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MAINTENANCE OPTIMIZATION OF A PRODUCTION SYSTEM WITH BUFFERCAPACITY

Stephan G. Vanneste

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Maintenance Optimization of a Production system with Buffercapacity

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Abstract

We consider the problem of planning preventive maintenance of a deteriorating machine which is part of a production system. The system is composed of several machines with intermediate buffers. The production is driven by a demand process (pull system). Not only the age of the machine but also the content of the subsequent buffer is important to the decision whether or not to start preventive maintenance. For this integrated maintenance-production problem, we analyse a class of control limit policies which are nearly optimal and easy to implement. The analysis is based on the embedding technique from Markov decision theory. In addition, we provide a characterization of the overall optimal policy and report on numerical comparisons with the best control limit policy.

1 Introduction

The reliability of production systems has become an important issue in todays industry. The success of process oriented manufacturing techniques, like JiT, and high levels of automation depends heavily on the reliability of the equipment. At the same time, a growing interest can be observed in the literature concerning the modelling of (un)reliability of production systems and the analysis of the impact of disruptions on the performance. However, most papers ignore the possibility of preventive maintenance, which can significantly reduce the occurrence of breakdowns and improve the performance of the system. On the other hand, papers dealing with preventive maintenance mostly do not take into account special characteristics of the production facility.

In this paper, we are interested in the preventive maintenance of a production system, which is composed of several machines with intermediate buffers (flow shop production line; see figure 1). To obtain insight into that problem, we analyse a subsystem, consisting of a machine, and the subsequent buffer. The machine produces parts, which can be temporarily stored in the buffer. The subsystem faces a certain demand (input for the next machine) which is satisfied from the buffer. Both the production and the demand rate are constant. We assume that the probability that the previous buffer empties or the subsequent machine fails is negligible. A relaxation of this assumption is discussed in section 4.

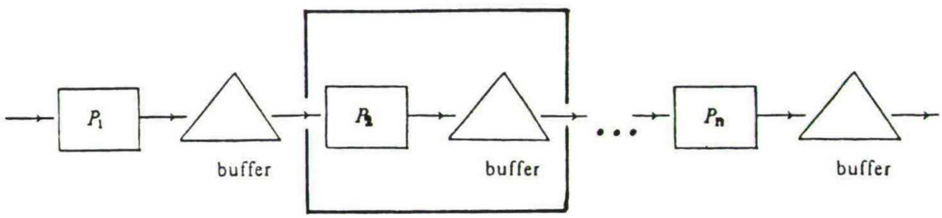

figure 1

The machine is subject to failures. Corrective maintenance is required to restore the condition of the failed machine. During the maintenance, the machine is inoperative, which may lead to a disruption in the production process in subsequent stations. To avoid frequent disruptions, preventive maintenance is allowed. Indeed, preventive maintenance also takes time, but it is assumed to be less time consuming than corrective maintenance. Another option could be to increase the buffercapacity, but this possibly accounts for high investment or inventory costs and may be undesirable from a management point of view. The purpose of this paper is to gain insight into the effect of preventive maintenance on the production process. The results can be used in a broader setting, both in the design phase as well as the operational phase of the production unit. In the design phase one has to balance the size of the buffer with the size of the maintenance crew. In the operational phase the issue is the optimal use of a given maintenance crew.

In our model, we keep the buffercapacity fixed and investigate the problem when to do preventive maintenance, as a function of both the age of the machine as well as the content of the buffer. A minor part of the paper deals with the optimal policy under the criterion of minimization of the average lost demand in the long-run. It turns out that the optimal policy may have a rather complicated form, which makes it difficult to implement. Moreover, the optimal policy may be hard to obtain numerically. Therefore we mainly focuss our attention on a class of policies with the property that they are (1) easy to characterize and to implement, (2) close to optimality, and (3) and allowing for a tractable analysis. A class of policies satisfying these properties, is the class of control limit policies characterized by three parameters n, N, k. For these policies, we do not only derive the average lost-demand, but also other performance characteristics relating to the production process. By varying the values n, N and k, and computing for each policy the desired performance measures, the decision maker can easily get insight in the performance and select the best policy according to his own preferences.

A detailed explanation of the model and the (n,N,k)-policy is given in the next section. In section 3 we analyse the (sub)system under a fixed (n,N,k)-rule and derive formulas for several performance characteristics. This is done by studying an appropriately embedded Markov chain. In section 4, we consider the special case of a fast production process in which the time needed to fill the buffer is negligible. Next, in section 5, we analyse the optimal policy and report on some results of our numerical investigations on the comparison with the best (n,N,k)-policy. Some conclusions and extensions are given in the final section.

Several papers deal with the reliability of a flow shop production line. A basic contribution is the paper of Wijngaard [9], who considers two machines with exponential up-and-down times and an intermediate buffer. Explicit formulas for various performance measures are obtained by solving a set of differential equations. A similar problem was considered more recently by Posner and Berg [4]. In a somewhat different setting, they applied level-crossing analysis to obtain several quantities of interest. The reliability aspect of production systems has also been studied in relation to lotsizing and batching decisions, see Groenevelt et al. [3] and references therein. Finally, we mention the paper of Doshi et al. [1], who consider a production-inventory system, where the machine can operate in a fast and a slow mode, depending on the inventory level.

2 Model and Strategy

Our production system consists of a machine with a subsequent buffer. The machine produces parts in order to satisfy a certain demand, generated by subsequent machines on the line. Excess production can be stored in the buffer, so that the demand can also be satisfied in case the production is temporarily stopped. The systems has the following characteristics:

(1) The demand has a fixed, constant rate, denoted by d (units/time)

(2) The buffer has finite capacity K (>0)

(3) As long as the buffercapacity is not reached, the machine operates at a constant rate of p units/time ($p>d$), and the excess production is stored in the buffer. When the buffer is full, the production slows down to d.

(4) The machine is subject to failures. The time until failure is a stochastic variable with known probability distribution function, which belong to the IFR class. Upon failure, corrective maintenance starts, bringing the condition of the machine back to (as good as) new. During the maintenance, which takes a stochastic amount of time, the machine is inoperative.

(5) Partial backlogging is allowed. With κ we denote the (nonpositive) bufferlevel, below which excess demand is lost.

To prevent frequent failures, or corrective maintenance (CM), preventive maintenance (PM) is allowed. It is assumed that PM lasts shorter than CM in a stochastic sense. Both types of maintenance incur a certain risk of disruption, either directly when the repair takes too much time, or indirectly because a reduced inventory level increases the risk that the next failure causes a shortage (this aspect is particularly of interest for unreliable production units). Both PM and CM are nonpreemptive, i.e. they cannot be interrupted, and after completion the machine starts with age equal to zero (the machine is as good as new).

The system is monitored at discrete, equidistant time epochs (say every hour). These epochs provide the only opportunities to start maintenance or resume production. During operation both the age and the buffercontent are monitored. The production and demand rate are assumed to be integers, so that the inventory position is also integer valued.

The lifetime distribution is assumed to be IFR and to have finite support.

Define:

L:= the lifetime of the machine, expressed in time units.

Let

$\mu := \mathrm{E}L,\ q_i := \mathrm{P}(L=i \mid L \geq i),\ i=0,..,m$

where

$m := \min\ \{j\colon \mathrm{P}(L \geq j+1)=0\}$, and $r_i := 1-q_i,\ i=0,..,m$

Assumption 1. $0=r_m<r_{m-1}\leq \ldots \leq r_0<1$ □

We say that at a certain moment the state of the machine is i if the machine has not failed and its age (i.e. the number of elapsed time units since the last renewal) equals i, $0\leq i\leq m$. When at an epoch the unit turns out to have failed during the last period, we say the state is CM. We assume that the bufferposition increases by p-d, during a period in which the machine fails (i.e. the machine fails at the end of the period).

Next, define the generic variables:

A:= the preventive repairtime, expressed in time units, and

B:= the corrective repairtime, expressed in time units.

Let

$\alpha = \mathrm{E}A$ and $a_i := \mathrm{P}(A=\mathrm{i}),\ i\geq 1,$

$\beta = \mathrm{E}B$ and $b_i := \mathrm{P}(B=\mathrm{i}),\ i\geq 1.$

We assume that the time needed for PM is stochastically smaller than for CM:

Assumption 2. $\Sigma_{i\geq j}\, a_i \leq \Sigma_{i\geq j}\, b_i$, $\mathrm{j}=1,2,\ldots$

Note that assumption 2 implies that the mean repairtime for PM is smaller than for CM ($\alpha\leq\beta$). We introduce the state PM to denote the situation that PM is performed on the machine.

A policy prescribes an action for each possible state of the system. There are three possible actions: 0 (do nothing), 1 (perform PM) and 2 (perform CM). We denote a policy by R. The behaviour of the system under a fixed policy R can be described by the stochastic process $\{X^R(t) = (X_1^R(t), X_2^R(t)), t = 0,1,2,..,\}$ on the state space

$$S = \{0,1,..,m\} \cup \{\text{PM}\} \cup \{\text{CM}\} \times \{\kappa,..,K\}$$

where $X_1^R(t)$ denotes the state of the machine at time t and $X_2^R(t)$ the content of the buffer. It is easily seen that $\{X^R(t), t \geq 0\}$ is a semi-Markov chain on S. Note further that the process is regenerative with regeneration state $\{0,\kappa\}$, irrespective of the type of policy used, provided that the following assumption holds:

Assumption 3. $j^* > (p-d)/d$, where $j^* := \sup\{j: b_j > 0\}$

To see this, note that the machine can fail during its first period of operation ($r_0 < 1$, see assumption 1). This results in an increase of the buffer by p-d. Suppose the subsequent corrective repair takes j periods, then the buffer decreases with jd. Due to assumption 3, there is a positive probability that a corrective repair results in a net decrease of the buffer. This means that after finitely many repetitions the bufferlevel κ will be reached with positive probability.

An (n,N,k)-policy prescribes to do preventive maintenance if and only if the buffercontent exceeds k and at the same time the age exceeds N, or, in case the buffer is full, if the age exceeds n, where $0 \leq n \leq N \leq m+1$, $\kappa \leq k \leq K$. Corrective maintenance is started upon failure. More precisely:

Definition 2.1 *An (n,N,k)-policy prescribes to do PM if and only if the age i and the buffercontent x satisfy $i \geq N$ and $k \leq x < K$, or $i \geq n$ and $x = K$.*

The near-optimality of this type of policies is discussed in section 5. The treshold values k for the buffercontent and N for the age are intuïtively appealing (when the age or buffercontent is too low, PM is not useful) although the straight lines and rectangles characterizing this policy (see figure 2) do not really occur in the optimal policy. In figure 2 below we depict the regions where action 0 (R_0) and 1 (R_1) are prescribed.

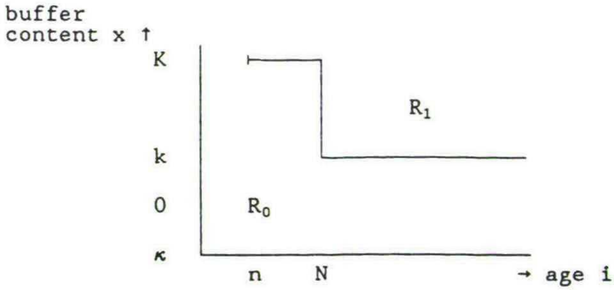

figure 2

The exceptional rule in the case of full buffer needs some explanation. First note that the difference between halting and continuing the production in case the buffer is full is less than when the buffercapacity has not been reached. Furthermore, performing PM at an early stage when the buffercapacity is reached, creates the possibility to remain in a situation of (practically) full buffer and low age, a comfortable and safe situation.

We conclude this section with a definition of the long-term average cost. For each state $s=(i,x)\in S$ and action $a\in\{0,1,2\}$ we introduce costs $c(s,a)$, which are incurred when in state s action a is chosen. For a fixed policy R, let:

$C^R(t)$: = the expected costs incurred in $[0,t]$ under policy R.

Then the long-term expected average cost under policy R, $g(R)$, is defined by

$$g(R):=\lim_{t\to\infty}\frac{C^R(t)}{t} \tag{2.1}$$

By an appropriate choice of the cost function, the average cost may represent various performance measures. An important performance measure of our system is the long-term average demand lost per unit time. This quantity is closely related to the amount of disruptions in the flow shop production line. Note that the fraction of demand lost follows directly from the average demand lost per unit time, since the demand rate is constant.

3 Analysis of (n,N,k)-policy

In this section we present an efficient method to compute several performance measures for a fixed (n,N,k)-policy, viz. the long-term average demand lost per unit time, the average amount of backorder, the average buffer content and the proportion of time associated with maintenance. This method is based on the embedding technique, which is discussed in Tijms [6]. For another succesful application of this method in the field of maintenance, we refer to van der Duyn Schouten and Vanneste [2].

We choose the following embedding set:

$$E = \{(0, x),\ \kappa \le x \le K\}$$

(We will also use the shorthand notation x for $(0, x)$ in the remainder)

Suppose the process $\{X(t),\ t \ge 0\}$ starts in (0,0) and define $T_0 \equiv 0$ and

$T_i :=$ the epoch of the i^{th} entrance into the set E, $i \ge 1$.

Then,

$Z_i := X(T_i),\ i \ge 1$

is an (embedded) Markov chain on E.

Following Tijms [6], we define for $\kappa \le x \le K$:

$c(x) :=$ the expected costs incurred until the first subsequent entry in the set E, starting in x,

$\tau(x) :=$ the expected time until the next entry in E, starting in x, and

$p(x,y) :=$ the probability that the next entry in E will be at state y, given the present state is x.

Then we have for the long-run expected average costs (see e.g. Tijms et al.[7]):

$$g(n,N,k) = \frac{\sum_{x \in E} c(x)\ \pi(x)}{\sum_{x \in E} \tau(x)\ \pi(x)} \tag{3.1}$$

where $\pi(\bullet)$ denotes the stationary distribution of the embedded Markov chain $(Z_i)_{i=0}^{\infty}$ and $g(n,N,k)$ the average expected costs under the policy (n,N,k).

Once the transition probabilities $p(x,y)$ for $x,y \in E$ are known, the stationary probabilities $\pi(x)$, $x \in E$ can be easily found, using standard procedures. It remains to derive expressions for $c(x)$, $\tau(x)$ and $p(x,y)$. We distinguish four cases: $\kappa \le x \le k\text{-}N$, $k\text{-}N < x \le K\text{-}N$, $K\text{-}N < x \le K\text{-}n$ and $K\text{-}N < x \le K$. Here, we restrict ourselves to the case $\kappa \le x \le k\text{-}N$. The other cases can be treated similarly. To simplify the analysis, we make the assumption (which is easy to relax) that $p\text{-}d=1$. For notational reasons we introduce the function $a:\mathbf{R}^+\to[0,1]$, defined as:

$$a(z):=\begin{cases} a_z & \text{if } z\in\mathbf{N}, \\ 0 & \text{otherwise} \end{cases} \tag{3.2}$$

Now choose x, such that $\kappa\le x\le k\text{-}N$. Then,

$$\tau(x)=\sum_{l=1}^{k-x-1} P(L=l)(l+\beta) + P(L\ge k-x)(k-x+\alpha)$$

From x, the only states that can be reached in one step by $(Z_t)_{t=0}^{\infty}$ are states y, $\kappa\le y\le k$. For $y\ne\kappa$ we find:

$$\begin{aligned} p(x,y) &= \sum_{l=1}^{k-x-1} P(L=l)P\left(CM \text{ takes } \frac{x+l-y}{d} \text{ time units, provided that } \frac{x+l-y}{d}\in\mathbf{N}\right) \\ &\quad + P(L\ge k-x)P\left(PM \text{ takes } \frac{k-y}{d} \text{ periods, provided } \frac{k-y}{d}\in\mathbf{N}\right) \\ &= \sum_{k-x>jd+y-x\ge 0} P(L=jd+(y-x))b_j + P(L\ge k-x)a\left(\frac{k-y}{d}\right) \end{aligned}$$

and,

$$p(x,\kappa)=\sum_{l=1}^{k-x-1} P(L=l)P\left(B\ge\frac{x+l-k}{d}\right) + P(L\ge k-x)P\left(A\ge\frac{k-\kappa}{d}\right)$$

Different performance measures can be found by specifying the appropriate cost function $c(x)$ and using (3.1). First we turn to the criterion of lost demand. Denote by

$c_1(x):=$ the expected lost demand, starting at $x\in E$, until the next entry in E.

The average demand lost is now given by $g(n,N,k)$ from (3.1). We introduce the auxiliary function:

$\gamma_A(y):=$ the expected lost sales during a preventive repair, when the repair starts at bufferposition y.

Note that,

$$\gamma_A(y)=\sum_{j>(y-\kappa)/d}^{\infty} a_j(\kappa-(y-jd)),\ \kappa\le y\le K \tag{3.3}$$

Similarly, we define $\gamma_B(y)$ in the case of corrective repair, which yields (3.3) with a_j replaced with b_j. Now we have, for $\kappa \leq x \leq k\text{-}N$,

$$c_1(x) = \sum_{l=1}^{k-x-1} P(L=l)\,\gamma_B(x+l) \; + \; P(L \geq k-x)\,\gamma_A(k)$$

Next, we turn to the average amount of backorders. Define:

$c_2(x)$: = the expected amount of backorders until the first subsequent entry in E, starting at $x \in E$.

Here we count the backorders at the moment of delivery. The average backlog can be found, using (3.1) with $c(x) = c_2(x)$. Clearly, $c_2(x) = 0$ for $x \geq 0$. If $x < 0$ we distinguish two cases. For $|x| < N$ we have:

$$c_2(x) = \sum_{l=1}^{|x|-1} P(L=l)\,l \; + \; P(L \geq |x|)\,|x|$$

and a similar formula applies to the case $|x| \geq N$.

The third quantity that we consider is the average buffercontent. For that purpose we define:

$c_3(x)$: = the expected cumulative buffercontent until the next entrance in E, starting at $x \in E$,

and the auxiliary functions:

$\delta_1(x,j)$: = accumulated buffercontent during j periods of production, when starting at bufferposition x, and

$\delta_0(x,j)$: = accumulated buffercontent during j periods of downtime (due to PM or CM) when starting at bufferposition x.

We have,

$$\delta_1(x,j) = \sum_{i=0}^{jmin} (x+i)^+ \; + \; (j - jmin)K, \quad \text{where } jmin := \min(j, K-x)$$

and,

$$\delta_0(x,j) = \sum_{i=0}^{jmin} (x - id), \quad \text{where } jmin := \min(j, [x/d])$$

(with $x^+ = \max(x,0)$). The summations can be simplified, using $\Sigma_{i=1}^{j}\, i = \tfrac{1}{2}\, j(j+1)$.

Choose again x, with $\kappa \le x \le k\text{-}N$. Then we obtain for $c_3(x)$:

$$c_3(x)=\sum_{l=1}^{k-x-1} P(L=l)\left\{\delta_1(x,l) + \sum_{j\geq 1} b_j \delta_0(x+l,j)\right\} + P(L\geq k-x)\left\{\delta_1(x,k-x) + \sum_{j\geq 1} a_j \delta_0(k,j)\right\}$$

As fourth and last performance characteristic we consider the proportion of the time during which maintenance is performed, or equivalently, the unavailability of the machine. For $\kappa \le x \le k\text{-}N$, this quantity follows from (3.1), using

$$c_4(x)=P(L<k-x)\,\beta + P(L\geq k-x)\,\alpha$$

4 High production rate

Suppose that the time needed to reach a full buffer during a production cycle is negligible (or small) compared to the mean lifetime of the machine. Then, it is very likely for maintenance activities (PM as well as CM) to start when the buffer is full. Therefore, the system is adequately described in terms of states of the machine (age), whereas the buffercontent is not relevant anymore. A single unit age-replacement model with non-zero repairtimes, which is analyzed in [8], can be used to describe the system in this case.

The single-state age-replacement model needs as input the (discretized) lifetime distribution, the repair time distributions and the costs associated with PM and CM. Like in the previous section, an appropriate specification of the cost parameters yields expressions for the desired performance measures. E.g., for the case of lost demand we take $\gamma_A(K)$ (formula (3.3)) for the costs of PM and $\gamma_B(K)$ for the costs of CM. In [8] it is shown that the optimal replacement policy is of the control limit type (i.e. there exists a treshold level, above which PM is done, cf. also section 5) and the average costs are a unimodal function of the control limit (without restrictions on the cost parameters). In [8] a simple and efficient algorithm is presented by which the optimal policy and the associated average costs can be determined. Apart from using the age-replacement model as an approximation for the production model in the case of fast production, it can also be used to provide a good starting policy in the search for the best values of n, N, k. We could e.g. initialize the search with (T,T,κ) with T the optimal control limit of the age-replacement model.

To a certain extent, the age-replacement model in [8] can be regarded as a special case of the model described in this paper, but it is more general with respect to the assumptions concerning the deterioration of the machine and the maintenance operations. Maintenance actions are allowed to be imperfect and are possibly restricted to opportunities. The latter assumption refers to the fact that preventive maintenance can only start at opportunities, which are generated according to a Poisson process, independently of the unit to be maintained. We mention two examples of opportunity processes, which are related to our production system. Both types of opportunities can be analyzed through the application of the age-replacement model, with the appropriate specification of the cost parameters.

Recall the description of the flow shop production line from the introduction (see figure 1). When the preceding buffer in the line gets empty, our machine cannot produce anymore, and while it is inoperative, one might as well perform preventive maintenance on it. Thus, the event that the preceding buffer gets empty provides an opportunity to perform PM on the machine. Another example of an opportunity is the failure of the subsequent machine in the line. The only reason to continue production in this case could be to fill the buffer. However, in this section we may assume that the buffer is already full, in view of the fast production. Note that both type of opportunities are associated with the failure of another machine in the line. Opportunities may be used to perform preventive maintenance on the machine, assuming enough repaircapacity is available to repair machines simultaneously.

5 Comparison with optimal policy

In this section we provide a characterization of the general form of the optimal policy for a particular optimization criterion, viz. the minimization of the average demand lost (or equivalently the fraction of demand lost). In addition we report on our numerical investigations concerning the comparison of the best (n,N,k)-policy with the overall optimal policy.

The optimal policy can be found by formulating the problem as a semi-Markov decision process and solving the resulting optimality-equations by either value-iteration or policy-iteration techniques. By introducing an additional assumption concerning the repair time distributions the model simplifies to a discrete time Markov decision process (MDP).

Assumption 4 The repairtime distributions for PM and CM are geometrically distributed with probability of success a and b respectively, i.e.: $a_i=(1-a)^{i-1}a$ and $b_i=(1-b)^{i-1}b$, $i\geq 1$, with $0<b<a\leq 1$.

The difference between the Markov model and the semi-Markov model is that a visit to state *PM* or *CM* is modeled differently. In the semi-Markov model the state *PM* (*CM*) is visited once, for a certain amount of time, whereas in the Markov case we model this as a number of subsequent visits of one-period. Being in state *PM* (*CM*) we move out with probability a (b) and return with the complementary probability, no matter how long we have been in state *PM* (*CM*) (here we use the memoryless property of the geometric distribution).

The state and action space are as before. Again we assume for notational convenience that p-$d=1$. Furthermore, we assume, without loss of generality, that $\kappa\equiv 0$ (we can renumber the states κ to K from 0 to K-κ, and obtain an equivalent system with the same limiting behaviour in terms of lost demand). Recall that (0,0) is a positive recurrent state under every stationary policy R. This implies that there is only one recurrent class under every policy R. Let $C(R)$ denote this recurrent class and $D(R)$ be the set of transient states under policy R. Note that state $(i,x)\in D(R)$ iff it cannot be reached from (0,0). Therefore, all states (i,x) with $i>x$ are transient under every policy. The process $\{X^R(t),\ t\geq 0\}$ is an aperiodic Markov chain on S. The aperiodicity of the process follows from the aperiodicity of the recurrent state (0,0), which is a consequence of assumption 1 ($r_0<1$) and 4 ($P(B=i)>0$, for all $i>0$).

The average cost optimality equations in terms of the relative values $v(s)$, $s\in S$ and the minimal long term average cost g are easily established (cf. Tijms [6]):

$$v(i,x)=\min\{-g+r_i v(i+1,x+1)+(1-r_i)v(CM,x+1),\ v(PM,x)\},\quad 0\leq i\leq m,\ 0\leq x\leq K$$

$$v(i,K)=\min\{-g+r_i v(i+1,K)+(1-r_i)v(CM,K),\ v(PM,K)\},\quad 0\leq i\leq m$$

$$v(CM,x)=(d-x)^+-g+b\,v(0,(x-d)^+)+(1-b)v(CM,(x-d)^+),\quad 0\leq x\leq K$$

$$v(PM,x)=(d-x)^+-g+a\,v(0,(x-d)^+)+(1-a)v(PM,(x-d)^+),\quad 0\leq x\leq K$$

(note that $r_m=0$)

The first term in braces corresponds to $a=0$, and the second term to $a=1$. When action 1 is chosen, there is an immediate transition from i to PM. The minimizing argument in each state yields an optimal policy, which we denote by R^*. Note that $(d\text{-}x)^+=\max(d\text{-}x,0)$ represents the one-period demand lost during maintenance, when the buffercontent equals x at the beginning of the period.

Theorem 1 *The relative values $v(i,x)$, $v(PM,x)$ and $v(CM,x)$, $0\le i\le m$, $0\le x\le K$, satisfy the following relations:*

(i) $v(i,x) \le v(i+1,x)$ *and* $v(m,x)\le v(CM,x)$ $,0\le i\le m\text{-}1;\ 0\le x\le K$

(ii) $v(i,x) \ge v(i,x+1)$, $0\le i\le m;\ 0\le x\le K\text{-}1$

(iii) $v(PM,x) \ge v(PM,x+1)$, $0\le x\le K\text{-}1$

(iv) $v(CM,x) \ge v(CM,x+1)$, $0\le x\le K\text{-}1$

(v) $v(PM,x) \le v(CM,x)$, $0\le x\le K$

The proof is given in the appendix. As a consequence, we obtain the following:

Corollary 1 *The average optimal maintenance policy R^* has the following property: For fixed bufferlevel x, $0\le x\le K$, the optimal action as a function of the age is a control limit rule. That is, for each bufferlevel x there exists a treshold level $i^*(=i^*(x))$ such that $R^*(i,x)=0$ (no PM) whenever $i<i^*$ and $R^*(i,x)=1$ (do PM) for $i^*\le i\le m$.*

It would, of course, be interesting to have in addition a characterization of the borderline $i^*(x)$, the control limit as a function of the bufferposition x, but we were not able to prove any analytical results related to this. Numerical investigations suggest the following general form of the optimal policy:

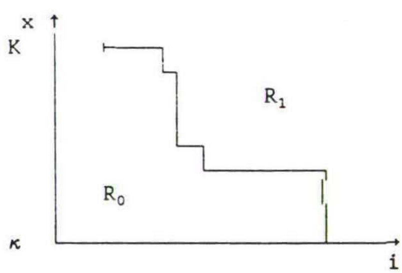

figure 3

When the bufferlevel increases the critical age for PM decreases, according to the picture, which is intuitively appealing. However, we did find a numerical example with $i^*(3)=5$ and $i^*(2)=4$. Although this happened on the transient region (which means that we can change the action at state (0,4) into 0 without affecting the average costs) it is still obtained as a (numerical) solution of the average cost equations.

The optimal policy of the MDP and the associated average costs were obtained using the value-iteration algorithm. For 91 numerical examples we computed the best (n,N,k)-policy as well as the associated average cost. In comparing this average costs with the average cost of the overall optimal policy, we found that the class of (n,N,k)-policies performed very well: in all cases the relative difference in average lost demand was below 1%, and in the majority of the cases (82) even below 0.1%. It appeared that in most cases the parameter k was only of minor importance (with respect to the criterion of lost demand): setting k equal to κ and then searching for the best values of n and N yielded only slight differences in the average cost.

In the table below we present five illustrative examples of our numerical investigations. We obtained results for several values of the parameters m, κ, K, d, a and b. The lifetimedistribution was obtained by discretizing Weibull$(1,\alpha)$-distributions into $m+1$ discrete intervals, for $\alpha=1.4$, 2, 2.5. The minimal average cost g^* is given and compared to the average cost of the best (n,N,k)-policy. The %-column gives the relative differences. The three parameters of the best (n,N,k)-policy are also given. In addition we give the average cost associated with the case of doing no preventive maintenance at all. For the fifth example, we computed the value of the optimal control limit T as it was obtained from the corresponding single-state approximation (section 4).

Table 1

m	κ	K	d	a	b	g	%	no PM	n,N,k
22	-3	10	4	1.0	0.4	0.096	0.8	0.262	2,6,-3
22	-3	20	4	1.0	0.4	0.070	0.2	0.184	2,6,-3
14	-2	4	1	0.5	0.2	0.127	0.0	0.239	1,3, 2
14	-2	8	2	1.0	0.33	0.192	0.0	0.282	1,3,-2
22	0	3	1	0.8	0.6	0.004	0.0	0.010	3,16,2 *)

*) $T=3$

Concluding remarks

We have seen that (n,N,k)-policies allow for a tractable analysis of a particular production problem, yielding a near-optimal policy which is easy to compute and to implement. The results were established, using Markov (decision) theory, a nice and flexible analytical tool to obtain insight into complex maintenance problems.

The area of integrated maintenance and production seems to be an interesting and challenging area of research. There are many variations and extensions to the problem considered here that require further research. In particular the interaction with other machines in the line should be taken into account. Thus far we considered our subsystem in isolation, and assumed that the probability that the preceding buffer gets empty or the subsequent machine fails is negligible (see the introduction). This assumption was made to simplify the analysis and only for the special case of fast production we considered a relaxation of this assumption. Note that in the general model we have to keep track of the decrease of the bufferposition when the production is stopped, for we have to know at which position the machine starts producing again.

Another aspect that has not yet been mentioned is that many modern automated production systems produce items of different quality, and deteriorate in the sense that the quality of the items produced decreases, rather than that the failure rate increases. We are currently studying this generalization.

Appendix Proof of Theorem 1

Theorem 1 follows from the corresponding results for the finite horizon problem with discounting by taking appropriate limits (Ross [5, Theorem 2.2 and Corollary 2.5]).

Let α $(\alpha<1)$ be the discount factor and define the function $V_t^{\alpha}(\bullet,\bullet)$ on S for $t\in\mathbb{N}$ recursively by:

$$V_0^\alpha(i,x)=0,\ 0\le i\le m,\ 0\le x\le K$$
$$V_0^\alpha(PM,x)=0,\ 0\le x\le K$$
$$V_0^\alpha(CM,x)=0,\ 0\le x\le K$$

and, for $t\ge 0$,

$$V_{t+1}^\alpha(i,x)=\min\{\alpha r_i V_t^\alpha(i+1,x+1)\ +\ \alpha(1-r_i)V_t^\alpha(CM,x+1),\ V_{t+1}^\alpha(PM,x)\},\quad 0\le i\le m,\ 0\le x\le K-1$$

$$V_{t+1}^\alpha(i,K)=\min\{\alpha r_i V_t^\alpha(i+1,K)\ +\ \alpha(1-r_i)V_t^\alpha(CM,K),\ V_{t+1}^\alpha(PM,K)\},\quad 0\le i\le m$$

$$V_{t+1}^\alpha(PM,x)=(d-x)^+\ +\ \alpha a V_t^\alpha(0,(x-d)^+)\ +\ \alpha(1-a)V_t^\alpha(PM,(x-d)^+),\ 0\le x\le K$$

$$V_{t+1}^\alpha(CM,x)=(d-x)^+\ +\ \alpha b V_t^\alpha(0,(x-d)^+)\ +\ \alpha(1-b)V_t^\alpha(CM,(x-d)^+),\ 0\le x\le K$$

(note that $r_m=0$)

The following theorem is the finite-horizon, discounted version of theorem 1:

Theorem 2 *For each $t\in\mathbb{N}$ we have that:*

- *(i)* $V_t^\alpha(i,x)\ \le\ V_t^\alpha(i+1,x)$ *and* $V_t^\alpha(m,x)\ \le\ V_t^\alpha(CM,x),\ 0\le i\le m\text{-}1;\ 0\le x\le K$
- *(ii)* $V_t^\alpha(i,x)\ \ge\ V_t^\alpha(i,x+1),\ 0\le i\le m;\ 0\le x\le K\text{-}1$
- *(iii)* $V_t^\alpha(PM,x)\ \ge\ V_t^\alpha(PM,x+1),\ 0\le x\le K\text{-}1$
- *(iv)* $V_t^\alpha(CM,x)\ \ge\ V_t^\alpha(CM,x+1),\ 0\le x\le K\text{-}1$
- *(v)* $V_t^\alpha(PM,x)\ \le\ V_t^\alpha(CM,x),\ 0\le x\le K$

Proof. By definition, the theorem holds for $t=0$. Suppose the theorem holds for t (≥ 0), then we have to show that it also holds for $t+1$. We will delete the discount factor α in the notation during the proof of the induction step. First we prove (iv) and (iii), then (v), then (ii) (using (iii)), and finally (i) (using (v)), all for $t+1$.

Part (iv). Note that, for $y\in\mathbf{R}$,

$$y^+=\max(y,0)\ge\max(y-1,0)=(y-1)^+ \tag{A.1}$$

Now, for $0\le x\le K\text{-}1$, we have

$$V_{t+1}(CM,x) = (d-x)^{+} + \alpha b V_t(0,(x-d)^{+}) + \alpha(1-b)V_t(CM,(x-d)^{+})$$

$$V_{t+1}(CM,x+1) = (d-(x+1))^{+} + \alpha b V_t(0,(x+1-d)^{+}) + \alpha(1-b)V_t(CM,(x+1-d)^{+})$$

Using (A.1) and inductionhypothesis, part (ii) and (iv), we conclude that $V_{t+1}(CM,x) \geq V_{t+1}(CM,x+1)$.

Part (iii) follows similarly.

Part (v). For $0 \leq x \leq K$,

$$\begin{aligned} V_{t+1}(PM,x) &= (d-x)^{+} + \alpha a V_t(0,(x-d)^{+}) + \alpha(1-a)V_t(PM,(x-d)^{+}) \\ &\leq (d-x)^{+} + \alpha a V_t(0,(x-d)^{+}) + \alpha(1-a)V_t(CM,(x-d)^{+}) \\ &= (d-x)^{+} + \alpha V_t(CM,(x-d)^{+}) - \alpha a\big(V_t(CM,(x-d)^{+}) - V_t(0,(x-d)^{+})\big) \\ &\leq (d-x)^{+} + \alpha V_t(CM,(x-d)^{+}) - \alpha b\big(V_t(CM,(x-d)^{+}) - V_t(0,(x-d)^{+})\big) \\ &= V_{t+1}(CM,x) \end{aligned}$$

The first inequality follows from inductionhypothesis, part (v). The second inequality follows from the assumption $a < b$, and

$$V_t(CM,(x-d)^{+}) - V_t(0,(x-d)^{+}) \geq 0$$

which is a consequence of part (i) of the inductionhypothesis.

Part (ii). Using inductionhypothesis, part (ii) and (iv), and part (iii) above, we obtain:

$$\begin{aligned} V_{t+1}(i,x) &= \min\{\alpha r_i V_t(i+1,x+1) + \alpha(1-r_i)V_t(CM,x+1),\ V_{t+1}(PM,x)\} \\ &\geq \min\{\alpha r_i V_t(i+1,x+2) + \alpha(1-r_i)V_t(CM,x+2),\ V_{t+1}(PM,x+1)\} \\ &= V_{t+1}(i,x+1),\quad 0 \leq i \leq m,\ 0 \leq x \leq K-1 \end{aligned}$$

Part (i). We have to show that

$$\begin{aligned} V_{t+1}(i,x) &\leq V_{t+1}(i+1,x),\ 0 \leq x \leq K,\ 0 \leq i \leq m-1, \text{ and} \\ V_{t+1}(m,x) &\leq V_{t+1}(CM,x),\ 0 \leq x \leq K \end{aligned} \tag{A.2}$$

The second part of (A.2) is easily established for $0 \leq x \leq K$-1, using part (v) above:

$$V_{t+1}(m,x) = \min\{\alpha V_t(CM,x+1), V_{t+1}(PM,x)\} \leq V_{t+1}(PM,x) \leq V_{t+1}(CM,x)$$

Similarly, we obtain the inequality for $x=K$.

Next, the case $0 \leq i \leq m-1$. We will show (A.2) for $0 \leq x \leq K-1$; the case $x=K$ is similar.

For $0 \leq x \leq K-1$ and $0 \leq i \leq m-1$, we have:

$$\begin{aligned} V_{t+1}(i,x) &= \min\{\alpha r_i V_t(i+1,x+1) + \alpha(1-r_i) V_t(CM,x+1),\ V_{t+1}(PM,x)\} \\ &= \min\{\alpha V_t(CM,x+1) - \alpha r_i (V_t(CM,x+1) - V_t(i+1,x+1)),\ V_{t+1}(PM,x)\} \\ &\leq \min\{\alpha V_t(CM,x+1) - \alpha r_{i+1} (V_t(CM,x+1) - V_t(i+2,x+1)),\ V_{t+1}(PM,x)\} \\ &= V_{t+1}(i+1,x) \end{aligned}$$

The inequality follows from assumption 1 ($r_i \geq r_{i+1}$) and the inequality

$$V_t(CM,x+1) - V_t(i+1,x+1) \geq V_t(CM,x+1) - V_t(i+2,x+1) \ (\geq 0)$$

which is implied by induction hypothesis (i).

This concludes the proof.

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