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CREDIT MARKET DISTORTIONS, ASSET PRICES AND MONETARY POLICY

By

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Credit Market Distortions, Asset Prices and Monetary Policy[*]

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Abstract

We study the conditions that ensure rational expectations equilibrium (REE) determinacy and expectational stability (E-stability) in a standard sticky-price model augmented with the cost channel. We allow for varying degrees of pass-through of the policy rate to bank-lending rates. Strong cost-side effects heavily constrain the policy rate response to inflation from above, so that inflation targeting policies may not be capable of ensuring REE uniqueness. In such cases, it is advisable to combine inflation responses with an appropriate reaction to the output gap and/or firm profitability. The negative reaction of real activity and asset prices to inflationary shocks adds a negative force to inflation responses that counteracts the borrowing cost effect and avoids expectations of higher inflation to become self-fulfilling.

JEL: E31; E32; E52
Keywords: Monetary Policy, Cost Channel, Asset Prices, Determinacy, E-stability.

1 Introduction

Financial intermediation and corporate credit play a central role in the transmission of monetary policy, determining the impact of interest rate changes on the prices of goods and assets. This paper examines a dynamic general equilibrium model in which bank-lending shapes the transmission of monetary policy on firm profitability, through the

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so-called cost channel.[1] We characterize the conditions that ensure rational expectations equilibrium uniqueness and stability under adaptive learning (E-stability) in the presence of varying degrees of pass-through from policy to bank-lending rates. Furthermore, we explore the interplay between asset prices and cost-side effects, as well as the implications of allowing the monetary authority to respond to asset prices, so as to enhance both price and financial stability.

Monetary policy supply-side effects classically arise from agency problems between producers and lenders. The importance of this channel crucially depends on the pass-through from policy to bank-lending rates. Chowdhury et al. (2006) show that heterogeneous financial systems can lead to major differences in the transmission of policy shocks: along with countries where the banking sector acts as an attenuator of changes in the risk-free rate (e.g., France and Germany), there are countries where bank-lending rates amplify movements in the policy rate (e.g., Japan and the US). The second case is central to our analysis.

Along with the traditional Taylor principle, the cost channel implies the emergence of an upper bound to inflation responses that prevents the central bank from being too aggressive in stabilizing inflation, if determinacy has to be attained. In contrast to previous studies, (e.g., Bruckner and Schabert, 2003, Surico, 2008 and Llosa and Tuesta, 2009), we show that the additional constraint may become a reason of concern for the policy maker when movements in the policy rate are amplified by the banking sector. In fact, the upper frontier may become so stringent that determinacy cannot be attained if the central bank acts as a pure inflation targeter. In such cases setting the policy rate in response to both inflation and the output gap may be desirable. Reacting to real activity adds a negative force to inflation responses that counteracts the borrowing cost effect operating through the direct influence of the lending rate on aggregate supply.

A main focus of the paper is to examine the role of monetary policy when the cost channel "matters" and affects firm profitability. Despite the increasing emphasis on the connection between financial frictions and macroeconomic fluctuations, the influence of cost-side effects on firm profits and asset prices has generally been neglected. We show that responding to asset prices helps at attaining determinacy when strong credit market distortions are at work. In this respect, two distinct effects are isolated. On the one hand, as shown by Carlstrom and Fuerst (2007) reacting to asset prices weakens the overall policy response to inflationary shocks, thus making the lower bound to inflation responses more stringent. On the other hand, a positive response to asset prices brings about much higher gains, in terms of increased chances to attain determinacy, outweighing the borrowing cost effect that operates in the model with the cost-channel. In turn, the second effect emerges as the outcome of two mutually reinforcing mechanisms that exploit the amplification induced by strong degrees of pass-through from policy to bank-lending rates. To see this, consider an increase in the nominal rate of interest aimed at offsetting the inflationary consequences of a demand or supply shock. When the central bank adjusts the policy rate in response to asset prices misalignments, the negative deviation of firm profits from their level under flexible prices exerts a direct impact on inflation that counteracts the borrowing cost effect on aggregate supply. In addition, the cost channel implies a direct influence of interest rate changes on firm dividends that dominates the

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[1]The literature on the cost channel has shown that, along with the usual demand-side transmission channel, monetary policy significantly affects the supply-side of the economy through the influence of the nominal rate of interest on firms' costs of production. See, among others, Christiano et al. (1997), Barth and Ramey (2000), Ravenna and Walsh (2006), Chowdhury et al. (2006) and Tillmann (2008).

negative correlation between the output gap and the dividend gap that generally arises in models with nominal rigidities. Together, these mechanisms avoid expectations of higher inflation to become self-fulfilling.

The remainder of the paper is laid out as follows: Section 2 introduces the theoretical setting; Section 3 shows that implementing rules that are exclusively aimed at stabilizing inflation may never ensure determinacy and E-stability in the presence of strong cost-side effects; Section 4 explores the connection between firm profitability and the cost channel, and shows that adjusting the policy rate in response to asset prices misalignments may help at alleviating the problems of dynamic instability highlighted in the previous section. The last section concludes.

2 The Model

The model economy is populated by households, firms and financial intermediaries. Households have preferences defined over a variety of consumption goods, supply labor to monopolistically competitive firms and deposit funds at the financial intermediaries. Firms utilize labor to produce goods and borrow from financial intermediaries to finance the wage bill, which has to be covered before revenues are collected. The decision problems of households and firms follow the standard treatment of Ravenna and Walsh (2006) and are outlined in Appendix A1. This section describes the role of financial intermediaries and reports the log-linearized model economy.

2.1 Financial Intermediation

We assume that financial intermediaries receive deposits M_t^d (remunerated at the gross rate R_t) from households and a cash injection X_t from the monetary authority. Moreover, they supply loans L_t to firms at the (gross) nominal rate R_t^l. These funds are used to finance the wage bill, $W_t N_t$, where W_t and N_t denote the real wage and the labor input, respectively. Following Chowdhury et al. (2006), we allow for varying degrees of interest rate changes to affect firms' cost of borrowing. For simplicity, we assume that this friction can be measured by an increasing function of the nominal rate of interest, $\Psi_t(R_t) \in (0, 1)$, which can be interpreted as a measure of defaults on loans.[2] Moreover, we assume an explicit cost to manage loans, which amounts to $\kappa^l (\geq 0)$ per unit of loan. Intermediaries operate in a competitive environment. Nominal profits in the banking sector are defined as:

$$\Pi^{int} = R_t^l \left[1 - \Psi(R_t) \right] L_t - R_t M_t^d - \kappa^l L_t. \tag{1}$$

The following bank balance sheet condition must hold in every period: $L_t = M_t^d + X_t$. From the maximization of (1) subject to this constraint we obtain a relationship that links deposit and loan rates. In log-linear terms such no-arbitrage condition reads as:[3]

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[2] As discussed by Stiglitz and Weiss (1981) this situation can be rationalized, in the presence of asymmetric information, by the willingness of a firm to invest in risky projects at high levels of the risk-free rate.

[3] Variables without a time subscript are evaluated at their steady state. For a generic variable X_t we denote with $\widehat{x}_t \equiv (X_t - X) X^{-1}$ $\left(\widehat{x}_t^f \equiv \left(X_t^f - X \right) X^{-1} \right)$ the percentage deviation of its value under sticky (flexible) prices from the steady state level. Moreover, log-linear "gap variables" are reported without superscripts, i.e. $x_t \equiv \widehat{x}_t - \widehat{x}_t^f$.

$\widehat{r}_t^l = (1 + \psi)\,\widehat{r}_t$, where $\psi(= \psi_1 - \psi_2)$ captures the elasticity of the contractual interest rate to percentage changes in the policy rate. This results from the combination of two components, $\psi_1 = \left[R\Psi'(R)/1 - \Psi(R) \right]$ and $\psi_2 = \left[\kappa^l / \left(R + \kappa^l \right) \right]$. A negative ψ indicates that a change in the risk-free interest rate is not completely passed through to the lending rate.[4] This is the case when managing costs are too high, and the cost channel is mitigated. Alternatively, Hannan and Berger (1991) attribute this effect to loan price rigidities.[5] When ψ is positive, a rise in the policy rate is even accelerated, so that the lending rate rises by more than one-to-one.[6] This can be viewed as a reduced-form relation based on financial market imperfections stemming from asymmetric information between borrowers and lenders, as advocated by the literature on the financial accelerator (see Bernanke et. al., 1999).

2.2 Log-linear System

The first order conditions characterizing the decisions of households and firms are log-linearized around the steady state. The linearized economy features a *IS* curve and an aggregate short-run aggregate supply (*AS*) schedule:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left(r_t - E_t \pi_{t+1}\right), \tag{2}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa\left(1 + \psi\right) r_t + \kappa\left(\sigma + \eta\right) y_t + \varepsilon_t, \tag{3}$$

where y_t is the output gap, π_t is the rate of inflation, r_t is the nominal interest rate gap and ε_t is a cost-push term that derives from assuming a log-stationary process from the elasticity of substitution in consumption, θ.[7] Moreover, β denotes households' discount factor, σ is the inverse of the elasticity of intertemporal substitution, η is the inverse of the Frisch elasticity of labor supply, while $\kappa = \left(1 - \omega\beta\right)\left(1 - \omega\right)\omega^{-1}$, where $1 - \omega$ is the probability that firms can re-optimize their prices at each given period, as in Calvo (1983).

From households' optimization problem we also retrieve a linearized relationship that describes the evolution of the stock price gap, q_t:

$$q_t = \left(1 - \beta\right) d_t + \beta E_t q_{t+1} - \beta\left(r_t - E_t \pi_{t+1}\right), \tag{4}$$

where d_t is the dividend gap. We assume that profits are fully transferred to the stockholders, so that dividends are $D_t = Y_t - R_t^l W_t N_t$ and the log-linear dividend gap reads as $d_t = \varsigma y_t - \mu r_t$, where $\varsigma = 1 - (\theta - 1)(\sigma + \eta)$ and $\mu = (\theta - 1)(1 + \psi)$. Note that ς is negative for a wide range of plausible parameterizations. The negative relationship between d_t and y_t is a characteristic feature of models with nominal rigidities and it represents the key to explain why adjusting the policy rate in response to movements in the price of assets may harm dynamic stability when only the demand channel of the monetary transmission mechanism is at work.[8] A novel feature in the specification of the dividend

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[4]According to Chowdhury et al. (2006), this is the case for France ($\psi = -0.8$) and Germany ($\psi = -0.04$).

[5]According to their evidence, financial intermediaries acting in imperfectly competitive environments do not fully adapt to changes in the policy rate, so as to increase their profit margins.

[6]Chowdhury et al. (2006) estimate a positive ψ for Italy ($\psi = 0.5$), Canada ($\psi = 0.1$), the UK and US ($\psi = 0.3$).

[7]See Steinsson (2003) and Ireland (2004).

[8]As explained by Carlstrom and Fuerst (2007), other things equal, an increase in the rate of inflation

gap stems from the direct influence of the interest rate gap on d_t. The magnitude of this effect – which is indexed by μ – increases in the degree of pass-through and is paramount to explain why responding to firm profitability may increase the chances to attain determinacy in the presence of strong supply-side effects. Section 4 will explore this point in close detail. We plug the dividend gap into (4) to get:

$$q_t = \beta E_t q_{t+1} + \beta E_t \pi_{t+1} + \epsilon y_t - \xi r_t, \tag{5}$$

where $\epsilon = (1 - \beta)\varsigma$ and $\xi = \beta + (1 - \beta)\mu$. Coefficient ξ suggests that, compared to the baseline setting with no cost channel, the effect of r_t on the asset price gap is reinforced.[9]

3   Determinacy and E-stability under Benchmark Interest Rate Rules

This section explores rational expectations equilibrium (REE) determinacy and stability under adaptive learning (E-stability). To close the model we alternatively consider Taylor-type rules that differ with respect to their timing and the information set available to the policy maker. The central bank may adjust the nominal rate of interest in response to movements in current (or expected) inflation, the output gap and the stock price gap. To build some intuition on the interplay between monetary policy and firm profitability in the presence of cost-side effects, we find instructive to first explore rules whereby the policy maker pursues nominal and real stability (Section 3), while postponing to Section 4 the analysis of rules that account for the dynamics of asset prices.

After substituting a specific policy rule into (2), (3) and (5), the resulting model can be reported as:

$$\boldsymbol{\Gamma}\mathbf{x}_t \;\;=\;\; \boldsymbol{\Phi} + \boldsymbol{\Omega}E_t\mathbf{x}_{t+1} + \boldsymbol{\Xi}\boldsymbol{\varpi}_t, \tag{6}$$

$$\boldsymbol{\varpi}_t \;\;=\;\; \boldsymbol{\rho}\boldsymbol{\varpi}_{t-1} + \boldsymbol{\epsilon}_t, \tag{7}$$

where $\mathbf{x}_t = [\pi_t, y_t, q_t]'$, $\boldsymbol{\varpi}_t$ is a vector shocks, $\boldsymbol{\Gamma}$, $\boldsymbol{\Omega}$, $\boldsymbol{\Phi}$ are matrices of structural parameters. Exogenous variables are assumed to follow a first-order stationary VAR with iid innovations and diagonal covariance matrix. It can be shown that REE uniqueness obtains if and only if the eigenvalues of $\boldsymbol{\Gamma}^{-1}\boldsymbol{\Omega}$ have real parts lying in the unit circle. Moreover, a necessary and sufficient condition for E-stability is that $\mathbf{J}\,(= \boldsymbol{\Gamma}^{-1}\boldsymbol{\Omega} - \mathbf{I}\,)$ has all roots with negative real parts (see Evans and Honkapohja, 2001).[10]

The model (6)-(7) falls within the class considered by McCallum (2007), according to which determinacy is a sufficient (though not necessary) condition for E-stability. Therefore, unique evolutionary stable RE solutions detected in this framework retain the property of E-stability. Moreover, we will also observe cases characterized by both indeterminacy and E-stability. These non-fundamental solutions will be briefly discussed, although most of the analysis will be restricted to the study of the fundamental solu-

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determines a higher marginal cost and thus lower dividends and share prices, so that the overall response to inflation falls as the response to asset prices increases.

[9]Moreover, it is interesting to note that an increase in the elasticity of substitution, θ, exerts a detrimental effect on the asset price gap along two directions: (i) via the output gap (y_t) and (ii) via the interest rate gap (r_t). The second effect is further amplified in the presence of strong cost-side effects ($\psi \gg 0$).

[10]More details on the conditions that ensure determinacy and E-stability are reported in Appendix B.

tions (i.e., MSV-type solutions). In addition, Section 3.1.1 will consider a rule based on expectations of contemporaneous data. It should be noted that under "nowcasting" the model features expectations of the endogenous state variables at both time t and $t+1$, so that (6) is replaced by $\mathbf{\Gamma}^n \mathbf{x}_t = \mathbf{\Phi} + \mathbf{\Lambda}^n E_t \mathbf{x_t} + \mathbf{\Omega}^n E_t \mathbf{x}_{t+1} + \mathbf{\Xi} \boldsymbol{\varpi}_t$, where "$n$" stands for nowcasting. As such, the model does not belong to the class examined by McCallum (2007) and E-instability may even characterize determinate equilibria.

3.1 Contemporaneous Data Rules

We start by assuming a central bank that adjusts the rate of interest in response to movements in the contemporaneous rate of inflation (i.e., $r_t = \chi_\pi \pi_t$, $\chi_\pi > 0$). The following proposition formalizes the conditions for determinacy in connection with the magnitude of cost-side effects. As such, it retains considerable importance for those monetary authorities that are primarily or exclusively concerned with inflation stability.[11]

Proposition 1 *Under the contemporaneous data rule $r_t = \chi_\pi \pi_t$ the following conditions are necessary and sufficient to ensure REE determinacy: i) $\chi_\pi > \overline{\chi}_\pi = 1$; ii) iff $\psi > \frac{\eta}{\sigma}$: $\chi_\pi < \widehat{\chi}_\pi = \frac{\sigma(1-\beta)}{\kappa(\sigma\psi-\eta)}$; iii) iff $\psi > \frac{\eta-\sigma}{2\sigma}$: $\chi_\pi < \widetilde{\chi}_\pi = \frac{2\sigma(1+\beta)+\kappa(\sigma+\eta)}{\kappa(\sigma(1+2\psi)-\eta)}$.* Proof: See Appendix C.

<center>Insert Figure 1 about here</center>

Thus, along with a lower bound to inflation responses ($\overline{\chi}_\pi$), depending on the magnitude of ψ different constraints may arise that bound inflation responses from above. To enhance a visual understanding of Proposition 1, we plot the conditions that ensure E-stability and determinacy, for different values of the pass-through parameter and alternative parameterizations of σ and κ: following Woodford (1999), we set $\sigma = 0.157$ and $\kappa = 0.0235/(\sigma + \eta)$, while McCallum and Nelson (1999) select $\sigma = 1/0.164$ and $\kappa = 0.3/(\sigma + \eta)$. As to the other structural coefficients, we set $\beta = 0.99$, $\eta = 2$ and $\theta = 6$. The left hand panel of Figure 1 shows that a strong degree of pass-through may imply that determinacy is never attained if the central bank acts as a pure inflation targeter. This is due to the intersection of the upper and lower frontiers that determinacy imposes on inflation responses.[12] Given our parameterization, Proposition 1 allows us to compute numerical ranges of the pass-through coefficient that are characterized by different properties in terms of dynamic stability. In fact, equilibrium uniqueness can never be attained for $\psi > \frac{\eta}{\sigma} + \frac{(1-\beta)}{\kappa}(\approx 0.60)$: this is the point where $\widehat{\chi}_\pi$ intersects $\overline{\chi}_\pi$. Otherwise, determinacy is always ensured for $\psi < \frac{\eta-\sigma}{2\sigma}$ (≈ -0.34), as long as $\chi_\pi > \overline{\chi}_\pi = 1$. Between these thresholds we determine two constraints that prevent the central bank from responding too strongly to inflation, $\widehat{\chi}_\pi$ and $\widetilde{\chi}_\pi$: the latter lies beyond the numerical range we consider in Figure 1,[13] while $\widehat{\chi}_\pi$ kicks in for $\psi \gtrsim 0.45$ in the sub-space examined. Such a situation has not been documented by previous studies, which only accounted for $\psi \leq 0$: in this case a moderate response to inflation would always ensure determinacy, as long as the Taylor principle is respected. By contrast, for $\psi > 0$ we need to explore the possibility of introducing additional targets in the policy maker's reaction function.

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[11] The conditions ensuring E-stability are reported in Proposition 2 (Appendix C).

[12] As reflected by Figure 1(b), this is not the case under the calibration proposed by Woodford (1999). However, in the next sections we will see how even under Woodford's calibration the conditions that ensure determinacy and E-stability may be severely affected when alternative rules are implemented.

[13] It can be shown that the locus $\widetilde{\chi}_\pi$ crosses $\widehat{\chi}_\pi$ from below. Thus, given the parameterization we use $\widetilde{\chi}_\pi$ is the relevant upper constraint to inflation responses for $\psi \in (-0.34, 0.33)$.

We first allow for the possibility of reacting to contemporaneous inflation and the output gap: $r_t = \chi_\pi \pi_t + \chi_y y_t$. Note that the analysis in the remainder of the paper will often be complemented by numerical simulations of the model over a parameter sub-space of the policy reaction coefficients.[14] Unless otherwise indicated, each numerical exercise is performed under three different values of the pass-through parameter, $\psi = \{-1, 0, 0.5\}$, so as to appreciate the effects induced by varying intensities through which the cost channel operates.

Insert Figure 2 about here

As displayed by Figure 2(a), ruling out the cost channel returns the condition embodied by the well-known Taylor principle, whereby determinacy is attained as long as $(\kappa(\sigma + \eta))^{-1}(1 - \beta)\chi_y + \chi_\pi > 1$. However, when cost-side effects are at work this principle may no longer be sufficient to ensure determinacy. Under a perfect degree of pass-through the area of indeterminacy expands [see Figure 2(b)]: the minimum bound to inflation responses shifts up along the χ_π axis and increases in χ_y. This situation is analogous to that analyzed by Surico (2008) and Llosa and Tuesta (2009). As the former first pointed out, higher inflation expectations may become self-fulfilling when the cost channel is at work. In fact, a central bank that assigns a positive response to real activity renders the economy more prone to multiple equilibria, as the output gap may not be "negative enough" to offset inflationary pressures.[15] However, it is possible to show that the analysis of Surico (2008) is only valid as long as movements in the policy rate are not amplified by the banking sector. As hinted by Figure 2(c), a higher degree of pass-through ($\psi = 0.5$) implies that reacting exclusively to the rate of inflation never ensures equilibrium uniqueness. Whenever the central bank does not react to the output gap, or the response is too weak $(\chi_y \approx 0)$, a region of indeterminacy can be detected along the χ_π axis, which stems from the intersection between the upper and lower bounds to inflation responses, as reported in Proposition 1. Interestingly, sunspot equilibria in this region are E-stable: this property has not been previously documented in standard New Keynesian models with contemporaneous data rules, as those examined by Bullard and Mitra (2002) and Honkapohja and Mitra (2004).[16] Even when accounting for cost-side effects, Llosa and Tuesta (2009) do not point to any discrepancy between the conditions that ensure E-stability and REE uniqueness under contemporaneous data rules [i.e., a

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[14]In the remainder we will mainly report numerical exercises under the parameterization proposed by McCallum and Nelson (1999). Simulations under alternative calibrations generally deliver similar results. Additional numerical exercises are available, upon request, from the authors.

[15]To provide an intuitive explanation of this statement, consider a situation in which the interest rate increases in response to a supply or demand shock, producing a positive ex-ante real interest rate. By responding to current inflation the central bank may trigger even stronger inflationary pressures through the direct impact of the nominal rate of interest on aggregate supply. In this case the negative output gap induced by the monetary tightening may offset the inflationary pressures arising from the shock. However, an explicit reaction to the output gap may weaken this counter-balancing force, thus rendering the system more prone to indeterminacy.

[16]Honkapohja and Mitra (2004) point out that E-stable sunspots may only occur when agents form expectations at time t for time $t + 1$, while just observing realizations of the endogenous state variables at period $t-1$. In this respect, sunspots that take the form of a martingale difference sequence are always E-unstable. By contrast, sunspots that take the form of a finite state Markov process may be E-stable for some parameterizations of the policy rule. In addition to finite state Markov sunspots, Evans and McGough (2005) show that in the plausible range of responses to the intermediate targets, E-stable sunspots assuming a common factor representation may be detected. As such, these sunspots represent a "threat" to monetary policy, as the public could potentially coordinate on them.

situation analogous to that pictured in Figure 2(b)]. However, a disconnection between determinacy and E-stability is highlighted when allowing for a positive pass-through.[17]

Given the impossibility of attaining a unique equilibrium under strong cost-side effects, the policy maker may need to combine inflation responses with an "appropriate" response to the output gap.[18] To provide some analytical intuition for this result, we find useful to recall the (sufficient and necessary) conditions for determinacy reported by Llosa and Tuesta (2009), suitably adapted to match our notation:[19]

$$(1 - \beta - (1 + \psi)\kappa)(\kappa(\sigma + \eta))^{-1}\chi_y + \chi_\pi > 1, \tag{8}$$

$$2\sigma(1 + \beta) + (1 + \beta + (1 + \psi)\kappa)\chi_y + (\eta + \sigma(1 - 2(1 + \psi)))\kappa\chi_\pi + \kappa(\sigma + \eta) > 0. \tag{9}$$

Condition (8) can be interpreted as a generalization of the Taylor principle. This is affected by the cost channel through the output gap response. Specifically, at standard calibrations the term multiplied by χ_y is negative, due to the presence of $\psi (> -1)$: this argument leads Surico (2008) to conclude that to avoid inducing multiple equilibria the central bank should not respond to output gap movements, as this would weaken the overall response to inflation. However, while condition (8) collapses to $\chi_\pi > 1$ (i.e., the standard Taylor principle, with no role for the cost channel) when $\chi_y = 0$, condition (9) is still affected by ψ. In fact, if we assume $\chi_y = 0$ and $\psi > (\eta - \sigma)(2\sigma)^{-1}$ condition (9) translates into $\chi_\pi < \widetilde{\chi}_\pi$, in agreement with Proposition 1. We should note that $\widetilde{\chi}_\pi$ decreases in ψ, implying that in the event of credit conditions becoming more tight, the maximum response to inflation beyond which determinacy cannot be attained is constrained further.[20] In the limit, credit market distortions may imply that the upper bound becomes so stringent that no determinate outcome is attained (this event occurs whenever $\widetilde{\chi}_\pi$ intersects $\overline{\chi}_\pi = 1$), unless the central bank responds to the output gap, in which case the left hand terms of both (8) and (9) increase in χ_y.

To gain further intuition on why responding to the output gap may be necessary to attain determinacy, it is useful to reparameterize the New Keynesian Phillips curve under $r_t = \chi_\pi \pi_t + \chi_y y_t$:[21]

$$\pi_t = \frac{\beta}{1 - \kappa(1 + \psi)\chi_\pi} E_t \pi_{t+1} + \frac{[(1 + \psi)\chi_y + (\sigma + \eta)]\kappa}{1 - \kappa(1 + \psi)\chi_\pi} y_t. \tag{10}$$

Recall that under strict inflation targeting determinacy may never be attained, no matter the central bank's aggressiveness in stabilizing inflation. To see this, consider an increase in the nominal rate of interest aimed at offsetting the inflationary pressures triggered by a demand or supply shock. The conventional prescription in this case is to generate a

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[17]The reason why this situation occurs is clear after inspecting Proposition 2 (Appendix C), where the thresholds for ψ in the analysis of E-stability do not match with those identified for the conditions that ensure determinacy. Otherwise, no discrepancy emerges when considering a less-than-perfect pass-through.

[18]This principle gains further relevance under a forward looking expectational rule, as it will be shown in Section 3.2.

[19]The conditions reported by Llosa and Tuesta (2009) extend those of Surico (2008) in that they consider no interest rate smoothing and a perfect pass-through between policy and bank-lending rates.

[20]Analogous considerations can be extended to the case of $\psi > (\eta/\sigma)$. In fact, also $\overline{\chi}_\pi$ decreases in ψ.

[21]We set, without loss of generality, $\varepsilon_t = 0$.

negative output gap. As hinted by Surico (2008), a positive χ_y may not allow the output gap to be negative enough. However, this view is crucially based on the assumption of a less than perfect pass-through. When ψ is high enough and $\chi_y = 0$, the borrowing cost effect always dominates the traditional real wage effect operating through the marginal rate of substitution between consumption and leisure, so that expectations of higher inflation become self-fulfilling. By contrast, reacting to y_t helps at counteracting the cost-side effect, reinforcing the impact of the negative output gap on current inflation through the direct influence of the rate of interest on aggregate supply. In fact, whereas reacting to the rate of inflation scales the impact of all the variables on the right hand side of (10) by the term $(1 - \kappa (1 + \psi) \chi_\pi)^{-1}$, responding to real activity avoids the upper bound to materialize. Importantly, the overall impact of y_t on current inflation is amplified by the magnitude of cost-side effects, as it increases in ψ.

3.1.1 Expectations of Current Data in the Policy Function

McCallum (1999) criticizes the use of rules that are not operational, i.e.: (i) rules that are expressed in terms of instrumental variables that can hardly be controlled on a high-frequency basis and (ii) rules that require information that cannot be plausibly possessed by the monetary authority. In this respect, contemporaneous data rules such as those explored in Section 3.1 are not operational. In response to this criticism, it is advisable to inspect policy functions based on the expectations of current data. Bullard and Mitra (2002) and Evans and McGough (2005) show that the analysis of determinacy under nowcasting produces results that fully conform to those observed under contemporaneous data rules. This result extends to the case under scrutiny.[22]

Insert Figures 3 and 4 about here

However, it is interesting to note that contemporaneous data rules and rules featuring nowcasting have rather different implications in terms of E-stability. In fact, Figure 3 shows that under $r_t = \chi_\pi E_t \pi_t$ E-stability may be compromised at rather low values of ψ, at least under the calibration proposed by McCallum and Nelson (1999). Most importantly, we note the existence of unique (locally) stationary but E-unstable equilibria.[23] Analogous evidence occurs when the central bank reacts to both expected current inflation and the output gap: under $r_t = \chi_\pi E_t \pi_t + \chi_y E_t y_t$ E-stability may never be attained for a certain range of χ_y, even when policy and bank-lending rates co-move on a one-to-one basis [see Figure 4(b)]. Thus, implementing a rule based on the expectations of current data in the presence of cost-side effects may seriously affect the chances to obtain E-stable equilibria, even when these are unique.

3.2 Forward Expectations in the Policy Function

Once again, we first consider a central bank that exclusively reacts to the expected rate of inflation: $r_t = \chi_\pi E_t \pi_{t+1}$. As in the case of a contemporaneous data rule, an upper bound to inflation responses can be detected, which prevents the monetary authority from being too aggressive.[24] Most importantly Figure 5, which reports the equilibrium properties

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[22]In addition, it is possible to prove that the conditions for E-stability under nowcasting are equivalent to those under the forward-looking rule, as we will see in Section 3.2.

[23]Moreover, as in Evans and McGough (2005) no indeterminate but E-stable equilibria are detected.

[24]Proposition 2 in Appendix C formalizes this result.

of the system in the $\{\psi, \chi_\pi\}$ sub-space, shows that a unique REE can be obtained just at low values of the pass-through. Compared to a contemporaneous data rule, forward expectations in the Taylor rule make the economy more prone to indeterminacy.

Insert Figure 5 about here

To overcome the limitations of a policy function that is exclusively focused on the stabilization of expected price changes, we next implement $r_t = \chi_\pi E_t \pi_{t+1} + \chi_y E_t y_{t+1}$.

Insert Figure 6 about here

Figures 6(b) and 6(c) confirm that reacting to both the expected inflation rate and the output gap is advisable to attain determinacy, at least when the cost channel "matters". The intuition underlying this result develops along the same lines we have detailed in Section 3.1 and rests on the observation that a strong pass-through may induce the upper and lower frontiers to intersect.[25]

Moreover, the region of indeterminacy on the sub-space considered – most of which emerges in correspondence with modest values of the response to the output gap – is greater than that obtained under the contemporaneous data rule. To gain some intuition on why forward looking rules make the system more prone to equilibrium multiplicity, it is useful to re-parameterize the New Keynesian Phillips curve under $r_t = \chi_\pi E_t \pi_{t+1}$:

$$\pi_t = \left[\beta + \kappa \left(1 + \psi\right) \chi_\pi\right] E_t \pi_{t+1} + \kappa \left(\sigma + \eta\right) y_t. \tag{11}$$

Note that responding to the expected rate of inflation reinforces the feedback from $E_t \pi_{t+1}$ to π_t – thus increasing the chances that expectations of higher inflation become self-fulfilling in the face of inflationary shocks – while leaving the impact of the forcing variable unaffected. Therefore, a shock that raises the nominal rate of interest may generate inflationary pressures that can hardly be offset by the negative output gap, even if $\chi_y = 0$. Importantly, such pressures increase in the degree of pass-through. By contrast, undesirable outcomes are less likely to occur under the contemporaneous data rule, $r_t = \chi_\pi \pi_t$, as in this case χ_π scales the impact of both y_t and $E_t \pi_{t+1}$ on current inflation. This can be checked by setting $\chi_y = 0$ in equation (10).

4 Asset Prices, the Cost Channel and Determinacy

So far we have shown that allowing for an amplification of movements in the policy rate on bank-lending rates has non-negligible implications for equilibrium dynamics: unlike the case of a less than perfect pass-through, reacting to both inflation and the output gap may be necessary to avoid indeterminacy. This is particularly important when the monetary authority reacts to forward expectations, as in this case the system is more sensitive to feedback effects from expected to current inflation. Concurrently, credit market distortions generally increase the chances to observe learnable sunspots, at least under rules based on contemporaneous data and forward expectations, while expectations of current data in the Taylor rule make the system more prone to determinate but E-unstable equilibria.

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[25]It is also possible to extend this reasoning to rules featuring an explicit response to the contemporaneous or the expected output gap.

This section highlights important effects emanating from the interplay between credit market distortions and firm profitability. We explore the implications of a central bank that, along with responding to (current or expected) inflation and the output gap, displays some concern for fluctuations in stock prices. We abstract from normative considerations on why the policy maker may want to react to asset prices, while merely relying on the evidence that supports this view (see, among others, Rigobon and Sack, 2003).

The general wisdom is that setting the policy rate in response to asset prices misalignments renders the system more prone to indeterminacy.[26] Carlstrom and Fuerst (2007) have explored the implications of responding to asset prices for equilibrium determinacy.[27] Their key insight is that in the face of inflationary shocks that lower firm profits (and asset prices) an interest rate rule reacting to stock prices adds a negative force to the overall response to inflation. If the share price response is large enough, indeterminacy cannot be avoided, as the Taylor principle is violated. It is important to stress that Carlstrom and Fuerst (2007) consider a standard situation in which responding to asset prices only affects the lower bound to inflation responses through the conventional demand channel of the monetary transmission mechanism. However, we have noted at different stages of the analysis that in the presence of relevant cost-side effects the upper constraint may represent a reason of concern.

Let us consider the following rule with forward expectations:[28]

$$r_t = \chi_\pi E_t \pi_{t+1} + \chi_y E_t y_{t+1} + \chi_q E_t q_{t+1}. \tag{12}$$

Insert Figure 7 about here

Figure 7 graphs the conditions that ensure determinacy over the sub-space $\{\chi_\pi, \chi_y\}$ and for $\psi = \{-1, 0, 0.5\}$. In each panel we consider different values of χ_q. Figure 7(a) accounts for the situation examined by Carlstrom and Fuerst (2007) and clearly shows that there are no benefits from reacting to asset prices, as the area of indeterminacy expands as χ_q increases. In the absence of cost-side effects, responding to firm profitability has no other implication but decreasing the chances to attain a unique equilibrium. Otherwise, when the cost channel is accounted for the policy maker needs to select combinations of χ_π and χ_y that fall in the region of determinate equilibria between the lower and the upper constraint to inflation responses. Section 3 has shown that the upper frontier may become an issue of concern for high values of the pass-through coefficient. Figures 7(b) and 7(c) clearly show that a positive reaction to asset prices raises both the lower and the upper bound. However, while increasing χ_q only exerts a negligible impact on the

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[26] A long-standing debate concerning the role and scope of central banks to stabilize asset prices has developed since the contributions of Bernanke and Gertler (1999, 2001) and Genberg et al. (2000). In connection with problems of dynamic stability induced by Taylor rules that respond to share prices, refer to Bullard and Schaling (2002) and Carlstrom and Fuerst (2007). More recently, Pfajfar and Santoro (2011) have shown that adjusting the policy rate in response to asset prices growth does not harm dynamic stability and may promote determinacy by inducing interest-rate inertia.

[27] We should stress that Carlstrom and Fuerst (2007) explore this situation in the presence of wage rigidity and different timings for money demand. In our setting such extensions are bound to be of marginal importance. In fact, unlike the sticky price model, in a sticky wage model profits will fall with positive interest rate innovations. Thus, sticky wages induce an effect on firm profits that works in the same direction as the cost channel. Concurrently, under typical calibrations according to which wages and prices are rather sticky, the money demand timing is almost irrelevant to the stability properties of the New Keynesian model explored by Carlstrom and Fuerst (2007).

[28] It is possible to show that analogous principles apply to contemporaneous data rules or under now-casting.

bottom frontier, shifts in the upper bound are by far more important. In fact, even a modest response to stock prices avoids the two frontiers to intersect. To provide an intuition of why the lower bound shifts upward, it is useful to explore the effect induced by an inflationary shock. In the model with the cost channel each percentage point of permanently higher inflation implies a permanent change in the dividend gap of:[29]

$$\frac{\mathrm{d}d}{\mathrm{d}\pi} = -\frac{1+\psi}{\sigma+\eta} - \frac{(1-\beta)\left[(\sigma+\eta)(\theta-1)-1\right]}{\kappa(\sigma+\eta)}, \tag{13}$$

which can be shown to be negative for a wide range of parameter values and to increase in ψ (in absolute value). Thus, as predicted by Carlstrom and Fuerst (2007), a trade-off between inflation and asset prices stabilization arises in the perspective of attaining a unique REE. Note that an analogous trade-off also emerges between inflation and output stabilization, as hinted by (8). In fact, a one percent permanent increase in inflation induces a (negative) change in the output gap of $[1-\beta-(1+\psi)\kappa]\left[\kappa(\sigma+\eta)\right]^{-1}$ percentage points (as opposed to $(1-\beta)\left[\kappa(\sigma+\eta)\right]^{-1}$ with no cost channel):[30] this translates into an upward sloping lower bound in the $\{\chi_\pi, \chi_y\}$ sub-space [see Figures 2(b,c)].

Nonetheless, the cost-channel urges us to account for the upper constraint to χ_π as well. In this respect, adjusting the rate of interest in response to asset prices misalignments has similar effects as responding to real activity: a positive χ_q channels a negative force on aggregate supply that outweighs the borrowing effect on inflation dynamics, thus avoiding expectations of higher inflation to become self-fulfilling. Importantly, such a negative effect is amplified by the direct impact of interest rate changes on firm profits: to see this, recall that the elasticity of the dividend gap to the nominal rate of interest, $\mu = (\theta-1)(1+\psi)$, increases in the intensity of cost-side effects. Overall, this translates into an upward shift in the upper bound to χ_π, so that any intersection with the bottom frontier is avoided.

The main point of departure from the result of Carlstrom and Fuerst (2007) lies in the role of the upper bound to inflation responses and its relevance in the presence of strong cost-side effects. It is true that the lower constraint becomes more stringent as χ_q increases, and more so when the cost-channel is accounted for [to see this, consider the term $-(1+\psi)(\sigma+\eta)^{-1}$ in (13), which is null for $\psi = -1$]. However, when cost-side effects are high enough there are considerably higher gains from responding to asset prices – at least in terms of increased chances to attain a determinate equilibrium[31] – in that the upper constraint is relaxed.

Thus, in the presence of strong cost-side effects that would otherwise prevent the attainment of REE equilibrium uniqueness, it is advisable to combine inflation responses with an explicit reaction to the output gap and/or asset prices. This allows the central bank to turn the cost channel at its own advantage, through the direct impact of the nominal rate of interest on aggregate supply. In this respect, reacting to asset prices proves to be quite efficient at avoiding the upper constraint to inflation responses to

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[29]This elasticity can be retrieved by setting, for a generic variable x_t, $E_t x_{t+1} = x_t = x$.

[30]While in the baseline scenario (i.e., under $\psi = -1$) any increment in the steady-state rate of inflation leads to a higher output gap ($\mathrm{d}y/\mathrm{d}\pi > 0$), under the cost channel we may assist to a permanent reduction in the output gap ($\mathrm{d}y/\mathrm{d}\pi < 0$), with the magnitude of this response increasing (in absolute value) in ψ.

[31]It could be noted that reacting to the stock price gap requires knowledge of asset prices under flexible goods prices. These are typically unobservable. However, it is important to stress that the conditions for determinacy and E-stability would not be qualitatively affected even if we were to consider a linearized model with variables expressed in percentage deviation from their steady state.

materialize, as it exploits the direct influence of interest rate changes on firm profitability.

5 Concluding Remarks

We have extended the cost channel framework of Ravenna and Walsh (2006) in two main directions: first, following Chowdhury et al. (2006), we allow for the introduction of varying degrees of interest rate changes to affect firms' cost of borrowing; second, we consider the direct influence of credit market distortions on firm profitability and stock price dynamics. The standard conditions ensuring REE uniqueness and E-stability are significantly altered in the presence of strong cost-side effects, i.e. when movements in the policy rate are amplified by the lending rate.

When changes in the policy rate are accelerated by the loan rate, conventional inflation targeting policies may not be effective at ensuring determinacy, regardless of the timing of the policy rule and the information set available to the policy maker. In contrast to much of the existing literature we show that, along with reacting to the rate of inflation, it may be necessary to adjust the policy rate in response to movements in real activity. Although responding to the output gap makes the lower bound to inflation responses more stringent, it produces greater benefits by relaxing the upper constraint. Along the same lines, we show that the policy maker may increase the chances to attain determinacy and E-stability when the cost channel matters, while reacting to asset prices. As in the case considered by Carlstrom and Fuerst (2007), firm profitability reacts negatively in response to inflationary shocks, and more so in the presence cost-side effects. In otherwise standard frameworks this effect reduces the overall response to inflation, thus decreasing the chances to attain determinacy. However, along with inducing this second-order effect, reacting to asset prices counteracts the borrowing cost effect operating in the model with the cost channel, ultimately avoiding the intersection between the frontiers that bound inflation responses from above and below.

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Figures

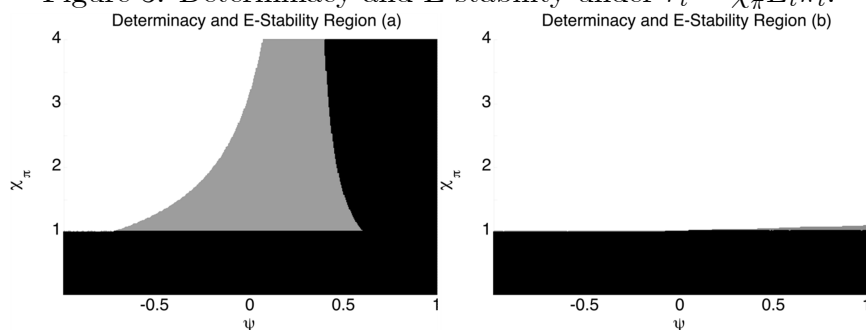## Figure 1: Determinacy and E-stability under $r_t = \chi_\pi \pi_t$.
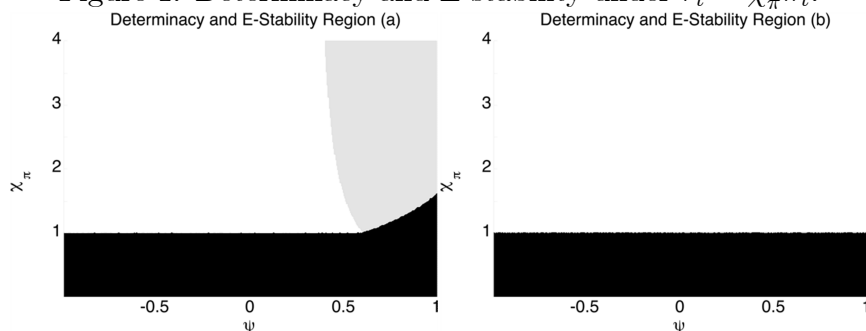
Notes. Figure 1(a) is obtained under the parameterization for σ and κ proposed by McCallum and Nelson (1999), while 1(b) is based on the parameters of Woodford (1999). Black: indeterminacy and E-instability; light grey: indeterminacy and E-stability; white: determinacy and E-stability.

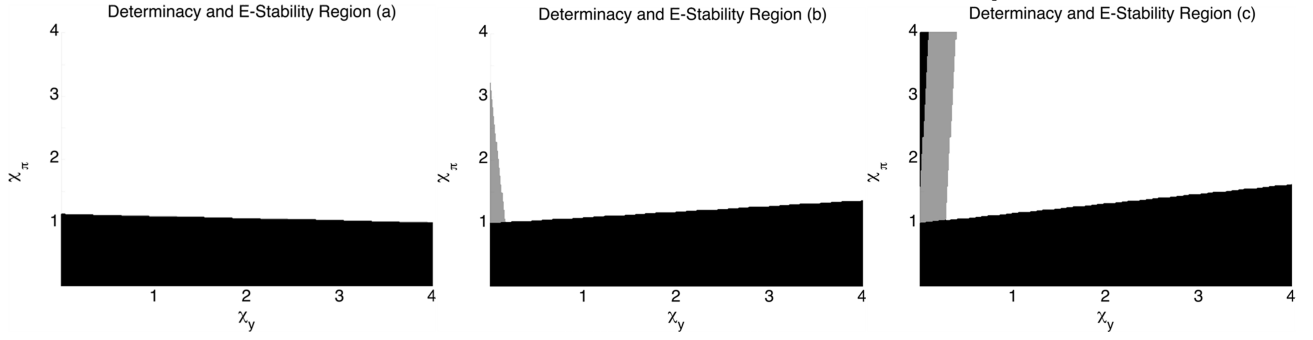## Figure 2: Determinacy and E-stability under $r_t = \chi_\pi \pi_t + \chi_y y_t$.
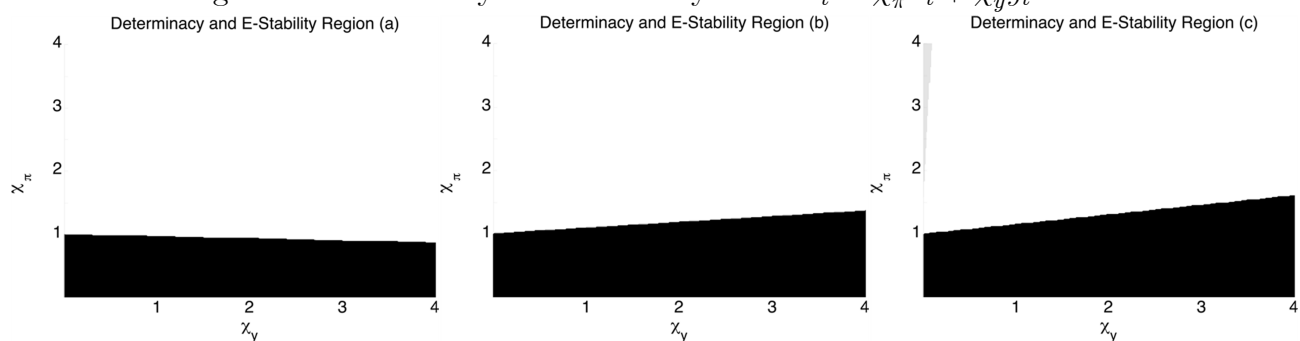
Notes. ψ is alternatively set to -1(a), 0(b), 0.5(c). Black: indeterminacy and E-instability; light grey: indeterminacy and E-stability; white: determinacy and E-stability.

Figure 3: Determinacy and E-stability under $r_t = \chi_\pi E_t \pi_t$.

Notes. Figure 3(a) is obtained under the parameterization for σ and κ proposed by McCallum and Nelson (1999), while 3(b) is based on the parameters of Woodford (1999). Black: indeterminacy and E-instability; dark grey: determinacy and E-instability; white: determinacy and E-stability.

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Figure 4: Determinacy and E-stability under $r_t = \chi_\pi E_t \pi_t + \chi_y E_t y_t$.

Notes. ψ is alternatively set to -1(a), 0(b), 0.5(c). Black: indeterminacy and E-instability; dark grey: determinacy and E-instability; white: determinacy and E-stability.

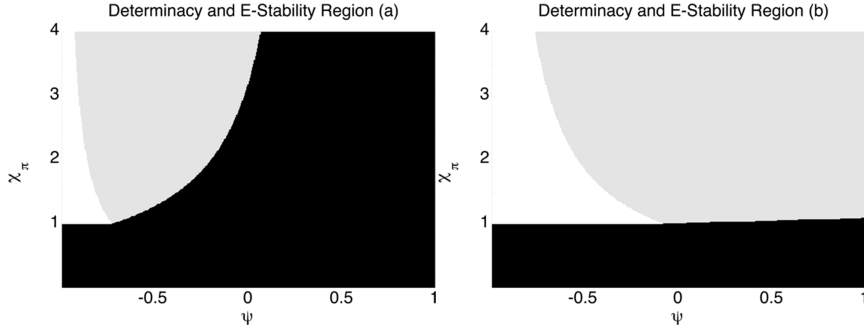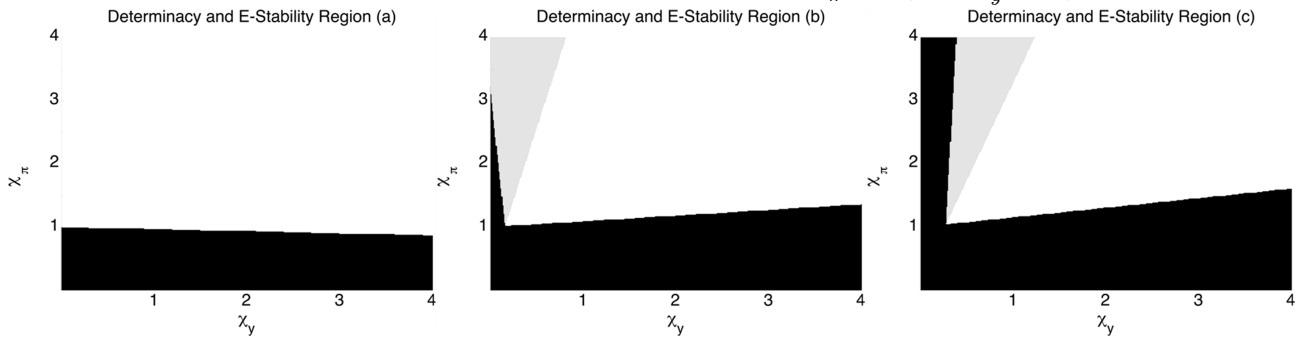Figure 5: Determinacy and E-stability under $r_t = \chi_\pi E_t \pi_{t+1}$.

Notes. Figure 5(a) is obtained under the parameterization for σ and κ proposed by Mc-Callum and Nelson (1999), while 5(b) is based on the parameters of Woodford (1999). Black: indeterminacy and E-instability; light grey: indeterminacy and E-stability; white: determinacy and E-stability.

Figure 6: Determinacy and E-stability under $r_t = \chi_\pi E_t \pi_{t+1} + \chi_y E_t y_{t+1}$.

Notes. ψ is alternatively set to -1(a), 0(b), 0.5(c). Black: indeterminacy and E-instability; light grey: indeterminacy and E-stability; white: determinacy and E-stability.

Figure 7: Determinacy under $r_t = \chi_\pi E_t \pi_{t+1} + \chi_y E_t y_{t+1} + \chi_q E_t q_{t+1}$.
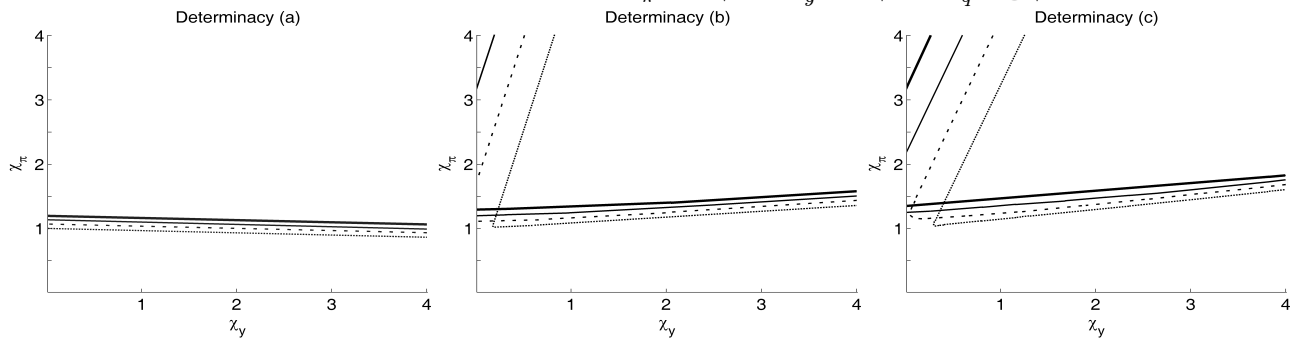
Notes: ψ is alternatively set to -1(a), 0(b), 0.5(c). In each panel, χ_q is alternatively set to 0 (dotted line), 0.05 (dashed line), 0.1 (thin continuous line) and 0.15 (thick continuous line).

Appendix A: The Model

Appendix A1: Decision Problems of Households and Firms

Households

Households have preferences defined over a composite consumption good, C_t, and leisure, $1 - N_t$. They maximize the expected present discounted value of their utility:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{H_{t+i} C_{t+i}^{1-\sigma}}{1 - \sigma} - \frac{N_{t+i}^{1+\eta}}{1 + \eta} \right], \tag{14}$$

where β is the intertemporal discount factor, $H_t = \exp(h_t)$ is a taste shock,[32] σ denotes the inverse of the elasticity of intertemporal substitution and η is the inverse of the Frisch elasticity of labor supply. The consumption composite is:

$$C_t = \left[\int_0^1 (C_{jt})^{1 - \frac{1}{\theta_t}} \, dj \right]^{\frac{\theta_t}{\theta_t - 1}}, \tag{15}$$

where C_{jt} is the consumption of the good produced by firm j. Following Steinsson (2003) and Ireland (2004), we allow for a log-stationary stochastic process to describe the evolution of the elasticity of substitution in demand (θ_t).

As to the budget constraint, we follow the setup of Ravenna and Walsh (2006) and assume that households, whose labor supply is remunerated at the real wage W_t, enter period t with cash holdings M_t. Before households enter the goods market, they deposit funds M_t^d at financial intermediaries, which in turn remunerate them at the gross interest $R_t(= 1 + i_t)$. Consumption expenditures are restricted by the following liquidity constraint:

$$P_t C_t \leq M_t - M_t^d + P_t W_t N_t. \tag{16}$$

We also assume that households enter period t with A_{t-1} shares of stock that sell at price Q_t and pay dividend D_t. As we deal with a representative-agent setting, households' investment does not affect their consumption through feedback effects from asset prices. We make this choice so as to enhance comparability between our results and those of Carlstrom and Fuerst (2007).

The intertemporal budget constraint reads as:

$$P_t C_t + P_t Q_t A_t + M_{t+1} + M_t^d \leq M_t + P_t A_t D_t + R_t M_t^d + P_t Q_t A_{t-1} + P_t W_t N_t. \tag{17}$$

Maximizing (14) subject to (16), and (19) leads to a set of first-order conditions that can be re-arranged to obtain:

$$\frac{N_t^{\eta}}{H_t C_t^{-\sigma}} = W_t, \tag{18}$$

$$H_t C_t^{-\sigma} = \beta E_t \left(\frac{R_t H_{t+1} C_{t+1}^{-\sigma}}{1 + \pi_{t+1}} \right), \tag{19}$$

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[32] A taste shock is introduced to account for the competing effects of supply and demand side innovations on the frictionless state of the economy.

$$H_t C_t^{-\sigma} (Q_t - D_t) = \beta E_t \left(H_{t+1} C_{t+1}^{-\sigma} Q_{t+1} \right), \tag{20}$$

where π_t denotes the rate of inflation. Equilibrium in the goods market requires $Y_t = C_t$. Note that equations (19) and (20) imply the usual no-arbitrage condition:

$$Q_t - D_t = \beta E_t \left(\frac{1 + \pi_{t+1}}{R_t} \right) E_t Q_{t+1} + \zeta_t, \tag{21}$$

where, following Smets and Wouters (2003), the term ζ_t accounts for the risk implied by the covariance between the stochastic discount factor and the nominal gross rate of return on stocks.

Firms

Following Ravenna and Walsh (2006), we assume that a generic firm j borrows an amount $W_t N_{jt}$ from intermediaries at the gross nominal rate R_t^l. It is assumed that firms are completely rationed on the equity market: although they could in principle issue equity to finance their production, this option is a priori ruled out, due to the possibility that new equity issues would be subject to adverse selection phenomena (see Myers and Majluf, 1984), thus resulting as too costly. At a given share price, only overvalued firms are willing to sell their shares. As potential shareholders anticipate this fact, no trade occurs on the equity market. Under these conditions, the announcement of an equity issue is generally interpreted as bad news by investors and, in extreme situations, the stock market becomes a market for lemons.[33]

As to price-setting, we follow Calvo (1983). The probability that a firm optimally adjusts its price in each period is $1 - \omega$. The remaining fraction of firms (ω) leave their price unchanged. If a firm sets its price at time t, it will do so by maximizing expected profits subject to the demand function for its good and a constant return to scale production technology $Y_{jt} = Z_t N_{jt}$, where Y_{jt} denotes firm-specific output and Z_t is a stochastic aggregate productivity factor. The cost minimization problem reads as:

$$\min_{N_{jt}} R_t^l W_t N_{jt} + \Phi_t \left[Y_{jt} - Z_t N_{jt} \right]. \tag{22}$$

The real marginal cost resulting from the cost minimization problem is $\Phi_t = R_t^l S_t$, where $S_t = W_t / Z_t$.

Appendix A2: Log-linearized System under Flexible Prices

We report the following set of log-linearized relationships describing the evolution of the nominal rate of interest, output, profits, and stock prices in the absence of nominal

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[33] Asymmetric information only affects the equity market. As to the credit market, it is assumed that the banking sector has perfect information, being capable to discriminate firms on the basis of their financial structure.

rigidities:

$$\widehat{R}_t^f = \sigma \left(E_t \widehat{y}_{t+1}^f - \widehat{y}_t^f + \frac{1}{\sigma} \left(h_t - E_t h_{t+1} \right) \right), \tag{23}$$

$$\widehat{y}_t^f = \frac{1}{\sigma + \eta} \left[\left(1 + \eta \right) z_t - \widehat{R}_t^f + h_t \right], \tag{24}$$

$$\widehat{d}_t^f = \widehat{y}_t^f, \tag{25}$$

$$\widehat{q}_t^f = \left(1 - \beta \right) \widehat{d}_t^f + \beta \left(E_t \widehat{q}_{t+1}^f - \widehat{R}_t^f \right). \tag{26}$$

Appendix B: Determinacy and E-stability

Let us write the model under the following state space form, after implementing a specific interest rate rule:

$$\mathbf{\Gamma} \mathbf{x}_t = \mathbf{\Phi} + \mathbf{\Omega} E_t \mathbf{x}_{t+1} + \mathbf{\Xi} \boldsymbol{\varpi}_t,$$
$$\boldsymbol{\varpi}_t = \boldsymbol{\rho} \boldsymbol{\varpi}_{t-1} + \boldsymbol{\epsilon}_t$$

where $\mathbf{x}_t = [\pi_t, y_t, q_t]'$ and $\boldsymbol{\varpi}_t$ is a vector of shocks. Exogenous variables are assumed to follow a first-order stationary VAR with *iid* innovations and diagonal covariance matrix. In the absence of any inertial effect in the model economy and the policy reaction function, REE uniqueness is simply attained if the matrix $\mathbf{\Gamma}^{-1} \mathbf{\Omega}$ has real parts of eigenvalues lying inside the unit circle (see Blanchard and Kahn, 1980).

To study the stability of the REE under adaptive learning, we follow Evans and Honkapohja (2001, Chapter 10) and assume that agents utilize a perceived law of motion (PLM) for $\mathbf{x}_t$ that corresponds to the minimal state variable (MSV) solution to the system:

$$\mathbf{x}_t = \mathbf{\Upsilon} + \mathbf{\Pi} \boldsymbol{\varpi}_t.$$

Agents are assumed to form expectations by relying on the perceived law of motion (PLM), $E_t \mathbf{x}_{t+1} = \widehat{\mathbf{\Upsilon}} + \widehat{\mathbf{\Pi}} \boldsymbol{\rho} \boldsymbol{\varpi}_t$. Consequently, the actual law of motion (ALM) reads as follows:

$$\mathbf{x}_t = \mathbf{\Gamma}^{-1} \mathbf{\Phi} + \mathbf{\Gamma}^{-1} \mathbf{\Omega} \left(\mathbf{\Upsilon} + \mathbf{\Pi} \boldsymbol{\rho} \boldsymbol{\varpi}_t \right) + \mathbf{\Gamma}^{-1} \mathbf{\Xi} \boldsymbol{\varpi}_t.$$

The T-mapping from the PLM to the ALM is:

$$T(\widehat{\mathbf{\Pi}}) = \mathbf{\Gamma}^{-1} \left(\mathbf{\Omega} \mathbf{\Pi} \boldsymbol{\rho} + \mathbf{\Xi} \right),$$
$$T(\widehat{\mathbf{\Upsilon}}) = \mathbf{\Gamma}^{-1} \left(\mathbf{\Phi} + \mathbf{\Omega} \mathbf{\Upsilon} \right).$$

According to Evans and Honkapohja (2001), the MSV-REE is E-stable when the following matrices, evaluated at the REE, have eigenvalues with real parts lower than 1:

$$DT_{\Pi}(\mathbf{\Pi}) = \boldsymbol{\rho}' \otimes \mathbf{\Gamma}^{-1} \mathbf{\Omega},$$
$$DT_{\Upsilon}(\mathbf{\Upsilon}) = \mathbf{\Gamma}^{-1} \mathbf{\Omega}.$$

Since $\boldsymbol{\rho}'$ has all roots with real parts less than 1, a necessary and sufficient condition for E-stability of the MSV-REE is that $\mathbf{J} (= \mathbf{\Gamma}^{-1} \mathbf{\Omega} - \mathbf{I})$ has all roots with negative real parts.

Appendix C: Proofs and Additional Propositions

Determinacy under a Contemporaneous Data Policy Rule

We assume a general structure for the interest rate rule:

$$r_t = \chi_\pi \pi_t + \chi_y y_t + \chi_q q_t. \tag{27}$$

To analyze determinacy and stability under adaptive learning, we report the linearized economy in compact form:

$$\mathbf{\Gamma}_c \mathbf{x}_t = \mathbf{\Phi}_c + \mathbf{\Omega}_c E_t \mathbf{x}_{t+1} + \mathbf{\Xi}_c \boldsymbol{\varpi}_t.$$

Proposition 1

Proof. If we set $\chi_y = \chi_q = 0$, the discount factor β turns out to be one of the three eigenvalues of $\mathbf{\Gamma}_c^{-1} \mathbf{\Omega}_c$. Under this setting, the NK Phillips curve and the IS curve constitute an autonomous system, in which the matrix of structural parameters associated with the forward looking vector is the cofactor:

$$\mathbf{J}_{c33} = \begin{bmatrix} \frac{\sigma\beta + \kappa\sigma + \kappa\eta}{\sigma + \kappa\eta\chi_\pi - \kappa\sigma\psi\chi_\pi} & \frac{\kappa\sigma^2 + \kappa\eta\sigma}{\sigma + \kappa\eta\chi_\pi - \kappa\sigma\psi\chi_\pi} \\ -\frac{\kappa\chi_\pi - 1 + \kappa\psi\chi_\pi + \beta\chi_\pi}{\sigma + \kappa\eta\chi_\pi - \kappa\sigma\psi\chi_\pi} & -\frac{\kappa\sigma\chi_\pi - \sigma + \kappa\sigma\psi\chi_\pi}{\sigma + \kappa\eta\chi_\pi - \kappa\sigma\psi\chi_\pi} \end{bmatrix}.$$

The necessary and sufficient conditions ensuring determinacy are as follows: $|B_c| < 1$, and $|A_c| < 1 + B_c$, where A_c and B_c are the coefficients of the characteristic polynomial of $\mathbf{J}_{c33}$ (i.e., $\lambda^2 + A_c\lambda + B_c = 0$):

$$B_c \equiv \frac{\beta\sigma}{\sigma + \kappa\chi_\pi (\eta - \sigma\psi)}, \tag{28}$$

$$A_c \equiv \frac{\kappa\sigma\chi_\pi (1 + \psi) - \sigma (1 + \beta) - \kappa (\sigma + \eta)}{\sigma + \kappa\chi_\pi (\eta - \sigma\psi)}. \tag{29}$$

Let us first focus on $|B_c| < 1$, which translates into:

$$\frac{\beta\sigma}{\sigma + \kappa\chi_\pi (\eta - \sigma\psi)} < 1 \tag{30}$$

and

$$\frac{\beta\sigma}{\sigma + \kappa\chi_\pi (\eta - \sigma\psi)} > -1. \tag{31}$$

We start from manipulating (30), multiplying both sides by $\sigma + \kappa\chi_\pi (\eta - \sigma\psi)$: this term is always positive for $\psi < \frac{\eta}{\sigma}$. By contrast, when $\psi > \frac{\eta}{\sigma}$ we need to introduce a restriction on κ to ensure that $\sigma + \kappa\chi_\pi (\eta - \sigma\psi)$ is positive, namely $0 < \kappa < \frac{\sigma}{\chi_\pi(\sigma\psi - \eta)}$.[34] To derive an explicit condition for χ_π we divide both sides of the resulting inequality by $\kappa (\eta - \sigma\psi)$. This term is negative for $\sigma\psi > \eta$: in this case we end up with $\chi_\pi < \frac{\sigma(\beta-1)}{\kappa(\eta-\sigma\psi)}$. Otherwise, when $\psi < \frac{\eta}{\sigma}$ we obtain $\chi_\pi > \frac{\sigma(\beta-1)}{\kappa(\eta-\sigma\psi)}$. Note that the term on the RHS of the last inequality

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[34] Note that this condition holds under different plausible parameterizations and is always satisfied under the set of calibrated parameters we consider in the paper.

is always negative. We now consider (31). Again, to isolate χ_π on the LHS we need to divide both sides of the inequality by $\kappa\left(\eta-\sigma\psi\right)$. Thus, if $\psi>\frac{\eta}{\sigma}$, we obtain:

$$\chi_\pi<\frac{\sigma\left(\beta+1\right)}{\kappa\left(\sigma\psi-\eta\right)}. \tag{32}$$

When $\psi>\frac{\eta}{\sigma}$ the term $\frac{\sigma(\beta+1)}{\kappa(\sigma\psi-\eta)}$ is always positive under the restriction characterizing the baseline parameterization. In the alternative case (i.e., $\psi<\frac{\eta}{\sigma}$), we obtain $\chi_\pi>\frac{\sigma(\beta+1)}{\kappa(\sigma\psi-\eta)}$: as the term on the RHS of the last inequality is always negative, this condition is nested in $\chi_\pi>1$. Finally, we turn our attention to the second condition for determinacy, $|A_c|<1+B_c$, which translates into:

$$\frac{\kappa\sigma\chi_\pi\left(1+\psi\right)-\sigma\left(1+\beta\right)-\kappa\left(\sigma+\eta\right)}{\sigma+\kappa\chi_\pi\left(\eta-\sigma\psi\right)} \;<\; 1+\frac{\beta\sigma}{\sigma+\kappa\chi_\pi\left(\eta-\sigma\psi\right)}, \tag{33}$$

$$\frac{\kappa\sigma\chi_\pi\left(1+\psi\right)-\sigma\left(1+\beta\right)-\kappa\left(\sigma+\eta\right)}{\sigma+\kappa\chi_\pi\left(\eta-\sigma\psi\right)} \;>\; -1-\frac{\beta\sigma}{\sigma+\kappa\chi_\pi\left(\eta-\sigma\psi\right)}. \tag{34}$$

Once again, we assume that the restriction on κ holds true: this allows us to write (33) as $\chi_\pi\kappa\left(\sigma\left(1+2\psi\right)-\eta\right)<2\sigma\left(1+\beta\right)+\kappa\left(\sigma+\eta\right)$. Thus, we have to evaluate the sign of $\kappa\left(\sigma\left(1+2\psi\right)-\eta\right)$: this turns out to be always positive iff $\psi<\frac{\eta}{\sigma}$. Otherwise, for $\psi\gtrless\frac{\eta-\sigma}{2\sigma}$ the relevant conditions are $\chi_\pi\gtrless\frac{2\sigma(1+\beta)+\kappa(\sigma+\eta)}{\kappa(\sigma(1+2\psi)-\eta)}$. However, for $\psi<\frac{\eta-\sigma}{2\sigma}$ the term $\frac{2\sigma(1+\beta)+\kappa(\sigma+\eta)}{\kappa(\sigma(1+2\psi)-\eta)}$ is negative, so that the resulting condition is nested in $\chi_\pi>1$. Finally, we consider (34). Algebraic manipulations similar to those followed for (33) lead us to show that the only relevant condition for determinacy is $\chi_\pi>1$. ∎

Corollary 1

The conditions reported in Proposition 1 can be re-stated to determine the following critical values of the pass-through coefficient:

1. For $\psi<\frac{\eta-\sigma}{2\sigma}$ the system is always determinate as long as $\chi_\pi>\overline{\chi}_\pi=1$;

2. For $\frac{\eta-\sigma}{2\sigma}\leq\psi<\frac{\sigma(1-\beta)+\left(\kappa\eta\sigma^{-1}+\kappa+3\beta+1\right)\eta}{\kappa\sigma+4\sigma\beta+\kappa\eta}$ the response coefficient to inflation has to lie in the region between the locus $\widetilde{\chi}_\pi$ and the bottom limit represented by $\overline{\chi}_\pi=1$;

3. For $\frac{\sigma(1-\beta)+\left(\kappa\eta\sigma^{-1}+\kappa+3\beta+1\right)\eta}{\kappa\sigma+4\sigma\beta+\kappa\eta}\leq\psi<\frac{\eta}{\sigma}+\frac{(1-\beta)}{\kappa}$ the response coefficient to inflation has to lie in the region between the locus $\widehat{\chi}_\pi$ and the bottom limit represented by $\overline{\chi}_\pi=1$;

4. For $\psi\geq\frac{\eta}{\sigma}+\frac{(1-\beta)}{\kappa}$ determinacy is never attained.

Proof. Note that $\overline{\chi}_\pi$ always represents the minimum response threshold on the relevant interval for the pass-through. Alternatively, $\widetilde{\chi}_\pi$ and $\hat{\chi}_\pi$ represent the maximum response thresholds. We examine the conditions reported in Proposition 1 over the $\{\chi_\pi,\psi\}$ sub-space. These can be written as $\chi_\pi\lessgtr f\left(\psi^{-1}\right)$ and generally behave as hyperbolae in the relevant space. As we search for a maximum response threshold, we are interested in those functions lying on the RHS of the asymptote to each curve. In this region all thresholds are strictly decreasing functions of the pass-through parameter. We should first note that $\frac{\eta-\sigma}{2\sigma}<\frac{\eta}{\sigma}$. Thus $\widetilde{\chi}_\pi$ will be first binding from the left. For $\psi\geq\frac{\eta}{\sigma}$ two conditions need to

be fulfilled. First, for $\psi > \frac{\eta}{\sigma}$ we can easily check that $\frac{(\beta-1)\sigma}{(\eta-\sigma\psi)\kappa} < \frac{\sigma(1+\beta)}{\kappa(\sigma\psi-\eta)}$, so that the term on the LHS of the inequality is the relevant threshold. We then compute the value of ψ at which $\widetilde{\chi}_\pi = \widehat{\chi}_\pi$: $\psi = \frac{\sigma(1-\beta)+\left(\kappa\eta\sigma^{-1}+\kappa+3\beta+1\right)\eta}{\kappa\sigma+4\sigma\beta+\kappa\eta}$. Finally, we compute the threshold for the pass-through parameter above which determinacy is never attained. This occurs at $\widehat{\chi}_\pi = \overline{\chi}_\pi = 1$. Straightforward algebra shows that this is the case whenever $\psi = \frac{\eta}{\sigma} + \frac{(1-\beta)}{\kappa}$. ∎

E-stability under a Contemporaneous Data Policy Rule

Proposition 2

Assume that the central bank implements the rule $r_t = \chi_\pi \pi_t$. Thus, along with $\chi_\pi > \overline{\chi}_\pi \equiv 1$, the following conditions are necessary to ensure E-stability:

1. For $\psi < 1 + \frac{2\eta}{\sigma}$:

$$\chi_\pi > \widehat{\chi}_\pi = \frac{(1-\kappa)\,\sigma - \sigma\beta - \kappa\eta}{(\sigma\,(\psi-1) - 2\eta)\,\kappa}$$

2. For $\psi > 1 + \frac{2\eta}{\sigma}$:

$$\chi_\pi < \widetilde{\chi}_\pi = \frac{(1-\kappa)\,\sigma - \sigma\beta - \kappa\eta}{(\sigma\,(\psi-1) - 2\eta)\,\kappa}$$

Proof. The following necessary and sufficient conditions ensure E-stability: $\bar{B}_c > 0$ and $\bar{A}_c > 0$, where $\bar{A}_c$ and $\bar{B}_c$ are the coefficients of the characteristic polynomial associated with $\mathbf{J}_{c33} - \mathbf{I}$ (i.e., $\lambda^2 + \bar{A}_c\lambda + \bar{B}_c = 0$):

$$\bar{B}_c \;\equiv\; \frac{\kappa\,(\sigma+\eta)\,(\chi_\pi-1)}{\sigma + \kappa\eta\chi_\pi - \kappa\sigma\psi\chi_\pi}, \tag{35}$$

$$\bar{A}_c \;\equiv\; \frac{\kappa\sigma\chi_\pi - \sigma + \kappa\sigma\psi\chi_\pi - \kappa\sigma + \sigma\beta + \kappa\eta}{\sigma + \kappa\eta\chi_\pi - \kappa\sigma\psi\chi_\pi} + 2. \tag{36}$$

Let us first focus on $\bar{A}_c > 0$. We multiply each side of this inequality by $\sigma + \kappa\chi_\pi\,(\eta - \sigma\psi)$. This term is always positive for $\psi < \frac{\eta}{\sigma}$. Otherwise, when $\psi > \frac{\eta}{\sigma}$ we need to impose a restriction on κ to ensure its positiveness, i.e. $0 < \kappa < \frac{\sigma}{\chi_\pi(\sigma\psi-\eta)}$. Thus, we can rearrange (36) as $(\kappa-1)\,\sigma + \sigma\beta + \kappa\eta + (\sigma\,(\psi-1) - 2\eta)\,\kappa\chi_\pi > 0$. There are three relevant cases to be considered: $1 + \frac{2\eta}{\sigma} < \psi$, $1 + \frac{2\eta}{\sigma} > \psi$ and $1 + \frac{2\eta}{\sigma} = \psi$. In the first case, the response to inflation must satisfy the following condition: $\chi_\pi > \frac{(1-\kappa)\sigma-\sigma\beta-\kappa\eta}{(\sigma(\psi-1)-2\eta)\kappa}$. In the second case, the maximum response to inflation is constrained from above by the following condition: $\chi_\pi < \frac{(1-\kappa)\sigma-\sigma\beta-\kappa\eta}{(\sigma(\psi-1)-2\eta)\kappa}$. Otherwise, note that for $\psi = 1 + \frac{2\eta}{\sigma}$ the relevant condition is $\kappa > \frac{(1-\beta)\sigma}{\eta+\sigma}$, which is easily satisfied at any conventional parameterization. Finally, we impose the condition $\bar{B}_c > 0$: as the denominator of (35) is always positive (given the restriction imposed on κ), this condition is simply satisfied for $\chi_\pi > 1$. ∎

Determinacy with Forward Expectations in the Policy Function

Proposition 3

Assume that the central bank implements the rule $r_t = \chi_\pi E_t \pi_{t+1}$. Thus, along with $\chi_\pi > \overline{\chi}_\pi = 1$, the following conditions are necessary to ensure equilibrium uniqueness:

1. For $\psi > -1$:

$$\chi_\pi < \widehat{\chi}_\pi = \frac{1 - \beta}{\kappa (\psi + 1)} \tag{37}$$

2. For $\psi < \frac{\eta - \sigma}{2\sigma}$:

$$\chi_\pi < \widetilde{\chi}_\pi = \frac{2\sigma (1 + \beta) + \kappa (\sigma + \eta)}{\kappa (\sigma + \eta) - 2\kappa\sigma (1 + \psi)} \tag{38}$$

Proof. As the central bank does respond neither to asset prices misalignments nor to the output gap, β is one of the three eigenvalues of $\mathbf{J}_f$. Furthermore, the NK Phillips curve and the IS constitute an autonomous system in which the matrix of structural parameters associated to the forward looking vector is represented by the following cofactor:

$$\mathbf{J}_{f33} = \begin{bmatrix} \beta + \kappa\chi_\pi (\psi + 1) - \frac{\kappa}{\sigma} (\chi_\pi - 1)(\sigma + \eta) & \kappa (\sigma + \eta) \\ -\frac{1}{\sigma} (\chi_\pi - 1) & 1 \end{bmatrix}.$$

The necessary and sufficient conditions ensuring determinacy are $|B_f| < 1$, and $|A_f| < 1 + B_f$, where A_f and B_f are the coefficients of the characteristic polynomial of $\mathbf{J}_{f33}$ (i.e., $\lambda^2 + A_f\lambda + B_f = 0$):

$$B_f \equiv \beta + \kappa\chi_\pi (\psi + 1), \tag{39}$$

$$A_f \equiv \left(\frac{\kappa}{\sigma} (\chi_\pi - 1)(\sigma + \eta) - \kappa\chi_\pi (\psi + 1) - \beta - 1 \right). \tag{40}$$

Let us first focus first on $|B_f| < 1$, which translates into $\beta + \kappa\chi_\pi (\psi + 1) < 1$ and $-\beta - \kappa\chi_\pi (\psi + 1) < 1$. We start by considering the first inequality, which can be rewritten as $\chi_\pi < \frac{1-\beta}{\kappa(\psi+1)}$. Otherwise, the second inequality can be expressed as $\chi_\pi > \frac{-1-\beta}{\kappa(\psi+1)}$: note that the term on the RHS is always negative. Let us now focus on $|A_f| < 1 + B_f$. This can be written as

$$\frac{\kappa}{\sigma} (\chi_\pi - 1)(\sigma + \eta) - \kappa\chi_\pi (\psi + 1) - \beta - 1 < 1 + \beta + \kappa\chi_\pi (\psi + 1) \tag{41}$$

and

$$\frac{\kappa}{\sigma} (\chi_\pi - 1)(\sigma + \eta) > 0. \tag{42}$$

After some rearrangements (41) is written as

$$\kappa\chi_\pi \left(\left(1 + \frac{\eta}{\sigma}\right) - 2(\psi + 1) \right) < 2 + 2\beta + \kappa + \eta\frac{\kappa}{\sigma}. \tag{43}$$

We then divide each side of (43) by the term $\kappa \left(1 + \frac{\eta}{\sigma}\right) - 2\kappa (\psi + 1)$, which is positive for $\psi < \frac{\eta - \sigma}{2\sigma}$, in which case we can determine the following constraint for the response

coefficient: $\chi_\pi < \frac{2+2\beta+\kappa+\eta\frac{\kappa}{\sigma}}{\kappa\left(1+\frac{\eta}{\sigma}\right)-2\kappa(\psi+1)}$. Otherwise, when $\psi > \frac{\eta-\sigma}{2\sigma}$ the relevant constraint is $\chi_\pi > \frac{2+2\beta+\sigma+\eta}{\kappa\left(1+\frac{\eta}{\sigma}\right)-2\kappa(\psi+1)}$: however the denominator of the term on the RHS is negative. Finally, if $\left(1+\frac{\eta}{\sigma}\right)-2\left(\psi+1\right)=0$ the term on the RHS of (43) is always positive and the inequality is always satisfied, no matter the value of χ_π. When exploring (42) it is immediate to show that $\chi_\pi > 1$ is the relevant constraint. $\blacksquare$

Corollary 2

The conditions in Proposition 3 can be re-stated to determine the following critical values of the pass-through:

1. For $-1 < \psi \leq \frac{(1-\beta)(\sigma+\eta)}{4\sigma+\kappa(\sigma+\eta)} - 1$ the response coefficient to inflation has to lie in the region between the locus $\widetilde{\chi}_\pi$ and the bottom limit represented by $\overline{\chi}_\pi = 1$.

2. For $\frac{(1-\beta)(\sigma+\eta)}{4\sigma+\kappa(\sigma+\eta)} - 1 < \psi < \frac{1-\beta-\kappa}{\kappa}$ the response coefficient to inflation has to lie in the region between the locus $\widehat{\chi}_\pi$ and the bottom limit represented by $\overline{\chi}_\pi = 1$.

3. For $\psi \geq \frac{1-\beta-\kappa}{\kappa}$ determinacy is never attained.

Proof. We first have to determine the points at which the thresholds implied by (37) and (38) cross $\overline{\chi}_\pi = 1$. It can easily be confirmed that $\widetilde{\chi}_\pi$ crosses $\overline{\chi}_\pi$ at $\psi^a = \frac{-1-\beta-\kappa}{\kappa}$, while $\widehat{\chi}_\pi$ crosses $\overline{\chi}_\pi$ at $\psi^b = \frac{1-\beta-\kappa}{\kappa}$. Note that $\psi^a < -1$ and $\psi^a < \psi^b$. Moreover, as $\widetilde{\chi}_\pi$ increases in ψ it will represent an upper bound to χ_π, from $\psi = -1$ up to the point where $\widetilde{\chi}_\pi$ crosses $\widehat{\chi}_\pi$, namely $\psi = \frac{(1-\beta)(\sigma+\eta)}{4\sigma+\kappa(\sigma+\eta)} - 1$. We need to show that this point lies on the LHS of the threshold implied by condition (37), namely $\frac{\eta-\sigma}{2\sigma}$. After some tedious algebra we can prove that this is always the case (under the restriction on κ we have imposed above). From this point onwards (37) bounds χ_π from above, up to the point in which $\widehat{\chi}_\pi$ crosses $\overline{\chi}_\pi$. From this point onwards determinacy can never be attained. $\blacksquare$

E-stability with Forward Expectations in the Policy Function

Proposition 4

Assume that the central bank implements the rule $r_t = \chi_\pi E_t \pi_{t+1}$. Thus, along with $\chi_\pi > \overline{\chi}_\pi = 1$, the following conditions are necessary to ensure E-stability:

1. For $\psi < \frac{\eta}{\sigma}$:

$$\chi_\pi > \widehat{\chi}_\pi = \frac{\kappa\left(\sigma+\eta\right)-\sigma\left(1-\beta\right)}{\kappa\left(\eta-\sigma\psi\right)}$$

2. For $\psi > \frac{\eta}{\sigma}$:

$$\chi_\pi < \widetilde{\chi}_\pi = \frac{\sigma\left(1-\beta\right)-\kappa\left(\sigma+\eta\right)}{\kappa\left(\sigma\psi-\eta\right)}$$

Proof. The following necessary and sufficient conditions ensure E-stability: $\bar{B}_f > 0$ and $\bar{A}_f > 0$, where $\bar{A}_f$ and $\bar{B}_f$ are the coefficients of the characteristic polynomial associated with $\mathbf{J}_{f33} - \mathbf{I}$ (i.e., $\lambda^2 + \bar{A}_f \lambda + \bar{B}_f = 0$):

$$\bar{B}_f \equiv \left(\frac{\kappa}{\sigma} (\chi_\pi - 1) (\sigma + \eta) - \kappa \chi_\pi (\psi + 1) - \beta + 1 \right), \tag{44}$$

$$\bar{A}_f \equiv \frac{\kappa}{\sigma} (\chi_\pi - 1) (\sigma + \eta). \tag{45}$$

It is immediate to check that $\bar{A}_f$ is always greater than zero if $\chi_\pi > 1$. As to (44), this can be rearranged as $(\eta - \sigma \psi) \kappa \chi_\pi > \kappa (\sigma + \eta) - \sigma (1 - \beta)$. Now, iff $\psi < \frac{\eta}{\sigma}$, the relevant condition reads as $\chi_\pi > \frac{\kappa(\sigma+\eta) - \sigma(1-\beta)}{\kappa(\eta - \sigma\psi)}$. Otherwise, when $\psi > \frac{\eta}{\sigma}$ the relevant condition is $\chi_\pi < \frac{\sigma(1-\beta) - \kappa(\sigma+\eta)}{\kappa(\sigma\psi - \eta)}$. Finally, if $\psi = \frac{\eta}{\sigma}$ the relevant condition is $\kappa > \frac{(1-\beta)\sigma}{\eta+\sigma}$, which is easily satisfied at any conventional parameterization. $\blacksquare$