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Generation Capacity Investments in Electricity Markets: Perfect Competition

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Abstract

In competitive electricity markets, markets designs based on power exchanges where supply bidding (barring demand-side bidding) is at the sole short run marginal cost may not guarantee resource adequacy. As alternative ways to remedy the resource adequacy problem, we focus on three different market designs in detail when demand is inelastic, namely an energy-only market with VOLL pricing (or a price cap), an additional capacity market, and operating-reserve pricing. We also discuss demand-side bidding (i.e., a price responsive demand) which can be seen as a categorically different alternative to remedy the resource adequacy problem. We consider a perfectly competitive market consisting of three types of agents: generators, a transmission system operator, and consumers; all agents are assumed to have no market power. For each market design, we model and analyze capacity investment choices of firms using a two-stage game where generation capacities are installed in the first stage and generation takes place in future spot markets at the second stage. When future spot market conditions are assumed to be known *a priori* (i.e., deterministic demand case), we show that all of these two-stage models with different market mechanisms, except operating-reserve pricing, can be cast as single optimization problems. When future spot market conditions are not known in advance (i.e., under demand uncertainty), we essentially have a two-stage stochastic game. Interestingly, an equilibrium point of this stochastic game can be found by solving a two-stage stochastic program, in case of all of the market mechanisms except operating-reserve pricing. In case of operating-reserve pricing, while the formulation of an equivalent deterministic or stochastic optimization problem is possible when operating-reserves are based on observed demand, this simplicity is lost when operating-reserves are based on installed capacities. We generalize these results for other uncertain parameters in spot markets such as fuel costs and transmission capacities. Finally, we illustrate how all these models can be numerically tackled and present numerical experiments. In our numerical experiments, we observe that uncertainty of demand leads to higher total generation capacity expansion and a broader mix of technologies compared to the investment decisions assuming average demand levels. Furthermore for the same VOLL (or price cap) level and under the assumptions of random demand with finite support and no forced outages, energy-only markets with VOLL pricing tend to lead to total generation capacity below the peak load with a

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certain probability whereas energy markets with a forward capacity market or operating-reserve pricing result in higher investments. Finally, the regulator decisions (e.g., reserve capacity target) in capacity markets and operating-reserve pricing can be chosen in such a way that results in very similar investment levels and fuel mix of generation capacities in both market designs.

Keywords: electricity markets, generation investment modeling, capacity market, operating reserve pricing, perfect competition equilibrium, stochastic optimization.

JEL codes: *C*61, *C*63, *D*41, *C*68, *L*94, *Q*48

1 Introduction

In the days of regulated monopolies, there was no issue of generation resource adequacy. Companies were obligated to serve the demand and had to invest accordingly; an optimization model was used to compute the expansion of generation capacity decisions that would satisfy demand at minimal investment and operations costs (subject to reliability constraints that we will not discuss here). In compensation for this obligation, electricity prices were regulated (often at average cost) in a way that guaranteed that the company could pay for its expenses (including reimbursement of long term debt) and make a reasonable profit on equity. The theory of peak load pricing for non-storable commodities, such as electricity, is the economic counterpart of these computational models; it can be seen as an economic interpretation of the capacity expansion model in terms of electricity prices that induce efficient investments and operations. Of particular importance, the theory of peak load pricing explains that the price of electricity in the highest demand period must embed a particular (peak load) component to induce an efficient capacity mix. Both the capacity expansion models and the theory of peak load pricing can be traced back to work conducted in Électricité de France in the fifties and sixties (see the collection of early papers treating both subjects in Morlat and Bessière (1971)).

Capacity expansion models were extensively developed during the regulatory periods before loosing some of their appeal after restructuring. After restructuring, generation and investments became the responsibility of companies who had to make a profit on the electricity market. This gave rise to the question whether energy-only electricity markets would provide incentives for adequate investments and thereby maintain security of supply. This discussion focuses especially on those power plants which will only be needed to meet demand at peak hours and therefore have to earn sufficient revenues in those hours to cover their investment costs. The theory of peak load pricing, which was initially developed for the regulated monopoly, was later proved equally relevant to perfectly competitive markets (in case of elastic demand, see Crew et al. (1995) for a survey). This theory has become crucial today to explain why competitive electricity markets may not spontaneously provide the right incentive to invest in generation capacity for peak load and to suggest remedies to this market failure.

In this paper, we consider three variations of competitive electricity market designs known as energy-only market with VOLL pricing (or a price cap), a forward capacity market, and operating-reserve pricing as possible remedies to a market failure of insufficient generation capacity investment. We also discuss demand-side bidding which is strictly speaking not a remedy to a market failure but an alternative way to remove this market failure. We formulate generation capacity investment decisions in these electricity market designs as two-stage equilibrium problems, which is a natural way of modelling problems with multiple decision makers in a competitive environment. We show that most of these equilibrium models can be cast in mathematical programming formulations that are not too far from the early capacity expansion models. Establishing the exact relations between the two-stage equilibrium problems and the early capacity expansion models under these market designs is one of the main objectives of this paper. We continue this introduction by formalizing the question of resource adequacy in competitive electricity markets. We do so by referring to the early capacity expansion models (in the most simplified setting) and to the interpretation of their dual solution in peak load pricing terms.

Consider the following simple generation capacity expansion model of a regulated monopoly where supply and demand are located at a single node. There is a finite set of plant types K and a finite set of time segments Ω (these can also be interpreted as states of the world, as we later do) each occurring with some duration $\pi(\omega)$ (that we later interpret as probabilities). $d(\omega)$ is the demand in time segment ω; κ_k and c_k are the unit capacity and unit generation cost of plant k; x_k is the capacity of plant type k and $y_k(\omega)$ is the generation of this plant in time segment ω. Assuming the monopoly firm is regulated in a way that motivates cost minimization, the capacity expansion model, together with its dual variables, $\beta(\omega)$[1] and $p(\omega)$[1], is stated as:

$$\begin{aligned}
&\min \sum_k \kappa_k x_k + \sum_\omega \pi(\omega) \sum_k c_k y_k(\omega) \\
&\text{s.t. } \; x_k - y_k(\omega) \geq 0 && \pi(\omega)\beta_k(\omega) \quad \forall \omega \quad \forall k \\
&\qquad \sum_k y_k(\omega) - d(\omega) \geq 0 && \pi(\omega) p(\omega) \quad \forall \omega \\
&\qquad y_k(\omega) \geq 0 \qquad x_k \geq 0 \quad \forall \omega \quad \forall k.
\end{aligned} \tag{1}$$

This model allows the monopoly firm to determine an optimal investment portfolio of generation capacity mix for satisfying various demand levels[2]. The closely related producer and consumer surplus maximization problem has a richer economic content. Let $P(\omega, d)$ be the inverse demand function of the market where the price is given as a function of quantity supplied in time segment $\omega \in \Omega$. The producer and consumer surplus maximization problem is written as:

$$\begin{aligned}
&\max \sum_\omega \pi(\omega) \left[\int_0^{d(\omega)} P(\omega, \xi) d\xi - \sum_k c_k y_k(\omega) \right] \quad - \sum_k \kappa_k x_k \\
&\text{s.t. } \; x_k - y_k(\omega) \geq 0 && \pi(\omega)\beta_k(\omega) \quad \forall \omega \quad \forall k \\
&\qquad \sum_k y_k(\omega) - d(\omega) \geq 0 && \pi(\omega) p(\omega) \quad \forall \omega \\
&\qquad y_k(\omega) \geq 0 \qquad x_k \geq 0 \quad \forall \omega \quad \forall k,
\end{aligned} \tag{2}$$

[1]Note that we multiply these dual variables with their probabilities in the models (1) and (2) for the purpose of scaling.

[2]In the formulation (1), the demand constraint is assumed to be always binding if $c_k > 0, \forall k$. In more general market models with transmission or unit commitment constraints, however, the optimal solution might not be binding since technical and economic reasons may drive generators to run even when power supply exceeds the demand. In these situations, generators may seek to maintain output by offering to pay wholesale buyers to take their electricity. This could yield negative price in some locations or periods.

where $\int_0^{d(\omega)} P(\omega,\xi)d\xi$ is known as the consumers' willingness to pay at $d(\omega)$. The KKT conditions of problem (2) provide the necessary relations to explain the resource adequacy problem. Specifically the KKT condition associated with a positive generation variable $y_k(\omega)$ is stated as:

$$c_k + \beta_k(\omega) = P(\omega, d(\omega)) = p(\omega), \quad \forall \omega. \tag{3}$$

Alternatively, the KKT condition associated with a positive investment variable x_k is stated as:

$$\sum_{\omega} \pi(\omega)\beta_k(\omega) = \kappa_k. \tag{4}$$

The economic interpretation of (3) is that a plant of type k which is operating in time segment ω generates a (scarcity or capacity) rent $\beta_k(\omega)$ in that time segment; this rent is equal to the difference between the electricity price $p(\omega)$ in that time segment and the plant's fuel cost c_k. The economic interpretation of (4) is that one invests in a plant of type k when the duration (π) weighted sum of the rents $\beta_k(\omega)$ over all time segments ω is equal to the investment cost κ_k. Departing from this pricing scheme induces inefficiencies, either at the consumption or generation side. The question of resource adequacy is whether restructured electricity markets lead to electricity prices that satisfy these relations. If not, there is a resource adequacy problem.

It is now recognized that the original restructured electricity markets do not spontaneously satisfy relations (3) and (4). Because electricity is not storable, the market (barring demand-side bidding) clears in the short run when demand is inelastic. The market therefore does not ensure (3), because it does not face a downward sloping demand curve in the short run. Instead, the organization of the market sets the price at the bid of the last plant selected to satisfy the demand, which used to be a widely used pricing scheme when liberalisation was introduced in many countries such as in Europe and in the US. Barring market power and demand-side bidding, this price is equal to the fuel cost of the last selected power plant in time segment ω. This implies that the most expensive unit in operations over the different time segments ω (the peak plant) does not make any margin and hence (3) is not satisfied for that unit. This can be interpreted as follows: the electricity price in the peak does not incorporate the necessary component, called as "scarcity rent", that pays for the capacity at peak demand. The peak plant therefore appears with a zero margin in (4) which is thus also never satisfied for that equipment. This missing margin is now commonly referred to as the missing money. Stoft (2002) was among the first ones to analyze investments by invoking insufficient payments for capacity. He explained how, barring well developed demand-side bidding, pools and power exchanges prevent prices on the market to reflect scarcity in generation capacity (see also Cramton and Stoft (2005, 2006)). Both Hogan (2005) and Joskow (2007, 2008) also offer enlightening and in depth analysis of the missing money. Oren (2007) provides a comprehensive description of the different techniques aimed at restoring resource adequacy; he also discusses various implementations and gives an extensive list of references.

Guaranteeing resource adequacy therefore requires eliminating the missing money by creating enough capacity rent $\beta_k(\omega)$ to cover the capital costs of an efficient generation system, including the capacity cost of the peak plant. This in turn requires changing the electricity pricing mechanism. In an energy-only system, the idea is to price electricity at a high value that is supposed to reflect the value of lost load (VOLL), known as VOLL pricing, when demand is curtailed. In perfectly competitive markets, Stoft (2002) shows that VOLL pricing results in optimal generation capacity investments. However, in reality VOLL is difficult to estimate and therefore a price cap (which is in general lower than VOLL) is used. If the price cap induced by the regulator is not high enough, this will constrain prices from rising up to their competitive levels at peak hours, yielding underinvestment in generation capacity (e.g., see Joskow (2008)). An alternative solution for avoiding market failure is to implement a capacity market where the regulator imposes some capacity target in line with historical data and expected demand and the firms contributing to the sufficient investment level receive numerations accordingly. In order to increase the robustness against market power, many variants of capacity markets have been proposed regarding the implementation of the market design and the treatment of demand response (e.g., Cramton and Stoft (2005), Joskow (2008), Hobbs et al. (2007), Cramton and Ockenfels (2011)). Here we assume a forward capacity market where capacity is auctioned before the investment decision is made and the resulting capacity payment is certain for the lifetime of the plant. A more sophisticated alternative remedy is to apply some form of reliability or operating-reserve pricing so that electricity price increases when the reserve margin decreases (Stoft (2002), Hogan (2005), Hogan (2009)). Lastly, as a categorically different alternative, ensuring a price responsive demand in the short-run as well as in the long-run may remedy the resource adequacy issue.

In recent years, more and more countries have implemented (e.g., US states) or are planning to implement a variety of these market mechanisms aimed at stimulating investments in new generation capacity, such as scarcity pricing when capacity is inadequate or capacity payments via additional capacity markets next to the electricity market. For policy analysis in real world problems, practical tools are needed to gain insights into the social implications of a market design or a policy target (see e.g., Schroeder (2012) and Allcott (2012) for examples of real world applications). Thus, we concentrate on these four mechanisms (including demand-side bidding briefly) in our analysis with the following contributions. We first expand on existing short run equilibrium models of restructured systems to include generation capacity decisions under these resource adequacy mechanisms and uncertainty about future electricity market conditions. The natural approach is to resort to complementarity formulations as this mathematical programming paradigm has been extensively used to model restructured electricity systems. We then assess the extent to which these models can be restated as optimization problems as solvers for optimization problems are now numerous and quite powerful. Furthermore, we provide insights about to what extent the investment incentives are affected by these different mechanisms under demand uncertainty. To this end, we illustrate how all these models can be numerically tackled and present some numerical experiments.

We assume price-taking firms and hence exclude market power. Even if real markets may depart from perfect competition, perfect competition models provide an essential benchmark for imperfect competition models. Moreover, real markets may suffer from inefficiencies as a result of regulatory intervention or market design. Utilizing perfect competition models still allows policy makers to gain insights into the social implications of a market design or a policy target (e.g., Allcott (2012)). In addition, there are computational reasons. Multi-stage imperfect competition models are difficult if not impossible to solve. This is already true for two-stage investment and operation models that are EPEC (equilibrium problem subject to equilibrium constraints) when involving market power (see Ralph and Smeers (2006), Hu and Ralph (2007) for more details on these and related complications). More general models involving a sequence of cycle of investment and operations are at this stage computationally unexplored. Last, the experience of reformed markets indeed shows that market power mitigation instruments are effective and that properly designed reformed markets function competitively and hence can be modeled under the perfect competition assumption. For example, the market monitoring results of PJM (2012) and CASIO (2012) indicate that market prices are at or near competitive levels most of the time in US states. Furthermore, the European Union and its Member States are underway to move towards a fully integrated European electricity market by 2014 with the aim to increase competition and maximize the economic welfare of all players. In some of the regions (e.g., Germany-Belgium-France-The Netherlands) where the integration has already taken place for some time, significant price convergence is observed between the countries in most of the hours, which is a good indicator for competitiveness (see DG ENERGY (2012)). To sum up, in this paper we will deal with a computable representation of the incentives to invest in power markets functioning under perfect competition.

In reality, generation capacity expansion is a multi-period process. The market induces the creation of new capacities and the retirement of old ones. A full version of the capacity expansion model therefore involves a sequence of successive cycles of investment and operations (e.g., Schroeder (2012)). We limit our analysis to simplified models that represent a single cycle of investment and operation: investment takes place in the first stage at some investment costs; the market operates in the second stage with generators collecting sales revenues and incurring fuel costs. This restriction is made for the sake of the presentation. In contrast with multi-period imperfect competition models, it is perfectly possible to implement the mechanisms considered here in a multi-period context since convexity is in general preserved under perfect competition.

Although we focus mainly on electricity demand being uncertain and/or fluctuating with a very low elasticity, our methodology and results summarized below can be easily generalized to spot markets with other uncertainties (i.e., fuel costs, transmission capacities, wind generation etc.). For given wind capacity and volatile wind generation, one can substitute demand with residual demand levels (i.e., demand minus wind). Under uncertainty about future electricity market conditions, real world problems including generation capacity investment decisions lead to stochastic equilibrium problems (e.g., Schroeder (2012) and Allcott (2012)) that are large scale and computationally more complex to solve than a stochastic optimization problem. Equilibrium problems are indeed broader than optimization problems which is addressed in detail by Gabriel et al. (2012). In this paper, we not only emphasize the link between optimization and equilibrium problems but also show that, regarding the problem of generation capacity investments in perfectly competitive electricity markets, most of the formulations of stochastic equilibrium problems can be cast as two-stage stochastic programs.

We consider a perfectly competitive market consisting of three types of agents namely generators, a transmission system operator (TSO), and consumers; all agents are price takers. Generators are assumed to be risk neutral and maximize their expected profits. The transmission system operator sells transmission services in order to maximize the value of its infrastructure and consumers are simply represented by an inelastic demand in most of the paper, except in Section 6 where we consider a price responsive demand. It may also be of interest to include risk averseness of generators by using "coherent risk measures" (e.g., Ehrenmann and Smeers (2011) and Ralph and Smeers (2011)). The resulting problems are still stochastic equilibrium problems which are somewhat modified versions of the stochastic equilibrium problems presented in this paper. When the markets are "perfectly competitive" and "complete", the formulation of an equivalent two-stage stochastic program is still possible as shown by Ralph and Smeers (2011). The main contributions of this paper can be summarized as follows:

- We expand the existing short-run models of restructured electricity system to include both uncertainty of spot market conditions (i.e., demand uncertainty) and resource adequacy mechanisms, such as VOLL pricing, capacity market, and operating-reserve pricing, and also demand-side bidding as an alternative.

- In perfect competition, we show that most of the formulations of these "equilibrium" models are in fact equivalent to or can be cast as optimization problems. In particular, in the stochastic setting this result indicates the prevalence of two-stage stochastic programming for providing solutions to stochastic equilibrium models. We believe that this result is helpful for making an economic assessment of the investment incentives in new generation capacity in real world systems for the following reasons: in Sections 8 and 9, we explain how we can solve a two-stage stochastic program as a nonlinear or linear optimization problem. Due to the availability of powerful and efficient nonlinear programming solvers and decomposition methods for two-stage stochastic programs, solving a two-stage stochastic program is computationally much faster than solving a stochastic equilibrium model for large scale systems, which we also observe in our numerical experiments. In only one case of operating-reserve pricing, we obtain a complementarity problem that is not equivalent to an optimization problem.

- In perfect competition, we show that single and two-stage representation of these "equilibrium" models are equivalent. In other words, the open loop equilibria and the closed loop equilibria of these models coincide.

- We use sample-path methods and provide detailed algorithmic approaches for numerically tackling all these models, which allows solving these computationally complex stochastic problems by utilizing deterministic off-the-shelf solvers. We also illustrate that these algorithmic approaches help further decrease the computational time compared to solving the two-stage equilibrium problem as a potentially very large MCP (Mixed Complementarity Problem) including all the first and second stage decision variables.

- Through numerical experiments, we gain insights on the impact of demand uncertainty and to what extend these different mechanisms can remedy the resource adequacy issue. In particular, we first find that uncertainty of demand leads to higher total generation capacity expansion and a broader mix of technologies compared to the investment decisions assuming average demand levels. Furthermore for the same VOLL (or price cap) level, energy-only markets with VOLL pricing tend to lead to total generation capacity below the peak load with a certain probability whereas energy markets with a forward capacity market or operating-reserve pricing result in higher investments assuming random demand with finite support and no forced outages[3]. Last, the regulator decisions (e.g., reserve capacity target) in capacity markets and operating-reserve pricing can be chosen in such a way that results in very similar investment levels and fuel mix of generation capacities in both market designs.

The rest of the paper is organized as follows. In Section 2, we give the set up and notation used throughout the paper. In Section 3, we first introduce the deterministic investment and operations model (in the context of a two-stage equilibrium problem under perfect competition) and present it in two different formulations. One is a single-stage (open loop) version of the model where generators simultaneously invest and decide operations knowing future market prices. The other formulation is a two-stage (closed loop) model in which investment and operation decisions are made sequentially. Generators operate the capacities inherited from the first stage to maximize their profits. This market operation results in marginal values of plants that generators take into account in the first stage in order to decide on their investments. The distinction between open and closed loop models is important when there is market power. We show that this distinction is irrelevant here: both models are equivalent and can be reformulated as a single optimization problem of the standard capacity expansion type. Although Section 3 begins with a deterministic model, the rest of the paper elaborates on stochastic models involving demand that is unknown at the time of investment. In Section 3.2 we extend the formulation to a stochastic energy-only equilibrium model and again find that it is equivalent to a stochastic capacity expansion model, which is a two-stage stochastic program. In Section 4, we take up the capacity market formulation, which we find again equivalent to a convex stochastic programming problem. In Section 5, we consider the more novel question of operating-reserve pricing for which we give two formulations that differ by the computation of the operating-reserves. One formulation refers the reserve to observed demand; the other refers it to the total capacity. The former one turns out to be a convex stochastic optimization problem, but the latter is not. In Section 6, we give a brief discussion of demand-side bidding which we treat by assuming price responsive demand; this can be thought of a completely different way of addressing resource adequacy issue. Section 7 outlines how all our results can be generalized for other uncertain elements in spot markets, such as unit generation costs and transmission capacities. We discuss various algorithmic approaches for handling these models numerically in Section 8. We report numerical results in Section 9 and provide insights to what extent the remedies to the missing money, discussed in Sections 3, 4, and 5, incite the generators to invest in generation capacity. Conclusions terminate the paper. Finally, Appendix contains the proofs of some theorems, lemmas, and propositions.

[3]Note that under different assumptions (e.g., forced outages, demand distribution without a finite support), VOLL pricing (with high values of VOLL) may also result in similar investment levels as the other market designs (see the result of Hobbs et al. (2001)).

2 Set-up and Notation

We consider a market with a regulator and three types of agents; namely generators, a transmission system operator (a TSO), and consumers. Generators and the TSO are price takers and maximize profits at given prices; consumers are represented by an inelastic demand. Generators and consumers are spatially distributed in an electricity transmission network which is operated by the TSO. Generation and transmission of electricity take place in the spot market and the locational marginal pricing is assumed to clear the spot market.

The regulator intervenes to remedy the lack of incentive to invest; his/her role differs depending on the market design. In an energy-only electricity market, he/she sets the price of the unserved energy (VOLL) or the price cap in case of demand curtailment. In an electricity market with a forward capacity market, he/she sets the capacity target to guarantee resource adequacy and rewards the firms who contribute to the sufficient investments to reach the target. Finally, in an electricity market with operating-reserve pricing, he/she sets the price of the operating-reserve and provides the firms with additional payments whenever the systems total reserve is scarce.

We consider agents interacting in a two-stage set-up: generators invest in their generation capacities in the first stage and the generation is dispatched in the second stage where the spot market clears to satisfy demand under transmission limitations. Under all market designs, the demand is first assumed to be constant during the whole year. Then we consider a random demand which varies over a year and extend our analysis under uncertainty of demand. The following notation would apply in a purely deterministic world:

Sets

N	:	set of all demand nodes
G	:	set of all firms
I_g	:	set of supply nodes of firm $g \in G$
I	:	set of all supply nodes ($I := \cup_g I_g$)
K_g	:	set of plant types of firm $g \in G$
L	:	set of electricity transmission lines in the network

Parameters

c_{ik}^{g}	:	unit generation cost of plant type $k \in K_g$ owned by firm $g \in G$ at supply node $i \in I_g$
κ_k	:	unit capacity cost of plant type $k \in K_g$
d_n	:	demand at node $n \in N$
$PTDF_{l,j}$	:	power transmitted through line $l \in L$ due to one unit of power injection from node $j \in \{N \cup I\}$ to an arbitrary hub[4] node
h_l	:	capacity limit of line $l \in L$
$VOLL$	:	the value of unserved energy or lost load

[4] $PTDF$ is calculated based on a hub node in $n \in N$ in a standard DC load flow model. The choice of hub node is arbitrary. That is,

Variables:

Second Stage:

y_{ik}^g	:	quantity of power generated by plant type $k \in K_g$ of firm $g \in G$ at supply node $i \in I_g$
f_j	:	net power flow dispatched by TSO from node $j \in \{N \cup I\}$
δ_n	:	unserved (curtailed) energy at node $n \in N$
p_j	:	locational market price (nodal price) at node $j \in \{N \cup I\}$ which corresponds to shadow price of market clearing constraint

First Stage:

x_{ik}^g	:	capacity of plant type $k \in K_g$ owned by firm $g \in G$ at node $i \in I_g$.

3 Energy-only Market

We start our analysis with an energy-only electricity market. In an energy-only market with inelastic (exogenous) demand, the price of electricity is set by the market at the bid of the most expensive plant generating unless the demand is curtailed. When demand is curtailed, the price is capped by the regulator at the value of loss load (VOLL) or a price cap. During the hours of curtailment, the peak plant obtains extra margin to compensate its missing money for the whole year. The demand is first assumed to be constant during the whole year in Section 3.1. In Section 3.2, we consider a random demand which varies over a year.

3.1 Two-stage Equilibrium Model with Constant Exogenous Demand

In Section 3.1.1, we give the exact formulation of the interactions between the agents in an energy-only market at both stages when demand is fixed and we show some characteristics of both the short run and the long run perfect competition equilibria. By using these characteristics, we show in Section 3.1.2 that solving a single optimization problem where all the generation capacities are determined by a central decision maker finds a perfect competition equilibrium of the two-stage game introduced in Section 3.1.1.

3.1.1 The Perfect Competition Equilibrium as Mixed Complementarity Problem

We next formulate each agent's problem in the two-stage game where firms give their investment decisions, x, simultaneously at the first stage and they decide on their optimal generation levels, y, in the spot market at the second stage. Note that demand is exogenous and known to all firms who are price takers at both stages.

Second Stage: At the second stage, each firm $g \in G$ maximizes its short term profit from the spot market by optimization problem (5):

the flows resulting from a power injection at one node and an equal withdrawal at another do not depend on the location of the hub.

$$
\begin{aligned}
\Pi_g^*(x^g) := \max_{y^g} \quad & \sum_{i\in I_g}\sum_{k\in K_g}(p_i - c_{ik}^g)y_{ik}^g \\
s.t. \quad & \\
Cons\_g(x^g): \quad & \boxed{\begin{array}{lll} y_{ik}^g \le x_{ik}^g & (\beta_{ik}^g) & \forall i\in I_g, k\in K_g \\ y_{ik}^g \ge 0 & & \forall i\in I_g, k\in K_g, \end{array}}
\end{aligned} \tag{5}
$$

where β^g is the vector of Lagrange multipliers associated with the capacity constraints ($y^g \le x^g$) and is referred as capacity/scarcity rent. Note that the nodal prices, p, enter as parameters to firms' second stage problems. Since all firms are price-takers under perfect competition, they act as if they cannot affect the value of p. As an important consequence, the nodal prices will also be taken as parameters in firms' first stage problems given in (10).

For a given set of generation decisions $\{y^g\}_{g\in G}$ of the firms and nodal prices $\{p_j\}_{j\in\{N\cup I\}}$, if there exists a price difference between any two nodes in the spot market, TSO decides on imports/export flows $\{f_j\}_{j\in N\cup I}$ as long as there are available transmission possibilities and it maximizes its profits from the transmission of electricity. Specifically, TSO effectively acts as an arbitrageur. TSO's problem is given by (6):

$$
\begin{aligned}
\max_f \quad & \sum_{j\in\{N\cup I\}} p_j f_j \\
s.t. \quad & \\
Cons\_TSO: \quad & \boxed{\begin{array}{lll} \sum_{j\in\{N\cup I\}} f_j = 0 & (\rho) & \\ \sum_{j\in\{N\cup I\}} PTDF_{l,j} f_j \le h_l & (\lambda_l^+) & \forall l \\ -\sum_{j\in\{N\cup I\}} PTDF_{l,j} f_j \le h_l & (\lambda_l^-) & \forall l, \end{array}}
\end{aligned} \tag{6}
$$

where ρ, λ_l^+, and λ_l^- are Lagrange multipliers of problem (6). In (6), $Cons\_TSO$ is the set of Kirchoff law based transmission constraints faced by TSO in the electricity network. TSO is also a price-taker and cannot affect the nodal prices, p, to maximize its profit.

Finally, the nodal prices are determined to clear the spot market when supply matches the demand minus possible curtailments. In case of a curtailment, the electricity is priced at VOLL by the regulator. The spot market clearance conditions are given in (7):

$$
MCP\_Market: \boxed{\begin{array}{llll} 0 \le \sum_{g\in G}\sum_{k\in K_g} y_{jk}^g + \delta_j + f_j - d_j & \perp & p_j \ge 0 & \forall j \in \{N\cup I\} \\ 0 \le VOLL - p_n & \perp & \delta_n \ge 0 & \forall n\in N, \end{array}} \tag{7}
$$

where p_j represents the locational marginal price of unit power (\$/MWh) at node $j \in \{N\cup I\}$ and δ_n is the curtailed energy. Note that node j may be both a supply and a demand node. If it is only a supply node, then δ_j and d_j equal to zero or if it is only a demand node then $(y^g_{jk})_{g\in G,k\in K}$ equal to zero.

The spot market equilibrium conditions consist of the KKT optimality conditions of problem (5) for all firms $g \in G$, the KKT optimality conditions of TSO's problem (6), and the market clearance conditions (7), which can altogether be formulated by the mixed complementarity problem (MCP) (8). An equilibrium point satisfying the conditions in MCP (8) consists of the optimal generation quantities y^* for all firms and the optimal import/export flow decisions f^* for TSO in the spot market:

$$
\begin{aligned}
&MCP\_Firms(x): \boxed{\begin{array}{lll} 0 \le c^g_{ik} - p^*_i + \beta^{*g}_{ik} & \perp \quad y^{*g}_{ik} \ge 0 & \forall g \in G, i \in I_g, k \in K_g \\ 0 \le x^{*g}_{ik} - y^{*g}_{ik} & \perp \quad \beta^{*g}_{ik} \ge 0 & \forall g \in G, i \in I_g, k \in K_g \end{array}} \\
&MCP\_TSO: \boxed{\begin{array}{lll} 0 \le h_l - \sum_{j\in\{N\cup I\}} PTDF_{l,j} f^*_j & \perp \quad \lambda^{*+}_l \ge 0 & \forall l \\ 0 \le h_l + \sum_{j\in\{N\cup I\}} PTDF_{l,j} f^*_j & \perp \quad \lambda^{*-}_l \ge 0 & \forall l \\ p^*_j - \rho^* + \sum_{l\in L} PTDF_{l,j}(\lambda^{*-}_l - \lambda^{*+}_l) = 0 & & \forall j \in \{N\cup I\} \\ \sum_{j\in\{N\cup I\}} f^*_j = 0 & & \end{array}} \\
&MCP\_Market: \boxed{\begin{array}{lll} 0 \le \sum_{g\in G}\sum_{k\in K_g} y^{*g}_{jk} + \delta^*_j + f^*_j - d_j & \perp \quad p^*_j \ge 0 & \forall j \in \{N\cup I\} \\ 0 \le VOLL - p^*_n & \perp \quad \delta^*_n \ge 0 & \forall n \in N. \end{array}}
\end{aligned}
\tag{8}
$$

Boucher and Smeers (2001) consider a competitive equilibrium of a game in spot market where none of the agents (firms, consumers, and TSO) has market power. The interactions between firms and TSO in (5) and (6), respectively, is an example of such a game. Boucher and Smeers (2001) also introduce an optimization problem referred to as Optimal Power Flow Problem (OPF). In our setting, for given x, the OPF problem with exogenous demand corresponds to a linear program (LP) as given in (9):

$$
\begin{aligned}
&\min_{\{y,\delta,f\}} && \sum_{g\in G}\sum_{i\in I_g}\sum_{k\in K_g} c^g_{ik} y^g_{ik} + VOLL \sum_n \delta_n \\
&s.t. && \\
&Cons\_Market: && \boxed{\begin{array}{lll} \sum_{g\in G}\sum_{k\in K_g} y^g_{jk} + \delta_j + f_j \ge d_j & (p_j) & \forall j \in \{N\cup I\} \\ \delta_n \ge 0 & & \forall n \in N \end{array}} \\
& && f \quad \text{satisfy } Cons\_TSO \\
& && y^g \quad \text{satisfy } Cons\_g(x^g) \qquad \forall g \in G.
\end{aligned}
\tag{9}
$$

Let (y^*, δ^*) and p^* be the optimal primal solution (generation quantities, curtailed demand) and optimal dual solution (nodal prices) of OPF problem (9) for a given x, respectively. Boucher and Smeers (2001) show that the solution (y^*, δ^*, p^*) is also a competitive equilibrium of the game between firms and TSO defined in (8) and vice versa. Indeed, it is easy to verify that the set of necessary and sufficient optimality conditions of the LP (9) is equivalent to the MCP (8). Therefore, we may solve the LP (9) directly and take its optimal solution as a perfect competition equilibrium of the spot market at the second stage.

First Stage: At the first stage, each firm $g \in G$ determines its optimal investment quantities x^g maximizing its long term profit which is equal to its optimal short term profit from the spot market at the second stage minus its investment cost. Since firms are price-takers, they act as if the market price is given, and hence the nodal prices, p, appear as parameters in firms' both first and second stage problems (10) and (5), respectively. Given p, each firm $g \in G$ maximizes its long term profit by optimization problem (10) at the first stage:

$$\max_{x^g \geq 0} \quad \sum_{i \in I_g} \sum_{k \in K_g} (p_i - c^g_{ik}) y^{*g}_{ik}(x^g) - \sum_{i \in I_g} \sum_{k \in K_g} \kappa_k x^g_{ik} \tag{10}$$

where $\{y^{*g}_{ik}(x^g), \forall g, i, k\}$ are the optimal generation quantities of firms in the spot market at equilibrium for given $(x^g)_{g \in G}$. Next, in Lemma 3.1, we show that each firm's optimal investment is equal to its optimal generation amount in the spot market at equilibrium when we have constant demand[5]. This is an intuitive result for deterministic investment problems with fixed demand level; however we are not aware of a formal proof.

Lemma 3.1. *Let x^{*g} be the vector of optimal investment quantities of each firm $g \in G$ for (10) and y^* be the vector of optimal generation quantities from OPF problem (9) for $x = x^*$. Then $y^{*g} = x^{*g}$ for all $g \in G$.*

Proof. There are two possibilities for y^{*g} as a solution of OPF problem for $x = x^*$: $y^{*g}_{ik} = x^{*g}_{ik}$, or $y^{*g}_{ik} < x^{*g}_{ik}$. The latter cannot hold at optimum of the first stage problem of firm $g \in G$, since one can always decrease x^{*g}_{ik} to the level of y^{*g}_{ik} and achieve a higher profit. In other words, the latter is always dominated by the former which achieves the same cost for OPF problem and a higher profit for firm $g \in G$. ∎

In the next lemma, we provide a characterization for the optimality conditions of each firm's capacity decisions at the first stage. This type of formulation for deterministic problems has also been formulated in the literature and explicitly illustrates the impact of the scarcity rents determined at the second stage on the firm's investment decisions at the first stage. We see that firms have an incentive to invest if the scarcity rents determined at the second stage offset their investment cost. This is a very intuitive result which can also be observed in early capacity expansion models developed during regulatory periods. A corresponding result (with the expectation of scarcity rents) will appear later when firms choose their capacity under demand uncertainty in Section 3.2.1, as well as when the firms have market power at the first stage as shown by Gürkan et al. (2012).

[5]Note that this result will not hold for all periods and all firms when we have multiple demand periods, see Section 3.2 for details.

Lemma 3.2. *Let $x^* = \{x^{*g}\}_{g\in G}$ be a point such that lower level problem (9) has a feasible solution and $\Pi_g^*(x^g)$ is finite for all $g \in G$ in the neighborhood of x^*. Then x^* is an equilibrium of the first-stage game if and only if there exists β^* such that*

$$0 \le -\beta_{ik}^{*g} + \kappa_k \quad \perp \quad x_{ik}^{*g} \ge 0 \quad \forall g \in G, i \in I_g, k \in K_g. \tag{11}$$

Proof. Firm g's problem (10) can also be formulated as follows:

$$\max_{x^g \ge 0} \quad \Pi_g^*(x^g) - \sum_{i\in I_g}\sum_{k\in K_g} \kappa_k x_{ik}^g$$

where $\Pi_g^*(x^g)$ is the optimal objective function value for problem (5) at a given x^g and given prices. Note that (5) is a linear program where x^g is the right hand side parameter. It is well known that $\Pi_g^*(\cdot)$ is a concave function of x^g. $\Pi_g^*(x^g)$ is also subdifferentiable at x^{*g} when $\Pi_g^*(x^g)$ is finite in the neighborhood of x^*. Hence, x^{*g} is an optimal solution of (10) for each firm $g \in G$ if and only if there exist $\beta_{ik}^{*g} \in \dfrac{\partial \Pi_g^*(x^{*g})}{\partial x_{ik}^g}$ satisfying the necessary and sufficient optimality conditions

$$0 \le -\beta_{ik}^{*g} + \kappa_k \quad \perp \quad x_{ik}^{*g} \ge 0 \quad \forall g \in G, i \in I_g, k \in K_g. \blacksquare$$

Therefore, a solution to this two-stage game, if it exists, should satisfy the optimality conditions given in (11) at the first stage and the optimality conditions given in (8) at the second stage simultaneously, and vice versa. Combining these with Lemma 3.1, we obtain the next lemma.

Lemma 3.3. *If there exists a solution x^* to the two-stage game then it satisfies the following complementarity conditions:*

$$\begin{array}{rcll} 0 \le c_{ik}^g - p_i^* + \kappa_k & \perp & x_{ik}^{*g} \ge 0 & \forall g \in G, i \in I_g, k \in K_g \\ 0 \le \sum_{g\in G}\sum_{k\in K_g} x_{jk}^{*g} + \delta_j^* + f_j^* - d_j & \perp & p_j^* \ge 0 & \forall j \in \{N \cup I\} \\ 0 \le VOLL - p_n^* & \perp & \delta_n^* \ge 0 & \forall n \in N \\ (f^*, p^*, \rho^*, \lambda^{*+}, \lambda^{*-}) & \textit{satisfy} & MCP\_TSO. & \end{array} \tag{12}$$

Moreover, if there exists a solution to the complementarity conditions in (12), then it is a solution to the two-stage game.

Proof. See Appendix.

3.1.2 An Equivalent Single Optimization Problem

It is easy to see that since we will always have $x^* = y^*$ in the two-stage game with constant demand, (10) and (5) reduce to the following single-stage formulation:

$$
\begin{aligned}
\bar{\Pi}_g^*(x^g) := \max_{x^g} \quad & \sum_{i\in I_g}\sum_{k\in K_g}(p_i - c_{ik}^g - \kappa_k)x_{ik}^g \\
s.t. \quad & x_{ik}^g \geq 0 \qquad \forall i \in I_g, k \in K_g.
\end{aligned}
\tag{13}
$$

The market clearance conditions can also be modified such that y^g in (7) is replaced with x^g (say in (7′)). Then, one can easily verify that the equilibrium conditions of the resulting single-stage game ((13), (6), and (7)′) are equivalent to the complementarity conditions given in (12); therefore by using Lemma 3.3, the solution of this single-stage game is equal to the solution of the two-stage game in Section 3.1.1. In other words, in perfect competition open loop and closed loop equilibria coincide.

Next, we show that a perfect competition equilibrium of the two-stage (or one-stage) game can be found by solving a particular single optimization problem which we introduce below in (14). This formulation is nothing but a variation of early capacity expansion models, given in (1), used for decisions of regulated monopolies. In this formulation, one can also think that the investment amounts of all firms are decided by a central decision maker or a regulated monopoly who is minimizing the total cost of the system (i.e., total generation, investment, and dispatch costs):

$$
\begin{aligned}
\min_{\{x,\delta,f\}} \quad & \sum_{g\in G}\sum_{i\in I_g}\sum_{k\in K_g}(c_{ik}^g + \kappa_k)x_{ik}^g + VOLL\sum_n \delta_n \\
s.t. \quad & \sum_{g\in G}\sum_{k\in K_g} x_{jk}^g + \delta_j + f_j \geq d_j \qquad (p_j) \quad \forall j \in \{N \cup I\} \\
& x, \delta \geq 0 \\
& f \text{ satisfy } Cons\_TSO.
\end{aligned}
\tag{14}
$$

Theorem 3.4. *A solution to the optimization problem (14), if it exists, is a solution of the two-stage game. Moreover, if there exists a solution of the two-stage game, then it is also a solution of the optimization problem (14).*

Proof. The necessary and sufficient optimality conditions of the linear program in (14) is equivalent to the complementarity conditions given in (12). By using Lemma 3.3, the result follows immediately.∎

By using Theorem 3.4, the uniqueness and existence of competitive equilibria for the two-stage game in Section 3.1.1 may also be established. The existence follows from the existence of a solution to the optimization problem (14), i.e., if it is feasible and bounded. Moreover, if the solution to (14) is unique, then clearly there is a unique perfect competition equilibrium to the two-stage game.

3.2 Two-stage Equilibrium Model with Stochastic Exogenous Demand

In Section 3.1, we focused on a constant demand load over a year and showed that single (open loop) and two-stage (closed loop) games are equivalent; furthermore, one can find the perfect competition equilibrium by solving a single optimization problem. We have done this to present the notation and the basic properties of the underlying mathematical model. Clearly, demand is seasonal and time varying in reality. Moreover, future demand is uncertain. Both seasonality and uncertainty of future demand affect the choice of the plant type. While a constant yearly demand would lead to selecting a single technology in the solution of (14), seasonality and uncertainty of demand imply a portfolio of technologies. We thus extend the preceding model and consider the more realistic case in which demand is uncertain and varies, say, over a year. First, we prove that one can find a perfect competition equilibrium to the two-stage game under demand uncertainty by solving a two-stage stochastic program. We then show that the equivalence of single and two-stage games still holds in a perfectly competitive market when demand is random and has finite number of possible scenarios. In other words, the open loop equilibria and the closed loop equilibria coincide.

In Section 3.2.1, we analyze the solution of the two-stage competitive game outlined in Section 3.1.1 under demand uncertainty. We then introduce in Section 3.2.2 a two-stage stochastic program where a central decision maker decides on the capacity levels of all firms minimizing the total expected cost at the upper level under demand uncertainty. He/she then chooses the optimal generation quantities of all firms as demand is observed at the lower level. The spirit of this stochastic program is not very far away from the early capacity expansion model given in (1). We end Section 3.2.2 with Theorem 3.7 by showing that a solution of this two-stage stochastic program is also a solution of the two-stage stochastic game. Finally, in Section 3.2.3 we give the single-stage formulation of the two-stage stochastic game when the random demand has discrete distribution. We show that an equilibrium of the single-stage formulation is also an equilibrium of the two-stage formulation

3.2.1 The Perfect Competition Equilibrium under Demand Uncertainty

Consider now the case when the investment decision at the first stage should be made before observing the uncertain demand at the second stage. The notation of the two-stage model under uncertainty will be almost identical to the notation given in Section 2 except we will utilize $\omega \in \Omega$ to denote the uncertainty in demand that can take different values in different states of the world $\omega \in \Omega$, each occurring with some probability. We will also denote the dependency of the second stage variables with respect to ω in order to facilitate the main distinction between the first stage variables and the second stage variables where the former do not depend on $\omega \in \Omega$. To derive our theoretical results, we will assume that the probability distribution of demand, $d(\omega)$, is known. This assumption is valid in situations where the dispatch of electricity and market clearance in the spot market repeats itself and the distribution of demand can be estimated from historical data. In practice, also as part of our numerical procedures in Section 8, we will need only a sample of $d(\omega)$ rather than the entire distribution of $d(\omega)$. Without knowing the exact distribution of the random variable, samples of $d(\omega)$ may be obtained simply from historical data or, for instance, from computation-based simulations (where it may be easier to estimate the so-called basic factors, but since these factors interact in nonlinear and/or non-smooth ways, numerical procedures are needed to draw the samples).

Second Stage: For now, we suppose that $d_n(\omega)$ indicates continuous random demand at node n with a general distribution Ψ_n. Let $(\Omega, \mathscr{F}, \Psi)$ denote the common underlying probability space where Ψ represents the joint probability distribution for the random demand vector $d(\omega) := \{d_n(\omega)\}_{n\in N}$ with $E[|d(\omega)|] < \infty$. Then for given $\omega \in \Omega$, we can write the second stage problem of each firm $g \in G$ in (5), TSO's problem in (6), and the market clearing conditions in (7) in the state of the world ω (i.e., with second stage variables depending on ω). For instance for given x^g, the second stage problem of each firm $g \in G$ in state of the world ω is given as:

$$\begin{aligned} \Pi_g^*(\omega, x^g) := \max_{y^g(\omega)} \quad & \sum_{i\in I_g}\sum_{k\in K_g}(p_i(\omega) - c_{ik}^g) y_{ik}^g(\omega) \\ s.t. \quad & \\ Cons\_g(\omega, x^g): \quad & \boxed{\begin{array}{lll} y^g(\omega) \le x_{ik}^g & (\beta_{ik}^g(\omega)) & \forall i \in I_g, k \in K_g \\ y_{ik}^g(\omega) \ge 0 & & \forall i \in I_g, k \in K_g, \end{array}} \end{aligned} \tag{15}$$

which is identical to (5) when there is one state of the world with constant demand. Note that since all firms are assumed to be price-takers, the nodal prices, $p(\omega)$, appear as parameters in firms' both first and second stage problems.

By similar arguments in Section 3.1.1, we know that we can find a perfect competition equilibrium of the spot market at each ω by solving OPF problem (16). For given x and each $\omega \in \Omega$:

$$\begin{aligned} Z^*(\omega, x) \quad & := \min_{\{y(\omega),\delta(\omega),f(\omega)\}} \sum_{g\in G}\sum_{i\in I_g}\sum_{k\in K_g} c_{ik}^g y_{ik}^g(\omega) + VOLL \sum_n \delta_n(\omega) \\ & s.t. \\ Cons\_Market(\omega) \quad & : \boxed{\begin{array}{lll} \sum_{g\in G}\sum_{k\in K_g} y_{jk}^g(\omega) + \delta_j(\omega) + f_j(\omega) \ge d_j(\omega) & (p_j(\omega)) & \forall j \in \{N \cup I\} \\ \delta_n(\omega) \ge 0 & & \forall n \in N \end{array}} \end{aligned} \tag{16}$$

$$
Cons\_TSO(\omega) \quad : \quad \boxed{\begin{array}{lll} \sum_{j \in \{N \cup I\}} f_j(\omega) = 0 & (\rho(\omega)) & \\ h_l - \sum_{j \in \{N \cup I\}} PTDF_{l,j} f_j(\omega) \geq 0 & (\lambda_l^+(\omega)) & \forall l \\ h_l + \sum_{j \in \{N \cup I\}} PTDF_{l,j} f_j(\omega) \geq 0 & (\lambda_l^-(\omega)) & \forall l \end{array}}
$$

$$
Cons\_g(\omega, x^g) \quad : \quad \boxed{\begin{array}{lll} x_{ik}^g - y_{ik}^g(\omega) \geq 0 & (\beta_{ik}^g(\omega)) & \forall g \in G, i \in I_g, k \in K_g \\ y_{ik}^g(\omega) \geq 0 & & \forall i \in I_g, k \in K_g. \end{array}} \quad \forall g \in G
$$

Remark 3.5. Note that (16) is almost identical to (9) in which we indicate the explicit dependence to ω for the variables that are affected by demand uncertainty. From Boucher and Smeers (2001), we know that at each ω a solution of (16) is also a perfect competition equilibrium of the spot market at the second stage.

First Stage: At the first stage, we assume risk neutral firms making decisions on the basis of the expectation of their short term profit in the spot market. Consequently, we can write the optimization problem of each risk neutral firm $g \in G$ maximizing its long term profit at the first stage:

$$
\max_{x^g \geq 0} \quad E_\omega[\sum_{i \in I_g} \sum_{k \in K_g} (p_i(\omega) - c_{ik}^g) y_{ik}^{*g}(\omega, x^g)] - \sum_{i \in I_g} \sum_{k \in K_g} \kappa_k x_{ik}^g, \tag{17}
$$

where $y^*(\omega, x)$ is the vector of generation quantities of firms in the spot market at equilibrium and, hence is the solution of Optimal Power Flow Problem (16) at the second stage for given x and $\omega \in \Omega$.

It is obvious that we no longer have the equality of first and second stage decision variables as in Lemma 3.1 when we move to the two-stage game under uncertainty. However, one can still use arguments similar to the ones in *Proof of Lemma 3.2* in order to formulate the impact of average scarcity rent received at the second stage on the investment decisions of the firms.

Lemma 3.6. *Let $\Pi_g^*(\omega, x^g)$ be finite at the neighborhood of a point $x^* = \{x^{*g}\}_{g \in G}$ for almost every $\omega \in \Omega$. For investment choice x^{*g}, let $E_\omega[\beta_{ik}^{*g}(\omega)]$ be the expected scarcity rent that firm $g \in G$ receives at the second stage for using technology $k \in K_g$ at node $i \in I_g$. Then x^* is a solution of the first stage game if and only if*

$$
0 \leq -E_\omega[\beta_{ik}^{*g}(\omega)] + \kappa_k \quad \perp \quad x_{ik}^{*g} \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g. \tag{18}
$$

Proof. See Appendix.

One can interpret the expected scarcity rent in (18) as the expected marginal revenue of firm g at node i for investing in technology k. If the expected marginal revenue of investing in technology k at node i is not enough to cover the firm's marginal cost of investment in technology k (κ_k), the firm chooses not to invest in that technology at node i. Otherwise, the firm invests in technology k at node i at a level where the expected scarcity rent is equal to the unit investment cost. (18) is essentially identical to the relation (4) of early capacity expansion models. For peak load generators, $\beta_{ik}^{*g}(\omega)$ is equal to zero unless demand is curtailed. Thus, peak load generators tend to underinvest so that total generation capacity is below the peak load with a certain probability and they cover their investment costs by VOLL during peak hours. Note that (18) represents the first stage equilibrium conditions of risk neutral generators. When generators are assumed to be risk averse, the formulation in (18) can be modified by replacing the statistical probabilities with the risk adjusted probabilities; see Ehrenmann and Smeers (2011) for the corresponding modification.

3.2.2 An Equivalent Two-stage Stochastic Program

In this section we show that one can find an equilibrium to the two-stage stochastic game by simply solving a stochastic program. As a consequence, the computational challenge of finding a solution of the two-stage game may be considerably reduced. The stochastic program we present below may be considered as the capacity expansion problem of a central decision maker who chooses the capacities (x) of all firms at the first stage in order to minimize the total expected cost of the system without knowing the future uncertain demand. He/she then determines the dispatch quantities $y(\omega)$ of all firms after observing the demand (possibly repeatedly) at the second stage by solving (16). The problem faced by the central decision maker at the first stage is formulated as

$$\min_{x\geq 0} \quad E_{\omega}[Z^*(\omega,x)] + \sum_{g\in G}\sum_{i\in I_g}\sum_{k\in K_g} \kappa_k x_{ik}^g. \tag{19}$$

Note that in Section 3.2.1 we have an equilibrium problem at the first stage consisting of $\mid G \mid$ optimization problems, each one given by (17) for $g \in G$. The stochastic program we introduce here consists of a single optimization problem, namely (19), at the first stage and the decision variables are the investment quantities of all firms. At the second stage, we have another single optimization problem for each realization which is formulated by (16).

Theorem 3.7. *Consider the two-stage stochastic program which consists of the problems* (19) *at the first stage and* (16) *at the second stage. Let x^* be the optimal solution of this two-stage stochastic program where $Z^*(\omega,x)$ and $\Pi_g^*(\omega,x^g)$ are finite in the neighborhood of x^* for almost every $\omega \in \Omega$. Then x^* is also a perfect competition equilibrium of the two-stage stochastic game given in Section 3.2.1 and vice versa.*

Proof. (16) is a linear program. Thus, we know that $Z^*(\omega,x)$ is a convex function of x for all $\omega \in \Omega$ which implies the convexity of the expectation $E_{\omega}[Z^*(\omega,\cdot)]$. By using an argument similar to the one in the *Proof of*

Lemma 3.6, one can show that $E_\omega[\beta^{*g}_{ik}(\omega)]$ is unique, $E_\omega[Z^*(\omega,\cdot)]$ is differentiable, and

$$\frac{\partial E_\omega[Z^*(\omega,\cdot)]}{\partial x^g_{ik}} = -E_\omega[\beta^{*g}_{ik}(\omega)]. \tag{20}$$

Since $E_\omega[Z^*(\omega,\cdot)]$ is convex and differentiable, x^* is optimal for the problem (19) if and only if

$$0 \leq \frac{\partial E_\omega[Z^*(\omega,x^*)]}{\partial x^g_{ik}} + \kappa_k \quad \perp \quad x^{*g}_{ik} \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g.$$

By using (20), we can rewrite the necessary and sufficient optimality conditions of (19) as

$$0 \leq -E_\omega[\beta^{*g}_{ik}(\omega)] + \kappa_k \quad \perp \quad x^{*g}_{ik} \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g. \tag{21}$$

The necessary and sufficient optimality conditions in (21) are identical to the equilibrium conditions of the first stage game given in Lemma 3.6. Besides, the necessary and sufficient optimality conditions of (16) are the equilibrium conditions of the second stage game. The solution $(x^*, y^*(\omega,x^*))$ to the two-stage stochastic program is feasible for the first stage game and an equilibrium of the second stage game since it satisfies the optimality conditions of (16). Moreover, it is also an equilibrium of the first stage game since it satisfies the equilibrium conditions at the first stage by equivalence of (21) and (18).

One can also make the same argument for the opposite direction. If $(x^*, y^*(\omega,x^*))$ is an equilibrium of the two-stage stochastic game, then it is a solution to the complementarity problem (18) and it satisfies the optimality conditions of (16). Since (18) is equivalent to (21) and the second stage optimality conditions of both the stochastic program and the stochastic game are derived from the same problem (16), $(x^*, y^*(\omega,x^*))$ will also be a solution of the two-stage stochastic program. ∎

As a conclusion, under the perfect competition assumption one can solve the stochastic optimization problem (19) of the central decision maker and take its optimal solution as equilibrium point of the two-stage stochastic game defined in Section 3.2.1.

3.2.3 Equivalence of Open and Closed Loop Equilibria with Finite Number of Scenarios

In this section, we assume that the demand distribution Ψ has a finite support (e.g., set of time segments) and takes values $d(\omega_1), d(\omega_2), \ldots, d(\omega_M)$ with respective probabilities $\pi_1, \pi_2, \ldots, \pi_M$ (e.g., duration of time segments). Obviously, one could also view $d(\omega_1), d(\omega_2), \ldots, d(\omega_M)$ as a particular sample of the random variable $d(\omega)$ with a general distribution. Then,

$$\begin{aligned} E_\omega[\Pi_g^*(\omega,x^g)] &= E_\omega[\sum_{i\in I_g}\sum_{k\in K_g}(p_i(\omega)-c_{ik}^g)y_{ik}^{*g}(\omega,x^g)] - \sum_{i\in I_g}\sum_{k\in K_g}\kappa_k x_{ik}^g \\ &= \sum_{m=1}^{M}\pi_m \sum_{i\in I_g}\sum_{k\in K_g}(p_i(\omega_m)-c_{ik}^g)y_{ik}^{*g}(\omega_m,x^g) - \sum_{i\in I_g}\sum_{k\in K_g}\kappa_k x_{ik}^g \end{aligned}$$

is the first stage objective function (17) of each firm $g \in G$ where $p(\omega_m)$ is given. Let $y^*(\omega_m, x)$ be the solution of the second stage OPF problem (16) for the scenario ω_m. The equilibrium conditions (18) of the first stage game can easily be restated as:

$$0 \leq -\sum_{m=1}^{M}\pi_m \beta_{ik}^{*g}(\omega_m) + \kappa_k \quad \perp \quad x_{ik}^{*g} \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g. \tag{22}$$

The equilibrium conditions of the second stage game can also be easily modified as the KKT optimality conditions of OPF (16) driven for each scenario. Next, we show that one can find an equilibrium point of the corresponding two-stage stochastic game with finite number of demand scenarios by solving a single-stage stochastic game. We first outline this single-stage game between the firms and TSO.

Each firm $g \in G$ chooses its optimal investment amount and generation quantities simultaneously such that it maximizes its total expected profit in (23):

$$\begin{aligned} \max_{\{x^g, y^g(\omega_m)\}} \quad & \sum_{m=1}^{M}\pi_m \sum_{i\in I_g}\sum_{k\in K_g}(p_i(\omega_m)-c_{ik}^g)y_{ik}^g(\omega_m) - \sum_{i\in I_g}\sum_{k\in K_g}\kappa_k x_{ik}^g \\ s.t. \quad & y^g(\omega_m) \text{ satisfy } Cons\_g(\omega_m, x^g) \qquad \forall m \in M \\ & x^g \geq 0, \end{aligned} \tag{23}$$

where $p(\omega_m)$ is the price observed by each firm in scenario ω_m. $p(\omega_m)$ is exogenous to each firm's problem and TSO's problem whereas it is endogenous to the MCP formulated by the KKT optimality conditions of all firms and TSO together with the market clearance conditions.

TSO's problem (24) is almost identical to (6) except it is formulated with explicit depends on ω_m since its import/export decisions depend on the state of the world (i.e., observed demand $d(\omega_m)$). For each scenario ω_m, TSO solves

$$\begin{aligned} \max_{f(\omega_m)} \quad & \sum_{j\in\{N\cup I\}} p_j(\omega_m) f_j(\omega_m) \\ s.t. \quad & f(\omega_m) \text{ satisfy } Cons\_TSO(\omega_m). \end{aligned} \tag{24}$$

Similarly, the spot market clearance conditions, $MCP\_Market(\omega_m)$, at each state of the world is almost identical to (7) except it is formulated with explicit depends on ω_m.

Next, we show in Theorem 3.8 that if the random demand has discrete distribution then we can find an equilibrium of the corresponding two-stage stochastic game in Section 3.2.1 by solving this single-stage stochastic game.

Theorem 3.8. *Let the demand distribution Ψ has a finite support and takes values $d(\omega_1), d(\omega_2), \ldots, d(\omega_M)$ with respective probabilities $\pi_1, \pi_2, \ldots, \pi_M$. Then an equilibrium of the single-stage stochastic game formulated by* (23), (24)*, and MCP_Market(ω_m), if it exits, is also an equilibrium of the corresponding two-stage stochastic game given in Section 3.2.1 (by* (17) *and* (16)*). Moreover if there exists an equilibrium of this two-stage stochastic game, then it is also an equilibrium of the single-stage stochastic game.*

Proof. See Appendix.

To summarize, the energy-only market model presented in Section 3 provides positive margin for peak load generators when demand is curtailed and electricity price is set at VOLL. As a result, energy-only markets tend to lead to total generation capacity below the peak load with a certain probability. In reality VOLL is difficult to estimate in a direct way. An alternative is to assess the impact of a particular VOLL value on the probability of not meeting the load and to revise this value if this probability is not satisfactory (too high or too low, see Stoft (2002)). In case price caps lower than VOLL are used in practice, this may lead to higher frequency of curtailments and may enhance the resource adequacy problem. Hence, it may be necessary to resort to additional market mechanisms as remedies to create better incentives for capacity investment and operation. Next, we consider two different market designs, namely a capacity market in Section 4 and operating-reserve pricing in Section 5, as potential remedies to resource adequacy problem.

4 Imposing a Capacity Market

An alternative way to avoid resource adequacy problem is to implement a capacity market where the regulator sets a total capacity target based on historic data and expected demand and rewards a side payment to the firms who contribute to reach this target. We continue our analysis by including such a capacity market in our basic model of Section 3. In this modified model, the regulator imposes a capacity constraint on the total capacity which needs to be fulfilled at the time of investment. To account for this, the following modifications are needed:

(i) We impose the following market condition of the regulator at the first stage:

$$0 \leq \sum_{g,i,k} x_{ik}^g - H \quad \perp \quad \lambda \geq 0, \tag{25}$$

where H is the capacity target which may be estimated by the regulator on the basis of the forecasts of the demand fluctuations and λ may be considered as the price of the capacity which is paid to the firms who contribute to the sufficient investment level.

(ii) We also add the corresponding side payments for capacity to each firm's profit at the first stage:

$$\lambda \sum_{i,k} x_{ik}^{g}.$$

Mathematically, these modifications constitute a straight forward extension of the two-stage game in Section 3. When we have constant demand, the equilibrium (KKT optimality) conditions of the first stage game given in Lemma 3.2 can easily be modified to:

$$\begin{array}{rcl} 0 \leq -\beta_{ik}^{*g} - \lambda^* + \kappa_k & \perp & x_{ik}^{*g} \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g, \\ 0 \leq \sum_{g,i,k} x_{ik}^{*g} - H & \perp & \lambda^* \geq 0. \end{array} \tag{26}$$

In case of implementation of a capacity market, the result in Lemma 3.1 (which is established for energy-only markets) holds if $H \leq \sum_j d_j$ whereas a similar type of result given in Theorem 3.4 always holds; that is, it is still possible to show that one can find an equilibrium of this two-stage game by solving a single optimization problem. This optimization problem is a slightly modified version of (14) and is given in (27):

$$\begin{array}{lll} \min_{\{x,y,\delta,f\}} & \sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} (c_{ik}^g y_{ik}^g + \kappa_k x_{ik}^g) + VOLL \sum_n \delta_n & \\ & & \\ s.t. & \sum_{g,i,k} x_{ik}^g - H \geq 0 & (\lambda) \\ & x \quad \geq 0 & \\ & (y,\delta,f) \quad \text{satisfy } Cons\_Market & \\ & f \quad \text{satisfy } Cons\_TSO & \\ & y^g \quad \text{satisfy } Cons\_g(x^g) & \forall g \in G. \end{array} \tag{27}$$

In case of uncertain demand, a similar modification of the equilibrium conditions (18) in Lemma 3.6 can be done for the first stage game:

$$\begin{array}{rcl} 0 \leq -E_\omega[\beta_{ik}^{*g}(\omega)] - \lambda^* + \kappa_k & \perp & x_{ik}^{*g} \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g, \\ 0 \leq \sum_{g,i,k} x_{ik}^{*g} - H & \perp & \lambda^* \geq 0. \end{array} \tag{28}$$

Note that the equilibrium conditions of the second stage game in the future spot market do not change and are thus still formulated by the optimality conditions of the OPF problem (16). Therefore, by similar arguments used for the energy-only market, we can again prove that an equilibrium of this two-stage stochastic game with capacity market can be found by solving a two-stage stochastic program. This stochastic program given in (29) is a slightly modified version of (19) in Section 3.2.2 with the additional deterministic constraint $\sum_{g,i,k} x_{ik}^g \geq H$:

$$\begin{aligned} \min_{x\geq 0} \quad & E_\omega[Z^*(\omega,x)] + \sum_{g\in G}\sum_{i\in I_g}\sum_{k\in K_g} \kappa_k x_{ik}^g \\ s.t. \quad & \sum_{g,i,k} x_{ik}^g - H \geq 0 \qquad\qquad (\lambda). \end{aligned} \tag{29}$$

Hence, the two-stage stochastic program consists of an optimization problem, namely (29), which minimizes the total expected cost of the system subject to a single constraint at the first stage and the OPF problem (16) at the second stage.

5 Operating-reserve Pricing

A more sophisticated approach for the regulator to provide extra capacity payments to the firms contributing to a sufficient level of investment is operating-reserve pricing. The principle of operating-reserve pricing is providing the firms with extra regulated payment whenever the system's total operating-reserve is scarce.

Let $rp(r,x,d)$ be the value of operating-reserves determined by the regulator for a given value of total operating-reserves $r = ex - ey$ where e is the transpose of the vector of 1's of appropriate dimension. Once x, y, and d are observed in the spot market, a mark-up price of $rp(r,x,d)$ is computed by the regulator and then taken as exogenous reserve price by the firms. For given x and d, we assume the following properties of $rp(r,x,d)$.

Assumption 5.1. *$rp(r,x,d)$ is a monotone decreasing and differentiable function of total operating-reserves r where*

- *If $ed = ey \ll ex$, there is ample operating-reserve and there is no markup; that is $rp(r,x,d) = 0$.*
- *If $ed = ey \leq ex$ and ey is close to ex, then the operating-reserve is scarce and the regulator charges consumers with the extra price of $rp(r,x,d)$ in addition to the equilibrium price p.*
- *If $ex = ey < ed$, there is curtailment and the regulator sets the price to $VOLL$ (or to a price cap).*

Next, we modify the formulation of the two-stage game in Section 3 by incorporating operating-reserve pricing scheme. In Sections 5.1 and 5.2, we give the corresponding two-stage model formulations in deterministic and stochastic settings respectively and derive the equilibrium conditions at the first stage. Note that choosing the function $rp(r,x,d)$ is an important issue since it may change the structure of the underlying mathematical formulation of the model and consequently applicable solution methods. In Section 5.3, we elaborate on that issue and give two possible formulations of $rp(r,x,d)$ function. We show that depending on the formulation, one cannot always preserve the simplicity of the single optimization formulation in deterministic setting and the two-stage stochastic program in stochastic setting.

5.1 The Perfect Competition Equilibrium under Constant Demand

In this section, we modify both the first and the second stage problems given in Section 3.1 by incorporating the operating-reserve pricing.

Second Stage: Let γ denote the unit price of operating-reserves. It is exogenous to the firms since they are price-takers and is set by the regulator at equilibrium in the spot market. For a given γ, firm $g \in G$ receives additional revenue, $\sum_{i \in I_g} \sum_{k \in K_g} \gamma(x^g_{ik} - y^g_{ik})$, for its operating-reserves at the second stage. This extra regulated payment received by firm $g \in G$ is added to its objective function at the second stage. The modified second stage problem of each firm $g \in G$ maximizing its short run profit is given as:

$$\begin{aligned} \Pi^{*R}_g(x^g) := \max_{y^g} \quad & \sum_{i \in I_g} \sum_{k \in K_g} (p_i - c^g_{ik} - \gamma) y^g_{ik} + \gamma \sum_{i \in I_g} \sum_{k \in K_g} x^g_{ik} \\ s.t. \quad & \\ Cons\_g(x^g): \quad & \boxed{\begin{array}{lll} y^g_{ik} \leq x^g_{ik} & (\beta^g_{ik}) & \forall i \in I_g, k \in K_g \\ y^g_{ik} \geq 0 & & \forall i \in I_g, k \in K_g, \end{array}} \end{aligned} \tag{30}$$

where p and γ are exogenous parameters to each firm's problem whereas they are endogenous to the whole system and are set at a level where the spot market clears itself. To emphasize, $\gamma^* = rp(ex - ey^*, x, d)$ is the unit operating-reserve price set by the regulator for given x, d and optimal generation dispatch y^* in the spot market at equilibrium. In addition, TSO's problem and the market clearing conditions remain identical to (6) and (7), respectively.

One can write the necessary and sufficient KKT optimality conditions of (30) for each firm $g \in G$, the optimality conditions of (6) for TSO, and the market clearing conditions (7) and set $\gamma^* = rp(ex - ey^*, x, d)$. The resulting KKT conditions are equivalent to the MCP (31). Hence, the solution to MCP (31) is a competitive equilibrium of the spot market. The first complementarity constraint in (31) ensures that when a firm produces positive amount $(y^{*g}_{ik} > 0)$, a price is paid, which covers its marginal cost plus the scarcity rent plus the operating-reserve price $rp(ex - ey^*, x, d)$:

$$
\begin{array}{lll}
0 \le c_{ik}^g - p_i^* + \beta_{ik}^{*g} + rp(ex - ey^*, x, d) & \perp & y_{ik}^{*g} \ge 0 \quad \forall g \in G, i \in I_g, k \in K_g \\
0 \le x_{ik}^g - y_{ik}^{*g} & \perp & \beta_{ik}^{*g} \ge 0 \quad \forall g \in G, i \in I_g, k \in K_g \\
 & & \\
(y^*, \delta^*, f^*, p^*, \rho^*, \lambda^{*+}, \lambda^{*-}) & \text{satisfy} & MCP\_Market \cap MCP\_TSO.
\end{array} \tag{31}
$$

As discussed in Section 3.1, we can formulate the corresponding OPF problem (32) whose solution gives perfect competition equilibrium in the spot market for given x:

$$
\begin{array}{lll}
\min\limits_{\{y,\delta,f\}} & \sum\limits_{g\in G}\sum\limits_{i\in I_g}\sum\limits_{k\in K_g} c_{ik}^g y_{ik}^g + VOLL \sum\limits_{n\in N} \delta_n - R(ex - ey, x, d) & \\
 & & \\
s.t. \quad (y, \delta, f) & \text{satisfy } Cons\_Market & \\
f & \text{satisfy } Cons\_TSO & \\
y^g & \text{satisfy } Cons\_g(x^g) & \forall g \in G,
\end{array} \tag{32}
$$

in which $R(ex - ey, x, d) := \int_0^{ex-ey} rp(s, x, d)ds$ can be interpreted as the additional welfare due to improved reliability, or willingness to pay for extra reliability; it is similar to the integral of the inverse demand function (e.g., in (2) or (42)) which is interpreted as consumers' willingness to pay. By Assumption 5.1, for given x and d, $rp(ex - ey, x, d)$ is a monotone decreasing function of $ex - ey$; therefore $R(ex - ey, x, d)$ is a concave function of $(ex - ey)$ and consequently it is concave in y. Hence OPF problem (32) is convex. The solution of OPF problem (32) is a competitive equilibrium of the modified game in the spot market since its necessary and sufficient KKT optimality conditions are equivalent to the MCP (31).

First Stage: Similar to Section 3.1, each firm $g \in G$ maximizes its long term profit at the first stage:

$$
\max_{x^g \ge 0} \quad \Pi_g^{*R}(x^g) - \sum_{i\in I_g}\sum_{k\in K_g} \kappa_k x_{ik}^g. \tag{33}
$$

When we consider the interaction between first and second stage problems, Lemma 3.1 does not hold anymore. Instead, we next show that optimality conditions of each firm's problem at the first stage now involves the scarcity rent and the operating-reserve price it receives at the second stage.

Theorem 5.2. *Let $x^* = \{x^{*g}\}_{g\in G}$ be such that OPF problem (32) has a feasible solution and $\Pi_g^{*R}(x^g)$ is finite for all $g \in G$ in the neighborhood of x^*. Then x^* is an equilibrium of the first-stage game if and only if there exists β^* and y^* such that*

$$
0 \le -\beta_{ik}^{*g} - rp(ex^* - ey^*, x^*, d) + \kappa_k \quad \perp \quad x_{ik}^{*g} \ge 0 \quad \forall g \in G, i \in I_g, k \in K_g. \tag{34}
$$

Proof. See Appendix.

5.2 The Perfect Competition Equilibrium under Demand Uncertainty

We next formulate the two-stage game including operating-reserve pricing scheme when demand, $d_n(\omega)$, at each node n is random having a general distribution Ψ_n. Let $(\Omega, \mathscr{F}, \Psi)$ denote the common underlying probability space where Ψ represents the joint probability distribution for the random demand vector $d(\omega) := \{d_n(\omega)\}_{n \in N}$ with $E[|d(\omega)|] < \infty$.

Second Stage: Let $\gamma(\omega)$ be the unit price of reserved capacity at demand realization $\omega \in \Omega$. As mentioned earlier, it is exogenous to the firms at both stages since they are price-takers and is set by the regulator at a level where the spot market clears itself: $\gamma^*(\omega) = rp(ex - ey^*(\omega), x, d(\omega))$.

For given $\omega \in \Omega$, the second stage game is identical to its deterministic formulation given in Section 5.1 except with explicit dependence on ω. Hence, the arguments in Section 5.1 hold when demand is stochastic as well. That is, for each ω, $\Pi_g^{*R}(\omega, x^g)$ is a concave function of x^g and $\beta_{ik}^{*g}(\omega) + \gamma^*(\omega)$ is a subgradient of $\Pi_g^{*R}(\omega, x^g)$ at a given x^g. Moreover, one can solve the OPF problem (35) to find an equilibrium of the spot market at the second stage.

For given x and $\omega \in \Omega$, we have $R(ex - ey(\omega), x, d(\omega)) = \int_0^{ex - ey(\omega)} rp(s, x, d(\omega)) ds$. Then the corresponding OPF problem at the second stage is formulated as:

$$\begin{aligned}
\min_{\{y(\omega), \delta(\omega), f(\omega)\}} \quad & \sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} c_{ik}^g y_{ik}^g(\omega) + VOLL \sum_{n \in N} \delta_n(\omega) - R(ex - ey(\omega), x, d(\omega)) \\
s.t. \quad & (y(\omega), \delta(\omega), f(\omega)) \quad \text{satisfy } Cons_Market(\omega) \\
& f(\omega) \quad \text{satisfy } Cons_TSO(\omega) \\
& y^g(\omega) \quad \text{satisfy } Cons_g(\omega, x^g) \qquad \forall g \in G.
\end{aligned} \tag{35}$$

First Stage: Assuming risk neutral firms, each firm $g \in G$ maximizes its long term expected profit at the first stage:

$$\max_{x^g \geq 0} \quad E_\omega[\Pi_g^{*R}(\omega, x^g)] - \sum_{i \in I_g} \sum_{k \in K_g} \kappa_k x_{ik}^g, \tag{36}$$

where, for given x^g, $\Pi_g^{*R}(\omega, x^g)$ is the optimal value of firm g's problem in the spot market(short term profit of firm $g \in G$) at realization $\omega \in \Omega$.

Similar to Lemma 3.6, it is possible to write the equilibrium conditions of the first stage game in terms of expected scarcity rent and operating-reserve prices that the firms receive at the second stage which is stated in the next theorem.

Theorem 5.3. *Let* $\Pi_g^{*R}(\omega, x^g)$ *be finite at the neighborhood of a point* $x^* = \{x^{*g}\}_{g\in G}$ *for almost every* $\omega \in \Omega$. *For investment choice* x^{*g}, *let* $E_\omega[\beta_{ik}^{*g}(\omega) + rp(ex^* - ey^*(\omega), x^*, d(\omega))]$ *be the expected marginal revenue that firm* $g \in G$ *receives at the second stage for using technology* $k \in K_g$ *at node* $i \in I_g$. *Then* x^* *is an equilibrium of the first stage game if and only if there exists* $\beta^*(\omega)$ *and* $y^*(\omega)$ *such that*

$$-E_\omega[\beta_{ik}^{*g}(\omega) + rp(ex^* - ey^*(\omega), x^*, d(\omega))] + \kappa_k \quad \perp \quad x_{ik}^{*g} \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g. \tag{37}$$

Proof. See Appendix.

5.3 Operating-reserve Price Curve

Operating-reserve prices are based on predetermined reserve target levels set by the regulator. How to determine these target levels is an important issue since they would have an effect on the investment incentives. Moreover, the formulation of the $rp(\cdot)$ function would also effect the mathematical properties of the resulting two-stage game. Next, we give two different formulations for the $rp(\cdot)$ function. In the first one, the reserve targets are set by predetermined ratios of observed demand. In the latter one, the reserve targets are set by the predetermined ratios of installed capacity. We show that while the two-stage game with the first formulation is still convertible to a single-stage optimization problem in the deterministic setting and to a two-stage stochastic program in the stochastic setting, this simplicity is lost when the target levels depend on the installed capacity.

5.3.1 Setting Reserve Targets Based on Observed Demand

In this case, the regulator determines the reserve targets based on observed demand. Therefore, we take the operating-reserve price function as

$$rp(ex - ey, x, d) := rp\_demand(\frac{ex - ey}{ed}).$$

An example of the corresponding price curve for a total fixed demand level (ed), taken from Hogan (2005), is depicted in Figure 1. The x-axis denotes the percentage of over-capacity (or operating-reserves) with respect to the observed demand. For given x and d, $rp\_demand(\frac{ex-ey}{ed})$ is a piecewise linear decreasing function of operating-reserves. In the operating-reserve price curve of Hogan (2005), the critical operating-reserve levels predetermined by the regulator are the minimum level of reserve (3% of demand) and the nominal reserve target (7% of demand). The minimum level of reserves is set by the regulator to prevent a catastrophic failure through a widespread and uncontrolled blackout in the system. The regulator would not go below this level of reserves even if this required curtailment of inflexible demand. Above this minimum level, there would be more flexibility up to nominal reserve target (7%). This is the price-sensitive part of the operating-reserve price curve illustrated in Figure 1. In the range between 3% – 7%, as reserve levels approach the nominal target, the operating-reserve price would be decreasing.

Note that Hogan's operating-reserve price curve given in Figure 1 is piecewise linear. In our analysis, in order to preserve twice differentiability in OPF problems (32) and (35), we assume a differentiable operating-reserve price curve which is a smooth approximation of Hogan's curve. The details of this approximation using a sigmoid function are given in Section 9 and an example of the corresponding differentiable operating-reserve price curve is illustrated in Figure 1.

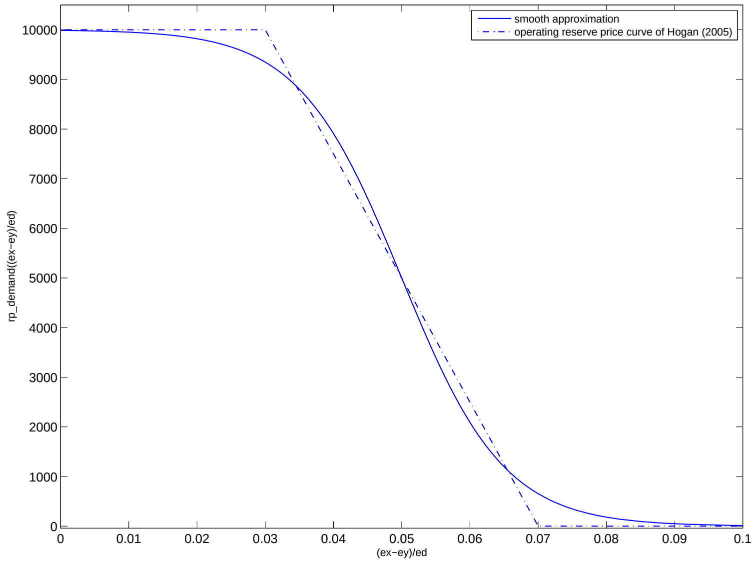


Figure 1: Operating-reserve price curve which is a smooth approximation of Hogan's curve on basis of observed demand

Next, for given x and d, we can calculate $R(ex-ey,x,d)$ as

$$\begin{aligned} R(ex-ey,x,d) &:= R\_demand(\tfrac{ex-ey}{ed}) \\ &= \int_0^{ex-ey} rp\_demand(\frac{s}{ed})ds = ed\int_0^{\frac{ex-ey}{ed}} rp\_demand(s)ds. \end{aligned}$$

We know from Assumption 5.1 that $rp\_demand(\frac{ex-ey}{ed})$ is a monotone decreasing function of $\frac{ex-ey}{ed}$; therefore $R\_demand(\frac{ex-ey}{ed})$ is concave in $\frac{ex-ey}{ed}$ which is an affine function of (x,y). As we mentioned above, we assume that $rp\_demand(\frac{ex-ey}{ed})$ is differentiable; hence $R\_demand(\frac{ex-ey}{ed})$ is twice differentiable. Moreover, it is concave in (x,y) with

$$\frac{\partial R\_demand(\frac{ex-ey}{ed})}{\partial x_{ik}^g} = rp\_demand(\frac{ex-ey}{ed}), \text{ and} \tag{38}$$

$$\frac{\partial^2 R\_demand(\frac{ex-ey}{ed})}{\partial (x^g_{ik})^2} = \frac{\partial rp\_demand(\frac{ex-ey}{ed})}{\partial(\frac{ex-ey}{ed})}\frac{1}{ed} \leq 0.$$

Remark 5.4. Joint concavity in (x,y) of $R\_demand(\frac{ex-ey}{ed})$ can be seen by constructing its Hessian matrix of and showing that it is negative semi-definite. The details of the proof are given in Appendix 10.

(38) indicates that a unit increase of x^g_{ik} indeed entails a marginal revenue for firm g from operating-reserves, which is equal to the operating-reserve price. By utilizing (38), we next show that a solution to (39) is a perfect competition equilibrium of the two-stage deterministic game:

$$\begin{aligned}
\min_{\{x,y,\delta,f\}} \quad & \sum_{g\in G}\sum_{i\in I_g}\sum_{k\in K_g}(c^g_{ik}y^g_{ik} + \kappa_k x^g_{ik}) + VOLL\sum_{n\in N}\delta_n - R_demand(\frac{ex-ey}{ed}) \\
s.t. \quad (y,\delta,f) \quad & \text{satisfy } Cons_Market \\
f \quad & \text{satisfy } Cons_TSO \\
y^g \quad & \text{satisfy } Cons_g(x^g) \qquad \forall g \in G \\
x \quad & \geq 0.
\end{aligned} \tag{39}$$

Theorem 5.5. *A solution to the optimization problem* (39)*, if it exists, is a solution of the two-stage deterministic game with operating-reserve pricing. Moreover, if there exists a solution of the two-stage deterministic game, then it is also a solution of the optimization problem* (39)*.*

Proof. The nonlinear program in (39) is convex in (x,y,δ,f). The necessary and sufficient optimality conditions of NLP (39) is equivalent to the first and second stage equilibrium conditions given in (34) and (31) respectively. The result follows immediately. ∎

Next, we obtain the corresponding result in the stochastic setting; that is, a solution to the two-stage stochastic program given in (40) is a perfect competition equilibrium of the two-stage stochastic game.

$$\min_{x\geq 0} \quad E_\omega[Z^{*R}(\omega,x)] + \sum_{i\in I_g}\sum_{k\in K_g}\kappa_k x^g_{ik}, \tag{40}$$

where

$$\begin{aligned}
Z^{*R}(\omega,x) = \min_{\{y(\omega),\delta(\omega),f(\omega)\}} \quad & \sum_{g\in G}\sum_{i\in I_g}\sum_{k\in K_g} c^g_{ik}y^g_{ik}(\omega) + VOLL\sum_{n\in N}\delta_n(\omega) - R_demand(\frac{ex-ey(\omega)}{ed(\omega)}) \\
s.t. \quad (y(\omega),\delta(\omega),f(\omega)) \quad & \text{satisfy } Cons_Market(\omega) \\
f(\omega) \quad & \text{satisfy } Cons_TSO(\omega) \\
y^g(\omega) \quad & \text{satisfy } Cons_g(\omega,x^g) \qquad \forall g \in G.
\end{aligned}$$

Theorem 5.6. *Let x^* be an optimal solution of the two-stage stochastic program formulated in* (40) *where $Z^{*R}(\omega,x)$ and $\Pi^{*R}(\omega,x^g)$ are finite in the neighborhood of x^* for almost every $\omega \in \Omega$. Then x^* is also a perfect competition equilibrium of the two-stage stochastic game with operating-reserve pricing and vice versa.*

Proof. By using the concavity of $R\_demand(\frac{ex-ey(\omega)}{ed(\omega)})$ in x, we know that $Z^{*R}(\omega,x)$ is a convex function of x for all $\omega \in \Omega$ which implies the convexity of $E_\omega[Z^{*R}(\omega,x)]$. One can compute the components of the gradient of $Z^{*R}(\omega,x)$ as:

$$\begin{aligned}\frac{\partial Z^{*R}(\omega,x)}{\partial x_{ik}^g} &= -\beta_{ik}^{*g}(\omega) - \frac{\partial R\_demand(\frac{ex-ey^*(\omega)}{ed(\omega)})}{\partial(\frac{ex-ey^*(\omega)}{ed(\omega)})} \cdot \frac{1}{ed} \\ &= -\beta_{ik}^{*g}(\omega) - rp\_demand(\frac{ex-ey^*(\omega)}{ed(\omega)}),\end{aligned}$$

except on a set L of Lebesgue measure zero.

By utilizing a similar argument in the *Proof of Theorem 3.7*, one can show that $E_\omega[Z^{*R}(\omega,x)]$ is differentiable and

$$\frac{\partial E_\omega[Z^{*R}(\omega,x^*)]}{\partial x_{ik}^g} = -E[\beta_{ik}^{*g}(\omega) + rp\_demand(\frac{ex^* - ey^*(\omega)}{ed(\omega)})].$$

Then one can easily derive the necessary and sufficient optimality conditions of the two-stage stochastic program (40) which are equivalent to the equilibrium conditions of the first stage game given in (37) and the optimality conditions of OPF problem (35). ∎

5.3.2 Setting Reserve Targets Based on Installed Capacity

Next, we deal with the case that the regulator determines the reserve targets based on total installed capacity. An example of the corresponding operating-reserve price curve for a fixed total installed capacity (ex) is given in Figure 2. This time the x-axis denotes the percentage of unused capacity with respect to the total available capacity. We assume the same minimum level of reserve (3% of total capacity) and the nominal reserve target (7% of total capacity) as in Figure 1. This time, working with an operating-reserve price curve like Figure 2 would mean that we take $rp(ey-ex,x,d) := rp\_capacity(\frac{ex-ey}{ex})$ which is again a differentiable function. Then, for given x, we can compute the corresponding willingness to pay for improved reliability as:

$$\begin{aligned}R(ex-ey,x,d) &:= R\_capacity(\frac{ex-ey}{ex}) \\ &= \int_0^{ex-ey} rp\_capacity(\frac{s}{ex})ds = ex\int_0^{\frac{ex-ey}{ex}} rp\_capacity(s)ds.\end{aligned}$$

Since $rp\_capacity(\frac{ex-ey}{ex})$ is differentiable, $R\_capacity(\frac{ex-ey}{ex})$ is twice differentiable. Moreover,

$R\_capacity(\frac{ex-ey}{ex})$ is concave in (x,y) with

$$\frac{\partial R\_capacity(\frac{ex-ey}{ex})}{\partial x_{ik}^{g}} = \int_0^{\frac{ex-ey}{ex}} rp\_capacity(s)ds + rp\_capacity(\frac{ex-ey}{ex})\frac{ey}{ex}, \text{ and}$$
$$\frac{\partial^2 R\_capacity(\frac{ex-ey}{ex})}{\partial (x_{ik}^{g})^2} = \frac{\partial rp\_capacity(\frac{ex-ey}{ex})}{\partial (\frac{ex-ey}{ex})}\frac{ey^2}{ex^3} \leq 0, \ \forall x \geq 0, \tag{41}$$

where ey^2 is the sum of squares for all the elements of a vector y and ex^3 is the sum of cubes for all the elements of a vector x.

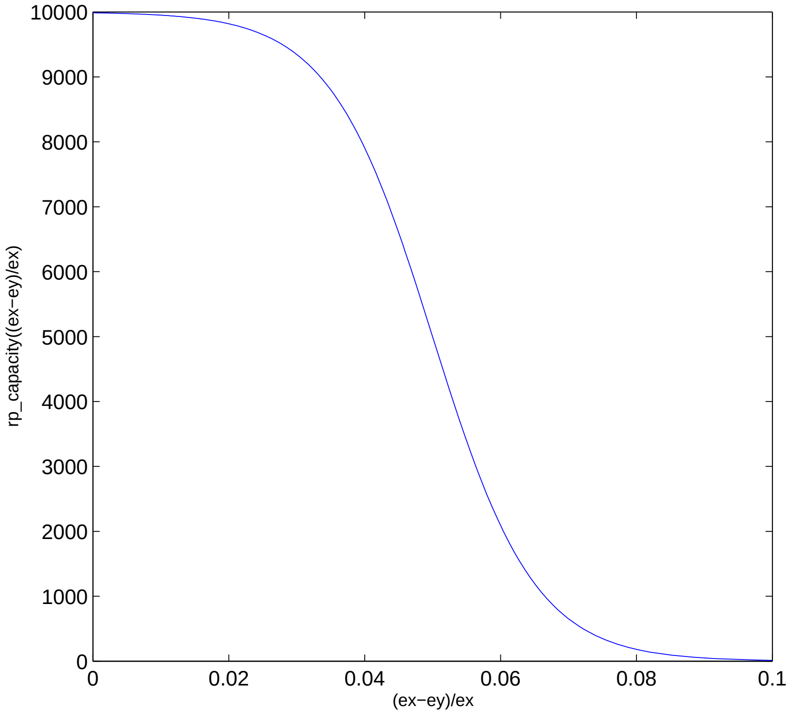


Figure 2: Operating-reserve price curve on basis of installed capacity

Remark 5.7. Similar to Remark 5.4, joint concavity in (x,y) of $R\_capacity(\frac{ex-ey}{ex})$ can be seen by constructing the Hessian matrix of $R\_capacity(\frac{ex-ey}{ex})$ and showing that it is negative semi-definite. The details of the proof are given in Appendix 10.

The first equation in (41) indicates that a unit increase of x_{ik}^{g} entails a marginal revenue from operating-reserves, which is different from the operating-reserve price. In other words, when we calculate the derivative of $R\_capacity$ with respect to x_{ik}^{g}, we do not get $rp\_capacity(\frac{ex-ey}{ex})$. Therefore, we cannot get the KKT conditions given in (11) from an optimization problem; hence, we can no longer reduce the two-stage game formulation to a single optimization problem in the deterministic setting and to a two-stage stochastic program in the stochastic setting. However, we can still formulate a mixed complementarity problem (MCP) which involves the equilibrium conditions of both first and second stage games and solve the corresponding MCP. We explain the details of solution methods for these problems in Section 8.

6 Demand-side Bidding (or an Endogenous Demand Curve)

In previous sections we consider three different ways of regulator's intervention to provide incentives for sufficient investment levels in electricity markets. Without the intervention of the regulator, the power systems tend to underinvest in generation capacity because of imperfections in the market due to unresponsive demand. When demand responds to price, regulator's intervention is needed less and the electricity market is more likely to operate normally such that the prices clear the market where supply meets the demand without any curtailments (e.g., Allcott (2012)). Therefore, if we consider "demand-side bidding" as a mechanism to ensure a price responsive demand, then it can be thought of as an alternative way of addressing the resource adequacy issue. In this section, we briefly comment on an electricity market where consumers can respond to prices.

When consumers respond to prices, we have an elastic demand and we represent the reaction of consumers to the prices by decreasing price-demand curves $p_n(d)$ with $p_n(0) < \infty$ for each demand node $n \in N$. On such a curve, consumers at demand node n choose their consumption level which maximizes their surplus at a given price p_n. We may represent the decision making process of the consumers at the second stage by incorporating, for each demand node n, an optimization problem which maximizes the consumer surplus. Let d_n^* denote the optimal consumption at demand node n such that

$$d_n^* := \arg\max_{d_n \geq 0} \{U_n(d_n) - p_n d_n\}, \tag{42}$$

where $U_n(d_n) = \int_0^{d_n} p_n(s)ds$ denotes the willingness to pay function.

Incorporating the consumers' problem will slightly change the equilibrium conditions of the second stage game. Note that $p(\cdot)$ is a decreasing function which leads to $U_n(\cdot)$ being a concave function. Therefore, we can still formulate a convex OPF problem whose solution maximizes the profit of firms, the surplus of consumers, and the profit of TSO in charge of operating the network as indicated in Boucher and Smeers (2001). The modified OPF problem will slightly be different from the ones introduced previously such that its objective functions will also include maximizing $U_n(\cdot)$. For example, the OPF problem involving price-demand curves with deterministic parameters will be formulated, for given x, as

$$Z^*(x) = \min_{\{y,f,d\}} \sum_{g\in G}\sum_{i\in I_g}\sum_{k\in K_g} c_{ik}^g y_{ik}^g - \sum_{n\in N} U(d_n)$$

$$s.t. \sum_{g\in G}\sum_{k\in K_g} y_{jk}^g + f_j \geq d_j \quad (p_j) \qquad \forall j \in \{N \cup I\} \tag{43}$$

$$\begin{aligned} d &\geq 0 \\ f &\text{ satisfy } Cons\_TSO \\ y^g &\text{ satisfy } Cons\_g(x^g) \qquad \forall g \in G, \end{aligned}$$

where d_n is the demand of consumers as a reaction of price p_n at node n.

When the parameters in the price-demand curve are assumed to be deterministic, we end up with a two-stage deterministic game in which the first and second stage optimization problems of firm $g \in G$ are still the same. Hence, the arguments in Lemmas 3.1 and 3.2 still hold. Moreover, $Z^*(x)$ in OPF problem (43) is a convex function of x. By using similar arguments in *Proof of Theorem 3.4*, one can still formulate a single optimization problem which finds an equilibrium of the corresponding two-stage deterministic game.

Furthermore, the approach in previous sections that assumes a fixed random demand can be extended to a random demand function where key parameters such as reference consumption for a given reference price and elasticities are random. Price elasticities are only known with very little accuracy. It thus makes sense to treat them as uncertain and to embed such parameters in the state of the world. When some parameters in price-demand curves are random, we have a two-stage stochastic game in which the first and second stage optimization problems of firm $g \in G$ in Section 3.2 remain the same. Similarly, the arguments in Lemma 3.6 and Theorem 3.8 still hold. One can write OPF problem (43) with explicit dependency on ω and the corresponding optimal value $Z^*(\omega, x)$ will be a convex function of x for every $\omega \in \Omega$. Therefore, $E_\omega[Z^*(\omega, \cdot)]$ is also convex. By using a similar argument in *Proof of Theorem 3.7*, one can still formulate a two-stage stochastic program which solves the corresponding two-stage stochastic game.

The standard view in the approach outlined so far is to assume a demand response with a particular functional form and possibly with some random parameters. This idea raises some questions though. The power generation part of the model discussed so far is long-term, in the sense that investments can change the capacity structure of the generation system; therefore, the response of the power system to price changes takes place both through modifications of plant operations and capacities. In contrast, the representation of consumption embedded in a demand function such as (42) does not offer that dual long and short term representation. Its most standard interpretation is to assume that it reflects demand-side bidding; that is, participation of the demand to the short-term power market. $p_n(d)$ is typically a short-run response of the demand to price with given capacity in the consuming sector. This creates a model inconsistency between the representation of the supply and demand sectors that can only be removed by introducing a more complex demand model that accounts for both the long run changes of capacity structure in the consuming sector and the short run response of demand with given capacity; for example a representation of consumer decision making including investments in durable energy, using equipment, and habit formation which results in a short run (very low) elasticity, but a longer term adjustment with higher long term elasticity (e.g., Celebi and Fuller (2012)). This discussion goes beyond the scope of this paper.

7 More General Forms of Uncertainties in Spot Markets

In this section, we give a summary of possible extensions to generalize the basic assumption of uncertain demand. In the previous sections, we assume that the demand in the spot market is possibly random. It is also possible to view some of the other parameters as random. These parameters include generation costs $c(\omega)$, transmission capacities $h_l(\omega)$, and power transfer distribution factors $PTDF(\omega)$ with $E_\omega[\| c(\omega) \|] < \infty$, $E_\omega[\| h_l(\omega) \|] < \infty$, and $E_\omega[\| PTDF(\omega) \|] < \infty$, respectively. When these parameters are random, one can prove that the results obtained throughout this paper still hold as argued briefly in the following propositions.

Proposition 7.1. *Let c, h, and $PTDF$ be random and let the second stage problem of each firm $g \in G$ and TSO be feasible and bounded in the neighborhood of a feasible point x^*. Then the equilibrium conditions of the first stage game given in Lemma 3.6 (energy-only market), MCP* (28) *(energy market with capacity requirements), and Theorem 5.3 (energy market with operating-reserve pricing) still hold.*

Proof: The second stage revenues of firm $g \in G$, $\Pi^*_g(\omega,\cdot)$ and $\Pi^{*R}_g(\omega,\cdot)$, are concave functions of x^g for every $\omega \in \Omega$ and $g \in G$ regardless of the random parameter. By similar arguments in *Proofs of Lemma 3.6* and *Theorem 5.3*, one can derive the equilibrium conditions (18), (28), and (37) for the first stage game. ▮

Proposition 7.2. *Let c, h, and $PTDF$ be random and let the corresponding lower level problem* (16) *be feasible and bounded in the neighborhood of a feasible point x^*. Then in case of an energy-only market, an energy market with capacity requirements, an energy market with operating-reserve pricing based on observed demand as given in Figure 1, or an energy market with demand bidding, the equivalence result with respect to the equilibrium of the two-stage game and the solution of a two-stage stochastic program established under random demand in previous sections (e.g., Theorem 3.7) still holds. That is, one can find a perfect competition equilibrium of these markets by solving the corresponding two-stage stochastic program (from the perspective of a central planner).*

Proof: The optimal value of the corresponding OPF problem, $Z^*(\omega,x)$, is a convex function of x for every $\omega \in \Omega$ regardless of the random parameter which implies the convexity of the expected system cost $E_\omega[Z^*(\omega,\cdot)]$ incurred at the second stage. By utilizing Proposition 7.1 and using a similar argument in *Proof of Theorem 3.7*, the results follow. ▮

8 Computational Methods for Solving Two-stage Equilibrium Models

In this section, we discuss the possible computational methods for solving the two-stage games numerically in four different market settings we have considered; namely energy-only markets, energy markets including forward capacity market, energy markets with operating-reserve pricing, and demand bidding.

8.1 Solving Two-stage Deterministic Equilibrium Models

MCP Approach

We showed that when we have constant demand, we can always write the equilibrium conditions of the two-stage game as a mixed complementarity problem; for instance the MCP (12) in energy-only markets, the MCP consisting of the KKT conditions of (27) in energy markets with forward capacity market, the MCP consisting of (31) and (34) in energy markets with operating-reserve pricing, and finally the MCP consisting of (11) and the KKT conditions of (43) in energy markets with demand bidding. We can solve these MCPs by using a state of the art MCP solver such as PATH (Dirkse and Ferris (1995) and Ferris and Munson (2008)). Our results ensure that a point satisfying such an MCP is indeed a solution to the original two-stage game.

(N)LP Approach

We showed in Section 3.1.2 that we can find an equilibrium of the two stage game in energy-only markets by solving the linear program (14). In Sections 4 and 5.3.1, we showed that we can extend this result to energy markets with forward capacity market and energy markets with operating-reserve pricing based on an observed demand. Hence instead of solving the MCPs of these two-stage games, we can solve the corresponding (N)LPs (14), (27), and (39) respectively and take the solution as an equilibrium point of the corresponding two-stage game in these market designs. Moreover as mentioned in Section 6, under the assumption of demand response we can again formulate a single optimization problem which is a nonlinear program, solve it using an off the shelf nonlinear programming solver, and take its optimal solution as an equilibrium point of the corresponding market. LPs may be much simpler to solve compared to MCPs, especially when we have realistic systems of large networks. Depending on the problem, NLPs may or may not be easier to solve than MCPs.

One should also note that it may not be possible to formulate a single optimization problem for every two-stage game under perfect competition as we elaborate this issue in Section 5.3.2 for energy markets with operating-reserve pricing based on installed capacities; in that case, one needs to resort to an MCP approach to provide numerical solutions.

8.2 Solving Two-stage Stochastic Equilibrium Models

In general, when we have demand uncertainty, we cannot observe the expected function values explicitly; hence we propose to approximate them by the corresponding sample average functions and solve the resulting approximate problem. The basic method we use is known as *sample-path method* or *sample average approximation method*; see for example Robinson (1996) and Shapiro and Homem-De-Mello (1998) for the theoretical background in optimization context. Roughly speaking, in sample-path methods the stochastic problem is observed for a fixed and long sample-path by fixing a large sample size and using the method of common random numbers. Since the sample path and length are fixed, the approximate problem actually becomes a deterministic problem. The resulting deterministic problem is solved by fast and effective solution methods available and its solution is taken as the approximate solution of the stochastic problem. We refer the interested reader to Gürkan et al. (1999) and Gürkan and Pang (2009) for the theoretical analysis of the sample-path method for solving stochastic equilibrium models.

Next, we explain how we use the basic idea of *sample-path methods* to solve the two-stage stochastic games in each market setting. We again propose two different solution approaches; in the solution approaches outlined below, we use a large, fixed sample size M and an i.i.d. sample point $\omega := \{\omega_1, \omega_2 \ldots, \omega_M\}$. Let $\beta(\omega_1), \beta(\omega_2), \ldots, \beta(\omega_M)$ be the vectors of scarcity rents corresponding to this sample.

MCP Approach

We can always formulate the approximate two-stage stochastic game as a potentially very large MCP. This MCP consists of the KKT conditions of every firm's first and second-stage problems for all realizations in the sample $\{\omega_m\}_{m=1}^M$. For instance, consider the energy-only market in Section 3. We cannot observe $E_\omega[\beta_{ik}^{*g}(\omega)]$; however using our random sample of size M, we can approximate it by a sample average function $-\frac{1}{M}\sum_{m=1}^M \beta_{ik}^{*g}(\omega_m)$. It follows from the strong law of large numbers that $-\frac{1}{M}\sum_{m=1}^M \beta_{ik}^{*g}(\omega_m)$ converges to $E_\omega[\beta_{ik}^{*g}(\omega)]$ with probability 1 as M gets large. We can then solve the MCP system which actually consists of (44) below and the KKT conditions of OPF problem (16) for all $\{\omega_m\}_{m=1}^M$ and take its solution as an approximate solution of the two-stage stochastic game in Section 3.2:

$$0 \le -\frac{1}{M}\sum_{m=1}^{M} \beta_{ik}^{*g}(\omega_m) + \kappa_k \quad \perp \quad x_{ik}^{*g} \ge 0 \quad \forall g \in G, i \in I_g, k \in K_g. \tag{44}$$

We can use the same approach for formulating the MCPs of the other three electricity market designs; namely energy markets including forward capacity market, with operating-reserve pricing, and with demand response. Then we can solve the corresponding MCP by a deterministic solver such as PATH. However, this approach has the following drawback: The size of the MCP increases rapidly with the sample size M and the size of the network. Therefore, straightforward construction of an MCP in the stochastic setting for realistic networks and solving this large scale system by using the currently available software may be a computational challenge, as we briefly illustrate in Section 9.

Implicit Function Approach

Energy-only Markets: We showed in Section 3.2.2 that we can find an equilibrium of the two-stage stochastic game in an energy-only market by solving a two-stage stochastic program. The first and second stages of this two-stage program are formulated in (19) and (16) respectively. As mentioned earlier, we cannot observe the expected cost function $E_\omega[Z^*(\omega,x)]$ in the first-stage problem (19); however we can again approximate it by a sample-average function using the sample $\{\omega_m\}_{m=1}^M$ and solve the approximate problem by using sample-path optimization.

We know that the following holds for the sample-average estimator of the expected cost and its (sub)gradient:

$$\begin{aligned}
\frac{1}{M}\sum_{m=1}^{M} Z^*(\omega_m,x) \quad &\to E_\omega[Z^*(\omega,x)] \text{ as } M\to\infty \text{ almost surely for given} x, \text{and} \\
\frac{1}{M}\sum_{m=1}^{M} \beta_{ik}^{*g}(\omega_m) \quad &\to E_\omega[\beta_{ik}^{*g}(\omega)] = \frac{\partial E_\omega[Z^*(\omega,x)]}{\partial x_{ik}^g} \text{ as } M\to\infty \text{ almost surely.}
\end{aligned} \tag{45}$$

We can construct the approximate problem (46) below to approximate (19):

$$\min_{x\geq 0} \quad \frac{1}{M}\sum_{m=1}^{M} Z^*(\omega_m,x) + \sum_{g\in G}\sum_{i\in I_g}\sum_{k\in K_g} \kappa_k x_{ik}^g. \tag{46}$$

Note that (46) is in general an NLP and we can try to solve it by using a standard NLP solver. An efficient NLP solver would require us to provide function values as well as (sub)gradient values of the objective function (46) as input. Although we do not have an explicit expression for $Z^*(\omega_m,x)$, given ω_m and x, we can solve the second stage problem (16) numerically (by using a standard LP solver) to obtain $Z^*(\omega_m,x)$ at that point. Once the optimal solution of (16) is found, we also have $\beta_{ik}^{*g}(\omega_m) \in \frac{\partial Z^*(\omega_m,x)}{\partial x_{ik}^g}$ for every g,i, and k as a by-product. Since x is a right-hand side parameter in (16), $\beta_{ik}^{*g}(\omega_m)$ is simply the associated multiplier.

To summarize, this approach would involve solving M consecutive LPs of format (16) at any point x that the NLP solver (used for solving (46)) would like to evaluate its optimality.

Energy Markets with a Forward Capacity Market: In Section 4, we showed that the main results of Section 3.2.2 can directly be extended to the two-stage game of Section 4, namely energy markets with forward capacity market. In this market setting, we can still find an equilibrium of the corresponding two-stage stochastic game by solving the two-stage stochastic program (29). By using the fixed sample point $\{\omega_m\}_{m=1}^M$, problem (29) can be approximated by (47):

$$\begin{aligned} \min_{x\geq 0} \quad & \frac{1}{M}\sum_{m=1}^{M} Z^*(\omega_m,x)+\sum_{g\in G}\sum_{i\in I_g}\sum_{k\in K_g}\kappa_k x_{ik}^g \\ s.t. \quad & \sum_{g,i,k} x_{ik}^g \geq H. \end{aligned} \tag{47}$$

The only difference between the approximate problems (46) and (47) is that a deterministic capacity regulation constraint is imposed in the latter. Hence, the arguments for solving (46) are also valid for (47).

Energy Markets with Operating-reserve Pricing: In Section 5.3, we showed that it is not always possible to formulate a two-stage stochastic program that finds an equilibrium of the two-stage stochastic game under perfect competition. If one can formulate a two-stage program as in Section 5.3.1, then it is possible to solve (40) in a similar way used for (19), as explained in *implicit function approach* for energy-only markets. If one cannot formulate a two-stage program as in Section 5.3.2, then in order to find an equilibrium of the corresponding two-stage stochastic game, one has to solve the stochastic complementarity problem (37) at the first stage and the optimization problem (35) at the second stage.

As explained before, one possible approach is solving the corresponding aggregate MCP system (that is, MCP (37) and KKT conditions of (35)) to find an approximate equilibrium point of the two-stage stochastic game, as described in MCP approach for stochastic systems. However, the size of the resulting aggregate MCP system grows rapidly with the sample size M. Hence, it may be computationally very time-consuming or impossible to solve such an MCP system for realistic networks.

We next propose another way for finding an approximate equilibrium of the two-stage stochastic game of Section 5.3.2 by implicit function approach. First, we approximate (37) by using sample average approximations of the expected functions. For given x, one can easily approximate $E_\omega[\beta_{ik}^{*g}(\omega)+rp\_capacity(\frac{ex-ey^*(\omega)}{ex})]$ by the sample average function $-\frac{1}{M}\sum_{m=1}^{M}[\beta_{ik}^{*g}(\omega_m)+rp\_capacity(\frac{ex-ey^*(\omega_m)}{ex})]$. Thus, we construct (48) below to approximate (37):

$$0 \leq -\frac{1}{M}\sum_{m=1}^{M}[\beta_{ik}^{*g}(\omega_m)+rp\_capacity(\frac{ex^*-ey^*(\omega_m)}{ex^*})]+\kappa_k \perp x_{ik}^{*g} \geq 0,\ \forall g\in G, i\in I_g, k\in K_g. \tag{48}$$

As mentioned, we would like to solve (48) reminiscent of the implicit function approach. Note that the size of (48) does not depend on the sample size M. In order to solve (48), we need to provide function and (sub)gradient values of $-\frac{1}{M}\sum_{m=1}^{M}[\beta_{ik}^{*g}(\omega_m)+rp\_capacity(\frac{ex-ey^*(\omega_m)}{ex})]$ at any x that PATH would like to explore. Given any x, we can solve the second stage OPF problem (35) M consecutive times and obtain the values of $\beta_{ik}^{*g}(\omega_m)$ and $rp\_capacity(\frac{ex-ey^*(\omega_m)}{ex})$ for each $m\in M$.

Unfortunately, to approximate the (sub)gradient of $E_\omega[\beta_{ik}^{*g}(\omega)+rp\_capacity(\frac{ex-ey^*(\omega)}{ex})]$, we cannot directly use the (sub)gradient values of $\beta_{ik}^{*g}(\omega_m)$. Although $E[\beta_{ik}^{*g}]$ is continuous and $\frac{1}{M}\sum_{m=1}^{M}\beta_{ik}^{*g}(\omega_m)$ becomes almost continuous for large M, $\beta_{ik}^{*g}(\omega_m)$ is actually a piecewise constant function of x. Thus, the (sub)gradient of $\beta_{ik}^{*g}(\omega_m)$ is either zero or undefined for a given x and ω_m. Hence, we use the following steps to obtain an sample average estimator of $\partial E_\omega[\beta_{ik}^{*g}(\omega) + rp\_capacity(\frac{ex-ey^*(\omega)}{ex})]/\partial x_{ik}^g$ for any $x_{ik}^g > 0$:

- By using (31), for any ω with $y_{ik}^*(\omega) > 0$:

$$E_\omega[\beta_{ik}^{*g}(\omega) + rp\_capacity(\tfrac{ex-ey^*(\omega)}{ex})] = E_\omega[(p_i^*(\omega) - c_{ik}^g)].$$

- Again, by (31), for any ω with $y_{ik}^*(\omega) = 0$, we have $\beta_{ik}^{*g}(\omega) = 0$. Thus:

$$E_\omega[\beta_{ik}^{*g}(\omega) + rp\_capacity(\tfrac{ex-ey^*(\omega)}{ex})] = E_\omega[rp\_capacity(\tfrac{ex-ey^*(\omega)}{ex})].$$

- Hence, by these two together, we can write:

$$\begin{aligned} E_\omega[\beta_{ik}^{*g}(\omega) + rp\_capacity(\tfrac{ex-ey^*(\omega)}{ex})] = \quad & E_\omega[(p_i^*(\omega) - c_{ik}^g) I_{\{y_{ik}^*(\omega)>0\}}] \\ & + E_\omega[rp\_capacity(\tfrac{ex-ey^*(\omega)}{ex}) I_{\{y_{ik}^*(\omega)=0\}}]. \end{aligned} \tag{49}$$

  By calculating the subgradients of (49), we get

$$\begin{aligned} \frac{\partial E_\omega[\beta_{ik}^{*g}(\omega) + rp\_capacity(\frac{ex-ey^*(\omega)}{ex})]}{\partial x_{ik}^g} = \quad & \frac{\partial E_\omega[(p_i^*(\omega) - c_{ik}^g) I_{\{y_{ik}^*(\omega)>0\}}]}{\partial x_{ik}^g} \\ & + \frac{\partial E_\omega[rp\_capacity(\frac{ex-ey^*(\omega)}{ex}) I_{\{y_{ik}^*(\omega)=0\}}]}{\partial x_{ik}^g}. \end{aligned} \tag{50}$$

- Note that for a fixed sample $\{\omega_m\}_{m=1}^M$, we can always perturb $x_{ik}^g > 0$ small enough such that $x_{ik}^g + \Delta x_{ik}^g > 0$ and the values of the indicator functions $I_{\{y_{ik}^*(\omega)>0\}}$ and $I_{\{y_{ik}^*(\omega)=0\}}$ do not change; hence they can be treated as constants.

- Next, we approximate the (sub)gradients of $E_\omega[(p_i^*(\omega) - c_{ik}^g) I_{\{y_{ik}^*(\omega)>0\}}]$ and $E_\omega[rp\_capacity(\frac{ex-ey^*(\omega)}{ex}) I_{\{y_{ik}^*(\omega)=0\}}]$ for any $x_{ik}^g > 0$ by using the following sample-average estimators. Let $u(\omega_m) = \dfrac{ex - ey^*(\omega_m)}{ex}$, then:

$$\frac{1}{M}\sum_{m=1}^{M}\frac{\partial p_i^*(\omega_m)}{\partial x_{ik}^g}I_{\{y_{ik}^*(\omega_m)>0\}} \rightarrow \frac{\partial E_\omega[(p_i^*(\omega)-c_{ik}^g)I_{\{y_{ik}^*(\omega)>0\}}]}{\partial x_{ik}^g}, \text{ and}$$

$$\frac{1}{M}\sum_{m=1}^{M}\frac{\partial rp\_capacity(u(\omega_m))}{\partial u(\omega_m)}\frac{\partial u(\omega_m)}{\partial x_{ik}^g}I_{\{y_{ik}^*(\omega_m)=0\}} \rightarrow \frac{\partial E_\omega[rp\_capacity(\frac{ex-ey^*(\omega)}{ex})I_{\{y_{ik}^*(\omega)=0\}}]}{\partial x_{ik}^g} \quad (51)$$

as $M \rightarrow \infty$ almost surely.

Given any x, we can calculate the the corresponding (sub)gradient values by using (50) and (51). Thus, one needs to calculate the values $(y^*(\omega_m), u(\omega_m))$ and the subgradients $\partial rp\_capacity(u(\omega_m))/\partial u(\omega_m)$, $\partial u(\omega_m)/\partial x_{ik}^g$, and $\partial p_i^*(\omega_m)/\partial x_{ik}^g$. Once the function $rp\_capacity(\cdot)$ is explicitly defined, all of these values, except subgradient of $p_i^*(\omega_m)$, are straightforward to calculate from the solution of OPF problem (35). For instance,

$$(u(\omega_m), \frac{\partial u(\omega_m)}{\partial x_{ik}^g}) = (\frac{ex-ey^*(\omega_m)}{ex}, \frac{ey^*(\omega_m)}{ex^2}).$$

For the calculation of the subgradient of $p_i^*(\omega_m)$, Castillo et al. (2006) gives an integrated approach which at once yields all the sensitivities of the optimal solution of an NLP problem to changes in the parameter values. They illustrate how to obtain the directional and partial derivatives of the optimal objective function value, optimal primal, and dual variable values with respect to the parameters of a general NLP problem by a single calculation. Once we solve the OPF problem (35), we utilize the approach of Castillo et al. (2006) to calculate the subgradient of $p_i^*(\omega_m)$.

Energy Markets with Demand-side bidding: Similar to the energy-only market, the first stage problem of the two-stage stochastic model can be approximated by (46). The only difference between the approximate problems of energy-only market and energy market with demand response is the second stage OPF problem. The second stage OPF problem of energy market with demand response would be almost identical to (43) with explicit depends on ω_m. Hence, the discussion related to solving (46) remain applicable in this case as well.

9 Numerical Illustration

The numerical experiments reported here have been conducted to serve two purposes. Firstly, we would like to compare the performance of the computational methods discussed in Section 8 and secondly we would like to see the impact of the market designs discussed in this paper on investment incentives of the firms. To this end, we apply in this section the methods proposed in Section 8 to solve the equilibrium for the electricity markets discussed throughout this paper under both deterministic and stochastic setting.

All the numerical experiments reported are performed by a Dell PC with Dual-Intel Xeon, 575 2.66 GHz processors, and 2 GB 266 MHz DDR Non-ECC SDRAM Memory using 576 Windows 2000. The solvers utilized for each problem depend on both the type of application problem and the method, which are summarized in Tables 1 and 2. Note that the methodologies discussed in Section 8 allow modularity; that is, one can always use any off-the-shelf solvers. In Table 2, (Non)Linear Program ((N)LP) and Mixed Linear/Nonlinear Complementarity Problem (MLCP/MNCP) are used to solve first and second stage problems simultaneously whereas Stochastic Program (SP) and MCP s.t. NLP are based on implicit function approach discussed in Section 8; that is, first stage problem and second stage problems at each realization are solved iteratively. Thus, one can use a separate solver for each stage. In our case, we use CONOPT(warm start) and SNOPT sequentially for SPs and PATH for MCPs to find an equilibrium of the first stage. These first stage solvers call the second stage solver M consecutive times at a point x to explore its optimality. The up-to-date information on the first stage solvers can be found in the online documentations available by GAMS (see GAMS (2012)). In addition, for solving the second stage OPF problem we use the deterministic nonlinear optimization code E04UCC of NAG C library, Mark 7, NAG (2002). E04UCC is designed to minimize an arbitrary smooth function subject to constraints, which may include simple bounds on the variables, linear constraints, and smooth nonlinear constraints. Essentially, it is a sequential quadratic programming method incorporating an augmented Lagrangian merit function and a BFGS quasi-Newton approximation to the Hessian of the Lagrangian.

Application Problem	**Remedy Mechanism**
EO	Energy-only with VOLL pricing
ECAP	Capacity markets
EORP1	Operating-reserve pricing based on demand
EORP2	Operating-reserve pricing based on capacity

Table 1: Overview of application problems

We consider a competitive power market of six nodes given by Chao and Peck (1998) in Figure 3, which has the following characteristics:

Method	Application Problem	Solver First Stage	Solver Second Stage
LP/NLP	All except EORP2	CPLEX/SNOPT(GAMS) (1^{st} and 2^{nd} stage)	with 1^{st} stage
MLCP/MNCP	All	PATH(GAMS) (1^{st} and 2^{nd} stage)	with 1^{st} stage
SP	All except EORP2	CONOPT(warm start) and SNOPT(GAMS)	E04UCC (NAG C)
MNCP s.t. NLP	EORP2	PATH(GAMS)	E04UCC (NAG C)

Table 2: Overview of solvers utilized for each method and problem type

- The nodes 1, 2, and 4 are supply nodes ($I := \{1,2,4\}$) and the nodes 3, 5, and 6 are demand nodes ($N := \{3,5,6\}$).

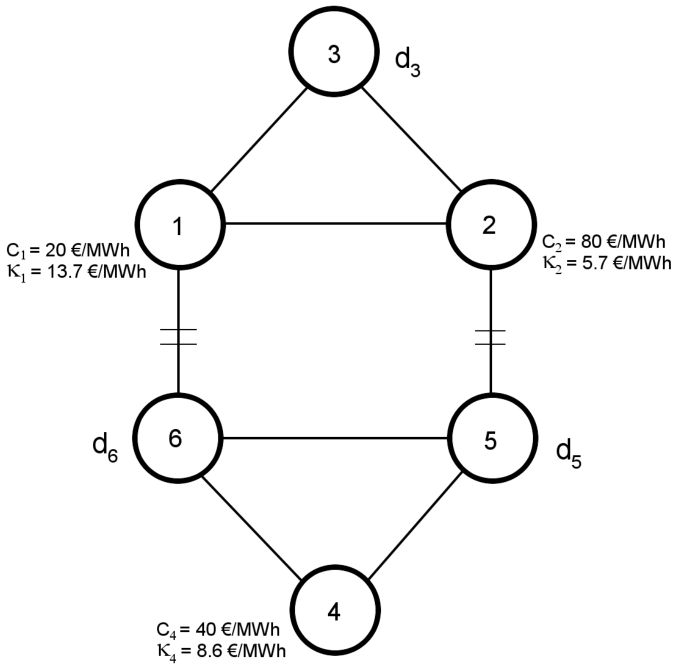
- Without loss of generality, at each supply node there is a single firm investing in one technology ($g = i = k$). The unit generation and investment costs of these firms are given in Figure 3. The corresponding data for marginal costs for these three generator types are taken from Schulkin et al. (2010).

- The demand in nodes 3, 5, and 6 are uniformly distributed with corresponding upper (D^{max}) and lower bounds (D^{min}) given in Figure 3.

- To analyze the electricity market with limited network capacity, we assume all lines have infinite capacities except for the lines (1-6) and (2-5) where $h_{16} = 8$ and $h_{25} = 8$. All line impedances are equal to 1 except for (1-6) and (2-5) that have impedances equal to 2. The PTDFs in Table 3 indicate the flows through lines (1-6) and (2-5) resulting from one unit injection at each node of the network and withdrawal at node 6, which is taken as the hub.

- VOLL is taken as 10,000 Euro/MWh which is a standard value of lost load used in the literature (e.g., Stoft (2002), Hogan (2005)).

- In case of operating-reserve pricing, both operating-reserve price curves introduced in Sections 5.3.1 and 5.3.2 are assumed to be smooth sigmoid functions as given in Figures 1 and 2, respectively. The mathematical representation of a sigmoid function is: $F(t) = \dfrac{F_{max}}{1+e^{k(t-F_{mid})}}$ where F_{max} is the maximum value of the function. F_{mid} gives the mid value of the function ($F(F_{mid}) = F_{max}/2$). Finally, k gives the curvature information centered on F_{mid}. Regarding the approximating function of the operating-reserve price curve offered by Hogan (2005) in Figure 1 and the operating-reserve price curve based on installed capacity in Figure 2, one should take $F_{max} = 10,000$ and $F_{mid} = 0.05$. In order to determine the value of k, the error between the real function and the approximating function is calculated for different values of k and $k = 133$ gives the best approximation with the smallest error. Therefore for the numerical results reported in this section, we use the formulation $rp(f(x,y,d)) = \frac{10000}{1+e^{133(f(x,y,d)-0.05)}}$ for the operating-reserve price curves, $rp\_demand(\frac{ex-ey^*}{ed})$ and $rp\_capacity(\frac{ex-ey^*}{ex})$, given in Figures 1 and 2, respectively. The unit of operating-reserve price in these figures are taken as Euro/MWh.



n	$D^{min}(GW)$	$D^{max}(GW)$
3	8	12
5	3	5
6	15	20

Figure 3: The data of 6-node example

Next, we solve two-stage deterministic and two-stage stochastic games by using the approaches proposed in Section 8.

Node	Line (1-6)	Line (2-5)
1	0.6250	0.3750
2	0.5000	0.5000
3	0.5625	0.4375
4	0.0625	-0.0625
5	0.1250	-0.1250

Table 3: Power distribution factors ($PTDF_{l,j}$)

Example 9.1. Perfect competition equilibria under deterministic demand

We first consider the situation in which the firms solve their first stage problem by simply taking the expected values of the demand: $d_3 = 10, d_5 = 4$, and $d_6 = 17.5$. Then the corresponding game is a two-stage deterministic game. In the energy-only (EO) market, energy market with forward capacity market (ECAP), and energy market with operating-reserve pricing based on the observed demand (EORP1), we find the equilibrium of the two-stage deterministic game by utilizing both MCP and (N)LP approaches. In the energy market with operating-reserve pricing based on installed capacity (EORP2), we find the equilibrium of the two-stage deterministic game by utilizing MCP approach. In the ECAP market, we impose that the capacity market requires 11.2% more capacity than the total expected demand ($H = 35.2$) which results in same reserve capacity as in EORP2 market. The results are presented in Tables 4 and 5. Furthermore, the results obtained by MCP approach and (N)LP approach for EO, ECAP, and EORP1 markets are identical and the computation time for both approaches takes less than a second.

Investment Incentives: Regarding the EO market result as reference, Tables 4 and 5 indicate that the investments increase when there is capacity market or operating-reserve pricing. The prices in both EORP markets remain identical whereas the prices in ECAP market are lower. However, there is an extra capacity price $\lambda^* = 5.7$ euro/MWh paid to the firms in ECAP market. If the consumers are paying this capacity price, then one can conclude that ECAP market results in identical prices as well.

Compared to EO market, total generation capacity investments are higher in ECAP and EORP markets; hence the system has higher investment costs in these markets. Regarding the comparison between different market designs, investment incentives are higher in EORP2 market than in EORP1 market. In addition, ECAP market results in identical investment levels as in EORP2 market since H is chosen to be equal to the total generation capacity in EORP2 market. If H were chosen to be equal to the total generation capacity in EORP1 market, then ECAP market would result in identical investment levels as in EORP1 market. Based on our extensive numerical experiments, we conjecture that for any operating-reserve function rp in EORP1 or EORP2 market, there is a corresponding H in ECAP market which results in identical total reserve capacity and mix of technologies. This observation is also made by Hobbs et al. (2001). Furthermore, if the “correct” extra capacity price ($\lambda^* = 5.7$) is paid by the consumers, then one can conclude that when H is equal to the total generation capacity in an EORP market, there is no difference between ECAP and the corresponding EORP market.

	GW			euro / MWh			Investment and
Market Design	x_1^*	x_2^*	x_4^*	p_3^*	p_5^*	p_6^*	Operational Cost (k Euro)
EO	31.5	0	0	33.7	33.7	33.7	1061.5
EORP1	31.5	3.3	0	33.7	33.7	33.7	1080.4
EORP2	31.5	3.7	0	33.7	33.7	33.7	1082.6
ECAP	31.5	3.7	0	28.0	28.0	28.0	1082.6

Table 4: Equilibrium in two-stage deterministic game with infinite transmission line capacities

	GW			euro / MWh			Investment and
Market Design	x_1^*	x_2^*	x_4^*	p_3^*	p_5^*	p_6^*	Operational Cost (k Euro)
EO	21.6	0	9.9	35.4	46.9	50.3	1208.9
EORP1	21.6	3.3	9.9	35.4	46.9	50.3	1227.9
EORP2	21.6	3.7	9.9	35.4	46.9	50.3	1230.1
ECAP	21.6	3.7	9.9	29.7	41.2	44.6	1230.0

Table 5: Equilibrium in two-stage deterministic game with limited network capacity

Example 9.2. Perfect competition equilibria under demand uncertainty

We now consider the situation in which the firms take the randomness of demand into account and the corresponding two-stage stochastic game needs to be solved. We first solve the two-stage stochastic games in EO, ECAP, and EORP2 markets by using both MCP and implicit function approaches discussed in Section 8 and compare the performance of these approaches. We use sample sizes of $M = 1000$ and $M = 8760$ and compare the computational performance of these two approaches in the energy-only market in Table 6, in EORP2 market in Table 7, and in ECAP market in Table 8, respectively. We report optimal capacities installed together with the solution time. We skip the comparison of the computational performance of these approaches for EORP1 market since the corresponding solution times are similar to that of EO market. Note that the particular problem being solved in the MCP approach is referred as mixed linear complementarity problem (MLCP) for energy-only and ECAP markets and mixed nonlinear complementarity problem (MNCP) for EORP2 market. Moreover, the particular problem being solved in the implicit function approach is referred as stochastic program (SP) for energy-only and ECAP markets and MNLCP s.t. NLP in EOPR2 market (since mixed nonlinear complementarity problem is solved at the upper level subject to the optimal solution set of the nonlinear program at the lower level). Next for $M = 8760$, we compare the corresponding generation capacity investments, average consumer prices, and average investment and operational (generation and curtailment) costs for EO, ECAP, and EORP markets for both unlimited and limited network in Tables 10 and 11, respectively.

Computational Performance: The comparisons between the MCP approach (MLCP or MNCP) and implicit function approach (SP or MNCP s.t. NLP) in all tables indicate that the computational time to solve the two-stage stochastic game by MCP approach increases rapidly with the sample size M. Since the implicit function approach involves solving the lower level (N)LP problem M consecutive times, the computational time increases with sample size M in a linear way. Hence, the MCP approach is more efficient in solving problems with smaller sample size (e.g., $M = 1000$) and implicit function approach is more efficient in solving problems with large sample size (e.g., $M = 8760$). We note that the solutions of the approximating problem with small sample size ($M = 1000$) are almost identical to the solutions of the approximating problem with large sample size ($M = 8760$) in Tables 6-8. This is mainly a consequence of using uniform distribution in which case the approximation functions converge to their limit very rapidly. Next, we consider an example in the energy-only market in which the stochastic demand in nodes $3, 5$, and 6 has triangular distribution with peak values $9, 3.5$, and 16, respectively. Table 9 contains the equilibrium points of this two-stage game. We see that the equilibria for different sample sizes show wider variations in this case. Therefore, depending on the variability of the underlying random data and the final accuracy desired, one may sometimes prefer to solve the two-stage game for larger sample sizes (e.g., $M = 8760$) which get closer to the true solution.

Investment Incentives: The comparison of investment capacities installed by each technology, average consumer prices, and total investment and operational cost of the system under each market design is given in Tables 10 and 11. The values in these tables represent the computations done with sample size of 8760. The results indicate that the total generation capacity in energy-only market is 1-2% less than the total peak demand, whereas ECAP and operating-reserve pricing in EORP1 and EORP2 markets result in total generation capacities that are about 4.5% - 5.0% above the total peak demand level. Thus, energy-only markets with VOLL pricing tend to lead to total generation capacity below the peak load with a certain probability whereas energy markets with a forward capacity market or operating-reserve pricing result in higher investments. VOLL pricing (with high values of VOLL) may also result in similar investment levels as the other market designs if forced outages are taken into account and the demand is assumed to have a distribution without a finite support (see the result of Hobbs et al. (2001)). In our experiments, since we do not consider forced outages and we assume demand distributions with finite support, the regulator's reserve target assumptions (in case of capacity markets or operating-reserve pricing) result in total generation capacity higher than the peak demand level which cannot be achieved by VOLL pricing.

As observed in the deterministic case, EORP2 market results in higher investment level of peak unit compared to EORP1 market. Different from the deterministic case, the average consumer prices in EORP2 market are slightly higher than the average prices in EORP1 market. In ECAP market, we used the total capacity requirement $H = 38.8$. As one can see in Tables 10 and 11, the investment levels obtained in ECAP market are close to the investment levels in EORP markets. Similar to the deterministic case, we conjecture a similar result of Hobbs et al. (2001) that for any operating-reserve function rp in EORP1 or EORP2 market, there is a corresponding H in ECAP market which results in identical total reserve capacity and mix of technologies.

Although the average prices in ECAP market are the lowest, there is again an extra capacity price $\lambda^* = 5.7$ euro/MWh which is likely be paid by the consumers. Furthermore compared to the EO market, the relative increase in system operational and investment cost in ECAP and EORP markets under uncertainty is lower than the increase in the total cost for the deterministic case. This can be explained by the impact of higher operational cost due to the curtailment at some realizations and lower investment cost in EO market versus lower operational and higher investment costs in the other market designs. Finally, the comparison of the results in deterministic and stochastic settings indicate that when uncertainty of future demand is taken into account by the risk neutral generators, their investments result in higher total generation capacity and a broader mix of technologies compared to the case when generators invest based on the expected demand.

Sample size M	Infinite network capacity (x_1^*, x_2^*, x_4^*) *CPU time*		Limited network capacity (x_1^*, x_2^*, x_4^*) *CPU time*	
	MLCP	SP	MLCP	SP
1000	(32.9, 2.0, 1.5) 00h 00’47”	(32.9, 2.0, 1.5) 00h 13’46”	(22.1, 1.3, 13.0) 00h 05’04”	(22.1, 1.3, 13.0) 00h 27’45’
8760	(32.9, 2.1, 1.5) 06h 16’ 49”	(32.9, 2.1, 1.5) 01h 36’ 25”	(22.0, 1.2, 13.3) 23h 01’ 09”	(22.0, 1.2, 13.3) 02h 32’ 38”

Table 6: Equilibrium of two-stage stochastic game in energy-only markets (EO)

Sample size M	Infinite network capacity (x_1^*, x_2^*, x_4^*) *CPU time*		Limited network capacity (x_1^*, x_2^*, x_4^*) *CPU time*	
	MNCP	MNCP s.t. NLP	MNCP	MNCP s.t. NLP
1000	(32.9, 4.4, 1.5) 00h 01’05”	(32.3, 3.6, 2.9) 01h 0’07”	(22.1, 3.7, 12.9) 00h 05’04”	(22.1, 3.7, 12.9) 01h 15’53”
8760	(32.9, 4.5, 1.5) 10h 46’ 22”	(32.9, 4.5, 1.5) 09h 31’ 38”	NA (time limit) 75h 48’ 19”	(22.0, 3.8, 13.0) 08h 19’ 42”

Table 7: Equilibrium of two-stage stochastic game in energy markets with operating-reserve price (EORP2)

Sample size M	Infinite network capacity (x_1^*, x_2^*, x_4^*) *CPU time*		Limited network capacity (x_1^*, x_2^*, x_4^*) *CPU time*	
	MNCP	MNCP s.t. NLP	MNCP	MNCP s.t. NLP
1000	(32.9, 4.4, 1.5) 00h 00’33”	(32.9, 4.4, 1.5) 00h 27’48”	(22.1, 3.7, 12.9) 00h 02’21”	(22.1, 3.7, 12.9) 00h 24’50”
8760	(32.9, 4.4, 1.5) 4h 44’ 11”	(32.9, 4.4, 1.5) 01h 54’ 11”	(22.1,3.6,13.1) 21h 38’ 34”	(22.1, 3.6, 13.1) 01h 54’ 49”

Table 8: Equilibrium of two-stage stochastic game in energy markets with forward capacity requirements (ECAP)

Sample size M	Infinite network capacity (x_1^*, x_2^*, x_4^*) *CPU time*		Limited network capacity (x_1^*, x_2^*, x_4^*) *CPU time*	
	MLCP	SP	MLCP	SP
1000	(31.5, 2.2, 1.1) 00h 00’51”	(31.5, 2.2, 1.1) 00h 14’18”	(21.5, 1.0, 12.3) 00h 02’21”	(21.5, 1.0, 12.3) 00h 23’48’
8760	(31.5, 2.4, 1.2) 10h 43’ 53”	(31.5, 2.4, 1.2) 02h 06’ 01”	(21.5, 0.8, 12.8) 34h 18’ 31”	(21.5, 0.8, 12.8) 04h 59’ 28”

Table 9: Equilibrium of two-stage stochastic game in EO markets with stochastic demand sampled from triangular distribution

Market Design	GW x_1^*	x_2^*	x_4^*	euro / MWh $E[p_3^*]$	$E[p_5^*]$	$E[p_6^*]$	Average Investment and Operational Cost (k Euro)
EO	32.9	2.1	1.5	33.6	33.6	33.6	1113.6
EORP1	32.9	4.3	1.5	33.5	33.5	33.5	1125.7
EORP2	32.9	4.5	1.5	33.8	33.8	33.8	1126.8
ECAP	32.9	4.4	1.5	28.0	28.0	28.0	1126.3

Table 10: Equilibrium in two-stage stochastic game with infinite transmission line capacities

Market Design	GW x_1^*	x_2^*	x_4^*	euro / MWh $E[p_3^*]$	$E[p_5^*]$	$E[p_6^*]$	Average Investment and Operational Cost (k Euro)
EO	22.0	1.2	13.3	36.0	46.5	49.5	1256.8
EORP1	22.1	3.5	13.1	35.4	46.6	49.8	1268.6
EORP2	22.0	3.8	13.0	38.5	50.2	53.6	1269.3
ECAP	22.1	3.6	13.1	29.8	41.0	44.3	1269.1

Table 11: Equilibrium in two-stage stochastic game with limited network capacity

10 Conclusions

We have considered alternative market designs which may remedy the resource adequacy problem in restructured electricity markets. Each market design corresponds to a different type of multi-agent model formulation depending on the remedy mechanism and the assumption on the market agents' behaviors. Taking into account the uncertainty or variability of parameters in these multi-agent models may lead to large-scale problems which are computationally complex to solve due to scarcity of resources (e.g., available memory and speed of computers). We show that, in perfectly competitive markets, most of the market models can be cast into deterministic or stochastic optimization problems similar to the early capacity expansion models of a regulated monopoly. This result also suggests that an equilibrium of a single-stage (open loop) model in which investment and operation decisions are made simultaneously coincides with an equilibrium of a two-stage (closed loop) model where investment and operation decisions are made sequentially. By using this result, we show that we can utilize sample-path methods together with the powerful available solvers for deterministic optimization problems, which provides computational simplicity for solving such models of realistic systems with stochastic elements.

By utilizing numerical experiments, we also provide insights on the impact of demand uncertainty and to what extend these market designs provide incentives to invest in generation capacities. Firstly, uncertainty of demand leads to higher investments in total generation capacity and a broader mix of technologies compared to the investment decisions assuming average demand levels. Furthermore in energy-only markets, peak-load generators tend to under-invest and curtail peak load so that they receive positive margin via high prices (e.g., VOLL) to cover their long-run marginal costs. In energy markets with a forward capacity market or with operating-reserve pricing, peak-load generators receive positive margin not only via curtailment but also by providing more capacity to the system. Therefore for the same VOLL (or price cap) level, energy-only markets with VOLL pricing tend to lead to total generation capacity below the peak load with a certain probability whereas energy markets with a forward capacity market or operating-reserve pricing result in higher investments under the assumptions of random demand with finite support and no forced outages. Moreover given similar regulator targets, operating-reserve pricing based on installed capacity provides higher incentives than operating-reserve pricing based on observed demand and it does not increase the total investment and operational cost in the system significantly. Lastly, the regulator decisions (e.g., reserve capacity target) in capacity markets and operating-reserve pricing can be chosen in such a way that results in very similar investment levels and fuel mix of generation capacities in both market designs.

Finally, the result of the prevalence of stochastic programming for providing solutions to stochastic equilibrium models can be extended to generation capacity investment strategies in perfectly competitive electricity markets with different regulatory mechanisms such as emission trading scheme (Gürkan et al. (2012)) or renewable obligations (Gürkan and Langestraat (2012)). In addition, risk aversion can also be included by using "coherent risk measures" in the first stage problem of the firms. This allows assessment of investment incentives of risk averse generators by utilizing a two-stage stochastic program when the markets are perfectly competitive and "complete" (Ralph and Smeers (2011)). However, this does not necessarily guarantee that every equilibrium model under perfect competition can be cast as two-stage stochastic program since equilibrium problems are indeed broader than optimization problems. The natural approach is to resort initially to complementarity formulations as to model competitive electricity markets. Depending on the structure of the market and regulatory intervention, market equilibrium may or may not be equivalent to the solution of a system optimization. For instance in one case of operating-reserve pricing, we obtain an equilibrium problem that is not equivalent to an optimization problem.

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Appendix: Proofs of Lemmas and Theorems

Proof of Lemma 3.3. The first stage problem (10) is concave in x^g and the second stage problem (9) is a linear program and therefore is convex. Hence, (11) and (8) are necessary and sufficient optimality conditions for all firms at both stages. By using the result, $x^* = y^*$, from Lemma 3.1, we can rewrite (8) by replacing y_{ik}^{*g} with x_{ik}^{*g}. Thus, the first two complementarity equations in (8) reduce to:

$$\begin{array}{lllll} (i) & 0 \leq c_{ik}^{g} - p_i^* + \beta_{ik}^{g*} & \perp & x_{ik}^{*g} \geq 0 & \forall g \in G, i \in I_g, k \in K_g \\ (ii) & \beta_{ik}^{g*} \geq 0 & & & \forall g \in G, i \in I_g, k \in K_g. \end{array}$$

In addition by Lemma 3.2,

$$(iii) \quad 0 \leq -\beta_{ik}^{*g} + \kappa_k \quad \perp \quad x_{ik}^{*g} \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g$$

should hold at the first stage. As a result, a solution x^* which satisfies *(i)-(iii)* would satisfy the following complementarity conditions as well:

$$0 \leq c_{ik}^{g} - p_i^* + \kappa_k \quad \perp \quad x_{ik}^{*g} \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g.$$

On the other hand, let x^* be a solution to the complementarity conditions in (12). Then it is an equilibrium of the two-stage game where $\beta_{ik}^{*g} = p_i^* - c_{ik}^{g} = \kappa_k$ for $x_{ik}^{*g} > 0$ and $0 \leq \beta_{ik}^{*g} \leq \kappa_k$ for $x_{ik}^{*g} = 0$.∎

Proof of Lemma 3.6. Similar to *Proof of Lemma 3.2*, first stage problem of each firm $g \in G$ can be formulated as

$$\max_{x^g \geq 0} \quad E_\omega[\Pi_g^*(\omega, x^g)] - \sum_{i \in I_g} \sum_{k \in K_g} \kappa_k x_{ik}^{g}, \tag{A1}$$

where $\Pi_g^*(\omega, x^g)$ is the optimal value of the firm g's second stage problem (15) at realization $\omega \in \Omega$. Similar to the arguments in *Proof of Lemma 3.2*, $\Pi_g^*(\omega, x^g)$ is a concave function of x^g for all fixed $\omega \in \Omega$; hence the expectation $E_\omega[\Pi_g^*(\omega, x^g)]$ is a concave function of x^g as well. By using the fact that (15) is a linear program, we know that at any given x^g if $\Pi_g^*(\omega, x^g)$ is finite then $\beta_{ik}^{*g}(\omega)$ is a subgradient of $\Pi_g^*(\omega, \cdot)$. Moreover, $\beta_{ik}^{*g}(\omega)$ is unique and $\Pi_g^*(\omega, \cdot)$ is differentiable except on a set L of Lebesgue measure zero. Thus,

$$\frac{\partial \Pi_g^*(\omega, \cdot)}{\partial x_{ik}^g} = \beta_{ik}^{*g}(\omega) \text{ except on } L.$$

Since $\beta_{ik}^{*g}(\omega)$ is a subgradient of $\Pi_g^*(\omega, \cdot)$, $E_\omega[\beta_{ik}^{*g}(\omega)]$ is a subgradient of $E_\omega[\Pi_g^*(\omega, \cdot)]$. Moreover $d(\omega)$ is a continuous random variable; hence L has a probability measure zero. Therefore, $E_\omega[\beta_{ik}^{*g}(\omega)]$ is unique and

$$\frac{\partial E_\omega[\Pi_g^*(\omega, \cdot)]}{\partial x_{ik}^g} = E_\omega[\beta_{ik}^{*g}(\omega)]. \tag{A3}$$

As a result, we can conclude that $E_\omega[\Pi_g^*(\omega, \cdot)]$ is concave and differentiable. Hence, x^{*g} is optimal for firm g's problem (17) if and only if it satisfies the optimality conditions of problem (A1) given as

$$0 \leq -\frac{\partial E_\omega[\Pi_g^*(\omega, x^{*g})]}{\partial x_{ik}^g} + \kappa_k \quad \perp \quad x_{ik}^{*g} \geq 0 \quad \forall i \in I_g, k \in K_g.$$

By using the equality in (A3), we get the following equilibrium conditions for the first-stage game:

$$0 \leq -E_\omega[\beta_{ik}^{*g}(\omega)] + \kappa_k \quad \perp \quad x_{ik}^{*g} \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g. \blacksquare$$

Proof of Theorem 3.8. Both (23) and (24) are convex problems. The necessary and sufficient KKT optimality conditions of (23) and (24) for all firms and TSO together with the market clearing conditions can be written as

$$0 \leq -\sum_{m=1}^{M} \pi_m \beta_{ik}^{*g}(\omega_m) + \kappa_k \quad \perp \quad x_{ik}^{*g} \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g; \tag{A4}$$

where for $m = 1, 2, \ldots, M$:
$(y^*(\omega_m), \beta^*(\omega_m), p^*(\omega_m))$ satisfy $MCP\_Firms(\omega_m, x)$
$(f^*(\omega_m), \rho^*(\omega_m), \lambda_l^{*+}(\omega_m), \lambda_l^{*-}(\omega_m))$ satisfy $MCP\_TSO(\omega_m)$
$(y^*(\omega_m), \delta^*(\omega_m), f^*(\omega_m), p^*(\omega_m))$ satisfy $MCP\_Market(\omega_m)$.

In (A4), the first line is equivalent to the equilibrium conditions of the first stage game given in (22) and the rest is KKT optimality conditions of the OPF problem (16) for every ω_m. Hence, a solution to the MCP (A4) is an equilibrium of the corresponding two-stage stochastic game with finite number of demand scenarios. Furthermore, since (A4) consists of necessary and sufficient optimality conditions of every firm's problem in both two-stage and single-stage games, an equilibrium of the two-stage stochastic game, if it exists, is also an equilibrium of the single-stage stochastic game.∎

Proof of Theorem 5.2. The second stage problem (30) of each firm $g \in G$ is a linear program where x^g appears both as a coefficient in the objective function and as a the right side parameter in the constraints. Since the objective function is a concave function of x^g and the corresponding constraints are convex in x^g, the optimal objective function value, $\Pi_g^{*R}(x^g)$, is a concave function of x^g in (30). When $\Pi_g^{*R}(x^g)$ is finite in the neighborhood of x^{*g}, it is also subdifferentiable at x^{*g} and $(\beta_{ik}^{*g} + \gamma^*)$ is a subgradient of $\Pi_g^{*R}(x^{*g})$. Hence, x^{*g} is an optimal solution of (33) for each firm $g \in G$ if and only if there exists $(\beta_{ik}^{*g} + \gamma^*) \in \frac{\partial \Pi_g^{*R}(x^{*g})}{\partial x_{ik}^g}$ satisfying the necessary and sufficient optimality conditions

$$0 \leq -(\beta_{ik}^{*g} + \gamma^*) + \kappa_k \quad \perp \quad x_{ik}^{*g} \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g.$$

After plugging in $\gamma^* = rp(ex^* - ey^*, x^*, d)$, we get the equilibrium conditions in first stage game as

$$0 \leq -\beta_{ik}^{*g} - rp(ex^* - ey^*, x^*, d) + \kappa_k \quad \perp \quad x_{ik}^{*g} \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g. \blacksquare$$

Proof of Theorem 5.3. $\Pi_g^{*R}(\omega, x^g)$ is the optimal objective function value of the second stage problem (30) in stochastic setting defined for each $\omega \in \Omega$. As argued in *Proof of Theorem 5.2*, $\Pi_g^{*R}(\omega, x^g)$ is a concave function of x^g; hence $E_\omega[\Pi_g^{*R}(\omega, \cdot)]$ is concave. By using an argument similar to the one in *Proof of Lemma 3.6*, we can conclude that $E_\omega[\Pi_g^{*R}(\omega, \cdot)]$ is differentiable and

$$\frac{\partial E_\omega[\Pi_g^{*R}(\omega, \cdot)]}{\partial x_{ik}^g} = E_\omega[\beta_{ik}^{*g}(\omega) + \gamma^*(\omega)].$$

Hence, x^{*g} is an optimal solution of (36) for each firm $g \in G$ if and only if it satisfies the optimality conditions

$$0 \leq -E_\omega[\beta_{ik}^{*g}(\omega) + \gamma^*(\omega)] + \kappa_k \quad \perp \quad x_{ik}^{*g} \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g.$$

After plugging in $\gamma^*(\omega) = rp(ex^* - y^*(\omega), x^*, d(\omega))$, we get the equilibrium conditions in (37). ∎

Proof of Remark 5.4. $(x, y) \in \Re_+^{M \times M}$ where $M := G \times I \times K$. $R\_demand(\frac{ex-ey}{ed})$ is concave in (x, y) iff its

Hessian $H := \Re^{2M\times 2M}$ is negative semidefinite. Next we calculate the Hessian of $R\_demand(\frac{ex-ey}{ed})$: Let $H^1 := \Re^{M\times M}$, then

$$\nabla^2_{xx} R\_demand(\frac{ex-ey}{ed}) = \nabla^2_{yy} R\_demand(\frac{ex-ey}{ed}) = H^1 \text{ and}$$

$$\nabla^2_{xy} R\_demand(\frac{ex-ey}{ed}) = \nabla^2_{yx} R\_demand(\frac{ex-ey}{ed}) = -H^1,$$

where $H^1_{ij} = h_1(ex,ey) = \dfrac{\partial rp\_demand(\frac{ex-ey}{ed})}{\partial(\dfrac{ex-ey}{ed})}\dfrac{1}{ed} \quad \forall i,j \in M$ and $h_1(ex,ey) \le 0$ by using Assumption 5.1. Then the Hessian matrix can be formulated as

$$H(x,y) = \begin{pmatrix} H^1 & -H^1 \\ -H^1 & H^1 \end{pmatrix}.$$

Let $z^T = [z_1 \quad z_2]$ where $z_1, z_2 \in \Re^M$. Next we show that H is negative semidefinite; that is, $z^T H(x,y) z \le 0$ for each $z \in \Re^{2M}$ and $(x,y) \in \Re^{M\times M}_+$:

$$\begin{aligned} z^T H(x,y) z &= [z_1 \quad z_2] \begin{pmatrix} H^1 & -H^1 \\ -H^1 & H^1 \end{pmatrix} [z_1 \quad z_2]^T \\ &= z_1^T H^1 z_1 + z_2^T H^1 z_2 = h_1(ex,ey)[(ez_1)^2 + (ez_2)^2]. \end{aligned}$$

Since $h_1(ex,ey) \le 0$, $z^T H(x,y) z \le 0$ for each $z \in \Re^{2M}$. Thus, H is negative semidefinite.∎

Proof of Remark 5.7. $(x,y) \in \Re^{M\times M}_+$ where $M := G \times I \times K$. $R\_demand(\frac{ex-ey}{ex})$ is concave in (x,y) iff its Hessian $H := \Re^{2M\times 2M}$ is negative semidefinite. Next we calculate the Hessian of $R\_demand(\frac{ex-ey}{ex})$: Let $H^1, H^2, H^3 := \Re^{M\times M}$, then

$$\nabla^2_{xx} R\_demand(\frac{ex-ey}{ex}) = H^1,$$

$$\nabla^2_{yy} R\_demand(\frac{ex-ey}{ex}) = H^2, \text{ and}$$

$$\nabla^2_{xy} R\_demand(\frac{ex-ey}{ex}) = \nabla^2_{yx} R\_demand(\frac{ex-ey}{ex}) = H^3,$$

where for $(x,y) \in \Re^{M\times M}_+$ and by using Assumption 5.1:

(i) $H^1_{ij} = \dfrac{\partial rp\_capacity(\frac{ex-ey}{ex})}{\partial(\dfrac{ex-ey}{ex})}\dfrac{ey^2}{ex^3} \le 0, \quad \forall i,j \in M.$

(ii) $H^2_{ij} = \dfrac{\partial rp\_capacity(\frac{ex-ey}{ex})}{\partial(\frac{ex-ey}{ex})}\dfrac{1}{ex} \le 0, \quad \forall i,j \in M.$

(iii) $H^3_{ij} = -\frac{\partial rp\_capacity(\frac{ex-ey}{ex})}{\partial(\frac{ex-ey}{ex})}\frac{ey}{ex^2} \geq 0, \quad \forall i,j \in M.$

Note that ey^2 denotes the sum of squares for all the elements of a vector y and ex^3 denotes the sum of cubes for all the elements of a vector x in the above equations. Then the Hessian matrix can be formulated as

$$H(x,y) = \begin{pmatrix} H^1 & H^3 \\ H^3 & H^2 \end{pmatrix}.$$

Let $z^T = [z_1 \quad z_2]$ where $z_1, z_2 \in \Re^M$. Next we show that H is negative semidefinite; that is, $z^T H(x,y)z \leq 0$ for each $z \in \Re^{2M}$ and $(x,y) \in \Re_+^{M\times M}$:

$$\begin{aligned} z^T H(x,y)z &= [z_1 \quad z_2]\begin{pmatrix} H^1 & H^3 \\ H^3 & H^2 \end{pmatrix}[z_1 \quad z_2]^T \\ &= z_1^T H^1 z_1 + z_2^T H^2 z_2 + z_1^T H^3 z_2 + z_2^T H^3 z_1. \end{aligned}$$

By using $H^1_{ij} = -\frac{ey}{ex}H^3_{ij}$, $H^2_{ij} = -\frac{ex}{ey}H^3_{ij}$, and $H^3_{ij} = h_3(ex,ey) \geq 0$, we have

$$\begin{aligned} z^T H(x,y)z &= -\frac{ey}{ex}z_1^T H^3 z_1 - \frac{ex}{ey}z_2^T H^3 z_2 + z_1^T H^3 z_2 + z_2^T H^3 z_1 \\ &= -\frac{ey}{ex}h_3(x,y)(ez_1)^2 - \frac{ex}{ey}h_3(x,y)(ez_2)^2 + 2h_3(x,y)ez_1 ez_2 \\ &= h_3(x,y)(ez_1 - \frac{ex}{ey}ez_2)(ez_2 - \frac{ey}{ex}ez_1). \end{aligned}$$

In the above equation, one of the three cases hold for ez_1, ez_2:

Case 1 ($ez_1 < \frac{ex}{ey}ez_2$): Then $ez_2 > \frac{ey}{ex}ez_1$ which implies

$$z^T H(x,y)z = h_3(x,y)(ez_1 - \frac{ex}{ey}ez_2)(ez_2 - \frac{ey}{ex}ez_1) < 0.$$

Case 2 ($ez_1 > \frac{ex}{ey}ez_2$): Then $ez_2 < \frac{ey}{ex}ez_1$ which implies

$$z^T H(x,y)z = h_3(x,y)(ez_1 - \frac{ex}{ey}ez_2)(ez_2 - \frac{ey}{ex}ez_1) < 0.$$

Case 3 ($ez_1 = \frac{ex}{ey} ez_2$): Then

$$z^T H(x,y)z = h_3(x,y)(ez_1 - \frac{ex}{ey} ez_2)(ez_2 - \frac{ey}{ex} ez_1) = 0.$$

Hence, $z^T H(x,y)z \leq 0$ for each $z \in \Re^{2M}$ and H is negative semidefinite. ∎

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