Essays on the valuation of discretionary liabilities and pension fund investment policy

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Dirk Broeders

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PROEFSCHRIFT

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Contents

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Introduction

The primary function of a pension is to maintain the standard of living after retirement. Pension systems around the globe are usually built around a 3-pillar structure. The first pillar comprises a mandatory state-sponsored old-age income insurance. A pension created by an employer for the benefit of an employee is commonly referred to as an occupational or employer pension. This is usually identified as the second pillar. Finally, there are possibilities for personal retirement saving in the third pillar.

This thesis focuses on the second pillar. Occupational pension funds are key in providing an adequate old age income to society. According to OECD 2008 data, pension funds have globally USD 15,800 billion in assets under management, which is roughly equal to 26% of global GDP and 14% of global market capitalization. On average these assets are allocated as follows: 41.5% in equities, 38.2% in fixed income securities, 2.7% in real estate and 17.6% in alternative asset classes. Approximately 62% of these assets serve defined benefit plans and 38% defined contribution plans. A defined benefit plan guarantees a certain payout at retirement, according to a fixed formula which usually depends on the member's salary, the accrual rate and the number of years of participation in the plan. A defined contribution plan will provide a payout at retirement that is dependent upon the cumulative amount of money contributed and the investments' performance.

The current funding deficits in defined benefit plans result from a 'perfect storm' with a simultaneous decline in equity prices and interest rates. Apart from these market risks, pension funds are also exposed to inflation risk and traditional insurance risks like mortality risk and longevity risk. It has become manifest that the key to managing these risks successfully requires a deep understanding of liability structures and financial markets dynamics. This is particularly true for defined benefit pension schemes that typically run a significant mismatch between assets and liabilities. The risks involved in such a strategy need to be shared across the different stakeholders. In a defined benefit scheme, the following three different stakeholders can be distinguished:

- Corporate shareholders
- Current participants
- Future participants

Corporate defined benefit pension schemes typically have an explicit or implicit risk sharing arrangement with the shareholders of the sponsor. This concordat is made explicit by fair value accounting under IAS 19: the accounting rule concerning employee benefits under the IFRS rules set by the International Accounting Standards Board. Chapters 1 and 2 in this thesis focus on the risk sharing arrangement between corporate shareholders and current participants.[1] One of the key elements to risk management is the investment policy of pension funds. This investment policy plays a central role in Chapters 3 and 4.

Defined benefit plans around the world are in decline as a combined result of demographic ageing, low interest rates and volatile investment returns. Therefore, the trend is away from defined benefit towards hybrid schemes and defined contribution schemes. A hybrid pension scheme is one which is neither a full defined benefit nor a full defined contribution scheme, but has some characteristics of each. Contingent liabilities play a key role in hybrid pension schemes as an efficient risk management tool. Career average defined benefit schemes with contingent indexation both during the accrual and the payout phase, are the foremost important example of hybrid plans. In these plans pension accrual is linked to income in a specific year, while the indexation of benefits, both during the accrual stage and the payout stage, is contingent on the funding ratio. It is important to study these contingent liabilities as they present significant economic value for the beneficiaries. This value depends on many factors, including the volatility of

[1] Intergenerational risk-sharing is described in Cui, de Jong and Ponds (2009) and Gollier (2009). Samuelson (1963) provides the theoretical argument of the gain of intergenerational risk sharing in an elegant way: a sequence of non-utile bets is never utile while the subdivision of non-utile bets may be utile. Although the risks as such are not reduced by intergenerational risk sharing, they are divided over a large group of subsequent generations. Consequently, shocks have less impact on the disposable income of participants in a pension compared to participants in individual schemes.

investment returns that follows from the asset allocation chosen by pension funds. Valuation and investment policy are key in this thesis.

Contribution of the thesis

The four essays collected in this thesis serve as a contribution to the broad field of pension finance. They focus on the economic understanding of liability valuation on the one hand and investment policy for pension funds on the other. Part I applies contingent claim analysis to determine the market-consistent value of guarantees and discretionary pension liabilities in defined benefit schemes. Market-consistent valuation relieves the necessity of specifying utility functions. This implies that when everything is traded and all market participants have recourse to the capital market, one can always take positions that will complement or set those resulting from pension funding decisions. Furthermore, we assume that it is possible to derive explicit rules for the implicit risk sharing arrangements between a pension fund and the corporate sponsor. Part II provides an empirical assessment of pension funds' investment behavior. We focus on two determinants of investment behavior: the relative performance of equities over bonds and the average age of participants.

Chapter 1, entitled 'Valuation of contingent pension liabilities and guarantees under sponsor default risk', concerns the impact of the sponsor's creditworthiness on the valuation of guarantees and contingent pension liabilities.[2] Although often legally independent, in practice an economic interaction exists between pension fund and company. This interface influences the pension fund's optimal investment policy and thereby the valuation of option-like features in pension benefits. Contingent claims in pension schemes have a long record in the literature, starting with the seminal paper by Sharpe (1976), followed by Treynor (1977), Bulow (1982) and more recently by Blake (1998), Steenkamp (1998) and Kocken (2006). The latter, e.g., distinguishes between the indexation option, the pension put and the parent guarantee option and takes the expected value of the payoffs under a risk neutral measure. Recently the valuation of contingent pension liabilities has received a lot of attention. Nijman and Koijen (2006) apply pricing kernels to value conditionally indexed pension liabilities. De Jong (2008a) employs models

[2] This Chapter is based on Broeders (2010).

for asset pricing in incomplete markets to value pension liabilities that have unhedgeable wage-indexation risk. This chapter offers a technique for optimizing the embedded risk sharing arrangement between a defined benefit pension scheme and its sponsor using contingent claims analysis. By reverse-engineering the applicable option valuation formulas, a pension scheme can infer the risk profile that maximizes the value for the beneficiaries. Furthermore, this optimization procedure takes account of the default risk of the pension plan sponsor. As such, a pension scheme can model the risk sharing arrangement in a fairly realistic manner. This is relevant for analyzing hybrid pension schemes. E.g., in a typical Dutch pension plan, indexation of accrued benefits is not guaranteed but depends on an annual discretionary decision made by the pension fund's trustees. According to Dutch pension regulation, pension funds are not obliged to asses the economic value of these discretionary liabilities. However, it shows that the market-consistent value of these contingent liabilities can be derived using the replication principle, including derivatives that mimic the contingency and sponsor default risk, to capture the risk sharing arrangement with the corporation.

Chapter 2, entitled 'Pension regulation and the market value of pension liabilities', considers the relationship between investment policy, regulatory environment and the valuation of contingent pension liabilities.[3] Being important financial institutions, pension funds are subject to governmental regulation. Key to this is the full funding requirement. However, as a 'run' on a pension fund seems inconceivable, often a grace period is given for reorganization and recovery before a premature closure is executed. This chapter fits into an emerging trend in the literature that uses derivatives to simulate regulation of financial institutions. Grosen and Jørgensen (2002) is one of the first papers to incorporate a regulatory mechanism into the market valuation of equity and liabilities at life insurance companies by using a regular down–and–out barrier feature to describe the regulatory intervention rule. However, they do not allow for a recovery term. This grace period in regulation can be captured by Parisian options, a particular type of barrier options as described in Chen and Suchanecki (2007). Both of these papers focus on the regulation of insurance companies. Insurance regulation offers only short recovery periods due to the limited liability of the insurance companies' shareholders. Recovery periods in pension regulation are often relatively

[3] This chapter is based on Broeders and Chen (2010).

long. Chapter 2 is the first to apply Parisian style options to mirror pension regulation. The presented framework is used to construct fair pension deals, in which the beneficiaries get value for money. It follows that beneficiaries should claim a fair stake in the pension funds surplus to compensate for the risk of premature closure. Furthermore, a utility analysis is performed to derive the optimal recovery period in pension regulation. We find that the optimal recovery period depends on the specific contract details, the pension fund's investment policy, the beneficiaries risk aversion and the regulatory features, such as the minimum funding requirement. For the numerical examples presented in this chapter the optimal recovery period ranges between 1 and 5 years. Knowledge of the optimal recovery period is important for designing future pension regulation.

Chapter 3, entitled 'Stock market performance and pension fund investment policy', describes how pension funds adjust their asset allocation in reaction to the performance of the stock market.[4] Many pension funds aim at maintaining a fixed asset allocation in terms of investment classes (strategic asset allocation). This requires a rebalancing strategy which promotes that changes in the relative value of financial assets give rise to offsetting purchases and sales, so that the relative weights in the portfolio remain fairly constant. However, it is also possible to accommodate value changes within defined bandwidths (tactical asset allocation).[5] It appears that pension funds do not perfectly rebalance their portfolios towards the strategic asset allocation. This first recording of this phenomenon shows that pension funds do not only show imperfect rebalancing behavior, they also adjust equity weightings asymmetrically. Pension funds are eager to rebalance after a period of relative underperformance of equities ('buy on the dip') but allow their asset allocation to free float after a period of equity outperformance ('the trend is your friend').[6] We find that, statistically, this behavior adds no (or destroys no) value to the overall performance of pension funds. Although few papers investigate the impact of market developments on investment policy, closely connected papers

[4] This chapter is based on Bikker, Broeders and de Dreu (2010).

[5] Dynamic asset allocation strategies are extensively described in many papers, e.g. Leibowitz and Weinberger (1982), Tilley and Latainer (1985), Brennan and Xia (2002) and Dai and Schumacher (2008). Gollier (2008) argues that the equity allocation should be drastically reduced when the financial health of a pension fund deteriorates.

[6] This pattern might have been interrupted during the recent credit crisis due to the excessive drop in equity prices. However, the crisis is not part of the period under consideration in this chapter.

are Grinblatt and Titman (1989, 1993), Lakonishok, Schleifer and Vishny (1992) and Blake, Lehmann and Timmermann (1999). The last paper, e.g., reports a negative correlation between asset class returns and net cash flows to the corresponding asset class for UK pension funds, which points to rebalancing. However, these authors also find that the asset allocation drifts toward asset classes that performed relatively well, in line with a free-float strategy. Therefore, the evidence in the existing literature is not conclusive.

Chapter 4, entitled 'Pension funds' asset allocation and participant age: a test of the life-cycle model', tests how pension funds account for the age of the participants in their investment policy.[7] It appears that, in line with life-cycle models, the equity allocation of pension funds is lower when the beneficiaries are on average older. Hereby age acts as a proxy for human capital. The key argument is that young workers have more human capital than older workers. As long as the correlation between labor income and stock market returns is assumed to be low, young workers may better diversify away equity risk with their large holding of human capital. This concept is described in Bodie, Merton and Samuelson (1992), Campbell and Viceira (2002), Cocco, Gomes and Maenhout (2005) and Ibbotson, Milevsky, Chen and Zhu (2007). Benzoni, Collin-Dufresne and Goldstein (2007) offer a contradicting view on age and asset allocation by arguing that labor income and capital income are highly correlated in the long run. Insightful empirical papers that focus on the asset allocation of institutional investors are Alestalo and Puttonen (2006), Gerber and Weber (2007) and Lucas and Zeldes (2009). The results presented in Chapter 4 support that pension funds invest according to the life-cycle hypothesis. Dutch pension funds with a higher average age of participants have significantly lower equity exposures than pension funds with younger participants. A non-linear age effect, as suggested by Benzoni *et al.* (2007), could not be confirmed. Another key contribution to the literature of this chapter is that the average age of active participants has a much stronger impact on equity allocation than the average age of all participants.

[7] This chapter is based on Bikker, Broeders, Hollanders and Ponds (2009).

Conclusion and future research agenda

This thesis is aimed at better understanding liability valuation and investment policy of pension funds. Based on the findings, this thesis suggests several directions to enhance pension deal design and risk management techniques and thereby improve the sustainability of old age provisioning. The following key points are made

- It is important to asses the economic value of discretionary liabilities and the interaction with a pension fund's investment policy. Contingent claims analysis can be used to determine the optimal investment strategy given the risk sharing arrangement with the pension fund's sponsor. The target indexation level and the default risk of the sponsor play a key role in the optimization procedure.

- Beneficiaries of a hybrid pension scheme should be sufficiently compensated for early termination exposure. The fair compensation takes the form of a higher claim on the pension fund's surplus. It is demonstrated that utility analysis can be used to determine the optimal recovery period in pension regulation. For the pension deals analyzed in this thesis, the optimal recovery period ranges form 1 to 5 years depending amongst others on investment policy, the level of risk aversion and other regulatory features.

- Pension funds do not show perfect rebalancing behavior. Equity reallocation is higher after underperformance of equity investments compared to outperformance. In particular, only 13 percent of positive excess equity returns is rebalanced, while 49 percent of negative shocks results in rebalancing. The latter can be indicated as a 'buy on the dip' strategy and the former as a 'the trend is your friend' approach. The rebalancing behavior does not add or destroy value.

- Pension funds follow a life-cycle approach in their investment policy. Dutch pension funds with a higher average age of participants have significantly lower equity exposures than pension funds with younger participants. It appears that the strategic equity allocation particularly strongly correlates with the average age of active participants. The effect is stronger for larger pension funds.

Looking forward, the interaction between investment policy, risk sharing and liability valuation reveals a clear need for additional research. I identify the following key research questions

- What is the optimal investment policy for hybrid pension schemes with contingent liabilities? Do these types of pension schemes invest accordingly?
- Are hybrid pension schemes designed to be economically fair for the beneficiaries and how to enhance incentive-compatible regulation for these type of schemes?
- Does rebalancing behavior of institutional investors support stable price formation on financial markets?
- Can an age-dependent investment strategy, where the young invest in risky assets and the elderly are safeguarded by risk-free claims, contribute to the sustainability of collective pension schemes in an ageing society?

Part I

Valuation of Discretionary Liabilities

Chapter 1

Valuation of Contingent Pension Liabilities and Guarantees under Sponsor Default Risk

This chapter is based on Broeders (2010)

1.1 Introduction

In its principles for the regulation of occupational pension schemes the OECD states that pension funds must be legally separated from the sponsoring company. This detachment is also prescribed by the Employee Retirement Income Security Act in the United States and the Pension Directive in Europe. Pension funds thus serve as special purpose vehicles to ensure that accrued pension rights are not subjected to the sponsor's default risk. In reality, however, there is no clean economic break between the sponsor and its pension fund. Particularly in the case of defined benefit schemes, subsequent situations of overfunding and underfunding may lead to additional cash flows between the two entities. Risk management at the pension fund level largely accounts for this. Pension funds in general do not, or are unable to, invest in the portfolio that exactly replicates the nature of their defined benefit liabilities. Generally there is a lack of suitable investment opportunities with guaranteed real returns. Under this restriction, guaranteeing inflation or wage indexed pensions might become infeasible at reasonable costs. Therefore, in many defined benefit pension schemes, part of the pension promise is contingent on the performance of the pension fund assets. In return for taking mismatch risk pension fund trustees accept the possibility of encountering strong or weak financial conditions.

This not only affects the pension fund and its beneficiaries, but also the sponsor. There is considerable evidence that the funding level of the defined benefit pension plan is reflected in the market value of the sponsor, see, e.g., Feldstein and Seligman (1981), Bulow, Morck and Summers (1987), Caroll and Niehaus (1998) and Coronado and Sharpe (2003). In addition to this value transparency argument, Lin, Merton and Bodie (2006) find that the market risk of the sponsor's equity reflects the risk level of the pension plan. The economic rationale for this is that in case of a (large probability of a) funding deficit, the sponsor may have the legal or moral obligation to increase contributions to the pension fund. On the other hand, surpluses in the pension fund tend to be, at least partially, claimed by the sponsor. Contribution holidays are a common phenomenon in prosperous times. These additional funding or refunding decisions can be considered as implicit options on the pension fund's assets. Through these contingent claims, there is a distinct financial connection between the pension fund and its sponsor.

These contingent claims in pension schemes have been described by Sharpe (1976), Treynor (1977), Bulow (1982) and recently by Blake (1998), Steenkamp (1998) and Kocken (2006). Contingent claims analysis can be used to show that a pension fund is a zero sum game in valuation terms amongst the relevant stakeholders: retirees, employees, future participants and corporate shareholders. If everything is traded and all stakeholders have recourse to the capital market, one can always take positions that will complement or offset those resulting from corporate pension funding decisions.[1]

This chapter focusses on the implicit contingent claims between a pension fund and its sponsor and the contingent indexation of pension liabilities. Typical of contingent indexed liabilities is that the indexation of benefits to inflation or

[1] Under this stringent assumption pension funds do not augment welfare. In reality individuals do not have unlimited access to capital markets and as such pension funds are potentially welfare enhancing, see, e.g., Cui, de Jong and Ponds (2006)

wage growth depends on a future decision to be taken by the pension funds' board. The fulfillment of the indexation in practice depends on the financial position of the pension fund. If financial resources are abundant, indexation is fully granted. However, if the financial resources are poor, the pension fund might choose not to fully index pension benefits. This contingency can be replicated by a series of financial options, that are linked to , e.g., the pension fund's funding ratio. The valuation of these options, amongst others, depends on the asset liability mismatch of the pension fund.

In practice, Dutch pension regulation does not require pension funds to value the contingent indexation promise, subject to two preconditions. First, the annual indexation level must be a discretionary decision by the trustees. Second, the pension fund must inform the beneficiaries adequately about the conditionality. Pension funds must however strive for consistency between the expectations raised, the level of financing achieved and the degree to which contingent claims are awarded to members, see Broeders and Pröpper (2010). This consistency needs to be grounded by the application of a long-term stochastic continuity analysis. The contingent indexation factor means that the beneficiaries are exposed to investment risk that they can not easily hedge if they wish to do so. For a true assessment of the financial wealth and the risk exposure, it is therefore important that these contingent claims are evaluated in a realistic manner. The value and the riskiness of their defined benefit pension savings is relevant for individuals since they need incorporate this in their optimal life-cycle saving and investment planning. Valuation is also relevant for accounting purposes as contingent claims in pension provisioning might be considered as constructive obligations for the sponsor. Under IAS a corporation should recognize the expected cost of profit-sharing and bonus payments when, and only when, it has a legal or constructive obligation to make such payments as a result of past events and a reliable estimate of the expected cost can be made.

Several recent papers discuss the valuation of different option features in pension liabilities. Sherris (1995) considers 'greater of' benefits in a multivariate contingent claims valuation framework. Lachance, Mitchell and Smetters (2003) analyze the option in some defined contribution plans to 'buy-back' a defined benefit plan at a pre-specified price. Kocken (2006) distinguishes between the indexation option, the pension put and the parent guarantee option and takes the expected value of the payoffs under a risk neutral measure. Nijman and Koijen (2006) use pricing kernels to value conditionally indexed pension liabilities. De Jong (2008a) employs models for asset pricing in incomplete markets to value pension liabilities that have unhedgeable wage-indexation risk.

The relation between the value of (contingent) pension liabilities and optimal investment policy has been documented by various studies. Sundaresan and Zapatero (1997) find that the optimal asset allocation is a mixture of a portfolio replicating the liabilities and an independent return portfolio. Inkmann and Blake (2007) show that the optimal asset allocation policy varies with the initial funding level of the pension plan, with severely underfunded pension plans preferring a large equity exposure. De Jong (2008b) argues that pension funds prefer equities as the market for index linked bonds is underdeveloped. Furthermore, pension funds deliberately take more risks than a pure defined benefit scheme would impose. They invest more in equities to chase the equity premium. to chase the equity risk premium. Dai and Schumacher (2008) solve for the optimal investment policy in a way that the expected utility of participants is maximized.

This chapter adds to these strands in the literature in two ways. *First*, by optimizing the embedded risk sharing arrangement between a defined benefit pension scheme and its sponsor applying contingent claims analysis. An important option valuation parameter is the volatility of the underlying assets. Option pricing models can also be used to back out the implied volatility. This principle is applied in this chapter in the context of a defined benefit pension scheme. By reverse-engineering the applicable option valuation formulas, a pension scheme can deduce the optimal risk profile that maximizes the value for the beneficiaries.[2] This risk profile relates to the mismatch between assets and liabilities. It will be shown that in the optimum, the marginal cost of acquiring insurance against underfunding equals the marginal reward for risk taking. *Second*, by explicitly taking into account sponsor vulnerability in this optimization procedure a pension scheme can model the risk sharing arrangement in a fairly realistic manner.

The chapter is structured as follows. Section 1.2 reviews preliminary concepts relevant for defined benefit pension fund risk management followed by an outline of a general framework for pension fund analysis in Section 1.3. Subsequent

[2] The net value for the beneficiaries should feedback in the contribution the sponsor is willing to pay to the pension fund. This feedback mechanism is not explicitly taken into account here.

sections consider the contingent liability valuation problem in relation to sponsor risk from the pension fund's perspective. Section 1.4 assumes that the sponsor unconditionally clears all deficits within the pension fund. Section 1.5 relaxes this assumption to the extent that the sponsor offers a limited guarantee or, alternatively, a partial loss insurance in Section 1.6. The next step in Section 1.7 is to include sponsor specific characteristics, specifically the financial ability to back the pension promises. This may be modelled as a vulnerable put option. Section 1.8 broadens the scope to a multiperiod analysis and Section 1.9 introduces the effect of volatility smiles. The final section summarizes the chapter and the appendices explain the technical details.

1.2 Environment and preliminary concepts

This section reviews some general issues on risk management for a defined benefit pension fund. The starting assumption is that asset prices follow a geometric Brownian motion

$$dA = \mu A dt + \sigma A dW \tag{1.1}$$

with μ the constant expected return per unit of time, σ^2 the constant variance of returns per unit of time and A the market value of the pension fund assets at time t. The time subscript is suppressed for ease of notation. The source of uncertainty is a Wiener process W. The distribution of the market value of the assets at maturity, A_T, is lognormal and the continuously compounded return until maturity is normally distributed. Using Itô's lemma, see Hull (2008), this implies that the change in the portfolio's value over time $(T-t)$ is:

$$\ln(A_T/A) \sim \mathcal{N}\left((\mu - \frac{1}{2}\sigma^2)(T-t), \sigma\sqrt{T-t}\right) \tag{1.2}$$

where $\mathcal{N}$ represents the normal distribution. In case of a nominal defined benefit, a payment $\underline{L}$ is guaranteed to the beneficiaries at maturity $t = T$. So, the market value of the pension fund's assets at maturity must be at least equal to $\underline{L}$. A case of default is defined as a situation in which the pension fund is underfunded and has insufficient assets to pay the beneficiaries in full at maturity ($A_T < \underline{L}$). Over the duration of the pension deal the pension fund trustees have to manage

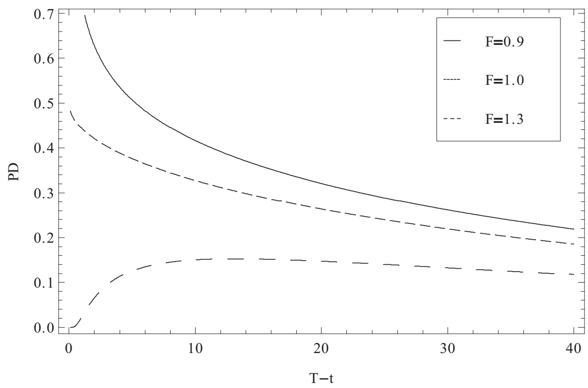

Figure 1.1: Probability of default (PD) in equation (1.3) for different initial funding ratios and time horizons, using $r = 0.05$, $\mu = 0.08$, $\sigma^2 = 0.02$ and $\underline{L}= 100e^{r(T-t)}$.

the assets in relation to their liabilities. Two related measures are important in managing the shortfall risk: the probability of a default and the expected loss given a default. The *probability of default* (PD) equals[3]

$$PD = P(A_T < \underline{L}) = N(-\mathbf{d_2}) \tag{1.3}$$

with N the cumulative normal distribution function and parameter $\mathbf{d}_2$ equal to

$$\mathbf{d_2} = \frac{\ln(A/\underline{L}) + \left(\mu - \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} \tag{1.4}$$

where bold face distinguishes this parameter from the equivalent risk-neutral parameter in the Black-Scholes-Merton framework. Figure (1.1) plots the probability of default as a function of the time to maturity for different initial funding ratios. The funding ratio is the market value of the assets divided by the market value of the liabilities discounted at the risk-free rate so $F = A/(\underline{L}e^{-r(T-t)})$. For instance, starting with a funding ratio of 130%, an annual expected return on assets of 8% with volatility 14.14% and a risk-free return of 5%, delivers a probability of default on a one year horizon of 2.3%.

This compares to the solvency test in Dutch pension regulation. The capital for pension funds is based on a confidence level of 97.5%, see Broeders and Pröpper

[3] This follows straigtforward from writing $P(A_T < \underline{L}) = P\left(\ln \frac{A_T}{A} < \ln \frac{\underline{L}}{A}\right)$ or $P\left(z < \frac{\ln(\underline{L}/A) - \left(\mu - \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}\right)$.

(2010). This means that, theoretically, the required buffer is at least enough to prevent the assets from falling below the level of the technical provisions with a level of probability of 97.5% in the subsequent year. Starting from a situation of overfunding ($F > 1$), the probability of default initially increases with time to maturity. Evaluated on a 5 year horizon the PD in (1.3) is 13.0%. This, however, is not a general result. After a certain time to maturity the default probability starts to decrease. The time to maturity at which the underfunding probability is at maximum value, given an initial funding ratio in excess of 100%, is given by

$$T - t^* = \ln(F) / \left(\mu - r - \frac{1}{2}\sigma^2 \right). \tag{1.5}$$

For instance, in the numerical example the probability of underfunding is the highest for a holding period of approximately 13 years. For longer maturities this risk measure decreases. This feature invalidates the measure for long term risk management and life-cycle planning, see Treussard (2005) for extensive considerations on this. In addition, from equation (1.5) it follows directly that a the turning point ($T - t^*$) is highly sensitive to the expected risk premium in the denominator. In the example the turning point doubles to 26 years if the expected risk premium is lowered by 1 percentage point.

A shortcoming of the probability of default is that it is a one-dimensional measure of risk. It does not take into account the severity of the shortfall. This aspect, however, can be quantified using another risk measure: *loss given default (LGD)*. The LGD can be derived using the conditional expectation of the market value of the pension fund's assets at maturity, given that these are less than the guaranteed pension benefit, or formally

$$E(A_T | A_T < \underline{L}) = \frac{\int_0^{\underline{L}} A_T f(A_T) dA_T}{P(A_T < \underline{L})} \tag{1.6}$$

where $f(A_T)$ is the log normal density function for A_T. Following Broeders (2006) the solution to equation (1.6) can be written as

$$E(A_T | A_T < \underline{L}) = A e^{\mu(T-t)} N(-\mathbf{d_1}) / N(-\mathbf{d}_2) \tag{1.7}$$

with parameter $\mathbf{d_2}$ in equation (1.4) and parameter $\mathbf{d_1}$ defined as

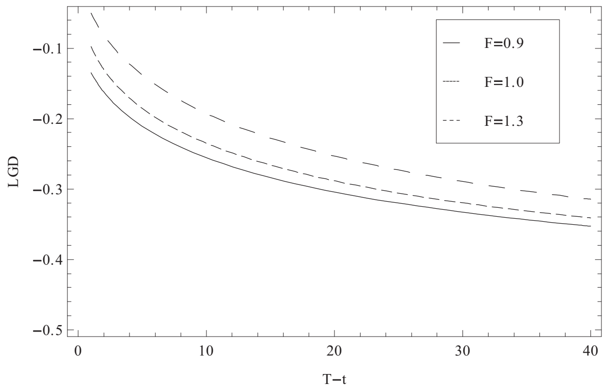

Figure 1.2: Present value of the loss given default (LGD) in equation (1.7) for different initial funding ratios and time horizons, using $r = 0.05$, $\mu = 0.08$, $\sigma^2 = 0.02$ and $\underline{L} = 100e^{r(T-t)}$.

$$\mathbf{d_1} = \mathbf{d_2} + \sigma\sqrt{T-t}. \tag{1.8}$$

The LGD is defined as $E(A_T|A_T < \underline{L})/\underline{L} - 1$. Figure (1.2) plots the present value of the loss given default for different initial funding ratios and maturities up to 40 years. The present value is taken for comparability of the loss given default over different horizons. A higher initial funding ratio lowers the expected value of the shortfall. However, for any initial funding ratio the present value of the loss is a monotonic increasing function of the time to maturity.

Both dimensions of risk (probability and severity) are taking into account in option pricing, see Bodie (1995) for a discussion on this. Option pricing for this reason may be a useful tool for pension fund risk management and therefore is central in the remaining of this chapter.

1.3 General framework for pension fund analysis

Pension benefits are often considered a form of deferred wage. A pension fund can be seen as a special purpose vehicle carrying forward the deferred income until it is payable. This way pension fund assets are bankruptcy remote from the sponsor. To decrease dependence on the sponsor the pension fund should always be fully funded and, e.g., by means of a buffer, derivatives or reinsurance contracts, be able to absorb market and actuarial risks. A clean break between a pension fund and its sponsor, however, is highly hypothetical. For analyzing the economic relation the following assumptions are made.

(Liabilities) The defined benefit is a single nominal cash flow of $\underline{L}$ at time $t = T$ to a homogenous cohort of beneficiaries, cf. Merton (1974) and Steenkamp (1998). This cash flow is equal to the present value of the annuity payments over the expected remaining life of the beneficiaries and is related to final or average pay and years of service, see Bodie (1990). One can also think of $\underline{L}$ representing the average of a sequence of cash flows for different cohorts with an equivalent duration. This assumption is justified by the observation in practice that pension funds often take the average participant as a benchmark in decision-making on funding and asset allocation. This one period approach is a simplification of reality. In practice a pension fund has a liability structure extending over multiple periods, where each periodical cash out-flow contains option features. By combining these cash flows in a single bullet on the basis of average characteristics a single period model can be used. Furthermore, all idiosyncratic mortality risk is assumed to be fully diversified and the expected improvement in life expectancy is included in $\underline{L}$. To express the market value of this guaranteed benefit it must be discounted at the risk-free rate r. That is to say, the defined benefit can be replicated by investing in a default free zero coupon bond with equivalent maturity.

(*Indexation policy*) The pension fund aims at providing an indexed pension $\overline{L}$ at maturity. If the ex ante indexation ambition is denoted by i, for instance 2% per annum, the relation between the nominal pension and the fully indexed pension is $\overline{L} = \underline{L}e^{i(T-t)}$.The indexation ambition could be linked to the expected inflation or wage growth over the maturity of the pension deal. The actual indexation is contingent on the funding ratio at maturity. If the funding ratio is below 100% ($A_T < \underline{L}$) there is no indexation at all, the beneficiaries still get the nominal pension $\underline{L}$. If the funding ratio is high enough as $A_T > \overline{L}$ full indexation is granted and the beneficiaries receive $\overline{L}$. In between the amount of indexation depends linearly on the funding ratio and the beneficiaries receive A_T. Such a rule is also known a policy ladder. To summarize at the maturity date T the payoff to the beneficiaries, ψ_B, depends as follows on A_T

$$\psi_B(A_T) = \begin{cases} \underline{L}, & \text{if } A_T < \underline{L} \\ A_T, & \text{if } L \leq A_T \leq \bar{L} \\ \bar{L}, & \text{if } A_T > \bar{L}. \end{cases} \tag{1.9}$$

(Funding decision) The funding decision entails the contribution level or the amount of assets (A) set aside and the investment policy characterized by the volatility of the return on the pension fund assets (σ).[4] Asset prices follow the geometric Brownian motion process in (1.1). The funding decision is made $t = 0$ and not changed afterwards.

(Risk sharing mechanisms) To insure the payment of $\underline{L}$ in the future, the sponsor will (partly) cover the deficit if at maturity the asset value is below the guaranteed defined benefit level $\underline{L}$. From the pension fund's perspective this resembles a long position in a put option with a strike price of $\underline{L}$. In return, the sponsor will claim all assets in excess of $\overline{L}$ at maturity. This represents a short call option with strike price $\overline{L}$. There are no intermediate cash outflows between the sponsor and the pension fund. This allows for the use of European options.

(Pension fund objective) It is assumed throughout the chapter that the pension fund's objective function is to maximize the market value of the beneficiaries' claim in the pension deal. For that, the trustees are able to make decisions independently of the sponsor. This might be the case if , e.g., the pension scheme is small compared to the corporation or if there are multiple independent sponsors for a single pension fund, like in an industry-wide pension scheme. However, the trustees may take into account the possibility that the sponsor defaults and can not complement shortages. It is also possible to envisage a multiple-stakeholder setting in which each stakeholder optimizes his objective function with respect to different criteria. This could be analyzed using a game-theoretic approach and is suggested as a subject for further research.

1.4 Unconditional guarantee of the defined benefit

In the current and next sections the embedded options are analyzed under different assumptions with respect to the ability of the sponsor to cover pension fund

[4] This implies that the pension fund continuously rebalances its portfolio. The realism of this assumption will be addressed in Chapter 3.

deficits. First, this section considers a situation in which the sponsor offers an unlimited guarantee to the pension fund. In the subsequent sections this assumption is relaxed. If the sponsor unconditionally covers all losses, the pension fund has implicitly a long position in a put option ($P_{\underline{L}}$) that gives the right to sell the assets to the company at maturity for $\underline{L}$. The pay-off of this put option is $\max(\underline{L} - A_T; 0)$. In return for providing insurance, it is assumed that the company has the right to withdraw any surpluses in the fund in excess of $\overline{L}$. The pension fund has implicitly written a call option ($C_{\overline{L}}$) on its assets with pay-off $\max(A_T - \overline{L}; 0)$. In absence of counterparty risk, the market value of the pension fund surplus I is given by

$$I = A + P_{\underline{L}} - C_{\overline{L}} - \underline{L}e^{-r(T-t)}. \tag{1.10}$$

The surplus in equation (1.10) can also be interpreted as the market value of the contingent indexation claim of the beneficiaries. Note that the pension fund has no influence on either $\underline{L}$ or $\overline{L}$ because they are given in the pension deal which is negotiated by employers and employees. Following Sharpe (1976) the only parameters to be influenced by the fund are the total amount of assets (A) and volatility of the surplus (σ). This surplus volatility is determined by the mismatch between assets and liabilities.

Figure (1.3) plots the market value of I as percentage of the present value $\underline{L}$ for different maturities and volatilities. The graph suggests there is an optimum with respect to σ for each time to maturity. Before analyzing the optimum, an assumption is made about the amount of total assets (A). Unless stated otherwise it is assumed throughout this chapter that the amount of assets is chosen such that the funding ratio (F) at the pension funds' inception is 100%. A funding ratio of 100% ($F = 1$) implies that the assets are exactly equal to the market value of the nominal liabilities or $A = \underline{L}e^{-r(T-t)}$. In this case the pension fund is not required to hold a solvency margin as the downside risks are covered by the sponsor guarantee.

Within this setting it is assumed that the pension fund trustees act in the best interest of the participants by choosing the asset allocation such that it maximizes the market value of the indexation contract. From (1.10), the pension fund therefore can derive its optimal investment policy by solving

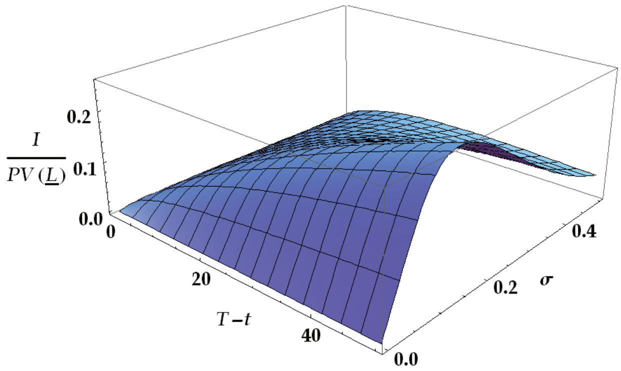

Figure 1.3: The market value of the pension fund surplus (I) in equation (1.10) divided by the market value of $\underline{L}$ using $r = 0.05$, $S = 100$, $i = 0.02$, $\underline{L}= 100e^{r(T-t)}$ and $\overline{L} = \underline{L}e^{i(T-t)}$.

$$\frac{\partial I}{\partial \sigma} = 0 \implies \frac{\partial P_{\underline{L}}}{\partial \sigma} = \frac{\partial C_{\overline{L}}}{\partial \sigma}. \tag{1.11}$$

The market value of the pension fund surplus is maximized when the sensitivities of the market values of both options for changes in volatility are equal. The put option provides downside insurance. The call option with the higher exercise price limits the upside potential for the beneficiaries. The economic interpretation of (1.11) is that, in the optimum, the marginal cost of insurance equals the marginal reward for risk taking.

Form Appendix 1.11.1 it follows that the market value of the surplus is maximized if the volatility of the pension fund is chosen equal to the square root of the fixed annual indexation ambition, so

$$\sigma^* = \sqrt{i} \tag{1.12}$$

where the asterisk denotes the optimal value. The interpretation of this result is straightforward. The optimal risk profile solely depends on the indexation ambition.[5] The only uncertainty for the participants in the pension fund is the value of the assets at $t = T$, within the following boundary conditions. The fund can always sell the assets at $\underline{L}$ if $A_T <\underline{L}$ and the sponsor will buy the assets for $\overline{L}$ if $A_T > \overline{L}$. This is also known as collar strategy. In this specific case, a longer

[5]Formula (1.12) immediately reveals that if the indexation target i is nil, the pension fund only has nominal guaranteed liabilities. The optimal investment strategy then is to replicate these liabilities by investing in the matching fixed income securities, implying $\sigma^* = 0$.

time to maturity does not change the optimal investment policy as $(T - t)$ does not appear in the optimal solution. Note that an option valuation model has been used to reverse-engineer the optimal volatility. The pension fund derives its optimal volatility given the indexation ambition and the perceived credit quality of the sponsor, which in this case is of the highest level. This translates into a particular asset allocation given the risk-return characteristics of the available investment opportunities. E.g., if the indexation target is 2%, than $\sigma^* = \sqrt{0.02}$ or approximately 14%.

The suitable asset allocation can now be found by choosing an investment portfolio that delivers the optimal volatility.[6] Numerous different asset allocations have the same volatility. As an example, Table (1.1) shows several asset allocations that offer a volatility ranging from 0.05 to 0.175. Panel A represents portfolios consisting of stocks and bonds only. Panel B presents portfolios including a 5% minimum allocation to real estate. And Panel C shows allocations that maximize expected return and cap the real estate allocation to 20%. Furthermore we can distinguish between several risk measures. The risk-neutral probability of the put option in (1.10) expiring in-the-money equals $N(-d_2)$. The true or physical probability of a funding deficit can be derived from (1.3) or Figure (1.1) taking $F = 1$. The loss given default follows from (1.6) or Figure (1.2).

For a funding ratio different from 100%, the solution to equation (1.11) is given by

$$\sigma^* = \sqrt{i - \frac{2\ln(F)}{T - t}} \tag{1.13}$$

see also Appendix 1.11.1. This implies that a higher funding ratio $(F > 1)$ lowers the optimal risk profile of the pension fund.[7] This can be explained through the fact that an increasing funding ratio will automatically increase the market value of the refunding option and lower the market value of the option to increase future premiums. Increasing mismatch risk in that case is not in the best interest of the beneficiaries of the pension fund. In fact, having a funding ratio in excess of

[6] Strictly speaking we should distinguish between the risk neutral volatility and the volatility in the physical world here. There is evidence that the risk neutral volatility implied by option prices is a biased upward predictor of the future realized volatility of returns on the underlying asset. See, e.g., Lamoureux and Lastrapes (1993) and Fleming (1998).

[7] It is straightforward to see that for $F \geq e^{i(T-t)}$ the optimal asset portfolio consists of risk-free fixed income securities that fully immunize the pension liabilities $(\sigma^* = 0)$.

		σ^*					
	Asset class	**0.05**	**0.075**	**0.100**	**0.125**	**0.150**	**0.175**
A	Stocks	0.070	0.326	0.479	0.616	0.747	0.874
	Bonds	0.930	0.674	0.521	0.384	0.253	0.126
B	Stocks	0.059	0.319	0.472	0.609	0.739	0.850
	Bonds	0.875	0.626	0.478	0.341	0.211	0.100
	Real estate	0.067	0.055	0.050	0.050	0.050	0.050
C	Stocks	0.062	0.284	0.440	0.578	0.715	0.850
	Bonds	0.888	0.516	0.360	0.222	0.100	0.100
	Real estate	0.050	0.200	0.200	0.200	0.188	0.050

Table 1.1: Asset allocations for different optimal volatilities. The standard deviation of stocks returns is 0.200, the standard deviation of bond returns is 0.055 and 0.120 for real estate returns. Furthermore, the correlation between stock and bond returns is assumed to be 0 and between stock and real estate returns and between bond and real estate returns 0.5. The expected return on equities is 0.090, on bonds 0.045 and on real estate 0.080.

100% is unnecessary since the downside risk is already fully insured through the put option and need not be covered by additional assets in the pension fund. At lower funding ratio ($F < 1$) risk taking optimally increases. This is sometimes also observed in practice as sponsors close to default are more likely to undertake riskier asset strategies as funding falls to make up the shortfall, and are more open to discussing de-risking their plans when the funding gap is reduced, see Inkmann and Blake (2007).

The same approach can be followed if the pension deal offers unconditional indexation. One additional feature compared to the previous setting, however, is that inflation is a stochastic variable and a such the exercise price of call option is uncertain. Assuming that inflation evolves according to a geometric Brownian motion process the option can be modelled as an exchange option, see Margrabe (1978), Fisher (1978) and Steenkamp (1998).

1.5 Limited sponsor guarantee

So far a loyal and solvent sponsor is assumed. This leads to a solution in which the optimal risk profile of the pension fund only relates to the indexation ambition expressed in the form of a fixed annual percentage. In reality the behavior of the sponsor will depend on the financial ability to guarantee the accrued pension rights of the retirees. The assumption that the sponsor will bear the full burden of subsequent deficits may be too strong. In the current and the following sections this assumption is relaxed in several ways.

This section assumes that the sponsor offers a guarantee, but only below a given percentage (κ) of the accrued benefits $\underline{L}$. In case of default of the pension fund, the beneficiaries lose $(1-\kappa)\underline{L}$ before the sponsor steps in and covers additional losses. This is a rudimentary way of sharing default risk between the stakeholders. Again, the sponsor successfully claims all assets in excess of $\overline{L}$, causing an asymmetric distribution of investment gains and losses between beneficiaries and the sponsor. The surplus at market value in this set-up is given by

$$I = A + P_{\kappa\underline{L}} - C_{\overline{L}} - \underline{L}e^{-r(T-t)}. \tag{1.14}$$

The annual indexation ambition is again fixed at i. The trustees of the pension fund act in the best interest of the beneficiaries by maximizing the market value of I as in equation (1.11). In Appendix 1.11.2 the optimal volatility is derived as

$$\sigma^* = \sqrt{\frac{i^2 - (\ln(1/\kappa)/(T-t))^2}{i + \ln(1/\kappa)/(T-t)}}. \tag{1.15}$$

For $\kappa = 1$, the sponsor fully guarantees nominal pensions, leading to the result in the previous section. Note that for $\kappa < e^{-i(T-t)}$, volatility should equal zero; the pension fund ought to confine its task to replicating the nominal liabilities in the capital market. If, e.g., $i = 2\%$ and $T - t = 15$ years, the sponsor should at least underwrite 74% of the nominal benefits to make it worthwhile for the pension fund to take on mismatch risk. In case of a limited guarantee (with boundary conditions $e^{-i(T-t)} < \kappa < 1$), optimal volatility is always less than in the fully assured situation. Also note that duration $(T - t)$ now influences the optimal solution.

Figure (1.4) shows the relationship between optimal volatility (σ^*), the fraction of defined benefits underwritten by the sponsor (κ) and time to maturity $(T - t)$. As can been seen from the graph, a lower κ can be partially offset by the time to maturity: a pension scheme with longer dated liabilities can engage somewhat more risk. Furthermore, the reader can easily check that the maximum volatility is reached for κ equal to one. In this case the pension fund can take advantage of

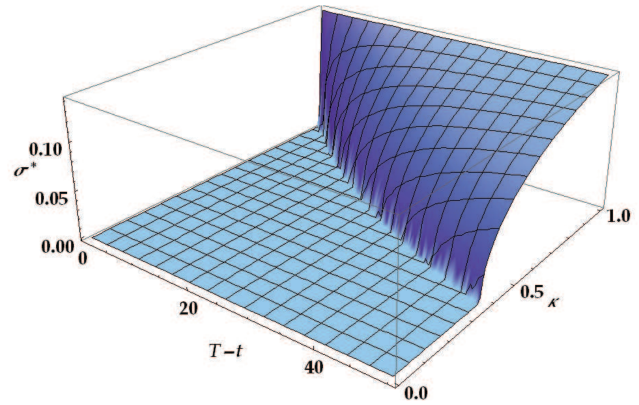

Figure 1.4: Optimal volatilities under limited guarantee for equation (1.15) using $r = 0.05$, $S = 100$, $i = 0.02$, $\underline{L}= 100e^{r(T-t)}$ and $\overline{L} = \underline{L}e^{i(T-t)}$.

the fact that the sponsor fully guarantees the nominal pensions ($\underline{L}$).

1.6 Partial loss insurance by sponsor

The next step is to consider a situation in which shortages in the pension fund are always partially shared between the beneficiaries and the sponsor. The pay-off of the put at maturity equals $\lambda \max(\underline{L}-A_T; 0)$ in case of default, with $0 \leq \lambda \leq 1$. Factor $1 - \lambda$ resembles a depreciation factor of the defined benefits for the beneficiaries in case of unforeseen cumulated investment losses at maturity. The sponsor finances the remainder of the loss. Again, the surplus at market value is given by

$$I = A + \lambda P_{\underline{L}} - C_{\overline{L}} - \underline{L}e^{-r(T-t)}. \tag{1.16}$$

Solving $\partial I/\partial\sigma = 0$ gives the following relationship between the indexation target (i), time to maturity ($T - t$) and loss sharing factor (λ)

$$\sigma^* = \frac{i}{\sqrt{i - 2\ln(\lambda)/(T-t)}}. \tag{1.17}$$

If the counterparty of the insurance contract covers all losses ($\lambda = 1$), again equation (1.12) results. Figure (1.5) shows the relationship between optimal volatility (σ^*), the loss sharing factor (λ) and time to maturity ($T - t$). The maximum optimal volatility is reached for λ equal to one. In the limit that the

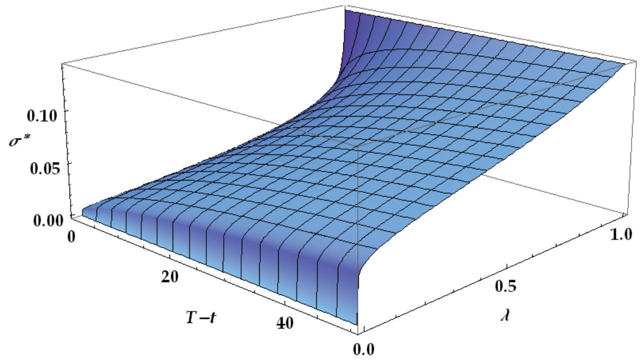

Figure 1.5: Optimal volatilities under partial loss insurance for equation (1.17) using $r = 0.05$, $S = 100$, $i = 0.02$, $\underline{L}= 100e^{rT}$ and $\overline{L} = \underline{L}e^{i(T-t)}$.

sponsor offers no loss compensation at all ($\lambda \to 0$), volatility should converge to zero.

The optimum under partial loss insurance in (1.17) differs from the optimum under a limited guarantee in (1.15) in the sense that a non-zero volatility is always optimal. This is based upon the assumption that in the first case any losses are to some extent always shared among the sponsor and the beneficiaries. Put differently, the beneficiaries can benefit from the fact that the sponsor will bear part of the downside risk. However, in the case of a limited guarantee there is a boundary condition because the sponsor bears all the risk only below a certain threshold level indicated by κ.

1.7 Sponsor default risk

The preceding sections assumed that loss absorption occurs according to fixed parameters and is known in advance. Typically, pension arrangements rarely include such explicit arrangements. The quality of the sponsor guarantee will generally depend on the sponsor's financial ability to underwrite losses. Extending the analysis further, this section introduces specific characteristics of the sponsor into the objective function. The main feature is that the put option in equation (1.10) is a so-termed *vulnerable option*: a derivative security with the risk of a defaulting counterparty. It is straightforward that options which are vulnerable to counterparty credit risk have lower market values than otherwise identical but non-vulnerable options. These options are described in Johnson and Stulz (1987), Hull and White (1995) and Klein (1996). For an overview see Ammann (2001).

This section applies the closed form formula from Klein (1996) as it allows for a correlation between the corporate and the pension fund's assets. Let V be the current market value of the sponsor (time subscript t is suppressed for ease of notation) following a geometric Brownian motion with volatility σ_V. Since A also follows a similar process, $\ln(A_T)$ an $\ln(V_T)$ are bivariate normally distributed. D_T represents the future total (fixed) liabilities of the sponsor including those potentially arising from underfunding at the pension fund level. All liabilities have the same maturity. Furthermore, Klein (1996) distinguishes deadweight losses associated with bankruptcy expressed as a percentage of the market value of the assets of the counterparty (α). These losses include the direct cost of the bankruptcy, reorganization expenses and the effects of distress on the business operations of the company. These costs are often minor but can go to 100% if the defaulting company is for instance a consultancy firm that only has intangible assets. Key in this set-up is that at $t = T$ default of the company is triggered if $V_T < D_T$.[8] The market value of the pension fund surplus is equal to

$$I = A + P^v_{\underline{L}} - C_{\overline{L}} - \underline{L}e^{-r(T-t)} \tag{1.18}$$

with the market value of the vulnerable put option P^v equal to

$$\begin{aligned} P^v_{\underline{L}} &= \underline{L}e^{-r(T-t)}N_2(-b_1, b_2, \rho) - AN_2(-a_1, a_2, \rho) \\ &+ (1-\alpha)\frac{V}{D_T}\left\{\underline{L}N_2(-d_1, d_2, -\rho) - Ae^{(r+\rho\sigma\sigma_V)(T-t)}N_2(-c_1, c_2, -\rho)\right\}. \end{aligned} \tag{1.19}$$

The first two terms on the right hand sight of this equation are basically similar to a regular, default-free, put option. The last term relates to the bankruptcy costs, to the current sponsor's financial position V/D_T and the interdependence between the sponsor and its pension fund. Symbol ρ represents the correlation between the sponsor's assets and the pension fund's assets and $N_2()$ is the cumulative bivariate normal density function. The remaining pricing parameters (a_1, a_2, b_1, b_2, c_1, c_2, d_1, d_2) are all defined in Appendix 1.11.4.[9]

[8] The possibility of a premature default is ruled out.

[9] Implicitly to this model is that if the company defaults at maturity and the pension fund has surplus assets, these assets are nonetheless transferred to the company or to the creditors of the

The optimal volatility can be found by numerical procedures. For the special case of zero correlation between the pension fund and the sponsor ($\rho = 0$), the optimal risk profile reduces to the following analytical solution, which is derived in Appendix 1.11.4.

$$\sigma^* = \frac{i}{\sqrt{i - 2\ln\left\{N(a_2) + (1-\alpha)\frac{V}{D_T}e^{r(T-t)}N(c_2)\right\}/(T-t)}}. \tag{1.20}$$

Note that for $V >> D_T$ there is virtually no default risk for the pension fund's beneficiaries. In that case, the optimum again equals $\sigma^* = \sqrt{i}$, which is also the upper limit of the feasible risk profiles.

Table (1.2) shows how volatility is conditional on distinct characteristics of the sponsor. As one would expect, the table shows that there is an apparent relationship between the volatility of the sponsor (σ_V) and optimal mismatch risk at the pension fund level (σ^*). If the sponsor has a high risk profile, the associated pension fund should reduce risk taking. This is also the case for the correlation between the sponsor and the pension fund. As already mentioned before, an increasing ratio of the market value of the sponsor to the notional value of all debt (V/D_T) provides the pension fund with additional risk taking resources. The quality of the sponsor guarantee increases for the beneficiaries because the sponsor has less outstanding debt relative to its own market value. The impact of bankruptcy costs (α) on σ^* is limited. Although ranging from $\alpha = 0$ to $\alpha = 1$, the average reduction in volatility may count for a few percentage points.

1.8 Multi period analysis

One of the elements in the previous sections that can be challenged is the single period assumption. This implies that indexation is only granted at maturity. In reality however, the indexation decision is made every consecutive year. Therefore, the frequency of indexation decisions might influence the optimal asset allocation. This will be explored in this section. For this purpose we introduce ratchet options, also known as cliquet options. A ratchet option is a series of options that

company.

	$\boldsymbol{\alpha}$				
Case	**0.000**	**0.250**	**0.500**	**0.750**	**1.000**
$\sigma_V = 0.10$	0.139	0.135	0.131	0.128	0.125
$\sigma_V = 0.25$	0.095	0.087	0.080	0.074	0.069
$\sigma_V = 0.50$	0.057	0.054	0.051	0.049	0.046
$\rho = -0.25$	0.096	0.090	0.086	0.082	0.079
$\rho = \quad 0.25$	0.081	0.075	0.070	0.066	0.062
$\rho = \quad 0.50$	0.067	0.065	0.063	0.061	0.059
$V/D_T = 2.0$	0.111	0.102	0.095	0.089	0.084
$V/D_T = 0.5$	0.071	0.064	0.059	0.054	0.049
$V/D_T = 0.2$	0.051	0.046	0.042	0.039	0.034

Table 1.2: Optimal volatility under sponsor default risk: using vulnerable option valuation formula from Klein (1996) with defaults $A = 100$, $r = 0.05$, $i = 0.02$, $T - t = 15$, $\underline{L} = Se^{r(T-t)}$, $\overline{L} = \underline{L}e^{i(T-t)}$ and the following default values $\alpha = 0$, $\sigma_V = 0.25$, $\rho = 0$ and $V/D_T = 1$.

allows for a frequent resetting of the strike price. The increase in the strike price is typically equal to the greater of a certain guarantee rate (g) and the increase in the underlying asset ($R_t = \ln[S_t/S_{t-1}]$). Moreover, the increase can be capped by ceiling rate (c). Ratchet options are being used to evaluate Equity Indexed Annuities, see Tiong (2000) and Hardy (2003). Dai and Schumacher (2008) apply the ratchet feature to analyze contingent indexation for pension funds. The contribution of this section is to determine the implications for the optimal investment policy of pension funds with contingent indexation.

Following Tiong (2000), the *per monetary unit* present value I of a compounded ratchet option equals[10]

$$I = E\left[e^{-rT}\prod_{t=1}^{T}\min(\max(e^{R_t}, e^g), e^c)\right].$$

Tiong (2000) provides a closed form formula for I, under the assumptions that the returns are identically and normally distributed with variance σ^2 and interest rates are constant. This formula is repeated here

[10] Tiong (2000) also identifies participation rate α ($0 \leq \alpha \leq 1$) in the valuation formula. This participation rate is typical for insurance contracts where policyholders do not in full participate in the return of the underlying assets. Part of the return accrues to the insurance companies' shareholders. For a pension contract the participation rate α is typically equal to 1 as there are no external shareholders and is therefore left out of the valuation formula here.

$$I = \left[e^{-(r-g)}N(d_1) - N(d_2) + e^{c-r}N(-d_3) + N(d_4)\right]^T \qquad (1.21)$$

where

$$\begin{aligned} d_1 &= \frac{g - r + \frac{\sigma^2}{2}}{\sigma}, \; d_2 = d_1 - \sigma \\ d_3 &= \frac{c - r + \frac{\sigma^2}{2}}{\sigma}, \; d_4 = d_3 - \sigma. \end{aligned}$$

Now we turn to the pension fund setting introduced in the previous sections. Under the assumption of a stationary pension fund, where the pension accrual exactly offsets the outflow of benefit payments, we know that the annual nominal growth rate of the pension fund's technical provision equals the risk-free rate (r). Here we assume a constant interest rate again. If the assets perform well ($R_t > r + i$), full indexation is granted and the growth rate will be ($r + i$). If the assets perform moderately ($r < R_t < r + i$) the growth rate equals R_t and if the asset return drops below the risk-free rate ($R_t < r$) no indexation is given and the growth rate equals r. Compounding all indexation decisions until maturity T of the pension contract effectively results in the following present value of the indexation policy

$$I = E\left[e^{-rT}\prod_{t=1}^{T}\min(\max(e^{R_t}, e^r), e^{r+i})\right].$$

It is now straightforward to identify the floor rate being equal to the risk-free rate ($g = r$) and the ceiling rate equal to the risk-free rate plus full indexation ($c = r+i$). Therefore, the value of the indexation contract in (1.21) can be written as follows

$$I = \left[2N(\frac{1}{2}\sigma) - 1 + e^i N(\frac{-i - \frac{1}{2}\sigma^2}{\sigma}) + N(\frac{i - \frac{1}{2}\sigma^2}{\sigma})\right]^T. \qquad (1.22)$$

Hereby implicitly an initial funding ratio of 100% is assumed. To maximize the value of I, and therefore to determine the optimal asset allocation, we take the partial derivative of I with respect to volatility. Appendix 1.11.5 shows that the optimal volatility equals

$$\frac{\partial I}{\partial \sigma} = 0 \implies \sigma^* = \sqrt{i}.$$

This result exactly matches the one period model in Section 1.4. Apparently the frequency of indexation decisions is not relevant in determining the optimal asset allocation under the prevailing assumptions.

1.9 Volatility smiles

So far in the analysis volatility is assumed to be constant when pricing the options. In practice however a phenomenon called volatility smile or volatility skew is observed. This refers to the observation that the (implied) volatility decreases as the strike price increases. This implies that the market would price the put option (with the lower strike price $\underline{L}$) at a higher volatility than the call option (with the higher strike price $\overline{L}$). In this section we allow for different volatility parameters when evaluating the options with different strike prices. For this we define σ as the volatility of the put option in (1.10) and σ' as the call options' volatility. We assume the following simple linear relation

$$\sigma' = \beta\sigma$$

where β is the smile parameter. Under this assumption it can be shown that the partial derivative of the call option in (1.10) with respect to σ is given by

$$\frac{\partial C}{\partial \sigma} = An(d_{1c})\beta\sqrt{T-t}$$

with A the pension fund's assets, and

$$d_{1c} = \frac{\ln\left(\frac{A}{\overline{L}}\right) + (r + \frac{1}{2}\beta^2\sigma^2)(T-t)}{\beta\sigma\sqrt{T-t}}. \tag{1.23}$$

The optimal asset allocation, that is the optimal choice of σ given the level of assets A and the smile parameter β, follows again from evaluating

$$\frac{\partial P_{\underline{L}}}{\partial \sigma} = \frac{\partial C_{\overline{L}}}{\partial \sigma}$$

or

$$n(d_{1p}) = \beta n(d_{1c}).$$

However, since the complex definition of d_{1c} in (1.23), an analytical solution is out of reach. Alternatively, Table (1.3) shows some numerical results for the optimal asset volatility (σ^*) for different combinations of the funding ratio ($F = A/(\underline{L}e^{-r(T-t)})$) smile parameter ($\beta$).

Funding	β						
Level	**1.00**	**0.99**	**0.98**	**0.97**	**0.96**	**0.95**	**0.90**
$F = 0.90$	0.185	0.192	0.200	0.209	0.219	0.229	0.291
$F = 0.95$	0.164	0.170	0.178	0.186	0.195	0.205	0.267
$F = 1.00$	0.141	0.147	0.154	0.161	0.169	0.178	0.239
$F = 1.05$	0.116	0.121	0.127	0.133	0.140	0.148	0.205
$F = 1.10$	0.085	0.089	0.094	0.100	0.105	0.111	0.162
$F = 1.15$	0.037	0.040	0.043	0.046	0.050	0.054	0.092
$F = 1.20$	0.004	0.004	0.004	0.004	0.004	0.004	0.004
$F = 1.25$	0.002	0.003	0.003	0.003	0.003	0.003	0.003
$F = 1.30$	0.001	0.001	0.001	0.001	0.001	0.001	0.001

Table 1.3: Optimal volatility under a linear volatility smile: using $A = 100$, $r = 0.05$, $i = 0.02$, $T - t = 15$, $\underline{L} = 100e^{r(T-t)}$, $\overline{L} = \underline{L}e^{i(T-t)}$.

Several observations follow from Table (1.3). First, for $\beta = 1$ the optimal asset volatility coincides with the result in (1.13). Second, if β decreases the call options is priced at a lower volatility and becomes less expensive. As a result the optimal volatility goes up. This means that the pension fund can maximize the wealth for the beneficiaries by holding a more risky portfolio. Third, as the funding ratio increases the optimal asset volatility goes down. In fact, if the funding level is high enough to buy the replicating portfolio of risk-free indexed linked bonds, that is if $A/\overline{L}e^{-rT} > 1$, there is no need to take investment risk.

1.10 Summary

The first chapter of this thesis analyzes the market value based balance sheet of a pension fund with contingently indexed defined benefit liabilities. It is assumed that the trustees of the pension fund act in the best interest of the beneficiaries by maximizing the market value of the pension fund surplus. This market value can be derived from the difference between an implicit put and call option on the assets of the pension fund. The put resembles future contribution increases to the pension fund, the call future refunding to the sponsor. When these derivatives are analyzed as being traded on regulated markets, the optimal risk profile depends only on the indexation ambition, expressed in the form of a fixed annual target rate. Traded derivatives are virtually free from counterparty credit risk through the clearing and settlement function of the exchange. In the context of a pension fund, sponsor default risk reduces the quality of the downside insurance for the beneficiaries of a defined benefit pension scheme. Unlike on regulated markets, between a pension fund and its sponsor there are no margin requirements and, in many cases, not even explicit financing arrangements. This gives the sponsor the upper hand in the game of sharing the residual risk at the pension fund level. Residual risk is a loss that cannot be absorbed and ultimately leads to a write-off of accrued pension benefits. In fact, the beneficiaries of the pension fund are confronted with counterparty credit risk. This is shown by correcting the market value of the option to increase future contributions for the financial ability of the sponsor to actually do so. This chapter suggests that given a situation in which the sponsor unconditionally claims surplus assets but is reluctant or unable to fully cover losses, there is an asymmetric allocation of the residual risk over the sponsor and the participants in the fund. In such a situation and under the assumption that the pension fund maximizes the market value of its surplus, it is optimal to reduce risk-taking, which means that the pension fund cannot fully pursue its indexation policy. However, as such it also reduces the risk that the guaranteed liabilities are not fulfilled. Furthermore we investigate a multiperiod model in which annual indexation decisions are made. However, it appears that the frequency of indexation decisions is not of influence on the optimal asset allocation. The existence of a volatility smile has some consequences for the optimal asset allocation. If volatility smile is more skewed, the optimal portfolio is more risky.

1.11 Appendix

1.11.1 Unconditional guarantee

The pension fund aims at maximizing the market value of the indexation contract. From equation (1.10) the fund therefore can derive its optimal risk profile by solving

$$\frac{\partial I}{\partial \sigma} = 0 => \frac{\partial P_{\underline{L}}}{\partial \sigma} = \frac{\partial C_{\overline{L}}}{\partial \sigma}.$$

The partial derivative of the option price with respect to the volatility of the underlying asset is known as vega, see Hull (2008). Vega is the change in the value of an option for a one-percentage point change in volatility. The market value of the pension surplus is maximized when the sensitivities of the market values of both options for changes in surplus volatility are equal. Or the vega of the put should equal the vega of the call, so

$$An(d_{1,P})\sqrt{T-t} = An(d_{1,C})\sqrt{T-t}$$

or

$$n(d_{1,P}) = n(d_{1,C}).$$

Using the definition of density function of standardized normal variable

$$n(d_1) = e^{-\frac{1}{2}d_1^2}/\sqrt{2\pi}$$

gives

$$d_{1,P}^2 = d_{1,C}^2.$$

Assuming a funding ratio of 100% ($F = 1$) or $A = \underline{L}e^{-r(T-t)}$, so that $\underline{L} = Ae^{r(T-t)}$ and $\overline{L} = \underline{L}e^{i(T-t)}$, the option valuation parameters are defined by

$$d_{1,P} = \frac{\ln\left(\frac{A}{Ae^{r(T-t)}}\right) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} = \frac{\frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}$$

and

$$d_{1,C} = \frac{\ln\left(\frac{A}{Ae^{(r+i)(T-t)}}\right) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} = \frac{-i(T-t) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}.$$

Therefore $d_{1,P}^2 = d_{1,C}^2$ equals

$$\left\{\frac{1}{2}\sigma^2(T-t)\right\}^2 = \left\{-i(T-t) + \frac{1}{2}\sigma^2(T-t)\right\}^2.$$

Note that this equality has the form $A^2 = \{B + A\}^2$ and has solutions for $B = 0$ and $B = -2A$. The reader can easily infer from the latter solution that the optimal volatility equals

$$\sigma^* = \sqrt{i}.$$

For $F \neq 1$ the option valuation parameters are given by

$$d_{1,P} = \frac{\ln(F) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}} \text{ and } d_{1,C} = \frac{\ln(F) - i(T-t) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}.$$

Solving $d_{1,P}^2 = d_{1,C}^2$ in this case results in the following expression for the optimum

$$\sigma^* = \sqrt{i - \frac{2\ln(F)}{T-t}}.$$

1.11.2 Limited sponsor guarantee

In the case of a limited guarantee the exercise price of the put option $\underline{L}$ is multiplied by factor κ to obtain

$$\frac{\partial I}{\partial \sigma} = 0 => \frac{\partial P_{\kappa \underline{L}}}{\partial \sigma} = \frac{\partial C_{\overline{L}}}{\partial \sigma}.$$

Where parameter d_1 is adjusted accordingly

$$d_{1,P} = \frac{\ln\left(\frac{A}{\kappa Ae^{r(T-t)}}\right) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} = \frac{\ln(1/\kappa) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}.$$

With this adjustment $d_{1,P}^2 = d_{1,C}^2$ results in

$$\left\{\frac{\ln(1/\kappa) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}\right\}^2 = \left\{\frac{-i(T-t) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}\right\}^2.$$

The risk profile maximizing the market value of surplus is given by

$$\sigma^* = \sqrt{\frac{i^2 - (\ln(1/\kappa)/(T-t))^2}{i + \ln(1/\kappa)/(T-t)}}.$$

1.11.3 Partial loss insurance by sponsor

In the case of partial loss insurance the problem is as follows

$$\frac{\partial I}{\partial \sigma} = 0 => \lambda \frac{\partial P_{\underline{L}}}{\partial \sigma} = \frac{\partial C_{\overline{L}}}{\partial \sigma}.$$

This can be expressed as

$$\lambda \exp(-\frac{1}{2}d_{1,P}^2) = \exp(-\frac{1}{2}d_{1,C}^2).$$

Taking the log of both sides and multiplying by 2 leads to

$$d_{1,P}^2 - 2\ln\lambda = d_{1,C}^2.$$

$$\left\{\frac{\frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}\right\}^2 - 2\ln\lambda = \left\{\frac{-i(T-t) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}\right\}^2.$$

This equation can be simplified to

$$\sigma^* = \frac{i}{\sqrt{i - 2\ln(\lambda)/(T-t)}}.$$

1.11.4 Sponsor default risk

The market value of a vulnerable put option is given in Klein (1996) as

$$\begin{aligned} P_{\underline{L}}^v &= \underline{L}e^{-r(T-t)}N_2(-b_1, b_2, \rho) - AN_2(-a_1, a_2, \rho) + \\ &(1-\alpha)\frac{V}{D_T}\left\{\underline{L}N_2(-d_1, d_2, -\rho) - Ae^{(r+\rho\sigma\sigma_V)(T-t)}N_2(-c_1, c_2, -\rho)\right\}. \end{aligned}$$

where $N_2()$ represents the cumulative bivariate normal density function. The valuation parameters and partial derivatives are

$$a_1 = \frac{\ln\left(\frac{A}{Ae^{r(T-t)}}\right) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \qquad \frac{\partial a_1}{\partial \sigma} = \frac{1}{2}\sqrt{T-t}$$

$$a_2 = \frac{\ln\left(\frac{V}{D_T}\right) + (r - \frac{1}{2}\sigma_V^2 + \rho\sigma\sigma_V)(T-t)}{\sigma_V\sqrt{T-t}} \qquad \frac{\partial a_2}{\partial \sigma} = \rho\sqrt{T-t}$$

$$b_1 = \frac{\ln\left(\frac{A}{Ae^{r(T-t)}}\right) + (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \qquad \frac{\partial b_1}{\partial \sigma} = -\frac{1}{2}\sqrt{T-t}$$

$$b_2 = \frac{\ln\left(\frac{V}{D_T}\right) + (r - \frac{1}{2}\sigma_V^2)(T-t)}{\sigma_V\sqrt{T-t}} \qquad \frac{\partial b_2}{\partial \sigma} = 0$$

$$c_1 = \frac{\ln\left(\frac{A}{Ae^{r(T-t)}}\right) + (r + \frac{1}{2}\sigma^2 + \rho\sigma\sigma_V)(T-t)}{\sigma\sqrt{T-t}} \qquad \frac{\partial c_1}{\partial \sigma} = \frac{1}{2}\sqrt{T-t}$$

$$c_2 = -\frac{\ln\left(\frac{V}{D_T}\right) + (r + \frac{1}{2}\sigma_V^2 + \rho\sigma\sigma_V)(T-t)}{\sigma_V\sqrt{T-t}} \qquad \frac{\partial c_2}{\partial \sigma} = -\rho\sqrt{T-t}$$

$$d_1 = \frac{\ln\left(\frac{A}{Ae^{r(T-t)}}\right) + (r - \frac{1}{2}\sigma^2 + \rho\sigma\sigma_V)(T-t)}{\sigma\sqrt{T-t}} \qquad \frac{\partial d_1}{\partial \sigma} = \frac{1}{2}\sqrt{T-t}$$

$$d_2 = -\frac{\ln\left(\frac{V}{D_T}\right) + (r + \frac{1}{2}\sigma_V^2)(T-t)}{\sigma_V\sqrt{T-t}} \qquad \frac{\partial d_2}{\partial \sigma} = 0.$$

The partial derivative of the value of the put with respect to surplus volatility is

$$
\begin{aligned}
\frac{\partial P^v}{\partial \sigma} = {} & An(-b_1)N\left(\frac{b_2+b_1\rho}{\sqrt{1-\rho^2}}\right)\frac{\partial - b_1}{\partial \sigma} \\
& - A\left\{n(-a_1)N\left(\frac{a_2+a_1\rho}{\sqrt{1-\rho^2}}\right)\frac{\partial - a_1}{\partial \sigma} + n(a_2)N\left(\frac{-a_1-a_2\rho}{\sqrt{1-\rho^2}}\right)\frac{\partial a_2}{\partial \sigma}\right\} \\
& + (1-\alpha)\frac{V}{D_T}Ae^{r(T-t)}n(-d_1)N\left(\frac{d_2-d_1\rho}{\sqrt{1-\rho^2}}\right)\frac{\partial - d_1}{\partial \sigma} \\
& - \rho\sigma\sigma_V(T-t)(1-\alpha)\frac{V}{D_T}Ae^{r(T-t)}e^{\rho\sigma\sigma_V(T-t)}N_2(-c_1,c_2,-\rho) \\
& - (1-\alpha)\frac{V}{D_T}Ae^{r(T-t)}e^{\rho\sigma\sigma_V(T-t)}\{n(-c_1)N\left(\frac{c_2-c_1\rho}{\sqrt{1-\rho^2}}\right)\frac{\partial - c_1}{\partial \sigma} \\
& + n(c_2)N\left(\frac{-c_1+c_2\rho}{\sqrt{1-\rho^2}}\right)\frac{\partial c_2}{\partial \sigma}\}.
\end{aligned}
$$

Solving for $\rho = 0$ gives the following reduced formula for the sensitivity with respect to volatility

$$\frac{\partial P^v}{\partial \sigma} = An(a_1)\sqrt{T-t}\left\{N(a_2) + (1-\alpha)\frac{V}{D_T}e^{r(T-t)}N(c_2)\right\}.$$

Deriving $\frac{\partial P^v_{\underline{L}}}{\partial \sigma} = \frac{\partial C_{\overline{L}}}{\partial \sigma}$ results in

$$n(a_1)\left\{N(a_2) + (1-\alpha)\frac{V}{D_T}e^{r(T-t)}N(c_2)\right\} = n(d_1).$$

which can be written as

$$\frac{-i^2(T-t) + i\sigma^2(T-t)}{\sigma^2} = 2\ln\left\{N(a_2) + (1-\alpha)\frac{V}{D_T}e^{r(T-t)}N(c_2)\right\}.$$

Given zero correlation the optimal volatility equals

$$\sigma^* = \frac{i}{\sqrt{i - 2\ln\left\{N(a_2) + (1-\alpha)\frac{V}{D_T}e^{r(T-t)}N(c_2)\right\}/(T-t)}}.$$

1.11.5 Multiperiod analysis

Following Tiong (2000) the value of the indexation contract can written as follows

$$I = \left[2N(\frac{1}{2}\sigma) - 1 + e^i N(\frac{-i - \frac{1}{2}\sigma^2}{\sigma}) + N(\frac{i - \frac{1}{2}\sigma^2}{\sigma}) \right]^T .$$

Taking the partial derivative with respect to volatility σ results in $\frac{\partial I}{\partial \sigma} = 0 \Longrightarrow$

$$n(\frac{1}{2}\sigma) + e^i n(\frac{-i - \frac{1}{2}\sigma^2}{\sigma}) \frac{i - \frac{1}{2}\sigma^2}{\sigma^2} + n(\frac{i - \frac{1}{2}\sigma^2}{\sigma}) \frac{-i - \frac{1}{2}\sigma^2}{\sigma^2} = 0.$$

Applying the density function of the standard normal distribution and after multiplying with $2\sigma^2\sqrt{2\pi}$ the following expression results

$$2\sigma^2 e^{-\frac{1}{2}(\frac{1}{2}\sigma)^2} + e^i e^{-\frac{1}{2}(\frac{-i-\frac{1}{2}\sigma^2}{\sigma})^2} \left[2i - \sigma^2\right] + e^{-\frac{1}{2}(\frac{i-\frac{1}{2}\sigma^2}{\sigma})^2} \left[-2i - \sigma^2\right] = 0.$$

This can be analytically solved by noting that $e^i e^{-\frac{1}{2}(\frac{-i-\frac{1}{2}\sigma^2}{\sigma})^2} = e^{-\frac{1}{2}(\frac{i-\frac{1}{2}\sigma^2}{\sigma})^2} = e^a$, where $a = -\frac{1}{2}(\frac{i-\frac{1}{2}\sigma^2}{\sigma})^2$. The expression can now be rewritten into

$$2\sigma^2 e^{-\frac{\sigma^2}{8}} + e^a(2i - \sigma^2) + e^a(-2i - \sigma^2) = 0.$$

Simplifying further yields

$$e^{-\frac{\sigma^2}{8}} = e^a.$$

Solving this expression delivers the optimal solution

$$\sigma^* = \sqrt{i}.$$

Chapter 2

Pension Regulation and the Market Value of Pension Liabilities

This chapter is based upon Broeders and Chen (2010)

2.1 Introduction

As shown in the previous chapter, defined benefit (DB) pension plans are often viewed as a combination of option contracts. The beneficiaries of such a pension plan are entitled to a prespecified amount related to years of service and salary. In some cases the beneficiaries also have a share in the pension fund's surplus. This surplus is to some extent also accruable to the sponsor, often via contribution holidays. Conversely, as part of a risk sharing arrangement, the sponsor might have the obligation to increase contributions to the pension fund in case the funding level is inadequate. All these claims can be considered as options on the pension fund's assets. Unlike most other financial contracts, pension plans have a peculiar legal status, i.e. they are in most cases not entirely legally enforceable. Contrary to, for instance, a life insurance contract, a defined benefit pension promise is not completely irreversible. Most current pension contracts implicitly enable their sponsors to terminate the deal prematurely or to convert it along the way. This implies that the sponsor can avoid the payment of recovery premiums by changing the nature of the pension liabilities from a DB to a defined contribution (DC) pension plan when the pension fund is unable to settle its original DB promises[1]. In reality, many sponsoring companies consider this as an ultimate escape route, via which they are able to discard DB pension obligations if the financial burden of maintaining them gets too high. For instance, in the early 2000s when funding ratios fell substantially after the stock market crash, some companies indeed changed the nature of their pension promise from DB to DC. Some closed the pension fund for new entrants and others transferred the liabilities to an insurance company. Furthermore, the extra burden related to improved longevity and lower interest rates makes DB plans expensive to maintain. Some other important drivers of this conversion are changes in pension regulation and accounting, exposing marked-to-market values of pension liabilities and asset-liability-mismatch risks. Aaronson and Coronado (2005) and Broadbent *et al.* (2006) provide a wider variety of reasons to convert, also with respect to the interests of employees. Yang (2005) analyzes the factors that influence the choice between DB and DC plans for individuals.

The conversion trend is most manifest in the US. According to the US Flow of Funds Accounts, the division between assets held in private DB plans and private DC plans was 60% versus 40% in 1987, whereas in 2007 this ratio was exactly reversed; the turning point appears to be 1995. Recently large companies such as Ford, General Motors, IBM and Sears made a (partial) shift towards DC plans. The changes in the UK reveal a similar development. In 1979, final salary DB plans constituted 92% of all pension funds. However, in 2005 the Government Actuary's Department observed that 41% of all active members accrue their DB rights in pension plans closed to new entrants. This closure has been accompanied by the emergence of DC plans and average pay DB schemes. Starting from a relatively wealthy position with a funding ratio around 200% at the turn of the millennium, the Netherlands have been able to avoid a bulky shift towards DC plans. The perfect storm (negative stock returns combined with decreasing market interest rates) generated a change from final pay DB schemes (66% in 1998) to conditionally indexed average pay DB schemes (85% in 2007), see Bikker and Vlaar (2007). Under such schemes, the benefit depends not on the final but on

[1] A detailed description on the differences between DB and DC plans can be found e.g. in Bodie, Marcus and Merton (1988).

the career average salary. Final pay systems are expensive to maintain due to the so called back-service liabilities. Instead, in a career average plan both during the accrual and the benefit stage the pension rights are indexed to price or wage inflation conditional upon a sufficient funding ratio of the pension fund. As such, inflation and investment risks are shifted in part to active fund members. All these changes imply that DB pension plans do *not* provide their participants with a guaranteed amount and regular (unconditional) options introduced by Sharpe[2], but the premature closure and conversion features of the pension plans have made the various claims of the participants on the pension fund's assets more exotic. With this feature, the pension plan participants are in fact exposed to more risks. When the pension fund's assets value falls below the applicable regulatory boundary (roughly speaking in case of extended underfunding), the guaranteed payment may be fulfilled only partially. This affects the economic value of the beneficiaries' claim.

The pension conversion feature has been pointed out by several empirical studies. For instance, Petersen (1992) examines three hypotheses concerning the motivation underlying pension plan reversion and finds that all the hypotheses are empirically supported by the US data. Niehaus and Yu (2005) analyze the conversion of DB plans to cash balance plans in the US in the nineties. From a regulatory point of view cash balance plans are treated as DB plans, however beneficiaries conceive it as DC plans. More on the nature of pension contracts can be found, e.g., in Treynor (1997), Bulow (1982) and Ippolito (1985).

However, the premature closing or converting feature of pension plans has never been investigated analytically and theoretically. This chapter aims to fill the gap. The objective is to incorporate the closing feature in the valuation of DB pension liabilities, i.e., the contract payoff of the DB plan depends on the entire evolution of the pension fund's assets. When the funding ratio deteriorates, the DB plan might be closed by the regulatory authorities or converted to a DC plan.[3] In this chapter, the emphasis is not placed on how to model the DC plan, but on how the premature closing feature affects the market value of the DB plans. Therefore, we assume that the DB contract is terminated upon conversion. We set ourselves in a contingent claim framework and use knock–out barrier options

[2] See Section (1.1) for a literature overview.

[3] The assets in the DB plan can also be used to buy deferred annuities from a life insurer.

to describe the closing feature. We distinguish between two procedures: "immediate closure procedure" and "delayed closure procedure". In an immediate closure procedure, when the assets value hits the regulatory boundary, the pension plan is terminated immediately. This immediate closure procedure does not reflect reality in all cases because pension funds are usually given time to reorganize and recover. The recovery period varies across jurisdictions, as will be shown in the next section. Therefore, in addition to the immediate closure procedure, the delayed closure procedure is analyzed to capture all possible regulatory situations. The main feature of this procedure is that the closure does not come into force immediately when default (or underfunding) occurs; instead, a grace period is given to enable recovery. Mathematically the immediate and delayed closure procedures can be mirrored by applying standard and Parisian down–and–out barrier options, respectively.

Barrier options belong to the family of exotic options and are first mentioned in Snyder (1969). The payoff of these products is not based on the final value of the underlying asset only, but linked to the additional conditions of the asset value evolution. Let us assume that we are interested in the modelling of a down–and–out barrier option. The option contract is knocked out if the underlying asset hits the barrier (from above) during the option life. The topic of barrier options has been studied very widely in the literature, e.g., Rubinstein and Reiner (1991) and Rich (1994), to mention just a few. Recently, Grosen and Jørgensen (2002) incorporate a regulatory mechanism into the market valuation of equity and liabilities at life insurance companies by using a down–and–out barrier feature to describe the regulatory intervention rule. Compared to standard barrier options, Parisian options do not have a long history in the literature on exotic options. They were introduced by Chesney *et al.* (1997) and subsequently developed by Moraux (2002), Anderluh and van der Weide (2004) and Bernard *et al.* (2005). In a standard Parisian down–and–out option, the contract is knocked out if the underlying asset value remains consecutively below the barrier for longer than some predetermined time d before the maturity date. In the context of with–profit life insurance contracts, Chen and Suchanecki (2007) apply the Parisian barrier option framework to incorporate more realistic bankruptcy procedures (Chapter 11 bankruptcy procedure) in the market valuation of life insurance liabilities. This chapter is the first to incorporate Parisian barrier options in a pension fund setting.

The remainder of this chapter is organized as follows. Section 2.2 reviews funding requirements in different countries and Section 2.3 describes the basic payoff structure of pension plans and the underlying contingent claim model setup. Additionally, we introduce the theoretical background of barrier and Parisian barrier options. The next section focuses on the valuation of the DB pension plans. Section 2.4 contains a variety of numerical analyses aiming to derive fair pension deals, while section 2.5 discusses the role of recovery periods in pension regulation. Section 2.6 concludes the chapter with a summary of the key findings.

2.2 Overview of funding requirements

One of the key parameters in our framework is the regulatory boundary, which represents the minimum funding requirement and will be captured by the parameter λ. It represents the minimum funding requirement. Following Grosen and Jørgensen (2002), $\lambda \geq 1$ describes a situation in which the regulator requires the financial institution to always maintain a *buffer* so that in an unforeseen event of default the beneficiaries do not experience a loss with respect to the marked-to-market value of their claims. Conversely, if $\lambda < 1$ the regulator allows temporary deficits which might lead to a marked-to-market loss in case of default. However, since we allow for extended recovery periods, the expected loss is not equal to $1 - \lambda$ but to the difference between assets and liabilities at the end of the recovery period (conditional on no recovery having occurred and the pension fund being liquidated). The importance of an adequate funding level is underlined by the OECD (2007) as it recognizes that the amount of pension fund assets should in any case be sufficient to meet accrued benefit payments. Furthermore, the OECD argues that the funding level should also take account of the plan sponsor's ability and commitment to increase contributions to the pension plan in situations of underfunding, the possibility of benefit adjustments or changes in retirement ages, as well as the link between the pension fund's assets and its liabilities. Funding requirements can be expressed as a function of the regulatory boundary λ, the amortization of funding deficits, the definition of pension liabilities, the valuation procedure (or discount rate) for pension liabilities and the valuation of pension assets.

Funding requirements differ across countries. Table (2.1) based upon Pugh

Country	Minimum required funding rate	Deficit amortization	Pension liabilities	Discount rate
US	100%	Amortize within 7 years by sponsor	ABO	Yield on high quality bonds
Canada	100%	Amortize within 5 years by sponsor	ABO	Market based
UK	No specific requirements	Whatever is reasonable to the sponsor	ABO or PBO	Rate should be chosen prudently
Germany[4]	104.3%	Unknown	ABO	2.25%
The Netherlands	104.3%	Amortize within 3 years	ABO	Swap rates
Japan	90%	Amortize within 7 years by sponsor	ABO	80 to 120% of average yield on 10 year government bonds
Switzerland	90%	Amortize 5 to 7 years by sponsor and/ or employees	PBO	Traditionally between 3.5 and 4.5%

Table 2.1: Funding requirements for pension funds.

(2006), Blome *et al.* (2007), Yermo (2007) and Pugh and Yermo (2008), presents an overview of funding requirements in several Western countries.[5] The Pension Protection Act of 2006 requires US pension funds to be fully funded (i.e. $\lambda \geq 1$).[6] Sponsors must amortize any funding shortage within seven years. Pension liabilities in US are defined as accrued benefit obligations (ABO)[7] and are typically discounted by the yield on high quality bonds. The UK situation is rather unique because many decisions on funding and valuation are left at the discretion of pen-

[5] Expect for the Netherlands, all countries in Table (2.1) have a pension guarantee system for defined benefit plans.

[6] Before the introduction of the Pension Protection Act the required funding level was 90%.

[7] Accumulated Benefit Obligations (ABO): pension liabilities are the accrued benefits up to the valuation date, based on completed service and salary. Allowances are made for early retirement or leaving service and life expectancy. Projected Benefit Obligations (PBO): is the same as ABO, but with expected salary increases between the valuation date and the normal pension age taken into account as well. Bodie (1990a) argues that PBO is not an appropriate measure of the guaranteed liabilities as it takes into account projected increases in salary between now and retirement.

sion fund trustees. Within reasonable boundaries, UK trustees decide on matters like the target funding level, actions to be taken in case of a shortage as well as the precise definition and discounting of the pension liabilities. The UK applies a flexible recovery period as it has explicitly stated the objective of preventing undue pressure on sponsors and protecting schemes from being wound up. In other words, there is a need to balance the ongoing viability of the employer against the long–term interests for the members and their DB pension provisions. One of the triggers for additional supervisory scrutiny in the UK is the duration of a recovery plan. Note that the discount rate indirectly influences the regulatory boundary λ. For instance in Germany, pension liabilities are discounted at a fixed rate of 2.25%. If the market risk-free interest rate is higher this implicitly means that German pension funds face a regulatory boundary $\lambda > 1$. Conversely, if the market risk-free interest rate is lower this corresponds to $\lambda < 1$. In the Netherlands the swap curve is used to discount pension liabilities.

The regulatory boundary is not always equal to 1. For instance Japanese pension funds with a funding ratio of at least 90% are considered to be sufficiently funded. In Switzerland, the pension fund's actuary develops a funding program together with the sponsor for a plan that must be at least 90% funded. The valuation of assets is omitted in Table (2.1) as this is usually done on a marked-to-market basis, although some countries (e.g., the US) also allow smoothed asset values. All in all, it seems that funding requirements in most countries are in line with the generally recognized standard developed by the OECD. More information on supervisory rules across European countries is provided in CEIOPS (2008).

2.3 Model

After the introduction of funding requirements, this section introduces the general framework for contingently indexed DB pension plans and particularly various regulatory procedures at default, distinguishing between immediate and delayed closure of the pension plan. A standard down–and–out barrier option framework is used to describe the immediate closure procedure, and a Parisian down–and–out option framework explains the delayed closure procedure.

2.3.1 Contract specification

Similar to Chapter 1, we consider the pension plan of a single representative participant who has to work another T years. Let us assume the pension plan is issued at time $t_0 = 0$. At time 0, the pension fund issues a conditionally indexed defined benefit pension plan to a representative beneficiary who provides an up-front contribution P_0. The pension fund also receives an amount of initial contributions S_0 from the sponsor at time 0. Consequently, the initial asset value of the pension fund is given by the sum of the contributions from both the beneficiary and the sponsor, i.e. $A_0 = P_0 + S_0$. From now on, we shall denote $S_0 = \alpha A_0$ with $\alpha \in [0,1]$. The pension fund invests the proceeds in a diversified portfolio of risky and non-risky assets.

At retirement T, the beneficiary receives a lump sum nominal pension of $\underline{L}$. In addition, the pension plan has the objective to increase pension rights by $i\%$ per annum, where $i\%$ might be related to, say, the average expected CPI or wage growth. Since the determination of this parameter should take into consideration many factors, in reality this procedure is fairly complicated. Here, for simplicity we assume i is deterministic and constant and a fully indexed pension is then $\bar{L} = \underline{L}e^{iT}$. However, it should be noted that the actual outcome of the pension plan is contingent on the funding ratio at maturity T, which is defined as the ratio of the pension fund's assets (A_T) to its liability. At maturity T, given that the assets are sufficiently high ($A_T > \bar{L}$), the beneficiary not only receives an indexed pension of $\bar{L}$, but is allowed to participate in the surplus of pension funds ($A_T - \bar{L}$) with a participation rate δ, where $\delta \in [0,1]$ is the surplus distribution parameter. For instance, $\delta = 0.75$ means that the pension beneficiary receive $3/4$ of the surplus. To entitle the beneficiary to share in the pension funds' surplus can be considered a reward for the fact that the beneficiary is exposed to the risk that the pension plan might be closed prematurely. When the assets of the pension fund do not perform well, we distinguish between two scenarios: $A_T < \underline{L}$ and $\underline{L} \leq A_T < \bar{L}$. In the latter case, the assets value A_T is assigned to the beneficiary, whereas in the former case the guaranteed amount $\underline{L}$ is paid out to the beneficiary.

Since a pension fund does not have external shareholders, there is instead the corporate pension plan sponsor, a pension guarantee fund or the government bearing the residual risk, when the assets are insufficient to cover the guaranteed

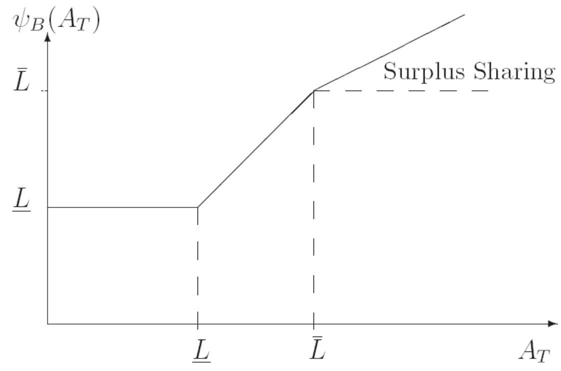

Figure 2.1: The payoff $\psi_B(A_T)$ to the beneficiary given no premature closure.

benefits $\underline{L}$. To sum up, at the maturity date T the payoff to the beneficiary is (assuming no early termination)[8]

$$\psi_B(A_T) = \begin{cases} \underline{L}, & \text{if } A_T < \underline{L} \\ A_T, & \text{if } L \leq A_T \leq \bar{L} \\ \bar{L} + \delta(A_T - \bar{L}), & \text{if } A_T > \bar{L} \end{cases}$$

as illustrated in Figure (2.1).

It is observed that this payoff differs from that of a with–profit life insurance contract in Grosen and Jørgensen (2002) and Chen and Suchanecki (2007). More specifically, when the final asset's value is not sufficiently high ($A_T < \underline{L}$), in a with-profit life insurance contract, the contract holder will obtain A_T due to the limited liability of the equity holder, whereas in a pension plan, a floor (here $\underline{L}$, provided by the sponsor) is ensured to the beneficiary by the pension plan sponsor. When the assets perform moderately ($\underline{L} \leq A_T \leq \bar{L}$), a with-profit life insurance provides its contract holder with the guaranteed amount $\underline{L}$, whereas in a pension plan, the entire assets value is assigned to the beneficiary. Finally, if the assets perform well ($A_T > \bar{L}$), the surpluses–sharing feature of a pension plan is quite similar to that of a with–profit life insurance contract. In addition to the promised amount implied by the guaranteed rate of return, policyholders and beneficiaries are entitled to a bonus if the assets return is sufficiently favorable. Therefore, the

[8] This differs from the payoff in (1.9) in Chapter one as now we allow for surplus sharing in case $A_T >\underline{L}$.

main difference between the payoff of the pension plan and that of a with–profit life insurance contract is observed when the assets do not perform well, i.e. when $A_T < \underline{L}$, the pension plan provides a better guarantee by ensuring the amount $\underline{L}$. More compactly, we can split the above payoff into two parts

$$\psi_B(A_T) = \min\{\max\{A_T, \underline{L}\}, \bar{L}\} + \delta \max\{A_T - \bar{L}, 0\}.$$

The first component on the right-hand side is capped by $\bar{L}$, i.e. it corresponds to the payoff of a traditional pension plan where the beneficiary sells off any payoffs above $\bar{L}$ and is not entitled to sharing in the surplus of the pension fund. The second component corresponds to the surplus participation which allows the beneficiary to share in the pension fund's surplus with a participation rate δ. Rephrasing the first component, we can rewrite this payoff to

$$\psi_B(A_T) = \underline{L} + [A_T - \underline{L}]^+ - (1-\delta)[A_T - \bar{L}]^+, \tag{2.1}$$

where we have used $[x]^+ := \max\{x, 0\}$. This payoff consists of three parts: a promised amount $\underline{L}$, a long call option on the assets with strike equal to the promised payment $\underline{L}$, and a short call option with strike equal to $\bar{L}$ (multiplied by $1-\delta$). The latter represents the money returned to the pension plan sponsor by the beneficiaries to cover the shortfall risk.

Analogously, as a compensation, the residual of surplus, if any, is provided to the pension plan sponsor. The total payoff to the pension plan sponsor at maturity $\psi_S(A_T)$, is given by

$$\psi_S(A_T) = \begin{cases} A_T - \underline{L}, & \text{if } A_T < \underline{L} \\ 0, & \text{if } \underline{L} \le A_T \le \bar{L} \\ (1-\delta)(A_T - \bar{L}), & \text{if } A_T > \bar{L} \end{cases}$$

or more compactly,

$$\psi_S(A_T) = (1-\delta)[A_T - \bar{L}]^+ - [\underline{L} - A_T]^+.$$

The payoff can be decomposed into two terms: a long call option which corresponds to the "bonus" received by the sponsor and a short put option reflecting the deficit which he covers in case of under-performance of the assets.

The payoff above can be regarded as a hybrid pension scheme. Such a scheme is neither a full DB nor a full DC scheme, but has some characteristics of each. $\underline{L}$ can be regarded as the DB element while the indexation and participation in the surpluses are DC elements. Hybrids offer some leeway as traditional DB pension plans appear difficult to maintain due to a combined result of increased life expectancy, aging of society, low interest rates and volatile investment returns. By adding DC elements the schemes' continuity might be improved, as these elements are an efficient way to manage risks. Of course these risks are then transferred to the beneficiaries. Still, there always remains a possibility that the funding level deteriorates significantly. In that case the sponsor might be tempted to try to close the pension plan or to convert it to a full DC plan. By doing so excessive premium increases can be avoided. Since equity returns are positively correlated with the macro-economic outlook, the need for premium increases often arises in less favorable economic situations. Therefore we will assume that in case of premature closure the sponsor will not cover the deficit in the pension plan. However, at maturity when the liabilities are due and there is a funding shortage it is unlikely that the sponsor can withdraw from its responsibilities. In that case we do presume that it will complement any deficiencies. After all, in return for writing this option the sponsor is compensated by owning a certain percentage in the pension fund's surpluses. This one period setup is of course a simplification of reality but matches, in principle, with the idea that the corporation also cannot claim any surpluses from the pension fund before the end of the term. At maturity the company covers any deficits or receives the surplus from the pension fund. Consequently the sponsor has a symmetric contract before and at maturity.

The premature closing feature implies that a mechanism similar to a knock–out barrier option framework must be incorporated when analyzing the pension plan. In case the pension fund's assets perform extremely poorly, the plan at some point in time is terminated prematurely and a rebate payment is provided to the beneficiary. The rebate proceedings can, e.g., be used to start a (collective) DC plan or to transfer the remaining liabilities to a pension guarantee fund or an insurance company. The role of the supervisor is to monitor the funding ratio during the regulatory grace period and either declare recovery if the funding ratio has rejuvenated or liquidate the pension fund if recovery has not occurred. The recovery is by and large financed by investment returns and the fact that indexation is not granted during the recovery period. This lowers the growth rate of the liabilities and as such releases additional resources to restore the financial position. In the following subsection, we describe two default and premature closing procedures: the immediate and the delayed closing procedure.

2.3.2 Default and premature closure formulation

We distinguish between an immediate closure and a delayed closure procedure. In an immediate closure procedure, a premature default (underfunding) leads to immediate termination of the pension fund, i.e., default and closure are treated as equivalent events. Since the emphasis of this chapter is to build in the early closure feature of the DB plans (to DC plans) in the market valuation of the DB plans, we assume that the DB contract is terminated by regulatory intervention and a rebate is paid to the beneficiary. So, we leave the complexity of modeling possible conversion to a DC plan at that point for further research.[9] In a delayed closure procedure, default and pension plan termination are distinguishable events. A chance is given for reorganization and recovery during some grace period. If the pension fund is unable to recover during this period, the DB plan is converted to a DC plan and the contract is terminated prematurely.

An immediate closure procedure can be mathematically realized by using standard knock–out barrier options, similarly as an immediate default and liquidation procedure in the life insurance literature (c.f. Grosen and Jørgensen, (2002)). A delayed closure procedure can be characterized by using the Parisian barrier option framework (similarly as Chen and Suchanecki (2007) in a life insurance context). In both articles, the regulatory intervention rule is introduced in the form of a boundary. Since the regulation objective is to provide the beneficiary with the guaranteed payment $\underline{L}$ at the maturity date, it is natural to assume an exponential barrier B_t which increases over time as follows:

$$B_t = \lambda \, \underline{L} \, e^{-r(T-t)} = B_0 e^{rt}, \quad t \in [0, T], \tag{2.2}$$

[9] This conversion could also be modeled as an option, more specifically as an exchange option, i.e., the pension plan sponsor has the right to exchange cash flows from the DB pension plan to cash flows from the DC pension plan. In fact, this must be a compound exchange option, because it is an exchange option on a combination of options which represent the terminal payoff. Furthermore, to include default risk and a possible delay in the conversion decision Parisian compound exchange options should be used.

where r is the prevailing market interest rate for maturity T, λ is the regulation parameter chosen by the regulator and has been introduced in Section 2.2. It holds that $B_0 = \lambda \underline{L} e^{-rT}$. Obviously, the specified contract contains standard down-and-out barrier options. Therefore, the requirement $A_0 > B_0 = \lambda \underline{L} e^{rT}$ must be satisfied initially and leading to a reasonable range for the regulation parameter λ, i.e. $\lambda \in (0, A_0 e^{rT}/\underline{L}]$. Next define τ as the liquidation time of the pension fund. In an immediate closure procedure, the pension fund is immediately liquidated when the assets reach this boundary, namely, $A_\tau = B_\tau$ if $\tau < T$. Hence, the premature default and closure coincide and the premature closure time is given by

$$\tau = \inf \{t \in [0, T] | A_t \leq B_t\} . \tag{2.3}$$

Upon premature closure, the contract is terminated and a rebate payment

$$\Theta_B(\tau) = \min \left\{ \underline{L} e^{-r(T-t)}, B_\tau \right\} = \min \{1, \lambda\} \underline{L} e^{-r(T-t)} \tag{2.4}$$

is offered to the beneficiary immediately at the closure time τ. For $\lambda < 1$, the (discounted) guaranteed amount is not fully returned to the beneficiary, whereas in case of $\lambda > 1$, the (discounted) guaranteed amount is ensured and there will be a residual. The residual can be used to cover expenses in case the liabilities are transferred to an insurance company or a guarantee fund. The sponsor is thus provided with the remaining assets as the rebate payment

$$\Theta_S(\tau) = B_\tau - \min \left\{ \underline{L} e^{-r(T-t)}, B_\tau \right\} = \max \{\lambda - 1, 1\} \underline{L} e^{-r(T-t)}. \tag{2.5}$$

The delayed closure procedure can be realized by adding a Parisian barrier option feature instead of the standard knock–out barrier option feature to the model (c.f. Chen and Suchanecki (2007)). This feature works as follows. Suppose a regulatory authority takes its bankruptcy filing actions according to a hypothetical default clock. The default clock starts ticking when the asset price process breaches the default barrier and the clock is reset to zero when the company recovers from the default. Thus, successive defaults are possible until one of these defaults lasts κ units of time. Earlier defaults which may last a very long time but not longer than

κ do not have any consequences for eventual subsequent defaults. In a standard Parisian barrier option framework, the closure of the pension fund is declared when the financial distress has lasted at least a period of length κ. Therefore, κ can be considered the maximum recovery period assigned to the pension fund to recover from the financial distress.[10]

Before we come to the mathematical formulation of standard Parisian barrier options, it is convenient to specify the underlying assets process. Under the equivalent martingale measure Q, the price process of the pension fund's assets $\{A_t\}_{t\in[0,T]}$ is assumed to follow a geometric Brownian motion

$$dA_t = A_t(rdt + \sigma dW_t) \tag{2.6}$$

in which σ denotes the deterministic volatility of the asset price process $\{A_t\}_{t\in[0,T]}$. We assume that pension funds continuously rebalance their investment portfolios such that the asset return volatility remains the same over time. As pension funds typically are long-term investors their investment portfolio is usually at – or within narrow margins around – the strategic asset allocation. The reason is that strategic asset allocation explains close to 100% of the variability of returns over time, see, e.g., Ibbotson and Kaplan (2000). Empirical research shows that pension funds indeed rebalance their portfolios. Blake, Lehmann and Timmermann (1999) find evidence that portfolio weights of UK pension funds revert over time to a (common) strategic asset allocation. Evidence from the Dutch market suggests that pension funds, on average, do not adjust their strategic asset allocation significantly through the cycle (see Figure 3.1 in the next chapter). In fact regulation allows pension funds to always rebalance the portfolio towards the strategic asset allocation even if the funding ratio has dropped below the minimum regulatory level. Only few pension funds follow the contingent immunization procedure suggested by Leibowitz and Weinberger (1982) where risks are reduced when the financial buffers decrease. However, recently this dynamic investment

[10] In reality the regulator will monitor the pension fund in default very closely and require measures to be taken to improve recovery. If the funding ratio deteriorates significantly during the recovery period it is likely that it will not await the remaining recovery time and takes action. It is therefore possible to envisage a combination of Parisian and regular barrier options and two regulatory boundaries. Suppose for instance that the regulatory clock starts ticking when the assets breach a first (higher) barrier level. If subsequently a second (lower) barrier level is crossed, the pension plan is terminated immediately.

procedure got revived interest at it was suggested by Frijns *et al.* (2010) as a promising tool to manage a critical lower boundary for the funding level. Furthermore, $\{W_t\}_{t\in[0,T]}$ in (2.6) is a Brownian motion under the risk-neutral measure. Solving this differential equation, we obtain $A_t = A_0 \exp\left\{\left(r - \frac{1}{2}\sigma^2\right)t + \sigma W_t\right\}$.

There are several special cases of Parisian barrier options (a Parisian down–and–out call option is taken as an example):

- $A_t > B_t$ and $\kappa \geq T - t$: In this case, it is impossible to have an excursion of A_t below B_t, between t and T, of length at least equal to κ. Therefore, the value of a Parisian down–and–out call just corresponds to the Black–Scholes (Black and Scholes (1973)) price of a regular European call option.

- $\kappa \geq T$: In this case, the Parisian option actually becomes a standard call option.

- $A_t > B_t$ and $\kappa = 0$: When the time window κ is set at 0, we are back in the immediate default and closure procedure.

Apart from these special cases, in the standard Parisian down–and–out option framework, the final payoffs $\psi_B(A_T)$, $\psi_S(A_T)$ are paid only if the following technical condition is satisfied

$$T_B^- = \inf\{t > 0 | (t - g_{B,t}^A)\mathbf{1}_{\{A_t < B_t\}} > \kappa\} > T \tag{2.7}$$

with $g_{B,t}^A = \sup\{s \leq t | A_s = B_s\}$, where $g_{B,t}^A$ denotes the last time before t at which the value of the assets A hits the barrier B. T_B^- gives the first time at which an excursion below B lasts more than κ units of time. In fact, T_B^- is the premature closure (or contract–termination) date if $T_B^- < T$. Figure (2.2) simulates a path of the asset evolution, which leads to premature closure of the DB plan under the Parisian option framework[11]. At $g_{B,t}^A$, the pension fund starts defaulting and the DB plan is closed at T_B^- because it does not recover from underfunding after κ units of time.

It is noted that the condition in (2.7) is equivalent to $T_b^- := \inf\{t > 0 | (t -$

[11] For simplicity, we have used a constant barrier level in the figure.

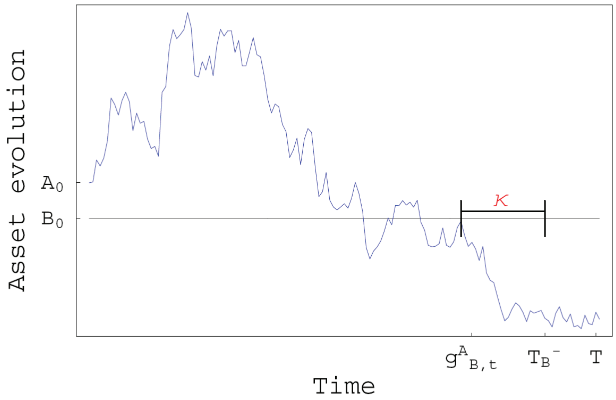

Figure 2.2: A path simulation of premature closure under the Parisian option framework.

$g_{b,t}^{Z})\mathbf{1}_{\{Z_t<b\}} > \kappa\} > T$ where

$$g_{b,t}^{Z} := \sup\{s \leq t | Z_s = b\}; \qquad b = \frac{1}{\sigma}\ln\left(\frac{B_0}{A_0}\right); \quad B_0 = \lambda \underline{L} e^{-rT}$$

and $\{Z_t\}_{0\leq t\leq T}$ is a martingale under a new probability measure $\tilde{Q}$ which is defined by the Radon–Nikodym density

$$\frac{dQ}{d\tilde{Q}} = \exp\left\{mZ_T - \frac{m^2}{2}T\right\}, \qquad m = -\frac{\sigma}{2}, \tag{2.8}$$

i.e. $W_t = Z_t - m\,t$.

Thereby, we transform the event "the excursion of the value of the assets below the exponential barrier $B_t = B_0 e^{rt}$" into the event "the excursion of the Brownian motion Z_t below a constant barrier $b = \frac{1}{\sigma}\ln\frac{B_0}{A_0}$". In other words, we deal with an excursion below a constant barrier under the new measure $\tilde{Q}$, which simplifies the entire valuation procedure. As specified, when $T_B^- = \inf\{t > 0|(t-g_{B,t}^{A})\mathbf{1}_{\{A_t<B_t\}} > \kappa\} < T$, the closure of the DB plan occurs and the contract is terminated prematurely. As already pointed out by Chen and Suchanecki (2007), we have to make a small change to the rebate term of the contract. The Parisian barrier option feature could lead to the result that at the closure time the asset price falls far below the barrier value, which makes it impossible for the pension fund to offer the rebate as in (2.5). Hence, a new rebate for the beneficiary is introduced to the model with the form

$$\Theta_B(T_B^-) = \min\{\underline{L}e^{-r(T-T_B^-)}, A_{T_B^-}\} \quad \text{with } A_{T_B^-} \leq B_{T_B^-} \tag{2.9}$$

where T_B^- is the closure time. The rebate term implicitly depends on the regulation parameter λ. Correspondingly, the new rebate for the pension fund sponsor can be expressed as follows:

$$\Theta_S(T_B^-) = A_{T_B^-} - \min\{\underline{L}e^{-r(T-T_B^-)}, A_{T_B^-}\} = \max\{A_{T_B^-} - \underline{L}e^{-r(T-T_B^-)}, 0\}, \tag{2.10}$$

i.e. the sponsor obtains the remaining assets value if there is any.

The strength of our model is that it is capable of analyzing the risk-sharing arrangement between a pension fund and its sponsor in a fairly realistic way. The option features described above are often implicit but play an important economic role in the relation between the two entities. The model also has a limitation. E.g., following Merton (1974) we assume that the pension fund acts on behalf of a single, homogeneous class of participants. As such, the financial policy and regulatory environment is tailored to the needs of the representative beneficiary. Although this resembles reality in many ways, it does not take into account that preferences might differ across age groups. For instance, a participant at a young age has a preference for a long recovery period. He has sufficient human capital to diversify losses on his financial capital, see Chapter 4. On the other hand, an older member with higher risk aversion and low human capital might opt for a short recovery period. We do not allow for mean reversion in equities returns, as suggested by Campbell and Viceira (2002). There is no unequivocal empirical support for this phenomenon, even in the long run, see Bodie (1995) and Jorion (2003). Instead it is well documented that the short-fall risk increases with the investment horizon. This is captured by option pricing models.

2.4 Valuation

We assume a continuous–time frictionless economy with a perfect financial market, no tax effects, no transaction costs and no other imperfections. Hence, we can rely on martingale techniques for the valuation of the contingent claims. Again a distinction is made between an immediate and a delayed closure procedure.

2.4.1 Immediate closure procedure

In a complete financial market, the price of a T–contingent claim with the payoff $\phi(A_T)$ corresponds to the expected discounted payoff under the risk–neutral probability measure Q, i.e.,

$$E_Q\left[e^{-rT}\phi(A_T)\mathbf{1}_{\{\tau>T\}}\right].$$

The market–consistent value of the payoff to the beneficiary is hence determined by the sum of the expected discounted payoff at maturity (in case of no premature liquidation) and the expected discounted rebate payment (when early liquidation does occur)

$$\begin{aligned} V_B(A_0,0) &= E_Q[e^{-rT}\Big(\underbrace{\underline{L}+[A_T-\underline{L}]^+-(1-\delta)[A_T-\bar{L}]^+}_{\text{payoff at maturity}}\Big)\mathbf{1}_{\{\tau>T\}}] \\ &\quad +E_Q[e^{-r\tau}\underbrace{\min\{1,\lambda\}\underline{L}e^{-r(T-\tau)}\mathbf{1}_{\{\tau\leq T\}}}_{\text{early liquidation rebate}}]. \end{aligned}$$

It is observed that the price of this contingent claim consists of four parts: a deterministic guaranteed or fixed part $\underline{L}$ which is paid at maturity when the value of the assets does not hit the barrier, a long down–and–out call option with strike $\underline{L}$, a shorted down–and–out call option with strike $\bar{L}$ (multiplied by $1-\delta$), and a rebate paid immediately upon premature closure.

The market–consistent value of the payoff to the sponsor is given by

$$\begin{aligned} V_S(A_0,0) &= E_Q[e^{-rT}\left((1-\delta)[A_T-\bar{L}]^+-(\underline{L}-A_T)^+\right)\mathbf{1}_{\{\tau>T\}}] \\ &\quad +E_Q[e^{-r\tau}\max\{\lambda-1,0\}\underline{L}e^{-r(T-\tau)}\mathbf{1}_{\{\tau\leq T\}}]. \end{aligned}$$

The valuation of each component relies on the valuation technique of knock-out barrier options and the resulting closed-form formulae for each component are given in Appendix 2.8.1.

2.4.2 Delayed closure procedure

For the valuation in the delayed closure procedure we require Parisian option valuation models. In the literature, various approaches are applied to valuing standard Parisian derivatives, such as Monte–Carlo algorithms (Andersen and Brotherton–Ratcliffe (1996)), binomial or trinomial trees (Avellaneda and Wu (1999); Costabile (2002)), partial differential equations (Haber et. al (2002)), finite–element methods (Stokes and Zhu (1999)) or the original inverse Laplace transform technique (initiated by Chesney *et al.* (1997)). However, most of these methods are very time–consuming if they are to obtain precise results. The inverse Laplace transform method is adopted here to price the standard Parisian claims. Further, as in Chen and Suchanecki (2007), the inverse procedure introduced by Bernard *et al.* (2005) is adopted to invert the Laplace transforms, which minimizes the computation time.

Under the new probability measure $\tilde{Q}$ (c.f. (2.8)), the value of the assets A_t can be expressed as

$$A_t = A_0 \exp\{\sigma Z_t\} \exp\{rt\}.$$

Under the risk–neutral probability measure Q, the price of a T–contingent claim with the payoff $\phi(A_T)$ corresponds to the expected discounted payoff

$$E_Q\left[e^{-rT}\phi(A_T)\mathbf{1}_{\{T_B^- > T\}}\right].$$

This can be rephrased as follows

$$e^{-(r+\frac{1}{2}m^2)T} E_{\tilde{Q}}\left[\mathbf{1}_{\{T_b^- > T\}}\phi(A_0 \exp\{\sigma Z_T\}\exp\{rT\})\exp\{mZ_T\}\right].$$

The market-consistent value of the payoff to the beneficiary under the delayed closure procedure is hence determined by

$$\begin{aligned} & V_B(A_0, 0) \\ = \; & E_Q[e^{-rT}\left(\underline{L} + [A_T - \underline{L}]^+ - (1-\delta)[A_T - \bar{L}]^+\right)\mathbf{1}_{\{T_B^- > T\}}] \\ & + E_Q[e^{-rT_B^-}\min\{\underline{L}e^{-r(T-T_B^-)}, A_{T_B^-}\}\mathbf{1}_{\{T_B^- \le T\}}] \end{aligned}$$

$$
\begin{aligned}
= \;& E_Q\left[e^{-rT}\underline{L}\mathbf{1}_{\{T_b^- > T\}}\right] + e^{-\frac{1}{2}m^2T}E_{\tilde{Q}}\left[\left(A_0e^{\sigma Z_T} - \underline{L}e^{-rT}\right)^+ e^{mZ_T}\mathbf{1}_{\{T_b^- > T\}}\right] \\
& -(1-\delta)e^{-\frac{1}{2}m^2T}E_{\tilde{Q}}\left[\left(A_0e^{\sigma Z_T} - \bar{L}e^{-rT}\right)^+ e^{mZ_T}\mathbf{1}_{\{T_b^- > T\}}\right] \\
& +E_{\tilde{Q}}\left[e^{-\left(r+\frac{1}{2}m^2\right)T_b^-}\exp\{mZ_{T_b^-}\}\min\{\underline{L}e^{-r(T-T_b^-)}, A_{T_b^-}\}\mathbf{1}_{\{T_b^- \leq T\}}\right]
\end{aligned}
$$

$$
\begin{aligned}
= \;& PDOC\left[A_0, B_0, \underline{L}e^{-rT}, r, r\right] - (1-\delta)\,PDOC[A_0, B_0, \bar{L}e^{-rT}, r, r] \\
& +E_Q\left[e^{-rT}\underline{L}\mathbf{1}_{\{T_b^- > T\}}\right] \\
& +E_{\tilde{Q}}\left[e^{-\left(r+\frac{1}{2}m^2\right)T_b^-}\exp\{mZ_{T_b^-}\}\min\{\underline{L}e^{-r(T-T_b^-)}, A_{T_b^-}\}\mathbf{1}_{\{T_b^- \leq T\}}\right]
\end{aligned}
$$

It is observed that the price of this contingent claim consists of four parts: a deterministic guaranteed part $\underline{L}$ which is paid at maturity when the value of the assets has not remained below the barrier for a time longer than κ, a long Parisian down–and–out call option with strike $\underline{L}e^{-rT}$, a shorted Parisian down–and–out call option with strike $\bar{L}e^{-rT}$ (multiplied by $1-\delta$), and a rebate paid immediately upon premature closure.

The present value of the payoff to the sponsor is given by

$$
\begin{aligned}
V_S(A_0, 0) \;=\; & E_Q[e^{-rT}\left((1-\delta)[A_T - \bar{L}]^+ - (\underline{L} - A_T)^+\right)\mathbf{1}_{\{T_B^- > T\}}] \\
& +E_Q[e^{-rT_B^-}\max\{\lambda - 1, 0\}\underline{L}e^{-r(T-T_B^-)}\mathbf{1}_{\{T_B^- \leq T\}}] \\
=\; & (1-\delta)\,PDOC[A_0, B_0, \bar{L}e^{-rT}, r, r] - PDOP\left[A_0, B_0, \underline{L}e^{-rT}, r, r\right] \\
& +E_{\tilde{Q}}\left[e^{-\left(r+\frac{1}{2}m^2\right)T_b^-}e^{mZ_{T_b^-}}\max\{A_{T_b^-} - \underline{L}e^{-r(T-T_b^-)}, 0\}\mathbf{1}_{\{T_b^- \leq T\}}\right].
\end{aligned}
$$

Concerning the rebate payment, the sponsor would possibly obtain a rebate payment in the case of $\lambda \geq 1$:

$$
\begin{aligned}
& E_{\tilde{Q}}\left[e^{-\left(r+\frac{1}{2}m^2\right)T_b^-}\exp\{mZ_{T_b^-}\}\max\{A_{T_b^-} - \underline{L}e^{-r(T-T_b^-)}, 0\}\mathbf{1}_{\{T_b^- \leq T\}}\right] \\
= \;& A_0E_{\tilde{Q}}\left[e^{-\frac{1}{2}m^2T_b^-}\exp\{(m+\sigma)Z_{T_b^-}\}\mathbf{1}_{\{T_b^- \leq T\}}\mathbf{1}_{\{k < Z_{T_b^-} < b\}}\right] \\
& -\underline{L}e^{-rT}E_{\tilde{Q}}\left[e^{-\frac{1}{2}m^2T_b^-}\exp\{mZ_{T_b^-}\}\mathbf{1}_{\{T_b^- \leq T\}}\mathbf{1}_{\{k < Z_{T_b^-} < b\}}\right].
\end{aligned}
$$

Detailed calculation of each component in the delayed closure procedure is carried out in Appendix 2.8.2.

2.5 Numerical analysis

In this section, we implement the valuation formulae obtained in Section 2.4 and the Appendix in Section 2.8. We carry out sensitivity analyses, i.e., calculate how the three policy parameters (the investment policy's riskiness level σ, the regulation parameter λ and the recovery period κ) affect the surplus participation rate δ based on a fair contract analysis. Put differently, how should the beneficiary and the sponsor divide the surplus for the pension deal to be fair given the investment policy of the pension fund and the regulatory environment[12]. The considered pension plan is a fair contract when:

$$E_Q[e^{-rT}\psi_B(A_T)1_{\{\tau>T\}}] + E_Q[e^{-r\tau}\Theta_B(A_\tau)1_{\{\tau\leq T\}}] \equiv P_0 = (1-\alpha)A_0 \tag{2.11}$$

where τ is the termination time of the pension fund under both the immediate and the delayed closure procedures. It says that a fair contract results when the initial market value of the pension liability equals the initial contributions made by the beneficiary. An alternative condition for a fair contract can be obtained from the viewpoint of the sponsor, i.e.

$$E_Q[e^{-rT}\psi_S(A_T)1_{\{\tau>T\}}] + E_Q[e^{-r\tau}\Theta_S(A_\tau)1_{\{\tau\leq T\}}] \equiv S_0 = \alpha\, A_0. \tag{2.12}$$

Equations (2.11) and (2.12) can both be used to conduct a fair contract analysis. For instance, if we are interested in determining the fair participation rate δ, these two equations lead to the same value for δ. For the following analysis, we fix the parameters

$$A_0 = 100;\ \alpha = 0.1;\ \underline{L} = 120;\ \bar{L} = 188.20;\ T = 15;\ \sigma = 0.15;\ r = 0.04;\ \ \kappa = 1.$$

A_0 has been chosen to be greater than $\underline{L}e^{-rT}$ (here the present value of the fixed payment discounted at the risk-free rate equals 65.86), which reflects the

[12] Fair contracts have been studied, e.g., in Grosen and Jørgensen (2002), Chen and Suchanecki (2007) and Døskeland and Nordahl (2008).

fact that the initial contribution should be at least equal to the present value of the nominal pension liability. $\bar{L}$ is the fully indexed pension, which is set equal to $\underline{L}e^{iT}$ with i being the parameter related to the expected CPI or wage growth etc. As a realistic i value must be smaller than the risk-free rate r, we choose $i = 3\%$ which leads to an $\bar{L}$ of 188.20. Furthermore, the parameter λ is chosen to ensure that the initial asset value lies above the barrier level, i.e. $\lambda \in (0, A_0 e^{rT}/\underline{L}]$ (for the chosen parameters λ shall be set in the interval $(0, 1.52]$). $T = 15$ is chosen because it is approximately the average duration of pension liabilities in reality. A volatility (σ) of 15% is also a reasonable number for a diversified portfolio.

Tables (2.2) and (2.3) demonstrate how the various components of the contract values (for both the beneficiary and the sponsor) are influenced by the recovery period κ and the regulation parameter λ (κ and λ are input parameters). In all rows the participation rate (δ) is chosen such that a fair contract results. For comparability, the simple case of DB plan where all the options expire at T (no barrier/Parisian barrier framework is involved) is also demonstrated in the first row. In this case, it is unnecessary to formulate the rebate payment, as we exclude the possibility of a premature default. Compared to the standard barrier option framework, the resulting fair participation rate is rather low (0.27), due to the fact that the beneficiary is assured of the defined benefit given in Equation (2.1) in a simple DB plan and therefore does not have to be compensated that much for downside risk. Next, we turn to the barrier framework. For $\kappa = 0$, there is no recovery time after default for the pension fund. Therefore, a standard Parisian option with $\kappa = 0$ in fact corresponds to a standard down–and–out barrier option.

Below, we observe the following three relations. First, a positive relation exists between the Parisian down–and–out call and the recovery period. The longer the allowed excursion, the larger the value of the option. In fact, the value of the call does not change much with the length of excursion when a certain level of κ is reached, i.e. the value of the Parisian down–and–out call is a concave increasing function of κ. Second, the value of the Parisian down–and–out call does not increase substantially in κ when the barrier level is extremely low or the strike of the call is fairly high. In the extreme case, if the regulation parameter λ is set at zero, which results in a barrier level of zero, it then follows that the recovery period κ has no effect on any of the components of the contract values (including the Parisian down–and–out call), because the asset price can never hit the barrier in this situation due to the log–normal assumption of the asset dynamics. For the given parameters which lead to a rather low barrier level and rather high strikes (both $\underline{L}$ and $\bar{L}$ are higher than A_0), the resulting Parisian down–and–out call values do not increase much in κ. Third, the fixed payment arises only when the asset price process does not remain below the barrier for a time longer than κ. Hence, as the size of κ goes up, the probability that the fixed payment will become due increases. Consequently, the expected value of the fixed payment rises with κ. Its magnitude is bounded from above by the payment $\underline{L}e^{-rT}$. In contrast, the rebate payment appears only when the pension fund is closed, i.e., when the asset price process stays below the barrier for a period longer than κ. Therefore, the longer the recovery period, the smaller the expected rebate payment. The Parisian put option changes with the length of excursion in a similar way as Parisian call option. It increases with κ but the extent to which it increases becomes smaller after a certain level of κ is reached.

Although κ has monotonic effects on the values of the call, the put, the fixed payment and the rebate payment (positive relation between the $PDOC$ and κ, between the expected fixed payment and κ, and negative relation between the shorted $PDOC$ and κ, between the rebate payment and κ), adding up their effects, a non-monotonic effect of κ is observed on the fair participation rate δ. However, we point out that this argument depends on the parameter choice. If the effects of the call and the fixed payments dominate, a negative relation between the fair participation rate δ and the recovery period κ results from fair contract analysis. The reversed effect is observed when the effects of the shorted call and the rebate payment dominate. Therefore, under certain circumstances, it is possible to observe a monotonic change of the contract value with respect to κ. Specifically, for the parameters under considerations, a decreasing effect of κ on the fair participation rate results. Concerning the regulation parameter λ, first of all, it is noted that different λ–values lead to different values of the barrier ($B_0 = \lambda \underline{L} e^{-rT}$). The higher the required funding ratio, the more likely that this barrier is hit (from above) and the values of the Parisian down–and–out call and put decrease, as does the value of fixed payment. In contrast, the expected value of the rebate increases with the barrier because the rebate payment is based on a countercondition as other components. The above non-monotonic effect of κ on the contract value for the beneficiary V_B can be observed in Table (2.4) (for

	κ	δ	C($\underline{L}$)	SC($\bar{L}$)	FP	RB	V_B	LC($\bar{L}$)	SP	RS	V_S
Simple case: all options expire at T											
Black&Scholes	–	0.27	45.39	-21.25	65.86	–	90	21.25	-11.25	–	10
$\lambda = 0.9 \Rightarrow B_0 = 59.27$											
Standard		0.52	37.25	-10.02	35.03	27.75	90	10.02	-0.02	0	10
Parisian	0.25	0.52	38.85	-10.17	40.74	20.46	90	10.17	-0.17	0	10
	0.50	0.52	39.29	-10.31	43.03	17.94	90	10.31	-0.31	0	10
	1.00	0.50	39.74	-10.60	46.19	14.79	90	10.60	-0.60	0	10
	3.00	0.45	40.31	-11.88	53.63	7.76	90	11.88	-1.88	0	10
	5.00	0.40	40.45	-12.97	57.83	4.69	90	12.97	-2.97	0	10
	10.00	0.28	40.51	-15.57	64.38	0.68	90	15.57	-5.57	0	10
$\lambda = 1 \Rightarrow B_0 = 65.86$											
Standard		0.52	34.14	-10.00	28.23	37.65	90	10.00	0	0	10
Parisian	0.25	0.52	36.99	-10.03	34.47	28.47	90	10.03	-0.03	0	10
	0.50	0.52	37.82	-10.08	37.00	24.99	90	10.08	-0.08	0	10
	1.00	0.52	38.71	-10.15	40.52	20.76	90	10.15	-0.15	0	10
	3.00	0.48	39.95	-11.19	49.24	11.97	90	11.19	-1.19	0	10
	5.00	0.44	40.31	-12.10	54.36	7.43	90	12.10	-2.10	0	10
	10.00	0.30	40.45	-15.13	63.10	1.53	90	15.13	-5.13	0	10

Table 2.2: Decomposition of the fair contract value with parameters: $A_0 = 100$; $\underline{L} = 120$; $\bar{L} = 188.20$; $T = 15$; $\sigma = 0.15$; $r = 0.04$. $C(\underline{\mathrm{L}})$ stands for the $PDOC(\underline{L})$, $SC(\bar{L})$ for the shorted $PDOC(\bar{L})$ (multiplied by $(1-\delta)$), FP the fixed payment, RB and RS the rebate payment of the beneficiary and the sponsor, SP the shorted put value, V_B and V_S the contract value of the beneficiary and the sponsor.

	κ	δ	$C(\underline{L})$	$SC(\bar{L})$	FP	RB	V_B	$LC(\bar{L})$	SP	RS	V_S
$\lambda = 1.1 \Rightarrow B_0 = 72.44$											
Standard		0.69	29.73	-5.57	21.71	44.14	90	5.57	0	4.43	10
Parisian	0.25	0.54	34.09	-9.22	28.27	37.26	90	9.22	0	0.78	10
	0.50	0.53	35.45	-9.63	31.00	33.35	90	9.63	-0.02	0.39	10
	1.00	0.52	36.97	-9.91	34.83	27.27	90	9.91	-0.08	0.17	10
	3.00	0.50	39.26	-10.63	44.28	17.94	90	10.63	-0.67	0.04	10
	5.00	0.49	39.99	-11.00	50.57	10.44	90	11.00	-1.02	0.02	10
	10.00	0.33	40.47	-14.49	61.15	2.87	90	14.49	-4.50	0.01	10
$\lambda = 1.2 \Rightarrow B_0 = 79.03$											
Standard		0.99	24.10	-0.15	15.66	50.20	90	0.15	0	9.85	10
Parisian	0.25	0.68	30.12	-5.91	22.28	43.52	90	5.91	0	4.09	10
	0.50	0.61	32.08	-7.40	24.81	41.00	90	7.40	0	2.60	10
	1.00	0.57	34.38	-8.68	29.23	35.40	90	8.68	-0.02	1.34	10
	3.00	0.53	38.07	-9.91	36.75	25.73	90	9.91	-0.30	0.39	10
	5.00	0.52	39.39	-10.30	46.77	14.14	90	10.30	-0.43	0.13	10
	10.00	0.37	40.39	-13.62	58.42	4.81	90	13.62	-3.65	0.03	10

Table 2.3: Decomposition of the fair contract value with parameters for $\lambda = 1.1$ and 1.2. Same parameters and notations are used as in Table 2.2.

$\delta = 0.75$). Furthermore, κ has a more apparent effect on the contract value if the regulation level (barrier level) is set higher.

κ	0.5	1	1.5	2	2.5	3
contract value ($\lambda = 0.9$)	93.06	93.16	93.96	94.80	95.20	95.31
contract value ($\lambda = 1.0$)	93.55	92.58	92.73	94.19	95.19	95.26
contract value ($\lambda = 1.1$)	94.16	93.36	91.13	92.57	95.10	95.96

Table 2.4: Contract value V_B as a function of κ and different λ values ($\delta = 0.75$).

Figures (2.3) and (2.4) depict the fair participation rate δ as a function of the investment policy σ for different recovery periods κ. As the volatility σ goes up, the value of the Parisian down–and–out call increases, while the value of the Parisian down–and–out put increases with the volatility at first and then decreases (hump–shaped). The value of the fixed payment goes down and the rebate term behaves similarly to the Parisian down–and–out put, i.e. goes up at first and then goes down after a certain level of volatility is reached. In all, as pension funds pursue a riskier investment policy, the beneficiaries should be given a higher share in the surplus to make it a fair pension deal.

Overall a positive relation between δ and σ is observed. As mentioned before, the recovery period κ does not necessarily affect the fair participation rate monotonically. For $\lambda = 0.9$, the higher the length of the regulatory recovery period, the lower the fair participation rate δ. Whereas for $\lambda = 1.1$, δ does not decrease in κ monotonically. In Figures (2.5) and (2.6), the relation between the fair participation rate δ and α is illustrated for different lengths of excursion. A negative relation between δ and α results, meaning the less money the sponsor contributes the higher the fair participation rate should be.

2.6 Regulation and recovery period

So far, we have analyzed the design of pension deals assuming that they are initially fair to both the customer and the company in valuation terms. Naturally, the question arises whether our framework can provide intelligence for the regulatory authorities to design policy on an optimal recovery period. This section deals with this question. Since the principal objective of the supervisory authorities is to protect customers, the optimal recovery period is determined by following two

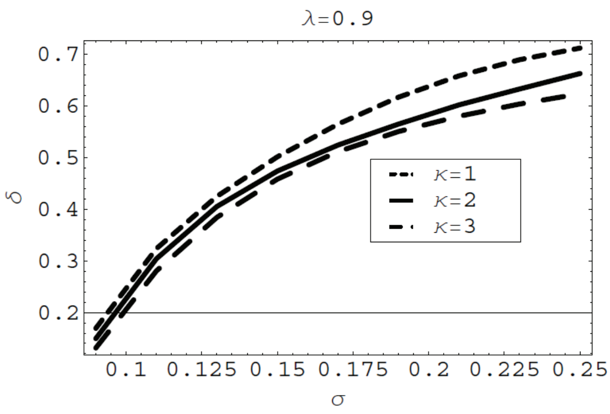

Figure 2.3: Fair combinations of δ and σ.

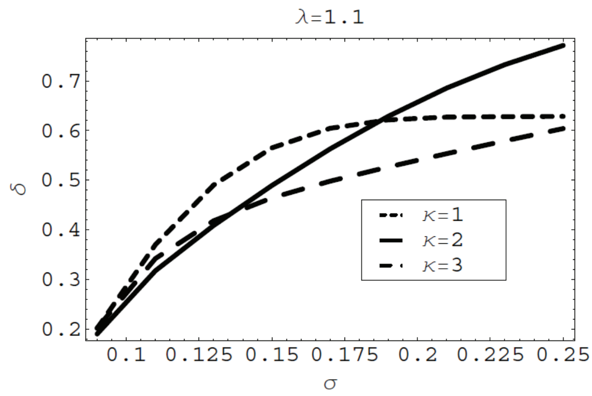

Figure 2.4: Fair combinations of δ and σ.

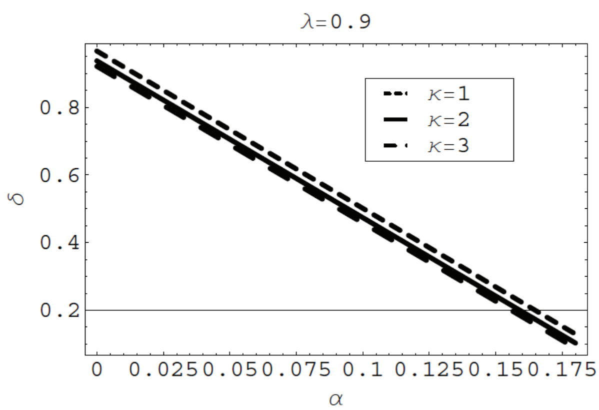

Figure 2.5: Fair combinations of δ and α.

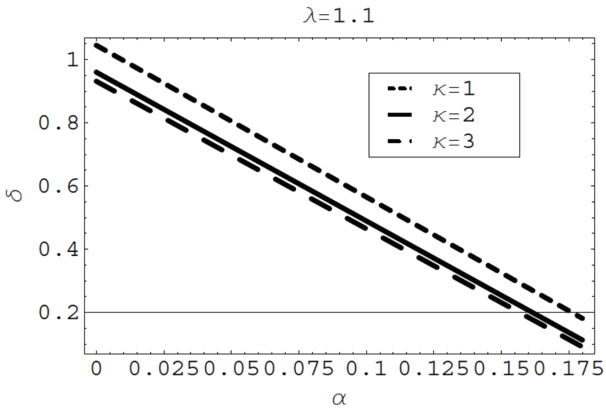

Figure 2.6: Fair combinations of δ and α.

strands. First, we determine the optimal recovery period using utility analysis. Hereby the supervisory authorities aim to maximize the utility of the beneficiary. Second, the recovery period is determined to control the regulatory liquidation probability to a low level, which provides stability to both customers and the company.

2.6.1 Welfare analysis and optimal recovery period

For the welfare analysis we assume that the regulator sets the optimal recovery period (κ^*) such that it maximizes the utility of the beneficiary.[13] The utility analysis is done by assessing the distribution of the terminal wealth. This analysis is carried out under the real world measure, because the liquidation probability constraints stated by regulatory authorities are set according to the market performance of the pension funds.[14] Since the recovery period always equals 0 in the immediate closure procedure, we only consider the delayed closure procedure in what follows. Due to the fact that the higher moments for Parisian options cannot be calculated in closed form, the results in this subsection are based on Monte Carlo simulation.

We assume the pension fund's assets value evolves according to a geometric

[13]Here we take the regulatory liquidation probability ε and the required funding level λ as given.
[14]This differs from the risk-neutral measure used for valuation purposes in the previous sections.

Brownian motion under the real world measure $\tilde{P}$

$$dA_t = A_t\,(\mu\,dt + \sigma\,d\tilde{W}_t),$$

where μ and $\sigma > 0$ are the instantaneous rate of return and the volatility of the assets. $\tilde{W}_t$ is a standard Brownian motion under the real world measure $\tilde{P}$.[15] The asset values are transformed in terminal wealth for the beneficiary in the following two-step approach. First, all scenarios are extracted that do not infer premature liquidation of the pension fund. This includes all scenarios in which one or more temporary shortfalls occur but are followed by recovery. In the delayed closure procedure, the terminal payment to the beneficiaries corresponds to the payment stated in (2.1). Second, all remaining scenarios are extracted where the pension fund is liquidated somewhere along the line and the rebate payment in (2.9) is provided to the beneficiaries. For time consistency reasons, we accrue the rebate payment at the risk-free rate r over the remaining time to maturity. Finally, we assume that the beneficiaries have a constant relative risk aversion (CRRA) utility function defined over terminal wealth. Let V_n denote the terminal wealth in scenario n, then the preferences are defined by

$$U[V] = \frac{1}{N}\sum_{n=1}^{N} e^{-rT}\frac{V_n^{1-\gamma}}{1-\gamma}$$

where γ is the risk aversion parameter and N is the number of scenarios. In the baseline setup, we set the risk aversion parameter equal to $\gamma = 0.05$ for the homogenous group of participants. As a robustness check we also take $\gamma = 0.03$ and $\gamma = 0.08$.

Table (2.5) illustrates the welfare analysis for different recovery periods (κ) and for the different levels of risk aversion (γ). The table is based on 2,500 scenarios where regulatory compliance is monitored every month. Recall from the results in Tables (2.2) and (2.3) that a longer recovery period reduces the fair level of the participation rate (δ). This is shown in the second column. PD, or probability of default in the third column, refers to the percentage of scenarios in which early

[15] A tilde is added to the Brownian motion $\widetilde{W}$ to distinguish it from the Brownian motion under the martingale measure Q in (2.6).

liquidation occurs. This is measured over the entire time horizon $(T-t)$ of the pension contract. As the recovery period is extended, the default probability diminishes due to the positive drift in asset returns. LGD refers to the loss given default. This is defined as the rebate payment over the regulatory boundary at premature liquidation. Obviously, this conditional expectation goes down if the recovery period is extended as some scenarios get worse and worse. Variable m_1 is defined as the mean of the terminal wealth for all scenarios given that the terminal wealth is below the guaranteed benefit level $(V < \underline{L})$. Similarly, m_2 is the mean of the terminal wealth contingent on the terminal wealth being larger than the guaranteed payout $(V \geq \underline{L})$. It follows that the conditional expected returns sink for longer recovery periods. This has several reasons that follow from the economic setting. First, as for longer recovery periods the fair participation rate goes down, more upside return accrues to the sponsor. Second, a longer recovery period shifts more defaults to the maturity date at which the pension fund participants benefit from the sponsor guarantee. In case of a premature liquidation no such sponsor support is given. The utility levels give insight into the optimal recovery period. For all levels of risk aversion the optimal recovery period appears at around 1 year.[16] However, this result is sensitive to the particular contract specification, the risk sharing arrangement and the other parameters being used. The result is therefore only indicative for this particular case.

Therefore, we also present some robustness checks. The optimal recovery period depends on the risk-return characteristics. For instance, if the volatility level is increased from 0.15 to 0.20 and the expected return from 0.08 to 0.093, the optimal recovery period doubles to 2 years. Alternatively, if $\underline{L}$ is increased from 120 tot 140, the optimal recovery period is at 3 to 5 years depending on the volatility level being used. In case the time-to-maturity is 15 years, lengthy recovery periods hamper an efficient analysis as there will be hardly any premature defaults. Therefore, Table (2.6) shows comparable results for a contract with a 30 year maturity. For computational reasons we switched from monthly to annual regulatory monitoring here. The optimal recovery period for all risk aversion levels is at 3 to 4 years. This result is conformed if the volatility level is increased from 0.15 to 0.20 or lowered to 0.10.[17] We conclude that the optimal recovery period depends

[16] Due to computational constraints we restricted the analysis to recovery periods of 1 year and beyond.

[17] Following Briec, Kerstens and Jokung (2007), we also looked at a utility function in which the

κ	δ	PD	LGD	m_1	m_2	$U[V]$ $\gamma = 0.03$	$U[V]$ $\gamma = 0.05$	$U[V]$ $\gamma = 0.08$
1	0.52	0.126	0.86	103.4	269.2	118.8	108.4	94.7
2	0.50	0.080	0.82	98.1	260.4	118.4	108.1	94.4
3	0.48	0.050	0.80	95.2	253.8	117.6	107.4	93.9
4	0.46	0.026	0.77	82.2	248.5	116.8	106.7	93.2
5	0.43	0.030	0.74	89.1	241.9	115.1	105.2	92.0
6	0.40	0.011	0.68	81.9	236.7	113.2	103.5	90.6
7	0.38	0.004	0.69	82.7	233.4	111.9	102.4	89.6
8	0.36	-	-	-	228.9	109.9	100.5	88.1
9	0.33	-	-	-	226.0	108.5	99.3	87.1
10	0.30	-	-	-	221.6	106.5	97.5	85.5

Table 2.5: Welfare analysis for different recovery periods κ, using $A_0 = 100$; $\underline{L} = 120$; $\lambda = 1$; $T = 15$; $r = 0.04$; $\mu = 0.08$; $\sigma = 0.15$. Based on 2,500 scenarios. Monthly monitoring of the regulatory boundary.

on the specific contract details, the investment policy, the risk aversion parameter and the other regulatory features. For the numerical examples presented the optimal recovery period ranges between 1 and 5 years.

2.6.2 Liquidation probability and recovery period

Another way to analyze recovery periods is to interconnect them with the regulatory liquidation probability. In case of a delayed closure procedure, the beneficiary is ultimately interested in the liquidation probability and not so much in the de-

beneficiary maps his preferences with respect to the first three moments of the terminal wealth distribution. The mean-variance-skewness utility function in this case reads

$$U[V] = \theta_1 E_{\tilde{P}}[V] - \theta_2 Var_{\tilde{P}}[V] + \theta_3 Sk_{\tilde{P}}[V],$$

where the parameters $(\theta_1, \theta_2, \theta_3) > 0$, and $\frac{\theta_2}{\theta_1} \geq 0$ is labeled the degree of absolute risk aversion and $\frac{\theta_3}{\theta_2} \geq 0$ is referred to as prudence. $E_{\tilde{P}}$, $Var_{\tilde{P}}$ and $Sk_{\tilde{P}}$ resemble the mean, variance and skewness under the real world measure respectively. Using $\theta_1 = 1$, $\theta_2 = 0.005$ and $\theta_3 = 10$ we find the optimal regulatory for the parameters used in Table (2.5) to be at 2 to 3 years.

κ	δ	PD	LGD	m_1	m_2	$U[V]$ $\gamma = 0.03$	$U[V]$ $\gamma = 0.05$	$U[V]$ $\gamma = 0.08$
1	0.52	0.299	0.91	105.5	429.0	86.1	77.9	67.2
2	0.50	0.232	0.86	98.8	410.1	87.5	79.2	68.3
3	0.48	0.190	0.82	94.4	396.2	87.9	79.6	68.6
4	0.46	0.156	0.79	90.8	384.0	87.7	79.5	68.6
5	0.43	0.132	0.77	88.7	371.8	86.8	78.7	68.0
6	0.40	0.110	0.75	86.1	360.5	85.8	77.8	67.2
7	0.38	0.092	0.73	84.1	352.0	85.1	77.2	66.7
8	0.36	0.073	0.72	82.4	341.6	83.9	76.1	65.9
9	0.33	0.059	0.70	80.7	334.3	83.1	75.4	65.3
10	0.30	0.049	0.69	79.7	325.6	81.7	74.2	64.3

Table 2.6: Welfare analysis for different recovery periods κ, using $A_0 = 50$; $\underline{L} = 120$; $\lambda = 1$; $T = 30$; $r = 0.04$; $\mu = 0.08$; $\sigma = 0.15$. Based on 10,000 scenarios. Annual monitoring of the regulatory boundary.

fault probability. The liquidation probability is given by[18]

$$\begin{aligned}
&\tilde{P}\left(T_B^- = \inf\left\{t > 0 | \left(t - g_{B,t}^A\right) 1_{\{A_t < B_t\}} > \kappa\right\} \leq T\right) \\
=&e^{-\frac{1}{2}\tilde{m}^2 T}\left(\int_{-\infty}^{b} h_2(T,y) e^{\tilde{m}y}\, d\, y + \int_{b}^{\infty} h_1(T,y) e^{\tilde{m}y}\, d\, y\right)
\end{aligned}$$

with $\tilde{m} = \frac{1}{\sigma}\left(\mu - r - \frac{1}{2}\sigma^2\right)$. In addition, $h_1(T,y)$ and $h_2(T,y)$ have the similar values as before (cf. Appendix 2.2). The supervisory rule is to control this liquidation probability to a maximum allowed probability of ε. In other words, the recovery policy is to choose the appropriate recovery period κ to satisfy this constraint

$$\kappa(\varepsilon) = \operatorname{argmin}\left\{\kappa > 0 | \tilde{P}(T_B^- \leq T) = \varepsilon\right\}.$$

Table (2.7) illustrates several matching recovery periods for diverse liquidation probability constraints. Similar to the previous section, the risk level σ is chosen to be either 10%, 15% or 20% and the corresponding rate of return μ is determined such that the market price of risk is equal for all volatility levels, i.e. $\frac{\mu - r}{\sigma} = \frac{4}{15}$.

Intuitively, as the liquidation probability constraint ε is set higher (or a lower confidence level), the recovery period can be adjusted downward. The longer an

[18] In the appendix of Bernard and Chen (2009), a detailed derivation is provided to this probability.

underfunded pension fund is allowed to recover, the less likely it is to be liquidated eventually.[19] Furthermore, Table (2.7) shows that more risky investment strategies should have a longer recovery period for a given probability constraint. A longer recovery period implies that regulators are less likely to intervene (and force the liquidation of the pension fund). In this sense, the regulation is less strict under longer recovery periods.

ε	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.10
$\kappa(\varepsilon)$; $\sigma = 0.10$	4.49	2.50	1.49	0.92	0.53	0.28	0.12	0.04	≈ 0
$\kappa(\varepsilon)$; $\sigma = 0.15$	8.70	6.98	5.40	4.40	3.54	2.96	2.46	2.01	1.38
$\kappa(\varepsilon)$; $\sigma = 0.20$	10.97	9.40	8.24	7.31	6.33	5.46	4.73	4.18	3.31

Table 2.7: Liquidation probability ε and recovery period $\kappa(\varepsilon)$ for different σ values with parameters: $A_0 = 100$; $L = 120$; $\lambda = 1$; $T = 15$; $r = 0.04$; $(\mu - r)/\sigma = 4/15$. The maximum liquidation probability ε is measured at a 15 year horizon.

This analysis shows that the supervisory authorities can design policy regarding the recovery period according to its rules for the liquidation probability regulation. For instance, for a one-year time horizon, Solvency II stipulates the maximum liquidation probability is 0.5% for insurance companies. It implies a liquidation probability constraint $1 - 0.995^{15} \approx 0.07$ for a 15-year time horizon and consequently an optimal recovery period of 0.12 ($\sigma = 0.10$), 2.46 ($\sigma = 0.15$) or 4.73 ($\sigma = 0.20$) years. Due to the fact that the liquidation probability increases in the risk level and decreases in the recovery period, a negative relation between σ and κ results. For the case $\sigma = 0.15$ and $\varepsilon = 0.2$, since ε value is set fairly low (for a 15-year time horizon), the resulting recovery period is rather high.

2.7 Conclusion

This chapter considers the interaction of pension fund regulation and pension fund investment policy on the market-consistent valuation of defined benefit pension liabilities. Typically, premature closure of a DB plan is triggered by a low funding ratio, e.g., if this ratio of assets to liabilities hits the applicable regulatory minimum. We assume that early termination leads to an unwinding of the

[19] This only shows that a longer recovery period lowers the liquidation probability, due to the positive drift in asset returns. However, this does not necessarily mean that a longer recovery period is always better, as we have seen in Section (2.6.1).

pension scheme and the assets are transferred to the beneficiaries. We distinguish between an immediate and a delayed closure procedure. In the former case, the moment the regulatory boundary is reached, the pension contract is immediately terminated. Whereas in the latter case, a grace period is given for reorganization and recovery. For both procedures, we derive closed–form formulae for the contracts which enable us to perform a fair contract analysis. A pension deal is defined economically fair if the initial contribution made by the participants to the pension fund equals the market–consistent value of the claim they get in return. Thereupon, the emphasis is placed particularly on how the interaction between the regulatory rules (required funding ratio and maximum recovery period) and the pension fund investment policy influences the optimal amount by which the beneficiaries should participate in the pension fund's surplus. This is relevant for the contemporary discussion on "who owns the pension fund's surplus". Several ceteris paribus insights follow from this analysis. First, as the pension fund pursues a more risky investment strategy, the beneficiary should claim a higher stake in the pension fund's surplus for the deal to be fair. Otherwise, the higher return volatility transfers value from the beneficiary to the sponsor. Second, a longer regulatory recovery period can be accompanied by a somewhat lower beneficiary's claim on the surplus. A longer recovery period increases the probability that the fixed defined benefit payment at maturity will become due. Third, a lower required funding ratio can be accompanied with a lower claim on the surplus as it lowers the probability of a premature closure thereby lowering the value of any early rebate payment but increasing the value of the fixed payment at maturity. Finally, we demonstrate that utility analysis can be used to determine the optimal recovery period in our particular contract setting. It ranges form 1 to 5 years depending on contract specification, investment policy, risk aversion and other regulatory features. We also show that under a longer regulatory recovery period for underfunded pension funds it is less likely that liquidation is going to occur, as such long recovery periods imply less stricter regulation.

2.8 Appendix

2.8.1 Valuation of each component in immediate closure procedure

The expected fixed payment at maturity can be expressed as follows:

$$E_Q\left[e^{-rT}\underline{L}\mathbf{1}_{\{\tau>T\}}\right]=\underline{L}e^{-rT}\left[N\left(d^-(A_0,B_0,T)\right)-\left(\frac{A_0}{B_0}\right)N\left(d^-(B_0,A_0,T)\right)\right].$$

The long down–and–out call option can be calculated further:

$$\begin{aligned}
& e^{-rT}E_Q\left[(A_T-\underline{L})^+_{\{\tau>T\}}\mathbf{1}_{\{\tau>T\}}\right] \\
= \;& A_0N\left(d^+\left(A_0,\max\left\{\underline{L}e^{-rT},B_0\right\},T\right)\right) \\
& -\underline{L}e^{-rT}N\left(d^-\left(A_0,\max\left\{\underline{L}e^{-rT},B_0\right\},T\right)\right) \\
& -\left(\frac{A_0}{B_0}\right)\frac{B_0^2}{A_0}N\left(d^+\left(\frac{B_0^2}{A_0},\max\left\{\underline{L}e^{-rT},B_0\right\},T\right)\right) \\
& -\underline{L}e^{-rT}N\left(d^-\left(\frac{B_0^2}{A_0},\max\left\{\underline{L}e^{-rT},B_0\right\},T\right)\right)
\end{aligned} \tag{2.13}$$

with $d^\pm(S,K,T)=\frac{\log\left(\frac{S}{K}\right)\pm\frac{1}{2}\sigma^2T}{\sigma\sqrt{T}}$ and $N(x)=\int_{-\infty}^x\frac{1}{\sqrt{2\pi}}e^{\frac{-u^2}{2}}du$. The shorted down–and–out call with strike $\bar{L}$ has the same form as (2.13) by replacing $\underline{L}$ by $\bar{L}$.

Finally, the expected rebate payment at the closure time has the form of

$$\begin{aligned}
& E_Q\left[e^{-r\tau}\min\{1,\lambda\}\underline{L}e^{-r(T-\tau)}\mathbf{1}_{\{\tau\le T\}}\right] \\
= \;& \min\{1,\lambda\}\underline{L}e^{-rT}\left[N\left(-d^-(A_0,B_0,T)\right)+\left(\frac{A_0}{B_0}\right)N\left(d^-(B_0,A_0,T)\right)\right].
\end{aligned}$$

In the valuation of the market-consistent value of the payoff to the sponsor, the long down–and–out call option is determined analogous to (2.13) and the shorted down–and–out put option's value and the rebate payment are computed as follows:

$$\begin{aligned}
& -E_Q[e^{-rT}(\underline{L}-A_T)^+\mathbf{1}_{\{\tau>T\}}] \\
= & -\mathbf{1}_{\{\lambda<1\}}\{\underline{L}e^{-rT}\left[N\left(-d^-(A_0,\underline{L}e^{-rT},T)\right)-N\left(-d^-(A_0,B_0,T)\right)\right] \\
& -A_0\left[N\left(-d^+(A_0,\underline{L}e^{-rT},T)\right)-N\left(-d^+(A_0,B_0,T)\right)\right] \\
& -\frac{A_0}{B_0}[\underline{L}e^{-rT}\left(N\left(-d^-(B_0^2,A_0^2\underline{L}e^{-rT},T)\right)-N\left(-d^-(B_0,A_0,T)\right)\right) \\
& -\frac{B_0^2}{A_0}\left(N\left(-d^+(B_0^2,A_0^2\underline{L}e^{-rT},T)\right)-N\left(-d^+(B_0,A_0,T)\right)\right)]\} \\
& E_Q[e^{-r\tau}\max\{\lambda-1,0\}\underline{L}e^{-r(T-\tau)}\mathbf{1}_{\{\tau\le T\}}] \\
= & \max\{\lambda-1,0\}\underline{L}e^{-rT}\left[N\left(-d^-(A_0,B_0,T)\right)+\left(\frac{A_0}{B_0}\right)N\left(d^-(B_0,A_0,T)\right)\right].
\end{aligned}$$

2.8.2 Valuation of each component in delayed closure procedure

It is well known that the price of a Parisian down–and–out call option can be described as the difference of the price of a plain–vanilla call option and the price of a Parisian down–and–in call option with the same strike and maturity date, i.e.,

$$PDOC[A_0,B_0,\underline{L}e^{-rT},r,r]=BSC[A_0,\underline{L}e^{-rT},r]-PDIC[A_0,B_0,\underline{L}e^{-rT},r,r].$$

Here the last component r in $PDIC$ and $PDOC$ is used to point out the fact that over time the barrier level increases by an exponential rate r. The price of the plain–vanilla call option is obtained by the Black–Scholes formula as follows:

$$\begin{aligned}
BSC\left[A_0,\underline{L}e^{-rT},r\right] &= E_Q\left[e^{-rT}\left(A_T-\underline{L}\right)^+\right] \\
&= A_0N\left(d^+(A_0,\underline{L}e^{-rT},T)\right)-e^{-rT}\underline{L}N\left(d^-(A_0,\underline{L}e^{-rT},T\right).
\end{aligned}$$

To calculate $PDIC$ we distinguish between $B_0\le e^{-rT}\underline{L}$ (i.e. $\lambda\le 1$) and $B_0 > e^{-rT}\underline{L}$ (i.e. $\lambda>1$) according to the relation between the barrier B_0 and the strike. For $B_0\le e^{-rT}\underline{L}$, $PDIC[A_0,B_0,\underline{L}e^{-rT},r,r]$ can be calculated as follows:

$$PDIC[A_0,B_0,\underline{L}e^{-rT},r,r]=e^{-\frac{1}{2}m^2T}\int_k^\infty e^{my}\left(A_0e^{\sigma y}-\underline{L}e^{-rT}\right)h_1(T,y)dy$$

with $k = \frac{1}{\sigma}\ln\frac{e^{-rT}\underline{L}}{A_0}$. The density $h_1(T,y)$ is uniquely determined by inverting the corresponding Laplace transform which is given by

$$\begin{aligned}
\hat{h}_1(\eta,y) &= \frac{e^{(2b-y)\sqrt{2\eta}}\psi(-\sqrt{2\eta\kappa})}{\sqrt{2\eta}\psi(\sqrt{2\eta\kappa})} \\
\text{with}\quad \psi(z) &= \int_0^\infty x\exp\left(-\frac{x^2}{2}+zx\right)dx = 1 + z\sqrt{2\pi}e^{\frac{z^2}{2}}N(z),
\end{aligned}$$

and η the parameter of Laplace transform, see Chen and Suchanecki (2007). For the case of $B_0 > e^{-rT}\underline{L}$, we have

$$\begin{aligned}
PDIC[A_0, B_0, \underline{L}e^{-rT}, r, r] &= e^{-\frac{1}{2}m^2T}\Big(\int_k^b e^{my}\left(A_0e^{\sigma y} - \underline{L}e^{-rT}\right)h_1(T,y)dy \\
&\quad + \int_b^\infty e^{my}\left(A_0e^{\sigma y} - \underline{L}e^{-rT}\right)h_1(T,y)dy\Big)
\end{aligned}$$

As before, $h_1(T,y)$ and $h_2(T,y)$ are calculated by inverting the corresponding Laplace transform. $\hat{h}_1(T,y)$ has the same value as before and the Laplace transform of $h_2(T,y)$ is given by

$$\begin{aligned}
\hat{h}_2(\eta,y) &= \frac{e^{y\sqrt{2\eta}}}{\sqrt{2\eta}\psi(\sqrt{2\eta\kappa})} \\
&\quad + \frac{\sqrt{2\eta\kappa}e^{\eta\kappa}}{\psi(\sqrt{2\eta\kappa})}\Big(e^{y\sqrt{2\eta}}\left(N\left(-\sqrt{2\eta\kappa} - \frac{y-b}{\sqrt{\kappa}}\right) - N(-\sqrt{2\eta\kappa})\right) \\
&\quad - e^{(2b-y)\sqrt{2\eta}}N\left(-\sqrt{2\eta\kappa} + \frac{y-b}{\sqrt{\kappa}}\right)\Big).
\end{aligned}$$

The second term, the Parisian down–and–out call option with strike $\bar{L}$ can be computed analogously by distinguishing between $B_0 \le e^{-rT}\overline{L}$ and $B_0 > e^{-rT}\overline{L}$. The cases $B_0 \le e^{-rT}\overline{L}$ and $B_0 > e^{-rT}\overline{L}$ are equivalent to $\lambda \le e^{iT}$ and $\lambda > e^{iT}$, respectively. The third term in the payoff function can be calculated as follows:

$$\begin{aligned}
E_Q[e^{-rT}\underline{L}\mathbf{1}_{\{T_b^- > T\}}] &= e^{-rT}\underline{L} - E_Q[e^{-rT}\underline{L}\mathbf{1}_{\{T_b^- \le T\}}] \\
&= e^{-rT}\underline{L}[1 - e^{-\frac{m^2T}{2}}\Big(\int_{-\infty}^b h_2(T,y)e^{my}dy \\
&\quad + \int_b^\infty h_1(T,y)e^{my}dy\Big)].
\end{aligned}$$

In the calculation of the expected rebate, distinction of cases becomes necessary again. For the case of $\lambda < 1$, the beneficiary will get $A_{T_b^-}$ upon early closure. Therefore, the expected rebate can be calculated as follows:

$$\begin{aligned}
& E_{\tilde{Q}}\left[e^{-\left(r+\frac{1}{2}m^2\right)T_b^-}\exp\{mZ_{T_b^-}\}\min\{L_{T_b^-}e^{-r(T-T_b^-)},A_{T_b^-}\}\mathbf{1}_{\{T_b^-\leq T\}}\right] \\
= & \; A_0E_{\tilde{Q}}\left[e^{-\frac{1}{2}m^2T_b^-}\exp\{(m+\sigma)Z_{T_b^-}\}\mathbf{1}_{\{T_b^-\leq T\}}\right] \\
= & \; A_0E_{\tilde{Q}}\left[e^{-\frac{1}{2}m^2T_b^-}\mathbf{1}_{\{T_b^-\leq T\}}\right]E_{\tilde{Q}}\left[\exp\{(m+\sigma)Z_{T_b^-}\}\right].
\end{aligned}$$

The last equality follows from the fact that T_b^- and $Z_{T_b^-}$ are independent, which is shown in the appendix of Chesney *et al.* (1997). Furthermore, the corresponding laws for these two random variables are given in Chesney *et al.* (1997), too. As a consequence, we obtain

$$E_{\tilde{Q}}\left[\exp\{(m+\sigma)Z_{T_b^-}\}\right]=\int_{-\infty}^{b}e^{(m+\sigma)x}\,\frac{b-x}{\kappa}\,\exp\left\{-\frac{(x-b)^2}{2\kappa}\right\}dx$$

and

$$E_{\tilde{Q}}\left[e^{-\frac{1}{2}m^2T_b^-}\mathbf{1}_{\{T_b^-<T\}}\right]=\int_{\kappa}^{T}e^{-\frac{1}{2}m^2t}\,h_3(t)\,dt,$$

where $h_3(t)$ denotes the density of the stopping time T_b^-. This density can be calculated by inverting the following Laplace transform, see Chen and Suchanecki (2007),

$$\hat{h}_3(\eta)=\frac{\exp\{\sqrt{2\eta}b\}}{\psi(\sqrt{2\eta}\kappa)}.$$

For the case of $\lambda \geq 1$, we obtain

$$\begin{aligned}
& E_{\tilde{Q}}\left[e^{-\left(r+\frac{1}{2}m^2\right)T_b^-}\exp\{mZ_{T_b^-}\}\min\{\underline{L}e^{-r(T-T_b^-)},A_{T_b^-}\}\mathbf{1}_{\{T_b^-\leq T\}}\right] \\
= & \; A_0E_{\tilde{Q}}\left[e^{-\frac{1}{2}m^2T_b^-}\exp\{(m+\sigma)Z_{T_b^-}\}\mathbf{1}_{\{T_b^-\leq T\}}\mathbf{1}_{\{Z_{T_b^-}\leq k\}}\right] \\
& +\underline{L}e^{-rT}E_{\tilde{Q}}\left[e^{-\frac{1}{2}m^2T_b^-}\exp\{mZ_{T_b^-}\}\mathbf{1}_{\{T_b^-\leq T\}1}\mathbf{1}_{\{k<Z_{T_b^-}<b\}}\right] \\
= & \; A_0E_{\tilde{Q}}\left[e^{-\frac{1}{2}m^2T_b^-}\mathbf{1}_{\{T_b^-\leq T\}}\right]E_{\tilde{Q}}\left[\exp\{(m+\sigma)Z_{T_b^-}\}\mathbf{1}_{\{Z_{T_b^-}<k\}}\right] \\
& +\underline{L}e^{-rT}E_{\tilde{Q}}\left[e^{-\frac{1}{2}m^2T_b^-}\mathbf{1}_{\{T_b^-\leq T\}}\right]E_{\tilde{Q}}\left[\exp\{mZ_{T_b^-}\}\mathbf{1}_{\{k<Z_{T_b^-}<b\}}\right].
\end{aligned}$$

with $k=\frac{1}{\sigma}\ln\frac{\underline{L}e^{-rT}}{A_0}$. All the expectations can be calculated in the same way as when $\lambda < 1$.

In the valuation of the payoff to the sponsor, the long Parisian down–and–out call option has been determined. The shorted Parisian down–and–out put option's value can be derived by the following in–out–parity:

$$PDOP[A_0, B_0, \underline{L}e^{-rT}, r, r] := BSP[A_0, \underline{L}e^{-rT}, r] - PDIP[A_0, B_0, \underline{L}e^{-rT}, r, r].$$

Here $BSP[A_0, \underline{L}e^{-rT}, r]$ gives the price of the plain–vanilla put option and the price of the Parisian down–and–in put option is given by $PDIP[A_0, B_0, \underline{L}e^{-rT}, r, r]$. $BSP[A_0, \underline{L}e^{-rT}, r]$ is derived by the Black–Scholes formula

$$\begin{aligned} BSP\left[A_0, \underline{L}e^{-rT}, r\right] &= E_Q\left[e^{-rT}\left(\underline{L} - A_T\right)^+\right] \\ &= \underline{L}e^{-rT} N\left(-d^-(A_0, \underline{L}e^{-rT}, T)\right) \\ &\quad -A_0 N\left(d^+(A_0, \underline{L}e^{-rT}, T)\right). \end{aligned}$$

Due to the different possible choices of the λ–value, different pricing formulas are obtained for the Parisian down–and–in put option. A $\lambda < 1$, which leads to the fact that the strike (here $\underline{L}e^{-rT}$) is larger than the barrier (B_0), results in

$$\begin{aligned} PDIP[A_0, B_0, \underline{L}e^{-rT}, r, r] &= e^{-\frac{1}{2}m^2 T}\Big(\int_{-\infty}^{b} e^{my}(\underline{L}e^{-rT} - A_0 e^{\sigma y}) h_2(T, y) dy \\ &\quad + \int_{b}^{k} e^{my}(\underline{L}e^{-rT} - A_0 e^{\sigma y}) h_1(T, y) dy\Big) \end{aligned}$$

with $k = \frac{1}{\sigma} \ln\left(\frac{\underline{L}e^{-rT}}{A_0}\right)$. As before, $h_1(T, y)$ and $h_2(T, y)$ are calculated by inverting the corresponding Laplace transform. $\hat{h}_1(T, y)$ and $\hat{h}_2(T, y)$ have the same values as before. Analogously, for the case of $\lambda \geq 1$, the Parisian down–and–in put option has the form of

$$PDIP[A_0, B_0, \underline{L}e^{-rT}, r, r] = e^{-\frac{1}{2}m^2 T} \int_{-\infty}^{k} e^{my}(\underline{L}e^{-rT} - A_0 e^{\sigma y}) h_2(T, y) dy.$$

Part II

Pension Fund Investment Policy

Chapter 3

Stock Market Performance and Pension Fund Investment Policy

This chapter is based on Bikker, Broeders and de Dreu (2010)

3.1 Introduction

The optimal equity allocation of pension funds is subject to considerable debate. A high percentage of assets invested in equities results in significant exposure of pension wealth to fluctuations in stock market prices. While nominal defined-benefit pension liabilities are best resembled by bond returns, considerable equity holdings may be optimal when indexation of benefits is contingent on the performance of the pension fund and the risks can be managed in an orderly fashion. In many Dutch defined benefit pension deals, indexation is contingent on the funding ratio of the pension fund. The market value of this contingent indexation can be derived using option pricing theory. During the nineties abundant equity returns led to premium reductions and even contribution holidays for pension plan sponsors. However, the risks of equity holdings surfaced after the collapse of the stock market in 2000-2002 and also in 2008, which resulted in large losses for pension funds. In reaction, pension benefits were curtailed and contributions steeply increased. This episode raised a debate on the investment strategies of Dutch pension funds and, particularly, on their exposure to equity markets.

The investment strategy of Dutch pension funds is of key importance to society, as it involves more than € 637 billion in assets, or close to € 40,000 per inhabitant. The way in which these assets are invested has a significant influence on the level of required premiums or final benefits. A one percent lower annual return over the life-cycle of a typical worker translates into 27 percent lower accumulated pension assets.[1] Consequently, one of the most important responsibilities of pension funds' trustees is to maximize the expected return on assets at an acceptable level of risk, e.g. measured in terms of the probability of underfunding.

This chapter investigates whether stock market performance influences pension funds' investment policies. In particular, two ways are examined in which stock market performance impacts the equity allocation of pension funds: (i) in the short term, as a result of market timing or imperfect rebalancing, and (ii) in the medium term, as a result of adjustments to strategic asset allocation.

Table (3.1) presents the asset allocation of Dutch pension funds over the following five broad classes: Equities, Bonds, Real Estate, Cash, and Other Assets. Pension fund investment policy includes the strategic asset allocation decision, which refers to choosing the investment percentages in each asset class. Of the aforementioned asset classes, equities have the highest expected return but also the highest volatility. For most pension funds it is the largest asset category together bonds. Consequently, equity allocation is one of the key policy variables determining the risk-return profile of a given pension fund.

Pension funds generally determine their strategic asset allocation policies using asset and liability management studies, in which they consider long-term expected returns, return variances and covariances of broad asset classes, given the size and characteristics of their pension liabilities, see, e.g., Campbell and Viceira (2002). The strategic asset allocation is typically set on a three to five year horizon. For many pension funds, the strategic asset allocation includes bandwidths for the actual asset allocation to drift. For this purpose a tactical risk budget can be made available. These bandwidths are chosen in such a way that the maximum ex ante tracking error does not exceed a given threshold. This tracking error (TE) is

[1] The three main components determining the costs of pensions are the quality of the pension scheme, the rate of return on investments and administrative and investment costs (see also Bikker and De Dreu, 2009).

Asset class	**Average strategic asset allocation**	**Standard deviation**	**Average actual asset allocation**	**Standard deviation**
Equities	42	15	41	15
Bonds	39	20	45	19
Real estate	10	6	10	6
Cash	1	11	1	10
Other	8	11	3	11
Total	100		100	

Table 3.1: Pension fund asset allocation 1999:I–2006:IV (in %): the asset allocations are averages across Dutch pension funds, weighted by total investments, source: De Nederlandsche Bank.

usually defined as $TE = w'\Sigma w$, where w is the vector of actual portfolio weights minus the vector of strategic portfolio allocation and Σ is the variance-covariance matrix. In this chapter, rebalancing is interpreted as a return to the midpoint of these bandwidths.[2]

This chapter examines the impact that higher or lower returns on stocks compared to the other asset categories have on the equity allocation of pension funds. To the best of our knowledge this is the first study that examines this relationship. Over the long term equity allocation is determined by a pension fund's strategic asset allocation. However, several factors influence asset allocation in the short to medium term. In the following three ways pension funds can respond to positive or negative stock market returns: rebalancing, free floating, and market timing.

Rebalancing refers to the investment process applied to ensure that a pension fund's actual equity allocation continuously equals its strategic equity allocation, which implies selling equities after relative high stock market returns and buying after relative low equity returns. This might also be indicated as a form of negative-feedback trading referring to buying past losers and selling past winners, see , e.g., Lakonishok, Schleifer and Vishny (1992). This form of trading is commonly a part of the argument that institutional investors stabilize asset prices. By contrast, *free floating* is indicated a passive investment strategy, in which pension funds allow their equity allocation to drift with market developments. Finally, as mentioned

[2] Detailed information on the bandwidth is not available.

above, *market timing* refers to a temporary higher or lower weighting of equities (or other asset classes) relative to the pension fund's strategic asset allocation, motivated by short-term return expectations. Note that where no equity trades are made, it is difficult to distinguish between free floating (passive management) and market timing (active management), as allowing the asset allocation to drift could be seen as an active investment decision.

A number of studies put forward that strategic asset allocation dominates portfolio performance. In particular, strategic asset allocation is shown to explain more than 90 percent of the variability in pension fund returns over time, while the additional variation explained by market timing is less than 5 percent (Blake, Lehmann and Timmermann, 1999; Brinson *et al.*, 1986, 1991; Ibbotson and Kaplan, 2000). Moreover, in line with the efficient market theory, evidence shows that pension funds are unsuccessful in exploiting market timing to generate excess returns. In particular, market timing is shown to cause an average loss of 20-66 basis points per year (Blake *et al.*, 1999; Brinson *et al.*, 1986, 1991; Daniel *et al.*, 1997).

A number of empirical papers examines the impact of investment policy on returns. The literature investigating the effectiveness of stock selection and market timing in improving investment performance is extensive. Most studies focus on US mutual funds and find that fund managers are not able to exploit selectivity and timing to generate excess returns (see, e.g., Fama (1972), Henriksson and Merton (1981), Kon and Jen (1979) and Kon (1983)). Agnew, Balduzzi and Sundén (2003) report that equity allocation of participants in 401(k) plans are positively related to the previous day's equity return (feedback trading). However, no significant correlation is found between changes in equity allocations and returns over the following three days suggesting the absence of market-timing abilities. Very few papers investigate the impact of market developments on investment policy. Blake *et al.* (1999) and Kakes (2008) report a negative correlation between asset class returns and net cash flows to the corresponding asset class, which points to rebalancing. However, Blake *et al.* (1999) also find that the asset allocation for UK pension funds drifts toward asset classes that performed relatively well, in line with a free-float strategy. Apparently, UK pension funds only partly rebalanced their investments in response to different returns across asset categories. Hence, the degree of rebalancing versus free floating in pension fund asset allocation re-

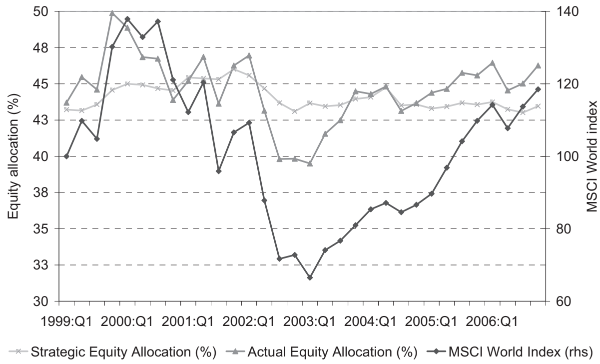

Figure 3.1: Stock market returns and equity investments (1999:I–2006:IV).

mains an open question.[3]

This chapter uses quarterly data from Dutch pension funds over 1999:I–2006:IV. Although this period is relatively short, it contains a significant stock market bubble as well as a burst. Figure (3.1) presents a preview of the empirical results, depicting the strategic and the actual equity allocation for Dutch pension funds, as well as the MSCI World Index. Three patterns stand out from this figure. First, the actual equity allocation tends to have a pattern similar to the MSCI World Index, but with some reversion to the strategic asset allocation. Generally, actual equity allocation increases when the stock market goes up, and vice versa. The main explanation for this pattern is that pension funds tend to rebalance their asset allocation only partly in response to changes in the value of their equity portfolio.

Second, Figure (3.1) points to interaction between stock market performance and strategic asset allocation. The strategic equity allocation appears to follow the performance of the equity market, although only gradually and with a time lag. Following the stock market boom in the second half of the 1990s, the strategic equity allocation increased until the end of 2001, but decreased from 2002 to 2003 in response to the fall of the stock market that started in 2000. A possible explanation is that pension funds adjust their investment policies based on recent stock market performance. Positive excess returns increases the pension fund's buffer, so that, as a consequence, regulatory rules also allow for a higher proportion

[3]De Haan and Kakes (2010) show that contrarian trading activities of Dutch pension funds are stronger during periods of market stress.

of the more risky equity investments. Apparently, pension funds make use of this opportunity and adjust their strategic asset allocation accordingly.

Third, the figure suggests that pension funds may have lost money from market timing over the business cycle. They seem to have gradually increased their equity allocation until the downturn of the stock market was well under way, confronting them with relatively large losses. Conversely, pension funds did not significantly increase their equity allocation portfolio investments to reap the full benefit of the subsequent upward stock market trend.

The structure of this chapter is as follows. Section 3.2 presents the data used in the analyses. Section 3.3 investigates the influence of market movements on asset allocation, whereas rebalancing is more closely examined in Section 3.4. Section 3.5 analyses the relationship between stock market returns and strategic asset allocation and 3.6 studies the additional return from market timing. Finally, the last section summarizes and concludes.

3.2 Description of the data

A detailed dataset is used with quarterly information on all Dutch pension funds for the 1999:I–2006:VI period. The data is from De Nederlandsche Bank, responsible for the prudential supervision of the 748 pension funds and their regulatory compliance. For each pension fund data is available on strategic asset allocation, asset sales and purchases, the market value of investments in different asset classes and their time-weighted returns. To assess the impact of stock market returns on actual and strategic equity allocation we either use data reported by pension funds or the MSCI World total return index.[4] All returns are in euros. The sample is an unbalanced panel, as not all pension funds reported data for the entire sample period due to new entrants, mergers, terminations, and reporting failures.[5] Since the aim is to study asset allocation over time, pension funds with less than two years of data are excluded. Finally, inconsistent observations and observations

[4] The MSCI World Index is a free float-adjusted market capitalization weighted index that is designed to measure the equity market performance of developed markets. The index contains some 1,500 different stocks and as such allows for comparison with large, globally diversified equity portfolios.

[5] We also ran regressions for a balanced sample of only 382 pension funds that reported at least seven years of data. The regression results were similar to those reported in Tables 3.4-7, suggesting that survivorship bias is not a significant issue.

with clear reporting errors are also excluded.

Our final sample includes data on 748 pension funds from 1999:I – 2006:IV, representing around 85% percent of total pension fund assets in the Netherlands. Table (3.2) presents summary statistics on the investment portfolios of pension funds in our sample. The size of pension funds in the sample is hugely divergent: the smallest pension funds have assets worth less than € 1 million, while the largest fund has assets of more than € 200 billion. The average and median sizes of pension fund assets equal € 799 million and € 53 million, respectively. Each period, distinction is made between size classes and types of pension funds and between types of pension plans. Small funds tend to invest relatively less in equity compared to larger funds, and more in bonds, reflecting lower risk appetite. Although large in number (70% of the sample), small pension funds administer only a minor share (less than 3%) of all pension fund investments.

Our sample includes 631 company pension funds, 95 industry-wide pension funds, and 10 professional group pension funds. Company pension funds provide pension plans to the employees of their sponsor company. They are separate legal entities, but are run by the sponsor company and employee representatives. Industry wide funds provide pension plans for employees working in an industry. Such pension plans are based on a collective labor agreement between an industry's companies and the labor unions, representing the employees in this industry. Finally, professional group pension funds offer pension schemes to specific professional groups (e.g., general practitioners, public notaries). Compulsory industry funds are largest in terms of investments. All pension fund categories invest on average between 41 and 45 percent in equity. Company funds and professional group funds invest relatively more in bonds than other types of funds, reflecting their stronger risk aversion. Industry funds invest substantially more in real estate. In terms of assets 70% is managed by industry wide pension funds, 27% by company pension funds and only 3% by professional group pension funds. Many industry wide pension funds operate under mandatory participation. The Act on mandatory participation in industry-wide pension funds empowers the Minister of Social Affairs and Employment, acting at the request of the employers' organizations and trade unions, to make membership of an industry-wide pension fund mandatory for corporations in a certain industry. On average, defined benefit (DB) pension funds have higher equity and lower bond investments than defined

Size classes based on total investment (mln euro)	Number of pension funds (1)	Average total investments (mln euro) (2)	Average bond investment (%) (3)	Average equity investment (%) (4)	Max-min actual equity investment over time (%) (5)	Max- min strategic equity investment over time (%) (6)	Investment gap[a] (%) (7)
0-100 (Small)	524	29	62	29	18	12	0.4
100-1000 (Medium)	177	320	51	37	18	13	0.2
>1000 (Large)	47	8,276	37	43	16	13	0.8
Total/average	748	799	39	42	16	13	0.8
Pension fund type[b,c]							
Industry (all)	95	3,798	35	41	14	12	1.1
- Compulsory	76	4,412	36	41	14	12	1.1
- Non-compulsory	19	1,099	35	45	16	15	1.7
Company	631	280	45	43	20	13	0.1
Professional group	10	2,292	51	42	18	12	0.5
Plan type[c]							
Defined benefit	592	926	39	42	16	13	0.8
Defined contribution	56	78	51	37	19	14	0.3

Table 3.2: Investments across size classes and pension fund types (1999:I – 2006:IV): all statistics are averages weighted by total investments except for the first two columns (Number of pension funds and Average total investments). [a]Investment gap is the absolute difference between the strategic and actual equity allocation; [b]Ten pension funds belong to other categories; [c]For some pension funds, type of pension fund or dominant plan type are unknown.

contribution (DC) pension funds, suggesting that DB funds may take higher risks since they can benefit from intergenerational risk sharing.

Columns 5 and 6 of Table 3.2 indicate how, respectively, the actual and strategic equity allocation vary over time. For the average pension fund, the range of the actual equity allocation is 16% and that of the strategic equity allocation is 13%. Thus, both actual and strategic equity allocation move significantly over time. The last column shows that the difference between strategic and actual equity allocation is, on average, 0.8 percentage point.

Table 3.3 shows that the strategic and actual equity allocation differs significantly across pension funds. A small majority of pension funds invest 20-40 percent of their assets in equities. A quarter of the funds invest more than 40 percent in equities, while around one-fifth invest less than 20 percent in equities.

Investments in equity classes	**Frequency distribution based on strategic asset allocation**	**actual equity allocation**
0 – 20	15.2	20.4
20 – 40	55.6	53.6
40 – 60	26.3	23.8
60 – 80	2.4	1.9
80 – 100	0.4	0.3
Total	100.0	100.0

Table 3.3: Frequency distribution of equity allocation across pension funds (1999:I – 2006:IV; in %).

3.2.1 Relative stock-market returns and short-term changes in equity allocation

To start our empirical analysis, this section examines the short-term impact of stock market performance on equity allocation. Over time, actual equity allocation may change either (i) due to excess returns on equities compared to other asset classes (free floating) or (ii) due to net purchases or net sales of equities (rebalancing and market timing). To investigate the impact of relative stock market returns on pension funds' equity allocation, the following equation is estimated

$$
\begin{aligned}
w_{i,t} &= \alpha_1 + \Sigma_{j=0}^{k}\beta_j \left(r^E_{t-j} - r^T_{i,t-j}\right) \\
&\quad + \gamma_1 Policy_{i,t-1} + \delta_1 Size_{i,t-1} + \theta_1 Funding_{i,t-1} + \varepsilon_{i,t}.
\end{aligned} \tag{3.1}
$$

The dependent variable $w_{i,t}$ is the actual percentage of the portfolio invested in equities of pension fund i ($i = 1, \ldots, N$) at quarter t ($t = 1, \ldots, T$). The variable $(r^E - r^T)$ is used to measure excess stock market returns compared to other investment categories on a quarterly basis. For stock market return (r^E), either the return on the MSCI World equity index or the pension funds' actual equity performance is used. For the average return on the pension fund portfolio's other asset categories (r^T) we multiply the strategic asset allocation of four key asset classes by representative broad market indexes.[6] Again, the alternative is to use the pension funds' actual performance on the respective asset classes. We consider two variants of Equation (3.1). The base model is without lagged stock market returns ($k = 0$), whereas alternatively, we include excess stock market returns with time lags ($k = 5$) to investigate the influence of past returns on pension funds' equity investments. The strategic equity allocation ($Policy$) also expressed as a percentage, is included to control for pension fund investment policy. $Size$, which is measured as the logarithm of the total investment portfolio, controls for the tendency of larger funds to invest relatively more in equities. $Funding$, or funding ratio is calculated as total investments over the discounted value of the pension liabilities, is included because funds with a higher buffer are allowed to invest more in equities. $Policy$, $Size$ and $Funding$ are included with one time lag to avoid endogeneity problems and since it may take some time before changes in these variables lead to changes in the equity portfolio investment. As stated before the panel is unbalanced, which implies that the number of observations varies across pension funds.

[6] We consider five investment categories: equities, bonds, real estate, money market instruments and other assets. For bonds we use the JP Morgan EMU bond index, for real estate we use the FTSE EPRA Netherlands real estate index and for money-market investments we use the 3-month Euribor interest rate. We assume that the fifth category 'other assets' is proportionally invested in the previous four investment categories (or has a similar return). We calculate excess returns as follows: excess return = return MSCI – [(return on bonds * bond investments + return on real estate * real estate investments + 3-months Euribor * money market investments) / (bond investments + real estate investments + money market investments)]

3.2.2 Empirical results of the impact of stock returns on actual equity allocation

Table (3.4) presents estimates of the impact of short-term excess stock returns on the percentage of equity portfolio investments, using Equation (3.1). The measure of excess stock returns in this table is based on the pension fund's actual asset returns. A one percentage point relative outperformance of the pension funds' equities leads to a significant increase in equity allocation of 0.12 percentage point in the subsequent quarter (first column). The second column shows that excess equity returns also have a (highly) significant impact on the equity allocation up to five quarters later. The impact decreases over time, indicating that pension funds rebalance gradually or infrequently. Since pension funds invest on average 42 percent in equity, a one percent rise of stock prices would imply an increase of the weight of stocks by 0.24 percentage point (being 42.42/100.42 minus 42/100), that is, as long as no adjustments are made. In this example, the observed 0.12 percentage point effect of excess returns on pension funds' equity implies that only half the excess return is rebalanced and that the other half of the equity weight moves in tandem with stock prices. The alternative measure of excess stock market returns based on the return on the MSCI World index provides quite similar results (not reported here).

Table (3.4) reveals also that a one percentage point increase in the strategic equity allocation causes a significant rise of around 0.90 percentage point in actual equity portfolio investments in the next period. As one would expect, pension fund investment managers adjust their equity portfolio investments almost fully in response to changes in the strategic equity allocation. The positive sign for the size of investments affirms that larger funds invest relatively more in equities (see also Table 3.1), except within the medium-sized pension funds class where the sign becomes negative. A possible explanation is that large pension funds tend to be less risk averse than small pension funds, which also holds within the classes of small and large funds. As large pension funds are typically mandatory industry wide pension funds, they have more opportunities for intergenerational risk sharing. As such they can accept a higher risk level, see Gollier (2008). Finally, in line with expectations, the funding ratio has a highly significant positive coefficient, indicating that pension funds with larger buffers invest more in equities.

	All funds				Small funds		Medium funds		Large funds	
	(1)		(2)		(3)		(4)		(5)	
Excess return[a]	0.118	***	0.103	***	0.094	***	0.109	***	0.125	***
Idem, lagged 1 quarter			0.067	***	0.068	***	0.069	***	0.056	***
Idem, lagged 2 quarter			0.053	***	0.055	***	0.052	***	0.054	***
Idem, lagged 3 quarter			0.031	***	0.023	***	0.037	***	0.042	***
Idem, lagged 4 quarter			0.023	***	0.020	***	0.024	***	0.037	***
Idem, lagged 5 quarter			0.018	***	0.014	**	0.020	***	0.028	**
Investment policy (t-1)	0.900	***	0.910	***	0.931	***	0.900	***	0.884	***
Size (t-1)	0.001	***	0.001	**	0.002	***	−0.004	***	0.005	***
Funding ratio (t-1)	0.025	***	0.016	***	0.011	***	0.025	***	0.011	***
Intercept	−0.009	***	0.009	***	−0.009		0.058	***	−0.043	**
Number of observations	11, 045		9, 358		4, 308		3, 855		1, 195	
R^2, adjusted	0.86		0.87		0.85		0.85		0.86	

Table 3.4: Estimates of the pension funds' equity investments model (1999:II – 2006:IV): ***, **, and * denote significance at the 1%, 5%, and 10% significance levels, respectively. The standard errors have been corrected for possible heteroskedasticity or lack of normality using the Huber-White sandwich estimators. [a]Estimates for excess returns are based on data reported by pension funds.

As equities are more risky, regulation requires larger buffers for this asset class.

If we consider the investment behavior across size classes (last three columns), where size classes are defined as in Table (3.2), we observe that the impact of excess stock market returns on equity allocation increases with the pension fund size, both immediately and in the long run. Apparently, large funds allow more free floating, whereas smaller funds rebalance more. In line with this result, larger funds react less to changes in the investment policy, compared to smaller funds. Table (3.5) presents the regression results of (3.1) for the different pension fund types. Apparently (compulsory) industry wide pension funds show less rebalancing behavior than company and professional group pension funds. Apart from that, the results are similar to Table (3.4).

As a robustness test we repeat the estimations of Table (3.4) and later tables with balanced samples with pension funds which have at least 28 quarters with all required data, instead of 8 quarters as in the current data set. For all tables, the results are fairly similar. As a second test, we re-estimated with fixed effects for pension funds and years. The Hausman test rejected random effects. The results are again fairly similar, except for Table (3.6). These results confirm that our outcomes are quite robust.

Furthermore we tested for cross-sectional dependence in the panel. Herding behavior and neighborhood effects across pension funds might cause dependence in the residuals. This may be due to the fact that pension funds copy each other's behavior, hire the same consultants to advise them or choose the same asset managers. Both the number of consultants and asset managers are relatively small compared to the number of pension funds. Driscoll and Kraay (1998) suggest standard errors that correct for spatial dependence. We have used this method to re-estimate Table (3.4), using Hoechle (2007). The results indicate somewhat higher standard errors, but overall the significance is not affected. This suggests that cross-sectional dependence is only minor.

3.3 Excess stock market returns and rebalancing

The positive impact of excess equity returns on equity allocation as found in the previous section may be (partly) due to imperfect rebalancing by pension funds. Excess equity performance will automatically lead to changes in equity allocation

	Compulsory industry wide (1)		Non-compulsory industry wide (2)		Company (3)		Professional (4)	
Excess return[a]	0.123	***	0.125	***	0.099	***	0.088	***
Idem, lagged 1 quarter	0.075	***	0.103	***	0.066	***	0.020	
Idem, lagged 2 quarter	0.061	***	0.123	***	0.050	***	0.070	**
Idem, lagged 3 quarter	0.014	*	0.108	***	0.032	***	0.011	
Idem, lagged 4 quarter	-0.005		0.049	*	0.026	***	0.019	
Idem, lagged 5 quarter	-0.001		0.050	*	0.020	***	0.007	
Investment policy (t-1)	0.984	***	0.886	***	0.906	***	0.708	***
Size (t-1)	0.000		0.001		0.000		0.002	
Funding ratio (t-1)	0.008	***	-0.004		0.019	***	0.007	
Intercept	0.002		0.040		0.009	**	0.067	
Number of observations	1, 524		263		7, 361		194	
R^2, adjusted	0.92		0.87		0.86		0.83	

Table 3.5: Estimates of the pension funds' equity investments model (1999:II – 2006:IV): ***, **, and * denote significance at the 1%, 5%, and 10% significance levels, respectively. The standard errors have been corrected for possible heteroskedasticity or lack of normality using the Huber-White sandwich estimators. [a]Estimates for excess returns are based on data reported by pension funds.

if pension funds do not actively rebalance their investment portfolios fully. This section presents an empirical rebalancing model, which is used to estimate to what extent pension funds rebalance, that is, re-adjust their asset allocation in response to excess equity returns. This model is derived as follows, starting from the definition of the actual equity allocation

$$w_{i,t} = E_{i,t}/TA_{i,t} \tag{3.2}$$

where $E_{i,t}$ represents the equity investments of pension fund i at time t, and TA stands for total assets. Taking first differences of this equation, we obtain

$$w_{i,t} - w_{i,t-1} = \frac{E_{i,t}}{TA_{i,t}} - \frac{E_{i,t-1}}{TA_{i,t-1}}$$

$$= \frac{E_{i,t-1}\left(1 + r^E_{i,t} + NCF^E_{i,t}\right)}{TA_{i,t-1}\left(1 + r^T_{i,t} + NCF^T_{i,t}\right)} - \frac{E_{i,t-1}}{TA_{i,t-1}}$$

$$= w_{i,t-1}\frac{\left(1 + r^E_{i,t} + NCF^E_{i,t}\right)}{\left(1 + r^T_{i,t} + NCF^T_{i,t}\right)} - w_{i,t-1}\frac{\left(1 + r^T_{i,t} + NCF^T_{i,t}\right)}{\left(1 + r^T_{i,t} + NCF^T_{i,t}\right)}$$

$$= w_{i,t-1}\frac{\left(r^E_{i,t} - r^T_{i,t} + NCF^E_{i,t} - NCF^T_{i,t}\right)}{\left(1 + r^T_{i,t} + NCF^T_{i,t}\right)}$$

where NCF^T is short for Net Class Flows converted into new investments as a fraction of total investments, NCF^E for new equity investments is also a fraction of equity investments, r^E for the return on equities over the last quarter, and r^T for the return on total assets (all for pension fund i and quarter t). Dividing both sides by $w_{i,t-1}$ results in

$$\frac{w_{i,t} - w_{i,t-1}}{w_{i,t-1}} = \frac{\left(r^E_{i,t} - r^T_{i,t}\right)}{\left(1 + r^T_{i,t} + NCF^T_{i,t}\right)} + \frac{\left(NCF^E_{i,t} - NCF^T_{i,t}\right)}{\left(1 + r^T_{i,t} + NCF^T_{i,t}\right)}. \tag{3.3}$$

This equation reveals that the percentage change in equity allocation can be contributed to: (i) excess equity returns, and (ii) net cash flows to equities, where both variables are scaled by the change in the total portfolio size. The first right-hand term is exogenous, since excess returns are determined by market developments and net cash flows into the pension fund are based on (previously made) decisions by employers and employees rather than on equity allocation. Given the small size of pension fund investments relative to total stock market capitalization, we can safely assume that changes in equity allocation do not affect stock market returns in general.[7] The second right-hand term, however, is endogenous. While net cash flows to equity investments directly influence the equity allocation of pension funds, the reverse can also be true: changes in the equity allocation may sway pension funds to adjust their net cash flows to equity investments. Thus, there is mutual causality between changes in equity allocation and net cash flows to equity investments. To estimate the impact of excess equity returns on equity allocation, we apply the above decomposition, ignoring the endogenous second right hand term. This results in the following empirical regression model

$$\frac{w_{i,t} - w_{i,t-1}}{w_{i,t-1}} = \alpha_2 + \beta_2 \left[\frac{\left(r_{i,t}^E - r_{i,t}^T \right)}{\left(1 + r_{i,t}^T + NCF_{i,t}^T \right)} \right] + \gamma_2 \left[\frac{\Delta Policy_{i,t-1}}{Policy_{i,t-2}} \right] + \varepsilon_{i,t}. \quad (3.4)$$

The percentage change or growth in the strategic equity allocation ($Policy$) is included to control for changes in investment policy. This variable is taken with a time lag of one quarter, since it may take some time before changes in policy lead to adjustments in the actual equity portfolio investments. In Equation (3.4), β_2 estimates the degree of free floating or market timing so that $1 - \beta_2$ assesses the rebalancing percentage. As an alternative model we split the excess equity return variable into positive and negative equity returns. This allows us to observe possible asymmetric effects in response to changes in excess equity returns.

3.3.1 Empirical results of rebalancing

Table (3.6) presents the estimated impact of excess equity returns on equity allocation from (3.4). The results show that pension funds rebalance, on average, around 39 percent of excess equity returns, leaving 61 percent for free floating. Thus 61 percent of excess equity returns translate into increases of the equity allocation in the next period. This is roughly in line with what we have observed in Table (3.4). Column (2) shows that pension funds rebalance differently in re-

[7] For individual, low-capitalization stocks this is not necessarily the case.

sponse to positive and negative equity returns. Only 13 percent of positive equity returns are rebalanced, against 49 percent of negative equity returns. Apparently, whereas pension funds do not automatically sell equities in bull markets, they do tend to buy additional equities in bear markets. In line with expectations, changes in policy affect the actual allocation positively (significant at the 1% significance level), with a lag of one quarter.

Columns (3) to (8) present the model estimates for the various size classes. In line with the results of Section 3.1, we observe that, in the symmetric model variant, large pension funds, at 30 percent, rebalance less than the small and medium-sized funds (around 40 percent). Consequently, large pension funds leave 70 percent for free floating. Changes in the one quarter lagged strategic equity allocation (Policy) affect actual allocation significantly (at the 1% significance level) for the small funds only. If we turn to the asymmetric effects on excess equity returns, we observe that the positive effects increase significantly with pension fund size, while the negative effects are similar across the size classes. The positive returns coefficient for the largest funds is, at 1.209, even above 1, indicating that large funds invest additional money in equities in response to positive excess returns in the last month. The appropriate t-statistic of this coefficient is (1.209 - 1) / 0.07478 = 2.80, which denotes significance at the 1% level. This suggests that excess equity returns are perceived by large pension funds to provide a positive signal for future returns, leading pension funds to increase their stakes. This is in line with results in Table (3.4), which indicate that large funds respond more strongly to excess equity returns than small ones. A possible explanation is that managers of large funds have more freedom to use market timing strategies in response to market developments. Quite remarkable, we observe that the strategic equity allocation (although increasing for small and medium-sized pension funds) is not increasing for large pension funds, e.g. compare 1999 to 2006. This holds also for the actual equity allocation. Hence, the overshooting for large funds, as we have estimated in this chapter, is apparently not due to an increase in the strategic asset allocation over time. Table (3.7) shows the estimation results per pension fund type. Apparently it are the industry wide pension plans that show overshooting with respect to positive excess returns.[8]

[8] One explanation might be that these large pension funds used the bull market to increase the equity allocation towards long term desirable levels. These target levels might not necessarily

	All funds		Small funds		Medium funds		Large funds	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Excess equity returns	0.613***		0.621***		0.586***		0.680***	
Positive excess returns		0.872***		0.839***		0.842***		1.209***
Negative excess returns		0.508***		0.532***		0.482***		0.483***
Change in policy (t-1)	0.075***	0.076***	0.109***	0.109***	0.020	0.022	−0.018	−0.015
Intercept	0.012***	0.003	0.014***	0.007***	0.010***	0.001	0.007***	−0.008***
Number of observations	12,010	12,010	5,889	5,889	4,705	4,705	1,416	1,416
R^2, adjusted	0.19	0.20	0.17	0.18	0.22	0.23	0.29	0.32

Table 3.6: Estimates of the equity allocation model: rebalancing versus free floating (1999:II – 2006:IV): ***, **, and * denote significance at the 1%, 5%, and 10% significance levels, respectively. The standard errors have been corrected for possible heteroskedasticity or lack of normality using the Huber-White sandwich estimators.

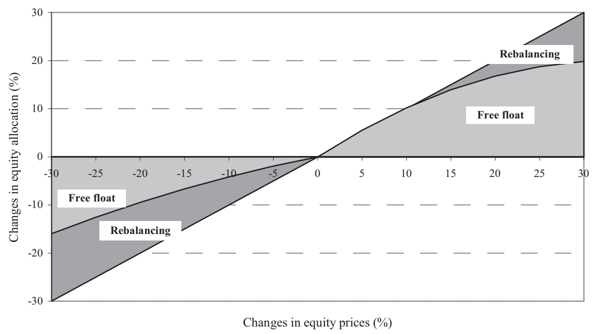

Figure 3.2: Reaction of pension funds to positive and negative excess equity returns: rebalancing and free floating.

Figure (3.2) presents the asymmetric relation between excess equity returns and rebalancing discussed above. To estimate this figure we adjusted Equation (3.5) by adding three additional terms: squared excess equity returns and excess and squared equity returns multiplied with 0-1 dummies indicating positive and negative returns. If pension funds used a free floating strategy and did not rebalance at all, excess equity returns would go in full to proportionate increases in equity allocation. This is represented by the diagonal line. Instead, with full rebalancing, excess equity returns would have no impact on equity allocation, marked off on the x-axis. The curvature dividing the free float and rebalancing areas reflects the actual rebalancing behavior of Dutch pension funds. We observe that rebalancing behavior depends on both the sign and size of excess equity returns. Positive equity returns (up to 10%) are not rebalanced at all, but the degree of rebalancing increases with the size of excess equity returns. Instead, negative returns are rebalanced half, although the degree of rebalancing decreases somewhat with the size of negative excess returns.

3.4 Excess stock market returns and medium-term changes in strategic equity allocation

The previous two sections described the effects of excess equity returns on actual equity allocation. This section investigates the impact of annual excess stock mar-

coincide with the reported current strategic asset allocation.

	Compulsory industry wide (1)		Non-compulsory industry wide (2)		Company pension funds (3)		Professional group pension funds (4)	
Positive excess equity returns	1.164	***	1.200	***	0.821	***	0.719	***
Negative excess equity returns	0.507	***	0.348	***	0.515	***	0.439	
Change in policy (t-1)	−0.017		0.028	*	0.072	***	0.091	
Intercept	−0.005	**	−0.008		0.005	**	−0.001	
Number of observations	1,832		380		9,505		240	
R^2, adjusted	0.29		0.31		0.18		0.18	

Table 3.7: Estimates of the equity allocation model per pension fund type: rebalancing versus free floating (1999:II – 2006:IV): the standard errors have been corrected for possible heteroskedasticity or lack of normality using the Huber-White sandwich estimators.

ket performance on pension funds' strategic equity allocation (*Policy*). Therefore, we estimate the following equation

$$Policy_{i,t} = \alpha_3 + \beta_3 \left(ar^E_{i,t} - ar^T_{i,t} \right) + \gamma_3 Policy_{i,t-1} + \delta_3 Size_{i,t-1} + \varepsilon_{i,t}. \quad (3.5)$$

The excess stock market performance has been taken on an annual basis, indicated by $(ar^E - ar^T)$, where ar refers to annual return. We assume that the pension fund trustees base their policy on longer-term measures of performance, as also reflected by the empirical results. Annual returns provide better results than quarterly returns. As above, *Size* controls for the tendency of larger pension funds to invest relatively more in equity portfolios. We also include a lag of the dependent variable *Policy*, as we expect only gradual changes in policy over time. Hence, the equation describes the quarterly adjustments in policy.[9]

3.4.1 Empirical results of the impact of stock market returns on strategic equity allocation

Table (3.8) shows the impact of excess stock market returns on strategic equity allocation. The investment policy is adjusted significantly in response to changes in equity returns, irrespective of whether they are measured by the MSCI or by the actual investment returns earned by pension funds. This shows that investment policy is not constant over time but, to some extent, follows market developments. The coefficient of the lagged dependent variable, 0.97, indicates that slowly the strategic equity allocation reacts only to a small extend to changes in the quarterly returns. On average, 97 percent of the equity investment policy is determined by the previous quarter's investment policy, whereas market developments account for the remaining 3 percent. These market developments, captured by the yearly excess return, have a small but very significant impact, both based on the MSCI and on the actual equity return of the pension fund. Their final impact on equity investment policy over time is 0.25 (= 0.007 / (1 – 0.972)). The size effect is also small but significant. The results across pension fund size classes are quite similar. While this equation shows how investment policy is influenced by market

[9] An alternative model, with first differences of Policy as the dependent variable, instead of gradual adjustment, leads to similar estimation results (not reported here).

developments, it does not provide a model of the underlying investment policy decisions, which are generally based on asset liability management studies.

3.5 The impact of market timing on pension fund returns

Sections 3.3 and 3.4 show that both actual and strategic asset allocation of pension funds are influenced by the relative performance of equity markets. Here we investigate whether the variations of actual and strategic asset allocation over time have generated excess returns. As investment opportunities change over time, deviations in expected returns from their long-term averages may warrant changes in the investment mix. Choosing actual portfolio weights that deviate from the strategic asset allocation is known as 'tactical asset allocation' or 'market timing'. Market timing refers to taking short-term (informed) bets on expected relative asset class returns. It can be implemented through actually buying and selling the underlying securities, although in practice, derivatives are also commonly used as an efficient manner to change a pension fund's asset allocation. However, the potential extra return through market timing is limited, as indicated by the so-called fundamental law of active management, see Grinold and Kahn (1999). This law is formulated as follows

$$IR \approx IC\sqrt{breadth}. \tag{3.6}$$

This law states that the information ratio (IR) approximately equals the information coefficient (IC) times the square root of the number of independent investment decisions (*breadth*). The information ratio is the risk-adjusted excess return over a passive investment strategy. An information ratio of 0.5, which is considered high, requires that asset managers earn a 50 basis points excess return ('alpha') per 1 percent of residual risk on a yearly basis. The information coefficient measures the skill of the asset manager, and is defined as the correlation between his forecasts on investment returns and the actual outcomes.

Now turn to the case of market timing. If an asset manager makes quarterly market timing decisions, the number of independent investment decisions is 4. To generate a market timing information ratio of 0.5 requires, in that case, an

	All funds				**Small funds**		**Medium funds**		**Large funds**	
	(1)		**(2)**		**(3)**		**(4)**		**(5)**	
Equity investment policy (t-1)	0.972	***	0.972	***	0.974	***	0.966	***	0.977	***
Yearly excess return MSCI	0.007	***			0.008	***	0.006	***	0.007	**
Yearly excess equity return[a]			0.007	***						
Size (t-1)	0.001	***	0.001	***	0.001	**	0.001		0.001	
Intercept	0.001		0.003	**	0.003		0.006		−0.003	
Number of observations	16,156		11,273		9,998		4,775		1,383	
R^2, adjusted	0.954		0.954		0.949		0.941		0.954	

Table 3.8: Estimates of the strategic equity allocation model (1999:II – 2006:IV): ***, **, and * denote significance at the 1%, 5%, and 10% significance levels, respectively. The standard errors have been corrected for heteroskedasticity using the Huber-White sandwich estimators. [a]Excess return reported by pension funds.

information coefficient of 0.25, which is considered extremely high. To see this a simplifying assumption is made. Suppose the asset manager only cares about the direction of the market, up or down, which can be described as $x = \pm 1$. The asset manager's forecast is equivalently given by $y = \pm 1$. Both x and y have mean 0 and standard deviation 1. If the asset managers makes N timing decisions, the covariance between $x(t)$ and $y(t)$ is

$$Cov\left[x_t, y_t\right] = \frac{1}{N}\sum_{t=1}^{N} x_t y_t. \tag{3.7}$$

Since the standard deviation of x and y is 1, this formula also represents the correlation. If the asset managers correctly forecasts the direction N_1 times ($x = y$), by definition the incorrect forecasts equals $N - N_1$ times ($x = -y$). So that the correlation between forecast and realization, or the information coefficient, equals

$$IC = \frac{1}{N}\left[N_1 - (N - N_1)\right] = \frac{2N_1}{N} - 1. \tag{3.8}$$

This shows that it would require the asset manager to forecast the direction of the stock market correctly 63 out of 100 times and adjust his portfolio likewise to achieve an information coefficient of 0.25 (since $2 * 63/100 - 1 \approx 0.25$). This is unlikely to be accomplished in a highly efficient market.[10] Although this is of course a simplification of the real world, the intuition behind the fundamental law of active management is that the potential added value of market timing is limited.[11] In addition, such a timing strategy would involve (substantial) transaction costs.

3.5.1 Empirical results of market timing

Although difficult, pension fund investment managers may profit from market timing in their decisions on the actual equity allocation, provided they have some ability in forecasting stock market trends. To earn higher risk-adjusted portfolio returns, skilled investors can create a positive information ratio through increasing equity allocation before the start of a bull market and conversely, decreasing them

[10] Furthermore, inevitable transactions costs will have a negative effect on the net performance.
[11] The law depends on the strong assumption of a diagonal covariance matrix of security returns. This assumption is challenged by De Silva, Thorley and Clarke (2006).

ahead of a bear market. Similarly, pension fund may gain from market timing in their decisions on the strategic equity allocation. This section examines whether pension funds indeed have profited from market timing during the sample period. We use the following three equations to split excess returns that can be attributed to market timing (ER_{MT}) into three sources[12]

$$ER_{MT}(1) = \sum_{t=1}^{T} \left(Policy_{i,t} - \overline{Policy_i}\right) \left(r_t^E - r_{i,t}^T\right) / T, \tag{3.9}$$

$$ER_{MT}(2) = \sum_{t=1}^{T} \left(w_{i,t} - \overline{w}_i\right) \left(r_t^E - r_{i,t}^T\right) / T, \tag{3.10}$$

$$ER_{MT}(3) = \sum_{t=1}^{T} \left(w_{i,t} - Policy_i\right) \left(r_t^E - r_{i,t}^T\right) / T. \tag{3.11}$$

By approximation (3.11) equals (3.10) minus (3.9) as, for each pension fund, the average $Policy_{i,t}$ is in line with the average $w_{i,t}$. The variable $(r^E - r^T)$ equals the quarterly excess return of pension fund i at time t as defined before. *Policy* and w are again, respectively, the strategic and actual equity weights in the asset portfolio. Equation (3.9) estimates the average excess return from varying the strategic equity allocation over time, Equation (3.10) measures the added value of varying the actual equity allocation over time[13], and Equation (3.11) determines the extra return from allowing the actual equity allocation to differ from the strategic equity allocation. The equations estimate the average quarterly return that would have been realized by applying a market timing strategy to investments in broad market indices.[14] Under the null hypothesis that the portfolio manager has no ability in forecasting expected stock market returns, the excess equity returns are uncorrelated to over- or underweighting of equity allocations relative to their mean and excess equity returns would be close to zero.

[12] Equations (3.9) and (3.10) are adapted from Grinblatt and Titman (1993). The performance measure proposed by Grinblatt and Titman (1993) is different from ours in two ways. First, they compare current portfolio weights to portfolio weights in the previous period instead of average portfolio weights. Second, they do not specifically focus on market timing but instead study whether active stock picking generated positive risk-adjusted returns.

[13] The bars above *Policy* and w indicate the respective averages over t (time).

[14] We calculate the average returns in three steps. For each pension fund, we first calculate the average of its log returns over time. Next, we convert these averages to simple returns. Finally, we calculate the average of these simple returns across pension funds. This procedure avoids a distortion in calculating average returns resulting from the fact that the log of an average is not equal to the average of logs.

Market timing measure	Average absolute weight	Average stdev of weights over time	Average excess equity return
(1) Varying strategic equity allocation over time			
a) Small funds	3.1	3.9	−0.05
b) Medium sized funds	3.1	3.9	−0.06
c) Large funds	3.1	3.8	−0.06
d) Full sample	3.1	3.8	−0.06
(2) Varying actual equity allocation over time			
a) Small funds	3.5	4.4	−0.07
b) Medium sized funds	3.6	4.5	−0.07
c) Large funds	3.2	4.0	−0.05
d) Full sample	3.2	4.1	−0.05
(3) Deviating actual from strategic equity allocation			
a) Small funds	2.5	3.2	−0.03
b) Medium sized funds	2.9	3.8	0.00
c) Large funds	2.2	2.9	0.01
d) Full sample	2.3	3.0	0.01

Table 3.9: The average impact of market timing on returns (1999:II – 2006:IV; in %): All statistics are averages weighted by total investments. The average absolute weights are calculated for the different measures as follows: (1) the average of the absolute deviation between the strategic equity allocation and the average strategic allocation calculated over time, (2) the same for actual equity allocation, and (3) the average of the absolute deviation between actual and strategic equity allocation.

Table (3.9) presents the estimation results of Equations (3.9) to (3.10). The first column shows that the average absolute weight, on which excess returns from market timing can be earned, is small. However, as the column in the middle indicates, the variation of the equity allocation is significant.[15] The last column presents both the variation of the strategic and actual equity allocation over time and shows that the average negative excess return is no less than between 5 and 7 basis points per euro invested per quarter, that is, 20-28 bps annually. In contrast, the effect from deviating actual from strategic equity allocation on excess returns has been close to zero. Note, however, that none of the results are significantly different from zero. Differences across size categories appear to be small.

[15] Significance is based on the annualized standard deviation of the calculated ‘excess returns’, which is around 0.9%, well above the ‘average excess returns’ in Table 3.9.

The costs of market timing are not fully internalized into the figures presented. The inclusion of transaction and operational costs would result in even more negative excess returns from market timing. Overall, these results show that for the average pension fund, market timing led to negative, non-significant excess returns during the sample period. Supporting the idea that timing is difficult in highly efficient markets.

3.6 Conclusions

This chapter finds that stock market performance influences the asset allocation of pension funds in two ways. In the short term, the outperformance of equities over bonds and other investment categories automatically results in higher equity allocation (and vice versa), as pension funds do not continuously rebalance their asset allocation. Each quarter, pension funds rebalance, on average, around 39 percent of excess equity returns. The remaining 61 percent leads to higher or lower equity allocation as a result of free floating, which are further rebalanced in subsequent quarters. In the medium term, outperformance of equities induces pension funds to increase their strategic equity allocation (and vice versa). Overall, our estimates indicate that the investment policy of pension funds is partially driven by the (cyclical) performance of the stock market. Apparently, pension funds suffer from myopic investment behavior: they tend to base investment decisions on recent stock market performance, rather than on long term trends.

We also find that pension funds react asymmetrically to stock market shocks. Equity reallocation is higher after underperformance of equity investments then after outperformance. In particular, only 13 percent of positive excess equity returns is rebalanced, while 49 percent of negative shocks results in rebalancing. The former can be indicated as a 'buy on the dip' strategy and the latter as a 'the trend is your friend' approach. Thus, pension funds limit any decline in equity allocation in response to underperformance but they allow higher exposures to equities when these outperform other investments. Apparently, equity portfolio managers have more funds available for investment, when they gain excess returns.

Large funds' investment behavior is different from that of small funds. They invest more in equity and their equity allocation is affected much more strongly by actual equity returns. The latter implies that large funds rebalance less, possibly because managers enjoy more freedom in implementing market timing strategies. We find asymmetric effects on excess equity returns, where the positive effects increase significantly with pension fund size. The coefficient of positive returns of the largest funds is, in fact, significantly above 1, reflecting ‘overshooting’ of free floating, or ‘positive feedback trading’. A possible explanation is that managers of large funds have more freedom to respond to market developments and, particularly in bull markets, demonstrate great risk tolerance. Finally, in line with most empirical evidence, we find that the market timing strategy of Dutch pension funds does not generate excess returns, indicating that Dutch pension fund managers are unable to predict market movements. Whether or not rebalancing behavior supports stable price development on financial markets is not tested and is left for further research.

Chapter 4

Pension Fund Asset Allocation and Participant Age: a Test of the Life-Cycle Model

This chapter is based on Bikker, Broeders, Hollanders and Ponds (2009)

4.1 Introduction

The aim of this final chapter is to assess whether Dutch pension funds' strategic investment policies depend on the age of their participants. The strategic investment policy reflects the objectives of pension funds, while the actual asset allocation may depart from the objective as a result of asset price shocks, since pension funds do not continuously rebalance their portfolios (see Chapter 3 in this thesis). In this chapter, we focus particularly on the strategic allocation of equities and bonds as representing, respectively, risky and safe assets. The argument for age-dependent equity allocation stems from optimal life-cycle saving and investing models (see, e.g., Bodie *et al.*, 1992; Campbell and Viceira, 2002; Cocco *et al.*, 2005; Ibbotson *et al.*, 2007). An important outcome of these models is that the proportion of financial assets invested in equity should decrease over the life-cycle, thereby increasing the proportion of the relatively safer bonds. The key argument is that young workers have more human capital than older workers. As long as the correlation between the return on human capital and stock market returns is low, a young worker may better diversify away equity risk with their large holding of human capital.[1]

Dutch pension funds effectively are collective savings arrangements, covering almost the entire population of employees. Pension funds take the characteristics of their participants on board in their strategic investment allocation. We investigate - against the background of the life-cycle saving and investing model – whether more mature pension funds pursue a more conservative investment policy, that is, whether they hold less equity in favor of bonds. An important feature of most Dutch pension funds is that they explicitly base their funding and benefit allocation decisions on intergenerational risk sharing, that is, nominal benefits are guaranteed, indexation is likely and pension premiums are adjusted, the latter two depending on the funding ratio. Effectively, intergenerational risk sharing extends the size of human capital in the risk bearing basis as human capital of current generations is pooled with that of future participants.

For pension funds' strategic asset allocation in 2007, we find that a rise in participants' average age reduces equity holdings significantly, as theory predicts. A cross-sectional increase of active participants' average age by one year appears to lead to a significant and robust drop in strategic equity exposure by around 0.5 percentage point. Considering this, the awareness of the optimal age-equity relationship for pension funds, and its incorporation in the strategic equity allocation, is remarkable. This negative equity-age relationship has been found in other studies as well. For pension funds in Finland, Alestalo and Puttonen (2006) report that a one-year average age increase reduced equity exposure in 2000 by as much as 1.7 percentage points. Likewise, for Switzerland in 2000 and 2002, Gerber and Weber (2007) report a negative relation between equity exposure and both short-term liabilities and age. The effect they find is smaller yet significant, as equity decreases by 0.18 percentage point if the average active participant's age increases by one year. For the US, Lucas and Zeldes (2009) did not observe a significant relationship between the equity share in pension assets and the relative share of active participants.

We also find that this equity-age relationship is not linear: active participants' average age has been incorporated much more strongly in investment behavior

[1] Note that the argument is not based on time diversification but on diversification of financial capital and human capital.

than that of retired and dormant participants. This is in line with the observation that in principle, employers and employees, who dominate pension fund boards, tend to show more interest in active participants. Furthermore, this is supported by the fact that retirees do not longer possess human capital.

The set-up of this chapter is as follows. Section 4.2 highlights the theoretical relationship between the average age of pension fund participants and the share of equity investments, stemming from the life-cycle saving and investing model. Next, Section 4.3 proceeds with a description of important characteristics of pension funds in the Netherlands. Section 4.4 investigates the age-dependency of asset allocation empirically using a unique dataset of 472 Dutch pension funds at end-2007. Section 4.5 presents a number of variants of our model, which act as robustness tests. Section 4.6 concludes.

4.2 The role of equity in pension fund investments

We start with discussing theoretical views on the suitability of equity in pension fund investment and thereafter consider the role of age as one of the determinants of the equity exposure. Two opposing views on optimal asset allocation by pension funds may be distinguished: the long-term investment strategy and the all-bonds strategy. After discussing these views, the life-cycle model is briefly reviewed.

4.2.1 Long-term investment strategy

Starting with the first one, we consider that a pension fund has to meet benefit promises to both current and future retirees. For a typical pension plan in the Netherlands, the duration of accrued benefits is between 15 and 20 years.[2] Campbell and Viceira (2002) argue that the risks of the various asset categories are different for varying time horizons. So, portfolio choices by long-term investors will differ from those of short-term investors. Both short-term and long-term investors benefit from risk diversification across asset classes. As risk is horizon-dependent, long-term investors also benefit from any time diversification within asset classes. Some empirical research suggests that stocks are less risky in the long run due to mean reversion: the annualized standard deviation halves over a 25 year horizon

[2] Amongst others, the duration depends on the interest rate level. For low interest rates the duration is larger. This is known as convexity.

(Campbell and Viceira, 2002; Hoevenaars, 2008). Besides, long-term investors may invest in less liquid assets such as private equity, real estate and infrastructure. Money market instruments are relatively safe for short-term investors, but not for long-term investors because of reinvestment risk, that is, uncertain future short-term interest rates. Apart from the favorable return-risk trade off in the long run, equities may partly hedge increasing wage- or inflation-indexed liabilities, due to the positive long-run correlation between stock returns, on the one hand, and wages and inflation on the other (Lucas and Zeldes, 2006). Amongst others, Bodie (1995) disputes that investment risk diminishes over time. He points out that prices of put options, insuring against a return below the risk-free rate of return, increase both theoretically and empirically with the lengthening of the horizon. This phenomenon also counts for option pricing models that take into account autocorrelation in stock returns (Lo and Wang, 1995).

4.2.2 All-bonds risk management strategy

Under the all-bonds risk management strategy defined benefit pension liabilities are by nature bond-like (Bodie, 1990; Bader and Gold, 2003; Gold, 2008). The value of these liabilities is equal to the value of the replicating portfolio consisting of a – usually indexed – bonds portfolio that matches timing and amount of the guaranteed benefits. Note that, of course, the funding decision does not change the liabilities, that is, the value of the promised benefits. A risky asset mix may have a high expected return, yet this comes with a mismatch risk, which has to be absorbed by one or more of the pension fund's stakeholders. In a perfect market setting, the cost of buying protection against that mismatch risk from the expected equity proceeds will leave the same return as an all-bonds strategy. Therefore, a pension fund cannot add value by changing the asset mix. Assets held in an all bonds strategy are equal in value to those in an all-equity strategy. Moreover a pension fund invests on behalf of the risk bearing stakeholders. In a perfect market, a pension fund can do nothing that individual stakeholders cannot do directly themselves. The best strategy would then be an all-bonds strategy with no mismatch risk at all.

If a pension fund's only purpose were to secure pension promises (at any cost), it would always be fully funded and fully immunized, that is, matched. This is clearly not the case. Cui *et al.* (2009) argue that although a pension fund is a zero sum game in valuation terms, a mismatch strategy might enhance welfare on account of the intergenerational risk sharing argument. The data reveal that, at end-2007, most pension funds in the Netherlands did not hold an all-bonds mix. Pension funds attempt to earn a risk premium on the fund's assets. Therefore, pension funds' balance sheets are exposed to considerable mismatch risk in the hope of earning a risk premium on pension assets. The risk associated with this can be spread across generations.

4.2.3 Introduction to the life-cycle model

The strategic allocation to equities differ between pension funds that can be explained by differences in risk appetite, determined by factors as size, type of industry, funding ratio, maturity, and the like. The degree of maturity can be measured by the average age of the plan participants. This chapter addresses the question what the impact is of participant age on the asset allocation. We put forward that the relationship between equity allocation and average age is negative in line with the life-cycle model.

In the late 1960s, economists developed the first life-cycle models which implied that individuals should optimally maintain constant portfolio weights throughout their lives (Samuelson, 1969; Merton, 1969). A restrictive assumption of these models was that investors are assumed to have no labor income (or human capital). As most investors do in fact have labor income, this assumption is unrealistic. If labor income is included in the portfolio choice model, individuals will optimally change their allocation of financial wealth over their life-cycle. The optimal allocation will therefore also depend on the risk-return characteristics of their labor income and the flexibility in their labor supply. Bodie *et al.* (1992) studied the impact of labor flexibility on investment strategy. They found that investors with safe labor income can invest in riskier assets. The preferred allocation to risky assets should be based, not only on the level of risk aversion, but also on total wealth, being the sum of financial wealth and human capital. As the size of human capital declines with age, the proportion of financial assets invested in equities should also decrease over the life-cycle, in favor of low-risk investments. In addition to higher human capital, the ability of the young to absorb an unexpected shock increases if time increases. This implies a young person with a long horizon can adjust more easily to financial shocks and therefore can invest a larger share of his wealth in risky assets. For an overview of life-cycle literature see Bovenberg *et al.* (2007).

Pension funds have participants in a wide range of ages, from just over 20 to over 100. In models of optimal life-cycle saving and investing, the age of the investor plays a key role. Therefore, the question is whether the average age of participants acts as a determinant of the asset allocation in the greater entity of pension funds. The rationale is that young workers possess more human capital than older workers, where younger workers can diversify investment risk, assuming that human capital is a relatively safe, so bond-like, asset. The age-dependency of human capital results in a negative age-dependency of equity exposure. A basic version of the life-cycle model under power utility and under the stringent assumption of risk-free human capital can be summarized by the following equation for the optimal fraction of stock investment, denoted α_t^*

$$\alpha_t^* = \frac{\mu + \frac{1}{2}\sigma^2}{\gamma\sigma^2}\frac{W_t + H_t}{W_t}. \tag{4.1}$$

Here H is the human capital (the total of current and discounted future wages) of an individual, and W is the person's current financial capital. The risk-premium of the stock market is given by μ, while γ and σ^2 denote, respectively, the individual's constant relative risk aversion and the variance of stock market returns. As can be seen, more human capital leads to higher optimal investment in stocks. The derivation of (4.1) is provided for in Appendix 4.1.

Not only do young workers have more human capital, they also have more flexibility to vary their labor supply – that is, to adjust the number of working hours or their retirement date – in the face of adverse financial shocks. They can also increase human capital through education. Flexible labor supply acts as a form of self-insurance for low investment returns. Bodie *et al.* (1992) show that this reinforces the optimal result, i.e., that young workers should have more equity exposure. Teulings and De Vries (2006) calculate that young workers should even go short in bonds equal to no less than 5.5 times their annual salary in order to invest in stock.[3] The negative age-dependency of asset holdings corresponds to the rule of thumb that an individual should invest $(100 - age)\%$ in stocks (see Malkiel, 2007).[4]

[3] A variant of this approach is to buy a house financed by a mortgage loan, as happens much more frequently. Though, this is not a well-diversified portfolio.

[4] Before the bull market of the nineties, the rule of thumb was claimed to be $(80 - age)\%$.

The negative relationship between age and equity exposure in the portfolio is usually derived under the assumption that human capital is close to risk-free, or at least is not correlated with capital return. Benzoni *et al.* (2007) put forward that in the short run, this correlation is indeed low while in the longer run, labor income and capital income are highly cointegrated, since the shares of wages and profits in national income are almost constant. This finding implies that the risk profile of young workers' labor income is equity-like and that they should therefore hold their financial wealth in the form of safe bonds to offset the high risk exposure in their human capital. Therefore, Benzoni *et al.* (2007) suggest that the optimal equity share in financial assets is hump-shaped over the life-cycle: cointegration between human capital and stock returns dominates in the first part of working life, whereas human capital becomes more bond like close to retirement. This, combined with a decline in human capital, accounts for the negative age-dependency of optimal equity holdings later in life.

All in all, economic theory suggests a negative relationship between participants' age and pension fund's equity exposure, although this relationship might be reversed in the early working years. It is generally accepted that human capital is less risky than equities, but not entirely risk-free. Palacios (2009) suggests that human capital can be replicated by a portfolio that, on average, consists of 65% invested in risk-free bonds and 35% in equities. Furthermore, Palacios (2009) estimates the weight of human capital in the overall wealth portfolio to be close to 85% on average, although in the literature often 70% is mentioned. Both the magnitude and risk of human capital are important in optimal portfolio choice.

4.3 Characteristics of Dutch pension funds

As in most developed countries, the institutional structure of the pension system in the Netherlands is organized as a three-pillar system. The first pillar comprises the public pension scheme financed on a pay-as-you-go base. It offers a basic flat-rate pension to all retirees. The benefit level is linked to the legal minimum wage. The second pillar provides retired workers with additional income from the supplementary scheme. The third pillar comprises tax-deferred personal savings, which individuals undertake on their own initiative. The Dutch pension system is unique as it combines a state run pay-as-you-go scheme in the first pillar with funded occupational plans in the second pillar. The first pillar implies that a young individual cedes part of its human capital to elder generations, in exchange for a claim on part of the human capital of future generations. Given the life-cycle hypothesis, this type of intergenerational risk sharing enforces the preference of younger people to invest in equity. The intuition is that the risk sharing arrangement in the first pillar allows for more risk taking in the second pillar (Heeringa, 2008). For that reason, we might expect a stronger age effect on equity exposure for Dutch pension funds.

The supplementary or occupational pension system in the Netherlands is organized mainly as a funded defined-benefit (DB) plan. The benefit entitlement is determined by years of service and a reference wage, which may be final pay or the average wage over the years of service. The defined-benefit formula takes into account the retirement benefit of the public scheme. The DB pension funds explicitly base their funding and benefits on intergenerational risk sharing (Ponds and Van Riel, 2009). Shocks leading to either a higher or lower funding ratio are smoothed over time, using the long-term nature of pension funds. Pension funds typically adjust contributions and indexation of accrued benefits as instruments to restore the funding ratio. Higher contributions weigh on active participants whereas lower indexation hurts older participants most.[5] The less flexible these instruments are, the longer it takes to adjust the funding level, and the more strongly will shocks be shared with future (active) participants. Effectively, intergenerational risk sharing extends the risk bearing basis in terms of human capital. The literature on optimal intergenerational risk sharing rules in pension funding concludes that intergenerational risk sharing within pension funds generally should lead to more risk taking by pension funds compared to individual pension plans (see, e.g., Gollier, 2008; Cui *et al.*, 2009). Thus Dutch pension funds, with their high call on intergenerational risk sharing, may be expected to invest relatively heavily in risky assets.

There are three types of pension funds in the Netherlands. The first is the industry-wide pension fund, organized for a specific sector of industry (e.g., construction, health care, transport). Participation in an industry-wide pension fund is mandatory for all firms operating in the sector. A corporate can opt out only

[5] In an average wage defined benefit scheme, the accrued pension rights of the active members are often also subject to conditional indexation. See Chapter 1 and 2 of this thesis.

if it establishes a corporate pension fund that offers a better pension plan to its employees than the industry-wide fund. Where a supplementary scheme exists, either as a corporate pension fund or as an industry-wide pension fund, participation by the workers is mandatory and governed by collective labor agreements. The third type of pension fund is the professional group pension fund, organized for a specific group of professionals such as physicians or notaries.

The Dutch pension fund system is massive, covering 94% of the active labor force. But whereas all employees are covered, the self-employed need to arrange their own retirement plans. As reported by Table (4.1), the value of assets under management at the end of 2007 amounted to € 637 billion, or 125% of Dutch gross domestic product (GDP). More than 85% of all pension funds are of the corporate pension fund type. Of the remaining 15%, most are industry-wide funds, besides a small number of professional group funds. The circa 95 industry-wide pension funds are the dominant players, in terms of their relative share in total active participants (> 85%) and in assets under management (> 70%). Almost 600 corporate pension funds encompass over a quarter of the remaining assets, serving 12% of plan participants. Professional group pension funds are mostly very small funds.

Pension fund type	**Number of pension funds** (%)	**AUM**^{b} (%)	**Active members** (%)	**DB**^{a} (%)	**DC**^{a} (%)
Corporate	85	27	12	90	10
Industry wide	13	71	87	96	4
Professional group	2	3	1	83	17
	(absolute numbers)				
Total	713	€ 684 bln	5,559,677		

Table 4.1: Pension funds in the Netherlands (end 2007): ^{a}Figures as per begin-2006, ^{b}Assets under management, source: De Nederlandsche Bank (DNB).

In the post-WW2 period, pension plans in the Netherlands were typically structured as final-pay defined benefit plans with (de facto) unconditional indexation. After the turn of the century, pension funds in the Netherlands, the US and the UK suffered a fall in funding ratios. In order to improve their solvency risk management, many pension funds switched from the final-pay plan structure

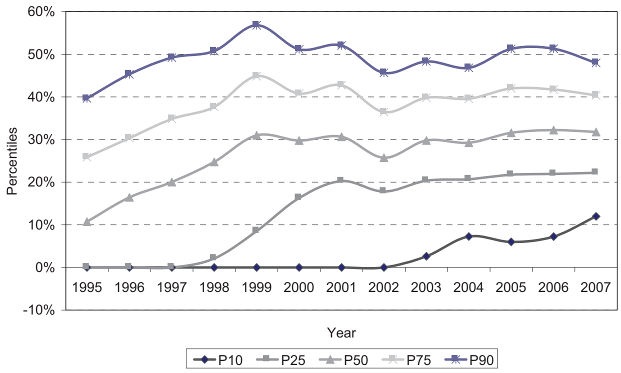

Figure 4.1: Development of equity exposure across Dutch pension funds, source: De Nederlandsche Bank.

to average-pay plans with conditional indexation. In many cases, indexation is ruled by a so-called policy ladder, with indexation and contribution tied one-to-one to the funding ratio (Ponds and Van Riel, 2009). Under an average-pay plan, a pension fund is able to control its solvency position by changing the indexation rate.

Figure (4.1) documents that Dutch pension funds increased their exposure to equities over time. Between 1995 and 2007 the median equity exposure tripled from 10.8% to 31.8%. This increase occurred mainly in the nineties. This increase over time is a combined effect of more pension funds choosing a positive equity exposure (see $P10$ and $P25$ indicating, respectively, the 10th and 25th percentile), and pension funds increasing their exposure.

4.4 Empirical results

4.4.1 Description of the data

Our dataset provides information on pension fund investments and other characteristics for the year 2007. The figures are from supervisory reports to De Nederlandsche Bank, the pension funds' prudential supervisor. Pension funds in the process of liquidation – that is, about to merge with another pension fund or to reinsure their liabilities with an insurer – are exempt from reporting to DNB. The original dataset covers 569 pension funds, of which 472 (or 83%) invest on behalf of the pension fund beneficiaries. As fully reinsured pension funds do not control the investments themselves, we excluded them from the analysis. Nineteen pension funds do not report the average age of their participants and 54 do not report their strategic asset allocation. Three pension funds with funding ratios higher than 250% were disregarded, as these are special vehicles designed to shelter savings from taxes and not representative of the pension fund population in which we are interested. Another three pension funds with assets worth over one million euros per participant were excluded for the same reason, as these are typically special funds serving a small number of company board members. These observations as well as fifteen pension funds, where one or more explanatory model variables were missing, were omitted from the regressions, so that our analysis is based on the remaining 378 pension funds, including all large pension funds.

Variable	**Mean**	**Percentile**		**Weigthed**
		10th	**90th**	**mean**^{b}
Average age of active participants	45.2	39.9	50.1	43.1
Average age of all participants	50.2	41.7	59.6	47.9
Strategic equity exposure (in %)	32.9	16.4	46.4	37.8
Actual equity allocation (in %)	33.2	17.6	46.9	37.6
Average assets of participants^c	81.2	11.7	155.4	42.3
Share of retired (in %)	20.9	4.0	41.5	15.6
Share of dormant members (in %)	42.3	23.3	65.7	50.5
Share of active participants (in %)	36.8	15.3	59.8	33.9
Funding ratio (in %)	139.4	120.2	163.9	142.3
Total assets (in € millions)	$1,791$	20.3	$2,153$	$55,400$
Total number of participants^d	42.3	0.4	43.3	$1,099$
Defined benefit schemes (in %)	0.97	1	1	1.00
Defined contribution schemes (in %)	0.03	0	0	0.00
Industry-wide^e (in %)	0.20	0	1	0.89
Corporate^e (in %)	0.78	0	1	0.11
Professional group^e (in %)	0.02	0	0	0.00

Table 4.2: Descriptive statistics of the dataset including 378 pension funds^a. ^{a}That is, the minimum number of pension funds included in the various regression analyses; ^{b}Weighted with the number of participants per pension funds, as in the weighted regressions, ^{c}in € 1,000s, ^{d}in thousands, ^{e}pension fund type. Source: DNB, own calculations.

Table (4.2) provides descriptive statistics of the dataset, with age and strategic equity allocation as key variables. One measure of age is the average age of all participants in the pension fund, including active participants, dormant members and retirees and equals 50, ranging widely across pension funds between 42 and 60 based on the 10th and 90th percentile, respectively. An alternative definition of age is the average age of active participants, which equals 45, varying across pension funds from 40 to 50. The shares of retired and dormant participants also vary strongly across pension funds, reflecting the various positions these pension funds occupy in the life-cycle or the dynamic development of their industry or their sponsor firm. The share of equity in the strategic asset allocation averages 32.9%, ranging across pension funds from 16% to 46%. The actual equity allocation differs from the strategic asset allocation due to free-floating and market timing, and appears to average 33.2%. Furthermore, Table (4.2) presents statistics on other pension fund characteristics, many of which act as control variables in the regression (see below). The 10% and 90% percentiles reveal that these characteristics tend to vary strongly. The right-hand column shows the mean values, weighted with the number of participants. For instance, larger funds tend to invest more heavily in the stock market than smaller ones, so that the percentage of all pension assets invested in equities equals 38%, against 33% for the average pension fund. Finally, the total assets and number of participants statistics explain that a small number of large pension funds dominate the pension market in terms of both total assets and number of participants.

4.4.2 Model specification

Most life-cycle theories suggest that the relationship between age and equity allocation is linear (Equation (4.1); see also Malkiel, 2007), while others postulate a non-linear or hump-shaped relationship (Benzoni *et al.*, 2007). Lucas and Zeldes (2009) investigate the relationship between the relative share of active participants and the equity allocation, also assuming a non-linear age pattern: a (constant) effect during the active years compared to the retirement years. Gerber and Weber (2007) regarded two definitions of average age: age of all participants and age of active participants, where the latter implies a non-linear functional form of age, due to the truncation at the retirement age.[6] Taking the various specifications in the literature into account, we focus on a non-linear specification. Our principal non-linear age-dependent model for the strategic equity allocation of pension

[6] Alestalo and Puttonen (2006) had data available on active participants only.

funds reads as

$$\begin{aligned} SAE_i &= \alpha + \beta_1 ageactive_i + \beta_2 shareretired_i + \beta_3 sharedormants_i \\ &+\gamma \log(size_i) + \delta fundingratio_i + \lambda log(accrual_i) \\ &+\zeta DB_i + \eta PGPF_i + \theta IPF_i + \varepsilon_i \end{aligned} \quad (4.2)$$

where *ageactive* represents the average age of pension fund's i active participants.[7] Following Lucas and Zeldes (2009) we include ratios of retired participants and dormant members (*shareretired* and *sharedormant*, respectively) to incorporate possible further non-linear effects of age on the equity allocation. This equation allows testing whether pension fund boards, populated by employers and employees, show equal versus more interest in active participants compared to dormant members and retirees.[8] A control variable size is included as larger pension funds tend to invest more in equity (see also Chapter 3 of this thesis). One argument may be that pension fund size will go hand in hand with degree of professionalism, investment expertise and willingness to exploit return-risk optimization. The pension fund's size is defined as its total number of participants, where we take logarithms of size to reduce possible heteroskedasticity. The funding ratio is a determinant of equity allocation as a higher funding ratio may stimulate higher risk taking as its provides a larger buffer against equity risk. A higher risk margin for equity is required under the Dutch supervisory regime (Bikker and Vlaar, 2007). Note that – unlike the actual equity allocation – the strategic equity allocation is not affected directly by shocks in the stock market, although gradually, over time, the strategic equity allocation may be influenced somewhat by trends in the stock market (see Chapter 3 of this thesis). Another explanatory variable is the average accrued wealth of the participants in a pension fund, defined as the total pension accrual per plan participant. This variable reflects, on average, the generosity of the pension plan.[9] Our hypothesis here is that generous

[7] Table 4.6 in appendix 4.7.1 provides the OLS estimates of a general model with average age, share and interaction terms as explanatory variables.

[8] Concerning the impact of age on asset allocation, we cannot distinguish between the life-cycle effect, on the one hand, and age dependent risk aversion, on the other hand. However, as the equity allocation is determined by the pension fund board, the life-cycle effect is more likely to dominate than the risk aversion of the elderly who are not represented in the board.

[9] This interpretation assumes a similar average duration of the participants' relationship with

pension schemes may go together with relatively higher equity allocations, in a manner similar to the behavior of private persons who, on average, invest more in equity the larger their savings are. We take logarithms of this variable to reduce possible heteroskedasticity. A set of dummy variables may reflect different behavior patterns related to different types of pension plan (DB versus DC) or pension fund (professional group pension funds ($PGPF$) and industry-wide pension funds (IPF) versus corporate pension funds).[10] Finally, ε_i denotes the error term.

4.4.3 Regression results

Figure (4.2) provides a preview of the findings and Table (4.3) reports estimation results of Equation (4.2), our key model. The age coefficient of the average age of active participants is significant and equals -0.44 (when unweighted; left-hand column), pointing to a negative relation between age and equity allocation. A one year higher average age is associated with a 0.44 percentage point lower equity exposure.[11] Unweighted estimation attaches equal informational value to each observation of a pension fund, irrespective of whether it has ten participants or 2.5 million. By contrast, weighted regression attributes similar importance to each participant, weighting pension funds proportionally according to size. Such a weighting regression would yield results which are more in line with economic reality. Dropping the largest two pension funds from the unweighted sample would not noticeably affect the regression results (representing less than 1% of the number of observations; result not shown here), whereas they include no less than 30% of participants.

Therefore, the right-hand column of Table (4.3) presents a weighted regression using the number of participants as weight. The rise in the adjusted R^2 from 0.21 (unweighted) to 0.52 (weighted) reveals that the variation in equity allocation is better explained by the larger pension funds, than by the smaller ones, confirming that weighting makes more sense economically. The age coefficient is both larger

the pension fund across pension funds, that is, the sum of the endured employment contract and the endured retirement period.

[10] Willingness of the sponsor company to compensate investment losses could be a relevant explanatory variable also. In practice however, we hardly observe this willingness, except for a few corporate pension funds. Industry wide pension funds service multiple corporations and it is unlikely that losses can be fairly distributed amongst those corporations.

[11] If we regress on the average age of *all* participants the unweighted coefficient equals -0.17, the squareroot weighted coefficient -0.18 and the full weighted coefficient -0.38. All three are significant at the 1% level.

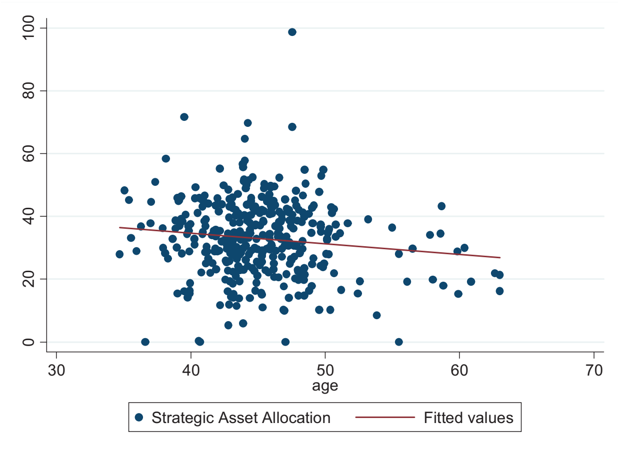

Figure 4.2: Strategic equity allocation and average age active participants (2007).

(at -0.56) and statistically more significant (at a t-value of 6.2). Apparently, the investment behavior of the larger pension funds is based more strongly on the age-related life-cycle argument. Weighting with the square root of the number of participants as weight takes an intermediate position for almost all coefficients. The unweigthed model represents the results for the average pension fund. The full weighted model shows the coefficients that are representative for the average participant. Our results are similar in direction but not in size to the findings of Gerber and Weber (2007, for Switzerland) and Alestalo and Puttonen (2006, for Finland), who find 'active-age' coefficients of, respectively, -0.18% and -1.73%.

The impact of retirees and dormant members is limited if not absent.[12] Only in the full weighting estimates of Table (4.3) we find a small reduction of the equity share for pension funds with relatively more retirees or dormant members. For retirees this is an intuitive result as they no longer possess human capital, so that the return on human capital is negligible also. One percentage point more retirees implies a 0.12 percentage point reduction in the equity allocation, while one percentage point more dormant members implies a 0.17 percentage point reduction in the equity allocation. Note that the signs of these variables are both in line with theory.[13] The absence of these effects in the unweighted or limited

[12]Table (4.5) in Appendix 4.7.2 reports the coefficients of a full regression model in which the average age of active participants, retirees and dormant members are taken separately. Apparently the age of retirees and dormant members have an offsetting impact on equity allocations.

[13]It is plausible that dormants are on average older than active participants because they already have a work history.

Variable	Unweighted		SQRT weighting		Full weighting	
	coeff.	t-value	coeff.	t-value	coeff.	t-value
Average age of active participants	−0.44	−2.88	−0.52	−4.28	−0.56	−6.20
Share of retired participants	0.04	0.89	0.01	0.14	−0.12	−2.60
Share of dormant members	0.09	2.09	0.03	0.62	−0.17	−4.73
Log of total number of participants	1.07	2.79	1.07	3.90	0.78	2.98
Funding ratio	0.20	6.89	0.23	8.41	0.27	9.46
Log of pension accrual	4.03	5.21	3.49	4.99	2.23	3.74
Dummy Defined benefit plans	0.37	0.10	3.80	1.08	6.00	1.27
Dummy Professional group funds	0.56	0.14	1.28	0.33	−0.95	−0.18
Dummy Industry-wide funds	0.37	0.18	−0.12	−0.08	0.89	0.60
Constant	−5.02	−0.51	−3.59	−0.43	9.48	1.13
Adjusted R^2	0.21		0.32		0.52	
Number of observations	378		378		378	

Table 4.3: Ordinary least squares estimates of the impact of the average age of active participants on the strategic equity allocation of pension funds (2007).

weight model variants implies that only the large pension funds incorporate (parts of) the optimal equity allocation associated with non-active participants. This is confirmed when we drop, as a robustness test, the two largest pension funds (30% of all participants): the two dependency ratios drop to near or total insignificance (results not shown here). Remarkably, in that case, the absolute value of the age effect increases further to 0.66. We conclude that while pension funds do incorporate the impact of their active participants' average age on the optimal investment portfolio in their strategic allocation of pension wealth to equities, they pay limited attention to the comparable impact of retirees and dormant members.

4.4.4 Further analysis

Turning to the other determinants of the equity allocation in Table (4.3), we observe that the effect of (the logarithm of) size appears to be positive and sizeable (with values around 1) which tallies with the stylized fact that large pension funds invest more in equity. The marginal effect of size – number of participants – on equity exposure depends on size itself, due to its logarithmic specification. An increase in the number of participants from 10 thousand to 100 thousand is associated with an increase of equity allocation roughly by 2.5 percentage points. One reason may be that larger funds have a more elaborated risk management function, an argument related to economies of scale. Another is that the largest pension funds are of the industry-wide type, which have better abilities to diversify risk over time, that is, over generations. That is particularly true as most of these funds are of the so called mandatory type, that is, corporations in the respective sector are obliged to join. One can also argue that pension funds deliberately take more risks than a pure defined benefit scheme might impose, see de Jong (2008b). This might especially be the case for large pension funds that are too 'political to fail'. We measure size as the total number of participants. The variable total assets would be an alternative size measure but we already included the per capita wealth which together with the total number of participants reflects total wealth. A drawback of total assets might be that this measure cannot safely be regarded as exogenous, because high equity returns would – for pension funds with a high equity allocation – enlarge both their size and their equity exposure. This is the more important given that pension funds do not constantly rebalance their asset portfolios, as shown in the previous chapter. In a robustness check (not shown here), we choose total assets as size measure and use instrumental least squares, but the size coefficient does not change much, and remains significant.[14]

Pension funds with higher funding ratios invest more in equity, because their buffers may absorb mismatch risks. This is also related to regulation, requiring that the probability of underfunding be less than 2.5%, which enables better funded pension funds to take more risks. The coefficient of around 0.25 implies that an increase of the funding ratio by 1% translates into an increase of the equity allocation of one quarter percentage point. Note that the funding ratio suffers less from endogeneity problems, as the dependent variable is the strategic – not the actual – equity allocation. Indeed, the actual equity exposure would be affected as high stock returns simultaneously increase both the funding ratio and the equity allocation (at least under 'free-floating'). Because the strategic equity allocation might nevertheless be adjusted to stock market developments, albeit gradually, we alternatively lag the funding ratio (that is, take 2006 figures) in our robustness analyses, see Section 4.5. As expected, the results turn out hardly different.

The coefficient of (the logarithm of) personal pension accrual consistently equals around 4 and is statistically significant. The marginal effect of an increase in personal wealth depends on its level, due to the logarithmic specification. Starting from the average value of 81 thousand, an increase by one standard deviation of 78 thousand is associated with an increase of stock allocation by 1.5 percentage points. This confirms that pension funds having a higher wealth per participant invest relatively more in equity, thereby accepting more risk, in line with expectations.

None of the dummy variables for types of pension plan or pension fund carry a statistically significant coefficient. Apparently, the incorporated model variables explain the differences in equity allocations so well that no systematic differences remain across types of pension plan or pension fund.

4.5 Robustness checks

The above specification rests on several assumptions regarding relevant covariates, variable definition and functional form. This section considers various departures

[14] Since size measured by total assets is highly correlated with size measured by total participants (0.87), the latter may be considered as a relevant and valid instrumental variable for the former.

from the assumptions underlying this regression.

In line with the literature, we have so far assumed the effect of the average age of active participants on the equity allocation to be linear. However, Benzoni *et al.* (2007) suggest that the relation between age and equity exposure may be hump-shaped rather than linear. They suggest that the age effect is positive in the younger age cohorts, due to the positive long-term correlation between capital returns and return on human capital (that is, the wage rate). Benzoni's age-equity relation reaches a maximum around a certain point (seven years before retirement), after which it is downward-sloping, as the long-term correlation of wages and dividends loses relevance. A simple but effective way to allow for a non-linear relationship is the inclusion of a quadratic age term in the regression, known as a second-order Taylor-series expansion, approximating an unknown, more complex relationship. The respective weighted regression model shows that the signs of both age coefficients are not in line with the assumption of Benzoni *et al.* (2007) about the investment behavior of pension funds (Table 4.4, first column), as the sign of the squared terms has the 'wrong' sign. Hence, we find no support for Benzoni's theory.

With regard to the dependent variable 'strategic equity allocation' several robustness checks may be considered. A small number (4) of pension funds have zero equity exposure. This runs counter to the OLS assumption that the dependent variable is of a continuous nature. In practice, equity exposure is censored at 0% and 100%. One may further argue that moving from zero equity allocation to a positive fraction is an intrinsically different decision than raising an already positive equity exposure. One way to address this is to omit zero observations for equity, restricting attention to funds with positive equity allocations. This does not alter the essence of the results (not shown here). A more elegant alternative approach is the Tobit model which takes this kind of censoring into account. Table (4.4), second column, reports the Tobit outcomes. The effect of age and the other OLS results from Table (4.3) do not change substantially.

Shocks in equity prices affect the funding ratio, but as observed in Section 4.4 they may also have a certain impact on the strategic equity allocation, which could create an endogeneity problem. For this reason, we lag the funding ratio, see the third column in Table (4.4). Although the sample is somewhat smaller, the results hardly change, particularly in terms of significance. The magnitude of

	Strategic equity allocation (incl. square age)		Tobit regression: equity exposure (censored at 0[a])		Strategic equity allocation		Actual asset allocation	
	coeff.	t-value	coeff.	t-value	coeff.	t-value	coeff.	t-value
Average age of active participants	−0.58	−6.34	−0.56	−6.22	−0.38	−2.91	−0.42	−3.61
Ditto, squared[b]	0.01	1.28						
Share of retired participants	−0.12	−2.61	−0.12	−2.62	−0.19	−3.53	−0.21	−4.38
Share of dormant members	−0.17	−4.85	−0.17	−4.75	−0.23	−5.17	−0.23	−5.79
Log of total number of participants	0.78	3.00	0.79	3.03	1.18	4.03	0.88	3.37
Funding ratio	0.27	9.23	0.27	9.50				
Funding ratio, lagged (2006)					0.19	5.72	0.16	5.31
Log of pension accrual	2.35	3.91	2.22	3.76	1.86	2.64	2.37	3.84
Dummy Defined benefit plans	5.88	1.24	5.95	1.26	3.41	0.60	5.10	1.09
Dummy Professional group funds	−1.44	−0.27	−0.91	−0.17	−0.95	−0.18	−14.23	−2.93
Dummy Industry-wide funds	0.91	0.62	0.90	0.62	0.89	0.60	−0.11	−0.07
Constant	10.72	1.28	9.35	1.13	9.48	1.13	24.16	2.49
Adjusted R^2	0.52		0.08[c]		0.43		0.48	
Number of observations	378		378		362		367	

Table 4.4: Alternative specifications of the weighted regression model as robustness tests (2007). [a] There are four censored observations, that is, four observations with zero equity exposure; [b]Expressed as the deviation from the average age of participants, allowing for easier interpretation of the coefficients; [c]This is the pseudo R^2.

the (lagged) funding ratio coefficient is slightly smaller here than in the unlagged regression.

The actual equity exposure of pension funds may differ from the strategic equity allocation where pension funds do not constantly rebalance their portfolio after stock price changes. The previous chapter documents that pension funds' assets are indeed partially free-floating, meaning that their asset allocation is not constantly adjusted. As strategic asset allocation reflects the real decision of the pension fund it is better suited for determining decision-making and behavior of pension funds. On the downside, however, this may affect comparability with other studies, such as Alestalo and Puttonen (2006) and Gerber and Weber (2007). Also, while the strategic asset allocation reflects intention, it does not give actual behavior. Table (4.4), right-hand columns, documents regression results for the actual stock allocation. To avoid endogeneity, we lag the funding ratio by one year. Sign and size of the coefficients hardly change, though the magnitude of the (lagged) funding ratio coefficient is slightly smaller than it is in the other regressions.

We also applied our model to the strategic bond allocation instead of the strategic equity allocation, where we expect a positive and not a negative sign for the age dependency. This is a slightly better approach because the distinction between risk-free and risky investments is made. The results (shown in Table 4.7 in Appendix 4.7.4) deviate as bonds are not exactly the complement of equity, due to other investment categories. These estimates confirm the age-bond relationship: the strategic bond exposure is significantly higher when the average age of active participants is higher.

Finally, we also tested for a possible outlier effect. From Figure (4.2) it might be argued that the negative relation between age and asset allocation is caused by a few outliers. To test this hypothesis we re-estimated (4.2) with the average age of active participants truncated at 50 years. The relationship between age and average equity allocation remains negative in all cases. Furthermore, weighted by size of pension funds the relation also remains significantly negative with a coefficient of -0.46 (t-value -3.31).

4.6 Conclusion

This chapter addresses the effect of the average age of pension funds' participants on their strategic equity allocation. Theory supports a negative relation between age and equity allocation. Our first and key finding is that Dutch pension funds with a higher average age of their participants indeed have statistical significant lower equity exposures than pension funds with younger participants. Although the effect appears only to be minor in economic terms. A one year higher average age reduces the strategic equity weight by 0.5 percentage points. This negative age-dependent equity allocation may be interpreted as an (implicit) application of the optimal life-cycle saving and investing theory. The basic version of this theory assumes a low correlation between wage growth and stock returns. It predicts that the vast amount of human capital of the young have an impact on asset allocation because of risk diversification considerations, as human capital has a different risk profile than financial capital.

A second finding is that we demonstrate that the strategic equity allocation particularly strongly correlates with the average age of the active participants. The available data can not answer the question whether this is due to the governance of pension funds.

A third result is that the age effect is much stronger in larger pension funds than in smaller funds. Apparently, larger funds' investment behavior is more precisely based on the age-dependency from the life-cycle hypothesis. A non-linear age effect allowing a hump-shaped pattern, as suggested by Benzoni *et al.* (2007), was not confirmed. However, other factors significantly influencing the strategic equity allocation are a pension fund's size, funding ratio, and average personal pension accrual of participants, which all have positive coefficients. We do not observe any effect of pension fund type or pension scheme type on funds' equity exposure.

This research provides valuable insights for contemporary policy issues concerning the ageing of society. As society grows older, pension funds will likely adapt their investment strategies to the needs of the average active participant who will get older over time. This may result in a safer investment strategy. According to the life-cycle saving and investing theory, this is not optimal for younger participants with low-risk human capital, who will not be fully able to utilize the diversification between human and financial capital. At the same time, this policy may not be conservative enough for retirees, whose interests are not weighted that heavily by the pension fund boards. One way to deal with this divergent interests is to replace the average age-based policy, as described in this chapter, by a cohort-specific investment policy as has been suggested by Teulings and De Vries (2006) and Ponds (2008).

4.7 Appendix

4.7.1 A simple life-cycle model

In this appendix we derive the simple life-cycle model in (4.1) with risk-free human capital based upon Campbell and Viceira (2002). Key to this approach is the following relation for the expectation of a lognormal random variable X

$$\log E_t X_{t+1} = E_t \log X_{t+1} + \frac{1}{2} Var_t \log X_{t+1} = E_t x_{t+1} + \frac{1}{2}\sigma^2_{x,t}. \tag{4.3}$$

First assume an investor with only financial capital, characterized by power utility over next period wealth W_t and risk aversion parameter γ. The investor maximizes next-period wealth in the usual manner

$$\max \frac{E_t W_{t+1}^{1-\gamma}}{1-\gamma} \tag{4.4}$$

under the budget constraint

$$W_{t+1} = (1 + R_{p,t+1})W_t$$

where $R_{p,t+1}$ represents the simple portfolio return. In log form the budget constraint reads

$$w_{t+1} = r_{p,t+1} + w_t. \tag{4.5}$$

Using (4.3), defining $\log W_t \equiv w_t$ and applying $\log X^a = a \log X$ we can rewrite (4.4) as follows

$$\max \log E_t W_{t+1}^{1-\gamma} = (1-\gamma)E_t w_{t+1} + \frac{1}{2}(1-\gamma)^2 \sigma^2_{w,t}. \tag{4.6}$$

If we divide (4.6) by $(1-\gamma)$ and use the budget restriction in (4.5) this problem can also be stated as

$$\max E_t r_{p,t+1} + \frac{1}{2}(1-\gamma)\sigma^2_{p,t}. \tag{4.7}$$

Next, consider a portfolio build up of two assets, one risky and one risk less. The simple portfolio return $R_{p,t+1}$ is equal to

$$R_{p,t+1} = \alpha_t R_{t+1} + (1 - \alpha_t) R_{f,t+1}$$

where α_t represents the portfolio share invested in the risky asset with return R_{t+1}. Consequently, $1 - \alpha_t$ of the portfolio returns the risk-free rate $R_{f,t+1}$. The variance of the portfolio return equals $\sigma_{p,t}^2 = \alpha_t^2 \sigma_t^2$. The log portfolio return $r_{p,t+1}$ is the log of this linear combination of simple returns. Campbell and Viceira (2002) show that the excess log portfolio return is approximately given by

$$r_{p,t+1} - r_{f,t+1} = \alpha_t \left(r_{t+1} - r_{f,t+1} \right) + \frac{1}{2} \alpha_t (1 - \alpha_t) \sigma_t^2.$$

The optimization problem in (4.7) can now be written as

$$\max_{\alpha_t} \alpha_t \left(E_t r_{t+1} - r_{f,t+1} \right) + \frac{1}{2} \alpha_t (1 - \alpha_t) \sigma_t^2 + \frac{1}{2} (1 - \gamma) \alpha_t^2 \sigma_t^2.$$

Deriving the solution results in the optimal allocation to the risky asset

$$\widehat{\alpha}_t = \frac{\mu + \frac{1}{2}\sigma^2}{\gamma \sigma^2}.$$

where we defined $\mu \equiv E_t r_{t+1} - r_{f,t+1}$. In the final step, we assume the investor has W in financial wealth and H in human capital. The amount to invest in risky assets is therefore $\widehat{\alpha}_t (W_t + H_t)$. The allocation to risky assets as percentage of financial wealth is given by

$$\alpha_t^* = \frac{\widehat{\alpha}_t (W_t + H_t)}{W_t} = \frac{\mu + \frac{1}{2}\sigma^2}{\gamma \sigma^2} \frac{W_t + H_t}{W_t}.$$

Furthermore, since H is an implicit investment in the risk-free asset, the allocation to the risk-free asset itself reads $(1 - \widehat{\alpha}_t)(W_t + H_t) - H_t$.

4.7.2 Full model

Table (4.5) shows the OLS estimation results for a general model specification. It includes average age per participants group and the share per participant group as explanatory variables. Following Brambor, Clark and Golder (2006) all interaction terms are also included as explanatory variables. The control variables are the same as in the regressions presented in Section 4.4. Again this table shows that the average age of active participants is a key explanatory variable.

Variable	**Unweighted**		**SQRT weighting**		**Full weighting**	
	coeff.	**t-value**	**coeff.**	**t-value**	**coeff.**	**t-value**
Interaction age and share, active participants	0.003	0.40	−0.003	−0.33	0.001	0.19
Interaction age and share, retired	−0.008	−0.81	−0.008	−0.69	0.005	0.34
Interaction age and share, dormants	0.004	0.58	0.000	0.05	−0.005	−0.74
Average age of active participants	−0.601	−2.38	−0.58	−2.75	−0.79	−4.02
Average age of retired participants	−0.099	−0.57	−0.16	−0.82	−0.61	−2.71
Average age of dormant members	0.052	0.12	0.46	1.01	1.00	2.28
Share of retired participants	0.78	1.21	0.41	0.57	−0.46	−0.53
Share of dormant members	0.049	0.11	−0.98	−0.23	0.13	0.30
Log of total number of participants	1.00	2.60	1.16	4.16	1.12	4.18
Funding ratio	0.19	6.55	0.23	8.12	0.27	9.41
Log of pension accrual	4.06	4.68	3.57	4.43	2.66	3.83
Dummy Defined benefit plans	1.77	0.49	5.13	1.42	7.08	1.47
Dummy Professional group funds	−2.07	−0.49	−1.74	−0.42	−4.20	−0.74
Dummy Industry-wide funds	0.68	0.32	−0.68	−0.40	−0.27	−0.17
Constant	−8.63	−0.31	−2.16	−0.08	4.63	0.16
Adjusted R^2	0.22		0.33		0.55	
Number of observations	377		377		377	

Table 4.5: Ordinary least squares estimates of the impact of general demographic variables on the strategic equity allocation of pension funds (2007).

4.7.3 Reduced model

The dummy variables for defined benefit pension plans, for professional group pension funds and for industry-wide pension funds do not appear to be significant. Therefore, in this appendix we estimate the results again in Table (4.3) without these dummy variables. The new results that are shown in Table (4.6) are largely consistent with earlier results found. To reduce the any potential endogenity effects, also the funding ratio is introduced with one period lag.

4.7.4 Strategic bond exposure and age

In Table (4.3) only the strategic equity allocation is used as the explanatory variable. It might be interesting to include all non-bond investments in the analysis. Therefore, Table (4.7) shows the comparable results of the impact of average age of active participants on 1 minus the strategic bond allocations. This regression implicitly takes into account all non-bond investments, like real estate, commodities, hedge funds, etc. If follows that the results are comparable, but the average age coefficient is somewhat lower in the square root and full weighting model.

Variable	**Unweighted**		**SQRT weighting**		**Full weighting**	
	coeff.	**t-value**	**coeff.**	**t-value**	**coeff.**	**t-value**
Average age of active participants	−0.43	−3.04	−0.28	−1.94	−0.44	−3.44
Share of retired participants	0.04	0.86	−0.04	−0.70	−0.16	−3.11
Share of dormant members	0.08	1.63	0.03	0.60	−0.21	−4.88
Log of total number of participants	1.09	3.56	1.29	5.70	1.16	5.51
Funding ratio, lagged (2006)	0.16	2.29	0.15	5.33	0.15	5.00
Log of pension accrual	4.02	5.03	4.07	5.88	1.94	2.87
Constant	1.48	0.16	−2.22	−0.23	27.55	2.76
Adjusted R^2	0.19		0.25		0.42	
Number of observations	362		362		362	

Table 4.6: Ordinary least squares estimates of the impact of the average age of active participants on the strategic equity allocation of pension funds (2007).

Variable	Unweighted		SQRT weighting		Full weighting	
	coeff.	t-value	coeff.	t-value	coeff.	t-value
Average age of active participants	−0.46	−2.75	−0.32	−2.40	−0.30	−2.99
Share of retired participants	0.076	1.45	0.003	0.06	−0.13	−2.73
Share of dormants	0.109	2.23	0.052	1.14	−0.016	−0.43
Log of total number of participants	2.51	5.88	2.62	8.67	1.92	6.68
Funding ratio	0.27	8.22	0.27	8.73	0.29	9.11
Log of pension accrual	4.29	5.17	4.71	6.22	4.68	7.26
Dummy Defined benefit plans	0.33	0.09	−0.98	−0.27	−3.49	−0.71
Dummy Professional group funds	−0.37	−0.08	−1.95	−0.46	−3.90	−0.68
Dummy Industry-wide funds	−0.24	−0.10	1.53	0.86	3.57	2.19
Constant	−16.86	−1.57	−21.34	−2.33	−9.95	−1.10
Adjusted R^2	0.32		0.44		0.55	
Number of observations	381		381		381	

Table 4.7: Ordinary least squares estimates of the impact of the average age of active participants on 1 minus the strategic bond allocation of pension funds (2007).

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Summury in Dutch

Dit proefschrift behandelt twee thema's die belangrijk zijn voor een krachtig financieel beleid van pensioenfondsen. Het eerste vraagstuk betreft dat van de waardering van pensioenverplichtingen. Dit staat centraal in deel I. Het tweede onderwerp is het beleggingsbeleid van pensioenfondsen en komt aan bod in deel II. Pensioen is voor veel mensen één van de belangrijkste financiële producten die ze bezitten. De wijze waarop het pensioenfonds de waarde van zijn verplichtingen vaststelt en de manier waarop het pensioengelden belegt, zijn daarmee cruciaal voor het financiële welzijn van pensioendeelnemers.

Hoofdstuk 1 behandelt de waardering van zogenaamde voorwaardelijke pensioenverplichtingen rekening houdend met de kredietwaardigheid van de sponsor. Dit is meteen bijzonder want een pensioenfonds staat juridisch los van de onderneming. In de praktijk is er echter economisch gezien wel een wisselwerking tussen pensioenfonds en onderneming. Het pensioenfonds ontvangt immers premies van de onderneming en als de pensioenbeleggingen onvoldoende renderen is er druk om de pensioenpremie te verhogen. Daar staat tegenover dat wanneer het pensioenfonds heel vermogend is de premie omlaag kan. Een fenomeen dat bekend staat als 'premievakantie'. In feite is er sprake van een risicoarrangement tussen beide partijen dat onder meer samenhangt met het beleggingsbeleid van het pensioenfonds. Bij risicovolle beleggingen is de kans op rijkdom én armoede groot en komt er sneller een extra geldstroom op gang tussen pensioenfonds en onderneming. Belegt het pensioenfonds voorzichtig dan treedt dit effect minder op. Hoewel het pensioenfonds juridisch zelfstandig opereert, is er dus wel degelijk een economische relatie met de onderneming.

Het eerste hoofdstuk werkt uit hoe deze wisselwerking in samenhang met het beleggingsbeleid van het pensioenfonds van invloed is op de waardering van voorwaardelijke pensioenverplichtingen. Het gaat hier om het indexatiebeleid van het pensioenfonds. In de meeste pensioenregelingen is een ambitie opgenomen het pensioen waarde- of welvaartsvast te houden door het jaarlijks te laten meegroeien met het algemene prijspeil of de loonontwikkeling in de betreffende bedrijfstak. Deze koppeling is echter geen garantie. Jaarlijks beoordeelt het pensioenfondsbestuur of er voldoende middelen beschikbaar zijn om de verhoging door te voeren. Veelal speelt de dekkingsgraad van het pensioenfonds hierbij een belangrijke rol. Dit is de verhouding tussen de bezittingen en de verplichtingen van het fonds. Doordat de indexatie afhankelijk is van dit kengetal is de economische waarde van het indexatiestreven te bepalen met behulp van de optiewaarderingsmodellen.

In deze modellen is het risicoprofiel (of 'volatiliteit') van de beleggingen van het pensioenfonds één van de belangrijkste variabelen. De techniek gepresenteerd in het eerste hoofdstuk gaat er daarom vanuit dat het pensioenfondsbestuur een beleggingsbeleid kiest dat de economische waarde van het indexatiebeleid voor de deelnemers maximeert. Het bestuur houdt hierbij simultaan rekening met de kredietwaardigheid van de onderneming die het pensioenfonds sponsort. In een ideale wereld, waarbij de sponsor altijd eventuele tekorten aanvult, hangt het optimale beleggingsbeleid alleen of van de indexatiedoelstelling van het pensioenfonds. In de praktijk is het echter zeer wel mogelijk dat de sponsor niet in staat is of niet bereid is deze tekorten aan te vullen. Dit terwijl de druk op een premievakantie groot is als de pensioenfondsbeleggingen floreren en de dekkingsgraad sterk oploopt. Bij een dergelijke asymmetrische verdeling van plussen en minnen dient het pensioenfonds, in het belang van zijn deelnemers, minder risicovolle beleggingen aan te houden.

Hoofdstuk 2 gaat op dit thema door en introduceert daarbij pensioenregulering en -toezicht. De Nederlandsche Bank (DNB) is, samen met de Autoriteit Financiële Markten (AFM) en de Nederlandse Mededingingsautoriteit (NMA), toezichthouder op pensioenfondsen. DNB ziet er op toe dat pensioenfondsen voldoende middelen in bezit hebben om de verplichtingen na te komen. In tegenstelling tot andere financiële instellingen hebben pensioenfondsen echter veelal minder korte termijn liquiditeitsbeperkingen, geen externe obligatiehouders die een faillissement kunnen afdwingen, zeer langlopende verplichtingen, en iets meer mogelijkheden om de financiële positie te herstellen als deze ontoereikend is. In het toezichtkader krijgen pensioenfondsen daarom enige tijd om te herstellen als er een tekort is. Als in die periode herstel niet optreedt, is het korten van opgebouwde pensioenrechten op enig moment onvermijdbaar. De deelnemers lopen daardoor een zeker gevaar. Dit toezichtraamwerk is na te bootsen met zogenaamde Parijse opties. Dit zijn optiecontracten die vroegtijdig aflopen als de onderliggende waarde (de dekkingsgraad) gedurende een bepaalde tijd beneden een zekere referentiewaarde (de minimaal vereiste dekkingsgraad) verkeert. Het tweede hoofdstuk kijkt specifiek naar de samenhang tussen beleggingsbeleid, toezichtregels en de waardering van voorwaardelijke pensioenverplichtingen. Hieruit volgt vervolgens wat de optimale compensatie is die aan pensioendeelnemers toekomt voor het geval het fonds te lang in een dekkingstekort blijft en rechten worden gekort (cq. ontbinding van het pensioenfonds volgt). Die compensatie behelst de claim die de deelnemers op de eventuele reserves in het pensioenfonds moeten leggen. Hierover zijn afspraken te maken bij de onderhandelingen over het pensioencontract.

Het blijkt bijvoorbeeld dat als pensioenfondsen meer beleggingsrisico nemen, deelnemers idealiter een groter deel van de reserves claimen. Als dat niet gebeurt, verschuift er waarde van de deelnemers naar de aandeelhouders van de onderneming die het pensioenfonds sponsort. Voor de gebruikte numerieke voorbeelden komt de claim neer op grofweg de helft van die reserves. Verder toont de analyse aan dat de maximale lengte van de hersteltermijn relevant is. Een langere hersteltermijn vergroot de kans dat het pensioenfonds zijn dekkingsgraad herstelt. Dit wordt gedeeltelijk teniet gedaan door de kans dat naarmate de tijd vordert de financiële positie van kwaad tot erger kan vervallen. Met behulp van een nutsanalyse is het mogelijk de optimale hersteltermijn vast te stellen. Voor de rekenvoorbeelden gepresenteerd in dit hoofdstuk varieert die termijn van 1 tot 5 jaar, afhankelijk van onder meer de aard van het pensioencontract, het beleggingsbeleid, de mate van risicoaversie van de deelnemers en de minimaal vereiste dekkingsgraad.

Uit de eerste twee hoofdstukken blijkt dat het beleggingsbeleid cruciaal is voor de waardering van voorwaardelijke pensioenverplichtingen. Hoofdstuk 3 analyseert daarom het beleggingsgedrag van Nederlandse pensioenfondsen als gevolg van de ontwikkelingen op de financiële markten. Het grillige verloop van aandelenkoersen beïnvloedt enerzijds het gewicht van aandelen in de beleggingsportefeuille van pensioenfondsen, terwijl anderzijds ook het strategische beleggingsbeleid enigszins met de aandelenkoersen meebeweegt. Het beleggingsbeleid van pensioenfondsen start meestal met het vaststellen van de strategische beleggingsportefeuille, ofwel de verdeling van het vermogen over de beleggingscategorieën. Dit gebeurt op basis van een 'asset liability management'-studie die de verwachte rendementen per beleggingscategorie en de onderlinge samenhang afzet tegen de aard en de omvang van de pensioenverplichtingen. De feitelijke beleggingsportefeuille kan, binnen bepaalde marges, afwijken van de aldus gevonden strategische portefeuillesamenstelling. Bij 'market timing' kiest het pensioenfonds actief voor hoger gewicht van een bepaalde beleggingscategorie als het verwacht dat deze categorie op korte termijn beter rendeert dan de overige beleggingscategorieën. Een dergelijk overgewicht ontstaat ook als de beleggingsportefeuille niet (volledig) wordt geherbalanceerd, nadat de portefeuillegewichten door relatieve koersontwikkelingen zijn verschoven. De term voor deze passieve beleggingsbeslissing is 'free floating'. Het verschil tussen market timing en free floating is in de praktijk overigens moeilijk te duiden, omdat het resultaat hetzelfde is, namelijk portefeuillegewichten die afwijken van de strategische portefeuille.

Een relatief hoog rendement op aandelen leidt op korte termijn systematisch tot een hoger gewicht van aandelen in de beleggingsportefeuille. Eén procentpunt extra aandelenrendement doet het aandelengewicht in de portefeuilles een kwartaal later met gemiddeld 0,12 procentpunt toenemen. Het omgekeerde geldt ook. Een relatief laag rendement op aandelen leidt automatisch tot een lager gewicht. Een hoog rendement op aandelen blijkt volgens de schattingen nog lang significant door te werken op het actuele aandelengewicht, tot zelfs vijf kwartalen terug. De oorzaak hiervan is dat pensioenfondsen hun beleggingsportefeuilles niet zo snel en ook niet volledig herbalanceren. Herbalanceren is het terugbrengen van de actuele beleggingen tot de strategische portefeuillegewichten. Het gevonden effect hangt samen met de fondsomvang. Grote pensioenfondsen laten hun aandelengewicht tot bijna een factor twee meer afhangen van het rendement op aandelen dan kleine pensioenfondsen. Pensioenfondsen reageren verder asymmetrisch op veranderingen in aandelenkoersen: herbalanceren is veel sterker na negatieve schokken. Bij een positief overrendement bedraagt de herbalancering gemiddeld slechts 13 procent, terwijl dat bij een negatief overrendement maar liefst 49 procent is. Bij dalende aandelenmarkten maken pensioenfondsen blijkbaar graag gebruik van lagere koersen om aandelen bij te kopen. In een stijgende aandelenmarkt verkopen ze echter minder snel aandelen, of kopen zelfs bij, waardoor hun risicopositie ten opzichte van het strategische beleggingsbeleid toeneemt. Het herbalanceren van portefeuilles van pensioenfondsen heeft mogelijk een stabiliserende werking op de aandelenmarkten (al is dit hier niet nader onderzoek). Verder blijkt in dit hoofdstuk dat op middellange termijn stijgende aandelenkoersen leiden tot een beperkte opwaartse aanpassing van de strategische aandelenallocatie (en vice versa).

Het vergt overigens veel voorspelkracht om extra rendement te genereren met market timing. De richting waarin de aandelenbeurs zich op korte termijn ontwikkelt is namelijk niet goed te voorspellen. De berekeningen laten dan ook zien dat als gevolg van het variëren van de feitelijke asset-allocatie door de tijd de beleggingsopbrengst gemiddeld per jaar 0,20 procent lager is ten opzichte van een vaste vermogensallocatie (perfect herbalanceren). Dit effect is overigens over de onderzochte (korte) periode van acht jaar statistisch gezien niet significant van nul verschillend en er is geen rekening gehouden met de kosten om de allocatie vast te houden. Dit alles laat wel zien dat het belangrijk is voor pensioenfondsen om een heldere herbalanceringsstrategie te hebben. Ieder tiende procent rendement werkt exponentieel door op het te bereiken pensioenresultaat of op de te betalen premie.

Het vierde en laatste hoofdstuk beschrijft de wijze waarop pensioenfondsen in hun beleggingsbeleid rekening houden met de leeftijd van de deelnemers. Het standaard levenscyclusmodel geeft zicht op het optimale beleggingsbeleid over de verschillende levensfasen. Het kernelement van dit model is dat het rekening houdt met de omvang én het risiconiveau van het menselijk kapitaal. Daarnaast is de correlatie tussen het rendement op menselijk kapitaal (loonontwikkeling) en op financieel kapitaal (koersontwikkeling) een belangrijke factor. Het menselijk kapitaal is de contante waarde van alle huidige en toekomstige looninkomsten. Op jonge leeftijd (na voltooiing van de studie) is het menselijk kapitaal het hoogst en bij pensionering is het menselijk kapitaal afgenomen tot nihil. Voor het financieel kapitaal geldt het omgekeerde. Bij de intrede in het arbeidsproces is dit minimaal terwijl het bij pensionering veelal op een maximum zit. Over de levenscyclus wordt het menselijk kapitaal dus geconsumeerd of omgezet in financieel kapitaal door besparingen. Onder de stringente voorwaarde dat het menselijk kapitaal een laag risico kent, en daardoor zwak is gecorreleerd met risicovolle beleggingen, is het vanuit oogpunt van risicodiversificatie optimaal voor jonge mensen om beleggingsrisico te nemen, teneinde daardoor een betere afruil tussen rendement en risico te bereiken. Dit effect is sterker als de jongere negatieve rendementen op zijn financieel kapitaal kan opvangen door zijn arbeidsaanbod te vergroten. Dit kan door meer uren per week te werken of later met pensioen te gaan. Ook kan het menselijk kapitaal toenemen door bijscholing. Ouderen kunnen daarentegen geen gebruik maken van het diversificatievoordeel tussen menselijk en financieel kapitaal, omdat ze eenvoudigweg geen menselijk kapitaal meer bezitten. Vandaar dat het standaard levenscyclusmodel een negatief verband tussen leeftijd en beleggingen in aandelen impliceert. Hoofdstuk 4 onderzoekt empirisch of en in welke mate dit negatieve verband bij Nederlandse pensioenfondsen is terug te vinden.

Het blijkt dat als de gemiddelde leeftijd van de actieve deelnemers met een jaar stijgt de strategische aandelenallocatie met 0,5 procentpunt daalt. Dit is een praktische toepassing van de optimale levenscyclustheorie. Een populaire vuistregel voor particuliere beleggers is om als percentage aandelen 100 minus de leeftijd aan te houden of bijvoorbeeld 80 minus de leeftijd. Bij een dergelijk aanpak neemt het aandelenpercentage per jaar met 1 procentpunt af. De coëfficiënt voor pensioenfondsen van -0,5 impliceert een half zo steile afname van het percentage aandelen. De meest waarschijnlijke verklaring hiervoor is dat pensioenfondsen in tegenstelling tot particuliere beleggers betere mogelijkheden hebben om risico's te spreiden in de tijd, dat wil zeggen over opeenvolgende generaties. Dat geldt in het bijzonder omdat veel bedrijfstakpensioenfondsen, de grootste pensioenbeleggers in Nederland, onder de wettelijke verplichtstelling opereren.

Voorgaande analyse suggereert dat het beleggingsbeleid vaak is afgestemd op de gemiddelde deelnemer. Naarmate de gemiddelde leeftijd van deelnemers toeneemt gaan pensioenfondsen daarom minder risicovol beleggen. Dit is voor de jonge deelnemer – met een menselijk kapitaal dat weinig risico bevat – niet optimaal omdat deze juist meer risico met zijn financiële vermogen kan lopen en daardoor een betere afruil tussen rendement en risico kan realiseren. Eveneens geldt dat het voor oudere deelnemers niet optimaal is, omdat het pensioenfonds voor deze groep nog altijd te veel risico neemt. Pensioenfondsen zouden daarom in hun beleid meer rekening kunnen houden met de uiteenlopende leeftijdskarakteristieken. Dit betekent, overeenkomstig het levenscyclusmodel, dat jonge deelnemers meer risico gaan lopen over hun pensioenopbouw terwijl ouderen meer zekere aanspraken verwerven.