Tilburg University

Model uncertainty in financial markets

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Publication date:
2014

Document Version
Publisher's PDF, also known as Version of record

Citation for published version (APA):
de Roode, F. A. (2014). *Model uncertainty in financial markets: Long run risk and parameter uncertainty*. CentER, Center for Economic Research.

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Model Uncertainty in Financial Markets: Long Run Risk and Parameter Uncertainty

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PROEFSCHRIFT

ter verkrijging van de graad van doctor aan Tilburg University op gezag van de rector magnificus, prof.dr. Ph. Eijlander, in het openbaar te verdedigen ten overstaan van een door het college voor promoties aangewezen commissie in de aula van de Universiteit op woensdag 8 oktober 2014 om 10.15 uur door

FLORIS ALEXANDER DE ROODE

geboren op 17 april 1987 te Roosendaal en Nispen.

PROMOTIECOMMISSIE:

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SAMENVATTING

Onzekerheid omtrent belangrijke parameters van de financiële markt, zoals de risicopremies voor inflatie en aandelen, brengt grote risico's met zich mee voor beleggers met een langetermijnhorizon.

Door de afwezigheid van lokale inflatie derivaten is het een uitdaging om de inflatie van beleggers adequaat af te dekken. Ik laat zien dat investeerders die hun inflatierisico willen afdekken profijt kunnen hebben van het houden van obligaties die gerelateerd zijn aan buitenlandse inflatie. Verder kunnen investeerders de afdekprestatie van hun portfolio verbeteren met behulp van langetermijninteracties tussen hun inflatie maatstaf en buitenlandse inflatie.

Voor de belangrijkste reële obligatie markten toon ik aan dat de inflatierisicopremie in de Engelse obligatie markt steeg gedurende de financiële crisis, terwijl deze in de Amerikaanse markt daalde. Gezien de grote parameteronzekerheid van de inflatierisicopremie, die is toegenomen gedurende de financiële crisis, presenteer ik een model waarmee de investeerder dit kan kwantificeren en meenemen in hun langetermijnbeslissingen.

Tot slot demonstreer ik dat de moeilijkheid van het schatten van de aandelenrisicopremie de belangrijkste bron van parameteronzekerheid is voor defined contribution pensioencontracten. Ik introduceer een manier om parameteronzekerheid te implementeren in de contributies, zodat deelnemers het risico van de vervangingsratio ten tijde van hun pensionering kunnen verbinden aan hun contributies.

Samenvattend demonstreert deze dissertatie robuuste methodes voor beleggers om parameter onzekerheid te implementeren in risicomodellen en biedt een nieuw inzicht op het effect van parameter onzekerheid in financiële modellen.

SUMMARY

Uncertainty surrounding key parameters of financial markets, such as the inflation and equity risk premium, constitute a major risk for institutional investors with long investment horizons.

Hedging the investors' inflation exposure can be challenging due to the lack of domestic inflation-linked securities. I show that inflation hedging investors can benefit from holding bonds that are linked to inflation in foreign countries. Investors can further improve their inflation hedge by incorporating the long term interactions between his own inflation exposure and the foreign inflation measures.

Focusing on the major inflation-linked security markets, I find an increase of the inflation risk premium over the financial crisis in the UK, whereas in the US it decreased. Since the parameter uncertainty of these estimates is large, and increased over the financial crisis in both the UK and US markets, I present a framework in which investors can quantify and integrate it in their long term investment decisions.

Finally, I demonstrate that the difficulty of estimating the equity risk premium is the largest source of parameter uncertainty in defined contribution pension contracts. I introduce a methodology to take parameter uncertainty into account, so that participants can set contributions that reflect the uncertainty about their replacement rate at retirement.

Overall, this thesis demonstrates robust methods to incorporate the effects of parameter uncertainty and contributes to the literature on how parameter uncertainty of financial models can substantially affect the investors' investment risk.

Acknowledgements

I am grateful to Netspar at Tilburg University for receiving their PhD grant to pursue my research interest. I am indebted to my advisors Ronald Mahieu and Theo Nijman for their help during the writing of this dissertation.

I appreciate the valuable comments and suggestions from Casper van Ewijk, Bertrand Melenberg, Peter Schotman, Alfred Slager and Marno Verbeek as members of the PhD committee. This dissertation has benefited from their comments.

Finally, I am obliged to Joseph McCahery for his comments and insights.

Alexander de Roode
Tilburg, July 2014

Contents

CHAPTER 1

INTRODUCTION

Uncertainty about the future development of financial markets requires investors to analyze their long term investment risk. To that end, investors rely on financial models to evaluate the investments with their liabilities. Since financial parameters, such as the risk premium of bond, equity and inflation are unknown, investors need to form a belief on these parameters. To establish an ex-ante view, estimates are typically derived from historical data. Estimates may not only differ substantially among sample periods, but also have wide confidence intervals within sample periods. This introduces parameter uncertainty, which will affect the risk analysis of investors and their portfolio strategy. Therefore, investors need to accommodate parameter uncertainty in their evaluation for their long term investments.

In this dissertation I examine three broad issues concerning parameter uncertainty of financial parameters. First, I investigate the risk for European investors if they acquire inflation derivatives on international financial markets to hedge their inflation exposure. Second, I analyze the uncertainty concerning the inflation risk premium in major inflation-linked bond markets. Third, I study the effects of parameter uncertainty on the replacement rate risk for Defined Contribution (DC) pension participants.

The literature on inflation hedging (see e.g., (Ang, 2012)) has pointed out that asset classes other than inflation derivatives are unable to adequately hedge inflation risk. Domestic inflation-linked securities, unfortunately, are scarce. Only few European countries issue bonds based on the local Consumer Price Index (CPI), whereas most issue inflation-linked bonds based on the aggregated European Harmonised Index of Consumer Prices (HICP). The opportunity for investors to acquire foreign inflation-linked securities has not received much attention. European institutional investors may turn to international markets not only because local inflation-linked derivatives are limited, but also because inflation derivatives have higher liquidity in developed financial markets. However, investors who are hedging their future liabilities indexed on local CPI, such as pension funds, will have to anticipate a mismatch in cash flows when investing in foreign inflation-linked securities. I investigate whether long run interdependency between foreign inflation measures and the investor's inflation exposure can improve the inflation hedge.

The price of inflation derivatives in financial markets is based on the market's expectation of inflation and an inflation risk premium. High inflation risk premia indicate that nominal debt holders are uncertain about future inflation and they demand compensation for inflation shocks. The inflation risk premium is also important for inflation hedging investors, as it determines their cost for hedging inflation. In case of large uncertainty about inflation, debt issuing countries are forced to issue inflation-linked government debt instead of nominal debt to raise long term capital, because investors demand immunization for large anticipated inflation shocks[1]. If inflation risk is low, however, debt issuers can profit from issuing inflation-linked bonds. Strict monetary policies to keep inflation stable will allow countries to reduce costs by issuing inflation-linked debt rather than nominal debt. I analyze the uncertainty about the estimate for the inflation risk premium in the UK and US, which are among the most liquid markets for inflation-linked derivatives.

Concerns about financial parameters is not unique to inflation hedging investors. For investors saving for retirement, the estimate of the equity risk premium is crucial in determining the saving rate. Although participants are compensated by higher expected portfolio returns, investing in equity substantially increases the risk of future pension wealth. If high estimates of the equity risk premium are used, participants may underestimate their replacement risk at retirement and set their pension contributions too low. I study the impact of parameter uncertainty in DC pension contracts. In particular, I investigate whether participants can incorporate the risk of parameter uncertainty in their contribution schemes.

The main contribution of this dissertation is to study how investors can in-

[1] Argentina and Chile are examples of the introduction of inflation-linked bonds for this purpose (García and Rixtel, 2007).

corporate parameter uncertainty about financial parameters in their decisions. The uncertainty in financial parameters is especially important for a long investment horizon. In the following sections, I specifically address how the findings of each chapter in this dissertation contribute to the literature.

1.1 Can investors exploit long term interrelation between inflation measures?

In Chapter 2, I analyze whether investors can benefit from acquiring foreign inflation-linked bonds on the international market. Moreover, I investigate whether investors can exploit long run interrelations between foreign inflation measures and their local inflation exposure to improve the inflation hedging portfolio.

The main finding of this chapter is that investors can improve their hedging portfolios by acquiring foreign inflation-linked bonds on international markets. If the Purchasing Power Parity (PPP) holds between two countries, then the currency exchange rates will account for the difference between the two inflation indices. As a result, the mismatch between the cash flow of the foreign inflation-linked derivative and the investor's inflation exposure will be compensated by the change in the currency exchange rate. While the PPP predicts that any inflation-linked bond on the international market can be used to hedge local inflation, European investors can mostly improve their hedging positions by investing in European inflation-linked bonds. Although investing in foreign inflation-linked bonds denominated in foreign currency introduces an additional risk due to movements in currency exchange rates, I find that these bonds can improve the inflation hedging portfolios.

To lower basis risk originating from investing in foreign inflation-linked derivatives, the investor has two options. First, he can hedge currency risk with forward contracts to limit his exposure to foreign currency exchange rates. This strategy allows investors to profit from possible differences in interest rates between countries. The second risk factor is the mismatch between the foreign inflation measure and the investors' inflation exposure. For European inflation-linked bonds, this factor is the only source of basis risk. To minimize this mismatch, investors can incorporate long run dynamics between their inflation exposure and the foreign inflation measure. Typically, movements of inflation measures for different countries will not diverge over longer horizons as this would violate the law of one price for tradeable goods.

I show that exploiting long run dynamics can enhance the effectiveness of hedging portfolios for investors, as long as the long run dynamics between inflation measures remain stable.

To derive this result, I estimate inflation replication portfolios using equity, nominal and real bonds for three European investors, namely Dutch, French and German investors. Since these investors do not have access to local inflation-linked bonds, they can acquire European, UK, and US inflation linked-bonds. To analyze the effects of currency dynamics, I also evaluate the replication strategies if currency exposure is fully hedged by forward contracts.

The main contribution to the literature is to extend the portfolio choice strategy of the investors to foreign inflation-linked bonds and incorporate long run dynamics to reduce the replication errors of these portfolios. Prior literature has shown that most asset classes except for inflation-linked derivatives are unable to hedge against inflation (Bodie, 1976; Fama and Schwert, 1977; Campbell, Chan, and Viceira, 2003; Hoevenaars, Molenaar, Schotman, and Steenkamp, 2008). As a result, most studies focus on the strategic asset allocation of local inflation-linked bonds without allowing investors to benefit from foreign developed inflation-linked bond markets. Moreover, I demonstrate how long term dynamics of inflation measures can be exploited by investors to hedge inflation, combining the insights from the PPP and the inflation hedging literature.

1.2 Uncertainty about the inflation risk premium

An important question of the inflation hedging literature is the cost of hedging inflation risk with inflation derivatives. Institutional investors may want to hedge their liabilities to lower their exposures to long term inflation. In Chapter 3, I investigate the inflation risk premium by using market data from UK and US government debt markets.

I show that large parameter uncertainty concerning the estimates of the inflation risk premium cannot be ignored by institutional investors and needs to be addressed in long term investment decisions. In particular, I find that the estimates are widely dispersed in both markets with 95% credibility intervals ranging from -95 to 88 basis points in the UK and -4 to 119 basis points in the US. These large intervals indicate that it is hard to capture the market premium for hedging inflation in the government debt market.

A complicating factor in the estimation of the inflation risk premium is the availability of data on inflation-linked derivatives. One concern is that market rates of inflation-linked bonds are substantially affected by liquidity shocks during the financial crisis. To address this issue, I use inflation swap rates which were reported to be less influenced by liquidity shocks during the financial crisis Haubrich, Pennacchi, and Ritchken (2011). To address a small sample bias, I use a Bayesian framework which allows me to take into account parameter uncertainty. Another advantage of this framework is that it offers the possibility to investigate the effect of the financial crisis by incorporating an informative prior on the inflation risk premium.

The financial crisis caused a sharp decline in both the nominal and real term structure of interest rates. Low nominal interest rates may lead to discontinuation of strict inflation targeting monetary policies by central banks. Therefore, the uncertainty about future inflation risk may increase. However, I find this risk is only reflected in the UK market. The financial crisis shifts the inflation risk premium upward in the UK, whereas the US premium decreased. The 95% credibility interval becomes -105 to 150 basis points in the UK market and -50 to 92 basis points in the US market. These results indicate that the impact of the financial crisis on the inflation risk premium can differ substantially among developed financial markets.

Recent literature has shown that affine term structure models are subject to small sample bias (Bauer, Rudebusch, and Wu, 2012). I contribute to the literature by using a Bayesian method to address this issue in affine nominal and real term structure models. This method is able to quantify large uncertainty about estimates for the inflation risk premium. Various estimates reported by prior literature fall within the range of my results, indicating that it is hard to discriminate between these estimates. Adding additional macroeconomic factors to the affine term structure only increases the uncertainty about the estimates, suggesting that it is hard to capture the inflation risk premium accurately with these types of models.

1.3 The equity risk premium and pension ambition

Parameter uncertainty is an additional source of risk for investors if they plan for their retirement. US DC pension plans are required to project replacement rate at retirement for their participants. Typically, such statements do not incorporate uncertainty about financial markets, which may misinform participants on their replacement rate risk at retirement. In chapter 4, I analyze the impact of financial parameter uncertainty on the replacement rate risk of DC pension contracts.

Assuming a typical US DC asset strategy, I find that the equity risk premium is the driving factor for parameter uncertainty in the financial market for the DC participants. Replacement rate risk increases substantially when parameter uncertainty of the equity risk premium is ignored, relative to extending parameter uncertainty to all financial parameters. Since participants of DC pension funds may be ill-informed about this additional risk, contributions are set too low. Therefore, parameter uncertainty concerning the equity risk premium is the most important uncertainty among all financial parameters for DC participants.

To limit the effects of economic and parameter uncertainties, the investor can employ a time-varying contribution scheme that targets a specific replacement rate at retirement. I show that a time-varying contribution scheme can partly compensate for parameter uncertainty if the investor's belief corresponds to the underlying equity return process. If his belief of the equity risk premium is inaccurate due to unexpected shifts in the equity risk premium, then the compensating effect diminishes and the replacement rate risk at retirement increases. For example, I show that a downward shift in the equity risk premium of 0.5% may already substantially affect the ability of the time-varying contribution scheme.

The literature on life cycle models has shown the importance of equity in the investment portfolio and has given insights on how optimal contributions may be set to limit risks for participants. I contribute to this literature by showing that life cycle models may underestimate the replacement rate risk, as financial parameters are typically assumed to be known. By incorporating parameter uncertainty participants can mitigate replacement rate risk.

CHAPTER 2

Basis Risk and Inflation Replication

2.1 Introduction

International investors acquire foreign inflation derivatives such as inflation-linked bonds in major markets to profit from high liquidity. At the US TIPS auctions about 40% of the total demand consists of foreign investors, suggesting that US inflation-linked bonds are popular assets for foreign portfolios (Fleckenstein, Longstaff, and Lustig, 2013). However, do these foreign inflation-linked assets protect against the local inflation risk of investors? The theory of Purchasing Power Parity (PPP) predicts that investors can acquire any foreign inflation-linked instrument without constistuting additional risk. The explanation is that the exchange rate will compensate the investor for the difference between the foreign inflation rate and his inflation exposure. Empirical studies, however, typically reject the PPP between countries, so that foreign investors will be exposed to basis risk of exchange rates and inflation (Roll, 1979). We examine this basis risk by means of inflation replication to investigate the risk impact if foreign investors acquire foreign inflation derivatives on the international markets.

In this chapter, we investigate the risk associated with foreign inflation derivatives for investors who are not able to acquire inflation-linked derivatives on the local market. Liquidity and high trading costs might also cause such limitations. A vast literature investigates the alternatives for such an investor, focusing on the hedging ability of various nominal asset classes (see e.g. Bodie (1976), Fama and Schwert (1977) for nominal bonds and equity, and for commodities, e.g. Campbell et al. (2003) and Hoevenaars et al. (2008)). They find that these nominal assets are unable to insure against inflation risk, which suggests that only real assets offer a long-run hedge against inflation risk. Consequently, the literature mostly focuses on including local inflation-linked securities in the asset mix (for a discussion see e.g., Ang, 2012). Despite its insights, the literature largely ignores the ability of investors to acquire foreign inflation derivatives on the international market. While inflation derivatives are the only asset class that could immunize the investor from inflation risk, assets generating equivalent payoffs similar to the inflation shocks experienced by an investor are not traded in the financial market. Consequently, even local inflation derivatives based on a national aggregated inflation measure may constitute a mismatch with the actual inflation experienced by an investor and requires a specific inflation replication strategy. Especially for pension funds with long term liabilities the differences between cost of living adjustments and the consumer price level inflations can attribute to substantial basis risk over the horizon (Boskin, Dulberger, Gordon, Griliches, and Jorgenson, 1998). Therefore, we analyze how investors can replicate his actual inflation with foreign inflation-linked derivatives by exploiting long run dynamics of inflation measures.

To study inflation hedging with foreign inflation derivatives, we construct portfolios that replicate the investor's inflation exposure with both local and foreign assets. To incorporate the horizon effect of the investments and the inflation risk, we estimate both the term structure of asset returns and inflation. Due to the dependence of the correlation structure on the investment horizon, the weights of the investor's inflation tracking portfolios may shift. Two important factors may alter the asset allocation as well, namely long run dependency between the inflation measures and currency risk. To investigate whether investors can utilize this long run dependency over various investment horizons, we analyze two types of investors. The ECVAR-type investor exploits long run dependency between his inflation exposure and foreign inflation measures, while the VAR-type investors ignores this information. To study these factors separately, we incorporate currency hedge strategies to study the effect of exchange rate movements separately. In our setting, the international market consists of EU, UK and US inflation derivatives, and includes equity and nominal bonds of Japan, UK, US and local markets. We take the perspective of a Dutch, French and German investor to analyze the impact on the hedging strategies for European investors from both large and small economies.

We first consider the question of whether investors can improve their inflation hedges by acquiring foreign inflation derivatives. In our sample period from 1999 to 2011, we show that on average investing in inflation derivatives from European, UK, US and Japanese markets is beneficial for investors exposed to either Dutch, French or German inflation. While Japanese inflation-linked securities would constitute a large mismatch with the investor's inflation risk, the exchange rate compensates for this effect. As a result, we exclude Japanese inflation-linked derivatives, although we do allow the investor to invest in nominal bonds to exploit the carry trade in our sample period[1]. Due to the fact that the exchange rates are quite volatile over time, the European inflation-linked bond is an important asset in the optimal portfolio for European investors. Not surprisingly, we show that European bond holdings are quite substantial, while the remaining weight of the portfolio is allocated to local nominal bonds. However, we find that over the investment horizon the optimal demand for European inflation-linked bonds reduces for all three investors. The attractiveness of the European inflation-linked bond diminishes, while the US inflation-linked bond holdings increase over the horizon. When currency risk is hedged, both the Japanese carry trade and the European inflation-linked bonds have an important weight in our optimal portfolios. To investigate whether investors can benefit from foreign inflation-linked bonds denoted in a foreign currency, we exclude European inflation-linked bonds from the asset choice. We find that the investor can still substantially improve the hedging portfolio by acquiring UK and US inflation-linked bonds. Over the investment horizon, we document that local nominal bond holdings together with UK inflation-linked bonds decrease, while the US inflation-link bond exposure increases. Thus, investors benefit from investing in foreign inflation-linked bonds, but currency risk and the investment horizon can substantially affect the portfolio weights.

To verify whether investors can exploit long run dynamics, we establish a cointegration relation between the investor's inflation exposure and foreign inflation measures. This cointegration relation captures long run dynamics enabling investors to adjust their strategy and incorporate long run dependency of the inflation measures. Investors incorporating such strategies, ECVAR-type investors, cannot necessarily benefit from incorporating long run risk in our sample. We find that Dutch and German ECVAR-type investor can improve the hedging error respectively 2% and 5% compared to the VAR-

[1]For more details on carry trade, see e.g., (Galati, Heath, and McGuire, 2007).

type investor at a 5 years investment horizon while hedging currency risk. Since the exchange rates can substantially influence the returns of the portfolios, we only find that the German ECVAR-type investor can exploit long run dynamics if exposed to currency risk. These results are mostly driven by our short sample period in which the German cointegration is most stable across subsample periods. Excluding European inflation-linked bonds from the asset mix allows us to use an extended sample period. In this period, the ECVAR-type investors outperforms the VAR-type investor in all three cases. If currency risk is hedged, the German ECVAR-type investors can improve his tracking error about 7% at a 5 years horizon while the Dutch and French investor can only improve 1.5% and 0.3%, respectively. This suggests that a stable cointegration relation across subsample period is important for inflation hedging strategies to exploit long run dynamics.

Since the estimation of long run dynamics of inflation measures may involve large parameter uncertainty, we employ a Bayesian methodology. This methodology allows us to explicitly take into account the uncertainty related to the estimate coefficients for the long run equilibrium between the inflation measures and its impact on the asset returns. Our Bayesian results suggest that parameter uncertainty substantially impacts the portfolio allocations. Again, we find a decline of European bond holdings over the investment horizon. In the Dutch and German cases this decline is more substantial, whereas in the French case the decline is less steep, if we compare the weights to the previous results without parameter uncertainty. For example, the German ECVAR-type investor holds 42% of his optimal portfolio in European inflation-linked bonds at a 1 month horizon whereas at a 5 years horizon the weight drops to 30%. Although the portfolio weights over the horizon differ among specifications, European inflation-linked bonds bear substantial weight in the portfolios across Dutch, French and German investors. Similarly, we find that all investors increase nominal bond holdings to about 25% if exposed to currency risk. For the French VAR-type investor, parameter uncertainty increases the nominal bond holdings from 20% at a 1 months horizon to about 27% at a 5 years horizon. Without taking into account parameter uncertainty, the French VAR-type investor decreases his optimal bond exposure from 14% of his total portfolio to 8%. Consequently, parameter uncertainty can substantially alter the portfolio weights for local bond holdings.

The interaction between inflation measures and exchange rates may influence the attractiveness of foreign inflation-linked bonds. Since exchange rates are more volatile than inflation rates in our sample, currency hedges with forward contract do alter the allocation of the hedging portfolios. In our model, the currency hedging investor assigns large weights to Japanese nominal bonds, exploiting the Japanese carry trade. Since nominal returns on Japanese nominal bonds are less volatile, these bonds hedged with forward currency contracts may offer an alternative hedging strategy to the investor in our sample. The attractiveness of the US inflation-linked bond diminishes due to the parameter uncertainty in case of unhedged currency risk. All three Bayesian investors decrease the weight of US inflation-linked bonds over the investment horizons, whereas the portfolio allocations without a Bayesian framework are upward sloping. Surprisingly, US inflation-linked bonds remain to have a more substantial role in our inflation hedging portfolios compared to UK inflation-linked bonds. Only for long investment horizons, the German investors attach a similar importance to UK inflation-linked bonds. The attractiveness of the US and UK inflation-linked bonds to hedge inflation Dutch, French or German inflation exposure is strongly influenced by currency risk over the horizon. Consequently, replicating inflation with foreign inflation-linked bonds requires investors to take into account such basis risk.

To evaluate how inflation hedging investors in our model can respond to the aftermath of the financial crisis in 2008, we investigate time-varying inflation replication portfolios. Our model reveals that during the crisis the demand for local nominal bonds substantially increased for all three investors on the short investment horizon. While these portfolio weights increase about 51% at 1 month investment horizon, at a 5 years horizon these holdings change on average about 2.7%. At the same time, these investors increase their UK inflation-linked portfolio weights, while decreasing the allocation to US inflation-linked bonds. Surpringly, all investors maintain similar portfolio weights for European inflation-linked bonds at a 5 years horizon. After the crisis, the dynamics reverse and all Bayesian inflation hedging investors decrease their nominal bond holdings. Consequently, our model confirms that the Bayesian inflation hedging investor switches their holdings to local nominal bonds. This flight home effect during the financial crisis was also documented in the debt market, where investors shift their demand to local assets (see e.g. Giannetti and Laeven (2012)). We add to this insight that a long run inflation hedging perspective may offer an explanation of why local nominal bonds were attractive during the financial crisis.

Our work extends the literature on inflation hedging in three ways. First, we build on the literature of the PPP for long run dynamics between inflation measures (see e.g., Roll, 1979) and apply this insight to inflation hedging portfolios. Unlike Bekaert and Wang (2010), we propose a method to include long run inflation dependency in the asset allocation portfolio to allow the investor to exploit long run dynamics. This chapter demonstrates how uncertainty involved with long run dynamics of inflation affects European inflation hedging investors. Under stable conditions of the long run dynamics, inflation hedging investors are likely to exploit these dynamics in their portfolios on longer investment horizons. Secondly, we confirm the importance of investment horizon as suggested by Schotman and Schweitzer (2000) and extend this insight to the asset class of foreign inflation-linked derivatives. In particular, we analyze the importance of European inflation-linked bonds for the European market. Thirdly, we extend the literature on home bias by offering an explanation in terms of inflation hedging to the question why investors resort to local assets during the financial crisis. Existing literature (see e.g. Popov and Udell (2010) and Cetorelli and Goldberg (2012)) mainly focus on the banking sector to capture the flight home effect in the debt market, we on the other hand offer an alternative explanation. Inflation hedging can drive the home bias effect in the governmental debt market by foreign investors.

The remainder of this chapter is organized as follows. Section 2 motivates our analysis of foreign inflation-linked securities and explains how the PPP affects inflation hedging in the international market. In Section 3 we define the portfolio choice problem of the investor and explain how investors can exploit the cointegration relation between inflation measures in our ECVAR model. Consequently, we are able to describe its effect on the long run term structure of asset returns. The empirical results are reported and discussed in Section 4. Our concluding remarks follow in Section 5.

2.2 Basis risk and foreign inflation-linked securities

A central theme motivating the use of foreign inflation-linked derivatives is that foreign inflation measures can relate to the inflation exposure of investors. The Purchasing Power Parity (PPP) states that as a result of the interaction of inflation rates and exchange rates the difference between price levels denominated in a common currency cannot differ between countries. The underlying idea of this hypothesis is that the law of one price should hold among countries for tradable goods. Under the PPP hypothesis the investor can hedge his inflation exposure by acquiring similar securities based on foreign inflation, since the spot exchange rate will compensate the mismatch between the inflation rates. Therefore, investments in foreign inflation-linked derivatives will not constitute basis risk given the investor is exposed to aggregated inflation.

Although short term violations of PPP can occur due to the slow adjustment rate of commodity prices, early studies observe empirical deviations from the PPP hypothesis (See e.g. (Roll, 1979), and (Huang, 1987)). On the other hand, McNown and Wallace (1989) find evidence that supports the PPP hypothesis for high inflation countries. Recent empirical studies suggest that the PPP with respect to the US dollar seems to hold for various countries over a longer horizon (See e.g. (Taylor, 2002) and (Wallace and Shelley, 2006)). However, deviations from the PPP might be persistent due to Balassa Samuelson effects (Samuelson, 1994). For example, differences in productivity between countries can lead to dissimilar price levels of nontradable goods. These deviations constitute a risk for the inflation hedging investor in the long run if mean reversion does not occur.

In our analysis we assume that the investor is exposed to inflation measured by the national consumer price index. In order to quantify basis risk for our sample period when using foreign inflation derivatives, we determine the mismatch between the foreign inflation measure that are traded on financial markets and the inflation to which the investor is exposed to. Only in a few financial markets inflation-linked securities based on a national consumer price index are traded. Examples of large markets are Japan, UK, and US. Several countries in Europe have introduced an inflation-linked bond that immunizes investors from European inflation. Consequently, Eurozone investors will be exposed to additional risk of a mismatch between their exposed inflation and the inflation that underlies their hedging derivatives. On the other hand investors will not be at risk for changes in the exchange rate. To quantify the basis risk in our sample, we use three European inflation exposures, namely Dutch, French and German consumer price index inflation, representing the perspective of a Dutch, French and German investor respectively. Among markets that offer inflation-linked bonds, we have chosen the relatively largest markets based on outstanding notional amounts in order to account for liquidity effects. These markets are: Europe, Japan, UK, and US[2]. While Japan, UK, and US issue inflation-linked bonds based on their national CPI inflation, European inflation-linked bonds are issued based on HICP Euro Area inflation measure.

[2]The report Barclays Capital (2005) suggests that the European, Japanese, UK, and US markets are in terms of notional amounts the largest inflation derivatives markets.

Table 2.1 reports the basis risk for our sample period for all three inflation exposures. We approximate basis risk in this table as the difference between two inflation measures denominated in the Euro currency. This approach allows us analyze the risk of foreign inflation-linked derivatives ignoring differences in real returns of both economies. On average the yearly mismatch is negative implying that investors would benefit from replication using foreign inflation-linked derivatives. Even though Japanese inflation is quite low and results in an average positive mismatch, the exchange rate dynamics increases the attractiveness of Japanese inflation derivatives[3]. As a results, the hedging ability of securities based on these inflation measures are influenced by currency dynamics. Since currency exchange spot rates are quite volatile, it introduces a large basis risk for the investor. This is, for example, reflected in the large standard errors of Japanese mismatch. Since our result on basis risk is mostly driven by currency movements, Japanese inflation-linked bonds might be less relevant for hedging inflation compared to European, UK and US inflation derivatives. Hence, we refrain from incorporating those securities in our framework. However, we do allow investors to benefit from the currency trade by adding nominal Japanese bonds to the asset choice.

Table 2.1 indicates that in terms of average mismatch, the UK inflation derivatives would yield the highest compensation. However, the volatility of this mismatch indicates that investing in UK inflation constitutes more basis risk than in the Japanese inflation derivatives. Surprisingly, the US mismatch has a lower volatility compared to the UK as well. This suggest that UK inflation-linked securities might not be optimal choice for an inflation hedging investor. Thus, currency dynamics are an important determinant in the asset allocation.

The volatility of the mismatch between the EU and the Netherlands is substantially larger than in the cases of France and Germany. The impact of inflation in both France and Germany on the HICP Eurozone inflation is larger than for Dutch inflation[4]. Consequently, the hedging capacity of European inflation-linked securities in terms of basis risk are more favourable for in-

[3]The appreciation of the Yen over the whole sample period is about 23.5%, so that an investor holding Japanese currency would profit substantially from a long position.

[4]The average weight between 1996 and 2011 used to determine the HICP Euroarea inflation measure is 20.6% for France and for Germany 29.8% whereas the Dutch weight is only 5.1%. The HICP Euroarea inflation measure consists of the weighted average of the invididual HICP inflation measures. The HICP meaure differs from the national consumer price indices. The country's weight in the HICP Euroarea is determined by the relative household consumption expenditure in comparison with the total expenditure of the Euro area.

vestors from Germany and France. The US inflation-linked securities are for all three investors promising considering the low volatility. Since the mismatch is positively skewed, larger positive yearly mismatches are more likely. Thus, investing in US inflation-linked securities constitutes more basis risk compared to European inflation derivatives due to exchange rate dynamics. Generally, European inflation-linked derivatives consistute less risk and hence will have an important role in the asset allocations for an investor hedging inflation.

Another component of inflation hedging is the correlation of the asset with the inflation measure. For example, if correlation between the inflation exposure of an investor and the hedging asset is sufficient, the investor can exploit the comovement by leveraging his position. However, the risk of changes in exchange rates can strongly influence the hedging ability of a foreign derivative. For example, an appreciating currency during the investment period can influence the attractiveness of assets denominated in other currencies. Therefore, we explicitly take currency risk into account in our model of the asset returns. To determine the effect of exchange rates on the asset allocation, we compare the asset allocation in which the investors are exposed to currency risk to an allocation in which currency risk is hedged with forward contracts.

2.3 Hedging inflation framework

In this section we derive the hedging portfolio framework and introduce a cointegration analysis between the investor's inflation exposure and foreign inflation measures in order to incorporate long run coherency.

2.3.1 Portfolio choice

We consider an investor who wants to hedge his inflation exposure with a buy and hold strategy over different horizons. The optimal asset allocation consists of a minimum variance portfolio that replicates the inflation to which the investor is exposed to. At time t the investor hedges his expected inflation exposure with specific traded assets for a certain time horizon s. Formally, the investor's problem can be denoted as

$$\begin{aligned}
\min_{\omega_t} \quad & \omega_t' \mathrm{Var}_t[R^p_{t+1\to t+s}]\omega_t \\
\text{s.t.} \quad & \omega_t' \left(\mathrm{E}_t[R^p_{t+1\to t+s}] + \frac{1}{2}\mathrm{dg}\left(\mathrm{Var}_t[R^p_{t+1\to t+s}]\right)\right) = \mathrm{E}_t[\pi^H_{t+1\to t+s}] \\
& + \frac{1}{2}\left(\mathrm{Var}_t[\pi^H_{t+1\to t+s}]\right) \\
& \omega_t'\mathbf{1} = 1, \qquad (2.1)
\end{aligned}$$

where ω_t is a time-dependent vector with portfolio weights, $R^p_{t+1\to t+s}$ are the returns of the traded assets determined over horizon s at time t, $\mathrm{dg}(A)$ is the matrix function that denotes the diagonal of matrix A, and $\pi^H_{t+1\to t+s}$ denotes the investor's expected inflation exposure over horizon s at time t, which can be Dutch, French or German inflation. The restriction in the optimization problem requires that the minimum variance portfolio is mimicking the arithmetic mean of inflation. Consequently, the solution of the optimization is a minimum variance portfolio of traded assets replicating the investor's inflation exposure. Since the investor cannot invest in securities that generate payoffs equivalent to his inflation exposure, he replicates his exposure using a portfolio from equity, nominal and real bonds traded on the financial markets. Although we ignore short selling constraints, our model can be easily adapted. For tractability we will assume that the monthly gross returns and inflation are lognormally distributed. We distinguish between the conditional and unconditional allocation problem. The conditional problem is stated as above, while the unconditional can be restated by dropping the time dependency of the expectation and variance. The log expected gross return, $\mathrm{E}_t[R^p_{t+1\to t+s}] + \frac{1}{2}\mathrm{dg}\left(\mathrm{Var}_t[R^p_{t+1\to t+s}]\right)$, denotes the arithmetic mean return. In order to investigate the effect of the holding periods, we scale both means and variances by horizon s and subsequently report these in our empirical section.

For the equity market allocation the investor can choose from the Nikkei, FTSE, and the Dow Jones, and his local market. The local markets consist of the AEX for the Dutch investor, the CAC for the French investor and the DAX for the German investor. International equity indices tend to show reversal in returns relatively to other equity markets. Richards (1997) and Balvers, Wu, and Gilliland (2000) find most evidence of this reversal on a horizon of three years. Therefore, it is important to include multiple markets available for the investor. The nominal bond market choice consists of 10 years government bonds from Japan, UK, US and the local market of the investor[5]. The

[5]The empirical hedging literature has documented that bonds with a short term maturity

inflation-linked bonds are from the UK and US with a maturity of 5 years. The European inflation-linked bond return is approximated by using the German nominal 5 year maturity bonds and the inflation swap rates for the same maturity. European inflation swaps rates are based on the HICP inflation rates and do not dependent on the issuing country. Due to the limitations of the available data for inflation-linked bonds, our data is sampled on a monthly frequency. Our sample periods range from January 1999 to December 2011 without European inflation-linked security and from May 2005 to December 2011 with European inflation-linked security.

In Table 2.2 we present the sample statistics of the inflation measures and the returns of the assets. We find that the dynamics of the returns are substantially influenced by hedging currency risk. With the use of forward contracts the investor can hedge this risk and reduce variability in the asset returns denominated his local currency. In our sample period, hedging exchange rates improves on average the returns for the US and Japan. This implies that the investor can benefit from the difference in the nominal interest rates between the two countries. However, hedging the UK pound is only beneficial for the investor to reduce variability in his returns. In particular, investors can reduce the standard deviation of the monthly returns of the UK equity market by 11.5% by hedging this risk. In Japan and the US, the effect on the variability of the asset returns is smaller. Consequently, in our empirical section we analyse two scenarios either with currency hedged asset returns and asset returns that are exposed to currency risk.

The analytic solution of our optimization problem in Equation (2.1) is equivalent to an optimization of an inflation tracking portfolio (See e.g. Bekaert and Wang (2010)). The latter portfolio minimizes the hedge error that consists of the exposed inflation and the assets returns. Our specification allows for horizon analysis and the incorporation of a cointegration relation between the inflation to which the investor is exposed to and national aggregated inflation measures. If the inflation exposure to which the investor is exposed to is tied together in the long run with the foreign inflation measures included in the model, then the ECVAR-type investor will incorporate this effect in his strategic asset allocation.

have higher correlation with expected inflation than long maturity bonds (Fama and Schwert, 1977). One disadvantage is that short term bonds do not generate term premia as long term bonds. An analysis of the optimal strategy using short term bonds is outside the scope of this study.

2.3.2 Asset returns and inflation

We describe the long run dynamics between the inflation exposure of the investor and the foreign inflation measures via the following cointegration relation

$$I_t^H = \alpha_0 + \alpha_1 t + \gamma_1 I_t^{EU} + \gamma_2 I_t^{JP} + \gamma_3 I_t^{UK} + \gamma_4 I_t^{US} + \epsilon_{\pi,t}, \tag{2.2}$$

where I_t^H denotes the logarithmic price level H to which the investor is exposed to, i.e. either Dutch, French or German inflation. The price levels of the foreign inflation measures are given on the right side of the equation and are denoted in Euros using the currency exchange rates. In case currency risk is hedged, we use the exchange rate implied by the currency forward contract used by the investor. As a consequence, the foreign inflation measures are affected by foreign exchange rates.

The random variable $\epsilon_{\pi,t}$ is stationary under this specification, such that $\epsilon_{\pi,t} \sim I(0)$. This equation implies that exposed monthly inflation $\pi_t^H = \Delta I_t^H$ is equivalent to $\alpha_1 + \gamma_1 \pi_t^{EU} + \gamma_1 \pi_t^{JP} + \gamma_1 \pi_t^{UK} + \gamma_1 \pi_t^{US} + \Delta\epsilon_{\pi,t}$. We include a time trend in our specification in order to capture a deterministic time trend between the price levels. Due to our specification the dynamics of the investor's exposure to monthly inflation is influenced in the long run by foreign inflation. Hence, the price levels share a common stochastic trend. We impose no restrictions on the parameters γ_i, so that price levels may have different exposures to underlying long run risks.

We motivate our cointegration specification based on the empirical literature related to the PPP (See e.g. Juselius and MacDonald (2004) and Chen, Choi, , and Devereux (2008)). The PPP literature has focused on price levels shifts in certain baskets of goods and service across countries in order to examine whether inflation shares a common stochastic trend with a base economy (see e.g. Taylor and Taylor (2004)). Empirical studies have used variety of base economies, where the US economy and the world economy receive much attention (Taylor, 2002). Closest to our specification is Chen et al. (2008), who extend the use of one base economy by analyzing the common stochastic trend of price levels in eleven developed countries.

Next, we describe the asset returns for the various investment horizons by modeling the single period returns together with the inflation state variables.

We estimate the following ECVAR

$$\begin{bmatrix} R_{t+1} \\ NB_{t+1} \\ RB_{t+1} \\ \pi_{t+1} \\ \epsilon_{\pi,t+1} \end{bmatrix} = \begin{bmatrix} a_R \\ a_{NB} \\ a_{RB} \\ a_\pi \\ a_\epsilon \end{bmatrix} + \begin{bmatrix} * & 0 & 0 & * & * \\ 0 & * & 0 & * & * \\ 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & * & * \end{bmatrix} \begin{bmatrix} R_t \\ NB_t \\ RB_t \\ \pi_t \\ \epsilon_{\pi,t} \end{bmatrix} + \begin{bmatrix} u_{R,t+1} \\ u_{NB,t+1} \\ u_{RB,t+1} \\ u_{\pi,t+1} \\ u_{\epsilon,t+1} \end{bmatrix}, \quad (2.3)$$

where a denotes the vector of constants, R_t the equity return, NB_t nominal bond return, RB_t the real bond return, and π_t the foreign inflation measures. The variable $\epsilon_{\pi,t}$ is the residual of the cointegration relation as described in Equation (2.2). We project the returns on their lags and associated national inflation measure. Following Bansal and Kiku (2011), we ignore interactions between the bond markets and the equity markets. By introducing vector $X_t = \begin{bmatrix} R'_{t+1} & NB'_{t+1} & RB'_{t+1} & \pi'_{t+1} & \epsilon_{\pi,t+1} \end{bmatrix}'$, we can rewrite our ECVAR in matrix notation as follows

$$X_{t+1} = a + BX_t + u_{t+1}. \quad (2.4)$$

The equity returns R_t and nominal bond return NB_t consist of the stock markets from Japan, UK, US and the local equity market of the investor, which is either the Dutch, French or German market. The real bond returns are taken from the EU, UK, and US. The variable π_t denotes the foreign inflation measures of the EU, Japan, UK, and US. In case of the VAR representation, the cointegration residual $\epsilon_{\pi,t+1}$ is replaced with the inflation, π_t^H, of France, Germany, or the Netherlands. The variable X_t is consequently a (16×1)-vector and u is a vector of error terms that follows a normal distribution with zero mean and variance-covariance matrix Σ_u.

Our approach differs from the hedging inflation literature by including the error-correction specification of the inflation measures in the asset returns. Typically, the standard framework is to estimate asset returns and inflation with a VAR model, see e.g. Campbell and Shiller (1988), Campbell et al. (2003), Kandel and Stambaugh (1996), Hodrick (1992), and Schotman and Schweitzer (2000). In our specification multi-period returns of the investor's local market will depend on the dynamics of the long-horizon local inflation measure. The interaction between the cointegration residual and the equity and bond markets can capture the effect of a disequilibrium between the inflation measures on the asset markets. Thus, the cointegration relation can influence the dynamics of the asset returns. Consequently, including the error correction in variable in the return dynamics may alter the inflation hedging portfolio allocation. To compare the implications of the ECVAR model with the standard

VAR model, we estimate both models. The VAR specification can be obtained by excluding the error-correction variable $\epsilon_{\pi,t}$ from Equation (2.3) and replace it with the inflation measure to which the investor is exposed to.

2.3.3 Expected returns and risks

The hedging portfolio allocation in Equation (2.1) depends on expectation and the variance of the multi-period distribution. First, we derive the solution to the unconditional problem and then the conditional returns and risk structure. In this derivation, we follow the arguments of Bansal and Kiku (2011).

The unconditional expectation of the returns over horizon s (scaled by their horizon) is constant, so that

$$\mathrm{E}[R^p_{t+1\to t+s}] = \frac{1}{s}\sum_{k=1}^{s} \mu = \mu, \tag{2.5}$$

where μ denotes the mean of the unconditional expectation of the asset returns. We estimate μ by its sample mean.

The unconditional variance of the returns at various horizons can be derived by expressing the ECVAR model as an infinite-order moving average. According to Wold's theorem we can decompose the state variables X_t as function of the coefficient B and the error term u_t. As a result, we can write the unconditional variance of X_t as

$$\Omega_0 = \sum_{k=0}^{\infty} B^k \Sigma_u B'^k. \tag{2.6}$$

Incorporating the time-horizon s we get the following expression

$$\Omega_s = \Omega_0 + \frac{1}{s}\sum_{k=1}^{s-k}(s-k)\left(B^k\Omega_0 + \Omega_0 B'^k\right), \tag{2.7}$$

where the matrix $B^k\Omega_0$ denotes the k-order autocovariance of X_t. Note that the covariance is scaled by the horizon s, so that measurement is per unit in time. The unconditional variance matrix can be partitioned in returns and inflation as follows,

$$\Omega_s = \begin{bmatrix} \Omega_{Rp,s} & * \\ * & \Omega_{\pi,s} \end{bmatrix}, \tag{2.8}$$

with $\mathrm{Var}[R^p_{t+1\to t+s}] = \Omega_{Rp,s}$ and $\mathrm{Var}[\pi^H_{t+1\to t+s}] = \Omega_{\pi,s}$. In the unconditional case, the expectation is not dependent on the horizon, while the unconditional variance-covariance matrix is dependent. As a result, return dynamics may be altered across horizons.

The conditional problem can be solved by using the structure of the ECVAR in Equation (2.3). The mean of the assets returns and inflation variables can be computed by

$$\mathrm{E}_t[R^p_{t+1\to t+s}] = \frac{1}{s}\sum_{k=1}^{s}\left(C_k A + B^k X_t\right), \tag{2.9}$$

where $C_k = C_{k-1} + B^{k-1}$ for $k = 1, ..., s$, and $C_k = 0$. Using the fact that summing s consecutive observations of state variables X_t subtracted with its mean is a function of the innovations u_t, i.e.

$$\sum_{k=1}^{s} X_{t+k} - \mathrm{E}_t\left[\sum_{k=1}^{s} X_{t+k}\right] = \sum_{k=1}^{s} C_k u_{t+1+s-k}, \tag{2.10}$$

we can derive the conditional variance-covariance matrix. We can exploit the fact that the errors are identically distributed and serially uncorrelated, so that

$$\Sigma_s = \frac{1}{s} C_s \Sigma_u C_s' + \left(1 - \frac{1}{s}\right)\Sigma_{s-1}, \tag{2.11}$$

with $\Sigma_0 = 0$. The conditional covariance is scaled by the associated horizon and is partitioned as follows

$$\Sigma_s = \begin{bmatrix} \Sigma_{Rp,s} & * \\ * & \Sigma_{\pi,s} \end{bmatrix}, \tag{2.12}$$

with $\mathrm{Var}_t[R^p_{t+1\to t+s}] = \Sigma_{Rp,s}$ and $\mathrm{Var}_t[\pi^H_{t+1\to t+s}] = \Sigma_{\pi,s}$. In the conditional setting both the horizon and the time dimension is incorporated. Consequently, we can analyze the impact of time varying economic conditions, so that investors can take the current levels of inflation into account when determining their hedging portfolio.

2.4 Empirical results

In this section we present our estimation results on the portfolio choice over various horizons implied by our ECVAR. We compare our results with a traditional VAR model for the estimation of the asset returns and inflation. First, we discuss the cointegration relation between the inflation measures. Subsequently, we describe the estimates for our model. Finally, we report the Bayesian portfolio strategies for various horizons to investigate the effect of parameter uncertainty.

2.4.1 Cointegration evidence

We estimate the cointegration relation as defined in Equation (2.2) by ordinary least squares (OLS) regression. For both Dutch and German inflation, the sample autocorrelations of the residuals determined by the cointegration relation decline rapidly within three lags and slightly increase in the subsequent lags, whereas for the residuals of the French cointegration equation exhibits a gradual decline in autocorrelation. We employ an augmented Dickey-Fuller test, which rejects the null hypothesis of a unit root at a 5% level for Dutch price levels and at a 1% level for the French and German price levels. Subsequently, we use a Johansen cointegration test to determine the number of cointegration relations in our sample. We find one cointegration relation in all three cases. This evidence supports our model specification for the long run dynamics of inflation exposure of the investor.

The estimates of the cointegration relation for the Dutch, French and German case indicate that the long run dynamics of the inflation measures are not similar. Although all three cases have a relatively high loading on the European price level series compared to other inflation series, the estimates differ substantially among the three presented cases. We report the estimates of the cointegration determined by OLS. These estimates are similar to cointegration coefficients implied by the Johansen ECVAR model. We find the following estimated equations for the cointegration relation

$$\begin{aligned}
I_t^{NL} &= \underset{(53.57)}{121.29} - \underset{(0.02)}{0.05}\, t + \underset{(0.15)}{1.63}\, I_t^{EU} - \underset{(0.08)}{0.19}\, I_t^{JP} - \underset{(0.10)}{0.19}\, I_t^{UK} - \underset{(0.07)}{0.46}\, I_t^{US} + e_{\pi,t}, \\
I_t^{FR} &= \underset{(0.07)}{0.89} - \underset{(0.00)}{0.00}\, t + \underset{(0.07)}{0.80}\, I_t^{EU} + \underset{(0.03)}{0.02}\, I_t^{JP} + \underset{(0.05)}{0.03}\, I_t^{UK} - \underset{(0.03)}{0.06}\, I_t^{US} + e_{\pi,t}, \\
I_t^{GER} &= \underset{(20.84)}{174.75} + \underset{(0.01)}{0.05}\, t + \underset{(0.08)}{0.34}\, I_t^{EU} + \underset{(0.06)}{0.28}\, I_t^{JP} + \underset{(0.05)}{0.06}\, I_t^{UK} + \underset{(0.03)}{0.12}\, I_t^{US} + e_{\pi,t},
\end{aligned} \tag{2.13}$$

where $e_{\pi,t}$ denote the residuals of the estimated relation. These estimates are based on the sample period from January 1999 to December 2011 as discussed in Table 2.2. The implications of our cointegration relation is that the inflation exposure of an investor has a stochastic trend with inflation measures of other economies. Although French and German economy have a large impact on the determination of European inflation, their long run dependency differs substantially. We find coefficients of about 0.80 and 0.34 for the French and German case, respectively. This observation suggest that economies with a large impact on Eurozone inflation, do not necessarily have similar and substantial long run dependency. In both the Dutch and German case, the foreign inflation measure have a substantial impact as well. For Dutch inflation, we observe that US inflation is an important component in the cointegration relation whereas for German inflation the Japanese inflation receives substantial weight. This suggests that the investor might exploit long run coherency of other foreign inflation-linked securities besides European inflation-linked securities to hedge his inflation exposure.

Since estimates of the coefficients in the cointegration relation might be unstable across sample periods and sampling frequencies, we test its sensitivity by reestimating these relations on two sample periods from 1996 to 2011 and from 2005 to 2011. We find similar estimates for the French and German cointegration equation. Surprisingly, we find an unstable cointegration relation across subsample periods for the Dutch case. The impact of European and US price levels on Dutch price levels in the cointegration equation is less stable across different samples. In the extended sample period the impact of European price level on the Dutch price levels decreases from 1.63 (SE of 0.15) as reported in Equation (2.13) to 0.28 (SE of 0.40). In the subsample of 2005 to 2011, the decrease is substantially smaller with an estimate of 0.84 (SE of 0.20). These variations are of concern for an investor on how to incorporate cointegration evidence in their investment decision. In order to address this issue, we incorporate a Bayesian approach to allow for parameter uncertainty in the asset allocation. By allowing parameter uncertainty, we do not rely on the OLS estimates of the cointegration relation in modeling the asset returns. Consequently, this approach will be only dependent on the observed data.

Our estimation of the cointegration relation suggests that the error correction may influence the hedging allocation. Therefore, we analyze the inflation exposures across various horizons and their associated variation. We follow Bekaert and Wang (2010) in the selection of investment horizons and report from 1 month up to 5 years. The inflation measures under different regimes are presented in Table 2.4. To analyze the implications of the different specifications on the inflation measures between the ECVAR and the VAR specification of our model, we report the inflation term structure in terms of expectation and volatility. The moments of European, Japanese, UK, and US inflation measures depend on the chosen investor's inflation exposure. Therefore, these results can differ due to the included cointegration relations in the ECVAR model. In Table 2.4 we report the moments using the Dutch cointegration relation as previously defined in Equation (2.13). Changing the cointegration relation to the French or German specification mostly affects the volatilities of the inflation measures, but does not alter our conclusions on the economic significance of the cointegration relation.

The arithmetic means of the inflation measures in Table 2.4 remain quite stable over the various horizons for the ECVAR and the VAR specification. Note that the arithmetic mean in the unconditional case is defined as the expected mean plus half of the scaled variance associated with the specific horizon as defined in Equation (2.1). Since the variance component in the arithmetic means depends on the specification used to model the dynamics of the returns, it can differ among the two specifications. The ECVAR specification mostly affects the volatilities of the inflation measures to which the investor is exposed to, namely Dutch, French and German inflation. Thus, predictability of the asset returns by incorporating the cointegration relation is economically quite small. One of the factors driving this result is that we report in monthly expectations. As a consequence of incorporating the cointegration relation, the Dutch and French inflation variability increases less sharply over the horizon and the German variability decreases more steeply compared to the VAR specification. In the German case, the volatility decreases from 0.33% for an one month horizon to 0.17% for a 5 years horizon in the ECVAR specification, but in the VAR model this remains 0.23% for the longer horizons. Although these changes on a monthly basis might be small, it can substantially affect portfolio consequences evaluated at a larger horizon. In addition, the correlation structure is altered because of the ECVAR specification. As a result, this can influence the ability of the ECVAR-type investor to exploit long run dynamics in his asset allocation.

Turning to implications of the cointegration on the asset returns, we find that the cointegration relation also influences the term structure of traded asset returns. In Table 2.3 we present the returns and volatility of the asset returns for both the ECVAR and VAR specification for a Dutch investor. Although the ECVAR specification alters the expected returns and the volatility, its economic effect on the monthly returns is not clearly evident in all three asset classes. As for most nominal bonds, the expected returns increase over the horizons in both specifications. The return on a nominal Dutch bond is 0.11% at a 1 month horizon, which increases to 0.12% at a 5 years horizon in the ECVAR specification. In the VAR specification the term structure remains flat, resulting in a expected 0.11% return at a 5 years horizon. A similar patern can be observed for the volatility structure. In the ECVAR specification the volatilities of the nominal bonds increase with respect to the VAR specification as well. Consequently, investing in nominal bonds will be more risky for a ECVAR-type of investor, yet result in higher expected returns.

The expected equity returns tend to increase only slightly in the ECVAR specification compared to the VAR. For example, the average monthly Dutch expected equity returns in the ECVAR specification is about -0.17% at a 1 month horizon whereas at a 5 year horizon the return increases to -0.14%. The VAR specification yields similar results. The differences between the two specifications for the foreign equity markets are hard to capture on a monthly basis. For the inflation-linked bonds, the cointegration does not largely impact the expected returns. However, the expected returns in Table 2.3 are influenced by currency risk. If the investor hedges currency risk with forward contracts, then the terms structure of expected returns will be affected. As previously discussed, this will mostly have an impact on the volatility structure of the asset returns, as described in Table 2.2. The cointegration relation will have less effect on the return dynamics of the assets, so that the returns of two specifications will be more similar. On the other hand, volatilities over the horizons remain different between the two specifications. The term structure of the expected asset returns is dependent on whether the Dutch, French or German cointegration is relation used. For the French and German cases, we observe similar effects for the expected nominal bond returns and volatilities as in the Dutch case. Therefore, the cointegration relation will also affect the asset allocation in these two cases.

To summarize our findings thus far, both currency hedging and the ECVAR specification lead to different expected returns and associated risks. Especially, the expected returns of the nominal bonds tend to increase more sharply in the ECVAR specification and become more volatile due to the influence of the cointegration relation. Consequently, the difference in volatility influences the demand of an investor for these assets, since the inflation exposure of the investor is less volatile in a ECVAR specification. Additionally, the correlations across horizons are influenced by the cointegration relation. Therefore, the investors will be able to exploit long run dynamics to hedge their actual experienced inflation.

2.4.2 Classical hedging allocation

By means of the return profiles of the assets and the inflation measures, we can determine the optimal hedging allocation for investors with different investment horizons. We report our benchmark case of an investor hedging currency risk and having access to European inflation-linked bonds. This benchmark case is evaluated for three investors, namely for Dutch, French and German investors. Throughout our analysis, we ignore short selling constraints since these restrictions do not alter our conclusions on basis risk and inflation-linked bonds. First, we focus on hedging with European inflation-linked bonds and the associated strategic hedging allocation. Second, we analyze the incorporation of foreign inflation-linked bonds traded in other currencies.

Unconditional strategy with European inflation-linked bonds

In Table 2.5, we report the portfolio allocation for a Dutch investor with access to European inflation-linked bonds and currency risk hedged by forward contracts. We find that in the optimal solution the investor allocates considerable wealth to European inflation-linked bonds. Nominal bonds have an important role in hedging inflation as well, whereas equity markets are less attractive in the inflation replicating strategy. Similarly, we find that if the investor hedges currency risk, UK and US inflation-linked bonds have only a small proportion of wealth allocated to them. For both the short and long investment horizon, the European inflation-linked bond allocation is quite substantial. However, the weight of the bond decreases over the horizon. For example, the Dutch ECVAR-type investor reduces his allocation from 45% at one month horizon to about 31% at a 5 years horizon. The Dutch VAR-type investor only lowers his proportion of European inflation-linked bonds to 33% at a 5 years horizon. This indicates that both type of investors hedging Dutch inflation with a longer horizon should incorporate a lower fraction of European inflation-linked bonds in their portfolios.

Another important component in the hedging allocation is the large weight for the Japanese nominal bond. This result can be explained by the Japanese carry trade during the sample period. Profiting from the interest rate differences used in the currency hedging strategy, the investor can benefit from including nominal Japanese bonds. Interestingly, both types of investors allocate about 10% of their wealth in nominal Dutch bonds at short maturity and increase their demand to 15% at a 5 years horizon. This suggests that there is a trade off between nominal bonds and European inflation-linked bonds over the holding investment horizon. Thus, long horizon investors shift part of their wealth from European inflation-linked bonds into nominal Dutch bonds.

Our results indicate that currency hedges may alter our conclusions. Therefore, we explore the optimal allocation for an investor exposed to currency risk. In case the investor is exposed to currency risk, the impact between the VAR and ECVAR specification is more substantial. In Table 2.6, we present the optimal allocation strategy for a Dutch investor exposed to currency risk.

Again, the European inflation-linked bond has a substantial role in hedging inflation in the ECVAR specification. The weight of the bond reduces from 0.46% at a 1 month horizon to 0.38% at a 5 year horizon. However, in the VAR specification the weight for European inflation-linked bonds reduces to 0.24%. So, the currency hedge causes an important shift in the allocation over the horizon. Moreover, both type of investors increase their nominal Dutch bond holdings, because the Japanese nominal bond returns are substantially lower when exposed to currency risk. As a result, the optimal demand of both type of Dutch investors for nominal Japanese bonds is reduced to zero at all horizons. Since the Dutch ECVAR-type investor incorporates the cointegration relation to exploit long run dynamics, he increases his proportion of European inflation-linked bonds. The VAR-type investors instead increases his nominal Dutch bond holdings. Surprisingly, both type of Dutch investors hold a substantial amount of US inflation-linked bonds compared to the Dutch investors who hedge currency risk. Part of this result is driven by the depreciation of the dollar in our sample. Since the US inflation-linked bond holdings are larger than the European bond holdings, other foreign inflation-linked bonds can have important role when the investor is faced with currency risk.

We repeat our analysis for French and German investors. We verify to which extend our previous conclusion concerning the asset allocation alter when investors are exposed to other European inflation measures. In Tables 2.7 and 2.8, we report the French case with and without currency exposure. We can conclude from Table 2.7 that the French case is similar to the Dutch case. European inflation-linked bonds have a large weight in the portfolio with the Japanese nominal bond. However, in case of currency risk, as reported in Table 2.8, the French investor has a much larger demand for European inflation-linked bonds than the Dutch investor. His demand in the ECVAR specification is about 58% at a 5 year horizon. Instead of increasing his demand for local nominal bonds as in the Dutch scenario, the French investor has a large exposure to equity. In addition, his demand for US real bonds is substantially lower, but increasing over the horizon. Thus, the French investor has, similar to the Dutch investor, a trade off between European and US inflation-linked bonds over the investment horizon.

The German case is similar to the previous cases, except that there is a large difference between the ECVAR-type and VAR-type investor in case currency is hedged. In Tables 2.9 and 2.10, we report the German case. At an investment horizon of 1 month both specifications allocate about 47% to European inflation-linked bonds, but at a 5 year horizon the ECVAR-type investor reduces to a weight of 43%. In contrast with the ECVAR-type investor, the VAR-type investor only allocates 33% at a 5 year horizon. Thus, a German ECVAR-type investor would allocate substantially more wealth to these bonds on a longer horizon to exploit the long run dependency. Both type of German investors exposed to currency risk will substantially reduce their European inflation-linked bonds and increase their allocations to US inflation-linked bonds at longer horizons as in the Dutch and French case. This indicates that US inflation-linked bonds have an important role in replicating his inflation exposure, although the ECVAR investor will incorporate less bonds at long run horizons. Similar to the Dutch investor, the German investor holds a large proportion of local nominal bonds. For example, if currency risk is not hedged, the Dutch and German proportion allocated to local nominal bond is on average about 25%, whereas French local nominal bond holding is 10%. Therefore, local nominal bond holdings can vary between the investors of different European countries.

In terms of performance, only the German ECVAR-type investors improves his replication strategy regardless of the currency hedge. At a 5 years horizon, the German ECVAR-type investor hedging currency risk reduces his hedging error by 3% compared to the VAR-type investor. Exposed to currency risk, the improvement is only 2.5% compared to the VAR-type of investor. This suggests that currency risk reduces the opportunity to exploit long run coherency. The Dutch ECVAR-type of investor is only able to improve his hedging error by 2% if currency risk is hedged. The French ECVAR-type of investor does not seem to improve his portfolio using long run dynamics, as the VAR-type of investor improves his hedging error by ignoring long run dynamics by 7% if currency risk is hedged. These results are mostly driven by the short sample period. Consequently, the estimated long run dynamics are less stable over such sample periods. Since the German cointegration relation remains quite stable across subsample periods, the German investor can improve his hedging portfolio. This shows that investors cannot necessarily exploit long run dynamics in their replicating portfolios with European inflation-linked bonds in the asset choice.

To summarize, we find that European inflation-linked bonds have an important role in the inflation hedging strategies for Dutch, French and German investors. The ECVAR-type investor cannot in all cases benefit from incorporating long run dynamics of basis risk. In addition, there is a demand for foreign inflation-linked bonds even though the investor is exposed to currency risk and can invest in European inflation-linked bonds. An important component in the allocations is whether currency risk is hedged. The improvement of the replication portfolio by incorporating the long run dynamics can especially be observed on longer investment horizons. Subsequently, we turn to the question whether only foreign inflation-linked bonds not denominated in Euros can improve the replicating portfolio of the investor. Excluding European inflation-linked bonds from the asset choice, allows us to use an extended sample period. In this way we can capture the long run coherency more accurately.

UK and US inflation-linked bonds only

To investigate the impact of UK and US inflation-linked bonds, we exclude European inflation-linked bonds from the asset choice of the investor. This setting allows us to use our largest sample period from 1999 to 2011. In Table 2.11, we only present the bond allocations of the nominal and the UK and US inflation-linked bonds in case the investor is exposed to currency risk. Although holdings in equity and foreign nominal bonds remain a part of the total portfolio, we focus on whether investors can exploit long run dynamics in our extended sample period. In particular, our previous results indicate a trade off between local nominal bonds and European inflation-linked bonds over the investment horizon. Table 2.11 shows that investors exposed to currency risk substantially allocate their wealth to foreign inflation-linked bonds denoted in foreign currencies, although nominal bonds holdings remain substantial in the optimal portfolios due to exchange rate risk.

The ECVAR-type investor holds across various horizons more wealth in his local nominal bonds than the VAR-type investor. For all three investors, nominal bond holdings decrease in the optimal portfolio over the investment horizon, although the holdings of ECVAR-type investors decrease less sharply. For example, the German ECVAR-type investor has about 58% of German nominal bonds in his portfolio at a 1 month horizon and reduces this weight to 56% at a 5 year horizon, whereas the VAR-type investor holds only 49% at a 5 year horizon. A similar effect can be seen in the Dutch and French case, although the effect is substantially smaller compared to the German investor. All three investors increase their holdings of UK inflation-linked bonds over the investment horizon, regardless of the long run dynamics. This shows that for long investment horizons UK inflation-linked bonds increase their attractiveness, while nominal local bond holdings are reduced. Compared to our previous results for our reduced sample period, the dynamics of the UK inflation-linked bonds over the investment horizon are reversed. Although parameter uncertainty of the cointegration relation is of a concern for the implementation of the hedging portfolio, UK and US inflation-linked bond holdings are substantial regardless of the sample period.

Turning to the question whether the ECVAR-type investors can exploit long run dynamics in the extended sample period, we find improvements for the inflation replicating portfolio in all three cases. The hedging portfolios improve on average by about 0.5% in case the investor is exposed to currency risk. Therefore, exploiting long run coherency remains difficult. Although we do not report the portfolio weights in case currency risk is hedged, we find more substantial improvements. In particular, the German ECVAR-type investor can substantially improve by 7%, while the Dutch and French ECVAR-type investor can improve their hedging portfolio by 1.5% and 0.3%, respectively. These results indicate that a longer sample period improves the ablity of the investor to exploit the long run dynamics. Although the economic significance of the improvement for the French ECVAR-type investor is small, compared to performance in the reduced sample period the improvement is quite substantial. Generally, exploiting long run coherency is more likely if currency risk is hedged. Since hedging currency risk reduces the asset volatility, the ECVAR-type of investor can benefit unconditionally from implementing long run coherency. Therefore, implementing the cointegration relation in the inflation hedging position of the investor can be beneficial, although the economic significance of the improvement could be less certain.

Next, we verify whether investors can benefit from adding UK and US inflation-linked bonds to their asset choice. To measure this improvement, we compare both the performance of the portfolio with and without these inflation-linked bonds, which allows us to use our extended sample period. We find that for all three investors regardless whether currency risk is hedged that the performance of the portfolios improve substantially by including UK and US inflation-linked bonds. Especially when exposed to currency risk, the impact of adding foreign inflation-linked bonds has large economic significance. The Dutch investors can improve their portfolio by about 45%, whereas French and German investors improve about 30% and 35%, respectively. If currency risk is hedged, the improvements reduce to about 13% for the Dutch and French investor and about 10% for the German investor. These results suggest that foreign inflation-linked bonds can be beneficial for all three investors. Subsequently, we will focus on how parameter uncertainty affects the portfolio choice of investors.

2.4.3 Hedging allocations with parameter uncertainty

To conclude our analysis, we investigate the effect of parameter uncertainty on the portfolio choice by using our Bayesian methodology. Given the uncertainty about long run dynamics and our small sample period, we employ a Bayesian approach using an uninformative prior. This ensures that the hedging allocations only rely on the actual data. In particular, we focus our analysis on the different dynamics of the local nominal bond and inflation-linked bond holdings between the VAR-type and ECVAR-type investor. Subsequently, we assess the optimal portfolio holdings during the financial crisis in a conditional framework for a Bayesian investor and observe a flight home effect in the allocations for all investors. We present the case that the investor is exposed to currency risk and can acquire European inflation-linked bonds. For details on the Bayesian methodology, we refer to Appendix 2.A.

Unconditional setting

Table 2.12 confirms our previous results on the importance of the European inflation-linked bonds. Both types of Dutch investors allocate a substantial weight to European inflation-linked bonds, which is again declining over the investment horizon. Similarly, we observe this decline in the French and German cases, which are reported in Table 2.13 and 2.14. Both the Dutch and German investor reduce their European inflation-lined bond holdings less sharply compared to previous results without parameter uncertainty. For example, at a 5 years maturity the German Bayesian investor holds about 30% European inflation-linked bonds, which is about 5% more. The French Bayesian investor, however, holds less European inflation-linked bonds for longer investment horizon. Therefore, parameter uncertainty influences the European inflation-linked bond holdings. Although the weights over the investment horizon differ, the declining pattern as observed in our previous setting without parameter uncertainty remains.

Parameter uncertainty has an impact on the nominal bond holdings of the investors as well. All investors increase their nominal bonds weights to a level of about 25%. Especially in the French case, parameter uncertainty reverses the dynamics over the investment horizon for the local nominal bond holdings. The French Bayesian VAR-type investor has French nominal bond holdings that increase from 20% at a 1 month horizon up to 27% at a 5 years horizon. However, the French VAR-type investor, ignoring parameter uncertainty, reduces his bond weights from from 14% to 8% over the same horizon. Parameter uncertainty also diminishes the attractiveness of the US inflation-linked bonds. While investors ignoring parameter uncerainty increase their US inflation-linked bond holding over the investment horizon, all three Bayesian investors decrease their weights. As a result, parameter uncertainty reduces the attractiveness of foreign inflation-linked bonds while local nominal bond holdings become more important.

Regarding the ECVAR-type and VAR-type investors, we find less differences in the German case than in the Dutch and French case. This observation is in line with our previous result regarding uncertainty of the Dutch and French cases. However, incorporating parameter uncertainty into the hedging strategy does not substantially alter the differences between the ECVAR-type and VAR-type investors. Next, we analyze parameter uncertainty in a conditional setting.

Conditional setting

We implement a conditional setting within our framework to allow for time-varying economic conditions in a Bayesian context. A conditional type of investor hedges his inflation exposure incorporating market timing, i.e. the conditional investors take into account the current level of their inflation exposure, the foreign inflation measures and the asset returns. To analyze the impact of the conditional setting, we present the optimal allocation of the inflation-linked bond holdings for the Dutch ECVAR-type and VAR-type investor exposed to currency risk in Figure 2.1 with a 1 year investment horizon.

Figure 2.1 suggests that the impact of time varying economic conditions on bond holdings for hedging inflation across our sample period is not quite substantial, except for the period of the financial crisis. Our results on the French and German case, reported in Figure 2.2 and 2.3, yield a similar conclusion. The conditional framework reveals similarly to the unconditional setting that investors hold a substantial fraction of inflation-linked bonds. The level of inflation-linked bond holdings is lower in the conditional setting. For example, in the Dutch and French case the portfolio weights of European inflation-linked bonds are about 10% lower than in the VAR specification. However, these results do not alter our previous conclusions.

Turning to the flight home bias, our results show that only in the financial crisis the conditional type of investors substantially alters his portfolio. To explore the impact of the financial crisis on the portfolios, we present in Tables 2.15, 2.16, and 2.17 respectively the Dutch, French, and German average portfolio weights for three periods. During the period of the financial crisis in 2008, the portfolio weights for nominal bonds of all three investors increase substantially on the short investment horizon. For example, the Dutch investor raises his holdings by 47% from a weight of 8.6% to 12.6%. The French and German investor increase their holdings respectively by about 60% and 46%. A similar result can be observed for investors with a longer investment scope. Since the term structure of asset returns for long maturity is less sensitive for monthly fluctuations, the magnitude of the impact between the two periods at the 5 year investment scope is smaller, about 2.7% on average. Meanwhile, the investors reduce their holdings of US inflation-linked bonds and increase their demand for UK inflation-linked bonds. Surprisingly, the European inflation-linked bond has the most stable allocation throughout all three periods regardless of investment horizon.

Our empirical evidence suggests a flight home effect occurs for the inflation hedging investors caused by the financial crisis. Evidence from the debt market shows that investors are more prone to select local assets rather than foreign assets during the financial crisis (see e.g. Giannetti and Laeven (2012)). Similarly, in our model the conditional Bayesian investor increases his demand for local nominal bonds to hedge his inflation risk. Since the allocations of the hedging portfolios are determined by market rates, the flight home effect occurs only due to changes in the international market. Therefore, the inflation hedging perspective may offer an explanation why investors resort to local assets during liquidity shocks.

After the financial crisis, our model reveals that the portfolio allocations return to precrisis levels for longer investment horizons. On average, all three conditional investors decrease their local nominal bond holdings by about 2.8% for a 5 years investment horizon, so that they hold a similar amount of bonds as before the financial crisis. For shorter investment horizons, the portfolio alters slightly after the financial crisis compared to allocations prior to the crisis. For example, both the Dutch and French conditional investor hold about 10% less local nominal bonds at a 1 month investment horizon compared to precrisis levels, whereas the German investor increases his weight by 18%. The European bond holdings slightly increase for all three investors, ranging from 2% for the Dutch and French investor to 0.2% for the German investor. Therefore, only the European inflation-linked bond holdings remain similar to precrisis levels after the financial crisis for all investment horizons.

Overall, our Bayesian analysis confirms our previous results that inflation-linked bonds have a substantial weight in inflation hedging portfolios. Over the investment horizon, we document a declining weight for the European inflation-linked bonds. Incorporating parameter uncertainty does not alter this trend, although the decline is less sharp. As a result, local nominal bond holdings are more attractive for longer investment horizons. Furthermore, our Bayesian conditional framework shows that European inflation-linked bonds holdings remain quite stable throughout the financial crisis for all investment horizons. During the financial crisis, the local nominal bond holdings of the investors increase substantially, suggesting a flight home effect for the inflation hedging investors.

2.5 Conclusion

This chapter presents a framework of how investors can replicate their actual inflation exposure by acquiring foreign inflation derivatives on the international market. We focus on two primary questions. First, we verify whether inflation hedging investors can benefit from entering the international inflation-linked securities market and second, we investigate if investors can exploit long run dynamics between their actual inflation risk and foreign inflation measures.

With regard to the first question, we find that European investors can substantially improve the hedging capacity of their portfolios by incorporating foreign inflation-linked derivatives. In particular, European inflation-linked bonds have a substantial weight in the portfolio allocations. However, these weights decline over the investment horizon, suggesting that other foreign inflation-linked derivatives can be attractive. Second, we show how inflation hedging investors can incorporate long run dynamics of inflation measures to improve their hedging portfolios. Under stable conditions of the long run inflation dynamics, the investors are able to exploit their allocations in the international market.

These results point to the importance of the international market for inflation-linked derivatives. Especially for the pension sector, inflation derivatives can be used to hedge inflation-linked liabilities. Early studies suggested that pension schemes could benefit from incorporating local inflation-linked bonds (Bodie, 1988). We extend this view by showing that investors can benefit from foreign inflation-linked bonds in case either the actual inflation experienced by investors substantial differs from the national aggregated inflation measure or the local market does not offer inflation-linked derivatives. Inflation replication with foreign inflation-linked bonds allows investors to profit from higher liquidity in major markets.

2.A Appendix A: Bayesian approach

In a Bayesian approach, the posterior density of the model is required. To obtain this distribution for the VAR model, we follow Bauwens, Lubrano, and Richard (1999) and Bansal and Kiku (2011) by rewriting the model into a system of seemingly unrelated regressions. Formally, define this system of equations as

$$y_i = X_i\beta_i + \epsilon_i, \tag{2.A.1}$$

for each $i = 1, .., n$ with n denoting the total number of state variables in the system. If the individual time series included in the model have dimension T, then y_i is a vector with $((T-1) \times 1)$ observations, X_i is a matrix with dimensions $((T-1) \times k_i$ with k_i independent variables, β_i consists of a coefficient vector with k_i elements, and ϵ_i is the vector with the associated errors for each observation $(T-1)$. We rewrite this model in two forms in order to draw parameters from the posterior density. By stacking all the observations for each equation i, we can express Equation (4.A.19) as

$$y = x\beta + \epsilon, \tag{2.A.2}$$

where $y = (y_1, .., y_n)$ is a vector with dimensions $((T-1)n \times 1)$, $\beta = (\beta_1, .., \beta_n)$ with a vector of k_n elements, $x = \text{diag}(x_1, .., x_n)$ with dimensions $((T-1)n \times k_n)$, and $\epsilon = (\epsilon_1, .., \epsilon_n)$. In the second approach, we write a VAR specification

$$Y = XB + E, \tag{2.A.3}$$

with $Y = (y_1...y_n)$ is a matrix with dimensions $((T-1) \times n)$, $X = (X_1...X_n)$ has dimensions $((T-1) \times k_n)$, $B = \text{diag}(\beta_1, .., \beta_n)$ is a matrix with dimensions $(k_n \times n)$ and $E = (E_1...E_n)$ is a matrix with dimensions $((T-1) \times n)$. Next, we derive the posterior density functions for the VAR and ECVAR framework.

2.A.1 VAR framework

In deriving the posterior density function for the VAR model, we assume a uninformative prior. This implies that the investor does not have any prior believe on the parameters of the model. Hence, the prior function is of the form

$$f(\beta, \Sigma) \propto |\Sigma|^{-(n+1)/2}, \tag{2.A.4}$$

where Σ denotes the variance-covariance matrix of the error in the VAR model. For this uninformative prior, the marginal posterior density of the parameters can be written as

$$\begin{array}{rcl}\beta|\Sigma & \sim & \mathrm{N}(\hat{\beta}, [x'(\Sigma^{-1} \otimes I_{T-1})x]^{-1}) \\ \Sigma|\beta & \sim & \mathrm{IW}(Q, T-1),\end{array} \tag{2.A.5}$$

with

$$\begin{array}{rcl}\hat{\beta} & = & [x'(\Sigma^{-1} \otimes I_{T-1})x]^{-1}x'(\Sigma^{-1} \otimes I_{T-1})y \\ Q & = & (Y - XB)'(Y - XB).\end{array}$$

Since the marginal posterior densities of the two parameters β and Σ are not available, we rely on the Block-Gibbs sampling algorithm (See e.g. Bauwens et al. (1999) and Bansal and Kiku (2011)). Conditional on a previous simulation of the variance-covariance matrix Σ_{j-1}, we can draw β_j from the conditional density function. Again, with the sampled β_j the variance-covariance matrix Σ_j can be drawn from the inverse Wishart distribution. This sequential sampling method is initialized with the ordinary least squares estimates of the model. To remove potential influence of the starting values, we remove the first 500 draws from the sequence of parameters. Additionally, we remove draws if any eigenvalues of matrix with the autoregressive coefficients of the included variables are larger than 0.98 in order to ensure stationarity as in Bansal and Kiku (2011).

Our final sequence consists of 20000 draws from the posterior density. Using these parameters, we calculate the associated means and variance-covariance matrices of the various horizons. For each of these moments, we determine the optimal allocation strategy. We report the average of portfolio holdings for various horizons and a 95% confidence bounds of these allocations. This procedure results in the optimal portfolio allocations that only rely on the observed data.

2.A.2 ECVAR framework

In deriving the posterior density functions for the ECVAR framework, we assume again that investors holds a uninformative believe on the ECVAR parameters and the cointegration relation. Assuming a uniform distribution for the coefficient cointegration vector γ, we denote this flat prior as $f_0(\gamma)$. Let Z denote the matrix with the price levels indices and the deterministic components of the cointegration relation such that its product with the coefficient cointegration vector, $Z\gamma'$, yields a vector with cointegration errors. We can formally define the prior distribution for the ECVAR framework as

$$f(\beta, \Sigma, \gamma) \propto f_0(\gamma)|\Sigma|^{-(n+1)/2}. \tag{2.A.6}$$

Following Bansal and Kiku (2011), we can write the conditional densities as

$$\begin{array}{rcl} \beta|\Sigma,\gamma & \sim & \mathrm{N}(\hat{\beta},[x'(\Sigma^{-1}\otimes I_{T-1})x]^{-1}) \\ \Sigma|\beta,\gamma & \sim & \mathrm{IW}(Q,T-1), \\ f_0(\gamma) & \propto & f_0(\gamma)\frac{|\gamma' W_0\gamma|^{l_0}}{|\gamma' W_1\gamma|^{l_1}}, \end{array} \tag{2.A.7}$$

with

$$\begin{array}{rcl} M_x & = & I - X(X'X)^{-1}X' \\ W_0 & = & Z'M_X Z \\ W_1 & = & Z'M_X(I_{T-1}(Y(Y'M_X Y)^{-1}Y')M_X Z \\ l_0 & = & (T-n-2)/2 \\ l_1 & = & (T-2)/2, \end{array}$$

and $\hat{\beta}$ and Q are defined as in the VAR specification. Since an analytical solution for the marginal posterior density is not available, we need to rely on simulation techniques. We apply the Griddy-Gibbs sampling technique to determine the marginal posterior density for each element of the coefficient cointegration vector γ. We obtain the marginal distribution density of each coefficient of the cointegration relation by evaluating the marginal function over a grid of points. For each grid point, we draw from the marginal function conditional on the remaining parameters and obtain the unconditional value. Next, we normalize the marginal density functions for each coefficients such that we can determine the cumulative density function. By drawing uniformly from inverted CDF, we can obtain simulations for γ^j. For each γ^j, we can apply the Block-Gibbs sampling technique to obtain the remaining parameters β and Σ as discussed in the previous section. We obtain a sequence of 20000 draws for each of the parameters and determine the associated mean and variance-covariance matrices for all horizons. Next, we calculate for each horizon the optimal strategy allocation. This allows us to obtain an allocation that does not rely on the ECVAR parameters, but depends on the observed data.

2.B Appendix B: Tables and figures

Table 2.1: Basis risk

This table presents the dynamics of basis risk for a Dutch, French and German investor. For foreign inflation measures, we use HICP Euro area, RPI of the UK , the All urban CPI of the US and Japanese CPI excluding fresh foods inflation measures. The inflation exposures of the investors are all the national CPI inflation measures. Basis risk is defined by subtracting the investor's exposed inflation with the foreign inflation measure denoted in Euros. A positive difference indicates the annual costs of the investor, whereas a negative sign implies the investor benefits from using securities linked to this inflation measure. Furthermore, this tables contains the summary statistics of the mismatches. The inflation rates are determined by yearly inflation rates from January to December. The sample period is from 1999 to 2011.

	Mean	St. dev	Skew	Kurt
Dutch inflation Exposure				
EU	-0.67%	0.7%	0.35	2.09
UK	-2.66%	13.3%	-1.14	2.90
US	-0.46%	7.9%	1.32	4.90
JP	-1.19%	11.2%	-0.15	1.43
French inflation Exposure				
EU	-0.62%	0.3%	0.47	2.11
UK	-2.60%	13.5%	-1.15	2.90
US	-0.40%	8.0%	1.19	4.51
JP	-1.14%	11.7%	-0.15	1.40
German inflation Exposure				
EU	-0.66%	0.2%	-0.57	2.13
UK	-2.64%	13.4%	-1.15	2.91
US	-0.45%	7.9%	1.25	4.80
JP	-1.18%	11.5%	-0.17	1.41

Table 2.2: Data summary

This table reports the descriptive statistics in percentages for the asset categories, namely the equity market, the nominal and real bond markets. Returns are determined by the difference of logarithmic price levels and are denominated in Euros. The currency hedged returns correspond to returns adjusted with an one month currency forward contract. The European inflation-linked bond returns are constructed with use of German nominal bonds and European inflation swap rates. All data are sampled at monthly frequency and denominated in Euros. The sample period covers January 1999 to December 2011 for all assets, except the European inflation-linked bond which covers May 2005 to December 2011.

	Equity markets				Nominal bond markets			
	With currency risk		Currency risk hedged		With currency risk		Currency risk hedged	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std
US	0.09	4.94	0.15	4.90	0.08	3.86	0.14	2.88
UK	-0.16	5.15	-0.16	4.56	0.01	2.58	0.01	2.29
JP	-0.16	6.02	-0.10	6.09	0.20	3.71	0.26	1.33
NL	-0.38	6.56			0.11	2.08		
FR	0.12	6.92			0.05	2.01		
GER	-0.15	5.90			0.09	2.10		

	Real bond markets			
	With currency risk		Currency risk hedged	
	Mean	Std	Mean	Std
US	0.26	2.99	0.33	1.64
UK	0.23	2.01	0.23	1.43
EU	0.22	1.31		

	Inflation measures			Inflation exposure	
	Mean	Std		Mean	Std
EU	0.18	0.33	NL	0.17	0.46
JP	-0.03	0.47	FR	0.15	0.27
UK	0.25	0.39	GER	0.14	0.33
US	0.20	0.41			

Table 2.3: Term structure of asset returns with currency risk

This table reports the expected asset returns and their associated risk across horizons for the investor exposed to currency risk. For both the ECVAR and the VAR model, the expected return and the volatility are presented. Since the arithmetic means and volatilities of the assets in the ECVAR model can differ due to the included cointegration relation, the reported moments are constructed using the Dutch cointegration relation. In the parenthesis the Bootstrap standard errors are reported.

	Expected return (%)				Volatility (%)			
	1	12	24	60	1	12	24	60
				ECVAR				
Equity US	0.207	0.229	0.230	0.230	4.93	5.35	5.38	5.39
	(0.424)	(0.424)	(0.425)	(0.425)	(0.31)	(0.55)	(0.57)	(0.58)
Equity UK	−0.030	−0.013	−0.012	−0.012	5.15	5.47	5.49	5.50
	(0.431)	(0.431)	(0.431)	(0.431)	(0.37)	(0.59)	(0.61)	(0.63)
Equity JP	0.026	0.063	0.066	0.067	6.02	6.61	6.65	6.67
	(0.529)	(0.530)	(0.530)	(0.530)	(0.37)	(0.70)	(0.73)	(0.75)
Equity NL	−0.168	−0.144	−0.143	−0.142	6.57	6.92	6.94	6.95
	(0.543)	(0.541)	(0.541)	(0.541)	(0.49)	(0.78)	(0.81)	(0.83)
NBond US	0.151	0.173	0.177	0.179	3.88	4.43	4.50	4.54
	(0.364)	(0.364)	(0.364)	(0.364)	(0.22)	(0.48)	(0.52)	(0.54)
NBond UK	0.039	0.039	0.039	0.039	2.57	2.56	2.57	2.57
	(0.203)	(0.203)	(0.203)	(0.203)	(0.15)	(0.25)	(0.27)	(0.28)
NBond JP	0.274	0.277	0.277	0.278	3.72	3.81	3.82	3.82
	(0.308)	(0.309)	(0.309)	(0.309)	(0.28)	(0.41)	(0.43)	(0.44)
NBond NL	0.110	0.116	0.117	0.117	2.10	2.37	2.41	2.43
	(0.194)	(0.194)	(0.194)	(0.194)	(0.13)	(0.26)	(0.28)	(0.30)
RBond US	0.309	0.304	0.304	0.304	2.99	2.82	2.81	2.81
	(0.231)	(0.231)	(0.231)	(0.231)	(0.20)	(0.27)	(0.27)	(0.28)
RBond UK	0.251	0.255	0.255	0.255	2.01	2.17	2.18	2.19
	(0.172)	(0.172)	(0.172)	(0.172)	(0.23)	(0.29)	(0.29)	(0.30)
				VAR				
Equity US	0.207	0.229	0.230	0.230	4.94	5.35	5.37	5.39
	(0.424)	(0.425)	(0.425)	(0.425)	(0.31)	(0.54)	(0.56)	(0.57)
Equity UK	−0.030	−0.015	−0.014	−0.013	5.16	5.45	5.46	5.47
	(0.427)	(0.426)	(0.426)	(0.426)	(0.37)	(0.58)	(0.60)	(0.61)
Equity JP	0.026	0.060	0.062	0.063	6.02	6.56	6.59	6.60
	(0.523)	(0.525)	(0.525)	(0.525)	(0.37)	(0.67)	(0.69)	(0.71)
Equity NL	−0.167	−0.145	−0.144	−0.144	6.58	6.91	6.92	6.93
	(0.534)	(0.534)	(0.534)	(0.534)	(0.51)	(0.76)	(0.78)	(0.79)
NBond US	0.150	0.165	0.167	0.168	3.87	4.24	4.27	4.29
	(0.341)	(0.342)	(0.342)	(0.343)	(0.22)	(0.43)	(0.45)	(0.46)
NBond UK	0.039	0.037	0.037	0.037	2.57	2.49	2.48	2.48
	(0.195)	(0.195)	(0.195)	(0.195)	(0.15)	(0.23)	(0.24)	(0.24)
NBond JP	0.274	0.277	0.277	0.277	3.72	3.80	3.80	3.80
	(0.309)	(0.310)	(0.310)	(0.310)	(0.30)	(0.43)	(0.44)	(0.45)
NBond NL	0.110	0.113	0.113	0.113	2.10	2.23	2.23	2.24
	(0.178)	(0.178)	(0.178)	(0.178)	(0.13)	(0.22)	(0.23)	(0.23)
RBond US	0.309	0.304	0.304	0.304	2.99	2.82	2.81	2.81
	(0.231)	(0.231)	(0.231)	(0.231)	(0.20)	(0.26)	(0.27)	(0.28)
RBond UK	0.251	0.255	0.255	0.255	2.01	2.17	2.18	2.18
	(0.170)	(0.170)	(0.170)	(0.170)	(0.23)	(0.29)	(0.29)	(0.30)

Table 2.4: Horizon of Inflation expectations

This table reports the profile of the inflation measures across horizons. For both the ECVAR and the VAR model, the expected return and the volatility are presented. Since the arithmetic means and volatilities of the EU, Japan, UK, and US in the ECVAR model can differ due to the included cointegration relation, the reported moments are constructed using the Dutch cointegration relation. The French and German inflation moments in this table are determined by their equivalent cointegration relation. In the parenthesis the Bootstrap standard errors are reported which are determined by a residual Bootstrap.

	Expected Inflation (%)				Volatility (%)			
	1	12	24	60	1	12	24	60
				ECVAR				
EU	0.179	0.179	0.179	0.179	0.33	0.36	0.36	0.36
	(0.030)	(0.030)	(0.030)	(0.030)	(0.13)	(0.17)	(0.18)	(0.18)
JP	−0.029	−0.028	−0.028	−0.028	0.47	0.65	0.66	0.67
	(0.056)	(0.056)	(0.056)	(0.056)	(0.17)	(0.32)	(0.34)	(0.35)
UK	0.247	0.248	0.248	0.248	0.40	0.47	0.48	0.48
	(0.038)	(0.038)	(0.038)	(0.038)	(0.15)	(0.23)	(0.23)	(0.24)
US	0.206	0.207	0.207	0.207	0.41	0.69	0.72	0.73
	(0.061)	(0.061)	(0.061)	(0.061)	(0.17)	(0.37)	(0.39)	(0.41)
NL	0.171	0.172	0.172	0.172	0.47	0.62	0.62	0.62
	(0.049)	(0.049)	(0.049)	(0.049)	(0.17)	(0.37)	(0.39)	(0.40)
FR	0.147	0.147	0.147	0.147	0.27	0.28	0.28	0.27
	(0.022)	(0.022)	(0.022)	(0.022)	(0.09)	(0.13)	(0.13)	(0.13)
GER	0.136	0.136	0.136	0.136	0.33	0.19	0.18	0.17
	(0.013)	(0.013)	(0.013)	(0.013)	(0.11)	(0.08)	(0.07)	(0.07)
				VAR				
EU	0.179	0.179	0.179	0.179	0.33	0.37	0.37	0.37
	(0.030)	(0.030)	(0.030)	(0.030)	(0.13)	(0.18)	(0.18)	(0.18)
JP	−0.029	−0.028	−0.028	−0.028	0.47	0.65	0.66	0.67
	(0.056)	(0.056)	(0.056)	(0.056)	(0.17)	(0.32)	(0.34)	(0.34)
UK	0.247	0.248	0.248	0.248	0.40	0.47	0.47	0.48
	(0.038)	(0.038)	(0.038)	(0.038)	(0.15)	(0.22)	(0.23)	(0.23)
US	0.206	0.207	0.207	0.207	0.41	0.65	0.67	0.68
	(0.058)	(0.058)	(0.058)	(0.058)	(0.17)	(0.35)	(0.37)	(0.38)
NL	0.171	0.172	0.172	0.172	0.46	0.64	0.65	0.65
	(0.055)	(0.055)	(0.055)	(0.055)	(0.15)	(0.31)	(0.32)	(0.33)
FR	0.147	0.147	0.147	0.147	0.27	0.29	0.29	0.29
	(0.023)	(0.023)	(0.023)	(0.023)	(0.09)	(0.12)	(0.13)	(0.13)
GER	0.136	0.136	0.136	0.136	0.33	0.23	0.23	0.23
	(0.023)	(0.023)	(0.023)	(0.023)	(0.12)	(0.09)	(0.09)	(0.09)

Table 2.5: Optimal allocation strategy for Dutch investor with currency hedge

This table reports the Dutch optimal allocation of the hedging portfolio in percentages across different horizons. The portfolio weights have been given for both the ECVAR and the VAR model. In this allocation the investors is not exposed to currency risk as exchange rate risk is hedged by forward contracts. The number in parentheses are the lower and upper bounds of the corresponding 95% Bootstrap confidence intervals.

	Investment horizon (months)			
	1	12	24	60
		ECVAR		
Equity US	−0.06 (−0.09,0.07)	−0.09 (−0.13,0.09)	−0.10 (−0.13,0.09)	−0.10 (−0.14,0.09)
Equity UK	−0.02 (−0.10,0.12)	0.06 (−0.10,0.18)	0.06 (−0.10,0.18)	0.06 (−0.10,0.19)
Equity JP	0.06 (−0.02,0.08)	0.07 (−0.02,0.09)	0.07 (−0.02,0.09)	0.07 (−0.02,0.09)
Equity NL	0.05 (−0.09,0.07)	0.02 (−0.09,0.08)	0.02 (−0.09,0.08)	0.02 (−0.09,0.08)
NomB US	−0.07 (−0.18,0.07)	−0.05 (−0.16,0.15)	−0.05 (−0.17,0.16)	−0.05 (−0.17,0.16)
NomB UK	−0.18 (−0.30,0.01)	−0.20 (−0.33,0.05)	−0.21 (−0.33,0.05)	−0.21 (−0.34,0.05)
NomB JP	0.85 (0.50,0.94)	0.94 (0.40,1.02)	0.94 (0.39,1.02)	0.95 (0.37,1.02)
NomB NL	0.10 (−0.17,0.22)	0.14 (−0.16,0.29)	0.15 (−0.17,0.29)	0.15 (−0.17,0.29)
RealB US	−0.10 (−0.22,0.10)	−0.08 (−0.24,0.19)	−0.08 (−0.24,0.20)	−0.08 (−0.24,0.21)
RealB UK	−0.09 (−0.11,0.31)	−0.12 (−0.14,0.34)	−0.12 (−0.14,0.35)	−0.12 (−0.14,0.36)
RealB EU	0.45 (0.17,0.58)	0.32 (0.00,0.55)	0.31 (−0.01,0.56)	0.31 (−0.03,0.56)
		VAR		
Equity US	−0.07 (−0.13,0.07)	−0.11 (−0.16,0.07)	−0.11 (−0.16,0.07)	−0.11 (−0.17,0.07)
Equity UK	−0.01 (−0.10,0.13)	0.05 (−0.09,0.20)	0.05 (−0.09,0.21)	0.06 (−0.09,0.21)
Equity JP	0.06 (−0.02,0.08)	0.06 (−0.02,0.08)	0.07 (−0.02,0.08)	0.07 (−0.02,0.08)
Equity NL	0.05 (−0.09,0.08)	0.03 (−0.09,0.08)	0.02 (−0.09,0.08)	0.02 (−0.10,0.08)
NomB US	−0.08 (−0.23,0.05)	−0.06 (−0.19,0.11)	−0.06 (−0.20,0.12)	−0.06 (−0.20,0.13)
NomB UK	−0.20 (−0.33,0.02)	−0.23 (−0.38,0.04)	−0.23 (−0.38,0.04)	−0.23 (−0.38,0.04)
NomB JP	0.84 (0.51,0.97)	0.96 (0.45,1.08)	0.96 (0.44,1.08)	0.96 (0.44,1.08)
NomB NL	0.12 (−0.16,0.24)	0.14 (−0.19,0.29)	0.14 (−0.20,0.29)	0.14 (−0.20,0.29)
RealB US	−0.09 (−0.22,0.14)	−0.06 (−0.21,0.24)	−0.05 (−0.21,0.25)	−0.05 (−0.20,0.25)
RealB UK	−0.08 (−0.10,0.29)	−0.12 (−0.13,0.32)	−0.12 (−0.13,0.32)	−0.12 (−0.13,0.32)
RealB EU	0.46 (0.17,0.61)	0.34 (0.01,0.59)	0.33 (−0.01,0.59)	0.33 (−0.01,0.60)

Table 2.6: Optimal allocation strategy for Dutch investor exposed currency risk

This table reports the Dutch optimal allocation of the hedging portfolio in percentages across different horizons. The portfolio weights have been given for both the ECVAR and the VAR model. In this allocation the investors is not exposed to currency risk as exchange rate risk is hedged by forward contracts. The number in parentheses are the lower and upper bounds of the corresponding 95% Bootstrap confidence intervals.

	Investment horizon (months)			
	1	12	24	60
	ECVAR			
Equity US	0.02	−0.01	−0.01	0.00
	(−0.02,0.19)	(−0.07,0.18)	(−0.08,0.18)	(−0.09,0.19)
Equity UK	−0.03	0.03	0.03	0.03
	(−0.15,0.13)	(−0.11,0.19)	(−0.11,0.19)	(−0.12,0.20)
Equity JP	0.00	−0.01	−0.01	−0.01
	(−0.10,0.04)	(−0.12,0.05)	(−0.12,0.05)	(−0.12,0.05)
Equity NL	0.12	0.05	0.05	0.04
	(−0.10,0.13)	(−0.09,0.11)	(−0.09,0.11)	(−0.09,0.11)
NomB US	−0.12	−0.19	−0.21	−0.22
	(−0.22,0.07)	(−0.22,0.09)	(−0.22,0.10)	(−0.22,0.10)
NomB UK	0.25	0.31	0.32	0.33
	(−0.21,0.35)	(−0.26,0.39)	(−0.26,0.39)	(−0.27,0.39)
NomB JP	−0.05	−0.04	−0.04	−0.03
	(−0.05,0.19)	(−0.05,0.18)	(−0.06,0.18)	(−0.06,0.18)
NomB NL	0.26	0.23	0.22	0.21
	(−0.16,0.38)	(−0.23,0.44)	(−0.25,0.44)	(−0.25,0.44)
RealB US	0.18	0.38	0.40	0.42
	(−0.03,0.28)	(0.00,0.49)	(0.00,0.51)	(−0.01,0.52)
RealB UK	−0.09	−0.14	−0.14	−0.15
	(−0.10,0.34)	(−0.16,0.35)	(−0.16,0.36)	(−0.16,0.35)
RealB EU	0.46	0.38	0.38	0.38
	(0.37,0.82)	(0.27,0.83)	(0.26,0.84)	(0.25,0.84)
	VAR			
Equity US	0.02	−0.02	−0.02	−0.02
	(−0.04,0.19)	(−0.09,0.19)	(−0.09,0.19)	(−0.09,0.19)
Equity UK	−0.02	0.11	0.12	0.12
	(−0.14,0.17)	(−0.10,0.23)	(−0.10,0.23)	(−0.10,0.23)
Equity JP	−0.02	−0.01	−0.01	−0.01
	(−0.12,0.04)	(−0.13,0.04)	(−0.14,0.05)	(−0.14,0.05)
Equity NL	0.12	0.04	0.03	0.03
	(−0.11,0.13)	(−0.10,0.11)	(−0.10,0.11)	(−0.10,0.11)
NomB US	−0.11	−0.11	−0.12	−0.12
	(−0.23,0.08)	(−0.24,0.09)	(−0.24,0.10)	(−0.24,0.10)
NomB UK	0.23	0.23	0.23	0.23
	(−0.24,0.37)	(−0.26,0.41)	(−0.26,0.42)	(−0.26,0.42)
NomB JP	−0.05	−0.08	−0.08	−0.08
	(−0.08,0.17)	(−0.09,0.16)	(−0.09,0.16)	(−0.09,0.16)
NomB NL	0.30	0.34	0.35	0.35
	(−0.14,0.35)	(−0.19,0.40)	(−0.19,0.39)	(−0.19,0.41)
RealB US	0.19	0.40	0.41	0.42
	(−0.02,0.30)	(0.03,0.56)	(0.02,0.58)	(0.02,0.60)
RealB UK	−0.08	−0.16	−0.16	−0.16
	(−0.12,0.33)	(−0.18,0.31)	(−0.18,0.31)	(−0.19,0.31)
RealB EU	0.43	0.26	0.25	0.24
	(0.33,0.81)	(0.21,0.82)	(0.20,0.82)	(0.19,0.83)

Table 2.7: Optimal allocation strategy for French investor with currency hedge

This table reports the French optimal allocation of the hedging portfolio in percentages across different horizons. The portfolio weights have been given for both the ECVAR and the VAR model. In this allocation the investors is not exposed to currency risk as exchange rate risk is hedged by forward contracts. The number in parentheses are the lower and upper bounds of the corresponding 95% Bootstrap confidence intervals.

	Investment horizon (months)			
	1	12	24	60
		ECVAR		
Equity US	−0.02 (−0.08,0.07)	−0.06 (−0.13,0.08)	−0.07 (−0.14,0.08)	−0.07 (−0.14,0.08)
Equity UK	0.07 (−0.09,0.14)	0.12 (−0.07,0.18)	0.13 (−0.07,0.19)	0.13 (−0.07,0.20)
Equity JP	0.06 (−0.01,0.08)	0.06 (−0.01,0.09)	0.06 (−0.01,0.10)	0.06 (−0.01,0.10)
Equity FR	−0.09 (−0.12,0.05)	−0.09 (−0.13,0.04)	−0.09 (−0.14,0.04)	−0.09 (−0.14,0.04)
NomB US	−0.08 (−0.16,0.07)	0.00 (−0.12,0.13)	0.00 (−0.12,0.14)	0.01 (−0.11,0.14)
NomB UK	−0.13 (−0.30,0.04)	−0.14 (−0.34,0.07)	−0.14 (−0.35,0.08)	−0.14 (−0.36,0.08)
NomB JP	0.89 (0.56,0.99)	0.95 (0.50,1.08)	0.95 (0.48,1.08)	0.95 (0.47,1.08)
NomB FR	−0.07 (−0.26,0.18)	−0.13 (−0.30,0.17)	−0.13 (−0.31,0.17)	−0.13 (−0.31,0.18)
RealB US	−0.11 (−0.22,0.08)	−0.08 (−0.22,0.17)	−0.07 (−0.22,0.19)	−0.07 (−0.22,0.19)
RealB UK	0.03 (−0.04,0.27)	0.01 (−0.08,0.31)	0.01 (−0.08,0.31)	0.01 (−0.09,0.32)
RealB EU	0.44 (0.17,0.63)	0.34 (0.03,0.60)	0.33 (0.01,0.60)	0.33 (0.01,0.60)
		VAR		
Equity US	−0.02 (−0.08,0.08)	−0.04 (−0.11,0.08)	−0.05 (−0.11,0.08)	−0.05 (−0.12,0.08)
Equity UK	0.08 (−0.09,0.14)	0.11 (−0.07,0.18)	0.11 (−0.06,0.18)	0.12 (−0.06,0.19)
Equity JP	0.06 (−0.01,0.08)	0.06 (−0.01,0.09)	0.06 (−0.01,0.09)	0.06 (−0.01,0.09)
Equity FR	−0.09 (−0.13,0.04)	−0.09 (−0.13,0.04)	−0.09 (−0.13,0.04)	−0.09 (−0.14,0.04)
NomB US	−0.08 (−0.17,0.06)	−0.01 (−0.12,0.13)	0.00 (−0.12,0.14)	0.00 (−0.12,0.14)
NomB UK	−0.14 (−0.30,0.03)	−0.14 (−0.34,0.06)	−0.14 (−0.35,0.06)	−0.14 (−0.35,0.06)
NomB JP	0.89 (0.55,0.98)	0.98 (0.51,1.07)	0.98 (0.50,1.07)	0.98 (0.50,1.07)
NomB FR	−0.05 (−0.25,0.18)	−0.15 (−0.30,0.15)	−0.15 (−0.31,0.15)	−0.16 (−0.31,0.15)
RealB US	−0.11 (−0.21,0.08)	−0.10 (−0.21,0.17)	−0.10 (−0.21,0.18)	−0.10 (−0.21,0.18)
RealB UK	0.04 (−0.04,0.28)	0.02 (−0.06,0.30)	0.02 (−0.06,0.30)	0.02 (−0.06,0.30)
RealB EU	0.42 (0.15,0.61)	0.35 (0.04,0.60)	0.35 (0.03,0.60)	0.35 (0.03,0.61)

Table 2.8: Optimal allocation strategy for French investor exposed to currency risk

This table reports the French optimal allocation of the hedging portfolio in percentages across different horizons. The portfolio weights have been given for both the ECVAR and the VAR model. In this allocation the investors is not exposed to currency risk as exchange rate risk is hedged by forward contracts. The number in parentheses are the lower and upper bounds of the corresponding 95% Bootstrap confidence intervals.

	Investment horizon (months)			
	1	12	24	60
		ECVAR		
Equity US	0.11 (−0.05,0.20)	0.08 (−0.09,0.19)	0.08 (−0.09,0.20)	0.08 (−0.10,0.20)
Equity UK	0.15 (−0.12,0.17)	0.20 (−0.09,0.20)	0.20 (−0.09,0.21)	0.20 (−0.09,0.21)
Equity JP	0.02 (−0.10,0.06)	0.01 (−0.12,0.05)	0.01 (−0.12,0.05)	0.01 (−0.12,0.05)
Equity FR	−0.16 (−0.19,0.11)	−0.16 (−0.18,0.11)	−0.16 (−0.17,0.12)	−0.16 (−0.17,0.12)
NomB US	−0.12 (−0.20,0.08)	−0.07 (−0.21,0.08)	−0.06 (−0.21,0.09)	−0.06 (−0.22,0.09)
NomB UK	0.05 (−0.21,0.15)	0.03 (−0.27,0.19)	0.03 (−0.27,0.19)	0.03 (−0.28,0.19)
NomB JP	0.04 (−0.05,0.17)	0.02 (−0.07,0.16)	0.03 (−0.07,0.17)	0.03 (−0.07,0.17)
NomB FR	0.11 (−0.19,0.31)	0.07 (−0.18,0.36)	0.06 (−0.18,0.37)	0.06 (−0.19,0.38)
RealB US	0.14 (−0.01,0.29)	0.23 (0.03,0.53)	0.23 (0.03,0.56)	0.23 (0.02,0.58)
RealB UK	0.04 (0.00,0.34)	0.01 (−0.05,0.32)	0.01 (−0.06,0.32)	0.01 (−0.06,0.32)
RealB EU	0.62 (0.31,0.83)	0.59 (0.19,0.80)	0.58 (0.18,0.80)	0.58 (0.17,0.80)
		VAR		
Equity US	0.12 (−0.04,0.20)	0.07 (−0.08,0.19)	0.07 (−0.09,0.19)	0.07 (−0.09,0.19)
Equity UK	0.15 (−0.12,0.17)	0.20 (−0.08,0.21)	0.21 (−0.08,0.21)	0.21 (−0.08,0.22)
Equity JP	0.01 (−0.10,0.05)	0.01 (−0.11,0.05)	0.01 (−0.12,0.05)	0.01 (−0.12,0.05)
Equity FR	−0.16 (−0.19,0.10)	−0.15 (−0.18,0.10)	−0.15 (−0.17,0.10)	−0.15 (−0.17,0.11)
NomB US	−0.13 (−0.21,0.07)	−0.08 (−0.20,0.08)	−0.07 (−0.20,0.09)	−0.07 (−0.21,0.09)
NomB UK	0.04 (−0.21,0.16)	0.04 (−0.26,0.18)	0.04 (−0.26,0.18)	0.04 (−0.26,0.18)
NomB JP	0.04 (−0.05,0.17)	0.03 (−0.07,0.16)	0.03 (−0.07,0.16)	0.03 (−0.07,0.16)
NomB FR	0.14 (−0.18,0.35)	0.08 (−0.17,0.38)	0.08 (−0.17,0.38)	0.08 (−0.18,0.38)
RealB US	0.14 (0.00,0.29)	0.26 (0.04,0.50)	0.27 (0.04,0.53)	0.27 (0.03,0.54)
RealB UK	0.04 (0.00,0.33)	0.01 (−0.05,0.32)	0.01 (−0.06,0.32)	0.00 (−0.06,0.32)
RealB EU	0.61 (0.31,0.83)	0.53 (0.19,0.81)	0.52 (0.18,0.81)	0.52 (0.18,0.82)

Table 2.9: Optimal allocation strategy for German investor with currency hedge

This table reports the German optimal allocation of the hedging portfolio in percentages across different horizons. The portfolio weights have been given for both the ECVAR and the VAR model. In this allocation the investors is not exposed to currency risk as exchange rate risk is hedged by forward contracts. The number in parentheses are the lower and upper bounds of the corresponding 95% Bootstrap confidence intervals.

	Investment horizon (months)			
	1	12	24	60
		ECVAR		
Equity US	−0.04 (−0.10,0.06)	−0.06 (−0.13,0.07)	−0.06 (−0.13,0.07)	−0.06 (−0.14,0.07)
Equity UK	−0.08 (−0.17,0.07)	−0.07 (−0.18,0.11)	−0.07 (−0.19,0.12)	−0.07 (−0.19,0.12)
Equity JP	0.04 (−0.01,0.07)	0.04 (−0.02,0.08)	0.05 (−0.02,0.08)	0.05 (−0.02,0.08)
Equity GER	0.09 (−0.03,0.15)	0.09 (−0.05,0.16)	0.10 (−0.05,0.16)	0.10 (−0.05,0.16)
NomB US	−0.06 (−0.16,0.07)	0.00 (−0.12,0.14)	0.00 (−0.12,0.15)	0.01 (−0.12,0.16)
NomB UK	−0.18 (−0.29,−0.01)	−0.21 (−0.35,0.00)	−0.22 (−0.36,0.00)	−0.22 (−0.36,0.00)
NomB JP	0.75 (0.50,0.91)	0.84 (0.46,1.00)	0.84 (0.46,1.01)	0.84 (0.46,1.01)
NomB GER	0.05 (−0.15,0.17)	0.04 (−0.21,0.20)	0.04 (−0.23,0.19)	0.04 (−0.23,0.19)
RealB US	−0.10 (−0.24,0.07)	−0.11 (−0.27,0.14)	−0.11 (−0.27,0.14)	−0.11 (−0.27,0.15)
RealB UK	0.04 (−0.05,0.28)	0.01 (−0.07,0.30)	0.01 (−0.07,0.31)	0.01 (−0.07,0.31)
RealB EU	0.47 (0.25,0.61)	0.42 (0.11,0.65)	0.42 (0.11,0.65)	0.43 (0.10,0.66)
		VAR		
Equity US	−0.05 (−0.10,0.06)	−0.08 (−0.14,0.06)	−0.08 (−0.14,0.06)	−0.08 (−0.15,0.06)
Equity UK	−0.07 (−0.17,0.07)	−0.03 (−0.15,0.13)	−0.03 (−0.15,0.14)	−0.03 (−0.15,0.14)
Equity JP	0.04 (−0.01,0.07)	0.04 (−0.01,0.08)	0.04 (−0.01,0.08)	0.04 (−0.01,0.08)
Equity GER	0.09 (−0.03,0.15)	0.08 (−0.06,0.14)	0.08 (−0.06,0.14)	0.08 (−0.06,0.14)
NomB US	−0.07 (−0.17,0.05)	−0.03 (−0.15,0.11)	−0.03 (−0.15,0.12)	−0.03 (−0.15,0.13)
NomB UK	−0.17 (−0.28,−0.01)	−0.20 (−0.34,0.01)	−0.20 (−0.35,0.01)	−0.21 (−0.35,0.01)
NomB JP	0.75 (0.48,0.89)	0.84 (0.44,0.99)	0.84 (0.43,0.99)	0.84 (0.43,1.00)
NomB GER	0.05 (−0.15,0.18)	0.08 (−0.17,0.23)	0.08 (−0.18,0.23)	0.08 (−0.18,0.23)
RealB US	−0.07 (−0.20,0.09)	−0.03 (−0.17,0.20)	−0.03 (−0.18,0.21)	−0.02 (−0.17,0.22)
RealB UK	0.04 (−0.05,0.27)	0.00 (−0.07,0.29)	−0.01 (−0.07,0.30)	−0.01 (−0.08,0.30)
RealB EU	0.46 (0.22,0.59)	0.34 (0.04,0.58)	0.34 (0.03,0.59)	0.33 (0.03,0.59)

Table 2.10: Optimal allocation strategy for German investor exposed to currency risk

This table reports the German optimal allocation of the hedging portfolio in percentages across different horizons. The portfolio weights have been given for both the ECVAR and the VAR model. In this allocation the investors is not exposed to currency risk as exchange rate risk is hedged by forward contracts. The number in parentheses are the lower and upper bounds of the corresponding 95% Bootstrap confidence intervals.

	Investment horizon (months)			
	1	12	24	60
	ECVAR			
Equity US	0.03 (−0.04,0.16)	−0.01 (−0.08,0.16)	−0.02 (−0.09,0.16)	−0.02 (−0.09,0.15)
Equity UK	−0.10 (−0.22,0.07)	0.00 (−0.20,0.14)	0.00 (−0.20,0.14)	0.01 (−0.21,0.15)
Equity JP	−0.03 (−0.10,0.04)	−0.03 (−0.11,0.04)	−0.03 (−0.11,0.04)	−0.03 (−0.11,0.04)
Equity GER	0.18 (0.02,0.20)	0.14 (0.00,0.20)	0.14 (−0.01,0.20)	0.14 (−0.01,0.20)
NomB US	−0.08 (−0.16,0.08)	−0.06 (−0.15,0.10)	−0.06 (−0.16,0.11)	−0.06 (−0.16,0.11)
NomB UK	0.08 (−0.16,0.14)	0.12 (−0.17,0.19)	0.12 (−0.18,0.19)	0.12 (−0.18,0.19)
NomB JP	0.03 (−0.04,0.16)	0.01 (−0.05,0.15)	0.01 (−0.05,0.16)	0.01 (−0.04,0.16)
NomB GER	0.23 (−0.09,0.26)	0.26 (−0.16,0.31)	0.26 (−0.17,0.31)	0.26 (−0.18,0.32)
RealB US	0.16 (−0.01,0.27)	0.30 (−0.01,0.44)	0.31 (−0.02,0.46)	0.32 (−0.03,0.47)
RealB UK	0.10 (0.04,0.33)	0.01 (−0.03,0.33)	0.01 (−0.04,0.33)	0.00 (−0.04,0.33)
RealB EU	0.41 (0.33,0.74)	0.27 (0.21,0.78)	0.26 (0.20,0.80)	0.25 (0.19,0.81)
	VAR			
Equity US	0.02 (−0.06,0.15)	−0.01 (−0.09,0.14)	−0.01 (−0.10,0.14)	−0.02 (−0.10,0.14)
Equity UK	−0.11 (−0.24,0.07)	−0.05 (−0.21,0.12)	−0.04 (−0.22,0.12)	−0.04 (−0.22,0.13)
Equity JP	−0.03 (−0.10,0.04)	−0.04 (−0.11,0.03)	−0.04 (−0.12,0.03)	−0.04 (−0.12,0.03)
Equity GER	0.19 (0.02,0.22)	0.17 (0.02,0.21)	0.17 (0.02,0.22)	0.17 (0.02,0.22)
NomB US	−0.09 (−0.18,0.07)	−0.09 (−0.20,0.07)	−0.09 (−0.20,0.08)	−0.09 (−0.21,0.08)
NomB UK	0.09 (−0.16,0.15)	0.11 (−0.18,0.20)	0.11 (−0.19,0.20)	0.12 (−0.19,0.21)
NomB JP	0.03 (−0.05,0.17)	0.01 (−0.07,0.17)	0.01 (−0.07,0.17)	0.01 (−0.07,0.17)
NomB GER	0.21 (−0.12,0.28)	0.25 (−0.15,0.31)	0.25 (−0.16,0.32)	0.25 (−0.16,0.32)
RealB US	0.17 (0.00,0.27)	0.35 (0.05,0.50)	0.37 (0.06,0.52)	0.38 (0.06,0.54)
RealB UK	0.10 (0.03,0.34)	0.02 (−0.04,0.33)	0.02 (−0.04,0.32)	0.02 (−0.05,0.32)
RealB EU	0.42 (0.32,0.75)	0.27 (0.18,0.75)	0.26 (0.16,0.75)	0.26 (0.16,0.76)

Table 2.11: Optimal foreign bond allocation with currency risk

This table reports the optimal bond allocation of the hedging portfolio in percentages across different horizons for all three investors, namely the Dutch, French and German. The asset choice of the investor excludes European inflation-linked bonds. The portfolio weights have been given for both the ECVAR and the VAR model. In this allocation the investors are exposed to currency risk since exchange rates are not hedged by forward contracts. The number in parentheses are the lower and upper bounds of the corresponding 95% Bootstrap confidence intervals.

	Investment horizon (months)			
	1	12	24	60
	ECVAR			
NomB NL	0.57 (0.38,0.73)	0.52 (0.31,0.72)	0.52 (0.30,0.72)	0.51 (0.30,0.73)
RealB UK	0.16 (0.05,0.38)	0.17 (0.04,0.41)	0.17 (0.03,0.41)	0.16 (0.02,0.42)
RealB US	0.30 (0.14,0.52)	0.26 (0.07,0.52)	0.26 (0.07,0.53)	0.26 (0.06,0.53)
NomB FR	0.63 (0.38,0.73)	0.52 (0.31,0.72)	0.51 (0.30,0.72)	0.51 (0.30,0.73)
RealB UK	0.18 (0.05,0.38)	0.22 (0.04,0.41)	0.22 (0.03,0.41)	0.22 (0.02,0.42)
RealB US	0.18 (0.14,0.52)	0.15 (0.07,0.52)	0.15 (0.07,0.53)	0.15 (0.06,0.53)
NomB GER	0.58 (0.34,0.68)	0.56 (0.28,0.69)	0.56 (0.28,0.70)	0.56 (0.28,0.70)
RealB UK	0.14 (0.03,0.33)	0.20 (0.06,0.40)	0.21 (0.06,0.40)	0.21 (0.06,0.40)
RealB US	0.22 (0.17,0.53)	0.15 (0.09,0.53)	0.15 (0.08,0.53)	0.15 (0.08,0.53)
	VAR			
NomB NL	0.56 (0.38,0.72)	0.50 (0.30,0.69)	0.50 (0.29,0.69)	0.50 (0.29,0.69)
RealB UK	0.16 (0.04,0.37)	0.20 (0.07,0.42)	0.20 (0.07,0.42)	0.20 (0.07,0.43)
RealB US	0.30 (0.14,0.53)	0.25 (0.09,0.53)	0.25 (0.09,0.53)	0.25 (0.09,0.53)
NomB FR	0.63 (0.39,0.77)	0.51 (0.28,0.71)	0.51 (0.28,0.71)	0.50 (0.27,0.71)
RealB UK	0.19 (0.04,0.38)	0.22 (0.09,0.43)	0.22 (0.09,0.44)	0.22 (0.09,0.44)
RealB US	0.18 (0.13,0.52)	0.15 (0.09,0.52)	0.15 (0.08,0.52)	0.14 (0.08,0.52)
NomB GER	0.58 (0.33,0.68)	0.50 (0.24,0.66)	0.50 (0.23,0.66)	0.49 (0.23,0.66)
RealB UK	0.14 (0.02,0.32)	0.16 (0.04,0.38)	0.16 (0.05,0.38)	0.16 (0.05,0.39)
RealB US	0.22 (0.16,0.54)	0.18 (0.11,0.54)	0.17 (0.11,0.54)	0.17 (0.11,0.54)

Table 2.12: Dutch Bayesian optimal allocation exposed to currency risk

This table reports the Dutch optimal bond allocation of the hedging portfolio in percentages across different horizons. The portfolio weights have been given for both the ECVAR and the VAR model. In this allocation the investors are exposed to currency risk since exchange rates are not hedged by forward contracts. The number in parentheses are the lower and upper bounds of the corresponding 95% confidence intervals.

	Investment horizon (months)			
	1	12	24	60
	ECVAR			
Equity US	0.11 (−0.04,0.26)	0.16 (−0.03,0.32)	0.16 (−0.04,0.32)	0.16 (−0.04,0.32)
Equity UK	−0.10 (−0.32,0.13)	−0.07 (−0.33,0.23)	−0.06 (−0.33,0.24)	−0.05 (−0.32,0.24)
Equity JP	−0.01 (−0.09,0.08)	−0.03 (−0.13,0.06)	−0.04 (−0.13,0.06)	−0.04 (−0.13,0.06)
Equity NL	0.06 (−0.09,0.21)	0.06 (−0.14,0.26)	0.05 (−0.14,0.26)	0.05 (−0.14,0.26)
NomB US	−0.34 (−0.54,−0.12)	−0.24 (−0.51,0.07)	−0.22 (−0.50,0.10)	−0.20 (−0.49,0.12)
NomB UK	0.26 (0.03,0.44)	0.15 (−0.18,0.44)	0.12 (−0.20,0.43)	0.11 (−0.22,0.42)
NomB JP	−0.02 (−0.13,0.09)	0.04 (−0.12,0.20)	0.04 (−0.12,0.21)	0.05 (−0.12,0.21)
NomB NL	0.37 (0.12,0.61)	0.32 (−0.03,0.68)	0.30 (−0.06,0.67)	0.29 (−0.08,0.66)
RealB US	0.29 (0.09,0.49)	0.21 (−0.08,0.48)	0.19 (−0.11,0.48)	0.18 (−0.13,0.48)
RealB UK	−0.12 (−0.30,0.09)	0.01 (−0.27,0.30)	0.03 (−0.26,0.33)	0.05 (−0.26,0.34)
RealB EU	0.48 (0.26,0.71)	0.40 (0.10,0.70)	0.40 (0.09,0.72)	0.41 (0.09,0.74)
	VAR			
Equity US	0.10 (−0.03,0.23)	0.15 (0.01,0.28)	0.15 (0.01,0.28)	0.15 (0.01,0.29)
Equity UK	−0.11 (−0.31,0.09)	−0.14 (−0.35,0.07)	−0.14 (−0.35,0.07)	−0.15 (−0.35,0.08)
Equity JP	−0.01 (−0.08,0.07)	−0.04 (−0.12,0.04)	−0.04 (−0.12,0.04)	−0.04 (−0.12,0.04)
Equity NL	0.09 (−0.03,0.21)	0.15 (0.01,0.28)	0.15 (0.01,0.29)	0.15 (0.01,0.29)
NomB US	−0.34 (−0.52,−0.15)	−0.29 (−0.51,−0.06)	−0.29 (−0.51,−0.05)	−0.29 (−0.51,−0.05)
NomB UK	0.30 (0.16,0.44)	0.27 (0.08,0.46)	0.27 (0.07,0.46)	0.27 (0.07,0.46)
NomB JP	−0.02 (−0.11,0.07)	0.03 (−0.07,0.13)	0.03 (−0.07,0.14)	0.04 (−0.07,0.14)
NomB NL	0.37 (0.15,0.59)	0.42 (0.15,0.69)	0.42 (0.14,0.70)	0.42 (0.14,0.70)
RealB US	0.29 (0.12,0.47)	0.23 (0.02,0.45)	0.23 (0.01,0.46)	0.22 (0.00,0.46)
RealB UK	−0.14 (−0.29,0.01)	−0.07 (−0.25,0.11)	−0.06 (−0.24,0.12)	−0.06 (−0.24,0.12)
RealB EU	0.46 (0.27,0.66)	0.30 (0.08,0.52)	0.29 (0.07,0.52)	0.29 (0.06,0.52)

Table 2.13: French Bayesian optimal allocation exposed to currency risk

This table reports the French optimal bond allocation of the hedging portfolio in percentages across different horizons. The portfolio weights have been given for both the ECVAR and the VAR model. In this allocation the investors are exposed to currency risk since exchange rates are not hedged by forward contracts. The number in parentheses are the lower and upper bounds of the corresponding 95% confidence intervals.

	Investment horizon (months)			
	1	12	24	60
	ECVAR			
Equity US	0.14 (0.02,0.25)	0.17 (0.01,0.31)	0.17 (0.01,0.31)	0.17 (0.00,0.31)
Equity UK	0.12 (−0.05,0.27)	0.10 (−0.13,0.33)	0.09 (−0.15,0.33)	0.08 (−0.16,0.33)
Equity JP	0.01 (−0.06,0.07)	−0.01 (−0.09,0.08)	−0.02 (−0.10,0.08)	−0.02 (−0.10,0.08)
Equity NL	−0.16 (−0.22,−0.09)	−0.12 (−0.24,0.02)	−0.11 (−0.24,0.05)	−0.10 (−0.24,0.07)
NomB US	−0.22 (−0.39,−0.05)	−0.19 (−0.41,0.07)	−0.18 (−0.42,0.09)	−0.17 (−0.41,0.10)
NomB UK	0.01 (−0.16,0.18)	−0.01 (−0.25,0.24)	−0.01 (−0.26,0.24)	−0.01 (−0.27,0.25)
NomB JP	0.05 (−0.04,0.14)	0.07 (−0.07,0.21)	0.07 (−0.08,0.21)	0.08 (−0.08,0.22)
NomB FR	0.19 (−0.02,0.39)	0.19 (−0.09,0.48)	0.19 (−0.11,0.49)	0.18 (−0.12,0.49)
RealB US	0.21 (0.04,0.38)	0.19 (−0.05,0.44)	0.19 (−0.07,0.44)	0.18 (−0.09,0.45)
RealB UK	0.03 (−0.13,0.19)	0.09 (−0.17,0.33)	0.09 (−0.17,0.34)	0.10 (−0.17,0.36)
RealB EU	0.64 (0.44,0.83)	0.52 (0.24,0.79)	0.51 (0.23,0.80)	0.51 (0.22,0.80)
	VAR			
Equity US	0.14 (0.03,0.24)	0.20 (0.10,0.31)	0.20 (0.10,0.31)	0.21 (0.10,0.32)
Equity UK	0.07 (−0.06,0.20)	0.13 (−0.01,0.27)	0.14 (0.00,0.28)	0.14 (0.00,0.28)
Equity JP	0.03 (−0.02,0.09)	0.00 (−0.06,0.07)	0.00 (−0.06,0.06)	0.00 (−0.06,0.06)
Equity FR	−0.14 (−0.19,−0.09)	−0.19 (−0.25,−0.13)	−0.19 (−0.25,−0.13)	−0.19 (−0.25,−0.13)
NomB US	−0.28 (−0.44,−0.13)	−0.28 (−0.46,−0.10)	−0.28 (−0.46,−0.09)	−0.28 (−0.46,−0.09)
NomB UK	0.10 (−0.04,0.24)	0.00 (−0.15,0.17)	0.00 (−0.16,0.17)	−0.01 (−0.17,0.17)
NomB JP	0.07 (−0.01,0.15)	0.12 (0.03,0.22)	0.12 (0.03,0.22)	0.13 (0.03,0.22)
NomB FR	0.20 (−0.03,0.43)	0.27 (0.01,0.54)	0.27 (0.01,0.54)	0.27 (0.00,0.54)
RealB US	0.19 (0.04,0.35)	0.18 (0.00,0.37)	0.18 (−0.01,0.38)	0.18 (−0.02,0.38)
RealB UK	0.00 (−0.14,0.14)	0.09 (−0.07,0.26)	0.10 (−0.06,0.27)	0.10 (−0.06,0.27)
RealB EU	0.61 (0.40,0.82)	0.46 (0.22,0.70)	0.46 (0.21,0.70)	0.46 (0.21,0.70)

Table 2.14: German Bayesian optimal allocation exposed to currency risk

This table reports the German optimal bond allocation of the hedging portfolio in percentages across different horizons. The portfolio weights have been given for both the ECVAR and the VAR model. In this allocation the investors are exposed to currency risk since exchange rates are not hedged by forward contracts. The number in parentheses are the lower and upper bounds of the corresponding 95% confidence intervals.

	Investment horizon (months)			
	1	12	24	60
		ECVAR		
Equity US	0.10 (0.00,0.20)	0.14 (0.01,0.26)	0.14 (0.01,0.27)	0.14 (0.00,0.28)
Equity UK	−0.26 (−0.42,−0.10)	−0.27 (−0.47,−0.06)	−0.27 (−0.48,−0.03)	−0.26 (−0.48,−0.01)
Equity JP	−0.02 (−0.08,0.04)	−0.04 (−0.10,0.03)	−0.04 (−0.10,0.03)	−0.04 (−0.10,0.03)
Equity GER	0.22 (0.11,0.34)	0.24 (0.06,0.41)	0.24 (0.04,0.42)	0.23 (0.03,0.41)
NomB US	−0.27 (−0.42,−0.11)	−0.23 (−0.43,−0.01)	−0.22 (−0.43,0.02)	−0.22 (−0.44,0.03)
NomB UK	0.17 (0.05,0.30)	0.11 (−0.09,0.29)	0.10 (−0.13,0.29)	0.09 (−0.15,0.29)
NomB JP	0.07 (−0.01,0.15)	0.09 (−0.04,0.20)	0.09 (−0.05,0.21)	0.09 (−0.05,0.21)
NomB GER	0.24 (0.05,0.43)	0.26 (−0.03,0.57)	0.26 (−0.06,0.59)	0.25 (−0.07,0.60)
RealB US	0.22 (0.07,0.37)	0.19 (−0.03,0.40)	0.18 (−0.05,0.40)	0.18 (−0.07,0.41)
RealB UK	0.11 (−0.03,0.25)	0.22 (0.01,0.44)	0.23 (0.01,0.46)	0.24 (0.01,0.48)
RealB EU	0.42 (0.24,0.58)	0.29 (0.05,0.53)	0.29 (0.04,0.55)	0.30 (0.03,0.57)
		VAR		
Equity US	0.09 (0.00,0.18)	0.13 (0.04,0.22)	0.13 (0.04,0.23)	0.13 (0.04,0.23)
Equity UK	−0.25 (−0.39,−0.11)	−0.27 (−0.41,−0.13)	−0.27 (−0.41,−0.13)	−0.27 (−0.42,−0.13)
Equity JP	−0.03 (−0.08,0.02)	−0.05 (−0.10,0.00)	−0.05 (−0.10,0.00)	−0.05 (−0.10,0.00)
Equity GER	0.22 (0.13,0.32)	0.24 (0.13,0.35)	0.24 (0.13,0.35)	0.24 (0.13,0.36)
NomB US	−0.25 (−0.39,−0.12)	−0.23 (−0.37,−0.09)	−0.23 (−0.37,−0.08)	−0.23 (−0.37,−0.08)
NomB UK	0.17 (0.06,0.27)	0.13 (0.00,0.25)	0.12 (0.00,0.25)	0.12 (−0.01,0.25)
NomB JP	0.07 (0.00,0.13)	0.11 (0.03,0.18)	0.11 (0.03,0.18)	0.11 (0.03,0.18)
NomB GER	0.23 (0.07,0.38)	0.25 (0.06,0.44)	0.25 (0.06,0.44)	0.25 (0.06,0.45)
RealB US	0.22 (0.10,0.35)	0.20 (0.06,0.35)	0.20 (0.05,0.35)	0.20 (0.05,0.35)
RealB UK	0.10 (−0.02,0.22)	0.20 (0.05,0.34)	0.20 (0.05,0.35)	0.20 (0.06,0.36)
RealB EU	0.43 (0.30,0.57)	0.30 (0.15,0.45)	0.29 (0.14,0.45)	0.29 (0.14,0.44)

Table 2.15: Conditional Dutch Bayesian optimal allocation with currency risk

This table reports the average of the Dutch optimal bond allocation of the replicating portfolio across different horizons. We split the sample in three periods: Prior to the Financial crisis is classified from June 2005 to August 2008, during is classified from September 2008 to December 2009, and post crisis from January 2010 to December 2011. The portfolio weights have derived for a conditional ECVAR-type of investor in a Bayesian context. In this allocation the investor is exposed to currency risk as exchange rate risk is not hedged by forward contracts.

	Investment horizon (months)			
	1	12	24	60
	Prior the financial crisis			
NomB NL	8.6 %	13.7 %	13.9 %	13.9 %
RealB UK	12.4 %	10.9 %	10.9 %	10.9 %
RealB US	14.7 %	14.9 %	14.9 %	15.1 %
RealB EU	63.4 %	50.1 %	49.7 %	49.7 %
	During the financial crisis			
NomB NL	12.6 %	15.8 %	14.9 %	14.2 %
RealB UK	13.4 %	13.4 %	12.1 %	11.2 %
RealB US	14.2 %	13.3 %	14.0 %	14.8 %
RealB EU	60.3 %	48.1 %	48.8 %	49.4 %
	Post the financial crisis			
NomB NL	9.4 %	13.6 %	13.8 %	13.8 %
RealB UK	11.8 %	11.2 %	11.0 %	10.9 %
RealB US	15.1 %	15.1 %	15.0 %	15.1 %
RealB EU	62.2 %	49.8 %	49.6 %	49.7 %

Table 2.16: Conditional French Bayesian optimal allocation with currency risk

This table reports the average of the French optimal bond allocation of the replicating portfolio across different horizons. We split the sample in three periods: Prior to the Financial crisis is classified from June 2005 to August 2008, during is classified from September 2008 to December 2009, and post crisis from January 2010 to December 2011. The portfolio weights have derived for a conditional ECVAR-type of investor in a Bayesian context. In this allocation the investor is exposed to currency risk as exchange rate risk is not hedged by forward contracts.

	Investment horizon (months)			
	1	12	24	60
	Prior the financial crisis			
NomB FR	9.4 %	13.7 %	13.9 %	13.9 %
RealB UK	12.3 %	10.8 %	10.8 %	10.8 %
RealB US	14.2 %	15.0 %	14.9 %	15.1 %
RealB EU	62.9 %	50.1 %	49.8 %	49.7 %
	During the financial crisis			
NomB FR	13.7 %	16.2 %	15.1 %	14.3 %
RealB UK	14.6 %	13.4 %	12.1 %	11.3 %
RealB US	13.3 %	13.0 %	13.9 %	14.7 %
RealB EU	58.7 %	47.9 %	48.8 %	49.4 %
	Post the financial crisis			
NomB FR	10.4 %	13.7 %	13.9 %	13.8 %
RealB UK	10.5 %	11.0 %	10.9 %	10.9 %
RealB US	15.8 %	15.1 %	15.0 %	15.1 %
RealB EU	61.5 %	49.8 %	49.7 %	49.7 %

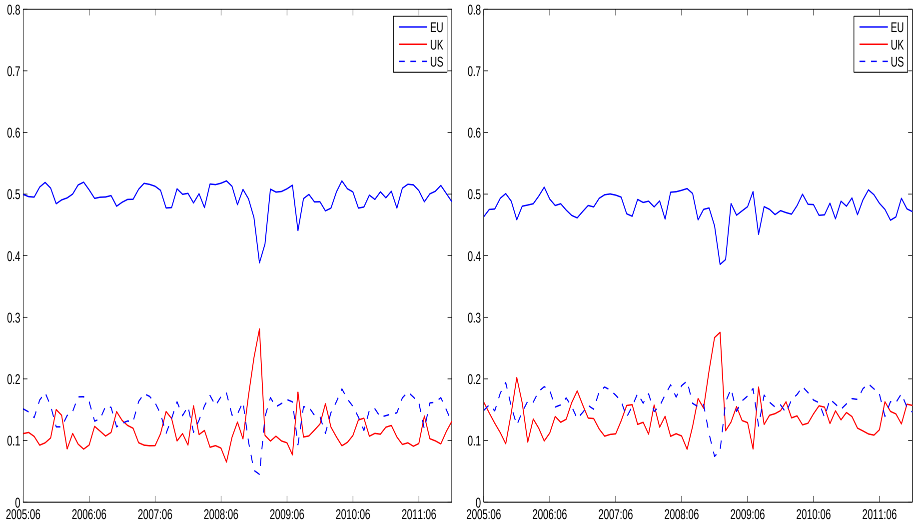
Table 2.17: Conditional German Bayesian optimal allocation with currency risk

This table reports the average of the German optimal bond allocation of the replicating portfolio across different horizons. We split the sample in three periods: Prior to the Financial crisis is classified from June 2005 to August 2008, during is classified from September 2008 to December 2009, and post crisis from January 2010 to December 2011. The portfolio weights have derived for a conditional ECVAR-type of investor in a Bayesian context. In this allocation the investor is exposed to currency risk as exchange rate risk is not hedged by forward contracts.

	Investment horizon (months)			
	1	12	24	60
	Prior the financial crisis			
NomB GER	3.4 %	9.0 %	10.4 %	11.3 %
RealB UK	10.2 %	11.2 %	11.4 %	11.5 %
RealB US	19.0 %	28.0 %	28.3 %	28.5 %
RealB EU	59.7 %	40.4 %	39.2 %	38.7 %
	During the financial crisis			
NomB GER	5.4 %	10.2 %	11.1 %	11.6 %
RealB UK	11.2 %	12.8 %	12.4 %	11.9 %
RealB US	18.5 %	27.1 %	27.7 %	28.2 %
RealB EU	58.3 %	39.1 %	38.4 %	38.4 %
	Post the financial crisis			
NomB GER	2.8 %	9.2 %	10.5 %	11.3 %
RealB UK	10.5 %	11.5 %	11.6 %	11.6 %
RealB US	18.7 %	28.0 %	28.3 %	28.5 %
RealB EU	59.6 %	40.0 %	39.0 %	38.6 %

Figure 2.1: Optimal conditional Bayesian allocation strategy for the Dutch investor

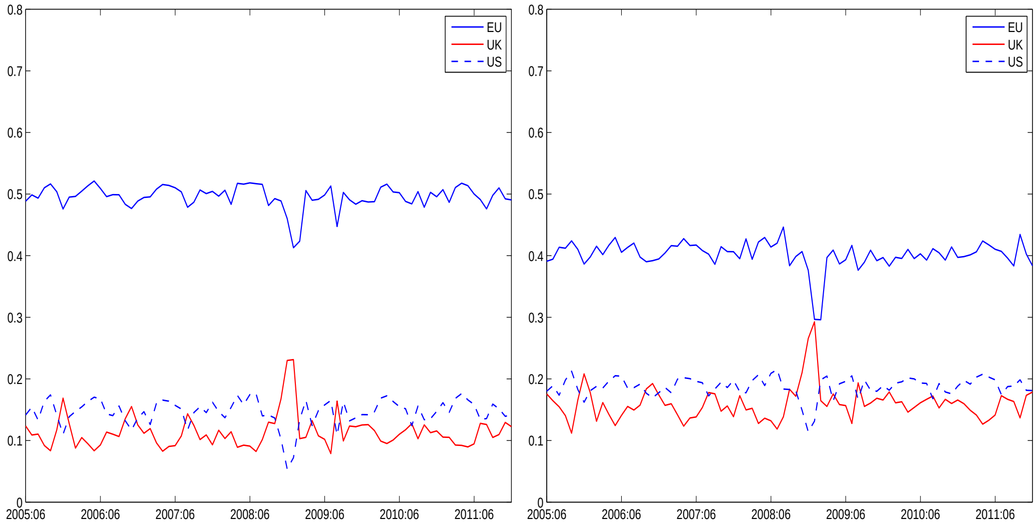
This figure presents the portfolio weights for EU, UK and US inflation-linked bonds for an investment horizon of 1 year. In this conditional framework the Dutch investor is exposed to currency risk. This allocation is the mean of all simulated strategies derived in our Bayesian framework.



(a) ECVAR (b) VAR

Figure 2.2: Optimal conditional Bayesian allocation strategy for the French investor

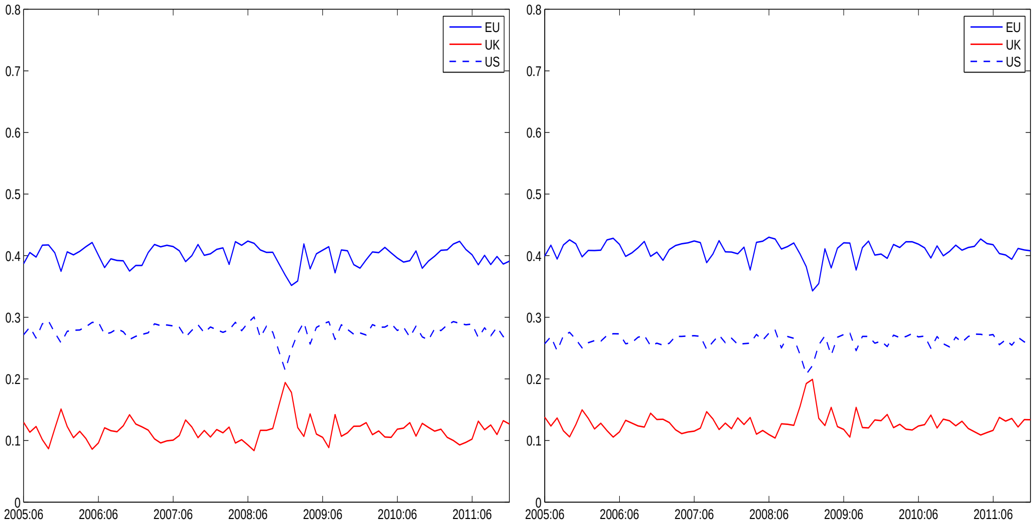
This figure presents the portfolio weights for EU, UK and US inflation-linked bonds for an investment horizon of 1 year. In this conditional framework the French investor is exposed to currency risk. This allocation is the mean of all simulated strategies derived in our Bayesian framework.



(a) ECVAR (b) VAR

Figure 2.3: Optimal conditional Bayesian allocation strategy for the German investor

This figure presents the portfolio weights for EU, UK and US inflation-linked bonds for an investment horizon of 1 year. In this conditional framework the German investor is exposed to currency risk. This allocation is the mean of all simulated strategies derived in our Bayesian framework.



(a) ECVAR (b) VAR

CHAPTER 3

The Inflation Risk Premium: The Impact of the Financial Crisis

3.1 Introduction

In most debt financing transactions, inflation risk is allocated to the debt holder instead of the debt issuer. In case of such nominal debt financing, the debt holder demands, in addition to the expected inflation rate, a premium associated to the expected inflation risk for that maturity. If uncertainty about future inflation is substantial, debt issuers are forced by markets to issue inflation-linked debt to immunize the holders from inflation risk[1]. For governments and institutional investors, the market inflation risk premium is an important factor in issuing long term debt or hedging liabilities that are exposed to inflation. In particular, the magnitude of the inflation risk premium determines whether governments can be cost efficient by issuing inflation-linked bonds. Due to the increasing market liquidity of inflation-linked bonds in the UK and US, studies have begun to empirically examine the inflation risk premium (e.g. Evans (2002) for the UK, whereas Ang, Bekaert, and Wei (2007b), and D'Amico, Kim, and Wei (2010) for the US). Since this premium cannot be directly observed in the market and relies on model specifications,

[1] Inflation-linked bonds have been introduced in South American countries to raise long term capital, e.g. Chile in 1951 and Argentina in 1973. Due to high uncertainty about future inflation in these countries, debt holders forced the issuers to bear inflation risk. For a further discussion, see e.g. García and Rixtel (2007).

the literature has offered a broad range of likely estimates for the inflation risk premium. However, less attention is given to how this uncertainty can be modeled for economic decision making.

In this chapter, we take an explicit approach to incorporate uncertainty about model specifications and examine the inflation risk premia in a Bayesian setting to correct for a small sample bias. To explain the large range of inflation risk premia reported in the literature, we determine empirical distributions of the inflation risk premium and present statistical ranges for these estimates. Our analysis starts with extracting the inflation risk premium implied by the nominal and real term structure of interest rates of both the UK and US markets in the period from 2004 to 2012. The typical approach in the literature is to identify real interest rates by inflation-linked bonds, but we use inflation swap rates to determine the real rates instead. Our approach is motivated by liquidity shocks observed in the TIPS market prior to 2003 and during the financial crisis (See e.g D'Amico et al. (2010)). As documented by Haubrich et al. (2011), US inflation swap rates responded less sharply to liquidity shocks in the market. Consequently, our estimation of the inflation risk premium is less influenced by liquidity issues. To enhance our model characteristics, we incorporate empirical nominal and real term premia to our measurement equations as well. This approach allows us to verify to what extent affine term structure models are able to capture the dynamics in real term premia.

Theory suggests a link between macroeconomic development and interest rates. Including macroeconomic factors in affine term structure models allowing to capture these dynamics and can improve the explanation of interest rates (Ang and Piazzesi, 2003). Prior literature has focused on the predictability of nominal bonds with the help of information from the macro economy (see e.g. Cochrane and Piazzesi (2005), Joslin, Priebsch, and Singleton (2010), Duffee (2011)). We contribute to the analysis by investigating the effect of macroeconomic factors on the inflation risk premium. The macroeconomic factors in our model have an impact on the unexpected inflation shock that is associated with the inflation risk premium. Consequently, the inflation risk premium depends on the included macroeconomic variables. We estimate a benchmark model with only inflation data and compare this to two models that add either macroeconomic data or survey data related to macroeconomic developments. In our Macro model, we use inflation factors that relate to the current condition of the macro economy and market volatility, whereas in the Survey model we incorporate expectations of the macroeconomic development such as inflation and economic confidence indicators. This approach allows us to link the inflation risk premium in these two models to macroeconomic developments and examine their impact on the inflation risk premium.

To evaluate these models, we first verify whether the Macro and Survey model improve the explanation of real interest rates compared to our benchmark. We confirm the importance of adding macroeconomic expectations to our model in the US during the financial crisis in 2008. Although our Survey model only slightly improves the measurement errors of the real rates across our whole sample, during the financial crisis the Survey model improves the fit by about 5% compared to the benchmark model. As a result of this improvement during the financial crisis, the Survey model leads to a better fit over the whole sample period. The Macro model, on the other hand, does not outperform the benchmark model. This suggests that expectations of the macroeconomy have more explanatory power for the real rate innovations in the US than actual macroeconomic development. Surpringly, we find that our Macro model improves the explanation of real rates in the UK rather than the Survey model. This result is driven by the timing difference of the impact of the financial crisis in the UK. While in the US interest rates started to decrease since 2007, the UK nominal rates rather sharply decreased in 2008. Since the macroeconomic factors improve the explanation of real rates, we investigate the effects on the inflation risk premium among these models.

To assess the small sample bias of the inflation risk premium, we first estimate our models by ignoring parameter uncertainty. Based on this methodology, we observe a declining trend for the inflation risk premium in all models for both markets. Prior to the financial crisis the inflation risk premia was positive in both markets, however during the financial crisis the inflation risk premia became negative. Although we find a decrease in the inflation risk premium during the crisis, it is hard to capture the magnitude of the inflation risk premium due to liquidity issues. After the financial crisis, we observe that rates increase but remain lower than pre-crisis levels. Where the US inflation risk premium returns to a positive level, the inflation risk premium in the UK remains negative. One of the factors leading to lower inflation risk premia in both markets is the drop in the nominal rates after the crisis. Many central banks implemented a zero interest rate policy after the financial crisis to stimulate macroeconomic development. It has been suggested that in order to recover from these low rates, central banks should refrain from deflationary measures. One of the consequences of such policy is an increase of inflation (see e.g. Krugman (1998) and Eggertsson and Woodford (2003)). The inflation risk associated with nominal rates remaining at zero bounds could explain the increasing inflation risk premia after the financial crisis. However, since the post crisis levels of the inflation risk premia remain quite low compared to pre-crisis levels, the markets do not fully reflect the inflation risk associated with low nominal rates.

To investigate the effect of parameter uncertainty on the inflation risk premium, we estimate our models for both markets using a Bayesian approach. Limited data on inflation-linked derivatives is likely to introduce a small sample bias. In particular, the high persistence of interest rates can aggravate this issue in the estimation of affine term structure models (Joslin, Singleton, and Zhu (2011)). Accordingly, we adopt a Bayesian approach that can reduce such biases and assess its impact on the inflation risk premium. Our findings suggest a wide range of likely estimates for the inflation risk premia in both the US and the UK. The 95% credibility intervals of our empirical distributions range from -95 to 88 basis points in the UK, whereas in the US we find a interval of -4 to 119 basis points over the sample period from 2004 to 2012. Although the mean of these distributions for the 5 year risk premium are about -8 basis points for the UK and 74 basis points in the US, the credibility intervals of these distributions include both positive and negative estimates. While these ranges quantify a large dispersion for the estimates of the inflation risk premium, we can conclude that the 5 year inflation risk premium in the US is positive with a probability of 97.2%. In the UK, we find a probability of 42 % that the 5 year inflation risk premium is negative. As a result, credibility intervals show wide ranges for estimates for the inflation risk premium. Hence, there is large uncertainty concerning the point estimates.

Our methodology and findings explain the wide range of estimates found by the affine term structure literature. For example, Ang et al. (2007b) find 115 basis points for the 5 year inflation risk premium, whereas D'Amico et al. (2010) find 36 basis points (see e.g. Bekaert and Wang (2010) for an overview in the US.). Since these US estimates fall within the credibility intervals for the inflation risk premia, it is hard to distinguish between the point estimates. Fewer studies have been conducted on the UK market. Evans (2002) estimates a negative inflation risk premium, although Risa (2001) and Joyce, Lildholdt, and Sorensen (2010) find substantial positive inflation risk premium of about 184 and 100 basis points. Our Bayesian methodology confirms their result of wide intervals as well. Since macroeconomic factors can improve the explanatory power of real rates, we examine the impact of the Macro and Survey model on the inflation risk premium. Our models reveal that the addition of macroeconomic factors leads to a wider dispersion of the inflation risk premium. The empirical distributions of the inflation risk premium are especially more platykurtic in the UK than in the US, resulting in larger credibility intervals than in the benchmark. For example, the results of our Macro model suggest a 95% credibility range of -131 to 143 basis points in the UK, whereas in the US our Survey model suggest an interval of -4 to 127 basis points. As a result, the impact of the macroeconomic variables leads to larger uncertainty about the inflation risk premium estimates in our models.

Given the wide dispersion for the estimate of the inflation risk premium, we investigate how the financial crisis impacts the uncertainty of the inflation risk premium. After the nominal interest rates decreased rapidly to relatively low levels for both the UK and US, both markets entered into a low nominal interest rate regime. By attaching more importance to the post crisis observations, we capture a shift in the inflation risk premium for the post crisis regime compared with the credibility intervals of our previous results. In the first part of our analysis we assign equal weights to each observation in our data period. The benefit of our approach is that we can use the entire sample period to identify our model, since discarding observations prior to the crisis and reestimating our model would be infeasible due to limited data. In the US we find a downward effect of about 36 basis points, shifting the mean of the distribution to 38 basis points. The 95% credibility interval ranges from -50 to 92 basis points, increasing the dispersion of the inflation risk premium estimate. As a consequence, negative estimates of the inflation risk premia are more likely. Surprisingly, we find in the UK an upward shift, increasing the mean to about 13 basis points. This leads to a 95% credibility interval from -105 to 150 basispoints. Again, we document a substantial dispersion for the estimate. This new empirical evidence suggests that the low interest rate regime after the financial crisis does not have a similar effect in both markets for the inflation risk premium. While an upward shift, as documented in the UK, would be expected due to the uncertainty of macro inflation risk and low nominal interest rates, our empirical evidence does not support this for the US.

Our contribution to the literature is threefold. First, we are the first to quantify the uncertainty associated with the inflation risk premium and quantify the impact of the financial crisis on the inflation risk premium. This chapter demonstrates how to extract the inflation risk premium from an affine term structure model of interest rates (Duffie and Kan, 1996) by using a Bayesian methodology. To this end, we introduce the Chi-squared estimation methodology of Hamilton and Wu (2012), which we apply to a term structure of interest rates with both nominal and real rates. This alternative approach to the typically Maximum Likelihood estimation allows us to employ the Bayesian methodology more easily. Since this methodology employs a two step estimation, we can easily address the issue of small sample bias in our sample. Second, we use a unique dataset of inflation swap rates to identify the real rate for both the UK and US markets. While inflation swap rates have been used by Haubrich et al. (2011), their study only focuses on the US market. Instead, we combine the insights of the macroeconomic literature on nominal interest rates (see e.g. Ang and Piazzesi (2003)) to study the inflation risk premium in both markets. In particular, we expand the literature on the inflation risk premium by showing how similar macroeconomic factors influence cross these markets. Finally, we contribute to the literature by explicitly including nominal and real term premia. To enhance identification of our structural parameters in our small sample, we use data-implied nominal and real term premia using Campbell-Shiller regressions, and exploit these in our model (Campbell and Shiller, 1991). While others rely on penalizing the maximum likelihood function to generate reasonable term premia (see e.g. Chernov and Mueller (2008)), we explicitly match our model implied and data implied term premia in our Chi-squared estimation methodology. Comparing our term premia to a study that mostly relates to our sample period (Haubrich et al., 2011), we find lower term premia for maturities of 5 and 10 years. As for the coefficients of the Campbell-Shiller regressions for the real returns, we find decreasing coefficients across maturity for both the UK and US market. This evidence is similar to the nominal pattern.

To summarize, we present a novel framework in which we formally quantify the uncertainty associated with estimating the inflation risk premium in affine term structure models. Overall, our results confirm the wide dispersion for the estimate of the inflation risk premium. As a result, governments and institutional investors need to incorporate uncertainty about the sign and magnitude of the inflation risk premium for economic implications of their policies.

The remainder of this chapter is organized as follows. Section 2 introduces the term structure model, estimation methodology and the describes the data used in our empirical analysis. Section 3 presents our estimation results without parameter uncertainty and describes the effect of model choice on the real term premia and inflation risk premia. Section 4 analyzes the impact of parameter uncertainty on the inflation risk premia. This section also reveals the shift on the inflation risk premia after the financial crisis. Our conclusions and policy implications follow in Section 5.

3.2 Methodology

In this section we introduce our Gaussian affine term structure model which is used to identify the inflation risk premium. Subsequently, we describe the minimum Chi-squared methodology to estimate our model. Lastly, we describe our data and the macro economic factors.

3.2.1 Discrete time Gaussian affine model

We use monthly frequency in our models. To estimate real interest rates, we incorporate both latent, X_t^L, and economic factors, X_t^{EC}, as state variables. We assume that the nominal bond price with maturity n at time t, $P_t^N(n)$, is exponentially affine in two latent state variables

$$P_t^N(n) = \exp(A_n^N + B_n'^N X_t), \tag{3.2.1}$$

where $X_t = [X_t^{EC}, X_t^L]$ denotes a vector with economic variables and latent state variables. We restrict $B_n'^N$ in such a way that only latent state variables can influence the nominal bond prices. Since real bond prices will be dependent on inflation in our framework, we need to incorporate economic factors that explain inflation. We assume that the state variables follow a vector autogressive model of order 1,

$$\begin{bmatrix} X_t^{EC} \\ X_t^L \end{bmatrix} = \begin{bmatrix} \Phi_0^{EC} \\ 0 \end{bmatrix} + \begin{bmatrix} \Phi_1^{EC} & \Phi_1^{EC,L} \\ 0 & \Phi_1^L \end{bmatrix} \begin{bmatrix} X_{t-1}^{EC} \\ X_{t-1}^L \end{bmatrix} + \begin{bmatrix} \Sigma^{EC} & 0 \\ 0 & I_2 \end{bmatrix} \epsilon_t, \tag{3.2.2}$$

where we let the economic factors to be correlated with the latent state variables through parameter $\Phi_1^{EC,L}$. We set Φ_0^L to zero and the variance covariance matrix of the latent state variables equal to the identity matrix, I_2, for identification purposes (see e.g. Duffee (2002) and Dai and Singleton (2000). Since we allow for the latent factors to influence the macroeconomic variables, we are able to capture the link between macroeconomic dynamics and the latent factors.

In order to derive the no-arbitrage nominal bond prices, we follow the literature of affine term structure models. We postulate the nominal affine pricing kernel as

$$M_{t+1}^N = \exp\left(-r_t^N - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\epsilon_{t+1}\right), \tag{3.2.3}$$

where r_t^N denotes the monthly nominal short rate and λ_t denotes the time dependent price of risk. Since we need to price real bonds as well, we need to establish a relation between the nominal and real pricing kernel. The difference between the nominal and real pricing kernel is determined by the impact of realized inflation, PI_t . Thus, the following condition needs to be satisfied between the nominal and real pricing kernel

$$M_t^R = M_t^N PI_t. \tag{3.2.4}$$

In order to derive the real pricing kernel, we need to define the inflation process. We specify the inflation process as

$$PI_{t+1} = \exp(\delta_\pi + \delta_{1,\pi}' X_t + \sigma_\pi' \epsilon_{t+1}). \tag{3.2.5}$$

The parameter $\delta_{1,\pi}'$ determines the impact of the state variables on inflation. By substituting this process in the definition of the real pricing kernel as given in Equation (3.2.4), we can derive the following real pricing kernel

$$M_{t+1}^R = \exp\left(-r_t^N + \delta_{0,\pi} + \delta_{1,\pi}' X_t - \frac{1}{2}\lambda_t'\lambda_t - (\lambda_t' - \sigma_\pi')\epsilon_{t+1}\right). \tag{3.2.6}$$

We can rewrite the real pricing kernel in its typical form as defined in Equation (3.2.3) but with equivalent real parameters instead. As a consequence of rewriting, the real short rate consists of four components, namely the nominal rate, expected inflation, an inflation risk premium and a convexity term. The inflation risk premium follows due to the unexpected inflation shock. The convexity term follows due to the lognormality assumption. In mathematical notation, the relation between the nominal and real short rate is described as

$$r_t^R = r_t^N - (\delta_{0,\pi} + \delta_{1,\pi}' X_t) + \sigma_\pi' \lambda_t - \frac{1}{2}\sigma_\pi' \sigma_\pi. \tag{3.2.7}$$

The term $\sigma_\pi' \lambda_t$ denotes the inflation risk premium for the short rate, whereas $\frac{1}{2}\sigma_\pi'\sigma_\pi$ is the convexity term. This equation is known as the modern Fisher equation[2].

Furthermore, we assume the monthly nominal short rate to be an affine function of latent state variables similarly as with the nominal bond prices. The nominal short rate can therefore be written as

$$r_t^N = \delta_{0,r}^N + \delta_{1,r}'^N X_t, \tag{3.2.8}$$

[2] In the derivation of Fisher the difference between the nominal and real yields is only influenced by the inflation component. To distinguish his contribution from the recent affine term structure models, this equation is typically denoted as the modern Fisher equation.

where δ_1^N is restricted to incorporate only the impact of the latent factors. Lastly, we assume the price of risk to be affine in the state variable, so that

$$\lambda_t = \Gamma_0 + \Gamma_1 X_t. \tag{3.2.9}$$

Substituting the affine assumptions of the short rate and the price of risk into the pricing kernel in Equation (3.2.3) leads to the typical form of pricing kernels used in affine term structure models.

Following the no-arbitrage relation between bond prices between two time periods, we can derive the no-arbitrage prices for a nominal bond with maturity n at time t. The no-arbitrage relation can be expressed as

$$P_t^N(n) = E_t\left[M_{t+1}^N P_{t+1}^N(n-1)\right]. \tag{3.2.10}$$

By substituting the nominal pricing kernel in Equation (3.2.3) and substituting the affine bond prices as defined in (3.2.1), a recursion for the coefficients of the bond prices can be derived, which is shown in Appendix 3.A. Using the relation between continuously compounded yields and bond prices, we can express the yield curve as a function of maturity n by the following function

$$Y_t^N(n) = \bar{A}_n^N + \bar{B}_n'^N X_t. \tag{3.2.11}$$

Equivalently, we can derive the real yields by using the real pricing kernel. For further details, we refer to Appendix 3.A.

3.2.2 Term premia

To enhance identification of the parameters in our model, we incorporate nominal and real term premia. The nominal term premium is defined as the difference between the expected return of the bond minus the nominal short rate. We can express term premium, TP, for a nominal bond in yields as follows

$$TP_t^N(n) \equiv \mathrm{E}_t\left[R_t^N(n)\right] - r_t^N = \left[Y_t^N(n) - \mathrm{E}_t\left[Y_{t+1}^N(n-1)\right]\right](n-1) + \mathrm{Slope}_t(n), \tag{3.2.12}$$

where $\mathrm{Slope}_t(n) = \left[Y_t^N(n) - r_t\right]$. Typically, the Campbell and Shiller (1991) long-horizon regressions are used to analyze bond return predictability and allow to identify the term premia (See e.g. Haubrich et al. (2011)). These regressions explain the changes of the bond yields by the slope of the term structure. Thus, for each nominal bond with maturity n, we can write the regression as

$$Y_{t+1}^N(n-1) - Y_t^N(n) = \beta_0 + \beta_1 \frac{\mathrm{Slope}_t(n)}{n-1} + \iota_{t+1}, \tag{3.2.13}$$

where ι_{t+1} is the error term. This equation allows us to compute the expectation for the bond and determine the nominal term premia as defined in Equation (3.2.12) for our sample. By substituting the nominal bonds for real bonds in Equations (3.2.12) and (3.2.13), we can equivalently determine the real term premia.

3.2.3 Estimation procedure

We estimate our model using the Chi-squared methodology as proposed by Hamilton and Wu (2012). To identify the latent state variables, we distinguish between yields measured with and without error as proposed by Chen and Scott (1992). In the first step, we use an ordinary least squares method (OLS) to estimate the VAR process of the implied state variables and to regress the implied latent state variables on the yields measures with errors. Subsequently, we link these OLS estimates with the coefficients implied by the structural parameters of our model and minimize the distance using a Chi-squared objective function.

Let Y_t^1 denote the vector containing the yields measured without error, and the remaining yields, Y_t^2, will be measured with error. The state and measurement equations can be written as the following system of equations

$$\begin{bmatrix} Y_t^1 \\ Y_t^2 \\ X_{t+1} \\ TP_t \end{bmatrix} = \begin{bmatrix} \bar{A}^1 \\ \bar{A}^2 \\ \Phi_0 \\ \bar{A}^4 \end{bmatrix} + \begin{bmatrix} \bar{B}'^1 \\ \bar{B}'^2 \\ \Phi_1 \\ \bar{B}'^4 \end{bmatrix} X_t + \begin{bmatrix} 0 \\ \Omega_2 \\ \Sigma \\ \Omega_4 \end{bmatrix} \eta_t, \tag{3.2.14}$$

where Ω_2 and Ω_4 are diagonal matrices denoting the standard error of the error measurement with $\eta_t \sim \mathrm{N}(0, I)$. Coefficients $\bar{A}^4$ and $\bar{B}'^4$ determine the model implied term premia and are reported in Appendix 3.B. We can rewrite this system of equations by substituting the affine relation between yields and factors as given in Equation (4.3.11). As a result, we can derive the reduced system of equations

$$\begin{array}{lcccccccc} Y_t^1 & = & A_1^* & + & \Phi_{11}^* \, Y_{t-1}^1 & + & \Omega_1^* \, \epsilon_{1t}^*, & & \\ Y_t^2 & = & A_2^* & + & \Phi_{21}^* \, Y_t^1 & + & \Phi_{2EC}^* \, X_t^{EC} & + & \Omega_2^* \, \epsilon_{2t}^*, \\ X_t^{EC} & = & \Phi_0^* & + & \Phi_{31}^* \, Y_t^1 & + & \Phi_{3EC}^* \, X_{t-1}^{EC} & + & \Sigma_{EC}^* \, \epsilon_{ECt}^* \\ TP_t & = & A_4^* & + & \Phi_{41}^* \, Y_t^1 & + & \Phi_{4EC}^* \, X_t^{EC} & + & \Omega_4^* \, \epsilon_{4t}^*, \end{array} \tag{3.2.15}$$

where the reduced parameters A^*, Φ^*, Ω and Σ^* have model coefficients implied equivalents, which are derived in Appendix 3.B. Applying OLS to these equations yields the estimates of the reduced form equations. These reduced form parameters can be used to derive the structural parameters by minimizing the distance between the OLS estimates and the coefficients implied the structural parameters.

To define the objective function, let π denote the vector containing the reduced OLS parameters. The estimates of the reduced form parameters are derived from the full information maximum likelihood, so that $\hat{\pi} = \arg\max_{\pi} L(\pi; Y)$ with the function $L(\pi; Y)$ denoting the log likelihood for the entire sample. Following Hamilton and Wu (2012), we define the minimum Chi-squared estimation as

$$\min_{\theta} T\,[\hat{\pi} - g(\theta)]'\,R\,[\hat{\pi} - g(\theta)], \tag{3.2.16}$$

The function $g(\theta)$ denotes the transformation of the structural parameters θ into a vector of the reduced form parameters as shown in Equation (3.2.15). In order to derive the asymptotic distribution of the estimate $\hat{\theta}$, we approximate it by using $T^{-1}\left(\hat{\Psi}'\hat{R}\hat{\Psi}\right)^{-1}$ with $\hat{\Psi} = \partial g(\theta)/\partial\theta'|_{\theta=\hat{\theta}}$[3]. In Appendix 3.C, we provide the details on the estimation methodology. Since the minimum Chi-squared estimation and the MLE are asymptotically equivalent, the minimum Chi-squared estimation has the equivalent asymptotic optimality properties of the MLE.

We motivate this estimation approach by the observation of the difficulties with numerical optimization of the ML function (Aït-Sahalia and Kimmel, 2010). One of the issues is the persistence of the latent factors causing the MLE function to be flat for autoregressive parameters of the latent factors. Consequently, grid searching techniques are quite vulnerable for path dependency and hence finding local minima (see e.g. for discussions Kim (2008) and Joslin et al. (2011)). The Chi-squared method allows to identify the structural parameters by their mapping into reduced form parameters.

Since the minimum Chi-squared approach relies on the initial parameters of the OLS estimation, parameter uncertainty enters at the first stage of our estimation procedure. In order to account for small sample bias, we adopt a Bayesian approach to the first step estimates. To draw from the marginal posterior distribution of the reduced form parameters, we employ a Gibbs sampler. For each set of obtained reduced form parameters, we estimate the associated structural parameters using the Chi-squared objective function. Consequently, we can determine the posterior density of the inflation risk premium.

[3] See for more details, Hamilton and Wu (2012)

We use an uninformative prior to analyze parameter uncertainty. Additionally, to study the impact of the post crisis period on the inflation risk premium, we adopt a Normal-diffuse prior that adds more weight to the observations in the post crisis period. For details on the Gibbs sampler applied to the first step reduced form parameters, we refer to Appendix 3.D.

3.2.4 Data

Our data sample ranges from July 2004 up to December 2012 for both markets. While inflation swap rates have been traded since 2001, the market became only mature since the beginning of 2004. In 2004 the aggregated notional amounts in the inflation swap market doubled to about 50 billion Euro, compared to 2003[4]. While Haubrich et al. (2011) use April 2003 as starting date, we exclude data from 2003 and early 2004 for potential liquidity issues.

Inflation swap rates provide a good alternative to the identification of real rates instead of relying on inflation-linked bonds. Although inflation-linked bonds in the US have been available since 1999, studies estimate that early interest rates are substantially affected by liquidity issues. Gürkaynak, Sack, and Wright (2010) and D'Amico et al. (2010) show that a liquidity premium of the TIPS is only negligible after 2004, substantially reducing the sample size. This evidence is in line with the observation of Roush and Ezer (2008) who reports that the TIPS markets increased substantially after 2004-2005. However, the bankruptcy of Lehman Brothers has induced a downward price pressure in the TIPS market as their inventory had to be unwinded (Campbell, Shiller, and Viceira (2009a)). As a result of liquidity the larger implied real rates will lead to a smaller inflation risk premium. The bid-ask spreads of inflation-swap rates on the other hand remained quite unaffected by the financial crisis, indicating that inflation swap rates offer a more accurate assessment of the real rate than the TIPS.

To enhance comparability of our results, we employ a similar sample period for the UK market. While not many studies have been conducted on the liquidity of the UK inflation-indexed Gilts, it is argued by Greenwood and Vayanos (2010) that the 2004 UK pension reform had a great effect on the liquidity in the UK inflation-linked bond market. Regulation required pension funds to discount their liabilities at long-term real rates, increasing demand for inflation-indexed Gilts to hedge their exposure. Since liquidity effects in

[4]Kerkhof (2005) reports that monthly volumes of inflation swaps traded in the broker market surpassed 3 billion Euro only in 2004.

the UK market may be affecting real rates, inflation swap rates can offer an alternative to identify the real rates.

For the US market we use 9 nominal yield series and 5 real yield series, namely the 1, 3 and 6 months Treasury Bill and the zero coupon bonds with maturities of 1, 2, 3, 5, 7 and 10 years. Our data of the US nominal government zero coupon yield curve is taken from Gürkaynak et al. (2010)[5]. To identify the real interest rates we rely on inflation swap rates as suggested by Haubrich et al. (2011). The differences between nominal rates and the inflation swap rates with equivalent maturities that matches those of the real interest rates. The real interest rates are determined using zero coupon inflation swap rates with maturities of 1, 3, 5, 7 and 10 years. For the UK market we employ a similar dataset with an equivalent sample period and maturities. The nominal zero coupon rates are obtained from the UK Central Bank[6]. The inflation zero coupon swap rates of the UK and US are obtained from Datastream/Bloomberg. In Table 3.1 we present the summary statistics of our sample.

In order to link expected inflation with macroeconomic developments, we employ two additional models with either macroeconomic or survey factors. For the macroeconomic factors we take consumer price inflation, commodity inflation, housing prices, and asset market volatility. We follow Ang and Piazzesi (2003) in selecting the inflation measures and add a measure to capture market volatility. Market volatility in the form of implied volatility can be linked to bond risk premia (see e.g. Rudebusch, Swanson, and Wu (2006)). For the US market, we use the all urban CPI inflation measure published by U.S. Bureau of Labor Statistics, and for the UK the RPI inflation measure. For the commodity prices, we use the average of the global World bank commodity inflation measured for spot prices of the energy and non-energy commodities[7]. For house pricing, we use the average change in monthly house prices. We rely on the US average price of new one-family houses sold during the month and for the UK market on the UK Nationwide Monthly average House price index. For the volatility factor, we MSCI US Minimum Volatility measure and the FTSE 100 Volatility index. These series are obtained from Datastream.

Our survey data consists of four factors in addition to inflation, namely

[5]The US nominal yield data are available on
http://www.federalreserve.gov/econresdata/researchdata.htm

[6]The UK nominal yield series can be found on
http://www.bankofengland.co.uk/statistics/pages/yieldcurve/default.aspx

[7]The World Bank publishes monthly data on commodity prices, see
http://data.worldbank.org/data-catalog/commodity-price-data

business conditions, consumer confidence, economic optimism, and inflation expectations. Several studies indicate that incorporating forward looking surveys on the state of the macro economy improve the forecast of interest rates (see e.g. Chernov and Mueller (2008) and Moench (2008)). For business conditions we rely on the US Empire State survey on the general business condition and on the UK Retail survey on price expectations. The expectations of the US Consumer Confidence index and UK Consumer Confidence Indicator are used for the consumer confidence factor. For economic optimism, we use data of US TIPP Economic Optimism Index and the UK ZEW indicator of Economic Sentiment. The data of the US University of Michigan Consumer Sentiment on mean expected inflation one year ahead is used for the US inflation expectations and for the UK we incorporate the data of the UK ZEW inflation rate expectation. All the survey data is obtained from Datastream.

While many studies have incorporated inflation surveys (D'Amico et al. (2010)), it is less clear whether those capture the inflation expectations implied by the bond market. As documented by Chernov and Mueller (2008), inflation surveys are prone to overpredict inflation. More specifically, Ang, Bekaert, and Wei (2007a) find that when inflation is low the SPF inflation survey tends to under predict inflation[8]. As a consequence of such biases, the inflation risk premium might be estimated less accurately. To measure the impact of survey information on the inflation risk premium, we compare our survey model and the benchmark model in the next section.

3.3 Empirical results

In this section we estimate both the Macro and Survey model together with a benchmark model. We discuss estimates of the models and their implications for the real rates. To further investigate the differences, we explore the out-of-sample performance of these three models. Subsequently, we assess the impact of the models on the term premia. Finally, we analyze the inflation risk premia implied by our models.

3.3.1 Parameter estimates

Tables 3.2 and 3.4 report the structural estimates for both the UK and US. We first examine the two market structures using the benchmark models of the

[8]The Survey of Professional Forecasters conducted by the Federal Reserve Bank of Philadelphia.

UK and US. Based upon our sample period from 2004 to 2012, we establish key differences in the market structures, indicating different dynamics for the inflation risk premia of the two markets. Since both markets are influenced by the financial crisis and experienced low nominal interest rates afterward, the correlation between the nominal short rate is quite substantial (about 85%). However, the level of the annualized nominal short rate in the US is substantially lower (about 1.6 %), whereas in the UK the nominal rate is on average about 2.6%. This difference can be partly explained by the higher level of inflation in the UK. For example, the annualized UK monthly inflation is about 3.3% whereas the US inflation rate is on average 2.3%. As a result of the low nominal interest rates, both markets experience negative real short rates in our sample. While both markets have similar autoregressive coefficients for the first factor ($\Phi_{1,11}$), the persistence of the second factor differs. The half-life in the UK market is about 2.5 years, whereas in the US it is 4 years. Consequently, shocks to the latent pricing factors are more likely to be persistent in the US market than in the UK market. Given the postive estimates for the impact of the pricing factors on the nominal short rate ($\delta_{1,r}$) in both markets, a positive shock to the latent factor will have an positive effect on the nominal short rates.

Tables 3.2 and 3.4 also show that the impact of the latent factors on the price of inflation risk differs in the two markets. In the US market the less persistent latent factor drives the price of inflation risk ($\sigma_{\pi,1}$), whereas in the UK there is an interaction between the factors. Although statistically the differences for the impact of the latent factors are hard to capture, inflation levels have a positive effect on the inflation risk premia. Due to the model specification the impact of inflation on the price of risk is estimated negatively ($\sigma_{\pi,CPI}$), resulting in a positive effect. Our model suggests that the inflation level in the US has a larger impact on the inflation risk premium than in the UK. Consequently, we document different dynamics in the prices of inflation risk among the two markets.

Next, we turn to the model selection within the two markets. To evaluate the impact of our benchmark and macro economic models on the estimation of real interest rates, we compare the in-sample measurement errors. Note that the specifications of the models are such that they differ in their ability to estimate the real interest rates. Since the financial crisis has substantially influenced the real rates, we evaluate the performance of our models in three periods, namely, prior, during, and post crisis. Tables 3.3 and 3.5 report our estimation results for these three periods. From these tables, we can conclude that in the UK the Macro model performs more accurately over the all three periods. In the US the difference between the Survey and Macro models are more difficult to capture. For maturities up to 5 year, the benchmark model outperforms both models in terms of smaller measurement errors. Although the Survey model performs more accurately for the maturities 7 and 10 year, these improvements are only about 1% compared to the Benchmark model. In the UK market the improvements of the UK Macro model are larger (about 5%). As a result, model selection has less impact on the fit of real rates in the US market than in the UK market.

Interestingly, during the financial crisis the Survey model in the US reduces measurement errors by about 5 %. This seems to support that adding surveys factors incorporating market expectations improves the ability of affine models to explain real interest rates. However, we observe in the UK that only for real rates with maturities of 1 and 3 years the Survey factors improve the fit of the real rates compared to the benchmark model. The survey factors in the UK add little value prior to the crisis. Our result of the Macro model in the UK improving the fit of the real rates is mainly driven by the improvement of the measurement errors during the financial crisis. Since the macro factors in our model include global indicators, the market timing of the financial crisis is more adequately captured in the UK market. While market rates in the US reacted prior to the bankrupcty of Lehman, the UK rates remained rather stable. While the nominal short rate had a declining trend since August 2007, the UK nominal rate declined rapidly after August 2008. Since these markets responded differently to the crisis, the impact of the economic variable differs substantially among the two markets.

While we find that the addition of survey and macro factors can improve the fit of the real rates, empirical evidence shows that out-of-sample performance of affine term structure models with additional macroeconomic variables is less strong (Ang et al., 2007a). To further investigate the effect of the model selection on the real interest rates, we perform an out of sample forecast in the post financial crisis period. Tables 3.6 and 3.7 show that the impact of additional macroeconomic factors is hard to capture. In the US all our indicators show that for short maturities (1 and 3 years) the Macro model outperforms whereas for longer maturities (5, 7, and 10 years) the Survey model improves the out-of-sample forecast. This result suggests that for long maturities real rates can benefit from incorporating market expectations through surveys. However, the Survey model in the UK does not improve the real rates for longer maturities. We do find evidence in the UK that short maturities (1 and 3 years) can benefit from incorporating macro factors, suggesting that short real rates are more influenced by actual macro developments rather than expectations. Similarly, we find in both markets that CPI inflation is more accurately predicted by our Macro model. However, the out-of-sample R-squared measure shows that only in the US the Macro model improves the historical mean.

To summarize, we show that the structures of the governmental bond markets of the US and UK differs, resulting in different underlying mechanics for the inflation risk premium. Consequently, the impact of our model selection may results in different dynamics for the inflation risk premium. While in the US the benchmark model performs reasonably well in-sample, our Macro model substantially outperform the benchmark model in the UK. Therefore, to extract the inflation risk premium we will focus on these models besides our benchmark model.

3.3.2 Campbell-Shiller regressions and term premia

To further investigate model selection, we investigate the ability of our models to replicate the Campbell and Shiller (1991) regressions. These regressions as defined in Equation (3.2.13) are an application of the Expectation Hypothesis and can therefore be used to test whether the hypothesis holds. The estimates of the impact of the slope on the bond return should be equal to one ($\beta_1 = 1$) according to the theory on the Expectation Hypothesis. However, the literature has documented the stylized fact these coefficients are decreasing with maturity and become negative for longer maturities. Nominal affine term structure models are known to be able to replicate the empirical coefficients of the Campbell-Shiller regression. While the pattern of the regression coefficients of nominal rates are frequently studied, less is known about characteristics of the coefficients implied by real rates. Therefore, we first estimate the Campbell-Shiller regressions implied by our data and subsequently compare them to our model implied regressions.

The estimates of the nominal coefficients in Campbell-Shiller regressions of both markets reported in Table 3.8 confirm the typical decreasing pattern of the coefficients across maturity. As expected, our estimates are decreasing and we reject the Expectation Hypothesis only for longer maturities. In the US, we reject the hypothesis for 7 and 10 year maturity at a 95% confidence level, while in the UK we only reject for the 10 year maturity. Due to our short sample, the uncertainty of the estimates remains fairly large. To confirm the accuracy of our estimates, we compare our results to Haubrich et al. (2011). Although they use a period of 1982-2010 for the nominal regression, we find similar estimates for shorter maturities although our estimates for 7 and 10 year maturity are half as large. Since they obtain large standard errors in their estimation as well, it is hard to statistically differentiate between the two sets of estimates. Consequently, our results of the nominal coefficients confirm the range documented in the literature.

Table 3.8 documents a similar decreasing characteristics for the real coefficients of the Campbell-Shiller regressions as observed in the nominal rates. This empirical observation sheds new light on the behaviour of real coefficients. Haubrich et al. (2011) report coefficients increasing with maturity in their sample of 2003-2010, while our estimates are declining with maturity. Our evidence is supported by the fact that we find the decreasing characteristics in both the UK and US. Table 3.8 shows that we cannot reject the Expectation hypothesis in the US for all real rates, while in the UK we can only accept the hypothesis for the 1 year maturity. One of the explanations of this results is that the standard errors in the US are substantially larger than in the UK. While standard errors remain large for the real rates, in both markets the uncertainty of the coefficients is larger than in the nominal regressions. Due to this uncertainty, it is hard to capture the link between the different patterns of the nominal and real coefficients within the two markets. For example, we find that the magnitude of nominal coefficients are not similar to the real coefficients. In particular, we observe low nominal coefficients in the US whereas we observe rather high nominal coefficients in the UK. A reversed pattern holds true for the real coefficients.

Table 3.9 shows that our estimated benchmark models are able to generate the equivalent decreasing pattern with maturity for both the nominal and real coefficients. Compared to Haubrich et al. (2011), our model performs more adequate on shorter maturities to reproduce the data implied coefficients. Similarly as in their study, it is hard for term structure models to capture both the short and long maturities. Consequently, for longer maturities our model is less able to adequately replicate the data implied estimates, although our estimates fall within the 95% confidence intervals.

Model selection has only a small impact on the ability to replicate the coefficients of the Campbell-Shiller regressions. While in the US the most promising models are the benchmark and the Survey model, both models are less able to capture the data implied coefficients for longer maturities. However, in the UK the benchmark model captures the longer maturities more adequately.

Although our previous results on model selection show improvements for the addition of macroeconomic factors, the benchmark model is more able to generate adequate estimates for the coefficients. Therefore, we observe a trade-off between in-sample fitting and the ability to replicate the data implied coefficients of the Campbell-Shiller regressions.

Next, we examine whether our model is able to replicate the data implied nominal and real term premia. Table 3.10 reports the mean absolute deviations (MAD) from the data implied term premia and shows that nominal term premia are better captured than the real term premia. For nominal term premia, we observe a smaller difference between short and long maturities than for the real term premia. While in the US we observe that short nominal term premia are more accurately replicated than the long term premia, in the UK our model is able to match both the short and long nominal term premia. Due to the different market structure of the UK, the nominal data implied term premia are less varying over the sample period. Consequently, the deviations of the model implied term premia are reduced.

Table 3.10 also shows the impact of macroeconomic factors on the estimation of term premia. For example, the benchmark and Survey model are able to replicate the real term premia on short maturities in the US. On the other hand, the Macro model has the ability to improve the long real term premia. This suggests that adding economic variables can influence the ability of the model to replicate real term premia. However, the Macro model in the UK is unable to improve the Benchmark or Survey model. In terms of fitting real rates, the Macro model was outperforming the other two models in the UK, but it is unable to replicate the data implied term premia. Regarding replicating term premia, affine term structure models perform more accurate for nominal term premia than real premia. Part of this can be explained by the uncertainty associated with the estimation of data-implied real term premia. Hence, it is not straightforward to determine whether affine term structure models are able to replicate time-varying real term premia.

Although our benchmark models are able to capture the main characteristics of the Campbell-Shiller regressions and term premia, it remains a challenge to accurately replicate all the empirical stylized facts. Model selection substantially affects the performance of the models. While the Benchmark model is able to perform rather well in both the UK and US to replicate the Campbell-Shiller regressions and term premia, our previous results indicated that macroeconomic factor improve the fit and forecasting of the real rates. Therefore, we will investigate the consequences of model selection for the inflation risk premia in the next section.

3.3.3 Inflation risk premium

An important component of modeling nominal and real interest rate is the inflation risk premium. Inflation risk is associated with unexpected inflation shocks in our model. As a result, we compute the inflation risk premium by taking the difference between the implied break-even inflation with and without unexpected inflation in the pricing kernel[9]. Therefore, we can write

$$IRP_t(n) = BEI_t^{\sigma_\pi}(n) - BEI_t^{\sigma_\pi=0}(n), \tag{3.3.1}$$

where $IRP_t(n)$ is the inflation risk premium for maturity n at time t, $BEI_t^{\sigma_\pi}(n)$ denotes the break-even inflation with unexpected inflation risk and $BEI_t^{\sigma_\pi=0}(n)$ denotes without. The break-even inflation curve can be determined by subtracting the real yield curve from the nominal yield curve.

Table 3.11 reports our estimates for the inflation risk premium for both the UK and US markets. For the US benchmark model we find for the 5 year maturity inflation risk premium of 72 basis points, whereas the UK has a negative premium of 45 basis points. Since the estimates for the UK market are negative, this implies that issuance of inflation-indexed Gilts would be costly in the UK. Investigating the impact of the model selection, we can conclude that in the US the models generate a similar inflation risk premium. For the UK market, we show adding macroeconomic factor can have a large impact on the estimate for the inflation risk premium. The Macro model that fits the real rates most accurately identifies a positive risk premium, whereas the Survey model suggests a substantial smaller risk premium than the Benchmark model.

Since the standard error of the average inflation risk premium are quite large, it suggests that there are level shifts in the inflation risk premium across our sample period. To understand the time-varying characteristics more adequately, we split the sample into three periods: prior to the Financial crisis in 2008, during and post crisis. We observe in all three models for both markets that the 5 year inflation risk premium was at its highest mark prior to the financial crisis. In our benchmark model, we find estimates in the US of about 115 basis points risk, whereas in the UK we find a positive premium of 30 basis points. During the financial crisis, we estimate negative premia

[9]This approach is similar to Chen, Liu, and Cheng (2010) and Haubrich et al. (2011).

for both markets across all our models. These negative rates do not necessarily correspond to inflation expectations. Although negative rates might imply that the market feared deflation due to the crisis, it could also suggest that the premium is affected by liquidity. Since inflation rates remained quite stable during the financial crisis, our results point at a liquidity shock rather than risk associated with inflation.

After the financial crisis, the inflation risk premium is again increasing but remains substantially lower than pre-crisis levels. For example, our benchmark model generates a 5 year inflation risk premium of 47 basis points in the US and about -104 basis points in the UK. These estimates suggest the UK market responded much stronger to shift in the inflation risk premium due to the financial crisis than the US market. Campbell, Shiller, and Viceira (2009b) argue that low inflation risk premia (or even negative risk premia) can be explained by positive correlation between asset returns and inflation in the observed period. Therefore, negative inflation risk premia might indicate low inflation expectations.

We observe similar movement in the inflation risk premia for both markets around the period of the financial crisis. Although there appears to be a different market timing in the drop of nominal interest rates across both markets, from August 2007 to January 2008 the inflation risk premium falls by 50% in both markets. While in the UK market the inflation risk premium again starts to drop around September 2008, the US market remains stable until March 2009. Afterward, the US inflation risk premium continues to drop and remains unstable. By the end of 2011 the US inflation risk premium starts to stabilize around pre-crisis levels (about 97 basis points), while the UK market stabilize around a substantial lower inflation risk premium (about -67 basispoints). Thus, in the UK market the recovery of the inflation risk premium is much less than in the US.

Our estimates for the inflation risk premia fall within the range suggested by earlier research. While the UK and US markets have never been compared with similarly estimated models, our US sample period and inflation-linked derivatives are most related to Haubrich et al. (2011). They find inflation risk premia are about twice as small for the 5 and 10 year maturity, yet their estimates are twice as large for the longer maturities (20 and 30 years). One the novelty of our approach is that we incorporate data-implied term premia. This suggests that they might underestimate unexpected inflation shock at shorter maturities, while they overestimate shocks for longer maturities. Other studies, e.g. D'Amico et al. (2010) report an estimate of 36 basis points for the 5 years inflation risk premium, whereas Ang et al. (2007b) estimate 115 basis points. Similar to our results, Buraschi and Jiltsov (2005) estimate a premium of 80 basis points for a 10 year premium. For UK, few comparable studies are available and their estimates of the inflation risk premium differ substantially. All these studies only use inflation-linked Gilts to identify the real interest rates. For example, Evans (2002) estimates a negative inflation risk premium, while on the other hand Risa (2001) reports for the period 1983 and 1999 an inflation risk premium of about 184 basis points. Joyce et al. (2010) on the other hand shows an estimate of about 100 basis points. As a result, the literature suggests a wide range of likely estimates for the inflation risk premium.

To summarize, we show that model selection is an important determinant for the inflation risk premium. Since our models perform differently across various criteria, we are unable to select the most appropriate model to identify the inflation risk premium. Our results might be influenced by a small sample bias. To address this issue and the uncertainty of the estimate of the inflation risk premium, we employ a Bayesian approach in the next section.

3.4 Parameter uncertainty

To investigate the uncertainty of the range of estimates for the inflation risk premium, we employ a Bayesian methodology. In this way, we can address the effect of a short sample period as well. First, we present posterior marginal distributions for the inflation risk premia, which we can use to calculate statistical intervals. These intervals allow us to explain the uncertainty concerning the estimates of the inflation risk premium. Secondly, we analyze on the impact of the financial crisis on the inflation risk premium.

3.4.1 A range of estimates for inflation risk premium

Figures 3.1 and 3.2 report our marginal posterior distributions for the 5 years inflation risk premia in the US and UK. We confirm our previous results on the different characteristics between the inflation risk premia in both markets. The uncertainty concerning the estimate of the inflation risk premium is larger in the UK than in the US. The 95% credibility interval of the UK distribution ranges from -95 to 88 basis points, whereas in the US we observe a range of -4 to 119 basis points. This implies that for both markets we cannot exclude negative inflation risk premia on a 95% credibility interval. However, our results suggests that the US inflation risk premium is substantially more likely to be positive than the UK inflation risk premium.

Next, we analyze the impact of parameter uncertainty on the inflation risk premia. The mean of the US distribution is about 74 basis points, which is about about 2 basis points larger than the our previous estimate ignoring parameter uncertainty. Therefore the effect of the parameter uncertainty is not quite large for the US market. In the UK, we observe a larger effect of parameter uncertainty, since the mean of the distribution is about -8 basis points. Since the inflation risk premium in the UK market was substantially affected by the financial crisis, the impact of parameter uncertainty is much larger. As a result, we observe large uncertainty about the point estimates for both markets.

To measure the impact of macroeconomic factors, we also determine the distributions of the inflation risk premium indicated by our Macro and Survey models. Figures 3.3, 3.5, 3.4, and 3.6 report the marginal distributions for both markets. Our results indicate that the distributions are not substantially altered, although the distributions are more platykurtic. As a result, the uncertainty of the estimates increases. For example, the US Survey model has a larger upper bound for the 95% credibility interval, namely an interval of -4 to 127 basis points. In the UK market the additional macroeconomic factors cause a broadening for the interval in both directions. While macroeconomic factors improve our model characteristics as previously shown, it generates larger credibility intervals for inflation risk premium. These results suggest that the identification of the inflation risk premium might not be helped by including such variables. Even though from an economic perspective a link between macroeconomic development and the inflation risk is preferred, empirically the addition of macro factors increase the uncertainty of the estimates.

The evidence presented by our Bayesian methodology raises questions about the ability of affine term structure models to adequately pinpoint the value of the inflation risk premium with high precision. Since our empirical distributions include the range of the inflation risk premium suggested by previous literature, it is hard to statistically distinguish between those estimates. To analyze the impact of the financial crisis in 2008 on the inflation risk premia, we extend our Bayesian methodology. In the next section, we investigate the consequences if we add more weight to the observations after the financial crisis in our Bayesian analysis. As a consequence, we can measure the shift in the inflation risk premium caused by the financial crisis.

3.4.2 The impact of the financial crisis

Since the financial crisis nominal interest rates have dropped to low levels. In the previous sections, we have observed a drop in the inflation risk premium during the financial crisis. In order to analyze the impact of the financial crisis on the range of the inflation risk premium, we extent our Bayesian analysis by adding more weight to post crisis observations. By using a Normal-diffuse prior estimated on a sample period from 2010 up to 2012, we shift the importance of the observations to the financial crisis while maintaining the original sample size. Reestimation of our model on the time period from 2010 up to 2012 is infeasible due to the short sample period. With our approach we can show shifts in the posterior probability distribution caused by the impact of the financial crisis.

Figures 3.1 and 3.2 show the impact of the financial crisis for both the UK and US 5 year inflation risk premium. In the US, we observe a downward shift in the distribution function for the premium, resulting in a mean of 38 basis points. Although we observe a lower estimate, the uncertainty of the estimate increases as well. The 95% credibility interval in the US ranges from -51 to 92 basis points. As a result, negative inflation risk premium are more likely due to the crisis. As uncertainty about future economic development increases, one would expect that the inflation risk premium would increase as well. Especially since low interest rates in the US would lead to uncertainty about future inflation ((Krugman, 1998) and (Eggertsson and Woodford, 2003)). However, the US governmental bond market does not reflect this uncertainty.

The impact of the financial crisis on the inflation risk premium in the UK is less obvious. We observe a slightly increase of the mean of the distribution to 13 basis points. Similarly, the 95% credibility interval of the inflation risk premium, ranging from -104 to 150 basis points, shifts upward, although the lower bound of the interval decreases as well. Due to the broadening of the interval, the uncertainty about the estimate of the inflation risk premium increases. As a result, it is hard to determine whether the inflation risk premium is positive of negative in the UK. About 42% of the probability mass in the UK is below 0, indicating that the inflation risk premium could be negative.

The impact of the financial crisis causes a wider range of estimates for the inflation risk premia in the UK and US. We observe that a large downward effect on the US estimates, whereas we show an upward effect for the UK market. In particular, our models seems to indicate that both markets have positive inflation risk premia. However, due to the large uncertainty about these estimates, we cannot statistically reject any of the estimates suggested by earlier studies.

3.5 Conclusion

Although debt markets with inflation-linked derivatives offer a possibility to capture the inflation risk, the identification remains problematic. Our study quantifies a wide interval of estimates for the inflation risk premium in both the UK and US. This large range suggests that for both markets it remains hard to pinpoint the value of the inflation risk premium with precision. In the US market we find evidence that a 95% credibility interval for the 5 year maturity inflation risk premium of -4 and 119 basis points. This range is even broader in the UK, where we observe an interval of about -94 to 88 basis points. The large uncertainty concerning these estimates are robust for different model specification, suggesting that affine term structure models are not able to accurately pinpoint the inflation risk premium.

This study contributes to research about the inflation risk premia by analyzing the inflation risk premium by means of marginal posterior distributions rather than point estimates. While prior research is mainly concerned about estimating the inflation risk premia, it ignores the large uncertainty associated to these estimations. We show using our Bayesian framework that the small sample bias can alter the estimates for the inflation risk premia. Our Bayesian methodology also allows to investigate the effect of the financial crisis on the inflation risk premia. We observe an upward effect in the inflation risk premium due to the financial crisis in the UK market, while we find downward shift in the US markets. Consequently, this shift results in more uncertainty in estimating the inflation risk premium.

Our findings raise a number of questions on the interpretation of inflation risk premium estimates and economic policy based on these estimates. Positive estimates of the inflation risk premia have frequently be used to validate the issuance of governmental inflation-linked bonds. Given the large uncertainty about these estimates, it is questionable whether debt policy should be based on such estimates. Our posterior probability distributions for UK inflation risk premium shows a large probability (42%) that the inflation risk premium is negative. Also, in the US we are unable to reject the hypothesis that the inflation risk premium is negative. Based on these results UK inflation-indexed gilts would be costly to issue. Further work should explore the impact of the financial crisis on the inflation risk premia, as our results point at a positive shift for the UK, while a negative shift in the US.

3.A Appendix A: Model derivations

In section A.1 we derive the no-arbitrage coefficients for the nominal yields and in section A.2 the derivation for the real yields coefficients are presented.

3.A.1 Coefficients for the nominal yields

In this section we derive the nominal bond yields for our model in a no-arbitrage framework. In this derivation we follow the typical noarbitrage framework in affine term structure models as derived for example by Duffie and Kan (1996). We substitute the affine bond prices, as defined in Equation (3.2.1), in the no-arbitrage relation of the expected bond price. For convenience, we restate this relation

$$P_t^N(n) = E_t\left[M_{t+1}^N P_{t+1}^N(n-1)\right]. \tag{3.A.1}$$

By substituting the affine bond prices and the dynamics of the nominal pricing kernel in this equation, we derive the following expression for the price of a bond,

$$P_t^N(n) = E_t\left[\exp\left(-r_t^N - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\epsilon_{t+1}\right)\exp\left(A_{n-1}^N + B_{n-1}'^N X_{t+1}\right)\right]. \tag{3.A.2}$$

Rewriting and determining the expectation, yields the following expression for the bond price

$$P_t^N(n) = \exp\Big(-\delta_{0,r} + A_{n-1}^N + B_{n-1}'^N\Phi_0 + (B_{n-1}'^N\Phi_1 - \delta_{1,r}')X_t - B_{n-1}'^N\Sigma\lambda_t \\ +\frac{1}{2}B_{n-1}^N\Sigma\Sigma' B_{n-1}'^N\Big). \tag{3.A.3}$$

Next, we need to substitute the affine function for the price of risk, λ_t. Substituting this, as defined in Equation (3.2.9), we arrive at the typical function for the no-arbitrage bond price,

$$P_t^N(n) = \exp\Big(-\delta_{0,r} + A_{n-1}^N + B_{n-1}'^N(\Phi_0 - \Sigma\Gamma_0) + \frac{1}{2}B_{n-1}'^N\Sigma\Sigma' B_{n-1}^N + \\ (B_{n-1}'^N(\Phi_1 - \Sigma\Gamma_1) - \delta_1')X_t\Big). \tag{3.A.4}$$

The last step is to determine the recursion for the coefficients by matching the coefficients of the left-hand side with the terms on the right-hand side. In this way we derive the recursion for the no-arbitrage coefficients of the bond prices

$$A_n^N = A_{n-1}^N + B_{n-1}'^N(\Phi_0 - \Sigma\Gamma_0) + \frac{1}{2}B_{n-1}'^N\Sigma\Sigma' B_{n-1}^N - \delta_{0,r}, \tag{3.A.5a}$$

$$B_n'^N = B_{n-1}'^N(\Phi_1 - \Sigma\Gamma_1) - \delta_{1,r}'. \tag{3.A.5b}$$

with the initial conditions $A_1^N = -\delta_0$ and $B_1^N = -\delta_1$. We restrict these initial parameters to exclude the impact of economic variables on the nominal yields, so that we have $\delta_{1,r} = [\delta_{1,r}^{EC}; \delta_{1,r}^{L}] = [0; \delta_{1,r}^{L}]$. As a consequence of this restriction, we can split for each maturity n $B_n'^N = [B_n'^{N,EC}; B_n'^{N,L}] = [0; B_n'^{N,L}]$, In order to derive the coefficients of the yields, using the relation between bond prices and continuously compounded yields, we adjust the coefficients of the bonds as follows

$$\bar{A}_n^N = -\frac{A_n^N}{n}, \tag{3.A.6a}$$

$$\bar{B}_n^N = -\frac{B_n^N}{n}. \tag{3.A.6b}$$

This function determines the no-arbitrage coefficients for the affine yields.

3.A.2 Coefficients for the real yields

In order to derive the coefficients for the real yields, we derive the real equivalent risk and short rate parameters. For convenience of notation, we state the real pricing kernel below again,

$$M_{t+1}^R = \exp\left(-r_t^N + \delta_\pi + \delta_{1,\pi}' X_t - \frac{1}{2}\lambda_t'\lambda_t - (\lambda_t' - \sigma_\pi')\epsilon_{t+1}\right). \tag{3.A.7}$$

First, we define the real pricing kernel in a similar notation,

$$M_{t+1}^R = \exp\left(-r_t^R - \frac{1}{2}\lambda_t'^R\lambda_t^R - \lambda_t'^R\epsilon_{t+1}\right), \tag{3.A.8}$$

where r_t^R denotes the instantaneous real rate, and $\lambda_t'^R$ the real price of risk vector. We need to find the equivalent parameters for the real pricing kernel. This can be done by matching the parameters of this kernel with the kernel implied by the relation between the nominal kernel and the inflation process. This relation is given in Equation (3.2.4).

We postulate the real price of risk as

$$\lambda_t^R \equiv \lambda_t - \sigma_\pi. \tag{3.A.9}$$

Consequently, the real price of risk parameters can be determined from the nominal parameters as follows

$$\Gamma_0^R = \Gamma_0 - \sigma_\pi, \tag{3.A.10a}$$
$$\Gamma_1^R = \Gamma_1, \tag{3.A.10b}$$

so that we have that the real price of risk is an affine function of the state variables

$$\lambda_t^R = \Gamma_0^R + \Gamma_1^R X_t.$$

In analogy of the nominal decomposition of the monthly short rate, as in Equation (3.2.8), we define the monthly real short rate as

$$r_t^R = \delta_{0,r}^R + \delta_{1,r}'^R X_t. \tag{3.A.11}$$

To find the relation between the nominal and real parameters of the short rate, we note that the product of the real price of risk can be written as

$$-\frac{1}{2}\lambda_t'^R\lambda_t^R = (\lambda_t - \sigma_\pi)'(\lambda_t - \sigma_\pi) = -\frac{1}{2}\lambda_t'\lambda_t + \sigma_\pi\Gamma_0 + \sigma_\pi\Gamma_1 X_t - \frac{1}{2}\sigma_\pi'\sigma_\pi.$$

By matching the real pricing kernel as given in Equation (3.A.8) with real pricing kernel implied by our relation between the nominal pricing kernel and inflation process, the following restrictions for coefficients of the instantaneous real rate are required

$$\delta_0^R = \delta_{0,r} - \delta_{0,\pi} - \sigma_\pi'\Gamma_0 - \frac{1}{2}\sigma_\pi'\sigma_\pi \tag{3.A.12a}$$
$$\delta_1'^R = \delta_{1,r}' - \delta_{1,\pi}' - \sigma_\pi'\Gamma_1. \tag{3.A.12b}$$

Since we have the real equivalent parameters Φ_0^R, Φ_1^R, $\delta_{0,r}^R$, and $\delta_{1,r}'^R$, we use a similar derivation as in the previous section for the recursion of the nominal coefficients. By simply substituting the real equivalent parameters, we obtain the coefficients for the real yields. These coefficients, $\bar{A}_n^R$ and $\bar{B}_n^R$, are the real equivalents of the nominal coefficients as defined in Equation (4.3.11).

3.B Appendix B: Reduced model derivations

The system of reduced equations, as given in Equation (3.2.15), will be derived in this section. We first focus on the equation of the VAR dynamics of the yields measured without error. We stack the nominal yields measured without errors in Y_t^1 and those measured with error in Y_t^2. Since we restrict the impact of economic variables on the nominal rates, we can write the pricing equation defined in Equation (4.3.11) for the yields measured without error as

$$Y^1 = \bar{A}_1 + \bar{B}_1' X_t^L, \tag{3.B.1}$$

where $\bar{A}_1$ is a vector with dimension 2 and $\bar{B}_1'$ is a 2×2 matrix. Both coefficients $\bar{A}_1$ and $\bar{B}_1'$ are constructed by stacking the coefficients of the yield state process as defined in Equation 4.3.11. To derive the reduced model, we use the state process as defined in Equation (3.2.2). Premultiplying this system with $\bar{B}_1'$ and adding $\bar{A}_1$ gives

$$\bar{A}_1 + \bar{B}_1' X_t^L = \bar{A}_1 + \bar{B}_1' \left(\Phi_1^L X_{t-1}^L + I_2 \epsilon_{1t} \right) \tag{3.B.2}$$

As a result, we can rewrite this equation to a VAR model of the yields measured without errors, Y_t^1,

$$Y_t^1 = \bar{A}_1 + \bar{B}_1' \Phi_1^L \bar{B}_1'^{-1} \left(Y_{t-1}^1 - \bar{A}_1 \right) + \bar{B}_1' \epsilon_{1,t}, \tag{3.B.3}$$

by use of the definition of Y_t^1 as given in Equation (4.3.11). Now we have expressed the time dynamics of the latent factors in yield series measured without error. Rewriting this equation yields the first reduced form regression,

$$Y_t^1 = \underbrace{(\bar{A}_1 - \bar{B}_1' \Phi_1^L \bar{B}_1'^{-1} \bar{A}_1)}_{\bar{A}_1^*} + \underbrace{(\bar{B}_1' \Phi_1^L \bar{B}_1'^{-1})}_{\Phi_{11}^*} Y_{t-1}^1 + \underbrace{\bar{B}_1'}_{\Omega_1^*} \epsilon_{1,t}. \tag{3.B.4}$$

In this equation the coefficients $\bar{A}_1^*$, Φ_{11}^*, and Ω_1^* will be obtained by OLS estimation.

The second reduced form equation is the impact of the latent factors on the yields measured with errors. For notional convenience, we repeat this equation

$$Y_t^2 = \bar{A}^2 + \bar{B}_2'^{EC} X_t^{EC} + \bar{B}_2'^L X_t^L + \Omega \epsilon_{2,t}. \tag{3.B.5}$$

Since we include real rate series in this equation we need to incorporate the effect of the economic factors as well. Next, we substitute the latent factors with inverse of the yields observed without error,

$$Y_t^2 = \bar{A}^2 + \bar{B}_2'^L \left(\bar{B}_1'^{-1} \left(Y_t^1 - \bar{A}_1 \right) \right) + \bar{B}_2'^{EC} X_t^{EC} + \Omega \epsilon_{2,t}. \tag{3.B.6}$$

Consequently, we derive the following reduced form regression,

$$Y_t^2 = \underbrace{(\bar{A}^2 - \bar{B}_2'^L \bar{B}_1'^{-1} \bar{A}_1)}_{A_2^*} + \underbrace{(\bar{B}_2'^L \bar{B}_1'^{-1})}_{\Phi_{21}^*} Y_t^1 + \underbrace{\bar{B}_2'^{EC}}_{\Phi_{2EC}^*} X_t^{EC} + \underbrace{\Omega}_{\Omega_2^*} \epsilon_{2,t}. \tag{3.B.7}$$

We denote the OLS estimates of the coefficients in this equation as $\bar{A}_2^*$, Φ_{21}^*, Φ_{2EC}^* and Ω_2^*.

The economic VAR equation is the third reduced form equation. We use the VAR process implied for the economic state variables as defined in Equation (3.2.2) and substitute the inverse of the latent state variables,

$$X_t^{EC} = \Phi_0^{EC} + \Phi_1^{EC} X_{t-1}^{EC} + \Phi_1^{EC,L} \bar{B}_1'^{-1} \left(Y_{t-1}^1 - \bar{A}_1 \right) + \Sigma_{EC} \epsilon_{3,t} \tag{3.B.8}$$

Rewriting this equation, yields

$$X_t^{EC} = \underbrace{\Phi_0^{EC} - \Phi_1^{EC,L} \bar{B}_1'^{-1} \bar{A}_1}_{A_3^*} + \underbrace{\Phi_{3EC}}_{\Phi_{3EC}^*} X_{t-1}^{EC} + \underbrace{\Phi_1^{EC,L} \bar{B}_1'^{-1} Y_{t-1}^1}_{\Phi_{3L}^*} + \underbrace{\Sigma_{EC}}_{\Sigma_{EC}^*} \epsilon_{3,t} \tag{3.B.9}$$

The last reduced form equation concerns the model implied term premia. To determine the model implied term premium, we substitute the affine yields in Equation (3.2.12) for the term premium and evaluate the expectation. Consequently, we can write for the nominal term premium

$$TP_t^N(n) = \left(n\bar{A}_n^N - (n-1)\bar{A}_{n-1}^N - \delta_{0,r} \right) + \left(n\bar{B}_n^N - (n-1)\bar{B}_{n-1}^N \Phi_1^L - \delta_{1,r} X_t \right) \tag{3.B.10}$$

Since the term premium is dependent on the latent factors, we substitute by the yields measured without measurement error. If we allow for measurement error between the model implied term premia and the observed term premia, we obtain the following equation for the nominal term premium

$$\begin{aligned} TP_t^N(n) = & \underbrace{\left(n\bar{A}_n^N - (n-1)\bar{A}_{n-1}^N - \delta_{0,r}^N \right) - \left(n\bar{B}_n^N - (n-1)\bar{B}_{n-1}^N \Phi_1^L - \delta_{1,r} \right) \bar{B}_1^{-1} \bar{A}_1}_{A_4^*} \\ & + \underbrace{\left(n\bar{B}_n^N - (n-1)\bar{B}_{n-1}^N \Phi_1^L - \delta_{1,r} \right) B_1^{-1}}_{\Phi_{4L}^*} Y_t^1 + \underbrace{\Omega_{TP}}_{\Omega_4^*} \epsilon_{4,t}. \end{aligned} \tag{3.B.11}$$

For the real term premium, we derive the following equation,

$$\begin{aligned} TP_t^R(n) = & \underbrace{\left(n\bar{A}_n^R - (n-1)\bar{A}_{n-1}^R - \delta_{0,r}^R \right) - \left(n\bar{B}_n^R - (n-1)\bar{B}_{n-1}^R \Phi_1^L - \delta_{1,r}^R \right) \bar{B}_1^{-1} \bar{A}_1}_{A_4^*} \\ & + \underbrace{\left(n\bar{B}_n^{EC,R} - (n-1)\bar{B}_{n-1}^{EC,R} \Phi_1^L - \delta_{1,r}^{EC,R} \right) B_1^{EC,-1}}_{\Phi_{4EC}^*} X_t^{EC} \\ & + \underbrace{\left(n\bar{B}_n^R - (n-1)\bar{B}_{n-1}^R \Phi_1^{EC} - \delta_{1,r}^R \right) B_1^{-1}}_{\Phi_{4L}^*} Y_t^1 + \underbrace{\Omega_{TP}}_{\Sigma_4^*} \Omega_{4,t}, \end{aligned} \tag{3.B.12}$$

where $\bar{B}^{EC,R}$ denotes the impact of coefficient $\bar{B}^{R}$ to the economic variables. The partitioning of the matrices is equivalent as defined in Equation (3.2.2).

3.C Appendix C: MSCE procedure

To minimize the distance between OLS estimates and the coefficients implied by the structural parameters, we employ the Minimum Chi-squared methodology. For notional convenience, we rewrite the reduced system of equations,

$$\begin{array}{lclclclcl} Y_t^1 & = & A_1^* & + & \Phi_{11}^* Y_{t-1}^1 & + & & + & \Omega_1^* \epsilon_{1,t}^* \\ Y_t^2 & = & A_2^* & + & \Phi_{21}^* Y_t^1 & + & \Phi_{2EC}^* X_t^{EC} & + & \Omega_2^* \epsilon_{2,t}^* \\ X_t^{EC} & = & \Phi_0^* & + & \Phi_{3EC}^* X_t^{EC} & + & \Phi_{3L}^* Y_t^1 & + & \Sigma_{EC}^* \epsilon_{3,t}^* . \\ TP_t & = & A_4^* & + & \Phi_{41}^* Y_t^1 & + & \Phi_{4EC}^* X_t^{EC} & + & \Omega_4^* \epsilon_{4,t}^* \end{array} \tag{3.C.1}$$

For details of the link between the structural parameters and the reduced form parameters, consult Appendix 3.B.

Applying the minimum Chi-squared estimator, we can directly match reduced form parameters with the coefficients implied by the structural parameters. Some of the parameters can directly be mapped into the OLS estimates, such as Φ_1^{EC} and the measurement error Ω. The other parameters need to be estimated using the Chi-squared estimation. Hence, the parameters used in the MSCE are reduced to

$$\hat{\pi} = \begin{bmatrix} \text{vec}\left(\hat{\Pi}_1\right) \\ \text{vech}\left(\hat{\Omega}_1^*\right) \\ \text{vec}\left(\hat{\Pi}_2\right) \\ \text{vec}\left(\hat{\Pi}_3\right) \\ \text{vec}\left(\hat{\Pi}_4\right) \end{bmatrix}, \tag{3.C.2}$$

and

$$\hat{R} = \begin{bmatrix} \hat{\Omega}_1^{*-1} \otimes T^{-1} \sum_{t=1}^{T} Z_{1t} Z_{1t}' \\ \frac{1}{2} D_2' \left(\hat{\Omega}_1^{*-1} \otimes \hat{\Omega}_1^{*-1}\right) D_2 \\ \hat{\Omega}_2^{*-1} \otimes T^{-1} \sum_{t=1}^{T} Z_{2t} Z_{2t}' \\ \hat{\Omega}_3^{*-1} \otimes T^{-1} \sum_{t=1}^{T} Z_{3t} Z_{3t}' \\ \hat{\Omega}_4^{*-1} \otimes T^{-1} \sum_{t=1}^{T} Z_{4t} Z_{4t}' \end{bmatrix} \mathbf{I}_n,$$

where

$$Z_{1t} = \begin{bmatrix} 1 \\ Y_{t-1}^1 \end{bmatrix} \text{ and } Z_{it} = \begin{bmatrix} 1 \\ Y_t^1 \\ X_t^{EC} \\ TP_t \end{bmatrix} \text{ for } i = 2, 3$$

$$
\begin{aligned}
\hat{\Pi}_i &= \left(\sum_{t=1}^{T} Y_t^i Z_{it}'\right)\left(\sum_{t=1}^{T} Z_{it} Z_{it}'\right)^{-1} \quad \text{for } i = 1, 2 \text{ and } 4 \\
\hat{\Pi}_3 &= \left(\sum_{t=1}^{T} X_{t+1}^{EC} Z_{it}'\right)\left(\sum_{t=1}^{T} Z_{it} Z_{it}'\right)^{-1} \\
\hat{\Omega}_1^* &= T^{-1}\sum_{t=1}^{T}\left(Y_t^1 - \hat{\Pi}_1' Z_{1t}\right)\left(Y_t^1 - \hat{\Pi}_1' Z_{1t}\right)' \\
\hat{\Omega}_i^* &= T^{-1}\text{dg}\left(\sum_{t=1}^{T}\left(Y_t^i - \hat{\Pi}_i' Z_{it}\right)\left(Y_t^i - \hat{\Pi}_i' Z_{it}\right)'\right) \\
\hat{\Omega}_3^* &= T^{-1}\sum_{t=1}^{T}\left(X_{t+1}^{EC} - \hat{\Pi}_1' Z_{3t}\right)\left(X_{t+1}^{EC} - \hat{\Pi}_1' Z_{3t}\right)'
\end{aligned}
$$

with the matrix function dg (A) is defined such that all elements outside the diagonal of matrix A are all zero and $\mathbf{I}_n$ is the identity matrix with dimension n. For notional convenience we use $Y_t^4 = TP_t$. By minimizing the MSCE, we find the estimates for the structural parameters.

3.D Appendix D: Bayesian approach

Parameter uncertainty enters in the first stage of the estimation namely the reduced form regressions. Therefore, we adopt a Bayesian methodology for these equations to address parameter uncertainty. In order to obtain the distribution for the reduced form of equations, we follow Bauwens et al. (1999) by rewriting these equations into a system of seemingly unrelated regressions. The reduced form of equations can easily be written in the following form,

$$y_i = X_i \beta_i + \epsilon_i, \tag{3.D.1}$$

for each $i = 1, .., n$ with n denoting the total number of state variables in the system. If the individual time series included in the model have dimension T, then y_i is a vector with $((T-1) \times 1)$ observations, X_i is a matrix with dimensions $((T-1) \times k_i$ with k_i independent variables, β_i consists of a coefficient vector with k_i elements, and ϵ_i is the vector with the associated errors for each observation $(T-1)$. We rewrite this model in two forms in order to draw parameters from the posterior density. By stacking all the observations for each equation i, we can express Equation (4.A.19) as

$$y = x\beta + \epsilon, \tag{3.D.2}$$

where $y = (y_1, .., y_n)$ is a vector with dimensions $((T-1)n \times 1)$, $\beta = (\beta_1, .., \beta_n)$ with a vector of k_n elements, $x = \text{diag}(x_1, .., x_n)$ with dimensions $((T-1)n \times k_n)$, and $\epsilon = (\epsilon_1, .., \epsilon_n)$. In the second approach, we write a VAR specification

$$Y = XB + E, \tag{3.D.3}$$

with $Y = (y_1 ... y_n)$ is a matrix with dimensions $((T-1) \times n)$, $X = (X_1 ... X_n)$ has dimensions $((T-1) \times k_n)$, $B = \text{diag}(\beta_1, .., \beta_n)$ is a matrix with dimensions $(k_n \times n)$ and $E = (E_1 ... E_n)$ is a matrix with dimensions $((T-1) \times n)$. Next, we use two prior distributions, namely a uninformative and a Normal-Diffuse prior.

3.D.1 Uninformative prior

In deriving the posterior density function of the OLS estimates, we assume an uninformative prior. This prior means that we do not impose any prior believe on the parameters of the model. Hence, the prior function is of the form

$$f(\beta, \Sigma) \propto |\Sigma|^{-(n+1)/2}, \tag{3.D.4}$$

where Σ denotes the variance-covariance matrix of the error in the VAR model. For this uninformative prior, the marginal posterior density of the parameters can be written as

$$\begin{array}{lcl} \beta|\Sigma & \sim & \text{N}(\hat{\beta}, [x'(\Sigma^{-1} \otimes I_{T-1})x]^{-1}) \\ \Sigma|\beta & \sim & \text{IW}(Q, T-1), \end{array} \tag{3.D.5}$$

with

$$\begin{array}{lcl} \hat{\beta} & = & [x'(\Sigma^{-1} \otimes I_{T-1})x]^{-1} x'(\Sigma^{-1} \otimes I_{T-1})y \\ Q & = & (Y - XB)'(Y - XB). \end{array}$$

Since the marginal posterior densities of the two parameters β and Σ are not available, we rely on the Block-Gibbs sampling algorithm (See e.g. Bauwens et al. (1999)). Conditional on a previous simulation of the variance-covariance matrix Σ_{j-1}, we can draw β_j from the conditional density function. Again, with the sampled β_j the variance-covariance matrix Σ_j can be drawn from the inverse Wishart distribution. This sequential sampling method is initialized with the ordinary least squares estimates of the model. To remove potential influence of the starting values, we remove the first 500 draws from the sequence of parameters. Additionally, we remove draws if any eigenvalues of matrix with the autoregressive coefficients of the included variables are larger than 0.99 in order to ensure stationarity.

Our final sequence consists of 1000 draws from the posterior density. Using these parameters, we determine the structural parameters of our model using the minimum Chi-squared estimation. For each of these set of structural parameters, we determine the implied inflation risk premium and the short rates. We report the distribution of the averages of these time series, which only rely on the observed data.

3.D.2 Informative prior

Next, we impose a Normal-diffuse prior on the parameters. Since the weight of recent observations bare more importance, we establish a prior on the impact of the OLS estimates. However, we hold a diffuse prior on Σ in Equation (4.A.19) or the covariance-variance matrix of the coefficients. Formally, we can write

$$\begin{aligned} f(\beta) &\sim \mathrm{N}(\beta_{Prior}, \Omega) \\ f(\Sigma) &\propto |\Sigma|^{-(n+1)/2}, \end{aligned} \tag{3.D.6}$$

where β_{Prior} denotes the estimates of the prior. Following Zellner (1971) we can write the marginal posterior distributions as follows,

$$\begin{aligned} \beta|\Sigma &\sim \mathrm{N}(\hat{\beta}, \hat{\Omega}) \\ \Sigma|\beta &\sim \mathrm{IW}(Q, T-1), \end{aligned} \tag{3.D.7}$$

with

$$\begin{aligned} \hat{\beta}_{OLS} &= [x'(\Sigma^{-1} \otimes I_{T-1})x]^{-1}x'(\Sigma^{-1} \otimes I_{T-1})y \\ \hat{\beta} &= \hat{\Omega}(\hat{\Omega}^{-1}\beta_{Prior} + [x'(\Sigma^{-1} \otimes I_{T-1})x]^{-1}\hat{\beta}_{OLS} \\ \hat{\Omega} &= (\Omega^{-1} + x'(\Sigma^{-1} \otimes I_{T-1})x)^{-1} \\ Q &= (Y - XB_{OLS})'(Y - XB_{OLS}) + (B - B_{OLS})'X'X(B - B_{OLS}). \end{aligned}$$

Again we rely on the Block-Gibbs sampling technique to derive the marginal posterior densities of the two parameters β and Σ. Conditional on a previous simulation of the variance-covariance matrix Σ_{j-1}, we can draw β_j from the conditional density function. Again, with the sampled β_j the variance-covariance matrix Σ_j can be drawn from the inverse Wishart distribution. This sequential sampling method is initialized with the ordinary least squares estimates of the model. To remove potential influence of the starting values, we remove the first 500 draws from the sequence of parameters. Additionally, we remove draws if any eigenvalues of matrix with the autoregressive coefficients of the included variables are larger than 0.99 in order to ensure stationarity. The prior estimates are derived from using the OLS estimates on a sample from January 2010 to December 2012. Our final sequence consists again of

1000 draws from the posterior density. Using these OLS parameters, we determine the structural parameters of our model using the minimum Chi-squared estimation. For each of these set of structural parameters, we determine the implied inflation risk premium and the short rates. We report the distribution of the averages of these time series, which only rely on the observed data.

3.E Appendix E: Tables and figures

Table 3.1: Summary Statistics Yields

This table presents the statistics on the main time series used in the chapter. The annualized nominal and real yields are presented for both the UK and US. The sample period ranges from July 2004 up to December 2012.

	Mean	St. dev	Min	Max
US				
	Nominal			
1m	1.68 %	1.87 %	0.00 %	5.13 %
3m	1.75 %	1.89 %	0.01 %	5.01 %
6m	1.87 %	1.89 %	0.05 %	5.04 %
1y	2.02 %	1.88 %	0.13 %	5.21 %
2y	2.16 %	1.76 %	0.19 %	5.13 %
3y	2.36 %	1.62 %	0.31 %	5.06 %
5y	2.82 %	1.36 %	0.63 %	5.01 %
7y	3.26 %	1.18 %	1.01 %	5.05 %
10y	3.77 %	1.00 %	1.55 %	5.17 %
	Real			
1y	0.28 %	1.63 %	-2.66 %	4.87 %
3y	0.26 %	1.28 %	-1.80 %	2.42 %
5y	0.50 %	1.14 %	-1.70 %	2.20 %
7y	0.79 %	1.05 %	-1.49 %	2.52 %
10y	1.14 %	0.92 %	-1.06 %	2.91 %
UK				
	Nominal			
1m	2.81 %	2.22 %	0.43 %	5.91 %
3m	2.78 %	2.21 %	0.39 %	5.83 %
6m	2.75 %	2.20 %	0.33 %	5.85 %
1y	2.72 %	2.12 %	0.18 %	5.83 %
2y	2.82 %	1.92 %	0.07 %	5.76 %
3y	3.00 %	1.75 %	0.16 %	5.69 %
5y	3.33 %	1.46 %	0.57 %	5.56 %
7y	3.60 %	1.22 %	1.02 %	5.48 %
10y	3.90 %	0.96 %	1.61 %	5.36 %
	Real			
1y	0.01 %	2.40 %	-3.72 %	4.43 %
3y	0.22 %	1.83 %	-2.61 %	2.82 %
5y	0.46 %	1.50 %	-2.20 %	3.09 %
7y	0.62 %	1.24 %	-1.80 %	2.50 %
10y	0.78 %	0.96 %	-1.31 %	2.16 %

Table 3.2: Estimation Results US

In this table we present a selection of the estimation results of three models using the Minimum Chi Squared approach for the US. For brevity, we only report the coefficients of the latent factors, other estimates are available upon request. All estimates are determined using the full sample ranging from August 2004 up to December 2012. The standard errors are determined by a weighted outer-product as described in Appendix 3.C.

	Benchmark		Macro		Survey	
Parameter	Estimate	Std error	Estimate	Std error	Estimate	Std error
Dynamics latent factors: $X_t^L = \Phi_1^L X_{t-1}^L + [0\ I_2]\epsilon_t$						
$\Phi_{1,11}$	0.9933	0.0019	0.9939	0.0016	0.9942	0.0002
$\Phi_{1,21}$	-0.0093	0.0015	-0.0083	0.0014	-0.0090	0.0006
$\Phi_{1,22}$	0.9858	0.0013	0.9864	0.0011	0.9857	0.0008
Price of Risk: $\Lambda_t = \Gamma_0 + \Gamma_1 X_t$						
$\Gamma_{0,EC}$	1.3164	0.3070	2.0016	0.3257	1.4412	1.4123
$\Gamma_{0,1}$	0.6435	0.0845	0.6452	0.0943	0.6228	0.0469
$\Gamma_{0,2}$	1.2409	0.3617	1.1414	0.3285	1.2893	0.1102
$\Gamma_{1,11}$	-0.0267	0.0042	-0.0211	0.0032	-0.0229	0.0017
$\Gamma_{1,12}$	0.0263	0.0064	0.0266	0.0055	0.0235	0.0028
$\Gamma_{1,21}$	-0.0457	0.0089	-0.0349	0.0067	-0.0432	0.0057
$\Gamma_{1,22}$	0.0228	0.0038	0.0188	0.0030	0.0199	0.0018
$\Gamma_{1,CPICPI}$	0.0000	0.7097	0.0000	1.1800	0.0000	0.8974
$\Gamma_{1,CPIEC1}$	-	-	0.0000	0.1074	-0.0737	0.0418
$\Gamma_{1,CPIEC2}$	-	-	0.0000	0.0877	-0.0447	0.0288
$\Gamma_{1,CPIEC3}$	-	-	0.0000	0.1064	0.0117	0.0570
$\Gamma_{1,CPIEC4}$	-	-	-	-	-0.0178	0.0299
$\Gamma_{1,CPI1}$	-0.0257	0.0133	-0.0394	0.0069	-0.0249	0.0829
$\Gamma_{1,CPI2}$	0.0353	0.0088	0.0471	0.0061	0.0366	0.0138
Short rate: $r_t^N = \delta_{0,r}^N + \delta_{1,r}'^N X_t$						
$\delta_{0,r}^N * 12$	0.0168	0.0018	0.0168	0.0018	0.0168	0.0018
$\delta_{1,r,1}^N * 12$	0.0010	0.0003	0.0011	0.0003	0.0010	0.0001
$\delta_{1,r,2}^N * 12$	0.0017	0.0002	0.0019	0.0002	0.0018	0.0001
Inflation rate: $\pi_t = \delta_{0,\pi} + \delta_\pi' X_t + \sigma_\pi' \epsilon_t$						
$\delta_{0,\pi}$	0.0001	0.0010	0.0000	0.0027	0.0003	0.0052
$\delta_{\pi,CPI}$	0.5064	0.0882	0.1770	0.1075	0.5521	0.1124
$\delta_{\pi,EC1}$	-	-	0.0456	0.0098	0.0108	0.0052
$\delta_{\pi,EC2}$	-	-	-0.0017	0.0080	0.0062	0.0036
$\delta_{\pi,EC3}$	-	-	-0.0098	0.0097	-0.0087	0.0071
$\delta_{\pi,EC4}$	-	-	-	-	-0.0020	0.0037
$\delta_{\pi,1} * 1000$	0.0402	0.0492	0.0427	0.0837	0.0361	0.2957
$\delta_{\pi,2} * 1000$	0.0063	0.0305	0.0160	0.0443	0.0140	0.0489
$\sigma_{\pi,CPI}$	-0.0036	0.0015	-0.0036	0.0003	-0.0033	0.0105
$\sigma_{\pi,1}$	0.0000	0.0027	0.0002	0.0014	0.0000	0.0042
$\sigma_{\pi,2}$	-0.0016	0.0035	-0.0002	0.0031	-0.0017	0.0201

Table 3.3: Results US Measurement errors

The measurement errors of three estimated models for the US are presented in this table. We report the full sample standard deviation of the measurement error across our entire sample period and the standard deviation of three periods. The first period, denoted as prior the financial crisis, ranges from August 2004 up to August 2008. As starting point for the Financial Crisis in 2008, we take the bankruptcy filling of Lehman Brother in September and we define the crisis period up to August 2009. The period afterward is defined as post crisis and ends in December 2012.

	Full sample	Prior crisis	During crisis	Post crisis
Benchmark				
Nominal				
ω_{1M}	0.0035	0.0038	0.0027	0.0014
ω_{3M}	0.0022	0.0019	0.0026	0.0011
ω_{6M}	0.0012	0.0012	0.0009	0.0007
ω_{2Y}	0.0009	0.0010	0.0010	0.0005
ω_{3Y}	0.0009	0.0009	0.0011	0.0005
ω_{7Y}	0.0009	0.0009	0.0013	0.0007
ω_{10Y}	0.0019	0.0018	0.0025	0.0016
Real				
ω_{1Y}	0.0099	0.0048	0.0168	0.0046
ω_{3Y}	0.0053	0.0026	0.0075	0.0038
ω_{5Y}	0.0043	0.0022	0.0057	0.0033
ω_{7Y}	0.0039	0.0021	0.0055	0.0030
ω_{10Y}	0.0035	0.0021	0.0048	0.0025
Macro				
Real				
ω_{1Y}	0.0106	0.0058	0.0190	0.0062
ω_{3Y}	0.0053	0.0030	0.0075	0.0041
ω_{5Y}	0.0043	0.0024	0.0061	0.0034
ω_{7Y}	0.0039	0.0023	0.0056	0.0030
ω_{10Y}	0.0035	0.0022	0.0048	0.0025
Survey				
Real				
ω_{1Y}	0.0100	0.0052	0.0163	0.0049
ω_{3Y}	0.0053	0.0029	0.0071	0.0039
ω_{5Y}	0.0043	0.0023	0.0056	0.0033
ω_{7Y}	0.0039	0.0022	0.0053	0.0029
ω_{10Y}	0.0034	0.0021	0.0046	0.0025

Table 3.4: Estimation Results UK

In this table we present a selection of the estimation results of three models using the Minimum Chi Squared approach for the UK. For brevity, we only report the coefficients of the latent factors, other estimates are available upon request. All estimates are determined using the full sample ranging from August 2004 up to December 2012. The standard errors are determined by a weighted outer-product as described in Appendix 3.C

	Benchmark		Macro		Survey	
Parameter	Estimate	Std error	Estimate	Std error	Estimate	Std error
Dynamics latent factors: $X_t^L = \Phi_1^L X_{t-1}^L + [0\ I_2]\epsilon_t$						
$\Phi_{1,11}$	0.9958	0.0016	0.9955	0.0016	0.9960	0.0009
$\Phi_{1,21}$	-0.0169	0.0023	-0.0171	0.0024	-0.0167	0.0011
$\Phi_{1,22}$	0.9785	0.0016	0.9783	0.0016	0.9784	0.0013
Price of Risk: $\Lambda_t = \Gamma_0 + \Gamma_1 X_t$						
$\Gamma_{0,EC}$	10.3594	4.3597	10.3457	4.0116	10.2048	8.8535
$\Gamma_{0,1}$	1.5049	0.4022	1.4927	0.3715	1.5340	0.2341
$\Gamma_{0,2}$	2.4425	0.8958	2.4586	0.8613	2.4999	0.4876
$\Gamma_{1,11}$	-0.0137	0.0035	-0.0144	0.0039	-0.0134	0.0017
$\Gamma_{1,12}$	0.0391	0.0045	0.0393	0.0047	0.0388	0.0035
$\Gamma_{1,21}$	-0.0427	0.0055	-0.0438	0.0061	-0.0424	0.0037
$\Gamma_{1,22}$	0.0227	0.0036	0.0234	0.0039	0.0224	0.0020
$\Gamma_{1,CPICPI}$	28.3748	13.5459	74.1052	18.7796	12.4202	13.8914
$\Gamma_{1,CPIEC1}$	-	-	0.0000	1.7080	-0.0001	0.4691
$\Gamma_{1,CPIEC2}$	-	-	0.0000	7.1958	0.0003	1.0564
$\Gamma_{1,CPIEC3}$	-	-	0.0001	0.4639	0.0000	1.0128
$\Gamma_{1,CPIEC4}$	-	-	-	-	-0.0002	0.8489
$\Gamma_{1,CPI1}$	-0.1261	0.0428	-0.1227	0.0391	-0.1203	0.0927
$\Gamma_{1,CPI2}$	0.1623	0.0513	0.1777	0.0542	0.1615	0.1084
Short rate: $r_t^N = \delta_{0,r}^N + \delta_{1,r}^{\prime N} X_t$						
$\delta_{0,r}^N * 12$	0.0282	0.0022	0.0282	0.0022	0.0282	0.0022
$\delta_{1,r,1}^N * 12$	0.0016	0.0002	0.0015	0.0002	0.0016	0.0000
$\delta_{1,r,2}^N * 12$	0.0021	0.0002	0.0020	0.0002	0.0021	0.0000
Inflation rate: $\pi_t = \delta_{0,\pi} + \delta_\pi' X_t + \sigma_\pi' \epsilon_t$						
$\delta_{0,\pi}$	-0.0034	0.0071	-0.0038	0.0056	-0.0031	0.0227
$\delta_{\pi,CPI}$	0.1703	0.1001	-0.0079	0.0933	0.0912	0.0992
$\delta_{\pi,EC1}$	-	-	0.0452	0.0084	0.0044	0.0040
$\delta_{\pi,EC2}$	-	-	0.0085	0.0357	0.0099	0.0091
$\delta_{\pi,EC3}$	-	-	0.0017	0.0023	0.0002	0.0087
$\delta_{\pi,EC4}$	-	-	-	-	0.0231	0.0073
$\delta_{\pi,1} * 1000$	0.0271	0.0918	0.0262	0.0713	0.0295	0.3114
$\delta_{\pi,2} * 1000$	-0.1470	0.1034	-0.1550	0.0877	-0.1458	0.3577
$\sigma_{\pi,CPI}$	-0.0020	0.0010	-0.0020	0.0008	-0.0020	0.0031
$\sigma_{\pi,1}$	-0.0021	0.0019	-0.0004	0.0014	-0.0024	0.0063
$\sigma_{\pi,2}$	0.0031	0.0014	0.0031	0.0006	0.0025	0.0042

Table 3.5: Results UK Measurement errors

The measurement errors of three estimated models for the UK are presented in this table. We report the full sample standard deviation of the measurement error across our entire sample period and the standard deviation of three periods. The first period, denoted as prior the financial crisis, ranges from August 2004 up to August 2008. As starting point for the Financial Crisis in 2008, we take the bankruptcy filling of Lehman Brother in September and we define the crisis period up to August 2009. The period afterward is defined as post crisis and ends in December 2012.

	Full sample	Prior crisis	During crisis	Post crisis
Benchmark				
Nominal				
ω_{1M}	0.0031	0.0026	0.0036	0.0020
ω_{3M}	0.0022	0.0020	0.0023	0.0014
ω_{6M}	0.0012	0.0013	0.0013	0.0006
ω_{2Y}	0.0009	0.0009	0.0014	0.0006
ω_{3Y}	0.0009	0.0007	0.0014	0.0007
ω_{7Y}	0.0009	0.0007	0.0015	0.0007
ω_{10Y}	0.0017	0.0014	0.0024	0.0017
Real				
ω_{1Y}	0.0116	0.0032	0.0183	0.0061
ω_{3Y}	0.0061	0.0028	0.0101	0.0035
ω_{5Y}	0.0044	0.0024	0.0090	0.0021
ω_{7Y}	0.0032	0.0022	0.0073	0.0016
ω_{10Y}	0.0027	0.0021	0.0054	0.0017
Macro				
Real				
ω_{1Y}	0.0116	0.0051	0.0219	0.0067
ω_{3Y}	0.0058	0.0031	0.0102	0.0035
ω_{5Y}	0.0041	0.0025	0.0084	0.0022
ω_{7Y}	0.0030	0.0023	0.0065	0.0017
ω_{10Y}	0.0026	0.0021	0.0047	0.0017
Survey				
Real				
ω_{1Y}	0.0110	0.0035	0.0172	0.0064
ω_{3Y}	0.0059	0.0028	0.0095	0.0037
ω_{5Y}	0.0042	0.0024	0.0086	0.0023
ω_{7Y}	0.0032	0.0022	0.0070	0.0018
ω_{10Y}	0.0027	0.0020	0.0052	0.0017

Table 3.6: Out of sample forecasts: US

This table reports the errors of the out-of-sample forecasts of the US using three methods, namely the Root mean squared error (RMSE), Mean absolute deviation (MAD) and the out-of-sample R squared (Out of Sample R^2). We forecast the rates using a fixed window from September 2009 up to December 2012. The bold numbers represent the model that performs the best for each time series.

	RMSE			MAD		
	Benchmark	Macro	Survey	Benchmark	Macro	Survey
1m	0.1942 %			0.1615 %		
3m	0.1597 %			0.1282 %		
6m	0.0844 %			0.0665 %		
1y	0.0839 %			0.0583 %		
2y	0.1495 %			0.1172 %		
3y	0.1861 %			0.1433 %		
5y	0.2450 %			0.1939 %		
7y	0.3052 %			0.2494 %		
10y	0.3617 %			0.3012 %		
1y real	0.8478 %	**0.8175** %	0.8329 %	0.7823 %	**0.6717** %	0.7295 %
3y real	0.4697 %	**0.4655** %	0.4851 %	0.4059 %	**0.3661** %	0.3988 %
5y real	**0.4473** %	0.4522 %	0.4510 %	0.3983 %	0.3743 %	**0.3736** %
7y real	0.4544 %	0.4614 %	**0.4529** %	0.4055 %	0.3912 %	**0.3825** %
10y real	0.4484 %	0.4541 %	**0.4442** %	0.4031 %	0.3913 %	**0.3789** %
CPI	0.2740 %	**0.1942** %	0.2991 %	**0.2113** %	0.2302 %	0.2450 %
	R^2 Out of Sample					
	Benchmark	Macro	Survey			
1m	99.13%					
3m	99.44%					
6m	99.86%					
1y	99.86%					
2y	99.53%					
3y	99.19%					
5y	98.24%					
7y	96.70%					
10y	94.12%					
1y real	79.61%	**82.84** %	82.19 %			
3y real	92.37%	**92.98** %	92.37 %			
5y real	92.23%	92.41 %	**92.45** %			
7y real	91.14%	91.17 %	**91.49** %			
10y real	89.43%	89.45 %	**89.90** %			
CPI	**20.85**%	14.63 %	5.68 %			

Table 3.7: Results UK Out of sample

This table reports the errors of the out-of-sample forecasts of the UK using three methods, namely the Root mean squared error (RMSE), Mean absolute deviation (MAD) and the out-of-sample R squared (Out of Sample R^2). We forecast the rates using a fixed window from September 2009 up to December 2012. The bold numbers represent the model that performs the best for each time series.

	RMSE			MAD		
	Benchmark	Macro	Survey	Benchmark	Macro	Survey
1m	0.1862 %			0.1531 %		
3m	0.1549 %			0.1283 %		
6m	0.1193 %			0.1034 %		
1y	0.1028 %			0.0799 %		
2y	0.1408 %			0.1194 %		
3y	0.1874 %			0.1552 %		
5y	0.2168 %			0.1638 %		
7y	0.2502 %			0.1898 %		
10y	0.2916 %			0.2280 %		
1y real	1.3176 %	**1.2622** %	1.2631 %	1.1621 %	**1.0901** %	1.1625 %
3y real	0.7072 %	0.7025 %	**0.6881** %	0.6183 %	**0.5813** %	0.6125 %
5y real	**0.4630** %	0.4928 %	0.4793 %	**0.3849** %	0.3963 %	0.4184 %
7y real	**0.3019** %	0.3447 %	0.3299 %	**0.2418** %	0.2780 %	0.2830 %
10y real	**0.2594** %	0.2955 %	0.2849 %	**0.2051** %	0.2408 %	0.2237 %
CPI	0.4161 %	**0.1862** %	0.4486 %	0.3416 %	**0.2854** %	0.3612 %

	R^2 Out of Sample		
	Benchmark	Macro	Survey
1m	99.60%		
3m	99.72%		
6m	99.83%		
1y	99.87%		
2y	99.72%		
3y	99.43%		
5y	98.96%		
7y	98.06%		
10y	95.85%		
1y real	87.06%	**88.12** %	88.11 %
3y real	93.54%	93.62 %	**93.88** %
5y real	**95.81**%	95.25 %	95.51 %
7y real	**97.34**%	96.53 %	96.82 %
10y real	**96.67**%	95.67 %	95.98 %
CPI	-33.79%	**-0.22** %	-55.51 %

Table 3.8: Data implied Campbell-Shiller Regressions

This table presents the estimates of the coefficients of the Campbell-Shiller regressions for both the UK and US interest rates. The regressions are defined in Equation (3.2.13). The reported standard errors are based on the Newey-West estimator. The p-value is shown for the test with the null hypothesis of a slope coefficient equal to one.

	US		UK	
Maturity	Coef	*p* Value	Coef	*p* Value
Nominal				
1y	0.39 (1.06)	0.57	2.86 (2.60)	0.48
2y	0.09 (1.58)	0.57	−0.07 (1.52)	0.48
3y	−0.89 (1.78)	0.29	−0.87 (1.30)	0.15
5y	−2.42 (1.89)	0.07	−1.44 (1.37)	0.08
7y	−3.35 (2.16)	0.05	−1.81 (1.53)	0.07
10y	−4.34 (2.62)	0.04	−2.31 (1.70)	0.05
Real				
1y	1.38 (0.95)	0.69	0.22 (0.46)	0.09
3y	0.85 (1.14)	0.90	−0.70 (0.79)	0.03
5y	0.61 (1.42)	0.78	−1.86 (0.85)	0.00
7y	0.45 (2.19)	0.80	−1.88 (0.82)	0.00
10y	−1.35 (2.36)	0.32	−2.53 (1.00)	0.00

Table 3.9: Model implied Campbell-Shiller Regressions

This table presents the model implied estimates of the coefficients of the Campbell-Shiller regressions for both the UK and US. The regressions are defined in Equation (3.2.13).

	US	UK
Maturity	Coef	Coef
Nominal		
Benchmark		
1y	0.06 (0.10)	−0.01 (0.10)
2y	−0.11 (0.07)	−0.25 (0.06)
3y	−0.28 (0.05)	−0.50 (0.02)
5y	−0.60 (0.02)	−0.97 (0.03)
7y	−0.89 (0.01)	−1.33 (0.06)
10y	−1.26 (0.04)	−1.63 (0.07)
Real		
Benchmark		
1y	0.94 (0.03)	0.78 (0.02)
3y	0.98 (0.01)	0.72 (0.06)
5y	0.99 (0.01)	0.13 (0.12)
7y	0.98 (0.03)	−0.44 (0.15)
10y	0.91 (0.07)	−0.92 (0.16)
Macro		
1y	1.62 (0.04)	1.79 (0.04)
3y	1.60 (0.03)	1.52 (0.07)
5y	1.59 (0.04)	1.23 (0.10)
7y	1.58 (0.05)	0.95 (0.12)
10y	1.53 (0.07)	0.63 (0.15)
Survey		
1y	0.83 (0.03)	1.01 (0.03)
3y	0.87 (0.02)	0.99 (0.04)
5y	0.88 (0.02)	0.70 (0.09)
7y	0.87 (0.03)	0.37 (0.13)
10y	0.83 (0.05)	0.00 (0.15)

Table 3.10: Fitting of Term premia

This table presents Mean absolute deviations (MAD) of the the model implied nominal and real term premia for both the UK and US. We show the MAD for all three models. We also report the standard error of the mean deviations.

	US		UK	
Maturity	MAD	Std	MAD	Std
Nominal				
Benchmark				
1y	0.02%	0.01%	0.05%	0.03%
2y	0.03%	0.02%	0.04%	0.03%
3y	0.05%	0.04%	0.05%	0.04%
5y	0.11%	0.09%	0.06%	0.05%
7y	0.19%	0.13%	0.08%	0.07%
10y	0.28%	0.19%	0.11%	0.08%
Real				
Benchmark				
1y	0.12%	0.06%	0.08%	0.07%
3y	0.11%	0.04%	0.14%	0.13%
5y	0.06%	0.04%	0.26%	0.23%
7y	0.11%	0.08%	0.28%	0.25%
10y	0.23%	0.21%	0.36%	0.33%
Macro				
1y	0.16%	0.16%	0.17%	0.17%
3y	0.14%	0.14%	0.18%	0.16%
5y	0.13%	0.13%	0.29%	0.25%
7y	0.15%	0.14%	0.31%	0.27%
10y	0.23%	0.18%	0.39%	0.34%
Survey				
1y	0.14%	0.08%	0.08%	0.06%
3y	0.13%	0.07%	0.15%	0.12%
5y	0.09%	0.06%	0.26%	0.23%
7y	0.14%	0.09%	0.28%	0.25%
10y	0.25%	0.21%	0.36%	0.33%

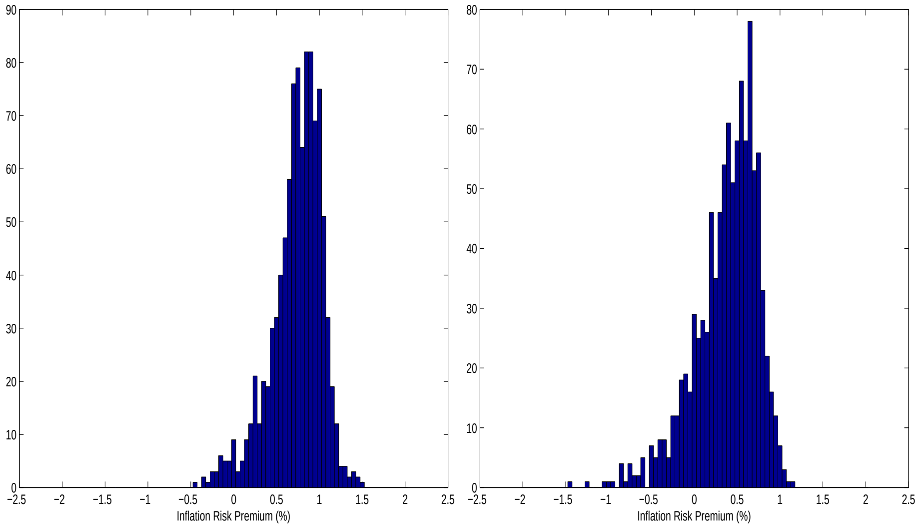
Table 3.11: Inflation risk premia

This table present the inflation risk premia of the UK and US. We report the mean and standard error of the series for the 5 and 10 year premium.

	US		UK	
Maturity	Mean	Std	Mean	Std
Benchmark				
5y	0.72%	0.65%	-0.45%	0.80%
10y	0.69%	0.58%	-0.26%	0.49%
Macro				
5y	0.82%	0.63%	0.13%	0.13%
10y	0.79%	0.56%	0.32%	0.32%
Survey				
5y	0.68%	0.64%	-1.08%	0.87%
10y	0.65%	0.56%	-0.90%	0.53%

Figure 3.1: Bayesian 5 year inflation risk premium US: Bench model

This figure presents the marginal posterior distribution of the mean of the US 5 year inflation risk premium using the Benchmark model. In the first graph, an uninformative prior is assumed, which assigns equal weights to the pre-crisis and post crisis periods. In the second graph a Normal-diffuse prior is used to add more weight to the post crisis period from January 2010 up to December 2012. In the first graph the distribution is centered around 0.74% with a standard deviation of 0.30% and in the second graph it is centered around 0.38% and a standard deviation of 0.37%.

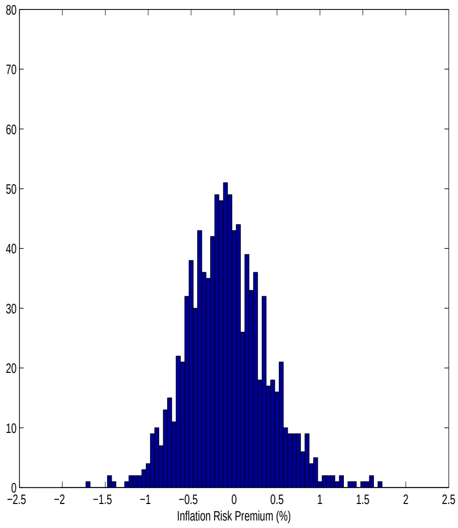


(a) Uninformative Prior

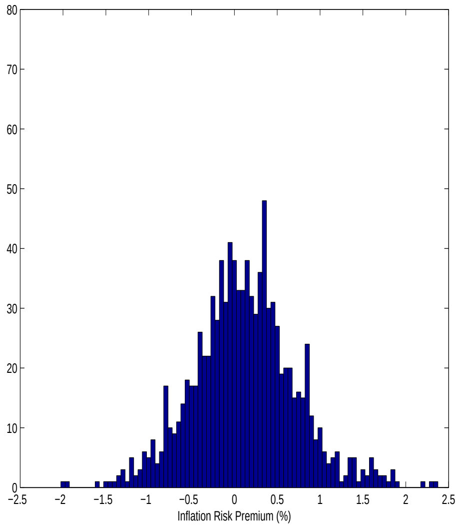
(b) Post Crisis Prior

Figure 3.2: Bayesian 5 year inflation risk premium UK: Bench model

This figure presents the marginal posterior distribution of the mean of the UK 5 year inflation risk premium using the Benchmark model. In the first graph, an uninformative prior is assumed, which assigns equal weights to the pre-crisis and post crisis periods. In the second graph a Normal-diffuse prior is used to add more weight to the post crisis period from January 2010 up to December 2012. In the first graph the distribution is centered around -0.08% with a standard deviation of 0.47% and in the second graph it is centered around 0.13% and a standard deviation of 0.61%.



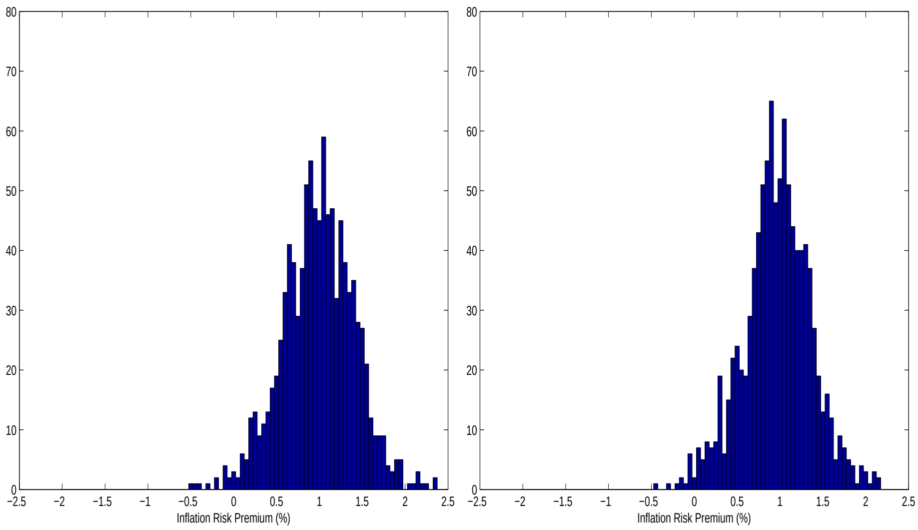
(a) Uninformative Prior



(b) Post Crisis Prior

Figure 3.3: Bayesian 5 year inflation risk premium US: Macro model

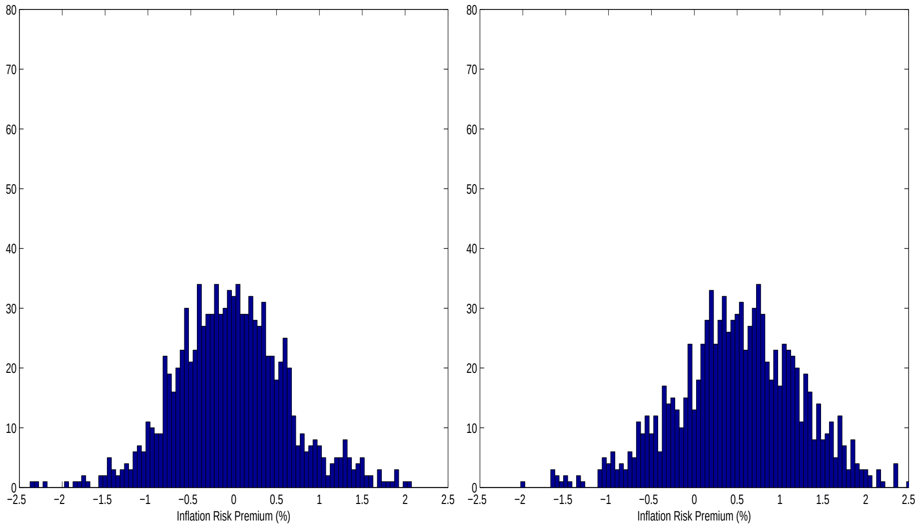
This figure presents the marginal posterior distribution of the mean of the US 5 year inflation risk premium using the Macro model. In the first graph, an uninformative prior is assumed, which assigns equal weights to the pre-crisis and post crisis periods. In the second graph a Normal-diffuse prior is used to add more weight to the post crisis period from January 2010 up to December 2012. In the first graph the distribution is centered around 0.99% with a standard deviation of 0.43% and in the second graph it is centered around 0.96% and a standard deviation of 0.40%.



(a) Uninformative Prior (b) Post Crisis Prior

Figure 3.4: Bayesian 5 year inflation risk premium UK: Macro model

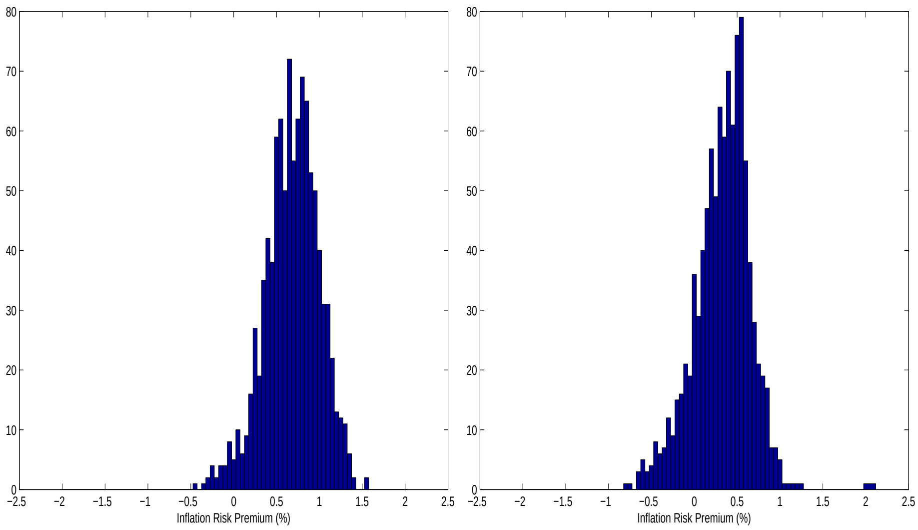
This figure presents the marginal posterior distribution of the mean of the UK 5 year inflation risk premium using the Macro model. In the first graph, an uninformative prior is assumed, which assigns equal weights to the pre-crisis and post crisis periods. In the second graph a Normal-diffuse prior is used to add more weight to the post crisis period from January 2010 up to December 2012. In the first graph the distribution is centered around 0.02% with a standard deviation of 0.67% and in the second graph it is centered around 0.49% and a standard deviation of 0.76%.



(a) Uninformative Prior (b) Post Crisis Prior

Figure 3.5: Bayesian 5 year inflation risk premium US: Survey model

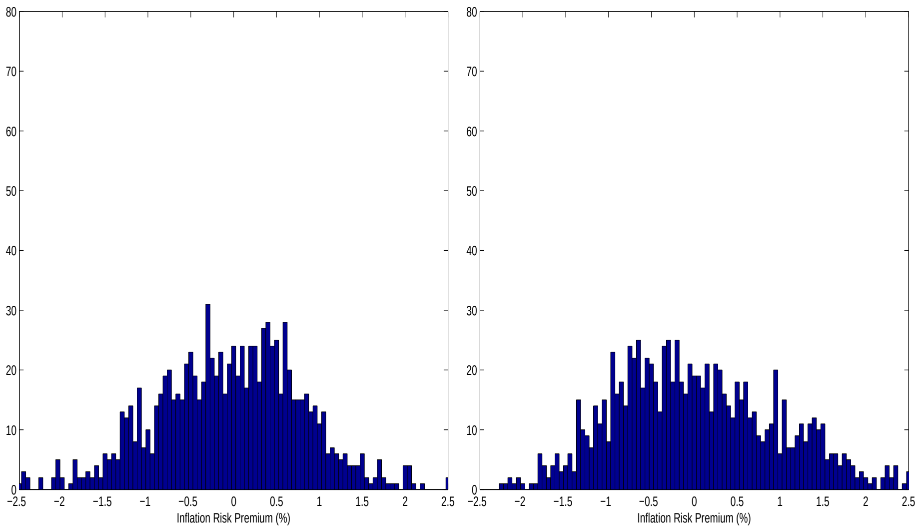
This figure presents the marginal posterior distribution of the mean of the US 5 year inflation risk premium using the Survey model. In the first graph, an uninformative prior is assumed, which assigns equal weights to the pre-crisis and post crisis periods. In the second graph a Normal-diffuse prior is used to add more weight to the post crisis period from January 2010 up to December 2012. In the first graph the distribution is centered around 0.68% with a standard deviation of 0.31% and in the second graph it is centered around 0.33% and a standard deviation of 0.32%.



(a) Uninformative Prior (b) Post Crisis Prior

Figure 3.6: Bayesian 5 year inflation risk premium UK: Survey model

This figure presents the marginal posterior distribution of the mean of the US 5 year inflation risk premium using the Survey model. In the first graph, an uninformative prior is assumed, which assigns equal weights to the pre-crisis and post crisis periods. In the second graph a Normal-diffuse prior is used to add more weight to the post crisis period from January 2010 up to December 2012. In the first graph the distribution is centered around -0.03% with a standard deviation of 0.87% and in the second graph it is centered around 0.03% and a standard deviation of 0.97%.



(a) Uninformative Prior (b) Post Crisis Prior

CHAPTER 4

THE EQUITY RISK PREMIUM AND PENSION AMBITION: THE EFFECT OF PARAMETER UNCERTAINTY

4.1 Introduction

A central theme of pension contracts is to accumulate sufficient wealth to realize the participants' pension ambition. In a defined contribution (DC) plan, the risk of attaining a lower replacement rate at retirement depends ex ante on the risk profile of the participant. A high risk profile will lead to higher expected returns, yet will increase the uncertainty about future replacement rates at retirement. In both public and private US pension plans, more than three quarters of pension wealth is invested in equity markets (Munnell and Soto, 2007). Such investment strategies will substantially increase uncertainty about future replacement rates. However, existing models to quantify uncertainty in replacement rates are typically based on the assumption that financial parameters such as the equity risk premium and the volatility in equity returns are known a priori.

In this chapter, we extend the analysis of uncertainty by introducing an additional source of risk, namely uncertainty of financial parameters. Since financial parameters are unknown and are typically estimated with large errors, participants have to form beliefs about ex ante projections for the financial market. Recent literature has shown that different beliefs about future projections can substantially affect the optimal investment strategy of the portfolio, because equity returns do not necessarily have decreasing volatility over the investment horizon (Pastor and Stambaugh, 2012; Hoevenaars, Molenaar, Schotman, and Steenkamp, 2013). Regardless of the investment strategy, ignoring parameter uncertainty will lead to an incorrect risk analysis of future replacement rates. As a result, participants may consider to invest in more risky portfolios although such investment strategies will not correspond with their risk perception.

This chapter highlights the complex interaction between real pension ambitions and minimum required contribution levels in a financial market with parameter uncertainty. To set a real ambition in our pension contract, the participant aims to acquire a real variable annuity at retirement with a desired replacement rate. Based on the risk preference of the participant, we establish an investment strategy to achieve this pension ambition. This approach is similar to the ambition of most DC plans in which participants can purchase an annuity at retirement.[1] Since DC contracts offer no guarantee for the desired replacement rate, sufficient capital accumulation to achieve the participant's ambition relies only on his pension contributions and the performance of his investment portfolio. Ignoring parameter uncertainty may introduce overly optimistic beliefs of future portfolio returns. As a result, participants will underestimate the uncertainty in replacement rates and will set lower contribution rates.

To analyze parameter uncertainty, we focus on two questions. First, we analyze the impact of parameter uncertainty on the replacement rates when the participant uses a fixed contribution scheme based on historical portfolio performance. We find that the major factor of parameter uncertainty driving the risk of the replacement rates in our pension contract is the equity risk premium. Although parameter uncertainty is an important additional source of risk for the participant, economic uncertainty causes the largest fluctuations in the replacement rates at retirement. Second, we verify whether a time-varying contribution scheme based on a combination of the term structure of interest rates and the historical equity risk premium estimated over a large sample period, allows the participant to respond to parameter uncertainty. We find that if parameter uncertainty observed in our sample period corresponds to

[1] An example of a DC contract with a pension ambition in the US is a target benefit plan. In these plans, contributions are set such that projected pension entitlements can be achieved, however the actual level of the participant's benefit depends on the accrued pension wealth at retirement.

future parameter uncertainty, the time-varying contribution scheme can partially compensate the participant for parameter uncertainty. However, when structural shifts occur in the financial parameters, the risk of replacement rates at retirement substantially increases in this contribution scheme.

Predictions of economic conditions to determine pension contributions rely on parameter and model assumptions. Ignoring parameter uncertainty in these projections will underestimate the risk involved with future pension entitlements. Estimates for the equity risk premium can be substantially different between various sample periods, suggesting large parameter uncertainty about the ex ante equity risk premium (Jagannathan, McGrattan, and Scherbina, 2001). We estimate the historical equity risk premium to be about 5.0% using an extensive time period of 1952 to 2012, yet uncertainty about this estimate is large with a standard error of 2.0%. When this historical equity risk premium is used to set contribution levels and uncertainty of the estimate is ignored, the participant will underestimate his risk of achieving a lower replacement rate at retirement.

To evaluate the effect of ignoring parameter uncertainty, we compare our pension contract in a setting with parameter uncertainty and without. We set the pension ambition of a participant to acquire a real variable annuity at retirement with a replacement rate of 70% relative to his end wage. We motivate this ambition by the findings of Munnell and Soto (2007) who report median replacement rates in the US for DB pension plans ranging from 76% for workers covered by social security and 81% for workers without social security. The participant uses a fixed pension contribution scheme based on historical portfolio performance, which requires him to yearly contribute 6.9% of his wage. While this contribution scheme is naive as he ignores both parameter uncertainty and economic shocks, it allows us to examine the effect of parameter uncertainty on the replacement rates. On average the participant achieves his ambition of 70%, however this strategy introduces large risk even without parameter uncertainty. With 2.5% probability, the participant will obtain a replacement rate of 35.3%. However, if parameter uncertainty of the equity risk premium is taken into account, his 2.5% percentile of replacement rate's distribution is far lower, namely a replacement rate of 26.6%. As a result, his risk perception of the replacement rate at retirement is substantially overestimated when parameter uncertainty about the equity risk premium is ignored.

Introducing parameter uncertainty to other financial parameters does not substantially affect the risk of the participant. The 2.5% percentile of replacement rate at retirement drops to 26.5% when parameter uncertainty is assumed for all financial parameters. This result shows that uncertainty of the equity risk premium is the key factor for parameter uncertainty in DC pension contracts. Therefore, ignoring uncertainty regarding the equity risk premium will lead to underestimation of the replacement rate's risk at retirement. As a consequence, the participant may set too low pension contributions due to his distorted risk perception.

Next, we examine whether a time-varying contribution scheme can compensate for the effects of parameter uncertainty. The intuition behind this contribution rate is to spread out differences between the present value of the price of the real variable annuity and current accumulated pension wealth over the remaining future expected wage of the individual. To set the required contribution level, we assume the participant sets the equity risk premium to our estimated historical equity risk premium. Although the estimate might be too optimistic or pessimistic about future equity return, this contribution scheme allows the participant to respond to economic shocks or parameter uncertainty. We find that the time-varying contribution scheme can partially compensate for the effects of uncertainty. Ignoring parameter uncertainty in this strategy results in a lower bound for the replacement rate of 45.2% at a 2.5% percentile. Allowing for parameter uncertainty of all financial parameters, the risk of the replacement rate at a 2.5% percentile drops to 42.1%. Incorporating parameter uncertainty causes a large shift in the risk perception of the participant. If his risk assessment is set at a 2.5% percentile of the replacement rate distribution with parameter uncertainty, then this corresponds to a shift to the 1.3% percentile of the distribution that ignores parameter uncertainty. Therefore, a 2.5% percentile of distributions ignoring parameter uncertainty of the equity risk premium to set the risk level of the participant may lead to substantial underestimation of replacement rate risk. To compensate for the effects of parameter uncertainty, the pension contribution rate needs to increase relatively to the case without parameter uncertainty at the start of the accumulation phase. For example, the contribution rate at age 25 increases from 5.8% to 6.5%, whereas at age 60 the contribution decreases from 11.6% to 10.7%. Due to the front loaded contribution, the participant requires less contribution at older ages compared to the contribution rate that ignores parameter uncertainty. While the time-varying contribution reacts to parameter uncertainty, it can only partially compensate for the replacement rate risk as it still affects the risk perception of the participant.

One of the factors explaining the impact of parameter uncertainty of the equity risk premium is the risk profile of the participant. The participant in our benchmark contract has a high risk profile with a fixed investment strategy of 60% equity and 40% bonds, which resembles the average US pension funds investment strategy.[2] To investigate the effects of a lower risk profile, we set the portfolio to 30% equity and the remaining invested in bonds.[3] While the absolute size of the effect of parameter uncertainty for the equity risk premium is smaller due to a lower exposure to the equity premium, parameter uncertainty causes a similar relative shift in risk perception. Similarly as in the benchmark case, the 2.5% percentile of the replacement rate's distribution at retirement drops from 51.0% to 48.4%, if parameter uncertainty is introduced and the time-varying contribution is employed. As a result, the risk perception of the participant shifts from a 2.5% percentile to a 1.4% percentile of the replacement rate's distribution that ignores parameter uncertainty. Hence, lowering the equity exposure in the portfolio leads to a similar shift in the participant's risk perception about the replacement rate at retirement.

A limiting factor of the time-varying contribution scheme's ability to compensate for parameter uncertainty is the occurrence of a permanent shock in our financial parameters. In that case, the historical equity risk premium might not be a good estimate for ex ante equity performance. Bansal and Lundblad (2002) report that the ex ante equity risk premium has globally decreased over time, so that the historical equity risk premium might therefore be an optimistic ex ante estimate. To account for the fact that economic regimes may shift our financial parameters, we investigate this effect by estimating a prior that is benchmarked on the period between 2000 and 2012. This period is not only distinct in that it features the financial crisis in 2008, it also includes the aftermath of the dot-com bubble in which partial recovery for the equity market and interest rates was observed. To incorporate the effect of this period in our predictions, we use a Normal-Diffuse prior calibrated to this period, so that our model adds more value to observations after the year 2000. As a consequence, both our estimates and parameter uncertainty are substantially affected by that subsample period. Our Bayesian model reveals

[2]An alternative to a fixed risk profile is to decrease the equity exposure over time. Using a linear decreasing exposure to equity during the accumulation phase, starting at age 25 with 90% equity and 10% 5-year maturity bonds and ending with 20% equity exposure at age 65 and the remainder in bonds, results in a similar impact of parameter uncertainty on the replacement rates for participants. Results are availible upon requests.

[3]This conservative approach is, for example, taken in the Netherlands, where the average invesment strategy of pension funds is a mix of 32% equity and 68% bonds portfolio. We obtain the Dutch and US percentages from the report of Watson (2013) by excluding other investments.

that the equity risk premium is negatively influenced by about 57 basis points, lowering the estimate from 5.0% to 4.5%. The expected inflation rate increases by 10 basis points to about 3.6%. The uncertainty regarding our financial parameters remains large, and only slightly increases if we incorporate our prior view. Consequently, the economic projections of the financial market are substantially affected by our prior view. However, the participant cannot observe this shift and continues to use the historical equity risk premium in his time-varying contribution scheme.

Allowing for unobserved shifts in financial parameters limits the ability of the time-varying contribution scheme to mitigate the effects of parameter uncertainty. Our benchmark results indicate that the 2.5% percentile of the replacement rate distribution at retirement is 42.1% when parameter uncertainty is incorporated with an uninformed prior. However, if we apply our prior view to our model, the 2.5% percentile of the replacement rate's distribution drops substantially to 38.0%. This replacement rate corresponds to a 1.2% percentile in the distribution that ignores the shift in financial parameters. Therefore, the shift in financial parameters has a large impact on the risk perception of the participant. Since the participant is uninformed about the regime shift in financial parameters, the belief to set his pension contributions based on the historical equity risk premium is too optimistic. The participant anticipates high future equity returns, so that for younger ages the contribution level is similar as in our model without a shift in financial parameters. After age 45, however, the time-varying contribution scheme responds to changes in the economic regime by increasing the contribution level since the accumulated pension wealth is not sufficient to reach the participant's ambition. For example, at age 45 and 55 the contributions increase by 7% and 17% respectively, due to the lower portfolio returns. This result shows that prudent estimation of the equity risk premium is required to absorb shocks using a time-varying contribution scheme, especially for younger age cohorts. Ignoring shifts in the financial parameters may, therefore, negatively affect the steering capacity of a time-varying contribution scheme.

This chapter contributes to the literature in three ways. First, we address the impact of parameter uncertainty on replacement rates in a DC pension setting with real variable annuities. It is widely observed that pension funds are switching from a DB structure to a DC contract without guarantees of a sponsor. Since insufficient capital at retirement will directly lower replacement rates for participants, risk analysis of replacement rate using future projections will receive considerably more attention. We show that parameter uncertainty can substantially affect these future projections and can lead to underestimation of replacement rate risk at retirement. Second, to implement parameter uncertainty, we extend the Minimum Chi-squared approach of Hamilton and Wu (2012) to the estimation of our financial market. This approach allows us to straightforwardly implement our no-arbitrage financial market which is a discrete time adaptation of the models used in Brennan and Xia (2002), and Campbell and Viceira (2001) and simplifies the estimation of the marginal posterior densities.

In terms of the broader pension literature, this chapter builds on the insights of the life cycle literature (see e.g., Cocco and Maenhout (2005), and Viceira (2001)) by considering parameter uncertainty. Our results indicate that the key source of parameter uncertainty for the replacement rate risk at retirement is the estimate of the equity risk premium. Since the literature has shown that high equity exposure at young age is optimal from a portfolio optimization perspective, the participant will be affected by parameter uncertainty of the equity risk premium. Life cycle models ignoring parameter uncertainty may therefore underestimate the risk of replacement rates at retirement and set inadequate contribution levels to achieve the participant's pension ambition.

The remainder of this chapter is organized as follows. Section 2 relates our approach to regulatory issues for US pension plans. Section 3 introduces our financial market, pension contract, estimation methodology and parameter uncertainty. Section 4 discusses the results of the pension contract if parameter uncertainty is ignored. Section 5 shows the impact of parameter uncertainty on the replacement rate and discusses whether a time-varying contribution scheme can compensate for this effect. Section 6 shows that shifts in the economic regime limits the ability of the time-varying contribution scheme to compensate for parameter uncertainty in terms of replacement rate risk. Our conclusions follow in Section 7.

4.2 Pension ambition and contribution

In this section, we discuss the intuition of our methodology to determine the required contribution rate for our DC pension contract. In particular, we explain how uncertainty affects the contribution rate to achieve the pension ambition and relate this to choices in US pension regulation.

To define the pension ambition in pension plans, the rights of the participant at retirement need to be analyzed. In our DC pension contract, we set the pension ambition of a participant to purchasing a real variable annuity at retirement with his desired replacement. This setting is similar to US target benefit plans which sets an ambition for the participant to accrue pension wealth, although the actual benefit of the participant depends on the accumulated pension wealth in their account at retirement.

An important factor in setting contribution rates is the equity risk premium. The life cycle literature has shown that the equity risk premium is an important driver of wealth accumulation (see e.g., Cocco and Maenhout (2005) and Viceira (2001)). Since most pension plans invest in equity to benefit from the higher expected return, the discount rate needs to account for equity return (Cochrane, 2011). When the discount rate ignores the higher expected equity returns, pension contributions will be larger than required based on the expected portfolio returns. Hence, the amount of pension wealth will overshoot the pension ambition in expectation. In order to account for equity returns, we raise the discount rate with the percentage invested in equity times the equity risk premium (see e.g., Nijman and Werker (2012)). When we allocate the required pension contribution based on this discount rate to a self-replicating portfolio, our portfolio will in expectation achieve the pension ambition at retirement based on the no-arbitrage argument.

Economic uncertainty of equity returns, however, can still substantially affect the replacement rate at retirement. When the equity risk premium is known, the previous approach allows us in expectation to accrue sufficient pension wealth to achieve the desired replacement rate. Only in case the participant invests in real bonds with a maturity corresponding to the remaining years before retirement, uncertainty about replacement rates can be eliminated. Since such an investment strategy requires high contribution levels, the participant may want to include equity in his portfolio to lower the contribution rate according to his risk averseness. An additional source of risk occurs if the future equity risk premium deviates from the estimated historical equity risk premium. For example, if the participant is optimistic about future equity returns and discounts using a high estimate of the equity risk premium, his portfolio return will be lower than expected. If the contribution rate is not adjusted during the accumulation phase, the participant will fall short of the desired amount of pension wealth at retirement. Consequently, a wrong risk perception of the participant may lead to the underestimation of the replacement rate at retirement and may lead to low contribution levels.

In DB funds, there is a similar risk in attaining the pension ambition for participants. DB funds typically offer nominal pension entitlements with additional Cost of Living Adjustments (COLAs) which are entitled to the participant in case the assets of the funds permit such benefits. Consequently, sufficient pension wealth accumulation is necessary to protect the participants from inflation risk and possible default risk of the fund. Newly accrued rights need to be valued taking into account the fund's portfolio allocation in order to determine the appropriate contribution rate for participants. Similar as in DC contracts, a wrong risk perception of the funds may lead to large exposure to the equity risk premium. Such a strategy will increase uncertainty about future replacement rates.

US pension regulation on contribution rates differs among types of pension plans. Since DC pension plans do not guarantee pension entitlements to participants, regulation concerning the level of contribution is typically not applicable to DC type of funds. However, DC funds are obliged under the Employee Retirement Income Security Act (ERISA) to provide benefit statements for participants.[4] These benefit statements are based on a number of actuarial assumptions. To determine benefit statements, the US department of Labor has set forward a few key assumptions which are based on long term projections on inflation and portfolio return.[5] These projections ignore uncertainty, so that the participants may form inadequate risk perceptions about the replacement rate at retirement. As a result, participants may contribute insufficiently to realize their pension ambition.

For DB funds, regulation is different for public and private funds. Regarding public US pension funds, the Governmental Accounting Standards Board[6] (GASB) holds the view that the pension contribution should be set using the long-term expected rate of return (GASB, 2012). Similarly, Canadian public state and local funds use a similar setting to set their contribution rate by investment return (CIA, 2010). While regulation on private US funds for the regulatory discount rate is split in accounting and contributions purposes, the regulator has chosen not to disentangle these two functions for public funds. Conceptually, regulation on accounting aims at transparency and comparison across funds, whereas guidelines on contribution influence the levels of con-

[4]Under ERISA section 105 all private funds that are governed by ERISA need to report the benefit levels to their participant periodically. Most DC funds fall into the scope of ERISA, except for some specific pension plans.

[5]A number of key assumptions are an inflation rate of 3%, nominal portfolio return of 7% and a nominal contribution increase of 3%. For more information, see http://www.dol.gov/ebsa/newsroom/fsanprm.html

[6]Public pension funds for the state and local government fall within the scope of the guidelines of the Government Accounting Standards Board (GASB).

tributions for the participant. These assumptions on long-term projections are set by the Board of Actuaries rather than observed market rates. As a result of using such projections in their contribution rates, these funds are also at risk for both economic uncertainty and parameter uncertainty of the equity risk premium.

Recent literature on DB pension funds have shown that public state pension plans are severely underfunded due to their reliance on discounting using high portfolio returns (Novy-Marx and Rauh, 2009, 2011). Due to perverse regulation, these funds have substantially increased their equity exposure, allowing participants to benefit from low contribution rates (Novy-Marx, 2013). The risk of not achieving sufficient pension wealth for the participant is transfered to the state, because public pension entitlements are guaranteed under federal and case law (Brown and Wilcox, 2009). As a consequence of the underfunding in these pension plans, the state may be less able to bear this risk. While the literature has pointed out that using actuarial parameters will introduce large economic uncertainty, these funds are also suspect to parameter uncertainty. In case of such underfunding, our framework can serve as a lower bound for the contribution rates if the pension ambition corresponds to a real variable annuity and the state is not offering guarantees.

Private US DB funds rely on the Internal Revenue Service (IRS) to determine their contribution levels, which publishes discount rates based on investment grade corporate bonds. To understand the regulatory choice for US private pension funds, the specification of the pension ambition in the contract is of importance. DB private pension funds entitlement are guaranteed by the Pension Benefit Guaranty Corporation (PBGC) which requires a mandatory contribution of the funds based on corporate bond yields to ensure the protection of the participants' rights. Since the pension ambition is guaranteed at retirement, the contribution levels follow the nominal term structure of interest rates. Such contribution scheme allows to capture economic shocks through the term structure of interest rates.

Regulation on projections in DC plans is less developed as in DB plans, which raises an important question about the adequate contribution rate in DC contracts. Since actuarial standards are typically based on long-term sample periods, we set a fixed contribution based on our historical portfolio performance to analyze the risk of economic uncertainty and parameter uncertainty. To contrast this approach, we use a time-varying contribution scheme based on the term structure of interest rates and the historical equity risk premium. The latter approach resembles the contribution scheme in private funds, except for that fact that equity risk is explicitly incorporated in the determining the contribution level.

4.3 Financial market and pension contract

4.3.1 Financial market

Before we introduce our Bayesian methodology, we first explain our financial market without parameter uncertainty. In this case the investor knows the parameters of the financial market, although he will experience economic shocks to his investment portfolio. We further develop our Bayesian methodology in section 3.3.

Our financial market consists of nominal bonds and equity. We use a discrete time model that relates to continuous time equivalents such as Brennan and Xia (2002), and Campbell and Viceira (2001). In line with this literature, we assume that two latent state variables capture the nominal interest rate movements. In addition, we assume the equity risk premium is constant. As a result, the expected equity return is dependent on our state variables.

To value the pension contracts of the participants, we focus on an asset choice of bonds and equity. In order to determine the no-arbitrage prices of these assets, we establish a nominal pricing kernel. First, we define the monthly nominal short rate, r_t, as a function of two latent state variables X_t

$$r_t = \delta_{0,r} + \delta_{1,r}' X_t, \tag{4.3.1}$$

with the restriction $\delta_{0,N} > 0$. Second, we postulate the inflation process in terms of changes in the price levels that is driven by monthly expected inflation, π_t as

$$\frac{I_{t+1}}{I_t} = \exp(\pi_t + \sigma_\pi' \epsilon_{t+1}). \tag{4.3.2}$$

As a result, the realized inflation process consists of a shock $\sigma_\pi' \epsilon_t$ and the expected inflation rate. The inflation process allows to link the nominal and real pricing kernels in the financial market. We define monthly expected inflation as an affine transformation of the state variables,

$$\pi_t = \delta_\pi + \delta_{1,\pi}' X_t, \tag{4.3.3}$$

restricting the parameter $\delta_\pi > 0$. The latent state variables, X_t, that determine the time dynamics in our financial market are assumed to be following a first order vector autoregressive model,

$$X_{t+1} = \Phi_1 X_t + \Sigma \epsilon_{t+1}, \tag{4.3.4}$$

where Σ is restricted to an identity matrix with dimension 2×2.

For the equity returns, we decompose the returns in the monthly short rate plus an equity risk premium. We assume the following process

$$R_t = r_t + \eta + \sigma_R' \epsilon_t, \tag{4.3.5}$$

where r_t denotes the monthly nominal short rate and η is the constant equity risk premium.

To derive the no-arbitrage bonds prices, we specify an affine nominal pricing kernel, M_t^N,

$$M_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\epsilon_{t+1}\right), \tag{4.3.6}$$

with λ_t denoting the price of risk. We assume that the price of risk is affine in the state variables,

$$\lambda_t = \Gamma_0 + \Gamma_1 X_t. \tag{4.3.7}$$

Since λ_t is dependent on the state variables, we establish time-varying bond risk premiums. For the equity risk premium, we have the following restriction on the parametrization

$$\eta = \sigma_s' \lambda_t. \tag{4.3.8}$$

To satisfy this restriction, the constraints $\sigma_s'\Gamma_0 = \eta$ and $\sigma_s'\Gamma_1 = 0$ need to hold. Following Koijen, Nijman, and Werker (2010), we choose the elements on the last row of the parameters Γ_0, Γ_1 and Γ_1 in such way that these constraints hold. The equity return depends on the state variables through the nominal short rate, whereas the equity risk premium is constant through time.

Lastly, we derive the no-arbitrage yield curve. We assume exponential affine bond prices in the state variables, i.e.

$$P_t(n) = \exp(A_n + B_n' X_t), \tag{4.3.9}$$

where n denotes the maturity for the bond at time t. The coefficients A_n and B_n' are restricted functions of maturity. Solving the no-arbitrage relation, we can establish a recursion for these coefficients. The no-arbitrage relation that has to hold in order to exclude arbitrage in the yield curve is

$$P_t(n) = E_t\left[M_{t+1} P_{t+1}(n-1)\right]. \tag{4.3.10}$$

Substituting the nominal pricing kernel as in Equation (4.3.6) and exponential affine bond prices of Equation (4.3.9), results in a recursion for the no-arbitrage coefficients for the yields. Therefore, we have the following expression for the yields

$$Y_t(n) = \bar{A}_n + \bar{B}_n' X_t, \tag{4.3.11}$$

where the coefficients $\bar{A}_n$ and $\bar{B}_n'$ are functions of maturity n. In Appendix 4.A, we derive the recursions for the yields in detail. To determine the real yield, $Y_t^R(n)$ and equity returns, we multiply the nominal yields by the inverse of the changes in price levels.

4.3.2 Pension contract

The objective of our pension contract is to ensure a sufficient standard of living after retirement for the participant. At retirement the participant will purchase an actuarial fair variable real annuity with his accumulated pension wealth. The real ambition of this pension contract is therefore to achieve an amount of pension wealth that is equivalent to the desired real replacement rate of the participant. At the start of the pension contract, the participant sets his retirement ambition at 70% of his real expected end wage at age 64. We set our ambition based on the findings of Munnell and Soto (2007), who report in the US median replacement rates of DB pension schemes ranging from 76% for workers covered by social security and 81 % for workers without social security.

The actuarial fair price of the real variable annuity at retirement age 65, PA_{65}, which entitles the participant to a replacement rate of 70% of the wage at age 64, Z_{64}, can be denoted as

$$PA_{65} = \sum_{k=65}^{99} 0.70 Z_{64} \exp\left(-k(Y_t^R(k) + 0.2\eta)\right) p_k. \tag{4.3.12}$$

The variable annuity is based on a 20% equity exposure, which is denoted by 0.2η. If the participant wants a real fixed anuity, then this exposure drops to 0%. To determine the surival probability p_k, we use the mortality data of the US for cohort 2010[7].

To achieve this real ambition, the participant accumulates pension wealth in his saving account, comparable to an Individual Retirement Account (IRA). Since the participant is not in a DC fund, he ignores mortality risk during the accumulation phase. The individual's risk preference will influence his investment strategy. A higher risk aversion leads to lower equity holdings,

[7] We obtained the dataset from www.mortality.org and used the 2010 cohort to calibrate our model.

so that the risk of not attaining the desired replacement rate decreases. We assume his risk aversion to be low by setting the investment strategy to a 60% equity and 40% bonds portfolio. We rely on data of the average equity and bonds holdings in the US to set his portfolio (Watson, 2013). They find that US pension funds' portfolios consist of about 66% equity with the remaining wealth of 34% invested in bonds. A lower risk profile is for example taken in the Netherlands, where the average portfolio consists of 32% equity and 68% bonds. This investment strategy will be used to analyze the consequences of a low risk profile. Each year the accrued pension wealth of a participant, W_t, is affected by the real portfolio return, R_t^P. In addition, the participant contributes a fraction of his wage Z_t, resulting in the following dynamics for the pension wealth

$$W_{t+1} = W_t R_t^p + \beta_t Z_t, \text{ for } t = 25, .., 64. \tag{4.3.13}$$

The pension contribution β_t is a fraction of the real wage of the individual. The participants starts without any pension wealth, rendering the initial condition $W_{25} = 0$.

We employ two contribution schemes to accrue sufficient pension wealth to purchase the variable annuity. First, we set a fixed contribution level based on the historical performance of the portfolio. To determine the fixed minimal contribution at age 25, the price of the annuity at retirement age 65 is divided by the sum of yearly salaries multiplied by the real portfolio return up to retirement, which yields the following contribution rate

$$\beta^{\text{fixed}} = \frac{PA_{65}}{\sum_{t=25}^{64} E\left[Z_t \exp\left((64-k)R_t^p\right)\right]}. \tag{4.3.14}$$

The average wage $E[Z_t]$ is assumed to be known, since the participant uses the average wage pattern known for his level of education. The underlying idea of this contribution scheme is that past historical performance, $E[R_t^p]$, is similar to future portfolio returns, so that in expectation the pension ambition can be achieved. Even when the parameters of our financial market are known to the participant, economic shocks occurring in the financial market can lead to large uncertainty for the replacement rate at retirement. This strategy will also ignore possible parameter uncertainty.

Second, we determine a time-varying contribution scheme that allows the participant to react to economic shocks or parameter uncertainty. Each year the participant verifies whether his pension wealth grows sufficiently compared to the present value of his annuity. To value the annuity for his desired replacement rate annuity, we use the real term structure of interest rates to discount its price. Since the participant invests in equity, we need to account for the equity risk premium in our contribution. Otherwise the contribution level will be too high, resulting in overshooting his pension ambition. Therefore, we add 60% of the equity risk premium to the real interest rates, 0.6η, since he will allocate 60% of his pension wealth to stocks. To determine the present value of the annuity during the accumulation phase, we use a similar methodology as in Equation (4.3.12). We first discount the pension payments during retirement, but use the real forward curve at time t instead of the yield curve, so that we are able to determine the amount of pension wealth required to purchase the real variable annuity at retirement. Next, we discount this required pension capital with the yield curve to determine its present value of the annuity at time t. Therefore, the present value of the real ambition before retirement is

$$PV_t^{\text{Annuity}} = \exp\Big(-(65-t)(Y_t^R(65-t)+0.6\eta)\Big) \sum_{k=65}^{99} 0.7E[Z_{64}]\exp\Big(-k(FR_t^R(65-t,k-t)+0.2\eta)\Big)\,p_k, \quad (4.3.15)$$

where $FR_t^R(65-t,k-t)$ denotes the real forward rate at time t for the pension payments from time period $65-t$ to $k-t$. Economics shocks will enter the term structure of interest rates, affecting the present value of the pension ambition.

Next, we determine the present value of the future expected wage pattern at time t. The present value is the sum of the expected wage, $E[Z_k]$, so that we can write

$$PV_t^{\text{Wage}} = \sum_{k=t}^{64} E[Z_k]\exp\Big(-k(Y_t^k+0.6\eta)\Big). \quad (4.3.16)$$

Combining the present value of the wage and the ambition, we can determine to the time-varying pension contribution as follows,

$$\beta_t = \frac{PV_t^{\text{Annuity}} - W_t}{PV_t^{\text{Wage}}},. \quad (4.3.17)$$

The underlying idea of the pension contribution is that the difference between the present value of the real ambition and current wealth will be spread out over the remaining future income of the participant. Thus, if pension wealth fall short compared to the present value of the annuity, the contribution will increase. Economics shocks to the term structure will affect the valuation as well. In case the yields decrease, the price of the annuity will go up.

Our specification of the pension contract allows for a number of individual choices. The underlying motivation is based on the global trend of DB shifting to DC schemes. For example, in 2012 the DC funds have grown 4% world wide relatively to DB funds in terms of assets since 2010. (see e.g., Watson (2011) and Watson (2013)). As a result, the participant needs to bear the risk about the replacement rate himself. Individual choices can help to overcome economic shocks and lower potential risk. For example, retirement age could be postponed in order to acquire a higher replacement rate at a later age. Also, lowering the risk profile may influence the risk of the replacement rate at retirement.

Real wage growth

We benchmark the real wage growth of the individual in our model to Cocco and Maenhout (2005). Using their characteristics for a High school individual, who did not obtain a college degree yet did obtain a high school degree, we calibrate the real income process and capture the income pattern over the lifetime for the participant in our framework. We assume the following polynomial function

$$g_t = \alpha_0 + \alpha_1 t + \alpha_2 \frac{t^2}{10} + \alpha_3 \frac{t^3}{100}, \tag{4.3.18}$$

where we set $\alpha_1 = 0.1682$, $\alpha_2 = -0.0323$, and $\alpha_3 = 0.0020$ which follow from Cocco and Maenhout (2005). We choose the parameter α_0 in such a way that the wage at age 25 for the individual is \$20,000. Cocco and Maenhout (2005) decompose the variance of the income process in transitory and permanent shocks with respectively the variances $\sigma_\nu = 0.0738$ and $\sigma_u = 0.0106$, where the permanent shock follows the following process $v_t = v_{t-1} + u_t$. As a result, we can write the income process

$$Z_t = \exp(g_t + v_t + \nu_t). \tag{4.3.19}$$

An important observation is that the real wage growth patterns differ among different levels of education. While all groups have an upward shaped salary pattern, the highest salaries are obtained at different ages. For example, the lowest educated level (No high school degree) reaches its highest real salary at age 38, whereas for the highest (College degree) and average educated groups reach this highest salary at a later age, at age 45 and 44 respectively. As a result, the lowest educational group has a rapid growing salary at young age. Moreover, the salary pattern of the average educational level remains relatively fairly high after reaching its peak compared to the other groups. Consequently, a pension ambition based on the end wage will be more costly compared to the other two educational groups.

Utility function

To compare the two different contribution schemes and the replacement rate at retirement, we introduce a Constant Relative Risk Aversion utility function. The expected utility, EU, of the participant at age 25 is defined as

$$EU = E\left[\sum_{t=t0}^{T} \frac{\chi^t}{1-\iota} C_t^{1-\iota}\right], \tag{4.3.20}$$

where χ denotes the subjective discount factor, ι denotes coefficient of the relative risk aversion, and C_t denotes the real consumption of the participant. The consumption of the participant is the remainder of the wage after the pension contribution, i.e.

$$C_t = (1-\beta_t)Z_t. \tag{4.3.21}$$

In our benchmark, we set the subjective discount factor χ to 0.96 and set the relative risk aversion coefficient to 5. To determine the impact of our pension contribution schemes, we calculate improvements in terms of certainty equivalent consumption. The certainty equivalent consumption, C_{CE}, can be determined as

$$C_{CE} = \left(\frac{EU}{\sum_{t=t0}^{T} \frac{\chi^t}{1-\iota}}\right)^{\frac{1}{1-\iota}}, \tag{4.3.22}$$

so that the participant has a constant consumption throughout his lifetime with a similar expected utility. To investigate the different contribution schemes, we compare the effects on the certainty equivalent consumption.

4.3.3 Estimation

Data

Our data sample ranges from January 1952 up to December 2012. We use 6 nominal yield series, namely the 3 and 6 months Treasury Bills and the US Treasury bonds with maturities 1, 2, 3, 5 and 10 year. The data on the 3-month and 6-month T-Bills are taken from the Federal Reserve Bank of St. Louis.[8]

[8] http://research.stlouisfed.org/fred2/

The other yields up to February 1991 are from the McCulloch and Kwon and extended using the data in Bliss (1997) to December 1998. From January 1999, we use the data from the Federal Reserve bank of New York.[9] Inflation data is obtained from the US CPI All urban price level from the Bureau of Labor Statistics. The monthly equity return is determined by the logarithmic return on the CRSP value-weighted NYSE/Amex/Nasdaq index data.

System of equations

We estimate our model using a system of reduced equations based on the methodology presented by Hamilton and Wu (2012). We invert the yields measured without error to identify the latent factors and subsequently we regress these factors on the remaining yields, inflation and equity return. Details on the derivation are provided in Appendix 4.A.

The reduced form of equations is used to estimate the reduced form parameter, which can be summarized as

$$\begin{aligned}
Y_t^1 &= \underbrace{(\bar{A}_1 - \bar{B}_1'\Phi_1\bar{B}_1'^{-1}\bar{A}_1)}_{\bar{A}_1^*} + \underbrace{(\bar{B}_1'\Phi_1^L\bar{B}_1'^{-1})}_{\Phi_{11}^*} Y_{t-1}^1 + \underbrace{\bar{B}_1'}_{\Omega_1^*} \epsilon_{1,t} \\
Y_t^2 &= \underbrace{(\bar{A}^2 - \bar{B}_2'\bar{B}_1'^{-1}\bar{A}_1)}_{\bar{A}_2^*} + \underbrace{(\bar{B}_2'B_1'^{-1})}_{\Phi_{21}^*} Y_t^1 + \underbrace{\Omega}_{\Omega_2^*} \epsilon_{2,t} \\
\tfrac{I_{t+1}}{I_t} &= \underbrace{\delta_{0,\pi} - \delta_{0,\pi}\bar{B}_1'^{-1}\bar{A}_1}_{A_3^*} + \underbrace{\delta_{1,\pi}\bar{B}_1'^{-1}Y_t^1}_{\Phi_{31}^*} + \underbrace{\Sigma_\pi}_{\Sigma_\pi^*} \epsilon_{3,t} \\
R_t &= \underbrace{\eta + \delta_{0,r} - \delta_{1,r}'\bar{B}_1'^{-1}\bar{A}_1}_{A_4^*} + \underbrace{\delta_{1,r}'\bar{B}_1'^{-1}Y_t^1}_{\Phi_{41}^*} + \underbrace{\sigma_R}_{\Sigma_R^*} \epsilon_{4,t}.
\end{aligned} \tag{4.3.23}$$

This system of equation can be estimated with OLS resulting in the reduced form estimates of our model. Subsequently, these estimates can be matched with the structural parameters of our model using a Minimum Chi-squared approach. Minimization of the Chi-squared value function gives the estimates for the structural parameters.

To estimate the effect of parameter uncertainty, we apply our Bayesian methodology to the reduced form of the system of equations in Equation (4.3.23). Our estimation strategy is to employ a Block-Gibbs sampling algorithm to draw from the marginal posterior probability distribution of the reduced form coefficients and subsequently link these coefficients with the associated structural parameters using the Minimum Chi-squared approach.

[9] The nominal yield data are available on http://www.federalreserve.gov/econresdata/researchdata.htm

Depending on the prior distribution, we can evaluate the pension contract for each set of parameters in our financial market incorporating the effect of parameter uncertainty. We refer to Appendix 4.A.3 for the details on the estimation strategy. Since we incorporate parameter uncertainty in the reduced form equations, our approach allows for a straightforward implication without having to determine the marginal posterior distributions of the structural coefficients. Therefore, our approach can be easily adopted to determine parameter uncertainty for pension contracts.

We estimate our model without an inflation risk premium due to limited data of inflation-linked derivatives. Recent empirical studies suggest a wide range for the magnitude of the inflation risk premium (See e.g., D'Amico et al. (2010) and De Roode (2013)). In addition, liquidity effects in the inflation-linked bond market, especially at the start of the issuance and during the financial crisis, substantially reduce the available data range to estimate the inflation risk premium from such derivatives. To refrain from the uncertainty about the magnitude of the inflation risk premium, we set the inflation risk premium to zero.

Financial market

Table 4.1 presents the estimates of our financial market based on our extensive sample from 1950-2012. Both our latent factors in our model exhibit a high degree of persistence, since the estimates on the diagonal of Φ_1 are fairly high. As a consequence, the half-life of the first factor is 7 years whereas the second factor has a shorter half-life of 1 year. Consequently, the second factor is less persistent than the first. Although our estimation suggests that the factors are negatively correlated, uncertainty remains an issue since the standard error of $\Phi_{1,21}$ is quite large. Moreover, we find that the monthly 5 year bond return is positively correlated with inflation, whereas the equity return is negatively correlated. However, since both assets are negatively correlated with inflation innovations, bonds do not offer a better hedge against inflation risk than equity.

To further analyze our financial market, we turn to the nominal risk premia. The average nominal yield curve is upward sloping with nominal term premia for a bond with maturities 1, and 5 year of about 45 basis points and 143 basis points respectively. As a result, the average nominal yield curve in our financial market is relatively high compared to recent periods observed after 2000. In Figure 4.1, we present the nominal yield for the 3 months and 5 year maturity. In addition, we show the relatively high average yield curve over our sample period in Figure 4.1. We observe a high average equity return during our sample as well, leading to an annual equity risk premium of about 5%. As a consequence of relatively high average equity rates during this sample period, our pension premium will be downward affected. Furthermore, we observe an average annual inflation of 3.5%, suggesting that inflation is an important factor in diminishing nominal returns.

In terms of model fit, our model is able to replicate the average yield curve within 95% confidence levels. The measurement errors in Table 4.1 show that our model is able to capture the long term nominal yields better than the short maturities. For example, we report a standard error of about 38 basis points for the 3 months yield, whereas for the 2 year maturity the error is about 12 basis points. Also, we find positive correlation between nominal bond and equity returns, similarly as in Sangvinatsos and Wachter (2005). While our model is able to capture the characteristics of the financial data rather well, the uncertainty about equity risk premium is large. Therefore, we use our Bayesian approach to verify the effect of parameter uncertainty on the replacement rate of our pension contract.

4.4 Pension contract with known financial market parameters

In this section, we analyze the uncertainty in the replacement rate at retirement, assuming there is no uncertainty about the parameters of our financial market. The participant will only be affected by economic uncertainty of the financial market, so that we can analyze the risk of achieving low replacement rates at retirement for our fixed and time-varying contribution schemes. To investigate the impact of risk profile, we also verify the replacement rate risk if the participant has a low equity exposure.

4.4.1 Benchmark pension contract

Table 4.2 shows that economic uncertainty in the equity returns affects the ability of the fixed contribution scheme to achieve the desired replacement rate of the real variable annuity. The fixed contribution based on the historical performance return of the participant's portfolio requires a yearly contribution of 6.85% and leads to an average replacement rate of 70.0%. However, the fixed contribution scheme introduces substantial uncertainty about the replacement rate. The lower bound of the 95% confidence interval of the replacement rate is about 35.2%. This result indicates that a fixed contribution scheme can harm the real ambition of the participant, because it prohibits the participant to respond to economic shocks. In bad economic scenarios in which equity returns are low, the participant remains optimistic about his future returns and refrains from increasing his pension contribution.

The time-varying contribution scheme can partially compensate for economic shocks. Table 4.2 shows that the average replacement rate is 75.8%, which is larger than the desired replacement rate. One of the reasons causing the higher replacement rate is the time-varying dynamics of the interest rates. In case the term structure of interest rates decreases early in the accumulation phase, the contribution will increase due to the high present value of the real ambition. If later on, during the accumulation phase, the pension wealth exceeds the present value of the real ambition, the time-varying contribution scheme would imply to withdraw pension wealth from the account. However, since we do not allow the participant to withdraw from his pension account, the participant slightly overshoots his desired pension ambition.

To achieve the pension ambition, the average time-varying contribution is increasing with the age of the participant. Since his investment strategy is not a self-replicating portfolio, the pension contribution needs to compensate this difference near the end of the accumulation phase. A self-replicating strategy would imply that the participant invests his pension contribution at age 25 in 40% bonds with a maturity of 40 years instead of a fixed maturity bond portfolio of 5 years. While the time-varying contribution scheme responds to shortages of pension wealth compared to the present value of the annuity due to equity shocks, the pension contribution is for young ages most sensitive to changes in the term structure. If pension wealth falls short due to low performance of equity returns at older ages, the contribution scheme will react by substantially increasing the contribution level. As a result, the 95% confidence intervals of the pension contributions are quite wide near retirement.

One important constraint for the time-varying contribution scheme is that the contributions are restricted to the interval of 0% and 30%. This implies that the participant is prohibited from withdrawing pension wealth during the accumulation phase and can only contribute at most 30% of his yearly wage to his pension account. These arbitrary fixed bounds are necessary, because contributions higher than 30% are economically unjustifiable. Otherwise these high contributions would have a large impact on the consumption of the participant. To analyse the impact of the contribution bounds, Table

4.3 compares the certainty equivalent wealth gains compared to the fixed contribution scheme for various bounds. This table denotes the improvement in terms of certainty equivalent consumption and suggests that the interval of 0% and 30%, receive larger certainty equivalent wealth gains than contribution scheme with a smaller interval. For example, for a neutral participant has utility gain of 5.8% when the interval is limited to 0% and 30%, whereas for an interval of 0% and 20% this gain is only 3.5%. Although certainty equivalent wealth gains are the largest for the interval of 0% and 40%, the marginal effects of allowing larger volatility to the contributions are quite small. Therefore, we restrict the interval to 0% and 30%.

As a result, these fixed bounds lower the volatility in the pension contribution, but also reduce the steering capacity of the time-varying contribution. If the participant is not restricted by these fixed bounds, the 2.5% percentile of the replacement rate at retirement would be 51.9%. This is 15% higher than when the contribution is restricted with fixed bounds, which results in a replacement rate of 45.2%. However, Figure 4.2 shows that in order to achieve this lower replacement rate risk, the contribution rates during the accumulation phase may become extremely high if the participant is not restricted by fixed bounds. For example, the 95% confidence interval of the contribution at age 55 is wider than the fixed interval of 0% and 30%, as it ranges from -34.1% to 54.6%. Such contribution rates are economically unjustifiable and lower expected utility substantially, so that fixed bounds to restrict contributions are neccessary.

To investigate whether the constraints of fixed bounds for the contribution are restrictive, Table 4.2 presents the percentage of scenarios in which the fixed interval is binding. At age 55, 43.4% of the scenarios hits the lower bound of the contraint at 0%, whereas the upper bound of 30% is binding in 14.3% of the scenarios. Near retirement, at age 64, this increase to a probability of 55% hitting the 0% lower bound and 38.8% the upper bound of 30%. These results suggest that the lower bound which restricts the participant from withdrawing pension wealth is more important than the upper bound. While the restriction of the lower bound is frequently binding, it can economically be justified since the additional accumulated pension wealth can be used either for a higher replacement rate or for smoothing economic shocks. However, the arbitary upper bound of the fixed interval would be dependent on the time preference of the individual for future consumption. Also, the probability of hitting the upper bound is a measure that indicates whether a pension amibition with a desired replacement rate is costly. For example, at age 64 with a probability of 14.3% the pension ambition with a real replacement rate of 70% is set too high. Therefore, in such scenarios the participant may want to reduce the pension ambition in favour of current consumption.

4.4.2 Alternative individual specifics

Our results on the risk of replacement rates are sensitive to three factors, namely level of education, risk profile and retirement age. Since our main findings are not materially affected, we only analyze the impact of a low risk profile. For the impact of education level and retirement age, we refer to Appendix 4.B.

Risk profile

An important factor driving the pension contribution level is the risk preference of the participant. To investigate its effect on the contribution level and the replacement rates, we switch the investment strategy to a low risk profile by altering the portfolio to 30% equity and 70% bonds.

Table 4.4 shows that a low risk profile decrease the risk of not achieving the desired replacement rate. To realize the pension ambition with lower equity holdings, the fixed contribution increases by 41.9% from a yearly premium of 6.85% to 9.72%. As a result of lower the equity exposure, the lower bound of the 95% confidence interval of the replacement rate increases to 43.2%. Not surprisingly, the lower risk profile leads to an improvement in the risk of low replacement ratee, since the 2.5% percentile of replacement rate is 35.2%. While a low risk profile improves the uncertainty about the replacement rate, it requires a substantial increase in contribution.

Regarding the effect of lower equity holdings, Table 4.4 shows that time-varying contribution scheme is less capable of improving the uncertainty about the replacement rate at retirement. The lower bound of the 95% confidence interval changes from a replacement rate of 44.5% to 48.3%. This result indicates that reducing the equity holdings from 70% to 30%, only slightly improves the worst-case scenario with a 2.5% probability, since the participant will only slightly reduce the uncertainty of the replacement rate at retirement. Investors with a low risk profile are therefore still substantially exposed to economic uncertainty.

Table 4.3 reports the certainty equivalent wealth gains if the participant alters his risk profile. This table suggests that switching from a high risk profile to a low risk profile is only beneficial for conservative-type participants. The neutral and aggressive-type investors will be associated to our benchmark risk profile.

4.5 Impact of parameter uncertainty on the pension contract

To assess parameter uncertainty in the pension contract, we first introduce the effects of parameter uncertainty on our estimates of the financial market. We show that the equity risk premium is the most important factor attributing to parameter uncertainty in our model. Therefore, we analyze this effect separately by first evaluating our pension contract with parameter uncertainty restricted to only the equity risk premium. Subsequently, we generalize our model to account for parameter uncertainty for all financial parameters.

4.5.1 Parameter uncertainty and the financial market

To incorporate parameter uncertainty in our financial market, we adopt a Bayesian approach with an uninformed prior. For details on the estimation approach, we refer to Appendix 4.A.3. By using an uninformative prior, we do not hold ex ante views on the projections of the financial market other than the observed data.

Figure 4.3 shows the dispersion of the equity risk premium by means of the estimated marginal posterior distributions. Since the estimate of the equity risk premium has a large standard error, these distributions have a wide support. For example, we find a 95% credibility interval ranging from 1.34% to 8.75 %. This result suggests that in projections of the financial market a wide range of estimates may be used. As a result of this parameter uncertainty, the mean of the stock return process can be substantial higher or lower in our scenarios. In particular, we are interested in the case that mean stock returns diminish and whether our time-varying contribution can partially compensate this effect on the portfolio returns.

The parameter uncertainty of inflation and the term structure of interest rates is far less substantial than the uncertainty regarding the equity risk premium. For example, Figure 4.4 shows the impact of parameter uncertainty on the average inflation rate is not widely dispersed. Since the inflation estimate has a lower uncertainty with a standard error of about 0.14%, the interval for the mean of the inflation process is much smaller. However, scenarios with a combination of a low equity risk premium and a high inflation level diminish real returns of the investor's portfolio more rapidly.

4.5.2 Parameter uncertainty restricted to the equity risk premium

Parameter uncertainty mostly affects the equity risk premium in our financial market. Therefore, we estimate our pension contract in this section with parameter uncertainty restricted to only the equity risk premium. As a consequence of parameter uncertainty, the average stock return in our model is uncertain for the participant. We assume that the participant is not aware of parameter uncertainty and only observes historical equity performance. Therefore, he relies on the estimated historical equity risk premium to set his minimal contribution level, which is for the fixed contribution scheme a yearly contribution of 6.85%. In case of a low equity risk premium, the participant is too optimistic about his future projections of the average stock return. However, at retirement the participant purchases a fair priced annuity based on the equity risk premium in the financial market. As a result, parameter uncertainty leads in this scenario to pension contributions that are too low.

Table 4.5 shows that employing parameter uncertainty about the equity risk premium puts the participant at risk, causing lower replacement rates at retirement, if a fixed contribution scheme is used. The 2.5% percentile of the replacement rate at retirement drops 24.4% compared to our previous results ignoring parameter uncertainty, from a replacement rate of 35.2% to 26.6%. This result shows that ignoring parameter uncertainty leads to the underestimation of risk in the replacement rate at retirement.

Next, we verify whether the time-varying contribution is able to partially subdue the effects of parameter uncertainty of the equity risk premium. Table 4.5 shows that the risk is far lower by employing a time-varying contribution scheme. For example, the lower bound of the 95% credibility interval decreases from 45.2% to only 41.3%, compared to model without parameter uncertainty. This results shows that only partial effects of the parameter uncertainty concerning the equity risk premium can be compensated by increasing the pension contribution.

One of the underlying factors are the restrictions on the pension contributions, requiring pension contributions to be between the fixed interval of 0% and 30%. The time-varying contribution scheme allows shocks in the portfolio return to be spread across the remaining years. As a consequence of the high present value of future wage at young years, the pension contribution is mostly affected by interest rate movements. At older ages, the present value of future wage dimishes more rapidly, so that the contribution mechanism steers more strongly. As a result, near retirement the restrictions become more important. For example, Table 4.5 shows that after age 55 the probability of hitting the fixed upper bound of 30% increases due to parameter uncertainty. As a result, near retirement, at age 64, he is less likely to hit the upper bound. This shows that parameter uncertainty forces the time-varying contribution scheme to react earlier because of the lower portfolio returns.

4.5.3 Parameter uncertainty for all financial parameters

Next, we verify whether parameter uncertainty about inflation and the term structure adds to the uncertainty of our previous reported results. We focus on the risk of the replacement rate and whether the time-varying premium can still partly compensate for these effects.

Table 4.6 shows that the uncertainty about the replacement rate at retirement increases only slightly compared to our previous result. Using a fixed contribution scheme lowers the 2.5% percentile from a replacement rate of 26.6% to a rate of 26.4%. This result shows that the uncertainty about the inflation and interest rates has a lower impact on the replacement rate. Thus, uncertainty about the equity risk premium is the most important factor for parameter uncertainty in our pension contract.

Adopting a time-varying contribution scheme actually decreases the risk about the replacement rate at retirement. Table 4.6 shows that the lower bound of 95% credibility interval increases from a replacement rate of 41.3% in case of restricting parameter uncertainty to only the equity risk premium to a lower bound of 42.1%. This increase can be explained by the higher pension contributions at the start of the accumulation phase. At younger ages, Table 4.6 shows that the participant increases his pension contribution due to the parameter uncertainty entering in the term structure of interest rates. For example at age 25, the average pension premium increases from 5.83 % to 6.51%, which is an increase of 11.7%. Simiarly, at age 35 the participant increases his contribution by 8.2% to 7.52% instead of 6.95%. As a result, near retirement the participant can lower his contribution rate.

The front loading of the pension contribution in the time-varying contribution scheme also leads to a lower probability of hitting the fixed upper bound of 30%. Table 4.6 shows the participant is less likely to hit the premium bound near retirement. With a probability of 35.6% the participant will hit the upper bound at age 64 when parameter uncertainty affects all financial parameters. In our previous result, this was 39.1%. This result suggests that incorporating only uncertainty about the equity risk premium may overestimate the effect of parameter uncertainty on the replacement rate for the time-varying contribution scheme.

Altering the risk profile of the investment portfolio leads to a similar conclusion. For the time-varying contribution scheme, the 2.5% percentile of the replacement rate at retirement is about 48.4% when parameter uncertainty affects all financial parameters. Ignoring parameter uncertainty results in a 2.5% percentile of 51.0%, which causes a similar shift in risk perception as in the benchmark case. In case of a low risk profile, the 2.5% percentile of the replacement rate's distribution that incorporates parameter uncertainty corresponds to a 1.4% percentile in case parameter uncertainty is ignored. A similar shift in risk perception can be observed for the benchmark case, where parameter uncertainty causes a shift from the 2.5% percentile to the 1.3% percentile. Therefore, the effect of parameter uncertainty has a similar effect on the risk perception regardless of the risk profile of the individual.

4.6 Economic regimes and parameter uncertainty

Our previous results are based on the underlying idea that uncertainty about financial parameters is captured by our full sample period. However, economic regimes may switch throughout our sample, substantially altering projections. For example, Figure 4.1 shows the declining trend of interest rates of US Treasury bonds after the year 2000. As a result, using average interest rates over the full sample period might lead to too optimistic projections. To analyze the impact of more recent economic regimes, we incorporate a prior view in this section. We first analyze the impact to our financial market and subsequently, verify its effect on the contribution schemes of our pension contract.

4.6.1 Impact of the 21^{st} century on market projections

During the first decade of the 21^{st} century, financial markets experienced rapid developments due to globalization. This period is characterized by two important events for the financial markets, namely the crash of the dotcom bubble in 2001 and the financial crisis in 2008. Due to these distinct features, we investigate the effect of this period on the projections of our financial market. By attaching more importance to observations from 2000 up to 2012 in our Bayesian framework, we are able to incorporate a shift in the economic regime for our projection without reducing the sample size. To this end, we calibrate a Normal-Diffuse prior in our Bayesian estimation to economic conditions observed in this period. The tightness of our Normal-Diffuse prior is calibrated to this period, so that the uncertainty of our prior view corresponds to the observed uncertainty in this period. This allows us to incorporate a market shift observed in this period without excluding observations.

Since the equity risk premium has the largest effect on our pension contract, we first analyze Figure 4.3 that presents the marginal posterior distribution of the equity risk premium. We show that incorporating our prior view leads to a decrease of 58 basis points in the average equity risk premium. This results suggests that projections of the average equity returns will be quite lower than the previous estimate. The dispersion of the estimate is slightly larger when incorporating a prior view, suggesting that the uncertainty about the estimate is similar compared to our previous result. For the other economic variables, the impact of a market shift is quite smaller. For example, Figure 4.4 shows that the mean inflation rate is shifted upward by 12 basis points. A higher inflation rate will cause that real returns on the investment portfolio decrease. In combination with a lower equity risk premium, this could lead to an increase in pension contributions. All together, these results imply more severe market conditions for the participant. Ignoring such developments in projections may cause that contribution scheme are too optimistic about future economic projections.

To measure the risk of using an ex post estimate of the equity risk premium for ex ante future stock returns, we assume that the participant's time-varying contribution is based on the historical equity risk premium as previously used. In case of a fixed contribution rate, the risk of the replacement rate at retirement will substantially increase. Table 4.7 shows that the lower bound of the 95% credibility interval of the replacement rate at retirement drops to 21.7%, which is lower than our previous result (26.5%), using the parameter uncertainty with an unformative prior. Since the fixed premium is based on historical performance, it substantially overestimates the future returns of the investment portfolio.

Next, we verify whether the time-varying contribution can partially compensate for the shift of the economic regime. In the time-varying premium, the participant uses the historical equity risk premium to value future stock returns. As a result of this overvaluation, the risk about the replacement rate at retirement increases. Table 4.7 shows that the time-varying contribution cannot compensate for the shift in the parameter uncertainty, since the lower bound of the 95% credibility interval drops from 42.1% to 37.6%. Consequently, the time-varying contribution scheme is less able to react to a shift in an economic regime.

To compensate for the lower portfolio return, the pension contribution is in general higher throughout the accumulation phase. However, Table 4.7 shows that at the start of the pension contract the economic projections are slightly better than in our previous case of parameter uncertainty. To summarize the contribution effects, Figure 4.5 shows the contributions for parameter uncertainty with an uninformative prior and with the informative prior. Incorporating the regime of the 21st century has strong upward impact on the average contribution after age 45. In particular, the upward effect can be seen for the median contribution rates. The median contribution based on the uninformative prior decreases to zero before age 60, whereas the median contribution using the prior view increases up to age 62 and starts declining after. Consequently, the ability of the time-varying contribution scheme to achieve the desired replacement rate decreases when economic regime alters.

4.7 Conclusion

Uncertainty about financial parameters complicates the participants' planning to achieve his pension ambition. When parameter uncertainty is ignored, the risk of lower replacement rates at retirement is underestimated. To investigate the main factor of uncertainty in financial parameters, we adopt a Bayesian methodology that captures uncertainty of the equity risk premium, inflation and term structure of interest rates. Based on our estimated financial parameters, we set a fixed yearly pension contribution that on average achieves the real pension ambition in our DC pension contract. The effects of parameter uncertainty affects our pension contract which allows us to analyze its effect on the replacement rate at retirement. Based on this framework, we demonstrate that uncertainty of the equity risk premium is the key factor driving the parameter uncertainty for the risk in the replacement rate at retirement.

Next, we verify whether a time-varying contribution scheme based on a combination of the term structure of interest rates and a historical estimate of the equity risk premium can partially compensate for parameter uncertainty. We show that although the participant either underestimates or overestimate future stock returns by using a historical equity risk premium, the time-varying contribution scheme partially compensates the risk of the replacement rate at retirement. However, our model reveals that the time-varying contribution scheme is less able to compensate for uncertainty in replacement rates if the equity risk premium is affected by a permanent unobserved shock. As a result of this shock, the participant's contributions are based on overly optimistic beliefs about future equity returns. Unobserved changes in financial parameters may, therefore, introduce risk in the time-varying contribution rate if the equity risk premium is set inadequately.

Our results imply that DC pension funds that rely on fixed yearly contributions to achieve a fixed real pension ambition, may introduce substantial replacement rate risk for participants. Even when time-varying contribution schemes are employed, risk in replacement rates can be substantial if the equity risk premium is set overly optimistically. To reduce replacement rate risk, regulators implementing contribution schemes similar to our time-varying contribution, need to address whether the ex post estimate of the equity risk premium is adequate to be used for future projections. Since pension wealth accumulation of young cohorts are particularly affected by inadequate estimates of the equity risk premium, regulators may want to impose upper limits of the equity risk premium to set pension contributions.

4.A Appendix A: Model derivations

4.A.1 Nominal yields

In this section we derive the nominal bond yields in a no-arbitrage framework. We substitute the affine bond prices, as defined in equation (4.3.9), in the no-arbitrage relation of the expected bond price. For convenience, we write this relation here

$$P_t(n) = E_t\left[M_{t+1}P_{t+1}(n-1)\right]. \tag{4.A.1}$$

By substituting the affine bond prices and the dynamics of the nominal pricing kernel in this equation, we derive the following expression for the price of a bond,

$$P_t(n) = E_t\left[\exp\left(-r_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\epsilon_{t+1}\right)\exp\left(A_{n-1} + B_{n-1}'X_{t+1}\right)\right]. \tag{4.A.2}$$

Substituting the short rate process as defined in Equation (4.3.1), and determining the expectation, yields the following expression for the bond price

$$\begin{aligned} P_t(n) = \exp\big(&-\delta_{0,r} + A_{n-1} + (B_{n-1}'\Phi_1 - \delta_{1,r}')X_t - B_{n-1}'\Sigma\lambda_t \\ &+\frac{1}{2}B_{n-1}\Sigma\Sigma'B_{n-1}'\Big). \end{aligned} \tag{4.A.3}$$

Next, we need to substitute the affine function of the price of risk, λ_t. Substituting this, as defined in Equation (4.3.7), we arrive at the typical function for the no-arbitrage bond price,

$$\begin{aligned} P_t(n) = \exp\Big(&-\delta_{0,r} + A_{n-1} + B_{n-1}' - \Sigma\Gamma_0 + \frac{1}{2}B_{n-1}'\Sigma\Sigma'B_{n-1} \\ &+(B_{n-1}'(\Phi_1 - \Sigma\Gamma_1) - \delta_{1,r}')X_t\Big). \end{aligned} \tag{4.A.4}$$

The last step is to determine the recursion for the coefficients by matching the coefficients of the left-hand side with the terms on the right-hand side. In this way, we derive the recursion for the no-arbitrage coefficients of the bond prices

$$A_n = A_{n-1} + B_{n-1}' - \Sigma\Gamma_0 + \frac{1}{2}B_{n-1}'\Sigma\Sigma'B_{n-1} - \delta_{0,r}, \tag{4.A.5a}$$

$$B_n' = B_{n-1}'(\Phi_1 - \Sigma\Gamma_1) - \delta_{1,r}'. \tag{4.A.5b}$$

with the initial conditions $A_1 = -\delta_{0,r}$ and $B_1 = -\delta_{1,r}$. Using the relation between bond prices and continuously compounded yields, we can derive the coefficients of the yields as follows

$$\bar{A}_n = -\frac{A_n}{n}, \tag{4.A.6a}$$

$$\bar{B}_n = -\frac{B_n}{n}. \tag{4.A.6b}$$

This function determines the no-arbitrage coefficients for the affine yields for each maturity n.

4.A.2 System of reduced equations

The system of reduced equations will be derived in this section. We first focus on the equation of the VAR dynamics of the yields measured without error. We stack the yields measured without errors in Y_t^1 and those measured with error in Y_t^2. Using Equation 4.3.11, we can write for yields measured without error

$$Y_t^1(n) = \bar{A}_1 + \bar{B}_1' X_t. \tag{4.A.7}$$

To derive the system of reduced equations, we start with the state process as defined in Equation (4.3.4). Premultiplying this system with $\bar{B}_1'$ and adding $\bar{A}_1$ gives

$$\bar{A}_1 + \bar{B}_1' X_t = \bar{A}_1 + \bar{B}_1' \left(\Phi_1 X_{t-1} + I_2 \epsilon_{1t}\right) \tag{4.A.8}$$

As a result, we can rewrite this equation to a VAR model of the yields measured without errors, Y_t^1,

$$Y_t^1 = \bar{A}_1 + \bar{B}_1' \Phi_1 \bar{B}_1'^{-1} \left(Y_{t-1}^1 - \bar{A}_1\right) + \bar{B}_1' \epsilon_{1,t}, \tag{4.A.9}$$

by use of the definition of Y_t^1 as given in Equation (4.3.11). Now we have expressed the time dynamics of the latent factors in yield series measured without error. Rewriting this equation yields the first reduced form regression,

$$Y_t^1 = \underbrace{(\bar{A}_1 - \bar{B}_1' \Phi_1 \bar{B}_1'^{-1} \bar{A}_1)}_{\bar{A}_1^*} + \underbrace{(\bar{B}_1' \Phi_1 \bar{B}_1'^{-1})}_{\Phi_{11}^*} Y_{t-1}^1 + \underbrace{\bar{B}_1'}_{\Omega_1^*} \epsilon_{1,t}. \tag{4.A.10}$$

In this equation the coefficients $\bar{A}_1^*$, Φ_{11}^*, and Ω_1^* will be obtained by OLS estimation.

The second reduced form equation is the impact of the latent factors on the yields measured with errors. For notional convenience, we repeat this equation

$$Y_t^2 = \bar{A}^2 + \bar{B}_2' X_t + \Omega \epsilon_{2,t}. \tag{4.A.11}$$

Since we include real rate series in this equation we need to incorporate the effect of the economic factors as well. Next, we substitute the latent factors with inverse of the yields observed without error,

$$Y_t^2 = \bar{A}^2 + \bar{B}_2' \left(\bar{B}_1'^{-1} \left(Y_t^1 - \bar{A}_1 \right) \right) + \Omega \epsilon_{2,t}. \tag{4.A.12}$$

Consequently, we derive the following reduced form regression,

$$Y_t^2 = \underbrace{(\bar{A}^2 - \bar{B}_2'^L \bar{B}_1'^{-1} \bar{A}_1)}_{\bar{A}_2^*} + \underbrace{(\bar{B}_2'^L B_1'^{-1})}_{\Phi_{21}^*} Y_t^1 + \underbrace{\Omega}_{\Omega_2^*} \epsilon_{2,t}. \tag{4.A.13}$$

We denote the OLS estimates of the coefficients in this equation as $\bar{A}_2^*$, Φ_{21}^*, and Ω_2^*.

The measurement equation for inflation follows a similar derivation. We start with the definition of inflation and substitute the model implied equivalents,

$$\frac{I_{t+1}}{I_t} = \delta_{0,\pi} + \delta_{0,\pi} \left(\bar{B}_1'^{-1} \left(Y_t^1 - \bar{A}_1 \right) \right) + \sigma_\pi \epsilon_{3,t} \tag{4.A.14}$$

Rewriting this equation, yields

$$\frac{I_{t+1}}{I_t} = \underbrace{\delta_{0,\pi} - \delta_{0,\pi} \bar{B}_1'^{-1} \bar{A}_1}_{A_3^*} + \underbrace{\delta_{0,\pi} \bar{B}_1'^{-1} Y_t^1}_{\Phi_{31}^*} X_{t-1} + \underbrace{\Sigma_\pi}_{\Sigma_\pi^*} \epsilon_{3,t} \tag{4.A.15}$$

We retrieve the OLS estimates of the coefficients in this equation as $\bar{A}_3^*$, Φ_{31}^*, and Σ_π^*.

Lastly, we need to retrieve the equity risk premium from the equity return process. For notational convenience, we repeat the definition of this process.

$$R_t = r_t + \eta + \sigma_R \epsilon_{4,t}, \tag{4.A.16}$$

where r_t denotes the monthly nominal short rate and η the equity risk premium. By substituting the definition of the nominal short rate and we derive the reduced form equation,

$$R_t = \eta + \delta_{0,r} - \delta_{1,r}' \bar{B}_1'^{-1} \bar{A}_1 + \delta_{1,r}' \bar{B}_1'^{-1} Y_t^1 + \sigma_R \epsilon_{4,t}. \tag{4.A.17}$$

Consequently, the OLS estimates can be mapped as follows,

$$R_t = \underbrace{\eta + \delta_{0,r} - \delta_1' \bar{B}_1'^{-1} \bar{A}_1}_{A_4^*} + \underbrace{\delta_1' \bar{B}_1'^{-1} Y_t^1}_{\Phi_{41}^*} X_t + \underbrace{\sigma_R}_{\Sigma_R^*} \epsilon_{4,t} \tag{4.A.18}$$

4.A.3 Bayesian approach

We apply a Bayesian methodology to our OLS estimates. In this step the parameter uncertainty enters since these reduced OLS estimates are used to estimate the structural parameters. To obtain the distribution for the reduced form of equations, we follow Bauwens et al. (1999) by rewriting these equations into a system of seemingly unrelated regressions. The reduced form of equations can easily be written in the following form,

$$y_i = X_i\beta_i + \epsilon_i, \tag{4.A.19}$$

for each $i = 1, .., n$ with n denoting the total number of state variables in the system. If the individual time series included in the model have dimension T, then y_i is a vector with $((T-1) \times 1)$ observations, X_i is a matrix with dimensions $((T-1) \times k_i$ with k_i independent variables, β_i consists of a coefficient vector with k_i elements, and ϵ_i is the vector with the associated errors for each observation $(T-1)$. We rewrite this model in two forms in order to draw parameters from the posterior density. By stacking all the observations for each equation i, we can express Equation (4.A.19) as

$$y = x\beta + \epsilon, \tag{4.A.20}$$

where $y = (y_1, .., y_n)$ is a vector with dimensions $((T-1)n \times 1)$, $\beta = (\beta_1, .., \beta_n)$ with a vector of k_n elements, $x = \text{diag}(x_1, .., x_n)$ with dimensions $((T-1)n \times k_n)$, and $\epsilon = (\epsilon_1, .., \epsilon_n)$. In the second approach, we write a VAR specification

$$Y = XB + E, \tag{4.A.21}$$

with $Y = (y_1 ... y_n)$ is a matrix with dimensions $((T-1) \times n)$, $X = (X_1 ... X_n)$ has dimensions $((T-1) \times k_n)$, $B = \text{diag}(\beta_1, .., \beta_n)$ is a matrix with dimensions $(k_n \times n)$ and $E = (E_1 ... E_n)$ is a matrix with dimensions $((T-1) \times n)$.

Uninformative prior

In deriving the posterior density function of the OLS estimates, we assume an uninformative prior. This prior means that we do not impose any prior belief on these parameters of the model. Hence, the prior function is of the form

$$f(\beta, \Sigma) \propto |\Sigma|^{-(n+1)/2}, \tag{4.A.22}$$

where Σ denotes the variance-covariance matrix of the error in the VAR model. For this uninformative prior, the marginal posterior density of the parameters can be written as

$$\begin{array}{rcl} \beta|\Sigma & \sim & \mathrm{N}(\hat{\beta}, [x'(\Sigma^{-1} \otimes I_{T-1})x]^{-1}) \\ \Sigma|\beta & \sim & \mathrm{IW}(Q, T-1), \end{array} \tag{4.A.23}$$

with

$$\begin{array}{rcl} \hat{\beta} & = & [x'(\Sigma^{-1} \otimes I_{T-1})x]^{-1}x'(\Sigma^{-1} \otimes I_{T-1})y \\ Q & = & (Y - XB)'(Y - XB). \end{array}$$

Since the marginal posterior densities of the two parameters β and Σ are not available, we rely on the Block-Gibbs sampling algorithm (See e.g., Bauwens et al. (1999)). Conditional on a previous simulation of the variance-covariance matrix Σ_{j-1}, we can draw β_j from the conditional density function. Again, with the sampled β_j the variance-covariance matrix Σ_j can be drawn from the inverse Wishart distribution. This sequential sampling method is initialized with the ordinary least squares estimates of the model. To remove potential influence of the starting values, we remove the first 500 draws from the sequence of parameters. Additionally, we remove draws if any eigenvalues of matrix with the autoregressive coefficients of the included variables are larger than 0.99 in order to ensure stationarity as in Bansal and Kiku (2011).

Our final sequence consists of 2000 draws from the posterior density. Using these parameters, we calculate the associated means and variance-covariance matrices of the various horizons. For each of these moments, we determine the optimal allocation strategy. We report the average of portfolio holdings for various horizons and a 95% confidence bounds of these allocations. This procedure results in optimal portfolio allocations that only rely on the observed data.

Informative prior

Next, we impose a Normal-diffuse prior on the parameters. Since the weight of recent observations bare more importance, we establish a prior on the impact of the OLS estimates. However, we hold a diffuse prior on Σ in Equation (4.A.19) or the covariance-variance matrix of the coefficients. Formally, we can write

$$\begin{array}{rcl} f(\beta) & \sim & \mathrm{N}(\beta_{Prior}, \Omega) \\ f(\Sigma) & \propto & |\Sigma|^{-(n+1)/2}, \end{array} \tag{4.A.24}$$

where β_{Prior} denotes the estimates of the prior. Following Zellner (1971) we can write the marginal posterior distributions as follows,

$$\begin{array}{rcl} \beta|\Sigma & \sim & \mathrm{N}(\hat{\beta}, \hat{\Omega}) \\ \Sigma|\beta & \sim & \mathrm{IW}(Q, T-1), \end{array} \tag{4.A.25}$$

with

$$\begin{aligned}
\hat{\beta}_{OLS} &= [x'(\Sigma^{-1} \otimes I_{T-1})x]^{-1}x'(\Sigma^{-1} \otimes I_{T-1})y \\
\hat{\beta} &= \hat{\Omega}(\hat{\Omega}^{-1}\beta_{Prior} + [x'(\Sigma^{-1} \otimes I_{T-1})x]^{-1}\hat{\beta}_{OLS} \\
\hat{\Omega} &= (\Omega^{-1} + x'(\Sigma^{-1} \otimes I_{T-1})x)^{-1} \\
Q &= (Y - XB_{OLS})'(Y - XB_{OLS}) + (B - B_{OLS})'X'X(B - B_{OLS}).
\end{aligned}$$

Again we rely on the Block-Gibbs sampling technique to derive the marginal posterior densities of the two parameters β and Σ. Conditional on a previous simulation of the variance-covariance matrix Σ_{j-1}, we can draw β_j from the conditional density function. Again, with the sampled β_j the variance-covariance matrix Σ_j can be drawn from the inverse Wishart distribution. This sequential sampling method is initialized with the ordinary least squares estimates of the model. To remove potential influence of the starting values, we remove the first 500 draws from the sequence of parameters. Additionally, we remove draws if any eigenvalues of matrix with the autoregressive coefficients of the included variables are larger than 0.99 in order to ensure stationarity. The prior estimates are derived from using the OLS estimates on a sample from January 2000 to December 2012.

4.B Appendix B: Alternative individual specifics

In this section, we analyze the impact of educational level and retirement age in case there is no parameter uncertainty.

4.B.1 Education level

We explore the differences of education level of the participant as it influences the wage pattern during the accumulation phase. In previous results, the educational level was set to high school level. In this section, we compare the results in case the participant has either a low educational level (no high school) and a high educational level (College level). We use the definitions of Cocco and Maenhout (2005) and calibrate our model using their coefficients $\alpha_1 = 0.1684$, $\alpha_2 = -0.0353$, and $\alpha_3 = 0.0023$ for No high school and $\alpha_1 = 0.3194$, $\alpha_2 = -0.0577$, and $\alpha_3 = 0.0033$ for College.

Table 4.8 shows that the low and high educational levels require a lower pension contribution than the average educated group in our benchmark model. The fixed contribution level is 5.76% for the low educated group, whereas it is 6.66% for the high educated group. While all groups share the characteristics that the end wage is lower than the average wage, the end wage of the average educated group is only 0.13% lower than the average wage. For the other two groups this difference is much larger, respectively 11.6% for the low educated group and 8.3% for the high educated group. As a result of the wage pattern, both groups require less pension contribution as their ambition level is relatively lower compared to the average wage.

Table 4.8 confirms our previous observation that a fixed pension contribution can lead to large uncertainty about the replacement rate at retirement. For both educational levels, the lower bound of the 95% confidence interval of the replacement rate is similar to the benchmark level. Likewise, the time-varying contribution scheme is only partial able to compensate for the effect of economic shocks. However, the 95% confidence interval of the contribution is less wide, suggesting that the low and high educational groups experience less volality in their contributions.

Regarding the importance of restrictions on the time-varying contribution, Table 4.8 shows that the high and low educated groups are less restricted by the fixed contribution bounds. However, the restriction of the fixed lower bound (at 0%) for the low educational level remains more important than in the benchmark case. For the low educated group, the fixed lower bound of 0% is binding in 58.2% of the scenarios at age 64, whereas in the benchmark this is 55.0%. This result indicates that restriction on withdrawal for pension wealth is more important for this group to achieve the pension ambition. Since lower educated groups might have less ability to address financial decisions, these resctriction are even more important to convey to this group.

4.B.2 Retirement age

Retirement age is an important factor since it gives the individual the opportunity to partially overcome economic shocks. By increasing the retirement age in case of bad scenarios, the participant will prolong the accumulation phase and purchase the real variable annuity with his desired replacement rate. To analyze the effect of retirement age, we extend the accumulation phase by two years in case the 70% replacement rate is not met. In case the desired replacement rate is achieved at age 66, we will assume the participant retires.

Our model shows that in 57.4% of the scenarios the participant can retire with a replacement rate of 70% or more. This result suggests that the decision to delay retirement is likely to occur. Within one year, the participant can retire in 20.7% of the scenarios at age 66 due to a lower annuity price and additional savings. Thus, the probability that the participant can retire at age 65 or 66 with the desired replacement rate is 78.1%. In 8.3% of the scenarios, the participant can retire at age 67 with a replacement rate larger than 70%. Thus, there is a probability of 13.6% the participant has to retire with a replacement rate lower than his desired ambition at age 67. In these bad scenarios, the average replacement rate is 59.5% when the participant retires at age 67. These results show that the participant can increase his probability to retire with a real replacement rate from 57.4% to 86.4% in case he is willing to work one or two additional years. In particularly, delaying retirement to age 66 increases the probability substantially with 20.7%, so that the marginal contribution of working one additional year after age 66 is far lower. This shows that delaying retirement can only partly compensate for the bad scenarios and that delaying retirement beyond age 66 does not have a substantial impact on the replacement rates.

4.C Appendix C: Tables and figures

Table 4.1: Estimation of financial market

This table presents the estimation results of the financial market based on a sample period of January 1952 to December 2012. For the short rate, inflation rate and the equity return process, we show the annualized terms.

Parameter	Estimate	Standard error
Dynamics latent factors: $X_t = \Phi_1 X_{t-1} + \Sigma\epsilon_t$		
$\Phi_{1,11}$	0.9917	0.0053
$\Phi_{1,21}$	-0.0279	0.0135
$\Phi_{1,22}$	0.9473	0.0127
Price of Risk: $\Lambda_t = \Gamma_0 + \Gamma_1 X_t$		
$\Gamma_{0,1}$	-0.0878	0.0230
$\Gamma_{0,2}$	-0.0119	0.0281
$\Gamma_{1,11}$	0.0109	0.0104
$\Gamma_{1,12}$	0.0332	0.0016
$\Gamma_{1,21}$	-0.0170	0.0127
$\Gamma_{1,22}$	-0.0294	0.0121
Short rate: $r_t^N = \delta_{0,r} + \delta_r' X_t$		
δ_r	0.0488	0.0026
$\delta_{r,1} \times 100$	0.5106	0.0520
$\delta_{r,2} \times 100$	0.1980	0.1263
Inflation rate: $\pi_t = \delta_{0,\pi} + \delta_\pi' X_t + \sigma_\pi' \epsilon_t$		
$\delta_{0,\pi}$	0.0354	0.0107
$\delta_{\pi,1} \times 100$	0.3471	0.0595
$\delta_{\pi,2} \times 100$	0.2092	0.0948
σ_π	0.0109	0.0009
Equity return: $R_t = r_t^N + \eta + \sigma_R' \epsilon_t$		
η	0.0504	0.0193
σ_R	0.1508	0.0012
Measurement error Yields: $Y_t^N = \bar{A}_n + \bar{B}_n' X_t + \Omega \eta_t$		
ω_{3M}	0.0040	
ω_{6M}	0.0021	
ω_{2Y}	0.0012	
ω_{10Y}	0.0028	

Table 4.2: Pension contract without parameter uncertainty

This table presents the replacement rates, and contribution for two contribution schemes. The fixed contribution scheme is set to a yearly rate of 6.85% and is based on the historical performance of the portfolio. The time-varying contribution scheme is determined by the term structure of interest rates and the estimated historical equity risk premium. The lower (L.B.) and upper bounds (U.B.) of the contributions are based on 95% confidence intervals (C.I.). The fraction bounds hit denotes the probability in which the fixed contribution bounds of 0% and 30% are restrictive.

Contribution Scheme		Fixed	Varying	
Replacement rate at 65	mean	70.0 %	76.8 %	
	median	65.3 %	73.4 %	
	L.B.	35.3 %	45.2 %	
	U.B.	124.2 %	127.5 %	
Average Contribution	25	6.85%	5.83 %	
	35	6.85%	6.96 %	
	45	6.85%	7.69 %	
	50	6.85%	8.57 %	
	55	6.85%	9.88 %	
	60	6.85%	11.61 %	
	64	6.85%	12.67 %	
			L.B.	U.B.
C.I. interval Contribution	25		4.07 %	8.01 %
	35		0.00 %	23.55 %
	45		0.00 %	30.00 %
	50		0.00 %	30.00 %
	55		0.00 %	30.00 %
	60		0.00 %	30.00 %
	64		0.00 %	30.00 %
			L.B. (0%)	U.B. (30%)
Fraction bounds hit	25		0.00 %	0.00 %
	35		10.40 %	0.90 %
	45		28.80 %	2.90 %
	50		35.80 %	6.40 %
	55		43.40 %	14.30 %
	60		48.60 %	26.80 %
	64		55.00 %	38.80 %

Table 4.3: Certainty equivalent wealth effects for bounds and risk profile

This table presents the certainty equivalent wealth gains for various regimes of contributions schemes and the low risk profile. Certainty equivalent wealth gains are determined relative to the benchmark risk profile with a fixed contribution scheme. To compare the impact of the low risk profile, the gains are determined relatively to the equivalent contribution scheme for the benchmark risk profile. The benchmark discount factor χ is 0.96. The risk aversion coefficients, ι, for the aggressive, neutral and conservative participant is 3, 5 and 7, respectively.

	Participant		
	Aggressive	Neutral	Conservative
Fixed contribution bounds of			
0%-20%	1.0%	3.5%	6.9%
0%-25%	1.6%	5.0%	9.9%
0%-30%	1.9%	5.8%	11.8%
0%-35%	2.0%	6.2%	12.9%
0%-40%	2.0%	6.2%	13.4%
Low risk profile			
0%-20%	-2.0%	0.0%	4.0%
0%-25%	-2.2%	-0.5%	3.0%
0%-30%	-2.3%	-0.9%	2.1%
0%-35%	-2.4%	-1.2%	1.3%
0%-40%	-2.5%	-1.5%	0.6%

Table 4.4: Impact of risk profile on the pension contract

This table presents the replacement rates, and contribution for two contribution schemes. The fixed contribution scheme is set to a yearly rate of 9.72% and is based on the historical performance of the portfolio. The low risk portfolio consists of 30% equity and 70% bonds. The lower (L.B.) and upper bounds (U.B.) of the contributions are based on 95% confidence intervals (C.I.). The fraction bounds hit denotes the probability in which the fixed contribution bounds of 0% and 30% are restrictive.

Contribution Scheme		Fixed	Varying	
Replacement rate at 65	mean	71.4 %	73.1 %	
	median	69.4 %	71.6 %	
	L.B.	42.4 %	51.0 %	
	U.B.	111.7 %	103.0 %	
Average Contribution	25	9.72%	8.65 %	
	35	9.72%	9.87 %	
	45	9.72%	10.11 %	
	50	9.72%	10.86 %	
	55	9.72%	11.37 %	
	60	9.72%	13.04 %	
	64	9.72%	13.16 %	
			L.B.	U.B.
C.I. Contribution	25		6.14 %	11.71 %
	35		0.00 %	30.00 %
	45		0.00 %	30.00 %
	50		0.00 %	30.00 %
	55		0.00 %	30.00 %
	60		0.00 %	30.00 %
	64		0.00 %	30.00 %
			L.B. (0%)	U.B. (30%)
Fraction bounds hit	25		0.00 %	0.00 %
	35		6.50 %	2.90 %
	45		22.00 %	5.50 %
	50		28.80 %	10.80 %
	55		35.60 %	18.30 %
	60		42.60 %	30.70 %
	64		53.40 %	40.50 %

Table 4.5: Pension contract with parameter uncertainty of the equity risk premium

This table presents the replacement rates, and contributions for two contribution schemes when the equity risk premium is uncertain. The parameter uncertainty is based on an uninformed prior using the sample period from 1952-2012. The time-varying contributions are based on the term structure of interest rates and the historical equity risk premium. The lower (L.B.) and upper bounds (U.B.) of the contributions are based on 95% credibility intervals (C.I.). The fraction bounds hit denotes the probability in which the fixed contribution bounds of 0% and 30% are restrictive.

Contribution Scheme		Fixed	Varying	
Replacement rate at 65	mean	73.1 %	80.2 %	
	median	64.0 %	74.1 %	
	L.B.	26.6 %	41.3 %	
	U.B.	170.2 %	154.8 %	
Average Contribution	25	6.85%	5.83 %	
	35	6.85%	6.95 %	
	45	6.85%	7.64 %	
	50	6.85%	8.53 %	
	55	6.85%	9.82 %	
	60	6.85%	11.54 %	
	64	6.85%	12.48 %	
			L.B.	U.B.
C.I. Contribution	25		4.08 %	8.01 %
	35		0.00 %	23.60 %
	45		0.00 %	30.00 %
	50		0.00 %	30.00 %
	55		0.00 %	30.00 %
	60		0.00 %	30.00 %
	64		0.00 %	30.00 %
			L.B. (0%)	U.B. (30%)
Fraction bounds hit	25		0.00 %	0.00 %
	35		10.41 %	0.89 %
	45		30.00 %	2.80 %
	50		37.99 %	6.61 %
	55		45.01 %	14.51 %
	60		49.84 %	26.89 %
	64		56.32 %	39.09 %

Table 4.6: The pension contract with parameter uncertainty of all financial parameters

This table presents the replacement rates, and contribution for two contribution schemes when all financial parameters are affected by parameter uncertainty using an uninformative prior. The lower (L.B.) and upper bounds (U.B.) of the contributions are based on 95% credibility intervals (C.I.). The fraction bounds hit denotes the probability in which the fixed contribution bounds of 0% and 30% are restrictive.

Contribution Scheme		Fixed	Varying	
Replacement rate at 65	mean	75.0 %	83.8 %	
	median	65.2 %	76.4 %	
	L.B.	26.5 %	42.1 %	
	U.B.	177.3 %	166.0 %	
Average Contribution	25	6.85%	6.51 %	
	35	6.85%	7.52 %	
	45	6.85%	7.97 %	
	50	6.85%	8.64 %	
	55	6.85%	9.42 %	
	60	6.85%	10.74 %	
	64	6.85%	11.31 %	
			L.B.	U.B.
C.I. Contribution	25		4.49 %	9.07 %
	35		0.00 %	25.52 %
	45		0.00 %	30.00 %
	50		0.00 %	30.00 %
	55		0.00 %	30.00 %
	60		0.00 %	30.00 %
	64		0.00 %	30.00 %
			L.B. (0%)	U.B. (30%)
Fraction bounds hit	25		0.00 %	0.00 %
	35		10.82 %	1.35 %
	45		30.68 %	3.76 %
	50		39.45 %	7.73 %
	55		47.57 %	14.52 %
	60		53.44 %	25.63 %
	64		60.18 %	35.61 %

Table 4.7: Prior view and parameter uncertainty

This table presents the replacement rates, and contribution for two contribution schemes. In this setting, parameter uncertainty affects all financial parameters and is incorporated using a Normal-Diffuse prior calibrated on the period from Jan 2000 to Dec 2012. The lower (L.B.) and upper bounds (U.B.) of the contributions are based on 95% credibility intervals (C.I.). The fraction bounds hit denotes the probability in which the fixed contribution bounds of 0% and 30% are restrictive.

Contribution Scheme		Fixed	Varying	
Replacement rate at 65	mean	65.1 %	76.6 %	
	median	56.0 %	70.5 %	
	L.B.	21.7 %	38.0 %	
	U.B.	160.4 %	150.4 %	
Average Contribution	25	6.85%	6.38 %	
	35	6.85%	7.65 %	
	45	6.85%	8.55 %	
	50	6.85%	9.65 %	
	55	6.85%	11.08 %	
	60	6.85%	13.17 %	
	64	6.85%	14.09 %	
			L.B.	U.B.
C.I. Contribution	25		4.59 %	8.60 %
	35		0.00 %	24.14 %
	45		0.00 %	30.00 %
	50		0.00 %	30.00 %
	55		0.00 %	30.00 %
	60		0.00 %	30.00 %
	64		0.00 %	30.00 %
			L.B. (0%)	U.B. (30%)
Fraction bounds hit	25		0.00 %	0.00 %
	35		8.36 %	1.04 %
	45		25.60 %	3.53 %
	50		32.99 %	8.01 %
	55		40.02 %	17.49 %
	60		44.84 %	32.80 %
	64		50.86 %	44.82 %

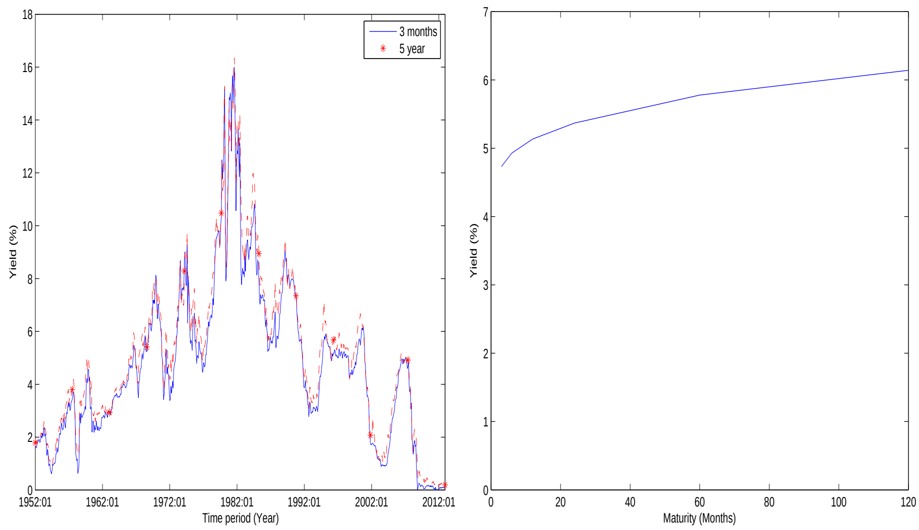
Table 4.8: Impact of educational level on the pension contract

This table presents the replacement rates (Repl. rate), and contribution for two contribution schemes when parameter uncertainty is ignored. The low educational level correspond to no high school and high educational level to College level. The fixed yearly contribution for the low educational level is 5.76%, whereas the contribution for the high educational level is 6.66%. The lower (L.B.) and upper bounds (U.B.) of the contributions are based on 95% confidence intervals (C.I.). The fraction bounds hit denotes the probability in which the fixed contribution bounds of 0% and 30% are restrictive.

		Educational level					
		Low			High		
Contribution Scheme		Fixed	Varying		Fixed	Varying	
Repl. rate at 65	mean	69.9 %	78.3 %		70.1 %	77.0 %	
	median	65.0 %	74.7 %		65.4 %	73.6 %	
	L.B.	35.2 %	45.8 %		36.0 %	44.9 %	
	U.B.	125.6 %	131.3 %		123.1 %	126.8 %	
Average Contribution	25	5.76%	4.88 %		6.66%	5.69 %	
	35	5.76%	5.97 %		6.66%	6.69 %	
	45	5.76%	6.79 %		6.66%	7.26 %	
	50	5.76%	7.71 %		6.66%	8.19 %	
	55	5.76%	9.09 %		6.66%	9.58 %	
	60	5.76%	10.83 %		6.66%	11.36 %	
	64	5.76%	11.69 %		6.66%	12.55 %	
			L.B.	U.B.		L.B.	U.B.
C.I. Contribution	25		3.38 %	6.76 %		4.02 %	7.76 %
	35		0.00 %	21.14 %		0.00 %	21.39 %
	45		0.00 %	28.29 %		0.00 %	27.49 %
	50		0.00 %	30.00 %		0.00 %	30.00 %
	55		0.00 %	30.00 %		0.00 %	30.00 %
	60		0.00 %	30.00 %		0.00 %	30.00 %
	64		0.00 %	30.00 %		0.00 %	30.00 %
			L.B. (0%)	U.B. (30%)		L.B. (0%)	U.B. (30%)
Fraction bounds hit	25		0.00 %	0.00 %		0.00 %	0.00 %
	35		12.40 %	0.50 %		7.40 %	0.50 %
	45		32.40 %	2.30 %		26.70 %	1.40 %
	50		40.50 %	5.70 %		34.00 %	5.50 %
	55		46.60 %	12.70 %		42.90 %	12.40 %
	60		52.00 %	24.10 %		49.10 %	25.50 %
	64		58.20 %	36.20 %		55.50 %	39.30 %

Figure 4.1: Yield curve

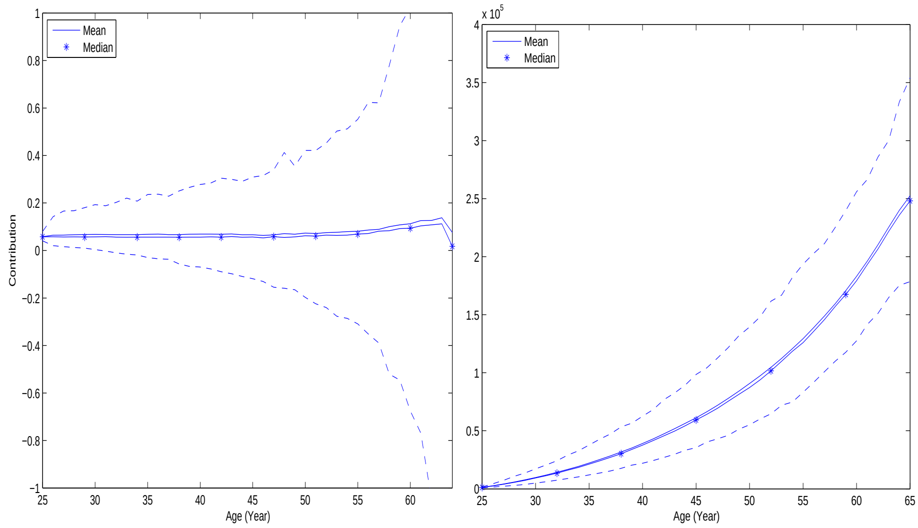
This figure present two time series of yields and the average yield curve for the sample period of Jan 1952 to Dec 2012. For the yields, we plot both the 3-months and 5 years bond yield.



(a) Yield rates (b) Average yield curve

Figure 4.2: Pension contribution without restrictions to the contribution level

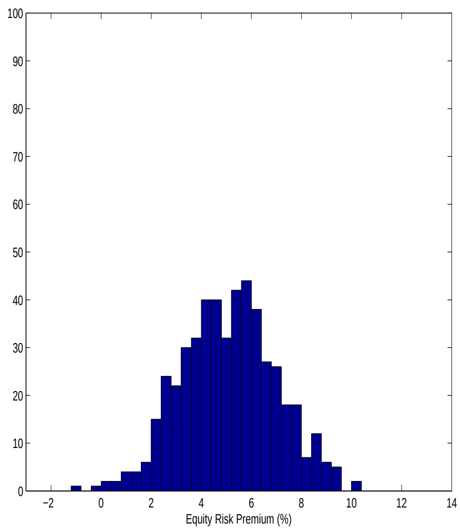
This figure presents the contribution during the accumulation phase if parameter uncertainty is ignored. Our second graph denotes the corresponding pension wealth over the life time. The time-varying contribution is not restricted by the fixed bounds of 0% and 30% as in the benchmark contract. Real wealth is denoted in dollars and the contribution in percentage of the annual wage.



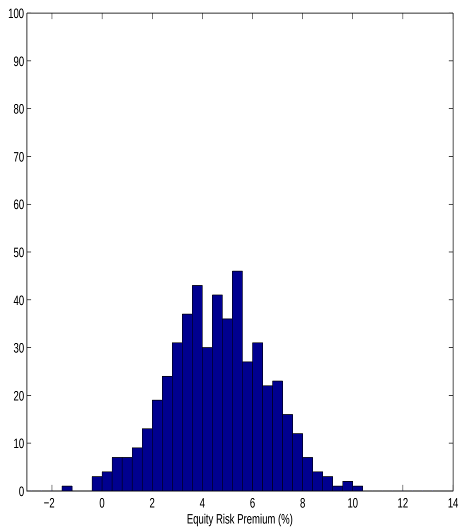
(a) Contribution (b) Real pension wealth

Figure 4.3: Equity risk premium

This figure presents the marginal posterior distribution of the equity risk premium. In the first graph, an uninformative prior is assumed, which assigns equal weight to all observations. In the second graph, the Normal-diffuse prior adds more weight to the period from Jan 2000 up to Dec 2012. The distribution in the first graph is centered around 5.09% with a standard deviation of 1.95% and in the second graph it is centered around 4.61% and has a standard deviation of 1.98%.



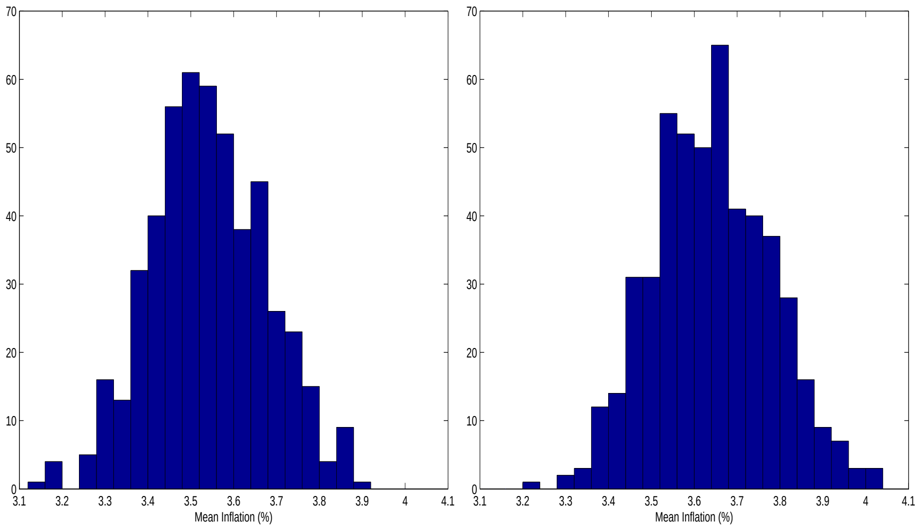
(a) Equal weight



(b) Post 2000 regime

Figure 4.4: Inflation rate

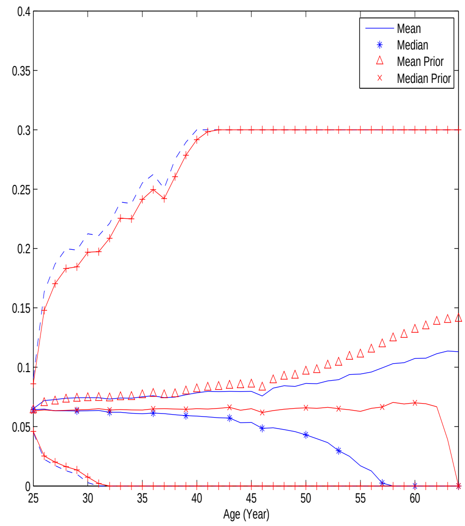
This figure presents the marginal posterior distribution of the mean inflation rate. In the first graph, an uninformative prior is assumed, which assigns equal weight to all observations. In the second graph, the Normal-diffuse prior adds more weight to the period from Jan 2000 up to Dec 2012. The distribution in the first graph is centered around 3.54% with a standard deviation of 0.14% and in the second graph it is centered around 3.61% and has a standard deviation of 0.14%.



(a) Equal weight (b) Post 2000 regime

Figure 4.5: The effect of an informative prior on the contribution levels

This figure presents the mean, median, and 95% credibility interval of the contributions levels for two different regimes on parameter uncertainty. In the first regime (blue line), parameter uncertainty is estimated with an uninformative prior, which holds equal weights to all observations. In the second regime (red line), an informative prior is used that assigns more weight to the period from Jan 2000 to Dec 2012.



CHAPTER 5

CONCLUSION

Parameter uncertainty of financial parameters affects investors in multiple ways. This dissertation shows that investors can exploit long run interrelations between foreign inflation measures and their inflation exposure in their inflation hedging portfolios. This allows investors to benefit from the higher liquidity of inflation-linked derivatives in international markets. Another factor that investors need to explicitly take into account is the substantial parameter uncertainty about the inflation risk premium. Similarly, investors targeting a certain pension wealth for retirement are susceptible to misestimates of the equity risk premium. My close study of parameter uncertainty leads to several policy implications.

European inflation-linked bonds are highly sought after by European inflation hedging investors. While international markets allow European investors to acquire inflation-linked bonds with higher liquidity, currency and foreign inflation risks may harm the hedging performance of the investors' portfolios. For European investors, an alternative to local inflation-linked bonds is European inflation-linked bonds. European governments issuing such bonds may profit from potential higher demand in the European inflation-linked bond market.

Issuing inflation-linked bonds to finance government debt has been argued as a way for governments to reduce their costs for issuing debt. Theory suggests that by switching from nominal debt to inflation-linked debt, governments can economize on the inflation risk premium when inflation is stable and predictable. However, I show that for most developed markets, there is large uncertainty about the estimate of the inflation risk premium. This uncertainty increases during the financial crisis. It is therefore unclear whether governmental debt financing can be less costly when financed with inflation-linked bonds. Such a cost assessment would require a model that explicitly takes parameter uncertainty into account.

Besides the inflation risk premium, parameter uncertainty regarding the equity risk premium represents a particularly large threat to DC pensions. Pension funds disregarding the uncertainty of the equity risk premium estimate may underestimate the replacement rate risk. As a result, participants may set their pension contributions inadequately. To ensure long term sustainability of pension funds, regulators may set prudent upper limits to the estimates of the equity risk premium which pension funds can use to project replacement rates at retirement. The framework presented in this dissertation can help determine prudential estimates for the equity risk premium to limit replacement rate risk.

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