Scale Returns of a Random Matching Model

Kultti, K.K.

Publication date:
1997

Citation for published version (APA):
Kultti, K. K. (1997). *Scale Returns of a Random Matching Model*. (CentER Discussion Paper; Vol. 1997-71). CentER, Center for Economic Research.

entER
for
omic Research

Discussion paper

[redacted]

Tilburg University

Center
for
Economic Research

No. 9771

SCALE RETURNS OF A RANDOM MATCHING MODEL

By Klaus Kultti

July 1997

ISSN 0924-7815

Scale returns of a random matching model

Klaus Kultti

Center for Economic Research, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, The Netherlands

Abstract

This note considers a random matching model in which the meeting probabilities can be derived from the basics of the model. We determine conditions for the matching technology to exhibit decreasing, constant, and increasing returns to scale.

Keywords: Random matching, scale returns

JEL classification: C78, J41

1. Introduction

Random matching and search models are popular ways to model the economy when Walrasian paradigm of complete markets is too restrictive. There are a multitude of applications ranging from job search and labour markets (eg. Mortensen, 1986), price formation in markets (eg. Rubinsteins and Wolinsky, 1985; Wolinsky, 1988; Gale, 1987), endogenous money (eg. Kiyotaki and Wright, 1993; Burdett et al., 1995), to stability analysis of various trading institutions (Lu and McAfee, 1996). Typically it is postulated that there are two types of agents in the economy, and that they are pairwise matched. A recurrent theme with the matching models is speculation about the matching technology. Results are often sensitive to whether the technology exhibits decreasing, constant, or increasing returns to scale; for instance, the results in Mac Namara and Collins (1990), and Bloch and Ryder (1997) hinge on a technology with increasing returns to scale. In this case the decisions to search or to get matched are omplementary. However, one can easily think of situations in which increasing the number of agents who search results in congestion that reduces the number of matches.

Since usually the environment where the agents operate, and consequently the manner matches come about, is not specified the modeller is free to choose the technology he wishes. Even though the actual process of matching is often left unspecified it is common to offer a verbal description about the process. For instance, Burdett et al. (1995) who study the decision to wait or search give two ways to picture the origin of matching probabilities. First it can be thought that agents who wait are in fixed locations, and searchers visit these locations. Second, one may imagine that the agents are colliding to each other like particles, and collisions indicate matches. If one is to model the matching technologies along either of these lines it is not immediately clear what kind of scale returns one should expect.

The aim of this note is to present a well known and well specified random matching model (see Lu and McAfee, 1996) and show that it exhibits constant returns to scale. We also present a variation of the model, and determine the conditions under which it exhibits decreasing, constant, and increasing returns to scale. The matching model itself is attractive since it is amenable to applications, and because the meetings are not restricted to be pairwise; an agent can meet any number of other agents. In applications that focus on price determination this means that both bargaining and auctions can be used.

2. The model

In the basic set-up there are B buyers and S sellers. The sellers are in fixed locations, and the buyers are randomly distributed on the sellers. Consequently, the number of buyers any particular seller expects to meet is binomially distributed with parameters B and $1/S$. Since binomial distributions are awkward to deal with we assume that B and S are large numbers, and approximate the binomial with a Poisson distribution with a parameter $\theta = \frac{B}{S}$. The approximation is exact if the numbers approach infinity while their ratio stays fixed.

To determine the number of matches that result from this matching technology it is enough to focus on one type of agents only, since a match is by definition a pairing of two agents. We study the sellers, and assume that the agents only objective is to get paired with an agent of the opposite type. The probability that a seller meets k buyers is $e^{-\theta}\frac{\theta^k}{k!}$, and as long as a seller meets any buyers a match comes about. We assume that if two or more buyers are distributed on any particular seller the seller chooses any one of them with equal probabilities. The probability that the seller is matched is then $1-e^{-\theta}$, i.e. one minus the probability of not meeting any buyers. As there are S sellers the expected number of matches is $S\left(1-e^{-\theta}\right)$.

Let f be the function that determines the expected number of matches $f: R_+ \times R_+ \to R_+$ such that $(S,B) \mapsto S\left(1-e^{-\theta}\right)$. The following result follows almost immediately.

Claim1. Function *f* is homogenous of degree one.

Proof. Consider $\alpha > 0$. $f(\alpha S, \alpha B) = \alpha S\left(1 - e^{-\frac{\alpha B}{\alpha S}}\right) = \alpha S\left(1 - e^{-\theta}\right) = \alpha f(S,B)$ ■

Thus, the matching technology exhibits constant returns to scale. The sellers and buyers are treated asymmetrically in this set-up, and it is not immediately clear whether agents would prefer to stay in fixed positions or to be distributed on the positions. Next we consider a slightly more complicated structure in which buyers and sellers can decide whether to stay or search. It is probably useful to think that there are two separate markets or locations; in the sellers' market sellers are in fixed positions and buyers are distributed on them, and in the buyers' market buyers are in fixed locations and sellers are distributed on them. In equilibrium both types of agents have to be indifferent between the markets.

We need to calculate the probability that a buyer is matched in the market where they are distributed on waiting sellers. A buyer always meets a seller for certain because buyers are distributed on the sellers. Thus, the probability that he is the only buyer is the same as the probability that no other buyers arrive at the same location, which is the same as the probability of a seller meeting no buyers, i.e $e^{-\theta}$. If k other buyers arrive at the location, an event that takes place with probability $e^{-\theta}\frac{\theta^k}{k!}$, any buyer is selected by the seller with probability $\frac{1}{k+1}$. Thus, a buyer is matched with probability

$$e^{-\theta}\left(1 + \frac{1}{2}\theta + \frac{1}{3}\frac{\theta^2}{2!} + \frac{1}{4}\frac{\theta^3}{3!} + \ldots\right) = \frac{e^{-\theta}}{\theta}\sum_{i=1}^{\infty}\frac{\theta^i}{i!} = \frac{e^{-\theta}}{\theta}\left(e^{\theta} - 1\right) = \frac{1 - e^{-\theta}}{\theta} \qquad (1)$$

From (1) we immediately see that when there are more buyers than sellers the buyers are better-off if they are in fixed positions and wait, and if there are less buyers than sellers the buyers prefer to be distributed on sellers.

In equilibrium proportion x of buyers and proportion y of sellers are in the sellers' market. The rest of the agents are in the buyers' market. Let us denote the rate in the sellers' market by $\phi = \frac{xB}{yS} = \frac{x}{y}\theta$ and the rate in the buyers' market by $\varphi = \frac{(1-y)S}{(1-x)B} = \frac{(1-y)}{(1-x)\theta}$. The buyers are indifferent between markets if

$1-e^{-\varphi}=\frac{1-e^{-\phi}}{\phi}$, and the sellers are indifferent if $1-e^{-\phi}=\frac{1-e^{-\varphi}}{\varphi}$. The equations are valid only if $\phi=\frac{1}{\varphi}$ which is equivalent to $x=y$. This means that $\phi=\theta$ and $\varphi=\frac{1}{\theta}$. Inserting this to either equilibrium condition yields $1-e^{-\theta}-\theta+\theta e^{-\frac{1}{\theta}}=0$. This equation has two solutions $\theta_1=1$ and $\theta_0\in(0,1)$. If the proportion of buyers to sellers happens to be either one of these, then there is an infinite number of equilibria with $x=y$. Otherwise both markets cannot exist simultaneously in equilibrium. Exactly as above we get the following result.

Claim2. When there are two active markets in equilibrium the matching technology exhibits constant returns to scale.

Two active markets can exists only for a set of parameters that is of measure zero. Thus, we ignore this case in the sequel. Either market by itself constitutes an equilibrium in the sense that no agent finds it profitable to go to the other market where he would be alone. As we are not that interested in equilibrium selection, we shall consider the variant where sellers are in fixed locations.

Unlike in many random matching models the buyers always meet a seller; they do not always end up in a match though. One may want to consider a situation where both buyers and sellers may end up without meeting anybody. The obvious way to accomplish this is to postulate that there may be more locations than there are sellers. If the buyers are distributed on the locations then some of them may end up in an empty location. Let us denote the number of locations by $L=L(S,B)\geq S$. Notice that since the important parameters of this model are the numbers of buyers and sellers we allow for the possibility that the number of locations depends on the number of both of them. However, not all cases about the number of locations turn out to be easy to interprete economically.

We denote the rate of the Poisson distribution by $\omega=\frac{B}{L}$. The expected number of matches is $f(S,B)=S\left(1-e^{-\omega}\right)$, and for $\alpha>0$ $f(\alpha S,\alpha B)=\alpha S\left(1-e^{-\frac{\alpha B}{L(\alpha S,\alpha B)}}\right)$. Now the matching technology exhibits constant returns to scale if $\alpha S\left(1-e^{-\frac{\alpha B}{L(\alpha S,\alpha B)}}\right)=\alpha S\left(1-e^{-\omega}\right)$ which is equivalent to $\frac{\alpha}{L(\alpha S,\alpha B)}=\frac{1}{L(S,B)}$ or $L(\alpha S,\alpha B)=\alpha L(S,B)$. But this is the condition for L to be a function homogenous of degree one. Similarly, the matching technology exhibits decreasing returns to scale if $\alpha S\left(1-e^{-\frac{\alpha B}{L(\alpha S,\alpha B)}}\right) < \alpha S\left(1-e^{-\omega}\right)$ which is equivalent to $L(\alpha S,\alpha B) > \alpha L(S,B)$. Increasing returns to scale requires $L(\alpha S,\alpha B) < \alpha L(S,B)$. This proves

Claim3. Let $L(S,B) \geq S$ be the number of locations. The matching technology has decreasing, constant, or increasing returns to scale if function L is homogenous of degree greater than one, one, or less than one, respectively.

The simplest choice in this framework is to equal the number of locations and the number of sellers so that the matching technology has constant returns to scale. If something else is wanted one has to say something about the function L. Both decreasing and increasing returns to scale are formally equally easy to implement, but we find only for the latter case an economically interesting interpretation.

Assume that L is constant, and the waiters in our model are firms, and the searchers are workers. Assume further that L is the total number of firms while S is the number of vacancies and B is the number of unemployed workers. If workers search firms, i.e. are distributed on firms, rather than vacancies then the matching technology exhibits increasing returns to scale. Of course, this happens only for a certain range of values since there cannot be more vacancies than there are firms. The job search interpretation seems to be in agreement with the observation that when both the number of vacancies and unemployed increase there are more matches, i.e. it is easier to find a job or an employee.

Motivating L to exhibit increasing returns to scale, and the matching technology decreasing returns to scale, requires a story for the number of locations to increase proportionately more than the number of agents. We do not know any plausible ones.

Acknowledgements

I thank Mikko Huhtamies and a referee for useful comments. Financial support by Yrjo Jahnsson Foundation, and in a form of a TMR grant from the European Commission is gratefully acknowledged.

References

Bloch, F. and H. Ryder, 1997, Two-sided search, marriages and matchmakers, manuscript.

Burdett, K., M. Coles, N. Kiyotaki and R. Wright, 1995, Buyers and sellers: should I stay or should I go?, American Economic Review 85, 281-286.

Gale, D., 1987, Limit theorems for markets with sequential bargaining, Journal of Economic Theory 43, 20-54.

Kiyotaki, N. and R. Wright, 1993, A search theoretic approach to monetary economics, American Economic Review 83(1), 63-77.

Lu, X. and R.P. McAfee, 1996, The evolutionary stability of auctions over bargaining, Games and Economic Behavior 15, 228-254.

Mac Namara J. and E. Collins, 1990, The job search problem as an employer-candidate game, Journal of Applied Probability, 28, 815-827.

Mortensen, D., 1986, Job search and labor market analysis in: O. Ashenfelter and R. Layard, eds., Handbook of labor economics, Vol. 2. (North-Holland, Amsterdam) 849-919.

Rubinstein, A. and A. Wolinsky, 1985, Equilibrium in market with sequential bargaining, Econometrica 53, 1133-1150.

Wolinsky, A., 1988, Dynamic markets with competitive bidding, Review of Economic Studies LV, 71-84

No.	Author(s)	Title
9689	T. ten Raa and E.N. Wolff	Outsourcing of Services and the Productivity Recovery in U.S. Manufacturing in the 1980s
9690	J. Suijs	A Nucleolus for Stochastic Cooperative Games
9691	C. Seidl and S.Traub	Rational Choice and the Relevance of Irrelevant Alternatives
9692	C. Seidl and S.Traub	Testing Decision Rules for Multiattribute Decision Making
9693	R.M.W.J. Beetsma and H. Jensen	Inflation Targets and Contracts with Uncertain Central Banker Preferences
9694	M. Voorneveld	Equilibria and Approximate Equilibria in Infinite Potential Games
9695	F.B.S.L.P. Janssen and A.G. de Kok	A Two-Supplier Inventory Model
9696	L. Ljungqvist and H. Uhlig	Catching up with the Keynesians
9697	A. Rustichini	Dynamic Programming Solution of Incentive Constrained Problems
9698	G.Gürkan and A.Y. Özge	Sample-Path Optimization of Buffer Allocations in a Tandem Queue - Part I: Theoretical Issues
9699	H. Huizinga	The Dual Role of Money and Optimal Financial Taxes
96100	H. Huizinga	The Taxation Implicit in Two-Tiered Exchange Rate Systems
96101	H. Norde, F. Patrone and S. Tijs	Characterizing Properties of Approximate Solutions for Optimization Problems
96102	M. Berg, A. De Waegenaere and J. Wielhouwer	Optimal Tax Reduction by Depreciation: A Stochastic Model
96103	G. van der Laan, D. Talman and Z. Yang	Existence and Approximation of Robust Stationary Points on Polytopes
96104	H. Huizinga and S.B. Nielsen	The Coordination of Capital Income and Profit Taxation with Cross-Ownership of Firms
96105	H. Degryse	The Total Cost of Trading Belgian Shares: Brussels Versus London
96106	H. Huizinga and S.B. Nielsen	The Political Economy of Capital Income and Profit Taxation in a Small Open Economy
96107	T. Dieckmann	The Evolution of Conventions with Endogenous Interactions
96108	F. de Jong and M.W.M. Donders	Intraday Lead-Lag Relationships Between the Futures-, Options and Stock Market
96109	F. Verboven	Brand Rivalry, Market Segmentation, and the Pricing of

No.	Author(s)	Title
		Optional Engine Power on Automobiles
96110	D. Granot, H. Hamers and S. Tijs	Weakly Cyclic Graphs and Delivery Games
96111	P. Aghion, P. Bolton and S. Fries	Financial Restructuring in Transition Economies
96112	A. De Waegenaere, R. Kast and A. Lapied	Non-linear Asset Valuation on Markets with Frictions
96113	R. van den Brink and P.H.M. Ruys	The Internal Organization of the Firm and its External Environment
96114	F. Palomino	Conflicting Trading Objectives and Market Efficiency
96115	E. van Damme and S. Hurkens	Endogenous Stackelberg Leadership
96116	E. Canton	Business Cycles in a Two-Sector Model of Endogenous Growth
9701	J.P.J.F. Scheepens	Collusion and Hierarchy in Banking
9702	H.G. Bloemen and E.G.F. Stancanelli	Individual Wealth, Reservation Wages and Transitions into Employment
9703	P.J.J. Herings and V.J. Vannetelbosch	Refinements of Rationalizability for Normal-Form Games
9704	F. de Jong, F.C. Drost and B.J.M. Werker	Exchange Rate Target Zones: A New Approach
9705	C. Fernández and M.F.J. Steel	On the Dangers of Modelling Through Continuous Distributions: A Bayesian Perspective
9706	M.A. Odijk, P.J. Zwaneveld, J.S. Hooghiemstra, L.G. Kroon and M. Salomon	Decision Support Systems Help Railned to Search for 'Win-Win' Solutions in Railway Network Design
9707	G. Bekaert, R.J. Hodrick and D.A. Marshall	The Implications of First-Order Risk Aversion for Asset Market Risk Premiums
9708	C. Fernández and M.F.J. Steel	Multivariate Student-*t* Regression Models: Pitfalls and Inference
9709	H. Huizinga and S.B. Nielsen	Privatization, Public Investment, and Capital Income Taxation
9710	S. Eijffinger, E. Schaling and M. Hoeberichts	Central Bank Independence: a Sensitivity Analysis
9711	H. Uhlig	Capital Income Taxation and the Sustainability of Permanent Primary Deficits

No.	Author(s)	Title
9712	M. Dufwenberg and W. Güth	Indirect Evolution Versus Strategic Delegation: A Comparison of Two Approaches to Explaining Economic Institutions
9713	H. Uhlig	Long Term Debt and the Political Support for a Monetary Union
9714	E. Charlier, B. Melenberg and A. van Soest	An Analysis of Housing Expenditure Using Semiparametric Models and Panel Data
9715	E. Charlier, B. Melenberg and A. van Soest	An Analysis of Housing Expenditure Using Semiparametric Cross-Section Models
9716	J.P. Choi and S.-S. Yi	Vertical Foreclosure with the Choice of Input Specifications
9717	J.P. Choi	Patent Litigation as an Information Transmission Mechanism
9718	H.Degryse and A. Irmen	Attribute Dependence and the Provision of Quality
9719	A. Possajennikov	An Analysis of a Simple Reinforcing Dynamics: Learning to Play an "Egalitarian" Equilibrium
9720	J. Jansen	Regulating Complementary Input Supply: Cost Correlation and Limited Liability
9721	J. ter Horst and M. Verbeek	Estimating Short-Run Persistence in Mutual Fund Performance
9722	G. Bekaert and S.F. Gray	Target Zones and Exchange Rates: An Empirical Investigation
9723	M. Slikker and A. van den Nouweland	A One-Stage Model of Link Formation and Payoff Division
9724	T. ten Raa	Club Efficiency and Lindahl Equilibrium
9725	R. Euwals, B. Melenberg and A. van Soest	Testing the Predictive Value of Subjective Labour Supply Data
9726	C. Fershtman and U. Gneezy	Strategic Delegation: An Experiment
9727	J. Potters, R. Sloof and F. van Winden	Campaign Expenditures, Contributions and Direct Endorsements: The Strategic Use of Information and Money to Influence Voter Behavior
9728	F.H. Page, Jr.	Existence of Optimal Auctions in General Environments
9729	M. Berliant and F.H. Page, Jr.	Optimal Budget Balancing Income Tax Mechanisms and the Provision of Public Goods
9730	S.C.W. Eijffinger and Willem H. Verhagen	The Advantage of Hiding Both Hands: Foreign Exchange Intervention, Ambiguity and Private Information
9731	A. Ridder, E. van der Laan and M. Salomon	How Larger Demand Variability may Lead to Lower Costs in the Newsvendor Problem
9732	K. Kultti	A Model of Random Matching and Price Formation

No.	Author(s)	Title
9733	J. Ashayeri, R. Heuts and B. Tammel	Applications of P-Median Techniques to Facilities Design Problems: an Improved Heuristic
9734	M. Dufwenberg, H. Norde, H. Reijnierse, and S. Tijs	The Consistency Principle for Set-valued Solutions and a New Direction for the Theory of Equilibrium Refinements
9735	P.P. Wakker, R.H. Thaler and A. Tversky	Probabilistic Insurance
9736	T. Offerman and J. Sonnemans	What's Causing Overreaction? An Experimental Investigation of Recency and the Hot Hand Effect
9737	R. Kabir	New Evidence on Price and Volatility Effects of Stock Option Introductions
9738	M. Das and B. Donkers	How Certain are Dutch Households about Future Income? An Empirical Analysis
9739	R.J.M. Alessie, A. Kapteyn and F. Klijn	Mandatory Pensions and Personal Savings in the Netherlands
9740	W. Güth	Ultimatum Proposals - How Do Decisions Emerge? -
9741	I. Woittiez and A. Kapteyn	Social Interactions and Habit Formation in a Model of Female Labour Supply
9742	E. Canton and H. Uhlig	Growth and the Cycle: Creative Destruction Versus Entrenchment
9743	T. Feenstra, P. Kort and A. de Zeeuw	Environmental Policy in an International Duopoly: An Analysis of Feedback Investment Strategies
9744	A. De Waegenaere and P. Wakker	Choquet Integrals with Respect to Non-Monotonic Set Functions
9745	M. Das, J. Dominitz and A. van Soest	Comparing Predicitions and Outcomes: Theory and Application to Income Changes
9746	T. Aldershof, R. Alessie and A. Kapteyn	Female Labor Supply and the Demand for Housing
9747	S.C.W. Eijffinger, M. Hoeberichts and E. Schaling	Why Money Talks and Wealth Whispers: Monetary Uncertainty and Mystique
9748	W. Güth	Boundedly Rational Decision Emergence -A General Perspective and Some Selective Illustrations-
9749	M. Lettau	Comment on 'The Spirit of Capitalism and Stock-Market Prices' by G.S. Bakshi and Z. Chen (AER, 1996)
9750	M.O. Ravn and H. Uhlig	On Adjusting the HP-Filter for the Frequency of Observations
9751	Th. v.d. Klundert and S. Smulders	Catching-Up and Regulation in a Two-Sector Small Open Economy

No.	Author(s)	Title
9752	J.P.C. Kleijnen	Experimental Design for Sensitivity Analysis, Optimization, and Validation of Simulation Models
9753	A.B.T.M. van Schaik and H.L.F. de Groot	Productivity and Unemployment in a Two-Country Model with Endogenous Growth
9754	H.L.F. de Groot and R. Nahuis	Optimal Product Variety, Scale Effects, and Growth
9755	S. Hochguertel	Precautionary Motives and Portfolio Decisions
9756	K. Kultti	Price Formation by Bargaining and Posted Prices
9757	K. Kultti	Equivalence of Auctions and Posted Prices
9758	R. Kabir	The Value Relevance of Dutch Financial Statement Numbers for Stock Market Investors
9759	R.M.W.J. Beetsma and H. Uhlig	An Analysis of the "Stability Pact"
9760	M. Lettau and H. Uhlig	Preferences, Consumption Smoothing, and Risk Premia
9761	F. Janssen and T. de Kok	The Optimal Number of Suppliers in an (s,Q) Inventory System with Order Splitting
9762	F. Janssen and T. de Kok	The Fill Rate Service Measure in an (s,Q) Inventory System with Order Splitting
9763	E. Canton	Fiscal Policy in a Stochastic Model of Endogenous Growth
9764	R. Euwals	Hours Constraints within and between Jobs
9765	A. Blume	Fast Learning in Organizations
9766	A. Blume	Information Transmission and Preference Similarity
9767	B. van der Genugten	Canonical Partitions in the Restricted Linear Model
9768	W. Güth and B. Peleg	When Will the Fittest Survive? -An Indirect Evolutionary Analysis-
9769	E. Rebers, R. Beetsma and H. Peters	When to Fire Bad Managers: The Role of Collusion Between Management and Board of Directors
9770	B. Donkers and A. van Soest	Subjective Measures of Household Preferences and Financial Decisions
9771	K. Kultti	Scale Returns of a Random Matching Model

P.O. BOX 90153, 5000 LE TILBURG, THE NETHERLANDS