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**VEHICLE PLANNING IN CROSS CHAIN  
CONTROL CENTERS**

By

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# Vehicle planning in Cross Chain Control Centers

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## Abstract

To satisfy the growing need for efficient supply chain orchestration, a Cross Chain Control Center (4-C) might be an effective concept. A 4-C is a center from which multiple supply chains are controlled simultaneously, thereby aiming to exploit synergetic potential. This article proposes optimization approaches for transport timetabling and vehicle routing of multiple shippers that are controlled by a 4-C. We associate inconvenience with the requirement that shippers typically need to deviate from their individual schedules in order to make collaboration possible. This brings up a natural trade-off between inconvenience and transportation costs, which forms the essence of the proposed optimization models. Next, we pay attention to a fair redistribution of exploited synergy to the collaborating shippers. By considering several test cases, we find perfectly intuitively clear results supporting the applicability of this new approach to supply chain collaboration.

*Keywords:* Supply Chain Collaboration, Integer programming, Logistics, Routing, Scheduling, Game theory

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## 1. Introduction

### 1.1. Problem setting

Due to continued globalization and a growing social demand for sustainable logistical activities, supply chains are becoming more comprehensive and complex. As a consequence, there is a growing need for efficient supply chain orchestration and configuration. The committee van Laarhoven – on behalf of the dutch logistics and supply chain industry – has identified today’s inability to fulfill this need as an opportunity to enhance the international logistical position of the Netherlands (Committee van Laarhoven,

2009). In fact, the committee set the ambition: “by 2020, let the Netherlands become European market leader in managing and controlling transnational product flows”, which boils down to a 200% increase of added value of chain orchestration and configuration.

In this respect, the committee van Laarhoven recognized the importance of Cross Chain Control Centers (4-C) in reaching the aforementioned ambition. A 4-C is able to “control multiple supply chains simultaneously by means of modern technology and top professionals in logistics” (Committee van Laarhoven, 2007). A common managerial approach to the movement of commodities, financial assets and information of multiple supply chains ought to result in significant costs savings and more environmental friendly logistical solutions.

As far as supply chain collaboration is concerned, a 4-C balances on the interface between vertical and horizontal collaboration. The former referring to collaboration between entities of the same supply chain, e.g. retailers or suppliers, while the latter includes cooperation of two or more unrelated or competing companies, e.g. in sharing transport or storage capacity, (Barratt, 2004). In literature, the notion of lateral collaboration is adopted to describe the concurrence of both vertical and horizontal collaboration (Simatupang and Sridharan, 2002). Some notable examples of lateral collaboration include Lean Logistics and Nistevro (Leavitt, 2000). These generally apply a tendering approach to make schedules of carriers and shippers compatible, and hence, are operational in nature. Different from this, a 4-C operates on the tactical and strategical level and attempts to establish long-term agreements among collaborating shippers. This allows for full harnessing of synergetic potential as a result of lateral collaborations.

A 4-C is an independent logistic service provider (LSP) aiming to establish and orchestrate lateral collaborations between its clients. In full operation, it is responsible for (part of) the logistical activities of shippers that are involved in a collaboration pact. In order to exploit synergetic potential, schedules of these shippers need to be synchronized. This typically requires shippers to deviate from their original schedules which causes them to experience inconvenience. Thus in essence, a 4-C faces the problem of finding the optimal balance between supply chain costs and shipper’s inconvenience. That is, shippers are only prepared to make large concessions if this results in significant costs savings. This central trade-off between supply chain costs and inconvenience forms the leading thread of our analysis. By introducing an inconvenience cost function, we use mathematical programming to find

optimal 4-C logistical solutions.

We suppose that a group of shippers has declared their willingness for collaboration and our analysis focuses on transportation. Thus effectively, collaboration boils down to consolidation of good flows of different shippers which should be arranged by the 4-C in question. In this setting, we associate for each shipper fixed pickup and delivery locations with corresponding preferred pickup and delivery times. As pointed out, shippers will generally have to deviate from these preferred times, i.e. inconvenience arises, in order to make bundling of goods possible. Against this background, we propose two modeling approaches: *Transport Timetabling* and *Vehicle Routing*. The former is a simple model that purely deals with the time-dimension of vehicle planning. Its job is to find an optimal tactical planning in a particular time-horizon for a given transport connection. The second modeling approach is more involved as it accounts for the routing aspect of vehicle planning as well.

The notion of inconvenience is of crucial importance in our analysis. In classical vehicle scheduling and routing problems, one often considers hard time window constraints to model a preferred just-in-time delivery. However, hard time windows clearly impede the possibility of freight consolidation, and hence, we soften these constraints. Instead, we suggest to use an inconvenience cost function that models the degree of discomfort due to an off-time delivery. Costs are typically associated with off-time delivery. Shippers will incur extra storage costs when delivery occurs too early and shippers have to reserve additional safety-stock to anticipate on possible stock-outs in case of too late deliveries. The idea of this approach is that even considerable inconvenience costs might be overcompensated by associated transportation costs savings, leading to efficient logistical solutions that never can be obtained by hard time window constraints.

Apart from determining optimal transport timetabling and vehicle routing, a 4-C is obliged with a fair distribution of obtained cost savings to shippers participating in a collaboration pact. That is, shippers that contribute more to total synergy should earn more from collaboration and vice versa. To this end, suitable gain-sharing models should be applied, which in fact forms an important driver of the success or failure of 4-C as a whole. It turns out that establishing horizontal collaboration is, among other factors, impeded by shipper's distrust in constructing a fair allocation mechanism of obtained savings (Cruijssen et al., 2007b). Due to its ease of practical use, it might seem sensible to apply simple rules of thumb based on usual indicators,

e.g. size of cargo or number of served customers, in determining a fair allocation. However, its simpleness may sooner or later lead to dissatisfaction of shippers feeling undervalued. It is therefore vital to deal with the notion of fairness more explicitly. Cooperative game theory is a field of research that describes fairness in a sensible way (Matsubayashi et al., 2005). Hence, we suggest, in line with Cruijssen et al. (2010), to use game theoretical methods in 4-C for suitable gain-sharing.

The remainder of this article is organized as follows. Section 1.2 provides a literature review of related topics in supply chain collaboration, transport timetabling, vehicle routing and gain sharing. Next, in section 2 we present the mathematical model formulations for transport timetabling and vehicle routing in Cross Chain Control Centers as well as theory with regard to gain-sharing. Then in section 3, several test cases are considered in order to establish the viability of proposed models. Finally, we summarize our main findings and suggest directions for future work in section 4.

### *1.2. Related literature*

Due to its recent introduction, to date no scientific literature is available on 4-C explicitly. As an LSP, it ought to establish and manage lateral collaborations, which can enhance logistic performance significantly. Mason et al. (2007) have conducted an assessment of three case studies to demonstrate the advantages of taking a collaborative approach in both the vertical and horizontal direction. In particular, factory gate pricing (FGP), i.e. the use of an ex-works prices and the obligation of purchasers to manage freight transport, gave (for example) UK retailers more control over inbound product flows (Potter et al., 2006). This enabled consolidation of inbound product flows of different suppliers, thereby reducing the mileage per product considerably. These developments are catalyzed by rapid advances in ICT that allow for coordination of multiple supply chains from a single point of control. In this respect, the use of telematics, i.e. external control and monitoring of vehicles via ICT, has gained ample attention (Pramatari et al., 2005).

As far as vehicle scheduling and routing is concerned, functionalities of 4-C have most prominent overlap with freight consolidation in horizontal partnership. Cruijssen et al. (2007a) consider a situation where multiple companies request freight distribution from a common center to different customer locations. The concerning companies are prepared to collaborate and order sets are known in advance that should be delivered in pre-specified hard time windows. The authors have developed a routing heuristic, based

on available VRP heuristics, to assess synergetic potential which is involved in situations like these. Indeed, they demonstrate a considerable reduction in transportation costs by joint route planning.

Transport timetabling concerns finding an efficient usage of resources in a given tactical planning horizon. In modeling terms, this boils down to discretizing a time-horizon and associating relevant decision variables with each time element. For example, Salema et al. (2010) have proposed an elaborate modeling framework that combines strategic decisions with associated planning of purchases, production, storage and distribution. The distribution planning determines optimal magnitudes of product flows in time while striving to satisfy a time-dependent demand. Other well-known examples of transport timetabling involve time-space networks, where the physical representation of a supply chain is replicated for each time element. Time-space models are popular in tactical decision making, in arranging e.g. public bus transport (Kliwer et al., 2006), airline scheduling (Hane et al., 1995) and disaster relief operations (Haghani, 1996).

In arranging vehicle routing, a 4-C should find optimal routing between various pickup and delivery locations. This job constitutes a classical Vehicle Routing Problem with Pickups and Deliveries (VRPPD). In fact, VRPPD is a generalization of VRP, where all customer request have a common pickup location. VRPPD can roughly be subdivided into three classes of problems, i.e. Pickup and Delivery Vehicle Routing Problem (PDVRP), Pickup and Delivery Problem (PDP) and Dial-A-Ride Problem (DARP) (Parragh et al., 2008). PDVRP refers to problems where pickup and delivery locations are unpaired. That is, each commodity that is picked can be used to satisfy customer's demand at delivery locations. In contrast, PDP and DARP suppose fixed pairs of pickup and delivery locations implicitly demanding that these are visited by the same vehicle. DARP deals with efficient design of vehicle routing for passenger transportation where for instance passenger's discomfort due to long travel times is taken into account (Cordeau and Laporte, 2003). On the other hand, the classical PDP describes transportation of cargo and its generalization accounting for time windows, i.e. PDPTW, is widely considered in operations research literature (Dumas et al., 1991; Ropke and Cordeau, 2009). Numerous variants of these three VRPPD descendants exists, e.g. with or without time windows, single or multiple vehicles, each with a specific model formulation. For a detailed overview of all these variants, see Parragh et al. (2008).

In the preceding, it was motivated to use cooperative game theoretical

methods in assessing a fair way of gain distribution in 4-C. Cooperative games are a special class of games where groups of players, called coalitions, may engage in cooperation. The players are the shippers that collaborate and each coalition is a subset of the grand coalition, i.e. the collection of all collaborating shippers. Each coalition is characterized by a specific extend to which profit can be achieved (Curiel, 1997). Allocation rules dictate how profit of the grand coalition should be subdivided to participants by taking the profit of all these coalitions into consideration. Some well-known allocation rules are the Shapley value (Shapley, 1953), the nucleolus (Schmeidler, 1969) and the compromise value (Tys, 1981). In identifying the appropriate allocation rule, several fairness criteria play an important role. *Individual fairness* states that a player should receive at least as much as he can generate on its own. This implies that a shipper will never have to pay for collaboration, otherwise he wont participate. *Symmetry* imposes that two players, who generate the same additional value to any coalition, should receive the same share. This makes sense since the only intrinsic difference between two shippers is how much savings they can generate. Finally, *strong monotonicity* states that a player’s pay-off solely depends monotonically on its marginal contribution when entering a coalition. This means that if for some reason the added value of a specific shipper increases for each coalition he is part of, his pay-off should also increase and vice versa. These three fairness criteria make perfect sense in the case of collaborating shippers and it is shown that the Shapley value is the only allocation rule that satisfies these criteria (Young, 1985). This makes it sensible to use the Shapley value as the appropriate gain-sharing model in 4-C.

## 2. Theory

Central to the study at hand is the notion of inconvenience, which models the degree of a shipper’s discomfort associated with deviating from it’s original schedule. In this respect, we introduce an inconvenience cost function to quantify this degree of inconvenience.

Let us suppose a group of shippers  $M$ , indexed by  $m$ , that have expressed their willingness for collaboration. Now consider a time horizon  $T$ , in which each shipper  $m$  requests a transport at time  $t \in T$  of size  $d_{t,m}$  to be delivered. The partnership is characterized by specific moments  $\tau \in T$  when the actual consolidated delivery occurs. In general for shipper  $m$ , time of actual delivery is unequal to time of this delivery’s request, i.e.  $t \neq \tau$ , giving rise to



inconvenience  $\text{In}_{\tau,t,m}$ , given by:

$$\text{In}_{\tau,t,m} = icf_m(\tau, t) \quad (1)$$

where  $icf_m(\tau, t)$  is the inconvenience cost function of shipper  $m$ . The exact form of  $icf_m(\tau, t)$  should be specified by the business modeler, however it is natural to assume a monotonically increasing function with the difference  $|\tau - t|$ .

### 2.1. Transport timetabling: TT4C

The problem of Transport Timetabling in Cross Chain Control Centers (TT4C) is based on the assumption that shippers have common pickup and delivery locations. Evidently, this is strictly an unrealistic assumption, however different pickup as well as delivery locations are considered to be sufficiently close to each other such that the distances among those can be neglected compared to the distance between pickup to delivery locations. This assumption eliminates the spatial dimension of the problem and we only have to determine when a vehicle should be used for transport. As a result, the required functionality can be described by a rather simple model formulation. In section 2.2 we propose a more elaborate model that is no longer based on this assumption.

In order to solve TT4C, we propose a mixed-integer linear programming formulation. Let the binary variable  $Z_{\tau,t,m}$  be equal to 1 if delivery request at time  $t$  of shipper  $m$  is delivered at time  $\tau$  and be equal to 0 otherwise. Secondly, the positive integer variable  $Y_\tau$  denotes the number of trucks that accommodates delivery at time  $\tau$ . Now, TT4C is formulated by:

$$\text{TT4C} = \min C_{\text{In}}^{\text{total}} + C_{\text{Tr}}^{\text{total}} \quad (2)$$

s.t.

$$\sum_{\tau} Z_{\tau,t,m} = 1 \quad \forall (t, m) | d_{t,m} \neq 0 \quad (3)$$

$$\sum_{t,m} Z_{\tau,t,m} d_{t,m} \leq QY_\tau \quad \forall \tau \quad (4)$$

$$Y_\tau \in \mathbb{Z}_+ \quad \forall \tau \quad (5)$$

$$Z_{\tau,t,m} \in \{0, 1\} \quad \forall \tau, t, m \quad (6)$$

The objective of TT4C (2) minimizes the sum of total inconvenience ( $C_{\text{In}}^{\text{total}}$ ) and transportation ( $C_{\text{Tr}}^{\text{total}}$ ) costs, which depend linearly on the decision variables. Constraint (3) ensures that each non-zero freight delivery request is

satisfied. Secondly, constraint (4) imposes that the transported freight is restricted to the vehicle capacity  $Q$ .

Total inconvenience costs  $C_{\text{In}}^{\text{total}}$  is related to  $\text{In}_{\tau,t,m}$ , by the definition of the inconvenience cost function one is using. That is, if total inconvenience does not depend on the delivered freight size, it is given by:

$$C_{\text{In}}^{\text{total}} = \sum_{\tau,t,m} \text{In}_{\tau,t,m} Z_{\tau,t,m}. \quad (7)$$

On the other hand, if inconvenience is due to a too early or tardy delivery per unit of freight size, the right-hand side of equation (7) should be multiplied by  $d_{t,m}$ .

For the sake of simplicity, it suffices to assume that total transportation costs depend linearly on the number of used vehicles. If  $C_v$  denotes the costs per vehicle, then:

$$C_{\text{Tr}}^{\text{total}} = C_v \sum_{\tau} Y_{\tau}. \quad (8)$$

In a more realistic approach, transportation costs are incurred per unit of freight. Total transportation costs as paid by shippers are typically given by a concave cost function due to economies of scale and volume discounting (Guisewite and Pardalos, 1990). The formulation of TT4C can readily be extended as to account for this feature by e.g. a piece-wise linear approximation (Stratila, 2002).

## 2.2. Vehicle routing: VR4C

In the previous section, it was shortly noted that the central assumption of treating pickup and delivery locations of different shippers as common begin and end points is strictly unrealistic. In this section, we relax this assumption and consider the routing aspect of vehicle planning in the context of 4-C. Again delivery requests are given by  $d_{t,m}$ , yet each shipper is associated with a specific pickup and delivery location and a 4-C has to determine the optimal routing between all those. The crux of this problem is that consolidation of goods generates costs savings, but from an individual shipper's perspective, it requires vehicles to make a detour in serving the other shippers as well. As a result, the individual shippers will typically experience inconvenience due to early or tardy delivery. Hence, the optimal routing strives to make delivery request times and travel times compatible as to prevent this inconvenience.

The problem at hand is referred to as Vehicle Routing in Cross Chain Control Centers (VR4C). It integrates elements of the previously addressed

TT4C and the classical PDP. As far as pure routing is concerned, we follow conventional notation (see e.g. Parragh et al. (2008); Cordeau (2006)).

Each shipper  $m \in M$  has a corresponding pickup and delivery location. We define the directed graph  $G = (N, A)$  with a node set  $N = \{0, 1, \dots, n, n+1, \dots, 2n, 2n+1\}$  and arc set  $A$ , thus  $n = |M|$ . The nodes  $1, \dots, n$  constitute the set of pickup locations  $P$  whereas the set of delivery locations  $D$  consists of the nodes  $n+1, \dots, 2n$ . Nodes 0 and  $2n+1$  represent an origin and a destination depot respectively. In addition, let  $P_m$  and  $D_m$  respectively include the pickup and delivery location belonging to shipper  $m$  and let  $\Theta_i \subset M$  return the shipper for which  $i$  is either a pickup or delivery location.

With each node  $i$ , we associate a service time  $S_i$  representing the time needed for loading or unloading freight at this particular node.

Let  $K$  denote the set of vehicles, indexed by  $k$ , where vehicles are assumed to be identical, each with capacity  $Q$ . Each arc  $(i, j) \in A$  is characterized by transportation costs  $c_{i,j}$  and travel time  $T_{i,j}$ , which are related to the Euclidean distance between nodes  $i$  and  $j$ .

The model VR4C is characterized by four types of descriptive decision variables. First, the binary  $x_{i,j,k}$  equals 1 if vehicle  $k$  travels directly from node  $i$  to node  $j$  and equals 0 otherwise. Then,  $Q_{i,k} \in \mathbb{R}_+$  denotes the size of the load of vehicle  $k$  upon leaving node  $i$ . Next, the binary variable  $W_{i,k,\tau}$  is equal to 1 if vehicle  $k$  visits node  $i$  and arrives at time  $\tau$  and 0 otherwise. Lastly, the binary  $U_{k,t,m}$  equals 1 if vehicle  $k$  serves delivery request  $d_{t,m}$  and 0 otherwise.

As with TT4C, the objective of VR4C is to minimize the sum of total inconvenience and transportation costs:

$$\text{VR4C} = \min \sum_{k,\tau,t,m} W_{i \in D_m, k, \tau} U_{k,t,m} \text{In}_{\tau,t,m} + \sum_{i,j,k} x_{i,j,k} c_{i,j,k} \quad (9)$$

We only consider inconvenience at the delivery locations, and hence, the time  $t$  of  $d_{t,m}$  corresponds to the preferred delivery time. The first term of the objective function is nonlinear, which can readily be linearized using standard reformulation techniques.

In order to ensure that every transport request is satisfied, the following constraint applies:

$$\sum_k U_{k,t,m} = 1 \quad \forall (t, m) | d_{t,m} \neq 0. \quad (10)$$

If a vehicle is used to serve a particular shipper  $m$ , it should visit this shipper's

pickup and delivery location:

$$\sum_t U_{k,t,m \in \Theta_i} \leq M \sum_{j|j \neq i} x_{i,j,k} \quad \forall i \in P \cup D, k \quad (11)$$

$$\sum_{j|j \neq i} x_{i,j,k} \leq \sum_t U_{k,t,m \in \Theta_i} \quad \forall i \in P \cup D, k \quad (12)$$

where  $M$  is a large number. Each route should start at node 0 and end at node  $2n + 1$ , which is imposed by:

$$\sum_j x_{0,j,k} \leq 1 \quad \forall k \quad (13)$$

$$\sum_i x_{i,2n+1,k} \leq 1 \quad \forall k. \quad (14)$$

As is typical for vehicle routing problems, each route should satisfy flow conservation and the possibility of subtour formation should be eliminated:

$$\sum_{j|j \neq i} x_{i,j,k} - \sum_{j|j \neq i} x_{j,i,k} = 0 \quad \forall i \in P \cup D, k \quad (15)$$

$$u_{i,k} - u_{j,k} + (|N| - 1) x_{i,j,k} \leq |N| - 2 \quad \forall (i, j) \in P \cup D, i \neq j, k \quad (16)$$

where  $u_{i,k} \in \mathbb{R}$  is a technical variable. There should be consistency between the arrival times at the different nodes of a specific route and the variable  $x_{i,j,k}$  describing this route:

$$\sum_j x_{0,j,k} = \sum_\tau W_{0,k,\tau} \quad \forall k \quad (17)$$

$$W_{j,k,\tau+T_{i,j}+S_i} \geq W_{i,k,\tau} x_{i,j,k} \quad \forall i, j, k, \tau. \quad (18)$$

Constraint (18) contains the product of two binary variables, which can be reformulated in a linearized way. Characteristic for PDP type problems is that corresponding pickup and delivery nodes are visited by the same vehicle and in the correct sequence, which is ensured by:

$$\sum_\tau \text{ord}_\tau W_{i \in D_m, k, \tau} \geq \sum_\tau \text{ord}_\tau W_{i \in P_m, k, \tau} + T_{i \in P_m, j \in D_m} + S_{i \in P_m} \quad \forall k, m \quad (19)$$

where  $\text{ord}_\tau$  returns the ordinal element of index  $\tau$  in set  $T$ . Finally, the routing given by  $x_{i,j,k}$  should be consistent with the loading variable  $Q_{i,k}$ , which in turn is restricted to the vehicle's capacity:

$$Q_{j,k} \geq Q_{i,k} + \sum_t U_{k,t,m \in \Theta_j} d_{t,m \in \Theta_j} - M(1 - x_{i,j,k}) \quad \forall i \in P \cup D \cup 0, j \in D, k \quad (20)$$

$$Q_{j,k} \geq Q_{i,k} - \sum_t U_{k,t,m \in \Theta_j} d_{t,m \in \Theta_j} - M(1 - x_{i,j,k}) \quad \forall i \in P \cup D, j \in D, k \quad (21)$$

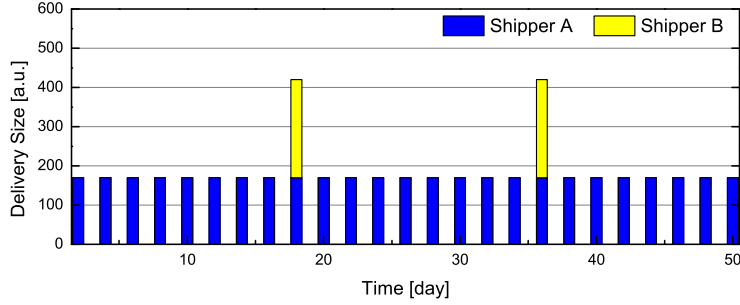
$$Q_{i,k} \leq Q \quad \forall i, k. \quad (22)$$

Constraints (20) and (21) are linearized reformulations of the usual loading consistency constraints, see e.g. Ropke and Cordeau (2009). Note that this linearized formulation strictly incorrectly models the actual loading at a particular node since, if  $x_{i,j,k} = 1$ , the load when leaving  $j$  can be larger than the load at  $i$  plus the picked up or delivered load at  $j$ . However, in combination with constraint (22) it introduces a new vehicle once a particular freight size can not be loaded anymore, which is its actual purpose rather than monitoring the actual load of a vehicle when leaving a node.

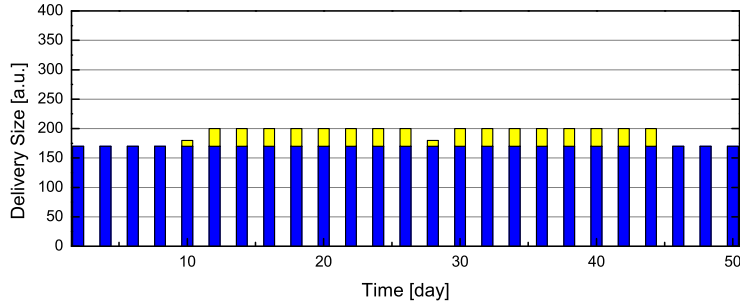
### 2.3. Extension: Convenience by a 4-C

Both TT4C and VR4C model the central trade-off between inconvenience and transportation costs. In this respect, we assume that deviating from an original schedule always generates inconvenience. However there are situations thinkable where this might not be the case. To see this, let us consider a simple situation where two shippers request transport according to Fig. 1a. We observe that shipper A delivers every two days a freight of about 85% of the vehicle capacity whereas shipper B delivers a large freight at just two moments in the time-horizon. There is only synergy associated with this situation if we allow the freight of shipper B to be split up. In this case, the large volume of B may be distributed over the rest capacity of A, thereby saving the four vehicles needed for B, see Fig. 1b.

In the combined schedule of Fig. 1b, the time discrepancy between request and actual delivery for shipper B is still considered as inconvenience. However, this may be questionable. Some shippers may actually prefer their freight being transported in smaller batches and at a higher frequency, e.g. because of lower inventory levels, but will not do this due to high associated transportation costs. Thus, rather than that shipper B experiences inconvenience, it may experience convenience, apart from the reduced transportation costs. In fact, this demonstrates a two-fold beneficial operation of a 4-C where transportation costs decreases and the delivery frequency increases.



(a) Original schedules.



(b) Combined schedule.

Figure 1: Simple two shipper situation, where vehicle capacity = 200.

In order to describe this convenience effect by 4-C, we subdivide the shipper set  $M$  into  $M_{\text{In}}$  and  $M_{\text{Co}}$ . The first still includes the shippers that experience inconvenience due to off-time delivery whereas the latter set includes the shippers that actually prefer delivery at higher frequency. The decision variable  $Z_{\tau,t,m}$  for  $m \in M_{\text{Co}}$  should be continuous in the interval  $[0, 1]$  since freight sizes should be able to be split up. If  $Z_{\tau,t,m}$  for  $m \in M_{\text{Co}}$  is non-zero, the indicator  $A_{\tau,t,m}$  equals 1 and 0 otherwise. Next, the binary variable  $B_{f,m}$  equals 1 if there are  $f$  number of deliveries for transport requests of shipper  $m$  and 0 otherwise.

Convenience measures the degree of comfort associated with delivery over more periods. Depending solely on the number of deliveries  $f$  and the shipper  $m$  in question, it is given by:

$$\text{Co}_{f,m} = cc f_m(f) \quad (23)$$

where  $ccf_m(f)$  is the convenience cost function of shipper  $m$ . This cost function typically decreases for small  $f$  due to reduced inventory costs and increases for large  $f$  due to costs associated per delivery, e.g. production-setup costs or shipment-handling costs.

Using the additional variables and the convenience parameter, both TT4C and VR4C can be extended as to account for the convenience effect. We will give its formulation and present results for TT4C explicitly, that is TT4C with Convenience is formulated by:

$$\text{TT4CwC} = \min \sum_{\tau,t,m} \text{In}_{\tau,t,m} Z_{\tau,t,m} + C_v \sum_{\tau} Y_{\tau} + \sum_{f,m} \text{Co}_{f,m} B_{f,m} \quad (24)$$

s.t.

$$\sum_{\tau} Z_{\tau,t,m} = 1 \quad \forall (t, m) \mid d_{t,m} \neq 0 \quad (25)$$

$$\sum_{t,m} Z_{\tau,t,m} d_{t,m} \leq QY_{\tau} \quad \forall \tau \quad (26)$$

$$Z_{\tau,t,m} \leq MA_{\tau,t,m} \quad \forall \tau, t, m \in M_{\text{Co}} \quad (27)$$

$$\rho A_{\tau,t,m} \leq Z_{\tau,t,m} \quad \forall \tau, t, m \in M_{\text{Co}} \quad (28)$$

$$\sum_{\tau,t} A_{\tau,t,m} = \sum_f \text{ord}_f B_{f,m} \quad \forall m \in M_{\text{Co}} \quad (29)$$

$$\sum_f B_{f,m} = 1 \quad \forall m \in M_{\text{Co}} \quad (30)$$

$$Y_{\tau} \in \mathbb{Z}_+ \quad \forall \tau \quad (31)$$

$$Z_{\tau,t,m} \in \{0, 1\} \quad \forall \tau, t, m \in M_{\text{In}} \quad (32)$$

$$Z_{\tau,t,m} \in [0, 1] \quad \forall \tau, t, m \in M_{\text{Co}} \quad (33)$$

$$A_{\tau,t,m} \in \{0, 1\} \quad \forall \tau, t, m \in M_{\text{Co}} \quad (34)$$

$$B_{f,m} \in \{0, 1\} \quad \forall f, m \in M_{\text{Co}} \quad (35)$$

where  $\rho$  indicates the minimum fraction of freight that should be transported in case of delivery. The objective (24) is extended with the convenience term. Note that we can still associate some form of inconvenience for shippers  $m \in M_{\text{Co}}$  in the first term of (24). Constraints (25) and (26) have the same functionality as in TT4C. Next, constraints (27) and (28) link  $Z_{\tau,t,m}$  correctly to its indicator  $A_{\tau,t,m}$  while ensuring a minimal fraction of transported freight. Then constraints (29) and (30) count the number of delivery moments serving  $m \in M_{\text{Co}}$  and set the variable  $B_{f,m}$  accordingly. The last five constraints mark the domains of the different decision variables.

The formulation of TT4CwC does not impose that the split up moments of delivery are equally spaced in the concerning time-horizon. From a practical point-of-view, this might be desirable otherwise the model may for in-

stance output to delivery at exactly consecutive days, which typically does not generate much convenience. In order to let the delivery moments be distributed uniformly, the objective of TT4CwC can be extended with the following additional term:

$$\sum_{\tau, t, m \in M_{Co}} \left( \frac{1}{2} |T| - \text{ord}_{\tau} \right) A_{\tau, t, m} \geq 0 \quad (36)$$

which via an additional constraint should be non-negative. By minimizing this term the average of the delivery moments tends to  $\frac{1}{2} |T|$ , and hence, delivery moments tend to be equally spaced.

#### 2.4. Gain Sharing

It was motivated that cooperative game theory provides a method to redistribute obtained savings in a fair way. To this end, we need some game theoretical terminology. Let  $2^M$  be the collection of all subsets of  $M$ , representing coalitions that may engage in cooperative behavior. The transportation costs savings that can be obtained by coalition  $S \in 2^M$  is given by  $v_{Tr}(S)$ . Following Cruijssen et al. (2010), that is:

$$v_{Tr}(S) = \max \left( \sum_{m \in S} C_m - C_S, 0 \right) \quad (37)$$

where  $C_m$  denotes the costs for shipper  $m$  if he operates individually and  $C_S$  denotes the joint costs for coalition  $S$ . Secondly, we let  $v_{In}(S)$  denote the total costs of inconvenience when shippers  $m \in S$  collaborate. In case shippers express to prefer delivery over more periods, a cost savings due to convenience can be obtained. Let  $v_{Co}(S)$  denote this source of cost savings, given by:

$$v_{Co}(S) = \max \left( \sum_{m \in S \cap M_{Co}} ccf_m(f_m^{ind}) - ccf_m(f_m^{col}), 0 \right) \quad (38)$$

where  $f_m^{ind}$  and  $f_m^{col}$  are the number of deliveries when shipper  $m$  respectively operates individually or engages in collaboration.

The Shapley value was recognized to be an appropriate allocation rule, determining a fair division of transportation and convenience costs savings



and inconvenience costs among all  $m \in M$  and is given by:

$$\phi_m = \sum_{S \subseteq M \setminus m} \frac{|S|! (|M|! - |S| - 1)}{|M|!} [v(S \cup m) - v(S)]. \quad (39)$$

For an elaborate discussion on the exact interpretation of the Shapley value, see e.g. (Shapley, 1953; Cruijssen et al., 2010). The Shapley value satisfies *additivity* (Curiel, 1997), implying that the nett savings distribution is simply given by:  $\phi = \phi_{\text{Tr}} + \phi_{\text{Co}} - \phi_{\text{In}}$ .

### 3. Results

In order to test the proposed models, we will consider several test cases. As pointed out, the inconvenience cost function should be determined by the modeler, perhaps by accounting for additional storage costs. The exact specification of  $icf_m(\tau, t)$  lies beyond the scope of this paper, however it was suggested to be a monotonically increasing function with  $|\tau - t|$ . In the test cases, we take  $icf_m(\tau, t) = \alpha_m (\tau - t)^2$ , with  $\alpha_m$  a shipper specific parameter. Thus, discomfort due to an early or tardy delivery is equally valued.

#### 3.1. Transport Timetabling in 4-C

In a 50-day time horizon, we simulated requests of freight transportation for three shippers (A, B and C). Both the interval length between deliveries and the freight size are randomly chosen from a log-normal distribution according to the specifications of Table 1.

Shipper	Interval	Freight size
A	3 (1)	95 (20)
B	8 (0.5)	115 (25)
C	14 (0)	125 (35)

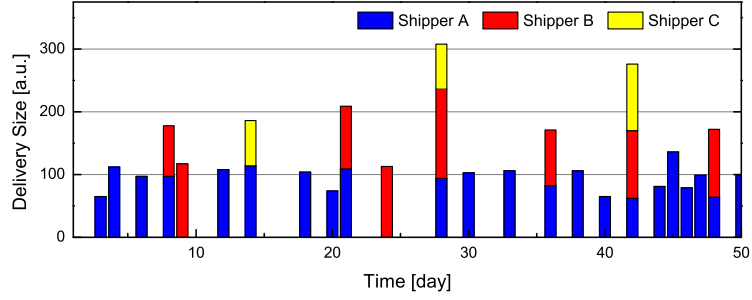
Table 1: Freight transport request specifications. The number in brackets gives the standard deviation.

##### 3.1.1. Optimal schedules

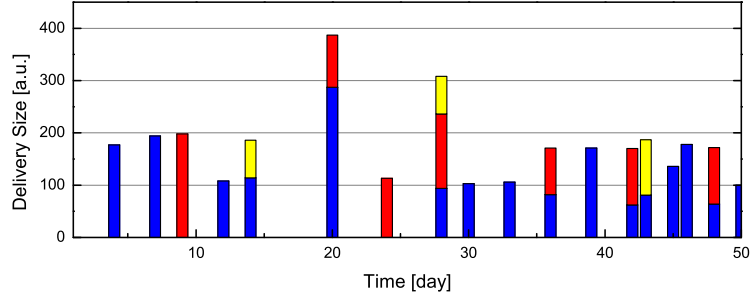
With  $d_{t,m}$  being simulated, we have determined the optimal combined schedules using TT4C, solved with CPLEX. We assume that inconvenience solely stems from a too early or too tardy delivery regardless of the freight

seize, hence equation (8) applies. Figure 2 shows obtained optimal schedules for three values of  $\alpha_m$ , i.e. inconvenience is equally valued by the three shippers in question.

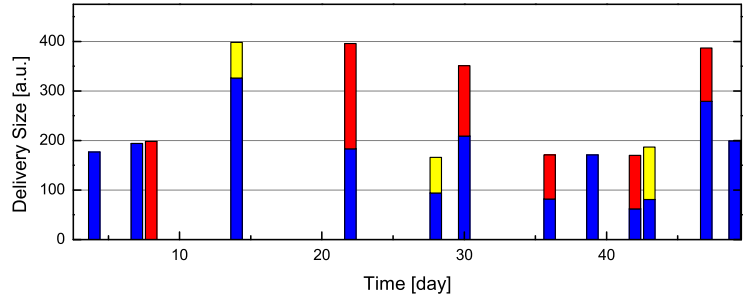
If  $\alpha_m$  is relatively high, inconvenience is dominant resulting in the optimal schedule of Fig. 2a. This schedule invokes no shipper's inconvenience, and hence, is effectively the same as input  $d_{t,m}$ . As  $\alpha_m$  decreases and inconvenience becomes relatively less important, it becomes advantageous to consolidate freight transport of different days, see Figs. 2b and 2c. As a result, transportation costs decreases accordingly and outweighs increasing inconvenience as to minimize the objective (eqn. (2)). Moreover, better vehicle capacity utilization is witnessed by delivery sizes tending to a multiple of  $Q$ .



(a)  $\alpha_m = 10$ .  $C_{Tr}^{total} = 270$ ,  $C_{In}^{total} = 0$



(b)  $\alpha_m = 1$ .  $C_{Tr}^{total} = 200$ ,  $C_{In}^{total} = 15$



(c)  $\alpha_m = 0.30$ .  $C_{Tr}^{total} = 170$ ,  $C_{In}^{total} = 18.9$

Figure 2: The optimal combined schedules for three different  $\alpha_m$ . Other parameters:  $C_v = 10$  and  $Q = 200$ .

Suppose a fourth shipper D joins the collaboration pact that transports once a freight of size 185 at day 25, but actually prefers delivery over more periods. Thus, shipper D forms a source of convenience cost savings, and

hence, we apply TT4CwC. It was motivated that the convenience cost function should decrease for small  $f$  and increase for larger  $f$ . In this case, we choose:  $ccf_D(f) = 100f^{-1} + 4f$ , which has its minimum at  $f = 5$ . Figure 3 shows the optimal combined schedule for  $\alpha_{m \in M_{C_0}} = 1$ .

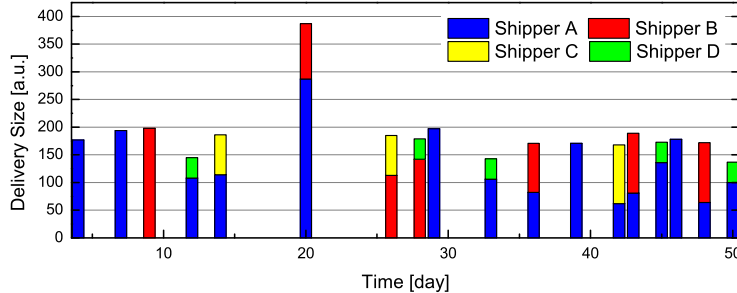


Figure 3: The optimal combined schedule found with TT4CwC for the situation of Fig. 2b plus shipper D that generates convenience cost savings.  $C_{Tr} = 190$ ,  $C_{In} = 25$  and  $C_{C_0} = 40$

In Figure 3 we observe that the single transport of shipper D is split up over 5 delivery moments, i.e. the number of deliveries where  $ccf_D(f)$  is minimal. As with Fig. 2b total inconvenience and transport costs add up to 215, however there is an additional convenience costs savings of  $ccf_D(1) - ccf_D(5) = 64$ . Moreover, one saves the additional truck originally needed for shipper D. Thus, the combined schedule of Figure 3 is clearly reminiscent of a two-fold beneficial operation of 4-C.

In the example of Fig. 3 the total transportation costs reduced from 340 when shippers operate individually to 190 when they operate jointly at the expense of 25 of inconvenience costs while in addition a convenience cost savings of 64 is obtained. In the next section, we determine how to redistribute the obtained savings and costs in a fair way.

### 3.1.2. Distributing savings and costs

For the test case of the previous section, Table 2 shows obtained transportation costs savings, inconvenience costs and convenience costs savings per coalition  $S$ . Evidently,  $v_{C_0}(S)$  can only be non-zero if  $D \in S$ . Apparently, in all but  $\{C, D\}$ , the optimal solution dictates to exploit convenience costs savings maximally, i.e.  $f_D^{col} = 5$ . In  $\{C, D\}$ , shipper C only delivers

three times, and hence,  $f_D^{col} = 3$ . Thus, exploiting convenience costs savings further does overcompensate the use of additional vehicles.

The Shapley values of the three concerning costs factors read via equation (39):  $\phi_{Tr} = (84.2; 29.2; 19.1; 17.5)$ ,  $\phi_{In} = (11.0; 7.0; 4.3; 2.7)$  and  $\phi_{Co} = (5.8; 5.8; 4.9; 47.5)$ . Thus, nett savings are distributed among the three shippers according to:  $\phi = (79.0; 28.0; 19.7; 62.3)$ . We observe that shippers A and D should receive most from obtained savings, which is in line with intuition as A provides the most bundling potential and D is a source of significant convenience cost savings. In accordance, shipper B receives more than C as the former provides more unused vehicle capacity to the collaboration pact.

$S$	$\{A\}$	$\{B\}$	$\{C\}$	$\{D\}$	$\{A, B\}$
$v_{Tr}$	0	0	0	0	120
$v_{In}$	0	0	0	0	20
$v_{Co}$	0	0	0	0	0
$S$	$\{A, C\}$	$\{A, D\}$	$\{B, C\}$	$\{B, D\}$	$\{C, D\}$
$v_{Tr}$	110	100	20	20	10
$v_{In}$	19	18	9	1	0
$v_{Co}$	0	64	0	64	58.7
$S$	$\{A, B, C\}$	$\{A, B, D\}$	$\{A, C, D\}$	$\{B, C, D\}$	$\{A, B, C, D\}$
$v_{Tr}$	130	130	110	30	150
$v_{In}$	15	20	13	9	25
$v_{Co}$	0	64	64	64	64

Table 2: Transportation costs savings, inconvenience costs and convenience costs savings for each  $S \subset 2^M$ .

It is interesting to investigate the effect of inconvenience on distributed savings. To this end, let us return to the three shipper situation of Figure 2. We make inconvenience for shipper A relatively more important by gradually increasing  $\alpha_A$  while keeping  $\alpha_B$  and  $\alpha_C$  fixed and investigating the effect on the Shapley value. Figure 4a shows the gross gain distribution  $\phi_{Tr}$  as a function of  $\alpha_A$  for A, B and C. As  $\alpha_A$  is gradually increasing, the optimal schedule tends to conform more and more to the original schedule of shipper A. The instantaneous jumps in Fig. 4b correspond to changes in the optimal combined schedule. From  $\alpha_A \sim 1.75$ , the original schedule of A is completely restored and the combined schedule does not change anymore. In addition,

the relative share of shipper A decreases at the expense of increasing share of B and C, see Fig. 4b. As A incurs more inconvenience costs, it contributes less to synergy and consequently should receive relatively less from obtained savings.

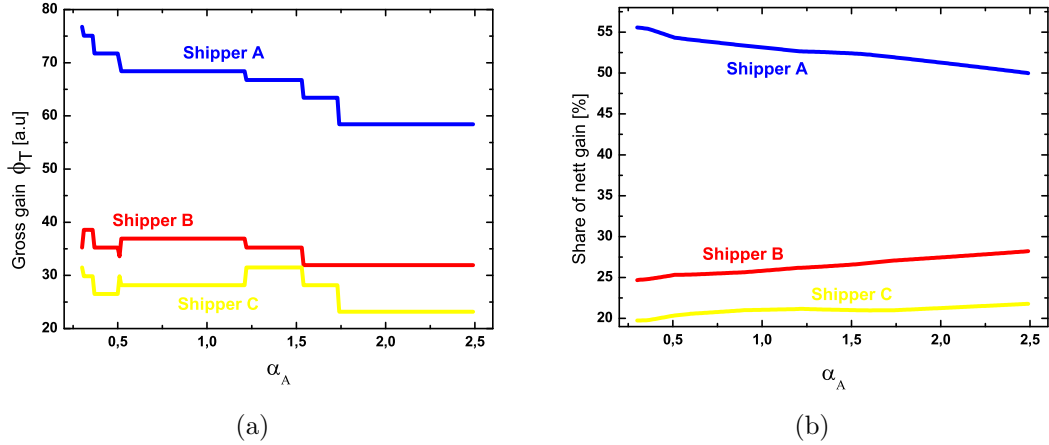


Figure 4: Gross gain (a) and share of nett gain distribution (b) as a function of  $\alpha_A$  for  $\alpha_B = \alpha_C = 0.3$ .

### 3.2. Vehicle Routing in 4-C

To demonstrate the proper operation of VR4C, let us consider a three shipper (A, B and C) situation which request freight transportation from the north to the south of the Netherlands. More precisely, the time horizon encompasses a 16-hour period (06.00 - 22.00) and the three shippers in question request transport as detailed in Table 3 and service at each locations is assumed to take 30 minutes.

Shipper	Pickup location	Delivery location	Request time(s)	Freight size
A	Groningen	Maastricht	11.00, 15.00	50, 50
B	Assen	Breda	12.30, 19.00	65, 65
C	Leeuwarden	Nijmegen	16.30	80

Table 3: Specifications of transportation requests of shipper A, B and C.

With the input transportation requests of the three shippers given by Table 3, we have determined the optimal routing and schedules for various values of  $\alpha_m$  and  $c_{i,j} = 10 \cdot T_{i,j}$ ,  $Q = 200$ , see Figs. 5 to 7. The caption of each routing indicates which transportation requests of Table 3 are served.

Figure 5 depicts the optimal solution for relatively high  $\alpha_m$ , i.e. inconvenience costs are dominant. As a result, it requires four vehicles for the five transportation requests to ensure on-time delivery. In fact, only one vehicle transports a consolidated freight invoking slight inconvenience for shipper B due to a too early delivery.

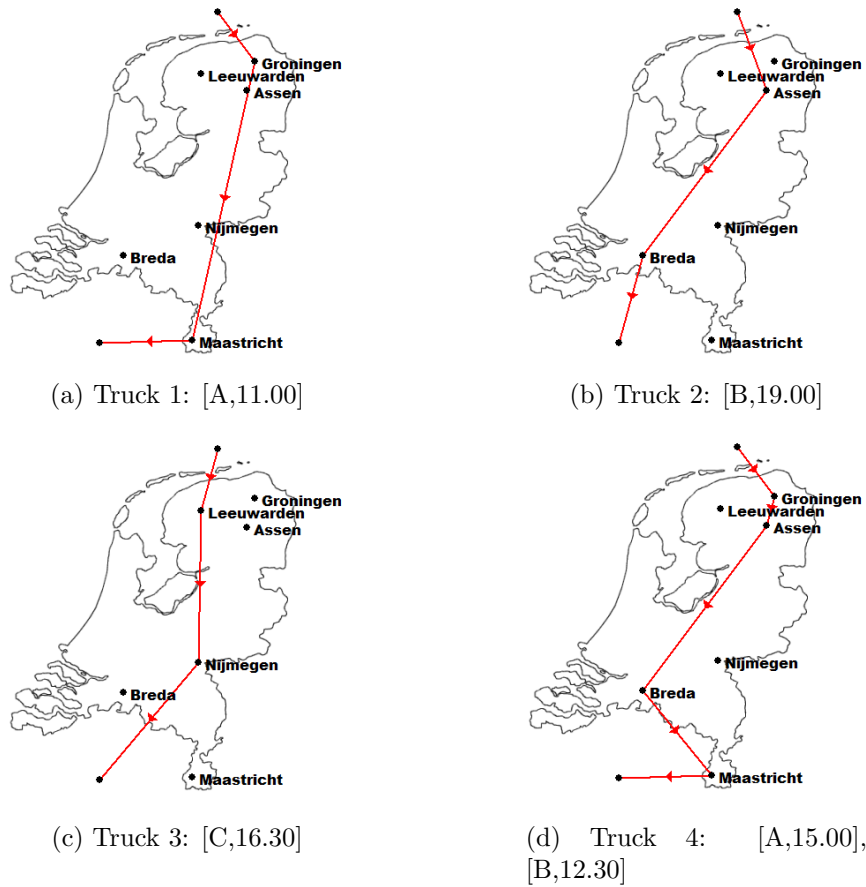


Figure 5: The optimal vehicle routing for  $\alpha_m = 20$  resulting in the use of four trucks.

If  $\alpha_m$  becomes smaller, transportation costs start to become more important, and hence, the routes of truck 3 and 4 of Fig. 5 are combined, see Fig. 6. At first sight, truck 3 seems to travel a rather striking route. It makes an evident detour in visiting successively Breda, Maastricht and Nijmegen. However, this particular route demonstrates in essence the strength of VR4C as it makes travel times and preferred arrival times more compatible. As a result it generates little inconvenience, but still requires the use of a single truck leading to an efficient logistical solution.

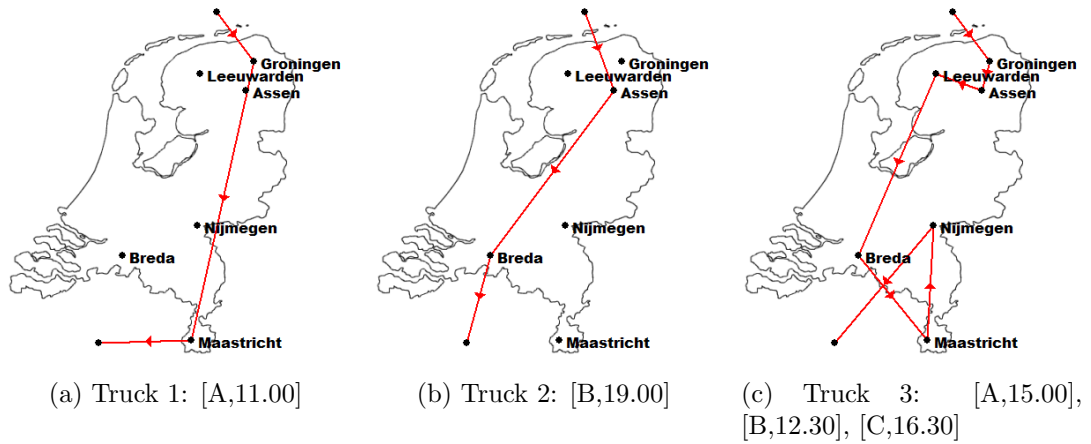


Figure 6: The optimal vehicle routing for  $\alpha_m = 10$  resulting in the use of three vehicles.

In the case  $\alpha_m = 1$  no further consolidation is possible due to the vehicle capacity resulting in the use of two trucks, see Fig. 7. As transportation costs are dominant, a truck travels the fastest route visiting the pickup and delivery locations, which are assigned to this truck. Consequently, the optimal solution tends to consolidate freight delivery requested by the same shipper. In Fig. 7, the two requests of shipper A are combined rather than the two requests of B. This is due an inconvenience effect as the difference in preferred arrival times of B is larger than of A, and hence, the two requests of B are delivered by two different trucks.



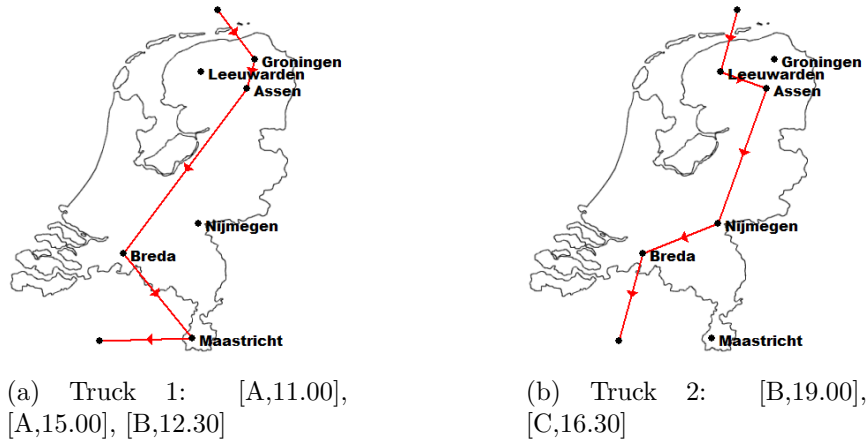


Figure 7: The optimal vehicle routing for  $\alpha_m = 1$  resulting in the use of two vehicles.

#### 4. Conclusion

In this paper, we have described the role of a Cross Chain Control Center (4-C) in satisfying the growing need for efficient supply chain orchestration. As an LSP, its task is to manage and control multiple supply chains simultaneously.

While considering transportation, we have proposed a methodology allowing a 4-C to arrange vehicle planning for a group of shippers that are willing to collaborate. As different shippers have typically different preferred delivery times, a 4-C should find a logistical solution requiring shippers to deviate from their original schedules in order to make freight consolidation possible. In this respect, we introduced the notion of inconvenience to capture the degree of this required deviation. It is believed that deviating causes shippers to incur costs which is modeled by an inconvenience cost function.

Central to the study at hand is the trade-off between transportation and inconvenience costs. We considered two optimization models that describe this trade-off. The first, Transport Timetabling in Cross Chain Control Centers (TT4C), is based on the assumption that shippers have the similar pickup and delivery locations thereby ignoring the spatial dimension of vehicle planning. This situation can be described by a simple and effective model to construct a common tactical planning, to which shippers should conform. In

a simple test case, we saw that allowing more inconvenience correctly results in more enforcement of synergetic potential and vice versa.

The second model is more complex as it accounts for vehicle routing as well. That is, the model produces the optimal routing that vehicles should travel between the different pickup and delivery locations of collaborating shippers. In line with intuition, we correctly find that the model aims to make preferred delivery times and travel times compatible as to minimize the extend of inconvenience.

We recognized the possibility of generating convenience by a 4-C. In a collaboration pact, it might be possible that the freight size of a shipper is split up and delivered over multiple periods which actually satisfies his preference. This effect allows for a two-fold beneficial operation of 4-C, i.e. reduced transportation costs and enhanced delivery frequencies. We captured the convenience effect by introducing a convenience cost function and demonstrated the two-fold beneficial operation via a test case.

An apparent downside of the proposed models is their mixed-integer nature, which makes them NP-hard. As a consequence, computation times may be impractical when input-data sets become larger. It may therefore be interesting to investigate whether heuristics can be developed, perhaps by integrating the notion of inconvenience into existing PDP-heuristics.

Lastly, we did not pay attention to the exact form of the inconvenience cost function. In order to let the proposed methodology work in practice the inconvenience cost function should be specified in agreement with the shipper in question. In this respect, a 4-C may suggest several different forms of the inconvenience cost function and present the corresponding cost savings that can be obtained, perhaps by accounting for additional inventory storage costs. A shipper may subsequently decide how much inconvenience it allows in return for prospected savings.

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