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DEMAND AND SUPPLY AS FACTORS DETERMINING ECONOMIC GROWTH

BY

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1 INTRODUCTION

In neoclassical theory of economic growth the long run rate of growth is determined by autonomous factors, such as the increase in working population and technical change. Demand for final products does not play an independent role, because it is assumed that supply creates its own demand. In this article we shall drop this assumption. Demand may differ from potential supply, which is reflected in the degree of capacity utilization. However, this does not undermine the proposition from neoclassical growth theory mentioned above. For a state of steady growth implies a constant degree of capacity utilization, so that population growth and technical progress still determine the rate of economic growth.

A completely different picture appears if it is assumed that technical change depends among other things on the degree of capacity utilization. In such a situation the factors regulating the demand for goods also influence the rate of growth in a steadily growing economy. The consequences of this will be analysed in the present article. In doing so we build heavily on two studies which appeared earlier in this review.[1]

The development of a number of economic variables in the course of time is determined by the intrinsic dynamics of the system, as represented by the fundamental relations between flows and stocks. Moreover, lags can be introduced on an *ad hoc* basis to deal with specific characteristics of human behaviour and the institutional organization of the economic process.[2] In a theoretical model such lags contain an element of arbitrariness. However, the assumed lag structure is essential to the dynamics of the system and may have

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1 S. K. Kuipers 'A Vintage Model of Growth, Employment and Inflation,' *De Economist*, CXXIII (1975), pp. 531–558. Th. van de Klundert and R. J. de Groof 'Economic Growth and Induced Technical Progress,' *De Economist*, CXXV (1977), pp. 505–524.

2 *Cf.* S. J. Turnovsky, *Macroeconomic Analysis and Stabilization Policy*, Cambridge, 1977, p. 68.

some influence on the results of the model in case of steady growth as will be shown. Because the analysis presented here is restricted to the steadily growing economy (in the absence of disturbances), only a few more or less obvious lags will be introduced when discussing the postulated relationships. This offers at the same time the possibility to show that the steady growth path is co-determined by factors, such as lags, which are important when studying situations of non-steady growth. However, the question under which circumstances the system is a stable one, will not be discussed. Dealing with more complicated models, such a question cannot usually be answered. At best simulation experiments can give an impression of the dynamic characteristics of the model. In the concluding observations we shall return to these problems.

The present study will be conducted as follows. In section 2 the model and the underlying assumptions will be presented. The formulas describing the steady growth path will be discussed in section 3. Next in section 4 various steady states will be compared with each other applying numerical examples. The intention is to trace the factors which determine growth and structure (the ratios between important variables) in the long run.

2 THE MODEL

Potential production, y_t^*, and potential demand for labour, a_t^*, are explained with the aid of a vintage model with two production factors (labour and capital) and fixed technical coefficients after installment of capital goods (clay-clay model). Technical progress is assumed to be labour saving only. The introduction of innovations is coupled with the installment of new capital goods (embodied technical progress). As it will appear, the rate of technical progress is determined endogenously. In addition technical obsolescence of equipment is taken into account.

On the basis of these assumptions the relation between (total) production capacity and the production factor capital can be written as follows:

$$y_t^* = \frac{1}{\kappa} \sum_{\tau=t-m_t}^{\tau=t-1} \frac{i_\tau}{(1+\delta)^{t-\tau}} \tag{2.1}$$

From this formula it follows, that production capacity is found by multiplying the constant capital productivity $(1/\kappa)$ with the volume of all vintages still in use. New investment contributes to production capacity after one period. Each period (year) $100\delta\%$ of the existing capital stock is scrapped for reasons of technical obsolescence. The economic life span of capital goods equals $(t-1) - (t-m_t) + 1 = m_t$ years.

The number of available jobs or the potential demand for labour depends on the volume of capital goods still in use and the year of their installation. Accordingly, as the vintage is younger, its capital intensity is higher. However, the change in capital intensity can differ from year to year. This is expressed in the following formula:

$$a_t^* = \frac{\alpha_0}{\kappa} \sum_{\tau=t-m_t}^{\tau=t-1} \frac{i_\tau}{(1+\delta)^{t-\tau} \prod_{\theta=1}^{\tau} (1+\mu_\theta)} \tag{2.2}$$

Where $1/\alpha_0$ is the labour productivity of the newest vintage in period $t = 0$, and μ_t symbolizes the rate of labour saving technical progress in period t.

Given the conditions of perfect competition it can be assumed that entrepreneurs operate vintages of capital equipment as long as the quasi-rent on them is not negative. This means that the labour productivity of the marginal vintage equals the real wage rate, w_t. Applying *realized* labour productivity the formal expression for the scrappage condition becomes:

$$w_t = \frac{y_{t,t-m_t}}{a_{t,t-m_t}} \tag{2.3a}$$

This formulation implies that cyclical movements in the labour productivity of the oldest vintage do influence the economic life time. It can be imagined that in a period of recession more marginal firms will close their doors than in a period with no cyclical disturbances. In order to fit this phenomenon into the model in a more adequate way the scrappage condition could be formulated differently. This could be done by introducing a distributed lag with regard to marginal labour productivity. For ease of survey however we restrict ourselves to the simple specification as laid down in formula (2.3a).

The degree of utilization of (total) production capacity equals:

$$q_t = \frac{y_t}{y_t^*} \tag{2.4}$$

It may be assumed, that the degree of utilization influences actual demand for labour:

$$a_t = a_t^* q_t^\zeta, \qquad 0 \leqslant \zeta \leqslant 1 \tag{2.5}$$

In case $\zeta = 1$ the relative utilization of potential demand for labour corresponds with the degree of utilization of capacity. If ζ is smaller than one, entrepreneurs employ more labour than necessary for actual production. The question arises whether for the long run this is a plausible statement. In the short-run such behaviour seems to be realistic. This view urges us to adapt formula (2.5). The parameter ζ should be set equal to one, whereas at the same time a (distributed) lag on the variable q should be introduced. For the analysis of steady growth paths however, such a lag is of no importance. For in case of steady growth the ratios a/a^* and y/y^* in equation (2.5) are constant.

It is assumed now that for every vintage the degree of capacity utilization and the ratio between actual employment and the number of available jobs are the same. From the relations (2.3a), (2.4) and (2.5) it can then be deduced:

$$w_t = \frac{y^*_{t,t-m_t}}{a^*_{t,t-m_t}} q_t^{1-\zeta}$$

or

$$w_t = \frac{1}{\alpha_0} \prod_{\theta=1}^{t-m_t} (1 + \mu_\theta) \cdot q_t^{1-\zeta} \tag{2.3}$$

Within the framework of perfect competition, equation (2.3) can be used to find a solution for the economic life time, m_t.

If labour becomes more scarce, the real wage rate will *ceteris paribus* rise faster. This induces additional scrappage of unprofitable vintages. As a consequence of this, potential employment decreases. It seems plausible to assume that the relative scarcity of labour also influences research and development efforts aimed at labour saving innovations. Technical change is often a matter of eliminating existing bottlenecks. In the absence of technical change, labour will probably form a principal bottleneck in the process of economic growth.[3] According to this idea it is assumed, that entrepreneurs increase the rate of technical progress, if the wage rate grows faster than labour productivity of the newest vintage in the previous period. This gives:

$$\mu_t - \mu_{t-1} = \psi(\dot{w}_t - \mu_{t-1}) \qquad 0 < \Psi < 1 \tag{2.6a}$$

$$\therefore \ \mu_t = (1 - \psi)\mu_{t-1} + \psi \dot{w}_t \tag{2.6b}$$

Demand may also play an important role in the process of technological innovation. Scale effects cannot be realised in case of an insufficient extent of the market. Moreover, a situation of excess demand will be a stimulus to increase the

3 See in this connection J. Hicks, *Economic Perspectives. Further Essays on Money and Growth*, Oxford, 1977, Chapter I.

level of production. The introduction of new products yields less risks in a sellers market. In short, there is every reason to take into account the pull effect of effective demand in addition to the push effect of labour scarcity.[4] This can be met by employing the degree of capacity utilization, in deviation from a chosen norm, as an explanatory variable in the equation for μ_t. In a state of steady growth with $\mu_t = \mu_{t-1}$, the lag in equation (2.6b) can be discarded. On the ground of these considerations we come to the following relationship for the explanation of the rate of labour saving technical progress:

$$\mu_t = \varepsilon_1 \dot{w}_t + \varepsilon_2 (q_t - \omega) + \mu_t^{au}, \qquad 0 < \varepsilon_1 < 1, \quad \varepsilon_2 > 0. \tag{2.6}$$

The degree of utilization has a positive impact on μ_t. The symbol μ_t^{au} represents autonomous technical change.

In literature one meets with studies connecting the rate of technical development with the volume of research and development efforts. In these studies the optimum size of the R&D activities is deduced from profit maximizing behaviour of individual entrepreneurs. This implies that the entrepreneur can gather the fruits of his research efforts. To guarantee this it may be assumed that innovations are protected during a certain period.[5] Both technical and juridical aspects can provide for the desired protection. In a macroeconomic model no distinction can be made between innovators and imitators. However, the effect of labour saving inventions will not always be compensated immediately by a proportional increase in real wages. In the situation of steady growth real wages increase according to the rate of technical change. In the short-run additional profits are enjoyed, if the increase in the wage rate lags behind the increase in labour productivity. The incurred expenses for research and development can then be earned back. It is not easy for entrepreneurs to estimate the impact of technical change on wages. However, by introducing an appropriate assumption it might be possible to determine the optimum research effort. Given the many complications which determine the innovations process, we prefer, in the framework of macroeconomic analysis, the more global approach with respect to the behaviour of entrepreneurs as expressed by our equation (2.6).

In case of perfect competition, constant returns to scale and full utilization of capacity, the volume of (gross) investment cannot be deduced easily from profit-

4 See for instance J. Schmookler, *Invention and Economic Growth*, Cambridge (Mass.), 1966, and R. J. de Groof, *Geïnduceerde technische ontwikkeling*, Unpublished doctoral thesis, Tilburg, 1977.

5 See W. Nordhaus, 'Theory of Innovation, An Economic Theory of Technological Change,' *American Economic Review*, LIX (1969), pp. 18–28.

maximising behaviour by entrepreneurs.[6] Instead it may be assumed that the volume of investment is determined by sales expectations and financial possibilities. Taking the imperfections on the capital market into account it can be maintained that realised profits are of considerable importance for the possibility to finance investments.[7] Sales opportunities are reckoned with by assuming that the degree of utilization influences the propensity to invest. The better the utilization the greater the volume of investment. If moreover lags of one year are introduced the above considerations lead to:

$$i_t = (\beta_1 + \beta_2 q_{t-1}) z_{t-1} + i_t^{au}, \qquad \beta_1, \beta_2 > 0 \tag{2.7}$$

The chosen specification admits, as it will appear, the possibility of steady growth. For the sake of completeness account is taken of autonomous investment.

The volume of consumption, c_t, depends on real income. It is assumed that the (constant) marginal propensity to consume out of labour income differs from that out of real non-wage income or real profits, z_t. It seems plausible that consumers react with some delay. The consumption function can then be written as:

$$c_t = \gamma_1 w_{t-1} a_{t-1} + \gamma_2 z_{t-1} + c_t^{au} \tag{2.8}$$
$$0 < \gamma_1,\ \gamma_2 < 1; \quad \gamma_1 > \gamma_2$$

where c_t^{au} stands for autonomous consumption.

Real aggregate demand is divided between consumption and investment. It is assumed that actual production equals aggregate demand, consequently

$$y_t = c_t + i_t \tag{2.9}$$

Real non-wage income is considered as a residual in the model:

$$z_t = y_t - w_t a_t \tag{2.10}$$

The real wage rate is per definition equal to the ratio of the nominal wage sum per worker, l_t, and the price level of final products, p_t:

$$w_t = \frac{l_t}{p_t} \tag{2.11}$$

6 See Th. van de Klundert, 'Winstmaximalisatie in het jaargangenmodel met vaste coëfficiënten; een inventarisatie van de problematiek,' Tilburg University, Reeks *Ter Discussie*, December 1977.
7 *Cf.* A. Wood, *A Theory of Profits*, Cambridge, 1975.

The relative change of the nominal wage rate will depend in general on the relative change of the price level, the relative rise of labour productivity and the state of the labour market. The latter can be measured by means of the rate of unemployment, u_t. Taking an autonomous factor into account, the wage equation can then be written as:

$$\dot{l}_t = v_1 \dot{p}_t + v_2(\dot{y}_t - \dot{a}_t) - v_3 u_t + \dot{l}_t^{au}, \qquad 0 < v_1 \leq 1, \quad v_2, v_3 > 0 \tag{2.12}$$

Price changes depend on the development of wage cost per unit of output and the situation on the market for final products. The situation on the market for goods is reflected in the degree of utilization of capacity. The development of wage cost per unit of output is determined by the rise of the wage sum per worker and the change in labour productivity. Of course autonomous price changes are also imaginable. This leads to:

$$\dot{p}_t = \eta_1 \dot{l}_t - \eta_2(\dot{y}_t - \dot{a}_t) + \eta_3 q_t + \dot{p}_t^{au}, \qquad 0 < \eta_1 < 1; \quad \eta_2, \eta_3 > 0 \tag{2.13}$$

The model has to be completed by the following definitions:

$$l_t = l_{t-1}(1 + \dot{l}_t) \tag{2.14}$$

$$p_t = p_{t-1}(1 + \dot{p}_t) \tag{2.15}$$

$$\dot{y}_t = \frac{y_t - y_{t-1}}{y_{t-1}} \tag{2.16}$$

$$\dot{a}_t = \frac{a_t - a_{t-1}}{a_{t-1}} \tag{2.17}$$

$$\dot{w}_t = \frac{w_t - w_{t-1}}{w_{t-1}} \tag{2.18}$$

$$u_t = \frac{a_t^s - a_t}{a_t^s} \tag{2.19}$$

Labour supply, a_t^s, is an exogenous variable in the model. Finally we add the definitions for the wage share, λ_t, and the macroeconomic savings or investment ratio, σ_t, to the whole.

$$\lambda_t = \frac{w_t a_t}{y_t} \tag{2.20}$$

$$\sigma_t = \frac{i_t}{y_t} \tag{2.21}$$

The model contains 21 variables, *i.e.* $y^*, a^*, y, a, c, i, z, \mu, m, q, w, l, p, \dot{l}, \dot{p}, \dot{y}, \dot{a}, \dot{w}, u$, λ and σ. These variables can be solved using the equations (2.1)–(2.21).

3 THE CORE SYSTEM OF STEADY GROWTH

In the situation of long-run equilibrium or steady growth the variables increase per definition with a constant rate. Eventually this rate may be equal to zero. From the model it appears, that steady growth is only possible if certain ratios are constant. This can be elucidated as follows. From equation (2.8) it follows that c_t grows with a constant rate if $w_t a_t$, z_t and c_t^{au} increase with the same rate. The variable i_t will also change with this rate, provided that i_t^{au} increases proportionally and q is constant. In case these conditions are not met, i_t will not increase with a fixed rate. From (2.9) it appears that y_t will also increase with the same rate. According to (2.20) and (2.21) both λ and σ are constant in this case. Because q is fixed, y_t^* increases *pari passu* with y_t. From (2.1) it then follows that m is constant. Next it can be derived from (2.2) that μ has a fixed value, because a_t has to change at a constant rate. The relative change of the demand for labour should be equal to the relative change of the supply of labour. For only in that case u will be constant. The nominal wage changes at a fixed rate if $\dot{l}_t^{au}$ is constant. The same provisio must be made with regard to $\dot{p}_t^{au}$.

The purpose of this section is to derive a sub-model (core system) to calculate the constant ratios σ, λ, q and u. Applying this core system of steady growth a solution is also found for the rate of growth, μ, and the economic life time, m.

If it is assumed henceforth, that the supply of labour does not change, investment, and consequently the other quantities too, should increase with $100\mu\%$ a year. Given a fixed supply of labour the demand for labour should be constant as well. This implies an unchanging number of available jobs. Given a constant life time of capital goods the number of eliminated jobs is a factor $(1 + \mu)^{m-1}$ greater than the number of jobs, that comes available by replacing the liquidated capacity. If investment increases with a factor $(1 + \mu)$ the difference is compensated for exactly.

Application of these conclusions to formula (2.1) yields:

$$y_t^* = \frac{1}{\kappa}\frac{1}{(1+\mu)(1+\delta)}\frac{1-\left[\dfrac{1}{(1+\mu)(1+\delta)}\right]^m}{1-\dfrac{1}{(1+\mu)(1+\delta)}} i_t \tag{3.1a}$$

From the equations (3.1a), (2.4) and (2.21) the following core relation can be derived:

$$\sigma = \frac{1}{q}\kappa\frac{(1+\mu)(1+\delta)-1}{1-\left[\dfrac{1}{(1+\mu)(1+\delta)}\right]^m} \tag{3.1}$$

In the case of steady growth formula (2.2) passes into:

$$a^* = \frac{\alpha_0}{\kappa}\frac{1-\left[\dfrac{1}{1+\delta}\right]^m}{\delta} i_0 \tag{3.2a}$$

Formula (2.3) is now replaced by:

$$w_t = \frac{1}{\alpha_0}(1+\mu)^{t-m}q^{1-\zeta} \tag{3.3a}$$

Substituting the relations (2.4), (2.5), (3.1a), (3.2a) and (3.3a) in formula (2.20) yields the second equation of the core system of steady growth:

$$\lambda = \frac{1}{(1+\mu)^{m-1}}(1+\delta)\frac{1-\left[\dfrac{1}{1+\delta}\right]^m}{\delta}\frac{1-\dfrac{1}{(1+\mu)(1+\delta)}}{1-\left[\dfrac{1}{(1+\mu)(1+\delta)}\right]^m} \tag{3.2}$$

In case of steady growth the share of labour income does not depend on the degree of capacity utilization, q. This happens to be so because the influence of the degree of capacity utilization on the (average) labour productivity is compensated, given the postulated scrappage condition, by a proportionally lower level of real wages. Apart from this it should be noted that the degree of utilization as a variable of the core system does influence λ indirectly.

From formula (3.3a) it appears, that in case of steady growth the real wage increases with the rate of labour saving technical progress. Moreover, if it is assumed that autonomous technical change is constant and is represented by the symbol ε_3, formula (2.6) can be transformed into:

$$\mu = \frac{\varepsilon_2}{1-\varepsilon_1}(q-\omega) + \frac{\varepsilon_3}{1-\varepsilon_1} \tag{3.3}$$

In case of steady growth the investment function (2.7) can be written as:

$$i_t = \frac{\beta_1 + \beta_2 q}{1+\mu} z_t + i_t^{au} \tag{3.4a}$$

Dividing both sides by y_t and taking into account (2.10), (2.20) and (2.21) it follows:

$$\sigma = \frac{\beta_1 + \beta_2 q}{1+\mu}(1-\lambda) + \underline{\sigma} \tag{3.4}$$

where $\underline{\sigma} = i_t^{au}/y_t$. This ratio should be fixed. In other words, autonomous investment should equal zero or increase with the growth rate of the national product. The appearance of the rate of growth μ in the denominator of (3.4) results from the assumption that investment responds with a lag of one year, as shown by equation (2.7).

From the relations (2.9) and (2.10) it turns out that factor-payments and expenditures are equal:

$$w_t a_t + z_t = c_t + i_t \tag{3.5a}$$

After substituting the relations (2.7) and (2.8) in the case of steady growth and dividing both sides by y_t formula (3.5a) passes into:

$$\frac{\gamma_1}{1+\mu}\lambda + \frac{\beta_1 + \beta_2 q + \gamma_2}{1+\mu}(1-\lambda) + \underline{\gamma} + \underline{\sigma} = 1 \tag{3.5}$$

where $\underline{\gamma} = c_t^{au}/y_t$ is assumed to be constant. From formula (3.5) the solution for the wage share can be found:

$$\lambda = \frac{(1+\mu)(1-\underline{\gamma}-\underline{\sigma}) - (\beta_1 + \beta_2 q + \gamma_2)}{\gamma_1 - (\beta_1 + \beta_2 q + \gamma_2)} \tag{3.5b}$$

subject to $\gamma_1 > \beta_1 + \beta_2 q + \gamma_2$, $(1+\mu)(1-\underline{\gamma}-\underline{\sigma}) > \beta_1 + \beta_2 q + \gamma_2$ and $\gamma_1 >$ $> (1+\mu)(1-\underline{\gamma}-\underline{\sigma})$.

The equations (3.1)–(3.5) make up a sub-system, determining the following five variables, σ, λ, μ, m and q. Some insight into the inter-relationship of these variables can be obtained by studying the special case, with $\beta_2 = 0$ and $\varepsilon_2 = 0$. In this case the rate of technical change is determined exogenously. In addition it is assumed that the degree of utilization does not influence investment. From formula (3.5) there results a solution for λ in accordance with the Kaldorian theory of income distribution. Substitution of this outcome for λ into (3.2) yields a solution for the economic life time m. Thereby the *capacity effect* of investment is determined, as described in (3.1a). Substituting the solution for λ in (3.4) gives the *expenditure effect* of investment:

$$y_t = \frac{1}{\dfrac{\beta_1}{1+\mu}(1-\lambda) + \underline{\sigma}} i_t \tag{3.4b}$$

Capacity and expenditure effect together determine the degree of utilization of the production potential.

The degree of utilization is not determined in the special case, where current wage income is fully consumed and current profits are fully invested.

$$(\gamma_1/(1+\mu) = 1, \quad \beta_2/(1+\mu) = 1, \quad \beta_2 = \gamma_2 = \underline{\gamma} = \underline{\sigma} = 0)$$

Equation (3.5) then disappears from the scene. There remain three equations, namely (3.1), (3.2) and (3.4), at least if it is assumed for sake of convenience that technical progress is constant. The number of unknowns then amounts to four: σ, λ, m and q. Because an assumption has already been made regarding investment behaviour, it is not logical to fix either σ or λ. The economic life time should not be fixed either. There remains the degree of utilization, which in this case has to be considered as an exogenous variable.

Under the assumption, that the autonomous wage and price changes are constant, the wage sum per worker and the price level grow with a fixed rate. In case of steady growth the equations (2.12) and (2.13) can then be written as:

$$\dot{l}_t = \nu_1 \dot{p} + \nu_2 \mu - \nu_3 u + \nu_4, \quad \text{where} \quad \dot{l}_t^{au} = \nu_4 \tag{3.6}$$

$$\dot{p}_t = \eta_1 \dot{l} - \eta_2 \mu + \eta_3 q + \eta_4, \quad \text{where} \quad \dot{p}_t^{au} = \eta_4 \tag{3.7}$$

Solution of this set of equations yields:

$$\dot{l}_t = \frac{-\nu_1\eta_2 + \nu_2}{1-\eta_1\nu_1}\mu - \frac{\nu_3}{1-\eta_1\nu_1}u + \frac{\nu_1\eta_3}{1-\eta_1\nu_1}q + \frac{\nu_1\eta_4 + \eta_4}{1-\eta_1\nu_1} \tag{3.6a}$$

$$\dot{p}_t = \frac{\eta_1 v_2 - \eta_2}{1 - \eta_1 v_1} \mu - \frac{\eta_1 v_3}{1 - \eta_1 v_1} u + \frac{\eta_3}{1 - \eta_1 v_1} q + \frac{\eta_1 v_4 + \eta_4}{1 - \eta_1 v_1} \tag{3.7a}$$

We introduce as an approximation[8]:

$$\dot{w}_t = \dot{l}_t - \dot{p}_t \tag{3.8}$$

From equation (3.3a) it appears, that in case of steady growth real wages increase with $100\mu\%$ per period:

$$\dot{w}_t = \mu \tag{3.9}$$

The rate of unemployment in the case of steady growth can now be found by using the equations (3.6a), (3.7a), (3.8) and (3.9):

$$u = \frac{\eta_2(1 - v_1) + v_2(1 - \eta_1) - (1 - \eta_1 v_1)}{v_3(1 - \eta_1)} \mu - \frac{\eta_3(1 - v_1)}{v_3(1 - \eta_1)} q - \\ - \frac{\eta_4(1 - v_1) - v_4(1 - \eta_1)}{v_3(1 - \eta_1)} \tag{3.10}$$

The level of the constant rate of unemployment is determined by three factors. The impact of technical change depends on the value of the coefficients in the wage and price functions. The degree of utilization has a negative effect, if $v_1 < 1$. In case $v_1 = 1$ q does not play a role, because price changes are then fully compensated in wages. Finally, u still depends on the autonomous components. An autonomous wage increase has a positive effect, whereas the opposite holds for an autonomous price increase (if $v_1 < 1$).

In the present model the rate of unemployment influences wages and prices only. It is of course imaginable that this variable has a greater impact on the whole. It could be assumed for instance that demand for and supply of labour also depend on the market situation itself. However, such complications are not dealt with here.

8 In case of discrete time the following formula applies:

$$\dot{w}_t(1 + \dot{p}_t) = \dot{l}_t - \dot{p}_t$$

Therefore, the neglected secondary order effect equals $\dot{w}_t\dot{p}_t$.

4 NUMERICAL EXAMPLES OF STEADY GROWTH

The characteristics of equilibrium solutions are usually explored by applying mathematical tools. The present model is less suited for practising this method. For that reason we resort to numerical examples. Starting point is a steady state, which serves as a *basic* path for the sake of comparison with alternative solutions. To obtain this basic solution the following values of the parameters have been chosen[9]:

capacity	*labour*	*technique*	*investment*	*consumption*	*wages*	*prices*
$\kappa = 2$	$\zeta = 1$	$\varepsilon_1 = 0.8$	$\beta_1 = 0.287$	$\gamma_1 = 0.9$	$v_1 = 1$	$\eta_1 = 0.5$
$\delta = 0.02$	$a^s = 3979$	$\varepsilon_2 = 0.05$	$\beta_2 = 0.287$	$\gamma_2 = 0.2$	$v_2 = 1$	$\eta_2 = 0.5$
		$\varepsilon_3 = 0.004$	$\underline{\sigma} = 0$	$\underline{\gamma} = 0.167$	$v_3 = 0.5$	$\eta_3 = 0.1$
		$\omega = 0.81$			$v_4 = 0.025$	$\eta_4 = -0.081$

The results of the computations for the basic path are given in column 1 of table 1. With exception of the economic life time the results are percentages. The life span is measured in years. The figures have, if necessary, been rounded off to one decimal point.

TABLE 1 – VARIANTS WITH REGARD TO THE SUPPLY OF GOODS

Variable \ Variant	Basic	(a) $\kappa = 2.5$	(b) $\delta = 0.03$	(c) $\varepsilon_3 = 0.006$
σ	14.3	14.9	14.8	13.6
μ	2.0	1.5	1.7	1.6
m	30.0	40.0	35.1	34.8
λ	71.9	70.5	70.9	72.6
q	81.0	79.1	79.7	75.6
u	5.0	5.0	5.0	5.0

In table 1 the results of some variants regarding the supply of goods have been recorded. These will be discussed in some detail below. In advance it should be noted, that the qualitative results are more important than the quantitative results. Moreover, experiments showed that the qualitative conclusions are not sensitive with regard to changes in the various parameters. An exception has to be made for changes in ε_2. We will return to this point later on.

9 The numerical values of β_1, β_2, $\underline{\gamma}$ and a^s are rounded off. These numbers have been determined in such a way that the solutions for $\underline{\mu}$, m, u and q yield predetermined values.

(a) An increase in the capital coefficient ($\Delta\kappa = 0.5$) leads to radical changes. The increase in κ may be interpreted as a capital-using technical change. Hereby the factor capital becomes scarcer in comparison with the factor labour. First this induces higher unemployment. As a consequence the real wage decreases and the economic life time increases. The rise in m induces on balance an increase in y_t^*, which forces the degree of utilization to a lower level. A lower degree of utilization is linked up with a smaller rate of technical progress. The distribution of income is determined by the Kaldorian mechanism. From equation (3.5b) it follows, taking into account the assumed restrictions on the parameters:

$$\frac{\partial\lambda}{\partial\mu} > 0 \text{ and } \frac{\partial\lambda}{\partial q} < 0$$

An increase in μ leads to a decrease in the marginal propensities to spend in the situation of steady growth. The effect on the marginal propensity to consume out of wage income is greater than the other effects. Therefore, in the case of an unchanged distribution of income, savings would become too high. This can be redressed by an increase in λ. A rise in q induces a higher level of investment. To obtain in this case a sufficient amount of savings the wage share has to decrease.

In the present case q decreases, so that λ increases. However, this effect is surpassed by the negative effect of a decrease in μ. Finally, the wage share decreases with 1.4 percentage-points. As a result the savings or investment ratio increases. The strong rise in the economic life time of capital goods is very marked. This depends among other things on the fact, that a slower rate of growth is accompanied by a smaller labour intensity of the relevant vintages. In such a case more vintages are needed to realize a given volume of labour. The rate of unemployment does not change, because the parameters of the wage and price functions have been chosen such that the coefficients preceding μ and q in formula (3.9) equal zero.

(b) An acceleration in the rate of technical depreciation of capital goods ($\Delta\delta = 0.01$) has, as might be expected, similar consequences as a decrease in the capital intensity of production. For that reason there is no need to explain the results in column 3 of table 1 any further.

(c) According to the table an acceleration in the rate of autonomous technical change ($\Delta\varepsilon_3 = 0.002$) leads to unexpected outcomes. The resulting value of μ appears to be lower than in the basic variant. On the other hand the wage share is higher. The key to a solution of these problems is connected with a decrease in the degree of utilization, which is relatively large. This variable is 5.4 percentage-points lower. As a result technical progress is slowed down to a considerable extent. The chosen value for ε_2 implies a domination of this effect, so that on balance technical progress decelerates. This has *ceteris paribus* a negative effect on the wage share. In contrast with this effect there is a much greater positive effect resulting from the decrease in q. The ultimate increase in the wage share is accompanied by a decrease in the savings or investment ratio. The economic life time also increases in this case because of the slower growth rate. However the question remains: why is the degree of capacity utilization so much lower? The answer to this question can be derived from equation (3.1a). With m constant, a greater μ implies a greater capacity effect of investment. The expenditure effect increases also according to formula (3.4b), but this does not counterbalance the increase in the capacity effect. The degree of utilization decreases under these circumstances. However there results, as mentioned before, a lower value of μ. Despite this, the degree of capacity utilization decreases is connected with the increase in the economic life time. *In summary* the following can be said. In the first instance the capacity effect of investment increases as a result of the initial rise in μ. The increase in the degree of utilization evokes an opposite change, which determines the scene in the second instance. Nevertheless the variable q remains lower than in the basic variant, because the increase in m ultimately leads to a greater production capacity.

TABLE 2 – VARIANTS WITH REGARD TO THE DEMAND FOR GOODS

Variable \ Variant	Basic	(d) $\beta_1 = \beta_2 = 0.3$	(e) $\gamma_1 = 0.91$	(f) $\underline{\gamma} = 0.175$
σ	14.3	16.3	15.7	15.9
μ	2.0	2.6	2.5	2.6
m	30.0	25.4	26.8	26.5
λ	71.9	69.6	69.3	69.0
q	81.0	83.5	83.0	83.3
u	5.0	5.0	5.0	5.0

Some variants with regard to the demand for goods are presented in table 2. For sake of comparability column 1 of this table again comprises the basic variant.

(d) An increase in the propensity to invest ($\Delta\beta_1 = \Delta\beta_2 = 0.013$) induces of course an increase in the macroeconomic investment ratio. The expenditure effect of the additional investment provides for an increase in the degree of capacity utilization, with as a consequence the rate of technical change being higher than in the basic variant. Given such a faster growth and the extensive investment effort accompanying it, the economic life time of capital goods is evidently considerably shorter. The wage share lies below the level which is reached in the basic variant. The negative effects as a result of an increase in β_1, β_2 and q dominate the positive effect of a higher μ.

(e) An increase in the propensity to consume out of wage income ($\Delta\gamma_1 = 0.01$) has the same qualitative effects as an increase in the propensity to invest. It is nevertheless remarkable, that a very small change in γ_1 leads to extensive results. However, the differences with the basic variant are smaller than in the case of variant (d). This notwithstanding the fact, that the demand pull turns out to be somewhat greater.[10] In the case (d) the decrease in the wage share is slightly less than in variant (e). This has a positive effect on expenditure and via q on the rate of technical progress. That the savings or investment ratio is nevertheless higher in the case of an increased propensity to invest needs no further comment.

(f) The results in case of an increase in the share of autonomous consumption ($\Delta\underline{\gamma} = 0.0077$) are similar to those obtained in case (e). The small differences arise from the slightly higher impulse in variant (f).

TABLE 3 – SOME OTHER VARIANTS

Variable \ Variant	Basic*	(g) $\zeta = 0.5$	(h) $\underline{\sigma} = 0.02$	(i) $\nu_4 = 0.035$
σ	14.3	14.0	20.1	14.3
μ	2.0	2.3	4.1	2.0
m	30.0	32.9	19.5	30.0
λ	71.9	72.6	65.4	71.9
q	81.0	82.1	89.5	81.0
u	5.8	5.8	6.0	7.8

* Adapted; see text.

In table 3 a few other variants are presented. The basic variant in this table differs in some respects from that in the preceding tables. To trace the influence of changes in the parameter ζ it has been assumed, that the degree of utilization plays no role in the scrappage formula. This means, that the variable q is discarded from equation (2.3). The parameter ζ now appears in the core system of

10 The increase of γ_1 implies an expenditure impulse in terms of y of

$$\lambda(\Delta\gamma_1)/(1 + \mu) = (0.01 \times 0.72)/(1.02) = 0.0071$$

whereas the increase of β_1 and β_2 yields an impulse in the size of

$$(\Delta\beta(1 + q)(1 - \lambda))/(1 + \mu) = (0.013 \times 1.81 \times 0.28)/(1.02) = 0.0065$$

the steady state. The right-hand side of equation (3.2) has to be multiplied in this case with the factor $q^{\zeta-1}$. However the results of the basic variant are not altered, because the basic path has been determined under the assumption $\zeta = 1$. In the new basic variant another modification has been incorporated. It is furthermore assumed that the price compensation in wages amounts to 80% ($\nu_1 = 0.8$). As a consequence of this the rate of unemployment increases with 0.8 percentage-points (see table 3, column 1). In case of the variants (g), (h) and (i) the influence of changes in μ and q on u should now be taken into account.

(g) A value of ζ lower than one implies, that also in the situation of steady growth, entrepreneurs employ more workers than seems to be justified in view of the sales potential. The resulting under-utilization losses lead to a higher wage share. Savings decrease, whereas the degree of utilization increases. The higher degree of utilization has a positive effect on the rate of technical change. On balance the rate of unemployment does not change. Opposite to the favourable outcome of a higher degree of utilization stands the adverse effect of a more rapid increase in labour savings.[11] These two opposite effects cancel out approximately. It can be concluded, that the intention of entrepreneurs to keep a certain labour reserve is favourable for the workers. The wage share increases, whereas the rate of unemployment does not change. In addition, the result is a more rapid growth.

(h) An increase in the share of autonomous investment with 2% entails, as might be expected, a substantial change in the variables. The qualitative effects evidently correspond with the results of variant (d). In the present case the influence on u is of special importance. The rate of unemployment is subject to a small increase. Again two forces counteract. The greater extent of labour savings has a negative effect. It must be admitted that employment declines under the influence of a better utilization of production capacity, but this effect is not high enough to compensate for the unfavourable influence of technical change.

(i) The last variant concerns a higher autonomous increase in wages ($\Delta\nu_4 = 0.01$). This leads to an increase in the rate of unemployment with 2-percentage-points. The result is rather high, because the Phillips-curve has a rather flat slope ($\nu_3 = 0.5$). To counterbalance an autonomous wage impulse under these circumstances calls for a considerable increase in unemployment. As noted in the previous section, the increase in u has no influence on the remaining variables.

11 After substitution of the parameter values equation (3.10) becomes:

$$u = \frac{0.1\mu - 0.02q + 0.0287}{0.25}$$

5 CONCLUDING REMARKS

The method of comparative dynamics employed in the preceding section gives some insight into the role played by demand and supply in the process of long-run economic growth. A stimulation of demand induces a faster rate of growth, at least as far as the degree of utilization can still be increased.

In the model introduced here the degree of utilization is not bounded from above. It is nevertheless conceivable, that the degree of utilization cannot increase above a certain level. The pressure on the market for goods is then demonstrated in other ways (long delivery delays, extensive back-logs, and so on). However this may be, in our approach it is assumed that technical change accelerates if demand increases in relation to production capacity.

The relationship explaining the rate of labour saving technical change is supposed to be linear. Such an apparently less realistic assumption serves as a crude macroeconomic approximation to complicated microeconomic processes. This may lead to unexpected results as in the 'perverse' case, where an increase (decrease) in autonomous technical change finally induces a smaller (higher) rate of growth (see variant (c)). However, a sufficient low value of ε_2 does not produce such a strange result.[12]

The variants, in which capital is used more intensively, do not give problems of interpretation. Notwithstanding the higher macroeconomic savings or investment ratio the pace of economic growth is slowed down in these cases. This is caused by a lower degree of utilization, which can be explained from the resulting shift in the distribution of income in favour of capital.

The rate of unemployment does not change in these variants. However, this is an accidental result. Other values of the parameters in the wage and price functions produce different results. In addition, the rate of unemployment then varies with μ and q. The outcome for u in case of steady growth can be conceived as the rate of unemployment, which is determined by the situation on the market for final products and for labour. (Eventually autonomous wage and price changes can be explained by taking segregated markets into account).

The different steady states are possible solutions of the model. If a development starting from an arbitrary initial position converges to such a solution, the system is (globally) stable. As observed in section 1, simulation experiments may give an impression of the dynamic characteristics of the model. In doing so a number of special problems have to be resolved. Because the model should cover a large number of periods it is important to avoid numerical noise, caused by rounding off or approximation. In this respect much is gained if the model can be made

12 In case $\varepsilon_2 = 0.01$ and $\varepsilon_3 = 0.005$ there results $\mu = 0.034$.

recursive by introducing appropriate lags. However, in this case the dynamics of the system becomes much more complicated, which is certainly a disadvantage.

Another problem is the handling of autonomous expenditure in situations of disequilibrium growth. In sections 3 and 4 the view has been taken, that the *share* of autonomous expenditure in total production is constant. In a dynamic simulation run however the *level* of autonomous expenditure has to be introduced as an exogenous variable. The problem can of course be solved, but not without the introduction of other arbitrary elements in the model.

A different approach to the analysis of dynamic systems is to look for local stability. In this case the model should be linearized in the neighbourhood of a steady growth path. Standard techniques can then be applied to derive the conditions for local stability.

More generally speaking, the question arises which transformations are allowed to analyse the characteristics of complex dynamic systems. This question cannot be answered here. Further research will have to show in which way the stability of the present model can be tested.

LIST OF SYMBOLS

Relative changes of variables are indicated by a dot above the symbol for the relevant variable. The suffix τ attached to a variable indicates that the variable belongs to vintage τ. The symbol t stands for discrete time.

Endogenous variables

a^* labour requirements (potential employment, available jobs)
a actual employment
c volume of consumption
i volume of investment
l nominal wage rate per worker
m economic life time of capital goods
p price level of goods
q degree of utilization of production capacity
u rate of unemployment
w real wage rate per worker
y^* production capacity (potential production)
y actual production
z volume of real profits
λ macroeconomic share of labour in income (wage share)
μ rate of labour saving technical progress
σ macroeconomic savings or investment ratio

Exogenous variables

a^s	supply of labour
c^{au}	autonomous consumption
i^{au}	autonomous investment
$\dot{l}^{au}$	autonomous wage change
$\dot{p}^{au}$	autonomous price change
μ^{au}	autonomous labour savings

Constants

α_0	labour coefficient of the newest vintage in period 0
$\beta_1, \beta_2, (\underline{\sigma})$	parameters in the investment function
$\gamma_1, \gamma_2, (\underline{\gamma})$	parameters in the consumption function
δ	rate of technical deterioration
$\varepsilon_1, \varepsilon_2, \varepsilon_3, \omega$	parameters in the function explaining labour savings
$\eta_1, \eta_2, \eta_3, (\eta_4)$	parameters in the price equation
κ	capital coefficient
$\nu_1, \nu_2, \nu_3, (\nu_4)$	parameters in the equation for the nominal wage rate
ζ	elasticity of demand for labour with respect to the degree of utilization

Summary

DEMAND AND SUPPLY AS FACTORS DETERMINING ECONOMIC GROWTH

In models of economic growth the long-run rate of growth is usually determined by exogenous factors like the increase in working population and technical progress. In this article the rate of technical progress is treated as an endogenous variable depending on the increase in real wages and the degree of capacity utilization. A clay-clay production model is presented. Moreover, consumption, investment, changes in wages and in prices are explained by additional equations. Numerical steady state solutions for different values of the parameters are discussed. In each case the specific role played by demand and supply is stressed.