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A CONSTRUCTIVE PROOF OF A UNIMODULAR TRANSFORMATION THEOREM FOR SIMPLICES

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A Constructive Proof of a Unimodular Transformation Theorem for Simplices *

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Abstract In a recent paper, the following theorem was proved by the author. Given an arbitrary n-dimensional simplex

$$P = \{ x \in R^n \mid Ax \leq c \}$$

where $A = (a_{ij})$ is an $n+1$ by n integer matrix and $c = (c_1, \cdots, c_{n+1})^T$ is an integer vector in the $(n+1)$-dimensional Euclidean space R^{n+1}, then there exists an $n \times n$ unimodular matrix U such that the matrix $B = (b_{ij}) = AU$ has the following properties:

(1) $b_{ii} > 0$ for all $i = 1, \cdots, n$;

(2) for each $i = 1, \cdots, n$, it holds that $b_{ij} \leq 0$ and $|b_{ij}| < b_{ii}$ for all $j \neq i$;

(3) $b_{(n+1)j} \leq 0$ for $j = 1, \cdots, n$.

The above theorem extends an earlier result of Scarf for $n = 2$. In this note a constructive proof of the theorem is proposed.

Keywords: Simplices, unimodular transformation, combinatorial theorem, integer linear programming.

1 Main results

Let R^n denote the n-dimensional Euclidean space. Z^n (Z^n_+) denotes the set of all (nonnegative) integer vectors in R^n. Moreover, we denote the identity matrix of rank n by I_n. Throughout the paper we consider an n-dimensional simplex

$$P = \{\, x \in R^n \,|\, Ax \leq c \,\},$$

where $A = (a_{ij})$ is an $(n+1) \times n$ integer matrix, and $c = (c_1, ..., c_{n+1})^\top$ is an integer vector of R^{n+1}.

A matrix U is unimodular if the entries of U are integral and the determinant of U is equal to 1 or -1.

Matrices which transform A as described by the following elementary column operations are unimodular:

(i) Interchange two columns.

(ii) Reverse the sign of a column.

(iii) Add an integral multiple of one column to another.

In [4], the following basic theorem was proved. This theorem extends an earlier result of Scarf [2] for $n = 2$ and has an important application in integer linear programming and economies with indivisibilities (see Scarf [2] and Yang [4]). A weaker version of an equivalent form of this theorem can be found in White [3][†]. Here a constructive proof of the following theorem is given.

[†] Formally White proves by induction that for any given n-dimensional simplex

$$P = \{\, x \in R^n \,|\, Ax \leq c \,\},$$

there exists an $n \times n$ unimodular matrix U such that the matrix $B = AU = (b_{ij})$ satisfies

(1) $b_{1i} \leq 0$ for all $i = 1, \cdots, n$;

(2) $b_{(i+1)i} > 0$ for all $i = 1, \cdots, n$;

Theorem 1.1

For any given n-dimensional simplex

$$P = \{ x \in R^n \mid Ax \leq c \},$$

there exists an $n \times n$ *unimodular matrix* U *such that the matrix* $B = AU = (b_{ij})$ *satisfies*

(1) $b_{ii} > 0$ *for all* $i = 1, \cdots, n$;

(2) *for each* $i = 1, \cdots, n$, *it holds* $b_{ij} \leq 0$ *and* $|b_{ij}| < b_{ii}$ *for all* $j \neq i$;

(3) $b_{(n+1)j} \leq 0$ *for all* $j = 1, \cdots, n$.

Proof:

Let a_i denote the i-th row of the matrix A for $i = 1, \cdots, n+1$. Notice that the origin of R^n is in the interior of the convex hull of the vectors $a_1,\ldots,a_{n+1}$. It implies that there are $n+1$ strictly positive convex combination coefficients $\lambda_1,\ldots,\lambda_{n+1}$ such that

$$\sum_{i=1}^{n+1} \lambda_i a_i = 0. \tag{1.1}$$

Moreover, the corresponding convex combination coefficients remain unchanged under any unimodular transformation of A. The subsequent proof consists of four phases.

Phase I:

We first implement the procedure I for A:

Step (0) Set $i = n + 1$.

Step (1) Set $j = i - 1$. If $a_{ij} = 0$, choose $a_{il} \neq 0$ for some $1 \leq l \leq j - 1$ and switch column j with column l. Set $k = j - 1$.

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(3) for each $i = 1, \cdots, n$, it holds $b_{(i+1)j} \leq 0$ for all $j \neq i$.

Step (2) If $a_{ik} = 0$, do nothing. Otherwise use the Euclidean algorithm to find the greatest common divisor of a_{ij} and a_{ik}, denoted by $r = g.c.d(a_{ij}, a_{ik})$, and p, q relatively prime such that $pa_{ij} + qa_{ik} = r$. Set $A' = AD$, where D is the indentity matrix I_n in all but columns k and j. In column k, we have $d_{jk} = -a_{ik}/r$, $d_{kk} = a_{ij}/r$, and $d_{sk} = 0$ otherwise. In column j, we have $d_{jj} = p$, $d_{kj} = q$, and $d_{sj} = 0$ otherwise. If $k > 1$, return to Step (2) with $k = k-1$. Otherwise check whether $a_{ij} \leq 0$. If not, reverse the sign of column j. Set $i = i - 1$ and $A = A'$. Go to Step (3).

Step (3) If $i = 1$, stop. Otherwise go to Step (1).

Let us consider Step (2) where $A' = (a'_{st}) = AD$. Note that

$$det D = pa_{ij}/r + qa_{ik}/r = 1.$$

Hence D is a unimodular matrix. Moreover, for $s = 1, \cdots, n+1$

$$a'_{st} = a_{st}, \; t \neq j, k$$

$$a'_{sj} = pa_{sj} + qa_{sk}$$

and

$$a'_{sk} = -a_{ik}a_{sj}/r + a_{ij}a_{sk}/r.$$

In particular, $a'_{ij} = r$ and $a'_{ik} = 0$. It is clear that all other operations are also unimodular transformations. Therefore, after a finite number of steps, the procedure I brings A into the form:

$$(a_{ij}) = \begin{bmatrix} + & ? & \cdots & ? & ? \\ - & ? & \cdots & ? & ? \\ 0 & - & \cdots & ? & ? \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & - & ? \\ 0 & 0 & \cdots & 0 & - \end{bmatrix} \tag{1.2}$$

where "+" stands for a positive entry, and "−" for a zero or negative entry. Notice that in the above matrix entries "−" can not be zero.

For $k = 2, \cdots, n+1$, we denote the submatrix of A obtained by taking rows 1 through k and columns 1 through $k-1$ of A by A_k, i.e.,

$$A_k = \begin{bmatrix} + & ? & \cdots & ? & ? \\ - & ? & \cdots & ? & ? \\ 0 & - & \cdots & ? & ? \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & - & ? \\ 0 & 0 & \cdots & 0 & - \end{bmatrix} \tag{1.3}$$

being a $k \times (k-1)$ matrix.

Phase II:

Suppose that for some k, $2 \le k \le n$, we have unimodularly transformed A_k into A'_k satisfying (1), (2), and (3). It implies that A_{k+1} is brought into the form:

$$\bar{A}_{k+1} = \begin{bmatrix} + & - & \cdots & - & ? \\ - & + & \cdots & - & ? \\ \vdots & \vdots & & \vdots & \vdots \\ - & - & \cdots & + & ? \\ - & - & \cdots & - & ? \\ 0 & 0 & \cdots & 0 & - \end{bmatrix}$$

of which the first k rows and $k-1$ columns form the matrix A'_k and the last column is the last column of A_{k+1}. Let a'_i denote the i-th row of A'_k for $i = 1, \cdots, k$. Note that the zero row vector of R^{k-1} is in the interior of the convex hull of all rows of A'_k. More precisely, the corresponding convex combination coefficients are $\lambda'_i = \lambda_i / \sum_{h=1}^{k} \lambda_h$ for $i = 1, \cdots, k$. It is known that A'_k is a productive Leontief matrix. Thus there exists a positive integral combination of columns one through $k-1$ of $\bar{A}_{k+1}$ for which the last element is zero, the k-th element is strictly negative and the other elements are strictly positive. These positive integral combination coefficients are solutions of the following system of linear equations:

$$\begin{aligned} a_1'x &\geq 1 \\ &\vdots \\ a_{k-1}'x &\geq 1 \\ x \in Z_+^{k-1}&. \end{aligned} \tag{1.4}$$

A solution of system (1.4) can be found by the basic algorithm in [4] within a finite number of steps from any starting point v in Z_+^{k-1}. More precisely, we adopt the following labeling rule:

To $y \in Z^{k-1}$, we assign y with the label $l(y) = i$ if i is the smallest index for which

$$a_i'y - 1 = \min\{\, a_h'y - 1 \mid a_h'y - 1 < 0,\ h \in \{1, \cdots, k-1\,\}\}.$$

If $a_h'y - 1 \geq 0$ for all $h = 1, \cdots, k-1$, then the label $l(y) = 0$ is assigned to y.

Furthermore, we define

$$q(i) = E(i),\ \ i = 1, \cdots, k-1$$

where $E(i)$ is the i-th unit vector of R^{k-1}.

By subtracting a large positive integral multiple of the above combination from the last column of $\bar{A}_{k+1}$, we can therefore transform A_{k+1} (hence A_{k+1}) into the form:

$$\hat{A}_{k+1} = (\hat{a}_{ij}) = \begin{bmatrix} + & - & \cdots & - & - \\ - & + & \cdots & - & - \\ \vdots & \vdots & & \vdots & \vdots \\ - & - & \cdots & + & - \\ - & - & \cdots & - & + \\ 0 & 0 & \cdots & 0 & - \end{bmatrix}. \tag{1.5}$$

Phase III:

Next we shall give a procedure to transform the matrix $\hat{A}_{k+1}$ to the form satisfying (1), (2) and (3). The procedure is described as follows:

Step (a) If there are indices i and j $(i \neq j)$ for some $1 \leq i, j \leq k$ with $\hat{a}_{ii} \leq |\hat{a}_{ij}|$, we can find a positive integer e and an integer $d \in \{0, 1, ..., \hat{a}_{ii} - 1\}$ such that $|\hat{a}_{ij}| = e\hat{a}_{ii} + d$, where e is the lower integer part of $|\hat{a}_{ij}|/\hat{a}_{ii}$, and then add e multiple of column i to column j.

Step (b) Repeat Step (a) until there are no indices i and j $(i \neq j)$ for $1 \leq i, j \leq k$ with $\hat{a}_{ii} \leq |\hat{a}_{ij}|$.

It is easy to see that the above operation is a unimodular transformation. It is shown in [4] that the above procedure will bring $\hat{A}_{k+1}$ to the desired form satisfying (1), (2) and (3), denoted by A'_{k+1}, within a finite number of steps.

Phase IV:

Step (0) Implement Phase I and obtain A as in (1.2). Set $k = 2$ and $A'_2 = A_2$ as in (1.3).

Step (1) Implement Phase II for A'_k and obtain $\hat{A}_{k+1}$ as in (1.5). Implement Phase III for $\hat{A}_{k+1}$ and obtain A'_{k+1}. Set $k = k + 1$.

Step (2) If $k = n + 1$, set $B = A'_{n+1}$ and stop. Otherwise, go to Step (1).

It is obvious that A_2 has the desired form and Step (1) always brings A_{k+1} to the desired form A'_{k+1} within a finite number of steps.

Hence the procedure produces the matrix $B = AU$ satisfying all conditions in the theorem where U is a unimodular matrix. This completes the proof. □

We shall conclude this note with three examples to illustrate the proof. In the sequel, $(d)f \rightarrow g$, $(-1)f$, and $f \leftrightarrow g$ denote the addition of d multiple of column f

to column g, the reversal of the sign of column f, and the interchange of columns f and g, respectively.

Example 1. We are given

$$A = \begin{bmatrix} 5 & 7 & -1 \\ -4 & 3 & 2 \\ 3 & -12 & 1 \\ -4 & 2 & -2 \end{bmatrix}.$$

Then

$$(1)3 \to 2 : A = \begin{bmatrix} 5 & 6 & -1 \\ -4 & 5 & 2 \\ 3 & -11 & 1 \\ -4 & 0 & -2 \end{bmatrix}$$

$$(-2)3 \to 1 : A = \begin{bmatrix} 7 & 6 & -1 \\ -8 & 5 & 2 \\ 1 & -11 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

$$1 \leftrightarrow 2 : A = \begin{bmatrix} 6 & 7 & -1 \\ 5 & -8 & 2 \\ -11 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

$$(11)2 \to 1 : A = \begin{bmatrix} 83 & 7 & -1 \\ -83 & -8 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

$$(-1)2 : A = \begin{bmatrix} 83 & -7 & -1 \\ -83 & 8 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

$$(-1)1 \to 2 : A = \begin{bmatrix} 83 & -90 & -1 \\ -83 & 91 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

$$(1)1 \to 2 : A = \begin{bmatrix} 83 & -7 & -1 \\ -83 & 8 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

$$(10)2 \to 1 : A = \begin{bmatrix} 13 & -7 & -1 \\ -3 & 8 & 2 \\ -10 & -1 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

$$(-1)1 \to 3 : A = \begin{bmatrix} 13 & -7 & -14 \\ -3 & 8 & 5 \\ -10 & -1 & 11 \\ 0 & 0 & -2 \end{bmatrix}.$$

Finally,

$$(-1)2 \to 3 : B = AU = \begin{bmatrix} 13 & -7 & -7 \\ -3 & 8 & -3 \\ -10 & -1 & 12 \\ 0 & 0 & -2 \end{bmatrix},$$

where

$$U = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -1 & 2 & 0 \end{bmatrix}.$$

Example 2. We are given

$$A = \begin{bmatrix} -7 & 3 & 10 & -6 \\ -8 & -2 & 1 & -5 \\ 9 & -6 & -7 & 6 \\ -1 & 6 & -3 & 4 \\ 7 & -1 & -1 & 1 \end{bmatrix}.$$

Then

$$U_1 = \begin{bmatrix} -5 & 1 & 0 & 0 \\ -31 & 6 & 0 & 0 \\ 165 & -31 & -1 & 0 \\ 169 & -32 & -1 & -1 \end{bmatrix}$$

brings A to

$$F_1 = \begin{bmatrix} 578 & -107 & -4 & 6 \\ -578 & 109 & 4 & 5 \\ 0 & -2 & 1 & -6 \\ 0 & 0 & -1 & -4 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

Then

$$U_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 4 & 0 & 0 \\ 10 & -11 & -1 & 0 \\ 9 & -14 & -1 & -1 \end{bmatrix}$$

brings A to

$$F_2 = \begin{bmatrix} 43 & -21 & -4 & 6 \\ -33 & 43 & 4 & 5 \\ -10 & -22 & 1 & -6 \\ 0 & 0 & -1 & -4 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

Then

$$U_3 = \begin{bmatrix} 0 & 1 & -1 & 0 \\ -1 & 4 & -3 & 0 \\ 10 & -11 & 0 & 0 \\ 9 & -14 & 4 & -1 \end{bmatrix}$$

brings A to

$$F_3 = \begin{bmatrix} 43 & -21 & -26 & 6 \\ -33 & 43 & -6 & 5 \\ -10 & -22 & 33 & -6 \\ 0 & 0 & -1 & -4 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

We now have to solve the following system of equations:

$$\begin{aligned} 43x_1 - 21x_2 - 26x_3 &\geq 1 \\ -33x_1 + 43x_2 - 6x_3 &\geq 1 \\ -10x_1 - 22x_2 + 33x_3 &\geq 1 \end{aligned} \tag{1.6}$$

$$x \in Z^3_+.$$

Some solutions of equation (1.6) found by the algorithm in [4] (see its computer code in [1]) are listed in Table 1.

Table 1. Some solutions of equation (1.6) found by the algorithm.

starting point	solution	number of steps
$(0,0,0)$	$(23,21,21)$	72
$(20,19,18)$	$(23,21,21)$	9
$(21,20,19)$	$(23,21,21)$	6
$(18,20,19)$	$(23,21,21)$	9
$(18,19,20)$	$(23,21,21)$	9
$(-10,5,10)$	$(23,21,21)$	65
$(39,1,-77)$	$(39,35,36)$	171
$(20,-43,33)$	$(39,35,36)$	104
$(55,31,16)$	$(56,51,51)$	69
$(-10,5,-10)$	$(23,21,21)$	89
$(34,19,-21)$	$(34,31,31)$	72
$(-10,-20,10)$	$(23,21,21)$	94
$(10,-20,-30)$	$(23,21,21)$	126
$(-100,-100,-100)$	$(23,21,21)$	401
$(-100,100,-200)$	$(110,100,101)$	533
$(200,300,-200)$	$(328,300,300)$	642
$(200,300,500)$	$(539,484,500)$	553
$(600,300,500)$	$(600,536,540)$	293
$(700,300,1)$	$(700,625,629)$	1170
$(1000,1000,1000)$	$(1661,1490,1542)$	9492

We choose $(23, 21, 21)$. Then

$$U_4 = \begin{bmatrix} 0 & 1 & -1 & 0 \\ -1 & 4 & -3 & 6 \\ 10 & -11 & 0 & 3 \\ 9 & -14 & 4 & 8 \end{bmatrix}$$

brings A to

$$F_4 = \begin{bmatrix} 43 & -21 & -26 & 0 \\ -33 & 43 & -6 & -49 \\ -10 & -22 & 33 & -9 \\ 0 & 0 & -1 & 59 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

Finally,

$$U = \begin{bmatrix} 0 & 1 & -1 & 1 \\ -1 & 4 & -3 & 10 \\ 10 & -11 & 0 & -8 \\ 9 & -14 & 4 & -6 \end{bmatrix}$$

brings A to

$$B = AU = \begin{bmatrix} 43 & -21 & -26 & -21 \\ -33 & 43 & -6 & -6 \\ -10 & -22 & 33 & -31 \\ 0 & 0 & -1 & 59 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

Example 3. We are given

$$A = \begin{bmatrix} -1 & -1 & -1 \\ 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}.$$

Then

$$U = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix}$$

such that

$$B = AU = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ -1 & 0 & 2 \\ -3 & -2 & -2 \end{bmatrix}.$$

References

[1] F.B. Janssen and Z. Yang, "An implementation of a simplicial algorithm for testing the integral property of polytopes", in preparation.

[2] H. Scarf, "Production sets with indivisibilities-part II: the case of two activities", *Econometrica* 49 (1981) 395-423.

[3] P.M. White, Discrete Activity Analysis, Ph.D. Thesis, Yale University, New Haven, 1983.

[4] Z. Yang, "A simplicial algorithm for testing the integral property of polytopes: Parts I and II", CentER Discussion Papers No. 9475 and No. 9489, Tilburg University, Tilburg, 1994.

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