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**WHEN DO REPRESENTATIVE AND DIRECT  
DEMOCRACIES LEAD TO SIMILAR POLICY  
CHOICES?**

By Alex Cukierman and Yossi Spiegel

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# When Do Representative and Direct Democracies Lead to Similar Policy Choices?\*

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# When Do Representative and Direct Democracies Lead to Similar Policy Choices?

*Abstract:* The paradigm of a direct democracy in which individuals vote directly on the issues is widely used in the recent political economy literature that explicitly models the interaction between economic and political behavior. Yet, in most existing democracies policy decisions are delegated to elected officials. This begs the question of whether direct democracy models are useful for understanding actual policy choices. This question is also of independent interest since until the eighteenth century, the possibility that legislatures might consist of elected representatives remained mainly outside the theory and practice of democratic government. To answer this question we posit a model in which elected officials are better informed than the public about exogenous circumstances, but at the same time, are also influenced by particular constituencies whose desired policies generally differ from those of the decisive voter in the population. The first and main objective of the paper is to identify conditions under which direct and representative democracies lead to similar policies and to characterize the determinants of systematic deviations between the two systems when they lead to different policies. The direction and magnitude of these deviations are fully characterized in terms of (i) the degree of political polarization, (ii) the degree of asymmetry between the parties, and (iii) the distribution of electoral uncertainty. The second objective is to compare the two systems in terms of their political uncertainty.

*Keywords:* direct democracy, representative democracy, median voter, policy bias, political uncertainty

## 1. Introduction

Direct democracy epitomizes the concept of democracy by giving each individual an equal and direct say in the choice of public policies. Indeed, in his classic book on democracy and its critics Robert Dahl (1989) argues that the ancient Greeks who were the fathers of democracy believed that: "citizens must be able to assemble and directly decide on the laws and decisions of policy. So deeply held was this view that the Greeks found it difficult to conceive of representative government much less accept it as a legitimate alternative to direct democracy" (p. 19). Yet, most modern democratic societies use representative institutions and delegate policy decisions to elected officials.<sup>1</sup>

The purpose of this paper is to compare policy choices under Representative Democracy (RD) and under Direct Democracy (DD). Such a comparison is interesting for at least two reasons. First, practically all modern democracies are representative, so it is natural to ask what is the extent to which they do, or do not, replicate the policy choices that would have been made in the purer DD system of government. Second, following the pioneering article of Meltzer and Richard (1981) on the growth of transfer payments in the US during the twentieth century, a large literature has evolved that uses the paradigm of direct democracy to study the interactions between economic and political behavior.<sup>2</sup> But since most existing democracies are representative rather than direct, one may wonder whether DD models are useful for understanding actual policy choices.

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<sup>1</sup> In fact, representation was developed as a medieval institution of monarchical and aristocratic government, mainly in England and Sweden, where assemblies were summoned by monarchs or nobles to deal with certain matters of the state such as revenues and wars. It was not until the eighteenth century that writers such as Montesquieu (1748) suggested that the democratic idea should be joined by the non-democratic practice of representation (Dahl, 1989).

<sup>2</sup> A non-exhaustive list includes Mayer (1984) who employs the DD paradigm to study the endogenous formation of tariffs; Cukierman and Meltzer (1989), Tabellini and Alesina (1990), and others, who use it to investigate some of the political and economic determinants of government debt and deficits; Tabellini (1991) who uses it to investigate the political forces that lead to intergenerational redistribution; Perotti (1993) who utilizes it to study the political interactions between growth and the distribution of income; and Saint-Paul (1996a, 1996b) who implicitly uses a DD framework to investigate the political economy of labor markets institutions.

There are two views on these questions. One is that political competition forces elected officials to converge towards the center of the political spectrum. According to this view, there should be no meaningful difference between policy choices in direct and in representative democracies. This idea, due to Downs (1957), finds a precise expression in the claim that if politicians are only office motivated, political competition will force them to implement the policies that would have been adopted by the decisive voter under DD. The other view, whose leading proponents are Shepsle and Weingast (1981), is that institutional detail matters for policy choices. This view implies that, in general, direct and representative democracies need not and typically will not lead to similar policy choices. This paper provides a sharper perspective on this controversy by identifying the circumstances under which each of them is correct.

What are the relative merits of direct and of representative democracy from the point of view of individual voters? DD has the obvious appeal of allowing each individual to participate **directly** in the decision making process, thus avoiding agency distortions that may arise when authority is delegated to elected officials. On the other hand, decision making under DD is likely to be based on less relevant information than under RD. Public policy issues are often quite complex and require careful studying. Most individuals do not have access to pertinent information or do not have the expertise and resources needed to evaluate it. And, even when they do possess these attributes, they still have little incentive to incur the cost needed to become seriously informed about the issues since the probability that any single individual will be decisive in a DD is negligible. This is the well-known “rational ignorance” idea which goes back to Downs (1957). Recent treatments of this idea appear in Lupia (1992), Gilligan (1993) and Matsusaka (1994). In view of this, it may be desirable for a majority of the public to delegate the choice of policy to elected officials or parties that incur the costs of becoming informed and therefore can achieve a better synchronization between exogenous circumstances and policy choices. The drawback of this delegation is that elected officials may be subject to the influence of particular constituencies whose desired policies generally differ from those of the decisive voter in the population.

In this paper we present a framework that captures this tradeoff between more informed decisions and possible systematic deviations of policies from those that would have been chosen by the decisive voter in the population under a DD. This framework involves a single issue

political system.<sup>3</sup> Under DD, voters choose policies directly, but they do so without exact knowledge of the true state of nature. The policy outcome then is the one that maximizes the expected utility of the median voter. RD is characterized by electoral competition between two large parties, or party blocks, that cater to two constituencies whose ideologies are located on opposite sides of the center of the political spectrum. Many modern democracies display such a pattern; obvious examples include the Republican and the Democratic parties in the US, the Conservative and the Labor parties in the UK, and the Likud and Labor parties in Israel. The two parties compete by announcing their respective platforms. Since there is electoral uncertainty, the **leaders of each party** must choose their platform by trading-off the benefits from moving towards the center of the political spectrum and securing more votes against the cost of moving away from the ideological position of their constituency and losing its support if they are elected. This support makes it easier for the leaders of the winning party to carry out their policies. These policies in turn are chosen on the basis of the platform that was announced during the elections and on the basis of new information about the realization of an external circumstances shock that elected officials learn once they are in office.

The main objective of the paper is to identify conditions under which DD and RD lead to similar policies and to characterize the determinants of systematic deviations between the two systems when they lead to different policies. In the latter case, we say that DD has a "policy bias" relative to the more prevalent system of RD.<sup>4</sup> From an ex ante point of view, both political systems yield uncertain policies. Under DD, this uncertainty arises because the location of the median voter in the population is stochastic. Under RD, there is an additional source of uncertainty due to the fact that the policy that is eventually implemented is based in part on the true state of nature that the leaders of the winning party observe once they are in office.

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<sup>3</sup> At least in the context of American politics, there is strong evidence that a one-dimensional policy space is an appropriate simplification. For a discussion of this, see Alesina and Rosenthal (1995), ch. 2.6, pp. 34-35. More specific statistical evidence appears in Poole and Rosenthal (1991, 1997).

<sup>4</sup> Although we take RD, which is the form of democracy used in practice, as a benchmark against which we compare the policy under DD, it is also possible to take DD as a benchmark as it represents the purer form of democracy.



Consequently we compare the *ex ante* distribution of policies that each system generates by looking at the expected policies and their variances.

Obviously, when the platforms of the two parties under RD fully converge on the center of the political spectrum, the expected policies under DD and RD coincide so DD has no policy bias. This "Downsian" benchmark arises whenever the ideal policies of the constituencies of the two parties are sufficiently close to the center of the political spectrum and/or the leaders of the two parties are sufficiently "office-motivated." But, even if the platforms do not fully converge, the expected policies under RD and DD may still coincide if the equilibrium platforms under RD are symmetrically located around the center of the political spectrum. This occurs for instance, when (i) the ideal policies of the two parties are equally distant from the center of the political spectrum, (ii) the leaders of the two parties have exactly the same "love of office," (iii) the distributions of median voter types and the external circumstances shock are both symmetric. Full symmetry of the political equilibrium under RD, however, is not a necessary condition for the expected policies under the two systems to be equal. In particular, we show that there may be no policy bias even if conditions (i) and (ii) above are violated, although the set of parameters for which this happens is of second order in comparison to the full set of parameters that determines political equilibrium under RD.

Although expected policies under DD and RD coincide for a rather limited set of parameters, this does not necessarily mean that the DD paradigm is completely useless as a guide for policy choices under the more prevalent system of RD. Rather, it suggests that it is important to distinguish between cases for which the resulting bias is small and cases for which it is large. The paper fully characterizes the determinants of the sign and magnitude of the policy bias in terms of the degree of political polarization between the two parties and their relative tendencies to converge towards the center of the political spectrum.

We show that when the political equilibrium under RD is asymmetric, there may exist a critical degree of political polarization along which there is no policy bias. This degree is referred to as the "asymmetric no-bias locus." Given the relative tendencies of the parties to converge towards the center of the political spectrum, the policy bias decreases continuously, in absolute value, as the degree of political polarization tends to the "asymmetric no-bias locus." The sign of the policy bias depends in turn on the interaction between the degrees of political

polarization and the relative tendencies of the parties to converge, which in turn affects their chances to win the elections. Specifically, expected policy under DD is biased towards the party which is a favorite to win the elections if the degree of political polarization is small and it is biased towards the party which is an underdog if the degree of polarization is high.

Political instability is believed to affect economic activity and growth (see for example Aizenman and Marion, 1993 and Forthcoming, and Alesina et al., 1996). Our analysis makes it possible to identify conditions under which political uncertainty is larger (or smaller) in a RD than in a DD. Other things the same, political instability in a RD is more likely to be smaller the smaller the political polarization of parties, the more office motivated are party leaders, and the higher is the difference in their degrees of “love of office.”

For RD, Laver and Schofield (1990) and Laver and Shepsle (1995) have emphasized the importance of intra party politics for the choice of national policies. Our model introduces elements of intra party politics into the analysis of RD by featuring a tension between the tendency of party leaders to move towards the center of the national political spectrum in order to boost their electoral prospects on one hand, and their tendency to cater to party interests in order to elicit the support of their respective constituencies on the other.

The paper is organized as follows. Section 2 lays down the basic structure and characterizes the political equilibrium under a DD. Derivation of the political equilibrium under a RD is presented in section 3. Since the mapping from policy choices to individual welfare is stochastic (as in Cukierman and Tommasi, 1998), and since there is electoral uncertainty, policy choices in both cases are stochastic. Section 4 compares policy choices under DD and RD and presents a preliminary discussion of the conditions for which expected policies under the two systems coincide. Using this set as a benchmark, Section 5 identifies the factors that determine the size and direction of policy biases that arise when expected policies under DD and RD do not coincide. This section contains some of the main results of the paper. Section 6 compares political uncertainty under the two systems, using the variance of policy as a metric for uncertainty. This is followed by concluding remarks.

## 2. The model

The economy consists of a continuum of individual voters who differ with respect to their preferences over a single policy issue. A main feature of the model is that at the time of the elections, voters are not yet sure what the true state of nature is and hence which policy is best for them. We capture this idea by assuming that the ideal policy of each voter depends on both his innate taste parameter,  $c$ , where  $c$  may differ across voters, and the realization of a common shock,  $\tilde{\gamma}$ , whose realization becomes known only after the elections.<sup>5</sup> Specifically, we assume that the ideal policy of a voter whose innate taste parameter is  $c$  is given by  $c + \tilde{\gamma}$ , where  $\tilde{\gamma}$  is a zero mean random variable, distributed on the interval  $[-\gamma, \gamma]$  according to a distribution function  $F(\tilde{\gamma})$  and a positive density function  $f(\tilde{\gamma})$ . The utility of a voter whose taste parameter is  $c$  from policy  $x$  when the state of nature is  $\tilde{\gamma}$ , is given by

$$-|x - (c + \tilde{\gamma})|. \quad (1)$$

The expected utility of the voter from policy  $x$ , given the voter's taste parameter, is therefore given by:

$$U(x|c) = -E_{\tilde{\gamma}}|x - (c + \tilde{\gamma})| = -\int_{-\gamma}^{x-c} (x - (c + \tilde{\gamma}))f(\tilde{\gamma})d\tilde{\gamma} + \int_{x-c}^{\gamma} (x - (c + \tilde{\gamma}))f(\tilde{\gamma})d\tilde{\gamma}. \quad (2)$$

Under direct democracy (DD), voters choose policies directly. Since at the time of the elections the voters still do not know the realization of  $\tilde{\gamma}$ , they choose policies with the objective of maximizing their expected utilities. Let  $x(c)$  be the optimal policy from the point of view of a voter whose innate taste parameter is  $c$ . This policy is determined implicitly by the following

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<sup>5</sup> To illustrate, consider the "land versus peace" issue which is at the heart of the Israeli-Syrian conflict. Israelis differ in their marginal rates of substitution between land and peace, and therefore have different opinions about how much territories to give up in return for peace. But the policy preferred by each individual (how much land to give up) also depends on the quality of the deal that Israel can work out with Syria. Virtually all Israelis advocate more dovish policies if that would lead to a "higher quality" peace ("warm" peace), although hawks are still willing to give up less territories than doves. In the context of this example,  $\tilde{\gamma}$  reflects the uncertainty that Israelis feel about the kind of deal that can be worked out with Syria.

first-order condition:

$$F(x-c) = \frac{1}{2}. \quad (3)$$

To interpret this condition, note that  $F(x-c)$  is the probability that  $x-c \geq \tilde{\gamma}$ , or  $x \geq c+\tilde{\gamma}$ . Thus, equation (3) says that ex post,  $x(c)$  turns out to be too high from the voter's point of view exactly half of the time, and too low in the other half. Solving equation (3) for  $x$ , yields

$$x(c) = c + \hat{\gamma}, \quad (4)$$

where  $\hat{\gamma}$  is the median of the distribution of  $\tilde{\gamma}$ . Recalling that  $\tilde{\gamma}$  has a zero mean, it follows that if the distribution of  $\tilde{\gamma}$  has no mean-median spread (e.g., it is symmetric) so that  $\hat{\gamma} = 0$ , each voter chooses a policy that coincides with his innate taste parameter; otherwise each voter adjusts his innate taste parameter by the median of  $\gamma$ . This ensures that, ex ante, he errs in either direction (setting  $x(c)$  too high or setting it too low) with equal probabilities.

We refer to the voter whose innate taste parameter is larger than those of exactly half of the voters as the *median voter*, and index him by a subscript  $m$ . From equation (3) it is clear that  $x(c)$  is monotonically increasing with  $c$ . Together with the fact that  $U(x|c)$  is a concave function of  $x$  and hence single-peaked, this implies that under a DD with a simple majority rule, the median voter is decisive in the sense that the policy that maximizes his expected utility,  $x_m \equiv x(c_m) + \hat{\gamma}$ , can defeat any other policy and will therefore win the election.

Under a representative democracy (RD), voters vote for parties, and the leader of the party that gets the majority of votes chooses a policy  $x$ .<sup>6</sup> A key assumption is that the leader of the elected party chooses  $x$  after observing the realization of  $\tilde{\gamma}$ . Hence compared to a DD, a RD has the advantage of leading to more informed decision-making in the sense that policies are based on the actual realization of  $\tilde{\gamma}$  rather than on its expected value. This assumption captures the idea that figuring out the true state of nature takes time, effort, and expertise which individual voters lack. Hence, delegating this task to an agent (the leader of the elected party)

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<sup>6</sup> Although we refer to a single leader, it is also possible to think of the party leadership as consisting of a narrow group of individuals who are candidates for major cabinet positions (this interpretation is perhaps more appropriate for parliamentary democracies).

is beneficial. But delegation has a drawback in that the objective function of the agent need not coincide with that of the median voter. Therefore, in a RD, decisions take full account of the true state of nature, but are not necessarily selected to maximize the median voter's utility.

The differences between RD and DD raises the question to what extent do RD and DD lead to similar policy choices. Policy choices under RD will almost surely diverge from policy choices under DD simply because the former is picked after the realization of  $\tilde{\gamma}$  has been revealed to decision makers while the latter is based only on the knowledge of the distribution of  $\tilde{\gamma}$ . Therefore, we focus on the set of conditions under which there are no **systematic** differences between policy choices under the two systems in the sense that they both lead to the same expected policies. Using these set as a benchmark, we then completely characterize the determinants of the direction and size of the difference between the expected policies under DD and under RD when they do not coincide.

To address these questions, we assume that there are two parties that compete for office: a right-wing party whose ideal policy is  $c_R + \tilde{\gamma}$ , and a left-wing party whose ideal policy is  $c_L + \tilde{\gamma}$ , where  $c_R > c_L$ . These ideal policies represent the policy preferences of the median voter within relatively well-organized, particular constituencies in the population. The leaders of both parties **do not have policy preferences of their own** - they simply act as political agents and embrace the ideal policies of their respective constituencies or parties in return for support in intraparty contests and political contributions.<sup>7</sup> This support increases as the policy implemented by the leader of the elected party when in office gets closer to the ideal policy of the party. As political entrepreneurs, party leaders like to be in office. The values that the leaders of the right-wing and the left-wing parties assign to holding office are  $h_R$  and  $h_L$ . When in office, the utility function of party  $j$ 's leader is given by

$$V_j(x, \tilde{\gamma}) \equiv h_j - |x - (c_j + \tilde{\gamma})|, \quad j = L, R. \quad (5)$$

The second term on the right side of (5) reflects the personal cost that the leader of party  $j$  pays in terms of dwindling party support for him and his policy, as  $x$  moves further away from  $c_j + \tilde{\gamma}$ ,

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<sup>7</sup> In other words, the party leaders are Downsian political entrepreneurs who do not have ideological preferences of their own.

which is the policy most preferred by his party's center.<sup>8</sup>

The sequence of events under RD is shown in Figure 1. First, the leaders of the two parties simultaneously choose platforms,  $y_R$  and  $y_L$ . Then, given  $y_R$  and  $y_L$ , voters decide which party to vote for and the leader of the party that gets elected becomes the chief executive officer (CEO) of the country. Once in office and in possession of the machinery of government, the CEO privately observes the realization of  $\tilde{\gamma}$  and chooses a policy,  $x$ , based on this observation. Finally, the realization of  $\tilde{\gamma}$  together with  $x$  determine the utilities of individuals in the economy.

Casual observation suggests that while platforms are not ironclad commitments, they are not empty statements either. Once the leader of a party embraces a platform and gets elected, he becomes, at least partially, personally committed to deliver the platform to the general public. One reason for this is that voters prefer national leaders who are honest and can be trusted. If a leader deviates from his campaign promises after being elected without a justifiable reason, voters realize that he is not trustworthy and may refrain from voting for him in the future.<sup>9</sup>

More precisely, when  $\tilde{\gamma} = 0$ , a voter whose taste parameter is  $c$  expects to obtain a utility level  $-|y_j - c|$  if party  $j$  is elected. When  $\tilde{\gamma} \neq 0$ , all voters understand that there is a justifiable reason for deviating from the platform but they expect it to be done by  $\tilde{\gamma}$  so as to exactly offset the external shock. The platform can therefore be viewed as a contingent rule that provides each voter with a fully predictable level of welfare,  $-|y_j - c|$ , in spite of the a priori uncertainty regarding  $\tilde{\gamma}$ . Therefore although individual voters do not necessarily observe the realization of  $\tilde{\gamma}$ , they can nevertheless detect ex post unjustifiable deviations by the leader in office from his campaign promises by simply comparing their realized utility levels with those they were expecting to get on the basis of the campaign platform. If a voter gets a utility level other than  $-|y_j - c|$ , he correctly infers that the leader in office acted dishonestly. We assume that

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<sup>8</sup> Since the general public is not as tightly organized as party constituencies, the leader of the elected party does not pay a personal cost if he implements policies that do not accord with the taste of individuals that are not members of his party.

<sup>9</sup> In other words, the majority of the public that has elected the leader expects him to implement his platform unless there is an unforeseen change in external circumstances as embodied in the realization of  $\tilde{\gamma}$ . If this expectation is not fulfilled, a majority of voters is disappointed and this reduces the future general electoral prospects of the party leader.

in that case the future electoral prospects of the leader are reduced to such an extent that it never pays him to behave in this way. This implies that given his platform, the leader of party  $j$  is committed, after being elected to office, to the following contingent rule:

$$x_j = y_j + \tilde{\gamma}. \quad (6)$$

Preelection platforms are typically rather vague; they are long on general descriptions of "national priorities," but short on specifics. Our formulation suggests that this vagueness is partly deliberate, that it is expected by voters, and that it is basically due to the fact that it is better for everybody to postpone the final choice of policy until after the resolution of exogenous uncertainty.

To reflect the uncertainty inherent in any electoral competition, we assume that the two parties do not exactly know the taste parameter of the median voter,  $c_m$ , and believe that it is distributed on the interval  $[c_0, c_1]$  according to a twice differentiable distribution function  $G(c_m)$  and a density function  $g(c_m)$ . Define  $\hat{c}_m$  as the median of the distribution of  $c_m$ . That is, the probability that  $c_m \leq \hat{c}_m$  is exactly 1/2. Hence, it is natural to refer to  $\hat{c}_m$  as the "center of the political spectrum." We now make the following assumptions on the distribution of the median voter's types and on the taste parameters of the two parties:

- A1:**  $M(c_m) \equiv G(c_m)/g(c_m)$  is increasing and  $H(c_m) \equiv (1-G(c_m))/g(c_m)$  is decreasing in  $c_m$ .
- A2:**  $h_L > 2M((c_L + c_R)/2)$  and  $h_R > 2H((c_L + c_R)/2)$ .
- A3:**  $c_0 < c_L < \hat{c}_m < c_R < c_1$ .

Assumption A1 is satisfied by standard continuous distributions (e.g., uniform, exponential, and normal). It ensures that the objective functions of the two parties are nicely behaved by putting some mild restrictions on the rate of change of the density function  $g(\cdot)$ . Assumption A2 ensures that the values that the party leaders assign to holding office are sufficiently large so that in equilibrium, both parties converge at least somewhat towards the center of the political spectrum. Assumption A3 states that some median voter types are more left-wing than the left-wing party, while others are more right-wing than the right-wing party. Hence, each party

would still have a chance to win the elections even if its leaders were to adopt a platform that coincides with the party's ideal policy. Absent this assumption, each party would be forced to move towards the center of the spectrum in order to have any chance of winning the elections.

In addition, Assumption A3 implies that more than half of the median voters' types are more right-wing than the left-wing party, and more than half of them are more left-wing than the right-wing party. Broadly interpreted, this implies that the political centers of organized parties are at least somewhat away from the center of the political spectrum. This assumption seems consistent with casual observation. Moreover, forming and maintaining a party as a going concern is a costly activity that requires real resources. The benefit from this activity is that a party can coordinate actions, transmit information to the general public, and bargain more effectively than a group of unorganized individuals. Consequently, party members have more influence on the decision making process than they would have by just participating in the general elections as separate individuals. To the extent that the cost of forming a party is independent of its location in the issue space, this implies that the formation of parties is relatively more advantageous for voters who are in the periphery of the political spectrum. For example, a right-wing group has relatively more to loose from a left-wing policy than individuals who are in the center of the political spectrum; hence such a group has a stronger incentive to become organized and incur the cost of collective action in order to reduce the likelihood that the left-wing policy will be implemented.

Since the party leaders are uncertain about the position of the median voter, the outcome of the elections from their point of view is random. Let  $P_j(y_L, y_R)$ , denote the probability that party  $j$  ( $j = R, L$ ) wins the elections given that the pair of platforms that was announced. Then, the expected payoff of party  $j$ 's leader, before committing himself to a given platform, is given by:

$$P_j(y_L, y_R) E_{\tilde{y}} V_j(x, \tilde{y}), \quad j = L, R, \quad (7)$$

where  $V_j(x, \tilde{y})$  is given by equation (5). Note that the party leaders do not care about the policy which is implemented when they lose the election: this reflects our assumption that party leaders are Downsian.



### 3. Political equilibrium under RD

To characterize the subgame perfect political equilibrium under RD, we solve the game backwards. We saw earlier that in the last stage of the game, the leader of the winning party selects the policy  $x_j = y_j + \tilde{\gamma}$ . Substituting for  $x_j$  into equation (5) and using equation (7), the expected payoff of party  $j$ 's leader becomes:

$$\pi_j(y_L, y_R) = P_j(y_L, y_R) [h_j - |y_j - c_j|], \quad j = L, R. \quad (8)$$

In a (subgame perfect) political equilibrium, the leader of each party chooses the platform of his party with the objective of maximizing his expected payoff, taking the platform of the rival party as given. The equilibrium platforms are denoted by  $y_L^*$  and  $y_R^*$ . Note that since the leader is expected to fully adjust the party's platform to the realized external circumstances shock once he is elected,  $\tilde{\gamma}$  vanishes from the expected payoffs of the party leaders. Hence, uncertainty with respect to external circumstances does not play any role in determining the equilibrium platforms. Equation (8) reveals that each leader has two considerations when he chooses his party's platform. First he takes into account the impact of the platform on his chances to be elected. Second, conditional on winning the elections, the party's leader wishes to minimize the deviation of the policy that he will implement,  $y_j + \tilde{\gamma}$ , from the party's ideal position,  $c_j + \tilde{\gamma}$ .

Next, consider the outcome of the elections. Sequential rationality implies that voters anticipate that if party  $j$  wins the elections, its leader will to carry out the policy  $x_j = y_j + \tilde{\gamma}$ . Substituting this policy in equation (2) reveals that the utility of a voter whose innate taste parameter is  $c$ , if party  $j$  is elected, is given by

$$U(x_j | c) = -|y_j - c|. \quad (9)$$

Equation (9) indicates that the utility of voters once a party has been elected does not depend on the realization of  $\tilde{\gamma}$ . This reflects the fact that under RD, elected officials can gather information on the external circumstances shock and fully adjust their policies accordingly. From equation (9) it is clear that if  $y_R = y_L$ , all voters are indifferent between the two parties, so they randomize their votes and  $P_R(y_L, y_R) = P_L(y_L, y_R) = 1/2$ . Otherwise, since  $U(x_j | c)$  is symmetric and single-peaked, each voter votes for the party whose platform is closer to his

innate taste parameter. Consequently, the party whose platform is closest to  $c_m$ , which is the innate taste parameter of the median voter, wins the elections. In order to derive the probability that each party wins the elections, we establish the following result:

**Lemma 1:** *The equilibrium platforms,  $y_L^*$ ,  $y_R^*$ , are such that either  $c_L \leq y_L^* < y_R^* \leq c_R$ , or  $y_L^* = y_R^* = \hat{c}_m$ .*

**Proof:** See the Appendix.

Lemma 1 shows that in equilibrium, the left-wing party never adopts a more right-wing platform than the right-wing party and vice versa. Hence in what follows, we shall restrict attention to cases where  $y_L \leq y_R$ . In addition, Lemma 1 shows that in equilibrium, the leader of each party either adopts the ideal policy of his party, or converges towards the center of the political spectrum. However, the platforms of the two parties fully converge only if both parties choose the platform  $\hat{c}_m$ , which represents the exact center of the political spectrum. It should be noted that Lemma 1 does not depend on the particular functional form of the utility function of the party leaders. All that is required for Lemma 1 to hold is that the utility of each leader decreases as the party's platform shifts away from the party's ideal policy.

Recalling that the party whose platform is closest to  $c_m$  wins the elections, and recalling that the distribution of  $c_m$  is  $g(c_m)$ , the probability that the left-wing party wins the elections is given by,

$$P_L(y_L, y_R) = \begin{cases} G(\hat{y}), & \text{if } y_L < y_R, \\ \frac{1}{2}, & \text{if } y_L = y_R, \end{cases} \quad (10)$$

where  $\hat{y} \equiv (y_L + y_R)/2$  is the ideal policy of the voter who is just indifferent between the two parties. The probability that the right-wing party wins is  $P_R(y_L, y_R) = 1 - P_L(y_L, y_R)$ . Figure 2 shows the probabilities of the two parties to win the elections when  $y_L < y_R$ . The probability that the left-wing party wins is given by the shaded area, and the probability that the right-wing party wins is given by the complementary area under the  $g(\cdot)$  curve.

Using equations (8) and (10) and recalling from Lemma 1 that  $y_L \geq c_L$  and  $y_R \leq c_R$ , the expected payoffs of the party leaders become:

$$\pi_L(y_L, y_R) = \begin{cases} G(\hat{y})[h_L - (y_L - c_L)], & \text{if } y_L < y_R \\ \frac{1}{2}[h_L - (y_R - c_L)], & \text{if } y_L = y_R \end{cases} \quad (11)$$

and

$$\pi_R(y_L, y_R) = \begin{cases} (1 - G(\hat{y}))[h_R - (c_R - y_R)], & \text{if } y_L < y_R \\ \frac{1}{2}[h_R - (c_R - y_L)], & \text{if } y_L = y_R \end{cases} \quad (12)$$

To characterize the equilibrium platforms,  $y_L^*$  and  $y_R^*$ , we first prove the following result:

**Lemma 2:** *The equilibrium platforms converge at least partially towards the center of the political spectrum, so in equilibrium either  $c_L < y_L^* < y_R^* < c_R$ , or  $y_L^* = y_R^* = \hat{c}_m$ .*

**Proof:** See the Appendix.

The equilibrium platforms when  $y_L^* < y_R^*$  (i.e., the equilibrium platform do not converge fully) are determined by the solution to the following pair of first-order conditions (in the Appendix we show that these conditions are sufficient for a maximum):

$$\frac{\partial \pi_L(y_L, y_R)}{\partial y_L} = \frac{g(\hat{y})}{2}[h_L - (y_L - c_L)] - G(\hat{y}) = 0, \quad (13)$$

and

$$\frac{\partial \pi_R(y_L, y_R)}{\partial y_R} = -\frac{g(\hat{y})}{2}[h_R - (c_R - y_R)] + (1 - G(\hat{y})) = 0. \quad (14)$$

The first term in  $\partial \pi_L(y_L, y_R)/\partial y_L$  represents the marginal benefit to the leader of the left-wing

party from shifting the party's platform to the right. This move enables the leader of the left-wing party to win the support of more median voter types and hence it increases the chances that the left-wing party will win the elections. The second term in  $\partial\pi_L(y_L, y_R)/\partial y_L$  represents the marginal cost to the leader of the left-wing party from shifting the party's platform away from  $c_L$ . Since the payoff of each party leader (conditional on being elected) is linear in the party's platform and the coefficient is equal to 1, the marginal cost to the leader is simply equal to the probability that he wins the elections. The interpretation of  $\partial\pi_R(y_L, y_R)/\partial y_R$  is similar, but since increasing  $y_R$  means that the right-wing party moves closer to  $c_R$ , now the first term represents a marginal cost, while the second term represents a marginal benefit.

More generally, the choices of  $y_L^*$  and  $y_R^*$  involve a tradeoff between the electoral concerns of party leaders that push the platform of each party closer to its rival's platform, and the ideological concerns of party members that induce each party leader to limit the distance between the party's platform and the party's ideal policy. These two factors are fully captured by the parameters  $a_L \equiv h_L - (\hat{c}_m - c_L)$  and  $a_R \equiv h_R - (c_R - \hat{c}_m)$  that reflect the combined impact of the intensity of each leader's love of office and the distance of his party's ideal policy from the center of the political spectrum. Hence, we shall refer to  $a_L$  and  $a_R$  as the "convergence parameters" of the left-wing and the right-wing parties. We now establish the following comparative statics result:

**Proposition 1:** *Suppose that the equilibrium platforms do not converge fully, i.e.,  $y_L^* < y_R^*$ . Then:*

- (i) *If  $a_L$  increases, then  $y_L^*$  increases and  $y_R^*$  decreases, but by less than the increase in  $y_L^*$ ; consequently, the probability that the left-wing party wins the elections increases.*
- (ii) *If  $a_R$  increases, then  $y_L^*$  increases and  $y_R^*$  decreases, but by more than the increase in  $y_L^*$ ; consequently, the probability that the right-wing party wins the elections increases.*

**Proof:** See the Appendix.

Proposition 1 indicates that if  $a_j$  increases, party  $j$  moves closer to its rival. This occurs either because the leader of party  $j$  becomes more office-motivated and hence eager to secure

more votes (at the cost of losing support from his party members if elected), or because the ideological position of the party shifts towards the center of the political spectrum (in which case moving towards the rival party is less costly for the leader of party j). The shift in the platform of party j triggers a counter move by the leader of the rival party in an attempt to mitigate the resulting decline in his electoral prospects. But since the platform of the rival party never shifts by as much as the platform of the first party, the latter ends up having a higher chance of winning the elections.

Since an increase in  $a_j$  pushes party j towards the center of the political spectrum more than it pushes its rival party and since it enhances the electoral prospects of party j, it seems reasonable to expect that the party whose convergence parameter is bigger will adopt a platform that is closer to the center of the political spectrum and will be the favorite to win the elections. The next proposition confirms this intuition.

**Proposition 2:** *Suppose that the equilibrium platforms do not converge fully, i.e.,  $y_L^* < y_R^*$ , and let  $d_L^* \equiv \hat{c}_m - y_L^*$  and  $d_R^* \equiv y_R^* - \hat{c}_m$  be the distances of the equilibrium platforms of the left-wing party and the right-wing party from the center of the political spectrum. Then  $d_L^* \leq d_R^*$  as  $a_L \geq a_R$  and  $G(\hat{y}^*) \geq 1/2$  as  $a_L \geq a_R$ .*

**Proof:** See the Appendix.

Proposition 2 implies that the equilibrium is symmetric in the sense that  $y_L^*$  and  $y_R^*$  are equally-distant from the center of the political spectrum and the parties have equal chances to win the elections only if the convergence parameters of the two parties,  $a_L$  and  $a_R$  are equal. Otherwise, the party whose leader is more eager to hold office or its constituency is more moderate will adopt a more moderate platform and will be a favorite to win the elections.

#### 4. Comparison of policy choices under DD and RD - a first look

This section focuses on the main issue of this paper which is the extent to which DD and RD lead to similar policy choices. This issue is interesting for at least two reasons. First,

representative democracy is a relatively new concept that was introduced only as late as the 18 century to overcome problems associated with the implementation of direct democracy in large societies (Dahl, 1989). Therefore, it is natural to examine the extent to which RD replicates the policy outcomes that would have been achieved under the purer system of DD. The second reason is due to the political economy literature that evolved over the last fifteen year. This literature has extensively used the direct democracy paradigm as a useful shortcut for studying the interactions between economic and political behavior. Given the size and importance of this literature, it is interesting to identify the conditions under which policy outcomes predicted by this literature would also emerge under the more prevalent setting of RD. And, when this is not the case, it is useful to know what are the factors that determine the direction and magnitude of systematic differences between the policies chosen under the two systems.

Before turning to a comparison between DD and RD, it should be noted that our model has the feature that under DD, policy is chosen by a median voter who does not know the realization of  $\tilde{\gamma}$ , whereas under RD the policy is chosen by elected officials who observe the realization of  $\tilde{\gamma}$ , but are already partially committed to a platform that was chosen before the ideal policy of the median voter was revealed. Hence, the policy under DD responds perfectly to electoral uncertainty but poorly to uncertainty regarding external circumstances, whereas the policy under RD has the opposite properties. Consequently, it is obvious that the policies under DD and RD will almost never coincide. Nonetheless, it is still conceivable that DD and RD will yield similar policies on average, after the informational asymmetries between the two systems are averaged out. Therefore we shall compare the expected policies across the two political systems and identify the circumstances under which DD and RD lead to the same expected policies.<sup>10</sup> When the expected policies under DD and RD do not coincide, we will say that the expected policy under DD has a "policy bias" relative to the expected policy under RD, and we shall examine the determinants of the direction and magnitude of these policy biases.

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<sup>10</sup> Another reason to look at the expected policies under the two political systems is that from an ex ante perspective, both systems induce a whole distribution of potential policies. Examining the differences between the expected policies then is a natural first step in the comparison of the two distributions.

#### 4.1 Policy biases - a first look

The policy adopted under DD is  $x_m = c_m + \hat{\gamma}$ . This policy maximizes the expected utility of the median voter. Hence, expected policy under DD is given by,

$$Ex_{DD} = E\left(c_m + \hat{\gamma}\right) \equiv \bar{c}_m + \hat{\gamma}. \quad (15)$$

That is, the expected policy under DD is equal to the sum of  $\bar{c}_m$ , which is the mean of the distribution of the median voter's taste parameter, and  $\hat{\gamma}$  which is the median of the external circumstances shock.

Under RD, the actual policy choice is  $y_L + \tilde{\gamma}$  if the left-wing party wins the elections and  $y_R + \tilde{\gamma}$  if the right-wing party wins. Recalling that  $\tilde{\gamma}$  has a zero mean, the expected policy under a RD is therefore given by:

$$\begin{aligned} Ex_{RD} &= G(\hat{y}^*) E\left(y_L^* + \tilde{\gamma}\right) + (1 - G(\hat{y}^*)) E\left(y_R^* + \tilde{\gamma}\right) \\ &= G(\hat{y}^*) y_L^* + (1 - G(\hat{y}^*)) y_R^*, \end{aligned} \quad (16)$$

where  $\hat{y}^* \equiv (y_L^* + y_R^*)/2$ , and  $G(\hat{y}^*)$  is the equilibrium probability that the left-wing party wins the elections.

Policy biases arise when  $Ex_{DD} \neq Ex_{RD}$ ; specifically, we will say that  $Ex_{DD}$  has a right-wing bias when  $Ex_{DD} > Ex_{RD}$ , and a left-wing bias when  $Ex_{DD} < Ex_{RD}$ . Recalling that  $d_L^* \equiv \hat{c}_m - y_L^*$  and  $d_R^* \equiv y_R^* - \hat{c}_m$ , it follows that

$$\begin{aligned} Ex_{DD} - Ex_{RD} &= \bar{c}_m + \hat{\gamma} - G(\hat{y}^*) y_L^* - (1 - G(\hat{y}^*)) y_R^* \\ &= (\bar{c}_m - \hat{c}_m) + \hat{\gamma} + (d_L^* + d_R^*) \left[ G(\hat{y}^*) - \frac{d_R^*}{d_L^* + d_R^*} \right]. \end{aligned} \quad (17)$$

Equation (17) reveals that there are four potential sources for policy biases. The first source depends on the mean-median spread of the distribution of median voter types,  $g(c_m)$  and it arise because  $Ex_{DD}$  depends on the mean of  $c_m$  whereas  $Ex_{RD}$  depends on the median of  $c_m$ . This implies for instance that if  $g(\cdot)$  is skewed to the right so that  $\bar{c}_m > \hat{c}_m$  (i.e., the extreme right-

wing is more extreme than the extreme left-wing), then  $Ex_{DD}$  tends to have a right-wing bias. The second source depends on the mean-median spread of the distribution of the external circumstances shock,  $\tilde{\gamma}$ . It arises because under DD, each voter (including the median voter) chooses a policy by adjusting his innate taste parameter by a constant, equal to the median of  $f(\tilde{\gamma})$ , whereas under RD, the policy is fully adjusted in line with the actual realization of  $\tilde{\gamma}$ . Consequently,  $Ex_{RD}$  depends on the mean of  $\tilde{\gamma}$ , which is zero by assumption, while  $Ex_{DD}$  depends on  $\hat{\gamma}$ .<sup>11</sup> Thus for instance, if the mean-median spread of the distribution of  $\tilde{\gamma}$  is positive, i.e.,  $\hat{\gamma} < 0$ ,  $Ex_{DD}$  tends to have a left-wing bias. The last source of policy bias in equation (17) arises when the equilibrium platforms are not equally-distant from the center of the political spectrum.<sup>12</sup> This may occur in turn when the leaders of the two parties are not equally office motivated or when the constituencies of the two parties are not equally-distant from the center of the political spectrum.

Recalling from Proposition 2 that the expression in the square brackets in equation (17) vanishes if  $a_L = a_R$ , we can state the following result:

**Proposition 3:** *A necessary condition for the existence of policy biases is that the political system has one of the following types of asymmetries:*

- (i) *the distribution of the external circumstances shock,  $f(\cdot)$  is skewed;*
- (ii) *the distribution of the median voter's taste parameter,  $g(\cdot)$  is skewed;*
- (iii) *the leaders of the two parties are not equally office-motivated (i.e.,  $h_L \neq h_R$ );*
- (iv) *the ideological positions of their respective constituencies are not equally-distant from the center of the political spectrum (i.e.,  $\hat{c}_m - \hat{c}_L \neq c_R - \hat{c}_m$ ).*

The impact of the first two types of asymmetries on policy biases is straightforward. In

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<sup>11</sup> Note that this source of policy bias disappears if individual voters have a quadratic utility function of the type  $-(x-(c+\tilde{\gamma}))^2$  rather than  $-|x-(c+\tilde{\gamma})|$  as we assume, because then the optimal policy from the median voter's point of view is  $x(c_m) = \hat{c}_m + \tilde{\gamma} = \hat{c}_m$ .

<sup>12</sup> When the equilibrium platforms are located at equal distances from the center of the political spectrum, the square bracketed term in equation (17) vanishes.



contrast, the impact of asymmetries in the office-motivation of party leaders and in the distances of the ideological positions of the parties from the center of the political spectrum are more complex because they affect both the equilibrium platforms of the parties as well as their electoral prospects. Since both factors are fully captured by the convergence parameters,  $a_L$  and  $a_R$ , we proceed by studying the impact of these parameters on policy biases.

#### 4.2 The effects of aggregate and relative convergence parameters on political equilibrium

Since  $\hat{y}^* = \hat{c}_m + (d_R^* - d_L^*)/2$ , the electoral prospects of the two parties, summarized by  $G(\hat{y}^*)$ , depend only on the relative distances of the equilibrium platforms from the center of the political spectrum. Proposition 2 suggests in turn that the latter may be related to the relative sizes of the convergence parameters but not to their absolute sizes. Moreover, Proposition 1 suggests that as the convergence parameters,  $a_L$  and  $a_R$ , increase, the equilibrium platforms move towards one another, thereby reducing political polarization. Therefore it seems natural to examine how the distance between the equilibrium platforms and the electoral prospects of the two parties are affected by the sum of the convergence parameters, denoted  $\Sigma \equiv a_L + a_R$ , and the difference between them, denoted  $\Delta \equiv a_L - a_R$ . We refer to  $\Sigma$  as the "aggregate convergence parameter," and to  $\Delta$  as the "relative convergence parameter."<sup>13</sup>

**Proposition 4:** *Suppose the equilibrium platforms do not converge fully, i.e.,  $y_L^* < y_R^*$ . Then,*

(i) *holding  $\Delta$  constant,*

$$\frac{\partial y_L^*}{\partial \Sigma} = \frac{1}{2}, \quad \frac{\partial y_R^*}{\partial \Sigma} = -\frac{1}{2}, \quad \frac{\partial \hat{y}^*}{\partial \Sigma} = 0;$$

(ii) *holding  $\Sigma$  constant and using  $|J_{(y_L^*, y_R^*)}|$  to denote the determinant of the Jacobian matrix corresponding to equations (13) and (14),*

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<sup>13</sup> Note that there is a one-to-one correspondence between  $(a_L, a_R)$  and  $(\Sigma, \Delta)$  in the sense that every pair  $(a_L, a_R)$  determines a unique pair  $(\Sigma, \Delta)$  and vice versa.

$$\frac{\partial y_L^*}{\partial \Delta} = \frac{g^2(\hat{y}^*) + g'(\hat{y}^*)}{8|J(y_L^*, y_R^*)|}, \quad \frac{\partial y_R^*}{\partial \Delta} = \frac{g^2(\hat{y}^*) - g'(\hat{y}^*)}{8|J(y_L^*, y_R^*)|}, \quad \frac{\partial \hat{y}^*}{\partial \Delta} = \frac{g^2(\hat{y}^*)}{8|J(y_L^*, y_R^*)|} > 0,$$

where  $|J(y_L^*, y_R^*)|$ , defined in equation (A-8) of the Appendix, is strictly positive.

**Proof:** See the Appendix.

Several conclusions can be drawn from Proposition 4. The first concerns the impact of the aggregate and relative convergence parameters on the extent to which the political equilibrium is symmetric. Under full symmetry, the distances of the equilibrium platforms from the center of the political spectrum,  $d_R^*$  and  $d_L^*$ , are equal and the parties have equal chances to win the elections. When the equilibrium is asymmetric, one of the parties adopts a more centrist platform than its rival and it becomes the favorite to win the elections. The more asymmetric the equilibrium becomes, the wider is the gap between the electoral prospects of the two parties. Since  $d_R^* - d_L^* = 2(\hat{y}^* - \hat{c}_m)$ , the equilibrium is fully symmetric if  $\hat{y}^* = \hat{c}_m$ , and as  $\hat{y}^*$  increases above  $\hat{c}_m$  or falls below it, the political equilibrium becomes more asymmetric. Proposition 4 shows that  $\hat{y}^*$  is independent of  $\Sigma$ , implying that the degree of symmetry of the political equilibrium depends only on  $\Delta$ . Figure 3 shows  $\hat{y}^*$  as a function of  $\Delta$ . As the figure shows,  $\hat{y}^*$  increases with  $\Delta$  and is equal to  $\hat{c}_m$  when  $\Delta = 0$ . Hence, the equilibrium is fully symmetric when  $\Delta = 0$  and it becomes increasingly more asymmetric as  $|\Delta|$  increases. This result is intuitive since an increase in  $|\Delta|$  means that the convergence parameters and hence the relative tendencies of the parties to move towards the center become more dissimilar.

Second, since the chances that the left-wing party wins the elections are given by  $G(\hat{y}^*)$  and since  $\partial \hat{y}^* / \partial \Delta > 0$ , it follows that increasing  $\Delta$  (i.e., increasing  $a_L$  relative to  $a_R$ ), boosts the electoral prospects of the left-wing party at the expense of the right-wing party. Although this result is related to Proposition 1, it is not quite the same because here the exercise involves a simultaneous increase in  $a_L$  and a decrease in  $a_R$  (to ensure that  $\Delta$  increases while  $\Sigma$  remains constant), whereas Proposition 1 examines the impact of changes in only one of the convergence parameters.

Third, Proposition 4 shows that raising the aggregate convergence parameter,  $\Sigma$ , while holding the relative convergence parameter,  $\Delta$ , constant, pushes the equilibrium platforms closer to one another. Hence, the political system becomes less polarized. Therefore, variations in  $\Sigma$  can be interpreted as reflecting changes in the degree to which the political system is polarized, with higher values of  $\Sigma$  being associated with less political polarization.

Fourth, Proposition 4 shows that the distance between  $y_L^*$  and  $y_R^*$  can either increase or decrease with  $\Delta$ , depending on the sign of  $g'(\hat{y}^*)$ . This implies that in general,  $\Delta$  has an ambiguous effect on the degree of political polarization. However, there are two special cases in which the impact of  $\Delta$  on political polarization is unambiguous. First, when  $g(\cdot)$  is uniform,  $g'(\cdot) = 0$ , so the distance between  $y_L^*$  and  $y_R^*$  depends only on  $\Sigma$  but not on  $\Delta$ . Hence changes in  $\Delta$  do not affect the degree of political polarization. Second, when  $g(\cdot)$  is symmetric and unimodal,  $-g'(\hat{y}^*) \cong 0$  as  $G(\hat{y}^*) \cong 1/2$ . Since Proposition 2 states that  $G(\hat{y}^*) \cong 1/2$  as  $\Delta \cong 0$ , it follows from part (ii) of Proposition 4 that the degree of political polarization is a U-shaped function of  $\Delta$  that attains a minimum at  $\Delta = 0$ . Since the political equilibrium becomes more asymmetric as  $|\Delta|$  increases, it follows that there is more polarization when the political equilibrium is more asymmetric. To better understand this result, suppose that  $\Delta > 0$ . Then Proposition 2 shows that  $y_L^*$  is closer to the center than  $y_R^*$  and  $G(\hat{y}^*) > 1/2$ . If  $g(\cdot)$  is symmetric and unimodal,  $G(\hat{y}^*) > 1/2$  implies that  $g'(\hat{y}^*) < 0$ , so when  $\Delta$  increases (i.e.,  $a_L$  increases and at the same time,  $a_R$  decreases)  $y_L^*$  shifts to the right by less than  $y_R^*$  shifts to the right. Consequently, the gap between  $y_L^*$  and  $y_R^*$  widens and there is more polarization. When  $\Delta < 0$ , the reverse holds: by Proposition 2,  $\Delta < 0$  implies that  $G(\hat{y}^*) < 1/2$  and since  $g(\cdot)$  is symmetric and unimodal, it follows that  $g'(\hat{y}^*) > 0$ . Consequently, when  $\Delta$  increases,  $y_L^*$  shifts to the right by more than  $y_R^*$  shifts to the right so there is less polarization.

The result that increasing  $\Sigma$  causes  $y_L^*$  and  $y_R^*$  to move closer to one another suggests that for sufficiently large values of  $\Sigma$ , the equilibrium platforms will fully converge. To find the critical value of  $\Sigma$  beyond which there is full convergence, suppose that  $y_L^* < y_R^*$ . Subtracting equation (14) from equation (13), noting that  $\Sigma \equiv a_L + a_R = h_L + h_R + c_L - c_R$ , and rearranging terms,

$$\frac{g(\hat{y}^*)}{2} [\Sigma + y_R^* - y_L^*] = 1. \quad (18)$$

Since Proposition 4 shows that  $\hat{y}^*$  and hence  $g(\hat{y}^*)$  are independent of  $\Sigma$ , it is clear that so long as  $y_L^* < y_R^*$ , equation (18) defines for every  $\Delta$ , a unique value of  $\Sigma$  that induces a given gap between  $y_L^*$  and  $y_R^*$ . Solving for  $\Sigma$  and recalling that  $y_R^* - y_L^*$  is monotonically decreasing with  $\Sigma$  (see Proposition 4), the upper bound on  $\Sigma$  such that there is still a positive gap between  $y_L^*$  and  $y_R^*$  is given by

$$\Sigma^{UB}(\Delta) \equiv \lim_{y_R^* - y_L^* \rightarrow 0} \left[ \frac{2}{g(\hat{y}^*)} - (y_R^* - y_L^*) \right] = \frac{2}{g(\hat{y}^*)}, \quad (19)$$

where "UB" stands for Upper-Bound. Hence,

**Lemma 3:** *Given  $\Delta$ , the equilibrium platforms,  $y_L^*$ ,  $y_R^*$ , are such that*

- (i)  $y_L^* = y_R^* = \hat{c}_m$ , whenever  $\Sigma \geq \Sigma^{UB}(\Delta)$  (full-convergence), and
- (ii)  $c_L < y_L^* < y_R^* < c_R$ , whenever  $\Sigma < \Sigma^{UB}(\Delta)$  (partial convergence),

where  $\Sigma^{UB}(\Delta) = 2/g(\hat{y}^*)$  is continuous in  $\Delta$ , and if  $g(\cdot)$  is symmetric and unimodal, then  $d\Sigma^{UB}(\Delta)/d\Delta \gtrless 0$  as  $\Delta \gtrless 0$ .

**Proof:** See the Appendix.

The next step is to examine the impact of  $\Delta$  and  $\Sigma$  on  $Ex_{RD}$  and hence on the direction and magnitude of the policy bias. Using equation (16) and Proposition 4, it follows that as long as  $y_L^* < y_R^*$ ,

$$\frac{\partial Ex_{RD}}{\partial \Sigma} = G(\hat{y}^*) - \frac{1}{2}, \quad (20)$$

and

$$\frac{\partial Ex_{RD}}{\partial \Delta} = \frac{g^2(\hat{y}^*) \left( 1 - g(\hat{y}^*) (y_R^* - y_L^*) \right) + 2g'(\hat{y}^*) (G(\hat{y}^*) - 1/2)}{8 |J(y_L^*, y_R^*)|}. \quad (21)$$

Equation (20) shows that, holding the relative convergence parameter constant, an increase in the aggregate convergence parameter lowers  $Ex_{RD}$  (i.e., makes it more left-wing) if the left-wing party is an underdog, and increases  $Ex_{RD}$  (i.e., makes it more right-wing) if the left-wing party is a favorite to win the elections. Recalling from Proposition 2 that  $G(\hat{y}^*) \gtrless 1/2$  as  $a_L \gtrless a_R$  and recalling that  $\Delta \equiv a_L - a_R$ , this implies the following result:

**Proposition 5:**  $\partial Ex_{RD} / \partial \Sigma \gtrless 0$  as  $\Delta \gtrless 0$ . Therefore, a small increase in  $\Sigma$  shifts the expected policy under RD towards the ideological position of the party which is an underdog in the political race.

To understand the intuition behind Proposition 5 suppose for instance that  $\Delta > 0$ . Then, the left-wing party is closer to the center and therefore a favorite to win the elections. Now, Proposition 4 shows that a small increase in  $\Sigma$  pushes  $y_L^*$  to the right and  $y_R^*$  to the left without changing the probability of each party to win the elections. But, since the left-wing party is the favorite to win, the shift of  $y_L^*$  to the right has a greater impact on  $Ex_{RD}$  than the corresponding shift of  $y_R^*$  to the left, so  $Ex_{RD}$  moves closer to the ideological position of the underdog right-wing party.

The impact of  $\Delta$  on  $Ex_{RD}$  is more complex since both terms in the numerator of equation (21) have ambiguous signs. To shed some light on this expression, we substitute for  $y_R^* - y_L^*$  from equation (18) into the numerator of equation (21) and rearrange terms:

$$\frac{\partial Ex_{RD}}{\partial \Delta} = \frac{g^3(\hat{y}^*) \left( \Sigma - \frac{1}{g(\hat{y}^*)} \right) + 2g'(\hat{y}^*) (G(\hat{y}^*) - 1/2)}{8 |J(y_L^*, y_R^*)|}. \quad (22)$$

To interpret this derivative, suppose that the distribution of median voter types is symmetric and unimodal. Then the second term in the numerator of equation (22) vanishes when  $G(\hat{y}^*) = 1/2$ ,

and is negative otherwise because  $G(\hat{y}^*)-1/2$  and  $g'(\hat{y}^*)$  have opposite signs. Therefore,  $\Sigma < 1/g(\hat{y}^*)$  is sufficient for  $\Delta$  to have a negative impact on  $Ex_{RD}$ . In other words, if the aggregate convergence parameter,  $\Sigma$ , is sufficiently small, then an increase in  $\Delta$  (raising  $a_L$  relative to  $a_R$ ) will shift  $Ex_{RD}$  closer to the ideological position of the left-wing party. In the symmetric equilibrium case where  $\Delta = 0$ , the second term in equation (22) vanishes, so  $\partial Ex_{RD}/\partial \Delta \gtrless 0$  as  $\Sigma \gtrless 1/g(\hat{c}_m)$ . Equation (19) indicates that the largest value of  $\Sigma$  for which  $y_R^* > y_L^*$  when  $\Delta = 0$  is  $\Sigma^{UB}(0) = 2/g(\hat{c}_m)$ . Hence, for  $\Delta = 0$ ,  $\partial Ex_{RD}/\partial \Delta > 0$  for values of  $\Sigma$  that exceed  $1/g(\hat{c}_m)$ , but once  $\Sigma$  falls below  $1/g(\hat{c}_m)$ , then  $\partial Ex_{RD}/\partial \Delta < 0$ .

## 5. The effects of polarization and of party asymmetries on the policy bias

Thus far we saw that when the model is completely symmetric in the sense that the distribution functions  $g(c_m)$  and  $f(\tilde{\gamma})$  are symmetric and  $\Delta = a_L - a_R = 0$ , there is no policy bias on average because  $Ex_{DD} = Ex_{RD}$ . Although complete symmetry is sufficient to ensure that  $Ex_{DD} = Ex_{RD}$ , it is not a necessary condition for this "no bias" result. To obtain a more complete view on the comparison between DD and RD, we shall now fully characterize the conditions under which there is no policy bias on average. Taking this set as a benchmark, we then determine the set of parameters for which DD is a reasonable approximation for RD in the sense that the policy bias is small. Since the impact of the shapes of the distribution functions  $g(c_m)$  and  $f(\tilde{\gamma})$  on the policy bias is already well-understood (see the discussion following equation (17)), we shall assume in what follows that  $g(\cdot)$  is symmetric and  $\hat{\gamma} = 0$ . Given this assumption,  $Ex_{DD} = \hat{c}_m$ , so the first two terms in equation (17) vanish and the direction and magnitude of policy biases depend only on  $d_L^*$  and  $d_R^*$ .

### 5.1 When is DD a reasonable approximation for RD and when is it not?

We begin by considering two special cases in which the political equilibrium is symmetric in the sense that  $d_L^* = d_R^*$ . First, when  $\Sigma \geq \Sigma^{UB}(\Delta)$ , Lemma 3 implies that  $d_L^* = d_R^* = 0$ . Second, when  $\Sigma < \Sigma^{UB}(\Delta)$  and  $\Delta = 0$  (i.e.,  $a_L = a_R$ ), Proposition 2 implies that  $d_L^* = d_R^* > 0$  and  $G(\hat{y}^*) = 1/2$ . In both cases, since  $g(\cdot)$  is symmetric and  $\hat{\gamma} = 0$ , it follows from equation (17) that  $Ex_{RD} = Ex_{DD}$  so there is no policy bias. The difference between the two cases is that

in the first case, the equilibrium platforms are both located at the center of the political spectrum, whereas in the second case, they are located at opposite sides of the center (though at equal distances). These observations are summarized in the following proposition:

**Proposition 6:** *Suppose that the distribution of median voter types,  $g(\cdot)$ , is symmetric and  $\hat{\gamma} = 0$ . Then there is no policy bias if one of the following conditions holds:*

- (i)  $\Sigma \geq \Sigma^{UB}(\Delta)$  (the equilibrium platforms fully converge);
- (ii)  $\Delta = 0$  and  $\Sigma < \Sigma^{UB}(0)$  (the equilibrium platforms converge only partially but they are equally-distant from the center of the political spectrum).

Next we consider cases where  $\Sigma < \Sigma^{UB}(\Delta)$  and  $\Delta \neq 0$ . In these cases, the platforms converge partially but not to the same extent. Hence, the political equilibrium is asymmetric and the left-wing party is a favorite to win the elections if  $\Delta > 0$  (in which case  $d_L^* < d_R^*$ ), whereas the right-wing party is a favorite to win if  $\Delta < 0$  (in which case  $d_L^* > d_R^*$ ).

**Proposition 7:** *Let the distribution of median voter types,  $g(\cdot)$ , be symmetric,  $\hat{\gamma} = 0$ , and  $\Sigma < \Sigma^{UB}(\Delta)$  (the equilibrium platforms converge only partially). Then:*

- (i) *As  $\Sigma$  increases towards  $\Sigma^{UB}(\Delta)$ ,  $Ex_{RD}$  increases if  $\Delta > 0$ , decreases if  $\Delta < 0$ , and is unaffected by  $\Sigma$  if  $\Delta = 0$ . In the limit as  $\Sigma$  approaches  $\Sigma^{UB}(\Delta)$  from below,  $Ex_{RD} \xrightarrow{\approx} Ex_{DD} = \hat{c}_m$  as  $\Delta \xrightarrow{\approx} 0$ .*
- (ii) *For sufficiently small values of  $|\Delta|$  (i.e., whenever the equilibrium is not too asymmetric), there exists for each  $\Delta$ , a unique value of  $\Sigma$ , denoted  $\Sigma^{NB}(\Delta)$ , for which  $Ex_{RD} = Ex_{DD} = \hat{c}_m$ , if and only if*

$$g(\hat{c}_m) > \frac{1}{c_1 - c_0}.$$

- (iii) *When  $\Sigma^{NB}(\Delta)$  exists and the distribution of median voter types is unimodal, then,  $\Sigma^{NB}(\Delta)$  is a symmetric, U-shaped, and smooth function that attains a minimum at  $\Delta = 0$ . Moreover,  $\Sigma^{NB}(0) = 1/g(\hat{c}_m) < 2/g(\hat{c}_m) = \Sigma^{UB}(0)$ , where  $\Sigma^{UB}(0)$  is the smallest value of  $\Sigma$  for which  $y_L^* = y_R^*$  when  $\Delta = 0$ .*

**Proof:** See the Appendix.

Proposition 7 provides the analytical skeleton for the characterization of the factors that determine the size and direction of the policy bias when the political equilibrium is asymmetric. In particular, it shows that even though the equilibrium is asymmetric, there exists a locus of points in the  $(\Sigma, \Delta)$  space, denoted  $\Sigma^{\text{NB}}(\Delta)$  ("NB" stands for No-Bias), along which the expected policies under RD and DD coincide, provided that (i) the distribution of median voter types, which reflects electoral uncertainty, has a sufficiently large density around its mean (since  $g(\cdot)$  is symmetric,  $\hat{c}_m$  is also the mean of the distribution), and (ii)  $|\Delta|$  is not too large (i.e., the political equilibrium is not too asymmetric). Otherwise, expected policies under RD and DD do not coincide unless the equilibrium is symmetric. To better understand the conditions for the existence of such an "asymmetric no bias" locus, note that  $1/(c_1 - c_0)$  is the density of a uniform distribution on the interval  $[c_0, c_1]$ . Thus, condition (i) says that the "asymmetric no bias" locus exists only if the distribution of median voter types has more weight around its mean than a uniform distribution; this is the case for example, in all symmetric and unimodal distributions. Condition (ii) in turn, requires the difference between the tendencies of the parties to converge towards the center to be sufficiently small, i.e., the equilibrium should not be "too asymmetric."

The main insights of the paper are now illustrated in Figures 4a and 4b which are based on Proposition 7. Figure 4a shows the situation when  $g(\hat{c}_m) < 1/(c_1 - c_0)$ , in which case, the distribution of median voter types is relatively flat around its mean. Then, Proposition 7 implies that  $Ex_{\text{DD}}$  has a right-wing bias if  $\Delta < 0$ , in which case the right-wing party is a favorite to win the elections, and a left-wing bias if  $\Delta > 0$ , in which case the left-wing party is a favorite to win the elections. In other words, when  $g(\hat{c}_m) < 1/(c_1 - c_0)$ ,  $Ex_{\text{DD}}$  is biased in the direction of the party which is the favorite to win the elections. This result is somewhat counterintuitive because it might be thought that if one party is a favorite to win the elections, the expected policy under RD will lean in its direction more than the expected policy under a DD which reflects only the preferences of the median voter. This intuition, however, fails to take into account the fact that a party can be a favorite to win the elections only if it adopts a more centrist platform than its rival party, and this has a moderating effect on  $Ex_{\text{RD}}$  in comparison with  $Ex_{\text{DD}}$ . For instance, when  $\Delta > 0$ , the left-wing party adopts a more centrist platform than



the right-wing party and is therefore a favorite to win the elections. As a result, the policy under RD is more likely to be selected by the left-wing party. But since this policy is also closer to the center of the political spectrum,  $Ex_{RD}$  can very well be less left-wing than  $Ex_{DD}$ , implying that DD has a left-wing bias. The size of the policy bias increases in absolute value as  $\Sigma$  approaches  $\Sigma^{UB}(\Delta)$ . But, once  $\Sigma \geq \Sigma^{UB}(\Delta)$ , the equilibrium platforms fully converge and the policy bias vanishes. Since an increase in  $\Sigma$  towards  $\Sigma^{UB}(\Delta)$  also means that there is less political polarization, we can conclude that the policy bias and the degree of political polarization are inversely related in this case.

Next, consider Figure 4b that shows the situation when  $g(\hat{c}_m) > 1/(c_1 - c_0)$ . If  $|\Delta|$  is sufficiently large so that one party is a clear favorite to win the elections, then once again,  $Ex_{DD}$  is biased in the direction of the electorally favorite party so that the policy bias and the degree of political polarization continue to be inversely related. Things are different however when  $|\Delta|$  is sufficiently small so that the lead margin of the favorite party is not too big. Now, there exists a curve,  $\Sigma^{NB}(\Delta)$ , along which the policy bias vanishes. The  $\Sigma^{NB}(\Delta)$  curve, together with the horizontal axis (along which  $\Delta = 0$ ), split the  $(\Sigma, \Delta)$  space into four distinct regions. In the first region where  $\Delta > 0$  and  $\Sigma > \Sigma^{NB}(\Delta)$ ,  $Ex_{RD} > Ex_{DD}$ , implying that  $Ex_{DD}$  has a left-wing bias. In the second region where  $\Delta < 0$  and  $\Sigma > \Sigma^{NB}(\Delta)$ ,  $Ex_{RD} < Ex_{DD}$ , implying that  $Ex_{DD}$  has a right-wing bias. Since the left-wing (right-wing) party is a favorite to win the elections when  $\Delta > 0$  ( $\Delta < 0$ ),  $Ex_{DD}$  is again biased in the direction of the favorite party. Unlike the first two regions, in the third and fourth regions where  $\Sigma < \Sigma^{NB}(\Delta)$ ,  $Ex_{DD}$  is biased in the direction of the party which is an underdog in the political race. In particular, in the third region where  $\Delta > 0$  and  $\Sigma < \Sigma^{NB}(\Delta)$ ,  $Ex_{RD} < Ex_{DD}$  implying that  $Ex_{DD}$  has a right-wing bias, whereas in the fourth region where  $\Delta < 0$  and  $\Sigma < \Sigma^{NB}(\Delta)$ ,  $Ex_{RD} > Ex_{DD}$  so that  $Ex_{DD}$  has a left-wing policy bias. In any event, since the absolute value of the policy bias is now decreasing with  $\Sigma$  when  $\Sigma < \Sigma^{NB}(\Delta)$  and increasing with  $\Sigma$  when  $\Sigma^{NB}(\Delta) < \Sigma < \Sigma^{UB}(\Delta)$ , the relationship between the (absolute value of the) policy bias and the degree of political polarization is no longer monotonic. When  $\Sigma < \Sigma^{NB}(\Delta)$ , both the degree of political polarization and the policy bias decrease with  $\Sigma$ . By contrast, when  $\Sigma^{NB}(\Delta) < \Sigma < \Sigma^{UB}(\Delta)$ , political polarization decreases with  $\Sigma$  while the absolute value of the bias increases with  $\Sigma$ .

The main conclusions from Figures 4a and 4b are now summarized as follows:

**Proposition 8:** *Let the distribution of median voter types be symmetric and unimodal,  $\hat{\gamma} = 0$ , and let  $\Sigma < \Sigma^{UB}(\Delta)$  (i.e., the equilibrium platforms converge only partially). Then,*

- (i) *If either  $g(\hat{c}_m) < 1/(c_I - c_D)$ , or  $g(\hat{c}_m) > 1/(c_I - c_D)$  and  $|\Delta|$  is relatively large, then  $Ex_{DD}$  is biased in the direction of the party that is the favorite to win the elections. Moreover, the policy bias and the degree of political polarization are inversely related.*
- (ii) *If  $g(\hat{c}_m) > 1/(c_I - c_D)$  and  $|\Delta|$  is not too large, then  $Ex_{DD}$  is biased in the direction of the party that is the favorite to win the elections provided that the degree of political polarization is sufficiently small (i.e.,  $\Sigma > \Sigma^{NB}(\Delta)$ ). Moreover, the policy bias and the degree of political polarization are inversely related in this case. In contrast, when the degree of political polarization is sufficiently large (i.e.,  $\Sigma > \Sigma^{NB}(\Delta)$ ),  $Ex_{DD}$  is biased in the direction of the party that is an underdog in the political race. Moreover, the policy bias and the degree of political polarization are positively related in this case.*

The general message of this subsection can be summarized as follows. Although there is a rather small set of parameters for which there is no policy bias at all, the set of parameters for which the policy bias is "small" in absolute value is considerably larger. The factors that determine the absolute size of the policy bias include the shape of the distribution of electoral uncertainty, the degree of political polarization, and the degree to which the political equilibrium is symmetric, which in turn determines the chances of the two parties to win the elections. Propositions 7 and 8 provide a full characterization of the policy bias for the case in which the distributions of electoral uncertainty and of the external circumstances shock are symmetric. In particular, when the former distribution has less weight around its mean than a uniform distribution, or when it has more weight and one party is a clear favorite to win the elections, the bias decreases as the degree of political polarization increases. When the distribution of electoral uncertainty has more weight around its mean than the uniform and no party is a clear favorite in the political race, the bias tends to zero in absolute value as  $\Sigma$  (which determines the degree of political polarization), tends to the "asymmetric no-bias" locus.

## 5.2 Examples

In order to gain further insights about the determinants of policy biases and to illustrate the

fundamental differences between cases in which the condition in part (ii) of Proposition 7 is satisfied and cases in which it is not, we now consider two examples that differ only with respect to the distribution of median voter types. Apart from this difference, the distributions of the external circumstances shock and the median voter types are symmetric in both examples. Consequently,  $\hat{\gamma} = \hat{c}_m = \bar{c}_m = 0$ , implying that policy biases can arise only for  $\Delta \neq 0$ , when the tendencies of the two parties to converge towards the center are not the same. In the first example,  $G(c_m)$  is uniform so the condition in part (ii) of Proposition 7 fails, whereas in the second example,  $G(c_m)$  is triangular so the condition in part (ii) of Proposition 7 holds for small values of  $\Delta$ . Consequently, in the first example,  $Ex_{DD} = Ex_{RD}$  only when the political equilibrium is symmetric, whereas in the second example, there exists an asymmetric no-bias locus along which  $Ex_{DD} = Ex_{RD}$ .

**Example 1:  $G(c_m)$  is uniform on the interval  $[-C/2, C/2]$**

Suppose that  $G(c_m) = 1/2 + c_m/C$ . To ensure that Assumption A2 is satisfied, suppose in addition that  $C - h_R < c_L + c_R < h_L - C$ . Since  $\bar{c}_m = \hat{\gamma} = 0$ , equation (15) implies that  $Ex_{DD} = 0$ . Under RD, the equilibrium platforms when there is only partial convergence, are given by

$$y_L^* = \frac{\Delta - 3(2C - \Sigma)}{6}, \quad y_R^* = \frac{\Delta + 3(2C - \Sigma)}{6}. \quad (23)$$

Equation (23) shows that the equilibrium platforms are positioned symmetrically around the center of the political spectrum (which is equal to 0 in this example) only when  $\Delta = 0$ . Otherwise, the left-wing (right-wing) party is closer to the center if  $\Delta > 0$  ( $\Delta < 0$ ). The equation also shows that as  $\Sigma$  increases, the equilibrium platforms converge so there is less political polarization. Since we have assumed that  $y_L^* < y_R^*$ , the equilibrium platforms are characterized by equation (23) only when  $\Sigma < 2C$ . Otherwise, there is full convergence, in which case Lemma 2 implies that  $y_L^* = y_R^* = \hat{c}_m = 0$ . As a result,  $Ex_{DD} = Ex_{RD}$ .

The comparison between  $Ex_{DD}$  and  $Ex_{RD}$  is illustrated in Figure 5. We already established that when  $\Sigma \geq 2C$ ,  $Ex_{DD} = Ex_{RD}$ . Hence, in what follows we consider the case where  $\Sigma < 2C$ . Then, substituting for  $y_L^*$  and  $y_R^*$  into equation (16), recalling that  $G(c_m) = 1/2 + c_m/C$  and simplifying, expected policy under a RD becomes:

$$Ex_{RD} = \frac{\Delta(\Sigma - C)}{6C}. \quad (24)$$

To determine the sign of  $Ex_{RD}$ , note that the assumption that  $C - h_R < c_L + c_R < h_L - C$  implies that  $2C < h_L + h_R$ , which in turn implies that  $2C - (c_R - c_L) < h_L + h_R - (c_R - c_L) \equiv \Sigma$ . But since by Assumption A3,  $c_R - c_L < C$ , the last inequality implies that  $\Sigma > C$ . Hence, the sign of  $Ex_{RD}$  is equal to the sign of  $\Delta$ . When  $\Delta = 0$ , the equilibrium is symmetric, so  $Ex_{DD} = Ex_{RD}$ . Otherwise, the equilibrium is asymmetric. When  $\Delta > 0$  ( $\Delta < 0$ ),  $y_L^*$  ( $y_R^*$ ) is closer to the center and the left-wing (right-wing) party is a favorite to win the elections. Then,  $Ex_{DD}$  has a left-wing (right-wing) bias, and this bias increases in absolute value as  $\Delta$  increases (the equilibrium becomes more asymmetric), and as  $\Sigma$  goes to  $2C$  (there is less political polarization).

**Example 2:  $G(c_m)$  is triangular on  $[-1, 1]$**

Now suppose that  $G(c_m) = (1 + c_m)^2/2$  if  $c_m < 0$  and  $G(c_m) = (1 + 2c_m - c_m^2)/2$  otherwise. As in the uniform distribution case,  $g(\cdot)$ , is symmetric around 0, so  $\bar{c}_m = \hat{\gamma} = 0$ . Hence,  $Ex_{DD} = 0$ . Under a RD, the pair of equilibrium platforms when  $y_L^* < y_R^*$  is given by:

$$(y_L^*, y_R^*) = \begin{cases} \left( \frac{-2 + 3\Delta - Z + 4\Sigma}{8}, \frac{-10 - \Delta + 3Z - 4\Delta}{8} \right), & \text{if } \Delta \leq 0, \\ \left( \frac{10 - \Delta - 3Z + 4\Sigma}{8}, \frac{2 + 3\Delta + Z - 4\Sigma}{8} \right), & \text{if } \Delta > 0, \end{cases} \quad (25)$$

where

$$Z \equiv \sqrt{36 - 4|\Delta| + \Delta^2}. \quad (26)$$

Equation (25) implies that  $\Sigma^{UB}(\Delta) = (-2 + |\Delta| + Z)/2$ . When  $\Sigma \geq (-2 + |\Delta| + Z)/2$ , the equilibrium platforms fully converge, and as Lemma 2 shows,  $y_L^* = y_R^* = \hat{c}_m$ ; hence, there is no policy bias. As  $\Sigma$  falls below  $(-2 + |\Delta| + Z)/2$ ,  $y_L^*$  and  $y_R^*$  move in opposite directions so the equilibrium features more political polarization.  $\Sigma$ , however, cannot be too small since Assumptions A2 and A3 ensure that  $-1 < y_L^* \leq y_R^* < 1$ . When  $\Delta > 0$ ,  $y_R^*$  is closer to 1 than  $y_L^*$  is to -1, so the lower bound on  $\Sigma$  is determined by setting  $y_R^* = 1$  and solving for  $\Sigma$ .

When  $\Delta < 0$ ,  $y_L^*$  is closer to -1 than  $y_R^*$  is to 1 so the lower bound on  $\Sigma$  is determined implicitly by  $y_L^* = -1$ . In both cases, the lower bound on  $\Sigma$  is given by  $(-6+3|\Delta|+Z)/4$ .

Assuming that  $(-6+3|\Delta|+Z)/4 < \Sigma < (-2+|\Delta|+Z)/2$ , equation (25) implies that

$$\hat{y}^* = \frac{\delta_\Delta(6+|\Delta|-Z)}{8}, \quad (27)$$

and

$$Ex_{RD} = \frac{\delta_\Delta [(6-|\Delta|)(16+2\Sigma+|\Delta|\Sigma) - Z(16+2\Sigma-|\Delta|\Sigma)]}{64}, \quad (28)$$

where  $\delta_\Delta = -1$  if  $\Delta < 0$ ,  $\delta_\Delta = 1$  if  $\Delta > 0$ , and  $\delta_\Delta = 0$  if  $\Delta = 0$ . Equation (27) indicates that  $\hat{y}^* \gtrless 0$  as  $\Delta \gtrless 0$ ; since  $G(\cdot)$  is symmetric around 0 this implies that  $G(\hat{y}^*) \gtrless 1/2$  as  $\Delta \gtrless 0$ . Hence, the left-wing party is a favorite to win if  $\Delta > 0$  and an underdog if  $\Delta < 0$ . Equation (27) also confirms that as  $|\Delta|$  increases, the equilibrium becomes more asymmetric in the sense that the chances of the parties to win the elections become more unequal.

The comparison between  $Ex_{RD}$  and  $Ex_{DD}$  is illustrated in Figure 6. When  $\Sigma$  is to the right of the dotted area,  $y_L^* = y_R^* = \hat{c}_m = 0$ , so there is no policy bias. Inside the dotted area, there is a positive gap between  $y_L^*$  and  $y_R^*$ . This gap increases as  $\Sigma$  decreases which confirms that lower values of  $\Sigma$  are associated with more political polarization. Values of  $\Sigma$  left of the dotted area are ruled out by Assumption A2. When  $\Sigma$  lies inside the dotted area and  $\Delta = 0$ , the equilibrium is symmetric and  $Ex_{RD} = Ex_{DD} = 0$ . In contrast, when  $\Sigma$  lies inside the dotted area and  $\Delta \neq 0$ , the political equilibrium is asymmetric and  $y_L^*$  ( $y_R^*$ ) is closer to the center of the political spectrum and the left-wing (right-wing) party is a favorite to win the elections if  $\Delta > 0$  ( $\Delta < 0$ ). If we fix  $\Delta > 0$  ( $\Delta < 0$ ) and pick a  $\Sigma$  just to the left of  $(-2+|\Delta|+Z)/2$ , then  $Ex_{RD} = \hat{y}^* > 0$ , ( $Ex_{RD} = \hat{y}^* < 0$ ), so  $Ex_{DD}$  has a left-wing (right-wing) bias. As we begin to lower  $\Sigma$ ,  $Ex_{RD}$  decreases (increases), but as long as  $|\Delta| > 2.764$ , the policy bias remains negative when  $\Delta > 0$  and positive when  $\Delta < 0$ , for all values of  $\Sigma$  inside the dotted area. In contrast, when  $|\Delta| \in (0, 2.764)$ , there exists for each  $\Delta$  a unique value of  $\Sigma$ ,  $\Sigma^{NB}(\Delta)$ , at which  $Ex_{RD} = 0$ . For values of  $\Sigma$  between  $\Sigma^{NB}(\Delta)$  and  $(-2+|\Delta|+Z)/2$ ,  $Ex_{DD}$  has a left-wing (right-wing) bias when  $\Delta > 0$  ( $\Delta < 0$ ), and for values of  $\Sigma$  between  $(-6+3|\Delta|+Z)/4$  and  $\Sigma^{NB}(\Delta)$ ,  $Ex_{DD}$  has a

right-wing (left-wing) bias when  $\Delta > 0$  ( $\Delta < 0$ ). Given  $\Delta$ , the policy bias vanishes when  $\Sigma = \Sigma^{\text{NB}}(\Delta)$ , and it increases monotonically, in absolute value, as  $\Sigma$  moves further away from  $\Sigma^{\text{NB}}(\Delta)$ .

## 6. Comparison of political uncertainty under DD and RD

This section compares the variability of policy choices under DD and under RD around their means. Such a comparison is interesting for at least two reasons. First, the variability of policy is one measure of political uncertainty. This uncertainty is believed to affect investment, economic activity and growth (see for example, Aizenman and Marion, 1993 and Forthcoming, and Alesina et al., 1996). In the spirit of "structure induced equilibrium" (Shepsle and Weingast, 1981), it is therefore useful to identify the circumstances under which one of these systems of government generates more policy uncertainty than the other.

Second, RD and DD generally induce different distributions of policy choices. Given the comparison between the first moments of these distributions, reported in Sections 4 and 5, a comparison of the variances of policy choices is a natural further step towards a fuller characterization of the differences between the policy choices that these two alternative systems of government yield.

Since the choice of policy is contingent on the realization of  $\bar{\gamma}$  under RD, but not under DD, it seems at first blush that political uncertainty should be larger under a RD as this system injects uncertainty about the state of nature into the choice of policy. On the other hand, under RD there is a small number of parties whose preferences are fixed up to the realization of  $\bar{\gamma}$ , so the impact of electoral uncertainty on the variability of policy is limited in comparison to DD. Hence, it is not obvious a priori which of the two systems generates a higher degree of political uncertainty. This section analyzes the factors that determine the relative magnitude of political uncertainty under the two systems, using the variance of policy as a metric for uncertainty.

Under DD the variance of policy is given by

$$V_{DD} = E_{c_m} [c_m + \hat{\gamma} - Ex_{DD}]^2 = \int_{c_0}^{c_1} (c_m - \hat{c}_m)^2 g(c_m) dc_m. \quad (29)$$

This expression shows that  $V_{DD}$  depends only on the electoral uncertainty regarding the preferences of voters. The larger is this uncertainty, the harder it is to predict in advance the location of the median voter and hence the policy that will be chosen under DD.

Recalling that  $\tilde{\gamma}$  has a zero mean, using equation (16) and rearranging terms, the variance of policy under RD can be expressed as:

$$\begin{aligned} V_{RD} &= E_{\tilde{\gamma}, c_m} [x - Ex_{RD}]^2 \\ &= G(\hat{y}^*) E_{\tilde{\gamma}} [y_L^* + \tilde{\gamma} - Ex_{RD}]^2 + (1 - G(\hat{y}^*)) E_{\tilde{\gamma}} [y_R^* + \tilde{\gamma} - Ex_{RD}]^2 \\ &= (y_R^* - y_L^*)^2 G(\hat{y}^*) (1 - G(\hat{y}^*)) + \sigma_{\tilde{\gamma}}^2, \end{aligned} \quad (30)$$

where  $\sigma_{\tilde{\gamma}}^2$  is the variance of  $\tilde{\gamma}$ . Equation (30) shows that  $V_{RD}$  has two components. The first component, represented by  $(y_R^* - y_L^*)^2 G(\hat{y}^*) (1 - G(\hat{y}^*))$ , reflects the electoral uncertainty under RD. For a given probability of winning the elections, this uncertainty increases with the distance between the two platforms, while for a given distance between the platforms, it increases when the electoral prospects of the parties become more equal, reaching a maximum when  $G(\hat{y}^*) = 1/2$ .

The second component of  $V_{RD}$ , represented by  $\sigma_{\tilde{\gamma}}^2$ , reflects uncertainty with respect to external circumstances. It arises because policy under RD is always adjusted by elected officials in line with the realization of  $\tilde{\gamma}$ . Although this adjustment is beneficial to all voters (see equation (1)), it nonetheless creates political uncertainty by making it harder to predict in advance which policy will be implemented. Under DD in contrast, policy choices are made by individual voters who do not observe the actual realization of  $\tilde{\gamma}$ , so  $V_{DD}$  is independent of  $\sigma_{\tilde{\gamma}}^2$ .

Clearly if  $\sigma_{\tilde{\gamma}}^2 > V_{DD}$ , there will be more political uncertainty under RD even if the two platforms fully converge on the center of the political spectrum. Otherwise things depend on the extent to which the two political systems are sensitive to electoral uncertainty. Recalling

from Proposition 4 that the distance between  $y_L^*$  and  $y_R^*$  decreases with the aggregate convergence parameter  $\Sigma$ , equation (30) suggests that all else equal, electoral uncertainty under RD is lower the larger is  $\Sigma$ . The next proposition sharpens this intuition.

**Proposition 9:** *Holding  $\Delta$  constant, electoral uncertainty under RD decreases with  $\Sigma$ . In the limit as  $\Sigma$  approaches  $\Sigma^{UB}(\Delta)$ , the equilibrium platforms fully converge so electoral uncertainty under RD vanishes. At the other extreme when  $\Sigma$  is sufficiently small so that  $y_L^*$  approaches  $c_0$  and  $y_R^*$  approaches  $c_1$ , electoral uncertainty under RD increases above that under DD.*

**Proof:** See the Appendix.

Proposition 9 indicates that by appropriately choosing the convergence parameters,  $a_L$  and  $a_R$ , it is possible to make electoral uncertainty in a RD smaller or larger than electoral uncertainty in a DD. When  $\Sigma$  is sufficiently large, this is clear since the two parties converge on the center of the political spectrum, and absent any uncertainty regarding external circumstances, the policy under RD would be perfectly predictable once the platforms are announced. At the other extreme when  $\Sigma$  is sufficiently small, the two platforms approach the boundaries of the support of  $c_m$ . Since the expression for  $V_{DD}$  assigns positive weights to all values in the support of  $c_m$  rather than just to its boundaries, it is clear that electoral uncertainty in DD is smaller than it is under RD. Since the aggregate convergence parameter,  $\Sigma$ , takes into account both the ideologies of the parties and the office motivation of the party leaders, Proposition 9 implies that we should expect more electoral uncertainty under RD as the two parties become more polarized, and as the party leaders become less office motivated.

### 6.1 A uniform distribution example

To illustrate the principles discussed above we consider the case where the distribution of median voters types is uniform on the interval  $[-C/2, C/2]$ . To ensure that  $y_L^* < y_R^*$ , we assume in addition that  $\Sigma < 2C$ . Then the variances of policy under DD and RD become



$$V_{DD} = \frac{C^2}{12}, \quad (31)$$

and

$$V_{RD} = \frac{(2C - \Sigma)^2 (9C^2 - \Delta^2)}{36C^2} + \sigma_{\hat{\gamma}}^2. \quad (32)$$

The first term on the right side of equation (32) represents electoral uncertainty under RD (and is therefore always positive), and the second term is the variance of the external circumstances shock. Comparing the two variances and recalling that  $C < \Sigma < 2C$ , yields the following result:

**Proposition 10:** *Suppose that  $\hat{\gamma} = 0$ ,  $G(c_m)$  is uniform on the interval  $[-C/2, C/2]$ , and  $\Sigma < 2C$ . Then,  $V_{RD}$  is always larger than  $V_{DD}$  if  $\sigma_{\hat{\gamma}}^2 \geq C^2/12$  and  $V_{DD}$  is always larger than  $V_{RD}$  if  $\sigma_{\hat{\gamma}}^2 < (\Delta^2 - 6C^2)/36$ . Otherwise,  $V_{RD}$  exceeds  $V_{DD}$  iff*

$$\Sigma < 2C - \sqrt{\frac{3C^2(C^2 - 12\sigma_{\hat{\gamma}}^2)}{9C^2 - \Delta^2}}.$$

Proposition 10 shows that if  $\sigma_{\hat{\gamma}}^2$ , which measures the uncertainty with respect to external circumstances, is larger than the uncertainty regarding the location of the median voter, then RD always generates more political uncertainty than DD, in the sense that the policy outcome under RD is less predictable than that under DD. But, if  $\sigma_{\hat{\gamma}}^2$  is sufficiently small, then the reverse is true (this case is possible however only if  $\Delta^2 > 6C^2$ ).<sup>14</sup> For intermediate values of  $\sigma_{\hat{\gamma}}^2$ , the comparison between  $V_{DD}$  and  $V_{RD}$  depends on the value of  $\Sigma$  which determines the extent to which the two platforms converge. When  $\Sigma$  is small, the equilibrium platforms are relatively

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<sup>14</sup> To check that there exists a nonempty set of parameters such that  $\Delta^2 > 6C^2$  and the equilibrium platforms converge partially, suppose that  $\Delta = 5$  and  $C = 2$ . Since  $C = 2$ , Assumption A3 implies that  $-1 < c_L < 0 < c_R < 1$ , so the equilibrium platforms converge partially provided that  $-1 < y_L^* < y_R^* < 1$ . Now, using equation (23), we get  $y_L^* = (-7+3\Sigma)/6$  and  $y_R^* = (17-3\Sigma)/6$ , so for all  $11/3 < \Sigma < 4$ , such an equilibrium exists.

far apart, so the political uncertainty under a RD is large; consequently,  $V_{RD} > V_{DD}$ . When  $\Sigma$  is large, the equilibrium platforms are relatively close to one another so the political uncertainty under RD is smaller than under DD.

## 7. Concluding remarks

There are several general lessons that emerge from this paper. First, the Downsian benchmark in which the platforms of the two parties fully converge on the center of the political spectrum arises only when party leaders are sufficiently office motivated and/or the parties are not too polarized. In this benchmark case, there are no systematic differences between policy choices in DD and in RD. Second, when there is only partial convergence, systematic policy differences between the two systems are likely to be the rule rather than the exception. In the case of partial convergence the sign and the size of the distance between the expected policy under DD and RD (the policy bias) depend on the degree of asymmetry in the ideal positions of the parties in relation to the center of the political spectrum, on differences in the extent to which the party leaders are office motivated, and on the skewness in the distributions of electoral uncertainty and of the external circumstances shock. Even when these two distributions are symmetric, the set of parameters for which there is no bias is rather narrow. Third, this set consists of a symmetric no-bias locus, along which the parties have the same tendency to converge, and (for distributions of electoral uncertainty with sufficiently salient modes) of an asymmetric no-bias locus along which the parties have different tendencies to converge. Fourth, given the relative tendency of the parties to converge, the magnitude of the policy bias is monotonically related to the divergence of the aggregate convergence parameter of the political system from the asymmetric no-bias locus. The results of the paper thus suggest under what conditions and where to look for "correction factors" for the results of politico - economic models that utilize the DD paradigm.

It would appear at first blush that if one party moves closer to the center of the political spectrum, the expected policy under RD would also shift in the same direction. This intuition however abstracts from the fact that when a party is closer to the center, it is a favorite to win the elections and implement its policy. This effect pushes the expected policy under RD in the

direction of that party and away from the center. One contribution of this paper is to evaluate the combined effect of the positions of the two parties and of their electoral prospects on expected policy and political uncertainty under RD, and consequently on the relationship between the policy outcomes generated by direct and representative democracies.

Cross country evidence indicates that political uncertainty reduces private investment and slows down economic growth (Aizenman and Marion, 1993 and Forthcoming, and Alesina et. al., 1996). This paper derives conditions which make it possible to compare political uncertainty under direct and representative democracies. In particular, it is shown that political uncertainty in a RD rises in comparison to this uncertainty in a DD the higher is the degree of polarization between parties.

The main strategic actors in our analysis of RD are party leaders. In spite of their ideology free Downsian disposition, those "political entrepreneurs" are sensitive to the preferences of the centers of their respective parties since they need their support. The paper takes as primitives the number of parties and their positions. Recent papers by Osborne and Slivinski (1996) and Besley and Coates (1997) consider models of "citizen-candidates," in which individuals choose whether or not to become candidates, and if they do, they pick their ideal positions as platforms (the "citizen-candidates" cannot precommit to policies that diverge from their ideal positions, so all voters correctly anticipate that once they are elected, they will implement their ideal policies). Their work suggests that in many cases, electoral competition will induce the emergence of exactly two candidates as we have assumed.

Our paper also assumes as a primitive that parties pick up ideology free leaders. A possible reason for this might be that political leaders who are not tainted by the ideology of a particular party have better nationwide electoral prospects. This element could be captured by modeling the party centers as strategic players too. Finally, our paper highlights the trade-off between the superior use of information on external circumstances under RD and the fact that DD better reflects the "will of the voters." Given this trade-off, it is natural to compare the two systems in terms of their welfare properties and identify the conditions under which one system dominates the other from a normative point of view. However, these two issues are beyond the scope of this already lengthy paper, so they are left for future research.

## Appendix

**Proof of Lemma 1:** The proof proceeds in three steps:

(i) Assume by way of negation that  $y_R^* < y_L^*$ . Then, there are three possible cases. First, if  $y_L^* \leq \hat{c}_m$ , then  $P_R < 1/2$  (all median voter types whose taste parameters exceed  $\hat{c}_m$  surely vote for the left-wing party). Now, by deviating to  $y_L^*$ , the leader of the right-wing party can increase his chances to be elected to  $1/2$ , and moreover, he moves closer to  $c_R$ . Hence, the deviation upsets the putative equilibrium. Second, if  $y_R^* \geq \hat{c}_m$ , the leader of the left-wing party can increase his expected payoff by deviating to  $y_R^*$ , because then his chances to be elected increase to  $1/2$ , and the party's platform is closer to  $c_L$ . Finally, if  $y_R^* < \hat{c}_m < y_L^*$ , both party leaders can increase their expected payoffs by moving slightly towards  $\hat{c}_m$ , thereby increasing their chances of being elected, while moving closer to their party's ideal policies. Thus, in equilibrium it must be the case that  $y_L^* \leq y_R^*$ .

(ii) Suppose by way of negation that  $y_L^* = y_R^* \neq \hat{c}_m$ . Since  $y_L^* = y_R^*$ , the probability of each party winning the elections is  $1/2$ . If  $y_L^* = y_R^* < \hat{c}_m$ , the leader of the right-wing party can increase his expected payoff by deviating to  $\hat{c}_m$ , in which case his chances to be elected exceed  $1/2$ , and his party's platform shifts closer to  $c_R$ . Similarly, if  $y_L^* = y_R^* > \hat{c}_m$ , the leader of the left-wing party can by moving to  $\hat{c}_m$ , increase his chances to be elected to more than  $1/2$  while moving closer to  $c_L$ . Hence the only equilibrium in which  $y_L^* = y_R^*$  is such that  $y_L^* = y_R^* = \hat{c}_m$ .

(iii) To prove that  $y_L^* \geq c_L$  and  $y_R^* \leq c_R$ , suppose by way of negation that  $y_L^* < c_L$ . Since by Assumption A3,  $c_L < \hat{c}_m$ , it is clear that  $y_L^* < y_R^*$  (from step 1 we know that  $y_L^* \leq y_R^*$ , while from step 2 we know that  $y_L^* = y_R^*$  only if  $y_L^* = y_R^* = \hat{c}_m$ ; hence the assumption that  $y_L^* < c_L$  rules out the possibility that  $y_L^* = y_R^*$ ). But now, the leader of the left-wing party can increase his chances to be elected by moving to  $c_L$ , while maximizing the support he gets from his party after being elected. Consequently,  $y_L^* < c_L$  cannot be an equilibrium choice for

the leader of the left-wing party. Similar argument establishes that  $y_R^* > c_R$  cannot be an equilibrium choice for the leader of the right-wing party. ■

**Proof of Lemma 2:** First consider  $y_L^*$ . Assumption A2 states that  $h_L > 2M((c_L + c_R)/2)$ . But since by Assumption A1,  $M'(\cdot) > 0$ , and since by Lemma 1,  $c_L < y_R^* \leq c_R$ , this implies that  $h_L > 2M((c_L + y_R)/2)$  for all  $c_L < y_R \leq c_R$ . Using the definition of  $M(\cdot)$ , this condition is equivalent to  $\partial \pi_L(c_L, y_R)/\partial y_L > 0$  for all  $c_L < y_R \leq c_R$ . Hence,  $c_L$  is a dominated strategy for the leader of the left-wing party. Next, consider  $y_R^*$ . Assumption A2 states that  $h_L > 2H((c_L + c_R)/2)$ . Recalling from Assumption A1 that  $H'(\cdot) < 0$  and recalling from Lemma 1 that  $c_L \leq y_L^* < c_R$ , it follows that  $h_R > 2H((y_L + c_R)/2)$  for all  $c_L \leq y_L < c_R$ . Using the definition of  $H(\cdot)$ , this condition is equivalent to  $\partial \pi_R(y_L, c_R)/\partial y_R < 0$  for all  $c_L \leq y_L < c_R$ . Hence,  $c_R$  is a dominated strategy for the right-wing party. Together with Lemma 1, this implies that in equilibrium, either  $c_L < y_L^* < y_R^* < c_R$ , or  $y_L^* = y_R^* = \hat{c}_m$ . ■

**Showing that the first-order conditions (13) and (14) are sufficient for a maximum:** The second derivatives of the payoff functions are given by:

$$\frac{\partial^2 \pi_L(y_L, y_R)}{\partial y_L^2} = \frac{g'(\hat{y})}{4} [h_L - (y_L - c_L)] - g(\hat{y}), \quad (\text{A-1})$$

and

$$\frac{\partial^2 \pi_R(y_L, y_R)}{\partial y_R^2} = -\frac{g'(\hat{y})}{4} [h_R - (c_R - y_R)] - g(\hat{y}), \quad (\text{A-2})$$

where  $\hat{y} \equiv (y_L + y_R)/2$  is the ideal policy of the voter who is just indifferent between the two parties. Substituting from equation (13) for  $h_L - (y_L - c_L)$ , using the definition of  $M(\cdot)$ , and rearranging terms, yields:

$$\frac{\partial^2 \pi_L(y_L^*, y_R^*)}{\partial y_L^2} = -\frac{g(\hat{y}^*)}{2} [1 + M'(\hat{y}^*)], \quad (\text{A-3})$$

which is negative by Assumption A1 (the superscript \* reflects the fact that the second derivatives are evaluated at the equilibrium values). Similarly, substituting from equation (14) for  $h_R(c_R - y_R)$ , using the definition of  $H(\cdot)$ , and rearranging terms yields

$$\frac{\partial^2 \pi_R(y_L^*, y_R^*)}{\partial y_R^2} = -\frac{g(\hat{y}^*)}{2} [1 - H'(\hat{y}^*)], \quad (\text{A-4})$$

which is negative by Assumption A1.

Next we characterize for later reference the determinant of the Jacobian matrix corresponding to equations (13) and (14). This matrix is given by,

$$J(y_L, y_R) \equiv \begin{vmatrix} \frac{\partial^2 \pi_L(y_L, y_R)}{\partial y_L^2} & \frac{\partial^2 \pi_L(y_L, y_R)}{\partial y_L \partial y_R} \\ \frac{\partial^2 \pi_R(y_L, y_R)}{\partial y_R \partial y_L} & \frac{\partial^2 \pi_R(y_L, y_R)}{\partial y_R^2} \end{vmatrix}. \quad (\text{A-5})$$

Evaluated at  $(y_L^*, y_R^*)$ , the diagonal terms are given by equations (A-3) and (A-4), and the off-diagonal terms are given by

$$\frac{\partial^2 \pi_L(y_L^*, y_R^*)}{\partial y_L \partial y_R} = -\frac{g(\hat{y}^*)}{2} M'(\hat{y}^*), \quad (\text{A-6})$$

and

$$\frac{\partial^2 \pi_R(y_L^*, y_R^*)}{\partial y_R \partial y_L} = \frac{g(\hat{y}^*)}{2} H'(\hat{y}^*), \quad (\text{A-7})$$

both of which are negative by Assumption A1. Using equations (A-3), (A-4), (A-6) and (A-7), the determinant of  $J(y_L^*, y_R^*)$ , is given by

$$\begin{aligned} |J(y_L^*, y_R^*)| &= \frac{3g^2(\hat{y}^*) + g'(\hat{y}^*)(1 - 2G(\hat{y}^*))}{4} \\ &= -\frac{g(\hat{y}^*)}{2} \left( \frac{\partial^2 \pi_L(y_L^*, y_R^*)}{\partial y_L \partial y_R} + \frac{\partial^2 \pi_R(y_L^*, y_R^*)}{\partial y_R \partial y_L} \right) + \frac{g^2(\hat{y}^*)}{4} > 0, \end{aligned} \quad (\text{A-8})$$

where the inequality follows since the two cross-partial derivatives are negative. This completes the proof. ■

**Proof of Proposition 1:** (i) Since  $c_L < y_L^* < y_R^* < c_R$ , the equilibrium platforms are given by the solution to equations (13) and (14). Differentiating these equations totally, the comparative statics matrix is given by

$$J(y_L^*, y_R^*) \times \begin{vmatrix} \partial y_L^* \\ \partial y_R^* \end{vmatrix} = \begin{vmatrix} -\frac{g(\hat{y}^*)}{2} \\ 0 \end{vmatrix} \times \partial a_L, \quad (\text{A-9})$$

where  $\hat{y}^* \equiv (y_L^* + y_R^*)/2$  and  $J(y_L^*, y_R^*)$  is the Jacobian matrix defined by equation (A-7) above, evaluated at the equilibrium platforms. Using Cramer's rule, and recalling from (A-8) that  $|J(y_L^*, y_R^*)| > 0$ , yields:

$$\frac{\partial y_L^*}{\partial a_L} = - \frac{g(\hat{y}^*) \frac{\partial \pi_R^2(y_L^*, y_R^*)}{\partial y_R^2}}{2 |J(y_L^*, y_R^*)|} > 0. \quad (\text{A-10})$$

and

$$\frac{\partial y_R^*}{\partial a_L} = \frac{g(\hat{y}^*) \frac{\partial^2 \pi_R(y_L^*, y_R^*)}{\partial y_R \partial y_L}}{2 |J(y_L^*, y_R^*)|} < 0. \quad (\text{A-11})$$

Finally, equations (A-4) and (A-6) reveal that  $\partial \pi_R^2(y_L^*, y_R^*) / \partial y_R^2 < \partial \pi_R^2(y_L^*, y_R^*) / \partial y_R \partial y_L$ . Thus it is clear from equations (A-10) and (A-11) that the increase in  $y_L^*$  outweighs the decrease in  $y_R^*$ . Hence the probability that the left-wing party wins the elections increases.

(ii) The proof of this part is analogous to the proof of part (i) and hence is omitted. ■

**Proof of Proposition 2:** Adding equations (13) and (14), recalling that  $d_L^* \equiv \hat{c}_m - y_L^*$ ,  $a_L \equiv h_L - (\hat{c}_m - c_L)$ ,  $d_R^* \equiv y_R^* - \hat{c}_m$ , and  $a_R \equiv h_R - (c_R - \hat{c}_m)$ , and rearranging terms yields,

$$\frac{g(\hat{y}^*)}{2} [(a_L - a_R) + (d_L^* - d_R^*)] = 2G(\hat{y}^*) - 1. \quad (\text{A-12})$$

Now let  $a_L > a_R$  and assume by way of negation that  $d_L^* > d_R^*$ . Then the left side of (A-12) is positive, while the right side is negative because  $d_L^* > d_R^*$  implies  $y_L^* + y_R^* < 2\hat{c}_m$ , so  $G(\hat{y}^*) < G(\hat{c}_m) = 1/2$ , a contradiction. Hence,  $a_L > a_R$  implies  $d_L^* < d_R^*$ .

Similarly, let  $a_L < a_R$  and suppose by way of negation that  $d_L^* < d_R^*$ . Then the left side of (A-12) is negative, while the right side is positive because  $d_L^* < d_R^*$  implies  $y_L^* + y_R^* > 2\hat{c}_m$ , so  $G(\hat{y}^*) > G(\hat{c}_m) = 1/2$ , a contradiction. Hence,  $a_L < a_R$  implies  $d_L^* > d_R^*$ .

Finally, when  $a_L = a_R$ , equation (A-12) can hold only if  $d_L^* = d_R^*$ , in which case  $y_L^* + y_R^* = 2\hat{c}_m$ , so  $G(\hat{y}^*) = G(\hat{c}_m) = 1/2$ . ■

**Proof of Proposition 4:** Using the definitions of  $\Delta$  and  $\Sigma$  it is possible to rewrite equations (13)



and (14) which determine the equilibrium platforms as follows:

$$\frac{g(\hat{y})}{2} \left[ \frac{\Sigma + \Delta}{2} + \hat{c}_m - y_L \right] - G(\hat{y}) = 0, \quad (\text{A-13})$$

and

$$-\frac{g(\hat{y})}{2} \left[ \frac{\Sigma - \Delta}{2} + y_R - \hat{c}_m \right] + (1 - G(\hat{y})) = 0. \quad (\text{A-14})$$

The two comparative statics matrices that correspond to this pair of equations are given by

$$J(y_L^*, y_R^*) \times \begin{vmatrix} \partial y_L^* \\ \partial y_R^* \end{vmatrix} = \begin{vmatrix} -\frac{g(\hat{y}^*)}{4} \\ -\frac{g(\hat{y}^*)}{4} \end{vmatrix} \times \partial \Delta, \quad (\text{A-15})$$

and

$$J(y_L^*, y_R^*) \times \begin{vmatrix} \partial y_L^* \\ \partial y_R^* \end{vmatrix} = \begin{vmatrix} -\frac{g(\hat{y}^*)}{4} \\ \frac{g(\hat{y}^*)}{4} \end{vmatrix} \times \partial \Sigma. \quad (\text{A-16})$$

Using Cramer's rule, and recalling from (A-8) that  $|J(y_L^*, y_R^*)| > 0$ , yields the comparative static results regarding  $y_L^*$  and  $y_R^*$ . Using these expressions we establish the results regarding the impact of  $\Delta$  and  $\Sigma$  on  $G(\hat{y}^*)$ . ■

**Proof of Lemma 3:** The discussion in the text implies that the equilibrium platforms converge fully when  $\Sigma \geq \Sigma^{\text{UB}}(\Delta)$ , and converge only partially when  $\Sigma < \Sigma^{\text{UB}}(\Delta)$ . Lemma 2 rules out the possibility that there is no convergence at all. To prove the properties of  $\Sigma^{\text{UB}}(\Delta)$ , note that since  $g(\hat{y}^*)$  is differentiable by assumption, it is also continuous. Clearly then,  $\Sigma^{\text{UB}}(\Delta)$  is also a continuous function of  $\hat{y}^*$ . But so long as  $y_L^* < y_R^*$  (recall that  $\Sigma^{\text{UB}}(\Delta)$  is defined for  $y_L^* \rightarrow y_R^*$ ), it follows from Proposition 4 that  $\hat{y}^*$  is a differentiable and hence a continuous function

of  $\Delta$ . Hence,  $\Sigma^{UB}(\Delta)$  is continuous in  $\Delta$ . Using Proposition 4 again,  $d\Sigma^{UB}(\Delta)/d\Delta = -g'(\hat{y}^*)/4|J(y_L^*, y_R^*)|$ . When  $g(\cdot)$  is symmetric and unimodal,  $g'(\hat{y}^*) \geq 0$  as  $G(\hat{y}^*) \leq 1/2$ ; since Proposition 2 implies that  $G(\hat{y}^*) \geq 1/2$  as  $\Delta \geq 0$ , it follows that  $d\Sigma^{UB}(\Delta)/d\Delta \geq 0$  as  $\Delta \geq 0$ . ■

**Proof of proposition 7:** (i) Suppose that  $\Delta > 0$  and fix the value of  $\Sigma$  at some level below  $\Sigma^{UB}(\Delta)$ . Then, Proposition 2 implies that  $d_L^* < d_R^*$ . But, since  $d_L^* \equiv \hat{c}_m - y_L^*$ ,  $d_R^* \equiv y_R^* - \hat{c}_m$ , and  $y_L^* < y_R^*$ , it follows that either  $\hat{c}_m < y_L^* < y_R^*$ , or  $y_L^* < \hat{c}_m < y_R^*$  (that is, we cannot have  $y_L^* < y_R^* < \hat{c}_m$  because then  $d_L^* > d_R^*$ ). If  $\hat{c}_m < y_L^* < y_R^*$ , then since  $\partial y_L^*/\partial \Sigma = 1/2 > 0$ , increasing  $\Sigma$  towards  $\Sigma^{UB}(\Delta)$  shifts  $y_L^*$  to the right, so as  $\Sigma$  approaches  $\Sigma^{UB}(\Delta)$ , we will have  $\hat{c}_m < y_L^* < y_R^*$ . Since  $Ex_{RD}$  is a weighted average of  $y_L^*$  and  $y_R^*$ , it is clear that  $Ex_{RD} > \hat{c}_m$ . If  $y_L^* < \hat{c}_m < y_R^*$ , then Proposition 4 implies that as  $\Sigma$  increases,  $y_L^*$  and  $y_R^*$  shift towards one another at the same rate; hence they will converge at  $(y_L^* + y_R^*)/2 = \hat{y}^*$ , which is halfway between their original positions. But since  $\Delta > 0$ ,  $\hat{y}^* > \hat{c}_m$  (see Figure 2), so again  $Ex_{RD} > \hat{c}_m$ . When  $\Delta < 0$ , the arguments are exactly the same but in the opposite direction so  $Ex_{RD} < \hat{c}_m$ .

(ii) To develop the necessary and sufficient condition for the existence of  $\Sigma^{NB}(\Delta)$ , we first prove the following Lemma:

**Lemma 4:** For each  $\Delta$  there exists a unique value of  $\Sigma$ , denoted  $\Sigma^{NB}(\Delta)$ , where  $\Sigma^{NB}(\Delta) < \Sigma^{UB}(\Delta)$ , for which  $Ex_{RD} = Ex_{DD} = \hat{c}_m$ , provided that either

- (i)  $\Delta > 0$  and given  $\Delta$ ,  $2G(\hat{y}^*)(c_1 - \hat{y}^*) > (c_1 - \hat{c}_m)$ , or
- (ii)  $\Delta < 0$  and given  $\Delta$ ,  $2(1 - G(\hat{y}^*))(\hat{y}^* - c_0) > (\hat{c}_m - c_0)$ .

**Proof of Lemma 4:** Recall from Proposition 5 that for all  $\Sigma < \Sigma^{UB}(\Delta)$ ,  $\partial Ex_{RD}/\partial \Sigma > 0$  if  $\Delta > 0$  and  $\partial Ex_{RD}/\partial \Sigma < 0$  if  $\Delta < 0$ . Since part (i) of Proposition 7 implies that at  $\Sigma^{UB}(\Delta)$ ,  $Ex_{RD} > 0$  if  $\Delta > 0$  and  $Ex_{RD} < 0$  if  $\Delta < 0$ , it is clear that we only need to find whether  $Ex_{RD}$  is larger or smaller than  $\hat{c}_m$  (which is equal to  $Ex_{DD}$ ) at the lowest feasible  $\Sigma$ . Since  $y_L^*$  shifts to the left as  $\Sigma$  decreases and  $y_R^*$  shifts to the right, both at equal rates,  $\Sigma$  can decrease until either  $y_R^*$  approaches  $c_1$  or  $y_L^*$  approaches  $c_0$  (note that since  $\Sigma \equiv h_L + h_R + c_L - c_R$ , we can continue to lower

$\Sigma$  until  $c_R$  approaches  $c_1$  and  $c_L$  approaches  $c_0$ . Now there are two case to consider.

First, if  $\Delta > 0$ , then Proposition 2 implies that  $d_L^* < d_R^*$ , so  $y_R^*$  is closer to  $c_1$  than  $y_L^*$  is to  $c_0$ . Hence  $\Sigma$  can decrease until  $y_R^*$  approaches  $c_1$ . Since  $y_L^*$  and  $y_R^*$  move away from  $\hat{y}^*$  at equal rates, it follows that as  $y_R^*$  approaches  $c_1$ ,  $y_L^*$  approaches  $2\hat{y}^* - c_1$ . Hence, at the lower bound on  $\Sigma$ ,  $Ex_{RD} = G(\hat{y}^*)(2\hat{y}^* - c_1) + (1 - G(\hat{y}^*))c_1 = c_1 - 2G(\hat{y}^*)(c_1 - \hat{y}^*)$ . If condition (i) in the lemma holds,  $c_1 - 2G(\hat{y}^*)(c_1 - \hat{y}^*) < \hat{c}_m$ , so at the lower bound on  $\Sigma$ ,  $Ex_{RD} < \hat{c}_m = Ex_{DD}$ . This prove the existence of  $\Sigma^{NB}(\Delta)$ , where  $\Sigma^{NB}(\Delta) < \Sigma^{UB}(\Delta)$ , at which  $Ex_{RD} = Ex_{DD} = \hat{c}_m$ .

Second, if  $\Delta < 0$ , then Proposition 2 implies that  $d_L^* > d_R^*$ , so  $y_L^*$  is closer to  $c_0$  than  $y_R^*$  is to  $c_1$ . Hence,  $\Sigma$  can decrease until  $y_L^*$  approaches  $c_0$ , at which point,  $y_R^*$  approaches  $2\hat{y}^* - c_0$ . Consequently,  $Ex_{RD} = c_0 + 2(1 - G(\hat{y}^*))(\hat{y}^* - c_0)$ . If condition (ii) in the lemma holds,  $c_0 + 2(1 - G(\hat{y}^*))(\hat{y}^* - c_0) > \hat{c}_m$ , so at the lower bound on  $\Sigma$ ,  $Ex_{RD} > \hat{c}_m$ . This prove the existence of  $\Sigma^{NB}(\Delta)$ , where  $\Sigma^{NB}(\Delta) < \Sigma^{UB}(\Delta)$ , at which  $Ex_{RD} = Ex_{DD} = \hat{c}_m$ . ■

Next, we prove that if  $|\Delta|$  is not too large, then  $g(\hat{c}_m) > 1/(c_1 - c_0)$  is necessary and sufficient for conditions (i) and (ii) in Lemma 4 to hold. To this end, note that since at  $\Delta = 0$ ,  $\hat{y}^* = \hat{c}_m$  and  $G(\hat{y}^*) = 1/2$ , then  $G(\hat{y}^*)(c_1 - \hat{y}^*) = (c_1 - \hat{c}_m)/2$ . Evaluated at  $\Delta = 0$ , the derivative of the left side of this equality with respect to  $\Delta$  is equal to  $(g(\hat{c}_m)/2)[2(c_1 - \hat{c}_m) - 1/g(\hat{c}_m)](d\hat{y}^*/d\Delta)$ , where  $d\hat{y}^*/d\Delta > 0$  by Proposition 4. Since the distribution of  $c_m$  is symmetric,  $2(c_1 - \hat{c}_m) = c_1 - c_0$ , so the derivative is positive if and only if  $g(\hat{c}_m) > 1/(c_1 - c_0)$ . When this inequality holds, condition (i) in Lemma 4 holds for small but positive values of  $\Delta$ .

Similarly, note that at  $\Delta = 0$ ,  $\hat{y}^* = \hat{c}_m$  and  $G(\hat{y}^*) = 1/2$ , so  $(1 - G(\hat{y}^*))(\hat{y}^* - c_0) = (\hat{c}_m - c_0)/2$ . Evaluated at  $\Delta = 0$ , the derivative of the left side of the equality with respect to  $\Delta$  is  $(g(\hat{c}_m)/2)[g(\hat{c}_m) - 2(\hat{c}_m^* - c_0)](d\hat{y}^*/d\Delta)$ . Since the distribution of  $c_m$  is symmetric,  $2(\hat{c}_m - c_0) = (c_1 - c_0)$ , so the derivative is negative if and only if  $g(\hat{c}_m) > 1/(c_1 - c_0)$ . When this condition holds, condition (ii) in Lemma 4 holds for small negative values of  $\Delta$ .

Before we turn to the properties of  $\Sigma^{NB}(\Delta)$ , we first prove the two following Lemmas:

**Lemma 5:** *Let  $\hat{y}^*(\Delta)$  be the value of  $\hat{y}^*$  given  $\Delta$  and define  $G(\Delta) \equiv G(\hat{y}^*(\Delta))$  and  $g(\Delta) \equiv g(\hat{y}^*(\Delta))$ . If  $g(\cdot)$  is symmetric and unimodal, then  $G(\Delta) = 1 - G(-\Delta)$  and  $g(\Delta) = g(-\Delta)$ .*

**Proof of Lemma 5:** Adding equations (A-13) and (A-14) and rearranging, yields

$$\hat{y}^*(\Delta) - \hat{c}_m = \frac{2}{g(\Delta)} \left[ \frac{1}{2} - G(\Delta) \right] + \frac{\Delta}{2}. \quad (\text{A-17})$$

Evaluating the same equation at  $-\Delta$  and adding to (A-17) yields:

$$[\hat{y}^*(\Delta) - \hat{c}_m] - [\hat{c}_m - \hat{y}^*(-\Delta)] = \frac{2}{g(\Delta)} \left[ \frac{1}{2} - G(\Delta) \right] + \frac{2}{g(-\Delta)} \left[ \frac{1}{2} - G(-\Delta) \right]. \quad (\text{A-18})$$

Now, assume by way of negation that the left side of equation (A-18) is positive. This implies that  $\hat{y}^*(\Delta)$  is further away from the center than  $\hat{y}^*(-\Delta)$ . Since  $g(\cdot)$  is symmetric and unimodal, this implies that  $G(\Delta) > 1-G(-\Delta)$  and  $g(\Delta) \leq g(-\Delta)$ . Hence,

$$\begin{aligned} \frac{2}{g(\Delta)} \left[ \frac{1}{2} - G(\Delta) \right] - \frac{2}{g(-\Delta)} \left[ \frac{1}{2} - G(-\Delta) \right] < \\ \frac{2}{g(\Delta)} [1 - G(\Delta) - G(-\Delta)] < 0, \end{aligned} \quad (\text{A-19})$$

a contradiction. Similarly we can prove that the left side of equation (A-18) cannot be negative. Hence,  $\hat{y}^*(\Delta) - \hat{c}_m = \hat{c}_m - \hat{y}^*(-\Delta)$ . Given that  $g(\cdot)$  is symmetric and unimodal, this implies in turn that  $G(\Delta) = 1-G(-\Delta)$  and  $g(\Delta) = g(-\Delta)$ . ■

**Lemma 6:** Let  $d_L^*(\Delta)$  be the value of  $d_L^*$  given  $\Delta$  and define  $d_R^*(\Delta)$  similarly. Then,  $d_L^*(\Delta) = d_R^*(-\Delta)$  and  $d_L^*(-\Delta) = d_R^*(\Delta)$ .

**Proof of Lemma 6:** Evaluating equation (A-13) at  $\Delta$ , recalling that  $d_L^*(\Delta) \equiv \hat{c}_m - y_L^*$ , and rearranging, yields

$$\frac{g(\Delta)}{2} \left[ \frac{\Sigma + \Delta}{2} + d_L^*(\Delta) \right] = G(\Delta). \quad (\text{A-20})$$

Similarly, evaluating (A-14) at  $-\Delta$ , recalling that  $d_R^*(\Delta) \equiv y_R^* - \hat{c}_m$ , and rearranging, yields

$$\frac{g(-\Delta)}{2} \left[ \frac{\Sigma + \Delta}{2} + d_R^*(-\Delta) \right] = (1 - G(-\Delta)). \quad (\text{A-21})$$

Subtracting equation (A-21) from (A-20), recalling from Lemma 5 that  $g(\Delta) = g(-\Delta)$ , and rearranging terms yields,

$$\frac{g(\Delta)}{2} [d_L^*(\Delta) - d_R^*(-\Delta)] = 0. \quad (\text{A-22})$$

Since  $g(\Delta) \neq 0$ , it follows that  $d_L^*(\Delta) = d_R^*(-\Delta)$ . The proof that  $d_L^*(-\Delta) = d_R^*(\Delta)$  is completely analogous. ■

We are now ready to prove the properties of  $\Sigma^{NB}(\Delta)$ :

□ **Symmetry:** To prove symmetry we need to show that  $\Sigma^{NB}(\Delta) = \Sigma^{NB}(-\Delta)$ . To this end, let  $Ex_{RD}(\Delta, \Sigma)$  be the expected policy under RD given  $\Delta$  and  $\Sigma$ , and recall that  $\Sigma^{NB}(\Delta)$  is defined by  $Ex_{RD}(\Delta, \Sigma^{NB}(\Delta)) = \hat{c}_m$ . Hence, we can prove that  $\Sigma^{NB}(\Delta)$  is symmetric by showing that  $Ex_{RD}(\Delta, \Sigma^{NB}(\Delta)) = Ex_{RD}(-\Delta, \Sigma^{NB}(-\Delta))$ . Using equation (16),

$$Ex_{RD}(\Delta, \Sigma^{NB}(\Delta)) = G(\Delta)y_L^*(\Delta) + (1 - G(\Delta))y_R^*(\Delta). \quad (\text{A-23})$$

Recalling from Lemma 5 that  $G(\Delta) = 1 - G(-\Delta)$ , this equation becomes,

$$Ex_{RD}(\Delta, \Sigma^{NB}(\Delta)) = (1 - G(-\Delta))y_L^*(\Delta) + G(-\Delta)y_R^*(\Delta). \quad (\text{A-24})$$

By Lemma 6,  $d_L^*(\Delta) = d_R^*(-\Delta)$ . Using the definitions of  $d_L^*(\cdot)$  and  $d_R^*(\cdot)$ , it follows that  $y_L^*(\Delta) = 2\hat{c}_m - y_R^*(-\Delta)$  and  $y_R^*(\Delta) = 2\hat{c}_m - y_L^*(-\Delta)$ . Substituting in equation (A-24) we get,

$$\begin{aligned} Ex_{RD}(\Delta, \Sigma^{NB}(\Delta)) &= 2\hat{c}_m - [(G(-\Delta))y_L^*(-\Delta) + (1 - G(-\Delta))y_R^*(-\Delta)] \\ &= 2\hat{c}_m - Ex_{RD}(-\Delta, \Sigma^{NB}(-\Delta)). \end{aligned} \quad (\text{A-25})$$

Hence,

$$\hat{c}_m - Ex_{RD}(\Delta, \Sigma^{NB}(\Delta)) = Ex_{RD}(-\Delta, \Sigma^{NB}(-\Delta)) - \hat{c}_m. \quad (A-26)$$

The left side of the equation vanishes by definition, so the right side must also vanish implying that  $Ex_{RD}(\Delta, \Sigma^{NB}(\Delta)) = Ex_{RD}(-\Delta, \Sigma^{NB}(-\Delta)) = \hat{c}_m$ .

□  **$\Sigma^{NB}(\Delta)$  is U-shaped:** Recall that  $\Sigma^{NB}(\Delta)$  is defined implicitly by  $Ex_{RD}(\Delta, \Sigma) = \hat{c}_m$ . Since  $\Sigma^{NB}(\Delta)$  is symmetric around 0, then if  $\Sigma^{NB}(\Delta)$  exists, there are exactly two values of  $\Delta$  that solve this equation, one positive and one negative. Now, consider the positive solutions and differentiate the equation fully to obtain:

$$\frac{d\Delta}{d\Sigma} = - \frac{\frac{\partial Ex_{RD}}{\partial \Sigma}}{\frac{\partial Ex_{RD}}{\partial \Delta}} = \frac{8(1/2 - G(\hat{y}^*)) |J(y_L^*, y_R^*)|}{-g^3(\hat{y}^*)(y_R^* - y_L^*) + g^2(\hat{y}^*) + 2g'(\hat{y}^*)(G(\hat{y}^*) - 1/2)}. \quad (A-27)$$

Differentiating this expression again with respect to  $\Sigma$  and using part (i) of Proposition 4 yields

$$\frac{d^2\Delta}{d\Sigma^2} = \frac{8g^3(\hat{y}^*)(1/2 - G(\hat{y}^*)) |J(y_L^*, y_R^*)|}{(-g^3(\hat{y}^*)(y_R^* - y_L^*) + g^2(\hat{y}^*) + 2g'(\hat{y}^*)(G(\hat{y}^*) - 1/2))^2}. \quad (A-28)$$

Since we consider positive values of  $\Delta$ ,  $G(\hat{y}^*) > 1/2$ , so (A-27) and (A-28) imply that along the  $\Sigma^{NB}(\Delta)$  curve,  $\Delta$  is increasing with  $\Sigma$  at a decreasing rate. When we take negative solutions of  $\Delta$ , then  $G(\hat{y}^*) < 1/2$ , so (A-27) and (A-28) imply that along the  $\Sigma^{NB}(\Delta)$  curve,  $\Delta$  is decreasing with  $\Sigma$  at a decreasing rate. Together, these properties imply that  $\Sigma^{NB}(\Delta)$  is a U-shaped function of  $\Delta$  that attains a unique minimum at  $\Delta = 0$  (the proof that  $\Sigma^{NB}(\Delta)$  is differentiable at  $\Delta = 0$  appears directly below).

□  **$\Sigma^{NB}(\Delta)$  is smooth:** To prove smoothness, we need to show that the slope of  $\Sigma^{NB}(\Delta)$  is equal to 0 when  $\Delta = 0$ , or alternatively, that the derivative in (A-27) goes to infinity as  $\Delta$  goes to 0. To this end, note from Proposition 2 that the numerator of equation (A-27) vanishes as  $\Delta$  goes to 0, and assume by way of negation that the denominator of (A-27) does not vanish as  $\Delta$  goes to 0. Given this assumption, the derivative in (A-27) vanishes at  $\Delta = 0$ . But since both

the numerator and the denominator of (A-27) are continuous functions and since by assumption the denominator does not vanish, the derivative in (A-27) is also a continuous function. Together with the fact that  $\Sigma^{NB}(\Delta)$  is symmetric this means that the derivative in (A-27) must go to infinity as  $\Delta$  goes to 0, a contradiction. Thus, the numerator of equation (A-27) must vanish at  $\Delta = 0$ . Since both the numerator and denominator of (A-27) vanish at  $\Delta = 0$ , we need to apply L'Hôpital's rule to determine the limit of this derivative as  $\Delta$  goes to 0. Recalling that  $\dot{y}^* = \hat{c}_m$  when  $\Delta = 0$ , and using the assumption that  $g(\cdot)$  is symmetric and unimodal so that  $G(\hat{c}_m) = 1/2$  and  $g^*(\hat{c}_m) = 0$ , we obtain:

$$\lim_{\Delta \rightarrow 0} \left( \frac{d\Sigma^{NB}(\Delta)}{d\Delta} \right) = \frac{6g^2(\hat{c}_m)}{0} = \infty. \quad (\text{A-29})$$

This implies in turn that  $d\Sigma^{NB}(\Delta)/d\Delta = 0$  as  $\Delta$  goes to 0, so  $\Sigma^{NB}(\Delta)$  is smooth.

□  $\Sigma^{NB}(0) = 1/g(\hat{c}_m)$ : Recall that  $\Sigma^{NB}(\Delta)$  is defined implicitly by  $\text{Ex}_{RD}(\Delta, \Sigma^{NB}(\Delta)) = \hat{c}_m$ . Differentiating the identity with respect to  $\Delta$ , evaluating at  $\Delta = 0$ , and using the fact that since  $g(\cdot)$  is symmetric and unimodal,  $G(\hat{c}_m) = 1/2$  and  $g'(\hat{c}_m) = 0$ , we obtain:

$$\frac{d\Sigma^{NB}(0)}{d\Delta} = \frac{-g^3(\hat{c}_m)(y_R^* - y_L^*) + g^2(\hat{c}_m)}{6g^2(\hat{c}_m)} = \frac{1 - g(\hat{c}_m)(y_R^* - y_L^*)}{6}. \quad (\text{A-30})$$

Substituting from equation (19) for  $y_R^* - y_L^*$ , this expression becomes,

$$\frac{d\Sigma^{NB}(0)}{d\Delta} = \frac{g(\hat{c}_m)}{6} \left[ \Sigma(0) - \frac{1}{g(\hat{c}_m)} \right]. \quad (\text{A-31})$$

Recalling that  $d\Sigma^{NB}(0)/d\Delta = 0$ , we get  $\Sigma^{NB}(0) = 1/g(\hat{c}_m)$ . ■

**Proof of Proposition 9:** Differentiating the last line of equation (30) with respect to  $\Sigma$  and using Proposition 4, we get

$$\begin{aligned}
\frac{\partial V_{RD}}{\partial \Sigma} &= 2(y_R^* - y_L^*) \frac{\partial (y_R^* - y_L^*)}{\partial \Sigma} G(\hat{y}^*) (1 - G(\hat{y}^*)) \\
&\quad + (y_R^* - y_L^*)^2 (1 - 2G(\hat{y}^*)) \frac{\partial G(\hat{y}^*)}{\partial \Sigma} \\
&= -2(y_R^* - y_L^*) G(\hat{y}^*) (1 - G(\hat{y}^*)) < 0.
\end{aligned}
\tag{A-32}$$

As for the comparison between electoral uncertainty under DD and under RD, the proof in the full convergence case is obvious. The proof in the other extreme case where  $\Sigma$  is sufficiently small (and  $\Delta$  is such that  $y_L^*$  and  $y_R^*$  approach the boundaries of the support of  $c_m$ ), relies on the fact that  $V_{DD}$  assigns positive weights to all values in the support of  $c_m$  while  $V_{RD}$  assigns a positive weight only to the two boundary points. ■



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Figure 1: The sequence of events under Representative Democracy (RD)

The leaders of the two parties simultaneously choose the platforms  $y_L$  and  $y_R$

Elections are held

The leader of the winning party observes the realization of  $\tilde{\gamma}$

The leader of the winning party chooses policy  $x$

The policy  $x$  and the realization of  $\tilde{\gamma}$  determine the utility level of individual voters; each voter compares his utility level to the one promised during the campaign



Figure 2: The probabilities that each party wins the elections when  $y_L < y_R$ .

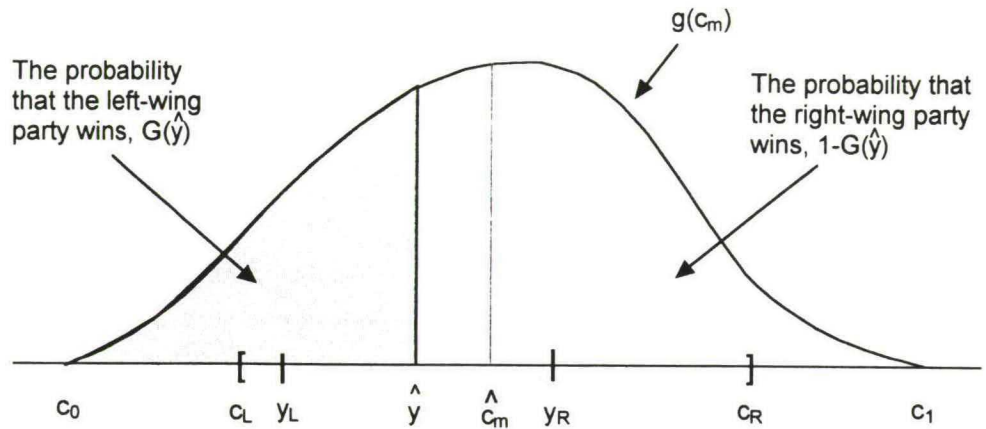
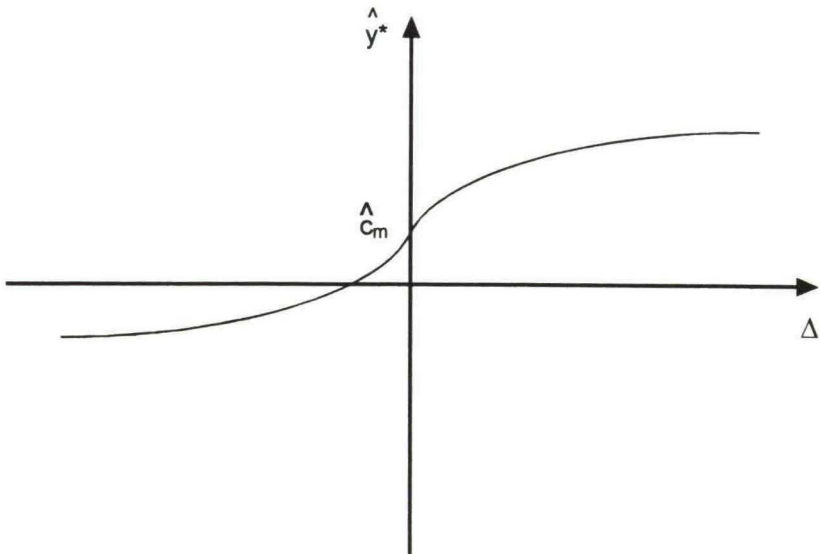


Figure 3:  $\hat{y}^*$  as a function of  $\Delta$



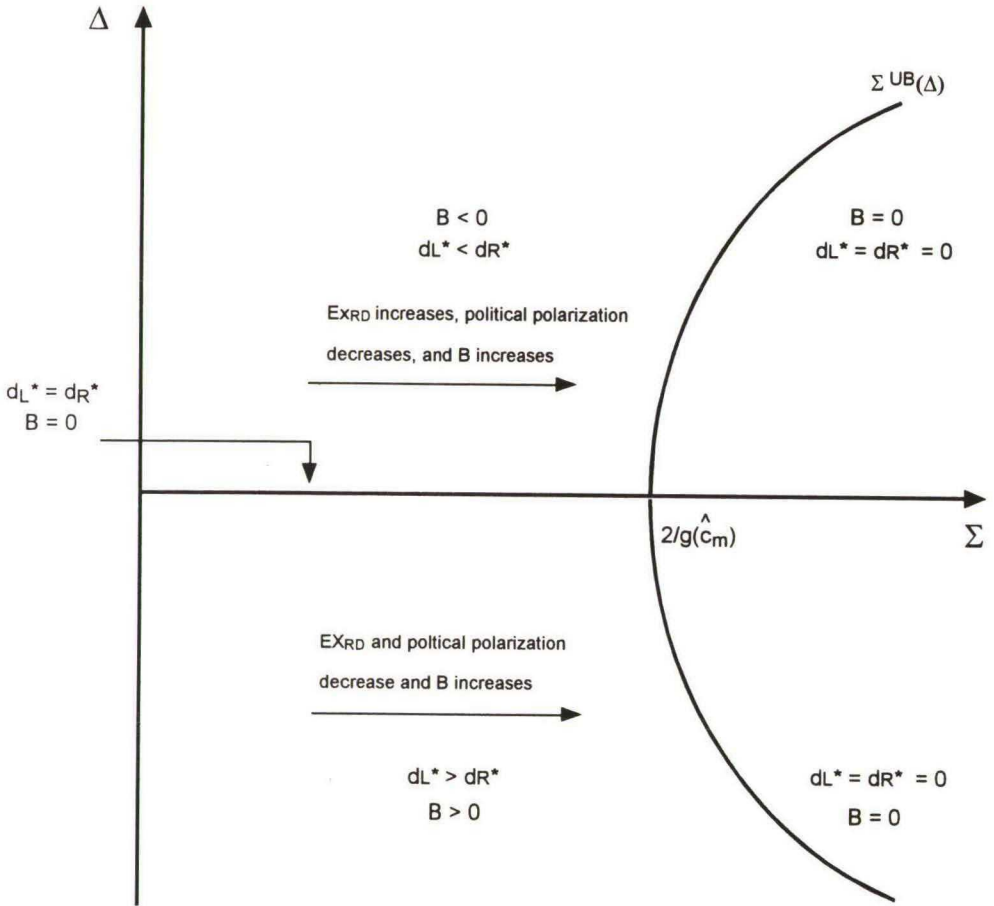


Figure 4a: Comparing  $EX_{RD}$  and  $EX_{DD}$  when  $g(\hat{C}_m) < 1/(C_1 - C_0)$

$B = EX_{DD} - EX_{RD}$  is the policy bias. There is a right-wing bias when  $B > 0$ , a left-wing bias when  $B < 0$ , and no bias when  $B = 0$ . The right-wing party is a favorite to win the elections when  $dR^* < dL^*$ , the left-wing party is a favorite to win when  $dR^* > dL^*$ , and when  $dR^* = dL^*$ , the political race is tied. When  $dR^* = dL^* = 0$ , the equilibrium platforms fully converge on the center of the political spectrum.

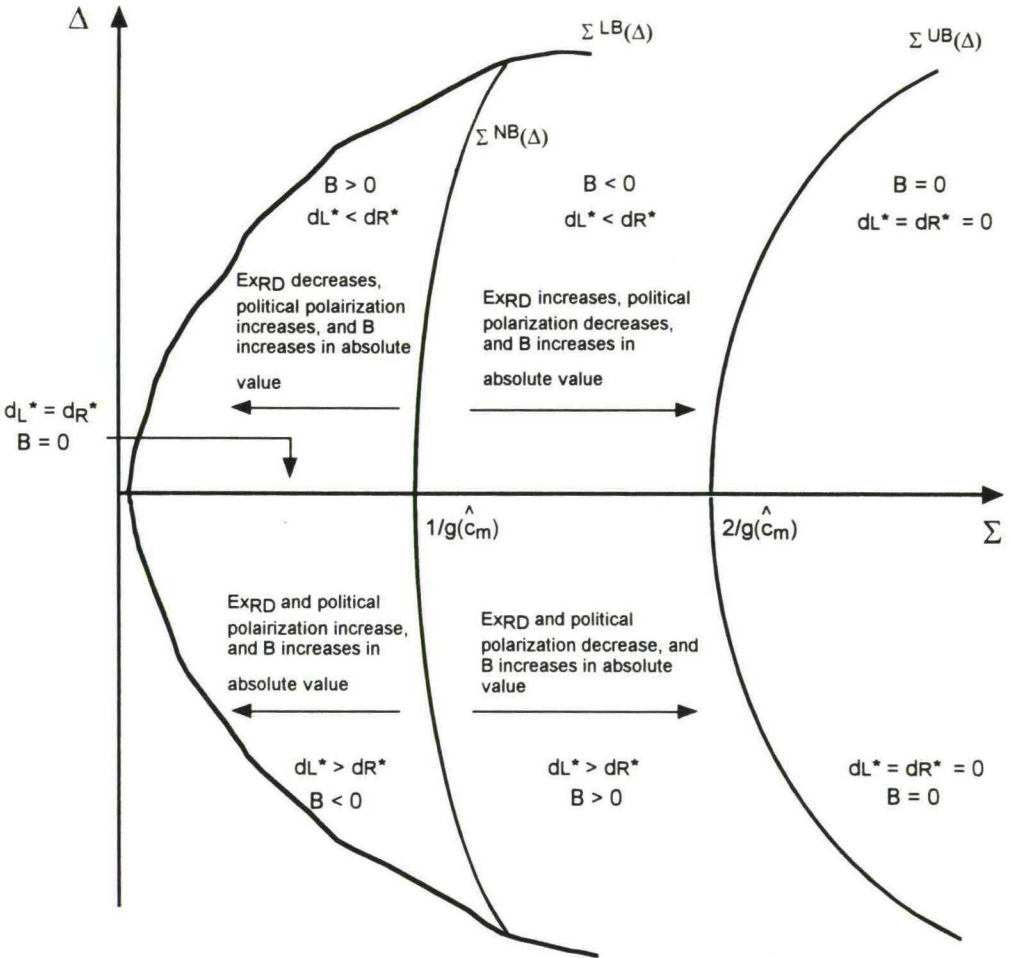


Figure 4b: Comparing  $Ex_{RD}$  and  $Ex_{DD}$  when  $g(\hat{c}_m) > 1/(c_1 - c_0)$

$B = Ex_{DD} - Ex_{RD}$  is the policy bias. There is a right-wing bias when  $B > 0$ , a left-wing bias when  $B < 0$ , and no bias when  $B = 0$ . The right-wing party is a favorite to win the elections when  $d_R^* < d_L^*$ , the left-wing party is a favorite to win when  $d_R^* > d_L^*$ , and when  $d_R^* = d_L^*$ , the political race is tied. When  $d_R^* = d_L^* = 0$ , the equilibrium platforms fully converge on the center of the political spectrum.

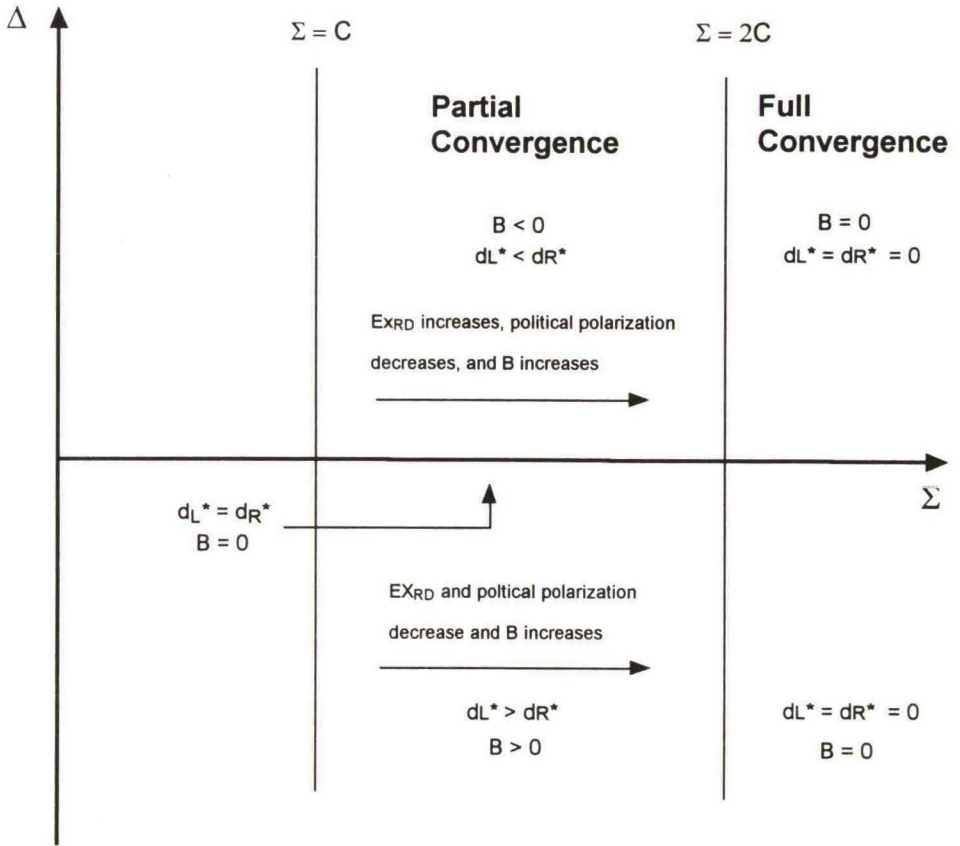


Figure 5: Comparing  $EX_{RD}$  and  $EX_{DD}$  when the distribution of median voter types is uniform on the interval  $[-C/2, C/2]$

$B = EX_{DD} - EX_{RD}$  is the policy bias. There is a right-wing bias when  $B > 0$ , a left-wing bias when  $B < 0$ , and no bias when  $B = 0$ . The right-wing party is a favorite to win the elections when  $d_R^* < d_L^*$ , the left-wing party is a favorite to win when  $d_R^* > d_L^*$ , and when  $d_R^* = d_L^*$ , the political race is tied. When  $d_R^* = d_L^* = 0$ , the equilibrium platforms fully converge on the center of the political spectrum.

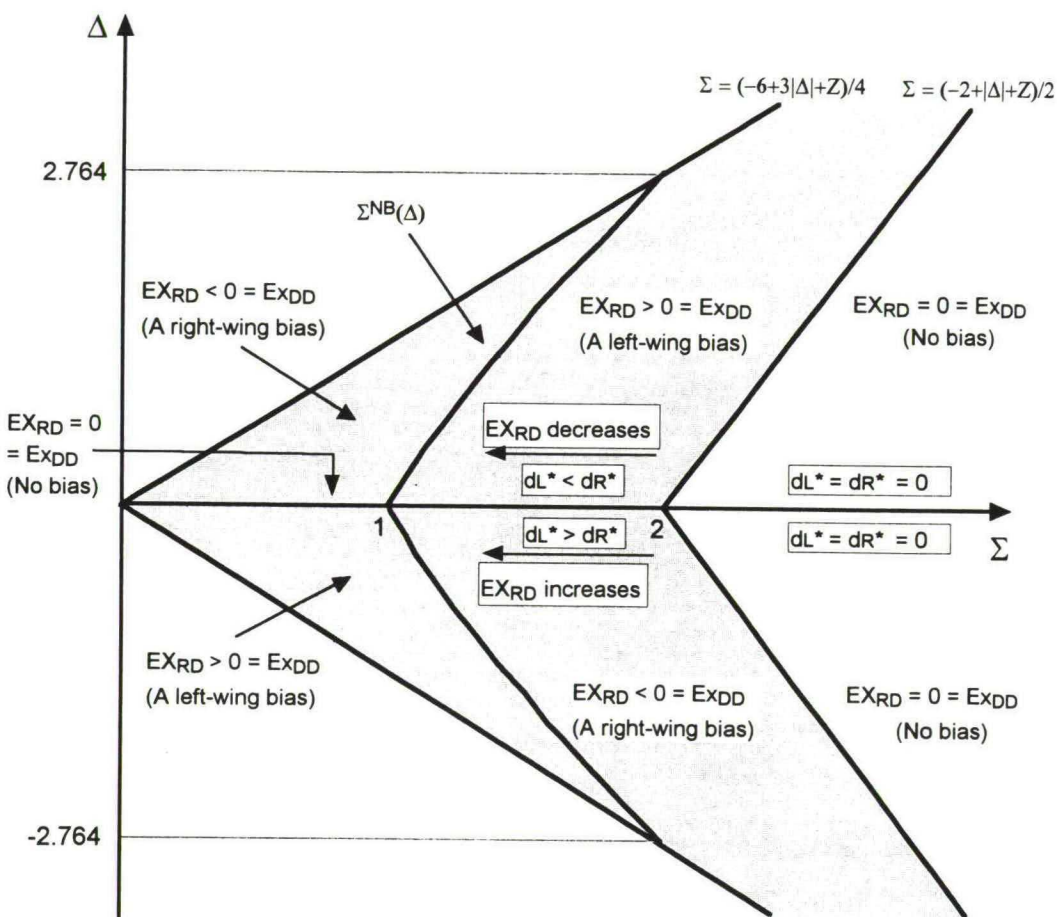


Figure 6: Comparing  $EX_{RD}$  and  $EX_{DD}$  when the distribution of median voter types is triangular on the unit interval



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