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A SIMULATION STUDY OF AN ASEAN MONETARY UNION

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A simulation study of an ASEAN Monetary Union

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Abstract:

This paper analyzes some pros and cons of a monetary union for the ASEAN¹ countries, excluding Myanmar. We estimate a stylized open-economy dynamic general equilibrium model for the ASEAN countries. Using the framework of linear quadratic differential games, we contrast the potential gains or losses for these countries due to economic shocks, in case they maintain their status-quo, they coordinate their monetary and/or fiscal policies, or form a monetary union. Assuming for all players open-loop information, we conclude that there are substantial gains from cooperation of monetary authorities. We also find that whether a monetary union improves upon monetary cooperation depends on the type of shocks and the extent of fiscal policy cooperation. Results are based both on a theoretical study of the structure of the estimated model and a simulation study.

Keywords: ASEAN economic integration, monetary union, linear quadratic differential games, open-loop information structure.

Jel-codes: C61, C71, C72, C73, E17, E52, E61, F15, F42, F47.

1 Introduction

This paper studies the pros and cons of further economic integration of the ASEAN countries. One of the major goals of the ASEAN countries is to achieve deeper regional integration. Since the launch of the ASEAN Free Trade Area (AFTA) in 1992², ASEAN has entered the first stage toward the process of reaching full economic integration. Full economic integration can be reached by subsequently forming a customs union, a common market, and an economic union. AFTA has reached its initial implementation in 2003. Systematic efforts to remove tariff and non-tariff barriers

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¹Association of Southeast Asian Nations, its members are: Brunei, Cambodia, Indonesia, Laos, Malaysia, Myanmar, Philippines, Singapore, Thailand, Vietnam.

²ASEAN has been founded in 1967 by Indonesia, Malaysia, Philippines, Singapore and Thailand, and now consists of ten countries mentioned above. The objective is to promote economic, cultural and political cooperation among the member countries. During a summit on the island of Bali in 2003, the "Bali Concord" was signed which foresees an economic union by 2020.

are being implemented, and all member countries are committed to make ASEAN a free trade and tariff zone by 2015. The current debates/challenges center around two questions whether further adjustments are required beyond the national borders and how to move toward creating an ASEAN Economic Community. Different views about the progress and the future of economic integration in ASEAN can be found in, e.g., the REPSF³ publications over the last decade, Soesastro (2005), Plummer (2006), Soesastro (2007) Hashmi and Lee (2008), Lim and Yi-Xun (2008), Heydon and Woolcock (2009), Park and Park (2009).

Following the seminal works of Mundell (1961) and McKinnon (1963) on optimum currency areas (OCAs), there have been numerous studies that assess the theoretical and practical implications of forming a monetary union (MU). In particular, the question whether countries form an OCA depends on the incidence of asymmetric shocks and the asymmetric transmission of shocks. Participating in a MU comes at the cost of losing monetary authority and exchange-rate adjustments as policy instruments. Additionally, being a member of a MU implies the need to comply with fiscal and other policy restrictions, like e.g. the Stability and Growth Pact (SGP) requirements in the European Monetary Union (EMU). On the other hand, alternative stabilization mechanisms may replace the role of the exchange-rate adjustment and there may be sizeable benefits outweighing the costs of participation to the MU. The OCA theory suggests that countries will establish a MU as soon as properly quantified economic benefits start to outweigh costs, see e.g. De Grauwe (2000). In a theoretical setting the question whether countries constitute an OCA can be assessed by comparing the effects of an asymmetric shock in a country, when it continues its independent monetary policy, with the effects of the same shock once it has entered the MU. For EMU, OCA issues have received a lot of attention in the literature; detailed surveys can be found in e.g. Buti and Sapir (1998) and Plasmans, Engwerda, van Aarle, Di Bartolomeo, and Michalak (2006)[Section 1.4.2].

Most of the studies on the potential of a monetary union in East Asia concentrate on the question whether these countries constitute an OCA or not - see e.g. Eichengreen and Bayoumi (1999), Bayoumi, Eichengreen, and Mauro (2000), Ng (2002), Zhang, Sato, and McAleer (2004), Ramayandi (2005), Ahn, Kim, and Chang (2006), Sato and Zhang (2006) Kawai (2008), Becker (2008) and Pontines and Rajan (2009). These studies along with a historical perspective suggest that there is a growing support for a (partial) MU, however the arguments are mainly based on the finding that the number of OCA criteria that are met by (some) of the countries have increased. In particular the study by Eichengreen and Bayoumi (1999) revealed that the value of their developed optimum currency index for ASEAN is not very different from what it was in Europe prior to the Maastricht Treaty. On the other hand, a number of these studies also report impediments to the formation of a (partial) MU (see, e.g., the REPSF reports, Kawai (2008) and Becker (2008)). These concern, e.g., the reluctance to lose national sovereignty over economic policymaking, the diversity of economic and political systems, the lack of a legal framework, too many bilateral trade agreements and the lack of mutual trust. The creation of a currency union by (some of) the ASEAN countries which can overcome the above mentioned obstacles is in general foreseen to be a natural step and a fly-wheel for the ultimate creation of an (East-Asian) MU.

Rather than evaluating the OCA criteria, in this paper we provide a rigorous simulation study in the framework of linear quadratic (LQ) differential games to illustrate the potential gains and losses that might incur from the establishment of a MU in the ASEAN countries (SEAMU thereafter), in the presence or absence of fiscal cooperation. Our simulation study is based on a small-scale

³The Regional Economic Policy Support Facility (REPSF) is one of the components of the ASEAN-Australia Development Cooperation Program.

general equilibrium model of the area. Assuming that the current status of economic policy making is marked by a non-cooperative setting, we show how the effect of cooperation between various (institutions of) countries can be analyzed. In particular we provide a framework for studying gains and losses from forming a monetary union among ASEAN countries. This framework is based on the following setting: under the assumption that SEAMU has been settled, we pose the question whether participating countries are better off in this union if an economic shock occurs compared to the current status quo. This typically involves a dynamic analysis. As shown in e.g. Levine and Brociner (1994), Hughes-Hallet and Ma (1996), Beetsma, Debrun, and Klaassen (2001), Debrun, Masson, and Pattillo (2005) and Michalak, Engwerda, and Plasmans (2009), the answer to the question whether the gains from being in a MU exceed its costs is not clear-cut, as it depends on the type of shocks, on the gains and losses from fiscal cooperation, and on the clash between monetary and fiscal objective functions under (partial) fiscal coordination. If a monetary union is formed, it was shown in a theoretical context that (assuming that the MU's Central Bank does not participate in fiscal coalitions) non-cooperation of fiscal authorities is sometimes the best local governments can do to cope with certain economic shocks. It is likely that under such conditions, not forming a MU would be preferable.

Therefore, it is paramount to study the dynamic effects of various shocks within a MU, and compare the potential gains and losses of countries to their status-quo without the formation of a MU. To that end, in this paper:

* We start by formulating a small-scale dynamic general equilibrium model of nine countries in ASEAN, which we estimate using recent data. Extensions of our framework for dynamic stochastic general equilibrium (DSGE) models can be found in Coenen, Lombardo, Smets, and Straub (2010) for two players, and for multiple players in Plasmans, Michalak, and Fornero (2006) in an econometric setting and in Michalak, Engwerda, and Plasmans (2009) in a more theoretical and numerical simulation setting. We chose to work with a simpler model mainly due to limited data availability, that restricts us from obtaining reliable parameter estimates when estimating a full-scale DSGE model.⁴

* We use the above model along with quadratic loss functions for monetary and fiscal players to formulate a linear quadratic differential game in continuous time. For dynamic optimization problems with more than one actor, the information each player possesses about the system plays an important role in choosing his/her actions. Here we assume that the information about the system is of the open-loop (OL) type, that is, it is known to all players *ex-ante*. This structure, while restrictive, is very useful for learning about the benefits of a MU, and is employed in several studies due to its analytic tractability.

* We solve the model and derive the properties of its solution. We find analytic conditions for the existence of equilibria in the above model (see e.g. Engwerda (2005) for a detailed discussion of challenges that arise in the OL setting).

* Given our theoretical insight, we analyze impulse response functions that provide guidelines for the consequences of different economic shocks in different economic settings and different levels of policy coordination. In our simulations, we use the numerical toolbox described in Michalak, Engwerda, and Plasmans (2011) which is especially suitable for OL type games, and find that there are substantial gains for monetary policy coordination, whether in the form of a MU or not. We also conclude that the question whether a monetary union improves upon monetary cooperation is not

⁴Note that the computational details are similar for our model, adapted to discrete time, and a small-scale DSGE model with two players, but for more than two players and a full-scale model, the existence and properties of the equilibria are not entirely known; such analyses are beyond the scope of our paper.

clear cut. Depending on the type of shocks and the extent of fiscal policy cooperation, while the aggregate costs of all countries in the block are similar, individual costs may differ, especially in the context of the grand coalition of monetary and fiscal authorities.

The rest of the paper is structured as follows. Section 2 describes the general equilibrium model, while Section 3 provides estimation results for this model. Based on these estimates, we analyze in Section 5 for the benchmark model two different scenarios for policy making, the non-cooperative and full cooperative case, respectively. For both, we first derive conditions for the existence of equilibria⁵ and study the analytic properties of our solutions. Comparing these properties with actual realizations, we conclude that current ASEAN economic policy making is best approached by a cooperative scenario. In Section 6 we perform a simulation study to quantify the effects on economic performance of forming a MU, in the absence or presence of fiscal policy and cooperation. Section 7 concludes.

2 The Basic Economic Framework

We consider a linear quadratic differential game model involving N countries participating in an economic union like the AFTA. With a slight notation abuse, we denote the set of countries by $\bar{N} := \{1, 2, \dots, N\}$. We begin by presenting the stylized analytical framework, a small-scale open-economy dynamic general equilibrium model, see e.g. Plasmans, Engwerda, van Aarle, Di Bartolomeo, and Michalak (2006)[Section 7.2].⁶

In the absence of a monetary union, each economy is described by its own aggregate demand (IS) curve and aggregate supply (AS) curve. The IS curves for each country j are:

$$y_j(t) = \gamma_j r_j(t) + \eta_j f_j(t) + \sum_{k \in \bar{N}/j} \phi_{jk} y_k(t) + \sum_{k \in \bar{N}/j} \delta_{jk} c_{jk}(t), \quad (1)$$

where the real output gap in a country, y_j , is a function of the domestic real interest rate $r_j(t) = i_j(t) - \dot{p}_j(t)$ - with $i_j(t)$ and $\dot{p}_j(t)$ being nominal interest rates and inflation in country j , respectively - of the domestic real fiscal deficit $f_j(t)$, of the foreign (real) output and of the competitiveness, $c_{jk}(t)$. Competitiveness, $c_{jk}(t) = e_{jk}(t) + p_k(t) - p_j(t)$, is measured by adjusting nominal exchange rates, $e_{jk}(t)$, for relative prices, $p_k(t) - p_j(t)$. All variables are in logarithms, except for interest rates which are in decimal points. A dot above a variable denotes its time derivative. The direct output and competitiveness spillovers are measured by ϕ_{jk} and δ_{jk} , respectively. The spillovers through the interest rate are determined by γ_j and the fiscal deficit spillovers by the direct effects of fiscal deficits, η_j . We treat the variables as deviations from their steady-states, which have been normalized to zero for simplicity. Nominal exchange rates⁷, adjusted for relative prices, measure the international competitiveness of the economy. We start by assuming that the nominal exchange rates

⁵Details can be found in Appendix C.

⁶See also van Aarle, Engwerda, and Plasmans (2002) and Neck and Döckner (1995). While attractive, DSGE models pose various difficulties in our setting. Estimation is possible but our data span is too short to provide reliable results. Calibration is not possible due to the fact that the literature on ASEAN countries is insufficiently informative. Related to the game-theoretic framework, a two-country DSGE model with and without monetary cooperation is presented in Justiniano and Preston (2010), and the only multi-country extension of a DSGE model that we are aware of is Michalak, Engwerda, and Plasmans (2009).

⁷As usual, nominal exchange rates are measured as the (logarithmic) price of one unit of foreign currency, expressed in domestic currency.

are determined according to the uncovered interest-rate parity (UIP) hypothesis, an assumption that we will relax later. That is, they adjust to corresponding interest-rate differentials:

$$\dot{e}_{jk}(t) = i_j(t) - i_k(t), \quad e_{jk}(0) = e_{jk0}. \quad (2)$$

The initial values of the exchange rates, e_{jk0} , represent (initial) level shocks that hit the exchange rate at time zero, reflecting e.g. (initial) shocks in international financial markets.

In case of a monetary union among some of the ASEAN countries - defined here by a common currency and a common interest rate - the IS curves in equation (1) become different across countries in and outside the monetary union:

$$\begin{aligned} y_j(t) &= \gamma_j r_j(t) + \eta_j f_j(t) + \sum_{k \in \bar{N}/j} \phi_{jk} y_k(t) + \sum_{k \in \bar{N}_U^c/j} \delta_{jk} c_{jk}(t) \\ &+ \sum_{k \in \bar{N}_U} \delta_{jk} c_{jU}(t), \quad j \in \bar{N}_U^c \end{aligned} \quad (3)$$

$$\begin{aligned} y_j(t) &= \gamma_j (i_U(t) - \dot{p}_j(t)) + \eta_j f_j(t) + \sum_{k \in \bar{N}/j} \phi_{jk} y_k(t) + \sum_{k \in \bar{N}_U^c} \delta_{jk} c_{jk}(t) \\ &+ \sum_{k \in \bar{N}_U/j} \delta_{jk} (p_k(t) - p_j(t)), \quad j \in \bar{N}_U, \end{aligned} \quad (4)$$

where \bar{N}_U consists of the countries which engage in the monetary union (and \bar{N}_U^c consists of the remaining countries), $c_{jU}(t) = e_{jU}(t) + p_k(t) - p_j(t)$, $\dot{e}_{Uk}(t) = i_U(t) - i_k(t)$, and $i_U(t)$ is the common interest rate in the monetary union. In this case $\dot{e}_{jk}(t) = 0$ for all t and for countries j, k both in the monetary union. The external exchange rate of the MU with non-MU countries together with the transmission mechanism of monetary policy and fiscal policy are the only shock absorbers. For simplicity, we assume that when there is a MU, all countries join this MU. This simplifies the analysis to some extent, since only equations (4) apply in that case.

Equations (5) are open-economy Phillips curves:

$$\dot{p}_j(t) = \zeta_j y_j(t) + \sum_{k \in \bar{N}/j} \psi_{jk} s_{jk}(t), \quad p_j(0) = p_{j0}. \quad (5)$$

where $s_{jk}(t) = \dot{e}_{jk}(t) + \dot{p}_k(t)$ reflects the ‘‘exchange rate pass-through’’ from country k to country j . In these Phillips curves, the inflation rates of the other countries play a role reflecting the effects of pass-through of foreign inflation on domestic currency. Since our focus is on short-run stabilization, the effectiveness of fiscal policy is limited to its transitory impact on output through the induced stimulus of aggregate demand. In this open-economy setting, monetary policy affects output not only via the interest-rate channel but also through the exchange-rate channel (i.e. via influencing international competitiveness).

In the absence of a monetary union, we assume that the loss functions of fiscal and monetary players are:⁸

⁸Most papers consider quadratic losses for monetary and fiscal authorities of similar type, see *inter alia* Dixit and Lambertini (2001), Benigno (2004) and Yeh (2009).

$$J_j^F = \int_0^\infty e^{-\theta t} \{ \alpha_j^F p_j^2(t) + \beta_j^F y_j^2(t) + \chi_j^F f_j^2(t) \} dt, \quad (6)$$

$$J_j^M = \int_0^\infty e^{-\theta t} \{ \alpha_j^M p_j^2(t) + \beta_j^M y_j^2(t) + \chi_j^M i_j^2(t) \} dt. \quad (7)$$

In (6) we follow the current literature on price level targeting - e.g. Buti and Sapir (1998) - and assume that the fiscal authorities are primarily concerned with the stabilization of the domestic nominal price, domestic real output gap and domestic real fiscal deficit. The parameter θ denotes the rate of time preference and α_j^ℓ , β_j^ℓ and χ_j^ℓ , where $\ell = F, M$, represent preference weights that are attached to the stabilization of inflation, output and fiscal deficits, respectively. Preference for a low fiscal deficit could reflect the costs of excessive deficits - which are in fact sanctioned in the EMU case by the SGP. Moreover, costs could also result from undesirable debt accumulation and inter-generational redistribution that high deficits bring about and, in that interpretation, χ_j^F could also reflect the priority attached to fiscal retrenchment and consolidation. Similarly, the loss functions J_j^M in (7) assume that the monetary authorities direct their policies at stabilizing the price level, the output (gap) and the interest rate. Moreover, it indicates that active use of monetary policy invokes costs for the monetary policymaker: other things equal he/she would like to keep his/her policy instrument constant, avoiding large swings.

In case of a monetary union, we define the loss function of the common central bank as:

$$J_U^M = \int_0^\infty e^{-\theta t} \{ \alpha_U^M p_U^2(t) + \beta_U^M y_U^2(t) + \chi_U^M i_U^2(t) \} dt,$$

where $p_U = \sum_{j \in \bar{N}_U} \omega_j p_j(t)$ is the loglinearized aggregate price level, $y_U(t) = \sum_{j \in \bar{N}_U} \omega_j y_j(t)$ is the loglinearized aggregate output (gap), and α_U^M and β_U^M indicate the relative preferences of the Central Bank (CB) of the MU concerning inflation and output of the MU as a whole. Parameter ω_j indicates the relative weight of country j in the MU ($\sum_{j \in \bar{N}_U} \omega_j = 1$). The minimization of the CB's loss function w.r.t. i_U is consistent with the derivation of a standard monetary policy rule, since it results in a linear function in its arguments - see e.g. Clarida, Gali, and Gertler (1999).

3 Model Estimation

The objective of this section is to estimate the parameters for our ASEAN model described in (1), (2), and (5), which will subsequently be used to determine the region's steady-state, and to perform simulations for revealing the potential gains from monetary and fiscal cooperation.

The estimation of the model as described in (1), (2) and (5) poses various difficulties, which we overcome by formulating a simplified model, performing robustness checks of our estimates across countries and providing a baseline specification (benchmark model) along with the corresponding parameter estimates. These are used in Sections 5 and 6 to perform simulations for analyzing pros and cons of an ASEAN monetary union, in a framework that is from a computational point of view quite similar to that of a DSGE model combined with game-theoretic behavior, except that it is cast in continuous time.⁹

⁹See Coenen, Lombardo, Smets, and Straub (2010) for a DSGE counterpart of our model, limited to two players, US and the Euro Area.

3.1 Structural equations and identifying restrictions

The model described in (1), (2), and (5) is an unbalanced multivariate panel data model with three dimensions: $N = 9$ countries, maximum $T = 12$ time observations per country, and $d = 3$ equations. It can be seen as a multivariate spatial panel with spatial heterogeneity across intercepts and slopes, and where endogeneity is present not only across equations, but also across countries. The latter is known as “spatially lagged dependent variables” in univariate models, see Elhorst (2003); to distinguish this from lagged dependent variables and emphasize endogeneity across the system of equations, we prefer to call it “spatial endogeneity”. To our knowledge, this is a nonstandard estimation problem due to the multivariate setting, but we show below that under some simplifications it can be reduced to settings already present in the literature. We begin by exposing all issues related to estimation.

First, we note that the UIP hypothesis does not hold upon testing, and its violation is not isolated to some countries. We therefore rewrite (2) in the spirit of linear exchange rate models - see Plasmans, Verkooien, and Daniëls (1998) and De Grauwe and Vansteenkiste (2007), assuming “partial” parity:

$$\dot{e}_{jk}(t) = \beta_{0j} + \beta_{1j}(i_j(t) - i_k(t)).$$

If we allow for fixed and/or random country-specific effects, our model becomes:

$$y_j(t) = \alpha_j + \gamma_j r_j(t) + \eta_j f_j(t) + \sum_{k \in \bar{N}/j} \phi_{jk} y_k(t) + \sum_{k \in \bar{N}/j} \delta_{jk} c_{jk}(t) + d_j^1 + u_j^1(t) \quad (8)$$

$$\dot{e}_{jk}(t) = \beta_j + \beta_{1j}(i_j(t) - i_k(t)) + d_j^2 + u_j^2(t) \quad (9)$$

$$\dot{p}_j(t) = \bar{\beta}_j + \zeta_j y_j(t) + \sum_{k \in \bar{N}/j} \psi_{jk} s_{jk}(t) + d_j^3 + u_j^3(t), \quad p_j(0) = p_{j0}. \quad (10)$$

where α_j, β_j and $\bar{\beta}_j$ are intercepts, d_j^i are country and equation specific effects, and $u_j^i(t)$ are idiosyncratic errors, for $i = 1, 2, 3$.

As can be noted from the model above, if we allow, say, ϕ_{jk} to be different across $k \in N \setminus j$, we would have, even in the absence of country-specific effects, at least $3(N - 1)$ parameters to estimate, with NT observations per equation. This gives rise to the well-known incidental parameter problem, and any estimation with both N and T small as is the case here would yield biased estimates. One solution is to ignore for each country j the dependence of parameters on other countries k , maintain parameter heterogeneity across countries, and estimate for each country a separate time series. Such a procedure would ignore spatial endogeneity as well as co-movement across countries and is thus not desirable - see Quah (1996). An alternative way to reduce the number of parameters is to join homogeneous countries in groups, and then have a separate equation for each group. Due to the small number of observations, we opt for starting with the most parsimonious model, in which $\alpha_j = \alpha$, $\beta_j = \beta$, $\bar{\beta}_j = \bar{\beta}$, $\phi_{jk} = \phi$, $\delta_{jk} = \delta$, $\gamma_j = \gamma$, $\eta_j = \eta$, $\zeta_j = \zeta$, $\psi_{jk} = \psi$, but allowing for random/fixed effects.¹⁰

We check the validity of the restrictions via specification checks, including re-estimation by excluding one country at a time.¹¹ The latter can be viewed as a jackknife procedure without replacement,

¹⁰This model is equivalent to a multivariate model with random intercepts but fixed slopes.

¹¹We also tried with excluding two and three countries at a time, but the results are qualitatively similar. They are available from the authors upon request.

to check for “outliers”. If the absence of country j changes the results dramatically, we take this into account by allowing the coefficients to differ for that particular country j .

Finally, the spatial endogeneity in a multivariate panel induces an additional complication related to setting up the estimation. The latter has been dealt with in univariate settings by rewriting the model in a reduced form as in Elhorst (2003)[equation (24)], and we adopt a similar approach here.

We first estimate each equation of the system separately, to explore the presence of random or fixed effects, and sensitivity to instruments. Then we average the variables in each equation of the system over the other countries and express the bilateral exchange rate as the ratio of the individual exchange rates with respect to the US dollar, after which we perform a 3SLS procedure.

Under our assumptions, the model in (8), (9) and (10) simplifies to:

$$y_j(t) = \alpha + \gamma r_j(t) + \eta f_j(t) + \phi y_j^*(t) + \delta c_j^*(t) + d_j^1 + u_j^1(t) \quad (11)$$

$$\dot{e}_j(t) - \dot{e}_j^*(t) = \beta_0 + \beta_1(i_j(t) - i_j^*(t)) + d_j^2 + u_j^2(t) \quad (12)$$

$$\dot{p}_j(t) = \bar{\beta}_0 + \zeta y_j(t) + \psi s_j^*(t) + d_j^3 + u_j^3(t) \quad (13)$$

where for any random variable x we used the shorthand notation

$$x_j^*(t) = \frac{1}{N-1} \sum_{k \in \bar{N} \setminus j} x_k(t),$$

to denote the sample average of all N random samples from x , excluding x_j .

3.2 Data

The dataset is drawn from ASEAN Statistical Yearbooks for 1995-2007, and from 2002-2007 through selected ASEAN Indicators. It contains yearly observations from 1995:2006, on the 10 ASEAN countries. Since budget deficit data is completely missing for Myanmar after 2002, we eliminate this country from the dataset. The data is an unbalanced panel containing $N = 9$ countries and maximum $T = 12$ years, and the data is detailed below.

Real GDP: y_j . It is the log of real GDP per capita measured in units of national currency. Although Cambodia, Vietnam and Indonesia have higher real GDP if expressed in US dollars, Figure 3.1 shows that the growth in GDP is quite similar, providing scope for estimating equation (11).

Interest Rates: i_j . These are nominal annualized short term interest rates, expressed in decimal points. Figure 3.2 indicates that the interest rates of Philippines, Indonesia and Laos are quite volatile, and often move in opposite directions; the remaining interest rates suggest a co-movement.

Inflation: \dot{p}_j . The inflation is measured as the annualized percentage increase in consumer price index (CPI), expressed in decimal points. Figure 3.3 shows that inflation is quite high for some countries, including Indonesia and Laos before 2000.

Exchange Rates: e_j . They are measured as the log of nominal exchange rates in units of national currency per US dollar. Note from Figure 3.4 the similar movement across ASEAN countries despite the removal of the currency pegs following the 1997 financial crisis.

Budget Deficits: f_j . They are measured in percentages of GDP.¹² From Figure 3.5, note that some countries are running a budget surplus over the period, while the others’ deficit is relatively small, suggesting that the fiscal tools might not be used intensively for policy purposes.

¹²Alternatively, one would want to include in (11) budget deficits in levels; the estimation results of Section 3 are qualitatively similar.

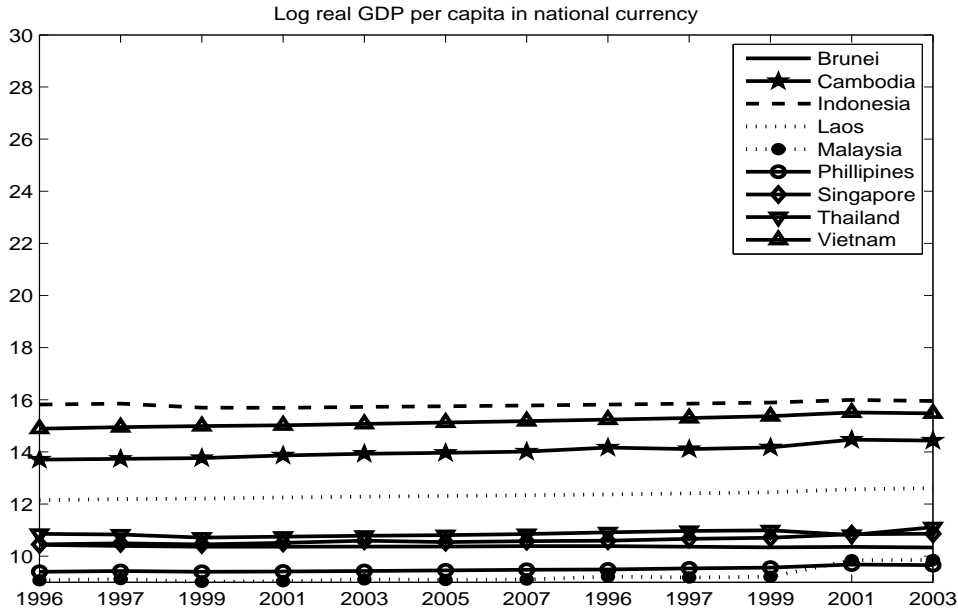


Figure 3.1: Log real GDP per capita in national currency

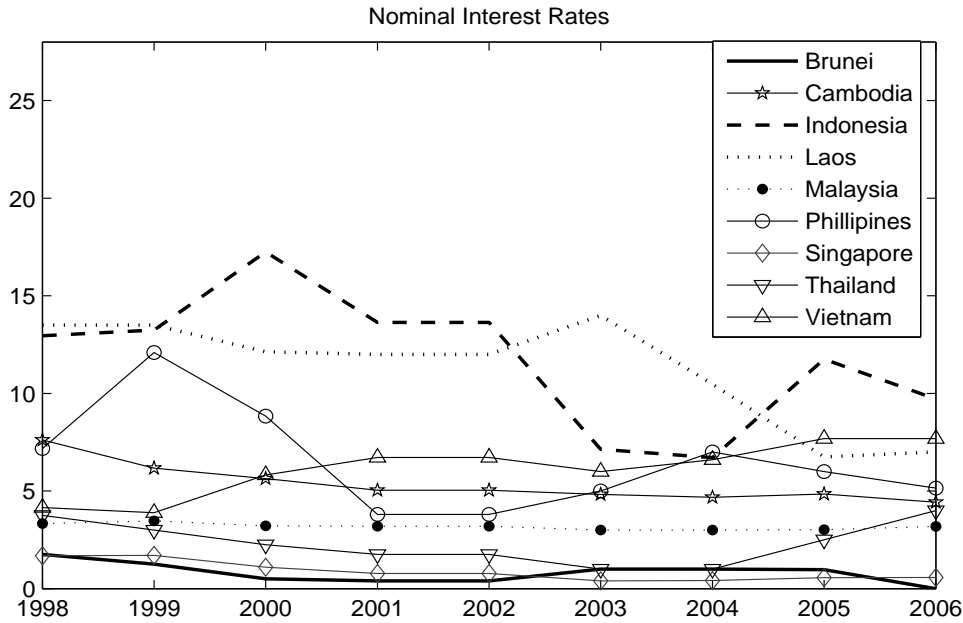


Figure 3.2: Nominal interest rates

3.3 Single Equation Estimation

In our model, $e_{jk}(t)$, $\dot{p}_j(t)$ and $y_j(t)$ are endogenous, and can be estimated jointly via instrumental variable methods for panel data. To find appropriate instruments, we start by separately estimating equations (11)-(13).

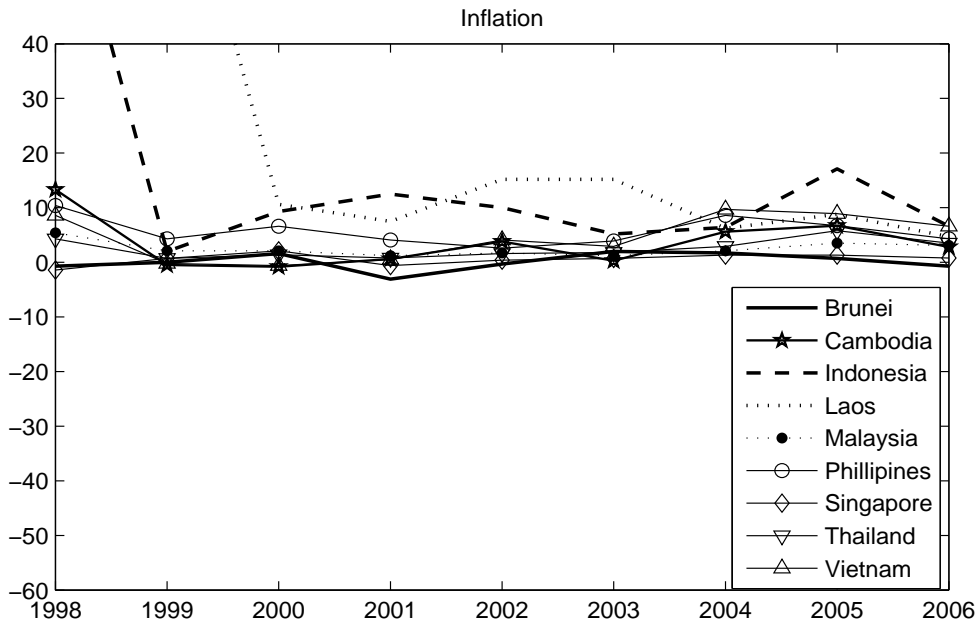


Figure 3.3: Inflation

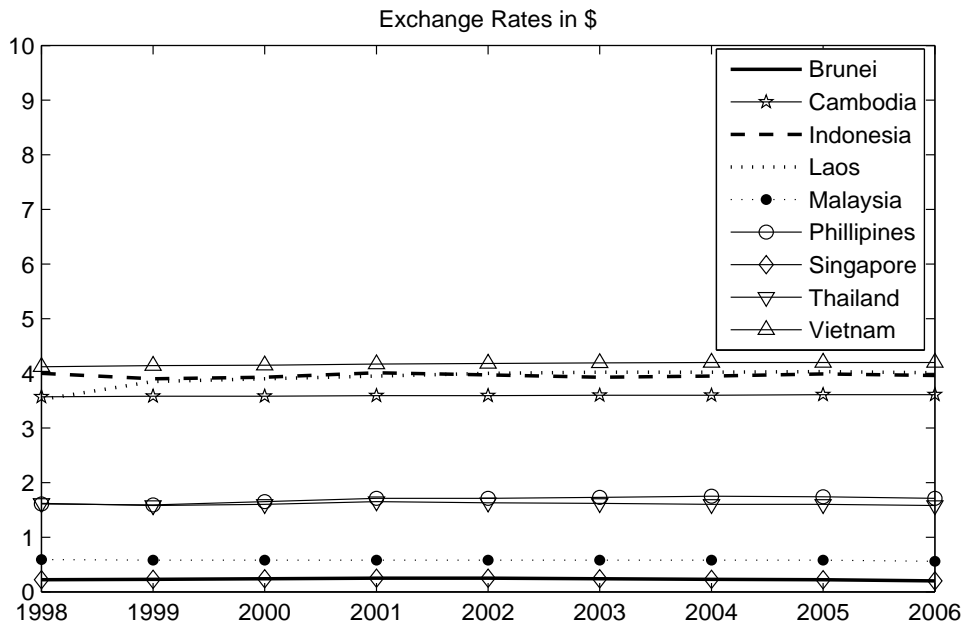


Figure 3.4: Exchange rates in \$

Aggregate Demand Equation (11). Estimation results suggest the presence of fixed effects, which we remove by estimating (11) in first differences rather than in levels. We use an Arellano-Bond estimator, with first differences and/or first lags as instruments. The results for all countries are in Table 1. Note that the results on the budget deficit and real competitiveness are of the wrong sign, but also insignificant. The next subsection shows that the single equation estimates for γ and

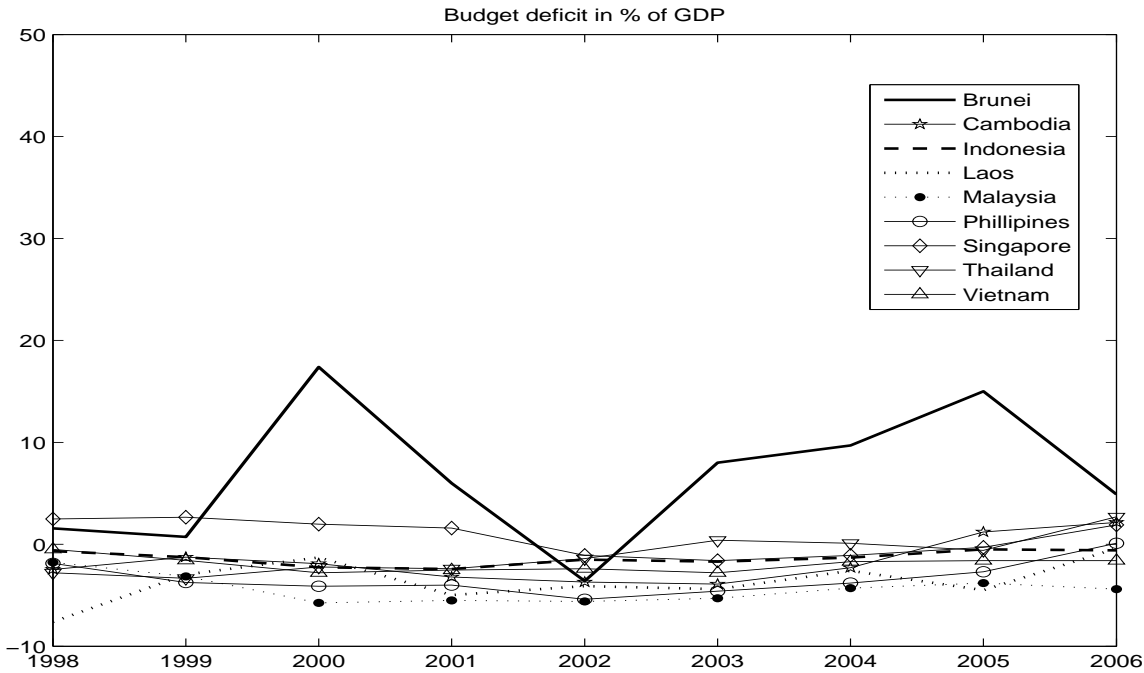


Figure 3.5: Budget deficits in % of GDP

Table 1: Estimation of (11) in first differences

α	γ	η	ϕ	δ
-2.5263**	-0.0004	-0.002	.4095**	-0.0058
(1.2184)	(.0012)	(.0035)	(.1600)	(.0070)

Subscripts *, ** and *** are used to indicate significance at the 10%, 5%, respectively 1% level. Standard errors are reported between parentheses.

δ are quite similar to the system estimates. The validity of instruments is assessed via Sargan tests, and the instrument set $\{y_j(t-1), y_j^*(t-1), \dot{r}_j(t), f_j(t), \dot{c}_j(t-1)\}$ is chosen.

Exchange Rate Equation (12). Estimation results do not suggest further dynamics in this equation. Since a Hausman test does not reject the null of no systematic differences between RE and FE estimates at the 5% level we report only RE estimates in Table 2 below. The instruments used for the nominal interest rate - which is endogenous because inflation is endogenous - are: $\{y_j(t-1), y_j^*(t-1), \dot{r}_j(t), f_j(t), \dot{c}_j(t-1)\}$.

Inflation Equation (13). This equation also shows no systematic difference between RE and FE estimation, with a Hausman test not rejecting at the 1% level. Several instrument sets yield similar estimates; the estimates with instruments $\{y_j(t-1), \dot{r}_j(t), f_j(t), \dot{c}_j(t-1), s_j^*(t-1), \dot{p}_j(t-1), \dot{p}_j^*(t-1)\}$, are summarized in Table 3 below.

Table 2: Single Equation Estimation of (12)

β_0	β_1
.0000	.0036***
(.0071)	(.0011)

Subscript *** is used to indicate significance at the 1% level. Standard errors are reported between parentheses.

Table 3: Single Equation Estimation of (12)

$\bar{\beta}_0$	ζ	ψ
-1.9313	.0417	.1961
(7.1808)	(1.6108)	(.5844)

3.4 System: Benchmark Model

The previous subsection shows that only the first equation has fixed effects (FE), which are removed by first differencing. The other two equations exhibit random effects (RE), and all of them pass further autocorrelation tests. Thus, a joint GMM estimation of the system, with (11) first-differenced, and (12)-(13) in levels, subject to strong instruments, will yield consistent and efficient estimates provided that heterogeneity and contemporaneous spatially correlated error structure are somehow correctly super-imposed in the estimation.¹³ However, we conjecture that after averaging each equation in the system with respect to the other countries, both a 2SLS and a 3SLS method will yield consistent estimates of the parameter values.¹⁴ The latter is the method used in the paper, and has the advantage that heteroskedasticity and contemporaneous spatial correlation among errors in different equations of the system are taken account of.

The baseline specification which we will use for system estimation is:

$$\dot{y}_j(t) = \gamma \dot{r}_j(t) + \eta \dot{f}_j(t) + \phi \dot{y}_j^*(t) + \delta \dot{c}_j^*(t) + v_j(t) \quad (14)$$

$$\dot{e}_j(t) - \dot{e}_j^*(t) = \beta_0 + \beta_2(i_j(t) - i_j^*(t)) + d_j^2 + u_j^2(t) \quad (15)$$

$$\dot{p}_j(t) = \bar{\beta}_0 + \zeta y_j(t) + \psi s_j^*(t) + d_j^3 + u_j^3(t) \quad (16)$$

where $v_j(t) = u_j(t) - u_j(t-1)$ refers to first differences, $\dot{f}_j(t) = f_j(t) - f_j(t-1)$, $\dot{r}_j(t) = i_j(t) - \dot{p}_j(t) - i_j(t-1) + \dot{p}_j(t-1)$, and the rest of the variables are in percentage growths. The instruments used are the exogenous variables, along with $y_j(t-1)$, $y_j^*(t-1)$, $\dot{p}_i(t-1)$, $\dot{p}_i^*(t-1)$, $c_j(t-1)$, $s_j^*(t-1)$, instruments which arise naturally when first differencing and pass the single equation Sargan tests.

The results from the baseline specification (14)-(16) are presented in Table 4; they are accompanied by results obtained when excluding a country from the system, one-at-a-time, to assess robustness of results for different countries.

¹³It is unclear how such a procedure would be pursued in our setting, but it is beyond the scope of our paper.

¹⁴The estimates for 2SLS and 3SLS are not very different, only their standard errors differ.

Table 4: Baseline Specification (14)-(16)

	Without									
	All	B	C	I	L	M	P	S	T	V
GDP Growth \dot{y} (14)										
\dot{r}	-.0031*** (.0013)	.0002 (.0005)	-.0003 (.0008)	-.0001 (.0006)	-.0017 (.0021)	.0000 (.0004)	.0000 (.0066)	.0003 (.0009)	-.0001 (.0005)	.0000 (.0006)
\dot{f}	-.0021 (.0031)	-.0121 (.0164)	.0021 (.0035)	-.0011 (.0030)	-.0026 (.0036)	-.0006 (.0020)	-.0015 (.0031)	-.0011 (.0029)	-.0007 (.0029)	-0.0033 (.0033)
\dot{y}^*	.7707 (1.0152)	1.0265 (.8324)	1.7398 (1.4377)	.8458** (.3676)	1.4143 (.9802)	.8303** (.3395)	1.0613*** (0.3496)	1.0191*** (.3409)	1.6069*** (.2628)	1.5532*** (.3667)
\dot{c}	-.0020 (.1420)	-.0170 (.9210)	.1163 (.7504)	-.4633 (.6319)	-1.0232 (1.0867)	-.5275 (.3755)	-.0166 (.5734)	-.0004 (.0010)	-.0010* (.0006)	-.0018*** (.0007)
Exchange Rates ($\dot{e} - \dot{e}^*$) (15)										
β_0	-.0003 (.0024)	.0000 (.0085)	.0233** (.0100)	.0033 (.0065)	-.0003 (.0071)	.0000 (.0090)	.0000 (.0089)	.0414** (.0152)	.0000 (.0084)	.0000 (.0083)
$i - i^*$.0013*** (.0005)	.0025 (.0015)	.0023 (.0015)	.0040*** (.0012)	.0030** (.0015)	.0024* (.0014)	.0031** (.0015)	.0014 (.0021)	.0013* (.0008)	.0012 (.0008)
Inflation \dot{p} (16)										
β_0	-5.8583** (2.4715)	-11.8776 (11.5782)	28.0274** (13.6566)	-5.5450 (10.6091)	-3.7917 (12.7638)	-12.5325 (12.2174)	-9.5172 (11.6498)	-5.0261 (10.6315)	-8.5201 (9.2500)	-11.3548 (9.7673)
y	.8298*** (.2020)	1.0382 (.8739)	-1.6917 (1.1474)	1.0440 (.8565)	1.0175 (.7756)	1.1525 (.9078)	1.2704 (.8809)	.9951 (.8296)	.8326 (.6748)	1.1049 (.7551)
s^*	.3369 (.0573)	.9088** (.4094)	4.6653*** (1.7045)	.0257 (.3966)	-.8534 (1.4562)	.7962* (.4244)	.2088 (.3409)	.1775 (.2325)	.8108** (.3495)	.8230** (.3290)
Model χ^2 Test p-value										
y	.8601	.0008	.3006	.1976	.2329	.0125	.0319	.0041	.0000	.0001
$(\dot{e} - \dot{e}^*)$.0097	.0249	.1376	.0015	.0448	.0827	.0404	.4862	.0984	.1217
\dot{p}	.0002	.0020	.0096	.4757	.2697	.0857	.0107	.3841	.0397	.0226

Here, 'All' indicates all countries, while B=Brunei, C=Cambodia, I=Indonesia, L=Laos, M=Malaysia, P=Phillippines, S=Singapore, T=Thailand, V=Vietnam, and *const* denotes the intercept. Subscripts *, ** and *** are used to indicate significance at the 10%, 5% and 1% level, respectively. Standard errors are reported between parentheses.

Overall, we find sizable and often significant effects of foreign demand on home demand, and also that an often significant part of exchange rate differentials is explained by interest rate differentials (note that a coefficient of say 0.002 on interest rate differentials in the exchange rate equation implies that a 1% difference in interest rates brings about a .2% change in exchange rate differentials). Our estimates suggest that exchange rate differentials contemporaneously influence inflation rates. Table 4 is particularly informative about the robustness of results to particular countries. Upon comparing coefficient estimates, we note that Singapore’s competitiveness and potentially their monetary policy may be different, that excluding Cambodia changes the fiscal policy and the overall conclusions from the Phillips curve, but that the exchange rate rule is quite similar across countries. Examining the data plots shows that Cambodia is among the ASEAN countries with high GDP growth and low inflation, which could explain the different behavior. To assess the plausibility of these comments above, we performed several specification checks; Table 5 presents results from the model with lowest mean squared error (MSE).

GDP Growth \dot{y}		
\dot{r}	.0026	(.0045)
\dot{f}	-.0002	(.0115)
$d_{Camb} \times \dot{f}$.1849	(.1319)
\dot{y}^*	.7553	(1.0385)
\dot{c}^*	.0821	(.0619)
$d_{Sing} \times \dot{c}^*$	-.8298*	(.4716)
Exchange Rates ($\dot{e} - \dot{e}^*$)		
intercept	-.0003	(.0024)
$i - i^*$.0019***	(.0005)
Inflation \dot{p}		
intercept	-12.2449***	(4.2495)
y	1.5039***	(.3803)
$d_{Camb} \times y$	-1.2832***	(.4688)
dep	.4773	(.8449)
Model χ^2 Test p-value		
y	.7728	
$(\dot{e} - \dot{e}^*)$.0001	
\dot{p}	.0012	

Table 5: Baseline Specification (14)-(16) with spatial heterogeneity.

Here, d_{Camb} , d_{Sing} indicate dummy variables for Cambodia, and Singapore, respectively.

Our estimation results suggest that fiscal instruments are not intensively used, and this is a result that seems to hold true for most countries, likely due to running budget surpluses; therefore, we drop fiscal deficits from the analysis. The finding related to small coefficients of the interest rates was investigated through alternative specifications of the aggregate demand as well as random coefficient models; the finding seems to be robust across specifications.¹⁵

The new model with no fiscal component is more robust to excluding different countries, although it still reveals issues in estimating the competitiveness parameter. However, such issues are well

¹⁵The interested reader can get these results from the authors.

documented in the empirical literature. We thus use model (14)-(16) and its estimates from the first column in Table 4, but we set $\delta = \eta = 0$ in Sections 5 and 6.1 to analyze gains from monetary cooperation. In Sections 6.2/6.3 we chose the parameter that models the effect of fiscal instruments on output, $\eta = 1/8$. This parameter is chosen here differently from zero in order to analyze gains from fiscal cooperation as well. It is chosen somewhat lower than its value observed in EU studies. We will see that the qualitative results from Sections 5/6.1 and 6.2, where we just compare again the full (non) cooperative case, are the same. In our preliminary study on the effects coalition formation within a monetary union can have on economic performance, studied in Section 6.3, it is important (and realistic) to assume that η differs from zero.

4 Limitations of the Simulation Model

In the previous section we estimated a small-scale open-economy dynamic general equilibrium model. This model is widely accepted in the literature as describing some key relationships between a number of important macro-economic variables. We estimated this model using historical data. This implies that our estimates depend on the pursued past policies and that it is assumed that these policies did not change over time. Further, only in case policy-makers will use the same strategy in the future it makes sense to use this model for forecasts. This observation is well-known in the literature as the Lucas critique. To obtain estimated models that also can be used to predict economic behavior under different policy regimes one could try to model the behavior of individual economic agents, estimate this, and then aggregate this on a macro-level. This would provide us then with a micro-founded macro model and might provide a model from which it is more clear to understand the economic mechanisms driving the results. We did not follow this track here for three reasons. First of all, this would require a much more detailed study and estimation, from which it is not clear at this moment whether this is feasible given the available data. Second, in the literature there is a large dispute whether this aggregation on micro-level makes sense at a macro-level. That is, there is some reasonable doubt whether these micro-founded models make sense at a macro-level. Third, it is not our intention to present accurate forecast of economic variables or present accurate optimal policies for the nearby future. We are merely interested in analyzing the consequences of future (non)-cooperative behavior on economic performance under different scenarios that are robust w.r.t. the underlying model assumptions.

It is often argued that policy rules that are linear functions of the underlying state variable of the model are used by actual policy makers (Taylor rule). We will make this assumption here too and assume that in our estimated model these policy rules have been used and restrict the analysis to a study of policy rules that are linear functions of the state variable. Based on the information structure the policymakers have about the system one arrives at different "equilibrium" policy rules. Since a number of application studies have shown that the differences in terms of performance between the most frequently used "equilibrium" rules is usually not that large, we will assume that the information structure policy makers have about the game is of the open-loop type (see e.g. Engwerda (2005)). The big advantage of this assumption is that one can often still derive analytic results, which is usually not the case under other information structures. As we will see in the next section this is also the case here. So in this way we can compensate somehow for the above mentioned shortcomings and obtain more robust conclusions. Of course also the assumption that the policy makers use policy rules based on an equilibrium concept is just another mathematical abstraction of reality and has as such its limitations too.

One of the interesting points of our estimation procedure is that it reports that at this moment economic policy in ASEAN countries is characterized by a lack of use of fiscal instruments. The parameter, characterizing the effect of fiscal policy on the economic variables, was estimated to be zero in our model. Obviously, when creating a SEAMU, countries cannot rely just on the monetary policy set by the common central bank to cope, e.g., with country specific economic shocks. So under a SEAMU fiscal policies will be needed to deal with economic shocks. This implies a structural change in the pursued policies and, in the light of the above mentioned Lucas critique, it might be that separate from the fiscal deficit parameter η estimates of the other parameters change somewhat too.

5 The Simulation Model

Consider thus the model (14) in levels, together with (15)-(16), as a benchmark model, with parameter values summarized in Table 6.

Equation (14)				
α	γ	η	ϕ	δ
0	-.31	0	.7707	0
Equation (15)				
β_0	β_1			
0	.0013			
Equation (16)				
β_0	ζ	ψ		
-5.8583	.8298	.3369		

Table 6: System Estimates

By definition we have that for an arbitrary variable x $\sum_{i=1}^N (x_i - x_i^*) = 0$. Using this, with $\tilde{e}_i(t) := e_i(t) - e_i^*(t)$, the basic empirically found model (hence, without the country and equation specific effects and idiosyncratic errors) can be rewritten as

$$y_j(t) = \alpha + \gamma(i_j(t) - \dot{p}_j(t)) + \eta f_j(t) + \phi y_j^*(t) + \delta(\tilde{e}_j(t) + p_j^*(t) - p_j(t)), \quad (17)$$

$$\dot{\tilde{e}}_j(t) = \beta_1(i_j(t) - i_j^*(t)), \quad (18)$$

$$\dot{p}_j(t) = \bar{\beta}_0 + \zeta y_j(t) + \psi(\dot{\tilde{e}}_j(t) + \dot{p}_j^*(t)), \quad j = 1, \dots, N. \quad (19)$$

Next, let $x = [\tilde{e}_1 \dots \tilde{e}_{N-1} \ p_1 \dots p_N]^T$. Then the above model can be rewritten in state-space form as outlined in Appendix A and B, with x as the state of the dynamic system. This model is used to derive both the analytical and simulation results presented here and in Section 6.

5.1 The general model lacking fiscal policies

In this subsection, we analyze the model in the absence of fiscal policies. For analyzing impulse responses, we need to establish the existence and multiplicity of equilibria for the two most important scenarios we focus on: no cooperation and cooperation of all monetary authorities.

Given the fact that both parameters η and δ were estimated to be zero in Section 3, we analyze here the following model:

$$y_j(t) = \alpha + \gamma(i_j(t) - \dot{p}_j(t)) + \phi y_j^*(t), \quad (20)$$

$$\dot{\tilde{e}}_j(t) = \beta_1(i_j(t) - i_j^*(t)), \quad (21)$$

$$\dot{p}_j(t) = \bar{\beta}_0 + \zeta y_j(t) + \psi(\dot{\tilde{e}}_j(t) + \dot{p}_j^*(t)), \quad j = 1, \dots, N. \quad (22)$$

At this moment we do not make any further assumptions on the numerical values of the remaining model parameters. Below, we recall some main conclusions of a theoretical study based on the structure of model (22), from which the details can be found in Appendices C and D. The study is about existence and behavior of equilibria under a non-cooperative and cooperative mode of play, respectively.

Noncooperation

In Appendix C, Corollary 7.3, we show that for an arbitrary choice of the model parameters, this game has at most one non-cooperative open-loop Nash (OLN) equilibrium.

Next consider the special case that monetary authorities have the same cost function across countries, yielding $\alpha_j^M = \alpha^M$, $\beta_j^M = \beta^M$ and $\chi_j^M = \chi^M$ for $j = 1, \dots, N$. Corollary 7.7 shows that if the variables (eigenvalues) λ_i , $i = 1, 2$, exist as a real number the game has an equilibrium and otherwise not. As to be expected all parameters that occur in the model as well as the combination of weights chosen by the monetary policymaker in the cost function and the used discount factor together determine whether a non-cooperative OLN exists or not. Only the weights chosen by the fiscal policy makers are irrelevant.

Theorem 7.9 describes the evolution of price and exchange rate differential paths in case an equilibrium exists. In case an equilibrium exists, and an asymmetric price shock occurs, the equilibrium price paths and exchange rate differentials are (generally) characterized by a constant growth which is smaller than half the discount factor ($\frac{1}{2}\theta$) used by the policy makers in their cost function. If a symmetric price shock occurs and an equilibrium exists a different equilibrium behavior may occur. If a symmetric shock occurs it may also happen that prices converge to a new constant level whereas exchange rate differentials are characterized by a linear growth. Which equilibrium behavior will occur under a symmetric price shock depends on the chosen weights in the cost function. If $\chi^M > d_1(d_2 - d_1)\beta^M$ (where $d_1(d_2 - d_1) = 0.2093$ for our estimated model parameters) one will observe a stabilizing price path. Otherwise, an inflationary adaptation regime occurs. So, the relative weight the monetary authority attaches to interest versus output stabilization determines which behavior occurs. If interest stabilization is the most important issue, the stabilizing price path occurs.

From Corollary 7.11 in Appendix C it follows that in case the game has an equilibrium, an increase in weight α^M or a decrease in either β^M or χ^M may result in a situation where equilibrium ceases to exist. Figure 5.1 plots for both $\theta = 0.05$ and $\theta = 0.1$, the set of β^M and χ^M parameters for which an equilibrium exists (assuming $\alpha^M = 1$). The figure illustrates that the smaller θ is, the smaller is the set of parameters for which an OLN equilibrium exists. So if policymakers care less about the future it is more likely that a non-cooperative equilibrium occurs.

Moreover, a direct consequence of Corollary 7.11 is that for the current model parameter estimates and $\chi^M > 0.2093\beta^M$ the monetary authority can obtain a faster convergence of price and exchange

rate differentials towards their new settings compared to the current status quo if it either attaches more weight to the stabilization of interest rate or output, or by giving less weight to the stabilization of prices. In case the current status quo is characterized by a situation where $\chi^M < 0.2093\beta^M$ such a faster convergence can be achieved by either giving more weight to output or price stabilization, or by giving less weight to interest rate stabilization.

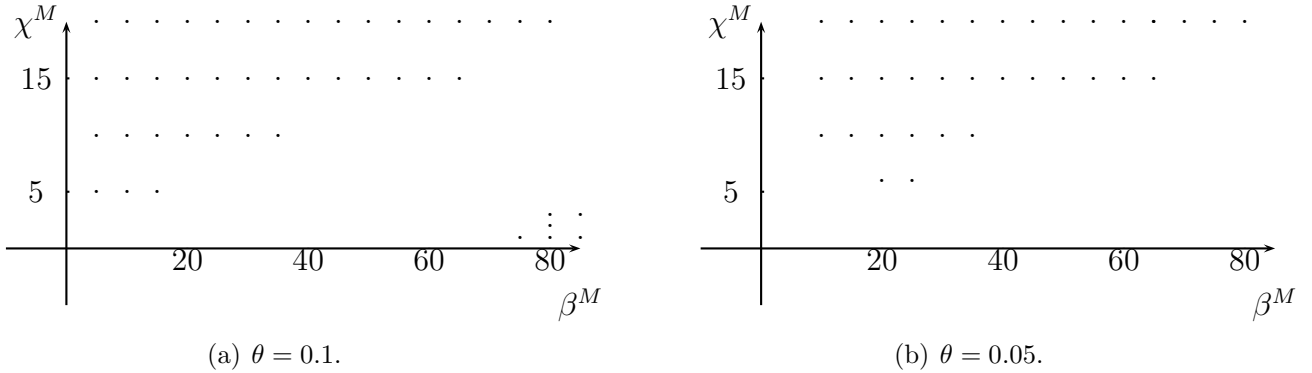


Figure 5.1: Dotted area: Cost weights yielding an OLN equilibrium with benchmark parameter estimates and $\alpha^M = 1$.

To assess the impact of the number of countries on the existence of an equilibrium, we perform a simulation experiment. The results are gathered in Table 7. In this experiment we used the estimated model parameters, $\alpha^M = \beta^M = 1$, $\chi^M = 2$ and $\theta = 0.1$ ¹⁶ The table indicates that either a too small or too large number of countries involved may lead to the non-existence of an equilibrium. So under those conditions the different policy-makers keep on reacting on each-other's policies without arriving at a final status quo.

N	equilibrium
6	no
7	no
8	no
9	yes
10	no

Table 7: Effect of number of countries on equilibrium existence

Monetary Cooperation

¹⁶These results are obtained by verifying whether the eigenvalues mentioned in Corollary 7.7, Appendix C, are both negative. Note that, while in principle simulations can be performed for any parameter configuration, we believe such results will not provide additional insights here.

Next, we analyze the monetary cooperation scenario: when all policy-makers cooperate to find the optimal policy to mitigate an economic shock. Since we restrict our setting, as before, to the symmetric case, we consider the social outcome (i.e. the case that all concerns of the different policy-makers are equally weighted).¹⁷ Even though fiscal policies are not used as an instrument to tackle shocks, monetary policy is affected by the concerns of fiscal authorities about the development of output and prices, and our scenario hypothesizes that monetary authorities give these concerns the same importance as their own concerns, except w.r.t. the use of fiscal instruments which they neglect.

An important distinction w.r.t. the non-cooperative case is that regardless what type of shock occurs, prices always converge to a new equilibrium point. Furthermore, independent of the number of participating countries and the choice of parameters, there is always a unique equilibrium. Finally, either an increase in α^i or an increase in β^i or χ^i will lead to faster convergence of prices to their new equilibrium values, ($i = F, M$).

Using the estimated parameter configurations, the resulting closed-loop cooperative model yields a small trend for the exchange rate differential of 0.002. Figure 3.3 shows that inflation shocks are only short-lived, whereas Figure 3.4 shows that exchange rates do not change much over time. Figures 3.3 and 3.4 therefore seem to support the idea that our cooperative model yields a good description of current ASEAN policy making.

6 Simulation Results

In this section we present simulation results in three different scenarios:

1. when there is no monetary union, countries pursue individual monetary policies, in the absence of fiscal policies;
2. when there is no monetary union, but both individual monetary and fiscal policies are used;
3. when there is a monetary union, but individual fiscal policies are being pursued.

6.1 Case 1: National Monetary Policies

For the nine countries with national monetary policies but no fiscal policies, there are nine central banks ($CB_i, i = 1, \dots, 9$) that play an LQ game. Every exhaustive and disjoint division of players into coalitions is called a coalition structure (CS). For 9 players there exist 21147 possible coalition structures; we only focus on the two structures, that are mostly studied in the literature due to the fact that they are most relevant for policy analysis:

- NC — the non-cooperative regime in which all the players play against each other:

$$\{\{CB_1\}, \dots, \{CB_9\}\}; \text{ and}$$

- C — the *grand coalition* in which all the players play together:

$$\{\{CB_1, \dots, CB_9\}\}$$

We consider two types of shocks - symmetric and asymmetric:

¹⁷Technical details can be found in Appendix D.

(a) Symmetric price level shock that hits all the countries with equal size.

This is modeled by choosing the initial state of the system as $x_{0S} := [0 \quad \dots \quad 0 \quad 1 \quad 1 \quad \dots \quad 1]$; and

(b) Asymmetric price level shock that hits only the first country.

This is modeled by choosing the initial state of the system as $x_{0A} := [0 \quad \dots \quad 0 \quad 1 \quad 0 \quad \dots \quad 0]$.

The loss functions are parameterized as follows: $\alpha_j^F = 1$; $\beta_j^F = 1$; $\chi_j^F = 2$; $\alpha_j^M = 1$; $\beta_j^M = 1$; $\chi_j^M = 1$; $\alpha_U^M = 1$; $\beta_U^M = 1$; and $\chi_U^M = 1$. Optimal losses for both a symmetric and asymmetric shock are presented in Table 8.

Player	N	C	N	C
CB_1	11238.3348	521.0502	11703.1069	498.6518
$CB_i, i = 2, \dots, 9$	11238.3348	521.0502	11048.4801	524.7885

Table 8: Optimal losses for the case of national monetary policies, symmetric shock (LHS) vs. asymmetric shock (RHS)

Table 8 shows that in a cooperative regime, losses of the country that is hit by an asymmetric price shock are lower than those for the countries that are not hit by this shock; such phenomenon does not occur under a non-cooperative regime.

6.1.1 Non-cooperative regime

Sample dynamics of exchange rates and price levels for the non-cooperative regime are presented in Figure 6.1. Like in all next figures the horizontal axis represents the time line (in years). We focus only on the exchange rates and price levels for country 1 and 2 in these figures, because in our asymmetric benchmark simulation country 1 is the country that is hit by the price level shock. So the graphs for country $i, i = 3, \dots, 9$, coincide with that for country 2. The graphs in Figure 6.1 illustrate an adjustment process that are in line with our findings of Section 5, that is, an exponential growth of prices and exchange rates if an asymmetric shock occurs and convergence of prices to some new level if a symmetric shock occurs. The corresponding sample dynamics of output gaps and control instruments for the same regime are presented in Figure 6.2. These graphs are again in line with the results of Section 5.

6.1.2 The grand coalition

Sample dynamics of exchange rates and price levels for the grand coalition are presented in Figure 6.3, whereas the sample dynamics of output gaps and control instruments for the same regime are presented in Figure 6.4. These graphs illustrate again the results from Section 5. That is, prices that now always converge towards a new equilibrium value, irrespective of the kind of shock that occurs.

6.2 Case of national monetary and fiscal policies

In our estimated model, there is no effect of fiscal instruments on output, likely due to the fact that in the past years fiscal instruments were not used to cope with economic shocks. If ASEAN countries

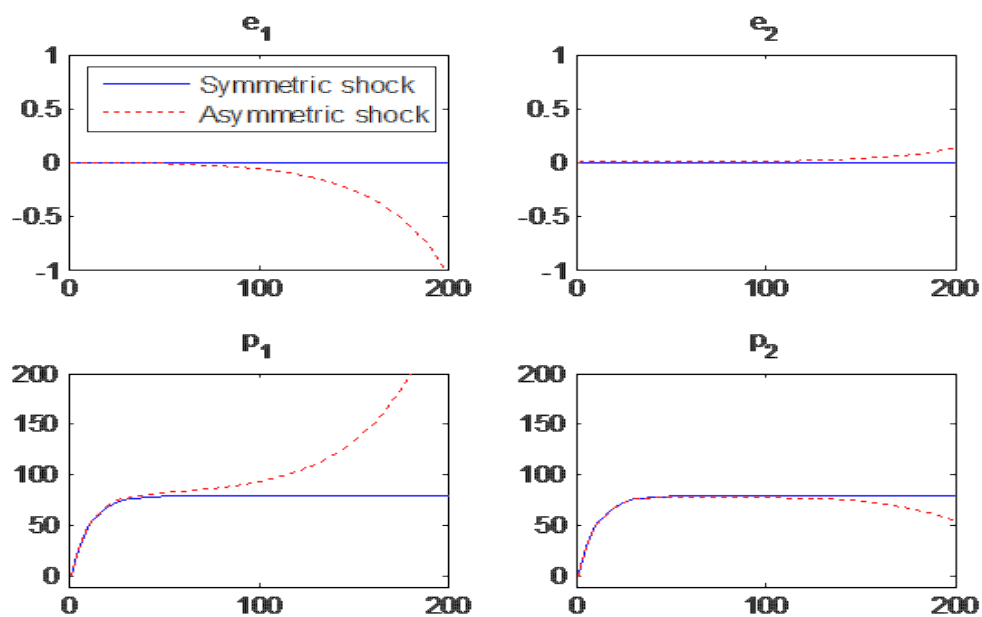


Figure 6.1: Sample exchange rates and price levels, national monetary policies: the non-cooperative regime

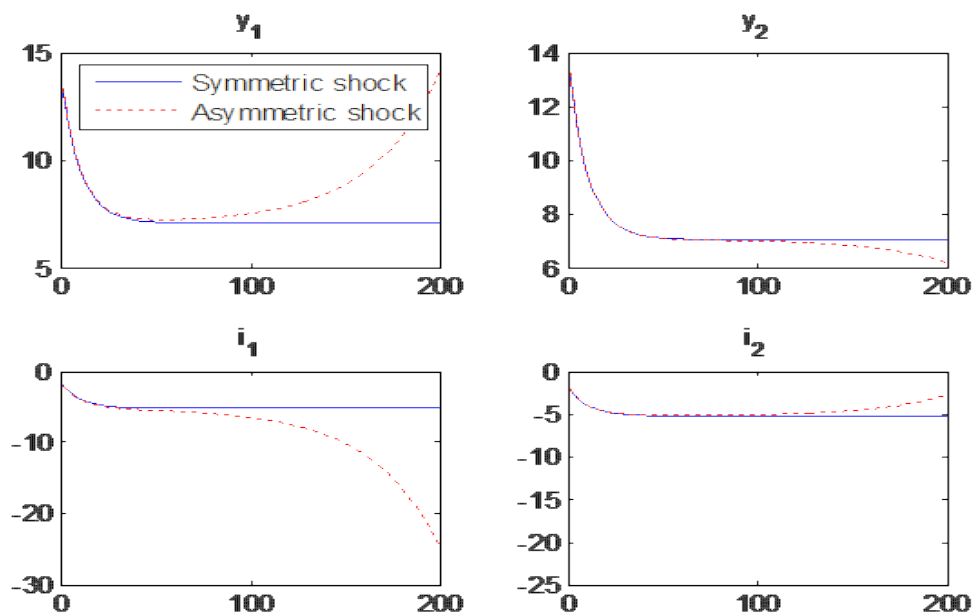


Figure 6.2: Sample output gaps and control variables, national monetary policies: the non-cooperative regime

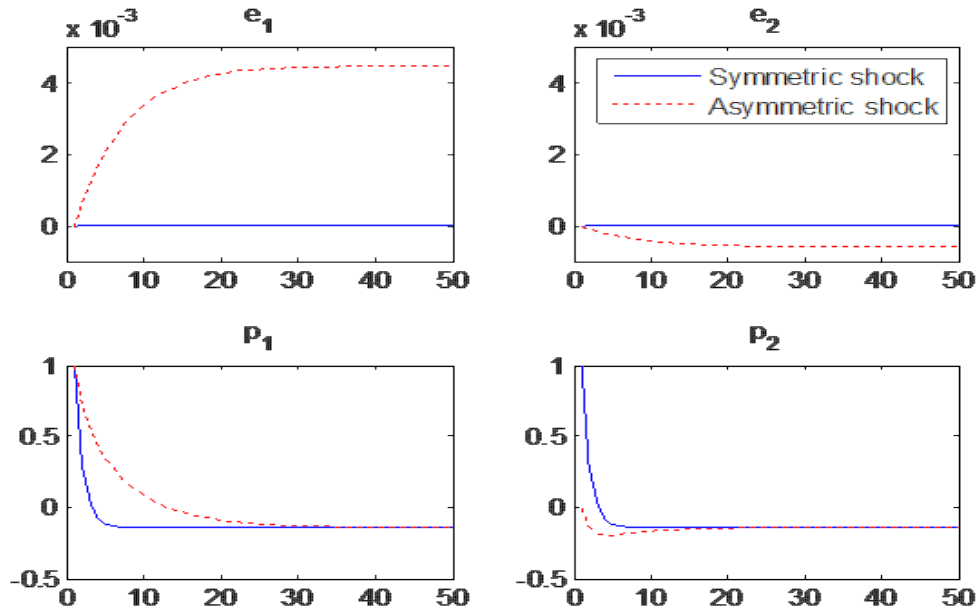


Figure 6.3: Sample exchange rates and price levels, national monetary policies: the grand coalition

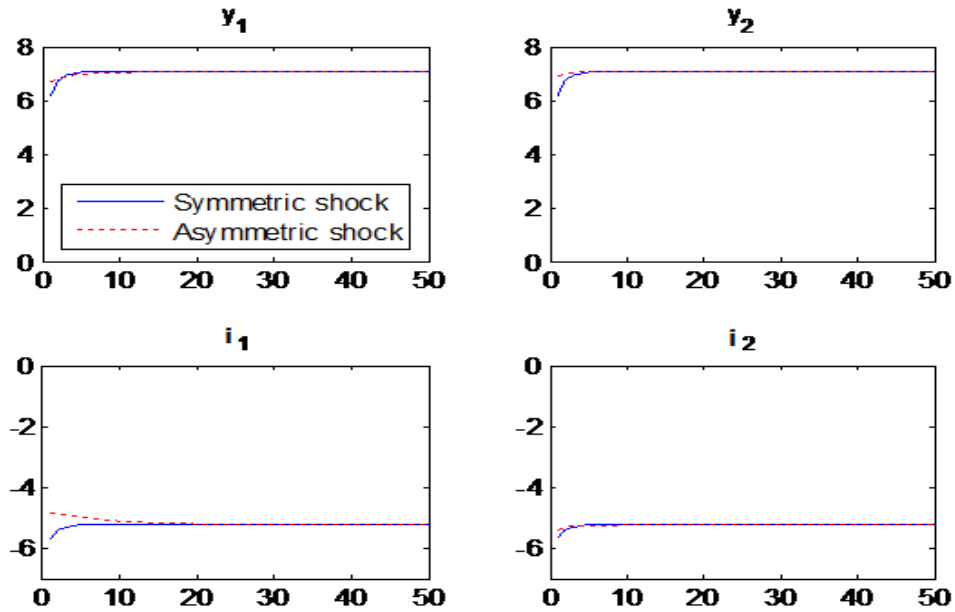


Figure 6.4: Sample output gaps and control variables, national monetary policies: the grand regime

would form a monetary union, it is hard to imagine that participating countries will not use their fiscal instruments if hit by a country specific price shock. Therefore, based on parameter values obtained for other countries such as European ones, we calibrate $\eta = \frac{1}{8}$.

Player	N	C	$\{F\}CB$	$F\{CB\}$	$\{F\}\{CB\}$
$F_i, i = 1, \dots, 9$	9363.7119	276.5691	2077.7552	244.1103	276.9911
$CB_i, i = 1, \dots, 9$	9458.7378	448.9673	276.1023	527.6308	448.8899
Player	N	C	$\{F\}CB$	$F\{CB\}$	$\{F\}\{CB\}$
F_1	9890.7752	272.2497	2000.6522	244.2389	273.9849
CB_1	9994.0481	428.7797	290.9468	504.8956	431.5522
$F_i, i = 2, \dots, 9$	9187.2802	281.5022	2071.7061	248.9385	281.7456
$CB_i, i = 2, \dots, 9$	9280.4469	453.4449	277.8544	531.4473	453.0123

Table 9: Optimal losses for the case of national monetary and fiscal policies, symmetric shock (top) vs. asymmetric shock (bottom)

With this calibration, we consider the consequences of countries involved in active fiscal policies. For 9 countries that maintain both national monetary and national fiscal policies, there are 18 players that play the LQ game - 9 central banks ($CB_i, i = 1, \dots, 9$) and 9 governments ($F_i, i = 1, \dots, 9$). Since there are too many possible coalitions, many of them being not realistic from a practical point of view, we focus on five important coalition structures:

- NC — the non-cooperative regime in which all the players play against each other:

$$\{\{CB_1\}, \{F_1\}, \dots, \{CB_9\}, \{F_9\}\}$$

- C — the *grand coalition* in which all the players play together:

$$\{\{CB_1, F_1, \dots, CB_9, F_9\}\}$$

- $\{F\}CB$ — all the fiscal players in a coalition playing against individual central banks:

$$\{\{F_1, \dots, F_9\}, \{CB_1\}, \dots, \{CB_9\}\}$$

- $F\{CB\}$ — all the central banks in a coalition playing against individual governments:

$$\{\{F_1\}, \dots, \{F_9\}, \{CB_1, \dots, CB_9\}\}$$

- $\{F\}\{CB\}$ — all the governments in a coalition playing against the central banks in a coalition:

$$\{\{F_1, \dots, F_9\}, \{CB_1, \dots, CB_9\}\}.$$

The same shocks as in the previous section are considered and the optimal losses are presented in Table 9.

From Table 9 we see that fiscal players gain when central banks cooperate. In case central banks cooperate we see that for any type of shock, it does not make much difference whether all players cooperate or there is just cooperation between fiscal and monetary policymakers separately. Furthermore, if monetary authorities cooperate fiscal players are best off if they choose not to cooperate. We also note that the country hit by an asymmetric shock suffers the least from it if some kind of cooperation occurs between either fiscal and/or monetary authorities, and it is in fact better off than the fiscal players that are not hit by this shock.

6.2.1 Non-cooperative regime

Sample dynamics of exchange rates and price levels for the non-cooperative regime are presented in Figure 6.5. We see a similar impulse response as in Figure 6.1 from Section 6.1, with exponential growth of prices and exchange rates in case of an asymmetric shock and convergence of prices to a new equilibrium level in case of a symmetric shock. Nevertheless, the growth under an asymmetric shock is larger, whereas the new equilibrium price level under a symmetric shock is lower. Thus, the effect of having additionally fiscal players in the non-cooperative setting seems to be that symmetric shocks are better absorbed, whereas the asymmetric shocks are dealt with worse. So, basically, the effects that were already observed in Section 5 are intensified. These effects are also confirmed by the adjustments of output gaps and control instruments shown in Figure 6.6. The asymmetric shock requires now by far more control effort compared to Figure 6.2.

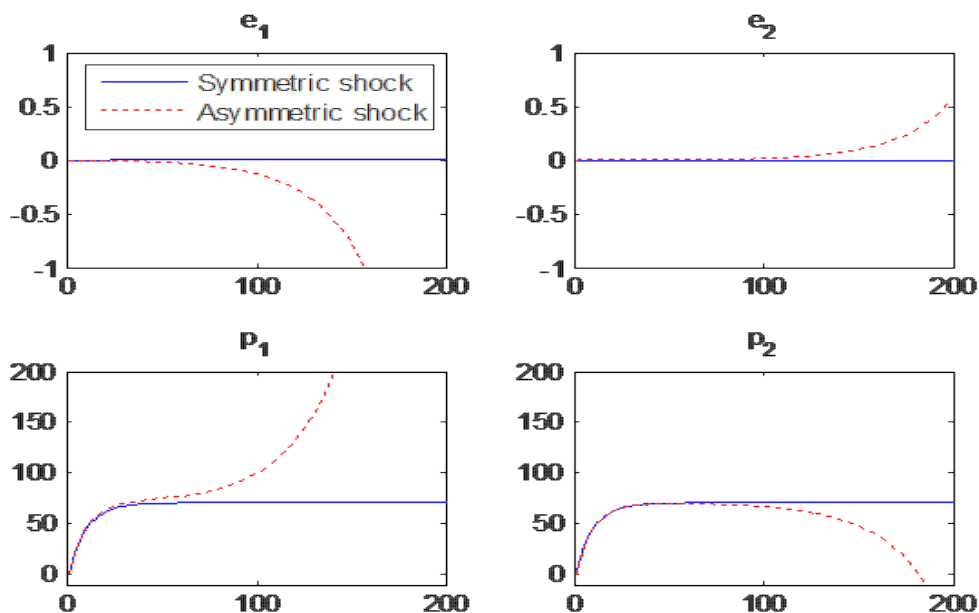


Figure 6.5: Sample exchange rates and price levels, national monetary and fiscal policies: the non-cooperative regime

6.2.2 The grand coalition

Sample dynamics of exchange rates and price levels for the grand coalition are presented in Figure 6.7. This figure shows that the adjustment towards the new (higher) equilibrium value under a symmetric and asymmetric shock almost coincide. Comparing Figures 6.3 and 6.7, we observe that with fiscal intervention, prices converge to an equilibrium value that differs more from the original value. In fact the adjustment process under a symmetric shock with and without the consideration of fiscal intervention almost coincide, whereas the adjustment process under an asymmetric shock is faster when countries use active fiscal policies. These effects are also visible in the graphs of output gaps and control instruments (Figure 6.8 vs. Figure 6.4).

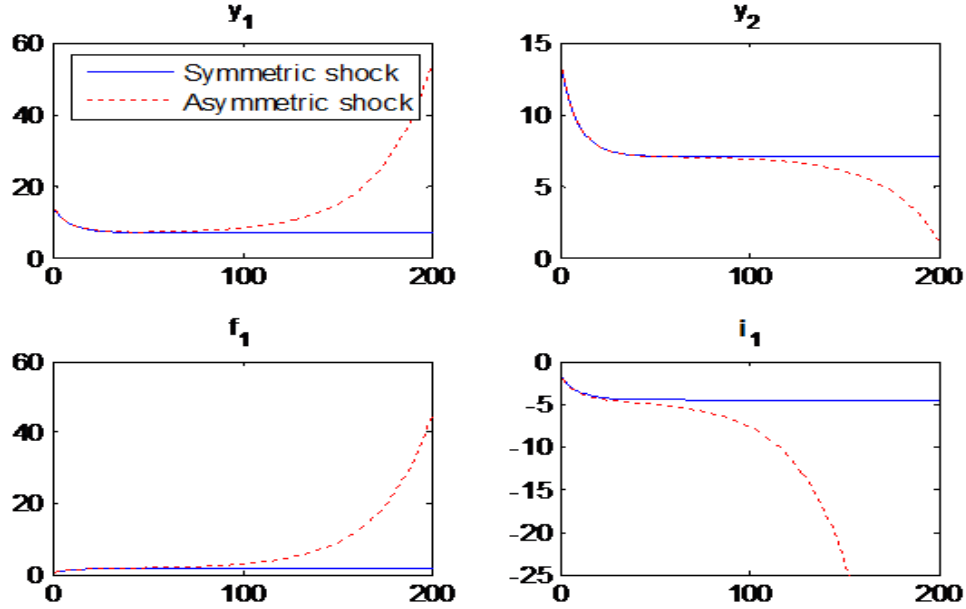


Figure 6.6: Sample output gaps and control variables, national monetary and fiscal policies: the non-cooperative regime

So grosso modo these simulations of Sections 6.2.1 and 6.2.2 suggest that even in case fiscal instruments are actively used one can expect a similar adaptation process of prices and exchange differentials like we observed in Section 5 under a non-cooperative and a full cooperative regime.

6.3 Monetary union

When a monetary union is created, there are 10 players altogether that play an LQ game (i.e. 9 governments and 1 union central bank). Consider as before the following three important coalition structures:

- NC — the non-cooperative regime in which all the players play against each other:

$$\{\{F_1\}, \dots, \{F_9\}, \{CB\}\}$$

- C — the *grand coalition* in which all the players play together:

$$\{\{F_1, \dots, F_9, CB\}\}$$

- $\{F\}CB$ — all the fiscal players in a coalition playing against the central bank:

$$\{\{F_1, \dots, F_9\}, \{CB\}\}.$$

The optimal losses for both shocks considered are presented in Table 10.

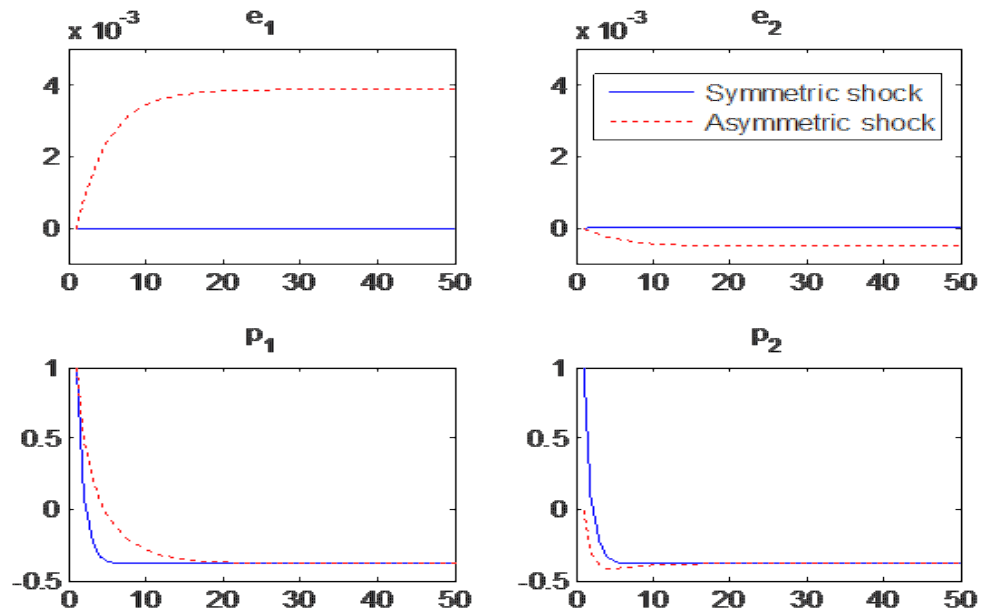


Figure 6.7: Sample exchange rates and price levels, national monetary and fiscal policies: the grand coalition

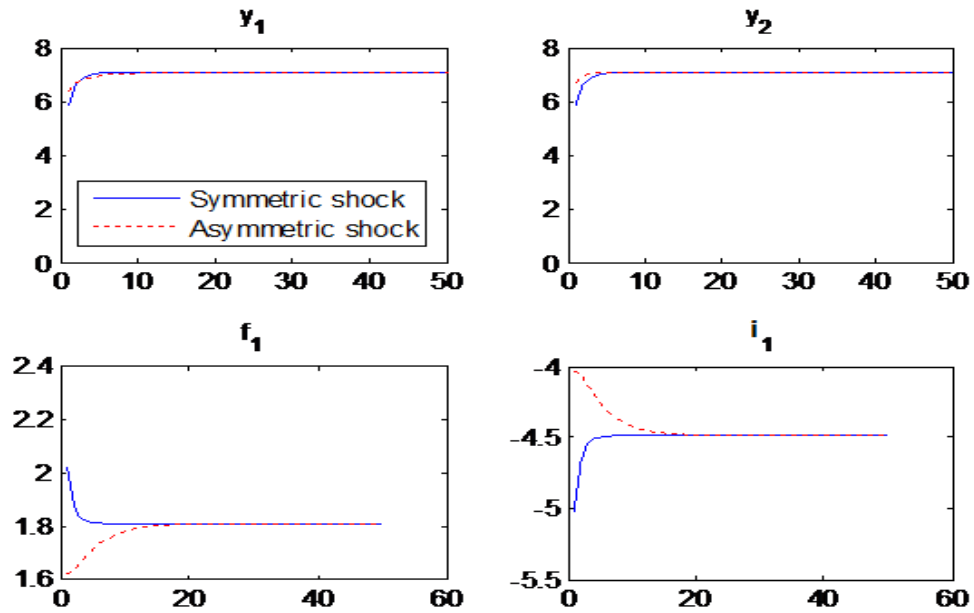


Figure 6.8: Sample output gaps and control variables, national monetary and fiscal policies: the grand coalition

Player	N	C	$\{F\}$
$F_i, i = 1, \dots, 9$	244.1103	243.4665	276.9911
CB_U	527.6308	512.0206	448.8899
Player	N	C	$\{F\}$
F_1	251.7005	244.0778	271.2793
$F_i, i = 2, \dots, 9$	248.4597	248.5424	282.3203
CB_U	528.3017	512.7220	450.4534

Table 10: Optimal losses for the case of monetary union, symmetric shock (top) vs. asymmetric shock (bottom)

From Table 10, we see that under a SEAMU fiscal authorities in fact loose when they cooperate against the central bank, which gains. Furthermore, under full cooperation there is only a small gain to be obtained for fiscal players under a symmetric shock. Under an asymmetric shock, the country that is hit by this shock and the central bank profit most from full cooperation. On the other hand the central bank can profit much more if it stays outside the fiscal coalition. So there is a conflict of interests between fiscal authorities and monetary authorities concerning the question whether they should cooperate or not when an asymmetric shock occurs.

Also notice again that in case fiscal authorities of a country hit by an asymmetric shock cooperate with other fiscal authorities, those countries will bear a larger cost than the country that is hit by the shock.

There are many differences to the non-SEAMU case we considered in Section 6.2, Table 9. First, if central banks cooperate in the non-SEAMU case, regardless of the coalition structure, there is no scenario where both fiscal players and central banks are better off. Regardless of the type of shock, either central banks gain or fiscal players gain, but not both simultaneously. If a grand coalition is formed, the fiscal players gain and the central banks loose as compared to SEAMU. If an asymmetric shock occurs, the fiscal player that is hit by the shock is under a SEAMU best off in the grand coalition, whereas in the non-SEAMU case he is best off in a situation where central banks cooperate and fiscal players act individually. The central bank(s) prefer to play against a fiscal coalition in both cases.

6.3.1 Non-cooperative regime

Sample dynamics of exchange rates and price levels for the non-cooperative regime are presented in Figure 6.9. Compared to the non-cooperative regime with no SEAMU, with active fiscal policies, we observe that if a symmetric shock occurs prices almost instantly adapt to almost the original prices. Furthermore, the price divergence in case an asymmetric shock occurs is less pronounced. Sample dynamics of output gaps and control instruments for the same regime are presented in Figure 6.10.

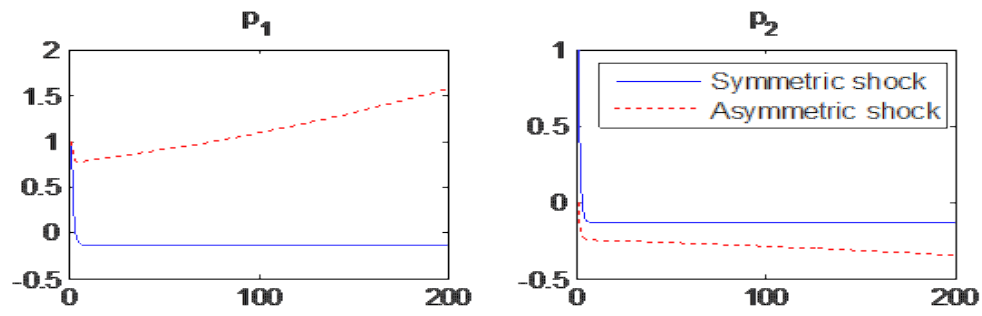


Figure 6.9: Sample exchange rates and price levels, monetary union: the non-cooperative regime

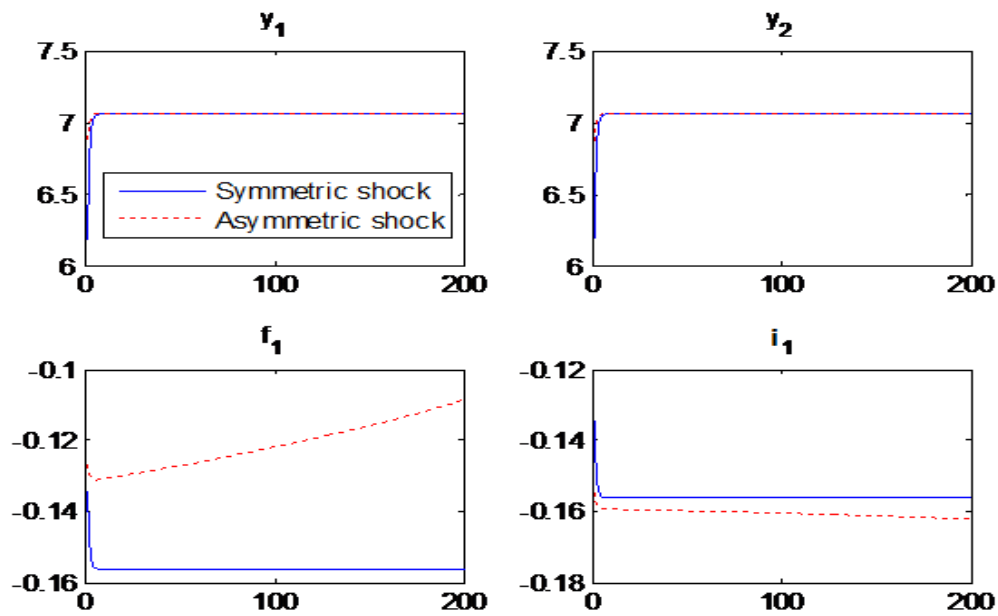


Figure 6.10: Sample output gaps and control variables, monetary union: the non-cooperative regime

6.3.2 The grand coalition

Sample dynamics of exchange rates and price levels for the grand coalition are presented in Figure 6.11. Sample dynamics of output gaps and control instruments for the same regime are illustrated in Figure 6.12. Compared to the non-cooperative scenario, we see that for a symmetric shock the convergence speed to the new price level is smaller and that this new price level also differs more from the original price level than in the non-cooperative situation. Compared to the non-SEAMU case in which countries use active fiscal policies, we observe almost the same price path in case a symmetric shock occurs. For an asymmetric shock we see that the price path to a new equilibrium level is much slower for the country that is hit by the shock, whereas for the other countries the differences in the price paths are much less pronounced. Therefore, the fact that the country hit by the shock cannot use its monetary instrument in this case implies that the adaptation process towards its new equilibrium price level is much slower.

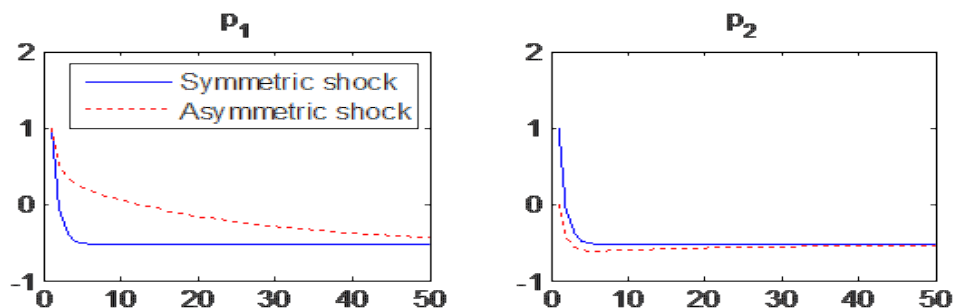


Figure 6.11: Sample exchange rates and price levels, monetary union: the grand coalition

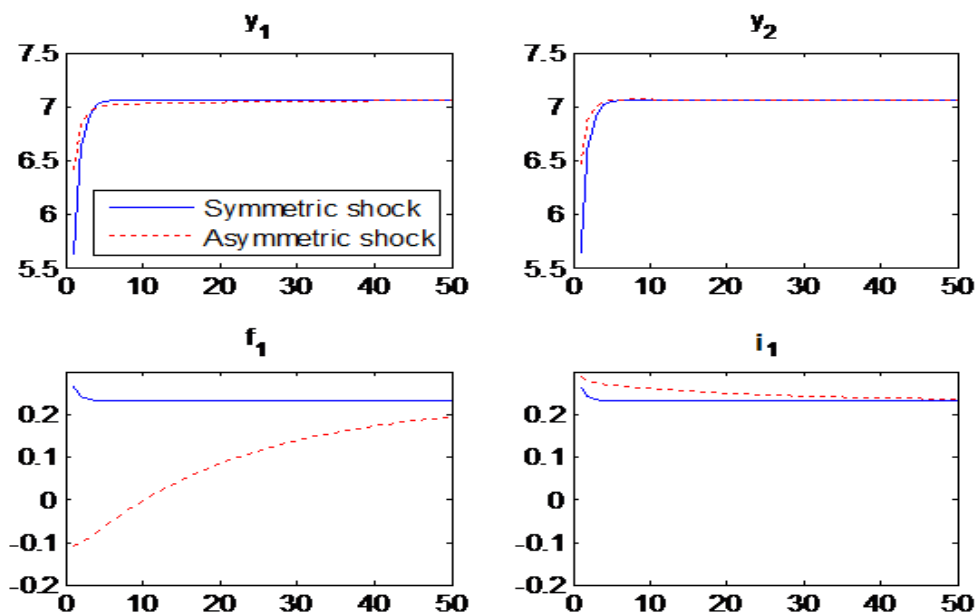


Figure 6.12: Sample output gaps and control variables, monetary union: the grand coalition

7 Concluding Remarks

In this paper we study pros and cons of further economic integration of the ASEAN countries in the framework of a monetary union. Our contribution is twofold. First, we estimate a small scale model for ASEAN countries except Myanmar.¹⁸ Second, we use it in a dynamic game setting to shed some light on the above issue.

An important conclusion from our estimation procedure is that ASEAN countries did not use fiscal instruments much in the considered estimation period to control their economies.

Based on the structure of the estimated model we next performed a theoretical study about the existence of equilibria and the closed-loop behavior of the model under different cooperation structures. The study shows that the model predicts a behavior of inflation and exchange rate differentials which seems to be in line with real observations, if we assume that ASEAN countries cooperate.

An additional simulation study shows that the global dynamic behavior of price and exchange rate differential paths is not changed if countries actively use fiscal instruments. This implies that the above conclusion that a cooperative mode of play in our model fits best with actual observations is robust w.r.t. the assumption whether fiscal policies have been actively used or not.

After that we continued our simulation study to assess the effects of different shocks in different coalition structures and the desirability of monetary cooperation. The impulse response functions for various shocks, symmetric or asymmetric, were calculated for different coalition structures with and without a monetary union using TOOLBOX, a numerical toolbox developed by Michalak, Engwerda, and Plasmans (2011) to calculate Nash equilibria for linear quadratic differential games if players have open-loop information.

¹⁸due to a lack of data for this country.

The main conclusion from this study is that there are substantial gains from cooperation between central banks in the ASEAN countries. Whether this should be in the form of a SEAMU or not is less clear cut. Our simulations suggest that the sum of the costs involved for all monetary and fiscal players is approximately the same under a SEAMU regime and a regime where central banks cooperate. The individually incurred cost may however differ, particularly under a grand coalition regime. A detailed study, complemented with welfare analysis, is required to evaluate the effects of different economic shocks and whether there are ways to smoothen negative effects that are solely due to the formation of a monetary union. Furthermore, it is of interest to explore to which extent the assumptions about the fiscal policy accelerator, the weights chosen in the cost functions and the exclusion of foreign trade partners affect the above conclusions.

In case ASEAN countries decide to cooperate on monetary policy we show that there is a conflict of interest on the issue whether fiscal players should cooperate with the central bank(s) or not. Fiscal authorities benefit most from a full cooperative scenario whereas cooperating monetary authorities benefit most in case they don't cooperate with fiscal authorities. The simulation study also shows that in case countries are hit by an asymmetric shock and there is some form of fiscal cooperation, the countries that are not hit by this shock suffer more than the country that is hit. So, in case this frequently happens there is probably little support to sustain such a cooperation from the point of view of the countries that are almost never hit by a shock. This supports of course the OCA idea that only ASEAN countries that have a similar economic structure and therefore are not hit too often by country specific shocks may gain from further economic cooperation.

Given this observation and the current political structure it might be good to consider, e.g., the pros and cons of a two-speed SEAMU.

Finally we like to stress the limitations of our approach again as we outlined in Section 4. Furthermore, in the context of the ASEAN countries, launching a monetary union is a hypothetical question as free trade is not yet fully established and there exists only a declaration of intent to move towards a customs union. As such our study is just one item in the process of getting insight into all expected advantages and disadvantages of forming a SEAMU (and where, for its formation, of course not only economic arguments play a role).

Appendix A

Introducing $x = [\tilde{e}_1 \cdots \tilde{e}_{N-1} p_1 \cdots p_N]^T$ model (17-19) can be rewritten as

$$y(t) = P_1 \dot{x}(t) + P_2 x(t) + P_3 y(t) + \sum_{j=1}^N P_{4j} f_j(t) + \sum_{j=1}^N P_{4(N+j)} i_j(t) + P_5 c \quad (23)$$

$$\dot{x}(t) = P_6 \dot{x}(t) + P_7 x(t) + P_8 y(t) + \sum_{j=1}^N P_{9j} f_j(t) + \sum_{j=1}^N P_{9(N+j)} i_j(t) + P_{10} c, \quad x(0) = x_0, \quad (24)$$

where $c = 1$ and, introducing $J = \frac{1}{N-1}(1_N 1_N^T - I_N)$, the $P_1 - P_{10}$ matrices are given by:

$$P_1 = \begin{bmatrix} 0_{N \times (N-1)} & -\gamma I_N \end{bmatrix}; P_2 = \begin{bmatrix} \delta I_{N-1} & -\delta(I_N - J) \\ -\delta 1_{N-1}^T & \end{bmatrix}; P_3 = \phi J; \quad (25)$$

$$P_4 = [\eta I_N \ \gamma I_N]; P_5 = \alpha 1_N \quad (26)$$

$$P_6 = \begin{bmatrix} 0_{(N-1) \times (N-1)} & 0_{(N-1) \times N} \\ \psi I_{N-1} & \psi J \\ -\psi 1_{N-1}^T & \end{bmatrix}; P_7 = [0_{(2N-1) \times (2N-1)}]; P_8 = \begin{bmatrix} 0_{(N-1) \times N} \\ \zeta I_N \end{bmatrix}; \quad (27)$$

$$P_9 = \begin{bmatrix} 0_{(2N-1) \times N} & \beta_1(I_{N-1} - J_{N-1}) & \frac{-\beta_1}{N-1} 1_{N-1} \\ & 0_{N \times (N-1)} & 0_N \end{bmatrix}, \text{ where } J_{N-1} = [I_{N-1} \ 0_{N-1}] J \begin{bmatrix} I_{N-1} \\ 0_{N-1}^T \end{bmatrix} \quad (28)$$

$$P_{10} = \begin{bmatrix} 0 \\ \bar{\beta}_0 1_N \end{bmatrix}. \quad (29)$$

Here e_j (e_j^{N-1}) is the j^{th} unit vector in \mathbb{R}^N (\mathbb{R}^{N-1}), $1_N \in \mathbb{R}^N$ denotes the vector $[1 \cdots 1]^T$, $0_N \in \mathbb{R}^N$ the vector $[0 \cdots 0]^T$, $0_{n \times m} \in \mathbb{R}^{n \times m}$ the zero matrix and $I_N \in \mathbb{R}^{N \times N}$ the identity matrix.

For analyzing the pros and cons of a monetary union, we consider only the case where all countries are in SEAMU. The model is:

$$y(t) = P_1 \dot{x}(t) + P_2 x(t) + P_3 y(t) + P_4 i(t) + P_5 c \quad (30)$$

$$\dot{x}(t) = P_6 \dot{x}(t) + P_7 x(t) + P_8 y(t) + P_9 i(t) + P_{10} c, x(0) = x_0. \quad (31)$$

Here $i(t)$ is the nominal interest rate set by the common central bank. Matrix $P_4 = \gamma 1_N$, matrix $P_9 = 0_{2N}$ and all other matrices are as specified in (23-24).

Appendix B

Let

$$\begin{aligned} \bar{c}_2 &= \frac{(N-1)\psi}{\psi + N - 1}; \hat{c}_2 := \frac{(N-2)\psi - (N-1)}{(\psi-1)(\psi + N - 1)}; \tilde{c}_2 = \frac{\psi(N-1)}{(\psi-1)(\psi + N - 1)}; \\ p_1 &= 1 + \gamma \zeta \hat{c}_2; p_2 := \gamma \zeta \tilde{c}_2 + \phi; \\ \rho_1 &= \frac{(N-2)p_2 - (N-1)p_1}{(p_2 - p_1) * ((N-1)p_1 + p_2)}; \rho_2 := \frac{(N-1)p_2}{(p_2 - p_1) * ((N-1)p_1 + p_2)}; \\ \hat{b}_1 &= \zeta \eta (\hat{c}_2 \rho_1 + \frac{\tilde{c}_2 \rho_2}{N-1}); \hat{b}_2 = \zeta \eta (\hat{c}_2 \rho_2 + \tilde{c}_2 \rho_1 - \frac{\tilde{c}_2 \rho_2 (N-2)}{N-1}); \\ \tilde{b}_1 &= \beta_1 \bar{c}_2 + \zeta \gamma (\hat{c}_2 \rho_1 + \frac{\tilde{c}_2 \rho_2}{N-1}) - \zeta \gamma \beta_1 \bar{c}_2 (\rho_1 + \frac{\rho_2}{N-1}) (\hat{c}_2 + \frac{\tilde{c}_2}{N-1}); \\ \tilde{b}_2 &= \beta_1 \bar{c}_2 + \zeta \gamma (\hat{c}_2 \rho_2 + \tilde{c}_2 \rho_1 - \frac{\tilde{c}_2 \rho_2 (N-2)}{N-1}) - \zeta \gamma \beta_1 \bar{c}_2 (\rho_1 + \frac{\rho_2}{N-1}) (\hat{c}_2 + \frac{\tilde{c}_2}{N-1}). \end{aligned}$$

Notice $\hat{c}_2 - \tilde{c}_2 = \frac{1}{1-\psi}$ and $\rho_1 - \rho_2 = \frac{1}{p_1 - p_2}$. Then

$$(I - P_6)^{-1} = \begin{bmatrix} I_{N-1} & 0_{(N-1) \times N} \\ \bar{c}_2 \begin{bmatrix} I_{N-1} \\ -1_{N-1}^T \end{bmatrix} & \hat{c}_2 I_N - \tilde{c}_2 J \end{bmatrix}.$$

Consequently,

$$\bar{P}_1 := I - P_1(I - P_6)^{-1}P_8 - P_3 = p_1I_n - p_2J, \text{ and } \bar{P}^{-1} = \rho_1I_N - \rho_2J.$$

If we define u as $u = [f_1 \cdots f_N i_1 \cdots i_N]^T$, the reduced form of the model (23-24) is then:

$$\dot{x}(t) = \hat{A}x(t) + \hat{B}u(t) + \hat{E}_1c, x(0) = x_0 \quad (32)$$

$$y(t) = \hat{C}x(t) + \hat{D}u(t) + \hat{E}_2c, \quad (33)$$

with the matrices \hat{A} - \hat{D} given by¹⁹

$$\begin{aligned} \hat{A} &= (I - P_6)^{-1}(P_8\hat{C} + P_7) = \delta\zeta(\rho_1 + \rho_2)(\hat{c}_2 + \frac{\tilde{c}_2}{N-1}) \begin{bmatrix} 0_{(N-1) \times (N-1)} & 0_{(N-1) \times N} \\ I_{N-1} & -I_N + J \\ -1_{N-1}^T & \end{bmatrix}; \\ \hat{B} &= (I - P_6)^{-1}(P_8\hat{D} + P_9) \\ &= \begin{bmatrix} 0_{(N-1) \times N} & \beta_1(I_{N-1} - J_{N-1}) & -\frac{\beta_1}{N-1}1_{N-1} \\ \hat{b}_1I_N - \hat{b}_2J & \tilde{b}_1I_N - \tilde{b}_2J & \end{bmatrix}; \\ \hat{E}_1 &= (I - P_6)^{-1}(P_8\hat{E}_2 + P_{10}) \\ &= \begin{bmatrix} 0_{N-1} \\ \frac{1}{1-\psi}(\bar{\beta}_0 + \zeta(\rho_1 - \rho_2)(\alpha - \frac{\gamma\bar{\beta}_0}{1-\psi}))1_N \end{bmatrix} = \begin{bmatrix} 0_{N-1} \\ \frac{\bar{\beta}_0(1-\phi) + \zeta\alpha}{\gamma\zeta + (1-\phi)(1-\psi)}1_N \end{bmatrix}; \\ \hat{C} &= \bar{P}_1^{-1}(P_1(I - P_6)^{-1}P_7 + P_2) = \delta(\rho_1 + \rho_2) \begin{bmatrix} I_{N-1} & -I_N + J \\ -1_{N-1}^T & \end{bmatrix}; \\ \hat{D} &= \bar{P}_1^{-1}(P_1(I - P_6)^{-1}P_9 + P_4) \\ &= \begin{bmatrix} 0_{N \times N} & \gamma\beta_1\bar{c}_2(\rho_1 + \frac{\rho_2}{N-1})(J - I_N) \end{bmatrix} + \begin{bmatrix} \eta(\rho_1I_N - \rho_2J) & \gamma(\rho_1I_N - \rho_2J) \end{bmatrix}; \\ \hat{E}_2 &= \bar{P}_1^{-1}(P_1(I - P_6)^{-1}P_{10} + P_5) = (\alpha + (\bar{c}_2 - \hat{c}_2)\bar{\beta}_0\gamma)(\rho_1 - \rho_2)1_N \\ &= \frac{\alpha(1-\psi) - \bar{\beta}_0\gamma}{\gamma\zeta + (1-\phi)(1-\psi)}1_N. \end{aligned}$$

Appendix C

In this appendix we consider the case that the model parameters η and δ are both zero. Substitution of this into the results obtained in Appendix B yields then, using the next notation,

$$\begin{aligned} d_1 &= \gamma\rho_1 - \gamma\beta_1\bar{c}_2(\rho_1 + \frac{\rho_2}{N-1}); \quad d_2 = \gamma\rho_2 - \gamma\beta_1\bar{c}_2(\rho_1 + \frac{\rho_2}{N-1}); \\ \hat{e}_1 &= \frac{\bar{\beta}_0(1-\phi) + \zeta\alpha}{\gamma\zeta + (1-\phi)(1-\psi)}; \quad \hat{e}_2 = \frac{\alpha(1-\psi) - \bar{\beta}_0\gamma}{\gamma\zeta + (1-\phi)(1-\psi)}, \end{aligned}$$

¹⁹For the relationship between the P_i matrices and the reduced form matrices one may consult e.g. Michalak, Engwerda, and Plasmans (2011).

the next expressions for the reduced form model matrices

$$\begin{aligned}
\hat{A} &= 0_{(2N-1) \times (2N-1)}; \\
\hat{B} &= \begin{bmatrix} 0_{(N-1) \times N} & \beta_1(I_{N-1} - J_{N-1}) & -\frac{\beta_1}{N-1}1_{N-1} \\ 0_{N \times N} & \tilde{b}_1 I_N - \tilde{b}_2 J & \end{bmatrix}; \\
\hat{E}_1 &= \begin{bmatrix} 0_{N-1} \\ \hat{e}_1 1_N \end{bmatrix}; \\
\hat{C} &= 0_{N \times (2N-1)}; \\
\hat{D} &= [0_{N \times N} \ d_1 I_N - d_2 J]; \\
\hat{E}_2 &= \hat{e}_2 1_N.
\end{aligned}$$

Remark 7.1 Notice that up to this moment the formal interpretation of \dot{x} is that it represents the value of $x(t) - x(t-1)$. However, since the parameter $\delta = 0$ and consequently both matrix \hat{A} and \hat{C} are zero, it can be easily shown (following Kwakernaak and R. (1972), see also Michalak (2009)[Section 4.7.3]) that the exact continuous time model that coincides at the discrete points in time with the discrete time model has the same parameters. For that reason we will interpret the above model in the continuous time setting where time is expressed in years. \square

To find the non-cooperative open-loop Nash (OLN) equilibria of this game, we follow the approach of Michalak, Engwerda, and Plasmans (2011) (see also Engwerda (2005) or Engwerda (2001)). Introducing the transformed state $\tilde{x}(t) = e^{-\frac{1}{2}\theta} [x^T(t) \ c]^T$, control $\tilde{u}(t) = e^{-\frac{1}{2}\theta} u(t)$ and output $\tilde{y}(t) = e^{-\frac{1}{2}\theta} y(t)$ the reduced form model dynamics can be rewritten as

$$\begin{aligned}
\dot{\tilde{x}}(t) &= A\tilde{x}(t) + B\tilde{u}(t), \quad \tilde{x}(0) = [x_0^T \ 1]^T; \\
\tilde{y}(t) &= [\hat{C} \ \hat{E}_2]\tilde{x}(t) + \hat{D}\tilde{u}(t),
\end{aligned}$$

where

$$\begin{aligned}
A &= \begin{bmatrix} \hat{A} - \frac{1}{2}\theta I_{2N-1} & \hat{E}_1 \\ 0_{1 \times (2N-1)} & -\frac{1}{2}\theta \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}\theta I_{2N-1} & \begin{bmatrix} 0_{N-1} \\ \hat{e}_1 1_N \end{bmatrix} \\ 0_{1 \times (2N-1)} & -\frac{1}{2}\theta \end{bmatrix} \text{ and} \\
B &= \begin{bmatrix} \hat{B} \\ 0_{1 \times (2N)} \end{bmatrix} = \begin{bmatrix} 0_{(N-1) \times N} & \beta_1(I_{N-1} - J_{N-1}) & -\frac{\beta_1}{N-1}1_{N-1} \\ 0_{N \times N} & \tilde{b}_1 I_N - \tilde{b}_2 J & \\ 0_{1 \times N} & & 0_{1 \times N} \end{bmatrix}.
\end{aligned}$$

Furthermore, the involved cost functions are given by

$$\begin{aligned}
J_i^F &= \int_0^\infty [\tilde{x}^T(s) \ \tilde{u}^T(s)] \{ \alpha_i^F e_{i+N-1}^{4N} e_{i+N-1}^{4N^T} + \beta_i^F \begin{bmatrix} \hat{C}^T \\ \hat{E}_2^T \\ \hat{D}^T \end{bmatrix} e_i^N e_i^{N^T} [\hat{C} \ \hat{E}_2 \ \hat{D}] + \chi_i^F e_{2N+i}^{4N} e_{2N+i}^{4N^T} \} \begin{bmatrix} \tilde{x}(s) \\ \tilde{u}(s) \end{bmatrix} ds \\
&=: \int_0^\infty [\tilde{x}^T(s) \ \tilde{u}^T(s)] M_i \begin{bmatrix} \tilde{x}(s) \\ \tilde{u}(s) \end{bmatrix} ds \\
J_i^M &= \int_0^\infty [\tilde{x}^T(s) \ \tilde{u}^T(s)] \{ \alpha_i^M e_{i+N-1}^{4N} e_{i+N-1}^{4N^T} + \beta_i^M \begin{bmatrix} \hat{C}^T \\ \hat{E}_2^T \\ \hat{D}^T \end{bmatrix} e_i^N e_i^{N^T} [\hat{C} \ \hat{E}_2 \ \hat{D}] + \chi_i^M e_{3N+i}^{4N} e_{3N+i}^{4N^T} \} \begin{bmatrix} \tilde{x}(s) \\ \tilde{u}(s) \end{bmatrix} ds \\
&=: \int_0^\infty [\tilde{x}^T(s) \ \tilde{u}^T(s)] M_{i+N} \begin{bmatrix} \tilde{x}(s) \\ \tilde{u}(s) \end{bmatrix} ds.
\end{aligned}$$

Here, with $\hat{d}_i = (\frac{d_2}{N-1} + d_1)e_i - \frac{d_2}{N-1}\mathbf{1}_N$,

$$M_i = \begin{bmatrix} 0_{(N-1) \times (N-1)} & & & & \\ & \alpha_i^F e_i e_i^T & & & \\ & & \beta_i^F \hat{e}_2^2 & 0_{1 \times N} & \beta_i^F \hat{e}_2 \hat{d}_i^T \\ & & 0_{N \times 1} & \chi_i^F e_i e_i^T & \\ & & \beta_i^F \hat{e}_2 \hat{d}_i & & \beta_i^F \hat{d}_i \hat{d}_i^T \end{bmatrix}, \quad i = 1, \dots, N;$$

and

$$M_{i+N} = \begin{bmatrix} 0_{(N-1) \times (N-1)} & & & & \\ & \alpha_i^M e_i e_i^T & & & \\ & & \beta_i^M \hat{e}_2^2 & 0_{1 \times N} & \beta_i^M \hat{e}_2 \hat{d}_i^T \\ & & 0_{N \times 1} & 0_{N \times N} & \\ & & \beta_i^M \hat{e}_2 \hat{d}_i & & \beta_i^M \hat{d}_i \hat{d}_i^T + \chi_i^M e_i e_i^T \end{bmatrix}, \quad i = 1, \dots, N,$$

where all non-specified matrix entries are zeros. Following the notation of Michalak, Engwerda, and Plasmans (2011) (see also Engwerda (2005) or Engwerda (2001)) we obtain from these M_i matrices the next matrices where, again, all non-specified entries are zero.

$$\begin{aligned} Q_i &= \begin{bmatrix} 0_{(N-1) \times (N-1)} & & \\ & \alpha_i^F e_i e_i^T & \\ & & \beta_i^F \hat{e}_2^2 \end{bmatrix}, \quad i = 1, \dots, N, \\ Q_{i+N} &= \begin{bmatrix} 0_{(N-1) \times (N-1)} & & \\ & \alpha_i^M e_i e_i^T & \\ & & \beta_i^M \hat{e}_2^2 \end{bmatrix}, \quad i = 1, \dots, N, \\ Z_i &= \begin{bmatrix} 0_{(2N-1) \times N} & 0_{(2N-1) \times N} \\ 0_{1 \times N} & \beta_i^F \hat{e}_2 \hat{d}_i^T \end{bmatrix}, \quad i = 1, \dots, N, \\ Z_{i+N} &= \begin{bmatrix} 0_{(2N-1) \times N} & 0_{(2N-1) \times N} \\ 0_{1 \times N} & \beta_i^M \hat{e}_2 \hat{d}_i^T \end{bmatrix}, \quad i = 1, \dots, N, \\ Z &= \begin{bmatrix} 0_{N \times (2N-1)} & 0_{N \times 1} \\ 0_{N \times (2N-1)} & d_1 \hat{e}_2 \begin{bmatrix} \beta_1^M \\ \vdots \\ \beta_N^M \end{bmatrix} \end{bmatrix}. \end{aligned}$$

Furthermore, we get

$$\begin{aligned} \tilde{B}_i^T &= 0_{2N \times 2N}, \quad i = 1, \dots, N; \\ \tilde{B}_{i+N}^T &= \begin{bmatrix} & 0_{(N+i-1) \times 2N} & & \\ \beta_1 e_i^{N-1T} (I_{N-1} - J_{N-1}) & e_i^T (\tilde{b}_1 I_N - \tilde{b}_2 J) & 0 & \\ & 0_{(N-i) \times 2N} & & \end{bmatrix}, \quad i = 1, \dots, N-1, \\ \tilde{B}_{2N}^T &= \begin{bmatrix} & 0_{(2N-1) \times 2N} & & \\ -\frac{\beta_1}{N-1} \mathbf{1}_{N-1}^T & e_N^T (\tilde{b}_1 I_N - \tilde{b}_2 J) & 0 & \end{bmatrix} \end{aligned}$$

and

$$G = \begin{bmatrix} \text{diag}(\chi_i^F) & 0_N \\ 0_N & \text{diag}(\chi_i^M) + d_1 \text{diag}(\beta_i^M)(d_1 I_N - d_2 J) \end{bmatrix}.$$

Using the above matrices one can calculate then the next matrix M , which eigenstructure determines the number of equilibria the game has.

$$M = \begin{bmatrix} A & & & \\ -Q_1 & -A^T & & \\ \vdots & & \ddots & \\ -Q_{2N} & & & -A^T \end{bmatrix} + \begin{bmatrix} -B \\ Z_1 \\ \vdots \\ Z_{2N} \end{bmatrix} G^{-1} [Z \tilde{B}_1^T \cdots \tilde{B}_{2N}^T].$$

Theorem 7.2 *Matrix M has N eigenvalues equal to $-\frac{1}{2}\theta$, $(4N - 1)N$ eigenvalues equal to $\frac{1}{2}\theta$ and the other $2N$ eigenvalues determined by an $2N \times 2N$ matrix \tilde{M} given by (34) below.*

Proof:

Since $\tilde{B}_i^T = 0_{2N \times 2N}$, $i = 1, \dots, N$ it follows that M has $2N * N$ eigenvalues $\frac{1}{2}\theta$.

Since the last column of $\tilde{B}_i^T = 0_{2N \times 1}$, $i = N + 1, \dots, 2N$ it follows that M has N eigenvalues $\frac{1}{2}\theta$.

Since row 1 until $(2N - 1)$ of Z_i is zero, $i = N + 1, \dots, 2N$ and all rows of Q_{i+N} , $i = 1, \dots, N$, except row $2N$ and $(N - 1 + i)$ are zero it follows that M has additionally $(2N - 2) * N$ eigenvalues $\frac{1}{2}\theta$.

Since column 1 until $(2N - 1)$ of Z is zero and column 1 until $(N - 1)$ of Q_i , $i = 1, \dots, 2N$ is zero it follows that M has $N - 1$ eigenvalues $-\frac{1}{2}\theta$.

Finally, since row $2N$ of B is zero it follows that M has 1 additional eigenvalue $-\frac{1}{2}\theta$.

Taking the above eigenvalues into account it follows that the remaining eigenvalues of M are obtained as the eigenvalues of the reduced matrix

$$\begin{aligned} \tilde{M} &= \begin{bmatrix} A(N : 2N - 1, N : 2N - 1) & & 0_{N \times N} \\ -Q_{N+1}(N, N : 2N - 1) & & \\ \vdots & & -\text{diag}(A^T(N, N), \dots, A^T(2N - 1, 2N - 1)) \\ -Q_{2N}(2N - 1, N : 2N - 1) & & \end{bmatrix} + \\ & \begin{bmatrix} -B(N : 2N - 1, :) \\ Z_{N+1}(N, :) \\ \vdots \\ Z_{2N}(2N - 1, :) \end{bmatrix} G^{-1} [Z(:, N : 2N - 1) \tilde{B}_{N+1}^T(:, N) \cdots B_{2N}^T(:, 2N - 1)] \\ &= \begin{bmatrix} -\frac{1}{2}\theta I_N & 0_{N \times N} \\ -\text{diag}(\alpha_i^M) & \frac{1}{2}\theta I_N \end{bmatrix} + \begin{bmatrix} -[0_{N \times N} \tilde{b}_1 I_N - \tilde{b}_2 J] \\ 0_{N \times 2N} \end{bmatrix} G^{-1} \begin{bmatrix} 0_{2N \times N} & [0_{N \times N} \\ \tilde{b}_1 I_N \end{bmatrix} \\ &= \begin{bmatrix} -\frac{1}{2}\theta I_N & H_1 \\ -\text{diag}(\alpha_i^M) & \frac{1}{2}\theta I_N \end{bmatrix}, \end{aligned} \tag{34}$$

where

$$\begin{aligned} H_1 &:= -[0_{N \times N} \tilde{b}_1 I_N - \tilde{b}_2 J] G^{-1} \begin{bmatrix} 0_{N \times N} \\ \tilde{b}_1 I_N \end{bmatrix} \\ &= -\tilde{b}_1 (\tilde{b}_1 I_N - \tilde{b}_2 J) [\text{diag}(\chi_i^M) + d_1 \text{diag}(\beta_i^M)(d_1 I_N - d_2 J)]^{-1}. \end{aligned}$$

□

By considering the eigenstructure of matrix \tilde{M} above in some more detail we arrive to the next conclusion.

Corollary 7.3 *The game has at most one OLN equilibrium.*

Proof. Notice that

$$\begin{aligned} \det(\tilde{M} - \lambda I_{2N}) &= \det\left(\begin{bmatrix} I_N & 0 \\ \frac{-1}{\frac{1}{2}\theta + \lambda} \text{diag}(\alpha_i^M) & I_N \end{bmatrix}\right) \det(\tilde{M} - \lambda I_{2N}) \\ &= \det\left(\begin{bmatrix} -(\frac{1}{2}\theta + \lambda)I_N & H_1 \\ 0 & \frac{-1}{\frac{1}{2}\theta + \lambda} \text{diag}(\alpha_i^M)H_1 + (\frac{1}{2}\theta - \lambda)I_N \end{bmatrix}\right) \\ &= \det(\text{diag}(\alpha_i^M)H_1 + (\lambda^2 - \frac{1}{2}\theta^2)I_N). \end{aligned}$$

So, we conclude that whenever λ is an eigenvalue of \tilde{M} , also $-\lambda$ is an eigenvalue of $-\tilde{M}$. So, \tilde{M} has at most N stable eigenvalues and therefore, by Theorem 7.2, matrix M has at most $2N$ stable eigenvalues. Consequently, see e.g. Engwerda (2001), the game has at most one OLN equilibrium. □

Next we concentrate on the symmetric case (i.e. all monetary policymakers share the same cost function). To derive some specific results for this case we first state some preliminary lemmas.

Lemma 7.4

$$\begin{aligned} [\alpha I_N + \beta \mathbf{1}_N \mathbf{1}_N^T]^{-1} &= \frac{1}{\alpha} I_N - \frac{\beta}{\alpha(\alpha + N\beta)} \mathbf{1}_N \mathbf{1}_N^T \\ [(\alpha + \beta)I_N - \beta \mathbf{1}_N \mathbf{1}_N^T]^{-1} &= \frac{-1}{(\alpha + \beta)(\alpha - (N-1)\beta)} [((N-1)\beta - \alpha)I_N - \beta \mathbf{1}_N \mathbf{1}_N^T] \\ [\alpha I_N - \beta J]^{-1} &= \frac{N-1}{(\beta - \alpha)(\beta + (N-1)\alpha)} [(1 - \frac{1}{N-1})\beta - \alpha] I_N - \beta J \\ (\alpha_1 I_N - \beta_1 J)(\alpha_2 I_N - \beta_2 J) &= (\alpha_1 \alpha_2 + \frac{\beta_1 \beta_2}{N-1}) I_N - (\alpha_1 \beta_2 + \alpha_2 \beta_1 - (1 - \frac{1}{N-1})\beta_1 \beta_2) J. \end{aligned}$$

□

Lemma 7.5 *Let $H_1 = (\alpha + \beta)I_N - \beta \mathbf{1}_N \mathbf{1}_N^T$ and $\theta, \mu \in \mathbb{R}$. Then the eigenvalues of matrix*

$$H = \begin{bmatrix} -\theta I_N & H_1 \\ -\mu I_N & \theta I_N \end{bmatrix}$$

are $\lambda = \pm \sqrt{\theta^2 + N\beta\mu - \mu(\alpha + \beta)}$ with multiplicity one and $\lambda = \pm \sqrt{\theta^2 - \mu(\alpha + \beta)}$ with multiplicity $N-1$.

Proof.

$$\begin{aligned}
\det(H - \lambda I_{2N}) &= \det\left(\begin{bmatrix} I_N & 0 \\ \frac{-\mu}{\theta+\lambda}I_N & I_N \end{bmatrix}\right) \det(H - \lambda I_{2N}) = \det\left(\begin{bmatrix} -(\theta + \lambda)I_N & H_1 \\ 0 & \frac{-\mu}{\theta+\lambda}H_1 + (\theta - \lambda)I_N \end{bmatrix}\right) \\
&= \det(-(\theta + \lambda)I_N) \det\left(\frac{-\mu}{\theta + \lambda}H_1 + (\theta - \lambda)I_N\right) = \det(\mu H_1 + (\lambda^2 - \theta^2)I_N) \\
&= \det(\mu((\alpha + \beta)I_N - \beta 1_N 1_N^T) + (\lambda^2 - \theta^2)I_N) \\
&= \det(-\mu\beta 1_N 1_N^T + (\lambda^2 - \theta^2 + \mu(\alpha + \beta))I_N) \\
&= (\lambda^2 - \theta^2 + \mu(\alpha + \beta)) - \mu\beta N (\lambda^2 - \theta^2 + \mu(\alpha + \beta))^{N-1},
\end{aligned}$$

from which the result follows now directly. \square

Corollary 7.6 *If $H_1 = (\alpha_1 I_N - \beta_1 J)(\alpha_2 I_N - \beta_2 J)^{-1}$ then the eigenvalues of matrix H are $\lambda = \pm\sqrt{\theta^2 + \mu\frac{\alpha_1 - \beta_1}{\beta_2 - \alpha_2}}$ with multiplicity one and $\lambda = \pm\sqrt{\theta^2 + \mu\frac{\alpha_1 - \beta_1}{\beta_2 - \alpha_2} - \frac{N\mu(\alpha_1\beta_2 - \alpha_2\beta_1)}{(\beta_2 - \alpha_2)(\beta_2 + (N-1)\alpha_2)}}$ with multiplicity $N - 1$.*

Proof. From Lemma 7.4 we have

$$\begin{aligned}
H_1 &= (\alpha_1 I_N - \beta_1 J) \frac{N-1}{(\beta_2 - \alpha_2)(\beta_2 + (N-1)\alpha_2)} \left[\left(1 - \frac{1}{N-1}\right)\beta_2 - \alpha_2 \right] I_N - \beta_2 J \\
&= \frac{N-1}{(\beta_2 - \alpha_2)(\beta_2 + (N-1)\alpha_2)} \left\{ \left[\alpha_1 \left(1 - \frac{1}{N-1}\right)\beta_2 - \alpha_2 + \frac{\beta_1\beta_2}{N-1} \right] I_N \right. \\
&\quad \left. - [\alpha_1\beta_2 + \left(1 - \frac{1}{N-1}\right)\beta_2 - \alpha_2)\beta_1 - \left(1 - \frac{1}{N-1}\right)\beta_1\beta_2] J \right\} \\
&= \frac{1}{(\beta_2 - \alpha_2)(\beta_2 + (N-1)\alpha_2)} \left\{ \alpha_1((N-2)\beta_2 - (N-1)\alpha_2 + \beta_1\beta_2) I_N - (\alpha_1\beta_2 - \alpha_2\beta_1)(1_N 1_N^T - I_N) \right\}.
\end{aligned}$$

So, following the notation from Lemma 7.5, we have that

$$\begin{aligned}
\theta^2 + N\beta\mu - \mu(\alpha + \beta) &= \theta^2 + \frac{\mu}{(\beta_2 - \alpha_2)(\beta_2 + (N-1)\alpha_2)} \left\{ (N-1)(\alpha_1\beta_2 - \alpha_2\beta_1) - \right. \\
&\quad \left. \alpha_1((N-2)\beta_2 - (N-1)\alpha_2) - \beta_1\beta_2 \right\} \\
&= \theta^2 + \mu \frac{\alpha_1 - \beta_1}{\beta_2 - \alpha_2}.
\end{aligned}$$

From this then straightforwardly the advertised result follows. \square

Corollary 7.7 The game has at most one equilibrium. In case there is an equilibrium, $\alpha_i^M = \alpha^M$, $\chi_i^M = \chi^M$ and $\beta_i^M = \beta^M$, $i = 1, \dots, N$, the eigenvalues of the closed-loop matrix are $-\frac{1}{2}\theta$ with multiplicity N ; $\lambda_1 = -\sqrt{(\frac{1}{2}\theta)^2 + \alpha^M \frac{\alpha_1 - \beta_1}{\beta_2 - \alpha_2}}$ with multiplicity one and $\lambda_2 = -\sqrt{(\frac{1}{2}\theta)^2 - \alpha^M \frac{\beta_1 + (N-1)\alpha_1}{\beta_2 + (N-1)\alpha_2}}$ with multiplicity $N - 1$, where $\alpha_1 = -\tilde{b}_1^2$, $\beta_1 = -\tilde{b}_1\tilde{b}_2$, $\alpha_2 = \chi^M + \beta^M d_1^2$ and $\beta_2 = \beta^M d_1 d_2$.

Proof. First notice that \tilde{M} is a Hamiltonian matrix. So its eigenvalues are symmetrically distributed w.r.t. the imaginary axis. Consequently it has at most N stable eigenvalues. From which we obtain the conclusion that the game has at most one equilibrium.

The other statement follows directly from Corollary 7.6, using the structure of matrix H_1 in (34) and the additional assumptions on the weighting matrices. \square

Lemma 7.8

1) The solutions K_i of the set of coupled algebraic Riccati equations have the following structure:

$K_i = e_{i+N-1}[0_{1 \times (N-1)} \ k_i^{1T}] + e_{2N}[0_{1 \times (N-1)} \ k_i^{2T}]$, where $k_i^j \in \mathbb{R}^N, j = 1, 2$ and $K_{i+N} = K_i, i = 1, \dots, N$.

2) The closed-loop system matrix $A_{cl} := A - BG^{-1}(Z + \tilde{B}^T K)$, where $K = [K_1^T \cdots K_{2N}^T]$, equals

$$\begin{bmatrix} -\frac{1}{2}\theta I_{N-1} & [V_{N-1} \ v1_{(N-1)}] & f_1 1_{N-1} \\ 0_{N \times (N-1)} & S & f_2 1_N \\ 0_{1 \times (N-1)} & 0_{1 \times N} & -\frac{1}{2}\theta \end{bmatrix}, \quad (35)$$

where, with λ_i as in Corollary 7.7, $S = \lambda_2 I_N + \frac{\lambda_1 - \lambda_2}{N} 1_N 1_N^T$ and $V_{N-1} = \tau_1 I_{N-1} + \tau_2 1_{(N-1)} 1_{(N-1)}^T$, for some $\tau_i \in \mathbb{R}$.

Proof. From the structure of the matrices K_i the structure of matrix A_{cl} follows, with $S = \mu_1 I_N + \mu_2 1_N 1_N^T$, for some $\mu_i \in \mathbb{R}$. From Corollary 7.7 it follows next that matrix S has the eigenvalues λ_1 with multiplicity one and λ_2 with multiplicity $N - 1$. Next notice that the trace of matrix S equals the sum of its eigenvalues and 1_N is an eigenvector of S corresponding with the eigenvalue $\mu_1 + N\mu_2$, with multiplicity one. From this we obtain the next two equations for μ_i : $\mu_1 + N\mu_2 = \lambda_1$ and $N(\mu_1 + \mu_2) = \lambda_1 + (N - 1)\lambda_2$. From which the advertised formula for S results. \square

Theorem 7.9 (symmetric country case)

Under the above mentioned parameter restrictions under a non-cooperative equilibrium regime price trajectories, $p(t) := [p_1(t) \cdots p_N(t)]^T$, and exchange rate differential trajectories, $\tilde{e}(t) := [\tilde{e}_1(t) \cdots \tilde{e}_N(t)]^T$ are given by:

$$\begin{aligned} p(t) &= (e^{(\lambda_2 + \frac{1}{2}\theta)t} I_n + \frac{1}{N}(e^{(\lambda_1 + \frac{1}{2}\theta)t} - e^{(\lambda_2 + \frac{1}{2}\theta)t}) 1_N 1_N^T) p_0 + f_2 \int_0^t e^{(\lambda_1 + \frac{1}{2}\theta)(t-s)} ds 1_N \\ \tilde{e}(t) &= [V_{N-1} \ 1_{N-1}] \int_0^t p(s) ds + f_1 t 1_{N-1} + \tilde{e}_0. \end{aligned}$$

Proof. Under the assumption that a non-cooperative equilibrium exists it follows from Lemma 7.8, item 2., that the equilibrium closed-loop dynamics of the transformed state, $\tilde{x}(t) = e^{-\frac{1}{2}\theta t} x(t)$, satisfies the differential equation

$$\dot{\tilde{x}}(t) = A_{cl} \tilde{x}(t), \quad \tilde{x}(0) = x_0.$$

So, the closed-loop dynamics of the untransformed state vector are given by

$$\dot{x}(t) = (A_{cl} + \frac{1}{2}\theta I)x(t), \quad x(0) = x_0.$$

So, $p(t)$ and $\tilde{e}(t)$ satisfy

$$\dot{p}(t) = (S + \frac{1}{2}\theta I_N)p(t) + f_2 1_N, \quad p(0) = p_0 \quad (36)$$

$$\dot{\tilde{e}}(t) = [V_{N-1} \ 1_{N-1}]p(t) + f_1 1_{N-1}, \quad \tilde{e}(0) = e_0. \quad (37)$$

Notice that the eigenvalues of matrix $S + \frac{1}{2}\theta I_N$ are $\lambda_1 + \frac{1}{2}\theta$ with multiplicity one and $\lambda_2 + \frac{1}{2}\theta$ with multiplicity $N - 1$.

From (36,37) it follows that

$$p(t) = e^{(S + \frac{1}{2}\theta I_N)t} p_0 + \int_0^t e^{(S + \frac{1}{2}\theta I_N)(t-s)} f_2 1_N ds$$

$$\tilde{e}(t) = [V_{N-1} \ 1_{N-1}] \int_0^t p(s) ds + f_1 t 1_{N-1} + \tilde{e}_0.$$

Due to the structure of matrix $S + \frac{1}{2}\theta I_N$ it follows that $e^{(S + \frac{1}{2}\theta I_N)t}$ has the same structure, i.e. $\mu_1(t)I_N + \mu_2(t)1_N 1_N^T$, for some $\mu_i(t) \in \mathbb{R}$ too. Following the lines of the proof of Lemma 7.8 it follows that $\mu_i(t)$ should satisfy the equations $\mu_1(t) + N\mu_2(t) = e^{(\lambda_1 + \frac{1}{2}\theta)t}$ and $N(\mu_1(t) + \mu_2(t)) = e^{(\lambda_1 + \frac{1}{2}\theta)t} + (N-1)e^{(\lambda_2 + \frac{1}{2}\theta)t}$, $i = 1, 2$. This yields $\mu_1(t) = e^{(\lambda_2 + \frac{1}{2}\theta)t}$ and $\mu_2(t) = \frac{1}{N}(e^{(\lambda_1 + \frac{1}{2}\theta)t} - e^{(\lambda_2 + \frac{1}{2}\theta)t})$. Substitution of this into the above equation for $p(t)$ yields then the result. \square

From our parameter estimates it follows that, under the assumption that $N = 9$,

$$\lambda_1 = -\sqrt{\left(\frac{1}{2}\theta\right)^2 + \alpha^M \frac{0.0342}{\chi^M - 0.2093\beta^M}} \quad \text{and} \quad \lambda_2 = -\sqrt{\left(\frac{1}{2}\theta\right)^2 - \alpha^M \frac{0.0324}{8\chi^M + 0.3125\beta^M}}.$$

So an OLN non-cooperative equilibrium exists if and only if λ_i exist as a real number. In that case $0 < \lambda_2 + \frac{1}{2}\theta < \frac{1}{2}\theta$. From Theorem 7.9 we conclude therefore that if there is a symmetric price shock (i.e. $p_0 = \alpha 1_N$) then all prices will converge to a new constant level of $\frac{-f_2}{\lambda_1 + \frac{1}{2}\theta}$ whereas exchange rate differentials have a linear trend if $\lambda_1 < \frac{-1}{2}\theta$ (which occurs iff. $\chi^M > 0.2093\beta^M$). In case either $\lambda_1 > \frac{-1}{2}\theta$ or there is an asymmetric price shock and there is an equilibrium, there will be a constant inflation rate of at most $\frac{1}{2}\theta$. The exchange rate differentials will behave like the price paths in this case.

Finally we consider the effect of the different weight parameters in the cost function on the game. From Corollary 7.7 we obtain after some elementary calculations the next result.

Lemma 7.10

$$\frac{\partial \lambda_1}{\partial \alpha^M} = \frac{1}{2\lambda_1} \frac{\alpha_1 - \beta_1}{\beta_2 - \alpha_2}; \quad \frac{\partial \lambda_2}{\partial \alpha^M} = -\frac{1}{2\lambda_2} \frac{\beta_1 + (N-1)\alpha_1}{\beta_2 + (N-1)\alpha_2}$$

$$\frac{\partial \lambda_1}{\partial \beta^M} = -\frac{1}{2\lambda_1} \frac{\alpha^M(d_1 d_2 - d_1^2)(\alpha_1 - \beta_1)}{(\beta_2 - \alpha_2)^2}; \quad \frac{\partial \lambda_2}{\partial \beta^M} = \frac{1}{2\lambda_2} \frac{\alpha^M(d_1 d_2 + (N-1)d_1^2)(\beta_1 + (N-1)\alpha_1)}{(\beta_2 + (N-1)\alpha_2)^2}$$

$$\frac{\partial \lambda_1}{\partial \chi^M} = \frac{1}{2\lambda_1} \frac{\alpha^M(\alpha_1 - \beta_1)}{(\beta_2 - \alpha_2)^2}; \quad \frac{\partial \lambda_2}{\partial \chi^M} = \frac{1}{2\lambda_2} \frac{(N-1)\alpha^M(\beta_1 + (N-1)\alpha_1)}{(\beta_2 + (N-1)\alpha_2)^2}.$$

\square

Corollary 7.11 For the estimated parameters we obtain from Lemma 7.10 the next results:

$$\frac{\partial \lambda_2}{\partial \alpha^M} > 0; \quad \frac{\partial \lambda_1}{\partial \beta^M} < 0; \quad \frac{\partial \lambda_2}{\partial \beta^M} < 0; \quad \frac{\partial \lambda_1}{\partial \chi^M} > 0; \quad \frac{\partial \lambda_2}{\partial \chi^M} < 0.$$

Furthermore, $\frac{\partial \lambda_1}{\partial \alpha^M} = \text{sgn}(\chi^M - 0.2093\beta^M)$. □

Appendix D

Like in Appendix C we consider here the symmetric model under the assumption that $\eta = \delta = 0$. We consider here the social outcome in some detail. That is the cooperative scenario where it is assumed that all countries cooperate and the involved cost function is the sum of the individual cost of the countries. That is, the policies are determined by solving the next optimization problem (using the notation of Appendix C)

$$\min_{\tilde{u}(\cdot)} \int_0^\infty [\tilde{x}^T(t) \tilde{u}^T(t)] M_c [\tilde{x}^T(t) \tilde{u}^T(t)]^T dt, \text{ subject to}$$

$$\dot{\tilde{x}}(t) = A\tilde{x}(t) + B\tilde{u}(t), \quad \tilde{x}(0) = [x_0^T \ 1]^T,$$

where $M_c = \sum_{i=1}^{2N} M_i$ is given by

$$\begin{bmatrix} 0_{(N-1) \times (N-1)} & (\alpha^F + \alpha^M)I_N & & & \\ & & N(\beta^F + \beta^M)\hat{e}_2^2 & 0_{1 \times N} & (\beta^F + \beta^M)\hat{e}_2(d_1 - d_2)1_N^T \\ & & 0_{N \times 1} & \chi^F I_N & \\ & & (\beta^F + \beta^M)\hat{e}_2(d_1 - d_2)1_N & & (\beta^F + \beta^M)(\gamma_1 I_N + \gamma_2 1_N 1_N^T) + \chi^M I_N \end{bmatrix}.$$

Here,

$$\gamma_1 := \left(\frac{d_2}{N-1} + d_1\right)^2 \text{ and } \gamma_2 := \frac{N-2}{(N-1)^2} d_2^2 - \frac{2d_1 d_2}{N-1}.$$

Next factorize M_c as $\begin{bmatrix} Q & V \\ V^T & R \end{bmatrix}$. To solve this problem we consider the with this problem corresponding Hamiltonian matrix, $Ham := \begin{bmatrix} A - BR^{-1}V^T & -BR^{-1}B^T \\ -(Q - VR^{-1}V^T) & -(A - BR^{-1}V^T)^T \end{bmatrix}$ (see e.g. Engwerda (2005)).

Theorem 7.12 *Matrix Ham has N eigenvalues equal to $-\frac{1}{2}\theta$, $(4N-1)N$ eigenvalues equal to $\frac{1}{2}\theta$ and the other $2N$ eigenvalues determined by the $2N \times 2N$ matrix \tilde{Ham} given by (38) below.*

Proof:

From the structure of matrix Ham it follows directly that this matrix has $N-1+1$ eigenvalues $-\frac{1}{2}\theta$ (see columns 1 until $N-1$ and row $2N$) and $N-1+1$ eigenvalues $\frac{1}{2}\theta$ (see row $2N+1$ until $3N$ and column $4N$). It is easily verified that the remaining eigenvalues from Ham are then obtained as the eigenvalues of the reduced matrix

$$\tilde{Ham} = \begin{bmatrix} -\frac{1}{2}\theta I_N & -(\tilde{b}_1 I_N - \tilde{b}_2 J)R_{22}^{-1}(\tilde{b}_1 I_N - \tilde{b}_2 J) \\ -(\alpha^F + \alpha^M)I_N & \frac{1}{2}\theta I_N \end{bmatrix}, \quad (38)$$

where $R_{22} := (\beta^F + \beta^M)(\gamma_1 I_N + \gamma_2 1_N 1_N^T) + \chi^M I_N$. \square

Notice that $\tilde{H}am$ has the same structure as matrix \tilde{M} we considered in (34) of Appendix C. Introducing

$$\begin{aligned} \nu_1 &= \gamma_1 * (\beta^F + \beta^M) + \chi^M; \quad \nu_2 = \gamma_2(\beta^F + \beta^M); \\ \nu_3 &= \frac{1}{\nu_1}(\tilde{b}_1 + \frac{\tilde{b}_2}{N-1})^2; \quad \nu_4 = -\frac{\nu_2(\tilde{b}_1 - \tilde{b}_2)^2}{\nu_1(\nu_1 + N\nu_2)} + \frac{\tilde{b}_2}{\nu_1(N-1)}(\frac{N-2}{N-1}\tilde{b}_2 - 2\tilde{b}_1), \end{aligned}$$

we have that $R_{22} = \nu_1 I_n + \nu_2 J$ and $-(\tilde{b}_1 I_N - \tilde{b}_2 J)R_{22}^{-1}(\tilde{b}_1 I_N - \tilde{b}_2 J) = -\nu_3 I_N - \nu_4 1_N 1_N^T$. Using Lemma's 7.4 and 7.5, respectively, we obtain then the next result.

Lemma 7.13 *The eigenvalues of matrix $\tilde{H}am$ are $\pm\frac{1}{2}\theta$ with multiplicity N , $\pm\lambda_1^c$ with multiplicity one and $\pm\lambda_2^c$ with multiplicity N . Here $\lambda_1^c = -\sqrt{(\frac{1}{2}\theta)^2 + (\alpha^F + \alpha^M)(\nu_3 + N\nu_4)}$ and $\lambda_2^c = -\sqrt{(\frac{1}{2}\theta)^2 + (\alpha^F + \alpha^M)\nu_3}$. \square*

Notice that $\nu_3 + N\nu_4 = \frac{(\tilde{b}_1 - \tilde{b}_2)^2}{\nu_1 + N\nu_2}$, $\gamma_1 + N\gamma_2 = (d_1 - d_2)^2$ and $\nu_3 > 0$. Consequently,

$$\lambda_1^c = -\sqrt{(\frac{1}{2}\theta)^2 + \frac{(\alpha^F + \alpha^M)(\tilde{b}_1 - \tilde{b}_2)^2}{(\beta^F + \beta^M)(d_1 - d_2)^2 + \chi^M}} < -\frac{1}{2}\theta \text{ and } \lambda_2^c < -\frac{1}{2}\theta.$$

From this we conclude in particular that $\frac{\partial \lambda_1^c}{\partial \alpha^j} < 0$, $\frac{\partial \lambda_1^c}{\partial \beta^j} > 0$ and $\frac{\partial \lambda_1^c}{\partial \chi^M} > 0$. Furthermore, we obtain along the lines of Lemma 7.8.

Corollary 7.14

The game always has a unique cooperative equilibrium. Using the cooperative strategies $\tilde{u}(t) = -R^{-1}(V^T + B^T K)$, where K is the stabilizing solution of the algebraic Riccati equation $A^T K + KA - (KB + V)R^{-1}(B^T K + V^T) + Q = 0$ (which can also be obtained from the stable graph subspace of matrix $\tilde{H}am$), the closed-loop system matrix $A_{cl}^c := A - BR^{-1}(V^T + B^T K)$ for the model equals

$$\begin{bmatrix} -\frac{1}{2}\theta I_{N-1} & [V_{N-1}^c \quad v 1_{(N-1)}^c] & f_1^c 1_{N-1} \\ 0_{N \times (N-1)} & S^c & f_2^c 1_N \\ 0_{1 \times (N-1)} & 0_{1 \times N} & -\frac{1}{2}\theta \end{bmatrix}, \quad (39)$$

where, with λ_i^c as in Lemma 7.13, $S^c = \lambda_2^c I_N + \frac{\lambda_1^c - \lambda_2^c}{N} 1_N 1_N^T$ and $V_{N-1}^c = \tau_1^c I_{N-1} + \tau_2^c 1_{(N-1)} 1_{(N-1)}^T$, for some $\tau_i^c \in \mathbb{R}$. \square

As a direct consequence of Corollary 7.14 we have that the original price and exchange rate differential trajectories are as in Theorem 7.12. However, since both λ_i^c are smaller than $-\frac{1}{2}\theta$, we conclude that whatever the disturbance will be, the prices will converge to a constant level of $\frac{-f_2^c}{\lambda_1 + \frac{1}{2}\theta}$ and exchange rate differentials will have a linear trend.

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