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AN INTEGRATED PROCEDURE FOR BUILDING A COMMON PERCEPTUAL SPACE BASED ON COMPLETELY INDIVIDUALIZED DATA COLLECTION

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1. INTRODUCTION

Perceptual mapping is one of the most important marketing research tools. It has received much attention in the literature and has been used extensively in commercial studies. The aim of a perceptual mapping study is to yield insight in the basic cognitive dimensions consumers use to distinguish between the 'products' in the category under investigation and the relative positions of the products in respect to these dimensions (Hauser and Koppelman 1979).
The major perceptual mapping techniques are factor analysis and multidimensional scaling.
Factor analysis usually requires a two-stage data collection procedure. First, a pilot-study is conducted to identify the attributes on which consumers base their perceptions. In the second stage, subjects rate each product on each attribute. The resulting aggregate data matrix serves as input to factor analysis. The perceptions of the products are represented by factor scores which are based on the attribute ratings. The dimensions are interpreted by examining the correlations between attribute ratings and the newly constructed dimensions.

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A major advantage of factor analysis is that the dimensions are usually readily interpretable in terms of the original attributes. Further, it can also be used if the number of products is small.

Major disadvantages are (1) it is a rather expensive method since qualitative research is usually required to generate the relevant attributes, (2) the attributes must be specified a priori and (3) all subjects evaluate the same set of attributes. Thus, the factor analysis approach to perceptual mapping implicitly assumes that all attributes used in the study are relevant to all subjects. However, an individual subject's set of relevant attributes may not be identical to the set of attributes presented (e.g. Williams and Langron 1984, Böcker and Schweikl 1986, Van den Heuvel 1986, Boivin 1986). Some of the attributes presented may not be relevant to the subject whereas he or she can base his or her perceptions on attributes which are not included in the set of attributes presented. Besides, subjects may attach different meanings to the same attribute and/or describe the same product aspect by different words (e.g. Williams and Arnold 1985).

A second approach to perceptual mapping involves multidimensional scaling. In multidimensional scaling (Schiffman et al. 1981) judgments are made with respect to the actual products rather than to specific attribute scales. The subject is asked to judge the perceived similarity or dissimilarity between (all) possible pairs of products. A perceptual configuration is constructed on the basis of the (dis)similarity judgments. A vast array of computer algorithms is available for this purpose. Similarly a perceptual configuration can be found from preference judgments of products.

The most important advantages of multidimensional scaling are (1) the subject is allowed to use his or her own attributes to discriminate between the products and (2) the data can be obtained in a single session.

Multidimensional scaling also suffers from a number of limitations. The most important limitations are (1) the judgment task becomes very time consuming and expensive if the number of products exceeds, say, 10 to 11, (2) the dimensions of the perceptual map are often very difficult to interpret without external information and (3) the number of products should at least be 7 or 8.

The limitations of the traditional approaches to the construction of perceptual maps can be overcome by allowing the subject to describe and evaluate the products explicitly in his or her own terminology. In this paper we present an efficient integrated procedure for constructing common perceptual maps on the basis of completely individualized data. The procedure is illustrated empirically by an application to consumer perceptions of meat cuts.

2. A NEW PROCEDURE FOR CONSTRUCTING PERCEPTUAL SPACES

2.1 Data collection

The data collection part of the procedure consists of two phases. In the first phase, attributes are generated by a technique called "natural grouping". In the second phase, each subject rates all products on all of his or her own attributes. Below, we shall describe each phase of the proposed procedure in more detail.

The subject is presented with a number of products. Verbal or pictorial description or actual products can be used. He or she is asked to split the set into two groups according to their perceived similarity, so that products which appear to the subject to be similar are placed in the same group. The subject is asked to verbalize the aspect or aspects on which

the two groups differ. Further, he or she is asked to indicate the pole of the aspect that describes each group best (e.g. the aspect is type of meat and the poles are beef for group 1 and pork for group 2). Experience with natural grouping revealed that prompting for attribute levels is frequentely not necessary as subjects show a natural tendency to describe groups with poles of attributes.
The attribute(s) and poles are written down by the interviewer.
The procedure is repeated for each of the two groups separately and continued until the subject indicates that no further partitions are possible because the products in a group are similar to him/her.
Subsequently, each product is rated on each attribute; a five-point semantic differential scale is used. The pairs of adjectives are the poles of the attributes used by the subject.

2.2 Data analysis

With the data collection procedure described above, a completely individualized perceptual configuration is obtained for each subject. Each product has a unique position in this configuration. The position of a product is given by the ratings of the product on the attributes. However, when the number of subjects is large, completely individualized perceptual representations make analysis and understanding of the results cumbersome and managerially less useful. It is preferable to search for communalities in the individual perceptions while retaining individual differences. Generalized Procrustes analysis (Gower 1975, Ten Berge 1977) is particularly suited for this purpose (Steenkamp et al. 1986) as it calculates a centroid or 'group' perceptual space, while allowing for individual variation. Generalized Procrustes analysis is a rather recent extension of the better known simple Procrustes analysis in which only two configurations are being matched (Hurley and Cattell 1962, Schönemann 1966, Browne 1967, Schönemann and Carroll 1970).
Generalized Procrustes analysis starts with a set of individual matrices X_i ($i=1,...,p$). X_i is of order $n \times m_i$ where n indicates the number of products and m_i is the number of attributes used by subject i. Thus, an element of X_i, x_{ijk} denotes subject i's rating of product j ($j=1,....,n$) on attibute k ($k=1,...,m_i$). It is assumed that the meaningful information of X_i is contained in the relative distances among the n products (cf. Lingoes and Borg 1978, Coxon 1982).
The centroid configuration Y of the order $n \times m$ ($m=\max_i(m_i)$) is derived from the X_i's. Zero-element columns are appended to each X_i that initially has fewer than m columns. Y is computed as the average of all X_i's after they have been fitted optimally to each other under the admissible transformations (i.e. transformations that leave the relative distances between the products unchanged). The optimization criterion is to minimize the residual sum of squares between Y and the X_i's. The individual configurations are brought into maximum congruence to one another by translation, rotation/reflection and central dilation. Each X_i is centered at the origin to neutralize effects due to different subjects scoring at different levels of the scale. Rotation/reflection is applied to account for the effect that subjects use different words or combinations of words to describe the same stimulus. Differences in the range of scores used by different subjects are adjusted for by central dilation.
Thus, the transformations used in generalized Procrustes analysis do not affect the relative distances among the products.
The overall communality in individual perceptions can be assessed by the average percentage of variance in the individual configurations that can be explained by the centroid configuration. The residual sum of squares can both be partitioned for products and for subjects (Gower 1975).

Product residuals are derived from the sum of squared distances (over all subjects) from the centroid position to actual individual subject's final product position for each stimulus. Similarly, subject residuals are derived from the sum of squared distances (over all products) for each subject.[1] A smaller sum of squares for any product indicates greater agreement among subjects with respect to the relative perceptual position of that stimulus. The residual sum of squares explained per subject provides information on the communality between the individual configuration and Y. Thus, subjects whose perceptions are poorly explained by Y can be identified.

For ease of interpretation, the centroid configuration may be referred to new orthogonal dimensions, accounting successively for decreasing amounts of variation in the data (Gower 1975, Williams and Langron 1984). These new uncorrelated dimensions may be interpreted in terms of each subjects original attributes by calculating the correlations of the original attributes with the new dimensions.

The computing procedure for the generalized Procrustes analysis used in this paper is described in more detail by Gower (1975).

In some aspects, generalized Procrustes analysis resembles INDSCAL (Carroll and Chang 1970). However, generalized Procrustes analysis has a number of substantial advantages over INDSCAL (Lingoes and Borg 1976, 1978, Borg en Lingoes 1978). Whereas generalized Procrustes analysis starts with a set of individual configurations, INDSCAL starts with scalar product matrices . This affects the meaning of the distances among points in the subject space. INDSCAL applies unadmissible transformations to the data, without providing information on the increase in explanatory power due to these unadmissible transformations. In INDSCAL, the relationship assumed between the group space and the individual perceptions is also relatively simple: negative weights and different points of perspective are not allowed for. Although not pursued in this paper, the above mentioned limitations of INDSCAL can be accommodated for within the generalized Procrustes framework (Lingoes and Borg 1976, 1978, Borg and Lingoes 1978). See also the Discussion section of this paper.

A simple example may clarify the way individual configurations are fitted optimally to each other by admissible transformations: translation, rotation/reflection and central dilation. Consider two subjects who rated four products A, B, C and D on two (self-generated) attributes using a five-point scale. The scores can be treated as coordinates of the products in a two dimensional space. Suppose subject 1 (S_1) gave product A a rating of three on both attibutes. This means that A is located at (3,3) in the two dimensional space. Further, if S_1 gave ratings of 5 and 3 for B, 5 and 1 for C and 3 and 1 for D, then the coordinates of B_1, C_1 and D_1 are (5,3), (5,1) and (3,1), respectively. Suppose further that subject 2 (S_2) located the four products at (4,2), (2,5), (1,3) and (2,1), respectively. The perceptual configurations, X_1 and X_2, of the two subjects at this stage are shown in figure 1a. Generalized Procrustes analysis was applied to bring X_1 and X_2 into maximum congruence to one another in order to obtain the best possible estimate (in a least squares sense) of Y.

First, X_1 and X_2 are brought to a common origin by subtracting the mean score on each attribute from the original scores (figure 1b). In the example $S_1(S_2)$ was on the average scoring higher on attribute 1 (attribute 2) than $S_2(S_1)$, so these differences are removed by translation. Second, the configurations are rotated/reflected (figure 1c).[2][3] Third, each configuration is centrally dilated (figure 1d). It can be seen that X_1 is somewhat expanded (the stretching factor was 1.16) and X_2 is somewhat contracted (the shrinking factor was 0.89) because S_2 was more extreme in her ratings than S_1. Finally,the centroid

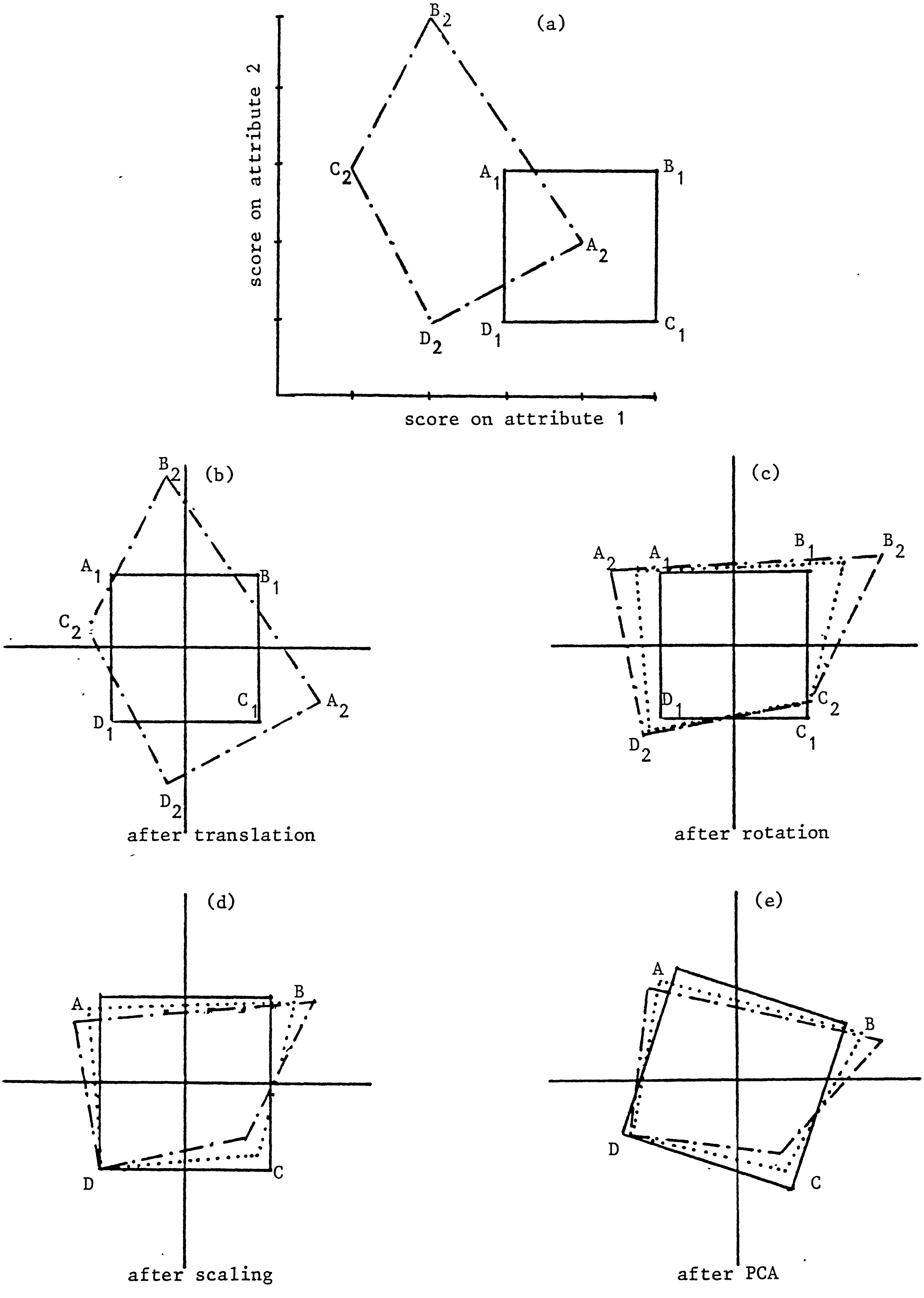

Figure 1. Illustration of the generalized Procrustes procedure for two subjects' configurations. Subject 1 (solid lines), subject 2 (dashed lines) and centroid configuration (dotted lines).

configuration is referred to new orthogonal dimensions by principal components analysis (figure 1e).
In this example the congruence of the individual configurations after the admissible transformations is nearly perfect. The residual sum of squares between Y and X_1 and X_2 is very small. In fact, Y accounted for 98.1% of the variance in both X_1 and X_2.

2.3 Comparison with traditional approaches

The proposed procedure for building a common perceptual map has several advantages to factor analysis and multidimensional scaling.
First, this procedure starts with a set of completely individualized perceptual spaces, that is, each individual's configuration is build on individually generated attributes and therefore is meaningful to this person. While searching for communalities among individuals, this individual information is maximally taken into account both for construction and for interpretation of the centroid configuration.
Second, the respondent's task is placed within the context of the total set of products under study. When generating attributes, the subject can derive information from all stimuli simultaneously.
Third, the procedure takes into account differences in perceptual acumen among subjects. Subjects that do not perceive much differences between products, are not forced to perform a time consuming task as they score only those attributes on which differences are perceived. On the other hand, maximum information is obtained from subjects that show more involvement in the product being investigated. Thus the proposed procedure is a very efficient one.
Fourth, the procedure allows for the identification of respondents whose perceptions are poorly explained by the common space. Factor analysis does not provide this possibility, as individual perceptions are not strictly kept separate from the group space (individual factor scores are dependent on factor loadings).
A potential weakness of the proposed procedure is its basic assumption that consumers are capable of verbalizing differences between products. However, up to date experience has not shown any indication that this problem is a serious one but more research is needed before this can be concluded with confidence.

3. APPLICATION

3.1 Data collection

Subjects and materials
Sixty-four female purchasers of meat were interviewed at two different facilities of a market research agency. None of them had prior experience with the task involved, and no training was conducted. All subjects were interviewed individually by an experienced interviewer of the market research agency.
Subjects were provided with color photographs of 15 different meat cuts. The photographs were made by a professional photographer who had prior experience with the photography of meat cuts. Table 1 provides a list of the 15 meat cuts used.

Table 1. Meat cuts used in this study.

Number	Meat cut
1	sirloin steak
2	brisket beef steak
3	fore rib steak
4	pork shoulder chop
5	minced beef
6	minced meat (pork with beef)
7	hamburger
8	pork steak
9	pork rib chop
10	pork sausages
11	pork belly steak
12	blade steak
13	pork fillet
14	rolled pork
15	roast beef

Procedure

Data collection consisted of two phases: natural grouping and rating of the products on attributes. Both phases will be described in more detail below.

For each subject the 15 color photographs of different meat cuts were spread out on a table in random positioning in such a way that the subject could overlook all photographs at one time. All photographs were provided with the name of the meat cut depicted. As described above, subjects were asked to divide the photographs into two homogeneous groups (not necessarily of equal size) and were asked to verbalize both the criterion used for the partition made and the poles describing the two groups formed best. Subjects continued doing so until no further partitions were possible, as the products in a group were similar to her. In practice, subjects varied in the number of attributes mentioned; it ranged from two to nine, with an average of five.

For each partition made, newly generated attributes and the poles associated with the groups were written down by the interviewer. After completion of the natural grouping task, each subject rated all 15 meat cuts on all attributes that she had generated during the natural grouping task. Thus a completely individualized m-dimensional configuration is obtained for each subject, whereby 'm' (i.e. the number of attributes used by a particular subject) may vary between subjects.

Subjects were allowed to work at their own pace. On average, the data collection procedure (comprising both natural grouping and rating) took about 20 minutes to complete.

3.2 Results

Data were analyzed using generalized Procrustes analysis. The computer program used was kindly provided to us by the Food Research Institute, Bristol, U.K. and is a slightly modified version of the Rothamsted Experimental Station-version.

As the maximum number of attributes used by any of the subjects was nine, a nine dimensional centroid configuration was obtained in first instance. The nine-dimensional centroid configuration accounted for 63.1% of the

total amount of variation in the data. This implies that 63.1% of the variance in all individual configurations can be explained by the centroid configuration.
The residual sum of squares has both been partitioned for products and for subjects. Product residuals are shown in table 2.

Table 2. Residual sum of squares in the centroid configuration for the fifteen meat cuts.

Meat cut		Residual SS
1	sirloin steak	1.548
2	brisket beef steak	1.372
3	fore rib steak	1.271
4	pork shoulder chop	1.607
5	minced beef	1.825
6	minced meat (pork with beef	1.492
7	hamburger	1.735
8	pork steak	1.584
9	pork rib chop	1.597
10	pork sausages	1.750
11	pork belly steak	1.681
12	blade steak	1.202
13	pork fillet	1.842
14	rolled pork	1.729
15	roast beef	1.365

Examination of the product residuals in table 2 reveals that subjects show most agreement with respect to their perceptions of beef meat cuts such as blade steak, fore rib steak, roast beef and brisket beef steak, while they show a relatively strong heterogeneity in respect to their perception of minced beef and pork fillet.

The residual sum of squares has also been partitioned for subjects. Information on how well each subject's perceptual space is represented by the perceptual positions of the meat cuts in the centroid configuration can be obtained from the subject residuals. Subject residuals are shown in table 3. As follows from the relatively large amount of variance in individual matrices that is accounted for by the centroid configuration (63,1%), the average residual sum of squares is small. Most subjects appear to perceive the meat cuts rather similarly, although using different words. However, a number of subjects have relatively large residual sum of squares. For 14 subjects the residual sum of squares was more than one standard deviation larger than the mean residual sum of squares (5 subjects even had a residual sum of squares that was more than two standard deviations larger). These subjects might attach differential weighting to the attributes or have a different point of perspective (see also the Discussion section of this paper). This might be an indication that not all 64 subjects form a homogeneous group with respect to their perception of meat cuts.

Table 3. Residual sum of squares (RSS) for each of the 64 subjects.

no	RSS	no	RSS	no	RSS	no	RSS
01	0.332	17	0.301	33	0.324	49	0.363
02	0.382	18	0.291	34	0.333	50	0.320
03	0.322	19	0.429	35	0.343	51	0.373
04	0.333	20	0.458	36	0.348	52	0.289
05	0.354	21	0.273	37	0.316	53	0.320
06	0.324	22	0.462	38	0.366	54	0.295
07	0.390	23	0.495	39	0.338	55	0.437
08	0.363	24	0.324	40	0.307	56	0.279
09	0.455	25	0.495	41	0.298	57	0.492
10	0.454	26	0.377	42	0.400	58	0.316
11	0.302	27	0.379	43	0.506	59	0.355
12	0.304	28	0.367	44	0.316	60	0.368
13	0.349	29	0.392	45	0.332	61	0.500
14	0.334	30	0.536	46	0.456	62	0.272
15	0.413	31	0.341	47	0.344	63	0.447
16	0.317	32	0.399	48	0.331	64	0.460

$\overline{RSS} = 0.369$ $s(RSS) = 0.067$

The nine-dimensional centroid configuration is subjected to principal components analysis. The first three principal components accounted for 92.6% of the variation in the centroid configuration. The fourth principal component accounted only for 4.9% of the variance. When relating the three-dimensional PCA-solution to the individual data, it accounts for 58.4% of the variance. A plot of the centroid configuration after PCA is given in figure 2.
The interpretation of this orthogonalized centroid configuration is based on the correlation coefficients obtained by correlating the scores on the principal components with each individual's original attribute scores. Thus, the centroid is interpreted on the basis of each individual's own vocabulary. Table 4 gives the most frequently used attributes for each principal component. Most frequently used was defined as: correlating higher than 0.70 for at least five subjects. Scale poles are reversed, where necessary, so that the poles first mentioned correlate possitively with the principal component in question).

Table 4. Interpretation of the centroid configuration in terms of subjects' original descriptions (number of times correlations > 0.70 were obtained).

Principal Component 1		Principal Component 2		Principal Component 3	
beef vs pork	(46)	fat vs lean	(23)	long time of preparation vs short time of preparation	(48)
lean vs fat	(23)	minced vs pure	(15)		
expensive vs cheap	(8)	poor quality vs good quality	(13)		
good quality vs poor quality	(7)	poor taste vs good taste	(11)		
		cheap vs expensive	(11)		
		rarely used vs often used	(6)		
		common vs exclusive	(5)		
		tough vs tender	(5)		

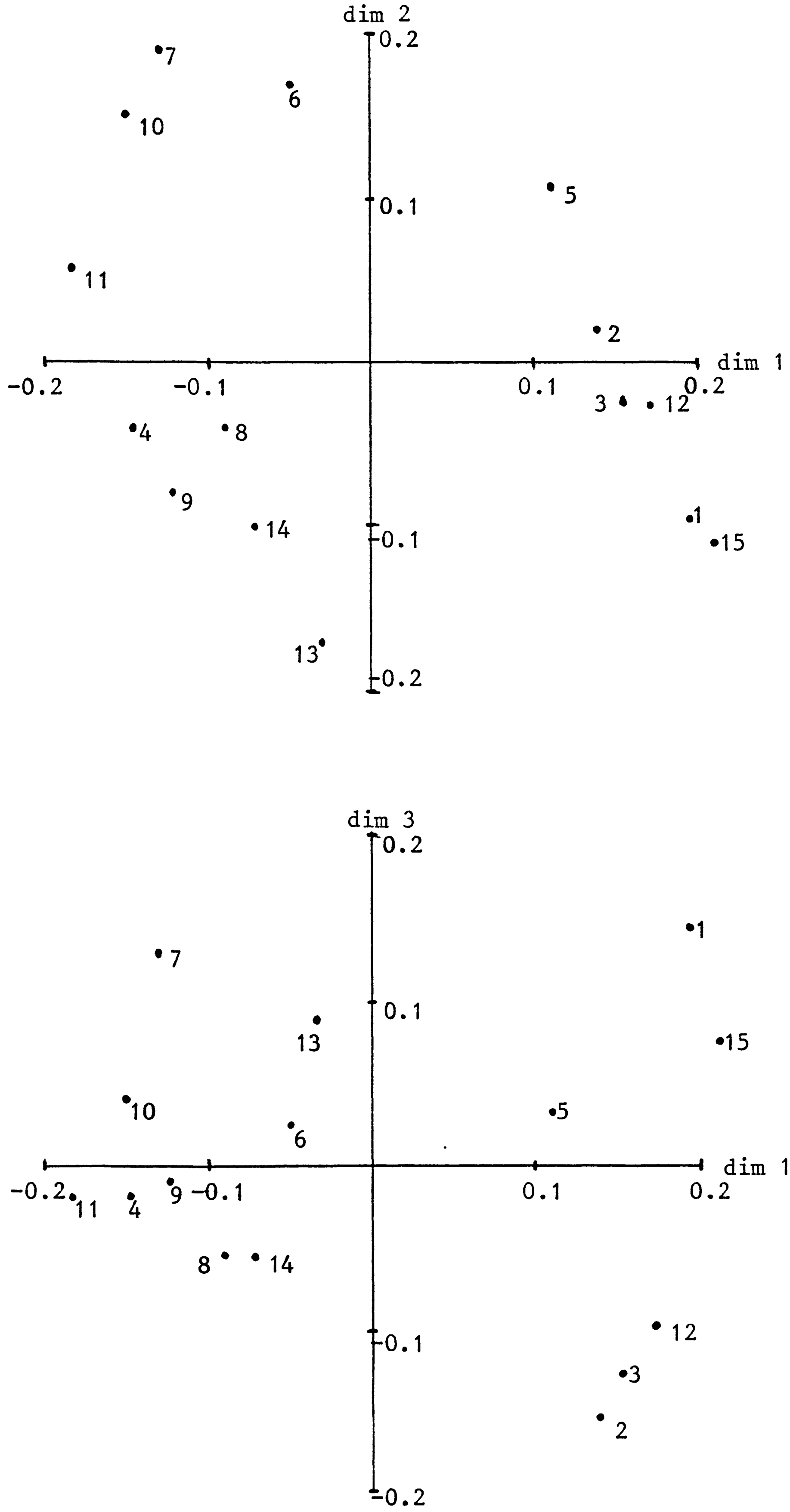

Figure 2. Centroid configuration after Principal Component Analysis.
For identification of the meat cuts see Table 1.

Table 4 reveals that the interpretation of the centroid configuration is straightforward. The first principal component refers to the type of meat, beef being considered leaner, of better quality and more expensive than pork. Other verbalizations used to describe this dimension were color (red vs not red), tenderness and appreciation of taste.
The second dimension refers to the perceived quality of the different meat cuts. Good quality is obviously associated with leanness, pure meat, good taste and being more expensive.
The third dimension is exclusively described by duration of preparation of the meat.

4. DISCUSSION

The integrated procedure presented in this study appears to be both a reliable and easy to handle method for the construction of a common perceptual space while retaining individual differences. Reliability is apparent from the consistency of the results obtained in this study with other studies concentrating on consumer behavior with respect to meat (Steenkamp and van Trijp 1986). Further, reliability was assessed by comparing the results of generalized Procrustes analysis to those obtainable from INDSCAL. Euclidean distances between the products were computed for each subject separately and analyzed via INDSCAL. The C-match index of fit (Cliff 1966) between INDSCAL's three-dimensional group space and generalized Procrustes analysis' centroid space was 0.966. This indicates high congruence between the two solutions. Moreover, INDSCAL's subject space revealed a relatively homogeneous group of subjects.
In the integrated procedure presented in this study, natural grouping was used for the generation of individually relevant attributes. As such, this method is comparable to other methods for eliciting attributes, Kelly's Repetory Grid-method (Kelly 1955) being the best known among these. When compared to Kelly's Repetory Grid, the natural grouping procedure has the advantage that it is less time-consuming and that partitions are being made within the context of the total group of products under study. In Kelly's Repetory Grid judgments are made within the context of only three products (though in varying combinations), which may lead to attributes which are not that relevant within a larger context. Further, within the natural grouping task, subjects that do not perceive much difference between products, are not forced to perform a long-winded - and therefore boring - task, and next they only score those attributes on which they perceive differences among the products. Subjects that show more involvement with the product under study will differentiate among the products in more detail and as such, a multi-layer organization of the relevant attributes is obtained. For these the maximum amount of information is obtained, without the respondent getting bored.
It remains a question whether natural grouping is equally useful in case the number of products is small. It is quite possible that Kelly's Repetory Grid would be the preferable method in that case. However, up to date natural grouping has only been applied to relatively large groups of stimuli.
The natural grouping is fast and easy to understand but also rather rigid. Once products are placed into different groups, they cannot be placed into the same group with respect to other attributes. Especially the first partition exerts a considerable influence on the outcome of the process. Possible modifications may make the natural grouping procedure more flexible.
Generalized Procrustes analysis is a versatile approach, of which in this study only the basic approach is used. Not all of the subjects are equally well represented by the centroid configuration, obtained on the

basis of admissible transformations only. The reasons for this can be investigated within the generalized Procrustes framework by extending the basic model to unadmissible transformations of the data. It can be investigated whether individuals attach greater salience to certain (fixed) aspects of the difference between products than to others (differential dimensional weighting) or whether individuals have idiosyncratic frames of reference (i.e. use different spatial directions as a result of using different attributes). Further, it can be investigated whether individuals have different points of perspective. Based on these additional individual parameters subjects could be partitioned into relatively homogeneous subgroups, if necessary. However, other research has shown that the admissible transformations generally account for the largest share of the total variance explained in the individual data, by far (Borg 1977, Coxon 1982).
Future research could apply the proposed data collection and analysis procedure to other products in order to investigate its possibilities and limitations. In the present study the products were, for most subjects, perceptually rather differentiated. It is worthwhile to study whether subjects are able to verbalize their attributes and whether the centroid configuration is stable in case the products are perceptually rather close to each other.
In sum, the results appear encouraging. It was possible to construct a readily interpretable common perceptual map on the basis of each subject's own vocabulary. The methods described in this study to obtain an individualized perceptual product map deserve further study.

Footnotes

1) In formula:
product residual sum of squares : $RSS_j = \sum_i \sum_k (y_{ik} - x_{ik})^2$

subject residual sum of squares : $RSS_i = \sum_j \sum_k (y_{jk} - x_{jk})^2$

2) In the present example only X_2 is rotated. Rotation of X_1 did not lead to a further reduction of the residual sum of squares. However, if more matrices are involved all matrices are usually rotated.

3) After translation, the individual matrices are uniformly rescaled to allow for different magnitudes of data. We have multiplied the matrices after rotation and scaling by the uniform scaling factor (which is another factor than the stretching/shrinking factor; see Gower (1975; 40-43)) to return to the original units of measurement in order to facilitate visual comparison between the figures. The results are in no way influenced by this scaling-back procedure.

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