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*Publication date:*  
2014

*Document Version*  
Early version, also known as pre-print

[Link to publication in Tilburg University Research Portal](#)

*Citation for published version (APA):*

Okullo, S. J., Reynes, F., & Hofkes, M. W. (2014). *Modeling Peak Oil and the Geological Constraints on Oil Production*. (CentER Discussion Paper; Vol. 2014-036). Economics.

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No. 2014-036

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CONSTRAINTS ON OIL PRODUCTION**

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4 June, 2014

ISSN 0924-7815  
ISSN 2213-9532

# Modeling peak oil and the geological constraints on oil production

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## Abstract

We propose a model to reconcile the theory of inter-temporal non-renewable resource depletion with well-known stylized facts concerning the exploitation of exhaustible resources such as oil. Our approach introduces geological constraints into a Hotelling type extraction-exploration model. We show that such constraints, in combination with initially small reserves and strictly convex exploration costs, can coherently explain bell-shaped peaks in natural resource extraction and hence U-shapes in prices. As production increases, marginal profits (marginal revenues less marginal extraction cost) are observed to decline, while as production decreases, marginal profits rise at a positive rate that is not necessarily the rate of discount.

A numerical calibration of the model to the world oil market shows that geological constraints have the potential to substantially increase the future oil price. While some (small) non-OPEC producers are found to increase production in response to higher oil prices induced by the geological constraints, most (large) producers' production declines, leading to a lower peak level for global oil production.

*Keywords:* Peak oil, Hotelling rule, Exploration, Reserve development, Geological constraints

*JEL:* Q30, Q47, C61, C7

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## 1. Introduction

Nonrenewable resource models of the Hotelling (1931) tradition are powerful tools for examining how economic and physical variables may interact, influencing production and price profiles. The underlying feature characteristic of these models is the optimal inter-temporal depletion of a

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stock of non-renewable resource. The original non-renewable resource depletion model proposed by Hotelling (1931), which also still happens to be the standard economic model in the field, predicts that as a non-renewable resource is depleted, production (resp. price) declines (resp. rises) monotonically and the marginal profit rises at the rate of discount. These predictions, however, are at odds with, firstly the observed stylized facts regarding price and quantity paths that (i) regional and aggregate nonrenewable resource production profiles usually exhibit bell-shaped peaks, and (ii) nonrenewable resource prices tend to follow U-shapes. Secondly, the result that marginal profits rise at the rate of discount has so far received limited empirical support from the data despite researchers' best efforts (cf. Slade and Thille, 2009; Livernois, 2009; Krautkraemer, 1998; Chermak and Patrick, 2002).

While some Hotelling-type models proposed in the literature attempt to reconcile the theory with the outlined empirical facts (cf. Holland, 2008; Livernois and Uhler, 1987; Campbell, 1980; Slade, 1982; Pindyck, 1978), none consider the contribution of geological constraints<sup>1</sup>. At the same time, those that introduce geological constraints (e.g. Cairns and Davis, 2001; Nystad, 1987; Thompson, 2001) focus on the intensive margin, ignoring the extensive margin that is key to explaining those facts. As emphasized in the oil engineering literature (e.g. Ahmed, 2010; Arps, 1945), geological constraints, as dictated by physical reservoir characteristics and pressure within the reservoir, limit the amount of oil a producer can extract at any moment in time. To overcome these constraints, the producer might be induced to explore for new reserves at the extensive margin so as to increase production. Nonetheless, since resources are ultimately limited, aggregate production may increase only for a while before declining. Such a view of increasing and subsequently declining oil production, due to the impacts of geological constraints, is the backbone for curve-fitting peak oil models like the Hubbert (1962) model that predominate the technical literature (cf. Berg and Korte, 2008; Kaufmann, 1991; Pesaran and Samiei, 1995; Mohr and Evans, 2007). However, because production in such models traces a pre-specified mathematical curve, there is no straightforward means to introduce economic information. Consequently, such models are generally of limited use for understanding how economics and geological constraints can interact to shape production profiles.

The aim of this paper is to investigate how geological constraints for pressure produced resources integrated into a Hotelling-type model, can alter the producer's optimal extraction decision under divergent assumptions about reserve size and cost. We propose a model for the exploitation of the nonrenewable resource at both the intensive and extensive margin, that introduces a standard engineering representation of the geological constraint. Using this model we show, without relying on demand shifts, technical progress, cost reducing exploration, or endogenous spatial extraction and additions, that for a finite resource base, a bell-shaped peak in production and hence a U-shape in price occurs provided initial reserves are small, and exploration and reserve development costs

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<sup>1</sup>For oil and natural gas that are of main concern in this article, such constraints are the pressure within the reservoir and the reservoir characteristics such as the porosity and permeability of the producing rock.

are strictly convex. If, however, initial reserves are large, say because first period exploration was highly successful, production is found to decline monotonically and price to rise monotonically, as in the basic Hotelling model. The model also generates interesting implications for the marginal profit (marginal revenues less marginal extraction cost), which predominates in the literature as the relevant measure of scarcity. Our principal model predicts that rather than marginal profit rising monotonically at the rate of discount — as is the case with typical Hotelling models — it declines (resp. rises) as production rises (resp. falls)<sup>2</sup>. Notably, the inclusion of geological constraints into the Hotelling extraction-exploration model allows for the reconciliation of the classic theory of non-renewable resource extraction with the empirical facts of oil extraction.

The rest of this paper is organized as follows. The next section provides a review of the literature on Hotelling-type models that attempt to explain the stylized facts as well as a review of the literature on other economic models that introduce geological constraints. Section 3 presents our proposed model of geological constraints and derives its optimality conditions. Section 4 characterizes the model’s equilibrium, while section 5 examines and discusses extensions to the model. More specifically, impacts of (i) spatial extraction and endogenous field opening and (ii) reserve degradation are discussed. In order to quantify the impact of geological constraints on prices and production, section 6 presents comparative results from a numerical simulation using a global oil market model that accounts for the geological constraint and another model that does not, that is, a Hotelling version of the model. Over the period of interest, it appears that geological constraints induce higher oil prices and alter the production profile in favor of increased production from currently marginal producers: Brazil, other South and Central America (excluding Venezuela and Ecuador), and other smaller producers in the global oil market labeled “Rest of the World”. Section 7 concludes.

## 2. Explaining stylized facts: related literature

Our review covers both Hotelling type models that try to explain the stylized facts, and inter-temporal economic models that introduce geological constraints. The former allows us to identify the mechanisms those models rely upon, and thus isolate how such mechanisms might differ from those based on geological constraints. The latter places our model in a wider context in relation to the fact that the introduction of geological constraints into a Hotelling style model is not entirely new, but is a concept that is gaining increasing traction in the field. Surprisingly, we find that none of the models introducing geological constraints seeks to explain the observed stylized facts. The main reason for this is their focus is on the intensive margin, ignoring the extensive margin that is also key to resource exploitation.

To explain the observed stylized facts, Pindyck (1978) and Slade (1982) rely on reductions in production costs. Pindyck shows that if producers carry out exploration with the aim of finding

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<sup>2</sup>Whereas other papers indicate that the marginal profit may decline as production increases (see e.g., Farzin (1992); Cairns (2001); Arrow and Chang (1982)), none of these has explicitly shown how geological constraints might induce this outcome.

lower cost reserves, then production will exhibit a bell-shaped path (and prices a U-shaped path) if initial extraction costs are high. In this instance, the producer's optimal strategy is to first invest in exploration. Then, as new less costly reserves are found, production costs are lowered, in turn increasing production and reducing prices. Nonetheless, since low cost reserves get increasingly difficult to find, extraction costs soon start to rise, discovery effort declines, production declines, and in turn price rises. Extraction finally ceases when marginal profits on the last unit extracted are choked off. Similarly, Slade (1982) shows in a model without exploration that if the rate of exogenous technical progress is sufficiently high, lower future costs will entice producers to shift their production forward in time, causing production to peak (and prices to trough) later. In Slade's model, production starts to decline when the scarcity rent begins to grow faster than technical progress. Production eventually ceases when the resource gets ultimately exhausted.

Despite the relevance of Pindyck's model to the peak oil literature, the effects of reserve degradation as implemented in his extraction-exploration model are contestable since deposits of lower cost tend to be found and extracted first. Accordingly, Livernois and Uhler (1987) and Holland (2008) propose alternative extraction-exploration models that explain the stylized facts. Livernois and Uhler (1987) show that if producers explore for reserves in new locations (i.e. the extensive margin) so as to offset cost escalations in old deposits (i.e. the intensive margin) then production will initially increase, and hence prices will fall, if the number of new deposits is sufficiently high to offset cost escalations and production declines in old deposits. Similarly, Holland (2008) shows that if producers explore for new reserves at the extensive margin to offset depletion at the intensive margin, then production will follow a bell-shape and prices a U-shape, provided new deposits are large enough to offset production declines and rent escalations at the old deposits.

Yet another explanation of how bell shaped peaks in resource extraction can arise is given by Campbell (1980) and Cairns (1998, 2001). These authors consider the impact of capacity constraints. By building capacity from initially low levels, Cairns (2001) shows that producers will match increases in production to the increase in capacity. This happens up to the point where sufficient capacity is held, whereafter production remains constant for a while before declining, if capacity does not depreciate, or declines immediately, if capacity depreciates. Production as a consequence exhibits a bell shaped peak and for a defined stationary demand function, price in turn exhibits a U-shape.

Finally, another attempt at explaining the stylized facts is by Chapman (1993) who relies on demand side implications. In his model, a positive demand shift leads to an initial increase in production, but since resources are ultimately limited, and when the scarcity rent eventually grows faster than demand, production reaches its peak and declines thereafter. Unlike the previous models, price in Chapman's model rises monotonically over time.

Clearly, the above models provide various plausible mechanisms that can explain the observed stylized facts. The first four, Pindyck (1978), Livernois and Uhler (1987), Holland (2008) and Slade (1982) rely on cost reductions either at the intensive or extensive margin. The next model by Cairns (2001) relies on capacity constraints, and the last model by Chapman (1993) on a positive demand

shift. None of these models explain the peak on the basis of geological constraints. Yet, geological constraints as argued by Hubbert curve proponents are an important, if not the main determinant of crude oil production profiles. Unlike capacity constraints that bind only in the initial stages of resource extraction (Holland, 2003a; Campbell, 1980; Holland, 2003b; Cairns, 2001), geological constraints influence how oil and gas reserves are extracted at each point in time (Chermak et al., 1999; Ahmed, 2010). Accounting for geological constraints is therefore crucial for the understanding of production and price profiles.

Nystad (1987), Cairns and Davis (2001) and Thompson (2001) are prominent works that propose optimal inter-temporal depletion models that integrate geological constraints. Their analyses are, however, focused on the time path for the rent or initial production level than on explaining the observed stylized facts. More specifically, Nystad (1987) introduces geological constraints into a Hotelling model and examines how this influences the initial level of capacity installed. He finds that these constraints lead to a smaller initial capacity than that predicted by the Hotelling model. Production in his model declines monotonically. Cairns and Davis (2001) formalize Adelman's hypothesis that because of the impact of geological constraints, producers value reserves at about half the level implied by the Hotelling Valuation Principle (cf. Miller and Upton, 1985). Pressure rather than the stock of oil is the scarce resource in their model. Their model predicts declining production and marginal profit (average profit as they define it) that always rises, but can for a while do so below the rate of discount. Thompson (2001) by contrast focuses on an empirical examination of whether the classic Hotelling extraction model or one that embeds geological constraints empirically explains industry oil and gas extraction practice better. He finds support that production is always constrained, consistent with the impacts of geological constraints. None of these works, and to the best of our knowledge, none in the literature explore the role of geological constraints in explaining the observed stylized facts described above.

In our analysis, we introduce geological constraints into a Hotelling type model of reserve extraction and augmentation. Like Cairns and Davis (2001); Nystad (1987) and Thompson (2001), we use the exponential decline curve to approximate the impact of geological constraints. This tractable representation is standard among oil and gas engineers (Ahmed, 2010; Brandt, 2007; Leach et al., 2011). In principle, exponential decline implies that production at any point in time is proportional to a constant and fixed share of developed reserves. This constant is the depletion/decline rate, which is normally a function of investments sunk at the time of opening a reservoir, continuous investments to maintain or enhance the reservoirs productivity, and physical characteristics such as pressure within the reservoir and the parent rock in the reservoir. Consistent with Thompson's finding that producers usually extract at capacity, the noted depletion rate can be interpreted as the maximum depletion rate the producer can attain under its optimal extraction choice. In the next section we present our model of resource exploitation under the influence of geological constraints.

### 3. The model

Consider an oil producer,  $i$ , who owns an initial amount of undeveloped resources,  $S_{i0}$  ( $S_{i0} > 0$ ), and developed reserves,  $R_{i0}$  ( $R_{i0} \geq 0$ ), where both amounts are known with certainty. Due to exploration and development<sup>3</sup>, the undeveloped resources,  $S_{it}$ , decline by the amount of additions,  $x_{it}$ , made to reserves,  $R_{it}$ . Reserves in turn decline by the amount,  $q_{it}$ , that is extracted. It follows that at any time  $t$ , the producer's remaining reserves and resources can be obtained using the following equations, respectively:

$$\dot{R}_{it} = x_{it} - q_{it} \quad (1)$$

$$\dot{S}_{it} = -x_{it} \quad (2)$$

The first equation states that the rate of change in reserves for any period,  $t$ , will be given by the difference between additions and extraction. If additions are greater than extraction, reserves grow, if equal, they remain constant, or otherwise, they decline. The second equation gives the resource dynamics; it states that resources over time decline by the size of additions that are made to reserves. We assume well defined property rights both for reserves and resources.  $i$  can thus be seen as country-level producers, or as regional producers who have a priori secured rights to explore parcels of land, and have the subsequent right to extract what is found, such as is the case in the United States.

As mentioned in the previous section, development investment is normally carried out in order to initiate extraction (Smith, 1995). In our simple model, development investment can be seen as embedded in the geological constraint parameter that sets the maximum depletion rate. Let  $\gamma$  ( $0 < \gamma < 1$ ),<sup>4</sup> denote the maximum depletion rate per unit time. For tractability, we assume that this maximal rate of depletion is exogenous. In a more realistic model, this rate would be an endogenous function of initial and any subsequent investments to enhance reservoir productivity (cf. Adelman, 1990; Cairns and Davis, 2001; Ahmed, 2010; Smith, 1995). Leach et al. (2011) for instance model  $\gamma$  as a function of the amount of CO2 that is sequestered into a reservoir during the enhanced recovery phase. Introducing these detailed attributes, however, would complicate our analysis whereas our goal is to illustrate (without misrepresenting too severely) how geological constraints can influence production and price profiles.

The maximal extraction at  $t$  in terms of remaining reserves and the geological constraints parameter, for the exponential decline path,<sup>5</sup> is given as  $q_{it} = \gamma R_{it}$  (cf. Cairns and Davis, 2001;

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<sup>3</sup>We use the word exploration in the context of identifying reserves, rather than as a process of learning about the size and cost characteristics of the unexplored resource base. The latter would require the explicit treatment of uncertainty which is beyond the scope of the current paper.

<sup>4</sup>The case  $\gamma = 0$  is uninteresting since in that case production will always remain at zero. Similarly, the case  $\gamma = 1$  is uninteresting since it excludes the impact of geological constraints as we define them in this paper.

<sup>5</sup>Other observed production decline paths include the Harmonic and Hyperbolic (see Ahmed 2010, pp 1237 ). These are, however, analytically less tractable than the exponential decline production path because of their explicit dependence on time. They are therefore not discussed here.



Thompson, 2001; Nystad, 1987; Adelman, 1990).<sup>6</sup> For simplicity we assume that  $\gamma$  is identical across producers and generalize the constraint to read as:

$$q_{it} \leq \gamma R_{it} \quad (3)$$

The producer may extract under the geological constraint for example when past exploration and reserve development was highly successfully, leading to an abundance of developed reserves. This can happen when marginal exploration and development costs are constant in production. In practice,  $q_{it} < \gamma R_{it}$  can also mean that some capacity may have been shut in, or is being maintained/repared. When the constraint is binding,  $1/\gamma$  is the reserve to production ratio per unit time. Data from leading mature oil provinces such as the US and North Sea basins indicate that  $\gamma$  hardly ever exceeds 20 percent per annum (corresponding to a minimum reserve to production ratio of 5 years).

Let us assume an oligopolistic (possibly asymmetric) market, and define the producer's decision problem as one of finding optimal levels for extraction and additions<sup>7</sup> that maximize inter-temporal profit. For a producer  $i$ , we formally write the decision problem as:

$$\max_{\{q_{it}, x_{it}\}} \pi_i = \int_0^\infty \{ (P(Q_t) q_{it} - C(q_{it}) - W(x_{it})) e^{-\delta t} \} dt \quad (4)$$

where,  $P(Q_t)$ , for  $Q_t = \sum_i q_{it}$ , is the market price of oil.  $C(q_{it})$  is the cost of extraction, and  $W(x_{it})$  the cost of building the reserve base, that in a more formal model would be split into the cost of development investment and the cost of exploration. To capture the tendency for costs to increase at an increasing rate with production or additions, we assume strict convexity in both extraction and additions, i.e.,  $C_q(q_{it}), C_{qq}(q_{it}), W_x(x_{it}), W_{xx}(x_{it}) > 0$ , unless otherwise stated. For now, costs are assumed to be independent of reserve and resource degradation; in subsection 5.2, however, this assumption is relaxed.

Defining revenues as,  $\mathcal{R}(q_{it}, P(Q_t)) = P(Q_t) q_{it}$ , we have by assumption that  $\mathcal{R}_q(q_{it}, P(Q_t)) > 0$  and  $\mathcal{R}_{qq}(q_{it}, P(Q_t)) \leq 0$ . The condition,  $\mathcal{R}_q(q_{it}, P(Q_t)) > 0$ , requires marginal revenues to be positive, while,  $\mathcal{R}_{qq}(q_{it}, P(Q_t)) \leq 0$ , requires marginal revenues to be non-increasing in the producer's production. We proceed with the analysis at a general level allowing for either  $\mathcal{R}_{qq}(q_{it}, P(Q_t)) < 0$ , or  $\mathcal{R}_{qq}(q_{it}, P(Q_t)) = 0$ , or both in the case of a price leadership model such as the cartel-fringe model of Salant (1976). In the rest of the analysis we drop the producer indices, where no confusion may arise.

The objective function (4) is maximized subject to Equations (1), (2), and (3), including non-

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<sup>6</sup>Defining  $\tau > t$ , it follows that in the absence of new discoveries, future extraction levels can be assessed using:  $q_{i\tau} = q_{it} e^{-\gamma(\tau-t)}$ , where  $q_{it}$  is extraction at an initial date  $t$ . Reserves in terms of production follow as:  $R_{it} = \int_t^\infty q_{it} e^{-\gamma(\tau-t)} d\tau = \frac{q_{it}}{\gamma}$ . That is,  $q_{it} = \gamma R_{it}$ .

<sup>7</sup>The assumption that one can select the optimal level of additions is a simplification. In reality, producers select optimal effort levels,  $E_t$ , and then additions arise as an interaction between the discovery factor,  $y_t$ , and the effort undertaken (e.g. as in Jakobsson et al. (2011)). Additions in this instance would be given by  $x_t = E_t y_t$ , where  $y_t$  could potentially follow a stochastic process. In our case however, we assume that  $y_t = 1$  and there is no uncertainty. This in turn dictates a one-one relationship between additions and effort, and simplifies the model.

negativity constraints on production, additions, reserves and resources. Because the objective function is set up to be strictly concave and the constraint set convex, the optimum to the defined problem exists and is unique (Léonard and Long, 1992, pp. 210-214, 288-289). To find the optimal solution to the problem (4) and its associated constraints, the current value Lagrangian to the producer's problem is formulated as:

$$\mathcal{L} = \mathcal{R}(q_t, P(Q_t)) - C(q_t) - W(x_t) + \nu_t(x_t - q_t) - \lambda_t x_t + \mu_t(\gamma R_t - q_t) \quad (5)$$

The first order optimality conditions and adjoint equations are:

$$\mathcal{R}_q(q_t, P(Q_t)) - C_q(q_t) - \nu_t - \mu_t \leq 0 \quad \perp \quad q_t \geq 0 \quad (6)$$

$$-W_x(x_t) + \nu_t - \lambda_t \leq 0 \quad \perp \quad x_t \geq 0 \quad (7)$$

$$\dot{\nu}_t - \delta\nu_t + \gamma\mu_t \leq 0 \quad \perp \quad R_t \geq 0 \quad (8)$$

$$\dot{\lambda}_t - \delta\lambda_t \leq 0 \quad \perp \quad S_t \geq 0 \quad (9)$$

$$\mu_t(\gamma R_t - q_t) = 0 \quad (10)$$

where  $\perp$  denotes the orthogonality condition for complementary slackness, and  $\lambda_t, \mu_t, \nu_t \geq 0$  are shadow prices.  $\nu_t$  is the shadow price on reserves,  $\mu_t$  the shadow price on the geologically extractable reserve base, and  $\lambda_t$  is the shadow price on the resource base.

Defining  $CSR_t = \nu_t + \mu_t$  as the composite scarcity rent, (6) states that whenever marginal profit is less than  $CSR_t$ , there will be no extraction. (7) gives the optimal size for additions. It requires that additions are determined such that marginal exploration and development costs,  $W_x(x_t)$ , equal the net marginal benefit from reserve additions ( $\nu_t - \lambda_t$ ). (8) and (9) give the dynamic efficiency conditions for  $\nu_t$  and  $\lambda_t$ , respectively. Observe that  $\nu_t$  grows (resp. declines) when the shadow price on the geological constraint,  $\mu_t$ , is small (resp. large), that is, when reserves are abundant (resp. scarce). When the geological constraint is slack,  $\nu_t$  and  $\lambda_t$  both rise at the rate of discount, giving the standard Hotelling result that the marginal profit rises at the rate of discount.

(10) is the complementary slackness condition on geologically constrained extraction. One of the following must hold at every instant: i) if  $\mu_t > 0$ , then  $q_t = \gamma R_t$ , otherwise, ii) if  $\mu_t = 0$ , then  $q_t \leq \gamma R_t$ . In the first case, the reserve base is generally small relative to the desired extraction level, hence extraction is bound to the geological limit. In this case  $\mu_t$  is negatively related to  $R_t$ ,  $\frac{d\mu}{dR} < 0$ .<sup>8</sup> If reserves are abundant relative to the desired extraction level, production could lie

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<sup>8</sup>Let  $V^*(R_t, S_t) = \max_{\{x_{it}\}} \int_0^\infty \{ (P(Q_t) \gamma R_{it} - C(\gamma R_{it}) - W(x_{it})) e^{-\delta t} \} dt$  denote the maximum value function associated to the problem in Equations (1) to (4). Since the extractable reserve base is taken as exogenous when setting production for any period  $t$ , by the envelope theorem:  $\frac{dV^*}{dR} = \frac{\partial \mathcal{L}}{\partial R} = \gamma \mu_t$  (cf. Léonard and Long, 1992, pp. 36). Taking a second total derivative with respect to  $R$ , we have that at the optimum  $\frac{d\mu}{dR} = \frac{1}{\gamma} \frac{d^2 V^*}{dR^2} < 0$ , because of the concavity of the value function in both  $q_t$ , and  $R_t$ .

below the geological bound and the geological constraint would command a zero shadow price.

On an interval where both extraction and additions are positive, the rent from resource and reserve depletion can be obtained by substituting (7) in (6) to give:

$$\mathcal{R}_q(q_t, P(Q_t)) - C_q(q_t) - W_x(x_t) = \lambda_t + \mu_t \quad (11)$$

Note that  $\lambda_t$  grows at the rate of interest, whereas  $\mu_t$  changes instantaneously depending on the size of extractable reserves. Equation (6) and (11) both define marginal profit, but in different ways. The first, simply as marginal revenue less marginal extraction costs, while the second additionally subtracts marginal exploration costs. For the sake of comparison with the classic Hotelling (1931) model, and since we are primarily interested in the time path for extraction, it will prove more convenient to use the definition of Equation (6), i.e. simply as marginal revenues less marginal extraction costs.

Terminal constraints associated with the above defined problem are: (i)  $\lim_{t \rightarrow \infty} e^{-\delta t} \lambda_t \geq 0$ ,  $\lim_{t \rightarrow \infty} e^{-\delta t} \lambda_t S_t = 0$  and (ii)  $\lim_{t \rightarrow \infty} e^{-\delta t} \nu_t \geq 0$ ,  $\lim_{t \rightarrow \infty} e^{-\delta t} \nu_t R_t = 0$ . Note that  $\lim_{t \rightarrow \infty} e^{-\delta t} \lambda_t S_t = 0$  holds with complementary slackness, as does  $\lim_{t \rightarrow \infty} e^{-\delta t} \nu_t R_t = 0$ . For positive additions at any point on  $t$ ,  $\lambda_0 > 0$ . Thus as  $t \rightarrow \infty$ ,  $\lambda_t \rightarrow \infty$ . Accordingly, the resource base must eventually become physically depleted. Similarly, if reserves were to get exhausted, then  $\lim_{t \rightarrow \infty} e^{-\delta t} \nu_t > 0$ . However, when the geological constraint is binding ( $\mu_t > 0$ ), we see from the restriction  $\lim_{t \rightarrow \infty} \frac{Q_t}{R_t} = \gamma$ , that reserves will not get exhausted. Thus, since  $\lim_{t \rightarrow \infty} R_t > 0$ , we shall have that  $\lim_{t \rightarrow \infty} e^{-\delta t} \nu_t = 0$ . Indeed, this is usually the case in practice since with increased depletion, the amount of oil that can be extracted from a reservoir becomes too small to be of any economic value.

#### 4. Equilibrium dynamics

As noted by Pickering (2008) and Thompson (2001) there is overwhelming support that extraction normally takes place at the geological bound for most producing reservoirs and regions, i.e.,  $q_t = \gamma R_t$ . Moreover, as we also show in Appendix A, such an extraction profile is consistent with an inter-temporally efficient process of reserve additions. In our analysis we therefore focus only on the geological constraints equilibrium.

Solving the first-order conditions for dynamic changes in additions and production gives:

$$\dot{x}_t = \frac{\delta W_x(x_t) - \gamma \mu_t}{W_{xx}(x_t)} \quad (12)$$

$$\dot{q}_t = \gamma(x_t - \gamma R_t) \quad (13)$$

Equation (12) is obtained by solving for time derivatives of (7) with  $\dot{\nu}_t$  substituted for (8),  $\dot{\lambda}_t$  for (9), and for  $W_x(x_t)$  using (7) itself. Equation (13) is obtained by simply differentiating  $q_t = \gamma R_t$  with respect to time and substituting for  $\dot{R}_t$  using (1).

From Equation (12), additions either grow or decline depending on the shadow price for the geological rate of extraction flows,  $\mu_t$ . In case of high short-term scarcity — i.e., when  $\mu_t \gg 0$  — additions will decline; this occurs when the reserve base is small: definitely during the final time periods of resource exploitation and in some cases in the initial time periods. By contrast, for low short term scarcity, the rate of decline in additions slows down and additions can grow if reserves are relatively large.

Equation (13) dictates the relationship between changes in production, the level of additions, and the size of the extractable reserve base. For sufficiently large (resp. small) additions relative to extraction, production increases (resp. declines). As shown in Appendix B, production declines monotonically if marginal exploration and development costs are constant since in such instances the producer can easily amass large reserves in the initial period(s). With strictly convex exploration costs, however, producers build up reserves gradually since large initial additions are constrained by marginal exploration and development cost escalations.

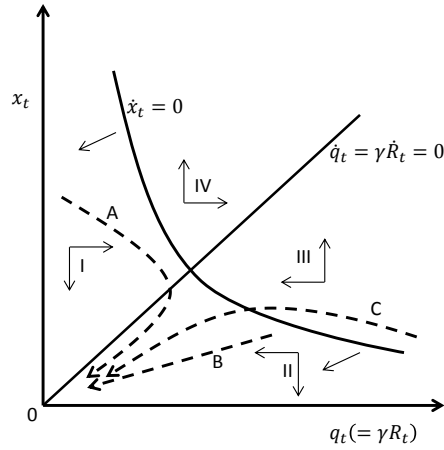


Figure 1: Equilibrium for additions against reserves.

To get a better understanding of the dynamics of the system represented by equations (12) and 13, we draw the phase portrait Figure 1. Because Equation (12) contains an endogenous variable,  $\mu_t$ , we must infer its behavior at the optimum. Setting  $\dot{x}_t = 0$  in (12) and substituting for  $\mu_t$  using Equation (11) gives:

$$\left(\frac{\gamma + \delta}{\gamma}\right) W_x(x_t) = (\mathcal{R}_q(q_t, P(Q_t)) - C_q(q_t) - \lambda_t) \quad (14)$$

We infer that the isocline  $\dot{x}_t = 0$  is downward sloping, and shifts inwards overtime since  $\dot{\lambda}_t = \delta \lambda_t > 0$ . To see that the isocline is downward sloping, let us for a moment assume that  $\lambda_t$  is fixed; by Equation (14), if production is low then additions are high establishing the downward sloping nature of the isocline. The curvature of the isocline depends on whether  $W_{xx}(x_t)$  is constant or changes in  $x_t$ . Our Figure assumes the latter. Along the isocline,  $\dot{x}_t = 0$ , additions remain constant; above it additions grow, while below it additions decline. The isocline for production is

straightforward to draw. From  $q_t = \gamma R_t$ , we see that  $\dot{q}_t = \gamma \dot{R}_t = 0 \Rightarrow x_t = q_t$ , a 45 degree line. Above the isocline production grows, while below it production declines.

The phase portrait is divided into four regions that each impose unique dynamics on the path for production and additions as indicated by the sets of perpendicular arrows. Paths that start in Region I or III will make their way into either region II or IV. Paths that have entered regions II and IV or that commence in these regions will not leave; these regions are the terminal quadrants. For a nonrenewable resource that exists in finite quantities, production and hence reserves ultimately decline, making terminal region II (and not terminal region IV) of most interest to the task at hand. It is the stable quadrant.

We see in the figure and in reference to path “A,” that production will exhibit a peak if initial reserves are small and initial additions are relatively large. This is the standard case that is normally observed over the lifetime of a resource. Large initial resources, relative to the initial reserve base, allow producers to make comparatively large additions that lead to reserve growth and hence initially increasing extraction. As additions continue to exceed extraction for a while, reserves grow, in turn increasing production. Eventually a point is reached where additions are just equivalent to production (on the isocline  $\dot{q}_t = 0$ ), where production peaks. Observe from the figure that additions attain a peak before production, as predicted by Hubbert (1962) and proponents. Because path “A” initially moves to the right, while the isocline  $\dot{x}_t = 0$  shifts leftwards, it is possible to witness a considerable slow-down in the decline rate for additions made to reserves. Production can exhibit either a sharp or flat peak before declining towards zero additions and zero reserves.

If on the other hand initial reserves are relatively large and initial resources are sufficiently small so as to limit the size of initial additions, additions, production, and hence reserves can decline monotonically as in path “B”. For considerably larger initial reserves, additions rise initially before declining, while production declines monotonically as indicated by path “C.” Additions initially rise because the net marginal value for new reserves,  $\nu_t - \lambda_t$ , is initially increasing.<sup>9</sup> Cases of large initial reserves can arise for example when vast amounts of the resource being exploited require minimal to no prospecting and have low development costs. For oil, this pattern typically arises at the pool level, rather than at an aggregate level. In the instances where production declines monotonically (paths “B” and “C”), the shadow price on the geological constraint rises monotonically because of increasingly scarce reserves, that is, new reserves can not offset depletion.

For constant marginal extraction costs ( $C_{qq}(q_t) = 0$ ), production can still follow paths “A,” “B,” or “C.” The difference with the case of increasing marginal extraction costs is that production rises to peak much faster, peaks at a higher level, and declines faster from peak. To see this, note that the production level is determined by the relation  $q_t = \gamma R_t$ , and that the dynamic Equations (14) and (13) still hold for constant marginal extraction costs as well. Marginal extraction costs

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<sup>9</sup>If marginal exploration costs are constant, producers following path “C,” can postpone additions entirely until when the cost of additions matches the discounted price for new reserves, that is, when  $\delta W_x(x_t) = \gamma \mu_t$  (refer to Appendix B).

therefore merely act to either increase or reduce the level of extraction at any particular point in time. Given that producers discount the future, they prefer to extract as soon as possible. And since extraction is directly increasing in the level of reserves, this directly translates into an urgency to build up reserves. With increasing marginal extraction costs, holding increasingly larger reserves so as to speed up extraction, leads to larger cost escalations from extraction. By contrast, constant marginal extraction costs allow producers to hold such (larger) reserves sooner, and hence extract more initially, without the cost escalations.

The above discussion leads to the following proposition.

**Proposition.** *For strictly convex exploration costs, and a finite resource base, one of the following must hold in a solution corresponding to the unique optimum:*

*i) Production grows to peak before declining. In this instance initial reserves must be small and initial additions sufficiently large.*

*ii) Production peaks in the initial period. Here initial reserves are relatively large compared to the initial resource endowment.*

*Proof.* Refer to Figure 1. For an explicit proof on production exhibiting a peak see Appendix C. As a special case of ii), with initially no resources, but with positive reserves, it is straight forward that production declines monotonically.  $\square$

**Corollary.** *When marginal extraction costs are non-increasing in production, production peaks at a higher level and in the case of a bell shaped production path, at an earlier date, as compared to the case where marginal extraction costs increase in production.*

What happens to marginal profit (marginal revenues less marginal extraction costs) as production increases or decreases? By taking time derivatives of Equation (6) we have that:

$$\dot{q}_t = \frac{C\dot{S}R_t}{(\mathcal{R}_{qq}(q_t, P(Q_t)) - C_{qq}(q_t))} \quad (15)$$

The denominator on the RHS has a negative sign, meaning that  $C\dot{S}R_t$  takes the opposite sign of  $\dot{q}_t$ . Thus, the time path for marginal profit, i.e., the composite scarcity rent  $C\dot{S}R_t$ , mirrors that followed by production. As production rises, marginal profits decline and as production declines marginal profits rise. Producers allocate extraction in line with changes in short-term scarcity of reserves. This result is in contrast to models of Slade (1982); Livernois and Uhler (1987); Chapman (1993); Holland (2008) that explain bell shaped peaks but cannot explain why marginal profits could fail to rise at (or close to) the rate of interest. Moreover, the results are also in contrast to the basic Hotelling extraction-exploration model<sup>10</sup> that explains neither bell-shaped extraction paths, nor the U-shaped prices, nor possibly declining marginal profits. Marginal profit in our proposed geological constraints model declines (resp. increases) because  $\mu_t$  declines (resp. increases) with

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<sup>10</sup>By basic Hotelling extraction-exploration model we refer to the model proposed in Section 3, but without the geological constraint on extraction. For such a model, we can show that production declines monotonically, additions rise monotonically, and marginal profit rises at the rate of discount. Additions rise monotonically, because the net marginal value for new reserves,  $\nu_t - \lambda_t$ , rises indefinitely. Additions only cease when the resource base is exhausted. Note that additions will jump down from their highest observed value to zero following the exhaustion of the resource base.

the growing (resp. declining) reserve base. As short-term scarcity declines because of reserve build-up, producers are more willing to increase production. On the contrary, as it increases with reserve draw-down, producers cut back on extraction.

## 5. Extensions

In this section, we extend the principle model in order to examine: (i) what happens when the model accounts for spatial exploration and extraction and (ii) how the time path for production and additions is altered when the model accounts for increasing extraction costs due to the depletion of high quality reserves. Here, we also restrict the analysis to the case of binding geological constraints. While these extensions do make the principal model more realistic but also more complex, we find that the main result about geological constraints inducing bell-shaped paths for production, U-shaped paths for prices, and U-shaped paths for marginal profits still hold.

### 5.1. Spatial extraction and exploration

In this case, we assume that producers have a fixed total land mass to explore. The producers' problem then becomes one of demarcating the optimal land/field sizes to be commissioned for exploration during each time period, optimal exploration to be made in demarcated fields during each time period, and optimal extraction to be carried out in those developed fields. Note that for the demarcated fields, producers commit to an exploration program that can entail the exploration of several other fields simultaneously. Infact, convex exploration costs in each field imply that it would be more profitable to spread such costs over several fields at any point in time. Similarly so for extraction, that is, extraction can take place over several fields simulatenously.

For such a program, it is possible to show that the optimal field size demarcated for exploration in every other period is declining over time (Holland, 2008). It also follows straightforwardly that for  $S_0^j$  representing any single field, the analytical results derived in the preceding section are generalizable over each field  $j$ . As illustrated using numerical simulations<sup>11</sup> (see: Figure 2), this implies that the life cycle of an extraction-exploration process for the described disaggregate model is as follows: (i) aggregate production exhibits a bell-shaped peak, (ii) marginal profits follow a U-shaped path that is symmetrically opposite to that followed by production, (iii) prices exhibit a U-shape, (iv) additions (sum of new discoveries) increase and then later decline, and (v) the optimal land size demarcated for exploration declines monotonically over time. Following the analysis in the previous section, (vi) production from each field exhibits a bell shaped production peak since initial reserves are zero for each field.

Additions increase because discoveries from new fields initially offset declines in discoveries in old fields. However, as fields for exploration become ever limited, additions soon reach a peak and then start to decline when new fields cannot offset declines in discoveries in old fields. This

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<sup>11</sup>The authors thank Stephen Holland for making available the Stata code used as a basis for developing the GAMS code used for this simulation.

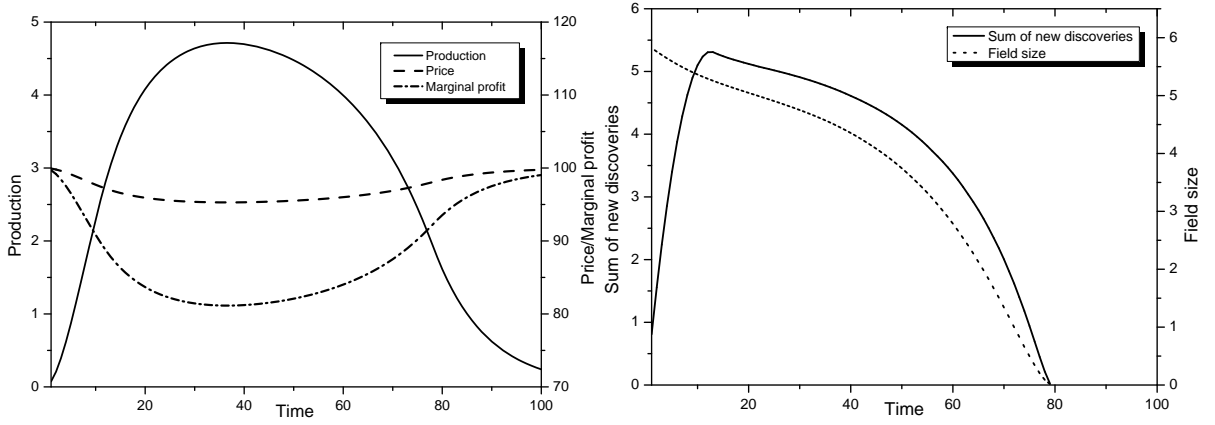


Figure 2: Trajectory for aggregate production, marginal profit (marginal revenues less marginal extractions), and price when additions are from multiple fields.

Observe that production exhibits a (asymmetric) bell-shaped peak while the composite rent and price initially decline before rising to the backstop price. Optimal field size demarcated for exploration and development declines monotonically, while additions (sum of new discoveries) rise to peak, and then decline. When there are no more fields to explore, additions decline to zero. For the simulation, the following parametrization is used:  $P(Q_t) = 100 - \sum_j q_t^j$ ;  $C(q_t^j) = 3(q_t^j)^2$ ;  $W(x_t^j) = 40(x_t^j)^2$ ;  $F(S_t^j) = 5(S_t^j)^2$ ;  $\gamma = 0.01$ ;  $\delta = 0.05$ ;  $A(0) = 300$  where  $A(0)$  is the initial endowment of land to be explored,  $S_t^j$  is the optimal field size demarcated for exploration and development, with associated demarcation/prospecting costs  $F(S_t^j)$ .

is in contrast to the principal model of section 3 where additions increase because incentives for exploration are initially low. Notably, predictions from the principal model about the nature of the production path and marginal profits path are upheld even in the spatial model. Note that in this spatial model, exploration ceases when expected value of reserves becomes less than the cost of extraction including exploration and development expenses.

### 5.2. Reserve degradation

As high quality reserves get depleted, costs often rise with depletion. To capture this, the cost function can be modified to vary in both the level of remaining reserves and the stock of remaining resources, i.e.  $C(q_t, R_t, S_t)$ , with the restriction that  $C_R(q_t, R_t, S_t) = C_S(q_t, R_t, S_t) < 0$  (cf. Swierzbinski and Mendelsohn, 1989; Boyce, 2003). Substituting for this extraction cost function in Equation (4), in the case of a binding depletion constraint, the equation describing production dynamics remains the same as that in Equation (13), while that for discoveries becomes:

$$\dot{x}_t = \frac{\delta W_x(x_t) - \gamma \mu_t + 2C_R(q_t, R_t, S_t)}{W_{xx}(x_t)} \quad (16)$$

Since  $C_R(q_t, R_t, S_t)$  is decreasing in reserves, the isocline of Equation (16) still takes the same form as that of Equation (12). This means that the same extraction paths as those in Figure 1, are observed even for this case of depletion dependent costs. The shapes of the trajectories, however, differ since the isocline of Equation (16) is steeper than that of Equation (12). Therefore for



trajectories such as “A,” the decline in additions will be faster and thus the peak in production will occur earlier. Also it can be easily verified that although marginal profits no longer follow a path that is symmetrically opposite to the production path, they must, however, first decline if production is to increase. Moreover, they continue to decline for a while even after production has peaked.

## 6. Modeling the world oil market

To assess the quantitative impacts of the above results, we numerically test the effect of the reserve depletion constraint (3) in a model for global oil<sup>12</sup> production and reserve augmentation. We base the model on Equations (1) to (4). We assume that OPEC producers<sup>13</sup> act as a perfect cartel — they make joint production decisions and thus share a common market power — that supplies global oil demand residual of non-OPEC oil supply. OPEC sets the global oil price, while non-OPEC producers<sup>14</sup> who form the competitive fringe choose their inter-temporally optimal paths for production and reserve additions, taking the price path as given. Here, the equilibrium used is the cartel-fringe price leadership model of Salant (1976). In OPECs residual demand curve, we specify a supply elasticity of 0.1 for non-OPEC oil supply (Horn, 2004). Our goal is to compare extraction in the above described price leadership with and without the geological constraint.

Because our model does not capture all the detailed complexities of oil exploration and extraction, the numerical results should be seen as merely illustrative. Nonetheless given all the available information, the model is calibrated to be as consistent with empirical data as possible. In line with preference for current extraction, both OPEC and non-OPEC producers are assumed to discount future revenues at the same rate of 5%. Additionally, for each producer, costs are specified to increase with cumulative production as estimated using regional costs curves given in Chakravorty et al. (1997). Resource endowments used in the simulation are taken from USGS (2000); WEC (2007)<sup>15</sup>, whereas reserve estimates are from BP (2009); OPEC (2009). Since not all proven reserves are developed reserves, we assume that as of 2005 (the base year) all producers only have 50%<sup>16</sup> of their 2005 proven reserves in the developed reserves category while the rest are

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<sup>12</sup>The model accounts for crude oil, natural gas liquids, and tar and bituminous sands.

<sup>13</sup>Although OPEC is modeled as perfect cartel, its members are still distinctively represented, allowing us to maintain their individual attributes such as their production profiles, cost profiles, and resource endowments during the simulations. The modeled OPEC producers are: Angola, Algeria, Ecuador, Kuwait, Iran, Iraq, Libya, Nigeria, Qatar, Saudi Arabia, United Arab Emirates (UAE) and Venezuela (combined with Ecuador). This distinction and the modeling of OPEC as a perfect cartel is achieved by ensuring that production is determined where marginal revenues (full marginal costs) are equal across members. Despite being better than aggregating all players into a singleton, this is still simplification of the OPECs actual market structure, which authors (see, e.g., Smith, 2005; Kaufmann et al., 2008) agree is more complex than that of mere oligopoly but, less stringent than that of a perfect cartel. Our focus here, however, is on exploring the impacts of geological constraints and not OPEC market structure.

<sup>14</sup>Non-OPEC is disaggregated into seven producers: North America, Brazil, Other South and Central America, Asia and the Pacific, Former Soviet Union, and Rest of the World.

<sup>15</sup>We use the high end (most optimistic) resource estimates.

<sup>16</sup>We adopt this simplified assumption because coherent data for a country by country assessment of the share of developed reserves are nonexistent and are often confidential. The lower (higher) this value is, the more (less)

added to the resources category.

On the demand side, a non-stationary isoelastic demand function adapted from Okullo and Reynès (2011) is used for two modeled regions: OECD and non-OECD, each with an assumed demand elasticity of -0.7 and -0.4,<sup>17</sup> and income elasticities of 0.56 and 0.53 (Gately and Huntington, 2002, Table 7). Income elasticity is assumed to change with gross domestic product<sup>18</sup>; this allows us to account for the impact of energy efficiency on dampening oil demand over time (see Medlock and Soligo, 2001, for a detailed explanation). Additional assumptions are also made on backstops — e.g. biofuels, gas to liquids, coal to liquids, and oil shale — that will increasingly become part of the liquid fuels supply profile. We assume that their percentage, as modeled through a Gompertz curve gradually rises from 1% in 2005 to 45% of total liquid fuels supply in 2100. The model is simulated in ten year time steps until the year 2100, however, only results up to 2065 are reported.

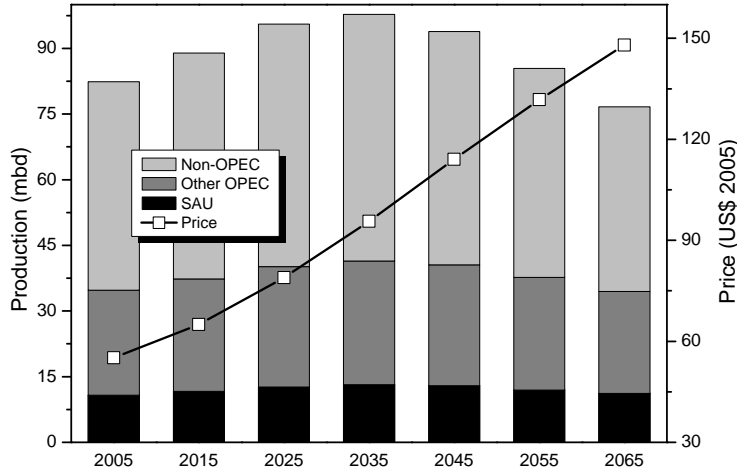


Figure 3: Global crude oil production profile and price in the classic Hotelling extraction-exploration problem (i.e., The “Hotelling” model).

Figure 3 shows global oil production disaggregated into Saudi Arabia, other OPEC, and non-OPEC production in the case that the reserve depletion constraint is absent, i.e., the classic Hotelling extraction-exploration model. We refer to this simulation set up as “Hotelling”. We see in this case that the oil price rises from US \$55 in 2005 to US \$175 in 2065. Prices increase monotonically because of a growing demand for oil but grow even faster because of the ever

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constraining geological constraints initially are. Thus, this value has an influence on the observed price and production paths. With the assumed rate of 50% and the depletion rates used in the analysis, most producers are observed to have positive shadow prices on their geological constraints right from the base year, i.e.,  $\mu_{t=2005} > 0$  and the rest (mainly OPEC producers) by 2015 (the second model period). This allows us to appropriately capture the constraining effects of geological constraints.

<sup>17</sup>Weighted by 2005 consumption shares, this gives a world oil demand elasticity of 0.58.

<sup>18</sup>Gross domestic product data projections are taken from IIASA (2009); medium level growth projections are used for the simulations.

increasing extraction costs and the Hotelling depletion rents. Because of a positive shift in demand in the initial years, supply initially increases, however, scarcity, energy efficiency, and the growing penetration of substitutes gradually reduces the availability of oil on the global market. Over the horizon, oil production is dominated by non-OPEC producers who increase production so as to compensate for OPEC withholding supply. Non-OPEC production, however, soon starts to decline because of increasing depletion and the penetration of the backstops. Saudi Arabia, on the other hand, marginally increases its production despite increasing demand and increasing resource depletion in non-OPEC. Such a production profile is consistent with a conservationist extraction policy by Saudi Arabia that is exercised in line with the market power that the cartel holds. Unlike Saudi Arabia, other OPEC producers initially increase their production, before the onset of a decline due to depletion from the smaller OPEC producers. The peak in global oil production comes in the year 2035 at nearly 98 millions barrels per day (mbd).

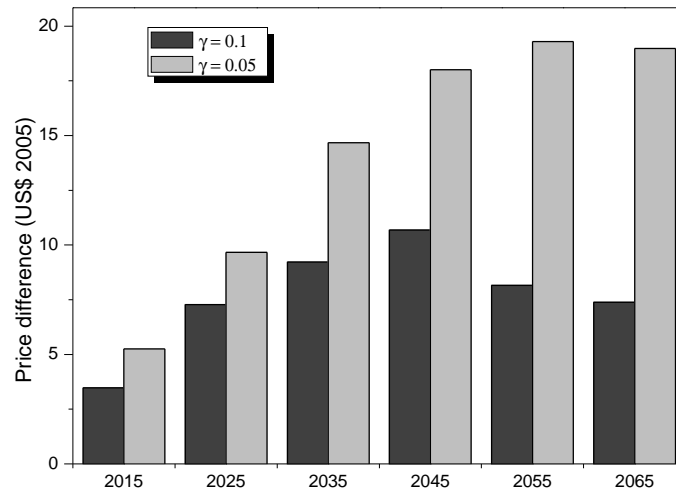


Figure 4: Difference in global oil prices for two depletion rate constrained production paths relative to the “Hotelling” production path.

Compared to the “Hotelling” scenario, the impact of the depletion constraint is to substantially raise prices (see Figure 4). The figure shows the price difference for two geologically constrained production paths ( $\gamma = 0.1$  and  $\gamma = 0.05$ )<sup>19</sup> relative to the “Hotelling” price path that is depicted in Figure 3. Notably, the more binding the depletion constraint, the higher prices are. This points to the significance of having to account for reservoir engineering constraints in particular context to increasing global oil depletion. Note that in the initial years, when production is not as constrained due to the ability for producers to augment reserves, prices from the  $\gamma = 0.1$  and  $\gamma = 0.05$  scenarios are more or less similar. However, as reserve additions diminish, the opportunity cost that the

<sup>19</sup>The former corresponds to a minimum reserve to production ratio of 20 years and the latter to 10 years. For those producers whose reserve to production ratios were lower than 20 or 10 years in 2005, their individual R/P ratios was used to calibrate the depletion rate.

constraints impose on production leads to faster growing prices in the  $\gamma = 0.05$  scenario with a stronger divergence observed after 2035. In 2065, these prices are separated by US \$11. Moreover, price in the  $\gamma = 0.1$  case — which is also the lower price of the two — is US \$8 higher than the Hotelling price. The impact of the depletion constraint is therefore unambiguous. It leads to higher prices by imposing an opportunity cost on current extraction.

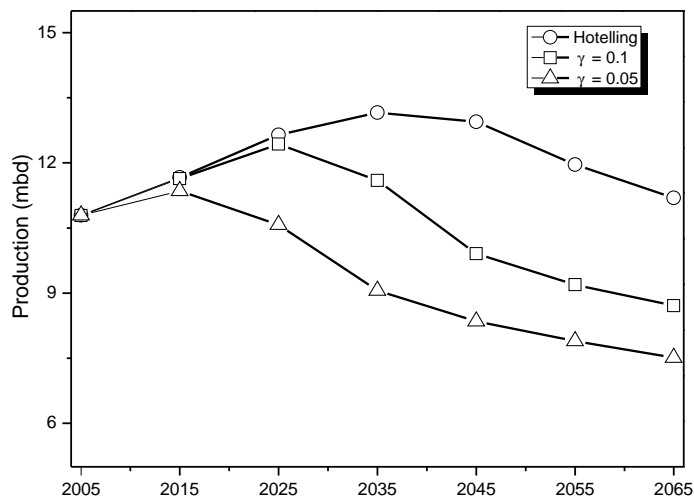


Figure 5: Oil production by Saudi Arabia in the “Hotelling” scenario; and when the depletion rate is constrained to 0.1 and to 0.05.

Geological depletion constraints also substantially affect producers’ extraction profiles. For most producers, this leads to more conservative levels of production than is predicted by the Hotelling model, which in turn gives rise to lower levels of global oil supply. Generally, the more binding the geological constraint, the earlier is a producer’s peak date and the lower their level of peak production. Take for instance Saudi Arabia’s production profile (see Figure 5). In the  $\gamma = 0.05$  case, its production is limited at just above 11 mbd and a peak is observed to occur by 2015. By 2035, Saudi Arabia’s production declines by 1.7 mbd relative to the 2005 levels. This is as opposed to the Hotelling path where production peaks in 2035, at a level of 13.17 mbd. Recall that on the Hotelling path, the production path is only influenced by (i) an exogenously changing demand path, (ii) increasing extraction costs, and (iii) marginal profits that rise at more or less at the rate of interest. In the geologically constrained models, however, the production path is additionally influenced by the opportunity cost of building the extractable reserve base. This opportunity cost slows the rate at which a majority of producers can extract their reserves.

For some currently marginal producers, however, the effect of the opportunity cost in raising the crude oil price induces them to build their reserve base faster. This allows them to increase their production level more rapidly than in the "Hotelling" scenario. Among these include Brazil, other producers in South and Central America (excluding Venezuela and Ecuador), and producers in the grouping Rest of the World. From Figure 6, the total sum of their production in the  $\gamma = 0.05$  scenario rises from 7.5 mbd to 8.4 mbd in 2035 before declining to 7.1 mbd in 2065. This is as

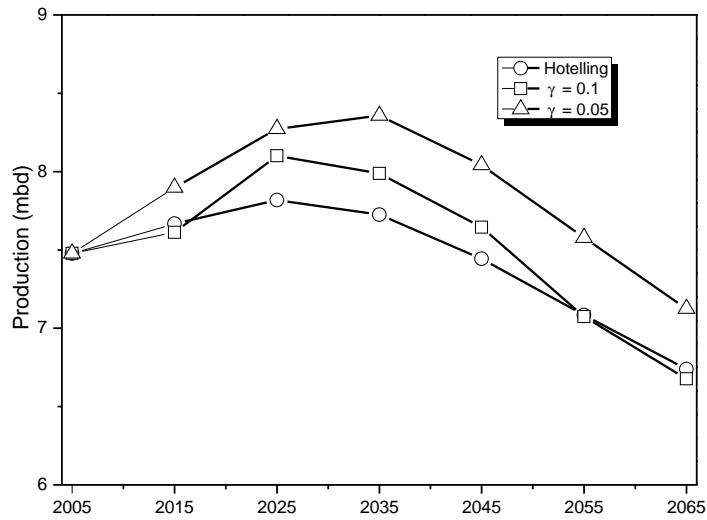


Figure 6: Oil production by currently marginal producers in the “Hotelling” scenario, and when the depletion rate is constrained to 0.1 and to 0.05.

compared to the “Hotelling” production path where their peak occurs in 2025, and at a lower level of 7.7 mbd. Given the small size of these producers, however, their increase in production is not sufficient to offset the general reduction in supply by the other major producers. This results, as can be seen in Figure 7, into lower global oil supply and a lower level for the global peak in oil production, when the depletion constraint is more binding.

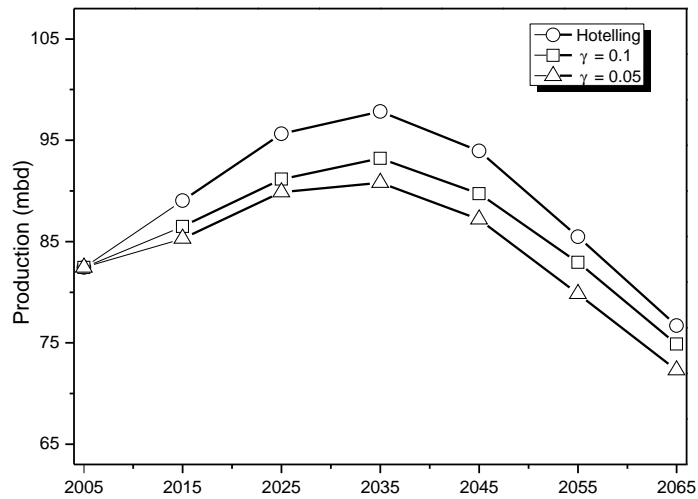


Figure 7: Global crude oil production in the “Hotelling” scenario and when the depletion rate is constrained to 0.1 and to 0.05.

## 7. Conclusions

This paper adds to the literature by proposing an additional framework through which peak oil can be studied. By assuming that geological constraints impose an upper bound on the amount producers can extract, we sought to characterize the optimal paths for production. Relative to the Hotelling model that does not account for a geological constraint, we find that the constraint substantially alters the optimal behavior of a producer. In the case that a producer is initially endowed with large reserve holdings, production is found to decline monotonically and marginal profits rise monotonically, however, at an ever increasing rate. In the alternative case where the producer initially has small reserves, production initially rises as producers build their reserves but then later declines as new reserves fail to sustain the high production levels. As production increases (decreases), marginal profits decline (increase) as if to reflect a period of declining (increasing) scarcity.

Two extensions were made to the main model. Extending the main model for endogenous field opening reinforces the result that the life cycle of resource exploitation can be described as one with a bell-shaped peak, and marginal profits and price that could decline over a large part of the resource's exploitation horizon. Allowing costs to rise with depletion of reserves still leads to the same basic paths as in the main model, discoveries however decline faster, leading production to decline faster over time, and in the case of a bell-shaped peak, to peak earlier. Marginal profits in this case continue to decline even after production has peaked and an initial decrease (in marginal profits) is necessary so as to induce a phase of increasing production.

In the final part of the paper, we compare projections for global crude oil prices and production when the numerical model accounts for the geological constraint (i.e., geologically constrained extraction paths) and when the constraint is absent (i.e., Hotelling extraction paths). Prices from the geologically constrained model are found to be substantially higher than those from the Hotelling model. Moreover, by comparing scenarios for production, it is observed that whereas the Hotelling model predicts that Saudi Arabia's production could start declining in 2035, the depletion constrained model indicates declining production starting from 2015. The depletion constraint induces Saudi Arabia to extract reserves more conservatively. Thus, although its peak occurs early, production declines more slowly over time. For smaller producers such as Brazil, however, the higher prices induced by the depletion constraint allows them to build up their reserves faster, and in turn extract faster than in the Hotelling production path. Their increase in production is, however, insufficient to offset declines from major oil producers.

In the context of model building, this paper shows the importance of paying attention to, and modeling the geophysical aspects that have so far been underrepresented in economic models. Our simple but tractable representation of geological constraints substantially improves the theoretical model's ability to explain industry stylized facts. Moreover, our simulation using real world data suggests that the impact of geological constraints is non-negligible, having observed their effects on prices and producers' production profiles. Accounting for geological constraints can thus improve the reliability of applied models as tools for influencing policy.

The study in this paper may benefit from a number of extensions. Endogenizing the reserve development decision when opening reserves for extraction is an extension we hope to carry out in future work. Here, two cases could be considered: (i) geologically efficient, and (ii) accelerated depletion programs. In the former, extraction is subject to a physical extraction constraint considered to be optimal from a geological perspective, while in the later, producers may extract above the geologically optimal rate, but would face reserve losses as a consequence. Also, empirically testing whether producer rents actually decline with reserve growth, and to what extent they do, would be useful in indicating the significance of additions in explaining trend-less oil prices over the past century.

### Acknowledgments

The authors acknowledge the financial support of the NWO's ACTS Sustainable Hydrogen research program. They also wish to thank, two anonymous referees, Erik Ansink, Christian Bogmans, Reyer Gerlagh, Matti Liski, Thomas Michielsen, Daan van Soest, Cees Withagen, Sjak Smulders, participants of the 2011 European Summer School in Resources and Environmental Economics, and participants the European Association of Environmental and Resource Economists 2012 conference for their useful comments. Remaining errors are the sole responsibility of the authors.

### AppendixA. Showing that the geological constraint binds under weak conditions

The Appendix proves that if a producer has to build up a reserve base over time, as is normally the case with natural resources, then a path with delayed reserve build up such that  $q_t = \gamma R_t$  will be the optimal production profile since it is the least costly plan. The proof is by contradiction. Suppose for some initial extraction plan  $\{q_t\}_{t=0}^{t=\infty}$ ,  $q_{t_1} < \gamma R_{t_1}$  at some  $t_1 \in [0, \infty)$ . Now consider an alternate plan in which extraction is unchanged for every  $t$ , but  $R_t$  is changed such that reserve additions are delayed so that  $q_{t_1} = \gamma R_{t_1}$ . Both extraction profiles are clearly feasible. However, the alternate profile is a lower cost plan since it allows the producer to delay making costly reserve additions to the future when the discounted cost of additions is much smaller. The initial profile therefore cannot be optimal, since the alternate profile generates a more efficient plan.

### AppendixB. Proving that production declines monotonically if marginal exploration costs are constant

From Equation (7), constant marginal exploration costs imply that  $\dot{\nu}_t = \dot{\lambda}_t$ , which in Equation (8) implies that  $\delta\lambda_t = \delta\nu_t - \gamma\mu_t \Rightarrow \delta W_x(x_t) = \gamma\mu_t$ . Since  $W_x(x_t)$  is constant,  $\mu_t$  will remain constant for as long as additions are positive. Notice that by re-arranging  $\delta W_x(x_t) = \gamma\mu_t$ , as  $W_x(x_t) = \frac{\gamma\mu_t + W_x(x_t)}{1+\delta}$ , we have that additions take place when the cost of additions matches the discounted price of expanding the geologically extractable reserve base. In order to show that production will be in decline even when additions are positive, we take time derivatives of Equation

(6) and solve knowing that  $\dot{\mu}_t = 0$ . We then see that  $\dot{q}_t = \frac{\delta\lambda_t}{\mathcal{R}_{qq}(q_t, P(Q_t)) - C_{qq}(q)} < 0$ . Production therefore declines monotonically even when additions are positive. On the interval where additions are zero, it is straightforward to see that production will be in decline.

### Appendix C. Ascertaining that the single-peaked production path is feasible for initially small reserves.

Firstly, we derive a precise expression for changes in production. From  $\dot{R}_t = x_t - q_t$ , consider a unit change of time such that  $R_t = x_t - q_t + R_{t-1} = x_t - \gamma R_t + R_{t-1}$ . Let  $R_{t-1} = \alpha_t R_t$  where  $\alpha_t \geq 0$ . Writing  $R_t$  in terms of  $x_t$  gives:  $R_t = \frac{x_t}{1 + \gamma - \alpha_t}$ . From  $\dot{q}_t = \gamma(x_t - \gamma R_t)$ , (Equation (13)) we have that  $\dot{q}_t = \frac{\gamma x_t (1 - \alpha_t)}{1 + \gamma - \alpha_t}$ . Production is increasing when  $\alpha_t < 1$ , but non-increasing when  $\alpha_t \geq 1$ . In the special case that reserves are initially zero ( $\alpha_t = 0$ ), it is clear that production increases. Moreover, as long as reserves are growing, production continues to increase. Along the optimal path, additions eventually reach zero since the marginal benefit of exploration and development ultimately declines to zero. This implies that production eventually goes through a decline phase since  $\dot{R}_t = -\gamma R_t < 0$ . With initially small reserves we thus ascertain that production goes through an increasing phase, but later also goes through a decline phase.

It now suffices to show that if additions are non-increasing as depicted in path “A” of Figure 1, then production exhibits a single peak. We proceed by proving by contradiction. Consider two points,  $t_1, t_2$  along  $t$  such that  $t_2 > t_1$ . Let  $x_{t_1} \leq q_{t_1} \Leftrightarrow x_{t_1} \leq \gamma R_{t_1}$ , and also let  $x_{t_2} > q_{t_2} \Leftrightarrow x_{t_2} > \gamma R_{t_2}$ . Assuming multiple peaks, let  $t_1$  by definition be the point where production attains a minimum before increasing again. We have that  $\gamma R_{t_1} < \gamma R_{t_2} \Rightarrow x_{t_1} < x_{t_2}$ ; a contradiction. Thus as long as additions are non-increasing, production will exhibit only a single peak. A numerical illustration for the feasibility of this path is available from the authors. In general, we can not prove whether additions will always be non-increasing whenever production exhibits a peak. However, Figure 1 of the feasible economic dynamics shows that only paths that start in region I will exhibit peaks, and along such paths additions are non-increasing.

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