting cascades and isolating feedbacks (Anderson, 1999; Kambhu et al., 2007; Haldane and May, 2011). Therefore, according to literature on networks, the observed modular scale-free architecture tends to make the financial networks under analysis robust and resilient, yet fragile.

The observed connective architecture contradicts the main assumptions of conventional research on financial contagion and financial stability (e.g. Allen and Gale, 2000; Freixas et al., 2000; Cifuentes et al., 2005; Nier et al., 2008; Gai and Kapadia, 2010; Battiston et al., 2012a). The modular and scale-free architecture of financial networks invalidates traditional homogeneous and non-hierarchical oversimplifying case models, and cautions about how prior beliefs regarding contagion and financial stability may be unfounded and potentially misleading.

Identifying a hierarchical connective structure in financial networks overlaps with two distinct literature strands. First, it concurs with evidence of hierarchies in the German, Italian, Dutch and UK interbank markets, as reported by Craig and von Peter (2014), Fricke and Lux (2014), in 't Veld and van Lelyveld (2014), and Wetherilt et al. (2010), respectively; however, our findings suggest that the hierarchical form may be modular scale-free, whereas prior works point to a core-periphery structure. Second, the generating process for a modular scale-free hierarchical architecture proposed by Assenza et al. (2011) may be considered a generalization of the trade-off between the benefits and costs of becoming a financial intermediary in trading relationships literature (e.g. Babus, 2012; Afonso, 2013; van der Leij et al., 2013), which yields hierarchical structures as well.

Our results entail several challenges related to financial stability. First, results urge a revision of how financial contagion is modeled: Assuming that financial networks are homogeneous and non-hierarchical is false, thus modeling and analyzing contagion based on these assumptions may be questionable. Second, due to the benefits of a modular scale-free architecture, namely the ability to limit cascades and isolate feedbacks, popular efforts to reduce financial markets' inhomogeneity by simply downsizing or dismantling systemically important financial institutions may backfire in the form of a less robust and less resilient financial system. Third, systemic-calibrated prudential requirements (e.g. capital, liquidity) should be designed and imposed to enhance the ability of the observed architecture to limit cascades and isolate feedbacks. These three challenges are consistent with a macro-prudential approach to systemic risk.

5.2. Literature: From Homogeneous to Modular Scale-Free Financial Networks

Real-world networks, both biological and social, tend to display inhomogeneous connective structures, in which connections are approximately distributed as a *power-law*, commonly known as scale-free networks after the seminal work of Barabási and Albert (1999). Moreover, not only are most real-networks inhomogeneous, but they also tend to display a particular hierarchical structure characterized by the existence of clusters or communities of dense interaction, also known as hierarchical modularity. Such modularity is at odds with the standard scale-free connective structure of Barabási and Albert (1999). Thus, both features constitute a particular type of networks introduced by Barabási (2003): modular scale-free networks.

To the best of our knowledge, the evidence attained in this paper contributes to the financial literature by documenting for the first time the presence of a modular scale-free architecture in financial networks. Furthermore, we contribute by linking the observed architecture to literature on financial stability.

5.2.1. From Homogeneous to Inhomogeneous Financial Networks

Most literature that models the interactions between financial institutions is based on the assumption of homogeneity, in which financial institutions tend to connect to each other in a dense and uniform manner. Under such assumption influential papers (Allen and Gale, 2000; Freixas et al., 2000; Cifuentes et al., 2005; Nier et al., 2008; Gai and Kapadia, 2010; Battiston et al., 2012a) converge -*ceteris paribus*- to diversification or absorption effects due to the dispersion of shocks within larger or denser financial networks.[1] Accordingly, as reported by Allen and Babus (2008), examining direct linkages generally results in more connections reducing the risk of contagion.

Some of these influential papers eventually arrive to a non-monotonic relation between financial connectedness and stability. Nevertheless, they do it after

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[1] For instance, Allen and Gale (2000) use a homogeneous four-bank structure to demonstrate that if the network is complete (i.e. all having exposures to each other) the impact of a shock is mitigated. Gai and Kapadia (2010) assume that interbank linkages form randomly and exogenously, with the probability of all links being independent and distributed as a Poisson process, akin to the core model by Nier et al. (2008). Freixas et al. (2000) analyze the "diversified lending" case, in which every bank gives credit lines uniformly to all other banks. Cifuentes et al. (2005) simulate banking linkages by fixing the number of possible counterparts for all banks. Battiston et al. (2012a) assume "uniform risk sharing", where all participants share the same number of counterparties.

examining *indirect balance-sheet linkages* (Allen and Babus, 2008), which consists of modeling the impact of other feedback effects, such as the mark-to-market of portfolio holdings, bank runs, next-period tighter funding conditions, and institutions' dissimilar initial endowments. Yet, at the core of conventional models there is a homogeneous network structure of direct linkages, as in early research by Allen and Gale (2000).

Traditionally, networks of complex topology have been described with the random graph theory of Erdös and Rényi (Barabási and Albert, 1999). Erdös and Rényi (1960) study a particular type of network in which connections are homogeneously distributed between participants due to the assumption of exponentially decaying tail processes for the distribution of links –such as the Poisson distribution. This type of network, also labeled as "random" or "Poisson", is –explicitly or implicitly- the main assumption of most literature on financial contagion.

However, as first documented by Barabási and Albert (1999), a particular type of inhomogeneity is ubiquitous in real-world networks, with the distribution of connections approximating a power-law distribution. In this type of network there are a few heavily connected participants and many poorly connected participants, in which there is no typical or representative participant; thus, it has no scale, it is scale-free or scale-invariant.

Besides documenting the inhomogeneity of real-world networks and their approximate scale-free nature, Barabási and Albert (1999) suggested *growth* and *preferential attachment* as the corresponding generating process. Against customary network models at that time, Barabási and Albert acknowledged that real-world networks are dynamic due to the addition and removal of vertexes, and that the new vertexes do not connect randomly to the existing ones, but depending on the actual degree (i.e. number of links) of existing vertexes. This is, growth and preferential attachment allow for early vertexes to have more time to acquire links, and allow for early vertexes to be selected more often and to grow faster than their younger and less connected peers (Barabási, 2003), akin to a "rich-get-richer" process with seniority as the main driver.

In financial networks growth and preferential attachment are marked and interrelated features as well. About growth, financial systems are not static: They are the result of a long evolutionary process in which new financial institutions, business niches or cognitive structures appeared, some old ones disappeared, and where some existing ones recombined (e.g. merged) in a new form, with such evolution modifying the main characteristics (e.g. pattern, intensity, direction) of the connections between financial institutions.

About preferential attachment, financial institutions' seniority (i.e. the advantage of older vertexes) may be a signal of endurance and resilience to changing market conditions, including extreme negative events (e.g. financial crises); in this sense, seniority captures how financial institutions have behaved, survived, and evolved amid real-life conditions. However, seniority may be limited for explaining preferential attachment.[2] A more general framework for preferential attachment in financial networks may be based on some broad metric of *fitness* (see Barabási, 2003), comprising efficiency, costs, size, connectedness, systemic importance, geographical location, market power, access to last-resort lending, reputation, etc. In this vein, in 't Veld and van Lelyveld (2014) relates preferential attachment in financial networks to institutions searching for reliable counterparties that are used by many other institutions.

Consistent with growth and preferential attachment in financial networks, literature documents that they display scale-free structures. Such inhomogeneous structure was well-documented by the time of the Global Financial Crisis (e.g. Renault et al. 2007; Soramäki et al., 2007; Becher et al., 2008; Cepeda, 2008; Iori

[2] For example, based on the assessment of systemic importance for local financial institutions by León et al. (2013) and León and Machado (2014), a handful of banks that are non-central were established about a century ago, whereas three out of the five most important were established less than fifty years ago. Therefore, seniority provides a partial rationale for preferential attachment in the Colombian case.

et al., 2008; May et al., 2008; Pröpper et al., 2008), and even well-before its arrival (e.g. Boss et al., 2004; Inaoka et al., 2004).

Whereas the relevance of the network structure prior to crisis was mentioned only infrequently, it has now caught the attention of both academics and policy makers (in 't Veld and van Lelyveld, 2014). The crisis spurred research aimed at identifying financial networks' observed connective structure, as in Haldane (2009), Schweitzer et al. (2009), Bech and Atalay (2010), Wetherilt et al. (2010), Arinaminpathy et al. (2012), Markose (2012), Markose et al. (2012), Martínez-Jaramillo et al. (2012), Craig and von Peter (2014), Fricke and Lux (2014), in 't Veld and van Lelyveld (2014), and León et al. (2014). Moreover, based on the evidence of inhomogeneity in financial networks, a rising literature is now encouraging the usage of network theory metrics of importance (e.g. centrality) for identifying "super-spreaders" (Haldane and May, 2011; Markose et al., 2012), systemically important financial institutions (as in Lovin, 2012; León et al., 2013; León and Machado, 2013; Soramäki and Cook, 2013) and systemically important financial market infrastructures (León and Pérez, 2014).

5.2.2. From Inhomogeneous to Modular Scale-Free Financial Networks

Not all scale-free networks are the same. Networks with degree distributions approximating a power-law may display a modular hierarchy as well. Modularity conveys a salient feature: Groups of participants have a high density of links within them, with a lower density of links between groups (Newman, 2003).

According to Simon (1962), the existence of clusters of dense interaction in social systems identifies well-defined hierarchical structures that may be defined as *nearly decomposable systems*. In such type of systems each cluster may be regarded as a subsystem composed of subordinates led by a boss, in whom the interactions among subsystems are weak, but not negligible, where intra-subsystem linkages are generally stronger than inter-subsystem linkages. This nearly decomposable architecture resembles that reported by Battiston et al. (2012b) for describing credit networks: Agents clustered in neighborhoods so that most of the time the action is at the local level, but with a few connections among neighborhoods that make the network sparse yet responsive to shocks hitting any participant.

Hierarchical modularity in real-world systems is by no means accidental. Hierarchical modularity has significant design advantages, such as making multitasking possible: While the dense connections within each module help the efficient accomplishment of specific tasks, the hubs coordinate the communication between the many parallel functions (Barabási, 2003). Moreover, as highlighted by Anderson (1999), as most components or subsystems receive inputs from only a few of the other components, change can be isolated to local neighborhoods. Henceforth, by limiting the potential cascades, modularity protects the systemic resilience of both natural and constructed networks (Haldane and May, 2011).

Regarding the rationale behind modularity, Assenza et al. (2011) propose a specific model of an adaptive network that generates modular scale-free architectures. Their model consists of two competing feedback mechanisms: *homophily* and *homeostasis*. Homophily is related to increasing the intensity of interactions with other similar institutions, whereas homeostasis consists of an intensity preservation mechanism that weakens prior interactions in favor of new ones. Together, these two feedback mechanisms lead to the emergence of real-world (e.g. social and neural systems) features such as scale-free distributions of linkages and their intensity, along with a strong modularity (Assenza et al., 2011).

The competition between homophily and homeostasis may be applied to financial systems as well. The selective trial and error process of Simon (1962) may explain increases in intensity of interactions between financial institutions, in which such process may be related to financial institutions' search for fitness; this is, there is a preference to link to fit counterparties in financial markets. On the other hand, the intensity preservation mechanism (i.e. homeostasis) may be related to financial institutions being forced to counter-balancing the intensity of their aggregated interactions due to finite resources (e.g. money, time, securities, risk limits).

Financial institutions' fitness-driven preferential attachment and homeostasis may be related to literature on trading relationships. For instance, the endogenous intermediation model for over-the-counter markets by Babus (2012) presents a theoretical model that explains the existence of intermediation (i.e. a tiered, core-periphery hierarchy) as a result from two competing forces: (i) Financial institutions avoiding the costs of maintaining relationships with many counterparties when informational frictions exist, akin to the homeostasis mechanism by Assenza et al. (2011), and (ii) financial institutions trying to become one of the few large and central intermediaries able to collect intermediation fees because of concentrating links in a "rich-get-richer" process. A similar argument is developed by van der Leij et al. (2013): In a heterogeneous network the trade-off between collecting intermediation benefits and the costs from assessing counterparties' risks results in large financial institutions benefiting from maintaining direct lending relationships with all other large financial institutions in the core, such that the core-periphery network becomes a stable equilibrium.

From a different perspective, Afonso et al. (2013) identify that small borrowers form stable and persistent relations with a small number of lenders in the U.S. interbank market because searching for liquidity insurance is costly. Similarly, Cocco et al. (2009) document that relationships (i.e. concentrating counterparties) in the Portuguese interbank market allow small financial institutions to insure liquidity risk in the presence of market frictions such as information costs. Likewise, Battiston et al. (2012b) link the costs of assessing credit worthiness and of establishing a trust/customer relationship to the sparse and clustered connective structure of credit markets, which yield sparsely interconnected neighborhoods. Again, concentrating counterparties due to informational frictions or transactions costs is a form of an intensity preservation mechanism or homeostasis.

To the best of our knowledge, evidence of financial networks describing modular scale-free architecture has not been reported in the literature. However, the work of Craig and von Peter (2010) may be considered as the first evidence of a non-flat hierarchical structure in financial networks, in which the tiering and core-periphery structure of the German interbank credit network contradicts the homogeneity assumption of traditional models, with such tiered structure resulting from economic reasons (e.g. size). Likewise, based on the approach of Craig and von Peter, Fricke and Lux (2014), and in 't Veld and van Lelyveld (2014) report a similar core-periphery hierarchical structure for the Dutch and Italian interbank markets, respectively. For the U.K. CHAPS Sterling interbank network, Wetherilt et al. (2010) find a core-periphery structure as well. Also, some features of the Italian interbank network reported by Bargigli et al. (2013) are consistent with a modular scale-free architecture.

Craig and von Peter (2014) and Fricke and Lux (2014) not only report that scale-free generated networks provide a better fit than Erdös and Rényi networks in the case of the observed interbank hierarchies, but they also acknowledge the limitations of scale-free generating models (i.e. growth and preferential attachment) for fitting the observed hierarchical nature. Those limitations are due to the standard scale-free network model's assumption of a few central participants well-connected to participants across the whole network, whereas modularity consists of well-interconnected central participants that are not well-connected to non-central participants in other modules than the one they lead. Those limitations are also the main drivers behind the works of Dorogovtsev et al. (2002), Barabási (2003), Ravasz and Barabási (2003), and Assenza et al.

(2011) about networks that combine scale-free structures and hierarchical modularity.

Our work is akin to those of Craig and von Peter (2014), Fricke and Lux (2014), and in 't Veld and van Lelyveld (2014) regarding the evidence of a hierarchical structure within the three studied financial networks. Yet, based on the literature on modular risk-free networks, we (i) characterize these networks as modular scale-free, and (ii) highlight some immediate challenges for financial stability. Hence, our work is related to Farmer et al. (2012) demand for updating what was thought of as the main features of financial networks (i.e. their density and connective homogeneity), which may be critical for modeling contagion and pursuing financial stability. Also, our work overlaps with literature on trading relationships on the formation of financial networks (Cocco et al., 2009; Babus, 2012; Afonso et al., 2013), in which scale-free modular generating processes based on preferential attachment and homeostasis are analogous to the trade-off between the benefits and costs of maintaining counterparties.

5.3. Network Analysis

Due to its interdisciplinary origin and recent use in economics and finance, network science's concepts and notation are worth stating first. Afterwards, the main statistics employed are classified according to their purpose, either identifying networks' connective pattern or their hierarchical structure.

5.3.1. Concepts and Notation

A network, or graph, represents patterns of connections between the parts of a system. The most common mathematical representation of a network is the *adjacency matrix*. Let n represent the number of vertexes or participants, the adjacency matrix A is a square matrix of dimensions $n \times n$ with elements A_{ij} such that

$$A_{ij} = \begin{cases} 1 \text{ if there is an edge between vertexes } i \text{ and } j, \\ 0 \text{ otherwise.} \end{cases} \quad (1)$$

A network defined by the adjacency matrix in (1) is referred as an undirected graph, where the existence of the (i, j) edge or link makes both vertexes i and j adjacent or connected, and where the direction of the edge is unimportant. However, the assumption of a reciprocal relation between vertexes is inconvenient for some networks. For instance, the deliveries of money between financial institutions constitute a graph where the character of sender and recipient is a particularly sensitive source of information for analytical purposes, in which the assumption of a reciprocal relation between both parties is unwarranted. Thus, the adjacency matrix of a directed network or *digraph* differs from the undirected case, with elements A_{ij} being referred as directed edges or arcs, such that

$$A_{ij} = \begin{cases} 1 \text{ if there is an edge from } i \text{ to } j, \\ 0 \text{ otherwise.} \end{cases} \quad (2)$$

It may be useful to assign real numbers to the edges. These numbers may represent distance, frequency or value, in what is called a weighted network and its corresponding weighted adjacency matrix (W_{ij}). For a financial network the weights could be the monetary value of the transaction or of the exposure.

Regarding the characteristics of the system and its elements, a set of concepts is commonly used. The simplest concept is the vertex *degree* (k_i), which corresponds to the number of edges connected to it. In directed graphs, where the adjacency matrix is non-symmetrical, *in degree* (k_i^{in}) and *out degree* (k_i^{out}) quantifies the number of incoming and outgoing edges, respectively (3).

$$k_i^{in} = \sum_{j=1}^{n} A_{ji} \qquad\qquad k_i^{out} = \sum_{j=1}^{n} A_{ij} \qquad (3)$$

In the weighted graph case the degree may be informative, yet inadequate for analyzing the network. *Strength* (s_i) measures the total weight of connections for a given vertex, which provides an assessment of the intensity of the interaction between participants. Akin to degree, in the directed graph case *in strength* (s_i^{in}) and *out strength* (s_i^{out}) sum the weight of incoming and outgoing edges, respectively (4).

$$s_i^{in} = \sum_{j=1}^{n} W_{ji} \qquad\qquad s_i^{out} = \sum_{j=1}^{n} W_{ij} \qquad (4)$$

5.3.2. Identifying Connective Patterns

Some metrics allow for determining the connective pattern of the graph. The simplest metric for approximating the connective pattern is *density* (d), which measures the cohesion of the network. The *density* of a graph with no self-edges is the ratio of the number of actual edges (m) to the maximum possible number of edges (5).

$$d = \frac{m}{n(n-1)} \qquad (5)$$

By construction, density is restricted to the $0 < d \leq 1$ range. Formally, Newman (2010) states that a sufficiently large network for which the density (d) tends to a constant as n tends to infinity is said to be *dense*. In contrast, if the density tends to zero as n tends to infinity the network is said to be *sparse*. However, as one frequently works with non-sufficiently large networks, they are commonly labeled as sparse when the density is much smaller than the upper limit ($d \ll 1$), and as dense when the density approximates the upper limit ($d \cong 1$). The term *complete network* is used when $d = 1$.

An informative alternative to density is to examine the degree probability distribution ($\mathcal{P}_k$); such distribution provides a natural summary of the connectivity in the graph (Kolaczyk, 2009). Akin to density, the first moment of the distribution of degree (μ_k) measures the cohesion of the network, and is usually restricted to the $0 < \mu_k \leq n - 1$ range. A sparse graph has an average degree that is much smaller than the size of the graph ($\mu_k \ll n - 1$). As the number of edges in a directed network is equal to the number of incoming edges and to the number of outgoing edges, there is a unique *average degree* for the network (6).

$$\mu_k = \frac{1}{n}\sum_{i=1}^{n} k_i^{in} = \frac{1}{n}\sum_{i=1}^{n} k_i^{out} = \frac{m}{n} \tag{6}$$

The second moment of the distribution (σ_k) indicates how disperse is the vertexes' degree around the average degree. The standard deviation of the in and out degree may not be the same (7).

$$\sigma_{k_{in}} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (k_i^{in} - \mu_k)^2} \qquad \sigma_{k_{out}} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (k_i^{out} - \mu_k)^2} \tag{7}$$

The third moment (i.e. skewness or asymmetry) of the degree distribution is particularly informative about the connective pattern of the network. If asymmetry is nil or negligible, the average degree is meaningful, and the majority of the vertexes display an average degree, and few vertexes are of low or high degree. In this case vertex degree is of a fairly similar order of magnitude across the graph (i.e. homogeneous), the corresponding degree distribution is concentrated, and typically decay exponentially fast in k (Kolaczyk, 2009). In the limiting case of a symmetric distribution the degree follows a Poisson process, in which the probability of observing a vertex with k edges becomes negligibly small when $k \ll \mu_k$ or $\mu_k \gg k$.

However, most real-world networks display right-skewed distributions, where the majority of vertexes are of very low degree, and few vertexes are of very high degree; hence inhomogeneous. Such right-skew of real-world network's degree distributions has been found to approximate a power-law distribution (Barabási and Albert, 1999). In traditional random networks, in contrast, all vertexes have approximately the same number of edges.

In the case of inhomogeneous networks the power-law (or Pareto-law) distribution of degree suggests that the probability of observing a vertex with k edges obeys the potential functional form in (8), where z is an arbitrary constant, and γ is known as the *exponent* of the power-law.

$$\mathcal{P}_k \propto z k^{-\gamma} \tag{8}$$

Besides degree distributions approximating a power-law, other features have been identified as characteristic of real-world networks. As explained below, these features are: (i) low mean geodesic distances; (ii) high clustering coefficients; and (iii) significant degree correlation.

Let g_{ij} be the *geodesic distance* (i.e. the shortest path in terms of number of edges) from vertex i to j, the mean geodesic distance for vertex i (ℓ_i) corresponds to the mean of g_{ij}, averaged over all reachable vertexes j in the network (Newman, 2010), as in (9). Respectively, the mean geodesic distance or average path length of a network (i.e. for all pairs of vertexes) is denoted as ℓ, and corresponds to the mean of ℓ_i over all vertexes. Consequently, the mean geodesic distance (ℓ) reflects the global structure; it measures how big the network is, it depends on the way the entire network is connected, and cannot be inferred from any local measurement (Strogatz, 2003).

$$\ell_i = \frac{1}{(n-1)} \sum_{j(\neq i)} g_{ij} \qquad \ell = \frac{1}{n} \sum_i \ell_i \tag{9}$$

The mean geodesic distance (ℓ) of random or Poisson networks is small, and increases slowly with the size of the network; therefore, as stressed by Albert and Barabási (2002), random graphs are small-world because in spite of their often large size, in most networks there is relatively a short path between any two vertexes. For random networks: $\ell \sim \ln n$ (Newman et al., 2006). This slow logarithmic increase with the size of the network coincides with the small-world effect (i.e. short average path lengths).

However, the mean geodesic distance for scale-free networks is smaller than $\ell \sim \ln n$. As reported by Cohen and Havlin (2003, 2010), scale-free networks with $2 < \gamma < 3$ tend to have a mean geodesic distance that behaves as $\ell \sim \ln\ln n$, whereas networks with $\gamma = 3$ yield $\ell \sim \ln n/(\ln \ln n)$, and $\ell \sim \ln n$ when $\gamma > 3$. For that reason, Cohen and Havlin (2003, 2010) state that scale-free networks can be regarded as a generalization of random networks with respect to the mean average geodesic distance, in which scale-free networks with $2 < \gamma < 3$ are "ultra-small".

The clustering coefficient (c) corresponds to the property of network transitivity. It measures the average probability that two neighbors of a vertex are themselves neighbors; the coefficient hence measures the frequency with which loops of length three (i.e. triangles) appear in the network (Newman, 2010). Let a *triangle* be a graph of three vertexes that is fully connected, and a *connected triple* be a graph of three vertexes with at least two connections, the calculation of the network's clustering coefficient is as follows: [3]

$$c = \frac{(\text{number of triangles in the network}) \times 3}{\text{number of connected triples}} \qquad (10)$$

Hence, by construction, clustering reflects the local structure. It depends only on the interconnectedness of a typical neighborhood, the inbreeding among vertexes tied to a common center, and thus it measures how incestuous the network is (Strogatz, 2003). Intuitively, in a random graph the probability of a connection between two vertexes tends to be the same for all vertexes regardless of the existence of a common neighbor. Therefore, in the case of random graphs the clustering coefficient is expected to be low, and to tend to zero in –the limit- of large random networks.

Contrarily, real-world complex networks tend to exhibit a large degree of clustering. Albert and Barabási (2002) report that in most –if not all- real networks the clustering coefficient is typically much larger than it is in a comparable random network. Accordingly, in inhomogeneous graphs, as those resulting from real-world networks, the probability of two neighbors of a vertex being themselves neighbors is reported to be in the 10% and 60% range (Newman, 2010). In this sense, scale-free networks combining particularly low mean geodesic distance and high clustering implies that the existence of a few too-connected vertexes plays a key role in bringing the other vertexes close to each other, also indicating that the scale-free topology is more efficient in bringing the vertexes close than is the topology of random graphs (Albert and Barabási, 2002).

[3] If three vertexes (i.e. a, b, c) exist in a graph, a triangle exists when edges (a,b), (b,c) and (c,a) are present (i.e. the graph is complete), whereas a connected triple exists if at least two of these edges are present. In this sense, a triangle occurs when there is transitivity (i.e. two neighbors of a vertex are themselves neighbors). The factor of three in the numerator arises because each triangle is counted three times when the connected triplets are counted (Newman, 2010).

Besides displaying low mean geodesic distances and clustering, real-world graphs also display non-negligible degree correlation between vertexes. They are characterized by either a positive correlation, where high-degree (low-degree) vertexes tend to be connected to other high-degree (low-degree) vertexes, or a negative correlation, where high-degree vertexes tend to be connected to low-degree vertexes. Positive degree correlation, also known as *homophily* or *assortative mixing by degree*, results in the core-periphery structure typical of social networks, whereas negative degree correlation (i.e. *dissortative mixing by degree*) is typical of technological, informational, and biological networks, which display star-like features that do not usually have a core-periphery but have uniform structures (Newman, 2010). In contrast, the degree of random (i.e. homogeneous) networks tends to be uncorrelated.

Degree correlation may be measured by means of estimating the assortativity coefficient (Newman, 2010). As before, let m be the number of edges, the degree assortativity coefficient of a network (r_k) is estimated as follows (11):

$$r_k = \frac{\sum_{ij}\left(A_{ij} - k_i k_j/2m\right)k_i k_j}{\sum_{ij}\left(k_i\delta_{ij} - k_i k_j/2m\right)k_i k_j} \tag{11}$$

Where

$$\delta_{ij} = \begin{cases} 0 \text{ if } i \neq j \\ 1 \text{ if } i = j \end{cases}$$

However, it should be noted that the assortativity coefficient is not limited to vertexes' degree. Other characteristics of vertexes (e.g. age, income, gender, ethnics, size) may condition their tendency to be connected. In this case, the characteristics of connected vertexes may be correlated, which results in *assortative mixing by scalar characteristics* (Newman, 2010). For financial networks it is important to assess the intensity of the interaction between participants. As highlighted by Barrat et al. (2004) and Leung and Chau (2007), the inclusion of weights and their correlations might consistently change our view of the hierarchical and structural organization of the network. Based on (11), it is possible to estimate the *assortative mixing by strength* (12).

$$r_s = \frac{\sum_{ij}\left(A_{ij} - k_i k_j / 2m\right) s_i s_j}{\sum_{ij}\left(k_i \delta_{ij} - k_i k_j / 2m\right) s_i s_j} \tag{12}$$

5.3.3. Identifying Hierarchies

Simon (1962, p.468) suggests a narrow definition of *hierarchical system* or *hierarchy*: *a system that is composed of interrelated subsystems, each of the latter being, in turn, hierarchic in structure until we reach some lowest level of elementary subsystem*. Correspondingly, Casti (1979) points out that the number of hierarchical levels in a given system represents a rough measure of its complexity.

Some authors link the hierarchical structure of networks to the existence of communities or modules. For instance, Newman (2003) defines that a network displays community structures when groups of vertexes have a high density of edges within them, with a lower density of edges between groups. Similarly, Simon (1962) portrays modularity as *nearly decomposable systems*: a collection of subsystems that are weakly interconnected among them, but that are heavily interconnected within them.

Correspondingly, Barabási (2003) labels modularity in real-world networks as an architecture where the more connected a vertex is, the smaller is its clustering coefficient. Moreover, Barabási (2003) pinpoints that such low clustering from central vertexes contradicts the standard scale-free model. Hence, in order to quantitatively measure the hierarchical modularity of a network, Barabási (2003) suggests assessing whether (or not) the most connected vertexes display low local (i.e. individual) clustering, as the real-world observed hierarchical modularity suggests. Newman (2010) defines local clustering as in (13):

$$c_i = \frac{(\text{number of pairs of neighbors of } i \text{ that are connected})}{(\text{number of pairs of neighbors of } i)} \quad (13)$$

If there is no dependence between degree and clustering (i.e. clustering is democratically distributed), then the network has no hierarchical modularity, as expected from both standard random and scale-free networks. However, quite commonly, there is an inverse relationship between local clustering and degree (Bargigli et al., 2013). Such inverse relationship suggests that high-degree vertexes serve as hubs that connect vertexes across different modules, thus on average they tend not to be incestuous and they display low local clustering. On the other hand, low-degree vertexes tend to be incestuous within their corresponding module as they share a common hub; hence they tend to display high clustering coefficients.

Accordingly, Dorogovtsev et al. (2002) and Barabási (2003) suggest that hierarchical modularity may be captured by fitting a power-law to the distribution of local clustering as a function of average degree ($\mu_{\mathscr{k}}$) (14):

$$\mathcal{P}_{c_i} \propto z\mu_{\mathscr{k}}^{-\gamma} \quad (14)$$

Barabási (2003) highlights that the existence of hierarchical modularity in real-world networks is a defining feature of most complex systems, but it is not caused and may not be explained by the mere presence of scale-free properties. Consequently, because the standard scale-free model presumes the existence of a few central vertexes connected to vertexes in numerous modules (i.e. against the evidence of modularity in real-world networks), Barabási (2003) introduces a new type of network: a modular scale-free network. According to Dorogovtsev et al. (2002) and Barabási (2003), the clustering coefficient of a network and its local distribution by degree may confirm –or reject- the presence of a hierarchy within the system.

5.4. The Datasets: Colombian Payment and Settlement Networks

Two main data sources have been used in the financial networks literature: (i) financial transactions (i.e. flows), and (ii) financial exposures (i.e. stocks).[4] Networks of financial transactions correspond to the delivery of money, securities or currencies, or to the corresponding trades among financial institutions, which are automatically registered and safeguarded by financial market infrastructures (e.g. large-value payment systems, clearing houses, securities settlement systems, central securities depositories, trading platforms, trade repositories) whenever a transaction occurs. As highlighted by some authors (e.g. Kyriakopoulos et al., 2009; Uribe, 2011a,b), the information conveyed in financial transactions is particularly valuable due to its (i) granularity, with informative details such as sender, recipient, amount, type of transaction, underlying asset, etc.; (ii) completeness, because all financial transactions ineludibly involve the delivery of money or securities, or a trade; (iii) reliability from a supervisory perspective because payments and settlements cannot be –easily- falsified; and (iv) opportunity, with data usually available in real-time (or with a minimal lag).

On the other hand, financial exposures ordinarily emerge from reports prepared and delivered by each financial firm to the corresponding authorities (e.g. financial statements), where the most commonly used for building financial networks are interbank credit and derivatives exposures. This type of information tends to be aggregated (i.e. details of individual exposures, counterparties, instruments, etc. are usually unavailable) and lagged, and its completeness, consistency and validity depend on accounting practices by each financial firm

[4] An incomplete list of research on financial transactions' networks comprises Inaoka et al. (2004), Soramäki et al. (2007), Becher et al. (2008), Cepeda (2008), Iori et al. (2008), Bech and Atalay (2010), Fricke and Lux (2014); on financial exposures networks, Boss et al. (2004), Markose et al. (2012), Bargigli et al. (2013), and Craig and von Peter (2014).

and the corresponding jurisdiction.[5] Yet, as highlighted by Craig and von Peter (2014), because exposures do not cease to exist (as payments do), they convey relevant information for financial stability purposes.

In order to analyze and understand the structure of the Colombian financial system three financial market infrastructures were selected as sources of transactions: the large-value payment system (CUD – *Cuentas de Depósito*), the sovereign securities settlement system (DCV – *Depósito Central de Valores*) and the spot foreign exchange settlement system (CCDC – *Cámara de Compensación de Divisas de Colombia*). The rationale behind this selection follows four facts: First, these three financial market infrastructures account for 88.4% of the gross value of the payments and deliveries within the local financial market infrastructure during 2012 (Banco de la República, 2013); second, based on León and Pérez (2014), they are the three most systemically important local financial market infrastructures; third, the sovereign securities settlement system (DCV) and the foreign exchange settlement system (CCDC) provide detailed data for the two largest local financial markets (i.e. local sovereign securities and foreign exchange); and, fourth, the large-value payment system (CUD) provides aggregated data for all financial transactions occurring in the local market (i.e. from all financial market infrastructures). Therefore, this selection may be considered comprehensive and representative, yet parsimonious.

Consequently, the three corresponding datasets –large-value payments, sovereign securities settlements and foreign exchange settlements- consist of daily transactions for year 2012, with each transaction containing the time (date, hour, minute, etc.), sender, receiver and amount. 236 working days are available during 2012. For the large-value payment system the original dataset (i.e. in edge list format) consists of 450.124 transactions during 2012, whereas for the sovereign

[5] Smith (2011) reports that there is strong evidence of non-trivial debt masking in Enron and Lehman Brothers audited financial statements prior to their failures, which may verify the lack of completeness, consistency and validity of reported exposures as a rigorous source of information for financial networks' building.

securities settlement system (DCV) and the foreign exchange settlement system (CCDC) datasets consist of 169.398 and 115.733 registries, respectively.

The period under analysis is particularly interesting for analytical purposes. Even though it is a rather tranquil period for global financial markets, 2012 witnessed the failure of Interbolsa (November 2nd), a major brokerage firm that was well-known for its connectedness and size within the local financial markets. Therefore, despite analyzing the impact of this event on the connective architecture of the three networks is beyond our scope, the behavior of daily network statistics (in Appendix A) is valuable for supporting the consistency and robustness of empirical findings below.

The cumulative weighted networks for the whole sample are visualized in Figure 1. Each vertex corresponds to a financial institution, whereas each arrow and its width represent the existence of a transaction between financial institutions and its monetary value, respectively.

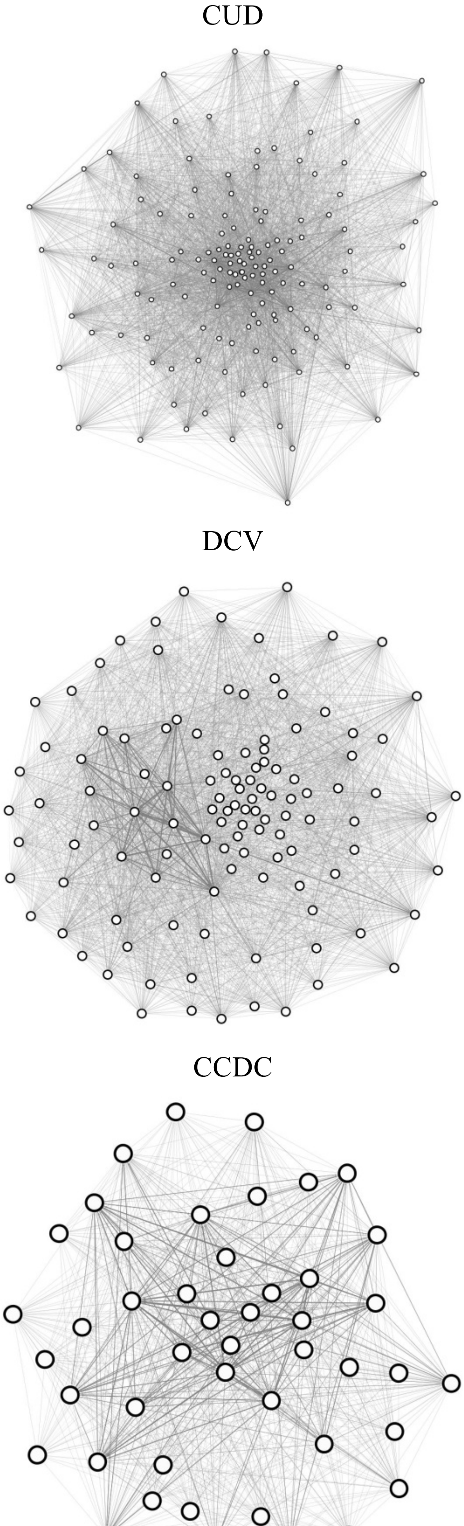


Figure 1. Weighted graphs. Each vertex corresponds to a financial institution, whereas each arrow and its width represent the existence of transactions between financial institutions and its 2012-aggregated monetary value, respectively.

5.5. Network Analysis on Colombian Selected Payment and Settlement Systems

Based on the concepts and statistics described before, Table 1 presents the main properties of the three networks under analysis, which jointly suggest that they describe a modular scale-free architecture. Statistics correspond to the estimated mean on the whole sample (236 working days, from January 3rd to December 28th 2012), with the expected values for random (i.e. homogeneous) networks included –in brackets- when feasible; no aggregation of daily networks was implemented for estimating the statistics.

Table 1
Basic statistics of the networks

Statistic	CUD		DCV		CCDC	
n	144		116		46	
d	0.07		0.05		0.24	
μ_{k}	9.75		5.93		10.66	
$\sigma_{k_{in/out}}$	13.45/13.45		8.96/8.84		8.47/8.43	
$\gamma_{k_{in/out}}$	2.26/2.23		2.26/2.28		3.10/3.12	
$\gamma_{s_{in/out}}$	1.93/1.94		1.83/1.83		2.42/2.43	
ℓ	2.20	[~4.97]	2.21	[~4.75]	1.83	[~3.82]
c	0.17	[~0.00]	0.15	[~0.00]	0.24	[~0.00]
c_w	0.25	[~0.00]	0.19	[~0.00]	0.28	[~0.00]
$r_{k_{in/out}}$	0.36/0.37	[~0.00]	0.33/0.31	[~0.00]	0.60/0.59	[~0.00]
$r_{s_{in/out}}$	0.15/0.13	[~0.00]	0.19/0.20	[~0.00]	0.34/0.37	[~0.00]
γ_{c_i}	3.51		3.01		4.37	

This table shows that the basic statistics of the networks approximate to those of a modular scale-free network. Statistics presented are: number of vertexes (n); density (d); average degree (μ_{k}); in/out degree standard deviation ($\sigma_{k_{in/out}}$); in/out degree Power-law exponent ($\gamma_{k_{in/out}}$); in/out strength Power-law exponent ($\gamma_{s_{in/out}}$); mean geodesic distance (ℓ); clustering coefficient (c); degree correlation ($r_{k_{in/out}}$); strength correlation ($r_{s_{in/out}}$); local clustering power-law exponent (γ_{c_i}). Expected values for large random networks are reported in brackets.

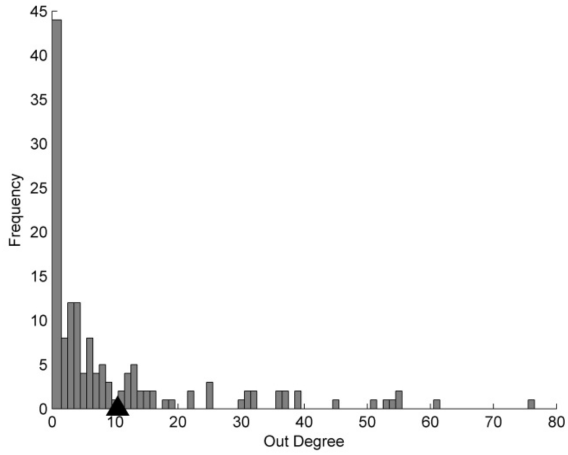
Using the sample mean for every statistic is safe because the sign and level of daily statistics is consistent along the whole sample. As exhibited in Appendix A, the daily evolution of all statistics is consistent with the sample mean reported in Table 1. Moreover, despite the failure of Interbolsa on November 2nd 2012 affected the evolution of some statistics, they all preserved the features that support the modular scale-free architecture of the networks under analysis. Consequently, not only the analysis and conclusions are robust to the frequency of data, but they are also robust to stressful conditions such as the failure of a major market participant.

CUD and DCV networks are particularly sparse, in which less than 10% of the potential links are observed, whereas CCDC network is sparse but with higher density. Likewise, the average degree of each network is much smaller than the number of participants ($\mu_k \ll n$), which verifies the sparse nature of the networks and the particularly high sparseness of CUD and DCV.

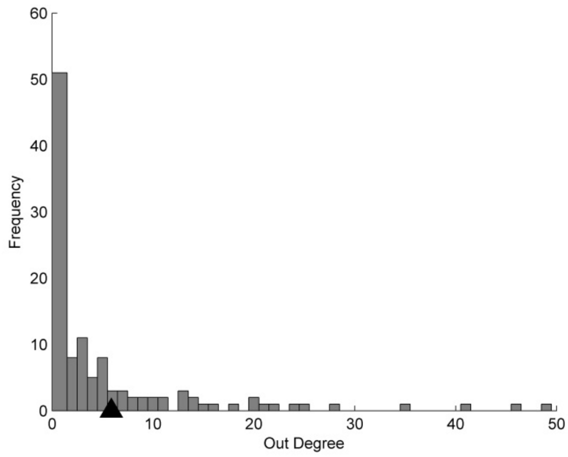
Because degree is limited to positive numbers by construction, the high dispersion around a low average degree suggests the presence of skewness and kurtosis.[6] The histogram of the degree distribution is the customary graphical test for the presence of right-skewed (i.e. heterogeneous) connective patterns. Figure 2 presents three out degree histograms for a single day (i.e. June 1st, 2012). As expected, the distributions are right-skewed, in which the estimated average degree (black triangle on x-axis) does not characterize the distribution of edges among the vertexes, especially for the CUD and DCV networks.

[6] Estimating the third and fourth moments of the degree distribution confirms such suggestion: the sample mean of out (in) degree skewness is 2.05 (1.92), 2.34 (2.38) and 0.46 (0.49) for CUD, DCV and CCDC, respectively, whereas the sample mean of out (in) degree kurtosis is 6.43 (13.45), 9.48 (8.84) and 2.23 (8.43), correspondingly. Kolmogorov-Smirnov normality tests were rejected at traditional significance levels for the distribution of degree and strength.

CUD



DCV



CCDC

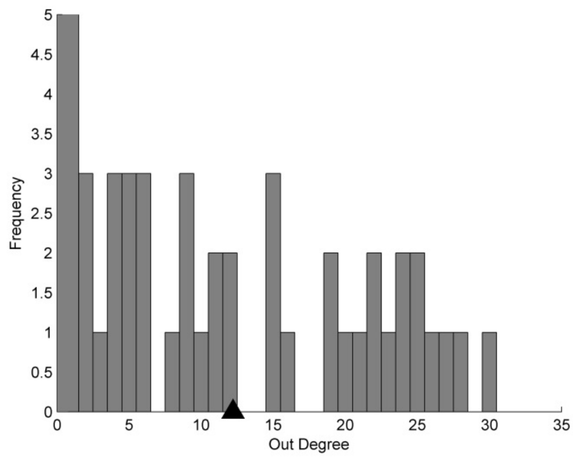


Figure 2. Out degree distribution for June 1st 2012. The triangle corresponds to the estimated mean of the distribution. The estimated average degree (black triangle on x-axis) does not characterize the distribution of edges among the vertexes, especially for the CUD and DCV networks.

As usual, consistent with Figure 2 and agreeing with Barabási and Albert's (1999) seminal findings, the right skew in the distribution of degree and strength approximates to a power-law distribution.[7] Estimated exponents for the three systems in Table 1 agree with typical values for real-world networks (i.e. $2 \leq \gamma \leq 3$).[8] However, it is evident that CUD and DCV exponents share a common (lower) level, whereas CCDC displays a higher exponent level; such difference suggests that connectedness in the CCDC is less heterogeneous. It is also evident that strength's power-law exponent tends to be lower than degree's; due to the functional form of the power-law distribution, this suggests that the distribution of the payments is more right-skewed (i.e. more heterogeneous) than the distribution of edges.

Based on the evidence previously reported, it is possible to characterize the networks under analysis as scale-free in the sense of Barabási and Albert (1999). Unlike any homogeneous network, CUD, DCV and CCDC networks lack characteristic vertexes, and exhibit structures in which most vertexes have very few connections and yet a few vertexes have many connections.

As formerly stated, other features have been identified as characteristic of real-world networks: low mean geodesic distances, high clustering coefficients, and significant degree correlation. Regarding the first issue, Table 1 shows that the mean geodesic distance is particularly low for the three networks. The CUD, DCV and CCDC networks have sample means about $\ell_{CUD} = 2.20$, $\ell_{DCV} = 2.21$

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[7] The simplest method for estimating the exponent of the power-law (γ) consists of an ordinary least squares (OLS) regression on a logarithmic transformation of (8): $\ln(p_k) = \ln(C) - \gamma ln(k)$. However, as stressed by Clauset et al. (2009), OLS fitting may be inaccurate due to large fluctuations in the most relevant part of the distribution (i.e. the tail). Therefore, all estimations of γ employed the maximum-likelihood algorithm developed by Clauset et al. (2009).

[8] Values in the range $2 \leq \gamma \leq 3$ are typical of scale-free networks, although values slightly outside it are possible and are observed occasionally (Newman, 2010). As the power-law distribution of links is an asymptotic property, a strict match between observed and expected theoretical properties for determining the scale-free properties of non-large networks may be impractical.

and $\ell_{CCDC} = 1.83$. The observed mean geodesic distances ($\ell \sim 2$) suggest that networks are tightly connected despite their sparseness, with most participants being two edges away from each other, with a single financial institution in between. Additionally, the observed mean geodesic distances suggest that some financial institutions fulfill an intermediation role within these sparse networks, presumably as in a core-periphery structure.

The observed mean geodesic distances are much lower than the expected for homogeneous networks of the corresponding size (i.e. $\ln \mathscr{n}_{CUD} = 4.97$; $\ln \mathscr{n}_{DCV} = 4.75$; $\ln \mathscr{n}_{CCDC} = 3.82$). They approximate to the characterization proposed by Cohen and Havlin (2003, 2010), which states that networks with $2 < \gamma_{\mathscr{k}} < 3$ (i.e. CUD and DCV) have a mean geodesic distance that behaves as $\ell \sim \ln\ln \mathscr{n}$, whereas networks with $\gamma_{\mathscr{k}} = 3$ (i.e. CCDC) yield $\ell \sim \ln \mathscr{n} / (\ln \ln \mathscr{n})$. Therefore, as the mean geodesic distance of the three networks is much lower than the homogeneous case ($\ell \sim \ln \mathscr{n}$), and it is closer to those typical of ultra-small networks in the Cohen and Havlin sense, the scale-free characterization is reinforced.

The second additional characteristic of real-world networks is the evidence of clustering. As previously stated, in a random graph the probability of two vertexes being connected tends to be the same for all vertexes regardless of the existence of a common neighbor. Thus, the clustering coefficient of a large random network should be close to zero. The estimated clustering coefficients are much larger than those expected for a homogeneous network, and larger than the probability of any two participants being connected in CUD and DCV.

The sample mean of the weighted clustering coefficient being higher than the non-weighted conveys relevant information about the structure of the networks. According to Barrat et al. (2004), this fact reveals that clusters are more likely formed by edges with larger weights, which further underlines the importance of clusters in the structure of the network. Likewise, Leung and Chau (2007) points out that this fact suggests that the topological (i.e. non-weighted) clustering underestimates the cohesiveness of the vertexes within their neighborhoods.

The third additional characteristic of real-world networks is the presence of significant degree correlation ($r_{k_{in/out}}$), as measured by the degree assortativity coefficient (11). In the case of homogeneous networks, in which edges are evenly distributed at random, correlation by degree should be nil. Nevertheless, as depicted in Table 1, degree correlation appears to be positive and significant for the three networks. The strength correlation ($r_{s_{in/out}}$) is also positive, but lower than the degree correlation, which may be interpreted as the degree being more relevant as an explanatory variable than strength for understanding vertexes' affinity to connect to others. More importantly, the presence of significant degree and strength correlation suggests that some preferential attachment exists within these networks.

The observed positive degree correlation, also known as *assortative mixing by degree*, in which high-degree (low-degree) vertexes have a larger probability of being connected to other high-degree vertexes (low-degree), concurs with the presence of core-periphery structures within a network (Newman, 2010).[9] In this sense, positive degree correlation suggests that there is a core of well-connected financial institutions that intermediate between numerous non-well-connected ones in the periphery, which agrees with most participants being two edges away from each other –as suggested by the observed mean geodesic distances ($\ell \sim 2$).

The existence of a core-periphery structure in financial networks is also documented by Craig and von Peter (2014), Fricke and Lux (2014), and in 't Veld and van Lelyveld (2014), who suggest using a *blockmodel*[10] to visualize the

[9] Evidence of assortative mixing in the selected Colombian payment and settlement networks contradicts the findings of Bech and Atalay (2010) for the Federal Funds Market network and of Soramäki et al. (2007) for Fedwire interbank network.
[10] As quoted by Craig and von Peter (2014), blockmodels are theoretical reductions of networks and have a long tradition in the analysis of social roles.

tiered structure of interbank markets. Figure 3 displays the observed blockmodel as intensity plots for the adjacency and weighted matrices. Adjacency matrices result from the mode of the (236) observed networks, whereas weighted matrices correspond to the arithmetic sum of the observed networks.[11] To facilitate visual inspection and analysis, the order of the participating financial institutions in the axis obeys their strength (i.e. high-strength vertexes appear in the upper-left corner). Each (i, j) element in the adjacency matrix corresponds to the existence of a local currency payment from i to j on a regular basis, whereas each (i, j) element in the weighted matrix represents the contribution of all i to j payments to all system's payments along the period under analysis.

A strict definition of a core-periphery network (see van der Leij et al., 2013) points out that the participants can be partitioned in a core and a periphery, such that all participants in the core are completely connected within and are linked to a peripheral participant, and all peripheral participants have at least one link to the core, but none to other peripheral participants. In our case financial institutions operate in a hierarchical manner, in which high-tier (i.e. core) institutions are densely interconnected in the upper-left corner of the blockmodel. However, the three networks in Figure 3 exhibit lower-tier (i.e. peripheral) financial institutions that deal with each other whilst dealing through high-tier (i.e. core) institutions. This not only is a violation of the main assumptions of the core-periphery model, but it may also be a preliminary evidence of modules of interconnected peripheral participants linked by core institutions.

[11] Using the mode for the adjacency matrix is convenient because aggregating edges across time results in artificially dense networks; this is, as adjacency matrices are binary, the mere existence of a single transaction during the analyzed period would result in a disproportionate bias towards admitting that such edge exists on a regular basis. On the other hand, aggregating weighted matrices is sound because adding monetary values preserves the true intensity of the network.

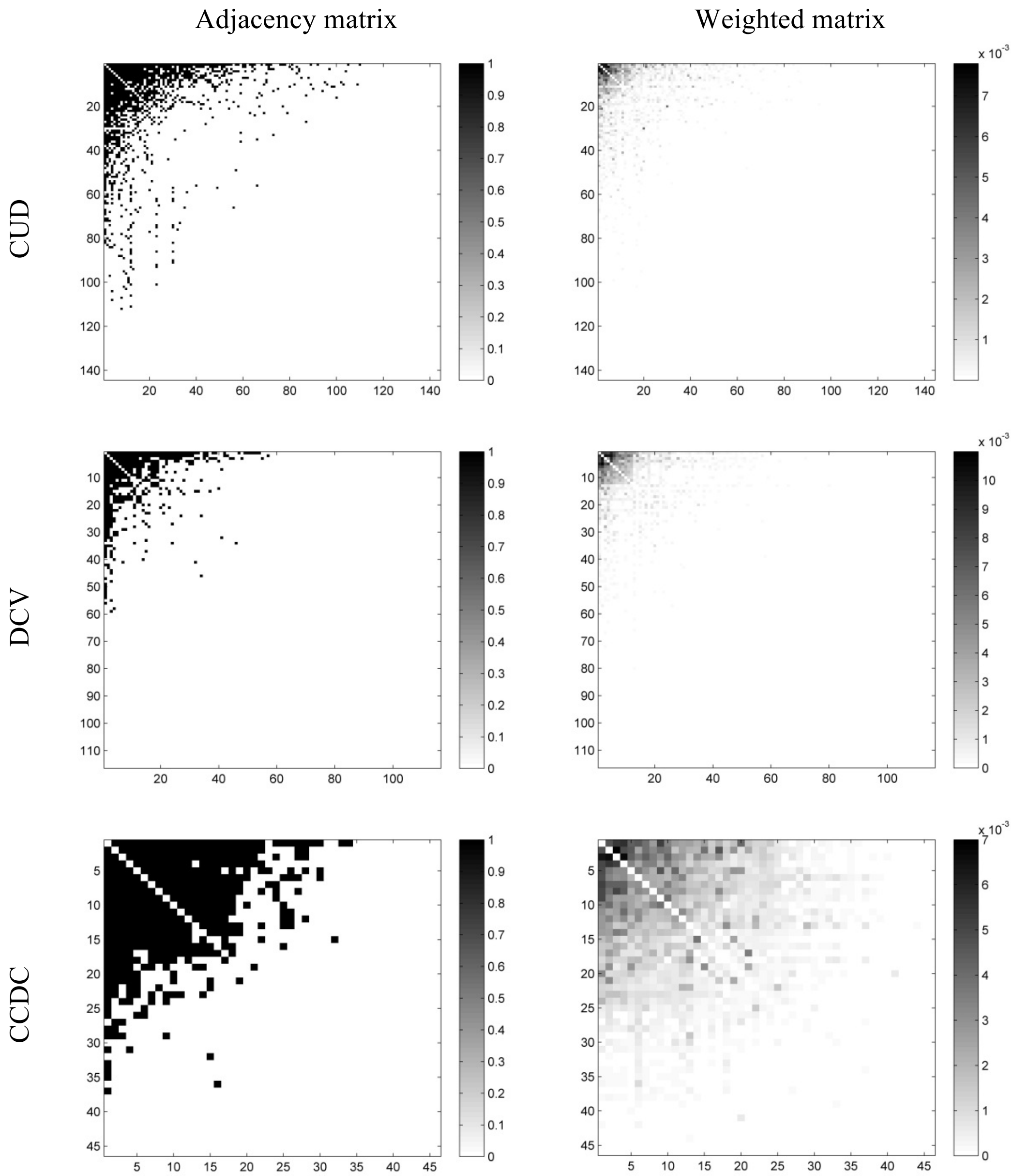


Figure 3. Blockmodel intensity plots. Blockmodels for the adjacency (left column) and weighted (right columns) matrices. The adjacency matrix corresponds to the mode of all daily adjacency matrices; the weighted matrix corresponds to the arithmetical sum of all daily weighted matrices. The order of the participating financial institutions in the axis obeys their strength (i.e. high-strength vertexes appear in the upper-left corner).

Regarding the identification of modular hierarchies, following Barabási (2003) and Newman (2010) about the information conveyed in the relation between degree and local clustering, Figure 4 exhibits the pair-wise relation between average degree (horizontal axis) and the local clustering coefficient (13) for all financial institutions, for the whole sample. It is evident that heavily connected vertexes are restricted to low clustering coefficients (i.e. less than 0.10 for CUD and DCV, and less than 0.20 for CCDC), whereas less connected vertexes may display a broad spectrum of clustering coefficients, including particularly high levels of local clustering (i.e. above 0.30).

Correspondingly, as suggested by Dorogovtsev et al. (2002) and Barabási (2003) when characterizing modular networks, γ_{c_i} in Table 1 verifies that there is an inverse relationship between average degree and local clustering. Therefore, numerical evidence supports that high-degree vertexes tend not to be incestuous as they serve as hubs that connect vertexes across different modules, whereas low degree vertexes tend to display higher clustering coefficients due to their incestuous relations within their corresponding module as they share a common hub. Our numerical evidence is consistent with the features reported by Bargigli et al. (2013) for the Italian interbank network, and agrees with the depiction of credit networks by Battiston et al. (2012b) as sparsely interconnected neighborhoods dominated by local interactions.

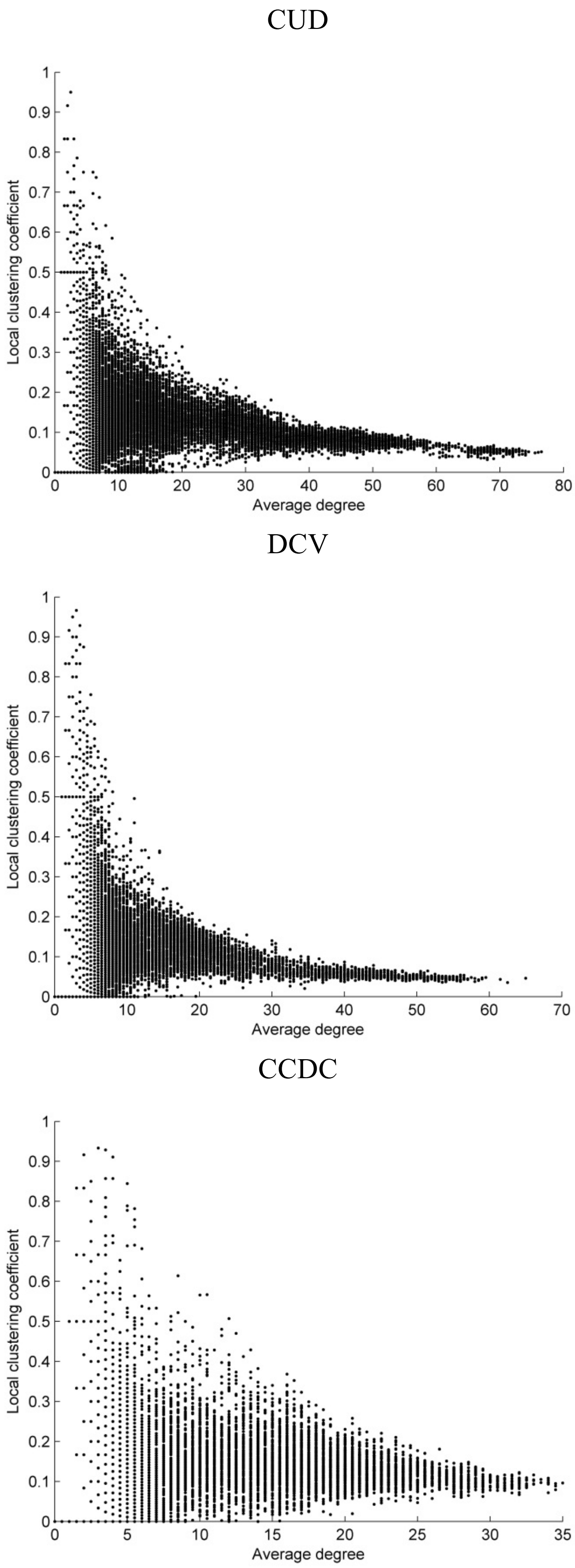


Figure 4. Degree and local clustering coefficient. Based on daily networks. There is evidence of an inverse relation between average degree and clustering.

Hence, the low clustering coefficient of central vertexes reveals that they are not connected to vertexes in numerous modules (as the standard scale-free model would suggest), whereas peripheral vertexes tend to share neighbors among them. Therefore, the three systems appear to be modular scale-free networks, where such modularity exceeds the framework of standard network models (i.e. Poisson and scale-free), with connections between peripheral financial institutions contradicting core-periphery models.

5.6. Pursuing Stability in a Modular Scale-Free Financial Network

Our findings of an inhomogeneous and hierarchical architecture agree to those of Craig and von Peter (2014), Fricke and Lux (2014), in 't Veld and van Lelyveld (2014), and Wetherilt et al. (2010) for the German, Italian, Dutch and UK interbank networks, respectively. However, the hierarchical architecture we document departs from their core-periphery structure by means of identifying that peripheral financial institutions do link among them and that there is an inverse relationship between clustering and degree. These two features are consistent with the modular scale-free networks of Barabási (2003), and agree with results attained by Bargigli et al. (2013) for the Italian interbank network. Hence, as envisaged by Fricke and Lux (2014), our findings support that the inhomogeneous and hierarchical connective structure is a new "stylized fact" of financial networks, but ours depart from the core-periphery structure, and suggest the presence of a modular scale-free network architecture instead.

To the best of our knowledge, numerical evidence of a modular scale-free architecture has not been documented for financial networks before. Our results verify that Colombian payment and settlement systems are modular scale-free networks. Accordingly, these networks consist of a few heavily connected financial institutions that serve as hubs for clusters of dense interaction that resemble nearly decomposable systems in the sense of Simon (1962). Therefore, the networks under analysis display a hierarchical structure that may be described as a system of subsystems, similar to the depiction of credit networks by Battiston et al. (2012b) as sparsely interconnected neighborhoods.

Such hierarchical architecture of financial institutions' linkages contradicts the traditional assumption of homogeneity in financial systems (à la Allen and Gale, 2000). Consequently, customary models of financial contagion based on simple, homogeneous and flat connective structures, along with the traditional emphasis on average or representative behavior of financial institutions, contradicts factual evidence in the Colombian case. As in Miller and Page (2007, p.84), *homogeneity is not a feature we often observe in the world but rather a necessity imposed on us by our modeling techniques*.

Regarding the quest for financial stability, several consequences, both theoretical and practical, arise from our findings of a modular scale-free architecture. From a theoretical point of view evidence vindicates contemporary calls for a new fundamental understanding of the structure and dynamics of financial networks (e.g. Kambhu et al., 2007; Schweitzer et al., 2009; Haldane, 2009; Haldane and May, 2011; Farmer et al., 2012). This is precisely the type of understanding that was absent before and during the Global Financial Crisis.

From a practical perspective, the modular scale-free architecture of financial networks conveys some long-ignored advantages and challenges. The main advantages of such architecture come in two forms. First, due to inhomogeneity approximating a power-law, financial networks are robust to random shocks, yet fragile to targeted attacks. In Haldane's (2009) words, financial networks are robust-yet-fragile. Second, as modularity limits cascades (Haldane and May, 2011) and isolates feedbacks (Kambhu et al., 2007), financial networks tend to be resilient; as reported by Battiston et al. (2012b), financial institutions' clustering in neighborhoods favors the regularity of local interactions. Therefore, the observed modular scale-free architecture tends to make the analyzed financial networks robust and resilient, yet fragile.

An empirical confirmation of the advantages arising from modularity may be found in the U.S. interbank market during the Global Financial Crisis. Afonso et al. (2013) reports that small (i.e. peripheral) borrowers in the U.S. interbank market concentrate their relationships to a few core financial institutions, and relates such concentration to their unexpected ability to expand their access to credit immediately after the Lehman Brothers collapse. In this case, it is reasonable to affirm that there is a homeostasis mechanism that averted small financial institutions from maintaining numerous trading partners throughout the network, which resulted in a clustered or modular connective structure that limited or isolated the effects of the sudden absence of a major participant. As highlighted by Kambhu et al. (2007), modularity can be an important part of robustness if it ensures that an affected component will be isolated from destabilizing feedbacks.

As suggested by Battiston et al. (2012a), network structure should be carefully taken into account when trying to devise policies that enhance the resilience of the financial system. In this sense, modularity may be convenient for authorities pursuing financial stability. The most obvious strategy is inherited from biology, more specifically from epidemic theory. As argued after the crisis (e.g. Kambhu et al., 2007; May et al., 2008; Haldane and May, 2011; Markose, 2012), preventive action should be on systemically important financial institutions or *super-spreaders*.

However, based on the evidence of a modular scale-free connective architecture of financial networks, dealing with systemically financial institutions is complex: They are the source of fragility in financial networks, but they also function as firewalls or circuit breakers against widespread contagion. In this sense, carelessly downsizing or dismantling systemically important financial institutions may backfire in the form of a less robust and less resilient financial system.

Accordingly, as suggested by Haldane and May (2009), protecting the financial system from future systemic events would require stronger systemically important financial institutions, but not necessarily fewer. Financial super-spreaders running with higher buffers of capital and liquid assets, proportional to the system-wide risk they contribute, would reinforce the benefits of financial networks' modular architecture. This is, prudential regulation has to be system-calibrated rather than institution-calibrated.

If core financial institutions are to be required with higher buffers of capital and liquid assets in order to profit from modularity, sound quantitative methods are required. Network centrality measures may be an interesting and objective approach to determining systemic-calibrated macro-prudential requirements, as in the eigenvector centrality-based "super-spreader tax" proposal by Markose (2012). An alternative approach is to recognize that the role of these super-spreaders is proximate to that of a central counterparty, a type of financial market infrastructure that concentrates systemic risk, and that is designed and regulated with stringent risk management requirements to serve as a source of financial stability.

5.7. Final Remarks

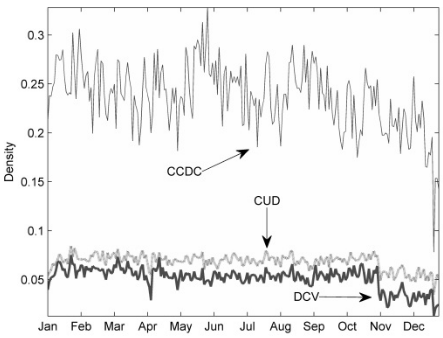
In this paper we contribute to the literature on financial stability by updating what we know about the connective structure of financial networks and by highlighting the main challenges that arise from such structure. Our results concur with evidence of non-flat hierarchies, but advocates for the modular scale-free architecture instead of the core-periphery. Even though our results are limited to the Colombian case, numerical results for the Italian interbank market reported by Bargigli et al. (2013) and the existence of non-negligible connections between peripheral financial institutions in core-periphery models for the Dutch, Italian and German interbank markets support the modular scale-free architecture as a feasible stylized fact for financial markets. We expect that new research will demonstrate whether our findings are stylized facts (as the overlap with Bargigli et al. (2013) preliminarily suggests) or they are just particular –isolated- results.

As already presented, the modular scale-free architecture entails advantages, namely its robustness due to the scale-free networks' ability to withstand random shocks, and its resilience due to modular architectures' ability to isolate cascades and feedback effects. It also conveys challenges, such as recognizing that systemically important financial institutions are the source of fragility in financial networks, but they also function as firewalls or circuit breakers against widespread contagion. In this sense, designing prudential regulation capable of enhancing inhomogeneity and modularity as the drivers of robustness and resilience in financial networks is essential for financial stability.

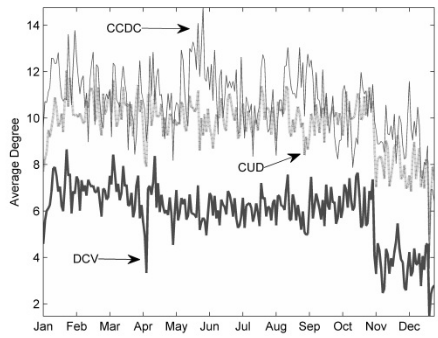
However, literature related to the modular scale-free architecture in the financial case is scarce. Several issues are waiting to be addressed. Extensions to our work may come in several forms. One line of further research is to study whether there is an optimal level of inhomogeneity and clustering that balances efficiency and stability for financial networks. Another is to further examine how this modular scale-free connective structure relates to the literature on trading relationships, an examination in which we found some preliminary overlapping elements worth evaluating in a rigorous fashion. Additionally, with financial stability in view, it is important to examine whether the modular scale-free architecture is robust to the inclusion of financial market infrastructures (e.g. central counterparties, settlement systems) and other critical infrastructures (e.g. power, communications) in financial networks. Finally, as a modular scale-free architecture is a common feature typical of complex adaptive systems (Simon, 1962; Gell-Mann, 1994; Holland, 1998; Anderson, 1999), a compelling research direction is to attain a new fundamental understanding of the structure and dynamics of financial markets that introduces non-linearity, non-homogeneity and adaptiveness, as vindicated by Krugman (1996), Haldane (2009), and Farmer et al. (2012).

5.8. Appendix

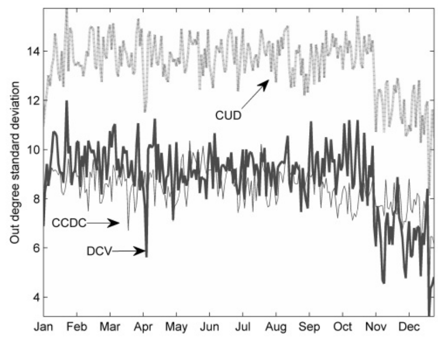
Density
(d)



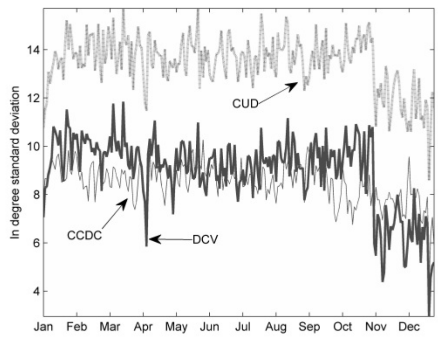
Average degree
(μ_{k})



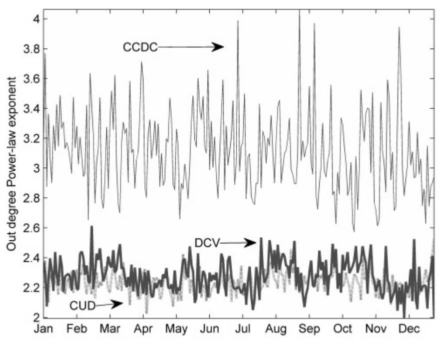
Out degree standard deviation
($\sigma_{k_{out}}$)



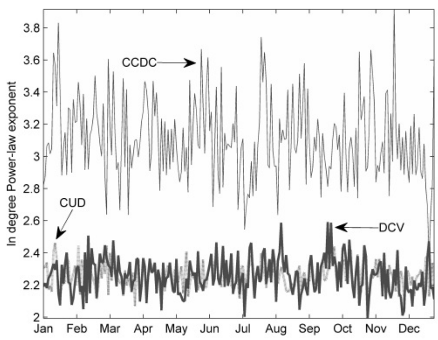
In degree standard deviation
($\sigma_{k_{in}}$)



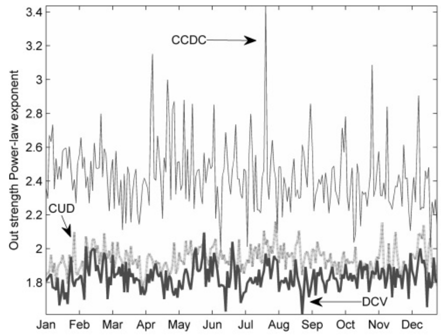
Out degree power-law exponent
($\gamma_{k_{out}}$)



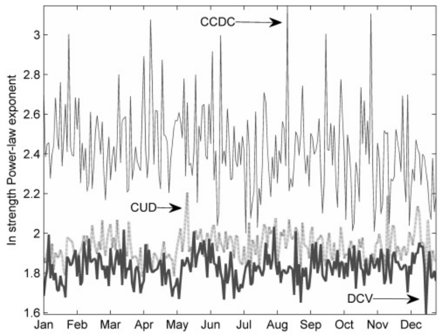
In degree power-law exponent
($\gamma_{k_{in}}$)



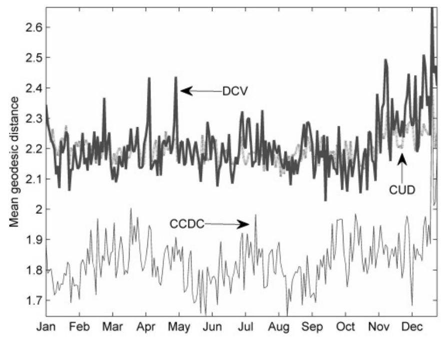
Out strength power-law exponent
($\gamma_{s_{out}}$)



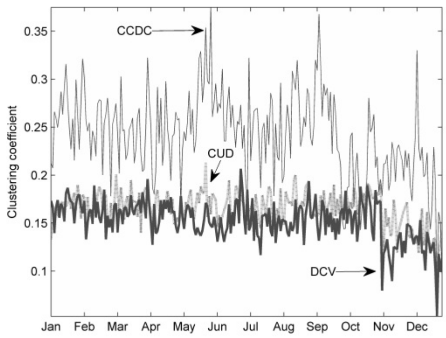
In strength power-law exponent
($\gamma_{s_{in}}$)



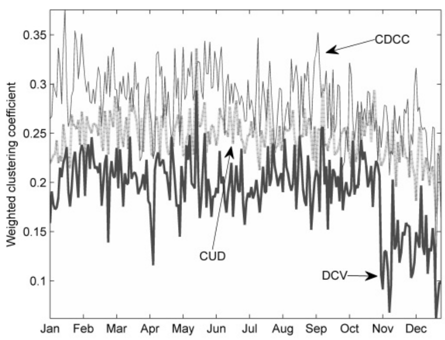
Mean geodesic distance
(ℓ)



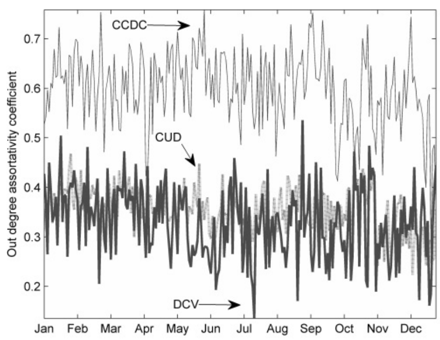
Clustering coefficient
(c)



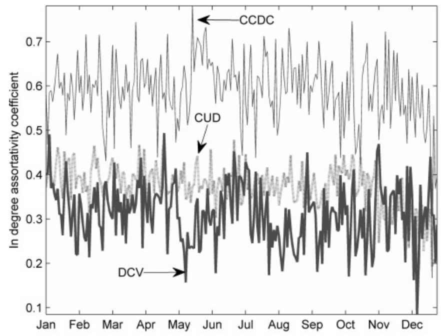
Weighted clustering coefficient
(c_w)



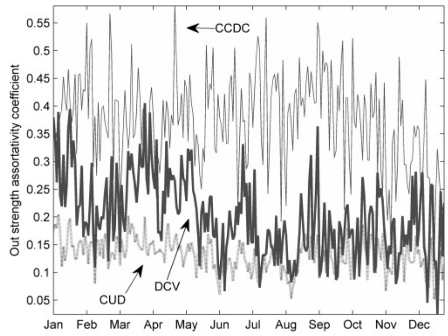
Out degree assortativity coefficient
($r_{k_{out}}$)



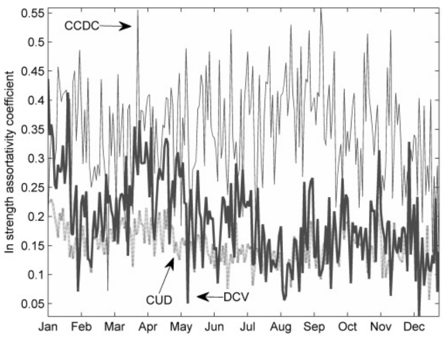
In degree assortativity coefficient
($r_{k_{in}}$)



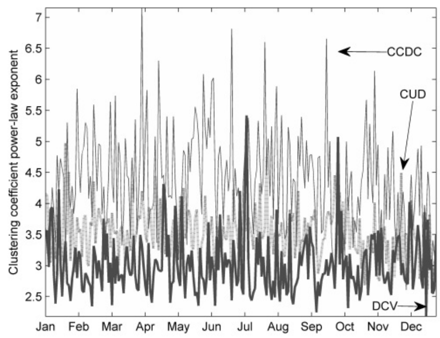
Out strength assortativity coefficient
($r_{s_{out}}$)



In strength assortativity coefficient
($r_{s_{in}}$)



Local clustering power-law exponent
(γ_{c_i})



Chapter 5: Rethinking Financial Stability: Challenges Arising From Financial Networks' Modular Scale-Free Architecture

6

Of course, hierarchy may appear in many forms, and what is necessary is to find a decomposition of the system that accounts for its intrinsic complexity.

John Casti (1979)

6. Financial Stability and Interacting Networks of Financial Institutions and Market Infrastructures

Abstract

An interacting network coupling financial institutions' multiplex (i.e. multi-layer) and financial market infrastructures' single-layer networks gives an accurate picture of a financial system's true connective architecture. We examine and compare the main properties of Colombian multiplex and interacting financial networks. Coupling financial institutions' multiplex networks with financial market infrastructures' networks removes modularity, which augments financial instability because the network then fails to isolate feedbacks and limit cascades while it retains its *robust-yet-fragile* features. Moreover, our analysis highlights the relevance of infrastructure-related systemic risk, corresponding to the effects caused by the improper functioning of financial market infrastructures or by financial market infrastructures acting as conduits for contagion.

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6.1. Introduction

Most transactions between financial institutions (FIs) require a financial market infrastructure (FMI) that settles the exchange of money, securities, foreign exchange and derivatives.[1] Therefore, directly connecting financial institutions (FIs) to each other in a network or graph may be convenient and illustrative, but it is incomplete as it ignores the role of additional networks of distinct financial and non-financial participants that are essential for the completion of financial transactions in terms of their settlement.[2] Modeling the settlement of transactions between FIs without accounting for FMIs comes down to making the assumption that FMIs always work.

Following Kurant and Thiran (2006), networks of FIs are *logical networks*, and the links between FIs are of a *logical* (i.e. virtual) nature. Additional layers of networks exist beneath logical networks, either of logical or physical nature. In the case of financial markets, due to their role in the settlement of financial transactions, FMIs may be considered as the financial system's "plumbing" (Bernanke, 2011), or the "medium" in which FIs interact in the sense of Gambuzza et al. (2014). In this vein, FMIs that settle transactions between FIs are the first logical layer beneath the traditional FIs' logical network. Accordingly, the well-functioning of those FMIs is not only crucial for financial markets and

[1] Financial institutions (FIs) correspond to depository institutions (e.g. banks), broker-dealers, investment companies (e.g. mutual funds), insurance companies, and credit unions. Financial market infrastructures (FMIs) correspond to multilateral systems providing trading, clearing, settling, recording, and compressing services for transactions between FIs. For the purpose of this paper we focus on FMIs providing settlement services.

[2] Correspondent banking is an alternative to the settlement role of FMIs, and thus can be modeled as a network of connected FIs. However, correspondent banking is only a minor channel compared to FMIs.

for financial stability, but FMIs should also be considered as *critical infrastructures.*[3]

In spite of the fact that the critical role of FMIs for financial systems has been stressed before (Bernanke, 2011; CPSS and IOSCO, 2012; Dudley, 2012a,b), most research on financial networks still focuses on single-layer FIs-only networks. Their linkages then correspond to transactions or exposures pertaining to a single market (e.g. interbank, foreign exchange, derivatives, etc.). Hence, the financial literature tends to ignore two sources of complexity in financial markets: (i) The simultaneous presence of FIs across different financial markets and their networks, and (ii) the coupling of financial markets and their networks by means of the settlements across different FMIs. Furthermore, these two sources of complexity yield two unmapped sources of systemic risk, respectively: (i) *Cross-system risk* (CPSS, 2008), corresponding to the potential effects caused by a FI experiencing problems across different markets or layers, and (ii) infrastructure-related systemic risk (Berndsen, 2011; p.15), corresponding to *the improper functioning of the financial infrastructure, or where the financial infrastructure acts as the conduit for shocks that have arisen elsewhere* (i.e. in another layer).

Therefore, in order to gain a comprehensive and enhanced understanding of financial systems' complex architecture, we implement the two existing approaches to interdependent or multi-layer networks modeling, namely *multiplex networks* and *interdependent networks* (D'Agostino and Scala, 2014). First, we build a financial multiplex network, consisting in a multi-layer network of FIs acting in different financial markets or environments. Second, by explicitly incorporating the role of the network of FMIs for the FIs' multiplex network, we build an interacting financial network. This way, based on a unique dataset, we examine how FMIs provide the medium that allows FIs to interact across distinct

[3] To consider FMIs as critical infrastructures means that they compose one of those systems that are essential for the maintenance of vital societal functions, health, safety, security, economy or social well-being of people (European Commission, 2008).

financial markets. Additionally, this paper examines whether, or not, the interaction between different layers of FIs and FMIs preserves the main connective and hierarchical features of single-layer financial networks, which have been reported to exhibit features corresponding to a modular scale-free architecture (Bargigli et al., 2013; León and Berndsen, 2014).

Five novel findings result from our paper. First, building a multi-market financial multiplex network has not been attempted before in the Colombian case. Second, to the best of our knowledge, coupling FIs and FMIs into an interacting network is a significant step forward in the examination of financial networks that has not been attempted before. Third, we verify that the multiplex network of FIs preserves the main connective and hierarchical features of single-layer or *monoplex networks* (i.e. their modular scale-free architecture), and we suggest that this is a byproduct of *positively correlated multiplexity* in the sense of Lee et al. (2014). Fourth, coupling FIs and FMIs yields a scale-free but non-modular architecture, an outcome with noteworthy implications for financial stability purposes. Fifth, in the sense of Gao et al. (2012), our results confirm that the connections between FIs and FMIs correspond to *dependence links* (i.e. links that are critical for participants' functions) instead of traditional *connectivity links* (i.e. links that enable to carry out functions), which emphasizes that the safe and efficient functioning of the FMIs' network is critical for FIs and for financial stability. Together, these five findings make a significant contribution to the literature on financial networks and financial stability, and differentiate our work from other related single- or multi-layer financial network research.

6.2. Literature Review: From Single- to Multi-Layer Financial Networks

Interdependent or coupled networks' modeling requires defining different networks (i.e. layers) and the interactions among them (D'Agostino and Scala, 2014). Two approaches to such multi-layer network modeling are available: multiplex networks and interacting networks. In the former case, each layer consists of a network containing distinct types of links but a common type of participant. In the interacting networks approach the different layers are explicitly modeled as separate networks and the links among them represent inter-layer interactions.[4]

While from a graph-theory point of view, a multi-layer network is just a larger network, networks in real life are governed and operated separately, and interactions are only allowed at well-defined boundaries (D'Agostino and Scala, 2014). In the case of financial systems, in which FIs and FMIs coexist and are mutually dependent, the resulting network will not only be larger than the traditional –logical- FIs' network, but it will also reveal how financial transactions are settled between FIs under the corresponding legal and operational framework. And, most importantly, it will reveal whether, or not, the main connective and hierarchical features of the constituent logical networks are preserved after their coupling.

Our research is related to two strands within financial network analysis, namely how financial networks couple, and on how coupling affects the connective and hierarchical architecture of financial networks. These two topics are critical for the understanding of the organization and behavior of financial systems, which enhances our understanding of financial stability. In addition, our work also contributes to the broad set of network science applications.

[4] Put another way, multiplex networks reveal how single-type participants (e.g. financial institutions) relate to each other in different environments (e.g. markets). On the other hand, interacting networks reveal how distinct layers, corresponding to different types of participants, couple between them. In the interacting networks the layers couple because their participants connect across layers, whereas in the multiplex the layers couple because of participants' overlapping across layers.

6.2.1. On Interdependent Financial Networks

Most efforts to characterize the topology of complex systems by means of network analysis have assumed that each system is isolated (i.e. non-coupled) from other networks. Yet, as highlighted by Cardillo et al. (2013; p.1), many biological and man-made networked systems are characterized by the simultaneous presence of different sub-networks organized in separate layers, with connections and participants of qualitatively different types. Such multi-layered nature of networks, also known as “interdependent networks” or “network of networks”, has been the focus of network scientists rather recently, and the work of Kurant and Thiran (2006) is among the first contributions to this field.

Most literature on financial networks deals with a single type of participant: financial institutions (FIs). A customary financial network consists of FIs pertaining to a single-layer or monoplex network, in which the network comprises one sort of participant and a single type of connection.[5] It is also possible to construct and analyze a financial monoplex composed by FMIs, as in León and Pérez (2014).

Research on multi-layer financial networks has appeared rather recently. The standard multi-layer framework in finance corresponds to the so-called multiplex network, which may be described as networks containing participants of one sort but several kinds of edges (Baxter et al., 2014).[6] Figure 1 displays a two-layer

[5] Literature on single-layer financial institutions networks is abundant. A short list includes Boss et al. (2004), Inaoka et al. (2004), Cifuentes et al. (2005), Renault et al. (2007), Soramäki et al. (2007), Cepeda (2008), May et al. (2008), Nier et al. (2008), Pröpper et al. (2008), Haldane (2009), Bech and Atalay (2010), Gai and Kapadia (2010), Arinaminpathy et al. (2012), Markose (2012), Markose et al. (2012), Martínez-Jaramillo et al. (2012), and León and Berndsen (2014).

[6] Several applications of multiplex networks have been documented for non-financial complex systems, such as transport systems (Kurant and Thiran, 2006; Cardillo et al., 2013), electrical networks (Pahwa et al., 2014), physiological systems (Ivanov and

network composed by layers X and Y, and the multiplex (Z) resulting from merging X and Y. Vertical lines connecting superimposed vertexes are the participants, whereas each vertex is a role in the corresponding layer.

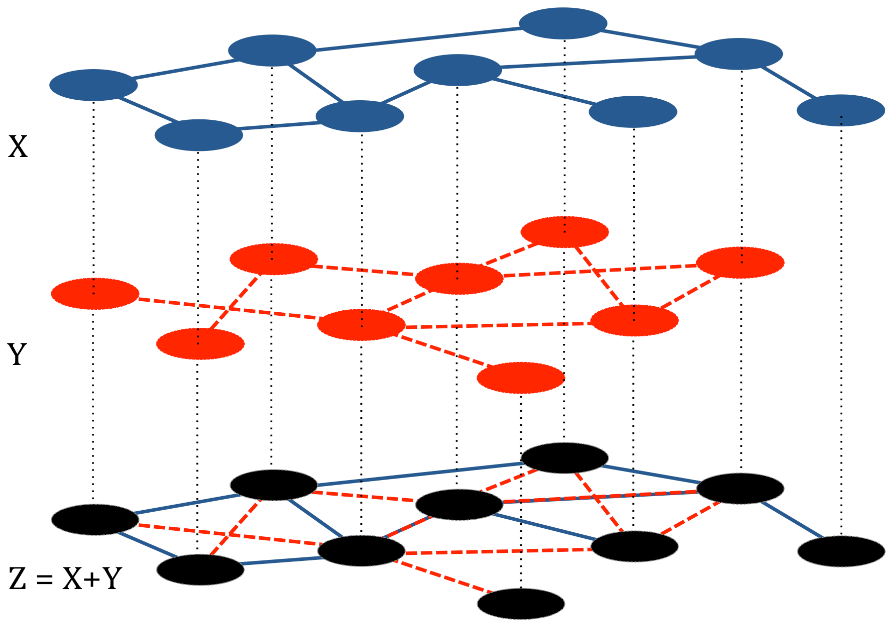

Figure 1. A multiplex network. Two-layer networks, X and Y, and the multiplex (Z) resulting from merging X and Y. Vertical lines connecting superimposed vertexes are the participants, whereas each vertex is a role in the corresponding layer.

Some financial networks have been treated as a multiplex network. Montagna and Kok (2013) model interbank contagion in the Eurozone market with a triple-layer multiplex network consisting of long-term direct bilateral exposures, short-term bilateral exposures and common exposures to financial assets. Bargigli et al. (2013) examine the Italian interbank multiplex network by transaction type (i.e. secured and non-secured) and by maturity (i.e. overnight, short-term and long-term). León et al. (2014) examine the connective properties of the multiplex network that results from the three different Colombian sovereign securities' trading and registering platforms. In this sense, recent efforts to examine financial

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Bartsch, 2014), critical infrastructures (Martí, 2014; Rome et al., 2014) and cooperation networks (Gómez-Gardenes et al., 2012).

multi-layer networks regularly correspond to the multiplex network, with FIs as the usual participant.

However, multi-layer networks models are not limited to the multiplex case. The coupled nature of layers of distinct participants may be decisively important. For instance, as depicted by Kurant and Thiran (2006), a first look at the World-Wide Web shows a network of interconnected IP vertexes, whereas a deeper examination will reveal that IP vertexes' linkages are possible via IP routers, and those linkages between IP vertexes and IP routers depend on a physical network (e.g. fiber optic, cable). As a result, Kurant and Thiran motivate a multi-layer model of distinct types of participants, in which the disruption or malfunction of concealed interacting layer(s) might destroy a substantial part of the most evident (i.e. the first) layer.

Our research overlaps with the original motivation of Kurant and Thiran (2006) for multi-layer networks, but departs from the standard multiplex network case. Our analysis acknowledges for the first time in financial networks' literature that FIs would not be able to settle most of their transactions in the absence of FMIs. Accordingly, our research considers the existence of the medium or plumbing that allows FIs to connect to each other across distinct financial environments.

Figure 2 depicts an analytical representation of a two-layer interacting network of FIs and FMIs. Let the multiplex Z from Figure 1 be Figure 2's first layer of FIs participating in two asset markets (e.g. money and foreign exchange), and let the second layer represent the settlement FMIs for both markets, in which the direction of the linkages has been omitted for practical purposes. In this case, each FI connects to the corresponding FMI, and both FMIs connect to each other in order to instruct the delivery of money and foreign exchange that would settle any transaction of this oversimplified financial system, say buying or selling foreign exchange.

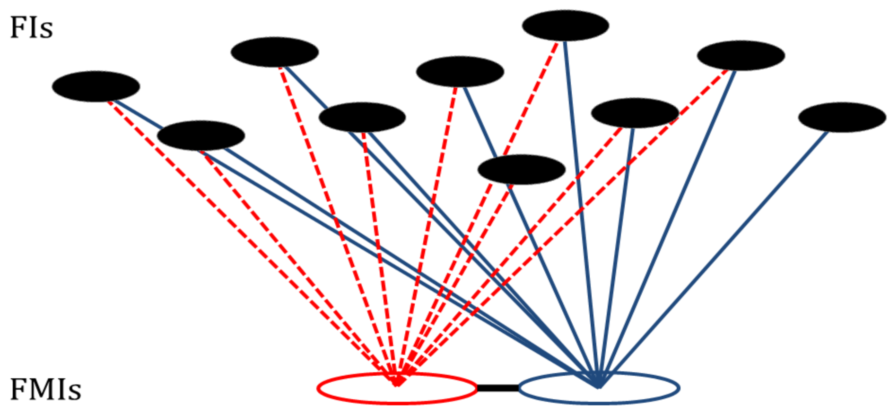

Figure 2. Coupling FIs' and FMIs' networks. This analytical representation shows a two-layer interacting network of FIs and FMIs. Each FI connects to the corresponding FMI, and both FMIs connect to each other in order to instruct the delivery of money and foreign exchange that would settle any transaction of this overly simplified financial system.

Unlike links in FIs' monoplex networks (Figure 1), which enable FIs to carry out their functions, the links in the interacting network in Figure 2 are of a critical nature for FIs: If an inter-layer link is removed, the corresponding FI would not be able to settle its transactions, whereas the absence of the link between FMIs would endanger the settlement of all FIs' transactions. The links in the FIs' and FMIs' interacting network are *dependence links*, whereas links in a FIs-only network are *connectivity links* (see Gao et al., 2012). This type of modeling emphasizes the critical role FMIs play in the financial system and the broader economy, as acknowledged by CPSS and IOSCO (2012). Furthermore, such dependence links are consistent with viewing the FMIs' network as a first layer of a financial system's *critical infrastructure*, and underscores the existence of a non-negligible *critical infrastructure dependency* in the sense of Rome et al. (2014).[7] Financial authorities in their quest to preserve financial stability should closely monitor the presence of dependence links and the fundamental role of the FMIs' network.

[7] Additional critical layers for financial networks, mostly of physical nature (e.g. communications, power), are not considered in this paper.

6.2.2. On The Connective and Hierarchical Features of Multi-Layer Networks

Even to date, the literature concentrates on an inhomogeneous connective structure of single-layer financial networks, typically in the form of the distribution of links –and their weights- approximating a *power-law* (e.g. Boss et al., 2004; Inaoka et al., 2004; Renault et al. 2007; Soramäki et al., 2007; Cepeda, 2008; May et al., 2008; Pröpper et al., 2008; Bech and Atalay, 2010; León and Berndsen, 2014).[8] The power-law or Pareto distribution of links within a network is commonly referred as a *scale-free network* (Barabási and Albert, 1999), and it corresponds to a particular case in which there are a few heavily linked participants (i.e. high *degree* vertexes) and many poorly linked participants (i.e. low *degree* vertexes). This is precisely the most documented type of network in real-world complex systems (e.g. social, biological, man-made).

The inhomogeneity in scale-free networks yields a structure that is robust to random shocks but fragile to targeted attacks, as in the nowadays celebrated *robust-yet-fragile* characterization of financial networks by Haldane (2009). In this sense, a scale-free connective structure provides everyday stability and efficiency for complex systems, but exposes them to rare massive transformations (Miller and Page, 2007), as in a power-law distribution of events or a *coevolution to the edge of chaos* (Anderson, 1999).[9] Consequently, a network approximating

[8] However, most literature that models the interactions between financial institutions is based on the assumption of a homogeneous connective structure, in which financial institutions tend to connect to each other in a dense and uniform manner that tends to diversify or disperse shocks (e.g. Allen and Gale (2000), Freixas et al. (2000), Cifuentes et al. (2005), and Battiston et al. (2012a)). Therefore, observed financial networks' inhomogeneous distribution of linkages and their weights contradicts the traditional assumptions of standard contagion models.

[9] The ubiquity of scale-free networks has been related to its robustness (Barabási, 2003; Strogatz, 2003) and to systems' self-organization (Bak, 1996; Krugman, 1996; Barabási and Albert, 1999) and adaptive features (Holland, 1998; Anderson, 1999). Economic and financial systems are particular cases of such structures and their corresponding features.

a scale-free connective structure tends to be robust because it is able to withstand random shocks, yet it is fragile if shocks target heavily connected participants.

Some real-world networks also display a particular modular hierarchical organization, a defining feature of most complex systems according to Barabási (2003). In a modular hierarchy there are densely connected clusters or communities that are sparsely connected to other clusters, resulting in systems composed by *nearly decomposable systems* in the sense of Simon (1962).

This nearly decomposable architecture resembles that reported by Battiston et al. (2012b) for describing credit networks: Agents clustered in neighborhoods so that most of the time the action is at the local level, but with a few connections among neighborhoods that make the network sparse yet responsive to shocks hitting any participant. Such modularity has been documented to be a non-accidental feature that makes a system resilient due to its ability to isolate feedbacks and to limit cascades (Kambhu et al., 2007; Haldane and May, 2011). As most components or subsystems receive inputs from only a few of the other components, change can be isolated to local neighborhoods (Anderson, 1999).

Yet, as acknowledged by Dorogovtsev et al. (2002), Ravasz and Barabási (2003), Assenza et al. (2011), and Craig and von Peter (2014) scale-free connective structures are not hierarchical in nature. Indeed, tiered or intermediated structures, namely in the form of a modular architecture, are not distinctive features of scale-free networks. Therefore, a network simultaneously displaying a scale-free connective structure and a modular hierarchical organization pertains to a particular type of network architecture, a modular scale-free architecture (Barabási, 2003), which tends to be robust and resilient, yet fragile.

Preliminary results show that analyzing complex systems as a network of coupled networks may alter the basic assumptions that network theory has relied on for single-layer networks (Kenett et al., 2014). The literature converges to non-additive and non-trivial effects arising from networks' coupling, with a remarkable finding: Coupling scale-free distributions may yield a less robust network (Buldyrev et al., 2010; Gao et al., 2012), unless the number of links (i.e. the degree) of interdependent participants coincides across the layers. This signifies that the scale-free networks' robustness is likely to be preserved if *positively correlated multiplexity* exists, such that a high-degree vertex in one layer likely is high-degree in the other layers as well (Kenett et al., 2014; Lee et al., 2014). On the other hand, to the best of our knowledge, there are no studies on how networks' coupling affects their hierarchical modularity and their resilience.

The literature on the effects of coupling networks is still preliminary, and particular cases are scarce. Our case is particular for two reasons. First, unlike most studies on multi-layer financial networks, our system consists of two distinct types of participants, FIs and FMIs; therefore, our case is not a multiplex network but an interacting network. Some examples of non-multiplex multi-layer financial networks are Kenett et al. (2014), who couple layers composed by financial institutions and assets for studying the U.S. banking system, whereas Fujiwara et al. (2009) analyze coupled layers of banks and non-financial firms for studying the structure of the Japanese credit network. Yet, to the best of our knowledge, networks composed by FIs and FMIs have not been examined in the literature.

Second, unlike most literature on multi-layer networks, one of the networks is not interconnected, and its participants depend on the other network for interacting. In our case FIs do not connect to each other due to the inevitable intervention of the FMIs' network as the plumbing or medium that allows settling financial transactions, as depicted in Figure 2. Gambuzza et al. (2014) examines and analyzes the main theoretical properties of such type of networks in physics (i.e. synchronization of oscillators), but no literature exists for financial networks.

6.3. Network Analysis

A network, or graph, represents patterns of connections between the parts of a system. The most common mathematical representation of a network is the *adjacency matrix*. Let n represent the number of vertexes or participants, the adjacency matrix A is a square matrix of dimensions $n \times n$ with elements A_{ij} such that

$$A_{ij} = \begin{cases} 1 \text{ if there is an edge between vertexes } i \text{ and } j, \\ 0 \text{ otherwise} \end{cases} \quad (1)$$

A network defined by the adjacency matrix in (1) is referred as an undirected graph, where the existence of the (i, j) edge makes both vertexes i and j adjacent or connected, and where the direction of the link or edge is unimportant. However, the assumption of a reciprocal relation between vertexes is inconvenient for some networks. For instance, the deliveries of money between financial institutions constitute a graph where the character of sender and recipient is a particularly sensitive source of information for analytical purposes, in which the assumption of a reciprocal relation between both parties is unwarranted. Thus, the adjacency matrix of a directed network or *digraph* differs from the undirected case, with elements A_{ij} being referred as directed edges or arcs, such that

$$A_{ij} = \begin{cases} 1 \text{ if there is an edge from } i \text{ to } j, \\ 0 \text{ otherwise} \end{cases} \quad (2)$$

It may be useful to assign real numbers to the edges. These numbers may represent distance, frequency or value, in what is called a weighted network and its corresponding weighted adjacency matrix (W_{ij}). For a financial network, the weights could be the monetary value of the transaction or of the exposure.

Regarding the characteristics of the system and its elements, a set of concepts is commonly used. The simplest concept is the vertex *degree* (k_i), which corresponds to the number of edges connected to it. In directed graphs, where the adjacency matrix is non-symmetrical, *in degree* (k_i^{in}) and *out degree* (k_i^{out}) quantifies the number of incoming and outgoing edges, respectively (3).

$$k_i^{in} = \sum_{j=1}^{n} A_{ji} \qquad\qquad k_i^{out} = \sum_{j=1}^{n} A_{ij} \tag{3}$$

In the weighted graph case the degree may be informative, yet inadequate for analyzing the network. *Strength* (s_i) measures the total weight of connections for a given vertex, which provides an assessment of the intensity of the interaction between participants. Akin to degree, in case of a directed graph *in strength* (s_i^{in}) and *out strength* (s_i^{out}) sum the weight of incoming and outgoing edges, respectively (4).

$$s_i^{in} = \sum_{j=1}^{n} W_{ji} \qquad\qquad s_i^{out} = \sum_{j=1}^{n} W_{ij} \tag{4}$$

Some metrics enable us to determine the connective pattern of the graph. The simplest metric for approximating the connective pattern is *density* (d), which measures the cohesion of the network. The *density* of a graph with no self-edges is the ratio of the number of actual edges (m) to the maximum possible number of edges (5).

$$d = \frac{m}{n(n-1)} \tag{5}$$

By construction, density is restricted to the $0 < d \leq 1$ range. Formally, Newman (2010) states that a sufficiently large network for which the density (d) tends to a constant as n tends to infinity is said to be *dense*. In contrast, if the density tends to zero as n tends to infinity the network is said to be *sparse*. However, as one frequently works with non-sufficiently large networks, networks are commonly labeled as sparse when the density is much smaller than the upper limit ($d \ll 1$), and as dense when the density approximates the upper limit ($d \cong 1$). The term *complete network* is used when $d = 1$.

An informative alternative measure for density is the degree probability distribution ($\mathcal{P}_k$). This distribution provides a natural summary of the connectivity in the graph (Kolaczyk, 2009). Akin to density, the first moment of the distribution of degree (μ_k) measures the cohesion of the network, and is usually restricted to the $0 < \mu_k < n - 1$ range. A sparse graph has an average degree that is much smaller than the size of the graph ($\mu_k \ll n - 1$).

Most real-world networks display right-skewed distributions, in which the majority of vertexes are of very low degree, and few vertexes are of very high degree, hence the networks are inhomogeneous. Such right-skewness of degree distributions of real-world networks has been documented to approximate a power-law distribution (Barabási and Albert, 1999). In traditional random networks, in contrast, all vertexes have approximately the same number of edges.[10]

The power-law (or Pareto-law) distribution suggests that the probability of observing a vertex with k edges obeys the potential functional form in (6), where z is an arbitrary constant, and γ is known as the *exponent* of the power-law.

$$\mathcal{P}_k \propto z k^{-\gamma} \tag{6}$$

Besides degree distributions approximating a power-law, other features have been identified as characteristic of real-world networks: (i) low mean geodesic distances; (ii) high clustering coefficients; and (iii) significant degree correlation, which we explain below.

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[10] Random networks correspond to those originally studied by Erdös and Rényi (1960), in which connections are homogeneously distributed between participants due to the assumption of exponentially decaying tail processes for the distribution of links –such as the Poisson distribution. This type of network, also labeled as "random" or "Poisson", was –explicitly or implicitly- the main assumption of most literature on networks before the seminal work of Barabási and Albert (1999) on scale-free networks.

Let g_{ij} be the *geodesic distance* (i.e. the shortest path in terms of number of edges) from vertex i to j. The mean geodesic distance for vertex i (ℓ_i) corresponds to the mean of g_{ij}, averaged over all reachable vertexes j in the network (Newman, 2010), as in (7). Respectively, the mean geodesic distance or average path length of a network (i.e. for all pairs of vertexes) is denoted as ℓ (without the subscript), and corresponds to the mean of ℓ_i over all vertexes. Consequently, the mean geodesic distance (ℓ) reflects the global structure; it measures how big the network is, it depends on the way the entire network is connected, and cannot be inferred from any local measurement (Strogatz, 2003).

$$\ell_i = \frac{1}{(n-1)} \sum_{j(\neq i)} g_{ij} \qquad \ell = \frac{1}{n} \sum_{i} \ell_i \tag{7}$$

The mean geodesic distance (ℓ) of random or Poisson networks is small, and increases slowly with the size of the network; therefore, as stressed by Albert and Barabási (2002), random graphs are small-world because in spite of their often large size, in most networks there is relatively a short path between any two vertexes. For random networks: $\ell \sim \ln n$ (Newman et al., 2006). This slow logarithmic increase with the size of the network coincides with the small-world effect (i.e. short average path lengths).

However, the mean geodesic distance for scale-free networks is smaller than $\ell \sim \ln n$. As reported by Cohen and Havlin (2003, 2010), scale-free networks with $2 < \gamma < 3$ tend to have a mean geodesic distance that behaves as $\ell \sim \ln\ln n$, whereas networks with $\gamma = 3$ yield $\ell \sim \ln n/(\ln\ln n)$, and $\ell \sim \ln n$ when $\gamma > 3$. For that reason, Cohen and Havlin (2003, 2010) state that scale-free networks can be regarded as a generalization of random networks with respect to the mean average geodesic distance, in which scale-free networks with $2 < \gamma < 3$ are "ultra-small".

The clustering coefficient (c) corresponds to the property of network transitivity. It measures the average probability that two neighbors of a vertex are themselves neighbors; the coefficient hence measures the frequency with which loops of length three (i.e. triangles) appear in the network (Newman, 2010). Let a *triangle* be a graph of three vertexes that is fully connected, and a *connected triple* be a graph of three vertexes with at least two connections, the calculation of the network's clustering coefficient is as follows: [11]

$$c = \frac{(\text{number of triangles in the network}) \times 3}{\text{number of connected triples}} \quad (8)$$

Hence, by construction, clustering reflects the local structure. It depends only on the interconnectedness of a typical neighborhood, the inbreeding among vertexes tied to a common center, and thus it measures how incestuous the network is (Strogatz, 2003). Intuitively, in a random graph, the probability of a connection between two vertexes tends to be the same for all vertexes regardless of the existence of a common neighbor. Therefore, in the case of random graphs the clustering coefficient is expected to be low, and tends to zero - in the limit – for large random networks.

Contrarily, real-world complex networks tend to exhibit a large degree of clustering. Albert and Barabási (2002) report that in most –if not all- real networks the clustering coefficient is typically much larger than it is in a comparable random network. Accordingly, in inhomogeneous graphs, as those resulting from real-world networks, the probability of two neighbors of a vertex being themselves neighbors is reported to be in the 10% and 60% range (Newman, 2010). In this sense, scale-free networks combining particularly low mean geodesic distance and high clustering implies that the existence of a few too-connected vertexes plays a key role in bringing the other vertexes close to

[11] If three vertexes (i.e. a, b, c) exist in a graph, a triangle exists when edges (a,b), (b,c) and (c,a) are present (i.e. the graph is complete), whereas a connected triple exists if at least two of these edges are present. In this sense, a triangle occurs when there is transitivity (i.e. two neighbors of a vertex are themselves neighbors). The factor of three in the numerator arises because each triangle is counted three times when the connected triplets are counted (Newman, 2010).

each other. It also indicates that the scale-free topology is more efficient in bringing the vertexes close than is the topology of random graphs (Albert and Barabási, 2002).

Besides displaying low mean geodesic distances and clustering, real-world graphs also display a non-negligible degree correlation between vertexes. They are characterized by either a positive correlation, where high-degree (low-degree) vertexes tend to be connected to other high-degree (low-degree) vertexes, or a negative correlation, where high-degree vertexes tend to be connected to low-degree vertexes. Positive degree correlation, also known as *homophily* or *assortative mixing by degree*, results in the core-periphery structure typical of social networks, whereas negative degree correlation (i.e. *dissortative mixing by degree*) is typical of technological, informational, and biological networks, which display star-like features that do not usually have a core-periphery but have uniform structures (Newman, 2010). In contrast, the degree of random (i.e. homogeneous) networks tends to be uncorrelated.

Degree correlation may be measured by means of estimating the assortativity coefficient (Newman, 2010). As before, let m be the number of edges, the degree assortativity coefficient of a network (r_k) is estimated as follows:

$$r_k = \frac{\sum_{ij}\left(A_{ij} - k_i k_j/2m\right)k_i k_j}{\sum_{ij}\left(k_i \delta_{ij} - k_i k_j/2m\right)k_i k_j} \tag{9}$$

Where

$$\delta_{ij} = \begin{cases} 0 \text{ if } i \neq j \\ 1 \text{ if } i = j \end{cases}$$

However, it should be noted that the assortativity coefficient is not limited to vertexes' degree. Other characteristics of vertexes (e.g. age, income, gender, ethnics, size) may condition their tendency to be connected. In this case, the characteristics of connected vertexes may be correlated, which results in *assortative mixing by scalar characteristics* (Newman, 2010). For financial networks it is important to assess the intensity of the interaction between participants. As highlighted by Barrat et al. (2004) and Leung and Chau (2007), the inclusion of weights and their correlations may consistently change our view of the hierarchical and structural organization of the network. Based on (9), it is possible to estimate the *assortative mixing by strength* (10).

$$r_s = \frac{\sum_{ij}\left(A_{ij} - k_i k_j/2m\right)s_i s_j}{\sum_{ij}\left(k_i \delta_{ij} - k_i k_j/2m\right)s_i s_j} \tag{10}$$

Regarding the existence of hierarchies in networks, Simon (1962, p.468) suggests a narrow definition of *hierarchical system*: *A system that is composed of interrelated subsystems, each of the latter being, in turn, hierarchic in structure until we reach some lowest level of elementary subsystem*.

Some authors link the hierarchical structure of networks to the existence of communities or modules. For instance, Newman (2003) defines that a network displays community structures when groups of vertexes have a high density of edges within them and a lower density of edges between groups. Similarly, Simon (1962) portrays modularity as *nearly decomposable systems*: A collection of subsystems that are weakly interconnected among them, but that are heavily interconnected within them.

Correspondingly, Barabási (2003) labels modularity in real-world networks as an architecture where the more connected a vertex is, the smaller is its clustering coefficient. Moreover, Barabási (2003) pinpoints that such low clustering from central vertexes contradicts the standard scale-free model. Hence, in order to quantitatively measure the hierarchical modularity of a network Barabási (2003) suggests assessing whether (or not) the most connected vertexes display low local (i.e. individual) clustering, as real-world observed hierarchical modularity suggests. Newman (2010) defines local clustering as in (11):

$$c_i = \frac{(\text{number of pairs of neighbors of } i \text{ that are connected})}{(\text{number of pairs of neighbors of } i)} \tag{11}$$

If there is no dependence between degree and clustering (i.e. clustering is democratically distributed), then the network has no hierarchical modularity, as expected from both standard random and scale-free networks. However, quite commonly, there is an inverse relationship between local clustering and degree (Bargigli et al., 2013). Such inverse relationship suggests that high-degree vertexes serve as hubs that connect vertexes across different modules, thus on average they tend not to be incestuous and they display low local clustering. On the other hand, low-degree vertexes tend to be incestuous within their corresponding module as they share a common hub; hence they tend to display high clustering coefficients.

Accordingly, Dorogovtsev et al. (2002) and Barabási (2003) suggest that hierarchical modularity may be captured by fitting a power-law to the distribution of local clustering as a function of average degree ($\mu_{\mathcal{k}}$) (12):

$$\mathcal{P}_{c_i} \propto z\mu_{\mathcal{k}}{}^{-\gamma} \tag{12}$$

Barabási (2003) highlights that the existence of hierarchical modularity in real-world networks is a defining feature of most complex systems, but it is not caused and may not be explained by the mere presence of scale-free properties. Consequently, because the standard scale-free model presumes the existence of a few central vertexes connected to vertexes in numerous modules (i.e. against the evidence of modularity in real-world networks), Barabási (2003) introduces a new type of network: a modular scale-free network. According to Dorogovtsev et al. (2002) and Barabási (2003), the clustering coefficient of a network and its local distribution by degree may confirm –or reject- the presence of a hierarchy within the system.

6.4. The Datasets

Two main data sources have been used in the financial networks literature: (i) financial transactions (i.e. flows), and (ii) financial exposures (i.e. stocks). Networks of financial transactions correspond to the delivery of money, securities or currencies, or to the corresponding trades among financial institutions, which are automatically registered and safeguarded by FMIs (e.g. large-value payment systems, clearing houses, securities settlement systems, central securities depositories, trading platforms, trade repositories) whenever a transaction occurs. As highlighted by some authors (e.g. Kyriakopoulos et al., 2009; Uribe, 2011a,b), the information conveyed in financial transactions is particularly valuable due to its (i) granularity, with informative details such as sender, recipient, amount, type of transaction, underlying asset, etc.; (ii) completeness, because all financial transactions ineludibly involve the delivery of money or a financial asset, or a trade; (iii) reliability from a supervisory perspective because payments and settlements cannot be –easily- falsified; and (iv) opportunity, with data usually available in real-time (or with a minimal lag).

On the other hand, financial exposures ordinarily emerge from reports prepared and delivered by each financial firm to the corresponding authorities (e.g. financial statements), where the most commonly used for building financial networks are interbank credit and derivatives exposures. This type of information tends to be aggregated (i.e. details of individual exposures, counterparties, instruments, etc. are usually unavailable) and lagged, and its completeness, consistency, and validity depend on accounting practices by each financial firm and the corresponding jurisdiction.[12] Yet, as highlighted by Craig and von Peter

[12] Smith (2011) reports strong evidence of non-trivial debt masking in audited financial statements of Enron and Lehman Brothers prior to their failures. This confirms the lack of completeness, consistency, and validity of reported exposures as a rigorous source of information for financial network building.

(2014), because exposures do not cease to exist (as payments do), they convey relevant information for financial stability purposes.

In order to analyze and understand the structure of the Colombian financial system three FMIs were selected as sources of financial transactions: the large-value payment system (CUD – *Cuentas de Depósito*), the sovereign securities settlement system (DCV – *Depósito Central de Valores*) and the spot market foreign exchange settlement system (CCDC – *Cámara de Compensación de Divisas de Colombia*). The rationale behind this selection follows four facts: First, these three FMIs account for 88.4% of the gross value of the payments and deliveries within the local financial market infrastructure during 2012 (Banco de la República, 2013); second, based on León and Pérez (2014), they are the three most systemically important local FMIs; third, the sovereign securities settlement system (DCV) and the foreign exchange settlement system (CCDC) provide detailed data for the two largest local financial markets (i.e. local sovereign securities and foreign exchange); and, fourth, the large-value payment system (CUD) provides aggregated data for all financial transactions occurring in the local market (i.e. from all financial market infrastructures). Therefore, this selection may be considered comprehensive and representative, yet parsimonious.

The dataset used for our research is unique, and particularly useful for extending the examination and understanding of financial markets' connective architecture. As emphasized by D'Souza et al. (2014), being able to use factual data related to the role of critical infrastructure networks has been elusive because they are independently owned and operated, with poor incentives for owners or operators to share data, and because the linkages between them are often only revealed during extreme events.

6.4.1. Financial Institutions' Monoplex and Multiplex Networks

The information obtained from these three FMIs serve the purpose of building three monoplex networks corresponding to three different environments or markets in which Colombian FIs interact: (i) sovereign securities market; (ii) foreign exchange market; (iii) other markets (i.e. equity, non-sovereign securities, derivatives, interbank funds). In that order, they represent 75.7%, 3.4% and 20.9% of the payments in the local financial market.[13] The three monoplex networks are displayed in Figure 3.[14]

Each monoplex network is a weighted directed graph that accumulates payments made throughout 2012. Every single vertex in a particular monoplex network corresponds to a FI fulfilling its role in the corresponding network, whereas each arrow (i.e. arc) and its width represent the existence of a payment between FIs and its monetary value, respectively.

[13] The construction of the networks to be analyzed here differs from that of León and Berndsen (2014). As their aim didn't include aggregating the three networks in a multiplex, León and Berndsen (2014) examined the networks corresponding to the three FMIs (i.e. CUD, DCV, CCDC) independently, disregarding that the CUD network contains data from DCV and CCDC. For the purpose of our paper, which includes building the multiplex by aggregating networks, it is imperative to work on non-overlapping networks. Accordingly, our results must differ from theirs.

[14] Yet, it is possible to successively decompose each single-layer network or monoplex into several single-layers. For example, the sovereign securities market monoplex network may be decomposed by type of transaction (e.g. buy/sell or repos), by term to maturity of the security, by origin of the transaction (e.g. over-the-counter or trading platform), etc. The existence of such interrelated subsystems is characteristic of complex systems (Simon, 1962) and a rough measure of their complexity (Casti, 1979).

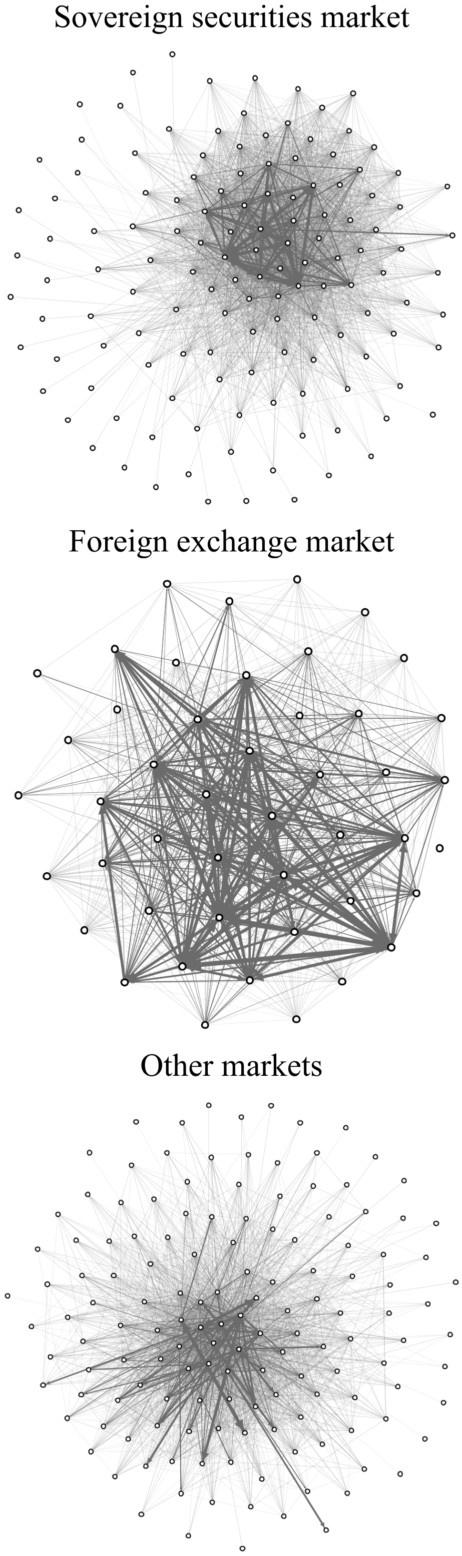

Figure 3. Monoplex networks. A vertex corresponds to a FI fulfilling its role in the corresponding network, whereas each arrow and its width represent the existence of a payment between FIs and its monetary value, respectively.

Aggregating the three monoplex networks of Figure 3 yields a network containing participants of one sort (i.e. FIs) but several kinds of edges (i.e. sovereign securities, foreign exchange, interbank, etc.). Therefore, according to Baxter et al. (2014), Figure 4 exhibits the Colombian financial multiplex network.

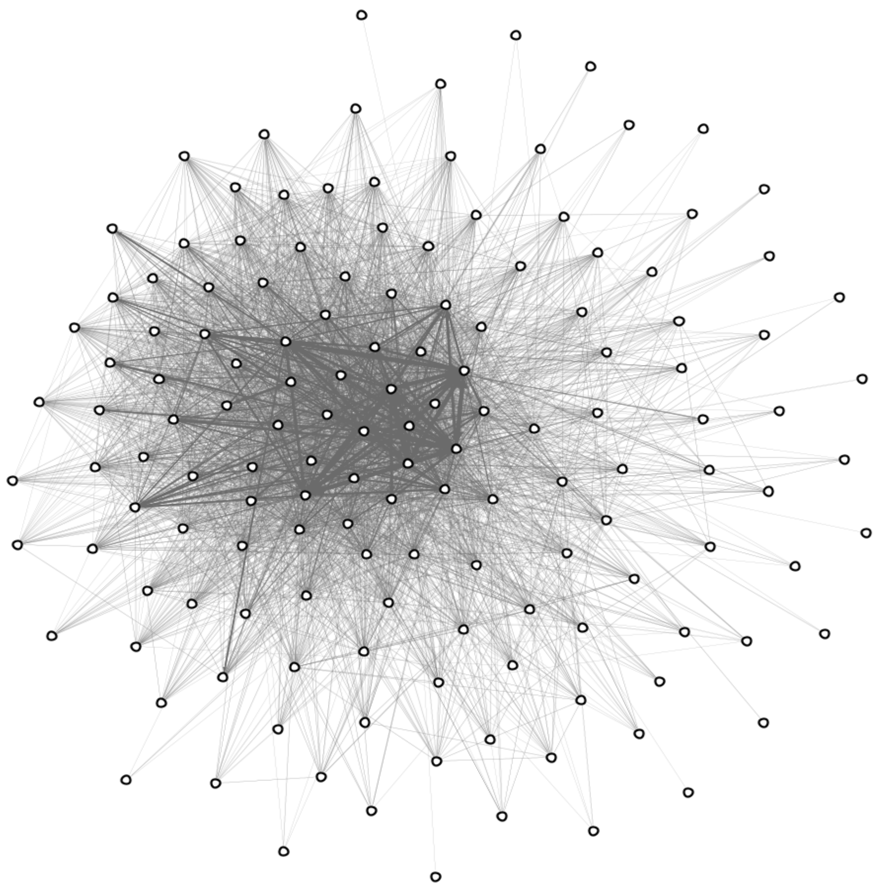

Figure 4. Multiplex network. Each vertex corresponds to a FI potentially fulfilling multiple roles in the Colombian financial system, whereas each arrow and its width represent the existence of a payment between FIs and its monetary value, respectively.

Unlike traditional monoplex financial networks that limit to a single market (e.g. interbank, derivatives), each vertex in Figure 4 corresponds to a FI potentially fulfilling multiple roles in the Colombian financial system. Hence, as the simultaneous presence of FIs across different markets is captured in the multiplex network, the potential effects of a FI failing across different markets (i.e. cross-system risk) are also considered.

6.4.2. Coupling Financial Institutions' and Financial Market Infrastructures' Networks

In the sense of Kurant and Thiran (2006), acknowledging the role of FMIs reveals that, from the settlement point of view, links between FIs are of a logical or virtual nature. Therefore, the FIs' and FMIs' interacting network consists of including the in-between role of FMIs for accomplishing the settlement of financial transactions. Accordingly, coupling FIs and FMIs entails breaking down each payment (i.e. each arrow) into its settlement constituents.

For example, the purchase of a sovereign security by FI_A from FI_B and the corresponding payment from FI_A to FI_B, which is customarily depicted as a single arrow from FI_A to FI_B, would consist of three different arrows[15]: (i) an arrow from FI_A to the sovereign securities system (DCV), corresponding to the order to buy the security;[16] (ii) an arrow from DCV to the large-value payment system (CUD), corresponding to an instruction to debit FI_A's money account and credit FI_B's money account after verifying the availability of securities and money; and (iii) an arrow from CUD to FI_B, corresponding to the payment received by FI_B for the sale. This breakdown corresponds to the typical involvement of FMIs in any

[15] Berndsen (2013) develops solutions to several types of settlement problems, including the case here depicted (i.e. a transaction involving two agents and two financial assets).
[16] Orders received by a settlement FMI usually arrive from other types of FMIs, such as trading or registering platforms. This additional layer is not considered in this paper; however, its role should not be underestimated.

payment in the local financial system, as described in Banco de la República (2013) and León and Pérez (2014).[17]

Breaking down all financial transactions in the multiplex network in Figure 4 yields the two-layer interacting network in Figure 5, in which the first (upper) and second (lower) layer corresponds to FIs and FMIs, respectively. The diameter of all vertexes is determined by their strength (i.e. the value of the ingoing and outgoing weighted connections). The number of links and the size of vertexes in each layer and across layers are particularly dissimilar, consistent with the inhomogeneous distribution of degree and strength of real-world networks.

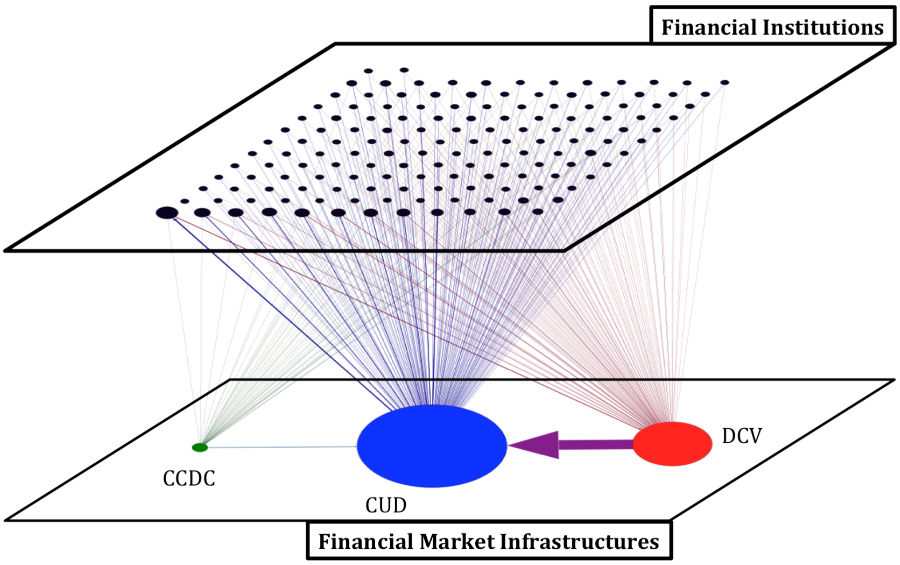

Figure 5. Interacting FIs' and FMIs' networks. Each vertex in the first (upper) layer corresponds to a FI potentially fulfilling multiple roles in the Colombian financial system, whereas each vertex in the second layer corresponds to a FMI. The diameter of all vertexes is determined by their strength (i.e. the value of the ingoing and outgoing weighted connections). Each arrow corresponds to a part of the settlement process.

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[17] Alternatively, this same transaction could be examined from the securities' delivery point of view –instead of the payment's. However, this would be impractical because payments with central bank's money is the numeraire for all financial transactions in the local market; this is, all transactions in Colombia ultimately involve the delivery of local currency irrespective of the financial asset involved (e.g. securities, foreign exchange, derivatives, etc.).

There are no intra-layer connections in the first layer.[18] Indirect connections between FIs are possible through FMIs in the second layer. Also, due to its role as the FMI responsible for the money settlement of all financial transactions, the diameter of CUD is much greater than any other FMI or FI; the second largest vertex is DCV, consistent with the relevance of the sovereign securities market in the Colombian financial system. Moreover, following Kurant and Thiran (2006), it is evident that the failure of CUD in the second layer would destroy a substantial part of the first layer, rendering the whole system useless in practice.

6.5. Main Results

Table 1 presents the main properties of the three monoplex networks and the multiplex exhibited in Figure 3 and 4, respectively. Statistics correspond to the estimated mean on the whole sample (January 3rd to December 28th 2012), with the expected values for random (i.e. homogeneous) networks included –in brackets- when feasible. Using the sample mean for every statistic is rather safe because the sign and level of daily statistics is consistent along the whole sample.

[18] Under Colombian regulatory framework securities and foreign exchange must be settled in a FMI. Thus, correspondent banking in the form of intra-layer connections in the first layer of Figure 5 is limited to marginal transactions (e.g. settlement of checks between banks pertaining to the same conglomerate).

Table 1
Basic statistics of the networks[a]

Statistic	Monoplex networks (in Figure 3)			Multiplex network (in Figure 4)
	Sovereign securities market	Foreign exchange market	Other markets	
n	134	46	143	143
d	0.04	0.24	0.05	0.07
μ_k	5.19	10.66	6.44	9.75
$\sigma_{k_{in/out}}$	8.60/8.51	8.47/8.43	10.23/10.17	13.41/13.44
$\gamma_{k_{in/out}}$	3.12/3.01	3.83/3.89	2.22/2.06	2.81/2.49
$\gamma_{s_{in/out}}$	1.77/1.75	3.76/3.69	2.22/2.20	1.96/1.99
ℓ	2.24 [~4.9]	1.83 [~3.8]	2.34 [~5.0]	2.19 [~5.0]
c	0.15 [~0.0]	0.24 [~0.0]	0.11 [~0.0]	0.17 [~0.0]
c_w	0.17 [~0.0]	0.28 [~0.0]	0.20 [~0.0]	0.25 [~0.0]
$r_{k_{in/out}}$	0.31/0.32 [~0.0]	0.59/0.60 [~0.0]	0.23/0.22 [~0.0]	0.38/0.36 [~0.0]
$r_{s_{in/out}}$	0.19/0.20 [~0.0]	0.34/0.37 [~0.0]	0.12/0.09 [~0.0]	0.15/0.13 [~0.0]
γ_{c_i}	4.72	4.24	5.43	5.67

This table shows that the basic statistics of the monoplex networks and the resulting multiplex approximate to those of a modular scale-free network. [a] Statistics presented are: number of vertexes (n); density (d); average degree (μ_k); in/out degree standard deviation ($\sigma_{k_{in/out}}$); in/out degree Power-law exponent ($\gamma_{k_{in/ou\square}}$); in/out strength Power-law exponent ($\gamma_{s_{in/out}}$); mean geodesic distance (ℓ); clustering coefficient (c); degree correlation ($r_{k_{in/out}}$); strength correlation ($r_{s_{in/out}}$); local clustering power-law exponent (γ_{c_i}). Expected values for large random networks are reported in brackets.

About the three monoplex networks, several features typical of real-world networks are evident: First, they are sparse, with $d \ll 1$ and $\mu_k \ll n-1$. Second, they are inhomogeneous, which is verified by degree's dispersion ($\sigma_{k_{in/out}}$) and approximate power-law distribution, with power-law exponents ($\gamma_{k_{in/out}}$) approaching typical values[19], hence consistent with the scale-free

[19] Values in the range $2 \leq \gamma \leq 3$ are typical of scale-free networks, although values slightly outside it are possible and are observed occasionally (Newman, 2010). As the

architecture of real-world networks. Third, the distribution of strength in the three monoplex networks approximates a power-law distribution. Fourth, agreeing with the scale-free features of the networks, the observed mean geodesic distances (ℓ) are much lower than the expected for random networks of the corresponding size, and they are consistent with the "ultra-small" characterization of Cohen and Havlin (2003, 2010). Fifth, there is evidence of positive degree correlation (i.e. *assortative mixing by degree,* $r_{k_{in/out}} > 0$) and positive strength correlation (i.e. *assortative mixing by strength,* $r_{s_{in/out}} > 0$), which suggests that high-degree (low-degree) vertexes have a larger probability to be connected to other high-degree vertexes (low-degree), and supports the presence of a core-periphery structure (Newman, 2010). Sixth, they display clustering coefficients (c and c_w) much larger than expected for random networks. Seventh, the distribution of local clustering coefficients as a function of average degree approximates a power-law distribution, which is consistent with the characterization of modular networks by Dorogovtsev et al. (2002) and Barabási (2003).

Accordingly, concurrent with León and Berndsen (2014), the three monoplex networks may be characterized as modular scale-free networks in the sense of Barabási (2003). Such characterization matches the description of credit networks as sparsely connected neighborhoods by Battiston et al. (2012b). Furthermore, this characterization is consistent with real-world biological and social networks, and it also agrees with the existence of hierarchies and nearly decomposable systems. As said, the literature points out that the modular scale-free architecture is by no means accidental, but follows the organization of systems towards structures that favor systemic resilience and robustness.

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power-law distribution of links is an asymptotic property, a strict match between observed and expected theoretical properties for determining the scale-free properties of non-large networks may be impractical. The exponents for the foreign exchange market display some departure from typical values, which may be due to the small number of participants. We use the algorithm developed by Clauset et al. (2009) for estimating the power-law exponents; this algorithm avoids several issues related to traditional estimation by ordinary least squares.

The multiplex network preserves the modular scale-free architecture of its constituent monoplex networks. Based on the literature on multiplex networks, this feature should be related to the existence of positively correlated multiplexity, in which a vertex with large degree in one layer likely has more links in the other layer as well (Kennet et al., 2014; Lee et al., 2014). In our case, the correlation matrix estimated on FIs' degree and strength across the three monoplex networks in Figure 3 verifies that there is significant evidence of positively correlated multiplexity (Figure 6).[20]

	k_{sov}	k_{fx}	k_{rest}	s_{sov}	s_{fx}	s_{rest}
k_{sov}	1					
k_{fx}	0.80	1				
k_{rest}	0.65	0.78	1			
s_{sov}				1		
s_{fx}				0.74	1	
s_{rest}				0.66	0.75	1

Figure 6. Monoplex networks' degree and strength correlation matrix. The correlation matrix was estimated based on the contribution of each FI to the total degree or strength in the whole samples.

Visual inspection of each FIs' degree and strength across the three monoplex networks is presented in Figure 7. Graphical inspection matches the numerical results in the correlation matrix presented in Figure 6: There is a clear tendency of high-degree and high-strength FIs overlapping across layers. Thus, Figures 6 and 7 together suggest that positively correlated multiplexity may explain why

[20] There would be a plausible explanation for the –hypothetical- case in which the multiplex network does not preserve the main features of the constituent monoplex networks: the role (i.e. importance) of financial institutions across networks is disparate. This could be a case for non-positive correlated multiplexity that would –presumably- make the multiplex less fragile (i.e. systemic importance would be more democratically distributed than in the monoplex networks) but also less robust (i.e. as systemic importance would be less concentrated, randomly extracting a participant may not be as innocuous as in the monoplex networks).

the modular scale-free features of the three monoplex networks is preserved in the resulting multiplex network.

By degree | By strength

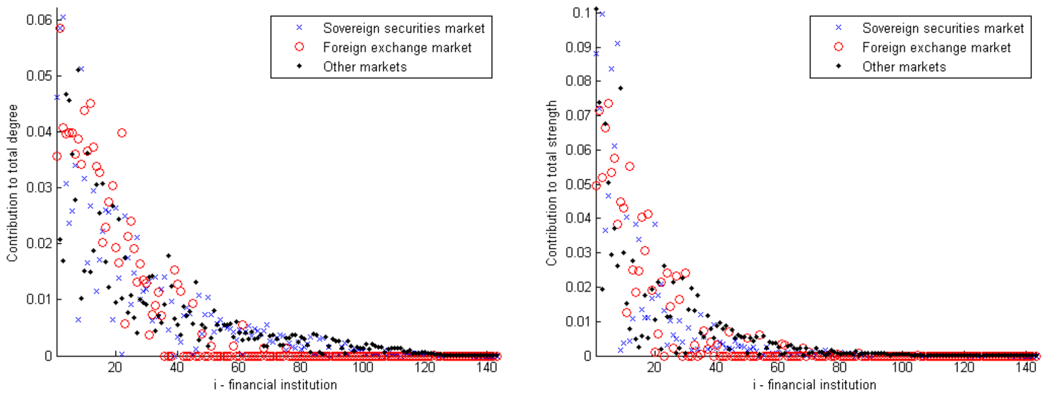

Figure 7. Positively correlated multiplexity. Participating FIs in each layer are ranked in decreasing order of degree (left) and strength (right) in the corresponding horizontal axis. High-degree and high-strength FIs in one layer tend to be the high-degree in the other two layers, which confirms the presence of positively correlated multiplexity.

Table 2 compares the main properties of the multiplex and the interacting networks. The multiplex corresponds to the network of FIs acting in different financial markets (Figure 4), whereas the interacting network incorporates the role of FMIs for the multiplex network (Figure 5). As before, the statistics correspond to the estimated mean on the whole sample (January 3rd to December 28th 2012), with the expected values for random (i.e. homogeneous) networks included –in brackets- when feasible.

Table 2
Basic statistics of the networks [a]

Statistic	Multiplex Network (in Figure 4)		Interacting Networks (in Figure 5)	
n	143		146	
d	0.07		0.02	
μ_{k}	9.75		2.57	
$\sigma_{k_{in/out}}$	13.41/13.44		13.04/9.75	
$\gamma_{k_{in/out}}$	2.81/2.49		3.42/3.36	
$\gamma_{s_{in/out}}$	1.96/1.99		1.78/1.77	
ℓ	2.19	[~5.0]	1.99	[~5.0]
c	0.17	[~0.0]	0.01	[~0.0]
c_w	0.25	[~0.0]	0.14	[~0.0]
$r_{k_{in/out}}$	0.38/0.36	[~0.0]	-0.43/-0.17	[~0.0]
$r_{s_{in/out}}$	0.15/0.13	[~0.0]	-0.31/-0.15	[~0.0]
γ_{c_i}	5.67		1.24×10^{14}	

This table shows that the basic statistics of the multiplex network approximate to those of a modular scale-free network, whereas the interacting network's to those of a scale-free network only. [a] Statistics presented are: number of vertexes (n); density (d); average degree (μ_{k}); in/out degree standard deviation ($\sigma_{k_{in/out}}$); in/out degree Power-law exponent ($\gamma_{k_{in/out}}$); in/out strength Power-law exponent ($\gamma_{s_{in/out}}$); mean geodesic distance (ℓ); clustering coefficient (c); degree correlation ($r_{k_{in/out}}$); strength correlation ($r_{s_{in/out}}$); local clustering power-law exponent (γ_{c_i}). Expected values for large random networks are reported in brackets.

Results reported in Table 2 verify that the FIs' and FMIs' interacting network is (i) sparse, with low density ($d \ll 1$) and low average degree ($\mu_{k} \ll n-1$); (ii) inhomogeneous and scale-free, with the distribution of degree and strength being disperse ($\sigma_{k_{in/out}} > \mu_{k}$) and approximating a power-law ($2 < \gamma < 3$); and (iv) approximately "ultra-small" ($\ell \sim \ln\ln n$). The three monoplex, the multiplex, and the interacting networks share these features.

However, there are three main differences between the architecture of the three monoplex networks, the multiplex network, and the FIs' and FMIs' interacting networks. First, degree correlation $r_{k_{in/out}}$ and strength correlation $r_{s_{in/out}}$ turned negative in the FIs' and FMIs' interacting network case. Second, clustering vanished as FMIs' role is considered. Third, as a byproduct of the lack of clustering, the distribution of local clustering (c_i) as a function of average degree is homogeneous and it does not distribute as a power-law, as exhibited in Figure 8. Therefore, based on Dorogovtsev et al. (2002) and Barabási (2003), the FIs' and FMIs' interacting network has no hierarchical modularity.

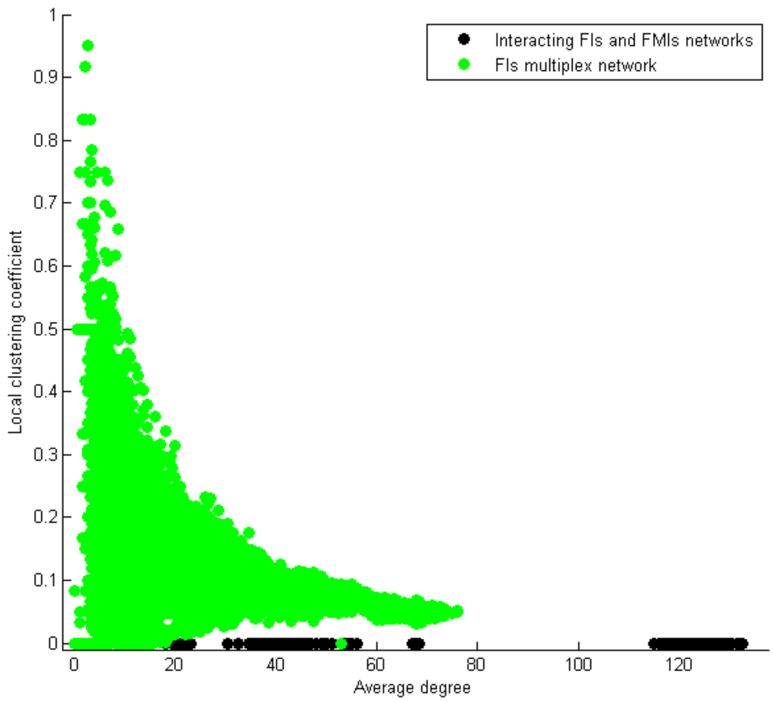

Figure 8. Distribution of local clustering as a function of average degree. Based on the entire data sample, there is evidence of an inverse relation between average degree and clustering for the FIs' multiplex, whereas such relation is absent in the FIs' and FMIs' interacting network.

All in all, comparing the numerical evidence between the FIs' and FMIs' interacting network and the FIs' monoplex and multiplex networks suggests that the modular scale-free architecture of FIs' networks fades as the role of FMIs is considered. Hence, including the next layer of complexity changed the architecture of the networks from displaying social network type features (e.g. assortative mixing, clustered, modular) to a network displaying technological networks' features (e.g. dissortative mixing) along with a non-clustered connective structure. As discussed next, the lack of modularity results in the interacting network's limited ability to isolate feedbacks and limit cascades, with noteworthy implications for financial stability.

6.6.The Critical Role of Financial Market Infrastructures in Financial Stability

The role of FMIs in financial stability is still an abstract issue. Despite the fact that financial authorities (i.e. supervisors, regulators) highlight the importance of FMIs, the literature has not addressed how the interaction between FIs and FMIs affects financial stability from the perspective of a rigorous quantitative framework. It is clear that the safe and efficient functioning of FMIs is critical for the functioning of financial markets (e.g. Bernanke, 2011; CPSS and IOSCO, 2012; Dudley, 2012a,b), but FMIs are typically disregarded when examining and analyzing the structure of financial markets.

The evidence here reported confirms that ignoring the connective role of FMIs within financial networks may mislead the analysis of the connective architecture of financial systems. The main consequence of coupling FIs' and FMIs' networks is the removal of modular hierarchy, which invalidates the presumption of a financial architecture that favors systemic resilience by means of limiting cascades (Haldane and May, 2011) and isolating feedbacks (Kambhu et al., 2007).

The absence of modularity in the FIs' and FMIs' interacting network contradicts the existence of sparsely connected financial neighborhoods that keep most of FIs' actions at the local level (as in Battiston et al., 2012b). In the absence of modularity, there are no subsystems of FIs, and they tend to receive inputs from all other FIs via FMIs, thus changes are not isolated and tend to spread across markets and their participants. As demonstrated by the failure of Bankhaus Herstatt in 1974, the technical problems of Fedwire (i.e. the U.S. large-value payment system) on October 20 1987, and Bank of New York's technological disruption in November 21 1985, a modest local shock may reverberate throughout financial markets by means of the connections between FIs and FMIs[21]. These cases showed that payments and settlement systems could play a major role in causing or amplifying financial shocks (Davis, 1995).

Explicitly incorporating the FMIs' role in the settlement of FIs transactions shows that the local financial system is not a nearly decomposable system in the sense of Simon (1962). This finding concurs with the economics behind settlement FMIs: The centralized extinction of claims between FIs. FMIs provide an alternative to the frictions that arise when money and financial securities are traded directly (Manning et al., 2009); hence their purpose is to stand as central participants that ensure the efficient and safe flow of money and financial securities to all FIs. Therefore, it is not surprising that FMIs remove modularity and may act as conduits for widespread contagion.

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[21] On June 26th 1974 the failure of Bankhaus Herstatt, a small German bank (i.e. around 50,000 customers and DM 2.0 billion in assets) caused an overseas chain reaction that forced the U.S. Clearing House Interbank Payments System (CHIPS) to halt and the U.S. clearing banks to barter checks. As documented by Davis (1995), this local event resulted in the collapse of the U.S. payments system. On Tuesday 20 October, 1987, Fedwire, the U.S. large-value payment system, had to shut down the day after black Monday (i.e. U.S. stock market largest one-day decline), causing uncertainty and the injection of funds by the U.S. Federal Reserve (Manning et al., 2009). Bank of New York's technological disruption in November 21 1985 led the U.S. Federal Reserve to provide liquidity to avoid widespread financial difficulties (Davis, 1995; Manning et al., 2009).

Accordingly, the benefits of the modular scale-free architecture of FIs' networks are of a logical or virtual nature, and depend critically on the functioning of the plumbing provided by the network of FMIs. In this sense, the FMIs' network should be considered a critical infrastructure for the financial system, and its contribution to financial stability should not be underestimated.

Furthermore, not only the well-functioning of the FMI network determines the extent to which the benefits of the modular scale-free architecture of FIs' networks apply, but it also determines whether the settlement of financial transactions is carried out or not. For instance, it is most likely that the malfunction of the FMI responsible for the settlement of the cash leg of financial transactions (i.e. payments) impedes the proper functioning of all financial markets; this is, if the buyers are unable to pay for the financial assets they are purchasing, no transactions could be completed and financial markets would halt.

Eliminating the large-value payment system (CUD) or the sovereign securities settlement system (DCV) would certainly impede the functioning of Colombian financial markets, and would threat the stability of the local financial system. This concurs with the main findings of León and Pérez (2014) regarding the systemic importance of CUD and DCV in the Colombian financial market, which results from their centrality and non-substitutability.

Yet, the malfunctioning of an ancillary or non-systemically important FMI may endanger the well-functioning of the FMIs network as well. As in León and Pérez (2014), non-systemically important are capable of endangering the markets they serve or stressing the system as a whole because of financial institutions simultaneously participating in several markets and various FMIs (i.e. cross-system risk). For instance, if an ancillary FMI is unable to clear and settle the transactions between market participants some of the latter could experience liquidity issues (e.g. not receiving securities or money in a timely manner) that could also impact their ability to fulfill their obligations in other markets and settlement systems, thus creating gridlocks in the settlement of the payments system as a whole. Consequently, due to the connective role of FMIs across markets, the role of non-systemically FMIs should not be overlooked.

Accordingly, following Kurant and Thiran (2006), Baxter et al. (2014), and Kennet et al. (2014), the interdependencies between FIs' and FMIs' networks make the financial system more fragile: Damage to one FMI (e.g. operational or financial) can trigger a catastrophic cascade of events that propagates across the global connectivity.[22] Moreover, due to the evidence of correlated multiplexity, the failure of one central FI may reverberate across financial markets by the linkages provided by FMIs, reinforcing the fragile nature (i.e. exposed to targeted attacks) of the examined financial system.

6.7. Final Remarks

Our research constitutes a step forward in the examination and understanding of local financial markets' connective architecture, and goes beyond the study of single-layer financial institutions' (FIs') networks. Our results confirm that aggregating three single-layer FIs' networks preserves their modular scale-free architecture due to the evidence of positively correlated multiplexity. This finding is essential for financial stability. First, it reveals that central FIs tend to overlap across financial networks, thus their systemic importance may be even greater than envisaged by studying each network in isolation due to cross-system risk. Second, the evidence of modularity within a scale-free connective structure suggests that the network is robust and resilient, yet fragile.

[22] Most FMIs are liable to operational risk only (e.g. technological failure, human error, terrorism, natural disasters). However, some FMIs are also exposed to financial risk. For instance, the safe and efficient functioning of central counterparties depend on the market value, liquidity, and creditworthiness of the assets comprising the margins and other layers of protection against the default of a member.

When we integrate the financial market institutions' (FMIs') network to the FIs' network, which gives a much more realistic picture consistent with FMIs' role in financial markets, we obtain a scale-free but non-modular architecture. This outcome is essential for financial stability as well. First, it stresses that the main benefit of modularity in FIs' networks, namely the resilience resulting from their ability to isolate feedbacks and limit cascades, is dependent on the well-functioning of FMIs. Second, it emphasizes the relevance of infrastructure-related systemic risk, corresponding to the effects caused by the improper functioning of FMIs or by FMIs acting as conduits for contagion.

Additional layers of complexity are readily available to expand our interacting network of FIs and FMIs. For instance, as acknowledged by D'Souza et al. (2014), global financial markets are increasingly intertwined and implicitly dependent on power and communication networks. Therefore, physical critical infrastructures are obvious candidates for examining the stability of financial systems from an operational perspective. As demonstrated by 9/11 terrorist attacks or 2011 Tohoku-Pacific Ocean Earthquake, the well-functioning of physical critical infrastructures should not be taken for granted.

Likewise, due to linkages between different countries' financial markets, multi-layer networks may not be limited by geographical or jurisdictional boundaries. For instance, the settlement of foreign exchange transactions in other countries' FIs or FMIs; local FIs with subsidiaries in other countries; local FIs being subsidiaries of foreign FIs; and the foreign component of investment portfolios, are some obvious examples of how local networks may be linked to other networks beyond national frontiers.

Chapter 5: Financial Stability and Interacting Networks of Financial Institutions and Market Infrastructures

7

No matter how many books the library holds, and makes available for retrieval, the size of your desk sets some processing limitations. Compression is vital to the performance of conscious work.

Nassim N. Taleb (2010)

7. Bibliography

Acharya, V.V., Gromb, D., & Yorulmazer, T. (2012). "Imperfect Competition in the Interbank Market for Liquidity as a Rationale for Central Banking". *American Economic Journal: Macroeconomics*, *2*, 184-217.

Afonso, G., Kovner, A., & Schoar, A. (2011). "Stressed, Not Frozen: The Federal Funds Market and the Financial Crisis". *The Journal of Finance*, *4*, 1109-1139.

Afonso, G., Kovner, A., & Schoar, A. (2013). "Trading Partners in the Interbank Lending Market". *Federal Reserve Bank of New York Staff Reports*, *620*, May.

Albert, R. & Barabási, A.-L. (2002). "Statistical Mechanics of Complex Networks". *Reviews of Modern Physics*, *74*, 47-97.

Allen, F. & Babus, A. (2008). "Networks in Finance". *Wharton Financial Institutions Center Working Paper*, *08-07*.

Allen, F. & Gale, D. (2000). "Financial Contagion". *Journal of Political Economy*, *108 (1)*, 1-33.

Allen, F., Carletti, E., & Gale, D. (2009). "Interbank Market Liquidity and Central Bank Intervention". *Journal of Monetary Economics*, *56*, 639–652.

Anderson, P. (1999). "Complexity Theory and Organization Science". *Organization Science*, *3 (10)*, 216-232.

Arinaminpathy, N., Kapadia, S., & May, R. (2012). "Size and Complexity in Model Financial Systems". *Proceedings of the National Academy of Sciences*, *45 (109)*, 18338-18343.

Assenza, S., Gutiérrez, R., Gómez-Gardañes, J., Latora, V., & Boccaletti, S. (2011). "Emergence of Structural Patterns out of Synchronization in Networks with Competitive Interactions". *Scientific Reports*, *99 (1)*, 1-8.

Babus, A. (2012). "Endogenous Intermediation in Over-The-Counter Markets". *Working Paper Series*, January.

Bak, P. (1996). *How Nature Works*. Copernicus.

Banco de la República (2012). *Reporte de Sistemas de Pago – 2012*. Banco de la República.

Banco de la República (2013). *Reporte de Sistemas de Pago – 2013*. Banco de la República.

Barabási, A.-L. (2003). *Linked*. Plume.

Barabási, A.-L. & Albert, R. (1999). "Emergence of Scaling in Random Networks". *Science*, *286*, 509-512.

Bargigli, L., di Iasio, G., Infante, L., Lillo, F., & Pierobon, F. (2013). "The Multiplex Structure of Interbank Networks". *Working Paper Series*, November. http://arxiv.org/pdf/1311.4798v1.pdf

Barrat, A., Barthélemy, M., Pastor-Satorras, R., & Vespignani, A. (2004). "The Architecture of Complex Weighted Networks". *Proceedings of the National Academy of Sciences of the United States of America (PNAS)*, *11 (101)*, 3747-3752.

Battiston, S., Delli, D., Gallegati, M., Greenwald, B., & Stiglitz, J.E. (2012a). "Default Cascades: When Does Risk Diversification Increase Stability?". *Journal of Financial Stability*, *8*, 138-149.

Battiston, S., Delli, D., Gallegati, M., Greenwald, B., & Stiglitz, J.E. (2012b). "Liaisons Dangereuses: Increasing Connectivity, Risk Sharing, and Systemic Risk". *Journal of Economic Dynamics & Control, 36, 1121-1141.*

Baxter, G.J. Dorogovtsev, A. Glotsec, V., & Mendes, J.F.F. (2014). "Avalanches in Multiplex and Interdependent Networks". In D'Agostino, G. & Scala, A. (Eds.), *Networks of Networks: the Last Frontier of Complexity* (37-52). Springer.

Bech, M. & Atalay, E. (2010). "The Topology of the Federal Funds Market". *Physica A*, *389*, 5223-5246.

Becher, C., Millard, S., & Soramäki, K. (2008). "The Network Topology of CHAPS Sterling". *Working Paper*, *355*, Bank of England.

Bernanke, B. (2011). "Clearinghouses, Financial Stability, and Financial Reform". *Remarks at the 2011 Financial Markets Conference (Stone Mountain, Georgia)*, April 4.

Berndsen, R. (2011). "What is Happening in Scrooge Digiduck's Warehouse?". *Inaugural Address Delivered at Tilburg University (Tilburg, The Netherlands)*, February 25.

Berndsen, R. (2013). "Toward a Uniform Functional Model of the Financial Infrastructure". *Journal of Financial Market Infrastructures, 2*, 77-108.

Bertay, A., Demirgüc-Kunt, A., & Huizinga, H. (2013). "Do We Need Big Banks? Evidence on Performance, Strategy and Market Discipline". *Journal of Financial Intermediation*, *22*, 532-558.

Bjelland, J., Canright, G., & Engo-Mønsen, K. (2008). "Web Link Analysis: Estimating Document's Importance from its Context". *Telektronikk*, *1*, 95-113.

Board of Directors of Banco de la República – BDBR (2003). *The Report to Congress by The Board of Directors*. Banco de la República.

Bonacich, P. (1972). “Factoring and Weighting Approaches to Status Scores and Clique Identification”. *Journal of Mathematical Sociology*, *2*, 113-120.

Boots, B.N. (1984). “Evaluating Principal Eigenvalues as Measures of Network Structure”. *Geographical Analysis*, *16 (3)*, 270-275.

Börner, K., Sanyal, S., & Vespignani, A. (2007). “Network Science”. *Annual Review of Information Science and Technology*, *1 (41)*, 537-607.

Boss, M., Elsinger, H., Summer, M., & Thurner, S. (2004). “The Network Topology of the Interbank Market”. *Quantitative Finance*, *6 (4)*, 677-684.

Brin, S. & Page, L. (1998). “Anatomy of a Large-Scale Hypertextual Web Search Engine”. *Proceedings of the 7th International World Wide Web Conference*.

Buldyrev, S.V., Parshani, R., Paul, G., Stanley, H.E., & Havlin, S. (2010). “Catastrophic Cascade of Failures in Interdependent Networks”. *Nature*, *15 (464)*, 1025-1028. doi:10.1038/nature08932

Cajueiro, D.O. & Tabak, B.M. (2008). “The Role of Banks in the Brazilian Interbank Market: Does Bank Type Matter?”. *Physica A*, *387*, 6825-6836.

Capera-Romero, L., Lemus-Esquivel, J., & Estrada, D. (2013). “Relaciones Crediticias y Riesgo de Contagio en el Mercado Interbancario No Colateralizado Colombiano”. *Temas de Estabilidad Financiera*, *77*, Banco de la República.

Cardillo, A., Gómez-Gardeñes, J., Zanin, M., Romance, M., Papo, D., del Pozo, F., & Boccaletti, S. (2013). “Emergence of Network Features From Multiplexity”. *Scientific Reports*, *3 (1344)*, 1-6.

Cardozo, P., Huertas, C., Parra, J., & Patiño, L. (2011). "Mercado Interbancario Colombiano y Manejo de Liquidez del Banco de la República". *Borradores de Economía*, *673*, Banco de la República.

Casti, J. L. (1979). *Connectivity, Complexity and Catastrophe in Large-Scale Systems.* John Wiley & Sons.

Castiglionesi, F. & Wagner, W. (2013). "On the Efficiency of Bilateral Interbank Insurance". *Journal of Financial Intermediation*, *22*, 177-200.

Cepeda, F. (2008). "La Topología de Redes como Herramienta de Seguimiento en el Sistema de Pagos de Alto Valor en Colombia". *Borradores de Economía*, *513*, Banco de la República.

Cifuentes, R., Ferrucci, G., & Shin, H.S. (2005). "Liquidity Risk and Contagion". *Journal of European Economic Association, 3*, 556-566.

Clauset, A., Shalizi, C.R., & Newman, M.E.J. (2009). "Power-Law Distributions in Empirical Data". *SIAM Review*, *4 (51)*, 661-703.

Cocco, J.F., Gomes, F.J., & Martins, N.C. (2009). "Lending Relationships in the Interbank Market". *Journal of Financial Intermediation*, *18*, 24-48.

Cohen, R. & Havlin, S. (2003). "Scale-Free Networks are Ultra-Small". *Physical Review Letters*, *5 (90)*, 1-4.

Cohen, R. & Havlin, S. (2010). *Complex Networks: Structure, Robustness and Function.* Cambridge University Press.

Committee on Payment and Settlement Systems – CPSS (2008). *The Interdependencies of Payment and Settlement Systems.* Bank for International Settlements (BIS).

Committee on Payment and Settlement Systems (CPSS) (1997). *Real-Time Gross Settlement Systems.* Bank for International Settlements (BIS).

Committee on Payment and Settlement Systems (CPSS) (2003). *Glosario de Términos Utilizados en los Sistemas de Pago y Liquidación*. Bank for International Settlements (BIS).

Committee on Payment and Settlement Systems (CPSS) and International Organization of Securities Commissions (IOSCO) (2012). *Principles for Financial Market Infrastructures*. Bank for International Settlements (BIS).

Craig, B. & von Peter, G. (2010). "Interbank Tiering and Money Center Banks". *BIS Working Papers*, *322*, Bank for International Settlements (BIS).

Craig, B. & von Peter, G. (2014). "Interbank Tiering and Money Center Banks". *Journal of Financial Intermediation, 23*, 322-347. dx.doi.org/10.1016/j.jfi.2014.02.003

Crockett, A. (2000). "Marrying The Micro- and Macro-Prudential Dimensions of Financial Stability". *Speech delivered at the Financial Stability Forum - Eleventh International Conference of Banking Supervisors (Basel, Switzerland)*.

D'Agostino, G., & Scala, A. (2014). *Networks of Networks: The Last Frontier of Complexity*. Springer.

D'Souza, R.M., Brummitt, C.D., & Leicht, E.A. (2014). Modeling Interdependent Networks As Random Graphs: Connectivity And Systemic Risk. In G. D'Agostino & A. Scala (Eds.), *Networks of Networks: The Last Frontier of Complexity* (73-94). Springer.

Davis, E.P. (1995). *Debt, Financial Fragility and Systemic Risk*. Oxford University Press.

De Nederlandsche Bank (2011). *Oversight*. De Nederlandsche Bank, August.

De Nicolò, G., Favara, G., & Ratnovski, L. (2012). "Externalities and Macroprudential Policy". *IMF Staff Discussion Note, SDN/12/05*, International Monetary Fund (IMF).

Dorogovtsev, S.N., Goltsev, A.V., & Mendes, J.F.F. (2002). "Pseudofractal Scale-Free Web". *Physical Review*, *65*.

Dudley, W.C. (2012a). "Reforming the OTC Market". *Remarks at the Harvard Law School's Symposium on Building the Financial System of the 21st Century (Armonk, New York).*

Dudley, W.C. (2012b). "What Does Interconnectedness Imply for Macroeconomic and Financial Cooperation?". *Remarks at the Swiss National Bank-International Monetary Fund Conference (Zurich)*.

Erdos, P. & Rényi, A. (1960). "On Random Graphs". *Publicationes Mathematicae*, *6*, 17-61.

Estrada, D. & Morales, P. (2008). "La Estructura del Mercado Interbancario y del Riesgo de Contagio en Colombia". *Reporte de Estabilidad Financiera*, Banco de la República.

European Commission (2008). Council Directive 2008/114/EC. *Official Journal of the European Union*, December 8.

Farmer, J.D., Gallegati, M., Hommes, C., Kirman, A., Ormerod, P., Cincotti, S., Sánchez, A., & Helbing, D. (2012). "A Complex Systems Approach to Constructing Better Models for Managing Financial Markets and the Economy". *The European Physical Journal – Special Topics*, *214*, 295-324.

Feynman, R. (1981). "The Pleasure of Finding Things Out". *Interview Delivered for the BBC Program Horizon*.

Feynman, R. (1985). “The Computing Machines of the Future”. *Speech Delivered as the Nishina Memorial Lecture (Gakushuin University, Tokyo)*, August 9.

Fiaschi, D., Kondor, I., & Marisli, M. (2013). “The Interrupted Power Law and the Size of Shadow Banking”. *unpublished manuscript*, http://arxiv.org/pdf/1309.2130v4.pdf

Freixas, X., Martin, A., & Skeie, D. (2011). “Bank Liquidity, Interbank Markets, and Monetary Policy”. *The Review of Financial Studies*, *24 (8)*, 2656-2692.

Freixas, X., Parigi, B.M., & Rochet J-C. (2000). “Systemic Risk, Interbank Relations, and Liquidity Provision by the Central Bank”. *Journal of Money, Credit and Banking*, *3 (32)*, 611-638.

Fricke, D. & Lux, T. (2014). “Core-Periphery Structure in the Overnight Money Market: Evidence From the e-MID Trading Platform”. *Computational Economics*, DOI 10.1007/s10614-014-9427-x.

Fujiwara, Y., Aoyama, H., Ikeda, Y., Iyetomi, H., & Souma, W. (2009). “Structure and Temporal Change of the Credit Network Between Banks and Large Firms in Japan”. *Economics: The Open-Access, Open-Assessment E-Journal, 3* (2009-7).

Furfine, C. (2001). "Banks Monitoring Banks: Evidence From the Overnight Federal Funds Market". *Journal of Business*, *74 (1)*, 33-58.

Gabaix, X., Gopikrishnan, P., Plerou, V., & Stanley, H.E. (2003). “A Theory of Power-Law Distributions in Financial Market Fluctuations”. *Nature*, *423*, 267-270.

Gai, P. & Kapadia, S. (2010). “Contagion in Financial Networks”. *Proceedings of the Royal Society A*, *466*. DOI 10.1098/rspa.2009.0410.

Gale, D. & Yorulmazer, T. (2013). “Liquidity Hoarding”. *Theoretical Economics*, *8*, 291-324.

Gambuzza, L.V., Frasca, M., & Gómez-Gardeñes, J. (2014). “Intra-layer synchronization in multiplex networks”. *Working Paper Series*, http://arxiv.org/abs/1407.3283.

Gao, J., Buldyrev, S.V., Stanley, H.E., & Havlin, S. (2012). “Networks Formed From Interdependent Networks”. *Nature Physics*, *8*, 40-48. DOI: 10.1038/NPHYS2180

Gell-Mann, M. (1994). “Complex Adaptive Systems”. In Cowan, G, Pines, D, & Meltzer, D. (Eds.) *Complexity: Metaphors, Models, and Reality* (17-45). Addison-Wesley.

Georg, C-P. & Poschmann, J. (2010). “Systemic Risk in a Network Model of Interbank Markets with Central Bank Activity”. *Jena Economic Research Papers*, *2013-033*.

Gómez-Gardenes, J., Reinares, I., Arenas, A., & Floría, L.M. (2012). “Evolution of Cooperation in Multiplex Networks”. *Scientific Reports*, *2 (620)*, 1-6. DOI: 10.1038/srep00620

González, C., Silva, L., Vargas, C., & Velasco, A.M. (2013). “Uncertainty in the Money Supply Mechanism and Interbank Markets in Colombia”. *Borradores de Economía*, *790*, Banco de la República.

Gould, P.R., (1967). “On the Geographical Interpretation of Eigenvalues”. *Transactions of the Institute of British Geographers*, *42*, 53-86.

Haldane, A.G. (2009). “Rethinking the Financial Network”. *Speech Delivered at the Financial Student Association (Amsterdam, Netherlands)*.

Haldane, A.G. & May, R.M. (2011). “Systemic Risk in Banking Ecosystems”. *Nature*, *469*, 351-355.

Holland, J.H. (1998). "The Global Economy as an Adaptive Process". In *SFI Studies in the Sciences of Complexity* (117-124). Perseus Books Publishing.

in 't Veld, D. & van Lelyveld, I., (2014). "Finding the Core: Network Structure in Interbank Markets". *Journal of Banking and Finance*, *49*, 27-40. DOI: http://dx.doi.org/10.1016/j.jbankfin.2014.08.006

Inaoka, H., Ninomiya, T., Tanigushi, K., Shimizu, T., & Takayasu, H. (2004). "Fractal Network Derived From Banking Transaction". *Bank of Japan Working Paper Series*, 04-E04, Bank of Japan.

International Monetary Fund (IMF) (2014). "How Big is the Implicit Subsidy for Banks Considered Too Important to Fail". *Global Financial Stability Report*, April, 101-132.

Iori, G., De Masi, G., Precup, O.V., Gabbi, G., & Caldarelli, G. (2008). "A Network Analysis of the Italian Overnight Money Market". *Journal of Economic Dynamics & Control*, *32*, 259-278.

Ivanov, P.Ch. & Bartsch, R.P. (2014). "Network Physiology: Mapping Interactions Between Networks of Physiologic Networks". In D'Agostino, G. & Scala, A. (Eds.), *Networks of Networks: The Last Frontier of Complexity* (203-222). Springer.

Kambhu, J., Weidman, S., & Krishnan, N. (2007). "New Directions for Understanding Systemic Risk". *Federal Reserve Bank of New York Economic Policy Review*, *13 (2)*, November.

Kenett, D.Y., Gao, J., Huang, X. Shao, S., Vodenska, I., Buldyrev, S.V., Paul, G., Stanley, E., & Havlin, S. (2014). "Network of Interdependent Networks: Overview of Theory and Applications". In D'Agostino, G. & Scala, A. (Eds.), *Networks of Networks: The Last Frontier of Complexity* (3-36). Springer.

Kleinberg, J.M. (1998). “Authoritative Sources in a Hyperlinked Environment”. *Proceedings of the ACM-SIAM Symposium on Discrete Algorithms*.

Kolaczyc, E.D. (2009). *Statistical Analysis of Network Data*. Springer.

Krugman, P. (1996). *Self-Organizing Economy*. Blackwell.

Kurant, M. & Thiran, P. (2006). “Layered Complex Networks”. *Physical Review Letters*, *96*, 138701. http://dx.doi.org/10.1103/PhysRevLett.96.138701

Kyriakopoulos, F., Thurner, S., Puhr, C., & Schmitz, S.W. (2009). “Network and Eigenvalue Analysis of Financial Transaction Networks”. *The European Physical Journal B*, *71*, 523-531.

Langville, A.N. & Meyer, C.D.D. (2012). *Google’s Pagerank and Beyond: The Science of Search Engine Rankings*. Princeton University Press.

Laverde, M. & Gutiérrez, J. (2012). “¿Cómo Caracterizar Entidades Sistémicas?: Medidas de Impacto Sistémico para Colombia*”*. *Temas de Estabilidad Financiera*, *65*. Banco de la República.

Lee, K-M., Kim, J.Y., Lee, S., & Goh, K.-I.. (2014). “Multiplex Networks”. In D’Agostino, G. & Scala, A. (Eds.), *Networks of Networks: The Last Frontier of Complexity* (53-72). Springer.

León, C. (2014). “Scale-Free Tails in Colombian Financial Indexes: A Primer”. *Borradores de Economía*, *812*, Banco de la República.

León, C. & Berndsen, R. (2014). “Rethinking Financial Stability: Challenges Arising From Financial Networks’ Modular Scale-Free Architecture”. *Journal of Financial Stability, 15*, 241-256. doi:10.1016/j.jfs.2014.10.006.

León, C. & Machado, C. (2013). “Designing an Expert-Knowledge-Based Systemic Importance Index for Financial Institutions”. *Journal of Financial Market Infrastructures*, *1 (2)*, 77-127.

León, C. & Pérez, J. (2014). "Assessing Financial Market Infrastructures' Systemic Importance with Authority and Hub Centrality". *Journal of Financial Market Infrastructures*, *3 (2)*, 67-87.

León, C., Machado, C., & Murcia, A. (2013). "Macro-Prudential Assessment of Colombian Financial Institutions' Systemic Importance". *Borradores de Economía*, *800*, Banco de la República.

León, C., Machado, C., & Sarmiento, M. (2014). "Identifying Central Bank Liquidity Super-Spreaders in Interbank Funds Networks". *CentER Discussion Papers*, *2014-037*, Tilburg University. https://pure.uvt.nl/portal/files/3170308/2014_037.pdf

León, C., Machado, C., Cepeda, F., & Sarmiento, M. (2012). "Systemic Risk in Large Value Payments Systems in Colombia: A Network Topology and Payments Simulation Approach". In Hellqvist, M. & Laine T. (Eds.), *Diagnostics for the Financial Markets – Computational Studies of Payment System* (267-313), Bank of Finland.

León, C., Pérez, J., & Renneboog, L. (2014). "A Multi-Layer Network of the Sovereign Securities Market". *Borradores de Economía*, *840*, Banco de la República.

Leung, C.C. & Chau, H.F. (2007). "Weighted Assortative and Disassortative Networks Model". *Physica A*, *378*, 591-602.

Lovin, H. (2012). "Systemically Important Participants in the ReGIS Payment System". In Hellqvist, M. & Laine T. (Eds.), *Diagnostics for the Financial Markets – Computational Studies of Payment System* (219-233), Bank of Finland.

Manning, M., Nier, E., & Schanz, J. (2009). *The Economics of Large-Value Payments and Settlement*. Oxford University Press.

Markose, S.M. (2012). "Systemic Risk From Global Financial Derivatives: A Network Analysis of Contagion and its Mitigation with Super-Spreader Tax". *IMF Working Paper*, *WP/12/282*, International Monetary Fund (IMF).

Markose, S.M., Giansante, S., & Rais Shaghaghi, A. (2012). "Too Interconnected to Fail Financial Network of U.S. CDS Market: Topological Fragility and Systemic Risk". *Journal of Economic Behavior & Organization*, *83*, 627-646.

Martí, J.R. (2014). "Multisystem Simulation: Analysis of Critical Infrastructures for Disaster Response". In D'Agostino, G. & Scala, A. (Eds.), *Networks of Networks: The Last Frontier of Complexity* (255-278). Springer.

Martínez-Jaramillo, S., Alexandrova-Kabadjova, B., Bravo-Benitez, B., & Solórzano-Margain, J.P. (2012). "An Empirical Study of the Mexican Banking System's Network and its Implications for Systemic Risk". *Working Papers*, *2012-07*, Banco de México.

May, R.M., Levin, S.A., & Sugihara, G. (2008). "Ecology for Bankers". *Nature*, *451*, 893-895.

Miller, J.H. & Page, S.E. (2007). *Complex Adaptive Systems*. Princeton University Press.

Montagna, M. & Kok, C. (2013). "Multi-Layer Interbank Model for Assessing Systemic Risk". *Kiel Working Papers, 1873*, Kiel Institute for the World Economy.

Newman, M.E.J. (2003). "The Structure and Function of Complex Networks". *SIAM Review*, *2 (45)*, 167-256.

Newman, M.E.J. (2010). *Networks: An Introduction*. Oxford University Press.

Newman, M.E.J., Barabási, A-L., & Watts, D.J. (2006). *The Structure and Dynamics of Networks*. Princeton University Press.

Nier, E., Yang, J., Yorulmazer, T., & Alentorn, A. (2008). “Network Models and Financial Stability”. *Working Paper*, *346*, Bank of England.

Pahwa, S., Youssed, M., & Scoglio, C. (2014). “Electrical Networks: An Introduction”. In D’Agostino, G. & Scala, A. (Eds.), *Networks of Networks: The Last Frontier of Complexity* (163-186). Springer.

Pröpper, M., Lelyveld, I., & Heijmans, R. (2008). “Towards a Network Description of Interbank Payment Flows”. *DNB Working Paper*, *177*, De Nederlandsche Bank (DNB).

Ravasz, E., & Barabási, A.-L. (2003). “Hierarchical Organization in Complex Networks”. *Physical Review E*, *67*.

Renault, F., Beyeler, W.E., Glass, R.J., Soramäki, K., & Bech, M.L. (2007). “Congestion and Cascades in Coupled Payment Systems”. *Paper Presented at the Joint European Central Bank-Bank of England Conference on ‘Payments and Monetary and Financial Stability’*.

Rochet, J-C. & Tirole, J. (1996). “Interbank Lending and Systemic Risk”. *Journal of Money, Credit and Banking*, *4 (28)*, 733-762.

Rome, E., Langeslag, P., & Usov, A. (2014). “Federated Modeling and Simulation for Critical Infrastructure Protection”. In D’Agostino, G. & Scala, A. (Eds.), *Networks of Networks: The Last Frontier of Complexity* (225-253). Springer.

Saade, A. (2010). “Estructura de Red del Mercado Electrónico Colombiano (MEC) e Identificación de Agentes Sistémicos Según Criterios de Centralidad”. *Reporte de Estabilidad Financiera*, Banco de la República.

Sarmiento, M. & Galán, J.E. (2014). “Heterogeneous Effects of Risk-Taking on Bank Efficiency: A Stochastic Frontier Model with Random Coefficients”. *WP 14-20*, Statistics and Econometrics Series (13), Universidad Carlos III Madrid.

Schweitzer, F., Fagiolo, G., Sornette, D., Vega-Redondo, F., Vespignani, A., & White, D.R. (2009). “Economic Networks: The New Challenges”. *Science*, *325*, 422-425.

Simon, H.A. (1962). “The Architecture of Complexity”. *Proceedings of the American Philosophical Society*, *6 (106)*, 467-482.

Smith, D. (2011). "Hidden Debt: From Enron’s Commodity Prepays to Lehman’s Repo 105s". *Financial Analysts Journal*, *5 (67)*, 15-22.

Soramäki, K. & Cook, S. (2013). “Sinkrank: An Algorithm for Identifying Systemically Important Banks in Payment Systems”. *Economics: The Open-Access, Open-Assessment E-Journal*, *2013-28 (7)*. http://dx.doi.org/10.5018/economics-ejournal.ja.2013-28

Soramäki, K., Bech, M., Arnold, J., Glass, R., & Beyeler, W. (2007). “The Topology of Interbank Payments Flow”. *Physica A*, *379*, 317-333.

Straffin, P.D. (1980). “Algebra in Geography: Eigenvectors of Networks”. *Mathematics Magazine*, *5 (53)*, 269-276.

Strogatz, S. (2003). *SYNC: How Order Emerges from Chaos in the Universe, Nature and Daily Life*. Hyperion Books.

Stumpf, M.P.H. & Porter, M.A. (2012). “Critical Truths About Power Laws”. *Science*, *335*, 665-666.

Tabak, B.M., Souza, R.S., & Guerra, S.M. (2013). “Assessing Systemic Risk in the Brazilian Interbank Market”. *Working Paper Series*, *318*, Banco Central do Brasil.

Taleb, N.N. (2010). *The Black Swan*. Random House.

The World Bank (2011). *Payment Systems Worldwide*. Financial Infrastructure Series. The World Bank.

Tinkler, K.J. (1972). "The Physical Interpretation of Eigenfunctions of Dichotomous Matrices". *Transactions of the Institute of British Geographers*, *55*, 17-46.

Trichet, J-C. (2009). "Systemic Risk". *Text of the Clare Distinguished Lecture in Economics and Public Policy (University of Cambridge, Cambridge).*

Uribe, J.D. (2011a). "Descifrando los Sistemas Bancarios Paralelos: Nuevas Fuentes de Información y Metodologías para la Estabilidad Financiera". *Revista del Banco de la República*, Banco de la República, April, 1-9.

Uribe, J.D. (2011b). "Lecciones de la Crisis Financiera de 2008: Cómo la Infraestructura Financiera puede Mitigar la Fragilidad Sistémica". *Revista del Banco de la República*, Banco de la República, June, 1-9.

van den Heuvel, M.P. & Sporns, O. (2011). "Rich-Club Organization of the Human Connectome". *The Journal of Neuroscience, 31 (44)*, 15775-15786.

van der Leij, M., in 't Veld, D., & Hommes, C. (2013). "The Formation of a Core Periphery Structure in Financial Networks". *Working Paper Series*.

Vargas, H. & Varela, C. (2008). "Capital Flows and Financial Assets in Colombia: Recent Behavior, Consequences and Challenges for the Central Bank". *BIS Papers*, *44*, Bank for International Settlements (BIS), 153-184.

Wetherilt, A., Zimmerman, P., & Soramäki, K. (2010). "The Sterling Unsecured Loan Market During 2006-08: Insights From Network Theory". *Working Paper*, *398*, Bank of England.