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**STRATEGIC GENERATION CAPACITY CHOICE
UNDER DEMAND UNCERTAINTY: ANALYSIS
OF NASH EQUILIBRIA IN
ELECTRICITY MARKETS**

By

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Strategic Generation Capacity Choice under Demand Uncertainty: Analysis of Nash Equilibria in Electricity Markets

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Abstract

We analyze a two-stage game of strategic firms facing uncertain demand and exerting market power in decentralized electricity markets. These firms choose their generation capacities at the first stage while anticipating a perfectly competitive future electricity spot market outcome at the second stage; thus it is a closed loop game. In general, such games can be formulated as an equilibrium problem with equilibrium constraints (EPEC) and examples have been posed in the literature that have multiple or no equilibria. Therefore, it is of interest to define general sets of conditions under which solutions exist and are unique, which would enhance the value of such models for policy and market intelligence purposes. In this paper, we consider various types of such a closed loop model regarding the underlying price-demand relations (elastic and inelastic demand), the assumed demand uncertainty with a broad class of continuous distributions, and any finite number of players with symmetric or asymmetric costs. We establish sufficient conditions for the random demand's probability distribution which guarantee existence and uniqueness of equilibria in most of the cases of this closed loop model. We identify a broad class of commonly used continuous probability distributions satisfying these conditions.

Keywords: electricity markets, strategic generation investment modeling, demand uncertainty, existence and uniqueness of equilibrium.

JEL codes: C62, C68, C72, D43, L94

1 Introduction

In this paper, we establish sufficient conditions which guarantee existence and uniqueness of equilibria in oligopolistic electricity markets where strategic electricity generators (anticipating a perfectly competitive spot

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market outcome) make capacity choices about future market conditions under demand uncertainty and the power generation is dispatched after the level of demand is realized; thus it is a closed loop game. In general, such games can be formulated as an equilibrium problem with equilibrium constraints (EPEC) which may have multiple or no equilibria. Therefore, it is of interest to define general sets of conditions under which solutions exist and are unique, which would enhance the value of such models for policy and market intelligence purposes (e.g., Schroeder (2012), Allcott (2012)). Furthermore, when firms anticipate a perfectly competitive spot market outcome, an equilibrium of the closed loop¹ model may not be found by solving a less complicated open loop model¹ (see Wogrin et al. (2012)). Thus, it is of interest to users of these models to know whether their assumptions satisfy some conditions which guarantee the existence and uniqueness of an equilibrium before solving such complicated closed loop models.

Short term aspects of market power (related to choices of production when capacities are given) within various specific market design assumptions have been extensively analyzed in the literature. Among these, equilibrium modeling of Nash games in quantities is a common approach (e.g., see Borenstein and Bushnell (1999), Wei and Smeers (1999), Hobbs (2001) for reviews). Long term aspects of market power related to generation capacity expansion have also received growing attention. Among the most important works in the literature, Kreps and Scheinkman (1983) analyze capacity choice at the first stage prior to price competition at the second stage (e.g., Bertrand) with deterministic demand. They show that there exists a unique equilibrium which is equivalent to the single stage Cournot outcomes. Gabszewicz and Poddar (1997) analyze capacity choices of two symmetric firms at the first stage prior to a Cournot market at the second stage with demand uncertainty. They prove existence of symmetric equilibrium and compare it with the deterministic solution of using expected demand. Murphy and Smeers (2005) move one step further and formulate open and closed loop market models with two asymmetric Cournot players and finite number of demand scenarios, each having a linear demand curve with respective probabilities. In their analysis, Murphy and Smeers (2005) conclude that the complexity of solving capacity expansion increases with a closed loop model even without considering the network limitations. They find closed loop examples which may not have an equilibrium. They show that if an equilibrium exists, then it is unique and falls between the perfect competition and open loop equilibria.

All the models mentioned above assume an elastic demand represented by a linear price-demand curve with random intercept and the structure of the assumed probability distribution is in general simple (i.e., uniform demand segments). Furthermore, their analysis are in general based on limited number of players (e.g., two players). In this paper, we consider both inelastic random demand and an elastic demand represented by a linear price-demand curve with random intercept having a general underlying continuous probability distribution. In addition, we analyze the capacity choices of both symmetric and asymmetric firms. We establish existence and uniqueness results under a broader scope in terms of the underlying price-demand relations (elastic and inelastic demand), the assumed demand uncertainty with a broad class of continuous distributions, and any finite number of players with symmetric or asymmetric costs.

¹In an open loop model, firms sell electricity production simultaneously with their investment decision, while in a closed loop model firms choose their capacity at the first stage and sell production at the second stage.

In a similar closed loop model with symmetric firms and elastic demand, Grimm and Zoettl (2008) analyze strategic capacity choices under more general assumptions of a monotone random demand curve and its underlying probability distribution. They establish existence and uniqueness results when firms engage in Cournot competition at the second stage. They also show existence of equilibrium when firms anticipate competitive spot market outcomes at the second stage; however the uniqueness of equilibrium cannot be established under their general assumptions. In our analysis, the closed loop game with symmetric firms facing a linear demand curve with random intercept is indeed a subclass of their model whereas the closed loop model with inelastic random demand constitutes another type. For both cases, we establish sufficient conditions for the random demand's probability distribution which guarantee a unique equilibrium for symmetric firms anticipating competitive spot market outcomes. Furthermore, we establish sufficient conditions for a unique equilibrium of such a closed loop game with asymmetric firms facing an elastic random demand (i.e., a linear demand curve with random intercept). We explain our analysis in more detail below and elaborate on our results and contributions.

In a single stage Cournot model where symmetric firms choose their production output under uncertainty about the intercept of a linear price-demand function, Lagerlöf (2006) shows that if the distribution of the demand has a monotone hazard rate then the uniqueness of equilibrium is guaranteed. In this paper, we establish an extension of that result for a two-stage game in oligopolistic energy-only electricity markets. As we mention before, we consider strategic firms facing uncertain demand and competing as Cournot players when they choose their generation capacities at the first stage while anticipating a perfectly competitive future spot market outcome based on their choices (for example they may expect regulatory intervention at the spot market). After a firm installs its capacity at the first stage, its production takes place in electricity spot market at the second stage which represents a competitive energy-only market. We look into both symmetric and asymmetric firms facing inelastic or elastic demand. In reality, electricity demand is uncertain and/or fluctuating with a very low elasticity. We focus on two cases of demand uncertainty: (i) an inelastic random demand drawn from a continuous distribution; or (ii) price-demand relationship that can be well approximated by a linear curve with a random intercept having a general underlying continuous probability distribution. Due to demand uncertainty, firms' capacity choices at the first stage may not be utilized for all the demand realizations at the second stage. For such a two-stage game, we characterize the class of problems and continuous probability distributions under which we can show that a unique equilibrium exists. We show that one general class of distributions having a monotone increasing hazard rate are logconcave distributions under which existence and uniqueness of equilibria is guaranteed for symmetric generators facing an inelastic or elastic random demand. When firms have asymmetric costs and the demand curve is elastic (represented by a linear demand curve with random intercept), uniqueness of equilibria can still be established under some sufficient conditions of demand's probability distribution which are similar to a standard assumption used for the demand function itself in the literature (see Sherali et al. (1983), Wolf and Smeers (1997), Grimm and Zoettl (2008), and Xu (2005)).

Finding optimal capacity choices of oligopolistic firms in a closed loop model is in general complex. In the absence of market power, the corresponding two-stage capacity investment problem is a convex problem and, as

shown by Gürkan et al. (2012), most of the two-stage capacity investment equilibrium problems in competitive markets can be cast into convex non(linear) optimization problems similar to the early capacity expansion models. However, convexity is in general lost when market power is introduced. The two-stage capacity investment problem of a strategic firm may be formulated as a bilevel problem, to be more precise as a stochastic MPEC (mathematical program with equilibrium constraints). We consider in this paper multiple strategic firms having an MPEC problem each, then one can speak of finding a Nash equilibrium to *an equilibrium problem with equilibrium constraints (EPEC)*. It is well known that EPECs are in general nonconvex problems and they may have multiple or no solutions (e.g., Hu (2003), Ehrenmann (2004), Hu and Ralph (2007), Gabriel et al. (2012)). As a consequence, an equilibrium for the corresponding two-stage game may not exist or, if exists, it is in general not unique. Thus, establishing a set of general conditions for existence and uniqueness is desirable for policy and market models including strategic firms so that users can have confidence in their solutions. Throughout the analysis in this paper, we aim to shed light on the characteristics of the random demand affecting the continuity, differentiability, and generalized concavity of each firm's expected payoff at the first stage which are crucial in establishing the existence and uniqueness of the equilibria of the two-stage strategic capacity choice game.

In the first part of the analysis, we consider an inelastic random demand and strategic firms anticipating a competitive energy-only spot market with VOLL pricing (or price cap). The idea of VOLL pricing is to price electricity at a high value that is supposed to reflect the value of lost load (VOLL) when demand is curtailed. In reality since VOLL is difficult to estimate, a high price cap (which is in general lower than VOLL) is used. As a preliminary, we first establish that whether strategic firms sell their power via central auction or bilateral contracts in the competitive spot market, the corresponding two-stage games yield the same equilibrium, if it exists. A similar result has also been established by Metzler et al. (2003) for a short-run Cournot competition in electricity spot markets where generation capacities are fixed. Metzler et al. (2003) show that a bilateral market model with Cournot producers and perfect arbitrageurs yields the same Nash-Cournot equilibrium (e.g., prices, generation) as a pool market with Cournot players. We extend their result for the two-stage capacity expansion equilibrium problem we consider here. As a result, we continue our analysis with the bilevel problem of each firm under the assumption of a stylized pool market model and in order to preserve analytical tractability we do not consider any network constraints. We note that the continuity of random demand distribution is a necessary condition for the rest of the analysis and the results we present below.

For the bilevel problem of each strategic firm maximizing its expected profit, the constraint set is closed and convex; however, the objective function is in general not concave. Hence, continuity and (strict) quasiconcavity of the objective function are desirable properties for establishing existence and uniqueness results for the corresponding two-stage stochastic game. We first show that each firm's expected profit is continuous provided that the probability distribution of the demand is continuous. Moreover, when firms are symmetric and the distribution of random demand has a monotone hazard rate, each firm's expected profit is strictly quasiconcave and hence the corresponding two-stage game has a unique equilibrium. One class of probability distributions having a monotone increasing hazard functions is logconcave probability distributions. The random demand

vector is log-concavely distributed if the logarithm of its probability density function is concave on its support. Logconcave probability distributions constitute a broad class (e.g., the uniform, normal, exponential, logistic, Weibull, gamma). Bagnoli and Bergstrom (2005) give a systematic treatment of the logconcave distributions and their crucial role in a wide variety of economic models. As a result, we show that in case of symmetric firms, random demand with a monotone increasing hazard function is sufficient to guarantee uniqueness of equilibrium. When firms are asymmetric, quasiconcavity of the expected profit function is lost for the ones investing in base-load technologies whereas the objective function of the firm investing in a peak load technology has similar characteristics to that of symmetric firms. Hence, the quasiconcavity of a firm's expected profit is dependent both on the characteristics of the underlying continuous probability distribution of the demand and on whether it is investing in a marginal unit or not. We also show by an example that even with two generators (e.g., peak-load and base-load) facing uniform exogenous demand, the first stage objective function of the base-load generator may not satisfy generalized concavity. Hence, an equilibrium may not exist.

In the second part of our analysis, instead of VOLL pricing we consider a linear price-demand curve with a random intercept, which implies random or fluctuating consumption levels for a given price. Under this setting, we can show that each firm's expected profit is differentiable provided that the probability density function of the intercept is continuous. Moreover when firms are symmetric and the underlying probability density function of the random intercept is logconcave, each firm's expected profit is strictly logconcave. Thus logconcavity of the underlying probability distribution is again a sufficient condition guaranteeing existence and uniqueness of the equilibrium for the two-stage capacity investment game between symmetric firms. When firms are asymmetric, the uniqueness of equilibrium is guaranteed for another condition (different than logconcavity) of probability distribution under which we prove strict concavity of all firms' expected profit functions. The corresponding condition is also similar to a standard assumption in the literature used for the demand function itself (see Sherali et al. (1983), Wolf and Smeers (1997), Grimm and Zoettl (2008), and Xu (2005)). We also conjecture that the logconcavity of probability distributions may also be sufficient to guarantee the existence of a unique equilibrium for closed loop games with asymmetric firms when demand is endogenous. Although we could not establish a theoretical proof for this conjecture, we observed it numerically. In a closed loop game with two asymmetric firms facing an elastic demand, we numerically computed the expected profit function of each firm under various logconcave probability distributions of random intercept (e.g., uniform, normal, exponential, Weibull, gamma, beta) and we observed that each firm's expected profit is strictly quasiconcave in all the cases.

To summarize, we take up a closed loop game in oligopolistic electricity markets which has received growing attention in the literature. In such a game, we formulate the two-stage investment model of multiple strategic firms which make generation capacity choices about future market conditions under demand uncertainty while anticipating a perfectly competitive spot market outcome. We contribute to existing literature along two dimensions:

- We show that whether strategic firms sell their power via central auction or bilateral contracts in the competitive spot market, the corresponding closed loop games yield the same equilibrium, if it exists.

Thus, when the spot market is perfectly competitive, different structures of buying and selling electricity do not affect the generation capacity choices of strategic firms.

- In general, such two-stage games can be formulated as an EPEC and examples have been posed in the literature that have multiple or no equilibria. We consider various types of the two-stage investment model of firms regarding the underlying price-demand relations, the assumed demand uncertainty, and any finite number of players with symmetric or asymmetric costs. In most of the cases, we establish sufficient conditions under which solutions exist and are unique and we identify a broad class of commonly used continuous probability distributions of the random demand satisfying these conditions, which will enhance the value of such models for policy and market intelligence purposes. In particular, establishing a set of general conditions for existence and uniqueness is desirable for policy and market models including strategic firms so that users can have confidence in their solutions before solving such complicated closed loop models.

The paper is organized as follows: In Section 2, we introduce the notation and some characteristics of short-run competitive equilibria in spot markets established in the literature. In Section 3, we formulate the two-stage game of multiple strategic firms in an energy-only electricity market with VOLL pricing in which we consider two different market structures of buying and selling electricity at the second stage. We first formulate the second stage game representing a spot market with a central auction (pool market) in Section 3.1. Then we consider, in Section 3.2, a bilateral market where firms trade bilaterally at the second stage. When the spot market is competitive, Boucher and Smeers (2001) show that for given capacities and fixed demand bilateral and pool markets yield the same equilibrium. By using their result, Section 3.3 concludes that the two-stage game with pool market and the two stage game with bilateral market yield the same equilibrium (e.g., capacities, prices, generation) when demand is random and capacities are endogenous. We analyze in Section 3.4 a simplified pool market model with exogenous random demand. For this simplified model, we show strict quasiconcavity of a firm's expected profit and existence and uniqueness of equilibria under symmetric costs and random demand having logconcave continuous probability distribution. Then in Section 4, we replace VOLL pricing with linear price-demand curves with random intercepts. Similar to the exogenous demand case, we show strict log-concavity of a firm's expected profit and existence and uniqueness of equilibria when firms have same costs and the corresponding probability density function is logconcave. When firms have different costs, a sufficient condition which grants existence and uniqueness of equilibria is also established. We finish the paper with our conclusions. Appendix contains the proofs of all theorems, lemmas, and propositions.

2 Set up

2.1 Notation

We consider an energy-only electricity market model in which there are three types of agents; namely generators, a transmission system operator (TSO), and consumers. We consider this model in a two-stage set-up: firms invest in generation capacity in the first stage and the market clears to satisfy demand in the spot market at the second stage. Firms maximize their profits at the first stage while anticipating a competitive spot market outcome at the second stage. They compete “a la Cournot” while choosing their generation capacities. That is, each firm chooses its generation capacity taking as given the investment strategies of its rivals. In reality, electricity demand is time varying and future demand is uncertain. We represent both phenomena by assuming that demand can take different values in different states of the world $\omega \in \Omega$, each occurring with some probability. The following notation would apply in state of the world $\omega \in \Omega$:

Sets

- N : set of all demand nodes
- G : set of all firms
- I_g : set of supply nodes of firm $g \in G$
- I : set of all supply nodes ($I := \cup_g I_g$)
- K_g : set of plant types of firm $g \in G$
- L : set of electricity transmission lines in the network

Parameters

- c_{ik}^g : unit generation cost of plant type $k \in K_g$ owned by firm $g \in G$ at supply node $i \in I_g$
- κ_k : unit capacity cost of plant type $k \in K_g$
- $d_n(\omega)$: demand at node $n \in N$
- $PTDF_{l,j}$: power transmitted through line $l \in L$ due to one unit of power injection from node $j \in \{N \cup I\}$ to an arbitrary hub² node
- h_l : capacity limit of line $l \in L$
- $VOLL$: the value of unserved energy or lost load

² $PTDF$ is calculated based on a hub node in $n \in N$ in a standard DC load flow model. The choice of hub node is arbitrary. That is, the flows resulting from a power injection at one node and an equal withdrawal at another do not depend on the location of the hub.

Variables:

Second Stage:

- $y_{ik}^g(\omega)$: quantity of power generated by plant type $k \in K_g$ of firm $g \in G$ at supply node $i \in I_g$
 $f_j(\omega)$: net power flow dispatched by TSO from node $j \in \{N \cup I\}$
 δ_n : unserved (curtailed) energy at node $n \in N$
 $p_j(\omega)$: locational market price (nodal price) at node $j \in \{N \cup I\}$ which corresponds to shadow price of market clearing constraint

First Stage:

- x_{ik}^g : the capacity of plant type $k \in K_g$ owned by firm $g \in G$ at node $i \in I_g$.

2.2 Characterizations of Perfectly Competitive Spot Markets

The spot market at the second stage may be considered under two different market structures, namely bilateral and pool spot markets. Boucher and Smeers (2001) consider a competitive equilibrium of a game in both pool and bilateral spot markets where all parameters are deterministic and none of the agents (firms, consumers, and TSO) has market power. Boucher and Smeers (2001) also consider, for pool and bilateral spot markets, a corresponding optimization problem referred to as Optimal Power Flow Problem (OPF) which minimizes the total system cost of the spot market. We will use their following result to formulate the second stage problem of each firm and to establish the equivalence of two-stage game in bilateral and pool markets.

Proposition 2.1. *[Theorem 1 of Boucher and Smeers (2001)] Consider a power system in a competitive pool or bilateral spot markets when capacities are given and all parameters are deterministic. If there exists an optimal dispatch, then there exists at least one equilibrium, which is a competitive equilibrium, for each of these markets. Moreover:*

- (i) *There is one-to-one correspondence between the equilibria of these spot markets; that is any equilibrium of the pool market can be written as an equilibrium of the bilateral spot market and vice versa.*
- (ii) *The equilibria of both pool and bilateral spot markets are equivalent to the solution of their associated OPF problems (which are specified later in Sections 3.1.1 and 3.2.1).*

3 Two-stage Capacity Choice Model under Exogenous Random Demand

In this section, we formulate the generation capacity investment model of a strategic firm anticipating the competitive energy-only spot market outcome under demand uncertainty. We focus on the *VOLL* pricing mechanism in the spot market which gives the firms the incentive to build peak capacity. Whenever there is a shortage of capacity, there is curtailment and the electricity price becomes a very high value (the *VOLL* or a price cap) set

by the regulator where

$$VOLL > \max\{c_{ik}^g | g \in G, i \in I_g, k \in K_g\}.$$

Hence, only when there is curtailment, the peak generator sells power at a price (the $VOLL$) higher than its marginal production cost. The difference between the $VOLL$ and the marginal cost of the peak generator operating contributes to covering its investment cost.

We also consider an exogenous random demand which varies, say, over a year. Let $(\Omega, \mathcal{F}, \Psi)$ denote the common underlying probability space where Ψ represents the joint probability distribution for the random demand vector $d(\omega) := \{d_n(\omega)\}_{n \in N, \omega \in \Omega}$ with $E[|d(\omega)|] < \infty$. Then each $d_n(\omega)$, demand at node $n \in N$, has a general distribution Ψ_n where $\Psi := \prod_{n=1}^N \Psi_n$.

Next, we formulate the two-stage game under two different market structures, namely bilateral and pool markets. Each of these market structures gives rise to different types of model formulations as outlined in Sections 3.1 and 3.2. As mentioned earlier, Boucher and Smeers (2001) address the equivalence of pool and bilateral markets in a single-stage perfectly competitive game when capacities are given and demand is deterministic. Section 3.3 is an extension of their result in our setting. First, instead of a single-stage perfectly competitive model we have a two-stage game and the firms may exert market power at the first stage. Second, the generation capacities are decision variables of the firms at the first stage. Finally, instead of deterministic demand we consider stochastic demand at the second stage. We show in Section 3.3 that the two-stage games with pool and bilateral markets, outlined in Sections 3.1 and 3.2, respectively, yield the same first and the second stage equilibria; hence they are equivalent.

3.1 Pool Market Model

We first consider the two-stage game in pool market in which competitive firms sell power to a central auction operated by the Transmission System Operator (TSO) at the price of their supply nodes and the TSO dispatches this power to the consumers at demand nodes. Next, we give details of the two stages in this setting.

3.1.1 Perfect Competition Equilibrium at Second Stage

By Proposition 2.1 (ii) we know that, in deterministic setting, competitive equilibria of the perfectly competitive pool market is equivalent to the solution of its associated OPF problem. Indeed, this result holds for each state of the world in our setting. For each state of the world $\omega \in \Omega$, one can find a competitive equilibrium of the

pool market at the second stage by solving the following OPF (Optimal Power Flow) problem:

$$\begin{aligned}
Z_{pool}^*(\omega, x) := & \min_{\{y(\omega), \delta(\omega), f(\omega)\}} \sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} c_{ik}^g y_{ik}^g(\omega) + VOLL \sum_n \delta_n(\omega) \\
& \text{s.t.} \\
& \boxed{\begin{aligned}
& \sum_{g \in G} \sum_{k \in K_g} y_{jk}^g(\omega) + \delta_j(\omega) + f_j(\omega) \geq d_j(\omega) \quad (p_j(\omega)) \quad \forall j \in \{N \cup I\} \\
& \sum_{j \in \{N \cup I\}} f_j(\omega) = 0 \quad (\rho(\omega))
\end{aligned}} \\
& \boxed{\begin{aligned}
& x_{ik}^g - y_{ik}^g(\omega) \geq 0 \quad (\beta_{ik}^g(\omega)) \quad \forall g \in G, i \in I_g, k \in K_g
\end{aligned}} \\
& \boxed{\begin{aligned}
& h_l - \sum_{j \in \{N \cup I\}} PTDF_{l,j} f_j(\omega) \geq 0 \quad (\lambda_l^+(\omega)) \quad \forall l \\
& h_l + \sum_{j \in \{N \cup I\}} PTDF_{l,j} f_j(\omega) \geq 0 \quad (\lambda_l^-(\omega)) \quad \forall l
\end{aligned}} \\
& y(\omega) \geq 0, \quad \delta(\omega) \geq 0, \tag{1}
\end{aligned}$$

where $\rho(\omega), p_j(\omega), \lambda_l^+(\omega), \lambda_l^-(\omega)$, and $\beta_{ik}^g(\omega)$ are Lagrange multipliers; in particular, $p_j(\omega)$ is the price of unit power (€/MWh) at node $j \in \{N \cup I\}$. In OPF problem (1), $Cons_Balance_{pool}(\omega)$ is the set of energy balance equations which state that the difference between total generation and demand at any node has to be balanced by injections into or withdrawals from the grid and the sum of all injections into and withdrawals from the grid should be zero. $Cons_PTDF(\omega)$ is the set of constraints for the technical network limits while the generation capacity constraints are given in $Cons_Cap(\omega)$ which indicate that firms cannot produce more than their maximum capacity. KKT conditions of the OPF problem (1), which is a linear program, yield the following equilibrium conditions of the pool market:

For each $\omega \in \Omega$ and x ,

$$\begin{aligned}
0 \leq c_{ik}^g - p_i^*(\omega) + \beta_{ik}^{*g}(\omega) & \perp y_{ik}^{*g}(\omega) \geq 0 & \forall g \in G, i \in I_g, k \in K_g \\
0 \leq VOLL - p_n^*(\omega) & \perp \delta_n^*(\omega) \geq 0 & \forall n \in N \\
0 \leq \sum_{g \in G} \sum_{k \in K_g} y_{jk}^{*g}(\omega) + \delta_j^*(\omega) - f_j^*(\omega) - d_j(\omega) & \perp p_j^*(\omega) \geq 0 & \forall j \in \{N \cup I\} \\
p_j^*(\omega) - \rho^*(\omega) + \sum_{l \in L} PTDF_{l,j}(\lambda_l^{*+}(\omega) - \lambda_l^{*-}(\omega)) = 0 & & \forall j \in \{N \cup I\} \\
\sum_{j \in \{N \cup I\}} f_j^*(\omega) = 0 & & (2) \\
0 \leq x_{ik}^g - y_{ik}^{*g}(\omega) & \perp \beta_{ik}^{*g}(\omega) \geq 0 & \forall g \in G, i \in I_g, k \in K_g \\
0 \leq h_l - \sum_{j \in \{N \cup I\}} PTDF_{l,j} f_j^*(\omega) & \perp \lambda_l^{*+}(\omega) \geq 0 & \forall l \\
0 \leq h_l + \sum_{j \in \{N \cup I\}} PTDF_{l,j} f_j^*(\omega) & \perp \lambda_l^{*-}(\omega) \geq 0 & \forall l.
\end{aligned}$$

3.1.2 First Stage Behavior with Market Power

If firm $g \in G$ operates at node $i \in I_g$, then it sells power to TSO in the spot market at the price of node i . Consequently, it receives a marginal profit equal to unit price ($p_i^*(\omega, \cdot)$) minus unit cost of its production (c_{ik}^g) at node i for realization $\omega \in \Omega$. The long run profit of each firm at the first stage is determined by its investment costs. Let x^{-g} be the set of investment decisions of the rival firms. For fixed but arbitrary x^{-g} , the profit of firm $g \in G$ at the first stage for each realization $\omega \in \Omega$ can be formulated as:

$$\Pi_{pool}^g(\omega, x^g, x^{-g}) = \sum_{i \in I_g} \sum_{k \in K_g} (p_i^*(\omega, x^g, x^{-g}) - c_{ik}^g) y_{ik}^{*g}(\omega, x^g, x^{-g}) - \sum_{i \in I_g} \sum_{k \in K_g} \kappa_k x_{ik}^g.$$

For given x^{-g} , firm $g \in G$ determines its optimal investment decision x^{*g} by solving its profit maximization problem:

$$\chi_{pool}^g(x^{-g}) := \{x^{*g} | x^{*g} = \underset{x^g \geq 0}{\operatorname{argmax}} E_\omega[\Pi_{pool}^g(\omega, x^g, x^{-g})]\}.$$

where $\chi_{pool}^g(x^{-g})$ is the set of solutions which maximize firm g 's long run profit for given x^{-g} .

3.2 Bilateral Market Model

In a bilateral market, competitive firms supply power by bilateral transactions and purchase transmission services for these transactions from TSO who prices transmission capacity based on a congestion pricing scheme. The TSO charges a congestion based wheeling fee v_j (€/MWh) for transmitting power from an arbitrary hub node to node j . Firm $g \in G$ pays $-v_i$ to get power to the hub from its supply node $i \in I_g$ and v_n to convey power

for sale from the hub to customers at node $n \in N$. The additional decision variables of the bilateral market at the second stage are the following:

$s_n^g(\omega)$: quantity of power sold by firm $g \in G$ at demand node $n \in N$ at demand realization $d(\omega)$

$v_j(\omega)$: wheeling fee for transmitting power from an arbitrary hub node to node $j \in \{N \cup I\}$ at demand realization $d(\omega)$.

Next, we formulate the two-stage model of each firm in a bilateral market.

3.2.1 Perfect Competition Equilibrium at Second Stage

Again by using Proposition 2.1 (ii), we know that the solution of OPF problem (3) is a competitive equilibrium of the bilateral spot market for each $\omega \in \Omega$. It can be easily observed that the only difference between OPF problems (1) and (3) is the energy balance constraints:

$$\begin{aligned}
Z_{bilateral}^*(\omega, x) := & \min_{\{s(\omega), y(\omega), \delta(\omega), f(\omega)\}} \sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} c_{ik}^g y_{ik}^g(\omega) + VOLL \sum_{n \in N} \delta_n(\omega) \\
& s.t. \\
& \sum_{g \in G} s_n^g(\omega) + \delta_n(\omega) \geq d_n(\omega) \quad (p_n(\omega)) \quad \forall n \in N \\
& \sum_{g \in G} \sum_{k \in K_g} y_{jk}^g(\omega) - \sum_{g \in G} s_j^g(\omega) - f_j(\omega) = 0 \quad (v_j(\omega)) \quad \forall j \in \{N \cup I\} \\
& \sum_{i \in I_g} \sum_{k \in K_g} y_{ik}^g(\omega) - \sum_{n \in N} s_n^g(\omega) = 0 \quad (\rho_g(\omega)) \quad \forall g \in G
\end{aligned} \tag{3}$$

$y(\omega)$ satisfy $Cons_Cap(\omega, x)$, $f(\omega)$ satisfy $Cons_PTDF(\omega)$

$y(\omega) \geq 0$, $\delta(\omega) \geq 0$, $s(\omega) \geq 0$.

Again, $\rho_g(\omega)$, $p_n(\omega)$, and $v_j(\omega)$, are Lagrange multipliers; in particular, $p_n(\omega)$ is the price of unit power (€/MWh) at demand node n and, as mentioned earlier, $v_j(\omega)$ is the congestion based wheeling fee (€/MWh) for transmitting power from an arbitrary node to node j . $\rho_g(\omega)$ is the dual variable of the energy balance equation of firm $g \in G$ at the hub; hence its standard interpretation is the marginal cost of firm g at the hub of the linearized DC network. The KKT conditions of the OPF problem (3) yield the following equilibrium conditions of the bilateral market:

For each $\omega \in \Omega$ and x ,

$$\begin{aligned}
0 \leq c_{ik}^g - \rho_g^*(\omega) - v_i^*(\omega) + \beta_{ik}^{*g}(\omega) & \perp y_{ik}^{*g}(\omega) \geq 0 & \forall g \in G, i \in I_g, k \in K_g \\
0 \leq \rho_g^*(\omega) - p_n^*(\omega) + v_n^*(\omega) & \perp s_n^{*g}(\omega) \geq 0 & \forall g \in G, n \in N \\
0 \leq VOLL - p_n^*(\omega) & \perp \delta_n^*(\omega) \geq 0 & \forall n \in N \\
0 \leq \sum_{g \in G} s_n^{*g}(\omega) + \delta_n^*(\omega) - d_n(\omega) & \perp p_n^*(\omega) \geq 0 & \forall n \in N \\
v_j^*(\omega) + \sum_{l \in L} PTDF_{l,j}(\lambda_l^{*+}(\omega) - \lambda_l^{*-}(\omega)) = 0 & & \forall j \in \{N \cup I\} \\
\sum_{i \in I_g} \sum_{k \in K_g} y_{ik}^{*g}(\omega) - \sum_{n \in N} s_n^{*g}(\omega) = 0 & & \forall g \in G \\
\sum_{g \in G} \sum_{k \in K_g} y_{jk}^{*g}(\omega) - \sum_{g \in G} s_j^{*g}(\omega) - f_j^*(\omega) = 0 & & \forall j \in \{N \cup I\} \\
0 \leq x_{ik}^g - y_{ik}^{*g}(\omega) & \perp \beta_{ik}^{*g}(\omega) \geq 0 & \forall g \in G, i \in I_g, k \in K_g \\
0 \leq h_l - \sum_{j \in \{N \cup I\}} PTDF_{l,j} f_j^*(\omega) & \perp \lambda_l^{*+}(\omega) \geq 0 & \forall l \\
0 \leq h_l + \sum_{j \in \{N \cup I\}} PTDF_{l,j} f_j^*(\omega) & \perp \lambda_l^{*-}(\omega) \geq 0 & \forall l,
\end{aligned} \tag{4}$$

where $\lambda_l^+(\omega)$ and $\lambda_l^-(\omega)$ are Lagrange multipliers of $Cons_PTDF(\omega)$ and $\beta(\omega)$ is the vector of Lagrange multipliers associated to $Cons_Cap(\omega, x)$.

3.2.2 First Stage Behavior with Market Power

At the first stage, each firm maximizes its revenue minus costs. The revenue of a firm from each demand node $n \in N$ is the unit price multiplied by the number of units sold by that firm at that demand node n . There are three components of the firm's total cost; namely investment, production, and shipment costs. The unit shipment cost consists of the wheeling fee for the generation at supply nodes ($-v_i$) and the wheeling fee for the sales at demand nodes (v_n). For fixed but arbitrary x^{-g} , the profit of firm $g \in G$ in the first stage at each realization $\omega \in \Omega$ is formulated as:

$$\begin{aligned}
\Pi_{bilateral}^g(\omega, x^g, x^{-g}) = & \sum_{n \in N} [p_n^*(\omega, x^g, x^{-g}) - v_n^*(\omega, x^g, x^{-g})] s_n^{*g}(\omega, x^g, x^{-g}) \\
& - \sum_{i \in I_g} \sum_{k \in K_g} [c_{ik}^g - v_i^*(\omega, x^g, x^{-g})] y_{ik}^{*g}(\omega, x^g, x^{-g}) - \sum_{i \in I_g} \sum_{k \in K_g} \kappa_k x_{ik}^g.
\end{aligned}$$

Thus for given x^{-g} , each firm $g \in G$ determines its optimal investment decision x^{*g} by solving its profit maximization problem:

$$\mathcal{X}_{bilateral}^g(x^{-g}) := \{x^{*g} | x^{*g} = \underset{x^g \geq 0}{\operatorname{argmax}} E_{\omega}[\Pi_{bilateral}^g(\omega, x^g, x^{-g})]\},$$

where $\chi_{bilateral}^g(x^{-g})$ is the set of solutions which maximize firm g 's long run profit for given x^{-g} .

3.3 Equivalence of Two-stage Bilateral and Pool Market Models

In this section, we show that the capacity choice set of the firms anticipating the competitive outcome of a pool market is equivalent to that of the firms anticipating the competitive outcome of a bilateral market. Proposition 2.1 (i) addresses the equivalence of the equilibria in bilateral and pool markets (i.e., same prices, production quantities, and scarcity rents) when capacities are given. By using this result in our setting, we can easily conclude that for given any x and $\omega \in \Omega$:

- $p^{*p}(\omega, x^g, x^{-g}) = p^{*b}(\omega, x^g, x^{-g})$,
- $y^{*pg}(\omega, x^g, x^{-g}) = y^{*bg}(\omega, x^g, x^{-g})$,
- $\beta^{*pg}(\omega, x^g, x^{-g}) = \beta^{*bg}(\omega, x^g, x^{-g})$,

where we put superscripts p and b to indicate the variables in pool and bilateral markets, respectively. By using the above equalities, we next show that when demand is stochastic and generation capacities are endogenously chosen at the first stage in which firms may exert market power, the first and the second stage problems still yield the same equilibria (i.e., same capacities, prices, production quantities, and scarcity rents) for both market models.

Proposition 3.1. *For a firm anticipating the competitive spot market outcome of the the second stage, $E_\omega[\Pi_{pool}^g(\omega, x^g, x^{-g})] = E_\omega[\Pi_{bilateral}^g(\omega, x^g, x^{-g})]$ holds for all $g \in G$ at any given (x^g, x^{-g}) . This implies that the two-stage game models with pool and bilateral markets not only yield the same second-stage equilibria at each ω but also the solution sets of the first stage game in these markets are equal, which implies both markets yield the same first stage equilibria.*

Proof. See Appendix.

Proposition 3.1 also holds when firms are Cournot players at the second stage by using the equivalence result of Metzler et al. (2003) (i.e., Theorem 1) for bilateral and pool spot market models. When capacities are given, Metzler et al. (2003) show that Cournot generators in bilateral spot markets with perfectly competitive arbitrageurs yield the same Nash equilibrium as Cournot competition in pool spot markets. Due to the equivalence of pool and bilateral market models, we continue our analysis with pool market model. Thus, we ignore the superscript p in the corresponding variables and use $\Pi^g(\cdot)$ to refer to the profit function of firm $g \in G$ in pool market model.

3.4 Characterization of the Two-stage Game

In this section, we investigate the characteristics of the expected profit function of a strategic firm anticipating a competitive market outcome under a continuous random demand with a general distribution. In order to preserve analytical tractability, we consider an electricity market without any network limitations; this in turn collapses the problem to a single node case. In such a single node market, there are several suppliers coping with an aggregated demand. Let $D = \sum_{n \in N} d_n$ denote the aggregated continuous random demand. The following assumptions hold for the two-stage model.

Assumption 3.2. D is a continuous random demand with a cumulative distribution Ψ and a continuous probability density function Ψ' whose support is on some interval $[\underline{D}, \bar{D}]$.

Assumption 3.3. (i) There is a single firm at each supply node and each firm invests in only one technology; that is $|G| = |I| = |K|$. Firm 1 owns the cheapest generator and firm K owns the peak generator where $c_1 < c_2 < \dots < c_K$ and $\kappa_1 > \kappa_2 > \dots > \kappa_K$.

(ii) For at least one technology $k \in K$, it holds that $VOLL > c_k + \kappa_k$.

From now on, we use $x_{-k} := (x_j)_{j=1, j \neq k}^K$ to denote the vector of strategies of the rival firms of firm $k \in K$ and $x_{-k}^* := (x_j^*)_{j=1, j \neq k}^K$ is the vector of their optimal decisions at equilibrium.

The continuity of the probability density function in Assumption 3.2 is crucial for establishing our results in this section. Furthermore, Assumption 3.3 (ii) guarantees that the equilibrium of the two-stage game is nontrivial. Let the total generation capacity be lower than the minimum demand ($\sum_{j=1}^K x_j < \underline{D}$). Then, the market prices would be equal to $VOLL$ for all demand realizations in the perfectly competitive spot market. According to Assumption 3.3 (ii), $VOLL$ is higher than the unit investment and operation cost for at least one technology, say $k' \in K$. This, in turn, implies that when there isn't sufficient capacity for any demand realization, the expected scarcity rent of at least one firm investing in technology k' , which is equal to $VOLL - c_{k'}$, is higher than its unit investment cost, $\kappa_{k'}$. Therefore, for given $x_{-k'} < \underline{D}$, $\frac{\partial E_\omega[\Pi^{k'}(\omega, x_{k'}, x_{-k'})]}{\partial x_{k'}} = VOLL - c_{k'} - \kappa_{k'} > 0$ for $x_{k'} \in [0, \underline{D} - x_{-k'})$. Thus, firm k' would be able to increase its expected profit by investing in technology k' until there is sufficient capacity at least for the minimum demand level. Thus, Assumption 3.3 (ii) guarantees that total investment capacity at equilibrium satisfies at least the minimum demand: $\sum_{j=1}^K x_j^* \geq \underline{D}$.

As given in Section 3.1.2, for each fixed and arbitrary x_{-k} , firm $k \in K$ obtains its expected profit $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ and determines its optimal strategy by solving its profit maximization problem:

$$\max_{x_k \geq 0} E_\omega[\Pi^k(\omega, x_k, x_{-k})]. \quad (5)$$

For any given x_{-k} , let $\chi^k(x_{-k})$ denote the solution set of problem (5). Then a Nash equilibrium is a point such that $x_k^* \in \chi^k(x_{-k}^*)$ for all $k \in K$. Next, we assess the properties of $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ in (5). Based on the corresponding properties, we will analyze the existence and uniqueness of equilibria $x_k^* \in \chi^k(x_{-k}^*)$ for all $k \in K$.

From the proof of Proposition 3.1 given in Appendix 5, one can easily see that, at the first stage, the expected profit function of each firm $k \in K$ for given x_{-k} can be formulated in terms of its expected scarcity rent:

$$E_\omega[\Pi^k(\omega, x_k, x_{-k})] = (E_\omega[\beta_k^*(\omega, x_k, x_{-k})] - \kappa_k)x_k.$$

For given (x_k, x_{-k}) at the second stage, Figure 1 illustrates the stepwise supply curve and some demand realizations within $[\underline{D}, \bar{D}]$ in a single-node spot market. As illustrated in Figure 1 unless the capacity of all firms are fully utilized at realization $\omega \in \Omega$, the market price ($p^*(\omega, x_k, x_{-k})$) is equal to one of the marginal generating cost values where the corresponding demand realization crosses the stepwise supply curve. Otherwise, the market price is equal to $VOLL$. Note that when demand is elastic with a random inverse demand curve, the market prices can also take values between c_k and c_{k+1} for all $k \in \{1, \dots, K-1\}$ or between c_K and $VOLL$ for some realizations $\omega \in \Omega$ as illustrated in Section 4.2 (See Figures 3 and 4).

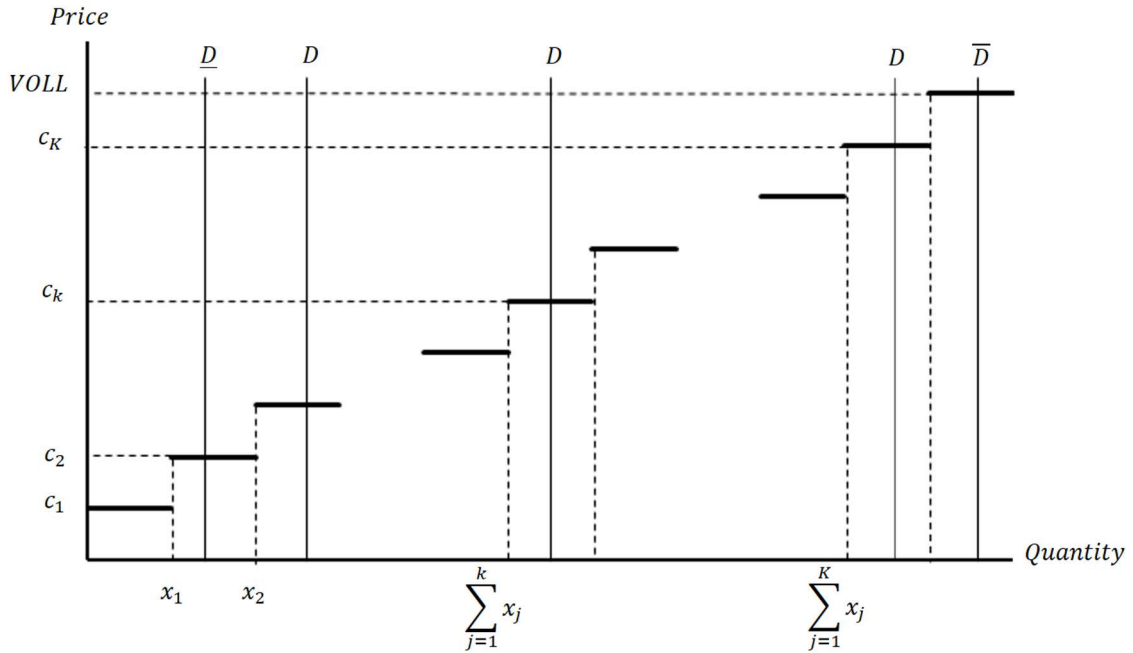


Figure 1: Illustration of a single-node market with stepwise supply function for given x and exogenous random demand

Next, we derive a closed form expression for the expected scarcity rent of firm k and establish two basic functional properties; namely its continuity and monotonicity in x_k . For given x_{-k} , we denote the total capacity

of the $(m - 1)$ cheapest firms excluding firm k in the market as $X_{m-1}(k) = \sum_{j=1, j \neq k}^m x_j$, for $k \leq m \leq K$. Then $X_{m-1}(k) + x_k$ is the total capacity of the m cheapest firms including firm k in the market.

Proposition 3.4. *Under Assumption 3.2, for given x_{-k} , the expected scarcity rent of firm $k \in K$ can be formulated in a closed form expression as follows:*

$$E_{\omega}[\beta_k^*(\omega, x_k, x_{-k})] = VOLL - c_k - (VOLL - c_K)\Psi(x_k + X_{K-1}(k)) - \sum_{m=k}^{K-1} (c_{m+1} - c_m)\Psi(x_k + X_{m-1}(k)). \quad (6)$$

Proof. See Appendix.

We may also write expression (6) in terms of probabilities (that are both driven from equation (20) given in *Proof of Proposition 3.4* in Appendix 5) which makes it easier to interpret:

$$E_{\omega}[\beta_k^*(\omega, x_k, x_{-k})] = \sum_{m=k}^{K-1} (c_{m+1} - c_k)P(X_{m-1}(k) + x_k \leq D \leq X_m(k) + x_k) + (VOLL - c_k)P(D \geq X_{K-1}(k) + x_k).$$

The above expression indicates the following: Firm k receives a positive scarcity rent either when there is at least one other firm operating with a higher marginal generation cost in the market or when demand exceeds the total generation capacity and the market price is set at $VOLL$. The price of the electricity is c_{m+1} with probability $P(X_{m-1}(k) + x_k \leq D \leq X_m(k) + x_k)$ in which case the firm operating with the highest marginal generation cost is using the technology $m + 1$, $m \geq k$. The price of the electricity may go up to $VOLL$ with probability $P(D \geq X_{K-1}(k) + x_k)$ in which case the demand is so high that there is not enough generating capacity in the market. Since the marginal generation cost of firm k is c_k , its scarcity rent from the second stage would be $(c_{m+1} - c_k)$ with probability $P(X_{m-1}(k) + x_k \leq D \leq X_m(k) + x_k)$, $m \geq k$, and $(VOLL - c_k)$ with probability $P(D \geq X_{K-1}(k) + x_k)$. Note that for given x_{-k} , the values of these probabilities depend on x_k which changes the corresponding ranges.

Lemma 3.5. *Under Assumption 3.2, the following properties hold for the expected scarcity rent function (6) of firm $k \in K$:*

(i) $E_{\omega}[\beta_k^*(\omega, x_k, x_{-k})]$ is a continuous function of $(x_k, x_{-k}) \in \mathcal{R}_+^{\mathcal{K}}$, and

(ii) $E_{\omega}[\beta_k^*(\omega, x_k, x_{-k})]$ is nonincreasing in $x_k \in \mathcal{R}_+$.

Proof. See Appendix.

Next, by using the functional properties of the expected scarcity rent function of firm $k \in K$, we will explore the functional properties - such as continuity, differentiability and concavity - of the expected profit function of firm k . In doing so, we will differentiate between two cases. The first one is the simpler case of symmetric firms using the same technology; hence having the same operational and investment costs. We show that when firms are symmetric, the expected profit of each firm is continuous and strictly quasiconcave under certain conditions and there exists a unique Nash equilibrium. Then we will consider asymmetric firms whose expected profit functions are more complicated and may be analytically intractable in general.

3.4.1 Symmetric Firms

In the symmetric case, the unit production and investment costs of all firms have the same value; that is, $c_1 = c_2 = \dots = c_K = c$ and $\kappa_1 = \kappa_2 = \dots = \kappa_K = \kappa$. This yields an identical profit function for all firms, $E_\omega[\Pi^k(\omega, x_k, x_{-k})] = (E_\omega[\beta_k^*(\omega, x_k, x_{-k})] - \kappa)x_k$, and (6) reduces to (7):

$$E_\omega[\beta_k^*(\omega, x_k, x_{-k})] = (VOLL - c)(1 - \Psi(x_k + X_{K-1}(k))), \quad (7)$$

where $X_{K-1}(k) = \sum_{j=1, j \neq k}^K x_j$ is the total capacity of all the rival firms. Next, we show in Lemma 3.6 that the expected scarcity rent function of each firm k in (7) is differentiable with respect to x_k almost everywhere except at three breakpoints $x_k \in \{0, \max(0, \underline{D} - X_{K-1}(k)), \max(0, \overline{D} - X_{K-1}(k))\}$. Among these breakpoints, $x_k^{min} = \underline{D} - X_{K-1}(k)$ is the investment level below which the probability of demand exceeding total investment capacity is 1 and $x_k^{max} = \overline{D} - X_{K-1}(k)$ is the investment level above which probability of demand exceeding total investment capacity is zero.

Lemma 3.6. *For given x_{-k} , if Ψ is differentiable on $[\underline{D}, \overline{D}]$, then $E_\omega[\beta_k^*(\omega, x_k, x_{-k})]$ in (7) is differentiable w.r.t. $x_k \in \mathcal{R}_+$ almost everywhere except $x_k \in \{0, x_k^{min}, x_k^{max}\}$.*

Proof. See Appendix.

In Theorems 3.8 and 3.10 we show that $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is a continuous function and (strictly) quasiconcave in x_k under certain conditions on the cumulative distribution function Ψ . It is well known that games possess a Nash equilibrium if (1) the strategy spaces are nonempty, convex and compact, and (2) players have continuous and quasiconcave payoff functions [e.g., Debreu (1952), Fudenberg and Tirole (1991)]. For completeness, we give the result by Debreu (1952) in the following proposition. This result can be used to establish existence of a Nash equilibrium for our symmetric game which is given by Theorem 3.13.

Proposition 3.7. [Debreu (1952)] *An n -persons strategic game has a Nash equilibrium if the strategy spaces*

$S_i, i = 1 \dots n$, in this game are non-empty, compact, convex subsets of a Euclidian space and the payoff functions u_i are continuous at s and quasiconcave in s_i where $s_i \in S_i$ is the strategy of player i .

Theorem 3.8. Let Ψ be twice continuously differentiable function on $[\underline{D}, \overline{D}]$ where Ψ' and Ψ'' are the first and second derivatives of Ψ . Then the expected profit function of each firm $k \in K$ has the following properties:

(i) $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is continuous at $(x_k, x_{-k}) \in \mathcal{R}_+^{\mathcal{K}}$;

(ii) For given x_{-k} , $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is differentiable w.r.t. $x_k \in \mathcal{R}_+$ almost everywhere except $x_k \in \{0, x_k^{\min}, x_k^{\max}\}$;

(iii) For given x_{-k} , $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is (strictly) concave for $x_k \in (x_k^{\min}, x_k^{\max})$ iff $-\Psi''(s+C)s - \Psi'(s+C) (<) \leq 0$ for $\forall s, C \in \mathcal{R}_+$ where C is some constant. Whenever this condition holds, $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is a (strictly) quasiconcave function of x_k on \mathcal{R}_+ .

Proof. See Appendix.

Remark 3.9. If the probability density function, Ψ' , is nondecreasing, then $-\Psi''(x_k + X_{K-1})x_k - \Psi'(x_k + X_{K-1}) \leq 0$ holds for all $x_k \in \mathcal{R}_+$. Therefore, the nondecreasing property of a probability density function is a sufficient condition for quasiconcavity of $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ on $[0, \infty)$. In particular, the uniform distribution satisfies this property.

The condition stated in (iii) of Theorem 3.8 is similar to a standard assumption used in the literature (see Sherali et al. (1983), Wolf and Smeers (1997), Grimm and Zoetl (2008), and Xu (2005)). In these references, the corresponding assumption is given for a random inverse demand function. In our setting with random exogenous demand, Theorem 3.8 indicates that a similar condition needs to be assumed for the cumulative probability function of demand. Aside from this standard condition, we identify, in the next theorem, a large class of probability distributions having a monotone increasing hazard function that guarantees the strict quasiconcavity of $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ w.r.t. x_k . In particular, logconcave probability distributions have a monotone increasing hazard function which is a milder condition than the condition stated in Remark 3.9. We will come back to this in Remark 3.14.

Theorem 3.10. Under Assumption 3.2, let the hazard function of demand, $H(s) = \Psi'(s)/(1 - \Psi(s))$, be monotone increasing on $[\underline{D}, \overline{D}]$. Then for any $k \in K$ and x_{-k} , $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is strictly quasiconcave for $x_k \in [x_k^{\min}, x_k^{\max}]$ which implies that it is also strictly quasiconcave w.r.t. x_k on \mathcal{R}_+ .

Proof. See Appendix.

The strategy space of each firm $k \in K$ can be represented by a nonempty, compact, and convex set which is

shown in the next lemma. Then in Lemma 3.12, we establish the boundaries for the total generation capacity at the first-stage equilibrium problem. Finally, in Theorem 3.13, we establish the existence and uniqueness result and characterize the Nash equilibrium for the generation capacity investment game of the symmetric firms.

Lemma 3.11. *The strategy space of each firm $k \in K$ is $S_k := [0, \bar{D}]$ which is nonempty, compact, and convex.*

Proof. See Appendix.

Lemma 3.12. *Assume that there exists an equilibrium, $(x_k^*)_{k=1}^K$, for the symmetric game. Then, under Assumption 3.3, it holds that $\underline{D} \leq \sum_{j=k}^K x_k^* \leq \bar{D}$.*

Proof. See Appendix.

Theorem 3.13 (Existence and Uniqueness). *Under Assumptions 3.3 and 3.2, let the hazard function of demand, $H(s) = \Psi'(s)/(1 - \Psi(s))$, be monotone increasing on $[\underline{D}, \bar{D}]$. Then for the games defined in Sections 3.1.2 and 3.2.2 with symmetric firms and unlimited transmission capacity, there exists a unique symmetric Nash equilibrium, $x^* = x_1^* = \dots = x_K^*$, which satisfies*

$$x^* = \frac{(VOLL - c)(1 - \Psi(Kx^*)) - \kappa}{(VOLL - c)\Psi'(Kx^*)} \quad (8)$$

and $\underline{D} \leq Kx^* \leq \bar{D}$.

Proof. See Appendix.

Remark 3.14. By Corollary 2 of Bagnoli and Bergstrom (2005), we know that if the probability density function $\Psi'(s)$ is logconcave on $[\underline{D}, \bar{D}]$, then the hazard function $H(s)$ is monotone increasing on $[\underline{D}, \bar{D}]$. Hence, the logconcavity of probability density function of demand implies strict quasiconcavity of $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ w.r.t. x_k for each k . By Remark 1 of Bagnoli and Bergstrom (2005), the converse is not true because there exists probability distributions with monotone increasing hazard functions but without logconcave density functions.

By Remark 3.14, we can identify a broad class of commonly used continuous probability distributions which guarantee the strict quasiconcavity of profit function of each firm at the first stage and hence the existence and uniqueness of Nash equilibria characterized as in Theorem 3.13. Bagnoli and Bergstrom (2005) list a number of distributions with logconcave density functions in their Table 1. These include several widely used distributions such as uniform, normal, exponential, Gamma (parameter ≥ 1), Weibull (parameter ≥ 1), Beta (both parameters ≥ 1), among others.

3.4.2 Asymmetric Firms

In the asymmetric case, firms may not have identical profit functions since each firm's marginal generation and investment costs are different. In the symmetric case, we have seen that the expected scarcity rent function of each firm is a continuous piecewise function with at most three breakpoints including zero. In the asymmetric case, for each firm $k \in K$, $E_\omega[\beta_k^*(\omega, x_k, x_{-k})]$ will be a piecewise continuous function which may have between 3 and $2(K-1) + 3$ breakpoints. The number of breakpoints depends on the position of the firm in the merit order in the spot market. A firm with the highest marginal generation cost (c_K) will have a similar expected scarcity rent function with at most 3 breakpoints as in the symmetric case whereas a firm with the lowest marginal generation cost (c_1) will have at most $2(K-1) + 3$ breakpoints. To generalize, a firm $k \in K$ with marginal generation cost c_k will have at most $2(K-k) + 3$ breakpoints which can be calculated by doing a careful accounting of the breakpoints in (6). Let $\underline{x}_k(m) = \max(0, \underline{D} - X_m(k))$ and $\overline{x}_k(m) = \max(0, \overline{D} - X_m(k))$. The breakpoints of the firm k 's expected scarcity rent function will be $0, \{\underline{x}_k(m)\}_{m=k-1}^{K-1}, \{\overline{x}_k(m)\}_{m=k-1}^{K-1}$.

As a result of expected scarcity rent being a piecewise continuous function, $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is also piecewise and continuous for each firm k . Moreover, $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ consists of at most $2(K-k) + 4$ function pieces with $2(K-k) + 3$ breakpoints. To establish conditions that guarantee quasiconcavity of $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ w.r.t. x_k for each firm k under a general distribution is more complicated here. This can be done for $k = K$,

$$E_\omega[\beta_K^*(\omega, x_K, x_{-K})] = (VOLL - c_K)(1 - \Psi(x_K + X_{K-1}(k)))$$

which is similar to (7). Therefore, the results in Theorems 3.8, 3.10, 3.13 and Remarks 3.9 and 3.14 also hold for the profit function of the most expensive firm ($k = K$) but not necessarily for the other firms ($k < K$).

Next, we give an example with two firms (a base-load and a peak generator) to illustrate the expected scarcity rent and expected profit functions of these firms when demand is uniformly distributed. The example clearly illustrates that for when the demand is uniformly distributed, the expected scarcity rent functions of the firms are piecewise linear. However even in this very simple case, the expected profit function of the base-load generator is not necessarily quasiconcave.

Assume that there are two generators: base-load ($g = 1$) and peak ($g = 2$) with $c_1 < c_2$ and $\kappa_1 > \kappa_2$. Let the transmission capacities be infinite ($h = \infty$) and the aggregated demand of all nodes be uniformly distributed on $[d_{min}, d_{max}]$; that is $d \sim U[d_{min}, d_{max}]$. Next, we give the formulation of the expected scarcity rent function of each firm given a investment strategy of its rival firm. Without loss of generality, we will give the formulations when the given investment strategies satisfy $x_1 < d_{min}$ and $x_2 < d_{min}$ since this will yield the maximum number of breakpoints for the expected scarcity rent and the expected profit functions for both firms. Depending on the values of x_1 and x_2 , the number of breakpoints may be lower but the formulations of corresponding function segments will remain the same.

Lemma 3.15. For given $x_2 \geq 0$, there are constants $a_{\beta_1}^1, a_{\beta_1}^2, a_{\beta_1}^3 \in R$ and $b_{\beta_1}^1, b_{\beta_1}^2, b_{\beta_1}^3 > 0$ which satisfy

$$E_{\omega}[\beta_1^*(\omega, x_1, x_2)] = \begin{cases} VOLL - c_1, & \text{if } 0 \leq x_1 < d_{min} - x_2 \\ a_{\beta_1}^1 - b_{\beta_1}^1 x_1, & \text{if } d_{min} - x_2 \leq x_1 < d_{min} \\ a_{\beta_1}^2 - b_{\beta_1}^2 x_1, & \text{if } d_{min} \leq x_1 < d_{max} - x_2 \\ a_{\beta_1}^3 - b_{\beta_1}^3 x_1, & \text{if } d_{max} - x_2 \leq x_1 < d_{max} \\ 0 & \text{otherwise.} \end{cases}$$

Similarly for given $x_1 \geq 0$, there are constants $a_{\beta_2} \in R$, and $b_{\beta_2} > 0$ satisfying

$$E_{\omega}[\beta_2^*(\omega, x_2, x_1)] = \begin{cases} VOLL - c_2, & \text{if } 0 \leq x_2 < d_{min} - x_1 \\ a_{\beta_2} - b_{\beta_2} x_2, & \text{if } d_{min} - x_1 \leq x_2 < d_{max} - x_1 \\ 0 & \text{otherwise.} \end{cases}$$

Furthermore, these constants can be explicitly found.

Proof. See Appendix.

Obviously, by using Lemma 3.15, the expected profit functions of firm 1 and 2 can also be explicitly formulated as follows (see the proof of Lemma 3.15 in Appendix 5 for the associated constants):

For given $x_2 \geq 0$,

$$E_{\omega}[\Pi_1(\omega, x_1, x_2)] = \begin{cases} (VOLL - c_1 - \kappa_1)x_1, & \text{if } 0 \leq x_1 < d_{min} - x_2 \\ (a_{\beta_1}^1 - \kappa_1)x_1 - b_{\beta_1}^1 x_1^2, & \text{if } d_{min} - x_2 \leq x_1 < d_{min} \\ (a_{\beta_1}^2 - \kappa_1)x_1 - b_{\beta_1}^2 x_1^2, & \text{if } d_{min} \leq x_1 < d_{max} - x_2 \\ (a_{\beta_1}^3 - \kappa_1)x_1 - b_{\beta_1}^3 x_1^2, & \text{if } d_{max} - x_2 \leq x_1 < d_{max} \\ -\kappa_1 x_1 & \text{otherwise.} \end{cases}$$

Similarly for given $x_1 \geq 0$,

$$E_\omega[\Pi_2(\omega, x_2, x_1)] = \begin{cases} (VOLL - c_2 - \kappa_2)x_2, & \text{if } 0 \leq x_2 < d_{min} - x_1 \\ (a_{\beta_2} - \kappa_2)x_2 - b_{\beta_2}x_2^2, & \text{if } d_{min} - x_1 \leq x_2 < d_{max} - x_1 \\ -\kappa_2x_2 & \text{otherwise.} \end{cases}$$

Lemma 3.16. $E_\omega[\Pi_1(\omega, x_1, x_2)]$ and $E_\omega[\Pi_2(\omega, x_2, x_1)]$ are continuous functions of $x_1 \in \mathcal{R}_+$ and $x_2 \in \mathcal{R}_+$, respectively; however $E_\omega[\Pi_1(\omega, x_1, x_2)]$ is not differentiable at points $x_1 \in \{d_{min} - x_2, d_{min}, d_{max} - x_2, d_{max}\}$ and $E_\omega[\Pi_2(\omega, x_2, x_1)]$ is not differentiable at points $x_2 \in \{d_{min} - x_1, d_{max} - x_1\}$. Moreover, $E_\omega[\Pi_2(\omega, x_2, x_1)]$ is quasiconcave in x_2 .

Proof. See Appendix.

Remark 3.17. For the above example, each function piece of $E_\omega[\Pi_1(\omega, x_1, x_2)]$ is concave. Let $\frac{\partial^+ E_\omega[\Pi_1(\omega, x_1, x_2)]}{\partial x_1}$ and $\frac{\partial^- E_\omega[\Pi_1(\omega, x_1, x_2)]}{\partial x_1}$ be the right-hand and left-hand side derivatives w.r.t. x_1 , respectively. Then, $E_\omega[\Pi_1(\omega, x_1, x_2)]$ is quasiconcave in x_1 if the following condition holds at each breakpoint $x_1 \in \{d_{min} - x_2, d_{min}, d_{max} - x_2, d_{max}\}$:

$$\text{If } \frac{\partial^- E_\omega[\Pi_1(\omega, x_1, x_2)]}{\partial x_1} < 0 \text{ then } \frac{\partial^+ E_\omega[\Pi_1(\omega, x_1, x_2)]}{\partial x_1} < 0, \quad (9)$$

which implies whenever $E_\omega[\Pi_1(\omega, x_1, x_2)]$ starts to decrease, it does not increase again.

Since we know the constants explicitly, we can calculate the right and left derivatives at each breakpoint and identify the cases when $E_\omega[\Pi_1(\omega, x_1, x_2)]$ will be quasiconcave. For the above example, all the breakpoints, except $x_1 = d_{max} - x_2$, always satisfy condition (9) regardless of the constant values. Thus for $x_1 = d_{max} - x_2$, the constants $a_{\beta_1}^2, a_{\beta_1}^3, b_{\beta_1}^2, b_{\beta_1}^3$ satisfying condition (9) guarantee the quasiconcavity of $E_\omega[\Pi_1(\omega, x_1, x_2)]$:

$$\begin{aligned} \text{If } \frac{\partial^- E_\omega[\Pi_1(\omega, d_{max} - x_2, x_2)]}{\partial x_1} &= (a_{\beta_1}^2 - \kappa_1) - 2b_{\beta_1}^2(d_{max} - x_2) < 0, \\ \text{then } \frac{\partial^+ E_\omega[\Pi_1(\omega, d_{max} - x_2, x_2)]}{\partial x_1} &= (a_{\beta_1}^3 - \kappa_1) - 2b_{\beta_1}^3(d_{max} - x_2) < 0. \end{aligned} \quad (10)$$

Since there are two firms in the example above, the expected profit functions of base-load and peak generators are piecewise continuous functions with maximum 5 and 3 breakpoints, respectively. Since the demand is uniformly distributed, each function piece is linear or quadratic. However, even for uniform demand the expected profit function of base-load generator may not be quasiconcave if it does not satisfy the condition given in (10). For instance, let $[d_{min}, d_{max}] := [100, 500]$ and $VOLL = 10000, c_1 = 10, c_2 = 200, \kappa_1 = 20$, then when $x_2 = 400$, condition (10) does not hold; hence $E_\omega[\Pi_1(\omega, x_1, x_2)]$ is not quasiconcave as shown in Figure

2. Thus, the conditions of Theorem 3.7 for existence of equilibrium are not satisfied.

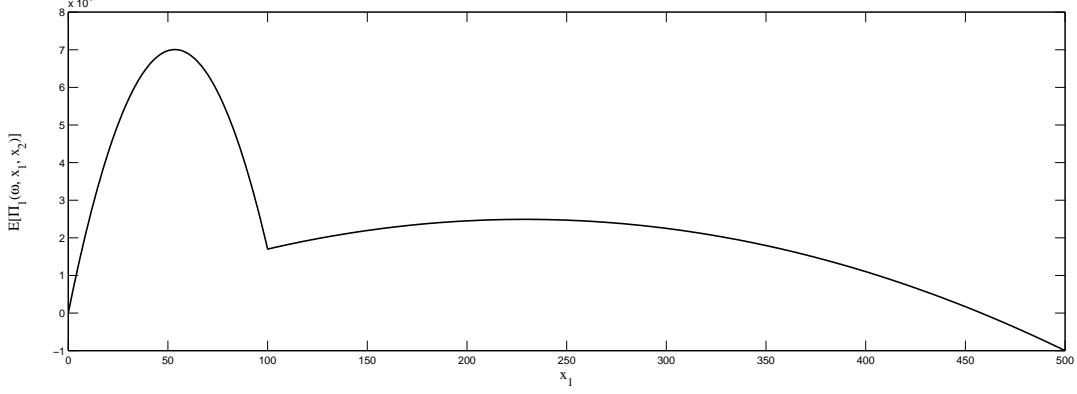


Figure 2: Expected profit function of firm 1 (base-load generator) at $x_2 = 400$

4 Two-stage Capacity Choice Model with Endogenous Random Linear Price-Demand Curve

In this section, instead of VOLL pricing we consider an electricity market where consumers can respond to prices and give firms an incentive to build generation capacity. When consumers respond to prices, we have an elastic demand and we represent the reaction of consumers to the prices by decreasing linear price-demand curves. The standard linear inverse demand function with random parameters may be represented by $P_n(\omega, d_n)$:

$$P_n(\omega, d_n) = \alpha_n(\omega) - \gamma_n(\omega)d_n,$$

where $P_n(\omega, 0) = \alpha_n(\omega) < \infty, \forall \omega$. For given ω , d_n is not fixed as in Section 3 but it is a decision variable which depends on the price of electricity at node n .

We focus on the case of random intercept: $P_n(\omega, d) = \alpha_n(\omega) - \gamma_n d_n$. This is a standard assumption in general in the literature (e.g., Gabszewicz and Poddar (1997), Xu (2005), Murphy and Smeers (2005), Lagerlöf (2006)). We denote the cumulative distribution of $\alpha(\omega)$ by Φ whose support is on some interval $[\alpha_{min}, \alpha_{max}]$.

Dealing with an elastic demand (without VOLL pricing) instead of a fixed demand with VOLL pricing will slightly change the equilibrium conditions of the second stage game and the corresponding OPF problem for pool and bilateral markets. For inelastic demand in Section 3, we showed that the first and second stage equilibrium of bilateral and pool market models are equivalent. This result will also hold when demand is endogenous. Hence, we will continue this section with pool market model formulation.

4.1 Pool Market Model

Perfect Competition Equilibrium at Second Stage: When demand responds to prices, the corresponding Optimal Power Flow Problem (OPF) will slightly be different from the OPF Problem (1). For any $\omega \in \Omega$ and x ,

$$\begin{aligned}
Z_{pool}^*(\omega, x) = & \min_{\{y(\omega), f(\omega), d(\omega)\}} \sum_{g \in G} \sum_{i \in I_g} \sum_{k \in K_g} c_{ik}^g y_{ik}^g(\omega) - \sum_{n \in N} \int_0^{d_n(\omega)} P_n(\omega, s) ds, \\
s.t. & \sum_{g \in G} \sum_{k \in K_g} y_{jk}^g(\omega) + f_j(\omega) \geq d_j(\omega) \quad (p_j(\omega)) \quad \forall j \in \{N \cup I\} \\
& y(\omega) \text{ satisfy } Cons_Cap(\omega, x) \\
& f(\omega) \text{ satisfy } Cons_PTDF(\omega) \\
& y(\omega) \geq 0, \quad d(\omega) \geq 0,
\end{aligned} \tag{11}$$

where $d_n(\omega)$ is the decision variable representing endogenous demand in the state of the world ω and $\int_0^{d_n(\omega)} P_n(\omega, s) ds$ can be interpreted as the consumer's willingness to pay in the state of the world ω . The endogenous demand assumption results in the second stage equilibrium conditions in which the following conditions are different compared to the equilibrium conditions given in (2). For each $\omega \in \Omega$ and x ,

$$\begin{aligned}
0 \leq c_{ik}^g - p_i^*(\omega) + \beta_{ik}^{*g}(\omega) & \perp y_{ik}^{*g}(\omega) \geq 0 \quad \forall g \in G, i \in I_g, k \in K_g \\
0 \leq p_n^*(\omega) - P(\omega, d_n^*(\omega)) & \perp d_n^*(\omega) \geq 0 \quad \forall n \in N \\
0 \leq \sum_{g \in G} \sum_{k \in K_g} y_{jk}^{*g}(\omega) - f_j^*(\omega) - d_j^*(\omega) & \perp p_j^*(\omega) \geq 0 \quad j \in \{N \cup I\}.
\end{aligned}$$

The above conditions imply that, at equilibrium, if any generator sells power at demand node n ($d_n^*(\omega) > 0$) then the market price at node n is equal to the price which consumers are willing to pay. Moreover, if any firm produces power at node i ($y_{ik}^{*g}(\omega) > 0$), then the market price at node i is equal to its marginal cost plus scarcity rent. Then for given x and $\omega \in \Omega$, the KKT conditions associated with a positive consumption and generation can be stated as:

$$\begin{aligned}
p_n^*(\omega, x) &= P(\omega, d_n^*(\omega, x)), \forall n \in N, \\
p_i^*(\omega, x) &= c_{ik}^g + \beta_{ik}^{*g}(\omega, x), \forall g \in G, i \in I_g, k \in K_g.
\end{aligned} \tag{12}$$

First Stage Behavior with Market Power: For given x^{*-g} , each firm's behavior and equilibrium conditions at first stage is the same. The profit function ($\Pi_{pool}^g(\omega, x^g, x^{-g})$) for firm $g \in G$ at each realization $\omega \in \Omega$ and the solution set $\mathcal{X}_{pool}^g(x^{-g})$ can be formulated similar to the first stage profit function and the solution set given in Section 3.1.2.

4.2 Characterization of the Two-stage Game

In order to preserve analytical tractability as in Section 3.4, we again consider a simplified model without any network limits. Then the model reduces to a single node electricity market where all demand and supply is concentrated at one node ($d^*(\omega) = \sum_n d_n^*(\omega)$) and Assumption 3.3 (i) holds. Combining Assumption 3.3 (i) and equation (12), the KKT condition associated with a positive generation $y_k^*(\omega, x) > 0$ in a single node market can be stated, for given x and $\omega \in \Omega$, as

$$p^*(\omega, x) = P(\omega, d^*(\omega, x)) = c_k + \beta_k^*(\omega, x), \forall k \in K. \quad (13)$$

According to (13), consumers' willingness to pay is equal to a single market price paid to each firm when the spot market is cleared. Furthermore, each generating firm receives scarcity rent being equal to the market price minus its marginal cost. Thus, from now on $p^*(\omega, x)$ and $P(\omega, d^*(\omega, x))$ will be used interchangeably when we formulate the first stage problem of the firms.

Besides Assumption 3.3 (i), we also make the following assumptions for the two-stage model with linear price-demand curve.

Assumption 4.1. α is a continuous random variable with a cumulative distribution Φ and a continuous probability density function Φ' whose support is on some interval $[\alpha_{min}, \alpha_{max}]$.

Assumption 4.2. For at least one technology $k \in K$, it holds that

$$\int_{\alpha_{min}}^{\alpha_{max}} P(s, 0) \Phi'(s) ds > c_k + \kappa_k.$$

When there is zero capacity ($\sum_{j=1}^K x_j = 0$), consumers are not supplied with any power ($d^*(\omega) = 0$) and are willing to pay $P(\omega, 0)$ for the first unit of power at all realizations, which yields

$$\int_{\alpha_{min}}^{\alpha_{max}} P(s, 0) \Phi'(s) ds = E[\alpha(\omega)]$$

as the expected price that consumers are willing to pay for the first unit of power. According to Assumption 4.2, there exists at least one technology, say $k' \in K$, such that the expected price that consumers are willing to pay for the first unit of power from technology k' is higher than its cost of investment and operation. Therefore, firm k' will be able to increase its expected profit by investing at a positive level in technology k' ; i.e., $x_{k'} > 0$. Thus, Assumption 4.2 guarantees that $\sum_{j=1}^K x_j^* > 0$.

Similar to the exogenous demand case, the expected profit function of firm $k \in K$ for given x_{-k} can be

formulated in terms of its expected scarcity rent:

$$E_{\omega}[\Pi^k(\omega, x_k, x_{-k})] = (E_{\omega}[\beta_k^*(\omega, x_k, x_{-k})] - \kappa_k)x_k.$$

Next, we derive a closed form expression for the expected scarcity rent function for cases with symmetric and asymmetric firms and establish the characteristics of the expected scarcity rent and the expected profit functions for each firm. Different from the case with exogenous demand, the expected scarcity rent and the expected profit function of each firm are differentiable everywhere. On the other hand, similar to the case with exogenous demand, when firms are symmetric we show that the expected profit of each firm is strictly quasiconcave under logconcavity assumption of underlying probability density function and there exists a unique symmetric Nash equilibrium. In case of asymmetric firms, we can still show existence and uniqueness of equilibria under a stricter condition than logconcavity of probability density function.

4.2.1 Symmetric Firms

As mentioned in Section 3 for the exogenous demand case, the assumption of symmetric firms implies that the unit costs are the same for all firms; that is, $c_1 = c_2 = \dots = c_K = c$ and $\kappa_1 = \kappa_2 = \dots = \kappa_K = \kappa$. This yields an identical expected scarcity rent and profit function for all firms as shown in Proposition 4.3.

For given (x_k, x_{-k}) , Figure 3 illustrates the competitive equilibrium in the spot market for a subset of realizations of the demand curve. As illustrated in the figure, there is a threshold value of intercept, say $\alpha(\hat{\omega})$, which depends on (x_k, x_{-k}) and from which all firms' capacities are fully utilized. For realizations $\alpha(\omega) \leq \alpha(\hat{\omega})$, firms' total available generation capacity is not fully utilized and the competitive market price is equal to the firms' marginal generating cost where the corresponding demand curve crosses the supply curve. According to (13), this yields zero scarcity rent for all firms. For realizations $\alpha(\omega) > \alpha(\hat{\omega})$, firms' total available generation capacity is fully utilized (e.g., $d^*(\omega) = X_{K-1}(k) + x_k$ where $X_{K-1}(k) = \sum_{j=1, j \neq k}^K x_j$) and the market price is higher than the firms' marginal generating cost, which yields a positive scarcity rent equal to $P(\omega, X_{K-1}(k) + x_k) - c$ for all firms.

As mentioned before, the value of $\alpha(\hat{\omega})$ depends on (x_k, x_{-k}) which can be easily derived from Figure 3 as:

$$\alpha(\hat{\omega}) = c + \gamma \cdot (X_{K-1}(k) + x_k).$$

Next, we derive the closed form expression of firm k 's expected scarcity rent function for given x_{-k} .

Proposition 4.3. *Under Assumption 4.1, for given x_{-k} , the expected scarcity rent of firm $k \in K$ can be formu-*

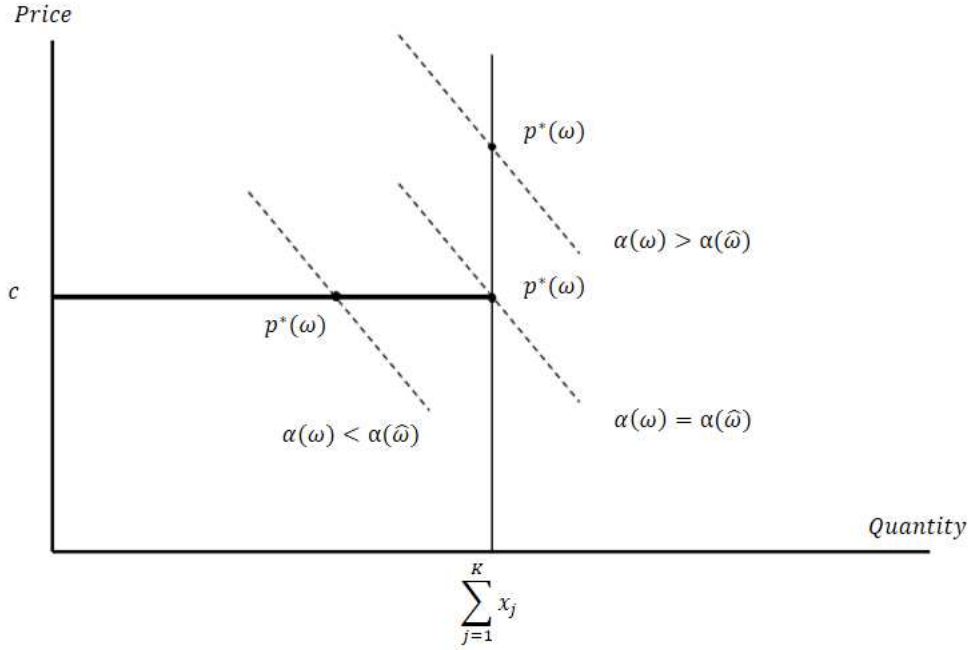


Figure 3: Illustration of a single-node market with symmetric firms and linear demand curve with random intercept for given x

lated as follows:

$$E_{\omega}[\beta_k^*(\omega, x_k, x_{-k})] = \int_{\alpha(\hat{\omega})}^{\alpha_{\max}} (1 - \Phi(s)) ds, \quad (14)$$

where $\alpha(\hat{\omega}) = c + \gamma \cdot (X_{K-1}(k) + x_k)$.

Proof. See Appendix.

In the following lemmas, we establish the differentiability of both expected scarcity rent and expected profit functions with respect to x_k which also imply their continuity. Then, we show in Theorem 4.5 that $E_{\omega}[\Pi^k(\omega, x_k, x_{-k})]$ is strictly logconcave under certain conditions on the support of cumulative distribution function Φ .

Lemma 4.4. *Let Φ be a differentiable function on its support. For given x_{-k} , the expected scarcity rent of each firm $k \in K$ given in (14) is differentiable and nonincreasing w.r.t. $x_k \in \mathcal{R}_+$ with:*

$$\frac{\partial E_{\omega}[\beta_k^*(\omega, x_k, x_{-k})]}{\partial x_k} = -\gamma \cdot (1 - \Phi(\alpha(\hat{\omega})) \leq 0. \quad (15)$$

Moreover, $E_\omega[\beta_k^*(\omega, x_k, x_{-k})]$ is a convex function of $x_k \in \mathcal{R}_+$.

Proof. See Appendix.

Theorem 4.5. Let Φ be a differentiable function on its support. For given $k \in K$ and x_{-k} ,

(i) $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is differentiable w.r.t. $x_k \in \mathcal{R}_+$, and

(ii) if the probability density function Φ' is logconcave, then $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is a strictly logconcave function of $x_k \in \mathcal{R}_+$.

Proof. See Appendix.

As in the exogenous demand case, we prove in the next lemmas that the strategy space of each firm is bounded and convex. Then in Theorem 4.7, we show existence and uniqueness of Nash equilibrium.

Lemma 4.6. The strategy space of each firm $k \in K$ is $S_k := [0, \frac{\alpha_{\max} - c}{\gamma}]$ which is nonempty, compact, and convex.

Proof. See Appendix.

Theorem 4.7 (Existence and Uniqueness). Let Φ be a differentiable function on its support and the probability density function Φ' be logconcave and Assumption 4.2 holds. Then for the game defined in Section 4.1 with symmetric firms and endogenous demand curve with random intercept, there exists a unique symmetric Nash equilibrium, $x^* = x_1^* = \dots = x_K^*$, which satisfies

$$x^* = \frac{\int_{\alpha(\hat{\omega})}^{\alpha_{\max}} (1 - \Phi(\omega)) d\omega - \kappa}{\gamma \cdot (1 - \Phi(\alpha(\hat{\omega})))}, \quad (16)$$

and $x^* > 0$ where $\hat{\omega}$ is such that $\alpha(\hat{\omega}) = c + K\gamma x^*$.

Proof. See Appendix.

4.2.2 Asymmetric Firms

For given (x_k, x_{-k}) , Figure 4 illustrates the competitive equilibrium in the spot market for a subset of realizations of demand curve. In comparison to Figure 3 of the symmetric case, the supply curve of asymmetric generators

is a piecewise constant function having more than one piece. Thus, there are multiple threshold values of intercept, $\alpha(\underline{\omega}_1) < \alpha(\overline{\omega}_1) < \alpha(\underline{\omega}_2) < \alpha(\overline{\omega}_2) < \dots < \alpha(\underline{\omega}_K) < \alpha(\overline{\omega}_K)$, at which the market price changes as explained below. For instance, let's take $k = 1$ in Figure 4 and consider the set of realizations $[\alpha(\underline{\omega}_1), \alpha(\overline{\omega}_1)]$

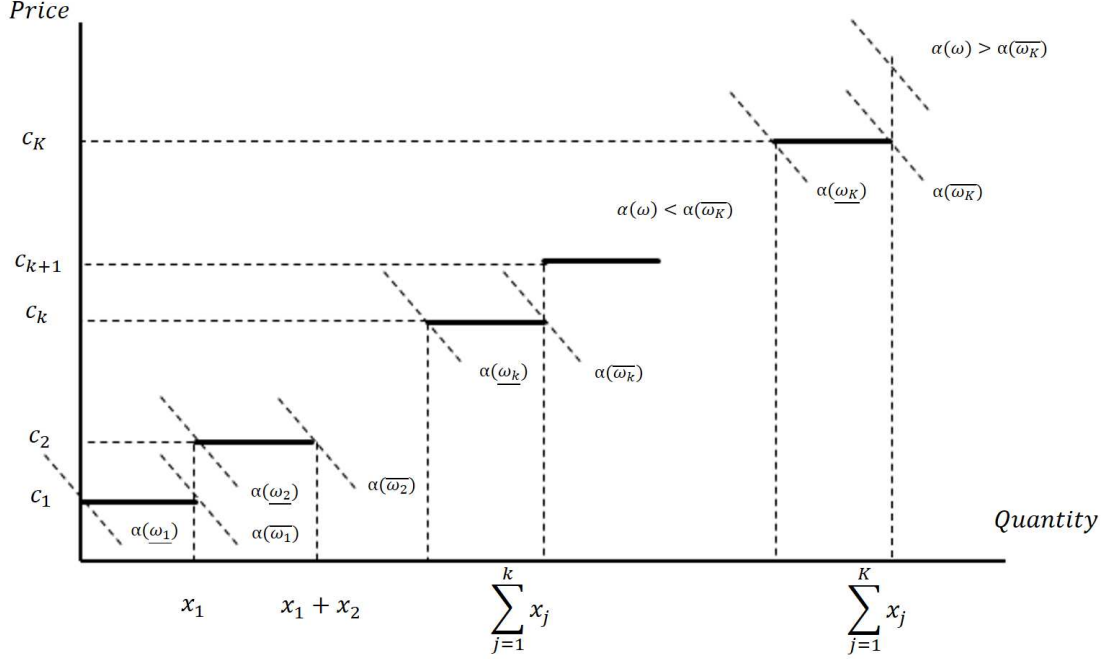


Figure 4: Illustration of a single-node market with asymmetric firms and linear demand curve with random intercept for given x

and $[\alpha(\overline{\omega}_1), \alpha(\underline{\omega}_2)]$ at which firm 1 is the marginal unit and the other firms do not generate at all. For given x_1 , the market price is equal to the sum of the marginal cost and the scarcity rent of firm 1 (c_1 and β_1^*) for the realizations within these sets:

- (a) for $\alpha(\omega) \in [\alpha(\underline{\omega}_1), \alpha(\overline{\omega}_1)]$, the capacity of firm 1 is not binding (i.e., $d(\omega) < x_1$), hence the market price is equal to c_1 and $\beta_1^*(\omega, x_1, x_{-1}) = 0$,
- (b) for $\alpha(\omega) \in [\alpha(\overline{\omega}_1), \alpha(\underline{\omega}_2)]$, the capacity of firm 1 is binding (i.e., $d(\omega) = x_1$) and the market price is equal to $P(\omega, x_1)$ and $\beta_1^*(\omega, x_1, x_{-1}) = P(\omega, x_1) - c_1$.

Similarly, let's consider the set of realizations $[\alpha(\underline{\omega}_2), \alpha(\overline{\omega}_2)]$ and $[\alpha(\overline{\omega}_2), \alpha(\underline{\omega}_3)]$ at which firm 1's capacity is fully utilized and firm 2 is the marginal unit. The other firms having higher marginal costs do not generate at all. Then for given x_1 , the market price is equal to the sum of the marginal cost and scarcity rent of firm 2 (c_2 and β_2^*) for the realizations within these sets:

- (c) for $\alpha(\omega) \in [\alpha(\underline{\omega}_2), \alpha(\overline{\omega}_2)]$, the capacity of firm 2 is not binding (i.e., $x_1 < d(\omega) < x_1 + x_2$) hence the market price is equal to c_2 and $\beta_1^*(\omega, x_1, x_{-1}) = c_2 - c_1$,

(d) for $\alpha(\omega) \in [\alpha(\bar{\omega}_2), \alpha(\underline{\omega}_3)]$, the capacity of firm 2 is binding (i.e., $d(\omega) = x_1 + x_2$) and the market price is equal to $P(\omega, x_1 + x_2)$ and $\beta_1^*(\omega, x_1, x_{-1}) = P(\omega, x_1 + x_2) - c_1$.

Next we will generalize the formulation of $\beta_k^*(\omega, x_k, x_{-k})$ for each k and $\alpha(\omega) \in [\alpha_{min}, \alpha_{max}]$. The value of $\beta_k^*(\omega, x_k, x_{-k})$ depends on the type of firm being the marginal unit at each $\alpha(\omega)$ as explained above. At a realization $\alpha(\omega)$, if firms $1, 2, \dots, m$ are generating, then firm m is called the marginal unit since it is the most expensive firm setting the market price. Furthermore, the scarcity rent of firm k is equal to the market price minus the marginal cost of firm k . For a given k , we will specify a range of $\alpha(\omega)$ where the marginal unit is firm m with a higher marginal cost ($m \geq k$). Otherwise when the marginal unit is with lower marginal cost ($m < k$), firm k is not generating at all and its scarcity rent is zero. Note that in the above example, (a) and (b) show the situation where $k = m = 1$ whereas (c) and (d) show the situation where $k = 1$ and $m = 2$.

For $k \leq m \leq K - 1$, let $\bar{\Omega}_m := [\alpha(\bar{\omega}_m), \alpha(\underline{\omega}_{m+1})]$ denote the set of realizations where the capacity of marginal unit m is binding; that is $d(\omega) = \sum_{j=1}^m x_j$. Similarly, $\bar{\Omega}_K := [\alpha(\bar{\omega}_K), \alpha_{max}]$ is the set of realizations where all firms' capacities are binding. In addition, for $k < m \leq K$, let $\underline{\Omega}_m := [\alpha(\underline{\omega}_m), \alpha(\bar{\omega}_m)]$ denote the set of realizations where the marginal unit m is generating but its capacity is not fully utilized; that is $\sum_{j=1}^{m-1} x_j < d(\omega) < \sum_{j=1}^m x_j$. Then for each firm $k \in K$, the following holds:

- (i) For $\alpha_{min} \leq \alpha(\omega) < \alpha(\bar{\omega}_k)$, the capacity of firm k is not fully utilized, hence $\beta_k^*(\omega, x_k, x_{-k}) = 0$.
- (ii) For $\alpha(\omega) \in \bar{\Omega}_m (m \geq k)$, firm k and marginal unit m generate at full capacity and market price is equal to $P(\omega, \sum_{j=1}^m x_j)$. Thus, $\beta_k^*(\omega, x_k, x_{-k}) = P(\omega, \sum_{j=1}^m x_j) - c_k \geq 0$.
- (iii) For $\alpha(\omega) \in \underline{\Omega}_m (m \geq k + 1)$, firm k generates at full capacity but marginal unit m generates less than its capacity. Then the market price is equal to the marginal cost of unit m . Thus, $P(\omega, d(\omega)) = c_m$ and $\beta_k^*(\omega, x_k, x_{-k}) = c_m - c_k > 0$.

Recall that for given x_{-k} , we denote the total capacity of the $(m - 1)$ cheapest firms excluding firm k in the market as $X_{m-1}(k) = \sum_{j=1, j \neq k}^m x_j$, for $k \leq m \leq K$. Then $X_{m-1}(k) + x_k$ is the total capacity of the m cheapest firms including firm k in the market. Then by using (i)-(iii) above and for given x_{-k} , we get:

$$\begin{aligned} \beta_k^*(\omega, x_k, x_{-k}) &= 0 I_{\{\alpha_{min} \leq \alpha(\omega) < \alpha(\bar{\omega}_k)\}} \\ &+ \sum_{m=k}^{K-1} (P(\omega, X_{m-1}(k) + x_k) - c_k) I_{\{\alpha(\bar{\omega}_m) \leq \alpha(\omega) \leq \alpha(\underline{\omega}_{m+1})\}} \\ &+ (P(\omega, X_{K-1}(k) + x_k) - c_k) I_{\{\alpha(\bar{\omega}_K) \leq \alpha(\omega) \leq \alpha_{max}\}} \\ &+ \sum_{m=k+1}^K (c_m - c_k) I_{\{\alpha(\underline{\omega}_m) < \alpha(\omega) < \alpha(\bar{\omega}_m)\}}. \end{aligned}$$

Then $E_\omega[\beta_k^*(\omega, x_k, x_{-k})]$ can be formulated as follows:

$$\begin{aligned}
E_\omega[\beta_k^*(\omega, x_k, x_{-k})] &= \sum_{m=k}^{K-1} \int_{\alpha(\bar{\omega}_m)}^{\alpha(\underline{\omega}_{m+1})} (P(s, X_{m-1}(k) + x_k) - c_k) \Phi'(s) ds \\
&\quad + \int_{\alpha(\bar{\omega}_K)}^{\alpha_{max}} (P(s, X_{K-1}(k) + x_k) - c_k) \Phi'(s) ds \\
&\quad + \sum_{m=k+1}^K \int_{\alpha(\underline{\omega}_m)}^{\alpha(\bar{\omega}_m)} (c_m - c_k) \Phi'(s) ds
\end{aligned} \tag{17}$$

By using the explicit formulation of the linear price-demand function, (17) can further be simplified as a closed form expression as given in the next proposition.

Proposition 4.8. *Under Assumption 4.1, for given x_k , the expected scarcity rent of firm $k \in K$ can be formulated in a closed form expression as follows:*

$$E_\omega[\beta_k^*(\omega, x_k, x_{-k})] = (c_K - c_k) + \int_{\alpha(\bar{\omega}_K)}^{\alpha_{max}} (1 - \Phi(s)) ds - \sum_{m=k}^{K-1} \int_{\alpha(\bar{\omega}_m)}^{\alpha(\underline{\omega}_{m+1})} \Phi(s) ds, \tag{18}$$

where, for $m = k, \dots, K-1$, the threshold values $\alpha(\bar{\omega}_m)$ and $\alpha(\underline{\omega}_{m+1})$ depend on the capacities of firms generating at $\bar{\omega}_m$ and $\underline{\omega}_{m+1}$, respectively and they can be easily derived from Figure 4:

$$\alpha(\bar{\omega}_m) = c_m + \gamma \cdot (X_{m-1}(k) + x_k) \text{ and } \alpha(\underline{\omega}_{m+1}) = c_{m+1} + \gamma \cdot (X_{m-1}(k) + x_k), \forall m \geq k. \tag{19}$$

Proof. See Appendix.

Note that when firms are symmetric $c_K = c_k = c$ and $\alpha(\bar{\omega}_m) = \alpha(\underline{\omega}_{m+1}), \forall m$. Then, (18) is reduced to (14). The expected scarcity rent function of each firm given in (18) is differentiable and decreasing as shown in the next lemma.

Lemma 4.9. *Let Φ be a differentiable function on its support. For given x_{-k} , the expected scarcity rent of each firm $k \in K$ given in (18) is differentiable w.r.t. $x_k \in \mathcal{R}_+$. Moreover, (18) is a decreasing function of $x_k \in \mathcal{R}_+$.*

Proof. See Appendix.

Next, we can show strict concavity of the expected profit function of each firm under a sufficient condition satisfied by the probability distribution function.

Theorem 4.10. *Let Φ be a differentiable function on its support. For given x_{-k} ,*

- (i) the expected profit function of each firm k is differentiable w.r.t. $x_k \in \mathcal{R}_+$, and
- (ii) the expected profit function of each firm k is (strictly) concave if

$$-\Phi'(s+C) - s \cdot \Phi''(s+C) < 0, \text{ for all } s, C \in \mathcal{R}_+, \text{ where } C \text{ is some constant.}$$

Proof. See Appendix.

The condition stated in Theorem 4.10 (ii) is again similar to the standard assumption used in the literature (see Sherali et al. (1983), Wolf and Smeers (1997), Grimm and Zoettl (2008), and Xu (2005)). Rosen (1965) shows that there is a unique equilibrium point for every strictly concave game. We next show that the strategy space of each firm is bounded and convex. Finally, in Theorem 4.12, we conclude with existence and uniqueness of a Nash equilibrium for the two-stage game with asymmetric firms by using the result of Rosen (1965).

Lemma 4.11. *The strategy space of each firm $k \in K$ is $S_k := [0, \frac{\alpha_{max} - c_k}{\gamma}]$ which is nonempty, compact, and convex.*

Proof. See Appendix.

Theorem 4.12. [Existence and Uniqueness] *Let Φ be a differentiable function on its support and $-\Phi'(s+C) - s \cdot \Phi''(s+C) < 0$, for all $s, C \in \mathcal{R}_+$, where C is some constant. Then, under Assumption 4.2, for the game with asymmetric firms and endogenous demand curve with random intercept, there exists a unique Nash equilibrium.*

Proof. See Appendix.

5 Conclusions

In this paper, we establish sufficient conditions which guarantee existence and uniqueness of equilibria in oligopolistic electricity markets where strategic electricity generators anticipate perfectly competitive spot market outcomes with demand uncertainty while choosing their capacities and their power generation is dispatched after the level of demand is realized. In case of symmetric firms, we show that a large class of continuous probability distributions guarantee uniqueness of equilibrium for the two-stage game. In case of asymmetric firms, equilibrium may not exist since the first-stage payoff functions of firms do not, in general, satisfy generalized concavity when demand is exogenous. When demand is endogenous, a condition on probability distribution function, which is similar to the standard assumption used in the literature for inverse demand curve, is sufficient to guarantee uniqueness of the equilibrium.

In general, two-stage closed loop models with strategic firms are nonconvex problems and examples have been posed in the literature that have multiple or no equilibria. Therefore, it is of interest to users of such closed loop models to know general sets of conditions, as we define here, under which the existence of a unique equilibrium is guaranteed. Availability of such conditions will enhance the value of these closed loop models for their implementation for policy and market intelligence purposes.

As a final remark, in a closed loop game with two asymmetric firms facing an elastic demand, we numerically computed the expected profit function of each firm under various logconcave probability distributions of random intercept (e.g., uniform, normal, exponential, Weibull, gamma, beta) and we observed that each firm's expected profit is strictly quasiconcave in all the cases. Thus, we conjecture that the logconcavity of probability distributions may also be sufficient to guarantee the existence of a unique equilibrium for closed loop games with asymmetric firms when demand is endogenous. However, we could not establish a theoretical proof for this conjecture which remains to be a topic for future research.

Appendix: Proofs of Propositions, Lemmas, and Theorems

Proof of Proposition 3.1. First, we look at the profit of firm $g \in G$ at realization $\omega \in \Omega$ in a bilateral market at a given x^{-g} :

$$\begin{aligned} \Pi_{bilateral}^g(\omega, x^g, x^{-g}) &= \sum_{n \in N} [p_n^{*b}(\omega, x^g, x^{-g}) - v_n^{*b}(\omega, x^g, x^{-g})] s_n^{*bg}(\omega, x^g, x^{-g}) \\ &\quad - \sum_{i \in I_g} \sum_{k \in K_g} [c_{ik}^g - v_i^{*b}(\omega, x^g, x^{-g})] y_{ik}^{*bg}(\omega, x^g, x^{-g}) - \sum_{i \in I_g} \sum_{k \in K_g} \kappa_k x_{ik}^g. \end{aligned}$$

From the KKT conditions in (4), we have:

If $y_{ik}^{*bg}(\omega, x^g, x^{-g}) > 0$, then $v_i^{*b}(\omega, x^g, x^{-g}) = c_{ik}^g + \beta_{ik}^{*bg}(\omega, x^g, x^{-g}) - \rho_g^{*b}(\omega, x^g, x^{-g})$.

If $s_n^{*bg}(\omega, x^g, x^{-g}) > 0$, then $v_n^{*b}(\omega, x^g, x^{-g}) = p_n^{*b}(\omega, x^g, x^{-g}) - \rho_g^{*b}(\omega, x^g, x^{-g})$.

Then by utilizing these together with the third constraint of OPF problem (3) we can simplify the profit function to:

$$\Pi_{bilateral}^g(\omega, x^g, x^{-g}) = \sum_{i \in I_g} \sum_{k \in K_g} \beta_{ik}^{*bg}(\omega, x^g, x^{-g}) y_{ik}^{*bg}(\omega, x^g, x^{-g}) - \sum_{i \in I_g} \sum_{k \in K_g} \kappa_k x_{ik}^g.$$

We also know that if $\beta_{ik}^{*bg}(\omega, x^g, x^{-g}) > 0$, then $y_{ik}^{*bg}(\omega, x^g, x^{-g}) = x_{ik}^g$; hence the profit function of the bilateral market model becomes:

$$\Pi_{bilateral}^g(\omega, x^g, x^{-g}) = \sum_{i \in I_g} \sum_{k \in K_g} \beta_{ik}^{*bg}(\omega, x^g, x^{-g}) x_{ik}^g - \sum_{i \in I_g} \sum_{k \in K_g} \kappa_k x_{ik}^g.$$

Now, we look at the profit of firm $g \in G$ at realization $\omega \in \Omega$ in pool market at a given x :

$$\Pi_{pool}^g(\omega, x^g, x^{-g}) = \sum_{i \in I_g} \sum_{k \in K_g} (p_i^{*p}(\omega, x^g, x^{-g}) - c_{ik}^g) y_{ik}^{*pg}(\omega, x^g, x^{-g}) - \sum_{i \in I_g} \sum_{k \in K_g} \kappa_k x_{ik}^g.$$

From the KKT conditions in (2), we have:

If $y_{ik}^{*pg}(\omega, x^g, x^{-g}) > 0$, then $p_i^{*p}(\omega, x^g, x^{-g}) = c_{ik}^g + \beta_{ik}^{*pg}(\omega, x^g, x^{-g})$.

Using this, we can again simplify the profit function to :

$$\begin{aligned} \Pi_{pool}^g(\omega, x^g, x^{-g}) &= \sum_{i \in I_g} \sum_{k \in K_g} \beta_{ik}^{*pg}(\omega, x^g, x^{-g}) y_{ik}^{*pg}(\omega, x^g, x^{-g}) - \sum_{i \in I_g} \sum_{k \in K_g} \kappa_k x_{ik}^g \\ &= \sum_{i \in I_g} \sum_{k \in K_g} \beta_{ik}^{*pg}(\omega, x^g, x^{-g}) x_{ik}^g - \sum_{i \in I_g} \sum_{k \in K_g} \kappa_k x_{ik}^g. \end{aligned}$$

The last equality follows with a similar reasoning used for the bilateral case.

The result given in Proposition 2.1 (i) indicates that at any given (x^g, x^{-g}) , $\beta_{ik}^{*pg}(\omega, x^g, x^{-g}) = \beta_{ik}^{*bg}(\omega, x^g, x^{-g})$.

Therefore, $\Pi_{pool}^g(\omega, x^g, x^{-g}) = \Pi_{bilateral}^g(\omega, x^g, x^{-g})$ which in turn implies $E_\omega[\Pi_{pool}^g(\omega, x^g, x^{-g})] = E_\omega[\Pi_{bilateral}^g(\omega, x^g, x^{-g})]$.

Thus, for given x^{-g} , if there exists a $x^{*g} \in \chi_{bilateral}(x^{-g})$ then it also holds that $x^{*g} \in \chi_{pool}(x^{-g})$ for all $g \in G$ and

vice versa. Hence, the optimal solution sets of the upper level problems in both market structures are identical which implies both markets yield the same first stage equilibria. ■

Proof of Proposition 3.4. We know that $\beta_k^*(\omega, x_k, x_{-k}) = p^*(\omega, x_k, x_{-k}) - c_k$. Thus, for given $\{X_{m-1}(k)\}_{m=k}^K$, the scarcity rent paid to firm $k \in K$ at realization $\omega \in \Omega$ can be formulated as follows:

$$\begin{aligned} \beta_k^*(\omega, x_k, x_{-k}) = & 0I_{\{\underline{D} \leq D \leq X_{k-1}(k) + x_k\}} \\ & + \sum_{m=k}^{K-1} (c_{m+1} - c_k) I_{\{X_{m-1}(k) + x_k < D \leq X_m(k) + x_k\}} \\ & + (VOLL - c_k) I_{\{X_{K-1}(k) + x_k < D \leq \bar{D}\}}. \end{aligned}$$

Then calculation of $E_\omega[\beta_k^*(\omega, x_k, x_{-k})]$ will yield the following:

$$\begin{aligned} E_\omega[\beta_k^*(\omega, x_k, x_{-k})] &= \sum_{m=k}^{K-1} \int_{X_{m-1}(k) + x_k}^{X_m(k) + x_k} (c_{m+1} - c_k) d(\Psi(s)) + \int_{X_{K-1}(k) + x_k}^{\bar{D}} (VOLL - c_k) d(\Psi(s)) \\ &= \sum_{m=k}^{K-1} (c_{m+1} - c_k) [\Psi(X_m(k) + x_k) - \Psi(X_{m-1}(k) + x_k)] \\ &\quad + (VOLL - c_k) (1 - \Psi(X_{K-1}(k) + x_k)). \end{aligned} \tag{20}$$

The expression (20) can be simplified to:

$$E_\omega[\beta_k^*(\omega, x_k, x_{-k})] = VOLL - c_k - (VOLL - c_K) \Psi(X_{K-1}(k) + x_k) - \sum_{m=k}^{K-1} ((c_{m+1} - c_m) \Psi(X_{m-1}(k) + x_k)). \blacksquare$$

Proof of Lemma 3.5.

(i) From (6), it is easy to see that $E_\omega[\beta_k^*(\omega, x_k, x_{-k})]$ is a continuous function of $(x_k, x_{-k}) \in \mathcal{R}_+^{\mathcal{K}}$ since Ψ is the cumulative distribution function of continuous random demand.

(ii) We know that Ψ is a nondecreasing function of its argument. Fix a k , then for all $0 \leq a < b$:

$$\begin{aligned} 0 &\leq \Psi(X_l(k) + a) \leq \Psi(X_l(k) + b) \leq 1, \quad k-1 \leq l \leq K-1 \text{ which implies} \\ 0 &\leq E_\omega[\beta_k^*(\omega, b, x_{-k})] = VOLL - c_k - (VOLL - c_K) \Psi(X_{K-1}(k) + b) - \sum_{m=k}^{K-1} (c_{m+1} - c_m) \Psi(X_{m-1}(k) + b) \\ &\leq VOLL - c_k - (VOLL - c_K) \Psi(X_{K-1}(k) + a) - \sum_{m=k}^{K-1} (c_{m+1} - c_m) \Psi(X_{m-1}(k) + a) = E_\omega[\beta_k^*(\omega, a, x_{-k})]. \blacksquare \end{aligned}$$

Proof of Lemma 3.6. By Lemma 3.5, we know that $E_\omega[\beta_k^*(\omega, x_k, x_{-k})]$ in (7) is a continuous function on $[0, \infty]$

where

$$E_\omega[\beta_k^*(\omega, x_k, x_{-k})] = \begin{cases} VOLL - c, & 0 \leq x_k < x_k^{min} \\ (VOLL - c)(1 - \Psi(x_k + X_{K-1}(k))), & x_k^{min} \leq x_k < x_k^{max} \\ 0, & \text{otherwise.} \end{cases} \quad (21)$$

Therefore, whenever Ψ is differentiable, then $E_\omega[\beta_k^*(\omega, x_k, x_{-k})]$ is also differentiable w.r.t. $x_k \in \mathcal{R}_+$ except at the breakpoints given in (21):

$$\frac{\partial E_\omega[\beta_k^*(\omega, x_k, x_{-k})]}{\partial x_k} = \begin{cases} 0, & x_k \in [0, x_k^{min}) \cup [x_k^{max}, \infty) \\ -(VOLL - c)\Psi'(x_k + X_{K-1}(k)), & x_k \in [x_k^{min}, x_k^{max}). \blacksquare \end{cases} \quad (22)$$

Proof of Theorem 3.8. Since $E_\omega[\Pi^k(\omega, x_k, x_{-k})] = (E_\omega[\beta_k^*(\omega, x_k, x_{-k})] - \kappa)x_k$, (i) and (ii) directly follow from Lemma 3.5 (i) and Lemma 3.6, respectively.

(iii) $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is continuous and differentiable w.r.t. x_k on $S := (x_k^{min}, x_k^{max})$. Therefore, we know that it is also (strictly) concave on S if and only if

$$\begin{aligned} & \frac{\partial^2 E_\omega[\Pi^k(\omega, x_k, x_{-k})]}{\partial x_k^2} (<) \leq 0, & \forall x_k \in S \\ \Leftrightarrow & \frac{\partial^2 E_\omega[\beta_k^*(\omega, x_k, x_{-k})]}{\partial x_k^2} x_k + 2 \frac{\partial E_\omega[\beta_k^*(\omega, x_k, x_{-k})]}{\partial x_k} (<) \leq 0, & \forall x_k \in S \\ \Leftrightarrow & -\Psi''(x_k + X_{K-1}(k))x_k - 2\Psi'(x_k + X_{K-1}(k)) (<) \leq 0, & \forall x_k \in S. \end{aligned}$$

Since $\Psi'(x_k + X_{K-1}(k))$ is nonnegative, the last inequality holds iff $-\Psi''(x_k + X_{K-1}(k))x_k - \Psi'(x_k + X_{K-1}(k)) (<) \leq 0$.

Outside of S , $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is equal to $(VOLL - c - \kappa)x_k$ on $(0, x_k^{min})$ and $-\kappa x_k$ on (x_k^{max}, ∞) which are increasing and decreasing linear functions, respectively. Therefore, whenever the stated condition holds, $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is a (strictly) quasiconcave function of x_k on \mathcal{R}_+ . \blacksquare

Proof of Theorem 3.10. $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is equal to $(VOLL - c - \kappa)x_k$ on $[0, x_k^{min})$ and $-\kappa x_k$ on (x_k^{max}, ∞) which are increasing and decreasing linear functions, respectively. Therefore, if $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is strictly quasiconcave on $[x_k^{min}, x_k^{max}]$, it is also strictly quasiconcave on \mathcal{R}_+ , which we prove next.

We know that

$$\frac{\partial E_\omega[\Pi^k(\omega, x_k, x_{-k})]}{\partial x_k} = \frac{\partial E_\omega[\beta_k^*(\omega, x_k, x_{-k})]}{\partial x_k} x_k + E_\omega[\beta_k^*(\omega, x_k, x_{-k})] - \kappa,$$

which can be formulated explicitly by using (21) and (22):

$$\frac{\partial E_\omega[\Pi^k(\omega, x_k, x_{-k})]}{\partial x_k} = \begin{cases} VOLL - c - \kappa, & x_k \in [0, x_k^{min}) \\ (VOLL - c)[1 - \Psi(x_k + X_{K-1}(k)) - \Psi'(x_k + X_{K-1}(k))x_k] - \kappa, & x_k \in [x_k^{min}, x_k^{max}) \\ -\kappa, & x_k \in [x_k^{max}, \infty). \end{cases}$$

- Then for $x_k \in (x_k^{min}, x_k^{max})$,

$$\frac{\partial E_\omega[\Pi^k(\omega, x_k, x_{-k})]}{\partial x_k} = -(VOLL - c)\Psi'(x_k + X_{K-1}(k))x_k + (VOLL - c)(1 - \Psi(x_k + X_{K-1}(k))) - \kappa,$$

is the gradient of $E_\omega[\Pi^k(\omega, x_k^{min}, x_{-k})]$.

- For the breakpoints $x_k \in \{x_k^{min}, x_k^{max}\}$,

$$\frac{\partial E_\omega[\Pi^k(\omega, x_k^{min}, x_{-k})]}{\partial x_k} := [VOLL - c - \kappa, (VOLL - c)(1 - \Psi'(\underline{D})x_k^{min}) - \kappa], \quad (23)$$

$$\frac{\partial E_\omega[\Pi^k(\omega, x_k^{max}, x_{-k})]}{\partial x_k} := [-\kappa, -(VOLL - c)\Psi'(\overline{D})x_k^{max} - \kappa] < 0.$$

are the subdifferentials of $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ at $x_k = x_k^{min}$ and $x_k = x_k^{max}$, respectively.

Furthermore, note that since the hazard function of demand is monotone increasing:

$$(i) 1 - \frac{\Psi'(x_k + X_{K-1}(k))}{(1 - \Psi(x_k + X_{K-1}(k)))} x_k \text{ is a decreasing function of } x_k, \text{ and}$$

$$(ii) \frac{\kappa}{(VOLL - c)(1 - \Psi(x_k + X_{K-1}(k)))} \text{ is a nondecreasing function of } x_k.$$

Next, we differentiate the three only possible cases :

Case (1) $x_k^{min} > 0$: We know that $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is continuous, and it is an increasing function on $[0, x_k^{min})$

and a decreasing function on (x_k^{max}, ∞) . Hence, there exists an $\tilde{x}_k \in [x_k^{min}, x_k^{max}]$ such that

$$0 \in \frac{\partial E_\omega[\Pi^k(\omega, \tilde{x}_k, x_{-k})]}{\partial x_k}.$$

Note that $\tilde{x}_k \neq x_k^{max}$ since $\frac{\partial E_\omega[\Pi^k(\omega, x_k^{max}, x_{-k})]}{\partial x_k} < 0$ from (23). For \tilde{x}_k , the following holds:

$$-(VOLL - c)\Psi'(\tilde{x}_k + X_{K-1}(k))\tilde{x}_k + (VOLL - c)(1 - \Psi(\tilde{x}_k + X_{K-1}(k))) = \kappa,$$

$$\text{which implies } 1 - \frac{\Psi'(\tilde{x}_k + X_{K-1}(k))}{(1 - \Psi(\tilde{x}_k + X_{K-1}(k)))}\tilde{x}_k = \frac{\kappa}{(VOLL - c)(1 - \Psi(\tilde{x}_k + X_{K-1}(k)))}. \quad (24)$$

Next we distinguish between the cases $\tilde{x}_k = x_k^{min}$ and $\tilde{x}_k > x_k^{min}$:

(1a) When $\tilde{x}_k = x_k^{min}$, by utilizing (i) and (ii), for $x_k^{min} = \tilde{x}_k < x_k < x_k^{max}$,

$$\begin{aligned} 1 - \frac{\Psi'(x_k + X_{K-1}(k))}{(1 - \Psi(x_k + X_{K-1}(k)))}x_k &< 1 - \frac{\Psi'(\tilde{x}_k + X_{K-1}(k))}{(1 - \Psi(\tilde{x}_k + X_{K-1}(k)))}\tilde{x}_k = \frac{\kappa}{(VOLL - c)(1 - \Psi(\tilde{x}_k + X_{K-1}(k)))} \\ &\leq \frac{\kappa}{(VOLL - c)(1 - \Psi(x_k + X_{K-1}(k)))}. \end{aligned}$$

Consequently,

$$-(VOLL - c)\Psi'(x_k + X_{K-1}(k))x_k + (VOLL - c)(1 - \Psi(x_k + X_{K-1}(k))) < \kappa,$$

which implies that

$$\frac{\partial E_\omega[\Pi^k(\omega, x_k, x_{-k})]}{\partial x_k} < 0.$$

Thus, $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is a decreasing function and hence strictly quasiconcave on $[x_k^{min}, x_k^{max}]$.

(1b) When $\tilde{x}_k > x_k^{min}$, again by utilizing (i) and (ii) for $x_k^{min} < x_k < \tilde{x}_k$,

$$\begin{aligned} 1 - \frac{\Psi'(x_k + X_{K-1}(k))}{(1 - \Psi(x_k + X_{K-1}(k)))}x_k &> 1 - \frac{\Psi'(\tilde{x}_k + X_{K-1}(k))}{(1 - \Psi(\tilde{x}_k + X_{K-1}(k)))}\tilde{x}_k = \frac{\kappa}{(VOLL - c)(1 - \Psi(\tilde{x}_k + X_{K-1}(k)))} \\ &\geq \frac{\kappa}{(VOLL - c)(1 - \Psi(x_k + X_{K-1}(k)))}. \end{aligned}$$

Consequently,

$$-(VOLL - c)\Psi'(x_k + X_{K-1}(k))x_k + (VOLL - c)(1 - \Psi(x_k + X_{K-1}(k))) > \kappa, \text{ and}$$

$$\frac{\partial E_\omega[\Pi^k(\omega, x_k, x_{-k})]}{\partial x_k} > 0.$$

Similarly, for $\tilde{x}_k < x_k < x_k^{max}$

$$1 - \frac{\Psi'(x_k + X_{K-1}(k))}{(1 - \Psi(x_k + X_{K-1}(k)))} x_k < 1 - \frac{\Psi'(\tilde{x}_k + X_{K-1}(k))}{(1 - \Psi(\tilde{x}_k + X_{K-1}(k)))} \tilde{x}_k = \frac{\kappa}{(VOLL - c)(1 - \Psi(\tilde{x}_k + X_{K-1}(k)))}$$

$$\leq \frac{\kappa}{(VOLL - c)(1 - \Psi(x_k + X_{K-1}(k)))},$$

which implies with the same argument that

$$\frac{\partial E_\omega[\Pi^k(\omega, x_k, x_{-k})]}{\partial x_k} < 0.$$

We can conclude in this case that $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is an increasing function for $x_k \in [x_k^{min}, \tilde{x}_k]$ and a decreasing function for $x_k \in [\tilde{x}_k, x_k^{max}]$. Then we can conclude that $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is strictly quasiconcave on $[x_k^{min}, x_k^{max}]$ by using the above arguments.

Case (2) $x_k^{min} = 0$ and $x_k^{max} > 0$: $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is continuous and decreasing on (x_k^{max}, ∞) . Moreover, from (23) and Assumption (3.3) (ii), we know that

$$\frac{\partial E_\omega[\Pi^k(\omega, x_k^{min}, x_{-k})]}{\partial x_k} = VOLL - c - \kappa > 0, \quad \frac{\partial E_\omega[\Pi^k(\omega, x_k^{max}, x_{-k})]}{\partial x_k} < 0.$$

Then there exists an $\tilde{x}_k \in (0, x_k^{max})$ such that $0 \in \frac{\partial E_\omega[\Pi^k(\omega, \tilde{x}_k, x_{-k})]}{\partial x_k}$ and we are back in Case (1). Thus, $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is a strictly quasiconcave function.

Case (3) $x_k^{max} = 0$: Then $E_\omega[\beta_k^*(\omega, x_k, x_{-k})] = 0$ and $E_\omega[\Pi^k(\omega, x_k, x_{-k})] = -\kappa x_k$ on $[0, \infty)$. Hence, $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is decreasing and strictly quasiconcave. ■

Proof of Lemma 3.11. One can easily see that the strategy space is nonempty since $x_k = 0$ is feasible for all firms. Moreover, there cannot be any optimal strategy $x_k^* > \bar{D}$ since $\frac{\partial E_\omega[\Pi^k(\omega, x_k, x_{-k})]}{\partial x_k} = -\kappa < 0$ for all $x_k > \bar{D}$. ■

Proof of Lemma 3.12. By Assumption 3.3 (ii), $\sum_{k=1}^K x_k^* \geq \underline{D}$. Let x^1 be the the vector of investment decisions of all firms where the total generation capacity is equal to the maximum demand level \bar{D} ; that is $x^1 \in \{x_k^1, k \in 1, \dots, K \mid \sum_{k=1}^K x_k^1 = \bar{D}\}$. Now assume any candidate x^2 for the equilibrium of all firms where $x^2 \in \{x_k^2, k \in 1, \dots, K \mid \sum_{k=1}^K x_k^2 > \bar{D}\}$. Then the following holds:

(i) At both x^1 and x^2 , the market price is equal to c for all demand realizations and the expected scarcity rents of all firms are zero which implies that all firms have negative profits equal to $-\kappa x_k^1$ and $-\kappa x_k^2$, respectively.

(ii) Since $x_k^1 < x_k^2$ for at least one firm k , the profit of at least one firm decreases from x^1 to x^2 ; that is, $-\kappa x_k^2 < -\kappa x_k^1$ for at least one firm k .

Then x^2 cannot be an equilibrium since at least one firm has incentive to deviate from this point to x^1 to increase its profit. Thus $\sum_{k=1}^K x_k^* \leq \bar{D}$ holds at equilibrium. ■

Proof of Theorem 3.13. From Lemma 3.11, we know that the strategy spaces of the firms are non-empty, compact and convex. Continuity and quasiconcavity of their payoff functions directly follow from Theorems 3.8 and 3.10, respectively. Then by utilizing Proposition 3.7, there exists a Nash equilibrium.

Each firm $k \in K$ is interested in finding a solution x_k^* which maximizes its expected profit for given x_{-k} :

$$x_k^* = \operatorname{argmax}_{x_k \geq 0} E_\omega[\Pi^k(\omega, x_k, x_{-k})].$$

Let $\mathcal{X}^k(x_{-k})$ denote the solution set of firm k for given x_{-k} . Then a Nash equilibrium is a point such that $x_k^* \in \mathcal{X}^k(x_{-k}^*)$ for all $k \in K$ and the following first-stage optimality conditions will hold for all firms at an equilibrium of the two-stage game:

$$0 \leq -\frac{\partial E_\omega[\Pi^k(\omega, x_k^*, x_{-k}^*)]}{\partial x_k} \perp x_k^* \geq 0 \quad \forall k. \quad (25)$$

Next we show that (i) no asymmetric equilibria exist for the symmetric game and (ii) for symmetric equilibria, denoted by x^* for each firm, (8) holds and $\underline{D} \leq Kx^* \leq \bar{D}$, (iii) finally symmetric equilibrium satisfying (8) is unique.

(i) First we assume that there is an asymmetric equilibrium. Any candidate x^* for an asymmetric equilibrium of arbitrary firms i and j can be ordered as $0 \leq x_i^* < x_j^*$. Next we show by contradiction that an asymmetric equilibrium cannot exist since the first-stage optimality conditions of firms i and j are not simultaneously satisfied at (x_i^*, x_j^*) :

- If $0 < x_i^* < x_j^*$, then the first-stage optimality conditions (25) for both firms i and j should satisfy the following:

$$\frac{\partial E_\omega[\Pi^i(\omega, x_i^*, x_{-i}^*)]}{\partial x_i} = \frac{\partial E_\omega[\Pi^j(\omega, x_j^*, x_{-j}^*)]}{\partial x_j} = 0.$$

We next show that whenever one of the equations above holds, the other cannot hold. Let

$$\frac{\partial E_\omega[\Pi^i(\omega, x_i^*, x_{-i}^*)]}{\partial x_i} = -(VOLL - c)\Psi'(\sum_{k=1}^K x_k^*)x_i^* + (VOLL - c)(1 - \Psi(\sum_{k=1}^K x_k^*)) - \kappa = 0.$$

If the above equation holds then we get

$$\frac{\partial E_\omega[\Pi^j(\omega, x_j^*, x_{-j}^*)]}{\partial x_j} = -(VOLL - c)\Psi'(\sum_{k=1}^K x_k^*)x_j^* + (VOLL - c)(1 - \Psi(\sum_{k=1}^K x_k^*)) - \kappa < 0 \text{ since } x_i^* < x_j^*.$$

- If $0 = x_i^* < x_j^*$, then the first-stage optimality conditions of firm i and j should satisfy

$$\frac{\partial E_\omega[\Pi^i(\omega, 0, x_{-i}^*)]}{\partial x_i} = (VOLL - c)(1 - \Psi(\sum_{k=1}^K x_k^*)) - \kappa \leq 0 \text{ and}$$

$$\frac{E_\omega[\Pi^j(\omega, x_j^*, x_{-j}^*)]}{\partial x_j} = 0.$$

By using a similar reasoning to the case above, whenever the first equation holds, then

$$\frac{\partial E_\omega[\Pi^k(\omega, x_j^*, x_{-j}^*)]}{\partial x_j} = -(VOLL - c)\Psi'(\sum_{k=1}^K x_k^*)x_j^* + (VOLL - c)(1 - \Psi(\sum_{k=1}^K x_k^*)) - \kappa < 0 \text{ since } x_j^* > 0.$$

Hence, no asymmetric equilibria exists.

(ii) Any symmetric equilibrium can be denoted by $x^* = x_1^* = \dots = x_K^*$. Next we identify the symmetric equilibrium by using Lemma 3.12 and the optimality conditions of the first stage:

- By Lemma 3.12, $\underline{D} \leq Kx^* \leq \bar{D}$. Thus, $\frac{\underline{D}}{K} \leq x^* \leq \frac{\bar{D}}{K}$.
- Since $\frac{\underline{D}}{K} \leq x^* \leq \frac{\bar{D}}{K}$, we are either in Case (1) or in Case (2) (i.e., $\underline{D} = 0$) of *Proof of Theorem 3.10*. Hence, there exists x^* which satisfies $\frac{\partial E_\omega[\Pi^k(\omega, x^*, (K-1)x^*)]}{\partial x_k} = 0$. By using the equality (24) given in *Proof of Theorem 3.10*, we get

$$x^* = \frac{(VOLL - c)(1 - \Psi(Kx^*)) - \kappa}{(VOLL - c)\Psi'(Kx^*)}.$$

(iii) From the above arguments we know that each firm's first stage problem (26) is equivalent and has at least one symmetric equilibrium $x^* = x_1^* = \dots = x_K^*$. Thus, at equilibrium, we know that x^* will be the optimal investment strategy of firm k when $x_{-k} := ex^*$ (where e is the vector of 1's of appropriate dimension). Therefore for given $x_{-k} := ex^*$, we would like to find x^* which is the solution to the following problem of any firm k :

$$x^* = \operatorname{argmax}_{x_k \in S} E_\omega[\Pi(\omega, x_k, ex^*)], \quad (26)$$

By Lemma 3.12, $S := \{x_k | \underline{D} - (K-1)x^* \leq x_k \leq \bar{D} - (K-1)x^*\}$ which is equivalent to $x_k \in [x_k^{min}, x_k^{max}]$. By Theorem 3.10, we know that if the hazard rate function $H(s) = \Psi'(s)/(1 - \Psi(s))$ is monotone increasing, then $E_\omega[\Pi(\omega, x_k, ex^*)]$ is strictly quasiconcave on S . Thus, there exists a unique x^* which maximizes the problem (26). ■

Proof of Lemma 3.15. From derivation of (20), recall that for given x_2 and realization ω ,

$$\beta_1^*(\omega, x_1, x_2) = 0I_{\{d(\omega) \leq x_1\}} + (c_2 - c_1)I_{\{x_1 < d(\omega) \leq x_1 + x_2\}} + (VOLL - c_1)I_{\{x_1 + x_2 < d(\omega)\}}$$

and hence:

$$\begin{aligned} E_\omega[\beta_1^*(\omega, x_1, x_2)] &= \int_{x_1}^{x_1 + x_2} (c_2 - c_1)d(\Psi(s)) + \int_{x_1 + x_2}^{d_{max}} (VOLL - c_1)d(\Psi(s)) \\ &= (c_2 - c_1)(\Psi(x_1 + x_2) - \Psi(x_1)) + (VOLL - c_1)(1 - \Psi(x_1 + x_2)) \\ &= VOLL - c_1 - (c_2 - c_1)\Psi(x_1) - (VOLL - c_2)\Psi(x_1 + x_2). \end{aligned}$$

When demand is uniformly distributed over $[d_{min}, d_{max}]$, $\Psi(s) = \frac{s - d_{min}}{d_{max} - d_{min}}$ is the cumulative distribution function. Therefore:

1) If $0 \leq x_1 < d_{min} - x_2$, then $\Psi(x_1) = \Psi(x_1 + x_2) = 0$ and $E_\omega[\beta_1^*(\omega, x_1, x_2)] = VOLL - c_1$.

2) If $d_{min} - x_2 \leq x_1 < d_{min}$, then $\Psi(x_1) = 0$. By plugging in $\Psi(x_1 + x_2) = \frac{x_1 + x_2 - d_{min}}{d_{max} - d_{min}}$, we get:

$$E_\omega[\beta_1^*(\omega, x_1, x_2)] = VOLL - c_1 - \frac{(VOLL - c_2)(x_2 - d_{min})}{d_{max} - d_{min}} - \frac{(VOLL - c_2)x_1}{d_{max} - d_{min}}.$$

It is easily seen that $E_\omega[\beta_1^*(\omega, x_1, x_2)]$ is a linear function where

$$VOLL - c_1 - \frac{(VOLL - c_2)(x_2 - d_{min})}{d_{max} - d_{min}} \text{ and } b_{\beta_1}^1 = \frac{(VOLL - c_2)}{d_{max} - d_{min}} > 0.$$

3) For $d_{min} \leq x_1 < d_{max} - x_2$, $\Psi(x_1) = \frac{x_1 - d_{min}}{d_{max} - d_{min}} \geq 0$ and $\Psi(x_1 + x_2) = \frac{x_1 + x_2 - d_{min}}{d_{max} - d_{min}} \geq 0$. Hence,

$$E_\omega[\beta_1^*(\omega, x_1, x_2)] = \frac{(VOLL - c_1)d_{max}}{d_{max} - d_{min}} - \frac{(VOLL - c_2)x_2}{d_{max} - d_{min}} - \frac{(VOLL - c_1)}{d_{max} - d_{min}}x_1.$$

$E_\omega[\beta_1^*(\omega, x_1, x_2)]$ is a linear function where

$$a_{\beta_1}^2 = \frac{(VOLL - c_1)d_{max}}{d_{max} - d_{min}} - \frac{(VOLL - c_2)x_2}{d_{max} - d_{min}} \text{ and } b_{\beta_1}^2 = \frac{(VOLL - c_1)}{d_{max} - d_{min}} > 0.$$

4) For $d_{max} - x_2 \leq x_1 < d_{max}$, $\Psi(x_1) = \frac{x_1 - d_{min}}{d_{max} - d_{min}} \geq 0$, and $\Psi(x_1 + x_2) = 1$. Then,

$$E_{\omega}[\beta_1^*(\omega, x_1, x_2)] = \frac{(c_2 - c_1)d_{max}}{d_{max} - d_{min}} - \frac{(c_2 - c_1)}{d_{max} - d_{min}}x_1,$$

which is again a linear function with $a_{\beta_1}^3 = \frac{(c_2 - c_1)d_{max}}{d_{max} - d_{min}}$ and $b_{\beta_1}^3 = \frac{(c_2 - c_1)}{d_{max} - d_{min}} > 0$.

5) If $x_1 \geq d_{max}$, then $\Psi(x_1) = \Psi(x_1 + x_2) = 1$ and $E_{\omega}[\beta_1^*(\omega, x_1, x_2)] = 0$.

Similarly, for a given x_1 and realization ω ,

$$\beta_2^*(\omega, x_2, x_1) = 0I_{\{d(\omega) \leq x_1 + x_2\}} + (VOLL - c_2)I_{\{x_1 + x_2 < d(\omega)\}} \text{ which yields}$$

$$\begin{aligned} E_{\omega}[\beta_2^*(\omega, x_2, x_1)] &= \int_{x_1 + x_2}^{d_{max}} (VOLL - c_2)d(\Psi(s)) \\ &= (VOLL - c_2)(1 - \Psi(x_1 + x_2)) \\ &= VOLL - c_2 - (VOLL - c_2)\Psi(x_1 + x_2). \end{aligned}$$

When we have $d \sim U[d_{min}, d_{max}]$, we get:

1) If $0 \leq x_2 < d_{min} - x_1$, then $\Psi(x_1 + x_2) = 0$ and $E_{\omega}[\beta_2^*(\omega, x_2, x_1)] = VOLL - c_2$.

2) For $d_{min} - x_1 \leq x_2 < d_{max} - x_1$,

$$E_{\omega}[\beta_2^*(\omega, x_2, x_1)] = \frac{(VOLL - c_2)d_{max}}{d_{max} - d_{min}} - \frac{(VOLL - c_2)x_1}{d_{max} - d_{min}} - \frac{(VOLL - c_2)}{d_{max} - d_{min}}x_2.$$

Clearly this is a linear function with

$$a_{\beta_2} = \frac{(VOLL - c_2)d_{max}}{d_{max} - d_{min}} - \frac{(VOLL - c_2)x_1}{d_{max} - d_{min}} \text{ and } \beta_{\beta_2} = \frac{(VOLL - c_2)}{d_{max} - d_{min}} > 0.$$

3) If $x_2 \geq d_{max} - x_1$, then $\Psi(x_1 + x_2) = 1$ and $E_{\omega}[\beta_2^*(\omega, x_2, x_1)] = 0$. ■

Proof of Lemma 3.16. We can write the expected profit functions as $E_{\omega}[\Pi_1(\omega, x_1, x_2)] = (E_{\omega}[\beta_1^*(\omega, x_1, x_2)] - \kappa_1)x_1$ and $E_{\omega}[\Pi_2(\omega, x_2, x_1)] = (E_{\omega}[\beta_2^*(\omega, x_2, x_1)] - \kappa_2)x_2$. From Lemma 3.15, we know that both $E_{\omega}[\beta_1^*(\omega, x_1, x_2)]$ and $E_{\omega}[\beta_2^*(\omega, x_2, x_1)]$ are continuous piecewise linear functions of $x_1 \in \mathcal{R}_+$ and $x_2 \in \mathcal{R}_+$, respectively; therefore the expected profit functions are continuous as well. They are also differentiable w.r.t. $x_1 \in \mathcal{R}_+$ and $x_2 \in \mathcal{R}_+$ in each region, respectively, due to the differentiability of expected scarcity rent functions in those regions. However, they are not differentiable at the breakpoints of the corresponding expected scarcity rent functions.

Given x_2 , $E_\omega[\Pi_1(\omega, x_1, x_2)]$ is concave in x_1 in each region $0 \leq x_1 < d_{min} - x_2$, $d_{min} - x_2 \leq x_1 < d_{min}$, $d_{min} \leq x_1 < d_{max} - x_2$, $d_{max} - x_2 \leq x_1 < d_{max}$, and $x_1 \geq d_{max}$ since it is a linear or quadratic function in these regions. However, it is not necessarily quasiconcave on \mathcal{R}_+ . On the other hand, given x_1 , $E_\omega[\Pi_2(\omega, x_2, x_1)]$ is an increasing linear function of x_2 on $(0, d_{min} - x_1)$, concave on $[d_{min} - x_1, d_{max} - x_1]$, and a decreasing linear function on $(d_{max} - x_1, \infty)$. Hence it is a quasiconcave function on \mathcal{R}_+ . ■

Proof of Proposition 4.3. From the illustration in Figure 3 and equation (13), we know that $\beta_k^*(\omega, x_k, x_{-k})$ has a positive value which is equal to $P(\omega, X_{K-1}(k) + x_k) - c$ when $\alpha(\omega) > \alpha(\hat{\omega})$:

$$\begin{aligned} \beta_k^*(\omega, x_k, x_{-k}) &= 0I\{\alpha_{min} \leq \alpha(\omega) \leq \alpha(\hat{\omega})\} \\ &+ [\alpha(\omega) - \gamma \cdot (X_{K-1}(k) + x_k) - c]I\{\alpha(\hat{\omega}) < \alpha(\omega) \leq \alpha_{max}\}. \end{aligned}$$

Then calculation of $E_\omega[\beta_k^*(\omega, x_k, x_{-k})]$ will yield the following:

$$\begin{aligned} E_\omega[\beta_k^*(\omega, x_k, x_{-k})] &= \int_{\alpha(\hat{\omega})}^{\alpha_{max}} [s - \gamma \cdot (X_{K-1}(k) + x_k) - c]d(\Phi(s)) \\ &= \int_{\alpha(\hat{\omega})}^{\alpha_{max}} sd(\Phi(s)) - [c + \gamma \cdot (X_{K-1}(k) + x_k)][1 - \Phi(\alpha(\hat{\omega}))]. \end{aligned}$$

By using integration by parts and the equality $\alpha(\hat{\omega}) = c + \gamma \cdot (X_{K-1}(k) + x_k)$,

$$\begin{aligned} E_\omega[\beta_k^*(\omega, x_k, x_{-k})] &= s\Phi(s)|_{\alpha(\hat{\omega})}^{\alpha_{max}} - \int_{\alpha(\hat{\omega})}^{\alpha_{max}} \Phi(s)ds - \alpha(\hat{\omega})[1 - \Phi(\alpha(\hat{\omega}))] \\ &= \alpha_{max}\Phi(\alpha_{max}) - \int_{\alpha(\hat{\omega})}^{\alpha_{max}} \Phi(s)ds - \alpha(\hat{\omega}). \end{aligned}$$

Since $\Phi(\alpha_{max})=1$, the above equation yields

$$\begin{aligned} E_\omega[\beta_k^*(\omega, x_k, x_{-k})] &= \alpha_{max} - \alpha(\hat{\omega}) - \int_{\alpha(\hat{\omega})}^{\alpha_{max}} \Phi(s)ds \\ &= \int_{\alpha(\hat{\omega})}^{\alpha_{max}} (1 - \Phi(s))ds. \blacksquare \end{aligned}$$

Proof of Lemma 4.4. Φ is a differentiable function and $\alpha(\hat{\omega}) = c + \gamma \cdot (X_{K-1}(k) + x_k)$, which is the limit of the integral in (14), is also differentiable w.r.t. $x_k \in \mathcal{R}_+$. Then $E_\omega[\beta_k^*(\omega, x_k, x_{-k})]$ is differentiable w.r.t. $x_k \in \mathcal{R}_+$.

By using Leibniz rule and the fundamental theorem of calculus for differentiation of integral, we get:

$$\begin{aligned}\frac{\partial E_\omega[\beta_k^*(\omega, x_k, x_{-k})]}{\partial x_k} &= -\frac{\partial \alpha(\hat{\omega})}{\partial x_k}(1 - \Phi(\alpha(\hat{\omega}))) \\ &= -\gamma \cdot (1 - \Phi(\alpha(\hat{\omega}))) \leq 0.\end{aligned}$$

Moreover, the second derivative of $E_\omega[\beta_k^*(\omega, x_k, x_{-k})]$ w.r.t. x_k will yield the following:

$$\frac{\partial^2 E_\omega[\beta_k^*(\omega, x_k, x_{-k})]}{\partial x_k^2} = \gamma^2 \Phi'(\alpha(\hat{\omega})) \geq 0.$$

Hence, $E_\omega[\beta_k^*(\omega, x_k, x_{-k})]$ is a non-increasing convex function of $x_k \in \mathcal{R}_+$. ■

Proof of Theorem 4.5. Since $E_\omega[\Pi^k(\omega, x_k, x_{-k})] = (E_\omega[\beta_k^*(\omega, x_k, x_{-k})] - \kappa_k)x_k$, (i) immediately follows from Lemma 4.4.

For (ii), note that

$$\log(E_\omega[\Pi^k(\omega, x_k, x_{-k})]) = \log x_k + \log(E_\omega[\beta_k^*(\omega, x_k, x_{-k})] - \kappa).$$

We know that $\log x_k$ is strictly concave. We next show that if the probability density function Φ' is logconcave then $B(x_k) := E_\omega[\beta_k^*(\omega, x_k, x_{-k})] - \kappa$ is logconcave.

By using Theorem 3 of Bagnoli and Bergstrom (2005), we know that log-concavity of Φ' implies the log-concavity of the right hand integral of the reliability function defined by

$$H(t) = \int_t^{\alpha_{max}} (1 - \Phi(s)) ds \quad \text{for } t \in (\alpha_{min}, \alpha_{max}),$$

which implies

$$\frac{\partial^2(\log H(t))}{\partial t^2} = \frac{\Phi'(t) \int_t^{\alpha_{max}} (1 - \Phi(s)) ds - (1 - \Phi(t))^2}{(H(t))^2} \leq 0$$

and

$$\bar{H}(t) := \Phi'(t) \int_t^{\alpha_{max}} (1 - \Phi(s)) ds - (1 - \Phi(t))^2 \leq 0.$$

By using the above inequality we can prove that $\frac{\partial^2 \log(B(x_k))}{\partial x_k^2} \leq 0$:

$$\begin{aligned} \frac{\partial^2 \log(B(x_k))}{\partial x_k^2} &= \gamma^2 \frac{\Phi'(\alpha(\hat{\omega})) \int_{\alpha(\hat{\omega})}^{\alpha_{max}} (1 - \Phi(s)) ds - (1 - \Phi(\alpha(\hat{\omega})))^2}{(B(x_k))^2} \\ &\quad - \gamma^2 \frac{\kappa \Phi'(\alpha(\hat{\omega}))}{(B(x_k))^2} \\ &= \gamma^2 \frac{\bar{H}(t)|_{t=\alpha(\hat{\omega})}}{(B(x_k))^2} - \gamma^2 \frac{\kappa \Phi'(\alpha(\hat{\omega}))}{(B(x_k))^2} \leq 0. \end{aligned}$$

Hence, $(E_\omega[\beta_k^*(\omega, x_k, x_{-k})] - \kappa)$ is a logconcave function of x_k . The sum of a strictly concave function, $\log x_k$, and a concave function, $\log(E_\omega[\beta_k^*(\omega, x_k, x_{-k})] - \kappa)$, is strictly concave. Thus, $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is strictly logconcave in x_k . ■

Proof of Lemma 4.6. The strategy space is nonempty since the strategy $x_k = 0$ is feasible for all firms. For any firm k , if $x_k > \frac{\alpha_{max} - c}{\gamma}$ then, by Proposition 4.3, $E_\omega[\beta_k^*(\omega, x_k, x_{-k})] = 0$ (note that $\alpha(\hat{\omega}) > \alpha_{max}$ for all $x_k > \frac{\alpha_{max} - c}{\gamma}$). Then for $x_k > \frac{\alpha_{max} - c}{\gamma}$, $\frac{\partial E_\omega[\Pi^k(\omega, x_k, x_{-k})]}{\partial x_k} = -\kappa < 0$ which implies that firm k 's profit for this set of investment strategies is lower than its profit gained by the strategies in $S_k := [0, \frac{\alpha_{max} - c}{\gamma}]$ and therefore can be excluded from its strategy space. ■

Proof of Theorem 4.7. From Lemma 4.6, we know that the strategy spaces are non-empty, compact, and convex. Continuity of the payoff functions directly follows from Theorem 4.5. Boyd and Vandenberghe (2004) gives a composition theorem which shows the preservation of quasiconcavity under monotonic functions (see Section 3.4.4 on pages 101/102 and Section 3.5 on page 104). Utilizing this result, we know that (strict) log-concavity implies (strict) quasiconcavity of firms' payoff functions. A similar result can also be found in Theorem 3.3 of Avriel (1972) under the notion of ρ -concavity. Then by utilizing Theorem 4.5 and Proposition 3.7, there exists a Nash equilibrium.

$E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is differentiable and strictly logconcave in x_k . Each firm $k \in K$ is interested in finding a solution x_k^* which maximizes its expected profit for given x_{-k} :

$$x_k^* = \operatorname{argmax}_{x_k \geq 0} E_\omega[\Pi^k(\omega, x_k, x_{-k})].$$

Let $\mathcal{X}^k(x_{-k})$ denote the solution set of firm k for given x_{-k} . Then a Nash equilibrium is a point such that $x_k^* \in \mathcal{X}^k(x_{-k}^*)$ for all $k \in K$ and the following first-stage optimality conditions will hold for all firms at an

equilibrium of the two-stage game:

$$0 \leq -\frac{\partial E_\omega[\Pi^k(\omega, x_k^*, x_{-k}^*)]}{\partial x_k} \perp x_k^* \geq 0, \forall k \in K, \quad (27)$$

where

$$\frac{\partial E_\omega[\Pi^k(\omega, x_k^*, x_{-k}^*)]}{\partial x_k} = \int_{\alpha(\hat{\omega})}^{\alpha_{max}} (1 - \Phi(s)) ds - \gamma \cdot (1 - \Phi(\alpha(\hat{\omega}))) x_k^* - \kappa \text{ and } \alpha(\hat{\omega}) = c + \gamma \sum_{j=1}^K x_j^*.$$

Next we show that (i) no asymmetric equilibria exist, (ii) for symmetric equilibria, denoted by x^* for each firm, (16) holds and $0 < x^* < \frac{\alpha_{max} - c}{K\gamma}$, and (iii) finally symmetric equilibrium satisfying (16) is unique.

(i) We first assume that there is an asymmetric equilibrium. Any candidate x^* for an asymmetric equilibrium of arbitrary firms i and j can be ordered as $0 \leq x_i^* < x_j^*$. Next we show by contradiction that an asymmetric equilibrium cannot exist since the first stage optimality conditions given in (27) are not simultaneously satisfied for firms i and j .

- If $0 < x_i^* < x_j^*$, then the first stage optimality conditions (27) for firms i and j should satisfy the following

$$\frac{\partial E_\omega[\Pi^i(\omega, x_i^*, x_{-i}^*)]}{\partial x_i} = \frac{\partial E_\omega[\Pi^j(\omega, x_j^*, x_{-j}^*)]}{\partial x_j} = 0.$$

We next show that whenever one of the equations above holds, the other cannot hold. Let

$$\frac{\partial E_\omega[\Pi^i(\omega, x_i^*, x_{-i}^*)]}{\partial x_i} = \int_{\alpha(\hat{\omega})}^{\alpha_{max}} (1 - \Phi(s)) ds - \gamma \cdot (1 - \Phi(\alpha(\hat{\omega}))) x_i^* - \kappa = 0.$$

If the above equation holds then since $x_i^* < x_j^*$,

$$\frac{\partial E_\omega[\Pi^j(\omega, x_j^*, x_{-j}^*)]}{\partial x_j} = \int_{\alpha(\hat{\omega})}^{\alpha_{max}} (1 - \Phi(s)) ds - \gamma \cdot (1 - \Phi(\alpha(\hat{\omega}))) x_j^* - \kappa < 0.$$

- If $0 = x_i^* < x_j^*$, then the first-stage optimality conditions of firm i and j should satisfy

$$\frac{\partial E_\omega[\Pi^i(\omega, x_i^*, x_{-i}^*)]}{\partial x_i} = \int_{\alpha(\hat{\omega})}^{\alpha_{max}} (1 - \Phi(s)) ds - \kappa \leq 0, \text{ and}$$

$$\frac{\partial E_\omega[\Pi^i(\omega, x_j^*, x_{-i}^*)]}{\partial x_i} = 0.$$

By using a similar reasoning to the case above, whenever the first equation holds then since $x_j^* > 0$

$$\frac{\partial E_\omega[\Pi^j(\omega, x_j^*, x_{-j}^*)]}{\partial x_j} = \int_{\alpha(\hat{\omega})}^{\alpha_{max}} (1 - \Phi(s)) ds - \gamma \cdot (1 - \Phi(\alpha(\hat{\omega}))) x_j^* - \kappa < 0.$$

Hence, no asymmetric equilibria exists.

(ii) Any symmetric equilibrium can be denoted by $x^* = x_1^* = \dots = x_K^*$. Next we identify the symmetric equilibrium by using Assumption 4.2 and the optimality conditions of the first stage:

- By Assumption 4.2, $x^* > 0$.
- At $x^* > 0$, the following optimality conditions are satisfied for each firm

$$\frac{\partial E_\omega[\Pi^k(\omega, x^*, (K-1)x^*)]}{\partial x_k} = \int_{\alpha(\hat{\omega})}^{\alpha_{max}} (1 - \Phi(s)) ds - \gamma \cdot (1 - \Phi(\alpha(\hat{\omega})))x^* - \kappa = 0.$$

Hence, $\int_{\alpha(\hat{\omega})}^{\alpha_{max}} (1 - \Phi(s)) ds - \kappa = \gamma \cdot (1 - \Phi(\alpha(\hat{\omega})))x^*$ which yields (16).

(iii) From the above arguments we know that each firm's first stage problem (28) is equivalent and has at least one symmetric equilibrium $x^* = x_1^* = \dots = x_K^*$. Thus, at equilibrium, we know that x^* will be the optimal investment strategy of firm k when $x_{-k}^* := ex^*$ (where e is the vector of 1's of appropriate dimension). Therefore for given $x_{-k} := ex^*$, we would like to find x^* which is the solution to the following problem of firm k :

$$x^* = \operatorname{argmax}_{x_k \geq 0} E_\omega[\Pi(\omega, x_k, ex^*)], \quad (28)$$

By Theorem 4.5 (ii), we know that if Φ' is logconcave, then $E_\omega[\Pi(\omega, x_k, ex^*)]$ is strictly logconcave for $x_k \in \mathfrak{R}_+$ which implies its strict quasiconcavity for $x_k \in \mathfrak{R}_+$. Thus, there exists a unique x^* maximizing $E_\omega[\Pi(\omega, x_k, ex^*)]$ in (28). ■

Proof of Proposition 4.8. By using explicit formulation of linear price demand curve, we get:

$$\begin{aligned} E_\omega[\beta_k^*(\omega, x_k, x_{-k})] &= \sum_{m=k}^{K-1} \int_{\alpha(\bar{\omega}_m)}^{\alpha(\underline{\omega}_{m+1})} (s - c_k - \gamma \cdot (X_{m-1}(k) + x_k)) \Phi'(s) ds \\ &\quad + \int_{\alpha(\bar{\omega}_K)}^{\alpha_{max}} (s - c_k - \gamma \cdot (X_{K-1}(k) + x_k)) \Phi'(s) ds \\ &\quad + \sum_{m=k+1}^K \int_{\alpha(\underline{\omega}_m)}^{\alpha(\bar{\omega}_m)} (c_m - c_k) \Phi'(s) ds. \end{aligned}$$

Next we simplify the above formulation:

$$\begin{aligned}
E_\omega[\beta_k^*(\omega, x_k, x_{-k})] &= \sum_{m=k}^{K-1} (\alpha(\underline{\omega}_{m+1})\Phi(\alpha(\underline{\omega}_{m+1})) - \alpha(\overline{\omega}_m)\Phi(\alpha(\overline{\omega}_m))) - \int_{\alpha(\overline{\omega}_m)}^{\alpha(\underline{\omega}_{m+1})} \Phi(s)ds \\
&\quad - \sum_{m=k}^{K-1} (c_k + \gamma \cdot (X_{m-1}(k) + x_k)) [\Phi(\alpha(\underline{\omega}_{m+1})) - \Phi(\alpha(\overline{\omega}_m))] \\
&\quad + \sum_{m=k+1}^K (c_m - c_k) [\Phi(\alpha(\overline{\omega}_m)) - \Phi(\alpha(\underline{\omega}_m))] \\
&\quad + (\alpha_{max}\Phi(\alpha_{max}) - \alpha(\overline{\omega}_K)\Phi(\alpha(\overline{\omega}_K))) - \int_{\alpha(\overline{\omega}_K)}^{\alpha_{max}} \Phi(s)ds \\
&\quad - (c_k + \gamma \cdot (X_{K-1}(k) + x_k)) [\Phi(\alpha_{max}) - \Phi(\alpha(\overline{\omega}_K))],
\end{aligned}$$

$$\begin{aligned}
E_\omega[\beta_k^*(\omega, x_k, x_{-k})] &= \sum_{m=k}^{K-1} (c_{m+1} - c_k)\Phi(\alpha(\underline{\omega}_{m+1})) - \sum_{m=k}^{K-1} (c_m - c_k)\Phi(\alpha(\overline{\omega}_m)) \\
&\quad + \sum_{m=k+1}^K (c_m - c_k)\Phi(\alpha(\overline{\omega}_m)) - \sum_{m=k+1}^K (c_m - c_k)\Phi(\alpha(\underline{\omega}_m)) \\
&\quad - \sum_{m=k}^{K-1} \int_{\alpha(\overline{\omega}_m)}^{\alpha(\underline{\omega}_{m+1})} \Phi(s)ds - \int_{\alpha(\overline{\omega}_K)}^{\alpha_{max}} \Phi(s)ds \\
&\quad - (c_K - c_k)\Phi(\alpha(\overline{\omega}_K)) + \alpha_{max} - (c_k + \gamma \cdot (X_{K-1}(k) + x_k))
\end{aligned}$$

$$\begin{aligned}
E_\omega[\beta_k^*(\omega, x_k, x_{-k})] &= (c_K - c_k)\Phi(\alpha(\overline{\omega}_K)) - (c_K - c_k)\Phi(\alpha(\overline{\omega}_K)) + \alpha_{max} - (c_k + \gamma \cdot (X_{K-1}(k) + x_k)) \\
&\quad - \sum_{m=k}^{K-1} \int_{\alpha(\overline{\omega}_m)}^{\alpha(\underline{\omega}_{m+1})} \Phi(s)ds - \int_{\alpha(\overline{\omega}_K)}^{\alpha_{max}} \Phi(s)ds,
\end{aligned}$$

$$E_\omega[\beta_k^*(\omega, x_k, x_{-k})] = \alpha_{max} - (c_k + \gamma \cdot (X_{K-1}(k) + x_k)) - \sum_{m=k}^{K-1} \int_{\alpha(\overline{\omega}_m)}^{\alpha(\underline{\omega}_{m+1})} \Phi(s)ds - \int_{\alpha(\overline{\omega}_K)}^{\alpha_{max}} \Phi(s)ds,$$

Finally, we add $c_K - c_K$ to the equation above, then we get:

$$E_\omega[\beta_k^*(\omega, x_k, x_{-k})] = (c_K - c_k) + \int_{\alpha(\overline{\omega}_K)}^{\alpha_{max}} (1 - \Phi(s))ds - \sum_{m=k}^{K-1} \int_{\alpha(\overline{\omega}_m)}^{\alpha(\underline{\omega}_{m+1})} \Phi(s)ds. \blacksquare$$

Proof of Lemma 4.9. $P(\omega, \cdot)$ and Φ are differentiable w.r.t. their arguments and Φ is independent from x_k . From (19) we know that, for $m \geq k$, $(\alpha(\overline{\omega}_m), \alpha(\underline{\omega}_{m+1}))$ are differentiable w.r.t. $x_k \in \mathcal{R}_+$. Then being a sum of

differentiable functions in (17), $E_\omega[\beta_k^*(\omega, x_k, x_{-k})]$ is differentiable w.r.t. $x_k \in \mathcal{R}_+$. By using Leibniz rule and the fundamental theorem of calculus for differentiation of integral in (17) and by utilizing (19) which simplifies $P(\underline{\omega}_{m+1}, X_{m-1}(k) + x_k) = c_{m+1}$ and $P(\bar{\omega}_m, X_{m-1}(k) + x_k) = c_m$, we get

$$\begin{aligned}
\frac{\partial E_\omega[\beta_k^*(\omega, x_k, x_{-k})]}{\partial x_k} &= \sum_{m=k}^{K-1} \left[\frac{\partial \alpha(\underline{\omega}_{m+1})}{\partial x_k} (c_{m+1} - c_k) \Phi'(\alpha(\underline{\omega}_{m+1})) - \frac{\partial \alpha(\bar{\omega}_m)}{\partial x_k} (c_m - c_k) \Phi'(\alpha(\bar{\omega}_m)) \right] \\
&+ \sum_{m=k}^{K-1} \int_{\alpha(\bar{\omega}_m)}^{\alpha(\underline{\omega}_{m+1})} \frac{\partial P(s, X_{m-1}(k) + x_k)}{\partial x_k} \Phi'(s) ds \\
&- \frac{\partial \alpha(\bar{\omega}_K)}{\partial x_k} (c_K - c_k) \Phi'(\alpha(\bar{\omega}_K)) + \int_{\alpha(\bar{\omega}_K)}^{\alpha_{\max}} \frac{\partial P(s, X_{K-1}(k) + x_k)}{\partial x_k} \Phi'(s) ds \\
&+ \sum_{m=k+1}^K \left[\frac{\partial \alpha(\bar{\omega}_m)}{\partial x_k} (c_m - c_k) \Phi'(\alpha(\bar{\omega}_m)) - \frac{\partial \alpha(\underline{\omega}_m)}{\partial x_k} (c_m - c_k) \Phi'(\alpha(\underline{\omega}_m)) \right].
\end{aligned} \tag{29}$$

The expression in (29) can be simplified to:

$$\begin{aligned}
\frac{\partial E_\omega[\beta_k^*(\omega, x_k, x_{-k})]}{\partial x_k} &= \sum_{m=k}^{K-1} \int_{\alpha(\bar{\omega}_m)}^{\alpha(\underline{\omega}_{m+1})} \frac{\partial P(s, X_{m-1}(k) + x_k)}{\partial x_k} \Phi'(s) ds \\
&+ \int_{\alpha(\bar{\omega}_K)}^{\alpha_{\max}} \frac{\partial P(s, X_{K-1}(k) + x_k)}{\partial x_k} \Phi'(s) ds \\
&= -\gamma \sum_{m=k}^{K-1} [\Phi(\alpha(\underline{\omega}_{m+1})) - \Phi(\alpha(\bar{\omega}_m)) - \gamma[1 - \Phi(\alpha(\bar{\omega}_K))] \\
&< 0.
\end{aligned}$$

Thus, $E_\omega[\beta_k^*(\omega, x_k, x_{-k})]$ is decreasing. ■

Proof of Theorem 4.10. Since $E_\omega[\Pi^k(\omega, x_k, x_{-k})] = (E_\omega[\beta_k^*(\omega, x_k, x_{-k})] - \kappa_k)x_k$, (i) follows immediately from Lemma 4.9. For (ii), we know that

$$E_\omega[\Pi^k(\omega, x_k, x_{-k})] = x_k(E_\omega[\beta_k^*(\omega, x_k, x_{-k})] - \kappa).$$

$E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is a differentiable function of x_k with

$$\frac{\partial E_\omega[\Pi^k(\omega, x_k, x_{-k})]}{\partial x_k} = x_k \cdot \frac{\partial E_\omega[\beta_k^*(\omega, x_k, x_{-k})]}{\partial x_k} + E_\omega[\beta_k^*(\omega, x_k, x_{-k})] - \kappa.$$

$\frac{\partial E_\omega[\Pi^k(\omega, x_k, x_{-k})]}{\partial x_k}$ can be explicitly formulated by using (18):

$$\begin{aligned} \frac{\partial E_\omega[\Pi^k(\omega, x_k, x_{-k})]}{\partial x_k} &= -x_k \cdot \gamma \sum_{m=k}^{K-1} \int_{\alpha(\bar{\omega}_m)}^{\alpha(\underline{\omega}_{m+1})} \Phi'(s) ds - x_k \cdot \gamma [1 - \Phi(\alpha(\bar{\omega}_K))] \\ &\quad + (c_K - c_k) + \int_{\alpha(\bar{\omega}_K)}^{\alpha_{max}} (1 - \Phi(s)) ds - \sum_{m=k}^{K-1} \int_{\alpha(\bar{\omega}_m)}^{\alpha(\underline{\omega}_{m+1})} \Phi(s) ds, -\kappa. \end{aligned}$$

which can further be rewritten as

$$\begin{aligned} \frac{\partial E_\omega[\Pi^k(\omega, x_k, x_{-k})]}{\partial x_k} &= c_K - c_k - \kappa + \underbrace{\int_{\alpha(\bar{\omega}_K)}^{\alpha_{max}} [(1 - \Phi(s)) - x_k \cdot \gamma \Phi'(s)] ds}_{A_1(x_k)} \\ &\quad - \underbrace{\sum_{m=k}^{K-1} \int_{\alpha(\bar{\omega}_m)}^{\alpha(\underline{\omega}_{m+1})} (\Phi(s) + x_k \cdot \gamma \Phi'(s)) ds}_{A_2(x_k)}. \end{aligned}$$

We know that $c_K - c_k - \kappa$ is constant. In addition under the condition $-\Phi'(s+C) - s \cdot \gamma \Phi''(s+C) (<) \leq 0$ for all $s, C \geq 0$, $A_1(x_k)$ and $A_2(x_k)$ are (decreasing) nonincreasing functions since $\frac{\partial A_1(x_k)}{\partial x_k} \leq 0$ and $\frac{\partial A_2(x_k)}{\partial x_k} (<) \leq 0$, which implies $\frac{\partial^2 E_\omega[\Pi^k(\omega, x_k, x_{-k})]}{\partial x_k^2} (<) \leq 0$. Hence, $E_\omega[\Pi^k(\omega, x_k, x_{-k})]$ is (strictly) concave if $-\Phi'(s+C) - s \cdot \gamma \Phi''(s+C) (<) \leq 0$ for all $s, C \geq 0$. ■

Proof of Lemma 4.11. The strategy space is nonempty since the strategy $x_k = 0$ is feasible for all firms. For any firm $k \in K$, if $x_k > \frac{\alpha_{max} - c_k}{\gamma}$ then $p_k^*(\omega, x_k, x_{-k}) = c_k$ for all $\omega \in \Omega$ when firm k 's generation is positive. Thus, $E_\omega[\beta_k^*(\omega, x_k, x_{-k})] = 0$ for all $x_k > \frac{\alpha_{max} - c_k}{\gamma}$. Then for $x_k > \frac{\alpha_{max} - c_k}{\gamma}$, $\frac{\partial E_\omega[\Pi^k(\omega, x_k, x_{-k})]}{\partial x_k} = -\kappa < 0$ which implies that firm k 's profit for this set of investment strategies is lower than its profit gained by the strategies in $S_k := [0, \frac{\alpha_{max} - c_k}{\gamma}]$ and therefore can be excluded from its strategy space. ■

Proof of Theorem 4.12. From Lemma 4.11, we know that the strategy spaces are non-empty, compact, and convex. Continuity of the payoff functions directly follows from Theorem 4.10. Then by Proposition 3.7, there exists a Nash equilibrium. Furthermore, Rosen (1965) shows that there exists a unique equilibrium point when the payoff function of every player is strictly concave. By using the result of Rosen (1965), there exists a unique equilibrium since the expected profit function of each firm is strictly concave under the given condition. ■

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