Financial Stability and Interacting Networks of Financial Institutions and Market Infrastructures

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Abstract

An interacting network coupling financial institutions' multiplex (i.e. multi-layer) and financial market infrastructures' single-layer networks gives an accurate picture of a financial system's true connective architecture. We examine and compare the main properties of Colombian multiplex and interacting financial networks. Coupling financial institutions' multiplex networks with financial market infrastructures' networks removes modularity, which enhances financial instability because the network then fails to isolate feedbacks and limit cascades while it retains its robust-yet-fragile features. Moreover, our analysis highlights the relevance of infrastructure-related systemic risk, corresponding to the effects caused by the improper functioning of FMIs or by FMIs acting as conduits for contagion.

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1. Introduction

Most transactions between financial institutions (FIs) require a financial market infrastructure (FMI) that settles the exchange of money, securities, foreign exchange and derivatives.[5] Therefore, directly connecting financial institutions (FIs) to each other in a network or graph may be convenient and illustrative, but is incomplete as it ignores the role of additional networks of distinct financial and non-financial participants that are essential for the completion of financial transactions in terms of their settlement.[6] Modeling the settlement of transactions between FIs without accounting for FMIs comes down to making the assumption that financial market infrastructures always work.

Following Kurant and Thiran (2006), networks of FIs are *logical networks*, and the links between FIs are of a *logical* (i.e. virtual) nature. Additional layers of networks exist beneath logical networks, either of logical or physical nature. In the case of financial markets, due to their role in the settlement of financial transactions, FMIs may be considered as the financial system's "plumbing" (Bernanke, 2011), or the "medium" in which FIs interact in the sense of Gambuzza et al. (2014). In this vein, FMIs that settle transactions between FIs are the first logical layer beneath the traditional FIs' logical network. Accordingly, the well-functioning of those FMIs is not only crucial for financial markets and for financial stability, but FMIs should also be considered as *critical infrastructures.*[7]

In spite of the fact that the critical role of FMIs for financial systems has been stressed before (CPSS and IOSCO, 2012; Dudley, 2012; Bernanke, 2012), most research on financial networks still focuses on single-layer FIs-only networks. Their linkages then correspond to transactions or exposures pertaining to a single market (e.g. interbank, foreign exchange, derivatives, etc.). Hence, the financial literature tends to ignore two sources of complexity in financial markets: (i) the simultaneous presence of FIs across different financial markets and their networks,

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[5] Financial institutions (FIs) correspond to depository institutions (e.g. banks), broker-dealers, investment companies (e.g. mutual funds), insurance companies, and credit unions. Financial market infrastructures (FMIs) correspond to multilateral systems providing trading, clearing, settling, recording, and compressing services for transactions between FIs. For the purpose of this paper we focus on FMIs providing settlement services.

[6] Correspondent banking is an alternative to the settlement role of FMIs, and thus can be modeled as a network of connected FIs. However, correspondent banking is only a minor channel compared to FMIs.

[7] To consider FMIs' critical infrastructures means that they compose one of those systems that are essential for the maintenance of vital societal functions, health, safety, security, economy or social well-being of people (European Commission, 2008).

and (ii) the coupling of financial markets and their networks by means of the settlements across different FMIs. Furthermore, these two sources of complexity yield two unmapped sources of systemic risk, respectively: (i) *cross-system risk* (CPSS, 2008), corresponding to the potential effects caused by a FI experiencing problems across different markets or layers, and (ii) infrastructure-related systemic risk (Berndsen, 2011; p.15), corresponding to "the improper functioning of the financial infrastructure, or where the financial infrastructure acts as the conduit for shocks that have arisen elsewhere" (i.e. in another layer).

Therefore, in order to gain a comprehensive and enhanced understanding of financial systems' complex architecture, we implement the two existing approaches to interdependent or multi-layer networks modeling, namely *multiplex networks* and *interdependent networks* (D'Agostino and Scala, 2014). First, we build a financial multiplex network, consisting in a multi-layer network of FIs acting in different financial markets or environments. Second, by explicitly incorporating the role of the network of FMIs for the FIs' multiplex network, we build an interacting financial network. This way, based on a unique dataset, we examine how FMIs provide the medium that allows FIs to interact across distinct financial markets. Additionally, this paper examines whether, or not, the interaction between different layers of FIs and FMIs preserves the main connective and hierarchical features of single-layer financial networks, which have been reported to exhibit features corresponding to a modular scale-free architecture (León and Berndsen, 2014; Bargigli et al., 2014).

Five novel findings result from our paper. First, building a multi-market financial multiplex network has not been attempted before in the Colombian case. Second, to the best of our knowledge, coupling FIs and FMIs into an interacting network is a significant step forward in the examination of financial networks that has not been made before. Third, we verify that the multiplex network of FIs preserves the main connective and hierarchical features of single-layer or *monoplex networks* (i.e. their modular scale-free architecture), and we suggest that this is a byproduct of *positively correlated multiplexity* in the sense of Lee et al. (2014). Fourth, coupling FIs and FMIs yields a scale-free but non-modular architecture, an outcome with noteworthy implications for financial stability purposes. Fifth, in the sense of Gao et al. (2012), our results confirm that the connections between FIs and FMIs correspond to *dependence links* (i.e. links that are critical for participants' functions) instead of traditional *connectivity links* (i.e. links that enable to carry out functions), which emphasizes that the safe and efficient functioning of the FMIs' network is critical for the FIs and for financial stability.

Together, these five findings make a significant contribution to the literature on financial networks and financial stability, and differentiate our work from other related single- or multi-layer financial network research.

2. Literature review: from single- to multi-layer financial networks

Interdependent or coupled networks' modeling requires defining different networks (i.e. layers) and the interactions among them (D'Agostino and Scala, 2014). Two approaches to such multi-layer network modeling are available: multiplex networks and interacting networks. In the former case, each layer consists of a network containing distinct types of links but a common type of participant. In the interacting networks approach the different layers are explicitly modeled as separate networks and the links among them represent inter-layer interactions.

While from a graph-theory point of view, a multi-layer network is just a larger network, networks in real life are governed and operated separately, and interactions are only allowed at well-defined boundaries (D'Agostino and Scala, 2014). In the case of financial systems, in which FIs and FMIs coexist and are mutually dependent, the resulting network will not only be larger than the traditional –logical- FIs' network, but it will also reveal how financial transactions are settled between FIs under the corresponding legal and operational framework. And, most importantly, it will reveal whether, or not, the main connective and hierarchical features of the constituent logical networks are preserved after their coupling.

Our research is related to two strands within financial network analysis, namely how financial networks couple, and on how coupling affects the connective and hierarchical architecture of financial networks. These two topics are critical for the understanding of the organization and behavior of financial systems, which enhances our understanding of financial stability. In addition, our work also contributes to the broad set of network science applications.

2.1. On interdependent financial networks

Most efforts to characterize the topology of complex systems by means of network analysis have assumed that each system is isolated (i.e. non-coupled) from other networks. Yet, as highlighted by Cardillo et al. (2013; p.1), many biological and man-made networked systems

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are characterized by the simultaneous presence of different sub-networks organized in separate layers, with connections and participants of qualitatively different types. Such multi-layered nature of networks, also known as "interdependent networks" or "network of networks", has been the focus of network scientists rather recently, and the work of Kurant and Thiran (2006) is among the first contributions to this field.

Most literature on financial networks deals with a single type of participant: financial institutions (FIs). A customary financial network consists of FIs pertaining to a single-layer or monoplex network, in which the network comprises one sort of participant and a single type of connection.[8] It is also possible to construct and analyze a financial monoplex composed by FMIs, as in León and Pérez (2014).

Research on multi-layer financial networks has appeared rather recently. The standard multi-layer framework in finance corresponds to the so-called multiplex network, which may be described as networks containing participants of one sort but several kinds of edges (Baxter et al., 2014).[9] Figure 1 displays a two-layer network composed by layers X and Y, and the multiplex (Z) resulting from merging X and Y. Vertical lines connecting superimposed vertexes are the participants, whereas each vertex is a role in the corresponding layer.

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[8] Literature on single-layer financial institutions networks is abundant. A short list includes León and Berndsen (2014), Chinazzi et al. (2013), Martínez-Jaramillo et al. (2012), Markose et al. (2012), Markose (2012), Arinaminpathy et al. (2012), Gai and Kapadia (2010); Haldane (2009), Bech and Atalay (2010), Nier et al. (2008); Pröpper et al. (2008), May et al. (2008), Cepeda (2008), Renault et al. (2007), Soramäki et al. (2007), Cifuentes et al. (2005), Inaoka et al. (2004), Boss et al. (2004).

[9] Several applications of multiplex networks have been documented for non-financial complex systems, such as transport systems (Cardillo et al., 2013; Kurant and Thiran, 2006), electrical networks (Pahwa et al., 2014), physiological systems (Ivanov and Bartsch, 2014), critical infrastructures (Martí, 2014; Rome et al., 2014) and cooperation networks (Gómez-Gardenes et al., 2012).
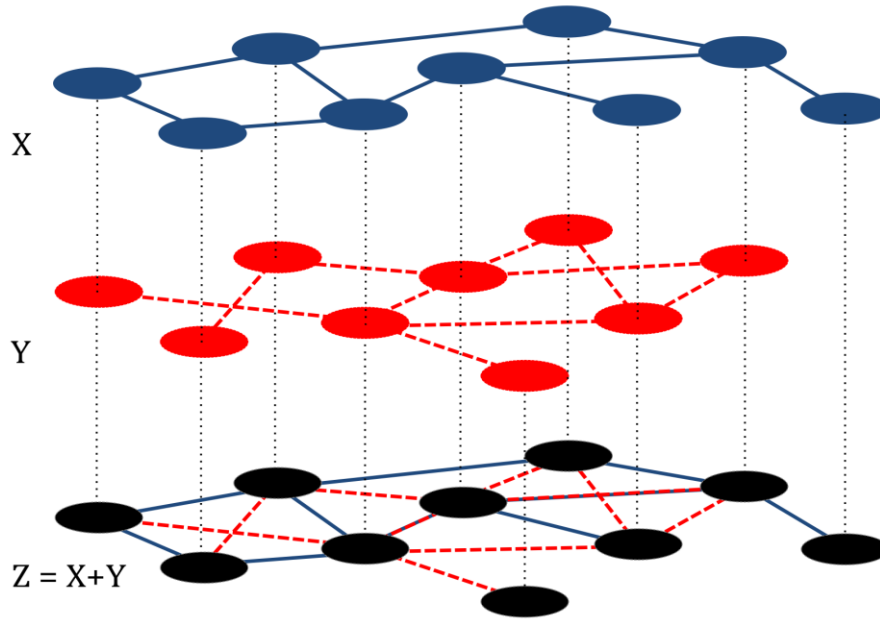
Figure 1. A multiplex network. Two-layer networks, X and Y, and the multiplex (Z) resulting from merging X and Y. Vertical lines connecting superimposed vertexes are the participants, whereas each vertex is a role in the corresponding layer.

Some financial networks have been treated as a multiplex network. Montagna and Kok (2013) model interbank contagion in the US market with a triple-layer multiplex network consisting of long-term direct bilateral exposures, short-term bilateral exposures and common exposures to financial assets. Bargigli et al. (2013) examine the Italian interbank multiplex network by transaction type (i.e. secured and non-secured) and by maturity (i.e. overnight, short-term and long-term). León et al. (2014) examine the connective properties of the multiplex network that results from the three different Colombian sovereign securities' trading and registering platforms. In this sense, recent efforts to examine financial multi-layer networks regularly correspond to the multiplex network, with FIs as the usual participant.

However, multi-layer networks models are not limited to the multiplex case. The coupled nature of layers of distinct participants may be decisively important. For instance, as depicted by Kurant and Thiran (2006), a first look at the World-Wide Web shows a network of interconnected IP vertexes, whereas a deeper examination will reveal that IP vertexes' linkages are possible via IP routers, and those linkages between IP vertexes and IP routers depend on a physical network (e.g. fiber optic, cable). As a result, Kurant and Thiran motivate a multi-layer model of distinct types of participants, in which the disruption or malfunction of

concealed interacting layer(s) might destroy a substantial part of the most evident (i.e. the first) layer.

Our research overlaps with the original motivation of Kurant and Thiran (2006) for multi-layer networks, but departs from the standard multiplex network case. Our analysis acknowledges for the first time in financial networks literature that FIs would not be able to settle most of their transactions in the absence of FMIs. Accordingly, our research considers the existence of the medium that allows FIs to connect to each other across distinct financial environments.

Figure 2 depicts an analytical representation of a two-layer interacting network of FIs and FMIs. Let the multiplex Z from Figure 1 represent a first layer of FIs participating in two asset markets (e.g. money and foreign exchange), and let the second layer represent the settlement FMIs for both markets, in which the direction of the linkages has been omitted for practical purposes. In this case, each FI connects to the corresponding FMI, and both FMIs connect to each other in order to instruct the delivery of money and foreign exchange that would settle any transaction of this oversimplified financial system, say buying or selling foreign exchange.
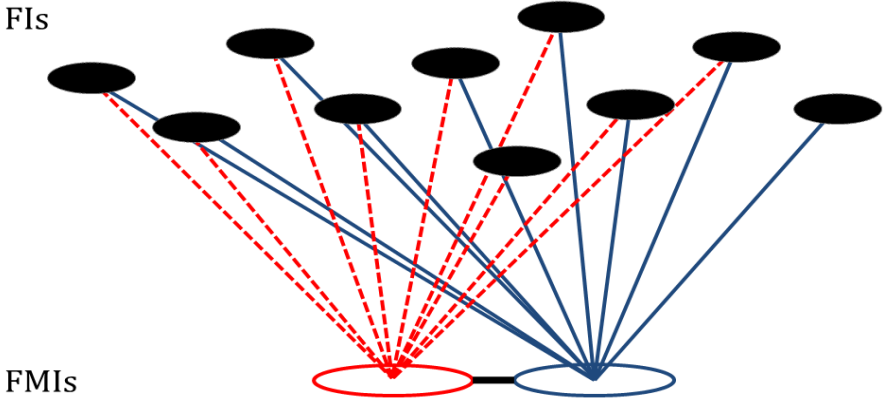
Figure 2. Coupling FIs' and FMIs' networks. This analytical representation shows a two-layer interacting network of FIs and FMIs. Each FI connects to the corresponding FMI, and both FMIs connect to each other in order to instruct the delivery of money and foreign exchange that would settle any transaction of this overly simplified financial system.

Unlike links in FIs' monoplex networks (Figure 1), which enable FIs to carry out their functions, the links in the interacting network in Figure 2 are of a critical nature for FIs: if an inter-layer link is removed, the corresponding FI would not be able to settle its transactions, whereas the absence of the link between FMIs would endanger the settlement of all FIs'

transactions. The links in the FIs' and FMIs' interacting network are *dependence links*, whereas links in a FIs-only network are *connectivity links* (Gao et al., 2012). This type of modeling emphasizes the critical role FMIs play in the financial system and the broader economy, as acknowledged by CPSS and IOSCO (2012). Furthermore, such dependence links are consistent with viewing the FMIs' network as a first layer of a financial system's *critical infrastructure*, and underscores the existence of a non-negligible *critical infrastructure dependency* in the sense of Rome et al. (2014).[10] The presence of dependence links and the fundamental role of the FMIs' network should be closely monitored by financial authorities in their quest to preserve financial stability.

2.2. On the connective and hierarchical features of multi-layer networks

Even to date, the literature concentrates on an inhomogeneous connective structure of single-layer financial networks, typically in the form of the distribution of links –and their weights-approximating a *power-law* (e.g. León and Berndsen, 2014; Bech and Atalay, 2010; Pröpper et al., 2008; May et al., 2008; Cepeda, 2008; Renault et al. 2007; Soramäki et al., 2007; Inaoka et al., 2004; Boss et al., 2004).[11] The power-law or Pareto distribution of links within a network is commonly referred as a *scale-free network* (Barabási and Albert, 1999), and it corresponds to a particular case in which there are a few heavily linked participants (i.e. high *degree* vertexes) and many poorly linked participants (i.e. low *degree* vertexes). This is precisely the most documented type of network in real-world complex systems (e.g. social, biological, man-made).

The inhomogeneity in scale-free networks yields a structure that is robust to random shocks but fragile to targeted attacks, as in the nowadays celebrated "robust-yet-fragile" characterization of financial networks by Haldane (2009). In this sense, a scale-free connective structure provides everyday stability and efficiency for complex systems, but exposes them to rare massive transformations (Miller and Page, 2007), as in a power-law

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[10] Additional critical layers for financial networks, mostly of physical nature (e.g. communications, power), are not considered in this paper.

[11] However, most literature that models the interactions between financial institutions is based on the assumption of a homogeneous connective structure, in which financial institutions tend to connect to each other in a dense and uniform manner that tends to diversify or disperse shocks (e.g. Battiston et al. (2012), Cifuentes et al. (2005), Freixas et al. (2000) and Allen and Gale (2000)). Therefore, observed financial networks' inhomogeneous distribution of linkages and their weights contradict the traditional assumptions of standard contagion models.

distribution of events or a *coevolution to the edge of chaos* (Anderson, 1999).[12] Consequently, a network approximating a scale-free connective structure tends to be robust because it is able to withstand random shocks, yet it is fragile if shocks target heavily connected participants.

Some real-world networks also display a particular modular hierarchical organization, a defining feature of most complex systems according to Barabási (2003). In a modular hierarchy, there are densely connected clusters or communities that are sparsely connected to other clusters, resulting in systems composed by *nearly decomposable systems* in the sense of Simon (1962).

This nearly decomposable architecture resembles that reported by Battiston et al. (2009) for describing credit networks: agents clustered in neighborhoods so that most of the time the action is at the local level, but with a few connections among neighborhoods that make the network sparse yet responsive to shocks hitting any participant. Such modularity has been documented to be a non-accidental feature that makes a system resilient due to its ability to isolate feedbacks and to limit cascades (Haldane and May, 2011; Kambhu et al., 2007). As most components or subsystems receive inputs from only a few of the other components, change can be isolated to local neighborhoods (Anderson, 1999).

Yet, as acknowledged by Assenza et al. (2011), Craig and von Peter (2014), Ravasz and Barabási (2003) and Dorogovtsev et al. (2002), scale-free connective structures are not hierarchical in nature. Indeed, tiered or intermediated structures, namely in the form of a modular architecture, are not distinctive features of scale-free networks. Therefore, a network simultaneously displaying a scale-free connective structure and a modular hierarchical organization pertain to a particular type of network architecture, a modular scale-free architecture (Barabási, 2003), which tends to be robust-yet-fragile and resilient.

Preliminary results show that analyzing complex systems as a network of coupled networks may alter the basic assumptions that network theory has relied on for single-layer networks (Kenett et al., 2014). The literature converges to non-additive and non-trivial effects arising from networks' coupling, with a remarkable finding: coupling scale-free distributions may yield a less robust network (Gao et al., 2012; Buldyrev et al., 2010), unless the number of links

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[12] The ubiquity of scale-free networks has been related to its robustness (Strogatz, 2003; Barabási, 2003) and to systems' self-organization (Barabási and Albert, 1999; Bak, 1996; Krugman, 1996) and adaptive features (Anderson, 1999; Holland, 1998). Economic and financial systems are particular cases of such structures and their corresponding features.

(i.e. the degree) of interdependent participants coincides across the layers. This signifies that the scale-free networks' robustness is likely to be preserved if *positively correlated multiplexity* exists, such that a high-degree vertex in one layer likely is high-degree in the other layers as well (Lee et al., 2014; Kenett et al., 2014). On the other hand, to the best of our knowledge, there are no studies on how networks' coupling affects their hierarchical modularity and their resilience.

The literature on the effects of coupling networks is still preliminary, and particular cases are scarce. Our case is particular for two reasons. First, unlike most studies on multi-layer financial networks, our system consists of two distinct types of participants, FIs and FMIs; therefore, our case is not a multiplex network but an interacting network. Some examples of non-multiplex multi-layer financial networks are Kenett et al. (2014), who couple layers composed by financial institutions and assets for studying the US banking system, whereas Fujiwara et al. (2009) analyze coupled layers of banks and non-financial firms for studying the structure of the Japanese credit network. Yet, to the best of our knowledge, networks composed by FIs and FMIs have not been examined in the literature.

Second, unlike most literature on multi-layer networks, one of the networks is not interconnected, and its participants depend on the other network for interacting. In our case FIs do not connect to each other due to the inevitable intervention of the FMIs' network as the plumbing or medium that allows settling financial transactions, as depicted in Figure 2. Gambuzza et al. (2014) examines and analyzes the main theoretical properties of such type of networks in physics (i.e. synchronization of oscillators), but no literature exists for financial networks.

3. Network analysis

A network, or graph, represents patterns of connections between the parts of a system. The most common mathematical representation of a network is the *adjacency matrix*. Let n represent the number of vertexes or participants, the adjacency matrix A is a square matrix of dimensions $n \times n$ with elements A_{ij} such that

$$A_{ij} = \begin{cases} 1 \text{ if there is an edge between vertexes } i \text{ and } j, \\ 0 \text{ otherwise.} \end{cases} \qquad (1)$$

A network defined by the adjacency matrix in (1) is referred as an undirected graph, where the existence of the (i, j) edge makes both vertexes i and j adjacent or connected, and where the direction of the link or edge is unimportant. However, the assumption of a reciprocal relation between vertexes is inconvenient for some networks. For instance, the deliveries of money between financial institutions constitute a graph where the character of sender and recipient is a particularly sensitive source of information for analytical purposes, in which the assumption of a reciprocal relation between both parties is unwarranted. Thus, the adjacency matrix of a directed network or *digraph* differs from the undirected case, with elements A_{ij} being referred as directed edges or arcs, such that

$$A_{ij} = \begin{cases} 1 \text{ if there is an edge from } i \text{ to } j, \\ 0 \text{ otherwise.} \end{cases} \qquad (2)$$

It may be useful to assign real numbers to the edges. These numbers may represent distance, frequency or value, in what is called a weighted network and its corresponding weighted adjacency matrix (W_{ij}). For a financial network, the weights could be the monetary value of the transaction or of the exposure.

Regarding the characteristics of the system and its elements, a set of concepts is commonly used. The simplest concept is the vertex *degree* (k_i), which corresponds to the number of edges connected to it. In directed graphs, where the adjacency matrix is non-symmetrical, *in degree* (k_i^{in}) and *out degree* (k_i^{out}) quantifies the number of incoming and outgoing edges, respectively (3); for undirected graphs, $k_i = k_i^{in} = k_i^{out}$.

$$k_i^{in} = \sum_{j=1}^{n} A_{ji} \qquad\qquad k_i^{out} = \sum_{j=1}^{n} A_{ij} \qquad (3)$$

In the weighted graph case the degree may be informative, yet inadequate for analyzing the network. *Strength* (s_i) measures the total weight of connections for a given vertex, which provides an assessment of the intensity of the interaction between participants. Akin to degree, in case of a directed graph *in strength* (s_i^{in}) and *out strength* (s_i^{out}) sum the weight of incoming and outgoing edges, respectively (4); for undirected graphs, $s_i = s_i^{in} = s_i^{out}$.

$$s_i^{in} = \sum_{j=1}^{n} W_{ji} \qquad\qquad s_i^{out} = \sum_{j=1}^{n} W_{ij} \qquad\qquad (4)$$

Some metrics enable us to determine the connective pattern of the graph. The simplest metric for approximating the connective pattern is *density* (d), which measures the cohesion of the network. The *density* of a graph with no self-edges is the ratio of the number of actual edges (m) to the maximum possible number of edges (5).

$$d = \frac{m}{n(n-1)} \qquad\qquad (5)$$

By construction, density is restricted to the $0 < d \le 1$ range. Formally, Newman (2010) states that a sufficiently large network for which the density (d) tends to a constant as n tends to infinity is said to be *dense*. In contrast, if the density tends to zero as n tends to infinity the network is said to be *sparse*. However, as one frequently works with non-sufficiently large networks, networks are commonly labelled as sparse when the density is much smaller than the upper limit ($d \ll 1$), and as dense when the density approximates the upper limit ($d \cong 1$). The term *complete network* is used when $d = 1$.

An informative alternative measure for density is the degree probability distribution ($\mathcal{P}_k$). This distribution provides a natural summary of the connectivity in the graph (Kolaczyk, 2009). Akin to density, the first moment of the distribution of degree (μ_k) measures the cohesion of the network, and is usually restricted to the $0 < \mu_k < n - 1$ range. A sparse graph has an average degree that is much smaller than the size of the graph ($\mu_k \ll n - 1$).

Most real-world networks display right-skewed distributions, in which the majority of vertexes are of very low degree, and few vertexes are of very high degree, hence the network is inhomogeneous. Such right-skewness of degree distributions of real-world networks has been documented to approximate a power-law distribution (Barabási and Albert, 1999). In traditional random networks, in contrast, all vertexes have approximately the same number of edges.[13]

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[13] Random networks correspond to those originally studied by Erdös and Rényi (1960), in which connections are homogeneously distributed between participants due to the assumption of exponentially decaying tail processes for the distribution of links –such as the Poisson distribution. This type of network, also labeled as "random" or "Poisson", was –explicitly or implicitly- the main assumption of most literature on networks before the seminal work of Barabási and Albert (1999) on scale-free networks.

The power-law (or Pareto-law) distribution suggests that the probability of observing a vertex with k edges obeys the potential functional form in (6), where z is an arbitrary constant, and γ is known as the *exponent* of the power-law.

$$\mathcal{P}_k \propto zk^{-\gamma} \tag{6}$$

Besides degree distributions approximating a power-law, other features have been identified as characteristic of real-world networks: (i) low mean geodesic distances; (ii) high clustering coefficients; and (iii) significant degree correlation, which we explain below.

Let g_{ij} be the *geodesic distance* (i.e. the shortest path in terms of number of edges) from vertex i to j. The mean geodesic distance for vertex i (ℓ_i) corresponds to the mean of g_{ij}, averaged over all reachable vertexes j in the network (Newman, 2010), as in (7). Respectively, the mean geodesic distance or average path length of a network (i.e. for all pairs of vertexes) is denoted as ℓ (without the subscript), and corresponds to the mean of ℓ_i over all vertexes. Consequently, the mean geodesic distance (ℓ) reflects the global structure; it measures how big the network is, it depends on the way the entire network is connected, and cannot be inferred from any local measurement (Strogatz, 2003).

$$\ell_i = \frac{1}{(n-1)} \sum_{j(\neq i)} g_{ij} \qquad\qquad \ell = \frac{1}{n} \sum_i \ell_i \tag{7}$$

The mean geodesic distance (ℓ) of random or Poisson networks is small, and increases slowly with the size of the network; therefore, as stressed by Albert and Barabási (2002), random graphs are small-world because in spite of their often large size, in most networks there is relatively a short path between any two vertexes. For random networks: $\ell \sim \ln n$ (Newman et al., 2006). This slow logarithmic increase with the size of the network coincides with the small-world effect (i.e. short average path lengths).

However, the mean geodesic distance for scale-free networks is smaller than $\ell \sim \ln n$. As reported by Cohen and Havlin (2010, 2003), scale-free networks with $2 < \gamma < 3$ tend to have a mean geodesic distance that behaves as $\ell \sim \ln\ln n$, whereas networks with $\gamma = 3$ yield $\ell \sim \ln n / (\ln\ln n)$, and $\ell \sim \ln n$ when $\gamma > 3$. For that reason, Cohen and Havlin (2010, 2003) state that scale-free networks can be regarded as a generalization of random networks with respect to the mean average geodesic distance, in which scale-free networks with $2 < \gamma < 3$ are "ultra-small".

The clustering coefficient (c) corresponds to the property of network transitivity. It measures the average probability that two neighbors of a vertex are themselves neighbors; the coefficient hence measures the frequency with which loops of length three (i.e. triangles) appear in the network (Newman, 2010). Let a *triangle* be a graph of three vertexes that is fully connected, and a *connected triple* be a graph of three vertexes with at least two connections, the calculation of the network's clustering coefficient is as follows: [14]

$$c = \frac{(\text{number of triangles in the network}) \times 3}{\text{number of connected triples}} \qquad (8)$$

Hence, by construction, clustering reflects the local structure. It depends only on the interconnectedness of a typical neighborhood, the inbreeding among vertexes tied to a common center, and thus it measures how incestuous the network is (Strogatz, 2003). Intuitively, in a random graph, the probability of a connection between two vertexes tends to be the same for all vertexes regardless of the existence of a common neighbor. Therefore, in the case of random graphs the clustering coefficient is expected to be low, and tends to zero - in the limit – for large random networks.

Contrarily, real-world complex networks tend to exhibit a large degree of clustering. Albert and Barabási (2002) report that in most –if not all- real networks the clustering coefficient is typically much larger than it is in a comparable random network. Accordingly, in inhomogeneous graphs, as those resulting from real-world networks, the probability of two neighbors of a vertex being themselves neighbors is reported to be in the 10% and 60% range (Newman, 2010). In this sense, scale-free networks combining particularly low mean geodesic distance and high clustering implies that the existence of a few too-connected vertexes plays a key role in bringing the other vertexes close to each other. It also indicates that the scale-free topology is more efficient in bringing the vertexes close than is the topology of random graphs (Albert and Barabási, 2002).

Besides displaying low mean geodesic distances and clustering, real-world graphs also display a non-negligible degree correlation between vertexes. They are characterized by either a

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[14] If three vertices (i.e. a, b, c) exist in a graph, a triangle exists when edges (a,b), (b,c) and (c,a) are present (i.e. the graph is complete), whereas a connected triple exists if at least two of these edges are present. In this sense, a triangle occurs when there is transitivity (i.e. two neighbors of a vertex are themselves neighbors). The factor of three in the numerator arises because each triangle is counted three times when the connected triplets are counted (Newman, 2010).

positive correlation, where high-degree (low-degree) vertexes tend to be connected to other high-degree (low-degree) vertexes, or a negative correlation, where high-degree vertexes tend to be connected to low-degree vertexes. Positive degree correlation, also known as *homophily* or *assortative mixing by degree*, results in the core-periphery structure typical of social networks, whereas negative degree correlation (i.e. *dissortative mixing by degree*) is typical of technological, informational, and biological networks, which display star-like features that do not usually have a core-periphery but have uniform structures (Newman, 2010). In contrast, the degree of random (i.e. homogeneous) networks tends to be uncorrelated.

Degree correlation may be measured by means of estimating the assortativity coefficient (Newman, 2010). As before, let m be the number of edges, the degree assortativity coefficient of a network (r_k) is estimated as follows:

$$r_k = \frac{\sum_{ij}(A_{ij} - k_i k_j / 2m) k_i k_j}{\sum_{ij}(k_i \delta_{ij} - k_i k_j / 2m) k_i k_j} \tag{9}$$

Where

$$\delta_{ij} = \begin{cases} 0 \text{ if } i \neq j \\ 1 \text{ if } i = j \end{cases}$$

However, it should be noted that the assortativity coefficient is not limited to vertexes' degree. Other characteristics of vertexes (e.g. age, income, gender, ethnics, size) may condition their tendency to be connected. In this case, the characteristics of connected vertexes may be correlated, which results in *assortative mixing by scalar characteristics* (Newman, 2010). For financial networks it is important to assess the intensity of the interaction between participants. As highlighted by Leung and Chau (2007) and Barrat et al. (2004), the inclusion of weights and their correlations may consistently change our view of the hierarchical and structural organization of the network. Based on (9), it is possible to estimate the *assortative mixing by strength* (10).

$$r_s = \frac{\sum_{ij}(A_{ij} - k_i k_j / 2m) s_i s_j}{\sum_{ij}(k_i \delta_{ij} - k_i k_j / 2m) s_i s_j} \tag{10}$$

Regarding the existence of hierarchies in networks, Simon (1962, p.468) suggests a narrow definition of "hierarchical system": "a system that is composed of interrelated subsystems, each of the latter being, in turn, hierarchic in structure until we reach some lowest level of elementary subsystem".

Some authors link the hierarchical structure of networks to the existence of communities or modules. For instance, Newman (2003) defines that a network displays community structures when groups of vertexes have a high density of edges within them and a lower density of edges between groups. Likewise, Barabási (2003) labels modularity in real-world networks as an architecture where the more connected a vertex is, the smaller is its clustering coefficient. Moreover, Barabási (2003) pinpoints that such low clustering from central vertexes contradicts the standard scale-free model.

Hence, in order to quantitatively measure the hierarchical modularity of a network Barabási (2003) suggests assessing whether (or not) the most connected vertexes display low local (i.e. individual) clustering, as real-world observed hierarchical modularity suggests. Newman (2010) defines local clustering as in (11):

$$c_i = \frac{(\text{number of pairs of neighbors of } i \text{ that are connected})}{(\text{number of pairs of neighbors of } i)} \tag{11}$$

If there is no dependence between degree and clustering (i.e. clustering is democratically distributed), then the network has no hierarchical modularity, as expected from both standard random and scale-free networks. However, if degree and clustering display an inverse relation (i.e. the higher the degree, the smaller the clustering coefficient), there is evidence of hierarchical modularity, in which central vertexes tend to be heavily connected to vertexes in their module and to other central vertexes in other modules, but not to non-central vertexes in other modules.

Barabási (2003) and Dorogovtsev et al. (2002) suggest that hierarchical modularity may be captured by fitting a power-law to the distribution of local clustering as a function of average degree (μ_k) (12):

$$\mathcal{P}_{c_i} \propto z\mu_k^{-\gamma} \tag{12}$$

Barabási (2003) highlights that the existence of hierarchical modularity in real-world networks is a defining feature of most complex systems, but it is not caused and may not be explained by the mere presence of scale-free properties. Consequently, because the standard scale-free model presumes the existence of a few central vertexes connected to vertexes in numerous modules (i.e. against the evidence of modularity in real-world networks), Barabási (2003) introduces a new type of network: a modular scale-free network. According to Barabási (2003) and Dorogovtsev et al. (2002), the clustering coefficient of a network and its local distribution by degree may confirm –or reject- the presence of a hierarchy within the system.

4. The data sets

Two main data sources have been used in the financial networks literature: (i) financial transactions (i.e. flows), and (ii) financial exposures (i.e. stocks). Networks of financial transactions correspond to the delivery of money, securities or currencies, or to the corresponding trades among financial institutions, which are automatically registered and safeguarded by FMIs (e.g. large-value payment systems, clearing houses, securities settlement systems, central securities depositories, trading platforms, trade repositories) whenever a transaction occurs. As highlighted by some authors (e.g. Uribe, 2011a,b; Kyriakopoulos et al., 2009), the information conveyed in financial transactions is particularly valuable due to its (i) granularity, with informative details such as sender, recipient, amount, type of transaction, underlying asset, etc.; (ii) completeness, because all financial transactions ineludibly involve the delivery of money or a financial asset, or a trade; (iii) reliability from a supervisory perspective because payments and settlements cannot be –easily- falsified; and (iv) opportunity, with data usually available in real-time (or with a minimal lag).

On the other hand, financial exposures ordinarily emerge from reports prepared and delivered by each financial firm to the corresponding authorities (e.g. financial statements), where the most commonly used for building financial networks are interbank credit and derivatives exposures. This type of information tends to be aggregated (i.e. details of individual exposures, counterparties, instruments, etc. are usually unavailable) and lagged, and its completeness, consistency, and validity depend on accounting practices by each

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financial firm and the corresponding jurisdiction.[15] Yet, as highlighted by Craig and von Peter (2014), because exposures do not cease to exist as payments do, they convey relevant information for financial stability purposes.

In order to analyze and understand the structure of the Colombian financial system three FMIs were selected as sources of financial transactions: the large-value payment system (CUD – *Cuentas de Depósito*), the sovereign securities settlement system (DCV – *Depósito Central de Valores*) and the spot market foreign exchange settlement system (CCDC – *Cámara de Compensación de Divisas de Colombia*). The rationale behind this selection follows four facts: first, these three FMIs account for 88.4% of the gross value of the payments and deliveries within the local financial market infrastructure during 2012 (Banco de la República, 2013); second, based on León and Pérez (2014), they are the three most systemically important local FMIs; third, the sovereign securities settlement system (DCV) and the foreign exchange settlement system (CCDC) provide detailed data for the two largest local financial markets (i.e. local sovereign securities and foreign exchange); and, fourth, the large-value payment system (CUD) provides aggregated data for all financial transactions occurring in the local market (i.e. from all financial market infrastructures). Therefore, this selection may be considered comprehensive and representative, yet parsimonious.

The dataset used for our research is unique, and particularly useful for extending the examination and understanding of financial markets' connective architecture. As emphasized by D'Souza et al. (2014), being able to use factual data related to the role of critical infrastructure networks has been elusive because they are independently owned and operated, with poor incentives for owners or operators to share data, and because the linkages between them are often only revealed during extreme events.

4.1. Financial institutions' monoplex and multiplex networks

The information obtained from these three FMIs serve the purpose of building three monoplex networks corresponding to three different environments or markets in which Colombian FIs interact: (i) sovereign securities market; (ii) foreign exchange market; (iii) other markets (i.e. equity, non-sovereign securities, derivatives, interbank funds). In that

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[15] Smith (2011) reports strong evidence of non-trivial debt masking in audited financial statements of Enron and Lehman Brothers prior to their failures. This confirms the lack of completeness, consistency, and validity of reported exposures as a rigorous source of information for financial network building.

order, they represent 75.7%, 3.4% and 20.9% of the payments in the local financial market.[16] The three monoplex networks are displayed in Figure 3.[17]

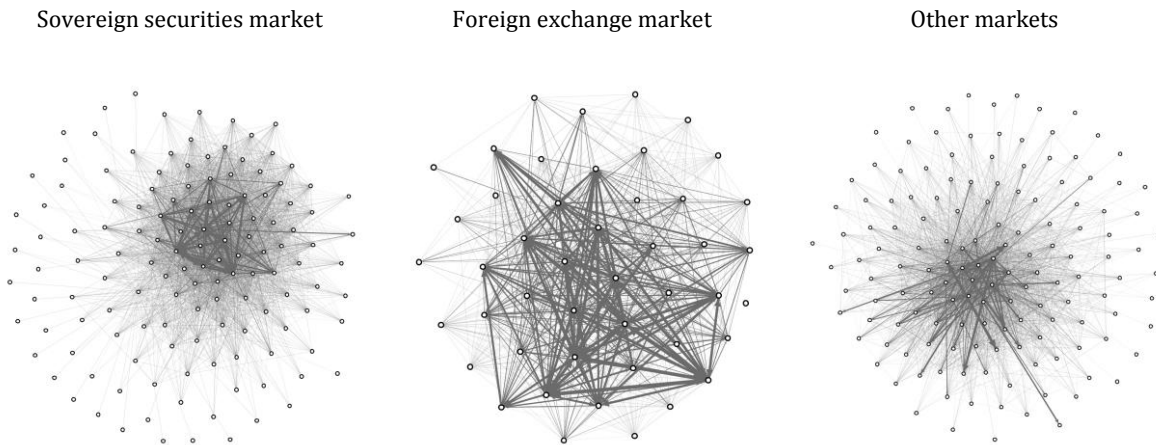| Sovereign securities market | Foreign exchange market | Other markets |

Figure 3. Monoplex networks. A vertex corresponds to a FI fulfilling its role in the corresponding network, whereas each arrow and its width represent the existence of a payment between FIs and its monetary value, respectively.

Each monoplex network is a weighted directed graph that accumulates payments made throughout 2012. Every single vertex in a particular monoplex network corresponds to a FI fulfilling its role in the corresponding network, whereas each arrow (i.e. arc) and its width represent the existence of a payment between FIs and its monetary value, respectively.

Aggregating the three monoplex networks of Figure 3 yields a network containing participants of one sort (i.e. FIs) but several kinds of edges (i.e. sovereign securities, foreign exchange, interbank, etc.). Therefore, according to Baxter et al. (2014), Figure 4 exhibits the Colombian financial multiplex network.

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[16] The construction of the networks to be analyzed here differs from that of León and Berndsen (2014). As their aim didn't include aggregating the three networks in a multiplex, León and Berndsen (2014) examined the networks corresponding to the three FMIs (i.e. CUD, DCV, CCDC) independently, disregarding that the CUD network contains data from DCV and CCDC. For the purpose of our paper, which includes building the multiplex by aggregating networks, it is imperative to work on non-overlapping networks. Accordingly, our results must differ from theirs.

[17] Yet, it is possible to successively decompose each single-layer network or monoplex into several single-layers. For example, the sovereign securities market monoplex network may be decomposed by type of transaction (e.g. buy/sell or repos), by term to maturity of the security, by origin of the transaction (e.g. over-the-counter or trading platform), etc. The existence of such interrelated subsystems is characteristic of complex systems (Simon, 1962) and a rough measure of their complexity (Casti, 1979).
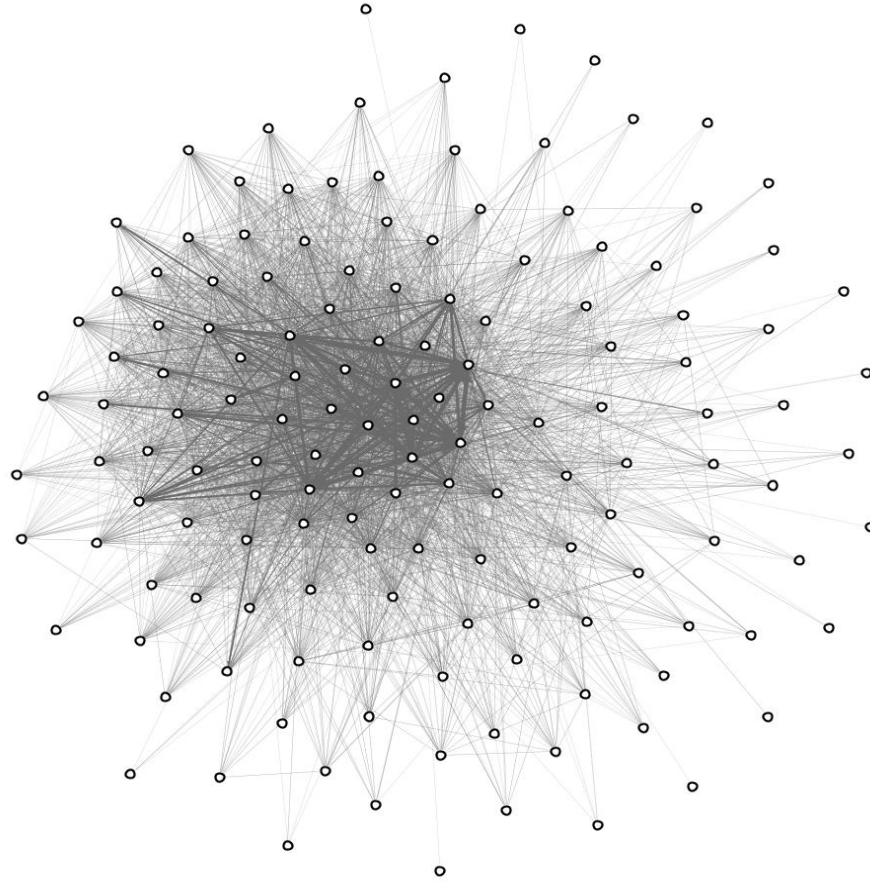
Figure 4. Multiplex network. Each vertex corresponds to a FI potentially fulfilling multiple roles in the Colombian financial system, whereas each arrow and its width represent the existence of a payment between FIs and its monetary value, respectively.

Unlike traditional monoplex financial networks that limit to a single market (e.g. interbank, derivatives), each vertex in Figure 4 corresponds to a FI potentially fulfilling multiple roles in the Colombian financial system. Hence, as the simultaneous presence of FIs across different markets is captured in the multiplex network, the potential effects of a FI failing across different markets (i.e. cross-system risk) are also considered.

4.2. Coupling financial institutions' and financial market infrastructures' networks

In the sense of Kurant and Thiran (2006), acknowledging the role of FMIs reveals that, from the settlement point of view, links between FIs are of a logical or virtual nature. Therefore, the FIs' and FMIs' interacting network consists of including the in-between role of FMIs for accomplishing the settlement of financial transactions. Accordingly, coupling FIs and FMIs entails breaking down each payment (i.e. each arrow) into its settlement constituents.

For example, the purchase of a sovereign security by FI_A from FI_B and the corresponding payment from FI_A from FI_B, which is customarily depicted as a single arrow from FI_A to FI_B, would consist of three different arrows[18]: (i) an arrow from FI_A to the sovereign securities system (DCV), corresponding to the order to buy the security;[19] (ii) an arrow from DCV to the large-value payment system (CUD), corresponding to an instruction to debit FI_A's money account and credit FI_B's money account after verifying the availability of securities and money; and (iii) an arrow from CUD to FI_B, corresponding to the payment received by FI_B for the sale. This breakdown corresponds to the typical involvement of FMIs in any payment in the local financial system, as described in Banco de la República (2013) and León and Pérez (2014).[20]

Breaking down all financial transactions in the multiplex network in Figure 4 yields the two-layer interacting network in Figure 5, in which the first (upper) and second (lower) layer corresponds to FIs and FMIs, respectively. The diameter of all vertexes is determined by their strength (i.e. the value of the ingoing and outgoing weighted connections). The number of links and the size of vertexes in each layer and across layers are particularly dissimilar, consistent with the inhomogeneous distribution of degree and strength of real-world networks.

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[18] Berndsen (2013) develops solutions to several types of settlement problems, including the case here depicted (i.e. a transaction involving two agents and two financial assets).

[19] Orders received by a settlement FMI usually arrive from other types of FMIs, such as trading or registering platforms. This additional layer is not considered in this paper; however, its role should not be underestimated.

[20] Alternatively, this same transaction could be examined from the securities' delivery point of view –instead of the payment's. However, this would be impractical because payments with central bank's money is the numeraire for all financial transactions in the local market; this is, all transactions in Colombia ultimately involve the delivery of local currency irrespective of the financial asset involved (e.g. securities, foreign exchange, derivatives, etc.).
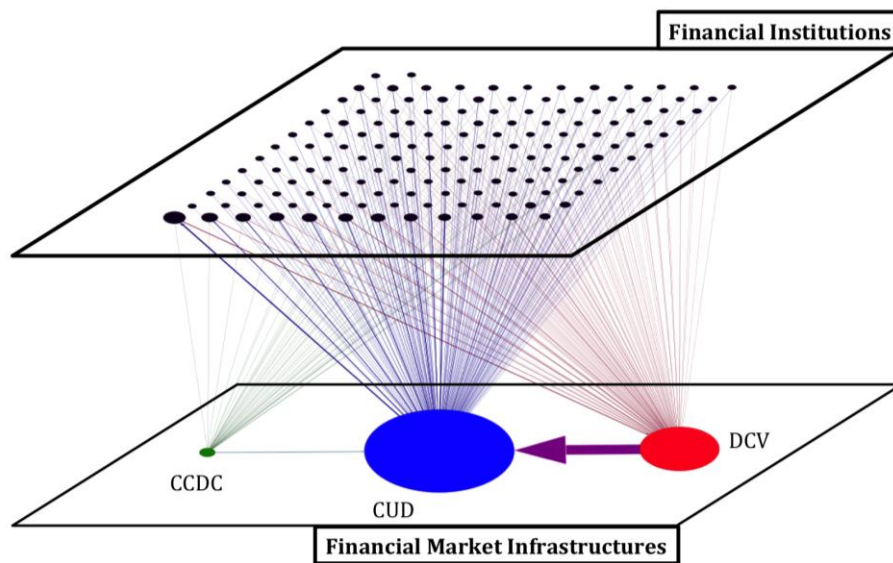
Figure 5. Interacting FIs' and FMIs' networks. Each vertex in the first (upper) layer corresponds to a FI potentially fulfilling multiple roles in the Colombian financial system, whereas each vertex in the second layer corresponds to a FMI. The diameter of all vertexes is determined by their strength (i.e. the value of the ingoing and outgoing weighted connections). Each arrow corresponds to a part of the settlement process.

There are no intra-layer connections in the first layer.[21] Indirect connections between FIs are possible through FMIs in the second layer. Also, due to its role as the FMI responsible for the money settlement of all financial transactions, the diameter of CUD is much greater than any other FMI or FI; the second largest vertex is DCV, consistent with the relevance of the sovereign securities market in the Colombian financial system. Moreover, following Kurant and Thiran (2006), it is evident that the failure of CUD in the second layer would destroy a substantial part of the first layer, rendering the whole system useless in practice.

5. Main results

Table 1 presents the main properties of the three monoplex networks and the multiplex exhibited in Figure 5 and 6, respectively. Statistics correspond to the estimated mean on the whole sample (January 3rd to December 28th 2012), with the expected values for random (i.e.

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[21] Under Colombian regulatory framework securities and foreign exchange must be settled in a FMI. Thus, correspondent banking in the form of intra-layer connections in the first layer of Figure 5 is limited to marginal transactions (e.g. settlement of checks between banks pertaining to the same conglomerate).

homogeneous) networks included –in brackets- when feasible. Using the sample mean for every statistic is rather safe because the sign and level of daily statistics is consistent along the whole sample.

Table 1
Basic statistics of the networks[a]

Statistic	Monoplex networks						Multiplex network	
	Sovereign securities market		Foreign exchange market		Other markets			
n	134		46		143		143	
d	0.04		0.24		0.05		0.07	
μ_k	5.19		10.66		6.44		9.75	
$\sigma_{k_{in/out}}$	8.60/8.51		8.47/8.43		10.23/10.17		13.41/13.44	
$\gamma_{k_{in/out}}$	3.12/3.01		3.83/3.89		2.22/2.06		2.81/2.49	
$\gamma_{s_{in/out}}$	1.77/1.75		3.76/3.69		2.22/2.20		1.96/1.99	
ℓ	2.24	[~4.9]	1.83	[~3.8]	2.34	[~5.0]	2.19	[~5.0]
c	0.15	[~0.0]	0.24	[~0.0]	0.11	[~0.0]	0.17	[~0.0]
c_w	0.17	[~0.0]	0.28	[~0.0]	0.20	[~0.0]	0.25	[~0.0]
$r_{k_{in/out}}$	0.31/0.32	[~0.0]	0.59/0.60	[~0.0]	0.23/0.22	[~0.0]	0.38/0.36	[~0.0]
$r_{s_{in/out}}$	0.19/0.20	[~0.0]	0.34/0.37	[~0.0]	0.12/0.09	[~0.0]	0.15/0.13	[~0.0]
γ_{c_i}	4.72		4.24		5.43		5.67	

This table shows that the basic statistics of the monoplex networks and the resulting multiplex approximate to those of a modular scale-free network. [a] Statistics presented are: number of vertexes (n); density (d); average degree (μ_k); in/out degree standard deviation ($\sigma_{k_{in/out}}$); in/out degree Power-law exponent ($\gamma_{k_{in/out}}$); in/out strength Power-law exponent ($\gamma_{s_{in/out}}$); mean geodesic distance (ℓ); clustering coefficient (c); degree correlation ($r_{k_{in/out}}$); strength correlation ($r_{s_{in/out}}$); local clustering power-law exponent (γ_{c_i}) Expected values for large random networks are reported in brackets.

About the three monoplex networks, several features typical of real-world networks are evident: First, they are sparse, with $d \ll 1$ and $\mu_k \ll n - 1$. Second, they are inhomogeneous, which is verified by degree's dispersion ($\sigma_{k_{in/out}}$) and approximate power-law distribution, with power-law exponents ($\gamma_{k_{in/out}}$) approaching typical values[22], hence consistent with the

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[22] Values in the range $2 \leq \gamma \leq 3$ are typical of scale-free networks, although values slightly outside it are possible and are observed occasionally (Newman, 2010). The exponents for the foreign exchange market display some departure from typical values, which may be due to the small number of participants. We use the algorithm

scale-free architecture of real-world networks. Third, the distribution of strength in the three monoplex networks approximates a power-law distribution. Fourth, agreeing with the scale-free features of the networks, the observed mean geodesic distances (ℓ) are much lower than the expected for random networks of the corresponding size, and they are consistent with the "ultra-small" characterization of Cohen and Havlin (2010, 2003). Fifth, there is evidence of positive degree correlation (i.e. *assortative mixing by degree,* $r_{k_{in/out}} > 0$) and positive strength correlation (i.e. *assortative mixing by strength,* $r_{s_{in/out}} > 0$), which suggests that high-degree (low-degree) vertexes have a larger probability to be connected to other high-degree vertexes (low-degree), and supports the presence of a core-periphery structure (Newman, 2010). Sixth, they display clustering coefficients (c and c_w) much larger than expected for random networks. Seventh, the distribution of local clustering coefficients as a function of average degree approximates a power-law distribution, which is consistent with the characterization of modular networks by Barabási (2003) and Dorogovtsev et al. (2002).

Accordingly, concurrent with León and Berndsen (2014), the three monoplex networks may be characterized as modular scale-free networks in the sense of Barabási (2003). Such characterization matches the description of credit networks as sparsely connected neighborhoods by Battiston et al. (2009). Furthermore, this characterization is consistent with real-world biological and social networks, and it also agrees with the existence of hierarchies and nearly decomposable systems. As said, the literature points out that the modular scale-free architecture is by no means accidental, but follows the organization of systems towards structures that favor systemic resilience and robustness.

The multiplex network preserves the modular scale-free architecture of its constituent monoplex networks. Based on the literature on multiplex networks, this feature should be related to the existence of positively correlated multiplexity, in which a vertex with large degree in one layer likely has more links in the other layer as well (Lee et al., 2014; Kennet et al., 2014). In our case, the correlation matrix estimated on FIs' degree and strength across the three networks (Figure 7) verifies that there is significant evidence of positively correlated multiplexity.

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developed by Clauset et al. (2009) for estimating the power-law exponents; this algorithm avoids several issues related to traditional estimation by ordinary least squares.

	k_{sov}	k_{fx}	k_{rest}	s_{sov}	s_{fx}	s_{rest}
k_{sov}	1					
k_{fx}	0.80	1				
k_{rest}	0.65	0.78	1			
s_{sov}				1		
s_{fx}				0.74	1	
s_{rest}				0.66	0.75	1

Figure 7. Monoplex networks' degree and strength correlation matrix. The correlation matrix was estimated based on the contribution of each FI to the total degree or strength in the whole samples.

Visual inspection of each FIs' degree and strength across the three monoplex networks is presented in Figure 8. Graphical inspection matches the numerical results in the correlation matrix presented in Figure 7: there is a clear tendency of high-degree and high-strength FIs overlapping across layers. Thus, Figures 7 and 8 together suggest that positively correlated multiplexity may explain why the modular scale-free features of the three monoplex networks is preserved in the resulting multiplex network.
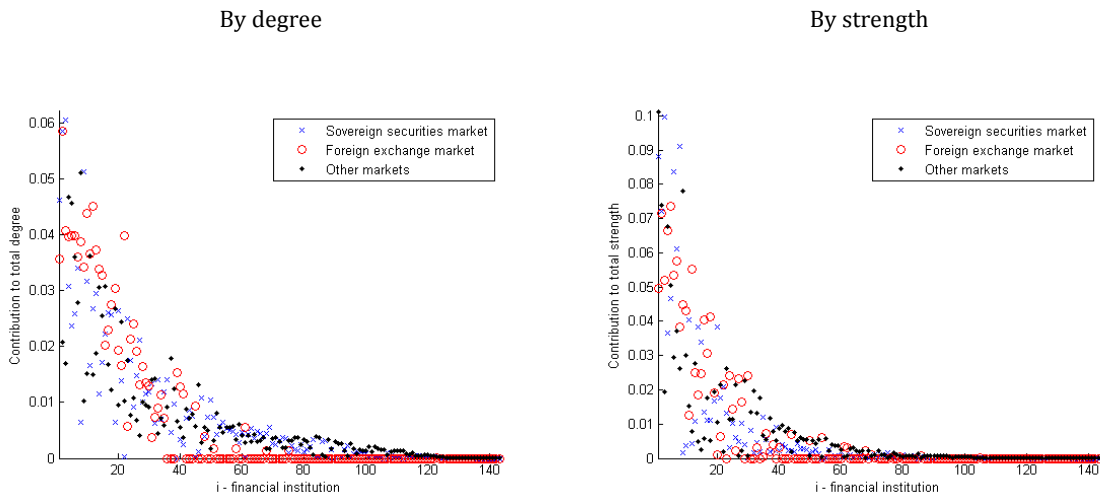

Figure 8. Positively correlated multiplexity. Participating FIs in each layer are ranked in decreasing order of degree (left) and strength (right) in the corresponding horizontal axis. High-degree and high-strength FIs in one layer tend to be the high-degree in the other two layers, which confirms the presence of positively correlated multiplexity.

Table 2 compares the main properties of the multiplex and the interacting networks. The multiplex corresponds to the network of FIs acting in different financial markets (Figure 4), whereas the interacting network incorporates the role of FMIs for the multiplex network (Figure 5). As before, the statistics correspond to the estimated mean on the whole sample (January 3rd to December 28th 2012), with the expected values for random (i.e. homogeneous) networks included –in brackets- when feasible.

Table 2

Basic statistics of the networks [a]

Statistic	Multiplex network		Interacting networks	
n	143		146	
d	0.07		0.02	
μ_k	9.75		2.57	
$\sigma_{k_{in/out}}$	13.41/13.44		13.04/9.75	
$\gamma_{k_{in/out}}$	2.81/2.49		3.42/3.36	
$\gamma_{s_{in/out}}$	1.96/1.99		1.78/1.77	
ℓ	2.19	[~5.0]	1.99	[~5.0]
c	0.17	[~0.0]	0.01	[~0.0]
c_w	0.25	[~0.0]	0.14	[~0.0]
$r_{k_{in/out}}$	0.38/0.36	[~0.0]	-0.43/-0.17	[~0.0]
$r_{s_{in/out}}$	0.15/0.13	[~0.0]	-0.31/-0.15	[~0.0]
γ_{c_i}	5.67		1.24×10^{14}	

This table shows that the basic statistics of the multiplex network approximate to those of a modular scale-free network, whereas the interacting network's to those of a scale-free network only. [a] Statistics presented are: number of vertexes (n); density (d); average degree (μ_k); in/out degree standard deviation ($\sigma_{k_{in/out}}$); in/out degree Power-law exponent ($\gamma_{k_{in/out}}$); in/out strength Power-law exponent ($\gamma_{s_{in/out}}$); mean geodesic distance (ℓ); clustering coefficient (c); degree correlation ($r_{k_{in/out}}$); strength correlation ($r_{s_{in/out}}$); local clustering power-law exponent (γ_{c_i}). Expected values for large random networks are reported in brackets.

Results reported in Table 2 verify that the FIs' and FMIs' interacting network is (i) sparse, with low density ($d \ll 1$) and low average degree ($\mu_k \ll n - 1$); (ii) inhomogeneous and scale-free, with the distribution of degree and strength being disperse ($\sigma_{k_{in/out}} > \mu_k$) and approximating a power-law ($2 < \gamma < 3$); and (iv) approximately "ultra-small" ($\ell \sim \ln\ln n$). These features are shared by the three monoplex and the multiplex networks.

However, there are three main differences between the architecture of the three monoplex networks, the multiplex network, and the FIs' and FMIs' interacting networks. First, degree correlation $r_{k_{in/out}}$ and strength correlation $r_{s_{in/out}}$ turned negative in the FIs' and FMIs' interacting network case. Second, clustering vanished as FMIs' role is considered. Third, as a byproduct of the lack of clustering, the distribution of local clustering (c_i) as a function of average degree is homogeneous and it does not distribute as a power-law, as exhibited in Figure 9. Therefore, based on Barabási (2003) and Dorogovtsev et al. (2002), the FIs' and FMIs' interacting network has no hierarchical modularity.
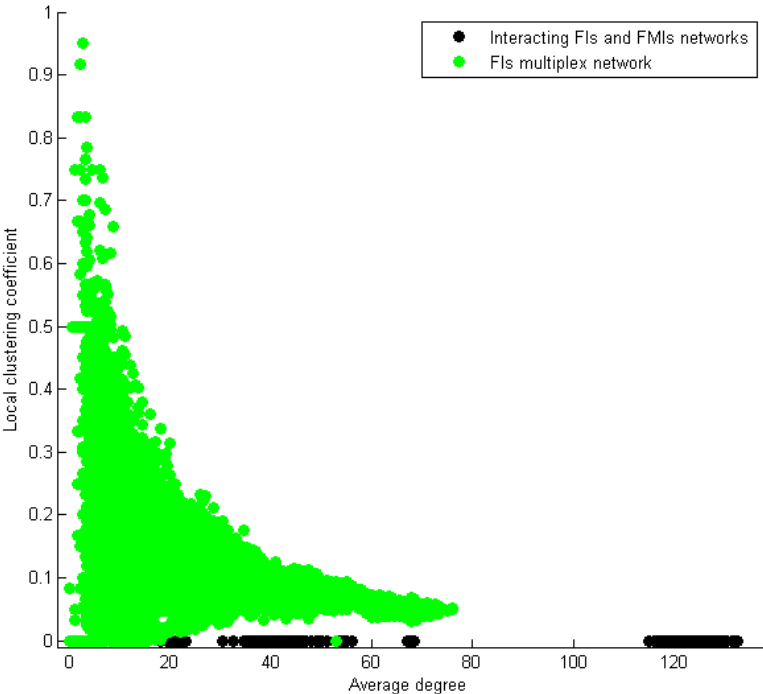

Figure 9. Distribution of local clustering as a function of average degree. Based on the entire data sample, there is evidence of an inverse relation between average degree and clustering for the FIs' multiplex, whereas such relation is absent in the FIs' and FMIs' interacting network.

All in all, comparing the numerical evidence between the FIs' and FMIs' interacting network and the FIs' monoplex and multiplex networks suggests that the modular scale-free architecture of FIs' networks fades as the role of FMIs is considered. Hence, including the next layer of complexity changed the architecture of the networks from displaying social network type features (e.g. assortative mixing, clustered, modular) to a network displaying technological networks' features (e.g. dissortative mixing) along with a non-clustered

connective structure. As discussed next, the lack of modularity results in the interacting network's limited ability to isolate feedbacks and limit cascades, with noteworthy implications for financial stability.

6. The critical role of financial market infrastructures in financial stability

The role of FMIs in financial stability is still an abstract issue. Despite the fact that financial authorities (i.e. supervisors, regulators) highlight the importance of FMIs, the literature has not addressed how the interaction between FIs and FMIs affects financial stability from the perspective of a rigorous quantitative framework. It is clear that the safe and efficient functioning of FMIs is critical for the functioning of financial markets (e.g. CPSS and IOSCO, 2012; Dudley, 2012; Bernanke, 2012), but FMIs are typically disregarded when examining and analyzing the structure of financial markets.

The evidence here reported confirms that ignoring the connective role of FMIs within financial networks may mislead the analysis of the connective architecture of financial systems. The main consequence of coupling FIs' and FMIs' networks is the removal of modular hierarchy, which invalidates the presumption of a financial architecture that favors systemic resilience by means of limiting cascades (Haldane and May, 2011) and isolating feedbacks (Kambhu et al., 2007).

The absence of modularity in the FIs' and FMIs' interacting network contradicts the existence of sparsely connected financial neighborhoods that keep most of FIs' actions at the local level (as in Battiston et al., 2009). In the absence of modularity, there are no subsystems of FIs, and they tend to receive inputs from all other FIs via FMIs, thus changes are not isolated and tend to spread across markets and their participants. As demonstrated by the failure of Bankhaus Herstatt in 1974, the technical problems of Fedwire (i.e. the U.S. large-value payment system) on October 20 1987, and Bank of New York's technological disruption in November 21 1985, a modest local shock may reverberate throughout financial markets by means of the

connections between FIs and FMIs[23]. These cases showed that payments and settlement systems could play a major role in causing or amplifying financial shocks (Davis, 1995).

Explicitly incorporating the FMIs' role in the settlement of FIs transactions shows that the local financial system is not a nearly decomposable system in the sense of Simon (1962). This finding concurs with the economics behind settlement FMIs: the centralized extinction of claims between FIs. FMIs provide an alternative to the frictions that arise when money and financial securities are traded directly (Manning et al., 2009); hence their purpose is to stand as central participants that ensure the efficient and safe flow of money and financial securities to all FIs. Therefore, it is not surprising that FMIs remove modularity and may act as conduits for widespread contagion.

Accordingly, the benefits of the modular scale-free architecture of FIs' networks are of a logical or virtual nature, and depend critically on the functioning of the plumbing provided by the network of FMIs. In this sense, the FMIs' network should be considered a critical infrastructure for the financial system, and its contribution to financial stability should not be underestimated.

Furthermore, not only the well-functioning of the FMI network determines the extent to which the benefits of the modular scale-free architecture of FIs' networks apply, but it also determines whether the settlement of financial transactions is carried out or not. For instance, it is most likely that the malfunction of the FMI responsible for the settlement of the cash leg of financial transactions (i.e. payments) impedes the proper functioning of all financial markets; this is, if the buyers are unable to pay for the financial assets they are purchasing, no transactions could be completed and financial markets would halt.

Eliminating the large-value payment system (CUD) or the sovereign securities settlement system (DCV) would certainly impede the functioning of Colombian financial markets, and would threat the stability of the local financial system. This concurs with the main findings of

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[23] On June 26th 1974 the failure of Bankhaus Herstatt, a small German bank (i.e. around 50,000 customers and DM 2.0 billion in assets) caused an overseas chain reaction that forced the U.S. Clearing House Interbank Payments System (CHIPS) to halt and the U.S. clearing banks to barter checks. As documented by Davis (1995), this local event resulted in the collapse of the U.S. payments system. On Tuesday 20 October, 1987, Fedwire, the U.S. large-value payment system, had to shut down the day after black Monday (i.e. U.S. stock market largest one-day decline), causing uncertainty and the injection of funds by the U.S. Federal Reserve (Manning et al., 2009). Bank of New York's technological disruption in November 21 1985, led the U.S. Federal Reserve to provide liquidity to avoid widespread financial difficulties (Davis, 1995; Manning et al., 2009).

León and Pérez (2014) regarding the systemic importance of CUD and DCV in the Colombian financial market, which results from their centrality and non-substitutability. Following Baxter et al. (2014), Kennet et al. (2014), and Kurant and Thiran (2006), the interdependencies between FIs' and FMIs' networks make the financial system more fragile: damage to one FMI (e.g. operational or financial) can trigger a catastrophic cascade of events that propagates across the global connectivity.[24] Moreover, due to the evidence of correlated multiplexity, the failure of one central FI may reverberate across financial markets by the linkages provided by FMIs, reinforcing the fragile nature (i.e. exposed to targeted attacks) of the examined financial system.

7. Final remarks

Our research constitutes a step forward in the examination and understanding of local financial markets' connective architecture, and goes beyond the study of single-layer financial institutions' (FIs') networks. Our results confirm that aggregating three single-layer FIs' networks preserves their modular scale-free architecture due to the evidence of positively correlated multiplexity. This finding is essential for financial stability. First, it reveals that central FIs tend to overlap across financial networks, thus their systemic importance may be even greater than envisaged by studying each network in isolation due to cross-system risk. Second, the evidence of modularity within a scale-free connective structure suggests that the network is robust-yet-fragile and resilient.

When we integrate the financial market institutions' (FMIs') network to the FIs' network, which gives a much more realistic picture consistent with FMIs' role in financial markets, we obtain a scale-free but non-modular architecture. This outcome is essential for financial stability as well. First, it stresses that the main benefit of modularity in FIs' networks, namely the resilience resulting from their ability to isolate feedbacks and isolate cascades, is dependent on the well-functioning of FMIs. Second, it emphasizes the relevance of infrastructure-related systemic risk, corresponding to the effects caused by the improper functioning of FMIs or by FMIs acting as conduits for contagion.

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[24] Most FMIs are liable to operational risk only (e.g. technological failure, human error, terrorism, natural disasters). However, some FMIs are also exposed to financial risk. For instance, the safe and efficient functioning of central counterparties depend on the market value, liquidity, and creditworthiness of the assets comprising the margins and other layers of protection against the default of a member.

Additional layers of complexity are readily available to expand our interacting network of FIs and FMIs. For instance, as acknowledged by D'Souza et al. (2014), global financial markets are increasingly intertwined and implicitly dependent on power and communication networks. Therefore, physical critical infrastructures are obvious candidates for examining the stability of financial systems from an operational perspective. As demonstrated by 9/11 terrorist attacks or 2011 Tohoku-Pacific Ocean Earthquake, the well-functioning of physical critical infrastructures should not be taken for granted.

Likewise, due to linkages between different countries' financial markets, multi-layer networks may not be limited by geographical or jurisdictional boundaries. For instance, the settlement of foreign exchange transactions in other countries' FIs or FMIs; local FIs with subsidiaries in other countries; local FIs being subsidiaries of foreign FIs; and the foreign component of investment portfolios, are some obvious examples of how local networks may be linked to other networks beyond national frontiers.

8. References

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