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## **Ownership structure and efficiency**

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## No. 8955

### OWNERSHIP STRUCTURE AND EFFICIENCY: AN INCENTIVE MECHANISM APPROACH

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# Ownership Structure and Efficiency: An Incentive Mechanism Approach

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#### Abstract

We derive optimal incentive mechanisms in two team production models involving problems of freeriding (team moral hazard) and (multi-dimensional) adverse selection. The models, formalized in the framework of a Bayesian game with public communication, differ only in the underlying ownership structures, one characterized by separation of ownership and control, the other by internal collective ownership. Explicit comparison of contractual solutions enables us to conclude that the underlying ownership structure of an organization matters for its aggregate efficiency: When the agents are risk neutral and possess pre-contractual private information, it can be socially more desirable to internalize the ownership rights. We also illustrate the intrinsic problem of "market for lemons" that may arise when the informed parties propose to purchase the ownership rights.

Key words: ownership, information leakage, multi-dimensional adverse selection, team moral hazard, Bayesian game with public communication, public communication equilibrium, Bayesian incentive compatible mechanism with public communication.

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### 1 Introduction

In this paper we reconsider a frequently asked question: "Given a firm that is viewed as a nexus of contracts, does the location of ownership have any import on the firm's aggregate efficiency of resource allocation?" A prevailing answer to this question, which we shall call the *Irrelevance Hypothesis*, is that insofar as the firm can be described as a comprehensive contract, where every contingency is thought of and built into the contract, the ownership is irrelevant to the issue of efficiency; location of ownership rights can only matter when it is impossible to write up such a comprehensive contract at the first date [See, e.g. Hart(1988), Tirole(1988)]. Some have thus turned to less standard models such as incomplete contracts for a better understanding of the role of ownership in firms' performance [e.g. Grossman and Hart (1987)]. However, it would be unfortunate if the important issues of ownership could only be sensibly discussed in non-standard framworks when standard principal-agent models have been fully developed and comprehensive techniques are available for standard analysis.

In fact the Irrelevance Hypothesis begs the important question why the bargaining parties necessarily always arrive at the same agreement concerning resource allocation (to be sure, not the income distribution) in comprehensive contracts when the underlying ownership structure alters. It might be because that the issues of ownership structure and the survival of organizational forms have been primarily discussed in settings with symmetrical pre-contractual information, namely in moral hazard settings, where it has been observed that the principal and the agent(s) always end up with an incentive scheme which is Pareto efficient, irrespective of who designs the incentive scheme See Hart and Holmström (1987)]. Thus saying that ownership does not matter is correct when the contracting parties have symmetrical information at the date when the contract is designed and signed. This statement is obviously incorrect if there exist pre-contractual informational asymmetries. For one thing, in principal-agent models with a single privately informed agent, the second-best incentive mechanisms (with adverse selection) do not lead to efficient resource allocation except for trivial cases. Therefore should the agent be vested with the exclusive authority to decide the sharing rules, he might use his superior knowledge to design a rule that could make both parties better off. This intuition will be the main theme of our discussion. Of course, in single-agent settings, once the agent has the power to set up a payment rule for himself, it no more makes sense to have a principal, since that would imply that the agent is the owner and the manager at the same time. However, when it concerns team production [in the sense of Alchian and Demsetz (1972)], even if the agents have the ownership rights, they still need a residual claimant to solve the budget balancing problem [Holmström(1982)].

We are thus motivated to investigate the ownership issues in more general, but stan-

dard, settings of organization. That is, we shall confine ourselves to viewing the organization or firm as a nexus of comprehensive contracts specifying the contingent profit sharing rules for the organizational members. More specifically, we consider two team production models where each agent pocesses two-dimensional private information, namely pre-contractual private information concerning his "type" and post-contractual private information concerning his "effort". Such models involve simultaneous (team) moral hazard and (multi-dimensional) adverse selection.

In the present context ownership is understood as the right to set up the rules concerning the distribution of the earnings.<sup>1</sup> The first model deals with a standard principal-agents problem in which the residual claimant is the owner who designs the incentive mechanism for the agents. In the second model the agents are the colletive owners of the firm who collectively design the incentive mechanism. In order to compare efficiency implications, we construct these models in such a way that they differ only in the underlying ownership structures.

The models are presented in the framework of a Bayesian game with incomplete information in the sense of Harsanyi (1967-68). In particular, they represent a sub-class of the "generalized principal-agent problems" formalized by Myerson (1982)(1985). In Myerson's model it is assumed that the principal has complete control over all possible means of communication among the players, and it is shown that there is no loss of generality for the owner (mediator) to limit his attention to direct incentive compatible communication mechanisms (DIC mechanism) where each agent reports separately and confidentially his true type to the owner and follows the effort recommendation from the owner.<sup>2</sup> Our model differs from Myerson's in that we assume public communication, i.e. all the messages sent or received by the players are common knowledge. Consequently, direct mechanisms in the sense of Myerson become inapplicable here and the equilibrium concept requires also some modification. We thus introduce the definition of Bayesian incentive compatible mechanism with public communication (PIC mechanism) and public communication equilibrium in next section. One understands that since any public communication equilibrium implies a communication equilibrium but not necessarily the reverse, the optimal solution derived using the former equilibrium concept may not be optimal in the class of all communication mechanisms. However, in the present context, we will show that our solutions are optimal in the general class of mechanisms. In the case of collective ownership, we will further show that the optimal PIC mechanisms can be first best [ex post efficiency in the

<sup>&</sup>lt;sup>1</sup>Indeed, the concept of ownership (or property rights) is always wanting a good definition. Traditional legal definitions of ownership have been either too specific or too vague that can be hardly useful for developing penetrating insights into the survival of ownership structures. Our conception of ownership, perhaps a bit narrow, is in line with that proposed by Grossman and Hart (1987).

<sup>&</sup>lt;sup>2</sup>Such a mechanism is also called canonical communication device by Forges (1986), or simply communication equilibrium by Myerson (1986).

sense of Holmström and Myerson (1983)], — that is, given that each agent chooses effort to maximize his own expected utility, there is no other mechanism that can lead to an equilibrium in which some agents are better off, without making any agent worse off even if the private information of all agents become public knowledge. This is an extension of the standard collective choice results [e.g. d'Aspremont and Gérard-Varet (1979)] with inclusion of a passive residual claimant and (team) moral hazard problems, apart from adverse selection problems.

To repeat, we have a twofold purpose in this paper. First, we are interested in how and what optimal comprehensive incentive mechanisms with public communication should be designed by the owner given a firm's ownership structure, when potential problems of team moral hazard and adverse selection exist. This is of interest in its own right as it can be viewed as a contribution to the incentive contract theory, which has recently attracted much attention. A major part of the paper will thus be devoted to derivations of the optimal incentive mechanisms. Secondly, we discuss normative implications of a firm's ownership structure for its aggregate efficiency viewing the firm as a nexus of comprehensive contracts, and suggest that collective ownership structures with proper insurance arrangement can be a superior organizational form, especially when the organizational members are willing and capable of taking risks.<sup>3</sup> We re-enforce this hypothesis by showing in Section 4 why, if collective ownership with passive residual claimants is superior, the agent do not come to the principal and purchase the ownership rights. We will illustrate that the fundamental reason is a mirror image of Akerlof's "market for lemons": the interest in purchasing the ownership rights by the agents would always convey good news to the principal, because only agents with higher types would have such interest; consequently, the principal would never sell his rights for any price so far as there are agents interested in buying the rights for that price.<sup>4</sup>

The paper proceeds as follows. We introduce the two team production models in next section, with the definition of incentive compatibility and equilibrium under public com-

<sup>&</sup>lt;sup>3</sup>In more concrete models, Pint (1989) uses a similar incentive mechanism approach to compare the efficiency effects of having a monopoly firm owned by the state or by the private shareholders; Roemer and Silvestre (1989) study the welfare implications of various possible ownership structures of a monopolistic firm, using also a similar incentive mechanism approach. In the school of so called "new institutional economics" [see Frey (1989), Pommerehne (1989), and Boardman and Vining (1989), for example], one is also interested in explaining the variety of institutional forms in terms of efficiency, which so far has been mainly descriptive or emperical.

<sup>&</sup>lt;sup>4</sup>Similar problems may arise when the agents jointly select an incentive mechanism without aid of an external representative: preference over a particular set of mechanisms may convey information about the private types. This information leakage problem has motivated Holmström and Myerson (1983) to study the "durable" decision rules. See also Crawford (1985) for a further analysis of the durable decision rules. It turns out that the optimal incentive mechanisms derived under collective ownership in the present context are durable because they are *ex post* Pareto efficient (see Section 4).

munication. In Section 3 we derive explicitly the optimal incentive mechanisms under the two ownership structures. This is done, in each case, by a sequential analysis of the adverse selection and moral hazard problems. We first assume that the agents' effort decisions were perfectly verifiable so that the problems reduce to particular multi-dimensional adverse selection problems. After characterizing the optimal solutions to the particular problems (Propositions 2 and 3), the optimal incentive mechanisms in the more general cases with nonverifiability of effort decisions are constructed using these solutions. We show that when the agents are risk neutral, the optimal sharing rules derived under perfect effort verifiability can be implemented via incentive mechanisms quadratic or linear in the realized joint output levels, hence the potential problems of moral hazard are circumvented (Propositions 4 and 5). In Section 4 we discuss the comparative ownership implications for efficiency of the two models, and illustrate the intrinsic costs which prevent the agents from purchasing the ownership rights when the rights rest initially with the outside investors. Section 5 concludes the paper.

#### 2 The model

Suppose an organization (or firm) is to be formed where *n* agents are to undertake a profitable activity. Each agent, indexed  $i \in N = \{1, \ldots, n\}$ , supplies nonobservable effort  $e^i \in A = [0, B]$  with a private (nonmonetary) cost or disutility  $V^i : (e^i, \theta^i) \in A \times \Theta^i \to V^i(e^i, \theta^i) \in R$ , where  $\theta^i \in \Theta^i = [\underline{\theta}^i, \overline{\theta}^i]$  is a private efficiency parameter, known as the *i*th agent's type. Assume  $V_e^i > 0, V_{ee}^i > 0, V_{ee}^i < 0$ , and  $V_{e\theta}^i < 0.5$  The agents are best thought of as specialized managers and workers, and we assume that none of them is dispensable for the activity. Assume also that it is the common knowledge that the agents' possible types are independently distributed, and their cumulative distribution and density functions are given by  $F^i(\cdot)$  and  $f^i(\cdot)$  on  $\Theta^i [f^i(\cdot) > 0$  on  $\Theta^i$ ]. Let  $e = (e^1, \ldots, e^n)$  and  $\theta = (\theta^1, \ldots, \theta^n)$ . The agents' effort together with some technology (or assets) I and random states of nature determine a monetary outcome. In order to focus on effort allocations we assume that the technology requires a lump sum monetary investment, and let x denote the earnings net of that lump sum capital investment, whose expected value is denoted  $\hat{x} : e \in A^n \to \hat{x}(e) \in R$ . Assume further that the variance of x is independent of e and equal to  $\sigma^2$ . We assume that  $\hat{x}(\cdot)$  is concave on  $A^n$ , increasing in each argument, and twice continuously differentiable.

An essential problem arises as how the profits are to be shared among the agents and the owner(s) of technology. We view this problem as the owner(s) designing the optimal incentive mechanisms (or sharing rules) under informational, individual rationality and, if required, budget balancing constraints. Let  $S^{i}$  denote the share of profits that goes to

<sup>&</sup>lt;sup>5</sup>When there is no ambiguity, we drop out the superscript of variables, e.g. the partial derivative of  $V^i$  w.r.t.  $\theta^i$  is written  $V^i_{\theta}$  instead of  $V^i_{\theta}$ .

the *i*th agent, whose preference functions are assumed to be additively separable in money and effort, and linear in money. The *i*th agent's utility thus writes as  $\pi^i(S^i, e^i, \theta^i) = S^i - V^i(e^i, \theta^i)$ . The profit net of payments to the agents, i.e.  $W = x - \sum_i S^i$ , goes to a risk neutral residual claimant, who is called the principal when he is the owner of the firm. All the agents and the principal are utility maximizers. We assume that the agents act noncooperatively in choosing their effort levels or reporting private types.

We consider incentive mechanisms (to be called incentive mechanisms with public communication) of the form  $(S, e) = ((S^1, e^1), \ldots, (S^n, e^n))$  where  $S^i : (x, \theta) \in R \times \Theta \rightarrow$  $S^i(x, \theta) \in R$  is the share of profits to the *i*th agent, called the incentive contract (or the share function), which is a function of the observed outcome and the reported parameters, and  $e^i : \theta \in \Theta \rightarrow e^i(\theta) \in R$  is the effort level recommended to the *i*th agent, called the effort recommendation.<sup>6</sup> We shall later justify the use of such form of incentive mechanisms by applying the well known "revelation principle" (see Subsection 3.1).

An incentive mechanism determines an internal "rule of the game". The game proceeds as follows. The owner first proposes to the agents an incentive mechanism (S, e). Then each agent anounces a type  $\hat{\theta}^i \in \Theta^i$  to the public. After the anouncement, the agents will know one another's share functions and effort recommendations, i.e. what incentive contract each agent has signed with the owner and what effort level each agent is recommended to make. We call this stage the stage of communication. We assume that when given an incentive mechanism, an agent must decide before the communication stage whether or not he wants to commit himself to playing the game, i.e. to signing the incentive contract.<sup>7</sup> Once the anouncement is made, both the owner and the agents are obliged to accept the incentive contract that become specified given all of the anounced types. In the following stage, each agent makes some private effort and receives a share of profits according to the sharing function when the output x is realized.

This game setting is close to but differs in an important aspect from the settings of "Bayesian game with incomplete information" proposed by Myerson (1982)(1985), where the agents are not allowed to anounce their types to the public but only to a mediator, who then recommends secretly an effort level to each agent. Here, after communication, uncertainty about the other agents' types and effort levels would disappear in a public communication equilibrium (to be defined shortly). With this consideration, it is less obvious that something parallel to the "revelation principle" would still hold which says that the optimal DIC mechanism is also optimal in the class of all possible mechanisms, i.e. we cannot immediately say that an optimal PIC mechanism (to be defined shortly) is

<sup>&</sup>lt;sup>6</sup>To save notation, we use the same term  $S^i$  to denote both an incentive contract and a particular amount of payment to the *i*th agent. Similarly,  $e^i$  may denote an effort recommendation to, or a particular effort level made by the *i*th agent. The context will make their meanings clear.

<sup>&</sup>lt;sup>7</sup>Implicit in our risk neutrality assumption is the absence of institutional constraints such as limited liability laws, thus all the players can commit to playing the game.

a generally optimal mechanism. However, in the present context, it is easy to verify using a round-about approach that there is indeed no loss of generality to restrict attention to PIC mechanisms (see the remark following Proposition 4).

We consider two polar types of ownership structures. Let  $O_p$  denote the firm with separated ownership and control where the owner is the principal.  $O_p$  can be interpreted as a private ownership organization where the owner(s) hires the agents who supply labor inputs. For instance, it may describe a classic capitalistic firm, the owners of which are exclusively the shareholders other than the agents.<sup>8</sup> The shareholders are viewed as a single legal person. The problems that may arise from the possible conflicts between the common shareholders and the decision making authorities, say, the board of directors, are abstracted away. That is, we suppose there is an honest representative who designs the sharing rules on behalf of the shareholders.

Let  $O_a$  denote the firm with overlapped ownership and control where the agents are the exclusive collective owners. The ownership forms nearest to  $O_a$  are partnerships, co-ops, labor managed firms, etc., but  $O_a$  may differ in that it allows outside non-owner residual claimants. Similar to  $O_p$ , we suppose there is a single honest representative of all the agents in  $O_a$  who designs the sharing rules for the organization.<sup>9</sup> Indeed, the problem of optimal incentive mechanism design under collective ownership structure ought to be analyzed as a cooperative game with incomplete information [Myerson(1984)]. This is a complicated issue. However, since our focus in this paper is not how the agents would reach a particular agreement concerning the optimal incentive mechanisms among feasible efficient choices (as will be clear, there are infinite such choices), but what optimal incentive mechanisms they would choose, the expected utility level of any individual agent is not important. The following scenario may justify this supposition.

"The agents first bargain over an incentive mechanism. Then an outsider, say, an unknown academic economist, will be invited to finally set up a sharing rule for their organization. The agents will pay the economist a constant amount of salary (presumably negligible, or equals the amount the principal would pay under  $O_p$ ) if he succeeds in designing a sharing rule which is *ex post* efficient and makes all agents at least weakly better off *ex ante* compared with their own bargaining solution (in the case of nonexistence of a bargaining solution, all the agents should be better off compared with their reserved expected utility levels for participation), otherwise he receives nothing. There may be some fairness criterion that the agents would agree to suggest, but if they cannot reach any agreement regarding this matter it will be up to the economist to decide, whose political

<sup>&</sup>lt;sup>8</sup>The case of mixed ownership with managers and workers being also shareholders is certainly an interesting subject which deserves being listed in the future research agenda. See the ending remarks.

<sup>&</sup>lt;sup>9</sup>In the incentive literature, such person is usually called a *government* or *central agency* [e.g. Groves and Ledyard (1977), d'Aspremont and Gérard-Varét (1979)].

viewpoint is presumably known by non of the agents. All the agents commit themselves to respecting the sharing rule the economist would design, provided the rule meets their provision. The economist will be isolated from all agents until he brings forward a sharing rule so that there is no chance for any one to influence his decision."

Since the agents utilities are transferable in this context, there is no difficulty for this economist to accomplish his task (see Propositions 3 and 5). Therefore, we may simply refer to the representative of the owner(s) the *owner* when the description is relevant to both organizational sturctures for ease of exposition.

Given an incentive mechanism (S, e), each agent's strategy is a pair of functions  $\hat{\theta}^i$ :  $\Theta^i \mapsto \Theta^i$  and  $\hat{e}^i : \Theta \times \Theta^i \mapsto A$  such that the *i*th agent having type  $\theta^i$  would choose to report  $\hat{\theta}^i(\theta^i)$  and choose effort level  $\hat{e}^i(\hat{\theta}, \theta^i)$  when  $\hat{\theta}$  is heard and  $e^i(\hat{\theta})$  is recommended. We will verify later (see the remark following Proposition 4) that there is no loss of generality in restricting attention to incentive mechanisms that induce each agent to report his type truthfully and choose the effort level recommended by the owner, i.e.  $\hat{\theta}^i(\theta^i) = \theta^i$ ,  $\hat{e}^i(\hat{\theta}, \theta^i) = e^i(\hat{\theta})$ . In other words, the owner only needs to consider sharing rules which are incentive compatible.

**Definition:** An incentive mechanism  $[S(\cdot), e(\cdot)]$  is Bayesian incentive compatible with public communication (abbreviated to PIC) if and only if it satisfies that for all  $\theta^i \in \Theta^i$ ,  $i \in N$ :

$$\theta^{i} \in \arg\max_{\hat{\theta}^{i}} E_{\theta^{-i}} E_{x|[\hat{e}^{i}(\hat{\theta}^{i},\theta), e^{-i}(\hat{\theta}^{i},\theta^{-i})]} [S^{i}(x, (\hat{\theta}^{i}, \theta^{-i})) - V^{i}(\hat{e}^{i}(\hat{\theta}^{i}, \theta), \theta^{i})]$$
(1)

where

$$\hat{e}^i(\hat{\theta}^i,\theta) \in \arg\max_{e^i} E_{x|(e^i,e^{-i}(\hat{\theta}^i,\theta^{-i}))}[S^i(x,(\hat{\theta}^i,\theta^{-i})) - V^i(e^i,\theta^i)];$$

and for all  $\theta \in \Theta$ ,  $i \in N$ ,

$$e^{i}(\theta) = \hat{e}^{i}(\theta^{i},\theta) \in \arg\max_{e^{i}} E_{x \mid [e^{i},e^{-i}(\theta)]}[S^{i}(x,\theta) - V^{i}(e^{i},\theta^{i})],$$
(2)

where  $e^{-i} = (e^1, \ldots, e^{i-1}, e^{i+1}, \ldots, e^n)$ ,  $\theta^{-i} = (\theta^1, \ldots, \theta^{i-1}, \theta^{i+1}, \ldots, \theta^n)$ ,  $E_{\theta}$  and  $E_{x|e}$  are the expectation operators over  $\theta$  and over x given e, respectively.

The above definition of PIC mechanism is a natural adaptation of the notion of *Bayesian Incentive Compatibility* first proposed by d'Aspremont and Gérard-Varet (1979) to the present Bayesian game setting with both private types and effort. It is a modification of Myerson's definition of *incentive compatible direct mechanisms* (1982)(1985) or Forges' definition of *canonical communication device* (1986)(1989). (1) says given that all the other agents report the true types and follow the recommended effort levels, no agent wants to misreport his type in the communication stage. (2) says given that all the agents

have reported their types honestly and all the other agents would follow the recommendation to choose effort levels, no agent can gain by first deviating his effort level from what is recommended. For a given PIC mechanism, we shall call a vector of type-effort pairs:  $(\theta, e(\theta)) = [(\theta^1, e^1(\theta)), \dots, (\theta^n, e^n(\theta))]$  satisfying (1) and (2) a PIC equilibrium. Put differently, PIC mechanisms induce PIC equilibria for the sub-game played among the agents. We shall call an optimal PIC mechanism a *public communication equilibrium*, which is a solution to the game when the owner is included as a player whose strategies are the incentive mechanisms. This latter equilibrium concept is parallel to Forges' canonical communication equilibrium (1986) or simply communication equilibrium [Myerson (1986)] where equilibrium is defined with respect to direct mechanisms.<sup>10</sup> In next section we shall investigate the optimal PIC mechanisms under the two organization structures of concern.

Voluntary participation gives each agent the opportunity to require a minimal expected utility level at the start which we normalize to zero.<sup>11</sup> We assume that the expected value of x is large enough such that it is optimal to let all agents, whatever their types happen to be, be satisfied to participate under the optimal sharing rules.<sup>12</sup> This implies that their individual rationality constraints at the PIC equilibria should be met, i.e.

$$E_{\theta^{-i}}E_{x|(c(\theta))}[S^{i}(x,\theta) - V^{i}(c^{i}(\theta),\theta^{i})] \ge 0 \quad \text{for all } \theta^{i} \in \Theta^{i}, \ i \in N.$$
(3)

In  $O_p$ , the principal-owner's problem can now be stated as

PP:

$$\max_{S,e} E_{\theta} E_{x|e(\theta)}[x - \sum_{i} S^{i}(x,\theta)]$$
(4)

subject to (1), (2) and (3).

<sup>&</sup>lt;sup>10</sup>PIC mechanisms random in S and/or in e can be defined analogously.

<sup>&</sup>lt;sup>11</sup>The *i*th agent's reservation level of utility may be allowed to depend on  $\theta^i$ , say,  $\tilde{\pi}^i(\theta^i)$ . But then we can replace  $V^i(e^i, \theta^i)$  by  $\tilde{V}^i(e^i, \theta^i) = V^i(e^i, \theta^i) - \tilde{\pi}^i(\theta^i)$  to transform the problem into the present one.

<sup>&</sup>lt;sup>12</sup>A more general analysis is to assign a probability  $r^i(\cdot)$  on  $\Theta$  of the acceptance of the *i*th agent. Then we need to specify the possible consequences of one or some agents' exclusion from cooperation, e.g. whether the project would be abandoned, or could still be carried out by the remaining agents, or the excluded agents could be replaced by someone else, etc. If no agent is dispensible for realizing the project, an unverified conjecture is that there is an acceptance level  $a \in \Theta$  such that  $r^i(\theta) = r(\theta)$  for all  $i \in N$  where  $r(\theta) = 1$  if  $\theta \ge a$  and  $r(\theta) = 0$  if for some i,  $\theta^i < a^i$ . This would be a generalization of the result in Baron and Myerson (1982). If this were true, then the lower bound of  $\Theta^i$ ,  $\underline{\theta}^i$ , could be taken as  $a^i$  to justify our assumption. If there are many candidates applying for each job, the owner may organize auctions to select one agent from each group. According to a result in [Laffont and Tirole (1987)] which builds the close link between auction design and incentive mechanism design, we conjecture that the auction in the present context will allow the owner to select the most efficient agent in each group, and the incentive contract designed in this context with  $\underline{\theta}^i$  modified as the second ith bidder's announced type for  $i \in N$ . It is less clear about the other cases. See the discussion in the last section.

In  $O_a$ , we assume that the agents' collective objective is to maximize the sum of all the agents' expected utilities (as will be justified by the efficiency result in Proposition 5), subject to the PIC and individual rationality constraints, and the expected budget balancing constraint

$$E_{\theta}E_{x|e(\theta)}[x-\sum_{i}S^{i}(x,\theta)]=0$$
(5)

The collective-owners' problem thus writes as

Pa:

$$\max_{S,e} E_{\theta} E_{x|e(\theta)} \sum_{i} [S^{i}(x,\theta) - V^{i}(e^{i}(\theta),\theta^{i})]$$
(6)

subject to (1), (2), (3) and (5).

It is clear that both models involve problems of moral hazard and adverse selection. The moral hazard problems stem from the nonobservability of effort which gives each agent the opportunity to reduce effort while not bearing the full consequence of the reduced outcome resulting from the reduction of his private effort.<sup>13</sup> The owner's ability to infer the true level of effort made by each agent or the agents as a group is further weakened by the randomness of the outcome, which contributes to collective as well as individual moral hazard. Adverse selection stems from the uncertainty of each agent's type, which enables the agents to enjoy some extra informational rents, as will be clear later.

The next section is devoted to solving problems  $\mathbf{P}^{\mathbf{p}}$  and  $\mathbf{P}^{\mathbf{a}}$ . The collection of the assumptions that we made in this section will be called A1.

#### 3 Optimal incentive mechanisms

We take two steps to solve the problems **PP** and **Pa**. We first derive the optimal mechanisms under the assumption that the agents' individual effort levels are perfectly observable. Then, by construction, we derive the optimal PIC mechanisms under nonobservability of effort.

#### 3.1 Optimal mechanisms under perfect effort observability

If the effort level of each agent is observable, hence enforceable, the agents do not have the freedom to choose the effort levels. An immediate consequence is that the incentive mechanisms need not depend on x, because for any incentive mechanism  $(S(x,\theta), e(\theta))$ depending on x, the mechanism  $(S(\theta), e(\theta)) = (E_{x|e(\theta)}S(x,\theta), e(\theta))$  will give each agent

<sup>&</sup>lt;sup>13</sup>Holmström (1982) uses the term "team moral hazard" to describe this free riding problem.

the same problem of utility maximization, hence induce the same equilibrium and give the owner the same expected return. $^{14}$ 

With effort observability our problem reduces to a Bayesian game with incomplete information in the conventional sense, i.e. the informational asymmetry only occurs before the contractual date. This is a pure adverse selection problem to which the standard revelation principle can be applied, thus the use of incentive compatible mechanisms is justified here [e.g., Harris and Townsend (1981)]. A Bayesian incentive compatible mechanism  $(S(\theta), e(\theta))$  (where  $e(\theta)$  is enforceable) should induce the agents to report their types truthfully, i.e. should satisfy [d'Aspremont and Gérard-Varet (1979)]

$$\theta^{i} \in \arg\max_{\hat{\theta}^{i}} E_{\theta^{-i}}[S^{i}(\hat{\theta}^{i}, \theta^{-i}) - V^{i}(e^{i}(\hat{\theta}^{i}, \theta^{-i}), \theta^{i})]$$
(7)

for all  $\theta^i \in \Theta^i$ ,  $i \in N$ .

Write the *i*th agent's optimal expected utility given a Bayesian incentive compatible mechanism as

$$\pi^{i}(\theta^{i}) \stackrel{\text{def}}{=} E_{\theta^{-i}}[S^{i}(\theta) - V^{i}(e^{i}(\theta), \theta^{i})]$$
(8)

The principal-owner's problem is

PPA:

$$\max_{S,e} E_{\theta}[\hat{x}(e(\theta)) - \sum_{i} S^{i}(\theta)]$$
(9)

subject to (7) and  $\pi^{i}(\theta^{i}) \geq 0$  for all  $\theta^{i} \in \Theta^{i}, i \in N$ .

And the collective-owners' problem is

PaA:

$$\max_{S,e} E_{\theta}[\hat{x}(e(\theta)) - \sum_{i} V^{i}(e^{i}(\theta), \theta^{i})]$$
(10)

subject to (7) and  $\pi^{i}(\theta^{i}) \geq 0$  for all  $\theta^{i} \in \Theta^{i}$ ,  $i \in N$ . Note that in **PaA** we have simplified the programme by inserting the expected budget balancing constraint (5) into the objective function in (6).

To ensure the existence of a (deterministic) solution, some more assumptions on the preference and distribution functions may be needed: for all  $i \in N$  and  $\theta^i \in \Theta^i$ ,

A2: i)  $\hat{x}_{e^{i}}(0, e^{-i}) > V_{e}^{i}(0, \theta^{i}) - [(1 - F^{i}(\theta^{i}))/f^{i}(\theta^{i})]V_{e\theta}^{i}(0, \theta^{i});$ ii)  $\hat{x}_{e^{i}}(B, e^{-i}) < V_{e}^{i}(B, \theta^{i})$  for all  $e^{-i} \in A^{n-1}$ . A3:  $k^{i'}(\theta^{i}) \leq 0$ , where  $k^{i}(\theta^{i}) = (1 - F^{i}(\theta^{i}))/f^{i}(\theta^{i})$ . A4:  $V_{ee\theta}^{i} \leq 0$ ;  $V_{e\theta\theta}^{i} \geq 0$ .

<sup>&</sup>lt;sup>14</sup>This amounts to saying as well that mechanisms random in S do not pay. Under proper assumptions to be introduced shortly, i.e. A1-A4, we can also show that there is no need to consider mechanisms random in e.

A2 is made to ensure the existence of an interior solution, i.e. to avoid the extreme cases where no effort or the maximum effort is provided. A3 is a monotone hazard rate property [e.g. Wilson(1983)]. It is a standard assumption made to avoid "bunching", i.e. several types of agents choosing the same level of effort [e.g. Laffont and Tirole(1987)], and to ensure a smooth solution. A4 is also made to ensure sufficient convexity of the problem.<sup>15</sup>

With these assumptions, we will be able to show the existence of a unique solution to  $\mathbf{P}^{\mathbf{p}\mathbf{A}}$  and  $\mathbf{P}^{\mathbf{a}\mathbf{A}}$  which is differentiable on  $\Theta$ . In the present context there is no loss of generality in restricting attention to differentiable mechanisms, provided such solutions exist.<sup>16</sup> Thus we may limit our attention to the differentiable solutions to  $\mathbf{P}^{\mathbf{p}\mathbf{A}}$  and  $\mathbf{P}^{\mathbf{a}\mathbf{A}}$ . Given a differentiable mechanism  $[S(\theta), e(\theta)]$ , let  $\pi^i(\hat{\theta}^i, \theta^i)$  denote the utility of the *i*th agent reporting  $\hat{\theta}^i$ . The first-order condition implied by (7) is

$$\pi^{i}_{\hat{\theta}^{i}}(\theta^{i},\theta^{i}) = \int_{\Theta^{-i}} [S^{i}_{\theta^{i}}(\theta) - V^{i}_{e}(e^{i}(\theta),\theta^{i})e^{i}_{\theta^{i}}]dF^{-i}(\theta^{-i}) = 0$$
(11)

where  $\Theta^{-i} = \Theta \setminus \Theta^i$  and  $dF^{-i}(\theta^{-i}) = dF^1(\theta^1) \dots dF^{i-1}(\theta^{i-1}) dF^{i+1}(\theta^{i+1}) \dots dF^n(\theta^n)$ . Note that (11) is evaluated at  $\hat{\theta}^i = \theta^i$ . By the Envelope Theorem, this implies<sup>17</sup>

$$\pi^{i'}(\theta^i) = -\int_{\Theta^{-i}} V^i_{\theta^i}(e^i(\theta), \theta^i) dF^{-i}(\theta^{-i}) \quad i \in N$$
(12)

(11) also implies  $\pi_{\hat{\theta}^i\hat{\theta}^i}^i = V_{e\theta}^i e_{\hat{\theta}^i}^i$  evaluated at  $\hat{\theta}^i = \theta^i$ . Thus  $e_{\hat{\theta}^i}^i(\theta) \ge 0$  with (12) is a necessary and sufficient condition for (7) to be satisfied with a differentiable mechanism (recall that  $V_{e\theta}^i < 0$ ).

All the above analysis is suitable for both problems of  $\mathbf{P}^{\mathbf{p}\mathbf{A}}$  and  $\mathbf{P}^{\mathbf{a}\mathbf{A}}$ . Now we concentrate on solving  $\mathbf{P}^{\mathbf{p}\mathbf{A}}$ .

Since condition (12) implies that the agents' optimal utility is an increasing function of their type, and the principal's utility is negatively correlated with S, hence with  $\pi$ , the individual rationality constraints can be replaced by

$$\pi^{i}(\underline{\theta}^{i}) = 0 \text{ for all } i \in N$$
(13)

In what follows we first neglect the requirement  $e^{i}_{\theta}(\theta) \geq 0$  and solve the following adverse selection problem:

<sup>&</sup>lt;sup>15</sup>See Zou (1989) for comments on this assumption.

<sup>&</sup>lt;sup>16</sup>See Guesnerie and Laffont (1984), Laffont and Tirole (1986),(1987), Zou (1989). Though those papers are concerned with single-agent models, the results can be readily extended to the present multi-agent situation.

 $<sup>^{17}\</sup>pi^{i'}$  denotes the derivative with respect to  $\theta^i$  viewing  $\pi^i$  as a compound function of  $\theta^i$ ;  $\pi^i_{\theta}$  denotes the partial derivative with respective only to  $\theta^i$  explicitly appearing in  $\pi^i$  (not those via other variables). The same rule applies to other variables.

PAS:

$$\max_{\pi,e} \int_{\Theta} \left[ \hat{x}(e(\theta)) - \sum_{i} \left[ \pi^{i}(\theta^{i}) + V^{i}(e^{i}(\theta), \theta^{i}) \right] \right] dF(\theta)$$
(14)

subject to (12) and (13). [Note that from (8), (14) is equivalent to (9)]. Then we just check that the solution to  $\mathbf{P}^{AS}$  satisfies  $e_{\theta^i}^i(\theta) \geq 0$  for all  $i \in N$ , and derive the optimal contract for  $\mathbf{PP}^A$  from (8).  $\mathbf{P}^{AS}$  appears to be a generalized control problem with n "time" dimensions  $(\theta^1, \ldots, \theta^n) \in \Theta$ . Such problems are known as multi-dimensional adverse selection problems and could become very complicated.<sup>18</sup> In the present model, it appears easiest to directly prove the existence of an optimal solution and give its characterization, though indirect methods may also exist. The following proposition draws inspiration from the existing theorems about the sufficient and necessary conditions for optimal control problems with one "time" dimension [e.g. Kamien and Schwarz(1971)].

**Proposition 1:** Under A1-A4, there exists a unique optimal solution to  $\mathbf{P}^{AS}$ . Denote this solution by  $(\tilde{\pi}(\theta), \tilde{e}(\theta)) = [(\tilde{\pi}^1(\theta^1), \tilde{e}^1(\theta)), \ldots, (\tilde{\pi}^n(\theta^n), \tilde{e}^n(\theta))]$ . It satisfies

$$L^{i} = \hat{x}_{e^{i}}(\tilde{e}(\theta)) - V_{e}^{i}(\tilde{e}^{i}(\theta), \theta^{i}) + \frac{1 - F^{i}(\theta^{i})}{f^{i}(\theta^{i})} V_{e\theta}^{i}(\tilde{e}^{i}(\theta), \theta^{i}) = 0 \quad i \in N, \ \theta \in \Theta.$$
(15)

$$\tilde{\pi}^{i}(\theta^{i}) = -\int_{\underline{\theta}^{i}}^{\theta^{i}} \int_{\Theta^{-i}} V_{\theta}(\tilde{e}^{i}(\tilde{\theta}), \tilde{\theta}^{i}) dF^{-i}(\tilde{\theta}^{-i}) d\tilde{\theta} \quad i \in N$$
(16)

Proof: See Appendix.

For deriving a solution to  $\mathbf{P}^{\mathbf{p}\mathbf{A}}$ , it remains to check whether the second-order condition for the incentive compatibility constraint (7), i.e.  $\tilde{e}^{i}_{\theta^{i}} \geq 0$ , is satisfied. Differentiating (15) w.r.t.  $\theta$  yields the matrix relation

$$\nabla_{\theta} \tilde{e} = -[\nabla_{e} L]^{-1} \nabla_{\theta} L$$

The components of the Jacobian matrix  $\nabla_{\theta} L$  are  $L^{i}_{\theta j}$ ,  $i, j \in N$ , which satisfy  $L^{i}_{\theta j} = 0$  for  $i \neq j$ , and  $L^{i}_{\theta i} = (k^{i'} - 1)V^{i}_{e\theta} + k^{i}V^{i}_{e\theta\theta} > 0$ . Since  $\nabla_{e}L$  is negative definite ( $\hat{x}$  is concave,  $V^{i}_{ee} > 0$ ,  $V^{i}_{ee\theta} < 0$ ), so is its inverse  $[\nabla_{e}L]^{-1}$ . Thus the Jacobian matrix  $\nabla_{\theta}\tilde{e}$  must be positive definite. But this implies that the diagonal components of  $\nabla_{\theta}\tilde{e}$ , i.e.  $\tilde{e}^{i}_{\theta i}$ ,  $i \in N$ , are all positive.

Finally, from (8) we obtain the optimal contract for PPA

$$\tilde{S}(\theta^{i}) = \tilde{\pi}(\theta^{i}) + V^{i}(\tilde{e}^{i}(\theta), \theta^{i}).$$
(17)

To summarize, we state the result in the following proposition.

**Proposition 2:** Under A1-A4, there exists an optimal solution to  $\mathbf{P}^{\mathbf{p}\mathbf{A}}$ , characterized by (15), (16) and (17).

<sup>&</sup>lt;sup>18</sup>See Laffont, Maskin and Rochet(1987) for the analysis of an optimal nonlinear pricing model with two-dimensional characteristics.

We next turn to solving problem PaA.

 $\mathbf{P}^{\mathbf{a}\mathbf{A}}$  can be viewed as a particular case of  $\mathbf{P}^{\mathbf{p}\mathbf{A}}$ , where the state variables  $\pi^i$  do not appear in the objective function. Thus by analogous derivation we have

**Proposition 3:** Under A1-A2, there exists an optimal solution to  $\mathbf{P}^{\mathbf{a}\mathbf{A}}$ . Denote this solution by  $(S^*(\theta), e^*(\theta))$ . It is characterized by

$$\hat{x}_{e^{i}}(e^{*}(\theta)) - V_{e}^{i}(e^{*i}(\theta), \theta^{i}) = 0 \quad i \in N, \ \theta \in \Theta.$$

$$(18)$$

$$\pi^{*i}(\theta^{i}) = -\int_{\underline{\theta}^{i}}^{\theta^{i}} \int_{\Theta^{-i}} V_{\theta}(e^{*i}(\tilde{\theta}), \tilde{\theta}^{i}) dF^{-i}(\tilde{\theta}^{-i}) d\tilde{\theta} + \alpha^{i} \quad i \in \mathbb{N}$$
(19)

$$S^{*i}(\theta^i) = \pi^{*i}(\theta^i) + V^i(e^{*i}(\theta), \theta^i).$$
<sup>(20)</sup>

$$\alpha^i \ge 0 \quad i \in N \tag{21}$$

$$\hat{x}(e^{\bullet}(\theta)) - \sum_{i} S^{\bullet i}(\theta) = 0$$
(22)

Proof: See Appendix.

Note that Proposition 3 holds under less restricted assumptions. (18), (19) and (20) are parallel to (15), (16) and (17) respectively. Our assumption that the joint output is high enough to allow substantial earnings to be shared by the agents can now be put more concretely by assuming (21) and (22) to hold at the optima. (It is sufficient to add a constant large enough amount to  $\hat{x}$ .)  $\sum \alpha^i$  is the difference between the joint output and the sum of the minimal amount needed to ensure that every agent, whatever his type happens to be, will participate in the cooperation at the outset. It is clear from (18) that the solution to  $\mathbf{P}^{\mathbf{aA}}$  attains the first-best allocation of effort, hence leads to the most efficient outcome (in the sense given in Section 4).

 $\mathbf{P}^{\mathbf{A}}$  may be viewed as a type of economy involving consumption and production of a public good  $\hat{x}(e)$ , where each agent with private type  $\theta^i$  provides a publicly observable input  $e^i$ . In the collective choice literature it has been recognized that incentive and budget balancing problems related with production and consumption of public goods when formalized as a Bayesian game with incomplete information, can be solved simultaneously without causing any loss of efficiency<sup>19</sup> via judicially designed incentive mechanisms [e.g. d'Aspremont and Gérard-Varet(1979)]. Call those mechanisms A-G mechanisms. A limitation of the mechanisms in our model as well as in A-G mechanisms is that the individual rationality constraint is not necessarily satisfied, hence these mechanisms do not necessarily work if the agents are not forced to participate. We have avoided this problem

<sup>&</sup>lt;sup>19</sup>In d'Aspremont and Gérard-Varet(1979) it is called outcome efficiency.

by simply assuming that the outcome is high enough such that the individual rationality constraints are not binding at the optimum.<sup>20</sup>

From Proposition 3, one can see that the efficiency solution will not change if a constant amount is added to or substracted from each agent's expected utility: The constant terms  $\alpha^i$  only need to be positive and satisfy the expected budget balancing constraint. How the agents may reach an agreement in the value of  $\alpha^i$  for  $i \in N$  is not of concern in this paper. It may depend on the internal organizational ideology, convention, consensus politics, decision rules and other particulars. Theoretically it would be a cooperative game, and the question for the agents would be to choose a residual sharing rule which is in the core. Necessarily this residual sharing rule of  $\alpha^i$  must not depend on the agents private information, for else the optimality of  $(S^*(x, \theta), e^*(\theta))$  would be jeopardized. Whatever the rule is, it is quite obvious that the agents will always agree to play cooperatively rather than breaking away. And since any division of the outcome is possible, they will always choose a sharing rule that leads to the first-best joint outcome  $\hat{x}(e^*(\theta))$  (characterized by (18), because for any sharing rule which is inefficient, all the agents can be made better-off if they shift to the first-best sharing rules. In other words, threats of breaking away are not credible.

### 3.2 Optimal incentive mechanisms under nonobservability of effort

The problems with nonobservability of effort have been given in Section 2 by  $\mathbf{PP}$  and  $\mathbf{P^a}$ . With the results in the above subsection, we are ready to show that the owner can achieve the optimal solution to  $\mathbf{PP^A}$ , or  $\mathbf{P^{aA}}$ , even when the only observable variable is the realized joint outcome x. This amounts to saying that under risk neutrality moral hazard can be effectively eliminated without the need for special arrangements, such as bonding or monitoring. Since  $\mathbf{PP}$  ( $\mathbf{P^a}$ ) can be viewed as a more restricted case of  $\mathbf{PP^A}$  ( $\mathbf{P^{aA}}$ ) where the choice of  $e(\cdot)$  is subject to an additional constraint (2), it is clear that if an incentive mechanism can guarantee the owner the same expected utility in  $\mathbf{PP}$  ( $\mathbf{P^a}$ ) as in  $\mathbf{PP^A}$  ( $\mathbf{P^{aA}}$ ), it must be an optimal solution to  $\mathbf{PP}$  ( $\mathbf{P^a}$ ). This is the spirit of the following two propositions.

<sup>&</sup>lt;sup>20</sup>However, the problem of participation seems to be less serious with our solution than with the class of A-G mechanisms (or Croves-Ledyard mechanisms, understood broadly). All the incentive mechanisms that can be classified as A-G mechanisms share a common property that they give each agent a payment which equals the whole joint output minus an amount independent of the agent's anounced type. It is easy to understand that such payment structures are sufficient but not necessary for achieving efficiency and incentive compatibility. Since the incentive compatibility conditions are not necessarily binding, the undesirable distributional effects, if any, may not be minimized in A-G mechanisms. In our model, the incentive compatibility constraints are binding at the optima, thus it is reasonable to surmise that the individual rationality constraints are less likely to be violated in our solution than in A-G mechanisms. It would be interesting to verify this conjecture.

**Proposition 4:** Under A1-A4, there exists an optimal solution to **P**<sup>p</sup>. Denote this solution by  $(S^{p}(x,\theta), e^{p}(\theta)) = [(S^{p1}(x,\theta), e^{p1}(\theta)), \dots, (S^{pn}(x,\theta), e^{pn}(\theta))]$ . It satisfies  $e^{p}(\theta) = \tilde{e}(\theta)$  and

$$S^{pi}(x,\theta) = \tilde{S}^{i}(\theta) + D(\theta)[x - \hat{x}(\tilde{e}(\theta)] - C[x - \hat{x}(\tilde{e}(\theta)]^{2} + C\sigma^{2},$$

where  $D(\theta) = \frac{V_{e}^{i}(\tilde{e}^{i}(\theta), \theta^{i}))}{\hat{x}_{e^{i}}(\tilde{e}(\theta))}$ ,  $(\tilde{S}(\theta), \tilde{e}(\theta))$  being the solution to  $\mathbf{P}^{\mathbf{p}\mathbf{A}}$  (Proposition 2). Proof: See Appendix.

**Remark:** In the proof of Proposition 4 we show that the incentive mechanism  $(S^p, e^p)$ induces a PIC equilibrium. When all the agents anounce their types truthfully and obey the effort recommendation, the expected payment to each agent equals to his expected payment under the optimal incentive mechanism  $(\tilde{S}, \tilde{e})$  derived under the effort observability assumption, and the effort level to be made by each agent equals his effort under  $(\tilde{S}, \tilde{e})$  for all  $\theta \in \Theta$ . Thus the principal's as well as all the agents' expected utilities are the same under  $(\tilde{S}, \tilde{e})$  and  $(S^p, e^p)$ .

This amounts to saying that when the agents are risk neutral, the optimal PIC mechanism can attain a solution in which the agents behave in a way as though their privately taken effort were publicly observable. Since that solution is optimal for the adverse selection problem  $\mathbf{P}^{pA}$ , reduced from  $\mathbf{P}^{p}$ , the PIC mechanism  $(S^{p}, e^{p})$  so derived must be optimal for  $\mathbf{P}^{p}$  in the class of all mechanisms.

The intuition is this: The difference between public communication and private communication (as under direct mechanisms) only becomes important when the agents have private decisions to make *after* the communication stage, because if there were no postcommunication private decision choices everything would be settled as soon as the agents have simultaneously reported their types. This is the case with our pure multi-dimensional adverse selection model. Moreover, the revelation principle can be applied to justify our formulation of the multi-dimensional adverse selection problem  $\mathbf{P}^{\mathbf{p}\mathbf{A}}$ , i.e. the solution  $(\tilde{S}, \tilde{e})$  is optimal not only among incentive compatible mechanisms but also among all mechanisms. To carry the argument a bit further, since the principal can always choose not to control the agents effort levels, the solution to  $\mathbf{P}^{\mathbf{p}\mathbf{A}}$  (with effort verifiability) must be always weakly superior to the solution to  $\mathbf{P}^{\mathbf{p}}$  (without effort verifiability). Therefore should the solution to  $\mathbf{P}^{\mathbf{p}}$  achieves the same result as the solution to  $\mathbf{P}^{\mathbf{p}\mathbf{A}}$ , the former solution must be optimal in the set of all mechanisms, not only in the set of PIC mechanisms.

From (16) it is clear that the agents, except for those whose types are the lowest, enjoy a strictly positive utility level. This amount is commonly perceived as the agent's informational rent. Following the convention, we call  $(S^p, e^p)$  the second-best incentive mechanism. It is second best first because the principal cannot extract all the agents' information rent as he can under perfect information, and because the agents do not provide the socially most efficient effort levels under this incentive mechanism (see the next section).

That the quadratic mechanism structure is optimal has been shown by Fishe and McAfee (1987) in a bidding model, and by Picard (1988) in a single agent model. The intuition here behind is that, when the agents are risk neutral, the principal can artificially create high individual risk linked with the joint outcome, in such a way that a PIC equilibrium is established at the desired effort levels, where moral hazard is eliminated at no costs. In other words, the optimal share-effort allocation can be implemented through the incentive contract  $S^p$ . Thus the crucial assumption is risk neutrality. An additional perhaps substantial limitation of the quadratic form of incentive contracts is the penalty it imposes for much better than expected outcomes. This perverse property would have the undesirable potential effect to induce *ex post* moral hazard if the agents could adjust their effort levels after observation of the states of nature. In that case, if the value of C is large and the states of nature happen to be favorable, the agents may find their income negatively correlated with the outcome to be realized, hence it will be in their interest to reduce effort (or even sabotage) in order to reduce the penalty. This drawback vanishes when C in  $S^p$  drops to zero, i.e. when  $S^p$  is a linear function of x.

Note that in our multi-agents model, it may be necessary for  $S^p$  to depend on x even if the joint outcome x is deterministic, because it may still be useful for assuring a PIC equilibrium. This is not true for single-agent models in which the agent's effort can be precisely inferred as soon as x is deterministic.

In the proof of Proposition 4, one can check that  $V_{ee}^i \tilde{e}_{\theta i}^i + V_{e\theta i}^i > 0$  is a sufficient condition for the optimal incentive contract to be linear in x, i.e. for  $C = 0.^{21}$  We can also deliminate the classes of cost and distribution functions which imply the linearity of the optimal incentive contracts. But the complexity of the expressions makes it hard to draw any meaningful economic insight.

Now we turn to problem  $\mathbf{P}^{\mathbf{a}}$ . We give the following result regarding the collectiveowners' problem which is parallel to Proposition 4.

**Proposition 5:** Under A1-A2, there exists an optimal solution to  $\mathbf{P}^{\mathbf{a}}$ . Denote this solution by  $(S^{f}(\theta), e^{f}(\theta))$ . It satisfies  $e^{f}(\theta) = e^{*}(\theta)$  and

$$S^{fi}(x,\theta) = S^{*i}(\theta) + [x - \hat{x}(e^*(\theta))], \qquad (23)$$

 $(S^*(\theta), e^*(\theta))$  being the solution to  $\mathbf{P}^{\mathbf{a}\mathbf{A}}$ .

<sup>&</sup>lt;sup>21</sup>In single-agent models, the linear property of the optimal incentive contract has been first recognized by Laffont and Tirole(1986), and later discussed in Picard(1987), Baron and Besanko(1987), etc. A similar property is also recognized in auction theories, e.g. Laffont and Tirole(1987), McAfee and McMillan(1987).

Proof: This is actually a corollary of Proposition 4 with C = 0, hence omitted.

Though in solving  $\mathbf{PP}$  and  $\mathbf{P^a}$  we have not directly employed the revelation principle, Proposition 4 and 5 do show that PIC mechanisms can be optimal. The above proposition extends the standard results in the public good literature (i.e. A-G mechanisms) into a model involving uncertainty of outcome, nonobservable private input levels, apart from private types. It provides another class of mechanisms that achieve the so called Groves result, i.e. under collective ownership the most efficient allocation of resources can be attained if (1) the agents can be subsidized by an outside community, or (2) the agents are forced to remain with and work for the community. Any one of these two conditions implies that the agents' individual rationality constraints need not be respected. In the present model, none of the above two conditions are required since we have assumed that the optimal expected joint outcome is larger than the expected payments to the agents.<sup>22</sup>

At this stage, one should realize that an outside residual claimant is still necessary . We have been considering only the expected budget balancing problems. Without an outside residual claimant an exact budget balancing problem which causes free riding may arise [Holmström (1982)]. In fact this residual revenue claimant does not have to be a residual control claimant, therefore does not have to be an owner. He only needs to serve the function of an insurer. The agents and the residual claimant only need to sign a contract which legalizes the residual claimant's right to receive  $\Delta = x - \sum_i S^{fi}$  when  $\Delta \ge 0$  and the obligation to pay the amount  $-\Delta$  when  $\Delta < 0$ . But will this residual claimant face moral hazard problems? The answer is No. A second look at the sharing rule  $(S^{f}, e^{f})$ reveals that the problem the residual claimant faces is quite benign. Under the sharing rules  $(S^{f}(x,\theta), e^{f}(\theta))$  defined in (23), the agents can shift the outcome distribution to the detriment of the residual claimant only by providing more effort than required, for the expected residual equals  $(n-1)[\hat{x}(e^*) - \hat{x}(e)]$ . But the marginal benefit from providing extra effort is a public good since it is shared by all the agents, while the marginal cost is borne individually. Thus no agent would like to increase the effort above the first-best level. It is also clear the agents do not have any incentive to make less than the first-best effort levels as well.23

<sup>&</sup>lt;sup>22</sup>It is important to remark that the indispensability of any agent for cooperation requires consensus for any internal sharing rule. The efficiency result may not hold if a sub-group of the agents can decide the rule. In that case, the problem becomes a new free reder-moral hazard-adverse selection model with the owner-group trying to exploit the rest of the agents as much as possible. For an analysis of this kind involving only adverse selection, see Boyd, Prescott and Smith (1988).

<sup>&</sup>lt;sup>23</sup>Here we find a very special type of residual claimants. Though their income takes the form of a residual return, it is actually negatively correlated to the net earnings of the firm. Whether such residual claimants exist in the real world is an empirical issue. Possible examples are insurance companies, shareholders having no voting rights, etc. Anyway, the theoretical possibility of such type of residual claimant remindes us that viewing the residual claimants as identical to the owners of a firm can be misleading. It is not necessary either that residual claimants would face incentive problems under asymmetric information.

### 4 Ownership structure and efficiency implications

The socially most efficient outcome (first-best) in the context of our model is naturally the one that maximizes the expected net surplus  $\hat{x} - \sum_i V^i$ . This first-best solution can only be achieved when each agent chooses the effort level which equates his marginal private cost with the marginal outcome, i.e.

$$\hat{x}_{e^{i}} - V_{e}^{i} = 0, \ i \in N.$$
 (24)

From (15) we see that in  $O_p$  the optimal (second-best) incentive mechanism does not attain the most efficient outcome since at the equilibrium the marginal outcome exceeds the marginal cost of effort for all the possible combinations of the types of agents, except for the unique case when *all* the agents are of the highest possible type; the first-best outcome is not attained even some of the agents are of the highest type. Nevertheless, the first-best outcome can be achieved under cooperative ownership structures with an outside risk-neutral residual claimant, as has been shown by Proposition 3 and 5. This observation drives home the proposition that ownership can matter for efficiency even if one views the firm as a complete comprehensive contract. One might then ask, in the spirit of "Coase Theorem", why in  $O_p$  the agents cannot persuade the principal via side payments or bribing to adopt a first-best sharing rule so that all the participants would be better off. To put the question more specifically, since the agents can calculate the expected earning to the principal in a second-best solution, why not let them purchase for a price equal to that value the right of ownership so that they can choose the sharing rules they prefer?

The answer to that question is not obvious, but has indeed been given by the "revelation principle". To see this point, imagine any possible sort of negotiation that may take place between the principal and the agents. It can but be a communication process whereby the participants receive messages, and form their strategies based on the messages received, as well as on their private information.<sup>24</sup> The communication process does not have to be a one-stage affair; with every one trying to drive a hard bargain, there can be bickering about on all sides. But what the revelation principle says is that in the present context, bickering notwithstanding, it does not pay for the principal to follow any long-winded communication process other than restricting himself to a single-stage communication process in which the agents are just required to report their types directly. On the other side of the coin, any final agreement is nothing but about a sharing rule, which ought to have reflected any consideration of side payments.

It might be instructive to show this point more clearly. Consider what will happen if the agents collectively decide to purchase the ownership right from the principal for a

<sup>&</sup>lt;sup>24</sup>See Myerson (1982) for a description of the communication process.

price equal to the principal's optimal expected return solved from  $\mathbf{P}^{\mathbf{P}}$ . Denote this price by  $\tilde{W}$ , as having been shown it is also the principal's optimal expected utility solved from  $\mathbf{P}^{AS}$ , which is given by

$$\tilde{W} = \int_{\Theta} [\hat{x}(\tilde{e}(\theta)) - \sum_{i} [\tilde{\pi}^{i}(\theta^{i}) + V^{i}(\tilde{e}^{i}(\theta), \theta^{i})]] dF(\theta)$$

where  $(\tilde{\pi}^i(\theta^i), \tilde{e}^i(\theta))$  is the optimal solution to  $\mathbf{P}^{AS}$  satisfying conditions (15) and (16).

In this case the agents can be perceived as playing a two-phase game. They play cooperatively in the first phase in deciding whether to take over the firm, and then choose an incentive mechanism solved from  $\mathbf{P}^{\mathbf{a}}$  under their collective ownership. By our assumption that no agent is dispensable for the cooperation, a unanimous agreement is necessary.<sup>25</sup> In the second phase, the agents will play noncooperatively in announcing their types and in choosing their private effort levels under the new incentive mechanism.

We have characterized the optimal PIC mechanisms under the agents' collective ownership in Proposition 5. This optimal solution ought to be known by the principal as well. Recall that in  $O_a$  the *i*th agent's expected utility is given by  $\pi^{*i}$ . Let  $\beta^i$ ,  $\sum^i \beta^i = \tilde{W}$ , be the share of the *i*th agent to be paid for the ownership right. We show that it is impossible for  $\pi^{*i}(\theta^i) - \beta^i$  to be positive for all  $i \in N$  and  $\theta^i \in \Theta^i$ . In fact, if on the contrary  $\pi^{*i}(\theta^i) - \beta^i \ge 0$  for all  $i \in N$  and  $\theta^i \in \Theta^i$ ,  $\pi^{*i}(\theta^i) - \beta^i$  and  $e^{*i}(\theta)$  would satisfy all the constraints in  $\mathbf{P}^{\mathrm{AS}}$ , and the following contradiction would result:

$$\tilde{W} > \int_{\Theta} [\hat{x}(e^*(\theta)) - \sum_i [\pi^{*i}(\theta^i) + V^i(e^{*i}(\theta), \theta^i)]] dF(\theta) = \tilde{W}$$

The above inequality results from that  $(\pi^{*i}(\theta^i), e^{*i}(\theta))$  differs from the unique optimal solution to  $\mathbf{P}^{AS}$ , i.e.  $(\tilde{\pi}^i(\theta^i), \tilde{e}^i(\theta))$ . The equality is due to the budget balancing condition (22), which implies

$$\hat{x}(e^*(\theta)) - \sum_i [\pi^{*i}(\theta^i) + V^i(e^{*i}(\theta), \theta^i)] = \tilde{W}$$

for all  $\theta \in \Theta$ . Since  $\pi^{*i}(\theta^i)$  increases in  $\theta^i$ , we can derive from the above observation that there exist  $\xi^i \geq \underline{\theta}^i$  with strict inequalities at least for some  $i \in N$ , such that for *i*th agent having type lower or equal to  $\xi^i$  he is better off not being the owner. Thus, only the agents with relatively high types may have an interest in purchasing the ownership rights. But should they come to the principal to propose a deal, the principal would infer that they have higher types and the principal's expected earning would thus shift upward, and so

<sup>&</sup>lt;sup>25</sup>We conjecture that a similar analysis can be carried over to the cases where one or some of the agents can take over the firm's technology. Only new considerations will emerge [see Myerson(1983)] and the analysis will be beyond the scope of the present model.

the agents would have to offer something more than the principal's originally expected return  $\tilde{W}$ . The problem is, at any price, the principal would never clinch a deal as far as there are agents who want to purchase his rights, because the fact that some agents are interested in his rights is always a signal that his expected return without selling the rights is higher than the price offered. Of course, such behavior ought to be anticipated by the agents, and they would rather pretend not to be interested in having the rights at any price above the principal's original expected earning. Thus the transfer of rights will never happen. This example is nothing but a mirror case of Akerlof's "market for lemons".

The impossibility of improving efficiency through ownership transactions under separated ownership and management allows us to conclude that in the presence of precontractual informational asymmetries (or transaction costs) among the members of a firm, the initial location of ownership rights matters for its efficiency in resource allocation.<sup>26</sup> In fact we have something more to say. Namely, given that agents are risk neutral, it can be more efficient to have the ownership of the firm rest with the agents as a group than with outside investors. To be sure, in order that the agents' cooperation is viable, or at least feasible, their financial capacity must be great enough to permit them to purchase, or take out loans within admissible cost limits to purchase the required assets. Of course, they may try to run a pure rental firm as well (if they can manage to do so), but then new moral hazard problems may arise [Jensen and Meckling (1976)].

Here we find a possible rationale for a third-party intervention. Suppose before the agents and the principal meet, a third party can impose an institutional rule that the agents have the right to decide a sharing rule, on condition that they do not abuse the right, i.e. the principal must be guaranteed a minimal expected return equal to what he would get if the right were vested with him. Then it is easy to see that both the principal and the agents are better off. We seem to have found a theoretical instance that mandatory codetermination can improve efficiency.<sup>27</sup>

#### 5 Conclusion

In this paper we have only considered two extreme cases of a firm's ownership structure: one with complete separation of ownership and management, another with complete overlap of ownership and management. We have derived explicitly the optimal PIC mechanisms under the two ownership structures, and shown that the public communication

<sup>&</sup>lt;sup>26</sup>The import of organizational structures on efficiency has actually been noticed by some authors in different adverse selection contexts, e.g. Miyazaki (1977), Boyd, Prescott and Smith (1988), though the issue of ownership has not been addressed explicitly.

<sup>&</sup>lt;sup>27</sup>For a recent property rights analysis of codetermination, see Furubotn(1988).

equilibrium derived under the former structure does not attain the socially most efficient output while it does under the latter structure. From the model analysis this implies that the ownership structure of an organization (even viewed as a comprehensive contract) matters for its resource allocation efficiency, and the collective ownership structure with the operating agents (managers, workers, etc.) being the owners of the firm can be an efficient form of organization. The caveat is the risk neutrality assumption, which implies that the agents have unlimited financial capacity to raise funds, and that there is no gain in risk diversification.

It would be interesting to continue the line of research to investigate the joint-ownership structure, where the firm is owned by outside investors and the employees [the case of Employee Stock Ownership Plans (ESOPs), for instance], with emphasis perhaps on the welfare implications of different power(voting right) distributions for each group, and the social efficiency implications. We may also examine the efficiency implications of the optimal incentive structure under collective ownership without external residual claimants, namely with strict budget balancing constraint. It is not clear whether the solution will still dominate [e.g. in terms of *interim* or *ex post* efficiency in the sense of Holmström and Myerson (1983)] the solution derived under separation of ownership and management.

Ownership structures of modern corporations or organizations have undergone, and are still undergoing, significant evolution. Our analysis in this paper might provide some explanation, even from merely a narrow point of view of incentive related resource allocational efficiency, for the real world phenomena such as company buyouts by management, and application of ESOPs, which are currently becoming quite fashionable.<sup>28</sup> Of course, the extent of the relevance between theory and practice is always subject to subsequent judgement.

This paper is only a modest attempt to investigate the efficiency implications of the ownership structure of a firm by explicit comparison of contractual solutions. We cite Klein (1983) to close: "Getting one's hands dirty to obtain a fuller description of the agency problems and contractual solutions in various forms at various times is likely to produce both a modified theoretical framework and further convincing evidence regarding the survival characteristics of alternative organizational arrangements. Certainly our view of large, complex organizations will continue to become much richer [...]."

<sup>&</sup>lt;sup>28</sup> Just recall that one of the biggest takeover battles in history, the case of RJR Nabisco, was initiated by the company's own management group. And see TIME, Feb.13,1989, for a report on some evidence of how ESOPs have enhanced efficiency, or company's value, in the United States. Of course, there are many other convincing explanations for these phenomena.

### 6 Appendix

**Proof of Proposition 1:** i) Existence and Uniqueness: Let  $L = (L^1, \ldots, L^n)$ . (15) gives n equations  $L^i = 0, i \in N$ . From A2 we can check that  $L^i > 0$  for  $e^i$  close to 0 and  $L^i < 0$  for  $e^i$  close to B. Thus by continuity of  $L^i$  there exists  $e = (e_1, \ldots, e_n) \in A^n$  which satisfies L = 0 for all  $\theta \in \Theta$ . Since from A1-A4  $\hat{x}$  is concave and  $V^i(e^i, \theta^i) - \frac{1-F^i(\theta^i)}{f^i(\theta^i)}V_{\theta}(e^i, \theta^i)$  is convex in e for all  $i \in N$ , the Hessian matrix of  $\hat{x}(e) - V^i(e^i, \theta^i) - \frac{1-F^i(\theta^i)}{f^i(\theta^i)}V_{\theta}(e^i, \theta^i)$  with respect to e must be negative definite, which implies that the Jacobian of L with respect to e is strictly negative. From the Implicit Function Theorem, this implies that there exists a unique vector valued function  $\tilde{e} : \Theta \to A^n$ , differentiable on  $\Theta$ . It is then easy to see that given  $\tilde{e}(\theta)$ , (12) and (13) determines a unique  $\tilde{\pi}(\theta)$ , which is given by (16). Such solved  $\tilde{e}(\theta)$  and  $\tilde{\pi}(\theta)$  obviously satisfy (12) and (13).

ii)Sufficiency:

Let  $G(\theta, \pi, e) = \hat{x}(e) - \sum_{i} [\pi^{i} + V^{i}(e^{i}, \theta^{i})]$  and let  $\nu^{i}(\theta^{i}) = \frac{1 - F^{i}(\theta^{i})}{f^{i}(\theta^{i})}$ . It is clear that  $\tilde{e}^{i}(\theta)$  is the solution to problem

$$\max_{e}[G(\theta, \pi, e) + \sum_{i} \pi^{i} + \sum_{i} \nu^{i} V_{\theta}^{i}(e^{i}, \theta^{i})]$$

for specific values of  $\theta$  and  $\pi$ . (Note that  $\sum_i \pi^i$  is cancelled out, hence  $\tilde{e}$  does not depend on  $\pi$ . And the concavity of the problem is ensured by A1-A4.)

Now choose a pair of functions  $(\pi(\theta), e(\theta))$  different from  $(\tilde{\pi}(\theta), \tilde{e}(\theta))$  which also satisfies conditions (12) and (13). We compare the principal's expected welfare with  $(\tilde{\pi}(\theta), \tilde{e}(\theta))$ and  $(\pi(\theta), e(\theta))$ , denoted  $\tilde{W}$  and W respectively:

$$\begin{split} & \Delta &= \tilde{W} - W \\ &= \int_{\Theta} [G(\theta, \tilde{\pi}, \tilde{e}) - G(\theta, \pi, e)] dF(\theta) \\ &= \int_{\Theta} [[G(\theta, \tilde{\pi}, \tilde{e}) + \sum_{i} \tilde{\pi}^{i} + \sum_{i} \nu^{i} V_{\theta}^{i}(\tilde{e}^{i}, \theta^{i})] \\ &- [G(\theta, \pi, e) + \sum_{i} \pi^{i} + \sum_{i} \nu^{i} V_{\theta}^{i}(e^{i}, \theta^{i})] \\ &- \sum_{i} \nu^{i} V_{\theta}^{i}(\tilde{e}^{i}, \theta^{i}) + \sum_{i} \nu^{i} V_{\theta}^{i}(e^{i}, \theta^{i}) - \sum_{i} (\tilde{\pi}^{i} - \pi^{i})] dF(\theta) \\ &> \int_{\Theta} [\sum_{i} \nu^{i} V_{\theta}^{i}(e^{i}, \theta^{i}) - \sum_{i} \nu^{i} V_{\theta}^{i}(\tilde{e}^{i}, \theta^{i}) - \sum_{i} (\tilde{\pi}^{i} - \pi^{i})] dF(\theta) \end{split}$$

Integrating by parts and using (12) and (13) we can derive

$$\begin{split} &\int_{\Theta} \sum_{i} (\tilde{\pi}^{i} - \pi^{i}) dF(\theta) \\ &= \sum_{i} \int_{\Theta^{i}} [\tilde{\pi}(\theta^{i}) - \pi(\theta^{i})] dF^{i}(\theta^{i}) \end{split}$$

$$\begin{split} &= \sum_{i} \int_{\Theta^{i}} [F^{i}(\theta^{i}) - 1] \int_{\Theta^{-i}} [V^{i}_{\theta}(e^{i}(\theta), \theta^{i}) - V^{i}_{\theta}(\tilde{e}^{i}(\theta), \theta^{i})] dF^{-i}(\theta^{-i}) d\theta^{i} \\ &= \int_{\Theta} \sum_{i} [\nu^{i} V^{i}_{\theta}(e^{i}, \theta^{i}) - \nu^{i} V^{i}_{\theta}(\tilde{e}^{i}, \theta^{i})] dF(\theta) \end{split}$$

Thus  $\Delta > 0$ . Q.E.D.

**Proof of Proposition 3:** i) Existence: By the concavety of  $\hat{x}$  and the convexity of  $V^i$ , and by assumption A2,  $e^{*i}(\theta)$ ,  $i \in N$  is uniquely determined by (18). Consequently  $\pi^{*i}(\theta^i)$  is given by (19) and  $S^{*i}(\theta^i)$  given by (20). In the statement of this proposition,  $\alpha^i$  is quite arbitary so far as it is not negative and the sum of them ensures the budgte balancing constraint (22). In assumption A1 we have assumed that the joint output is high enough such that it is suboptimal to give up cooperation for whatever reported types. To put it more concretely, we assume  $\hat{x}(e^*(\theta)) - \sum_i S^{*i}(\theta) \ge 0$  for  $\alpha^i = 0$ ,  $i \in N$ . Note that it is sufficient to add a constant amount to  $\hat{x}$  to ensure this assumption.

It remains to check if the constraints in  $\mathbf{P}^{\mathbf{A}\mathbf{A}}$  are satisfied.  $\pi^{*i}(\theta^i)$  obviously is nonnegative given that  $\alpha^i \geq 0$ , and from the preceeding analysis it has been shown that (7) is equivalent to (12), which  $\pi^{*i}(\theta^i)$  clearly satisfies for  $i \in N$ . Thus the existence of  $(S^*(\theta), e^*(\theta))$  is proved.

ii)Sufficiency:

Let  $G(\theta, e) = \hat{x}(e) - \sum_{i} V^{i}(e^{i}, \theta^{i})$  be the integrand of the objective functional in  $\mathbf{P}^{\mathbf{a}A}$ . It is clear that  $e^{*}(\theta) = (e^{*1}(\theta), \dots, e^{*n}(\theta))$  is the solution to problem

 $\max G(\theta, e)$ 

for specific values of  $\theta$ . This implies that for any other function  $e(\theta)$ ,  $G(\theta, e^{\bullet}(\theta)) \geq G(\theta, e(\theta))$  for all  $\theta \in \Theta$ . Thus

$$\int_{\Theta^{-i}} G(\theta, e^*(\theta)) dF^{-i} \ge \int_{\Theta^{-i}} G(\theta, e(\theta)) dF^{-i}$$

Q.E.D.

**Proof of Proposition 4:** Under A1-A4, and by Proposition 2,  $(S^p, e^p)$  is well defined since  $\tilde{S}$  and  $\tilde{e}$  are well defined. Given  $(S^p, e^p)$ , let  $\pi^i(\hat{\theta}^i, e^i, \theta^i)$  be the expected utility of agent *i* reporting  $\hat{\theta}^i$  and choosing effort  $e^i$  when the other agents play the PIC equilibrium strategies. Let  $\overline{\pi}^i(\hat{\theta}^i, e^i, \theta)$  denote the *i*th agent's utility whose type is  $\theta^i$  and who has anounced  $\hat{\theta}^i$ , after the stage of communication when the other agents have announced  $\theta^{-i}$ and are expected to follow the mediator's recommended effort levels. For neatness let  $\tilde{x} = \hat{x}(\tilde{e}(\hat{\theta}^i, \theta^{-i}))$  and  $\hat{x} = \hat{x}(e^i, \tilde{e}^{-i}(\hat{\theta}^i, \theta^{-i}))$ .  $\overline{\pi}^i(\hat{\theta}^i, e^i, \theta)$  writes as

$$\begin{split} &\overline{\pi}^{i}(\hat{\theta}^{i}, e^{i}, \theta) \\ &= \tilde{S}^{i}(\hat{\theta}^{i}, \theta^{-i}) + D(\hat{\theta}^{i}, \theta^{-i})(\hat{x} - \tilde{x}) - CE_{x|(e^{i}, \hat{e}^{-i}(\hat{\theta}^{i}, \theta^{-i}))}[x - \hat{x} + \hat{x} - \tilde{x}]^{2} + C\sigma^{2} - V(e^{i}, \theta^{i}) \\ &= \tilde{S}^{i}(\hat{\theta}^{i}, \theta^{-i}) + D(\hat{\theta}^{i}, \theta^{-i})(\hat{x} - \tilde{x}) - C(\hat{x} - \tilde{x})^{2} - V(e^{i}, \theta^{i}) \end{split}$$

and  $\pi^{i}(\hat{\theta}^{i}, e^{i}, \theta^{i})$  writes as

$$\pi^i(\hat{\theta}^i,e^i,\theta^i) = \int_{\Theta^{-i}} \overline{\pi}^i(\hat{\theta}^i,e^i,\theta) dF^{-i}$$

We first check that  $S^p$  induces a PIC equilibrium [See (1) and (2)]. The first and second-order conditions for (2) given ith agent's announcement  $\hat{\theta}^i$  should be

$$\overline{\pi}_{e^{i}}^{i}(\hat{\theta}^{i}, e^{i}, \theta) = [D(\hat{\theta}^{i}, \theta^{-i}) - 2C(\hat{x} - \bar{x})]\hat{x}_{e^{i}} - V_{e}(e^{i}, \theta^{i}) = 0.$$
(25)

for all  $\hat{\theta}^i \in \Theta^i$ ,  $\theta \in \Theta$  and  $i \in N$ , and

$$\overline{\pi}_{e^i e^i}^i = [D - 2C(\hat{x} - \tilde{x})]\hat{x}_{e^i e^i} - V_{ee}^i - 2C\hat{x}_{e^i}^2 \le 0$$
(26)

for all  $\hat{\theta}^i \in \Theta^i$ ,  $\theta \in \Theta$  and  $i \in N$ . (26) is obviously satisfied for  $e^i = \tilde{e}^i(\hat{\theta}^i, \theta^{-i})$  and for C sufficiently large.

Let  $\overline{e}^i(\hat{\theta}^i, \theta)$  denote the solution to (25). By the definition of  $D(\theta)$ , it is easy to check that when  $\hat{\theta}^i = \theta^i, \overline{e}^i(\hat{\theta}^i, \theta) = \overline{e}^i(\theta)$ .

Given  $e^i = \overline{e}^i(\hat{\theta}^i, \theta)$ , by the Envelope Theorem or using (25) the first-order condition for (1) should be

$$\begin{aligned} \pi^{i}_{\hat{\theta}^{i}}(\hat{\theta}^{i}, \vec{e^{i}}(\hat{\theta}^{i}, \theta), \theta^{i}) &= \int_{\Theta^{-i}} \tilde{S}^{i}_{\hat{\theta}^{i}}(\hat{\theta}^{i}, \theta^{-i}) - V^{i}_{e}(\tilde{e^{i}}(\hat{\theta}^{i}, \theta^{-i})\tilde{e^{i}}_{\theta^{i}}(\hat{\theta}^{i}, \theta^{-i}) \\ &+ [D_{\theta^{i}}(\hat{\theta}^{i}, \theta^{-i}) + 2C\tilde{x}_{e^{i}}\tilde{e^{i}}_{\theta^{i}}(\hat{\theta}^{i}, \theta^{-i})](\hat{x} - \tilde{x}) \\ &+ [D(\hat{\theta}^{i}, \theta^{-i}) - 2C(\hat{x} - \tilde{x})] \nabla_{e^{-i}}(\hat{x} - \tilde{x}) \nabla_{\theta^{i}}\tilde{e^{-i}}(\hat{\theta}^{i}, \theta^{-i}) dF^{-i} = (\mathfrak{P}7) \end{aligned}$$

where  $e^i = \overline{e}^i(\hat{\theta}^i, \theta)$  in  $\tilde{x}$ .<sup>29</sup>

When  $\hat{\theta}^i = \theta^i$ , from (12) and (17)

$$\int_{\Theta^{-i}} \tilde{S}^i_{\hat{\theta}^i}(\hat{\theta}^i, \theta^{-i}) - V^i_e(\tilde{e}^i(\hat{\theta}^i, \theta^{-i})\tilde{e}^i_{\hat{\theta}^i}(\hat{\theta}^i, \theta^{-i})dF^{-i} = 0,$$

thus (27) is satisfied because  $\hat{x} = \tilde{x}$  and  $\nabla_{e^{-i}} \hat{x} \nabla_{\hat{\theta}^i} \tilde{e}^{-i} = \nabla_{e^{-i}} \tilde{x} \nabla_{\hat{\theta}^i} \tilde{e}^{-i}$ .

The second-order condition for (1) requires

$$\pi^{i}_{\hat{\theta}^{i}\hat{\theta}^{i}}(\hat{\theta}^{i},\overline{e}^{i}(\hat{\theta}^{i},\theta),\theta^{i}) \leq 0$$

We show that this condition is also met for  $\hat{\theta}^i = \theta^i$ . Differentiating  $\overline{\pi}^i_{e^i}(\theta^i, \tilde{e}^i(\theta), \theta)$  and  $\overline{\pi}^i_{\hat{e}^i}(\theta^i, \tilde{e}^i(\theta), \theta)$  w.r.t.  $\theta^i$  we derive

$$\overline{\pi}^{i}_{\hat{\theta}^{i}\hat{\theta}^{i}} + \overline{\pi}^{i}_{\hat{\theta}^{i}e^{i}}\tilde{e}^{i}_{\theta^{i}} = 0$$

 $<sup>{}^{29} \</sup>nabla_{e^{-i}} \hat{x}(\hat{x})$  denotes the gradient vector of  $\hat{x}(\hat{x})$  with respect to  $e^{-i}$ , similarly  $\nabla_{\hat{\theta}}, \hat{e}^{-i}$  denotes the gradient vector of  $\hat{e}^{-i}$  with respect to  $\hat{\theta}^i$ . We neglect the transposition signs for neatness.

and

$$\overline{\pi}^{i}_{e^{i}\hat{\theta}^{i}} + \overline{\pi}^{i}_{e^{i}e^{i}}\tilde{e}^{i}_{\theta^{i}} + \overline{\pi}^{i}_{e^{i}\theta^{i}} = 0.$$

Twice differentiating  $\overline{\pi}^i$  yields, for C sufficiently large,

$$\begin{aligned} \overline{\pi}_{e^i e^i}^i &= D(\theta) \hat{x}_{e^i e^i} - V_{ee}^i - 2C \hat{x}_{e^i}^2 < 0 \\ \\ \overline{\pi}_{e^i \theta^i}^i &= -V_{e\theta}^i > 0 \end{aligned}$$

It follows that, for C sufficiently large (since  $\pi^i_{e^i e^i} \to -\infty$  as  $C \to \infty$ ),

$$\overline{\pi}^{i}_{e^{i}\hat{\theta}^{i}} = -\overline{\pi}^{i}_{e^{i}e^{i}}\tilde{e}^{i}_{\theta^{i}} - \overline{\pi}^{i}_{e^{i}\theta^{i}} > 0.$$

Thus

$$\pi^i_{\hat{\theta}^i\hat{\theta}^i} = \int_{\Theta^{-i}} \overline{\pi}^i_{\hat{\theta}^i\hat{\theta}^i} dF^{-i} = -\int_{\Theta^{-i}} \overline{\pi}^i_{\hat{\theta}^ie^i} \tilde{e}^i_{\theta^i} dF^{-i} < 0$$

for all  $\theta^i \in \Theta^i$  and  $i \in N$ .

Thus  $S^p$  induces a PIC equilibrium. When all the agents play PIC equilibrium strategies, their individual rationality constraints are clearly satisfied, and the principal obtains the expected utility as though he could observe the agents' effort. This implies that  $S^p$  is optimal to P<sup>P</sup>. Q.E.D.

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