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STRATEGIC INFORMATION MANIPULATION IN DUOPOLIES

ABSTRACT

This paper studies a duopoly market in which firms are uncertain about demand and can draw inferences concerning market demand from observations of their production quantities and market price. We are especially interested in the incentives for the firms to experiment, or adjust their outputs away from myopically optimal levels to affect the informativeness of the market price. We show that experimentation occurs if information affects future optimal actions and if current actions also affect information. We then develop conditions under which experimentation introduces incentives for firms to either increase or decrease output. Our primary departure from previous work on strategic information transmission arises out of the fact that firms' quantities are observed in our model. As a result, experimentation allows firms to manipulate not the direction in which beliefs are revised (as in signal-jamming models) but the extent to which belief revision occurs. Firms will adjust initial quantities to make prices either more or less informative, and hence to increase or decrease the extent of belief revision, depending upon whether information has positive or negative value. We present examples showing that firms may adjust output in order to reduce the informativeness of market price.

STRATEGIC INFORMATION MANIPULATION IN DUOPOLIES

I. Introduction

This paper studies a duopoly market with uncertain demand. Firms may draw inferences concerning market demand from observations of their production quantities and the market price. Given this ability to deduce information about demand from the market, the firms may experiment, or adjust their outputs away from myopically optimal levels to affect the informativeness of the market outcome. Experimenting firms sacrifice current profits in the hope of affecting the information content of the market outcome in such a way as to increase future expected profits.

We investigate the following questions. Under what conditions will firms experiment? If they do experiment, will they attempt to increase or decrease the informativeness of market price? Equivalently, when will additional information have a positive or negative value for the firm? Will incentives to experiment create incentives for firms to increase or decrease output? Will incentives to experiment produce equilibrium quantities that are higher or lower than their myopically-optimal levels?

We consider two firms which produce a homogeneous product over two periods. These firms face a demand function that has a parameter which is unknown to both firms as well as random noise. The latter masks the true value of the unknown parameter so that neither firm can obtain perfect information about the parameter from observing the period-one price. In period one, each firm chooses a level of production and a market price is then realized. We assume that both firms observe this price as well as industry output. This information is used to update prior beliefs.

Our work can be compared with four related types of analysis. First, Kamien, Tauman and Zamir (1987) have examined the value of information in a game. In particular, they examine the value to an agent of information held by that agent alone, where the agent is not a player in the game and may reveal information to one or more players. This value must always be nonnegative, since the agent retains the option of ignoring the information, and the interesting questions concern when the value is positive and how this value is to be measured. In contrast, we are concerned with the incentives for a player to manipulate information that is available to all players. A player in a game may be made worse off by an increase in public information.

Second, the value of information in oligopoly games has been the subject of intensive research.[1] These studies typically assume either that firms transmit information by means of "certifiable announcements" or that information is received from exogenously generated signals. Our model differs in that the amount of information generated is determined endogenously by the choice of first-period actions. Varying the amount of information in our model is then costly. Firms can affect information flows only by bearing the short-run reductions of profit caused by deviations from myopically optimal actions.[2] In addition, any information that is produced in our model is available to all firms, so that firms always receive identical information. This contrasts with previous models, in which firms receive private signals, and causes information to exhibit some of the properties of a public good.

Third, studies have appeared of how a *monopolist* might vary quantity to affect the informativeness of price.[3] In a single-agent decision problem, such as that facing a monopoly, more information is always better (or at least no worse). The only question facing a monopolist then concerns how to adjust quantity so as to increase the flow of information. In a game, such as a

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[1]For a survey of this literature and a list of references, see Okuno-Fujiwara, Postlewaite, and Suzumura (1990).

[2]Gal-Or (1988) examines a two-period duopoly model in which first period choices affect second-period information concerning cost. The signals that firms receive about costs in her model are privately observed, in contrast to the publicly observed price signal in our model. In her model, the (random) mapping from present choices to future beliefs is exogenously specified, with larger outputs assumed to yield more information. In our model, that mapping is endogenously determined through Bayes' rule.

[3]See Mirman, Samuelson, and Urbano (1989a) for an example and references.

duopoly, more information can be detrimental. The possibility then arises that duopolists will adjust their quantity so as to decrease the informativeness of market price.

Fourth, the examples closest in spirit to a model of duopoly experimentation are the "signal-jamming" models of Riordan (1985), Fudenberg and Tirole (1986), and Mirman, Samuelson and Urbano (1989b). In these models, like ours, no firm is perfectly informed about the state of nature and each firm may have an incentive to manipulate the inferences drawn by rival firms. The principal difference between these signal jamming models and ours lies in the observability of actions. In signal-jamming models, firms do not know, even *ex post*, the actions (for example, the quantities) chosen by rivals. Our analysis is motivated by the observation that firms often may be able to verify the actions taken by their rivals, and we assume that each firm can observe its rival's output.[4]

This observability assumption has significant consequences for the way in which a firm manipulates its rival's inferences. In signal jamming models, firms attempt to influence the *direction* in which a rival updates its beliefs. For example, if quantity is the choice variable, then a firm may use an (undetected) increase in output to lower price and convince a rival that demand is low. In our model, outputs are observed, so that the foregoing manipulation of beliefs cannot occur. Instead, firms affect the informative-

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[4]Aghion, Espinosa, and Jullien (1990) provide a complementary analysis of duopoly experimentation. However, we examine a homogeneous-product, quantity setting duopoly while they work with heterogeneous-product price setters. We examine the ability of firms to affect the distribution of likely market prices, and hence information, by adjusting quantities. Their model is constructed so that price *dispersion* is required for the revelation of information. We devote considerable attention to the question of when information is valuable, while they often work with the presumption that information is valuable and examine the steps taken by firms in light of this value. Their example of why the value of information may be negative in a duopoly game is also quite different from ours, with Aghion, Espinosa and Jullien working with a model in which the complete-information equilibrium is qualitatively different across the two possible states of nature, with the firms competing in one state and being effectively isolated in the other.

ness of the commonly observed price signal. Firms in our model may then affect the *degree* to which a rival is likely to update expectations, but not the direction.

We begin with the question of when firms will have an incentive to experiment. We show that firms will experiment if optimal second-period actions depend on second-period information (i.e., if information matters) and if alterations in period-one quantity affect the informativeness of period-one price (i.e., if information can be manipulated). We show by examples that cases can arise in which duopoly firms will face incentives to experiment but a monopoly (or colluding duopolists) would not; and vice versa.

We next turn to the question of whether experimentation introduces incentives for firms to increase or decrease their period-one outputs. We develop sufficient conditions for each case. We demonstrate that an increase in information in our duopoly game can make players worse off, in contrast to a single-agent decision problem, presenting an example in which the duopolists adjust their quantities so as to decrease information. In contrast, if a monopolist experiments, it is always in the direction of *increasing* information.

Our analysis shows that the effect of experimentation on first-period quantities is determined by the sign of the sum of four terms. One term captures the incentive to experiment in order to revise a firm's own beliefs; two terms capture the incentive to influence revision of the rival firm's beliefs; and a fourth term captures the interaction between these first two forces. The sign of each term is in general ambiguous, though we are able to establish signs for some interesting special cases.

These results describe the effect of experimentation on the best response map of a single firm. The question remains of how the effects of experimentation on firms' best-response maps interact to affect equilibrium outputs. We demonstrate that if prior beliefs are the same and if information is valuable, then industry output will be adjusted in the direction which generates more information. We then develop conditions under which information is valuable and under which the direction of increasing information can be identified. At the same time, we demonstrate by example that it is possible that industry output adjusts so that the price signal is less informative.

Finally, we address the question of existence of equilibrium. A subgame perfect equilibrium in pure strategies may not exist in this model because the objective function (after the two-period game is "rolled back" into a single period game) may not be not quasiconcave. Hence, in general the pure strategy best response will be a correspondence that is not convex-valued, so that standard existence arguments are inapplicable. On the other hand, we give sufficient conditions for existence of a mixed-strategy equilibrium and provide mixed strategy versions of our main findings.

Section II constructs the two-period duopoly model. Section III develops conditions under which firms will experiment and develops conditions under which this experimentation introduces incentives for firms to increase or decrease quantity. Section IV examines the effect of experimentation on equilibrium quantities. Section V considers the existence of equilibrium. Section VI summarizes the results and concludes.

II. The Model

We consider a two-period, homogeneous-products duopoly. For simplicity, we assume that production costs are zero (or that P is price net of a constant marginal cost that is the same for both firms). The inverse demand function is given by $P = g(Q,\gamma) + \varepsilon$, where P is market price, Q is industry output, γ is an unknown parameter, and ε is the realization of a random variable, $\tilde{\varepsilon}$, whose distribution is characterized by a continuously differentiable density function $f(\cdot)$ with $\int_{-\infty}^{\infty}\varepsilon f(\varepsilon)d\varepsilon = 0$. We assume that

(A1) γ takes on one of two possible values, $\bar{\gamma}$ or $\underline{\gamma}$;

(A2) $g(0,\gamma) > 0$ for $\gamma = \bar{\gamma}$ and $\gamma = \underline{\gamma}$;

(A3) $g(Q,\gamma)$ is nonincreasing on $\mathbb{R}_+$ and strictly decreasing on $\{Q: g(Q,\gamma)>0\})$;

(A4) $g(Q,\gamma)$ yields a unique myopic equilibrium;

(A5) $g(Q,\bar{\gamma}) \geq g(Q,\underline{\gamma})$ for all $Q \in \mathbb{R}_+$, so that $\gamma = \bar{\gamma}$ corresponds to the "good" state of demand.

Each firm begins period 1 with a prior probability distribution over $\{\bar{\gamma},\underline{\gamma}\}$. Let ρ_i^0 denote firm i's prior belief that $\gamma = \bar{\gamma}$ for $i = 1,2$. We assume that these prior beliefs are common knowledge.

We will generally assume that the firms have common priors, or $\rho_1^0 = \rho_2^0$. However, in Section III and one example in Section IV we allow the possibility that $\rho_1^0 \neq \rho_2^0$. This allows the firms to have different prior expectations, even though these priors are common knowledge. It is important to note that we use differing priors for expositional purposes. In particular, none of the results depend upon this assumption. However, the forces driving the results are often easier to isolate by provisionally allowing priors to differ. This is especially the case in Section III.

In period one, firm i chooses an output Q_i. After these first period quantities are chosen, a value of $\tilde{\varepsilon}$ and hence P is realized. Firm i's expected profit in period one is then

$$\pi^1(Q_1,Q_2,\rho_1^0) \quad = \quad \Big(g(Q,\bar{\gamma})\rho_1^0 + g(Q,\underline{\gamma})(1-\rho_1^0)\Big)Q_1, \tag{1}$$

where $Q = Q_1 + Q_2$. We assume that firms observe first period price and outputs, but not the realization of $\tilde{\varepsilon}$. Consequently, firms may not be able to determine the value of γ after the first period. The observation of P, together with knowledge of aggregate output, however, leads each firm to revise its beliefs regarding the value of γ. We assume that such revisions proceed according to Bayes' rule, so that firm i's posterior belief that γ =

$\bar{\gamma}$, denoted ρ_1, is given by

$$\rho_1 \quad = \quad \frac{\rho_1^0 f(P-g(Q,\bar{\gamma}))}{\rho_1^0 f(P-g(Q,\bar{\gamma})) + (1-\rho_1^0)f(P-g(Q,\underline{\gamma}))} \tag{2}$$

where Q is first period industry output and P is the realization of price in period one.

We shall restrict the density function $f(\cdot)$ to ensure that larger realizations of P lead to higher posterior beliefs that $\gamma = \bar{\gamma}$ and hence that the state of demand is "good". To do this, we impose the requirement that f satisfy the monotone likelihood ratio property (MLRP), namely that

$$\frac{f'(\varepsilon)}{f(\varepsilon)} \tag{3}$$

is strictly decreasing on supp(f) (i.e., the closure of $\{\varepsilon: f(\varepsilon)\neq 0\}$). It is easily shown that (3) implies $d\rho_1/dP > 0$ on $\{(P,Q): 0<\rho_1<1\}$.

In period 2, each firm again chooses a level of production Q_i^*. Period 2 expected profit is therefore $\pi^i(Q_1^*, Q_2^*, \rho_i)$ (where Q_i^* denotes second period output for firm i and ρ_i is calculated from (2)).

We are interested in a subgame perfect equilibrium of this two-period game. As usual, we analyze subgame perfect equilibria by first examining the second period. By assumption, the second period subgame possesses a unique Nash equilibrium for each vector of posterior beliefs (ρ_1, ρ_2). Let (Q_1^*, Q_2^*) now denote this equilibrium. Firm i's period 2 value function, giving the expected value of period-two profits as a function of posterior beliefs, is then given by

$$V^i(\rho_1,\rho_2) \quad \equiv \quad \Big(g(Q^*,\bar{\gamma})\rho_i + g(Q^*,\underline{\gamma})(1-\rho_i)\Big)Q_i^*,$$

where $Q^* = Q_1^* + Q_2^*$ and Q_i^* and Q^* are functions of (ρ_1, ρ_2).

In period 1, the posterior beliefs (ρ_1, ρ_2) are random variables whose distribution depends upon first-period industry output (as well as the distribution of P induced by $\tilde{\varepsilon}$). We may therefore write each firm's period-one expected profit as a function of first period output:

$$\Pi^1(Q_1, Q_2, \rho_1^0, \rho_2^0) = \pi^1(Q_1, Q_2, \rho_1^0) + \delta\int V^1(\rho_1(P,Q), \rho_2(P,Q))h_1(P)hP \qquad (5)$$

$$\equiv \pi^1(Q_1, Q_2, \rho_1^0) + \delta W_1(Q)$$

where δ is the (common) discount factor and

$$h_1(P) = \rho_1^0 f(P-g(Q,\bar{\gamma})) + (1-\rho_1^0)f(P-g(Q,\underline{\gamma})). \qquad (6)$$

Let Γ denote the single-period game whose payoff functions are given by (5) and let $\sigma_1(Q_j) = \{Q_1 \in \text{argmax}_{Q_1} \Pi^1(Q_1, Q_2, \rho_1^0, \rho_2^0)\}$ denote firm i's best reply correspondence.[5] A Nash equilibrium of Γ is a pair $(Q_1^{**}, Q_2^{**}) \in \sigma_1(Q_2^{**}) \times \sigma_2(Q_1^{**})$. Any such equilibrium will correspond to a subgame perfect equilibrium of the original two-period game.

III. The Value and Manipulation of Information

This section investigates the incentives, created by the effect of period-one quantities on the informativeness of price, for firms to adjust period-one quantities away from their myopically optimal levels. We compare firm i's best reply mapping of Γ with its "myopic" reaction function, i.e., its reaction function when its payoff function is simply $\pi^1(Q_1, Q_2, \rho_1^0)$, the first term in (5). The two best reply mappings differ in that the mapping derived in Γ takes into account the effects of first period choices on the flow of information for period two, whereas the myopic best-reply functions ignore such effects. A comparison of the two best reply mappings then determines whether the presence of such information flows introduce incentives

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[5] σ_1 is a correspondence because the second term in Π^1 need not be concave and hence $\sigma_1(Q_j)$ need not be a singleton.

for a firm to increase or decrease first period output (for a given output level of the other firm).

(3.1) Best Reply Mappings

The relationship between the myopic and nonmyopic best reply mappings is determined by $W_1(Q)$ (defined in (5)). Specifically, if $W_1(Q)$ is increasing [decreasing] in Q, then for any given period-one output of the other firm, firm i's myopic period-one best response will be less than [greater than] the output specified by $\sigma_i(Q_j)$, the best reply map for Γ.

We accordingly derive dW_1/dQ. To simplify our calculation, and to facilitate interpretation of the resulting expression, we find it helpful to use (2) to substitute one of the two posterior beliefs out of $V^1(\rho_1, \rho_2)$. It is perhaps most natural to eliminate ρ_2 from firm 1's value function. Hence, we rewrite (2) as

$$\rho_1(P,Q) = \frac{\rho_1^0 \bar{f}}{\rho_1^0 \bar{f} + (1-\rho_1^0)\underline{f}} \tag{7}$$

$$\rho_2(P,Q) = \frac{\rho_2^0 \bar{f}}{\rho_2^0 \bar{f} + (1-\rho_2^0)\underline{f}} \tag{8}$$

where $\bar{f} = f(P-g(Q,\bar{\gamma}))$ and $\underline{f} = f(P-g(Q,\underline{\gamma}))$. We then solve (7) for $\underline{f}$ to obtain

$$\underline{f} = \frac{\rho_1^0(1-\rho_1)\bar{f}}{\rho_1(1-\rho_1^0)}$$

and substitute this expression into (8) to obtain

$$\rho_2 = \frac{\rho_2^0\rho_1(1-\rho_1^0)}{\rho_2^0\rho_1(1-\rho_1^0) + (1-\rho_2^0)\rho_1^0(1-\rho_1)} \equiv \varphi(\rho_1). \tag{9}$$

The mapping $\varphi(\rho_1)$ gives firm 2's posterior beliefs as a function of 1's posterior beliefs (and each firm's prior beliefs). Note that

$$\varphi'(\rho_1) = \frac{\rho_2^0(1-\rho_2^0)\rho_1^0(1-\rho_1^0)}{\Omega^2} > 0, \tag{10}$$

where Ω is the denominator in (9). Thus the two posterior beliefs are perfectly positively correlated. The magnitude $\varphi'(\rho_1)$ identifies the rate at which firm 2 revises beliefs as firm 1 revises.[6] If $\rho_1^0 = \rho_2^0$, then $\varphi'(\rho_1) = 1$, since in that case $\rho_1 = \rho_2$ for all realizations of P.

We now define $U_1: [0,1] \to \mathbb{R}$ by

$$U_1(\rho_1) = V^1(\rho_1, \varphi(\rho_1)), \tag{11}$$

so that

$$W_1(Q) = \int U_1(\rho_1(P,Q))h_1(P)dP. \tag{12}$$

Differentiating (12) with respect to Q then yields

$$\frac{dW_1}{dQ} = \int\left(U_1'(\rho_1)\frac{d\rho_1}{dQ}h_1(P) + U_1(\rho_1)\frac{dh_1}{dQ}\right)dP. \tag{13}$$

Expression (13) can be manipulated to give (details of the derivation of (14) are given in Section VII):

$$\frac{dW_1}{dQ} = (\bar{g}'-\underline{g}')\int U_1''(\rho_1)(1-\rho_1)\rho_1^0\frac{d\rho_1}{dP}\bar{f}dP. \tag{14}$$

Moreover, from (11),

$$U_1''(\rho_1) = \frac{d^2V^1}{d\rho_1^2} + 2\frac{d^2V^1}{d\rho_1 d\rho_2}\varphi'(\rho_1) + \frac{d^2V^1}{d\rho_2^2}[\varphi'(\rho_1)]^2 + \frac{dV^1}{d\rho_2}\varphi''(\rho_1). \tag{15}$$

(3.2) Interpretation

The sign of the expression for dW_1/dQ given in (14) reveals whether the experimenting firm will have an incentive to increase output ($dW_1/dQ > 0$) or decrease output ($dW_1/dQ < 0$) compared to the myopic best response. Some

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[6]As ρ_2^0 converges to 0 or 1, φ' converges to zero. Hence, the more certain firm 2 initially is of the value of γ, the smaller will be the change in firm 2's beliefs relative to changes in firm 1's beliefs. On the other hand, as ρ_1^0 converges to 0 or 1, then φ' converges to $+\infty$. Hence, the more certain firm 1 is initially of the value of γ, the larger will be variations in firm 2's beliefs relative to firm 1's.

insight into the forces determining the optimal output level for the experimenting firm can then be obtained by interpreting the terms contained in (15). (The remaining terms in (14) will be considered in Section (3.3).) It is here that it is helpful to have allowed firms to hold different priors, since this helps us disentangle the forces present in (15) by allowing us to separate firm 1's incentive to affect its own beliefs from the incentive to affect firm 2's beliefs.

First, note that if all but the first term in (15) were absent, then the resulting expression for dW_1/dQ would match the analogous expression derived for the monopoly case by Mirman, Samuelson, and Urbano (1989a). The first term in (15) thus captures the incentive for firm 1 to produce information in order to revise its own beliefs.

To assess the sign of the first term in (15), it may be helpful to recall the logic underlying the result that in a single agent problem, the value of information is nonnegative. The value function for a single agent problem, being the supremum of a collection of linear functions, is always a convex function of the probabilities. Moreover, as demonstrated by Fusselman and Mirman (1990), an increase in information produces a mean-preserving increase in the riskiness of an agent's posterior beliefs (as seen by the agent before the relevant signal is realized). The desired result then immediately follows because the expected value of a convex function always rises with a mean-preserving increase in risk. This single-agent result suggests that the first term in (15) should similarly be positive, or at least nonnegative. However, the convexity of V^1 in ρ_1 (equivalently, $d^2V^1/d\rho_1^2 > 0$, or a positive first term in (15)), which implies that firm 1 prefers more to less information holding ρ_2 constant, does *not* follow from the standard argument for the single-agent problem. Even though ρ_2 and hence firm 2's second period reaction function is held constant in the calculation of $d^2V^1/d\rho_1^2$, firm 2's *equilibrium* output varies in ρ_1. If this variation of firm 2's output is sufficiently adverse to firm 1, then firm 1 might prefer less to more information, even though such information affects only firm 1's beliefs. The sign of $d^2V^1/d\rho_1^2$ is thus potentially indeterminate, though it can be shown that it is nonnegative for all vectors of posterior beliefs if both $\bar{g}(Q)$ and $\underline{g}(Q)$ are linear.[7]

Consider now the last two terms in (15). These terms reveal whether firm 1 prefers firm 2 to be better informed, holding ρ_1 constant. These terms thus capture the incentive for firm 1 to increase or decrease information to influence revision of firms 2's beliefs.[8] Notice that $\varphi'' = 2A\Omega^{-3}(\rho_1^0-\rho_2^0)$, where Ω is defined in (10) and A is the numerator in (10), so that φ'' has the same sign as of $\rho_1^0-\rho_2^0$. Thus, if priors are common, the fourth term of (15) vanishes. In this case, an increase in the information of firm 2 will cause a mean-preserving increase in the riskiness of firm 2's posterior beliefs. Hence, if V^1 is strictly convex (concave) in ρ_2, then firm 1 will prefer firm 2 to be better (worse) informed and the sign of $d^2V^1/d\rho_2^2$ determines firm 1's valuation of increased information to firm 2. If priors are not common, however, then an increase in information will not result in a *mean preserving* increase in the riskiness of firm 2's posterior beliefs. Instead, the mean of firm 2's beliefs, so firm 1 believes, will be displaced towards firm 1's

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[7]If $g(Q,\bar{\gamma})$ and $g(Q,\underline{\gamma})$ are linear, then $g(Q,\bar{\gamma}) = \bar{\alpha}-\bar{\beta}Q$ and $g(Q,\underline{\gamma}) = \underline{\alpha}-\underline{\beta}Q$, where we assume $\underline{\alpha}$, $\bar{\alpha}$, $\underline{\beta}$ and $\bar{\beta}$ are such that an interior solution appears for all (ρ_1,ρ_2). In this case the value function for firm 1 is $V^1(\rho_1,\rho_2) = (4\alpha_1^2\beta_2^2 + \beta_1^2\alpha_2^2 - 4\alpha_1\alpha_2\beta_1\beta_2)/9\beta_1\beta_2^2 = [4\alpha_1^2\beta_1^{-1} + \beta_1\alpha_2^2\beta_2^{-2} - 4\alpha_1\alpha_2\beta_2^{-1}]/9$ where $\alpha_1 = \bar{\alpha}\rho_1 + \underline{\alpha}(1-\rho_1)$ and $\beta_1 = \bar{\beta}\rho_1 + \underline{\beta}(1-\rho_1)$. Straightforward calculations then yield $d^2V^1/d\rho_1^2 > 0$.

[8]To verify this, fix the first argument of V^1 at some point, say $\hat{\rho}_1$. Firm 1's value function then becomes $\hat{U}(\rho_1) = V^1(\hat{\rho}_1,\varphi(\rho_1))$; (as an expositional matter, one may interpret ρ_1 here as what firm 2's posterior beliefs would be if firm 2 had the same prior as firm 1). $\hat{U}''(\rho_1)$ in this case is precisely the last two terms of (15), so that these terms measure firm 1's valuation of providing firm 2 with a more informative signal, holding its own information constant.

prior. The value to firm 1 of this shift is captured by the fourth term of (15). The sign of φ'' indicates whether, from firm 1's viewpoint, the mean of firm 2's posterior beliefs will increase or decrease, and the sign of $dV^1/d\rho_2$ indicates the effect of such changae on firm 1's second period payoff.

It may be useful to contrast the manipulation of a rival's updating represented by this third term with that in models of signal jamming such as Riordan (1985), Fudenberg and Tirole (1986), and Mirman, Samuelson and Urbano (1989a). In signal-jamming models, the output of rivals is not observed. Firms in these models thus have an incentive to vary output with a view toward pushing rivals' posterior beliefs in a particular direction. For example, an undetected increase in output on the part of firm 1 in the first period will lower P for any realization of $\tilde{\varepsilon}$, and thus reduce firm 2's belief that the state demand is "good" for each ε. In our model, however, outputs are observed, so that a firm cannot be assured that an increase in its output will make its rival more pessimistic. Instead, varying output varies the informativeness of the price signal. This allows firms to affect the likely *degree* to which a rival updates but not necessarily the direction in which the rival updates.

The middle term in (15), $d^2V^1/d\rho_1 d\rho_2$, captures the interaction between experimenting to revise one's own beliefs and experimenting to influence the beliefs of one's rival. Observe that $d^2V^1/d\rho_1 d\rho_2 > 0$ implies that upward [downward] revisions in ρ_1 are more valuable, the higher [lower] is ρ_2. Thus, a positive $d^2V^1/d\rho_1 d\rho_2$ encourages firm 1 to induce larger variations in beliefs, that is, to produce more information. In this case, we might say that the two motives for experimentation are strategic complements. Similarly, a negative $d^2V^1/d\rho_1 d\rho_2$ discourages the production of information: upward [downward] revisions in ρ_1 are *less* valuable the higher [lower] is ρ_2. In this case, we might say that the two motives for experimentation are strategic substitutes. Equation (16) and Example 2 demonstrate that if the slope only or intercept only is unknown for a linear demand curve, then $d^2V^1/d\rho_1\rho_2 < 0$ and the two motives are strategic substitutes.

Now consider the combined effect of the terms in (15). The first term captures the incentive for firm one to vary its own information, and it is the only term not weighted by either φ' or φ''. If these two weights are "small", then this first term in (15) will dominate. It is easy to show that both φ' and φ'' converge to 0 as ρ_2^0 converges to either 0 or 1. Thus, if firm 2 is relatively certain of the state of demand, then firm 1's princiopal concern becomes whether to produce a more informative signal for itself. On the other hand, if *firm 1* is virtually certain of the state of demand, then its principal concern becomes whether to increase firm 2's information. In this case, the *fourth* term in (15) will dominate. In particular, as ρ_1^0 converges to 0, φ'' will converge to $-\infty$ (and the fourth term will dominate both the second and third terms).[9] If firm 1 is relatively certain that the state of demand is bad, he will also be relatively certain that firm 2 will revivse downward, so that if $dV^1/d\rho_2$ is negative, firm 1 would prefer to hasten that revision by giving firm 2 a more informative signal. On the other hand as ρ_1^0 converges to 1, φ'' will converge to $+\infty$ (and the fourth term will again dominate), since firm 1 is relatively certain that firm 2 will revise upward. In that case, a positive $dV^1/d\rho_2$ implies that firm 1 prefers giving firm 2 a more informative signal. Finally, observe that if priors are common ($\rho_1^0 = \rho_2^0$), then $\varphi'' = 0$ and $\varphi' = 1$, so that the fourth term in (15) vanishes and the remaining three are weighted equally.

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[9]To see this, multiply and divide (15) by $[\varphi']$ to obtain $\{(V_{11}^1/[\varphi']^2) + V_{12}^1/\varphi' + V_{22}^1 + V_2^1\varphi''/[\varphi']^2\}[\varphi']^2$. The first two terms in braces converge to zero as ρ_1^0 converges to either 0 or 1, since φ' converges to $+\infty$ in either case. The coefficient on V_2^1 equals $2(\rho_1^0-\rho_2^0)/\Omega$, where Ω is defined in (10). Since Ω converges to zero as firm 1's beliefs converge to certainly, the claim is established.

An alternative interpretation of the decomposition may be helpful. An increase in information, as we have noted, changes the distribution of posterior beliefs. This change is decomposible, from the viewpoint of firm 1, into two parts: first, a mean-preserving increase in the riskiness of (ρ_1, ρ_2); and second, a displacement of the mean of firm 2's posterior belief towards firm 1's prior. Since the first three terms of (15) are a quadratic form involving the second cross partial of V^1 (and hence will be positive of V^1 is convex in (ρ_1, p_2)), the sum of the first three terms captures the value of firm 1 of the first part of this shift. The value of the second part is then captured by the fourth term.

(3.3) Value of Information

We now establish some necessary conditions for $dW_1/dQ_1 \neq 0$ or, equivalently, necessary conditions for experimentation to occur. Let $G_1(\rho_1, Q)$ denote the cumulative distribution function of ρ_1 induced by f and Q. Let $Q_1^*(\rho_1, \rho_2)$ and $Q_2^*(\rho_1, \rho_2)$ denote the equilibrium outputs for the second period subgame. Then

Lemma 1. $dW_1/dQ = 0$ if one of the following holds:

(1.1) $dG_1(\rho_1, Q)/dQ = 0$ for all (ρ_1, Q);

(1.2) $dQ_i^*/d\rho_j = 0$ for all $(\rho_1, \rho_2) \in (0,1)^2$ and all i and j;

where sufficient conditions for (1.1) include

(1.3) $\bar{g}'(Q) - g'(Q) = 0$ for all Q;

(1.4) $d\rho_1/dP = 0$ for all $P \in \text{supp}\,(h_1)$.

Conditions (1.1) and (1.2) are thus sufficient to induce firms to not experiment (so that their negation is necessary for experimentation). We provide an intuitive sketch of the proof; the details are immediate. To motivate (1.1), notice that if $dG_1/dQ = 0$, then variations in quantity cannot affect beliefs. There is then no benefit to experimentation, giving $dW_1/dQ = 0$ and precluding experimentation. Observe that dG_1/dQ may be zero for two reasons. First, we could have $\bar{g}'(Q) - \underline{g}'(Q) = 0$ for all Q, or (1.3). If this equality holds, then the two inverse demand functions have the same slope at each quantity, so that $\bar{g}(Q) - \underline{g}(Q)$ is constant. For any realization of $\tilde{\varepsilon}$, ρ_1 is then independent of Q (cf. (2)), so varying Q does not affect the informativeness of price. Second, $dG_1(\rho_1, Q)/dQ = 0$ if $d\rho_1/dP = 0$ for all $P \in \text{supp}(h_1)$, or (1.4). This latter condition holds under the MLRP if the support of f is "small enough" so that the price distributions corresponding to the two possible mean demand curves do not overlap and, for any Q, the firm learns the value of γ with probability one. For example, suppose that $g(Q, \bar{\gamma}) = 10 - Q$ and $g(Q, \underline{\gamma}) = 5 - 2Q$ and that $\text{supp}(f) = [-1, 1]$. Then the firm learns the value of γ after observing the realization of P in period one. If we assume that f has support on the entire real line, then this second way to yield $dG_1/dQ = 0$ is precluded.

Condition (1.1) thus indicates that a necessary condition for experimentation is for the firm to be able to alter information by adjusting quantity. A second necessary condition for $dW_1/dQ \neq 0$ and hence experimentation is that alterations in information must be useful for the firm. We shall say that information is *useless* if $dQ_i^*/d\rho_j = 0$ for all $(\rho_1, \rho_j) \in [0,1]^2$ and all i and j, or (1.2). Hence, information is useless if optimal second-period actions do not depend upon information. Lemma 1 then states that the firm will not experiment if information is useless. Straightforward calculations show that $U''(\rho_1) = 0$ on [0,1] if information is useless, giving $dW_1/dQ = 0$.

A comparison of the conditions under which information is useless in duopolistic and monopolistic markets may help clarify the role of the usefulness of information in creating incentives to experiment:[10]

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[10] If firm 1 is the only firm in the industry, so that the analysis of monopoly experimentation in Mirman, Samuelson and Urbano (1989a) applies, then information is useless if $dQ_1^*/d\rho_1 = 0$.

Example 1. Let $g(Q,\bar{\gamma}) = 60 - 10Q$ and $g(Q,\underline{\gamma}) = 27 - Q^2$. It is easily verified that the optimal output for a monopolist is 3 for either demand curve (and for any expectation concerning the likely demand curve), so that information is useless for a monopolist. Suppose now that there are two firms in the market and that they have a common belief ρ that $\gamma = \bar{\gamma}$. Then it is easy to verify that each firm would produce an output of 2 if $\rho = 1$ and an output of $(3/2)^{3/2}$ if $\rho = 0$, so that equilibrium outputs are not independent of beliefs and hence information is useful in the duopoly setting. We thus have a demand structure which makes information useful to duopolists but not to a monopoly, creating incentives for the former but not the latter to experiment. However, if we replace $g(Q,\underline{\gamma})$ with $32-Q^2$, then duopolists would find information useless and shun experimentation, whereas a monopolist would find information useful and would experiment. ■

An implication of Example 1 is that the usefulness of information is not solely a characteristic of the demand specification but also depends on the number of firms and whether they cooperate. In particular, firms which behave noncooperatively may find information useful, whereas information may become useless if they collude. This is not the result of the firms gaining information if they collude, since prior beliefs are assumed to be known by both firms and perhaps to be identical. Rather, it is due to the different strategic forces and hence output levels under cooperation and noncooperation.

(3.4) Quantity Manipulation

We now examine the conditions under which $dW_1/dQ > [<] \ 0$, so that experimentation introduces incentives for the firm to increase [decrease] quantity. To interpret the conditions, it is helpful to observe that if $\bar{g}'(Q) - \underline{g}'(Q) > 0$ for all Q, then larger quantities yield a more informative price signal. Formally, if $\hat{Q} > Q$, then the information structure induced by $\hat{Q}$ is *sufficient* for that induced by Q in the sense of Blackwell (1951). Intuitively, if $\bar{g}'(Q) > g'(Q)$, then an increase in Q spreads the two (expected) demand curves apart, so that it is easier to distinguish between them. Similarly, if $\bar{g}'(Q) - g'(Q) < 0$ for all Q, then smaller outputs yield a more informative price signal. Then:

Proposition 1. Let the MLRP (cf. (3)) hold. Then $dW_1(\hat{Q})/dQ > [<] \ 0$ if

(1.1) $\bar{g}'(\hat{Q}) - g'(\hat{Q}) > [<] \ 0$;

(1.2) $\text{supp } f(P-\bar{g}(\hat{Q})) \cap f(P-g(\hat{Q})) \neq \emptyset$; and

(1.3) $U_1''(\rho_1) > 0$ for all ρ_1;

where a sufficient condition for (1.3) is

(1.4) V^1 is convex in (ρ_1, ρ_2) with at least one nonzero second partial and $(\rho_1^0-\rho_2^0)\ dV^1/d\rho_2 \geq 0$.

Condition (1.3) ensures that information is valuable by requiring the period-two value function to be strictly convex. Condition (1.2) ensures that price does not always reveal the true state of demand with certainty, so that experimentation potentially increases the flow of information. Condition (1.1) indicates that the firm will then experiment by adjusting quantity in a direction which pushes the mean demand curves further apart and hence makes price more informative.

Proof. The monotone likelihood ratio property implies that $d\rho_1/dP \geq 0$ for all P. Condition (1.2), the MLRP and the continuous differentiability of f then imply that $d\rho_1/dP > 0$ on a set of prices with positive probability. The result now follows from (1,1), (1.2), and equations (14) and (15). To see that (1.4) is a sufficient condition for (1.3), note that the first three terms of (15) are a quadratic form involving second derivatives of V^1, and that the sign of φ'' is the same as the sign of $(\rho_1^0-\rho_2^0)$. ■

In the monopoly case, the requirement that the second period value function be convex (given by (1.3)) is superfluous. As we have noted, the firm's second period value function, being the supremum of a collection of linear functions of ρ, is always convex and is strictly convex whenever information is useful. In the duopoly case, however, each firm's second period value function is derived from payoff functions that are linear in (ρ_1, ρ_2) but is no longer the *supremum* of a collection of such functions, since the other firm's output enters as an argument in its payoff function. It is thus possible for information to be useful and for $U_1''(\rho_1)$ to be positive for some values of ρ_1 and negative for other values. Hence, it is possible for information to be useful, conditions (1.1) and (1.2) of Proposition 1 and the MLRP to hold, and yet for (1.3) to fail and hence for $dW_1/dQ = 0$ for some value of Q_1.[11]

To illustrate the conditions under which (1.4) holds, we can note that the simplest specification which gives convexity of V^1 is $g(Q,\gamma) = \gamma - Q$, where $\bar{\gamma} > \underline{\gamma}$ and $\bar{\gamma} < 2\underline{\gamma}$. (The latter inequality ensures an interior solution for both firms.) For this case of parallel, linear demand curves with intercept uncertainty, it is easy to verify that

$$V^1(\rho_1, \rho_2) \quad = \quad (2\gamma_1 - \gamma_2)^2/9, \tag{16}$$

where $\gamma_i = \bar{\gamma}\rho_i + \underline{\gamma}(1-\rho_i)$. (16) is clearly convex in (ρ_1, ρ_2) with $d^2V^1/d\rho_i^2 > 0$ for $i = 1,2$.[12] Moreover, $dV^1/d\rho_2 < 0$, so that (1.4) holds for $\rho_2^0 \le \rho_1^0$.

The convexity of V^1 in (ρ_1, ρ_2), required by (1.4), holds for the preceding example but is a stringent assumption. The following example shows that the convexity of V^1 may fail in our duopoly game.

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[11] An example may be constructed illustrating this possibility using our specification in Example 2 below.

[12] Notice, however, that while information is valuable in this specification, changing quantity cannot effect the informativeness of price under our assumption of an additive noise disturbance that is independent of Q. Hence, neither firm would adjust its best reply map from the myopic case. Experimentation may occur with these mean demand curves if the noise disturbance is not independent of Q. See Creane (1989).

Example 2. Let $g(Q,\gamma) = \alpha - \gamma Q$, where $\bar{\gamma} < \underline{\gamma}$, so that demand curves are linear with a common vertical intercept and unknown slope. Letting $\gamma_j = \bar{\gamma}\rho_j + \underline{\gamma}(1-\rho_j)$, it is easily verified that period-two equilibrium outputs are given by

$$Q_1^* = \frac{\alpha(2\gamma_2 - \gamma_1)}{3\gamma_1\gamma_2} \tag{17}$$

$$Q_2^* = \frac{\alpha(2\gamma_1 - \gamma_2)}{3\gamma_1\gamma_2}, \tag{18}$$

whenever $2\gamma_2 \geq \gamma_1 \geq \gamma_1/2$, so that

$$V^1(\rho_1,\rho_2) = \frac{\alpha^2}{9\gamma_1\gamma_2^2}(2\gamma_2-\gamma_1).$$

Thus,

$$\frac{d^2V^1}{d\rho_1^2} = \frac{\alpha^2}{9}(\bar{\gamma} - \underline{\gamma})^2 8\gamma_1^{-3} > 0$$

$$\frac{d^2V^1}{d\rho_1 d\rho_2} = -\frac{\alpha^2}{9}(\bar{\gamma} - \underline{\gamma})^2 2\gamma_1^{-3} < 0$$

and

$$\frac{d^2V^1}{d\rho_2^2} = \frac{2\alpha^2}{9}(\bar{\gamma} - \underline{\gamma})^2\gamma_2^{-4}(3\gamma_1 - 4\gamma_2) > [<] \quad 0$$

as $(3\gamma_1 - 4\gamma_2) > [<] \; 0$. Observe now that if priors are identical, so that $\rho_1 = \rho_2$, then $d^2V^1/d\rho_2^2 < 0$, so that V^1 is not convex on $[0,1]^2$. ■

It is straightforward to use the results of Proposition 1 to construct cases in which experimentation leads firms to increase or decrease output. However, Proposition 1 treats only cases in which information is valuable, so that the firm is always attempting to increase the informativeness of price. An interesting question is then whether a firm would ever adjust its output to reduce information. The next example demonstrates that a firm may reduce its output in order to reduce information. In the process, it should be clear that a counterpart to Proposition 1 for cases in which information has negative value is easily constructed by reversing the inequality signs in the first line of Proposition 1 and in (1.3) and suitably altering (1.4). The equilibrium analysis of Section IV develops conditions under which information has a positive or negative value.

Example 3. Consider the random slope specification of Example 2. In this case, $\bar{g}'(Q) > \underline{g}'(Q)$, so that, if the MLRP holds, a higher output leads to a more informative price signal. Suppose that $\tilde{\varepsilon}$ is uniformly distributed on $[-1,1]$.[13] Then each firm's posterior beliefs are given by

$$\rho_1(P,Q) = \begin{cases} 1 & \text{if } P \in (\alpha-\underline{\gamma}Q+1,\ \alpha-\bar{\gamma}Q+1] \\ \rho_1^0 & \text{if } P \in [\alpha-\bar{\gamma}Q-1,\ \alpha-\underline{\gamma}Q+1] \\ 0 & \text{if } P \in [\alpha-\underline{\gamma}Q-1,\ \alpha-\bar{\gamma}Q-1) \end{cases}$$

provided that $\alpha - \bar{\gamma}Q - 1 < \alpha - \underline{\gamma}Q + 1$, or $(\underline{\gamma}-\bar{\gamma})Q < 2$. (This inequality may be assured in a neighborhood of the myopic equilibrium industry output by choosing a sufficiently small α.) From (18), $V^1(\rho_1,\rho_2) = \alpha^2(2\gamma_2-\gamma_1)^2/9\gamma_1\gamma_2^2$. Since $\tilde{\varepsilon}$ is uniformly distributed on $[-1,1]$, we have from (12) that

$$W_1(Q) = \frac{\alpha^2}{9\gamma_1^0(\gamma_2^0)^2}(2\gamma_2^0-\gamma_1^0)^2 + \frac{\alpha^2}{18}(\underline{\gamma}-\bar{\gamma})Q\left[\frac{\rho_1^0}{\bar{\gamma}} + \frac{(1-\rho_1^0)}{\underline{\gamma}} - \frac{(2\gamma_2^0-\gamma_1^0)^2}{\gamma_1^0(\gamma_2^0)^2}\right], \tag{19}$$

where $\gamma_j^0 = \rho_j^0\bar{\gamma} + (1-\rho_j^0)\underline{\gamma}$. Since $\underline{\gamma} - \bar{\gamma} > 0$, (19) is decreasing in Q whenever the bracketed expression is negative. The simplest way to make this expression negative is to let $\gamma_2^0 = 2\gamma_1^0$. From (17), this implies that the myopic equilibrium involves firm 2 producing zero output. Under this assumption, the bracketed expression in (19) becomes

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[13] The uniform distribution does not satisfy the strong version of the MLRP we use, but it does satisfy a weaker version and can be approximated arbitrarily closely by distributions that satisfy the version we use.

$$\frac{\rho_1^0}{\bar{\gamma}} + \frac{(1-\rho_1^0)}{\underline{\gamma}} - \frac{9}{4\gamma_1^0} = \frac{4\rho_1^0\underline{\gamma}\gamma_1^0 + (1-\rho_1^0)\bar{\gamma}4\gamma_1^0 - 9\bar{\gamma}\underline{\gamma}}{4\bar{\gamma}\underline{\gamma}\gamma_1^0}. \tag{20}$$

Hence, we will have our example in which the experimenting firm reduces output in order to reduce information if we can find $\bar{\gamma}$, $\underline{\gamma}$, α, ρ_1^0 and ρ_2^0 such that (20) is negative, $\gamma_2^0 = 2\gamma_1^0$, and $\alpha < 4\bar{\gamma}/(\underline{\gamma}-\bar{\gamma})$. (The final inequality ensures that the myopic equilibrium industry output satisfies $Q < 2/(\underline{\gamma}-\bar{\gamma})$ and hence that an increase in Q will increase information.) The following values satisfy these constraints:

$$\begin{aligned} \alpha &= 1/3 & \qquad \rho_1^0 &= 2/3 \\ \underline{\gamma} &= 5 & \qquad \rho_2^0 &= 1/12 \\ \bar{\gamma} &= 1. \end{aligned}$$

■

Example 3 is our first result to make use of noncommon priors. Example 4 below shows that an analogous result can be obtained with common priors in a somewhat more complex demand specification.

IV. Equilibrium: Characterization

The preceding analysis has examined the effect of the ability of firms to manipulate the informativeness of price on their first-period best reply mappings. We now investigate the effect of such manipulations on equilibrium period-one quantities.

(4.1) Quantity-Increasing Experimentation

We restrict attention to the case of equal prior beliefs. Second period beliefs will then also be identical, so that the (assumed unique) second-period equilibrium will be symmetric. This in turn implies that the payoff functions given in (5) for the game Γ will be symmetric, so that the best reply mappings of the two firms are identical. Let σ_s denote this common best reply mapping for Γ when $\rho_1^0 = \rho_2^0$. Let Q_s^{**} denote the (symmetric) period-one equilibrium output for each firm in the two-period game and let $\tilde{Q}_s$ denote the myopically-optimal period-one output.

Lemma 2, which follows immediately from Proposition 1, now implies that if each firm's second-period value function is strictly convex in the (common) posterior belief parameter, then in a symmetric equilibrium, the presence of intertemporal information flows will push industry output in whichever direction generates more information. Lemma 2 also contains conditions under which this direction is known to be either an increase or decrease in quantity.

Lemma 2. Let $0 \neq \rho_1^0 = \rho_2^0 \neq 1$ and let the MLRP (as given in (3)) hold. Then $\tilde{Q}_s < [>] Q_s^{**}$ for all $Q_s^{**} \in \sigma_s(Q_s^{**})$ if

(2.1) $\bar{g}' > [<] \underline{g}'$ on supp $(\bar{g})$;

(2.2) supp $(f(P-\bar{g}(2\tilde{Q}_s)) \cap$ supp $(f(P-\underline{g}(2\tilde{Q}_s)) \neq \emptyset$;

(2.3) $d^2V^i(\rho,\rho)/d\rho^2 > 0$ for all $\rho \in (0,1)$ and $i = 1,2$.

Condition (2.3) imposes the requirement that the second-period value function is strictly convex and hence that information is useful. Condition (2.2) ensures that the price does not always reveal the true state of demand with certainty in the myopic equilibrium, so that there is some scope for experimentation. Condition (2.1) then indicates that the firm will adjust output in that direction which spreads the two mean demand curves farther apart.

The following result demonstrates that, if $\bar{g}(Q)$ and $\underline{g}(Q)$ are both linear in Q and have different horizontal intercepts, then condition (2.3) holds and hence information is valuable.

Proposition 2. If $\bar{g}(Q)$ and $\underline{g}(Q)$ are both linear in Q with $\bar{g}(0)/\bar{g}'(0) \neq \underline{g}(0)/\underline{g}'(0)$, then (2.3) of Lemma 2 holds. Hence, given the MLRP and (2.2), the sign of $Q_s^{**} - \tilde{Q}_s$ for any $Q_s^{**} \in \sigma_s(Q_s^{**})$ mataches the sign of $\bar{g}'(Q) - \underline{g}'(Q)$.

Proof. If both demand functions are linear, they must be of the form $\bar{g}(Q) = \bar{\alpha} - \bar{\beta}Q$ and $\underline{g}(Q) = \underline{\alpha} - \underline{\beta}(Q)$. It is then readily verified that, under identical beliefs, the second period value function for each firm is

$$V^1(\rho,\rho) = \frac{(\bar{\alpha}\rho + \underline{\alpha}(1-\rho))^2}{9(\bar{\beta}\rho + (1-\rho)\underline{\beta})},$$

which is strictly convex in ρ if $\bar{\alpha}/\bar{\beta} \neq \underline{\alpha}/\underline{\beta}$ (equivalently, $\bar{g}(0)/\bar{g}'(0) \neq \underline{g}(0)/\underline{g}'(0)$). ∎

Hence, information has positive value when demand curves are linear with different horizontal intercepts. Since Aghion, Espinosa and Jullien (1990) analyze price-setting duopolists with identical priors, Lemma 2 is the homogeneous-products, quantity-setting analogue to their Propositions 2 and 5.

(4.2) Quantity-Decreasing Experimentation

Lemma 2 deals with the case in which information has positive value, as does Proposition 1. As is the case with Proposition 1, we can create a version of Lemma 2 for the case in which information has negative value by reversing the inequalities in the second line of the Lemma and in (2.3). We illustrate this result by presenting an example in which equilibrium period 1 output in the two-period game is lower than the myopic equilibrium output, even though lower output implies a less informative price signal. The example is reconciled with Proposition 2 by noting that the example contains nonlinear demand curves, contrary to the assumptions of Proposition 2.

Example 4. Let $b \in \{\underline{b},\bar{b}\}$. Let $b = \bar{b}$ with probability ρ. Then let expected demand be given by

$$P(Q) = \frac{1}{Q^2} + \frac{1}{Q} - \hat{b}Q$$

where $Q = Q_1 + Q_2$ and $\hat{b} = \rho\bar{b} + (1-\rho)\underline{b}$. If ρ is the common posterior then firm i's first order condition for period-two profit maximization is given by

$$\frac{1}{Q^2} + \frac{1}{Q} - \hat{b}Q - Q_1\left[\frac{2}{Q^3} + \frac{1}{Q^2} - \hat{b}\right] = 0.$$

This condition immediately implies that if there is an equilibrium, it must be symmetric, and there is at most one symmetric equilibrium. Solving this first order condition under the hypothesis of symmetry gives

$$Q_1^* = .5(3\hat{b})^{-.5}.$$

One readily verifies that $\pi^1(Q_1,Q_j^*,\rho_1)$ is strictly concave in Q_1, and hence Q_1^* is an equilibrium, if $\hat{b} < 1/12$. The value function can then be calculated to be

$$V(\rho) = 1/3 + .5(3\hat{b})^{.5}.$$

This is globally concave in ρ. This creates an incentive for each firm to decrease output in order to decrease information, leading to (symmetric) period-one outputs which are lower than their myopic counterparts. ■

It is interesting to note that we can generate an analogous result with linear demand curves if we allow prior beliefs to differ:

Example 5. Consider the random slope specification of Example 3 with $\tilde{\varepsilon}$ distributed uniformly on [-1,1]. The myopic period one outputs are

$$\tilde{Q}_1 = \frac{\alpha(2\gamma_2^0 - \gamma_1^0)}{3\gamma_1^0\gamma_2^0} \qquad \tilde{Q}_2 = \frac{\alpha(2\gamma_1^0 - \gamma_2^0)}{3\gamma_1^0\gamma_2^0}$$

where $\gamma_j^o = \bar{\gamma}\rho_j^0 + \underline{\gamma}(1-\rho_j^0)$, so that aggregate output is

$$\tilde{Q}_1 + \tilde{Q}_2 = \frac{\alpha(\gamma_2^0 + \gamma_1^0)}{3\gamma_1^0\gamma_2^0}.$$

The payoff functions for the game Γ are given by

$$\Pi^1(Q_1,Q_2) = [\alpha - \gamma_1^0(Q_1+Q_2)]Q_1 + \delta A^1 + \delta(Q_1+Q_2)B^1$$

$$\Pi^2(Q_1,Q_2) = [\alpha - \gamma_2^0(Q_1+Q_2)]Q_2 + \delta A^2 + \delta(Q_1+Q_2)B^2,$$

where

$$A^1 = \frac{\alpha^2(2\gamma_2^0 - \gamma_1^0)^2}{9\gamma_1^0(\gamma_2^0)^2}$$

$$A^2 = \frac{\alpha^2(2\gamma_1^0 - \gamma_2^0)^2}{9\gamma_2^0(\gamma_1^0)^2}$$

$$B^1 = \frac{\alpha^2}{18}(\underline{\gamma}-\bar{\gamma})\left[\frac{\rho_1^0}{\bar{\gamma}} + \frac{(1-\rho_1^0)}{\underline{\gamma}} - \frac{(2\gamma_2^0-\gamma_1^0)^2}{\gamma_1^0(\gamma_2^0)^2}\right]$$

$$B^2 = \frac{\alpha^2}{18}(\underline{\gamma}-\bar{\gamma})\left[\frac{\rho_2^0}{\bar{\gamma}} + \frac{(1-\rho_2^0)}{\underline{\gamma}} - \frac{(2\gamma_1^0-\gamma_2^0)^2}{\gamma_2^0(\gamma_1^0)^2}\right].$$

The period-one equilibrium outputs for this game are

$$Q_1^{**} = \frac{2(\delta B^1+\alpha)\gamma_2^0 - (\delta B^2+\alpha)\gamma_1^0}{3\gamma_1^0\gamma_2^0}$$

$$Q_2^{**} = \frac{2(\delta B^2+\alpha)\gamma_1^0 - (\delta B^1+\alpha)\gamma_2^0}{3\gamma_1^0\gamma_2^0},$$

so that equilibrium industry output is

$$Q_1^{**} + Q_2^{**} = \frac{\alpha(\gamma_2^0+\gamma_1^0) + \delta B^1\gamma_2^0 + \delta B^2\gamma_1^0}{3\gamma_1^0\gamma_2^0}$$

$$= \tilde{Q}_1 + \tilde{Q}_2 + \frac{\delta B^1\gamma_2^0 + \delta B^2\gamma_1^0}{3\gamma_1^0\gamma_2^0}.$$

Thus $Q_1^{**} + Q_2^{**} < \tilde{Q}_1 + \tilde{Q}_2$ if $B^1\gamma_2^0 + B^2\gamma_1^0 < 0$, where $B^1\gamma_2^0 + B^2\gamma_1^0$ is given by

$$\frac{\alpha^2(\underline{\gamma} - \bar{\gamma})}{18\bar{\gamma}\underline{\gamma}\gamma_1^0\gamma_2^0}\left[\gamma_1^0\gamma_2^0\left[(\rho_1^0\underline{\gamma}+(1-\rho_1^0)\bar{\gamma})\gamma_2^0+(\rho_2^0\underline{\gamma}+(1-\rho_2^0)\bar{\gamma})\gamma_1^0\right]-\underline{\gamma}\bar{\gamma}\left[(2\gamma_2^0-\gamma_1^0)^2+(2\gamma_1^0-\gamma_2^0)^2\right]\right].$$

One can verify that the following values yield the desired result:

$$\alpha = 1\backslash 10 \qquad \bar{\gamma} = 1$$

$$\rho_1^0 = 2/3 \qquad \underline{\gamma} = 5$$

$$\rho_2^0 = 1/12.$$

■

Examples 4 and 5 demonstrate that Proposition 2 is not robust to relaxation of the linear demand and common prior assumptions, so that (2.3) can fail and firms seek to decrease information if demand is nonlinear or priors not common.

V. Equilibrium: Existence

We now investigate the conditions under which an equilibrium exists. Existence in pure strategies is problematic. Without severe restrictions on the demand functions and the distribution of the noise term, the value functions V^i may not be concave in first period output. As a result, the payoff functions in (5) may not be quasiconcave in the firms' outputs (even though the first period payoff function is concave), which in turn may preclude existence.

Since the first period best reply mappings will not in general be single-valued (because V^i may not be concave in first period output), we examine mixed strategies. To reformulate our model accordingly, let D denote the space of (Borel) probability measures on $\mathbb{R}_+$. A mixed strategy for firm i is an element $\mu_i \in D$. (As we show in the proof of Proposition 3 below, we may restrict our attention to probability measures on a compact subset of $\mathbb{R}_+$.) Firm 1's payoff function is now a mapping from D x D into $\mathbb{R}$ given by

$$\iint \pi^1(Q_1, Q_2, \rho_1^0) d\mu_1(Q_1) d\mu_2(Q_2) + \delta \iint W_1(Q_1+Q_2) d\mu_1(Q_1) d\mu_2(Q_2) \tag{20}$$

where π^1 is given in (1), W_1 is given in (12), and $(\mu_1, \mu_2) \in DxD$. A *mixed strategy equilibrium* for Γ is a pair $(\mu_1^*, \mu_2^*) \in DxD$ such that,

$$\iint \pi^1(Q_1, Q_2, \rho_1^0) d\mu_1^*(Q_1) d\mu_2^*(Q_2) + \delta \iint W_1(Q_1+Q_2) d\mu_1^*(Q_1) d\mu_2^*(Q_2)$$

$$\geq \quad \iint \pi^1(Q_1, Q_2, \rho_1^0) d\mu_1(Q_1) d\mu_2^*(Q_2) + \delta \iint W_1(Q_1+Q_2) d\mu_1(Q_1) d\mu_2^*(Q_2),$$

for all $\mu_1 \in D$, with a similar condition for firm 2.

We have the following existence theorem:

Proposition 3. Suppose that there exists a $\hat{Q}$ such that, for $g = \bar{g}$ and $g = \underline{g}$,

(3.1) supp $(g) \subseteq [0,\hat{Q}]$,

(3.2) g is concave and strictly decreasing on $[0,\hat{Q}]$,

(3.3) g is twice continuously differentiable on $(0,\hat{Q})$,

(3.4) g is constant on $(\hat{Q},\infty)$.[14]

Then Γ has a subgame perfect equilibrium.

Proof. The result follows from standard existence proofs after establishing continuity of the payoff functions and compactness of the strategy sets (since the payoff function (21) is quasi-concave on the set of mixed strategies, D).

To demonstrate compactness of strategy sets, observe that $\bar{g}$ and $\underline{g}$ are nonpositive on $(\hat{Q},\infty)$ and that changes in output cannot affect the informativeness of price on $(\hat{Q},\infty)$. We may thus without loss of generality take each firm's pure strategy set to be the closed interval $[0,\hat{Q}]$.

To demonstrate continuity of the payoff functions given in (5) in first period quantities, note first that (3.1)-(3.4) imply the existence of a unique second-period equilibrium for each (ρ_1,ρ_2), say $Q_1^*(\rho_1,\rho_2)$ and $Q_2^*(\rho_1,\rho_2)$. By the conditions imposed on g, Q_i^* is continuous in (ρ_1,ρ_2) for $i = 1,2$.[15]

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[14]By supp (g) we mean the set of quantities on which g specifies positive prices. To achieve the conditions of Proposition 3 it may be necessary to allow $\bar{g}$ or $\underline{g}$ to specify some negative expected prices (recall that price can be thought of as being net of marginal cost) and to become horizontal (at a zero or negative price) after Q.

[15]If $Q_1^*(\rho_1,\rho_2)$ is not continuous, then there exists a converging sequence of values of (ρ_1,ρ_2), with limit (ρ_1',ρ_2'), for which the associated sequence of

Moreover, since h is continuous, ρ_1 and ρ_2 are continuous functions of first period output, so that V^i is continuous in first-period output. Hence, Π^i in (5) is continuous, for i = 1,2.

The proposition now follows from an application of the existence theorem in Glicksberg (1952), who demonstrates that continuity of the payoff functions and compactness of the strategy sets are sufficient for the existence of a mixed strategy equilibrium. ∎

In order to simplify the exposition, sections III and IV considered only pure strategies. Given that we can only ensure existence with the help of mixed strategies, it is useful to note that the previous analysis may be modified to accommodate mixed strategies. For example, in Proposition 1 we demonstrated that if W_i is increasing (say) in Q, then every element of firm i's best reply mapping for Γ must exceed firm i's (single-valued) myopic best reply. If we permit mixed strategies, observe that a firm's myopic best reply is a degenerate probability measure. Then if W_i is increasing in Q, every element in the support of firm i's best (mixed strategy) reply for Γ must exceed firm i's myopic best reply (for a given mixed strategy of the other firm). Hence, the extension of Proposition 1 to mixed strategies is immediate.

In Lemma 2, we demonstrated that if prior beliefs are identical, then the equilibrium industry output for Γ in a symmetric equilibrium exceeds industry output in the myopic game if each firm's second period value function is strictly convex in the common belief parameter and if increasing industry output increases the informativeness of the price signal. The generalization to mixed strategies is that, under the foregoing conditions, the support of

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Q_i^* does not converge. Then given the compactness of strategy sets, the latter sequence must contain two converging subsequences with distinct limits. Each limit is then part of a period-two equilibrium for beliefs (ρ_1', ρ_2'), contradicting our uniqueness assumption (cf. A4).

the probability distribution for industry output under any symmetric mixed strategy equilibrium of Γ cannot lie entirely below the industry output in the one-period game. If $\bar{g}$ and $\underline{g}$ are both linear, then the *expected* industry output under any symmetric mixed strategy equilibrium of Γ cannot be less than the equilibrium output of the single period game. This conclusion follows from the mixed-strategy version of Proposition 1 after observing that, if the demand functions are linear, then the single-period best response depends only on the *expected* output of one's rival.[16]

VI. Conclusion

We have examined a duopoly market in which firms can experiment, or adjust their period-one quantities away from myopic optima in order to alter the informativeness of price. Our most significant departure from previous signal-jamming studies lies in our assumption that firms can observe their rival's outputs. We think that this is a realistic assumption in at least some markets.

This assumption has a dramatic effect on the incentives facing the firms. In traditional models of this type, firms alter their quantities in order to push their rival's posterior beliefs in a particular direction. In our analysis, firms cannot systematically affect the direction of rival belief

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[16]To see this, let $\tilde{Q}_s$ denote the first-period myopic equilibrium output of each firm. (There is a unique symmetric myopic equilibrium of the single-period game given common priors.) Let (μ_1^*, μ_2^*) denote a symmetric mixed strategy equilibrium of Γ. If $\int Q_s d\mu_s^*(Q_s) < \tilde{Q}_s$, then the myopic best response of firm 1 to μ_s^* is to produce some $Q_1' > \tilde{Q}_1$ with probability 1 if the demand functions are linear (since the single-period best response is a decreasing function of the expected output of firm 2 if the demand functions are linear). Moreover, under the conditions listed in the text, $W_1(Q)$ is increasing in Q. Consequently, by the extension of Proposition 1 to mixed strategies, the support of firm 1's best reply to μ_s^* in Γ must lie above $\tilde{Q}_1$ and therefore above $\int Q_s d\mu_s^*(Q_s)$, implying that (μ_1^*, μ_2^*) is not an equilibrium.

revision, but can affect the degree of belief revision by affecting the informativeness of price.

We examine a duopoly market, unlike previous models of monopoly experimentation. The effect of this difference is again dramatic. Information always has nonnegative value to a monopolist. In contrast, firms in a duopoly market may strictly prefer to have less information.

We first establish necessary conditions for experimentation to occur (Lemma 1), including that second period actions must depend upon information and information must in turn depend upon first-period price. While these conditions are straightforward, we present examples in which duopoly firms may face incentives to experiment but a monopoly (or colluding duopoly) in the same market would not experiment, and vice versa. We then establish conditions under which experimentation will introduce incentives for firms to either increase or decrease quantity, under the presumption that information has positive value (Proposition 1). Intuitively, the firm will adjust output so as to spread the mean demand curves farther apart, thus making price more informative. The presumption that information has positive value is nontrivial, and we present examples to show that information may have negative value and that firms may decrease output in order to decrease the informativeness of price. Finally, Lemma 2 and Proposition 2 extend Proposition 1 to equilibrium outputs, in the process developing conditions under which information is valuable.

Our results contribute to the understanding of how informational concerns can affect strategic interactions between firms. Our results suggest that the nature of these informational concerns can depend critically on the structure of the market. In particular, the incentives to produce more or less information can differ depending upon how many firms occupy the market and on the variables these firms can observe.

VII. Supplemental Notes: Derivation of (14)

To begin, note from (6) that

$$dh_1/dQ \quad = \quad -\rho_1^0\bar{f}'\bar{g}' - (1-\rho_1^0)\underline{f}'\underline{g}', \tag{22}$$

so that (13) becomes

$$dW_1/dQ \quad = \quad \int\left(U_1'(\rho_1)\frac{d\rho_1}{dQ}h_1(P) - U_1(\rho_1)[\rho_1^0\bar{f}'\bar{g}'+(1-\rho_1^0)\underline{f}'\underline{g}']\right)dP. \tag{23}$$

After integrating the second term in (23) by parts, we obtain

$$dW_1/dQ \quad = \quad \int U_1'(\rho_1)\left[\frac{d\rho_1}{dQ}h_1(P) + [\rho_1^0\bar{f}\bar{g}'+(1-\rho_1^0)\underline{f}\underline{g}']\frac{d\rho_1}{dP}\right]dP. \tag{24}$$

The next task is to simplify the expression in brackets. Observe from (7) that

$$\rho_1 h_1 \quad = \quad \bar{f}\rho_1^0. \tag{25}$$

Differentiating (25) with respect to P and with respect to Q yields

$$\frac{d\rho_1}{dP} \quad = \quad \frac{\rho_1^0\bar{f}' - (dh_1/dP)\rho_1}{h_1}, \tag{26}$$

and

$$\frac{d\rho_1}{dQ} \quad = \quad \frac{-\rho_1^0\bar{f}'\bar{g}' - \rho_1(dh_1/dQ)}{h_1}. \tag{27}$$

Since $dh_1/dP = \rho_1^0\bar{f}' + (1-\rho_1^0)\underline{f}'$, we have from (22) that

$$\frac{dh_1}{dQ} \quad = \quad -\bar{g}'\frac{dh_1}{dP} + \underline{f}'(1-\rho_1^0)(\bar{g}'-\underline{g}'). \tag{28}$$

Inserting (28) into (27) and then using (26) gives

$$\frac{d\rho_1}{dQ} \quad = \quad -\bar{g}'\frac{d\rho_1}{dP} - [\rho_1\underline{f}'(1-\rho_1^0)(\bar{g}'-\underline{g}')]/h_1. \tag{29}$$

Inserting (29) into (24) now gives

$$\frac{dW_1}{dQ} \quad = \quad \int U_1'(\rho_1)\frac{d\rho_1}{dP}\left[-\bar{g}'(\rho_1^0\bar{f}+(1-\rho_1^0)\underline{f}) + \rho_1^0\bar{f}\bar{g}' + (1-\rho_1^0)\underline{f}\underline{g}'\right]dP$$

$$- \int U_1'(\rho_1)\rho_1 \underline{f}'(1-\rho_1^0)(\bar{g}'-g')dP$$

$$= \quad -(\bar{g}'-g')\left[\int U_1'(\rho_1)\frac{d\rho_1}{dP}(1-\rho_1^0)\underline{f}dP + \int U_1'(\rho_1)\rho_1\underline{f}'(1-\rho_1^0)dP\right]. \tag{30}$$

We can now integrate the second term in brackets in (30) by parts to give

$$\int U_1'(\rho_1)\rho_1\underline{f}'(1-\rho_1^0)dP \quad =$$

$$U_1'(\rho_1)\rho_1\underline{f}(1-\rho_1^0)\Big]_{-\infty}^{\infty} - \int U_1''(\rho_1)\frac{d\rho_1}{dP}\rho_1\underline{f}(1-\rho_1^0)dP - \int U_1'(\rho_1)\frac{d\rho_1}{dP}\underline{f}(1-\rho_1^0)dP$$

$$= \quad -\int U_1''(\rho_1)\frac{d\rho_1}{dP}\rho_1\underline{f}(1-\rho_1^0)dP - \int U_1'(\rho_1)\frac{d\rho_1}{dP}\underline{f}(1-\rho_1^0)dP. \tag{31}$$

Inserting (31) into (30) yields (14). ∎

REFERENCES

Aghion, P., M. Espinosa, and B. Jullien (1990), "Dynamic Duopoly Games with Learning Through Market Experimentation," mimeo.

Blackwell, D. (1951), "Comparison of Experiments," in J. Neyman (ed.), *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, University of California Press.

Creane, A. (1989), "Monopoly Experimentation with Heteroskedastic Noise," mimeo.

Fudenberg, D. and J. Tirole (1986), "A 'Signal-Jamming' Theory of Predation," *Rand Journal of Economics* 17, 366-376.

Gal-Or, E. (1988), "The Advantages of Imprecise Information," *Rand Journal of Economics* 19, 266-275.

Glicksberg, I. (1952), "A Further Generalization of the Kakutani Fixed Point Theorem, with Application to Nash Equilibrium Points," *Proceedings of the American Mathematical Society* 38, 170-174.

Kamien, M. I., Y. Tauman and S. Zamir (1987), "The Value of Information in a Strategic Conflict," *Games and Economic Behavior* 2, 129-153.

Mirman, L., L. Samuelson and A. Urbano (1989a). "Monopoly Experimentation," mimeo.

Mirman, L., L. Samuelson and A. Urbano (1989b), "Duopoly Signal Jamming," mimeo.

Okuno-Fujiwara, M. A. Postlewaite and K. Suzumura (1990), "Strategic Information Revelation," *Review of Economic Studies* 57, 25-47.

Riordan, M. (1985), "Imperfect Information and Dynamic Conjectural Variations," *Rand Journal of Economics* 16, 41-50.

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