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Abstract

Using techniques drawn from recursive function theory, the computability of rational expectations equilibria in infinite horizon economies is investigated. The concept of a recursive economy is defined. An example of an overlapping generations model is given which is recursive but possesses only monetary equilibria which are non-computable.

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1 Introduction

When a rational expectations equilibrium exists it is normal to assume that agents are able to find it and solve for it. In this paper it will be shown that this may not be possible.

Our notion of what can be solved for will be the standard concept of a computable function in recursive function theory. Roughly speaking, a function is computable if some finite set of instructions - an "algorithm" or "effective procedure" - can be designed which computes the function at each point of its domain.

It might be thought that if the description of the economy itself is in some sense computable or effective then an equilibrium which is computable would be expected. Nevertheless an example will be given of an overlapping-generations model whose description is effective, in a sense to be made precise, but which has only non-computable monetary equilibria.

In a different context Lewis (1985) has considered the computability of choice functions (see also Rustem and Velupillai, 1990), and Rabin (1957) has analysed a game in which the winning strategy is not computable. Binmore (1977) and Anderlini (1990) amongst others have analysed what happens when players in games are modelled as Turing machines. Anderlini (1988) has analysed the problem of trying to match a given price sequence when the costs of computation matter. Finally Spear (1989) considers whether effective procedures exist which enable agents to learn a rational expectations equilibrium [1].

[1]The approach taken differs from that taken in the current paper in a number of important respects. In particular Spear assumes that a computable equilibrium exists, whereas it is this very existence

2 Computable Functions and Recursive Economies

Brief definitions of relevant concepts will be given. For more extensive discussion the reader should refer, for example, to Spear (1989), Minsky (1967, ch.5) or Cutland (1980). We characterize computability in terms of Turing machines, where a Turing machine is one particular formulisation of an algorithm. Church's thesis claims that Turing computability corresponds to the intuitive notion of an effective procedure (see Anderlini, 1989). Using Church's thesis we will restrict arguments to informal descriptions of algorithms as opposed to detailed descriptions of Turing machines. We shall consider functions defined on the natural numbers, $\mathbb{N}$, with range in $\mathbb{N}$. Suppose we have a Turing machine that takes inputs from the natural numbers, and given an input, it runs it program until it eventually halts, outputing a number, or it never halts. Then we can think of this machine computing the function with domain equal to those inputs for which the computation halts, and which takes values equal to the outputs from the computation. Alternatively, if for a given function f, there exists a Turing machine which computes f, then f is said to be *computable*, and if no such machine exists, f is *non-computable*. As mentioned above, the domain of a function need not be the whole of $\mathbb{N}$, but when it is, we say the function is *total*.

Computability over natural numbers can straightforwardly be extended to computability over $\mathbb{Q}$ where $\mathbb{Q}$ is the set of rational numbers by use of a *coding* device

question which is analysed here. Also, he considers only stationary equilibria, though with a potentially uncountable state space. While the concept of a rational expectations equilibrium is then mathematically equivalent to that given here, it is not possible to interpret the learning mechanisms which he studies in the current context where attention is not restricted to stationary equilibria.

$\alpha : \mathbb{N} \rightarrow \mathbb{Q}$ where α is an explicit and effective bijection (which exists). (The extension to computability over $\mathbb{Q}^k$ where $k \in \mathbb{N}$ is similarly straightforward.)

Turning to the description of an infinite horizon economy, suppose without loss of generality that there is a single variable, say the price level, $q(t) \in \mathbb{N}$ to be determined each period t, $t = 0, 1, 2, \ldots$. It is normally the case that the equilibrium conditions can be expressed in the form of a sequence of conditions, one set of conditions for each date, involving current variables $q(t)$ and variables at other dates $\{q(t)\}_{t=0}^{\infty}$. Formally let F be the set of all (not necessarily total) functions from $\mathbb{N}$ to $\mathbb{N}$. Then an economy Γ is a mapping, $\Gamma : F \rightarrow F$. Thus Γ allows the determination of prices at t, $\Gamma(q)(t)$ from knowledge of an input function q. We want to express the idea of an economy which can be *effectively* described, though the meaning of the word effective here is not so clear as we are considering the manipulation of infinite objects (the price sequences). The usual idea of effectiveness in this context would require that prices in any period can be effectively calculated in a finite time using only a *finite part* of the input sequence. Formally we say that a function γ is *finite* if its domain is finite, and that γ is a *finite part* of q if they coincide in value at all points where γ is defined, i.e. $\gamma(t) = q(t)$ for all $t \in$ domain (γ). It is easy to show that any finite function can be coded by a natural number in a one-to-one, effective manner, and we denote this code by $c(\gamma)$. So given any finite function γ the code can effectively be found, and given any value for c, γ can be effectively derived. The notation $\simeq$ means "equal when defined". Then

DEFINITION (Cutland, 1980, p. 183): $\Gamma : F \rightarrow F$ is a *recursive operator* if there is a computable function $\phi(x,t)$ such that for all $q \in F$ and $t \in \mathbb{N}$, $y \in \mathbb{N}$, $\Gamma(q)(t) \simeq y$ if and only if there exists a finite part of q, say γ, such that $\phi(c(\gamma),t) \simeq y$.

We shall say that an economy is *recursive* if there exists a recursive operator Γ such that a *total* $q()$ is a rational expectations equilibrium if and only if $\Gamma(q) = q$. [2]

Notice that the requirement is only that some representation of the equilibrium conditions exists in the form of a recursive operator Γ. It need not hold for every representation. The extension to prices belonging to $\mathbb{Q}$ or $\mathbb{Q}^k$ is achieved by interpreting y, ϕ and $\theta(t)$ as natural numbers which code for points in $\mathbb{Q}$ or $\mathbb{Q}^k$. For an extensive discussion of recursive operators the reader is referred to Rogers (1967) [3]. The following characterization of recursive operators will prove useful below:

THEOREM *(Cutland, 1980, p.185): Γ is a recursive operator if and only if*

(a) Γ is continuous: for any $q \in F$ and all $t \in \mathbb{N}$, $y \in \mathbb{N}$, $\Gamma(q)(t) \simeq y$ if and only if

[2] This turns out by the Myhill-Sheperdson Theorem to be exactly equivalent to the class of economies studied by Spear (1989) if one restricts his mapping g in Assumption 2.3 to *extensional* functions. While he does not make this restriction, its violation would imply that two different representations of the same forecast function could lead to two different temporary equilibrium price functions, which is against the spirit of his analysis.

[3] It may be thought that the definition of recursiveness is unduly restrictive as it appears to require a unique value for $q(t)$ given other prices, while there may be multiple (temporary) equilibria. This is not however the case. If at time t there is a condition of the form $f_t(q(0), q(1), \ldots, q(t), \ldots) = 0$ this can always be expressed as $\Gamma(q)(t) = f_t(q(0), q(1), \ldots, q(t), \ldots) + q(t)$. Recursiveness can therefore be interpreted as saying that given knowledge of a proposed price sequence q, *it is possible to check* at each time t if the necessary equilibrium condition holds.

there is a finite part of q, say γ, with $\Gamma(\gamma)(t) \simeq y$;

(b) the function $\phi(x,t)$ given by $\phi(c(\gamma),t) \simeq \Gamma(\gamma)(t)$ for $\gamma \in F$ (undefined for other x) is computable.

We give two examples of recursive economies, where $q(t) \in \mathbb{Q}$.

Example 1:

$$q(t) = \alpha q(t-1) + \beta q(t+1), \quad t \geq 1, \text{ where } \alpha, \beta \in \mathbb{Q},$$

$$q(0) = \mu.$$

Given any input sequence, $q, \Gamma(q)(t)$ is found simply from $q(t-1)$ and $q(t+1)$. This is clearly continuous, and $\phi(c(\gamma),t) \simeq \alpha\gamma(t-1) + \beta\gamma(t+1), t \geq 1$, with $\phi(c(\gamma),t) = \mu$, is a computable function.

Example 2:

$$q(t) = \sum_{\tau=0}^{t-1} \delta^{\tau} q(t-\tau), \quad t \geq 1, \text{where } \delta \in \mathbb{Q},$$

$$q(0) = \mu.$$

Again, $\Gamma(q)(t)$ is found from the first t values of q, and is therefore continuous while $\phi(c(\gamma),t) \simeq \sum_{\tau=0}^{t-1} \delta^{\tau}\gamma(t-\tau), t \geq 1$, with $\phi(c(\gamma),t) = \mu$, is a computable function.

3 A Non-Computable Equilibrium

Even the restriction to a recursive economy does not guarantee that the equilibrium will be computable if $\Gamma(q)(t)$ requires knowledge of future variables, $q(\tau)$, for $\tau > t$ [4]. (If on the other hand the determination of prices at t involves only backward looking variables then a trivial recursive procedure will produce the equilibrium, which is therefore computable.) To show that all equilibria may be non-computable, the concept of a simple set will be used. Two preliminary ideas are needed. Firstly, let A be a subset of $\mathbb{N}$. Then A is *recursively enumerable* (henceforth r.e.) if the function f given by

$$f(x) = \{ \begin{array}{ll} 1 & \text{if } x \in A \\ \text{undefined} & \text{if } x \notin A \end{array}$$

is computable. Secondly, it is possible to effectively number all Turing machines (such an enumeration, not uniquely defined, is a "Gödel numbering"). Let ϕ_x be the function computed by Turing machine with number x.

A set A is *simple* if (a) A is recursively enumerable, (b) $\bar{A}$ (the complement of A in $\mathbb{N}$) is infinite, and (c) $\bar{A}$ contains no infinite recursively enumerable subset. The following simple set will be used below. First define a (partial) computable function a: $a(x)$ is computed by computing in turn $\phi_x(0), \phi_x(1), \phi_x(2), \ldots$ (do not proceed to the computation of $\phi_x(r)$ unless and until $\phi_x(r-1)$ has been successfully computed); stop as soon as (and only if) an r is found such that $\phi_x(r) > 2x$ and put $a(x) = \phi_x(r)$. Clearly $a(x)$ need not be defined. Then the range of $a()$ is a simple set (Rogers, 1967, Theorem

[4]If we drop the requirement that the equilibrium price function must be total then existence of a computable solution can be proved using the recursion theorem. See McAfee (1984).

8.1.7). Call this set A. Moreover, because it is r.e. it is also the range of some total computable function g (Cutland, Theorem 7.2.7).

The main idea used below is very general and could be applied to show the possibility of non-computable equilibria in other areas of economics such as non-cooperative game theory [5]. The model used is no more than an example. The critical idea is to construct an economy in which equilibrium can only exist if the price sequence generates an increasing sequence of numbers which belongs entirely to $\bar{A}$. By definition of A being a simple set, the set of numbers contained in this latter sequence, which has an infinite number of distinct elements, is not r.e. and consequently cannot be generated by a computable function. This implies in turn that any equilibrium price sequence is not computable, despite the fact that equilibria exist (because $\bar{A}$ is infinite). To be able to construct such an economy, there must firstly be sufficient indeterminacy *in equilibrium* so that an arbitrary increasing sequence of real numbers can be generated by the price sequence. In our example it will be the case that prices can take on one of two values an infinite number of times. The second major difficulty is to ensure that the process of checking that the numbers generated belong to $\bar{A}$ can be done effectively.

Consider the following simple stationary overlapping generations model. Such a model has been extensively studied in recent years (see for example Cass, Okuno and Zilcha, 1979; Azariadis and Guesnerie, 1986; Grandmont, 1985). Individuals live for two periods, the same number being born each period. An individual born at time t is young at time t when he consumes c_{1t} of the single perishable good, and old at time $t+1$, consuming

[5] I would like to thank Joseph Greenberg for alerting me to this possibility.

c_{2t+1}, and has utility function $u(c_{1t}, c_{2t+1})$. Each individual has endowment e_1 when young and e_2 when old. The exception is the old at $t = 0$, who live for one period and possess one unit of money. Holding money is the only way of saving. Let p_t be the price of the good in terms of money at t and define $z_t = 1/p_t$. Government monetary policy involves at each date t the determination of the money stock M_t (per capita). It is assumed that changes are brought about by making transfers to the current young (only increases in M_t will be considered). Let $(1 + \theta_t)$ be the proportional change in M_t, where $\theta_t \in [0, \infty)$, so $M_t = (1 + \theta_t)M_{t-1}$ and each young individual at t receives a transfer $\theta_t M_{t-1}$. We assume $M_0 = 1$. The money supply rule is a function of previous prices: $\theta_t = \theta_t(z_0, z_1, \ldots, z_{t-1})$. The rate of return on savings is $R = z_{t+1}/z_t$. Let $s(R, \omega) = \text{argmax}_{0 \leq s \leq e_1 + \omega} u(e_1 + \omega - s, e_2 + Rs)$ be the real savings of the representative household (assuming a unique maximum exists) where $\omega = \theta_t M_{t-1} z_t$ is the real money transfer received at t. A rational expectations monetary equilibrium must satisfy

$$s\left(\frac{z_{t+1}}{z_t}, \theta_t M_{t-1} z_t\right) = (1 + \theta_t)M_{t-1}z_t, \quad t \geq 0. \tag{1}$$

It will be assumed that s is defined and computable on $\mathbb{Q}^2_{++}$. Additionally we shall require that s has three special properties. First note that $\{(s(R,0), Rs(R,0)) : R \geq 0\}$ is the usual "reflected offer curve"; we require this to have a certain shape: firstly it must initially be below the 45° line, be backward bending and cross the 45° line with slope smaller in absolute value than unity. Consider for any value of $z \geq 0$ the following set: $\{Rs(R,0) : \text{there exists } R \geq 0 \text{ such that } s(R,0) = z\}$. This has at most two values and define $\bar{h}(z), \underline{h}(z)$ to be respectively the larger and smaller of the two (equal if there is a single value). Then in the usually considered case where the money stock is constant ($M_t = 1$ for all t) the "forward" solution to the model satisfies $z_{t+1} \in \{\bar{h}(z_t), \underline{h}(z_t)\}$ for all $t \geq 0$. Consider the (unique) stationary monetary equilibrium in this case: real money z^* is given by the solution to $z^* = \bar{h}(z^*)$. The second special property is that there exists an interval Ω around z^* such that for $z \in \Omega$, an n' exists such that $\bar{h}^n(\underline{h}(z)) \in \Omega$ for $n \geq n'$. A reflected offer curve with these two properties is drawn in Figure 1. These two properties will be satisfied for example if the offer curve has a slope less than unity in absolute value for a sufficient wide range of values either side of z^* and there exists a cycle of period 3 in the case where the money stock is held constant (Grandmont (1986) has shown that a sufficient condition for this is that the utility function be additively separable into two constant relative risk aversion functions and the coefficient of risk aversion in the second period be sufficiently high). The third property is that there exists a $\hat{\omega}$ such that $s(R, \omega) \leq \omega$ for $R \geq 0, \omega \geq \hat{\omega}$. This merely says that for sufficiently large real transfers the individual will consume at least his endowment in his first period, no matter what the return on savings is. Notice that if such a transfer were to take place, (1) could not hold. None of these conditions requires a pathological utility function.

The construction works briefly as follows. Take a candidate monetary equilibrium sequence $\{z_t\}$. Suppose initially $z_o \in \Omega$, so z_1 can be chosen either equal to $\underline{h}(z_0)$ or $\bar{h}(z_0)$. Let this choice generate a binary digit, 0 or 1 respectively. If $\bar{h}(z_0)$ was chosen, $z_1 \in \Omega$ and the process can be repeated, generating the next digit. Otherwise either $z_{\tau+1} = \bar{h}(z_\tau)$ for $\tau \geq 1$ until $z_{\tau+1} \in \Omega$ again, when the next binary digit is generated as before, or this is not the case, in which case set the money stock too large to be consistent with equilibrium. So if the money stock is to stay constant, an infinite sequence of binary digits will be generated. This is converted into a sequence of natural numbers by a given way of "cutting up" the binary sequence, and the money rule will effectively specify that if ever one of these numbers belongs to A the money supply is again set too large to be compatible with equilibrium. So only a non-computable sequence can be a monetary equilibrium.

More precisely, the money supply rule will be specified as follows. A finite sequence up to $T, (z_0, z_1, \ldots, z_T)$, is defined as an *information sequence* if it satisfies $z_0 \in \Omega, z_{t+1} \in \{\bar{h}(z_t), \underline{h}(z_t)\}$, and $z_t \notin \Omega$ implies $z_{t+1} = \bar{h}(z_t)$, for $t \leq T-1$. If, at any $T, \{z_t\}_0^T$ fails this condition, $\theta_{T+1} = \hat{\omega}/\underline{h}(z_T)$. Any infinite information sequence produces a sequence of binary digits $\{\mu_n\}_{n=0}^{\infty}$: suppose at $t, z_t \in \Omega$ for the $n-th$ time; then at $t+1, \mu_{n-1}$ is produced:

$$\mu_{n-1} = \begin{cases} 1 \text{ if } z_{t+1} = \bar{h}(z_t) \\ 0 \text{ if } z_{t+1} = \underline{h}(z_t). \end{cases}$$

Notice that the maximum number of periods elapsing between μ_{n-1} and μ_n is $1 + min\{r : \bar{h}^r(\underline{h}(max\{z \in \Omega\})) \in \Omega\}$, which is finite by definition of Ω, so $\{\mu_n\}$ is an infinite sequence. This sequence of binary numbers is next cut up to convert it into a sequence of natural numbers $\{\sigma(v)\}_{v=0}^{\infty}$, where $\sigma(v)$ is the number represented by a block of binary numbers of length N satisfying $2^N > 2v \geq 2^{N-1}$ with $N = 1$ when $v = 0$ (so $\sigma(v) = (\mu_{q+1}\mu_{q+2}\ldots\mu_{q+N})$ where μ_q was the last digit used in computing $\sigma(v-1)$). If at time $T, (v+1)$ such numbers, $\{\sigma(0), \ldots, \sigma(v)\}$, have been produced from $\{z_0, \ldots, z_T\}$, define the set $G(v) = \{g(0), g(1), \ldots, g(v)\}$. Then

$$\theta_0 = 0$$

$$\theta_{T+1} = \begin{cases} 0 \text{ if } \sigma(i) \notin G(v), \sigma(i) \leq 2i \text{ and } \sigma(i+1) > \sigma(i), \ i = 0, \ldots, v, \text{ or} \\ \quad \text{if } \sigma(0) \text{ has not yet been produced} \\ \frac{\hat{\omega}}{\underline{h}(z_T)} \text{ otherwise} \end{cases}$$

for $T > 0$.

This fully defines the money supply rule. Notice that it is entirely backward looking and is computed by an effective procedure using only previous values of z_t.

Define

$$\Gamma(z)(t) \simeq s\left(\frac{z_{t+1}}{z_t}, \theta_t M_{t-1} z_t\right) / (1+\theta_t) M_{t-1}, \quad t \geq 0,$$

where θ_t and M_{t-1} are functions of $z_0, z_1, \ldots, z_{t-1}$ as described. Γ is a recursive operator: because the calculation of $\Gamma(z)(t)$ only involves $\{z_\tau\}_0^{t+1}$ it is continuous; the function $\phi(c(\gamma), t) \simeq \Gamma(\gamma)(t)$ for $\gamma \in F$ is computable since given any finite horizon γ with domain $\{0, 1, \ldots, t+1\}$ we have a finite algorithm given by the money supply rule to calculate θ_t and M_{t-1}, and by assumption $s(.,.)$ is computable; consequently by Church's thesis ϕ is computable (ϕ is undefined for γ with different domain).

Now for a monetary equilibrium to exist we must have $z_{t+1} \in \{\underline{h}(z_t), \bar{h}(z_t)\}$ and $\theta_t = 0$ all $t \geq 0$. Thus $\{z_t\}$ must be of the type which generates an information sequence. However, it cannot be computable. To see this, suppose first that $\sigma()$ is computable. Since $\sigma(v) > \sigma(v-1), \sigma()$ enumerates an infinite r.e. set, which cannot be a subset of $\tilde{A}$; then there exists $\hat{v}$ such that $\sigma(\hat{v}) \in A$. Since $g()$ enumerates A, there exists some $v\prime$ such that $g(v\prime) = \sigma(\hat{v})$, hence $\sigma(\hat{v}) \in G(v)$ for $v \geq v\prime$. So m will be non-zero (at the latest) from the period after $\sigma(max\{\hat{v}, v\prime\})$ is produced. Now if $\{z_t\}$ is a computable sequence, we have an effective procedure for producing $\sigma()$, which is therefore computable implying a monetary equilibrium does not exist.

On the other hand there exist sequences which work. By construction of A a total $\sigma()$ can be found satisfying $\sigma(v) < \sigma(v+1), \sigma(v) \leq 2v$ and $\sigma(v) \in \bar{A}$ for all $v \geq 0$. Notice that $\sigma(v)$ can be expressed in binary using no more than 2^N digits, where N is defined as before, so there exists an information sequence which will produce $\sigma(v)$, and any information sequence with $\theta_t = 0$ for all t is a monetary equilibrium. This completes the argument.

Even if, as has been shown, no computable function exists which is an exact equilibrium, it may be thought that this is unimportant in the sense that some computable price sequence can be found which is "close" enough to an equilibrium so as not to matter. This is not true however, at least in this example. Whatever computable sequence is chosen, there will come a time when θ_t is set positive and at this point any short-sighted monetary equilibrium will break down dramatically; presumably before this happens money will have zero value (the barter equilibrium being the only equilibrium from this point on). The usual backward induction arguments then imply that money will have zero value at all dates. More precisely, it can be shown that with standard assumptions on the utility function, for ϵ small enough, there is no strictly positive computable $\{z_t\}$ sequence and market clearing trades such that each generation's utility function is within ϵ of its maximum utility at the given prices.

While the non-existence of a computable monetary equilibrium has been shown, there will of course exist a non-monetary, in this case autarkic, equilibrium which is trivially computable ($z_t = 0$ for all t). It would be straightforward but tedious to extend the model to rule out the existence of a non-monetary equilibrium: see Cass, Okuno and Zilcha (1979) who give an example with a monetary equilibrium but no non-monetary equilibrium. In this case there would exist no computable equilibrium.

References

Anderlini, L. (1988), Forecasting Errors and Bounded Rationality, mimeo, University of Cambridge.

Anderlini, L. (1990), Some Notes on Church's Thesis and the Theory of Games, *Theory and Decision*, 29, 19-52.

Azariadis, C. and R. Guesnerie (1986), Sunspots and Cycles, *Review of Economic Studies*, 53, 725-737.

Binmore, K.G. (1987), Modeling Rational Players, *Economics and Philosophy*, 3, 179-214.

Cass, D., M. Okuno and I. Zilcha (1979), The Role of Money in Supporting the Pareto Optimality of Competitive Equilibrium in Consumption-Loan Type Models, *Journal of Economic Theory*, 20, 41-80.

Cutland, N.J. (1980), *Computability: An Introduction to Recursive Function Theory*, Cambridge University Press, Cambridge.

Grandmont, J.M. (1985), On Endogenous Competitive Business Cycles, *Econometrica*, 53, 995-1045.

Lewis, A.A. (1985), On Effectively Computable Realizations of Choice Functions, *Mathematical Social Sciences*, 10, 43-80.

McAfee, R.P. (1984), Effective Computability in Economic Decisions, mimeo, University of Western Ontario.

Minsky, M. (1967), *Computation: Finite and Infinite Machines*, Prentice Hall, Englewood Cliffs, NJ.

Rabin, M.O. (1957), Effective Computability of Winning Strategies, in *Contributions to the Theory of Games, Vol. 3*, Annals of Mathematics Studies No. 39, Princeton University Press, Princeton.

Rogers, H. (1967), *Theory of Recursive Functions and Effective Computability*, McGraw-Hill, New York.

Rustem, B. and K. Velupillai (1990), Rationality, Computability and Complexity, *Journal of Economic Dynamics and Control*, 14, 419-432.

Spear, S.E. (1989), Learning Rational Expectations under Computability Constraints, *Econometrica*, 57, 889-910.

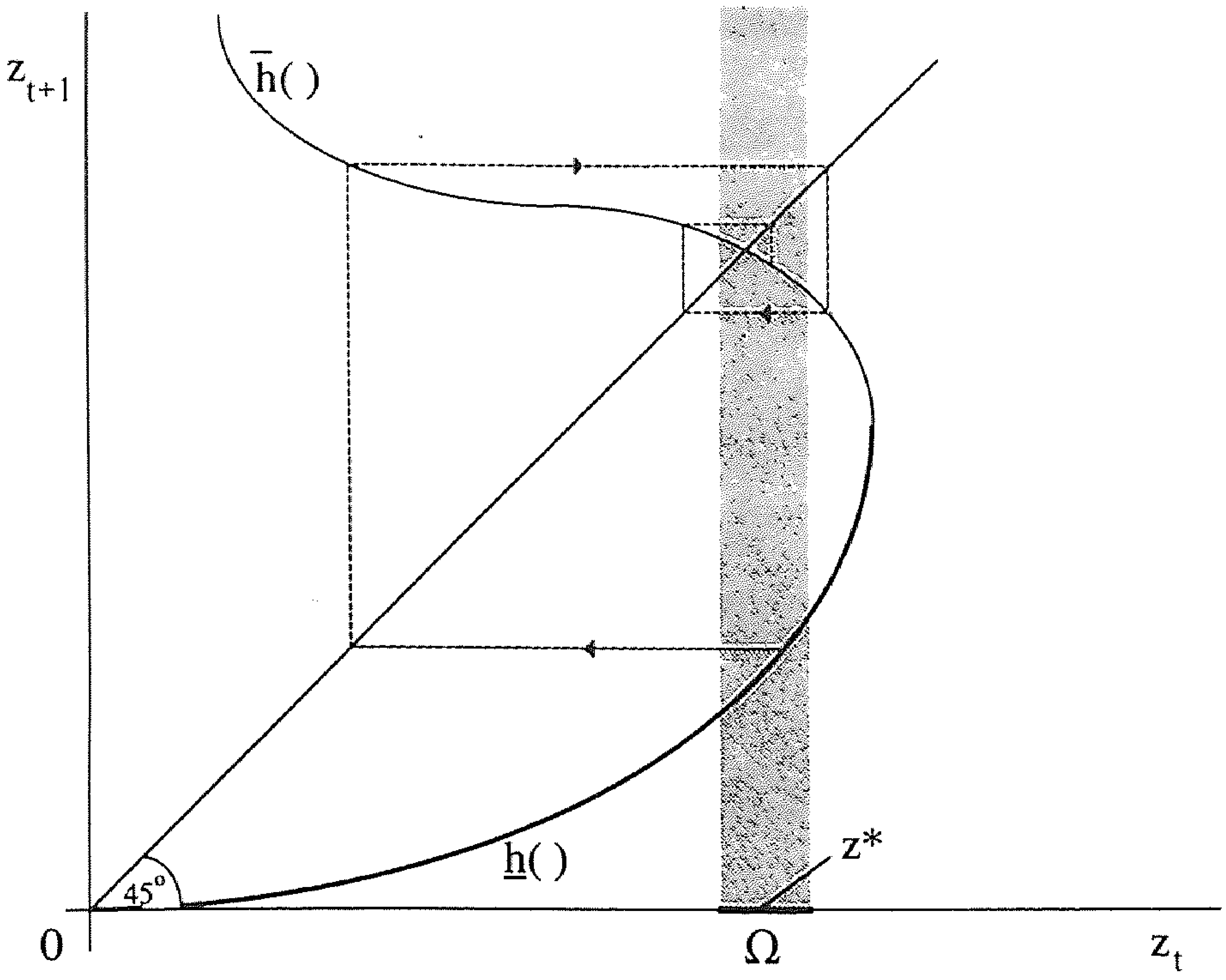

FIGURE 1

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