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IS THERE ROOM FOR
CONVERGENCE IN THE E.C.?
by R.C. Douven and
J.C. Engwerda

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# Is there room for convergence in the E.C.? 

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#### Abstract

In this paper we develop a theoretical framework which makes it possible to analyze several aspects of convergence between E.C. countries. The analysis is done in a dynamic game context, where countries, apart from minimizing individual cost functions, minimize cooperatively a convergence function, which represents the convergence conditions which are elaborated in the Maastricht treaty (1991). The aspect of convergence is modeled as a dynamic constraint on the individual cost functions. We show that the maximum degree of convergence is completely determined by the non-cooperative outcome of the game. The framework is illustrated in a theoretical example. The example shows that: (1) the goals with respect to convergence can seriously influence the outcome of the game. If these goals are set too ambitious the outcome can be that countries are not willing to cooperate anymore. (2) the costs involved to obtain convergence can differ substantially between countries. (3) a minor deviation from a Pareto optimal solution can increase convergence considerably. An algorithm is devised how to obtain solutions of the game which are politically more feasible than the Nash bargaining solution and improve on the non-cooperative solution.


[^0]
## 1 Introduction

Due to the integration process within the E.C. countries there is an increasing demand for price stability. To that end the European Council decided, at the Maastricht meeting in 1991, to start, at least in 1999, with irreversibly fixed exchange rates and to establish a European Central Bank. This final step towards the realization of the EMU sets out, however, that uneven developments in the process of integration are set aside. Therefore, greater convergence of economic performance is needed (see the report of Delors Committee [3]). Another aspect of the integration process is that as a consequence of the strengthened economic interdependence between member countries the room for independent policy manoeuvre is reduced and that cross-border effects of developments originating in each member country become more and more important. So, the stages towards an economic and monetary union involve on the one hand a process of closer convergence, and on the other hand coordination of the macroeconomic policies of the various countries. Important to note is that this of course does not imply that if there is convergence of economic policies and/or coordination of macroeconomic policies between countries, the integration process will be balanced and thus the establishment of a single market is justified. In other words convergence and coordination are prerequisites for obtaining a single market but don't guarantee a succesful establishment of it. Now, there is a general consensus amongst the participating countries that convergence and coordination of policies is needed for moving towards an economic and monetary union. There is, however, much less consensus how far and how fast this process should take place. This has, of course, everything to do with the internal forces working on the markets of each individual country. The possibly long run significant increases in economic welfare in the Community are much less tangible than the short term welfare loss effects incurred at various domestic markets. Therefore, a natural reaction one can expect from participating countries is that they do strive for convergence in economic variables, but that they are only willing to pay a price (in terms of welfare loss) for it if the additional increase in the degree of convergence will be significant. Studies with respect to macroeconomic policy coordination in a dynamic games context appear frequently in economic literature, see e.g. Brandsma [2], McKibbin and Sachs [6], Hughes Hallet [4]. However, the influence of the aspects of convergence, analysed in a dynamic games setting, on the effects of macroeconomic policy coordination are not studied before. This motivates the study of this paper.

Starting from the point of view that each country has its own individual welfare loss function it wants to minimize in cooperation with the other countries, we develop a theoretical framework to analyze the trade off between extra welfare loss and more convergence. The analysis will be done in a dynamic games framework. We assume that each policymaker has an individual objective function, he/she wants to minimize and that there is some common sense on a convergence function which they want to minimize simultaneously. In the case of the EMU this convergence function may e.g. represent the convergence conditions which are specified in the Maastricht treaty (1991). In particular the two conditions of
convergence in consumer price inflation and convergence in long term interest rates that are necessary for admitting a country to the monetary union (see e.g. Bean [1]) can be incorporated in such a function. Under the assumption that all policymakers like to cooperate, we analyze the set of solutions which are obtained by the policymakers when they simultaneously minimize their welfare loss functions and convergence function. We assume that the degree of convergence, which is represented by the value of the convergence function, depends on the agreements of the outcome of a negotiating process between countries. In particular we will show that if reducing welfare loss is the primary interest of countries, the degree of convergence countries can obtain is limited. So, if countries strive for a degree of convergence which is set too ambitious, the result can be that (some) countries will show non-cooperative behaviour. In a theoretical example we illustrate two additional aspects the game may have.
(1) The price (in terms of individual welfare loss) that countries have to pay will for some countries be higher than for other countries.
(2) There are situations in which by a minor deviation from the Pareto solution, a large increase in convergence degree is possible. In other words, by paying a small price (in terms of individual welfare loss) high revenues (in terms of convergence) can be obtained.

The organization of the paper is as follows. In section two we will introduce the theoretical framework. We consider $N$ countries which cooperatively agree on minimizing a convergence function and, moreover, all have their own individual objective function they like to minimize. The aspect of convergence is modeled as a dynamic constraint on the joint social welfare function. Under the assumption that all of these functions are convex and (some mild regularity conditions) we show the above mentioned aspects. Furthermore we show that the cooperative outcome wich yields the largest degree of convergence coincides with the Nash solution of the game. To help the reader to understand the basics of the presented theory we illustrate the approach in section three by means of a simple theoretical example. In section four we present the conclusions.

## 2 Incorporating convergence criteria: a theoretical framework

We consider an integrated economy of the European Community with $N$ interdependent economies, where the policymakers in each country face a dynamic economic model which connects the endogenous variables (denoted by $y$ ), instrumental variables (denoted by $u$ ) and other noncontrollable variables. Each country has control over a set of instruments for economic policy: denoted by $u_{i}$. In stacked form $u^{\prime}=\left(u_{1}^{\prime}, \ldots, u_{N}^{\prime}\right)$. We assume that each policymaker has a convex objective function, which we specify by $J_{i}$, which he/she wants to minimize. We denote the set of Pareto optimal solutions in the $J_{1}, \ldots, J_{N}$-plane
by $P$. The point $N_{\mathrm{c}}$ corresponds to the non-cooperative (Nash) solution, which is used as a bargaining threat-point, denoted by $N^{c}:=\left(J_{1}^{N}, \ldots, J_{N}^{N}\right)$. Furthermore we assume that the countries agree to strive for a certain amount of degree of convergence for some of their economic (endogenous and/or instrumental) variables. This agreement will be reflected in a convex convergence function, denoted by $C$, which is included in the optimization process. It is important to stress that the convergence function differs from the countries objective functions in a way that the latter contains only variables which belong to its own country whereas the convergence function contains variables of all the countries. Thus minimizing a costfunction is something that can, in principle, be done by a country alone whereas minimizing the convergence function has to be done simultanously.

The decision-making process of the policymakers concerning what strategy to follow, will depend on the following set:

$$
\begin{equation*}
\left\{\left(J_{1}(u), \ldots, J_{N}(u), C(u)\right) \mid u \in U\right\} \tag{1}
\end{equation*}
$$

where we suppose that the strategy-space $U$ is a convex set. The policymakers have to find a cooperative strategy which results in a point in (1) which is acceptable for them all. Now note that whenever two different strategies yield the same individual costs $J_{i}, i=1, \ldots, N$, but different values for the convergence function, only the strategy yielding the lowest value for the convergence function is of interest to all policymakers. So, the set of relevant control strategies consists of:

$$
\bar{U}=\left\{u \in U \mid \forall \bar{u} \in U \quad\left(J_{1}(u), \ldots, J_{N}(u)\right)=\left(J_{1}(\bar{u}), \ldots, J_{N}(\bar{u})\right) \Rightarrow C(u) \leq C(\bar{u})\right\} .
$$

This observation makes it possible to consider the decision problem from the following point of view. By varying the strategies over the whole set $\bar{U}$, we obtain the set of all possible objective outcomes in the $J_{1}, \ldots, J_{N}$-plane. To each point in this set is attached a unique value for the convergence function. The problem for the decision makers is now to select cooperatively a point within this set that is acceptable for everyone. Now, as mentioned in the introduction we will assume that minimizing their own cost function is the primary interest of countries and that striving for convergence is of secundary interest. In that case the aspect of convergence acts as a dynamic constraint on joint social welfare. If we, furthermore, refrain from the possibility of side-payments and assume that the axiom of individual rationality holds (see e.g. [7] ${ }^{1}$ ), then countries will cooperatively minimize the joint convergence function as long as their individual costs will be lower than their non-cooperative costs. So, the set of possible objective outcomes will then restricted on the one hand by the non-cooperative Nash threatpoint $N^{c}$, and on the other hand by the set of Pareto solutions. We will call this set in the sequel the negotiation area (see figure 1 for an illustration in a two player context). The basic question is now of course, how we can determine this negotiation area and its corresponding convergence function values

[^1]

Figure 1: Representation of the negotiation area in a two player context.
in an efficient way. We will not give a complete answer to this question, but present a solution which we expect will work for the applications we are aiming at (i.e. situations in which the set of Pareto-solutions and the Nash-threatpoint are situated not too far from each other). The solution we will present has a number of nice properties. First of all it attaches to every point in the negotiation area a unique control strategy that can be obtained by minimizing a strict convex combination of the individual object functions and the convergence function. Secondly, we will show that this control strategy is parametrized by $N$ parameters and that this parametrization is a continuous function of its parameters. By varying the parameters between 0 and 1 , the whole negotiation area can then be covered (in general (see note above)).

The solution is motivated by our assumption that each policymaker is primarily interested in minimizing his own objective function in a cooperative setting and that convergence plays a minor role. We model this aspect by rewriting the convex combination of individual cost and convergence cost in a special way. Consider

$$
\bar{\alpha}_{1} J_{1}+\ldots+\bar{\alpha}_{N} \cdot J_{N}+\bar{\alpha}_{N+1} C, \text { with } \sum_{i=1}^{N+1} \bar{\alpha}_{i}=1 .
$$

This is equivalent with (in the non-trivial case $\bar{\alpha}_{N+1} \neq 1$ ):

$$
(1-\lambda)\left(\alpha_{1} J_{1}+\ldots+\alpha_{N} J_{N}\right)+\lambda C, \text { where } \lambda=\bar{\alpha}_{N+1}, \text { and } \alpha_{i}=\bar{\alpha}_{i} /\left(1-\bar{\alpha}_{N+1}\right),
$$

which has the nice property that $\sum_{i=1}^{N} \alpha_{i}=1$. If we minimize this second convex combination of the individual object functions and the convergence function then we have the property that $\lambda=0$ resembles the case that countries completely ignore the convergence
goal (and because $\sum_{i=1}^{N} \alpha_{i}=1$ we find the Pareto optimal solutions), and that $\lambda=1$ corresponds with the case that countries only pay attention to their mutual convergence interests. We will show (under some smoothness conditions) that the set of cooperative optimal strategies corresponding with these adapted object functions for each of the $N$ countries, can be parametrized by the $N-1$ parameters $\alpha_{1}, \ldots, \alpha_{N-1}$ and $\lambda$, and that this parametrization is a continuous differentiable function of all these parameters. By varying these parameters, in particular $\lambda$, it is then possible to analyze the trade off between the costs individual countries have to pay and more convergence. First, we present a preliminary result. The next theorem shows that if one considers a certain convex combination of all object functionals $J_{i}, \quad i=1, \ldots, N$ and $C$, the optimal strategy minimizing this combination will be a continuous differentiable function of $N$ out of $N+1$ parameters.

Theorem 2.1 Suppost $l^{\prime}$ is a convex set, $J_{i}(u), i=1, \ldots, N$ and $C(u)$ are strictly convex functionals which are twice continuously differentiable in $u \in U$. Consider

$$
J\left(u, \alpha_{1}, \ldots, \alpha_{N}, \lambda\right):=(1-\lambda)\left(\sum_{i=1}^{N} \alpha_{i} J_{i}(u)\right)+\lambda C(u)
$$

for $u \in l . \lambda \in[0,1]$ and $\alpha_{i} \in[0,1]$ for $i=1, \ldots, N$, with $\sum_{i=1}^{N} \alpha_{i}=1$. Let

$$
u^{*}:=\arg \min _{u} J\left(u, \alpha_{1}, \ldots, \alpha_{N}, \lambda\right)
$$

Then. for every $\lambda \in[0,1]$ and $\alpha_{i} \in[0,1]$ for $i=1, \ldots, N$, with $\sum_{i=1}^{N} \alpha_{i}=1$, $u^{*}$ is uniquely determined as a function of the parameters $\alpha_{1}, \ldots, \alpha_{N-1}, \lambda$, i.e. $u^{*}=u^{*}\left(\alpha_{1}, \ldots, \alpha_{N-1}, \lambda\right)$. Moreorer. this function $u^{*}$ is a continuously differentiable function in $\left(\alpha_{1}, \ldots, \alpha_{N-1}, \lambda\right) \in$ $[0.1] \times \ldots \times[0.1]$, with $\sum_{i=1}^{N-1} \alpha_{i} \leq 1$.

Proof. Let $\bar{\alpha}:=\left(\bar{\alpha}_{1}, \ldots, \bar{\alpha}_{N}, \bar{\lambda}\right) \in[0,1] \times \ldots \times[0,1]$ be fixed numbers, with $\sum_{i=1}^{N} \bar{\alpha}_{i}=1$. The strictly convex properties of $J_{1} \ldots, J_{N}, C$ imply that the function $J(u)$ is strictly convex in $u \in l$. So for every $\bar{\alpha}$. $J$ has a unique global minimum on $U$. Denote this element in $U$, which depends on $\bar{\alpha}$. by $u_{\bar{\alpha}}$. Since $J$ is differentiable we conclude that the derivative of $J$ with respect to $u$. evaluated at the point $u_{\bar{a}}$ is zero. So,

$$
F\left(\bar{\alpha}_{1}, \ldots, \bar{\alpha}_{N}, \bar{\lambda}, u\right):=\frac{\partial J(u)}{\partial u}=(1-\bar{\lambda}) \bar{\alpha}_{1} \frac{\partial J_{1}(u)}{\partial u}+\ldots+(1-\bar{\lambda}) \bar{\alpha}_{N} \frac{\partial J_{N}(u)}{\partial u}+\bar{\lambda} \frac{\partial C(u)}{\partial u}=0
$$

evaluated at the point $u=u_{\bar{N}}$. Note that, since $J$ is by assumption twice continuous differentiable, the functional $F$ is continuous differentiable in $\left(\alpha_{1}, \ldots, \alpha_{N-1}, \lambda, u\right) \in[0,1] \times$ $\ldots \times[0,1] \times\left[\right.$. Furthermore since $J_{1}, \ldots, J_{N}, C$ are strictly convex functionals in $u$, we have that

$$
\forall \bar{\alpha} \in[0,1] \times \ldots \times[0,1] \quad \operatorname{det} \frac{\partial F\left(\bar{\alpha}, u_{\bar{\alpha}}\right)}{\partial u} \neq 0
$$

Applying the implicit function theorem yields then that there is an unique continuous differentiable function, say $f$, such that for all $\alpha:=\left(\alpha_{1}, \ldots, \alpha_{N}, \lambda\right) \in[0,1] \times \ldots \times[0,1]$,
$F(\alpha, f(\alpha))=0$. with $f(\alpha)=u_{0}$. So. $u^{*}:=u^{*}\left(\alpha_{1}, \ldots, \alpha_{,}, \lambda\right):=f\left(\alpha_{1}, \ldots, \alpha_{N}, \lambda\right)$ is a continuous differentiable function in $\alpha \in[0,1] \times \ldots \times[0.1]$. Using the fact that $\sum_{i=1}^{N} \alpha_{i}=1$ gives $u^{*}=u^{*}\left(\alpha_{1} \ldots, \alpha_{-1}, \lambda\right)$.

Remark. In the sequel we will use the notation $\left(\alpha_{1}, \ldots, \alpha_{N-1}\right) \in[0,1] \times \ldots \times[0,1]$, but, by doing so, we implicitly assume that the $\alpha_{i}, i=1, \ldots, V-1$ satisfy the constraint $\sum_{i=1}^{N-1} \alpha_{i} \leq$ 1.

Using the previous result we show now that the set of control strategies defined in theorem 2.1.. parametrized by

$$
\bar{I}^{\bar{L}}:=\left\{u^{*}\left(\kappa_{1}, \ldots . \alpha_{N_{-1}}, \lambda\right) . \mid\left(\alpha_{1}, \ldots . \alpha_{N-1}, \lambda\right) \in[0,1] \times \ldots \times[0,1]\right\}
$$

has the advertised properties. Formally the result reads as follows:

Theorem 2.2 There exists a bijective mapping between the set of unique points

$$
\left\{u^{*}\left(\alpha_{1} \ldots, \alpha_{v_{-1}}, \lambda\right) \mid\left(\alpha_{1}, \ldots, \alpha_{v_{-1}}, \lambda\right) \in[0,1] \times \ldots \times[0,1]\right\}
$$

and the set

$$
\left\{\left(J_{1}\left(u^{*}\right), \ldots . J_{N}\left(u^{*}\right), C\left(u^{*}\right)\right) \mid\left(\Omega_{1}, \ldots, \Omega_{N-1}, \lambda\right) \in[0.1] \times \ldots \times[0,1]\right\} .
$$

Furthermore $J_{1}\left(u^{*}\right)$..... $J_{\mathrm{N}}\left(u^{*}\right) .\left(^{\prime}\left(u^{*}\right)\right.$ are continuous functions. in $\left(\alpha_{1} \ldots . ., \alpha_{N-1}, \lambda\right) \in[0,1] \times$ $\ldots \times[0,1]$.

Proof. Because $(1-\lambda)\left(\sum_{i=1}^{\lambda} \alpha_{i}\right)+\lambda=1$, with $\lambda \in[0.1]$ and $\alpha_{i} \in[0,1]$,for $i=1, \ldots, N$, the unique solution $u^{*}$ of $J(u)$ is a Pareto solution for the objective function $J(u)$ which represents a game with $V+1$ players. where each player minimizes the objective function represented by $J_{2}$ for player $i .(i=1 \ldots . . V)$ and $($ for player $N+1$. According to .e.g., de Zeeuw [9. lemma 3.4.2 and 3.4.3] there is a bijective mapping between the Pareto solutions for $J_{1} \ldots . J_{3}$. $C$ and the optimal solution for $J$. The set of Pareto solutions can be found by varying the parameters $\left(\alpha_{1} \ldots . \alpha_{N}, \lambda\right)$ between $[0,1] \times \ldots \times[0,1]$ with $\sum_{i=1}^{N} \alpha_{i}=1$.
Because $u^{*}$ is a contmums function in $\left(\alpha_{1} \ldots, \alpha_{,-1}, \lambda\right) \in[0.1] \times \ldots \times[0,1]$ it is straightforward that $J_{1}\left(u^{*}\left(\alpha_{1} \ldots . \alpha_{-1}, \lambda\right)\right) \ldots . J_{v}\left(u^{*}\left(\alpha_{1} \ldots . \alpha_{\gamma_{-1}}, \lambda\right)\right) . C^{\bullet}\left(u^{*}\left(\alpha_{1}, \ldots, \alpha_{v_{-1}}, \lambda\right)\right)$ are continuous functions in $\left(\alpha_{1}, \ldots, a_{1-1}, \lambda\right) \in[0,1] \times \ldots \times[0.1]$.

Using the theorem, the set of control strategies $\overline{=}$ gives us the following subset of (1):

$$
\begin{equation*}
\left\{\left(J_{1}\left(u^{*}\right) \ldots . . J_{N}\left(u^{*}\right) \cdot\left(C\left(u^{*}\right)\right) \mid u^{*} \in \bar{L}^{\dot{L}}\right\}\right. \tag{2}
\end{equation*}
$$

To see that this reduction of the set in (1) still contains all the interesting points, we analyze the set in (2) in combination with $J$ more specifically. We have that:
(2) the set in (2) contains the whole set of points $\left(J_{1} \ldots \ldots J_{v}\right)$ which belong to the Pareto optimal solutions. To find these solutions we substitute $\lambda=0$ in $\overline{\bar{C}}$ and fill in the resulting
control strategies in (2).
(22) the set in (2) contains the points where ( is minimal. To find these points we substitute $\lambda=1$ in $\bar{l}$ and fill in the resulting strategies in (2).
Furthermore, from theorem 2.2 . we have that the set of points in (2) form a continuous surface in the $J_{1} \ldots . . J_{N}$. ('plane, which indicates that we have parametrized all the interesting points between (1) and ( 11 ) as well. These points can be found by varying $\lambda$ between 0 and 1.

From now on we will skip the $u^{*}$ in the notation and describe the set in (2) as:

$$
\begin{equation*}
\left\{\left(J_{1} \ldots \ldots J_{N} \cdot() \mid\left(\Omega_{1}, \ldots, \alpha_{N-1}, \lambda\right) \in[0,1] \times \ldots \times[0,1]\right\} .\right. \tag{3}
\end{equation*}
$$

We will now define some sets of interesting points. A projection of the set in (3), on the $J_{1}, \ldots . J_{N}$-plane is:

$$
S:=\left\{\left(J_{1}, \ldots . J_{N}\right) \mid\left(\Omega_{1}, \ldots \alpha_{N-1}, \lambda\right) \in[0,1] \times \ldots \times[0,1]\right\}
$$

The subset of $S$ :

$$
P:=\left\{\left(J_{1}, \ldots . J_{N}\right) \mid\left(\alpha_{1}, \ldots, \alpha_{N-1}, 0\right) \in[0,1] \times \ldots \times[0,1]\right\}
$$

represents the set of Pareto solutions. Iso-convergence lines. i.e. lines with the same degree of convergence. are defined as follows:

$$
I_{\gamma}:=\left\{\left(J_{1} \ldots . J_{N}\right) \mid C\left(\Omega_{1} \ldots . \alpha_{N-1} \cdot \lambda\right)=\gamma . \quad\left(J_{1}, \ldots . J_{v}\right) \in S, \gamma \in \mathbb{R}^{+}\right\}
$$

Note that a small value of ? corresponds with much convergence (and vice versa). The negotiation area is defined by:

$$
\mathcal{N}:=\left\{\left(J_{1}, \ldots . J_{N}\right) \mid J_{1} \leq J_{1}^{N} \ldots . . J_{N} \leq J_{3}^{*} .\left(J_{1}, \ldots . J_{N}\right) \in S^{\prime}\right\}
$$

Using the axiom of individual rationality it is clear that policymakers will not agree to a certain degree of convergence, denoted by $\gamma$. if $\Omega, \cap \Omega=\emptyset$. Moreover, the largest degree of convergence policymakers are willing tw accept is given by:

$$
\gamma^{*}:=\min _{\gamma}\{\gamma \mid I, \cap \mathcal{V}\} \neq 0
$$

So. in general policymakers should set their degree of convergence with care because if this degree is set too ambitions policymakers are not willing to cooperate anymore. In the next theorem we will prove fact that the point in the negotiation area which yields the largest degree of consergence is the non-cooperative threat point.

Theorem 2.3 If $\mathcal{N} \subset$, then the point in the negotiation area. $\mathcal{N}$, represented by a $x \in \overline{\bar{U}}$, for which $C(x)=\gamma^{*}$ tquals. 1 .

Proof. According to Theorem 2.2.. $\bar{l}$ is the set of Pareto solutions which represents a game of $N+1$ players. where each player minimizes the objective function represented by $\left.J_{\text {, for player } i . ~}^{\text {f }}=1 \ldots . . V\right)$ and $C$ for player $N+1$. Suppose that $x \in \bar{U}$ yields a point in the negotiation area .1 which not equals $V^{c}$ but for which $C(x)=\gamma^{*}$. Since $x$ yields a point $\left(J_{1}(x), \ldots, J_{v}(x)\right)$ in the negotiation area it satisfies the property that $J_{i}(x) \leq J_{i}^{\mathrm{N}}$. Because $x$ yields a point which not coincides with $V^{c}$ there is an $i \in 1, \ldots, N$ for which $J_{2}(x)<J_{i}^{V}$. Making use of the special properties of $V$ and the assumption $N \subset S$, it is now always possible to take a point on the boundary of $N$, represented by a $y \in \overline{\bar{U}}$, for which $J_{1}(y)=J_{1}(x) \ldots \ldots J_{i}(y)=J_{i}^{N} \ldots \ldots J_{N}(y)=J_{N}(x)$. The fact that $x$ and $y$ both belong to the set of the Pareto optimal solutions of the extended game yields that $C(y)<C(x)$. This fact. however, violates the assumption that $C(x)=\gamma^{*}$.

It is important to indicate here that the non-cooperative strategy which results in the point $V^{c} \in S$ in general differs from the cooperative strategy which results in the point $N^{c}$. In general, the convergence outcome of the non-cooperative strategy and the cooperative strategy will differ in the sense that the convergence value for the cooperative strategy will be lower than the convergence value for the non-cooperative strategy. So, the gains in convergence policymakers will receive by playing cooperatively will be at most $\gamma^{*}-C\left(x_{N^{c}}\right)$, where $x_{N^{c}}$ represents the non-cooperative strategy which yields $N^{c}$. Thus, in general, Pareto solutions will not yield maximal convergence. Therefore. if policymakers want a certain degree of convergence. it will usually not be possible to keep up with the Pareto optimal solutions. I'sefull Pareto solutions will only coincide with solutions with a certain degree of convergence. say $\gamma$, if $I_{\gamma} \cap . \backslash \cap P \neq \emptyset$. Note. furthermore, that the price to be payed for reaching convergence of a certain degree will not be the same for every country. We will illustrate this in an example in the next section.

## 3 An illustrative example

We consider a theoretic example in a (discrete time) deterministic linear quadratic difference game framework with two players (comotries). The dynamic behaviours of player 1 and player 2 are described by:

$$
\begin{array}{ll}
y_{1}(t)=y_{1}(t-1)+u_{1}(t)+0.3 y_{2}(t-1) . & y_{1}(0)=1 . \\
y_{2}(t)=y_{2}(t-1)+u_{2}(t)+0.6 y_{1}(t-1) . & y_{2}(0)=0 .
\end{array}
$$

where for $i=1.2 . \quad y_{2}(t) \in \mathbb{R}$ is the target variable and $u_{i}(t) \in \mathbb{R}$ is the instrumental variable. From the interaction terms $\left(0.3 y_{2}(t-1)\right.$ for player 1 and $0.6 y_{1}(t-1)$ for player 2$)$ follows that each player faces a different dymamical structure. Player 2 is more influenced by player 1 than vice versa. Each player makes his plans for the future. We assume that each player has a planning period of 2 and chooses his desired paths for the future, as
follows:

$$
\text { desired paths }\left\{\begin{array}{ll}
\text { player } 1: & y_{1}^{*}(1)=2 .
\end{array} \quad y_{1}^{*}(\cdot 2)=3 .\right.
$$

These desired paths reflect the policymakers own wishes of the future and are obtained independently from each wther. In this example the players have different preferences but, as can be seen from the ideal paths. both players are striving for convergence of their target variables in period 2 . It is of course not necessary to choose desired paths wich converge but by doing so we will be able to demonstrate the fact that Pareto optimal solutions do not coincide with convergence solutions, even if policymakers strive for convergence in their desired values. We represent the costfunctions $J_{1}, J_{2}$ for every individual player by:

$$
\begin{aligned}
& J_{1}=0.5\left(\left(y_{1}(1)-2\right)^{2}+\left(y_{1}(2)-3\right)^{2}+u_{1}(1)^{2}+u_{1}(2)^{2}\right), \\
& J_{2}=0.5\left(\left(y_{2}(1)-1.5\right)^{2}+\left(y_{2}(2)-3\right)^{2}+u_{2}(1)^{2}+u_{2}(2)^{2}\right) .
\end{aligned}
$$

Each player wants to play a strategy, during his planning period, which minimizes his costs. So the control problem for every individual player $(i=1,2)$ is:

$$
\min _{n_{1}(1), u_{1},(2)} J_{2} \text {. }
$$

Because the target variable (and indirectly the instrumental variable) of each player is directly related to those of the other player, the control problem of each player depends on the actions undertaken by the other plater. This gives rise to various solution concepts. From the non-cooperative solutions we will just consider the open loop Nash solution, which we denote by 1 . The cooperatise solutions are represented by the set of Pareto solutions which can he fonnd by solving:

$$
\min _{u} \alpha \cdot J_{1}+(1-\alpha) \cdot J_{2}
$$

for $a \in[0.1]$, where $u:=\left(u_{1}(1), u_{1}(2), u_{2}(1), u_{2}(2)\right)$.
However, before playing the game both players want to be sure that there will be some degree of consergence of their target variables. In this example we assume that both players want to converge to the aterage of their target variables. We take as a measure for the degree of convergence the following convergence function:

$$
{ }^{\prime}=\sum_{i=1}^{2}\left(y_{i}(1)-\bar{y}(1)\right)^{2}+4\left(y_{i}(\underline{2})-\bar{y}(\underline{2})\right)^{2} .
$$

where $\bar{y}(t):=0 . \bar{y}\left(y_{1}(t)+y_{2}(t)\right)$ for $t=1.2$. So. both players agree that they want to minimize the variance of their target variables in each period. Moreover. minimizing the variance in period 2 is given more weight than minimizing the variance in period 1 , which is represented by the weights of 1 in period 1 and 4 in period 2 . These weights indicate that both plavers find it more important that there is convergence at the end of the planning period than during the planning period.

Now. together. the players have to take a derision about the strategy they are going to follow. In order to choose a strategy they have to weigh out all possible strategies. So, ultimately they have to find a strategy which is "optimal" in some sense. In the next subsection we demonstrate the solution concepts developed in section 2 and analyze the space of interesting untcomes. After that we give one possible interpretation of "optimal" and give a proposal to determine a feasible degree of convergence, 9 , for both players.

### 3.1 Analysis of the possible outcomes

As stressed in section 2. the decision abont what strategy to follow, will depend upon the following set:

$$
\begin{equation*}
\left\{\left(J_{1}(u), J_{2}(u) \cdot C(u)\right) \mid u \in \mathbb{R}^{4}\right\} \tag{4}
\end{equation*}
$$

Because $J_{1}, J_{2}, C$ are strictly convex functions which are twice differentiable in $u$, the set $i^{2}$ can be found by solving the following problem:
Let $\alpha, \lambda \in[0,1]$, and

$$
J(u):=(1-\lambda)\left(\alpha \cdot J_{1}+(1-\alpha) \cdot J_{2}\right)+\lambda C
$$

Find now for every o. $\lambda \in[0.1]$ :

$$
u^{*}:=\arg \min _{u} . J(u)
$$

From section 2. the set of control strategies $l^{\prime \prime}$ is given by:

$$
\left\{u^{*}(a, \lambda) \mid(a, \lambda) \in[0,1] \times[0,1]\right\}
$$

Substituting these control strategies in ( 4 ) gives the following set (compare with (3)):

$$
\begin{equation*}
\left\{\left(. J_{1}(\Omega, \lambda) . I_{2}(\alpha, \lambda) \cdot((\Omega, \lambda)) \mid(\Omega, \lambda) \in[0.1] \times[0.1]\right\} .\right. \tag{5}
\end{equation*}
$$

In the sequel we will analyse this set of points for the given example.
Remark. ('omputing the ontcomes for $\lambda=1, \Omega=0 . \Omega=1$ gives some difficulties because in that case we have a singular system of equations. Howerer. we are not particularly interested in those situations so we used in our calculations values which are close to these points.

A projection of the surface in (5). on the $J_{1} . J_{2}$-plane is drawn in figure 2. This set of points is denoted by . $\mathrm{S}^{\text {. like in section } 2 \text {. }}$

$$
S=\left\{\left(J_{1}(\alpha, \lambda) . J_{2}(\alpha, \lambda)\right) \mid(\Omega, \lambda) \in[0,1] \times[0,1]\right\}
$$

The black lines in figure 2 represent the edges of $S$. One of these edges is the set of Pareto


Figure 2: The parametrized area $S$, the most left curve represents the pareto solutions $P$, the small triangle on this curve represents the negotiation area $N$.
solutions, which is gisen by the left black line. It is obtained by computing for various $\alpha$ :

$$
P=\left\{\left(J_{1}(\Omega, 0), J_{2}(\Omega, 0)\right) \mid \sigma \in[0.1]\right\}
$$

Points on the upper part of the Pareto line correspond with a high value of $\alpha$ and points on the lower part to a low value of $\alpha$. The edge $L_{1}$ in figure 2 is obtained by computing for various $\lambda \in[0,1]:\left(J_{1}(1, \lambda), J_{2}(1, \lambda)\right)$ and the edge $L_{2}$, $y$ computing for various $\lambda \in[0,1]$ : $\left(J_{1}(0, \lambda), J_{2}(0, \lambda)\right)$. The edge in the figire which corresponds to $\left(J_{1}(\alpha, 1), J_{2}(\alpha, 1)\right)$ for $\alpha \in[0.1]$ is reduced to one point in the figure. We denoted this point by $L_{3}$. The small triangle on the Paretu line denotes the negotiation area $N$ as defined in section 2. Note that the negotiation area $X$ is completely covered by s.

To get an idea of the degree of convergence in every point of the set in (3) we plotted figure 3. This figure shows a three dimensional plot of the following surface:

$$
\left\{C^{\prime}(\alpha, \lambda) \mid(\alpha, \lambda) \in[0,1] \times[0,1]\right\}
$$

Note that the points where $\lambda=0$ give the degree of convergence for the Pareto solutions. As can be seen in the figure points on the Pareto line where a almost equals 1 or 0 , give a very high ( value. which indicates that in those points the degree of convergence is rather small. Moreover we marked in this figure the point with the largest degree of


Figure 3: A three dimensional plot, where for each $\alpha, \lambda \in[0,1] \times[0,1]$ the corresponding convergence outcome is plotted. The curve on the back, where $\lambda=0$, represents the Pareto solutions $P$.

|  | $J_{1}$ | $J_{2}$ | $C$ | $a$ | $\lambda$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Cooperation |  |  |  |  |  |
| $N^{c}$ | 0.505 | 0.370 | 0.1059 | 0.080 | 0.270 |
| $A$ | 0.497 | 0.370 | 0.1258 | 0.625 | 0 |
| $B$ | 0.505 | 0.359 | 0.1483 | 0.523 | 0 |
| $D$ | 0.472 | 0.451 | 0.0896 | 0.836 | 0 |
|  |  |  |  |  |  |
| $N^{c}$ N-Cooperation | 0.505 | 0.370 | 0.1365 | - | - |

Table 1: Characteristics of some interesting points.
convergence on the Pareto line. It is denoted by $D$ and it corresponds with $\alpha=0.836$. In table 1 the corresponding $\left(J_{1}, J_{2}, C\right)$ of point $D$ is given. Moving away from the Pareto line, by increasing $\lambda$ just a little bit, we see that the degree of convergence increases also. This means that if the players choose an outcome outside the Pareto line $P$ their costs will increase but in return for these increasing costs the value of the convergence function will decrease, which means more convergence. In this example, if $\lambda$ increases to 1 , the convergence function will go to 0 , which means complete convergence (in period 1 and period 2). Zooming in on figure 2, around the negotiation area $N$, gives us figure 4.


Figure 4: Zooming in on figure 2 around the negotiation area. Iso-convergence lines are drawn.

Specific information about the points $A, B$, and $N^{c}$ can be found in table 1 . In table 1 the cooperative outcome $N^{c}$ corresponds with a strategy which yields a lower convergence value than the non-cooperative outcome $N^{c}$. The reason for this is that the non-cooperative strategy differs from the cooperative strategy. Playing the cooperative strategy gives, with the same individual costs, an increase in convergence of 0.306 ! In figure 4 we draw some iso-convergence lines, as defined in section 2 . In the figure for each iso-convergence line the corresponding convergence value is given. The degree of convergence on the Pareto line increases from $B$ to $A$. As proven in section 2 and visible in the figure, the point with the largest degree of convergence in the negotiation area lies on the edge of the negotiation area and is exactly the $N^{c}$ point which belongs to the iso-convergence line $I_{0.1059}$. So, the $\gamma^{*}$, as defined in section 2 , equals 0.1059 .

Zooming in on figure 3, from a different viewpoint, we get an indication of which values of $\alpha, \lambda$ belong to the negotiation area $N$, which is drawn on the surface in figure 5 . The corresponding $\alpha, \lambda$-values for $A, B$, and $N^{c}$ are given in table 1 .


Figure 5: Zooming in on figure 3 around the negotiation area ( $0.05 \leq \alpha \leq 0.65$ ) and $(0 \leq \lambda \leq 0.4)$. The curve on the back $(\lambda=0)$ represents a subset of the Pareto solutions $P$. The interior of the curve drawn on the surface represents the negotiation area $N$.

### 3.2 Fixing the degree of convergence

In this subsection we assume that both players agree they want a degree of convergence of at least $\gamma$. So, the players will play a strategy which results in a point in (4) which belongs to the iso-convergence line $I_{\gamma}$. In figure 6 we have drawn for three different values of $\lambda$ the convergence values for all $\alpha \in[0,1]$. On the one hand it gives an indication of how quickly convergence declines when increasing $\lambda$, and on the other hand it illustrates how the convergence depends on a for constant $\lambda$. Again, $\lambda=0$ corresponds with the Pareto
optimal strategies. Moreover. in figure 6 four different levels for $\gamma$ are drawn. Each level distinguishes a group of solutions with different properties which we will analyze below.


Figure 6: For various $\lambda$, and varying $\alpha \in[0,1]$ the corresponding convergence outcome is plotted.
(a) $\gamma<0.0896$

Both players play a strategy which results in a point on the iso-convergence line $I_{\gamma}$. However (see figure 4), for this $\gamma, I_{\gamma} \cap P=\emptyset$, and $I_{\gamma} \cap N=\emptyset$, which implies that the chosen strategy is not a Pareto optimal strategy and that the corresponding $\left(J_{1}, J_{2}\right)$ point falls outside the negotiation area. So, at least one of the players will have higher costs than when he plays the non-cooperative open loop Nash strategy. Such an ambitious setting of the degree of convergence is very unrealistic, it means that convergence prevails over individual costs. Therefore we excluded this possibility in section 2 by our assumption of rational behaviour.
(b) $0.0896 \leq \gamma<0.1059$

For this $\gamma$ (see figure 4). $I_{\gamma} \cap N=\emptyset$, but $I_{\gamma} \cap P \neq \emptyset$. So, a strategy can be played which coincides with the Pareto optimal strategies. From figure 6 follows that there are two possible Pareto optimal strategies, which corresponds in both cases wih an $\alpha>0.625$. So player 2 will have higher costs than when he plays the open loop Nash strategy. On the other hand, playing one of the two Pareto optimal solutions will be very profitable for player 1. Without any other additional agreements between the players (see (a)), player 2 will never accept such an outcome.
(c) $0.1059 \leq \gamma<0.1258$

In this case, $I_{\gamma} \cap N \neq \emptyset$, and $I_{\gamma} \cap P \neq \emptyset$, but $I_{\gamma} \cap N \cap P=\emptyset$. If the players decide to play a Pareto optimal strategy, player 2 will again have higher costs than when playing the open loop Nash strategy. More likely is an outcome within the negotiation area $N$. For instance if the players agree on a $\gamma=0.1104$ then they have to find a point in $I_{0.1104} \cap N$. Looking again at figure 4, we see that there is a whole range of possible outcomes. A unique outcome may be obtained using bargaining theory $[7,9]$. As already can be seen in figure 4, any outcome of such a game will be that player 2 does not gain much in a bargaining situation whereas the gains for player 1 can be considerable.
(d) $0.1258 \leq \gamma$

Now, $I_{\gamma} \cap N \cap P \neq \emptyset$, which means an outcome can be played on the Pareto line between $A, B$. Also here, bargaining theory can be applied to select a unique outcome.

Concluding we see that if the desired degree of convergence is set too high the players have to pay a price for that and can not obtain Pareto optimal solutions. Furthermore, if they can not obtain solutions within the negotiation area the player(s) will have an incentive to deviate towards the threatpoint and forget about any degree of convergence at all. Moreover, we observe that in almost all cases player 1 can gain more than player 2. In the figures this depends on the shape of the iso-convergence lines and ultimately is traced back to the fact that player 1 has more influence on player 2 than vice versa.

### 3.3 An approach to determine a reliable degree of convergence

In this section we present an algorithm to determine a feasible degree of $\gamma$. The previous subsection states that, without any other agreements between the players, a degree of convergence which has no correspondending outcome in the negotiation area is unlikely to happen. The question remains, however, which degree of convergence within this negotiation area ultimately will be selected by the players. In fact without making any further assumptions on the negotiation process, every point in the negotiation area is possible. One way to come to a unique point within the negotiation area is by axiomatizing the negotiation game. We shall not elaborate this subject here, since for the moment we are more interested in qualitative rather than quantitative statements. All we will do is sketch how a feasible degree of $\gamma$ can be determined, using some heuristic arguments. First we will give an example and then we will present two algorithms which illustrate the approach in general.

In figure 7 the convergence value is plotted against the costs of player 1 , along the line $A$ to $N^{c}$, where the costs of player 2 remain constant. Starting at point $A$ and moving towards $N^{c}$, the convergence value declines rapidly. This continues untill the point where $\left(J_{1}, C\right)=(0.4987,0.1098)$. After that point the derivative of the slope of the curve gets larger than -1 . In figure 7 we denoted this point by $E$. From that point on, towards $N^{c}$, the costs increase more rapidly than the degree of convergence. If player 1 has to choose an


Figure 7: For the edge of the negotiation area, from $A$ to $N^{c}$, the convergence value is plotted. Point $E$ is the point where the derivative of the tangent of the curve is -1 .
outcome on the line in figure 7, he will start in point $A$ where his costs are minimal. From thereon, if player 1 wants to increase convergence, he will have to weigh out costs against convergence. For instance, if player 1 starts in $A$ and moves towards $N^{c}$ and accepts only points where the slope $\partial C / \partial J_{1} \leq-1$, the result will be the outcome $E$.

The general idea expressed in the above example is that players accept an increase in convergence only if the corresponding costs stay within a prespecified region. So, a sketch of a numerical approach for determining a feasible degree of $\gamma$ would be the following:
(1) Start from a point $\left(\bar{J}_{1}, \bar{J}_{2}\right)$ on the Pareto line between $A, B$. It seems reasonable to start at a bargaining outcome $[7,9]$.
(2) Determine the direction $v=\left(v_{1}, v_{2}\right)$, for which there is a $t>0$ for which $\left(\bar{J}_{1}, \bar{J}_{2}\right)+t\left(v_{1}, v_{2}\right)$ $\in N$, and convergence increases maximal.
(3) Choose a small $t>0$.
(4) Calculate $\chi_{c}=-\partial C / \partial v$. Check if $\partial J_{i} / \partial v<\chi_{i}\left(\chi_{c}\right)$, for $i=1,2$ where $\chi_{i}\left(\chi_{c}\right), i=1,2$ are (decreasing) functions of $\chi_{c}$ which indicate the weight players want to assign to the tradeoff between convergence and costs. That is, if the additional increase in convergence (reflected by a smaller value for $C$ ) equals $\chi_{c}$ then the additional increase in costs for each player seperately should be less than $\chi_{i}\left(\chi_{c}\right)$ for $i=1,2$.
(5) If (4) holds then use this new point as a starting point and start again in (2). Stop, if no point in $N$ can be found for which (2) and (4) hold.

A drawback of this approach is that it is timeconsuming, even for small models. The reason is that the functions $J_{1}, J_{2}, C$ are parametrized in $\alpha$ and $\lambda$ and therefore calculating
 step (2). take a lot of time.

A good alternative which is strongly related with the previons algorithm. but is easier to compute. is the following algorithm:
(1) start in some feasible point between A. B. With this point there corresponds an uniquely determined a.
(2) Fix $a$.
(3) Increase $A$ from (1 10 1 by using a stepsize of for instance. 0.01 . ('heck if the point stays in the nequtiation area. 1 .
(4) ('heck for exery $\lambda$ whether -i)( $/ i) \lambda>0 . I_{1} / \partial \lambda$ and $-\partial C / \partial \lambda>\partial \cdot I_{2} / \partial \lambda$.
(5) Stop if no $\lambda$ can he fomul for which (3) and (4) holds.

The condition in - (ep (4) of the alentithm (an be compared with the conditions in step (4) of the previons algonthm. These comblions state that if for each player seperately costs rise less than combergence falls when $\lambda$ increases bye mat both players are willing to accept more consergence (as long an they stay within the negotiation area). Note that for our convenience we took $\backslash=\backslash_{1}=1_{2}$. We used the last algorithm to determine a feasible ontcome in onn example. Av a starting point we choose the axiomatic Nash bargaining sohtion (sere.g. [7, 9]). This solution corresponds with $a=0.575$ and lies on the Pareto curve appoximaty in the midhe leetwen $A$ and $B$. In figures we have drawn for $0 \leq \lambda \leq 1$ the curves -i $(\% / i) \lambda . \partial I_{1} / \partial \lambda$ and $\partial J_{2} / \partial \lambda$. The figure shows some interesting facts. First of all the conditions of atep) ( $t$ ) of the algorithm are violated when $\lambda>0.26$. Secondly: when increasing $\lambda$ the costs of player 1 fall! This lasts till $\lambda=0.4$ where $\partial . J_{1} / \partial \lambda$ gets pusitive. ()n the wher hath plavers ? additional costs for increasing convergence are for all $\lambda$ higher than the additional costs phater 1 is faced with. Finally in figure 9 we have drawn a small part of the corte:

$$
\left.\left.\left\{\left(. I_{1}(1)-5, \lambda\right) . I_{2}(1)-5, \lambda\right)\right) \mid \lambda \in[0.1]\right\}
$$

The curve temb to way very fore to the Pareto conve and croses the negotiation area al-
 Remarkable is that plaser 1 (the stomery phaver) has lower conts than he would have in the Nash hargamine sulution, a sulation which wonld be anceptable if the players did not have to reckon with and convergence asperta at all.

## 4 Conclusions

In this paper we presented a theoretimal apmach how to deal with the issue of consergence between E. (`. comatries. Based on the asmmption that the primary interest of the countries is minimizing their wwn individual welfare lass. we considered the question how cooperative strategies yielding maximal comergence ran be determined. We showed that for a large


Figure s: For increasing $\lambda$. the three curves $-\partial\left(/ \partial \lambda, \partial \cdot J_{1} / \partial \lambda\right.$ and $\partial J_{2} / \partial \lambda$ are drawn.


Figure 9: Starting from the Nash bargaining solution, fixing $\alpha$ and slowly increasing $\lambda$ gives a curve which tends to stay very close to the Pareto solutions.
class of problems. i.e. problems were the individual costfunctions and convergence function are twice differentiable and convex, a parametrization for a large set of cooperative strategies can be determined. This set covers the Pareto optimal solutions by construction and, in general (see note in section 2 ), covers all the cooperative strategies which improve over the non-cooperative solution. Using this approach a number of interesting questions can be considered. For instance whether it is possible that for a particular time horizon the E.C. countries can satisfy the convergence conditions in such a way that for every country the corresponding costs are acceptable, and how these costs differ among countries. In section three we showed in a simple theoretical example how to analyze such questions. The next step should be to use the same approach on more realistic dynamic (macro)econometric country models. or just on a part of these models where the interaction between countries is most essential. e.g. the monetary sector. In dealing with that problem countries should realize that
(1) it must be clear where one should converge to [ 8 ]. Should they converge to the lowest, the highest or the average rates of their target/instrumental variables? In our approach this means that comntries should agree on a common convergence function $C$.
(2) the preferences of countries should be finetuned on each other. It is clear that if these preferences differ strongly among countries, convergence will be a very tough issue. In the dynamical game approach this can be analysed with the desired paths and choice of weights for the target/instrumental variables [5]. The theoretical example was chosen in such a way that in the last period of the planning horizon the countries, at least, strive for convergence. which was implemented by choosing equal values for the corresponding desired paths.
(3) the time-horizon, necessary for reaching the convergence conditions within a limited period. plays a crucial role too. This aspect is strongly related to the determination of the degree of convergence. We expect that for a short planning period the costs for convergence can be very costly and this may ultimately result in non-cooperative behaviour of some countries. This subject remains, however, a topic for future research.
(4) costs for convergence differ among comntries. The example in the paper gives a way how to determine these costs for any given degree of convergence. In general these differences will depend on the ecommical structures of the participating countries. The theoretical example gives already an indication that these costs could be much higher for countries which have less influence in the ('ommmity.

The approach designed here for analysing convergence can be used for many other problems as well. If players in a dynamic game have common objectives. apart from their usual costfunctions. the approarh can be used as lung as we take twice differentiable convex functions. If we stay in a multicumtry setting, common objectives appear in e.g. environmental issues and trade issums.

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[^0]:    *This research was sponsored by the Economics Research Foundation, which is part of the Netherlands Organization for Scientific Research (NWO).

[^1]:    ${ }^{1}$ This axiom states that policymakers, if they behave rational, will never accept an outcome for their individual object function which is worse than the one a policymaker can obtain by acting independently (which is represented by the non-cooperative outcome $N^{c}$ ).

